

The behavior of the `factorial()` function when called with a negative value or a large value depends on the implementation of the function. Generally, a factorial function is implemented recursively or iteratively, and let's consider these cases.

## Recursive Implementation

Let's examine the behavior of the provided `factorial` function:

```
public static long factorial(int n) {  
    if (n == 1) return 1;  
    return n * factorial(n - 1);  
}
```

## With a Negative Value

If you call `factorial` with a negative value, there are two primary issues:

1. **Infinite Recursion:** The function does not handle negative values and will continue to call itself indefinitely, eventually causing a `StackOverflowError` due to infinite recursion.
2. **No Base Case for Negative Values:** There is no base case for `n <= 0`, so the recursion does not terminate properly for negative values.

## With a Large Value (e.g., 35)

For a large value like 35, the function will compute the factorial recursively. Since the return type is `long`, it can handle larger values than `int`, but `35!` exceeds the range of `long`.

- **Overflow for Long:** The maximum value for a `long` in Java is  $2^{63} - 1$  (approximately  $9.22 \times 10^{18}$ ). The factorial of 35 is approximately  $1.03 \times 10^{40}$ , which far exceeds the maximum value of `long`. This will result in an overflow, causing the function to return an incorrect result.

## Improved Implementation

To handle both negative values and large values properly, you can improve the function:

1. **Check for Negative Values:** Add a check to handle negative input values.
2. **Use BigInteger:** Use `BigInteger` to handle large values without overflow.

Here's an improved implementation:

```
import java.math.BigInteger;  
  
public static BigInteger factorial(int n) {
```

```

    if (n < 0) throw new IllegalArgumentException("Negative value:
" + n);
    if (n == 0 || n == 1) return BigInteger.ONE;
    return BigInteger.valueOf(n).multiply(factorial(n - 1));
}

```

## With a Negative Value

- The function will throw an `IllegalArgumentException`, properly handling negative input values.

## With a Large Value (e.g., 35)

- The function will compute `35!` accurately without overflow since `BigInteger` can handle arbitrarily large integers.

## Summary

- **Negative Value:** In the original implementation, the function will cause a `StackOverflowError`. In the improved implementation, it throws an `IllegalArgumentException`.
- **Large Value (e.g., 35):** In the original implementation, the function will cause an overflow and return an incorrect result. In the improved implementation using `BigInteger`, the function will return the correct result.

## With a Large Value (e.g., 35):

- For large values like 35, the function will compute the factorial recursively. However, since Java `int` type has a maximum value of `2,147,483,647`, the result for `35!` (which is approximately  $1.03 \times 10^{40}$ ) will exceed this limit and cause an integer overflow. This means the function will not return a correct result and instead will return a negative or otherwise incorrect value due to overflow.

## Iterative Implementation

Here's a typical iterative implementation of the factorial function in Java:

```

public static int factorial(int n) {
    if (n < 0) throw new IllegalArgumentException("Negative value:
" + n);
    int result = 1;
    for (int i = 1; i <= n; i++) {
        result *= i;
    }
    return result;
}

```

## With a Negative Value:

- The function will throw an `IllegalArgumentException`, as in the recursive case.

### With a Large Value (e.g., 35):

- Similar to the recursive case, this implementation will also result in integer overflow for values like 35, producing an incorrect result due to the overflow.

## Using a Larger Data Type

To handle larger values without overflow, you can use `long` or `BigInteger`. Here is an example using `BigInteger`:

```
import java.math.BigInteger;

public static BigInteger factorial(int n) {
    if (n < 0) throw new IllegalArgumentException("Negative value:
" + n);
    BigInteger result = BigInteger.ONE;
    for (int i = 1; i <= n; i++) {
        result = result.multiply(BigInteger.valueOf(i));
    }
    return result;
}
```

### With a Negative Value:

- The function will throw an `IllegalArgumentException`.

### With a Large Value (e.g., 35):

- The function will compute `35!` accurately without overflow since `BigInteger` can handle arbitrarily large integers.

## Summary

- **Negative Value:** Both implementations should throw an exception indicating that factorial is not defined for negative integers.
- **Large Value (e.g., 35):**
  - Recursive and iterative implementations using `int` will cause overflow and return incorrect results.
  - Using `BigInteger` or `long` (to a lesser extent) can handle larger values correctly, with `BigInteger` being the most reliable for very large values.