

$$\left. \begin{array}{l} F + f_s = Ma \\ -rF + Rf_s = I\alpha \end{array} \right\} \Rightarrow -rF + Rf_s = \frac{MR}{2}a$$

• says something about the direction of  $f_s$ .

• "Yapın aşağıdan ve yukarıdan gelecek menin funkcisi!"  
(Kablolara yukarıdan gelebiliyor ki  $f_s$  bizi bekliyor)

Remember always though!:

$$|f_s| \leq \mu_s |N|$$

## Superball Bounces (will be in HWs)

- Perfect spheres that conserve energy, momentum and angular momentum.

• "Do Newton's Laws have an inverse in time?"

$$\vec{F}(r) = m\vec{a} = m \frac{d^2\vec{r}}{dt^2}$$

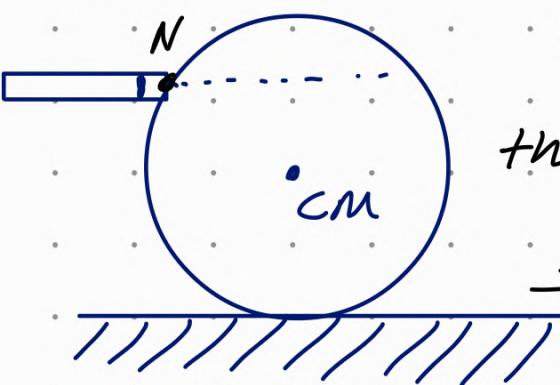
Yes.

"Does the reverse playback of a movie possible?"

entropy works only in one way ( $E_k$  not conserved) { → Different in entropic/complex system (Glass of water breaking so I can understand the film is played backwards)  
→ Not so for mechanic systems.

EX:

(solution is not trivially obvious)



"What happens when you hit the ball from different points?"

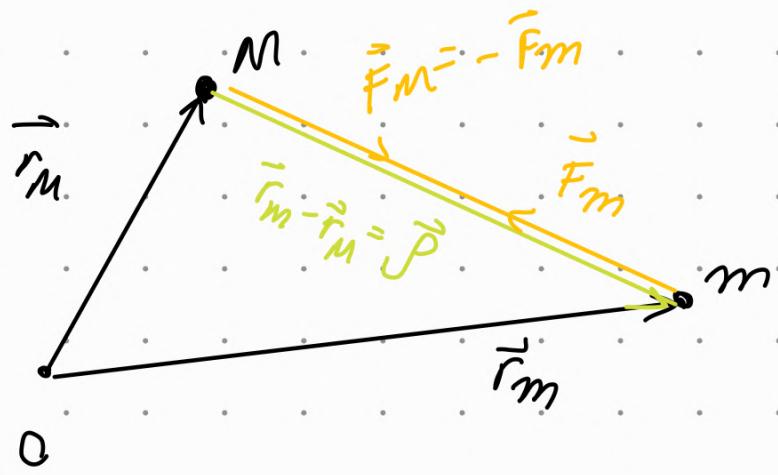
→ You can't simply say "oh there's  $F_N$  at N!"

• There is an impulse.

$$\int F_{\text{tot}} dt = \text{Impulse.}$$

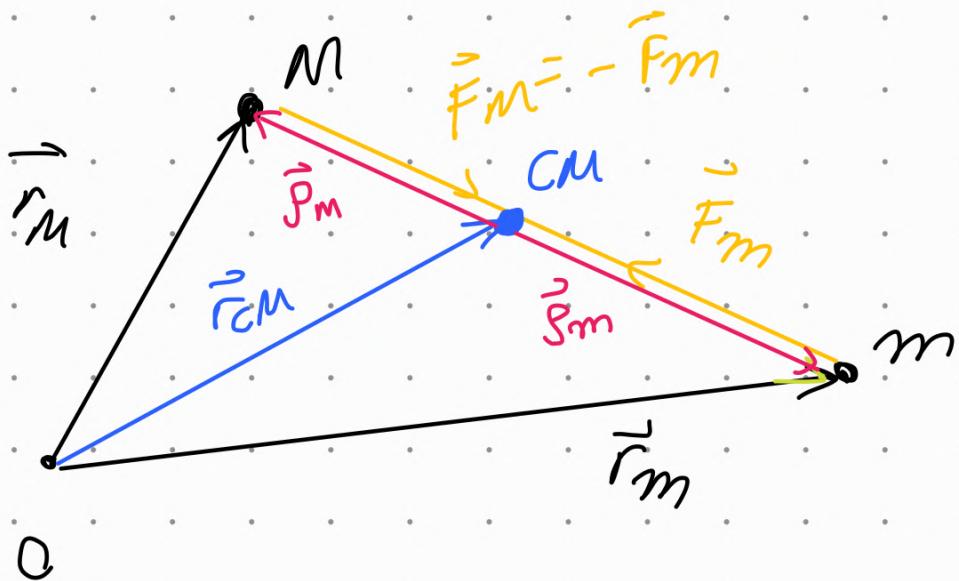
You should work with that.

## Kepler Problem



$$\vec{F}_M = -\frac{GMm}{|\vec{r}_M - \vec{r}_m|^2} \hat{p}$$

## Center of Mass coordinates



$$\bullet \vec{r}_M = \vec{r}_{CM} + \vec{p}_M \quad \vec{r}_m = \vec{r}_{CM} + \vec{p}_m \quad \Rightarrow$$

$$\boxed{\vec{r}_{CM} = \frac{M\vec{r}_M + m\vec{r}_m}{M+m}}$$

$$\bullet \frac{Md^2\vec{r}_M}{dt^2} = \vec{F}_M \quad ; \quad m \frac{d^2\vec{r}_m}{dt^2} = \vec{F}_m$$

$$M \frac{d^2 \vec{r}_{CM}}{dt^2} + m \frac{d^2 \vec{p}_m}{dt^2} = -\hat{\vec{p}}_M \frac{GMm}{|\vec{p}_M - \vec{p}_m|^2}$$

$$m \frac{d^2 \vec{r}_{CM}}{dt^2} + m \frac{d^2 \vec{p}_m}{dt^2} = -\hat{\vec{p}}_m \frac{GMm}{|\vec{p}_M - \vec{p}_m|^2}$$

$$\Rightarrow M \frac{d^2 \vec{r}_{CM}}{dt^2} + \underbrace{\frac{d}{dt^2} [M \vec{p}_M + m \vec{p}_m]}_0 = 0$$

$$\frac{d^2 \vec{r}_{CM}}{dt^2} = 0.$$

CM doesn't move.

$$\vec{p}_M - \vec{p}_m \equiv \vec{r}_\mu : \text{relative mass.}$$

$$\Rightarrow M \frac{d^2 \vec{p}_M}{dt^2} - m \frac{d^2 \vec{p}_m}{dt^2} = (-\vec{p}_M + \vec{p}_m) \frac{GMm}{|\vec{p}_M - \vec{p}_m|^3} = -\vec{r}_\mu \frac{GMm}{|\vec{r}_\mu|^3}$$

$$M \frac{d^2 \vec{r}_M}{dt^2} - m \frac{d^2 \vec{r}_m}{dt^2} = -\vec{r}_\mu \frac{GMm}{|\vec{r}_\mu|^3}$$

$$M \vec{r}_M - m \vec{r}_m = A \vec{r}_{CM} + B \vec{r}_\mu$$

$$(M+m) \vec{r}_{CM} = M\vec{r}_M + m\vec{r}_m$$

$$\mu (\vec{r}_\mu = \vec{r}_M - \vec{r}_m)$$

$$(M+m) \vec{r}_{CM} + m$$

∴ (missed some steps)

$$\vec{r}_M = \vec{r}_{CM} + \frac{m}{M+m} \vec{r}_\mu$$

$$\mu = \frac{Mm}{M+m}$$

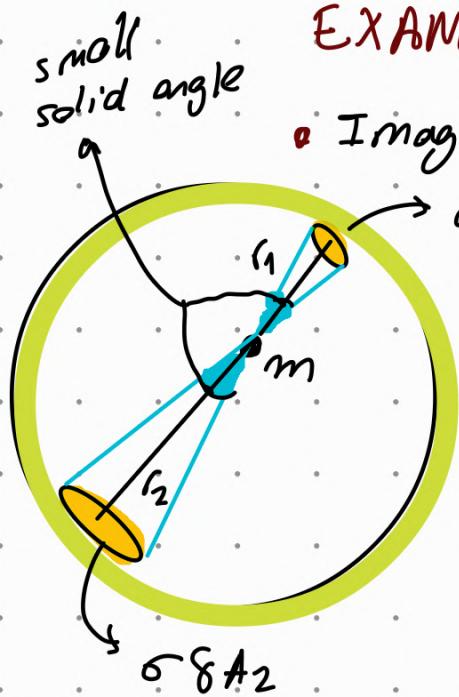
$$\vec{r}_m = \vec{r}_{CM} - \frac{M}{M+m} \vec{r}_\mu$$

$$\frac{2Mm}{M+m} \frac{d^2 \vec{r}_\mu}{dt^2} = -\vec{r}_\mu \frac{GMm}{|\vec{r}_\mu|^3}$$

You study motion of 2 particles. But you can ignore there's 2 particles, because they "act" like 1.

### EXAMPLE / NOTE

Imagine you are inside a uniform shell sphere



$$mG\alpha \left( \frac{\delta A_1}{r_1^2} - \frac{\delta A_2}{r_2^2} \right)$$

$$\delta A_1 = r_1^2 \delta \Omega \quad \text{solid angle}$$

$$\delta A_2 = r_2^2 \delta \Omega$$



This forces cancel each other.

So when you sum everything up, you'll see everything will sum up to 0.

This shows Gravitation inside is 0. (You float)

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### General Central Force Problem

$$\mu \frac{d^2\vec{p}}{dt^2} - \hat{p} f(|\vec{p}|) \vec{F} \quad | \quad \vec{F}(\vec{p}) = -\vec{\nabla} V(|\vec{p}|)$$

$$E_{\text{Total}} = \frac{1}{2} \mu |\vec{p}|^2 + V(|\vec{p}|)$$

$$\frac{dE_{\text{Total}}}{dt} = 0 \quad \mu \vec{p} \times \dot{\vec{p}} \equiv \vec{L} \rightarrow \frac{d\vec{L}}{dt} = 0$$

$$\frac{d}{dt} V(x(t), y(t), z(t)) = \dot{x} \frac{\partial V}{\partial x} + \dot{y} \frac{\partial V}{\partial y} + \dot{z} \frac{\partial V}{\partial z}$$

"Motion is constraint on a plane"

→ Defined by ( $L$ )

- Pergacığın belirli bir yere boyduktan sonra  $\vec{v}_0 / \vec{x}_0$  pergacığın hareket edeceğini düzleme belirterek ( $\mu \vec{p} \times \dot{\vec{p}} = \vec{L}$ ) (Düzen etti olmadığını)