Energy

$$\ddot{x} = f(x)$$

multiply both sides with $\dot{x}$.

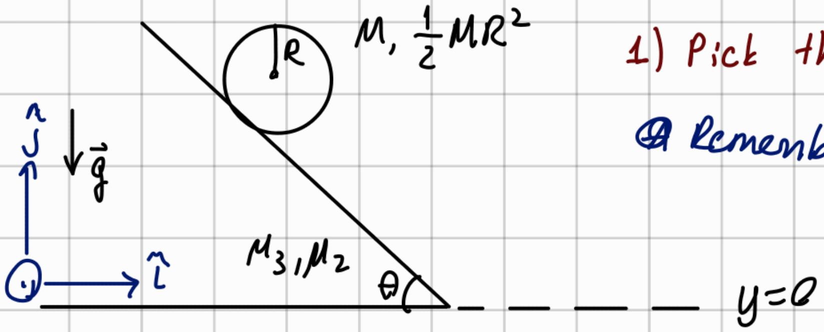
$$\underbrace{\dot{x}\ddot{x}} = \dot{x} f(x)$$

$$\frac{1}{2}\frac{d}{dt}(\dot{x}^2) = \frac{d}{dt}(-g(x))$$

$$(\vdots)$$

- missed this part -

⊛ Mechanical energy is not conserved when friction is "operative"



1) Pick the right coordinate system

⊛ Remember, distance itself isn't a coordinate

Remember:

Force of a conservative function:

$$\vec{F} = -\vec{\nabla} V$$

Then potential can be found:

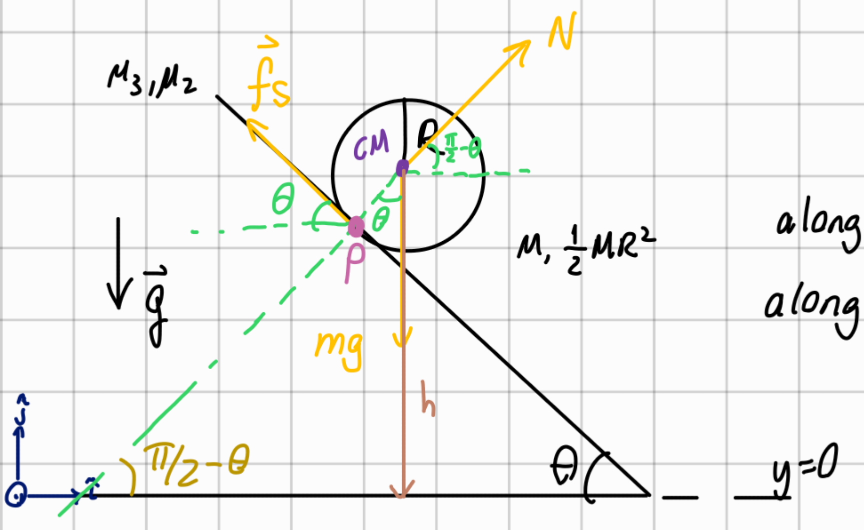
$$\underset{\text{path}}{\int_{\vec{r}_a}^{\vec{r}_b}} \vec{F}\,dr = \underset{\text{path}}{\int_{\vec{r}_a}^{\vec{r}_b}} -\vec{\nabla} V\,dr = -\int_{\vec{r}_a}^{\vec{r}_b} dV = \underset{\text{path independent.}}{-\vec{V}(\vec{r}_b) + \vec{V}(\vec{r}_a)}$$

Since it's path independent, I can choose anything.

$$V(\vec{r}_b) = \vec{V}(\vec{r}_a) - \underbrace{\int \vec{F}\,dr}_{\text{any path from } \vec{r}_a \text{ to } \vec{r}_b}$$

→ This prevents sign mistakes.

Returning to the question...



along $\hat{\imath}$: $N\sin\theta - f_s\cos\theta = M a_x$

along $\hat{\jmath}$: $N\cos\theta + f_s\sin\theta = M a_y$

CM has coordinates
x & y.

Now consider torque:

$$\text{Torque about CM} = -R f_s \hat{k} = I \underbrace{\alpha}_{\text{about } \hat{k}} \hat{k}$$

(?)

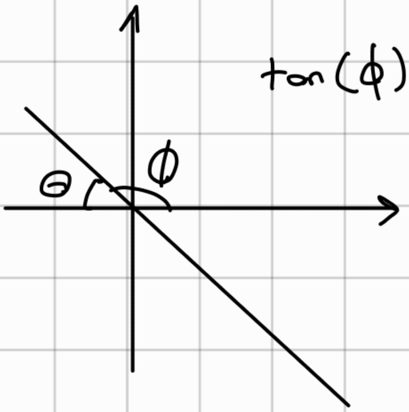
$$\vec{r}_P = -R\sin\theta\hat{\imath} - R\cos\theta\hat{\jmath}$$

$$\vec{\omega} = \omega\hat{k}$$

Still not enough to resolve the system.

⊛ constraints on the system.

- inclined plane:



$$\tan(\phi) = -\tan(\theta) = \tan(\pi-\theta) = \frac{y}{x}$$

- rolling without slipping:
  - point P has zero velocity!!!

$$\vec{V}_P = \vec{V}_{cm} + \vec{\omega} \times \vec{r}_P$$

↳ in relation to CM

$$\vec{r}_P = -R\sin\theta\hat{\imath} - R\cos\theta\hat{\jmath}$$

$$\vec{\omega} = \omega\hat{k}$$

$$\vec{V}_P = 0$$

$$= \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + (-R\sin\theta\hat{\jmath} + R\cos\theta\hat{\imath})\omega$$

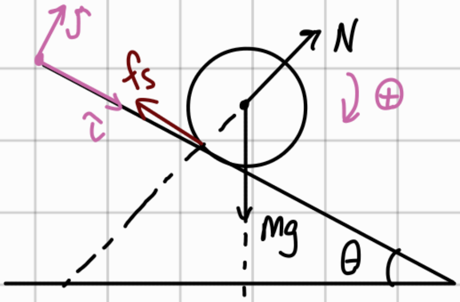
$$= (\dot{x} + R\cos\omega)\hat{\imath} + (\dot{y} - R\sin\theta\,\omega)\hat{\jmath}$$

$\dot{x} = -R\cos\theta\omega$ $\qquad$ $\dot{y} = R\sin\theta\omega$

$\ddot{x} = -R\omega\cos\theta\alpha$ $\qquad$ $\ddot{x} = R\sin\theta\alpha$

Now you can resolve it.

Let's Solve With another Coordinate System:



$$N - mg\cos\theta = 0$$
$$mg\sin\theta - f_s = ma$$

"what happens to rolling without slipping condition?"

$$R f_s = I\alpha = \frac{1}{2} m R^2 \frac{a}{R}$$

$$a = R\alpha$$

$h = R\theta$
$v = R\omega$
$a = R\alpha$

$$\Rightarrow f_s = +\frac{ma}{2}$$

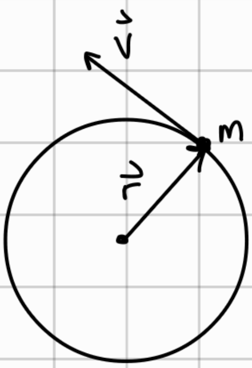
$$\frac{2}{3} g\sin\theta = a$$

See here the friction force does the "work" → gives object rotational KE, But! Linear acc. is reduced by f_s.

So V_{cm} will be less compared to absence of f_s.

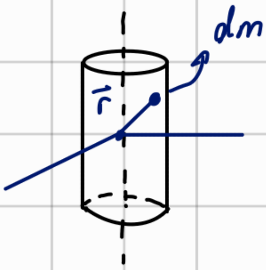
Rotational Kinetic Energy

Pure rotation about CM. of a point particle.



$$\vec{v} = \vec{\omega} \times \vec{r} \qquad KE = \frac{1}{2} m v^2 = \frac{1}{2} m\omega^2 r^2$$

KE of a rotating cylinder:



mass density

$$\int_{body} \left[\frac{1}{2} dm\, \omega^2 r^2\right] \qquad dm = \rho \, d\underbrace{V_{ol}}$$

$dx\,dy\,dz$

$r\,dr\,d\theta\,dz$

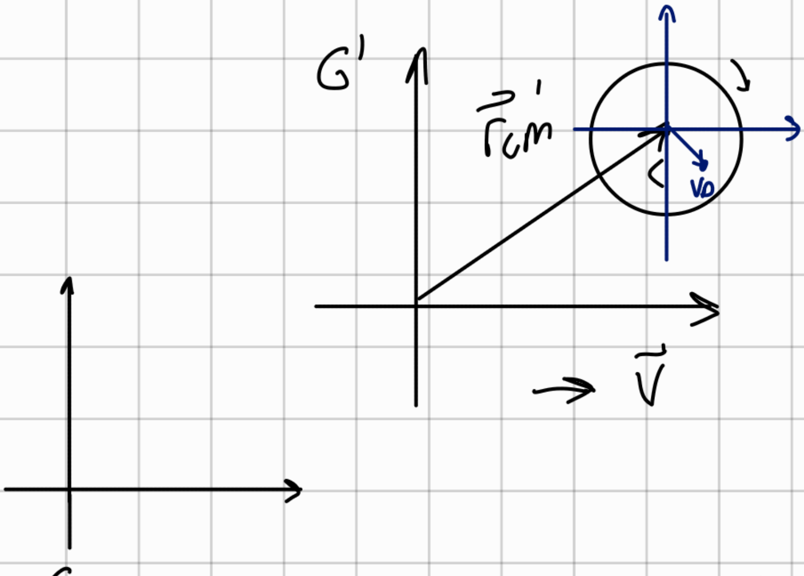
$$KE = \frac{1}{2} I \omega^2$$

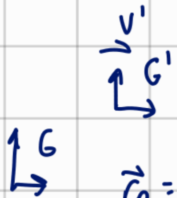
Motion along the CM and motion of CM:

(A drum which rolls)

- You are always told they are "added".

- Add another observer:





$\vec{r}_p = \vec{r}_p - \vec{V} t$

$\vec{r}_p = \vec{r}_p' + \vec{V} t$

$\vec{v}_p = \vec{v}_p' + \vec{V}$

$\vec{\omega}_p \times \vec{r}_p'$ CM coordinates

$$\frac{1}{2} m_i v_p^2 = \frac{1}{2} m_i (\vec{\omega} \times \vec{r}_p' + \vec{V}) \cdot (\vec{\omega} \times \vec{r}_p' + \vec{V})$$

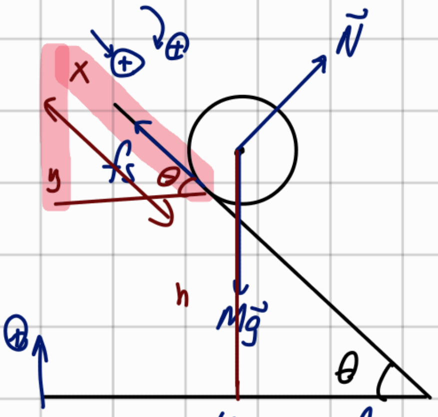
$$\sum_{i=1}^{N} \frac{1}{2} m_i v_p^2 = \frac{1}{2} \sum_{i=1}^{N} m_i (\,\text{"}\,)$$

all particles

$$\begin{bmatrix}\text{full kinetic} \\ \text{energy in the} \\ \text{G frame}\end{bmatrix} = \underbrace{\sum_{i=1}^{N} \frac{1}{2} m_i V^2}_{\frac{1}{2} M V^2} + \underbrace{\sum_{i=1}^{N} \frac{1}{2} m_i (\vec{\omega} \times \vec{r}_p')(\vec{\omega} \times \vec{r}_p')}_{\frac{1}{2} I \omega^2}$$

$$+ \sum_i m_i (\vec{\omega} \times \vec{r}_i) \cdot \vec{V}$$

$$= [\vec{\omega} \times (\Sigma m_i \vec{r}_i)] \vec{V}$$



$h_0 - y$ = distance from the bottom

$$KE = \frac{1}{2} M_{TOT} V_{cm}^2 + \frac{1}{2} I_{cm}\, \omega^2$$

$$N - Mg\cos\theta = 0$$

$$-f_s + Mg\sin\theta = Ma$$

$$Rf_s = I\alpha \qquad f_s = \frac{I\alpha}{R} = \frac{Ia}{R^2}$$

$$a = R\alpha$$

$\longrightarrow$ Do we have mechanical energy conservation?

Yes.

$$(KE + V) = \frac{1}{2} M v^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} M v^2 + \frac{1}{2} M v^2 + \frac{1}{2}\frac{M\cancel{R}^2}{2}\underbrace{\frac{v^2}{\cancel{R^2}}}_{\omega}$$

$$= \frac{3}{4} M v^2 + Mgh$$

$$\frac{dE_{TOT}}{dt} = \frac{3}{2} Mva + Mg\dot{h}$$ one can relate it to v via $\dot{x}$

$$0 \stackrel{?}{=} Mv\left(\frac{3}{2}a - g\sin\theta\right)$$

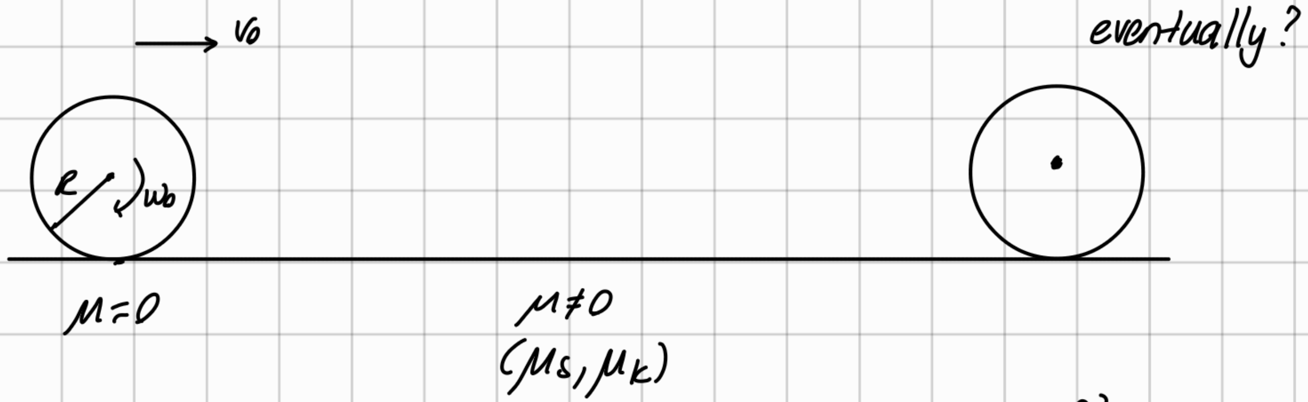
$$\rightarrow \left[-\frac{I}{R}a + Mg\sin\theta = Ma\right]\dot{x}$$

$$-\frac{1}{2}\frac{I}{R^2}\frac{d}{dt}(\dot{x}^2) + \frac{d}{dt}(Mgx\sin\theta)$$

$$= \frac{d}{dt}\left(\frac{1}{2}M\dot{x}^2\right)$$

$$\rightarrow \boxed{-\frac{1}{2}I\omega^2 + V = \frac{1}{2}M V_{cm}^2}$$ conserved for all times

Other problems ...



eventually, rolling without slipping $(v_f = R\omega_f)$

$$\frac{1}{2} M v_0^2 + \frac{1}{2} M \frac{R^2}{2} \omega_0^2$$

$$= \frac{1}{2} M v_f^2 + \frac{1}{2} M \frac{R^2}{2} \omega_f^2$$

+ lost to friction