

In a Binary Search Tree (BST), for any given node x , the properties are such that:

1. All keys in the left subtree of x are smaller than the key of x .
2. All keys in the right subtree of x are larger than the key of x .

For a leaf node x , its parent p can have either a left or right child as x . The statement suggests that p is either the smallest key larger than x or the largest key smaller than x . However, this is not necessarily true for a BST.

Explanation:

- If x is the left child of p , then p is indeed larger than x , but it is not guaranteed to be the smallest key larger than x . There could be other nodes in the right subtree of p that are smaller than p but larger than x .
- If x is the right child of p , then p is smaller than x , but it is not guaranteed to be the largest key smaller than x . There could be other nodes in the left subtree of p that are larger than p but smaller than x .

Therefore, the parent p of a leaf node x is not necessarily the smallest key larger than x or the largest key smaller than x in the entire BST. The statement does not hold true in general for all BSTs.

Let's clarify the conditions under which the parent node p of a leaf node x might be the smallest key larger than x or the largest key smaller than x .

Case 1: x is the Rightmost Child of p

- **Smallest Key Larger than x :** If x is the rightmost child of p , then p might be the smallest key larger than x , provided there is no right subtree of p .
- If p has a right subtree, the smallest key larger than x would be in that subtree, specifically the leftmost node of the right subtree of p .

Case 2: x is the Leftmost Child of p

- **Largest Key Smaller than x :** If x is the leftmost child of p , then p might be the largest key smaller than x , provided there is no left subtree of p .
- If p has a left subtree, the largest key smaller than x would be in that subtree, specifically the rightmost node of the left subtree of p .

Conclusion

The statement that the parent p of a leaf node x is either the smallest key larger than x or the largest key smaller than x is only conditionally true. It depends on whether x is the leftmost or rightmost child of p and whether p has any subtrees that could contain a smaller or larger key. Thus, the statement is not universally true for all BST configurations.