In [7]:

```
import matplotlib.pyplot as plt
import numpy as np
import scipy.stats as sps
import math
%matplotlib inline
```

In [ ]:

```
sample_size = 100
alpha = 0.95
```

Генерируем выборки

In [72]:

```
uniform_sample = sps.uniform.rvs(loc=0, scale=1, size=sample_size)
cauchy_sample = sps.cauchy.rvs(loc=0, scale=1, size=sample_size)
poisson_sample = sps.poisson.rvs(mu=1, size=sample_size)
gamma_sample = sps.gamma.rvs(10, 1, size=sample_size)
```

Воспользуемся теоритическими выкладками с семинаров

Доверительные интервалы для равномерного распределения:

Для $\overline{X}$:

In [16]:

```
conf_interv_left1 = [2.*uniform_sample[: i + 1].mean()/(1+2.*((1./12/(i+1)/(1.-alp
ha)))**(1./2)) for i in range(sample_size)]
conf_interv_right1 = [2.*uniform_sample[: i + 1].mean()/(1-2.*((1./12/(i+1)/(1.-al
pha)))**(1./2)) for i in range(sample_size)]
```

Для $X_{(1)}$:

In [17]:

```
conf_int_left2 = [np.min(uniform_sample[: i + 1]) for i in range(sample_size)]
conf_int_right2 = [conf_int_left2[i]*1./(1-alpha**(i + 1)) for i in range(sample_s
ize)]
```

Для $X_{(n)}$:

In [22]:

```
conf_int_left3 = [np.max(uniform_sample[: i + 1]) for i in range(sample_size)]
conf_int_right3 = [conf_int_left3[i]*1./(1-alpha**(i + 1)) for i in range(sample_s
ize)]
```

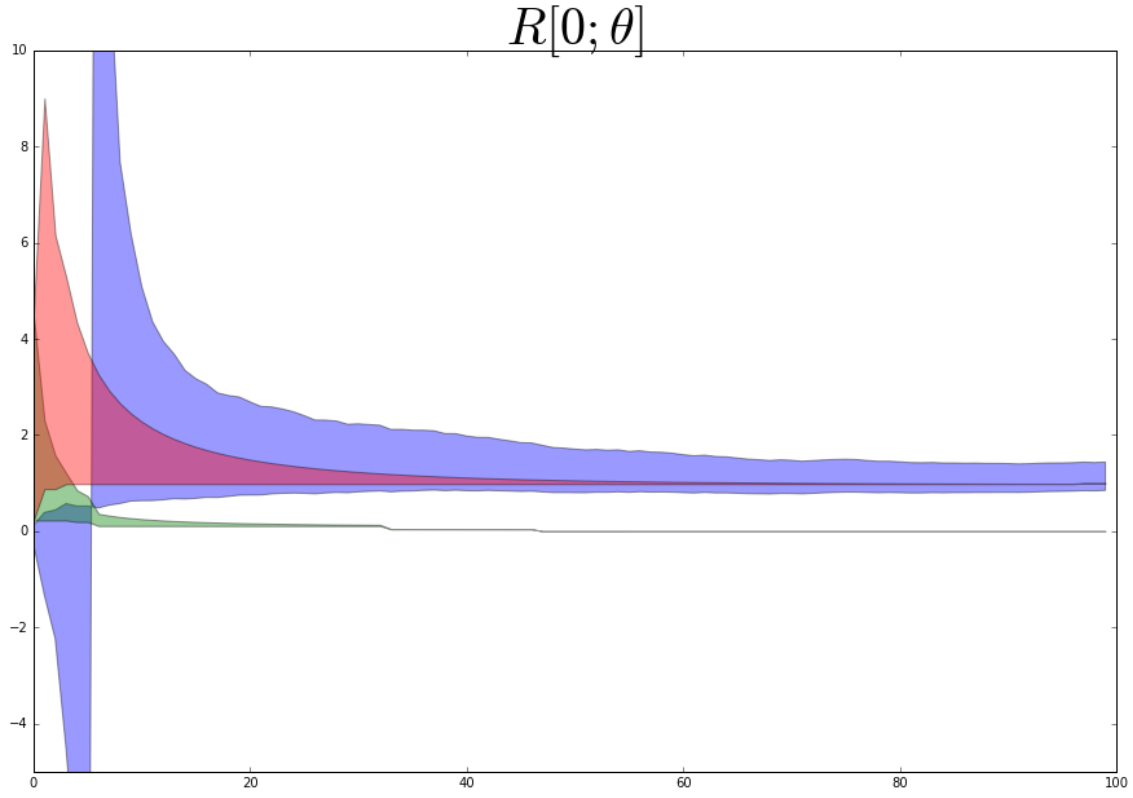
Построим графики доверительных интервалов

In [78]:

```
plt.figure(figsize = (15, 10))
plt.ylim(-5, 10)
plt.title(r'$R[0; \theta]$', fontsize=40)
plt.matplotlib.pyplot.fill_between(range(sample_size), conf_interv_left1, conf_int
erv_right1, facecolor='blue', alpha = 0.4)
plt.matplotlib.pyplot.fill_between(range(sample_size), conf_int_left2, conf_int_ri
ght2, facecolor='green', alpha = 0.4)
plt.matplotlib.pyplot.fill_between(range(sample_size), conf_int_left3, conf_int_ri
ght3, facecolor='red', alpha = 0.4)
```

Out[78]:

```
<matplotlib.collections.PolyCollection at 0x1eea1247b00>
```



Для распределения Коши

Посчитаем медиану

In [38]:

```
med = []
for i in range(sample_size):
    arr = np.sort(cauchy_sample[: i + 1])

    if ((i+1)%2 == 0):
        med.append((arr[i/2] + arr[i/2 + 1])/2)
    else:
        med.append(arr[i/2])
```

```
C:\Program Files\Anaconda3\lib\site-packages\ipykernel\__main__.py:8:
DeprecationWarning: using a non-integer number instead of an integer
will result in an error in the future
C:\Program Files\Anaconda3\lib\site-packages\ipykernel\__main__.py:6:
DeprecationWarning: using a non-integer number instead of an integer
will result in an error in the future
```

Посчитаем квантиль уровня $\frac{1+\alpha}{2}$:

In [63]:

```
distr_func = np.array(sps.norm.cdf(np.arange(-10, 10, 0.01)))
for i in range(distr_func.size):
    if (distr_func[i] >= (1 + alpha) / 2) :
        q = i * 0.01 - 10
        break
```

Строим доверительные интервалы

In [64]:

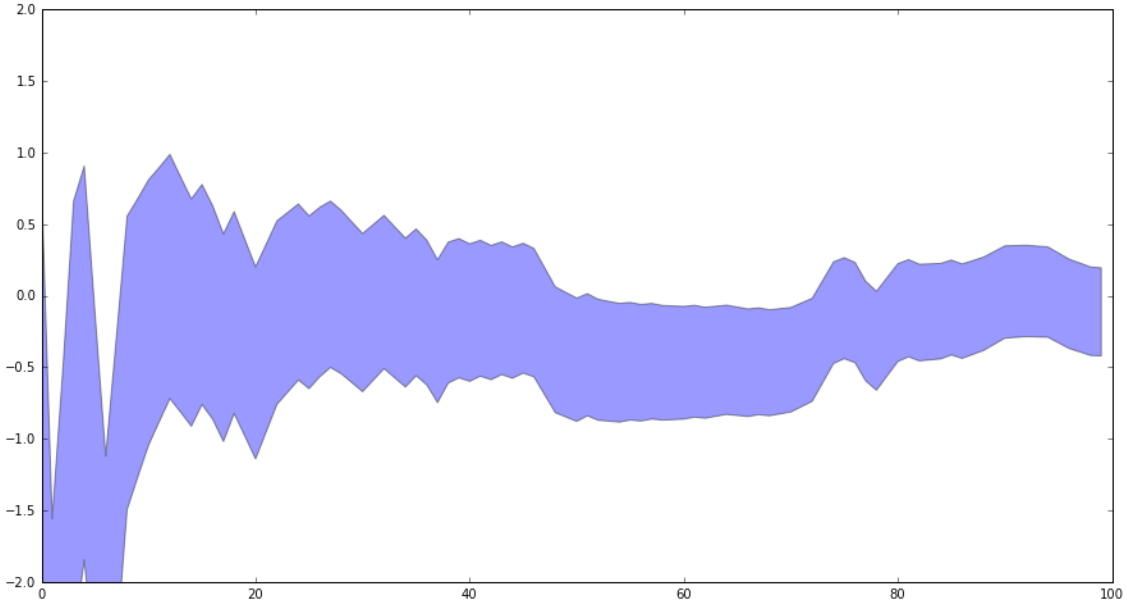
```
conf_interv_left4 = [med[i] - q * (math.pi / 2) / math.sqrt(i + 1) for i in range
(sample_size)]
conf_interv_right4 = [med[i] + q * (math.pi / 2) / math.sqrt(i + 1) for i in range
(sample_size)]
```

In [65]:

```
# строим график
plt.figure(figsize = (15, 8))
plt.ylim(-2, 2)
plt.matplotlib.pyplot.fill_between(range(sample_size), conf_interv_left4, conf_int
erv_right4, facecolor='blue', alpha = 0.4)
```

Out[65]:

```
<matplotlib.collections.PolyCollection at 0x1ee9d5fa3c8>
```



Пуассоновское распределение

In [67]:

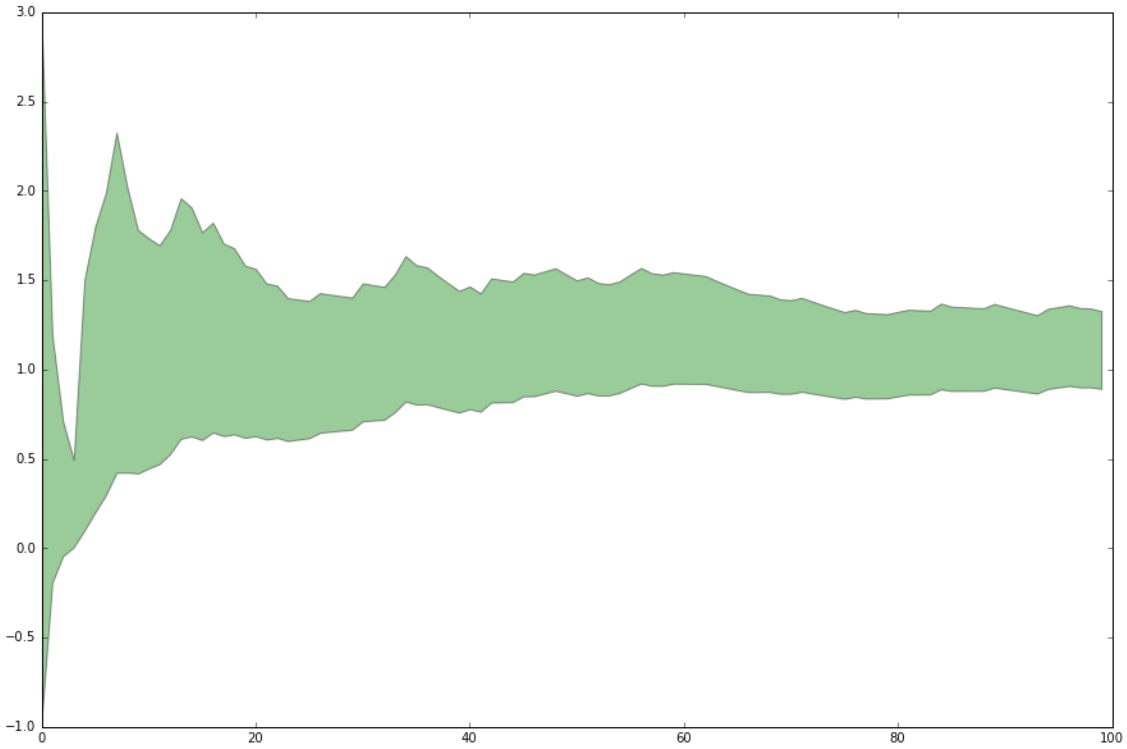
```
conf_interv_left5 = [poisson_sample[:i+1].mean() - q * poisson_sample[:i+1].mean()
/ math.sqrt(i+1) for i in range(sample_size)]
conf_interv_right5 = [poisson_sample[:i+1].mean() + q * poisson_sample[:i+1].mean
() / math.sqrt(i+1) for i in range(sample_size)]
```

In [71]:

```
plt.figure(figsize = (15, 10))
plt.ylim(-1, 3)
plt.matplotlib.pyplot.fill_between(range(sample_size), conf_interv_left5, conf_int
erv_right5, facecolor='green', alpha = 0.4)
```

Out[71]:

```
<matplotlib.collections.PolyCollection at 0x1eea0880b70>
```



Для гамма распределения

In [73]:

```
lambda_ = 10

conf_interval_left6 = [lambda_ / gamma_sample[:i+1].mean() - q * ((lambda_ / gamma
_sample[:i+1].mean()) ** 3) / math.sqrt(i + 1)
                       for i in range(sample_size) ]
conf_interval_right6 = [lambda_ / gamma_sample[:i+1].mean() + q * ((lambda_ / gamm
a_sample[:i+1].mean()) ** 3) / math.sqrt(i + 1)
                        for i in range(sample_size) ]
```

In [74]:

```
plt.figure(figsize = (15, 10))
plt.ylim(0,2)
plt.matplotlib.pyplot.fill_between(range(sample_size), conf_interval_left6, conf_i
nterval_right6, facecolor='red', alpha = 0.4)
```

Out[74]:

```
<matplotlib.collections.PolyCollection at 0x1eea0784048>
```