The general formula for the dot product of two vectors $\mathbf{u}$ and $\mathbf{v}$ is given by:

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos(\theta)$$

where $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$ are the magnitudes (norms) of the vectors $\mathbf{u}$ and $\mathbf{v}$, and θ is the angle between them.

Given that $\mathbf{u}$ and $\mathbf{v}$ are unit vectors, we have:

$$\|\mathbf{u}\| = 1 \quad \text{and} \quad \|\mathbf{v}\| = 1$$

So for unit vectors, the formula simplifies to:

$$\mathbf{u} \cdot \mathbf{v} = 1 \cdot 1 \cdot \cos(\theta) \, \mathbf{u} \cdot \mathbf{v} = \cos(\theta)$$

Detailed Proof

Let $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$ be two unit vectors. The dot product $\mathbf{u} \cdot \mathbf{v}$ is given by:

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2$$

Since both $\mathbf{u}$ and $\mathbf{v}$ are unit vectors, their magnitudes are:

$$\|\mathbf{u}\| = \sqrt{u_1^2 + u_2^2} = 1 \, \|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2} = 1$$

The cosine of the angle θ between the vectors $\mathbf{u}$ and $\mathbf{v}$ is given by:

$$\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

Since $\|\mathbf{u}\| = 1$ and $\|\mathbf{v}\| = 1$, this becomes:

$$\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{1 \cdot 1} \cos(\theta) = \mathbf{u} \cdot \mathbf{v}$$

Therefore, for unit vectors $\mathbf{u}$ and $\mathbf{v}$:

$$\mathbf{u} \cdot \mathbf{v} = \cos(\theta)$$

This confirms that the dot product of two two-dimensional unit vectors is the cosine of the angle between them.