

To demonstrate that a log-log plot of the function  $cn^b$  has a slope  $b$  and x-intercept  $\log c$ , we follow these steps:

## Log-Log Plot of $cn^b$

Given the function  $f(n) = cn^b$ :

1. Take the logarithm of both sides:  $\log(f(n)) = \log(cn^b)$
2. Use the properties of logarithms to separate the terms:  
 $\log(f(n)) = \log(c) + \log(n^b)$
3. Apply the power rule for logarithms ( $\log(a^b) = b\log(a)$ ):  
 $\log(f(n)) = \log(c) + b\log(n)$

This equation is in the form  $y = mx + b$ , which is the equation of a straight line where:

- $y = \log(f(n))$
- $x = \log(n)$
- $m = b$  (the slope)
- $\log(c)$  is the y-intercept.

Since this is a log-log plot, the slope of the line is  $b$  and the x-intercept occurs when  $y = 0$ :

- To find the x-intercept, set  $\log(f(n)) = 0$ :  $0 = \log(c) + b\log(n)$

$$-\log(c) = b\log(n)$$

$$\log(n) = -\frac{\log(c)}{b}$$

Therefore, the x-intercept is:  $\log(n) = \log(c^{-1/b})$

## Application to $4n^3(\log n)^2$

Now let's apply this to the function  $f(n) = 4n^3(\log n)^2$ :

1. Take the logarithm of both sides:  $\log(f(n)) = \log(4n^3(\log n)^2)$
2. Use the properties of logarithms to separate the terms:  
 $\log(f(n)) = \log(4) + \log(n^3) + \log((\log n)^2)$
3. Apply the power rule for logarithms:  $\log(f(n)) = \log(4) + 3\log(n) + 2\log(\log(n))$

This equation can be interpreted as a sum of linear terms on a log-log plot:

- The term  $3\log(n)$  indicates a slope of 3 for the  $n^3$  part.

- The term  $2\log(\log(n))$  does not affect the slope of the line on a standard log-log plot because it introduces a secondary logarithmic term.
- The term  $\log(4)$  is a constant and contributes to the y-intercept.

## Slope and X-Intercept

- **Slope:** The slope  $b$  is dominated by the term  $3\log(n)$ , so the slope is 3.
- **X-Intercept:** The x-intercept is found by setting  $y = 0$  (where  $\log(f(n)) = 0$ ) and solving for  $n$ :  $0 = \log(4) + 3\log(n) + 2\log(\log(n))$

This equation is complex due to the  $2\log(\log(n))$  term. However, for large  $n$ , the  $3\log(n)$  term dominates, and the x-intercept is primarily influenced by the term involving  $n^3$ :

Setting  $\log(4) + 3\log(n) = 0$  for simplicity:  $3\log(n) = -\log(4)$   $\log(n) = -\frac{\log(4)}{3}$

Therefore, the x-intercept is:  $\log(n) = \log(4^{-1/3})$   $\log(n) = \log(2^{-2/3})$  (since  $4 = 2^2$ )

So, the x-intercept in terms of  $n$  is  $n = 2^{-2/3}$ .

In conclusion:

- The **slope** of the log-log plot for  $4n^3(\log n)^2$  is 3.
- The **x-intercept** is influenced by the constant 4 but primarily dominated by the term involving  $n$ , simplifying to  $\log(n) = -\frac{\log(4)}{3}$ , or equivalently  $n = 2^{-2/3}$  in base 2 terms.