PHYS 311 – Lecture 4 02.10.2024

- Energy is additively seperated in Newtonian physics.

Non-relativistic kinetic E : $\tilde{E} = \frac{p^2}{2m}$

$$\tilde{E} = \frac{p^2}{2m} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}$$

- In relativistic physics, it is not additive seperated.

$$\boxed{E^2 = p^2c^2 + m^2c^4} \qquad \boxed{\vec{u} = \frac{\vec{p}\,c^2}{E}}$$

- Important question: what if $m=0$? (Can massless particles exist?)

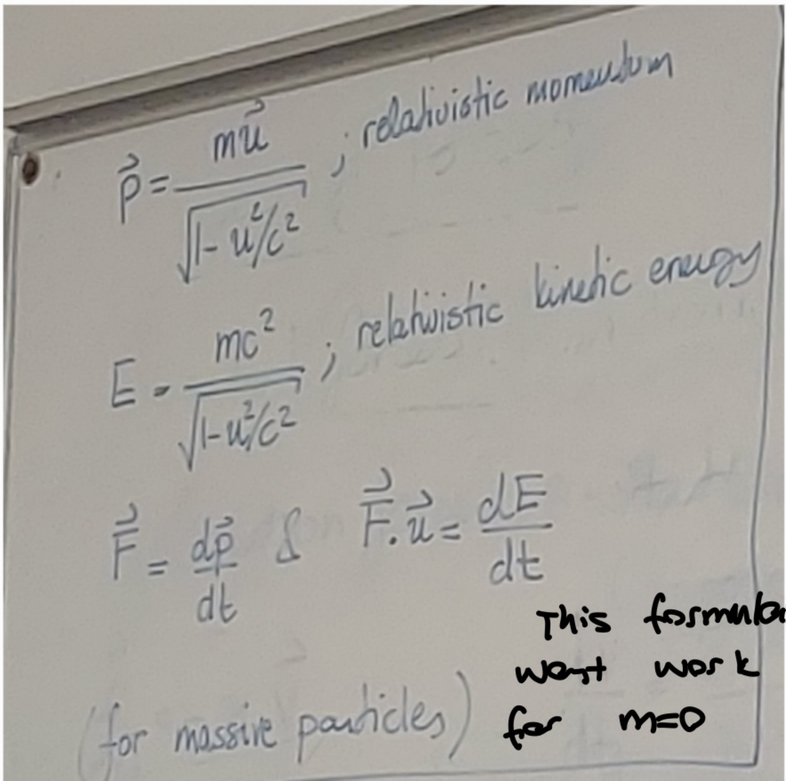
$$E^2 = p^2c^2 \quad \& \quad \vec{u} = \frac{\vec{p}\,c^2}{E}$$

$$E = |\vec{p}|c \Rightarrow \vec{u} = \frac{\vec{p}\,c^2}{|\vec{p}|c} = \hat{p}\,c = \hat{u}\,c$$

$$\Rightarrow |\vec{u}| = c$$

! Massless particles may exist. And they move with the speed of light.

Remember The Formulas :



This formulas won't work for $m=0$

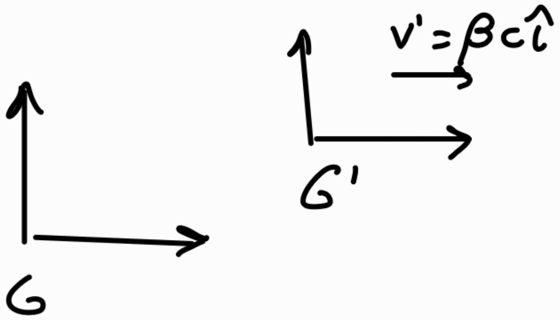
$$\frac{d\vec{p}}{dt} = \vec{F} \qquad \frac{dE}{dt} = \vec{F}\cdot\vec{u}$$

- How can you find the energy of a massless particle
- The sun bends light (Eclipse experiment)

① Doppler Effect for Light

- E and $\vec{p}$ were elements of a 4-vector.
  → So, they should be able to transform under Lorentz transformations.

- $$\frac{E'}{c} = \gamma\left(\frac{E}{c} - \beta p_x\right)$$
- $$p_x' = \gamma\left(p_x - \beta \frac{E}{c}\right)$$
- $$p_y' = p_y$$
- $$p_z' = p_z$$



If $m = 0$

$$E = |p_x| c \iff E' = |p_x'| c$$

$$E' = \gamma(E - \beta p_x c)$$

$$= \gamma(E - \beta \underbrace{|p_x|}_{E/c} \hat{p}_x c)$$

$$E' = \gamma(E - \beta E \hat{p}_x) \approx \frac{E}{\sqrt{1-\beta^2}}(1 \mp \beta)$$

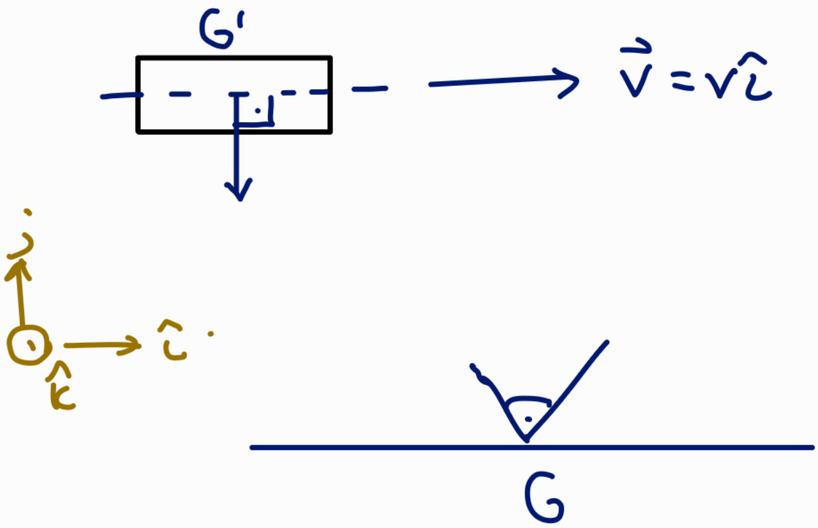
$$E' = \frac{E(1 \mp \beta)}{\sqrt{(1-\beta)(1+\beta)}} \qquad \boxed{E' = E\sqrt{\frac{1-\beta}{1+\beta}}}$$

How is this a doppler effect?

$$\boxed{E' = E\sqrt{\frac{1-\beta}{1+\beta}}} \Rightarrow f' = f\sqrt{\frac{1-\beta}{1+\beta}}$$

- See how dealing with a 4-vector made our life way easier!

Vertical Doppler Effect



Lorentz Transformation

- $\dfrac{E'}{c} = \gamma\left(\dfrac{E}{c} - \beta p_x\right)$
- $p_x' = \gamma\left(p_x - \beta \dfrac{E}{c}\right)$
- $p_y' = p_y$
- $p_z' = p_z$

$E' = E_0 \qquad \vec{p} = -\hat{j} p_0 \qquad E_0 = p_0 c$

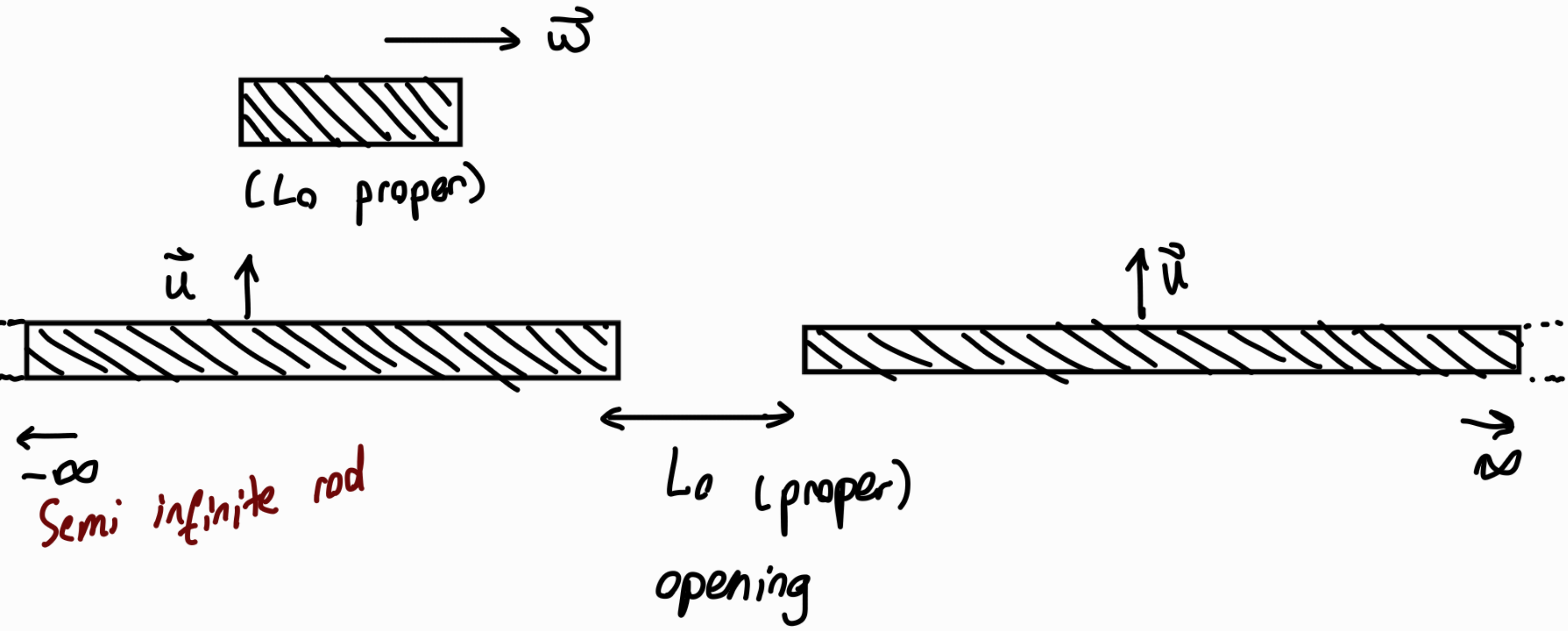
$$\frac{E}{c} = \gamma\left(\frac{E_0}{c} + 0\right) \Rightarrow f = \frac{f_0}{\sqrt{1-\beta^2}}$$

$$p_x = \gamma\beta\frac{E_0}{c}$$

$$p_y = -p_0 = -\frac{E_0}{c}$$

$$p_z = p_z = 0$$

Exercise (Drawn for us, 3rd person observer)



Can L_0 pass from the opening? Using Lorentz trans. you can show.

- Remember Lorentz transformations are not intuitive

→ **Answer:** Semi-infinite rods get tilted!

- "Group commutative" "Consequitive lorentz transformations"
  - ↳ you may end up being rotated

Example: Infinite Submarine Paradox : The submarine sinks

$\circledast$ $m_0 \neq 0$ motion under $\vec{F} = F_0 \hat{\imath}$ (can also solve this for massless particles)

$$\vec{p}(t) = \vec{p}_0 + F_0 t \hat{\imath}$$

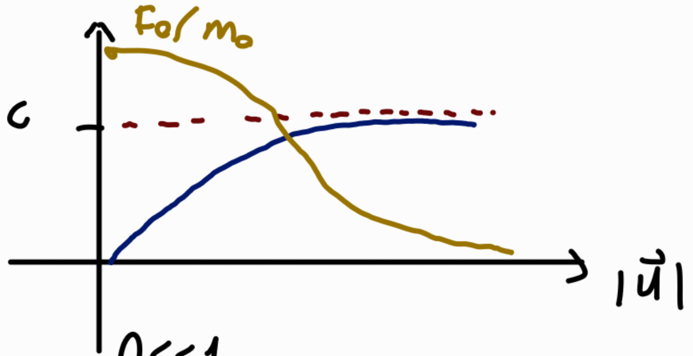
① $\vec{p}_0 = 0$ $\quad \vec{p}(t) = F_0 t \hat{\imath}$

$$p^2 = \frac{m_0^2 u^2}{1 - u^2/c^2} \Rightarrow p^2 = (m_0^2 + p^2)\frac{u^2}{c^2}$$

$$\frac{u^2}{c^2} = \frac{F_0^2 t^2}{m_0^2 c^2 + F_0^2 t} = \frac{\frac{F_0^2 t^2}{m_0^2 c^2}}{1 + \frac{F_0^2 t^2}{m_0^2 c^2}}$$

$$\boxed{\theta \equiv F_0 t / m_0 c}$$

$$\vec{u} = c \frac{\theta}{\sqrt{1+\theta^2}} \hat{\imath}$$



$\theta << 1$

$$|\vec{u}| \simeq c\theta = \frac{F_0}{m_0} t$$

$$\vec{a} = \frac{d\vec{u}}{dt} = \frac{d\vec{u}}{d\theta} \cdot \frac{d\theta}{dt} = \hat{\imath} \frac{F_0}{m_0} \frac{d}{d\theta}\left[\frac{\theta}{\sqrt{1+\theta^2}}\right]$$

$$= \hat{\imath} \frac{F_0}{m_0} \frac{1}{(1+\theta^2)^{3/2}}$$

$$\frac{1}{\sqrt{1+\theta^2}} - \frac{\theta \cancel{\frac{1}{2}} \cancel{2}\theta}{\sqrt{1+\theta^2}(1+\theta^2)} = \frac{1}{\sqrt{1+\theta^2}}\left(1 - \frac{\theta^2}{1+\theta^2}\right)$$

Example (Rindler Horizon)

Can you dodge a laser? There will be a distance that you can.

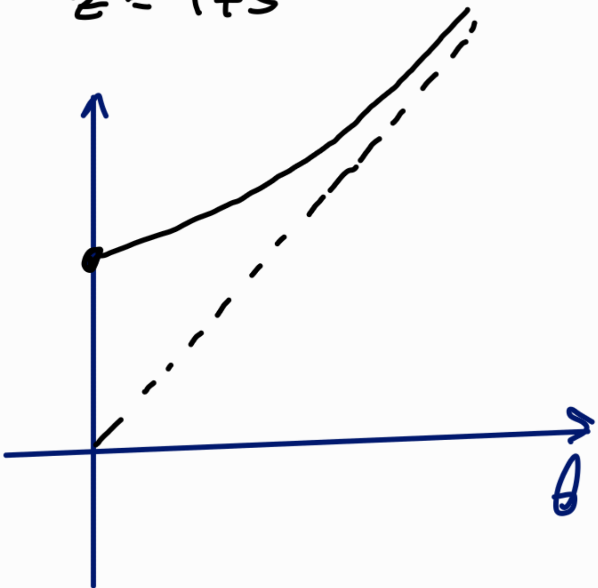
$$x = \int u\,dt = \int u \frac{m_0 c}{F_0} d\theta$$

$$= \frac{m_0 c^2}{F_0} \int_0^\theta \frac{\theta\, d\theta}{\sqrt{1+\theta^2}} = \frac{m_0 c^2}{2F_0} \int \frac{d(\theta)^2}{\sqrt{1+\theta^2}}$$

$$= \frac{m_0 c^2}{2F_0} \int \frac{ds}{\sqrt{1+s}} \qquad dz = ds \quad z = 1+s$$

$$= \frac{m_0 c^2}{2F_0} \int \frac{dz}{\sqrt{z}}$$

$$x_s(\theta) = \frac{m_0 c^2}{F_0}\left[\sqrt{1+\theta^2} - 1\right]$$



$$x_{ship} = \frac{m_0 c^2}{F_0}\left[\sqrt{1+\theta^2} - 1\right] + d_0$$

$$x_{laser} = \frac{m_0 c^2}{F_0} \theta$$

$$\eta_{ship} = \sqrt{1+\theta^2} - 1 + \eta_0$$

$$\eta_{laser} = \theta$$

$$\eta_{ship} - \eta_{laser} > 0$$

$$\underbrace{\sqrt{1+\theta^2} - \theta}_{>0} + \eta_0 - 1 > 0$$

$$\eta_0 > 1$$

$$\boxed{d_0 > \frac{m_0 c^2}{F_0}}$$

No light can reach you!

- You'd need infinite fuel (motion should stay)
- You'd need infinite mass (not happening