单腿运动控制原理

单腿正运动学

单腿基坐标系设立在髋关节出轴处

设三段长为 $l_1=l_{pelvic}$, $l_2=-l_{hip}$, $l_3=-l_{knee}$ 初始位型沿坐标负方向延伸，故取负值

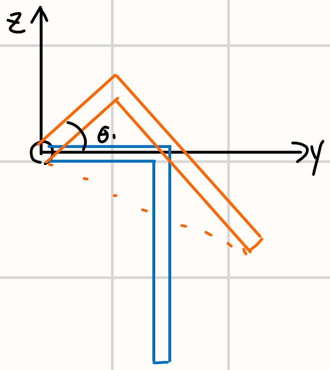
$$
\begin{cases}
x=-l_2 s_2 - l_3 s_{23} \\
y=l_1 c_1 + l_2 s_1 c_2 + l_3 s_1 c_{23} \\
z=l_1 s_1 - l_2 c_1 c_2 - l_3 c_1 c_{23}
\end{cases}
$$

单腿逆运动学

① 首先把二、三段连杆简化，最后等效成一段连杆，设其长度为 L.

则有如下表达.

$$
\begin{bmatrix} y_p \\ z_p \end{bmatrix} = \begin{bmatrix} c_1 & -s_1 \\ s_1 & c_1 \end{bmatrix}\begin{bmatrix} l_1 \\ -L \end{bmatrix} \quad \text{其中 } L=\sqrt{y_p^2+z_p^2-l_1^2}
$$



展开整理得 $\tan\theta_1 = \dfrac{z_p l_1 + y_p L}{y_p l_1 - z_p L}$

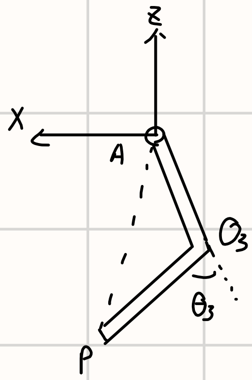
即 $\theta_1 = \operatorname{atan2}(z_p l_1 + y_p L,\ z_p l_1 - z_p L)$

② 随后展开计算二、三级连杆

$$
\cos\angle AO_3P = \frac{|O_3A|^2+|O_3P|^2-|AP|^2}{2\cdot|O_3A|\cdot|O_3P|}
$$

$$
\theta_3 = -\pi + \angle AO_3P = -\pi + \operatorname{acos}\left(\frac{|O_3A|^2+|O_3P|^2-|AP|^2}{2\cdot|O_3A|\cdot|O_3P|}\right)
$$

其中 $|AP| = \sqrt{x_p^2+y_p^2+z_p^2-l_1^2}$



以上已知 θ_1, θ_3，求 θ_2 尽可能使用atan2求得整圆角度

③ 从正运动学公式中拆分出 θ_2 的部分

$$
\begin{cases}
x_p = -(l_3 c_3 + l_2) s_2 - l_3 s_3 c_2 \\
y_p = l_3 s_1 c_{23} + l_1 c_1 + l_2 c_2 s_1 \\
z_p = -l_3 c_1 c_{23} + l_1 s_1 - l_2 c_1 c_2
\end{cases}
\Rightarrow
\begin{array}{l}
y_p s_1 = l_3 s_1^2 c_{23} + l_1 s_1 c_1 + l_2 c_2 s_1^2 \\
z_p c_1 = -l_3 c_1^2 c_{23} + l_1 s_1 c_1 - l_2 c_2 c_1^2
\end{array}
$$

$$
\Rightarrow y_p s_1 - z_p c_1 = l_3 c_{23} + l_2 c_2
$$

$$
\Rightarrow \frac{y_p s_1 - z_p c_1}{x_p} = \frac{l_3 c_{23} + l_2 c_2}{-(l_3 c_3 + l_2) s_2 - l_3 s_3 c_2} = \frac{-l_3 s_3 s_2 + (l_3 c_3 + l_2) c_2}{-l_3 s_3 c_2 - (l_3 c_3 + l_2) s_2} = \frac{-l_3 s_3 \tan\theta_2 + (l_3 c_3 + l_2)}{-l_3 s_3 - (l_3 c_3 + l_2)\tan\theta_2}
$$

令 $\begin{cases} a_1 = y_p s_1 - z_p c_1 \\ a_2 = x_p \\ m_1 = -l_3 s_3 \\ m_2 = -(l_3 c_3 + l_2) \end{cases}$ 则 $\dfrac{a_1}{a_2} = \dfrac{m_1 \tan\theta_2 + m_2}{m_1 - m_2 \tan\theta_2}$

$$
\tan\theta_2 = \frac{a_1 m_1 + a_2 m_2}{a_2 m_1 - a_1 m_2} \qquad \theta_2 = \operatorname{atan2}(a_1 m_1 + a_2 m_2,\ a_2 m_1 - a_1 m_2)
$$

单腿雅可比

① 对正运动学公式两边求一阶微分可得雅可比矩阵

$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{z}_p \end{bmatrix} = \begin{bmatrix} 0 & -l_2C_2-l_3C_{23} & -l_3C_{23} \\ -l_1S_1+l_2C_1C_2+l_3C_1C_{23} & -l_2S_1S_2-l_3S_1S_{23} & -l_3S_1S_{23} \\ l_1C_1+l_2S_1C_2+l_3S_1C_{23} & l_2C_1S_2+l_3C_1S_{23} & l_3C_1S_{23} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

以上可实现单腿速度映射

② 对于单腿静力学

假设腿静止，则总功率为0，有如下等式

$$\tau_1\dot{\theta}_1+\tau_2\dot{\theta}_2+\tau_3\dot{\theta}_3 = F_x\dot{x}_p+F_y\dot{y}_p+F_z\dot{z}_p$$

即 $[\tau_1\ \tau_2\ \tau_3]\cdot\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = [F_x\ \ F_y\ \ F_z]\cdot\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{z}_p \end{bmatrix}$

$$\tau^T\cdot\dot{\theta} = F^T\cdot J\cdot\dot{\theta} \Rightarrow \tau = J^T\cdot F$$