

Understanding McCarthy's 91 Function

The function is defined as:

```
public static int mcCarthy(int n)
{
    if (n > 100) return n - 10;
    return mcCarthy(mcCarthy(n + 11));
}
```

Key Observations

1. If $n > 100$, the function returns $n - 10$.
2. If $n \leq 100$, the function calls itself recursively with $n + 11$.

Determining the Value for mcCarthy(50)

Let's break down the recursion for $n = 50$:

1. *mcCarthy*(50) calls *mcCarthy*(61) because $50 + 11 = 61$.
2. *mcCarthy*(61) calls *mcCarthy*(72).
3. *mcCarthy*(72) calls *mcCarthy*(83).
4. *mcCarthy*(83) calls *mcCarthy*(94).
5. *mcCarthy*(94) calls *mcCarthy*(105) because $94 + 11 = 105$.

Since 105 is greater than 100: 6. *mcCarthy*(105) returns $105 - 10 = 95$.

Now, we need to resolve the inner call *mcCarthy*(95):

7. *mcCarthy*(95) calls *mcCarthy*(106).

Since 106 is greater than 100: 8. *mcCarthy*(106) returns $106 - 10 = 96$.

This process continues: 9. *mcCarthy*(96) calls *mcCarthy*(107). 10. *mcCarthy*(107) returns $107 - 10 = 97$. 11. *mcCarthy*(97) calls *mcCarthy*(108). 12. *mcCarthy*(108) returns $108 - 10 = 98$. 13. *mcCarthy*(98) calls *mcCarthy*(109). 14. *mcCarthy*(109) returns $109 - 10 = 99$. 15. *mcCarthy*(99) calls *mcCarthy*(110). 16. *mcCarthy*(110) returns $110 - 10 = 100$. 17. *mcCarthy*(100) calls *mcCarthy*(111). 18. *mcCarthy*(111) returns $111 - 10 = 101$.

Finally: 19. *mcCarthy*(101) returns $101 - 10 = 91$.

Thus, *mcCarthy*(100) calls *mcCarthy*(111) which returns 101, and eventually returns 91.

Base Case Reachability

To prove that the base case is reached for all $n \leq 100$:

- For $n > 100$, it immediately returns $n - 10$.
- For $n \leq 100$, each recursive call increases n by 11 until $n > 100$, ensuring it reaches a point where $n > 100$.

Recursive Calls Count

To compute $mcCarthy(50)$:

1. First series of calls until $mcCarthy(105)$: 6 calls.
2. Second series of calls from $mcCarthy(95)$ to $mcCarthy(101)$: 7 calls.

Therefore, the total number of recursive calls is 6 (outer) + 7 (inner) + 1 (initial call) = 14.

Conclusion

- The value of $mcCarthy(50)$ is 91.
- The number of recursive calls used by $mcCarthy(50)$ is 14.
- The function reaches the base case for all positive integers n , as each recursive step increases n by 11, eventually making $n > 100$. There are no values of n for which the function goes into an infinite recursive loop.