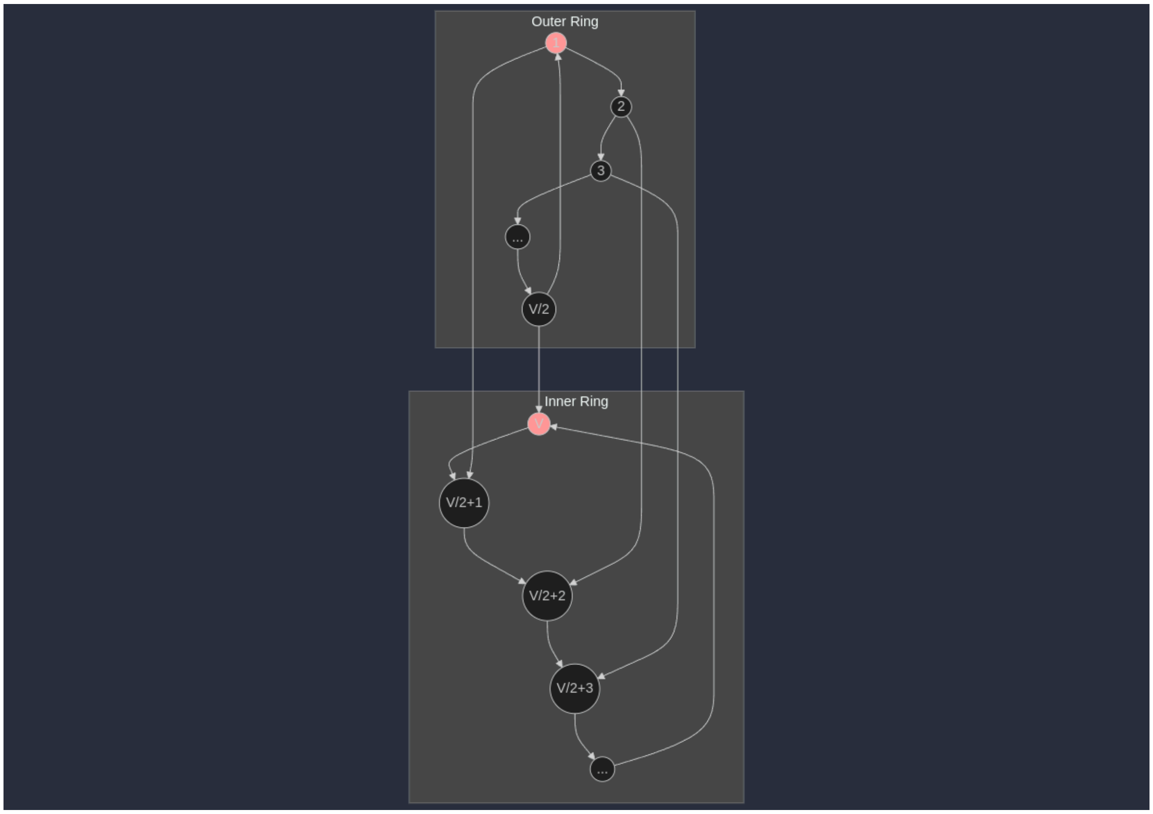
In a 2-ring graph on V vertices, each vertex is connected to its two nearest neighbors in a circular (cyclic) pattern. This means every vertex v_i is connected to vertices v_{i-1} and v_{i+1}, modulo V (accounting for the cyclic structure).

To show that the diameter of the graph is approximately $V/4$, we first define the **diameter** of a graph. The diameter is the maximum distance (shortest path) between any pair of vertices. Imagine the graph as a network of roads. The diameter is the longest distance one can travel by taking the shortest possible route between any two points.

1. **Cyclic structure**: The graph is a ring, so the distance between two vertices can be computed in terms of moving clockwise or counterclockwise around the ring.
2. **Distance between vertices**: For any two vertices v_i and v_j, the shortest path between them is the minimum number of steps to reach v_j from v_i, either by going clockwise or counterclockwise around the ring.



Graph Analysis

This diagram shows a 2-ring graph on V vertices. Here's an explanation of the key elements:

1. The graph consists of two concentric rings, each containing $V/2$ vertices.
2. The outer ring has vertices numbered from 1 to $V/2$.
3. The inner ring has vertices numbered from $V/2+1$ to V.

4. Each vertex in the outer ring is connected to its two adjacent vertices in the same ring and one vertex in the inner ring.
5. The same connection pattern applies to the inner ring vertices.

Diameter Calculation

To find the diameter, we need to consider the following:

1. **Maximum distance within a ring**: The maximum distance between any two vertices within a single ring is $V/2$ (half the number of vertices in the ring).

2. **Maximum distance between rings**: The worst-case scenario occurs when we need to traverse from one ring to the other at the farthest point from the connecting edges.

3. **Path of maximum length**: The longest path will start at a vertex in one ring, traverse to the farthest point in that ring, cross to the other ring, and then reach the farthest point in the second ring.

Formula for Diameter

Given these considerations, we can express the diameter as:

$$Diameter = \left\lfloor \frac{V}{2} \right\rfloor + 1 + \left\lfloor \frac{V}{2} \right\rfloor$$

Here's the breakdown of this formula:

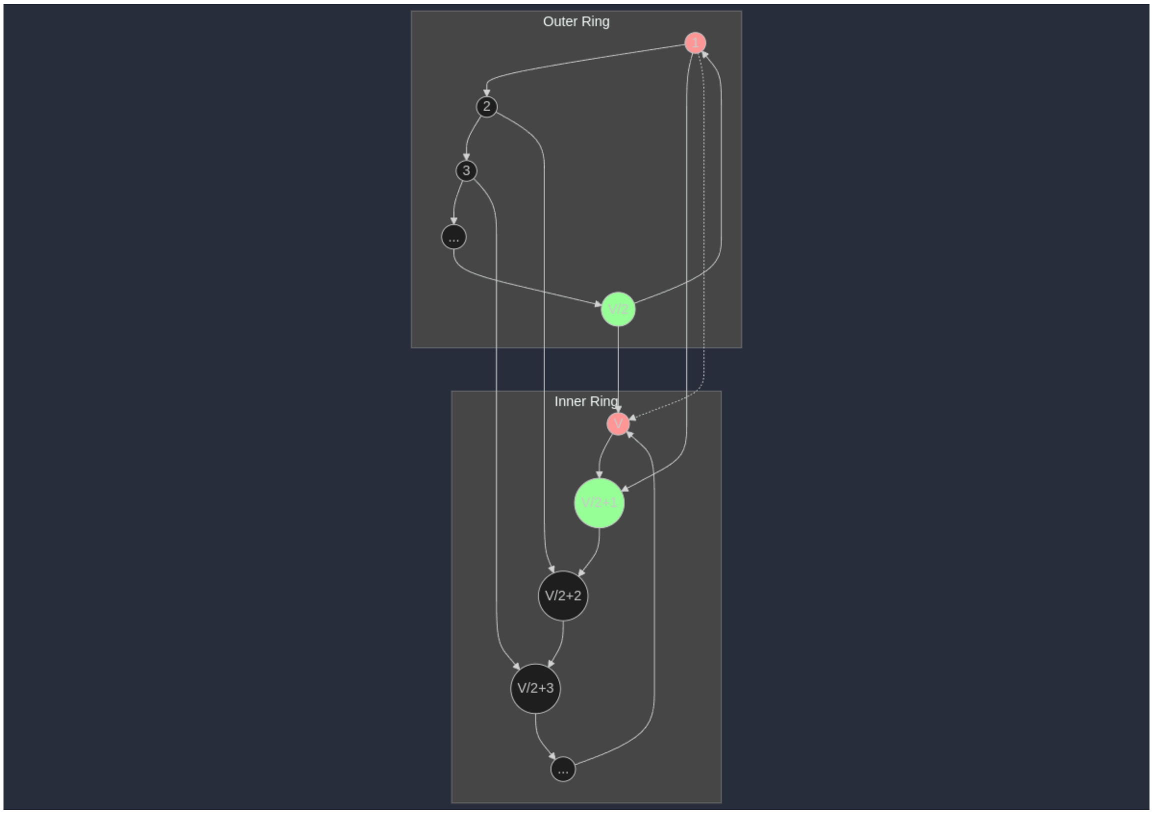
- $\left\lfloor \frac{V}{2} \right\rfloor$: Maximum distance within the first ring
- 1 : The edge connecting the two rings
- $\left\lfloor \frac{V}{2} \right\rfloor$: Maximum distance within the second ring

The floor function $\lfloor\rfloor$ is used to ensure we get an integer result for odd values of V.

Adding an Edge Between Antipodal Vertices in a 2-Ring Graph

Understanding the Impact

When you add an edge connecting two antipodal vertices (vertices that are farthest apart) in a 2-ring graph, you essentially create a shortcut between the two rings. This can significantly reduce the diameter of the graph.



Before the Addition:

- The diameter of the original 2-ring graph is typically determined by the sum of the maximum distances within each ring, plus the length of the bridge.

After the Addition:

- The new edge provides a direct path between the two antipodal vertices, bypassing the need to traverse the entire circumference of both rings.

1. **Outer Ring**:
   - The outer ring consists of $V/2$ vertices, connected cyclically.
   - The diameter of the outer ring alone would be $V/4$, as discussed before, since you must traverse around half the vertices to reach the farthest one.
2. **Inner Ring**:
   - Similarly, the inner ring consists of $V/2$ vertices, connected cyclically.
   - The diameter of the inner ring would also be $V/4$.
3. **Inter-ring Connections**:
   - The inter-ring connections significantly reduce the maximum distance between vertices.
   - Notably, there is a direct edge connecting two "antipodal" points (one on the outer ring and one on the inner ring), providing a shortcut across the graph.

Effect of Inter-ring Connections on the Diameter:

- The inter-ring edges effectively "cut" the graph in half, because they provide a shorter path between vertices on different rings. Without these edges, the maximum distance between vertices would be $V/4$ as you would have to traverse half the ring.
- With the inter-ring edges, vertices on one ring can reach the other ring quickly, potentially reducing the maximum distance to roughly $V/8$. This is similar to the case of adding an antipodal edge in a single-ring graph, which reduces the diameter from $V/4$ to $V/8$.

Conclusion:

Given the structure of the graph with two rings and inter-ring connections, especially the antipodal shortcut, the **diameter of this graph is approximately** $V/8$. This is because the inter-ring connections provide shortcuts, significantly reducing the maximum distance between any two vertices.