Your Thesis Title

Benjamin Williams

School of Computer Science

College of Science

University of Lincoln

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Supervisor Dr. Mantis Toboggan

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Chapter 1

Notations

Consider a simple, non-empty Graph $G = (V, E)$ comprising a set of Edges E and vertices E. For specificity, $E(G)$ and $V(G)$ is used to refer to the set of edges and vertices associated with Graph G respectively, where required.Further, denote by $|V|$ the cardinality of set of vertices V, let G be a Graph with $|V(G)| = |\{v_1, v_2, \ldots, v_n\}| = n$ vertices and $|E(G)| = m$ edges with $E \subset \binom{V}{2}$; where $\binom{V}{2} = \{\langle u, v\rangle \, |u, v \in V, u \neq v\}$, that is, the set of all possible unordered pairs created from V. For each $e = \langle u, v\rangle \in E$, e is said to be incident on $u, v \in V$, whereas u and v are adjacent to each other.

The Neighbor set $\mathcal{N}(v)$ of $v \in V$, defined as $\mathcal{N}(v) =: \{w \in V(G)|v \neq w, \exists e \in E(G)\}$ is the set of vertices (other than v) adjacent to $v\tilde{\mathcal{V}}\bar{\mathcal{V}}$. Given a Graph $\mathcal{G}$, $\mathcal{G} \subseteq G$ denotes that $\mathcal{G}$ is a subgraph of G, if it holds that $\forall e \in E(\mathcal{G}),\ u, v \subseteq V(\mathcal{G})$, $V(\mathcal{G}) \subseteq V(G)$ and $E(\mathcal{G}) \subseteq E(G)$.

The adjacency matrix of a weighted graph G is a symmetric matrix $A \in \mathbb{R}^{n\times n}$ with ith row and jth column containing the number of edges incident on vertex v_i and v_j. More precisely, let $\omega : \{0, 1\} \mapsto \mathbb{R}_0^+$ for $i, j \in \{1, 2, \ldots, n\}$

$$A_{i,j} = \begin{cases} \omega(e_i), & e_i = \langle v_i, v_j\rangle \in E(G) \\ 0, & \text{otherwise.} \end{cases} \tag{1.1}$$

By restricting (1.1) $\omega \in \{0, 1\}\ \ \forall\omega(e_i),\ e_i \in E$, we recover the adjacency matrix of an unweighted graph. The volume $vol(F)$ of a a subset of nodes $F \in V$ is the sum of weight of edges adjacent to the nodes of F and given as $vol(F) = \sum_{v_i\in F} \delta(v_i)$; where

$\delta(v_i)$ is the degree of vertex v_i obtained as the sum of the weights of edges incident on it: $\delta(v_i) = \sum_{i=1}^{n} A_{i,j}$.

Denote the number of components in of graph G by $\kappa(G)$. For the a set $V^* \subseteq V(G)$, V^* is a cut-vertex if $\kappa(G[V(G)\backslash V^* = \tilde{V}]) > \kappa(G)$; where $G[\tilde{V}]$ is the graph induced by $\tilde{V}$

Question 1.1 (Node Selection) *Is it possible to ensure that the selection of critical nodes in the network is based on both articulation points and their respective changing weights? (Where the weight on each critical (or cut) node is the total amount of traffic load generated from non-articulation points).*

Approach 1.0.1 *We would have two optimization objectives base on:*

1. *maximum volume*
2. *cut vertices.*

Problem 1.1 *Given a Graph $G = (V, E)$, let $\tilde{V} = V(G)\backslash V^*$. Denote by $\vartheta(G)$ the set of cut-vertices associated with Graph G defined by*

$$\vartheta(G) = \left\{(v_i, v_j) \in V^* \mid \kappa(G[\tilde{V}]) > \kappa(G)\right\}. \tag{1.2}$$

Further, let the set $\vartheta(\tilde{G})$ partition $\vartheta(G)$, that is, $\vartheta(\tilde{G}) = \bigcup_{i=1}^{n} \vartheta(G_i)$ where each $\vartheta(G_i)$ represents each vertex-cut associated with $\vartheta(G)$. The traffic on $\vartheta(G_i)$ is represented by its volume of edges $e \in E(G)$ incident on it and expressed as

$$vol(\vartheta(G_i)) = \sum_{\vartheta(G_i)\in G} \delta(\vartheta(G_i)) \tag{1.3}$$

Chapter 2

Related Work

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Chapter 3

Method

Here is a sentence, and you can see a nice picture in Figure 3.1.

Figure 3.1: A picture of the Brayford from Google Images.

Also, a table can be found in Table 3.1. You should use a LaTeX table generator like `https://www.tablesgenerator.com/` if you want to make your life easier.

Table 3.1: Here is a table. The caption goes above like this.

First name	Last name	Age
Bob	Bobbington	24
Benth	Wavies	49
Joe	Bloggs	37
Billy	Bob	10

Chapter 4

Conclusions

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