To prove by mathematical induction that the number of iterations for the `count` method in the `Foursum` class can be represented by the combination formula:

$$=\binom{n}{4} = \frac{n(n-1)(n-2)(n-3)}{24}$$

Basis Step

For (n = 4):

$$=\binom{4}{4} = \frac{4!}{4!0!} = 1$$

The loops would iterate as follows:

- `i = 0`, `j = 1`, `k = 2`, `l = 3`

There is exactly one combination of four indices (0, 1, 2, 3) when (n = 4), which matches $=\binom{4}{4} = 1$.

Induction Step

Assume the formula holds for some arbitrary (n = k). That is, assume:

$=\text{Number of iterations} = \binom{k}{4} = \frac{k(k-1)(k-2)(k-3)}{24}$ We need to prove that the formula holds for (n = k+1):

$$=\binom{k+1}{4} = \frac{(k+1)k(k-1)(k-2)}{24}$$

Induction Hypothesis

We assume the following holds true:

$$=\binom{k}{4} = \frac{k(k-1)(k-2)(k-3)}{24}$$

Induction Step Calculation

We need to show that adding one more element (to get (n = k+1)) fits the combination formula.

The total number of ways to pick 4 indices from (k+1) elements is the sum of two parts:

1. Choosing 4 elements from the first (k) elements.
2. Choosing 3 elements from the first (k) elements and the ((k+1))-th element.

First part:

$$=\binom{k}{4} = \frac{k(k-1)(k-2)(k-3)}{24}$$

Second part:

$$=\binom{k}{3} = \frac{k(k-1)(k-2)}{6}$$

Adding the ((k+1))-th element to any of the combinations of 3 from the first (k):

$$=(k+1) \times \binom{k}{3} = (k+1) \times \frac{k(k-1)(k-2)}{6}$$

Thus, combining both parts:

$$=\binom{k+1}{4} = \binom{k}{4} + (k+1) \times \binom{k}{3}$$

Substituting the known formulas:

$$=\binom{k+1}{4} = \frac{k(k-1)(k-2)(k-3)}{24} + (k+1) \times \frac{k(k-1)(k-2)}{6}$$

Simplifying:

$$== \frac{k(k-1)(k-2)(k-3)}{24} + \frac{(k+1)k(k-1)(k-2)}{6}$$

Finding a common denominator (24):

$$== \frac{k(k-1)(k-2)(k-3)+4(k+1)k(k-1)(k-2)}{24}$$

Factoring out (k(k-1)(k-2)):

$$== \frac{k(k-1)(k-2)[(k-3)+4(k+1)]}{24}$$

$$== \frac{k(k-1)(k-2)[k-3+4k+4]}{24}$$

$$== \frac{k(k-1)(k-2)[5k+1]}{24}$$

Which simplifies to:

$$== \frac{(k+1)k(k-1)(k-2)}{24}$$

Thus, we have shown that:

$$=\binom{k+1}{4} = \frac{(k+1)k(k-1)(k-2)}{24}$$