PHYS312 Lecture - 1

12.02.2025

- Biraz Vector a. gerekecek. (Divergence / Laplacian)

%30 HW

%20 Final Öncesi MT (Finale girmek için)

%50 Final

→ $i\hbar \frac{\partial \Psi}{\partial t} = H \Psi$

H: hermitian differential operator (Hamiltonian)

- eigenvalues always real.
- energy
- "summable"

"extensive" :

- entropy
- volume

- $H = H_{①} + H_{②}$ (etkileşmediklerinde) (seperable and seperable parts commute)

→ What if ... $[H_{①}, H_{②}] \stackrel{?}{=} 0$

★ $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) X(x)\, Y(y) = 0$$

$$\rightarrow \quad Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} = 0 \quad / XY$$

$$\boxed{\underbrace{\frac{1}{X} \frac{\partial^2 X}{\partial x^2}}_{\substack{f(x) \\ = -a}}} + \underbrace{\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2}}_{\substack{g(y) \\ = +a}} = 0$$

Sadece bunu çalışabiliriz.

• "Vektör uzaylarının çarpımı?"

$$A \otimes I + I \otimes B = C$$

$$(?, ?) \qquad\qquad A \otimes B$$

• İma edilen şey:

$$\frac{\partial^2}{\partial x^2} \otimes I + I \otimes \frac{\partial^2}{\partial y^2}$$

• Yani, Schrödinger denklemini H_{1} ve H_{2} ayrı ayrı çözülüyorsa $H = H_{1} + H_{2}$ çözülebilir.

* Şimdi 3D kutudaki parçacık için çözebiliriz.

$$\rightarrow \frac{1}{\sqrt{L_1 L_2 L_3}} \sin\left(\frac{n_1 x \pi}{L_1}\right) \sin\left(\frac{n_2 y \pi}{L_2}\right) \sin\left(\frac{n_3 z \pi}{L_3}\right)$$

$$E = \frac{\hbar^2 \pi^2 n_1^2}{2mL_1^2} + \frac{\hbar^2 \pi^2 n_2^2}{2mL_2^2} + \dots$$

! 2 Particle System in 3D

$$H_{1} = \frac{P_{1}^2}{2m_{1}} \qquad H_{2} = \frac{P_{2}^2}{2m_{2}}$$

Remember:

(1) $[P_{①}^{i}, P_{①}^{j}] = 0 \qquad i = 1, 2, 3$

(2) $[X_{①}^{i}, P_{①}^{j}] = 0 \qquad$ "

(3) $[X_{①}^{i}, P_{①}^{j}] = i\hbar \delta^{ij} = \begin{Bmatrix} 1 & \text{if} & i=j \\ 0 & \text{if} & i \neq 0 \end{Bmatrix}$

Same applies for ②

& $[①, ②] = 0$

Question

" What would happen if we put two non-interacting particles on a 1-D box?"

$$\sqrt{\frac{2}{L}} \sin\left(\frac{n_{①} \pi X_{①}}{L}\right) \cdot \sqrt{\frac{2}{L}} \sin\left(\frac{n_{②} \pi X_{②}}{L}\right)$$

$E =$

Increased particles in const. dimension or increased box dimension with same number of particles $\rightarrow$ SAME MATH!

$$H = H_{1} + H_{2} + H_{12} = \frac{P_{1}^{2}}{2m_{1}} + \frac{P_{2}^{2}}{2m_{2}} + V(|\vec{X}_{2} - \vec{X}_{1}|)$$

Let us define:

$$\vec{X}_{CM} \equiv \frac{m_{1}\vec{X}_{1} + m_{2}\vec{X}_{2}}{m_{1} + m_{2}}$$

$$\vec{P}_{TOT} = \vec{P}_{1} + \vec{P}_{2}$$

Question:

"Do these objects obey the commutation relations (1), (2) and (3)"?

Remember:

$$[A+B, C] = [A, C] + [B, C]$$

$$[A, B] = AB - BA$$

$$[X_{cm}^{i}, X_{cm}^{j}] = 0$$

$$[P_{tot}^{i}, P_{TOT}^{j}] = 0$$

$$[X_{CM}^{i}, P_{TOT}^{j}] = \left[\frac{m_{1} X_{1}^{i} + m_{2} X_{2}^{i}}{m_{1} + m_{2}}, P_{1}^{j} + P_{2}^{j}\right] = i\hbar\, \delta^{ij}$$

$$= \frac{m_{i}}{\quad} \quad \ldots$$

Question

$X_R^i = X_{②}^i - X_{①}^i$ • Does this commute with $\vec{P}_{Tot} = \vec{P}_{①} + \vec{P}_{②}$?

$\longrightarrow$ Any relative coordinate measurement is okay with any total momentum measurement.

• $P_R^i \equiv a\, P_{①}^i + b\, P_{②}^i$

$$\rightarrow [a P_{①}^i + b P_{②}^i,\ m_{①} X_{①}^j + m_2 X_{②}^j] = 0$$

$$\boxed{-i\hbar m_{①} a\, \delta^{ij} - i\hbar m_{②} b\, \delta^{ij} = 0}$$

$$\rightarrow [X_{②}^i - X_{①}^i,\ a P_{①}^j + b P_{②}^j] \equiv i\hbar\, \delta^{ij}$$

$$b i\hbar\, \delta^{ij} - a i\hbar\, \delta^{ij} = i\hbar\, \delta^{ij}$$

$$b = 1 + a$$

$$m_{①} a = - m_{②} b$$

$\rightarrow b = 1 + a$

$$a = -\frac{m_{②}}{m_{①}} b \qquad b = 1 - \frac{m_{②}}{m_{①}} b$$

$$b\left(1 + \frac{m_{②}}{m_{①}}\right) = 1$$

$$\boxed{b = \frac{m_{①}}{m_{①} + m_{②}}} \qquad \boxed{a = -\frac{m_{②}}{m_{①} + m_{②}}}$$

What happens to Hamiltonian?

$$H = \frac{P^2_{TOT}}{\underbrace{2(m_1 + m_2)}_{m_{Total}}} + \frac{P_R^2}{2\mu} + V(|\vec{X}_R|)$$

where, $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$

...

Now we see:

$$H_{CM} = \frac{P^2_{Tot}}{2(m_1+m_2)}$$

$$H_R = \frac{P^2_R}{2\mu} + V(|\vec{X}_R|)$$

Remember:

- seperability depends on the form of the hamiltonian.

$\boxed{H_{CM} \text{ and } H_R \text{ commute}}$!

→ Next week | More general forms of this.

Note: There's an extra d. symmetry (Rotationally invariant hamiltonian)