PHYS 311 Lecture 17

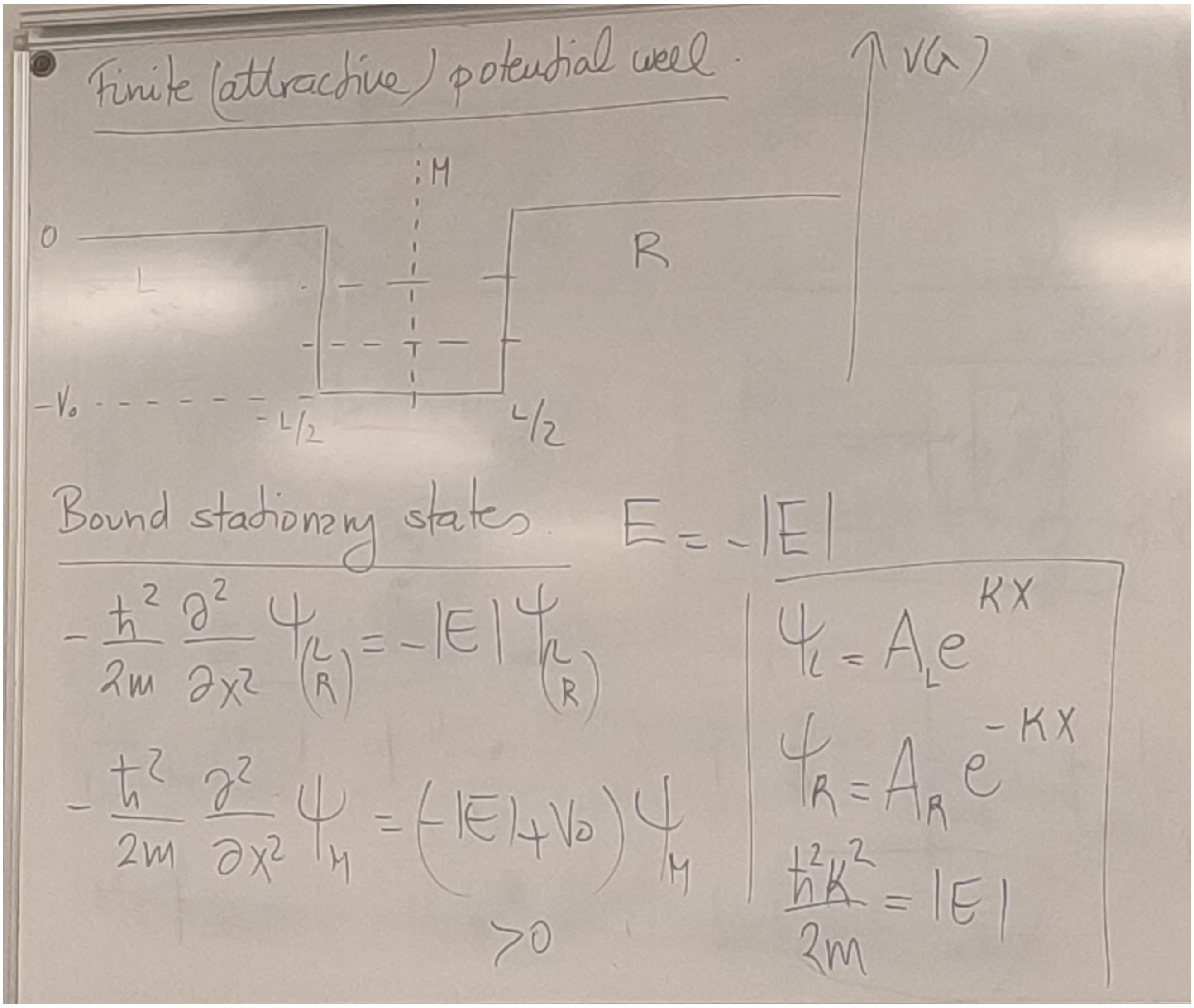
#physics #phys311 #lecturenotes

02.02.2024

Finite Potential Well (Finite Square Well)

More can be found in the book.

Finite Attractive Potential Well



Bound Stationary States

Should be: $E = -|E|$

$$\frac{-\hbar}{2m}\frac{\partial^2}{\partial x^2}\psi_{LR} = -|E|\psi_{LR}$$

$$\frac{-\hbar}{2m}\frac{\partial^2}{\partial x^2}\psi_M = (-|E| + V_0)\psi_M$$

We see that $-|E| + V_0 > 0$

$$\psi_L = A_L e^{\kappa x}$$
$$\psi_R = A_R e^{-\kappa x}$$
$$\frac{\hbar\kappa}{2m} = |E|$$

We see that wave function and its derivative should be continuous.

$$\psi_M = \alpha\sin(kx) + \beta\cos(kx),\ \frac{\hbar^2 k^2}{2m} = V_0 - |E|$$

Since this is an **even potential** we can write this to apply the boundary condition easier.

$$\psi^{even}(x) = \begin{cases} Ae^{\kappa x} \\ \\ \beta\cos(kx) \\ Ae^{-\kappa x} \end{cases}$$

$$\psi^{odd}(x) \begin{cases} \tilde{A}e^{\kappa x} \\ \alpha\sin(kx) \\ -\tilde{A}e^{-\kappa x} \end{cases}$$

Thus

$$\frac{\hbar k^2}{2m} + \frac{\hbar\kappa^2}{2m} = V_0$$

even:

$$Ae^{\kappa L/2} = \beta\cos(k\frac{L}{2})$$
$$\kappa Ae^{-\kappa L/2} = -k\beta\sin(k(-\frac{L}{2})) = k\beta\sin(k\frac{L}{2})$$

even

$$\tan(\frac{kL}{2})\frac{kL}{2} = \frac{\kappa L}{2}$$

See A/β:

$$= e^{\kappa L/2}\cos(kL/2) = \frac{k}{\kappa}e^{\kappa L/2}\sin(k\frac{L}{2})$$

odd:

$$\tilde{A}e^{kL/2} = -\alpha\sin(kL/2)$$
$$\kappa\tilde{A}e^{kL/2} = \alpha k\cos\left(\frac{kL}{2}\right)$$
$$\frac{\tilde{A}}{\alpha} = -e^{-\kappa L/2}\sin\left(\frac{kL}{2}\right)$$
$$\frac{\tilde{A}}{\alpha} = \frac{k}{\kappa}\cos\left(\frac{kL}{2}\right)e^{-\kappa L/2}$$

Summarizing all the equations

even: $\tan(\frac{kL}{2})\frac{kL}{2} = \frac{\kappa L}{2}$
odd: $-\cot(\frac{kL}{2})\frac{kL}{2} = \frac{\kappa L}{2}$

and for all

$$\frac{\hbar^2k^2L^2}{4.2m} + \frac{\hbar^2\kappa^2L^2}{2m.4} = \frac{V_0L^2}{4}$$

$tan(x)x = y$ even
$-cot(x)x = y$ odd

$$x^2 + y^2 = \frac{2mV_0L^2}{???}$$

{PUT GRAPH HERE}

???

"Bir yere kadar odd diye bir sey yok!"

"Sonsuz tane bound state olamaz!" Hidrojen atomu buna tam uymuyor. r=0'dan itibaren sonsuz potansiyel var. Potansiyel sonlu kaliyorsa bound state'leri de sayilir olmak zorunda

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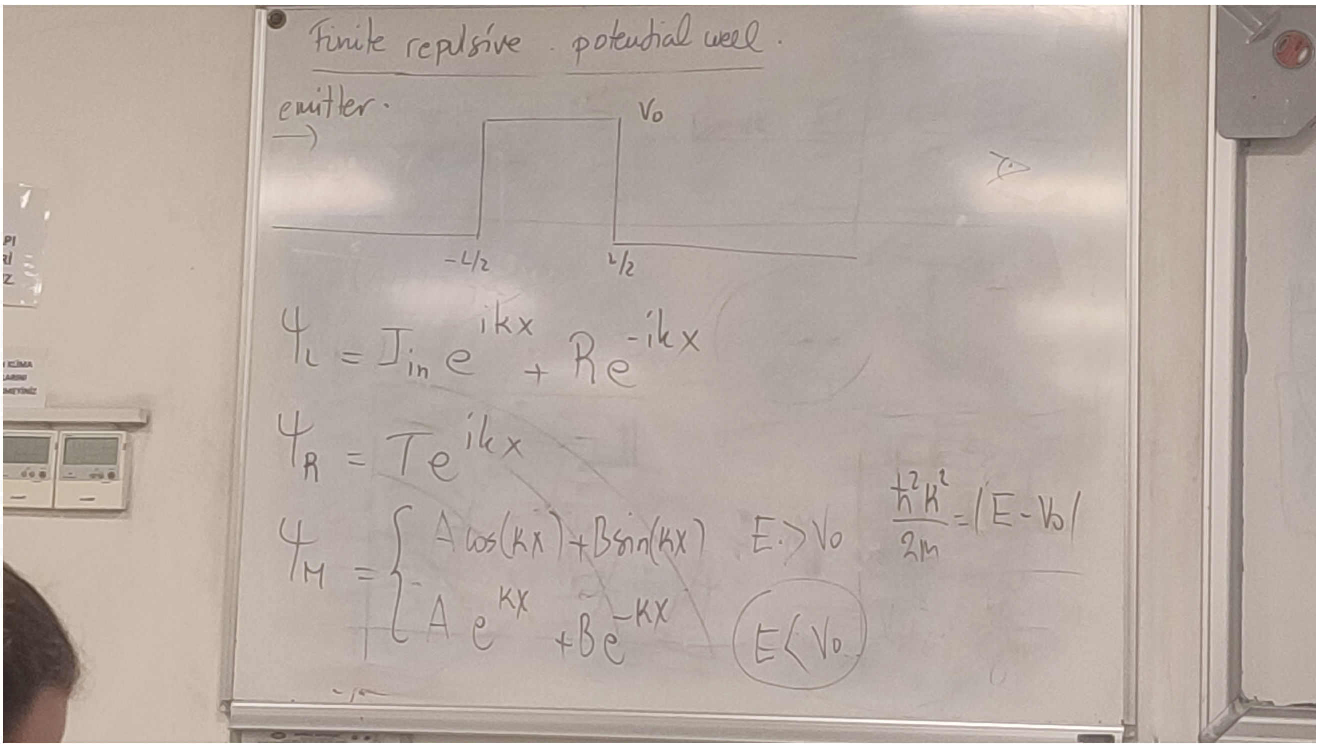
Finite Repulsive Potential Well



$$\psi_L = I_{in}e^{ikx} + Re^{-ikx}$$
$$\psi_R = Te^{ikx}$$
$$\psi_M = \begin{cases} A\cos(\kappa x) + B\sin(\kappa x), E > V_0 \\ Ae^{\kappa x} + Be^{-\kappa x}, E < V_0 \end{cases}$$

Remember: $\frac{\hbar^2\kappa^2}{2m} = |E - V_0|$

(I_{in} is on me -> 4 equations, 4 unknowns.)

Then:
At (x = -L/2):

$$I_{\text{in}}e^{-ikL/2} + Re^{ikL/2} = Ae^{kL/2} + Be^{-kL/2}$$
$$k\left(I_{\text{in}}e^{-ikL/2} - Re^{ikL/2}\right) = k\left(Ae^{kL/2} - Be^{-kL/2}\right)$$

At (x = L/2):

$$Te^{ikL/2} = Ae^{kL/2} + Be^{-kL/2}$$
$$kTe^{ikL/2} = k\left(Ae^{kL/2} - Be^{-kL/2}\right)$$

Then we did a shit ton of calculations for too long...
Final: 6 soru cevap
Next Time: Spin and Angular Momentum