PHYS 311 Lecture 14

13.11.24

Rest of this class was mathematica demo.

Particle in a box

$$\int_0^L \rho\, dx = 1 \Leftrightarrow \sum_{n=1}^{\infty} |c_n|^2 = 1$$

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

Stationary states

$$\psi_n(x,t) = \sqrt{\frac{2}{L}}\, e^{-i\frac{E_n t}{\hbar}} \sin\left(\frac{n\pi x}{L}\right)$$

Most general solution: $\psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x,t)$ $\quad c_n \in \mathbb{C}$ (& time independent)

Probability to measure x within $[x, x+dx] = \underbrace{|\psi(x,t)|^2}_{\rho(x,t)} dx$

$$j(x,t) = \frac{\hbar}{2mi}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial x} = 0$$

$$\langle x^n \rangle(t) = \int_0^L dx\, \psi^*(x,t)\, \psi(x,t)\, x^n$$

$$\langle p^n \rangle(t) = \int_0^L dx\, \psi^*(x,t) (-i\hbar)^n \frac{\partial^n \psi(x,t)}{\partial x^n}$$

$$E_n = E_1 n^2$$

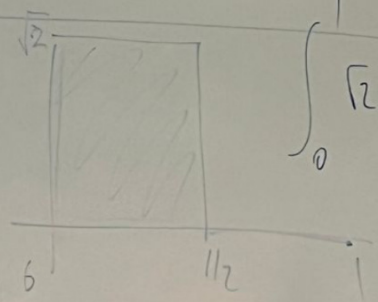
$$\frac{x}{L} \Rightarrow x \in [0,1]$$

$$\frac{E_n t}{\hbar} = \frac{E_1 t}{\hbar} n^2$$

$$\Downarrow$$

$$t\ [-\infty, \infty]$$

$\psi(x,0) =$



$$\int_0^1 \sqrt{2} \sin(m\pi x)\, \psi(x,0) = \int_0^1 \sum_n c_n \underbrace{\sqrt{2} \sin(n\pi x) \cdot \sin(m\pi x) \sqrt{2}}$$

$$\int_0^1 \psi^*(x,0)\, \underbrace{\psi_m(x,0)} = c_m$$

$$\boxed{c_m = \sqrt{2} \int_0^{1/2} dx\, \psi_m(x,0)}$$