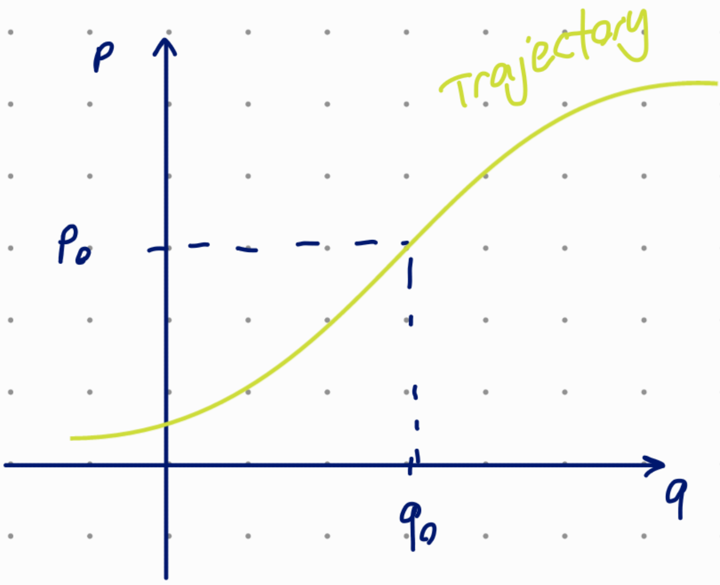
PHYS 311 Lecture 11 04.11.2024

Behr - Sommerfeld Quantization Prescription

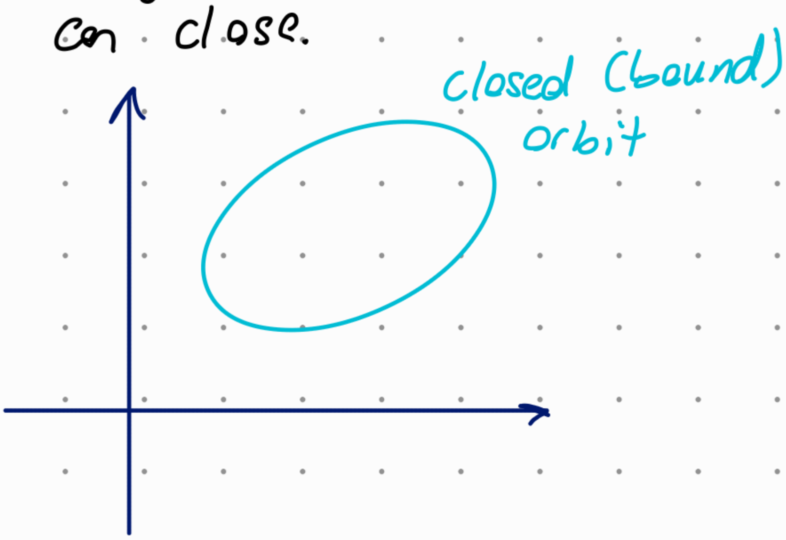
Phase Space



- $p = m\frac{dq}{dt}$
- $\frac{dp}{dt} = F(q)$
- $\frac{d}{dt}\begin{pmatrix} q \\ p \end{pmatrix}$ is known.

Closed (Bound) Orbits

- Trajectories can't cross themselves, but they can close.



Heisenberg's Uncertainty Principle

$$\Delta q \Delta p \geq \frac{\hbar}{2}$$

- He used wave motion

- "Exact freq = Exact wavelength" but you can't tell "where" the wave is.

- The moment you used superposition theorem to confine the wave, you can't know the exact frequency.

Remember:

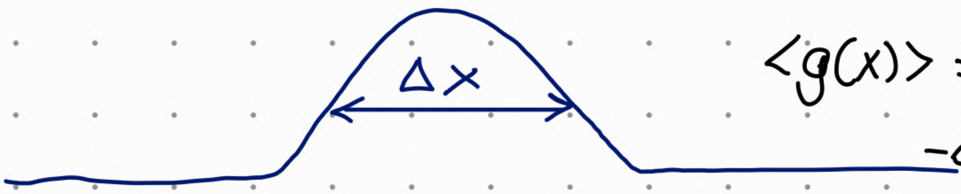
$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \, e^{ikx} f(x)$$ (Fourier Transform)

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \, e^{-ikx} \tilde{f}(k)$$ (inverse FT)

$f(x)$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

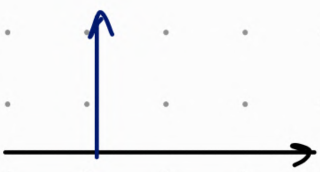
$$\langle g(x) \rangle = \int_{-\infty}^{\infty} dx \, g(x) f(x)$$



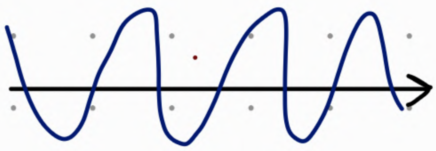
"If Δx is small, Δp is large and vice versa"

THIS WAS KNOWN!

What Heisenberg did:



don't know where



don't know frequency

"You can superpose various wavelength to any weirdo shape"

• If there's a wave (linear) it must obey these

$\rightarrow \Delta k \Delta x > \frac{1}{4\pi}$

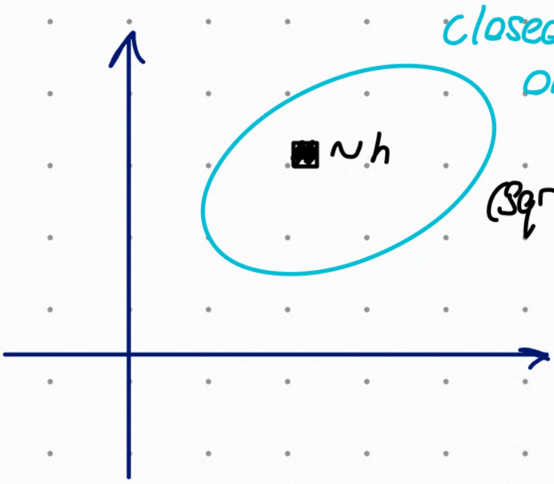
$\hbar = \frac{h}{2\pi}$

$\lambda = \frac{h}{p}$

$k = \frac{p}{h}$

} de Broglie's ideas

closed (bound) orbit



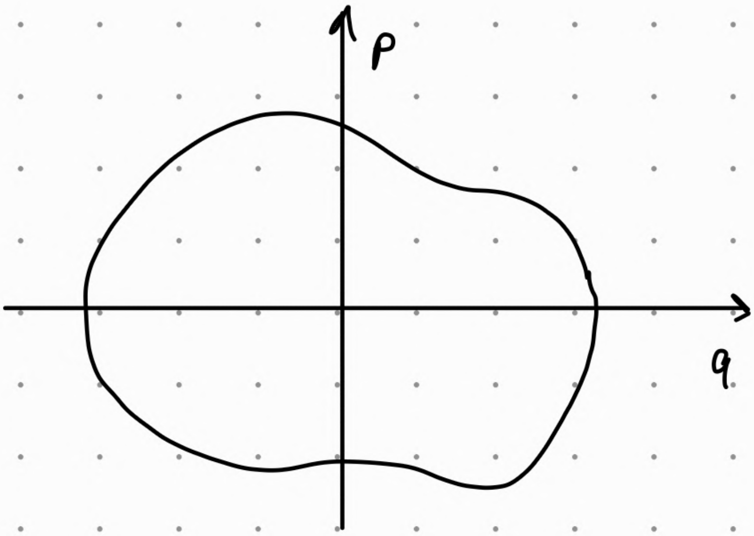
(Sqr area is ~ planck's constant)

→ Calculate the area by summing little area's ■

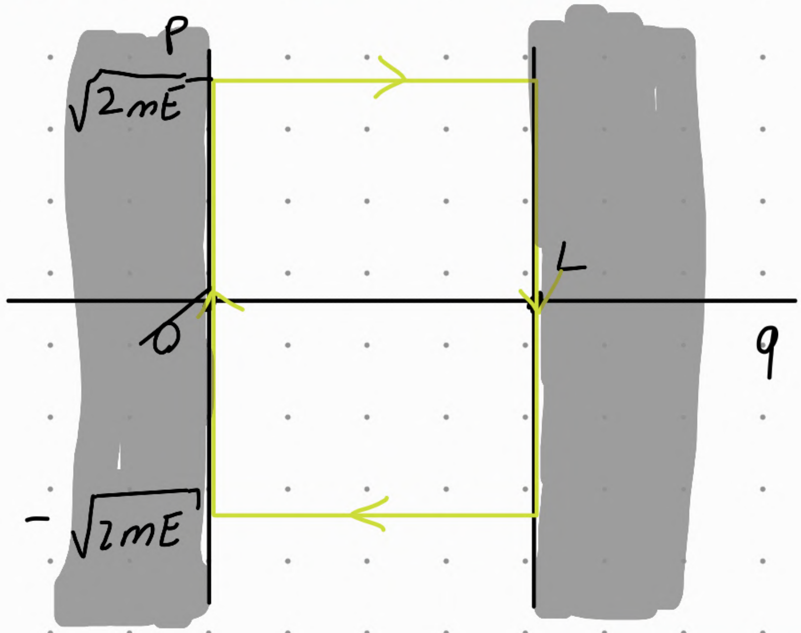
→ Larger the area, less the error.

$E = \frac{p^2}{2m} + V(q) \rightarrow p = \sqrt{2mE - V(q)}$

$\oint_{orbit} p\, dq$



Particle In A Box



Area:

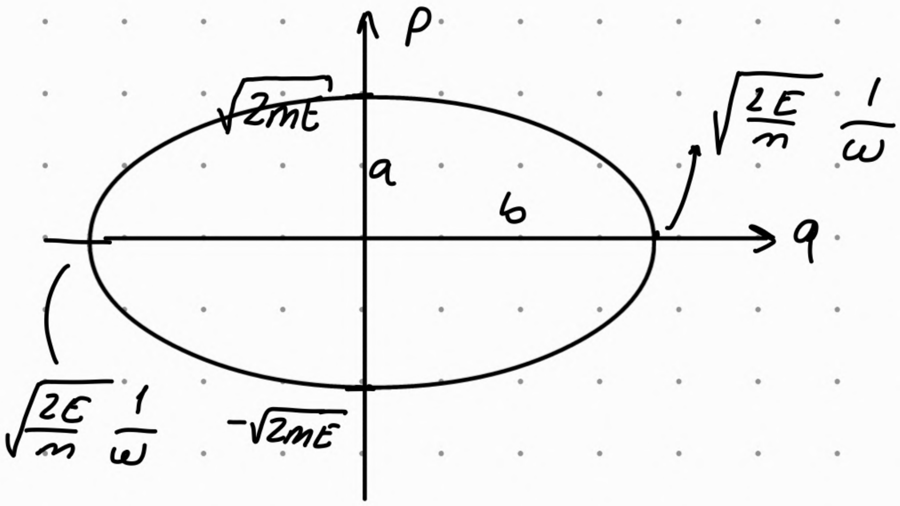
$$2\sqrt{2mE}\cdot L = A$$
$$= nh$$

$$\hbar = \frac{h}{2\pi}$$

$$E = \frac{n^2 h^2}{8mL^2} \Rightarrow \boxed{E = \frac{n^2 \hbar^2 \pi^2}{2mL^2}}$$

Harmonic Oscillator

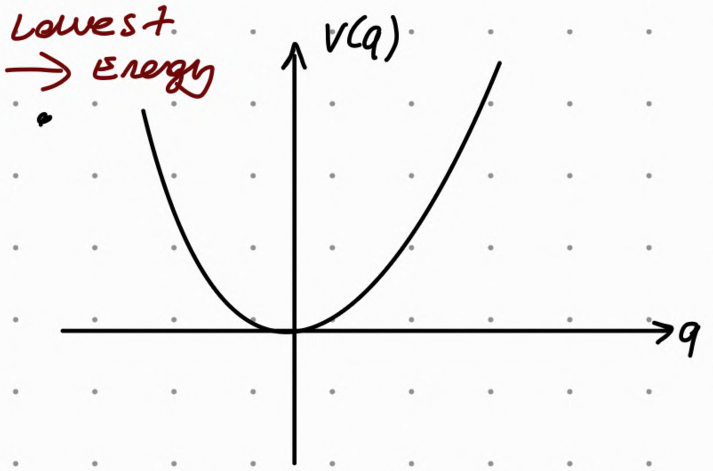
$$E = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2 \quad \text{(ellipse)}$$



$$A = \pi ab$$

$$\frac{1}{\omega}\pi\sqrt{2mE}\sqrt{\frac{2E}{m}}$$
$$= nh$$

$$\boxed{E = n\hbar\omega}$$



$$E = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2$$

In the ground state:

$\Delta p \sim p$ (bcuz its not well defined)

$\Delta q \sim q$

$\downarrow$

$pq \sim \hbar/2$

Now we can minimize the equation.

$$p = \hbar/2q \qquad E = \frac{\hbar^2}{8mq^2} + \frac{1}{2} m\omega^2 q^2$$

$$\frac{dE}{dq} = 0 = -\frac{\hbar^2}{4mq^3} + m\omega^2 q$$

$$q^4 = \frac{\hbar^2}{4m^2\omega^2} \qquad q^2 = \frac{\hbar}{2m\omega}$$

$$E = \frac{\hbar^2 \, 2m\omega}{8m\hbar} + \frac{1}{2} m\omega^2 \frac{\hbar}{2m\omega} = \boxed{\frac{\hbar\omega}{2}}$$

- In groundstate it's <u>never</u> 0!

We are Ready to do Things The Right Way

$\Delta q \Delta p \geq \frac{\hbar}{2}$ $\Delta \omega \Delta t \geq \frac{1}{4\pi}$

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int dx \, e^{ikx} f(x)$$

$\Delta k \Delta x \geq \frac{1}{4\pi}$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int dk \, e^{-ikx} \tilde{f}(k)$$

- Can do the same in frequency domain.

$$\tilde{F}(\omega) = \frac{1}{\sqrt{2\pi}} \int dt \, e^{i\omega t} F(t)$$

$\Delta \omega \Delta t \geq \frac{1}{4\pi}$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int d\omega \, e^{-i\omega t} \tilde{F}(\omega)$$

(It has all the properties of a linear vector(?) space)

- Now by using Planck's idea $\hbar\omega$, we can write:

$$\Delta E \Delta t \geq \hbar/2$$

- "You must think of momentum as an operator, not a number."
  ↪ But time is just a parameter.

- So, long lived spaces (Δt ∞) It may have precise energy.

- An operator is a function over a space of physical states onto another space of states. The simplest example of the utility of operators is the study of symmetry (which makes the concept of a group useful in this context). Because of this, they are useful tools in classical mechanics. Operators are even more important in quantum mechanics, where they form an intrinsic part of the formulation of the theory.

$$-i \frac{d}{dx} f(x) = \boxed{\frac{1}{\sqrt{2\pi}} \int dk\, e^{ikx}} \underbrace{k \tilde{f}(k)}$$

$k = \frac{2\pi}{\lambda}$ (wave number)

$= \frac{p}{\hbar}$

Momentum Operator

$$\hat{P}_{op} \equiv -i\hbar \frac{d}{dx}$$

"How do I take p from it?"

$\rightarrow \hat{P}_{op}$!

$$e^{ikx} = e^{i\frac{px}{\hbar}}$$

$$\boxed{\hat{P}_{op} \left(e^{i\frac{px}{\hbar}} \right) = p \left(e^{i\frac{px}{\hbar}} \right)}$$

(eigenvalue)

STATE

non relativistic

$$(\hat{P}_{op})^2 e^{\frac{ipx}{\hbar}} = p^2 e^{\frac{ipx}{\hbar}} = 2mE e^{\frac{ipx}{\hbar}}$$

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} = E - V(x)$$

The Energy Eigenvalue Equation

$$\boxed{\underbrace{\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right)}_{\text{Hamiltonian operator}}\psi_E(x) = E\,\psi_E(x)}$$

No time!

- How to get time?

$$\psi_E(x)\, e^{\frac{-iEt}{\hbar}} \equiv \psi(x,t)$$

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = E\,\psi_E(x)\, e^{\frac{-iEt}{\hbar}}$$

$$\boxed{i\hbar\frac{\partial\psi(x,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2+V\right)\psi(x,t)}$$

$$\rho \equiv |\psi|^2 \qquad \vec{J} \equiv \frac{\hbar}{2mi}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)$$

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\vec{J} = 0$$

$$\frac{d}{dt}\underbrace{\int\rho\, d^3\vec{r}}_{\text{everywhere}} + \underbrace{\int\vec{J}\,d\vec{s}}_{\text{over boundry}} = 0$$

$=0$ (if $\psi \to 0$)

Because ψ is linear, this part can be made 1.

↳ "Here comes the probability..."

- $|\psi|^2 d^3\vec{r}$ is the probability to observe the location in $d^3\vec{r}$ at $\vec{r}$

Why do they make this?

- Maxwell's Eqn.

→ Because there's a probability we can't talk about only one particle.

- We need an ensemble;
  - Probability is NOT associated with a single event, it's associated with an ensemble.