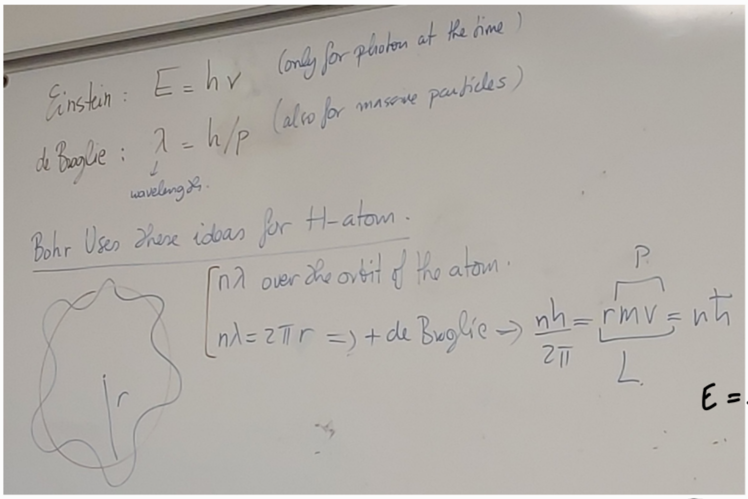
PHYS 311 Lecture 10

23.10.2024



$$\underbrace{\frac{mv^2}{r}}_{\text{centripetal}} = \underbrace{\frac{e^2}{4\pi\epsilon_0}\frac{1}{r^2}}_{\text{coulomb}} \Rightarrow \frac{e^2}{4\pi\epsilon_0 m}$$

$$E = \frac{1}{2}mv^2 - \frac{e^3}{4\pi\epsilon_0}\frac{1}{r} \qquad v = \frac{n\hbar}{mr}$$

$$E = \frac{1}{2}m\frac{n^2\hbar^2}{m^2r^2} - \frac{e^2}{4\pi\epsilon_0}\frac{1}{r} \qquad \frac{n^2\hbar^2}{m^2} = \frac{1}{r}\frac{e^2}{4\pi\epsilon_0 m}$$

$$E = -\frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2}\frac{1}{n^2} \qquad \boxed{r = \frac{n^2\hbar^2 4\pi\epsilon_0}{me^2}}$$

Bohr Orbitals

if n is large one has almost continuum levels.

$$\frac{1}{2}m\frac{n^2\hbar^2}{m^2}\frac{m^2e^4}{n^4\hbar^4(4\pi\epsilon_0)^2}$$

$$= -\frac{e^2}{4\pi\epsilon_0}\frac{me^2}{n^2\hbar^2 4\pi\epsilon_0}$$

$$\rightarrow -\frac{mc^2}{2}\cdot\underbrace{\frac{e^4}{(4\pi\epsilon_0)^2c^2\hbar^2}}_{\text{dimensionless}}\cdot\frac{1}{n^2} \rightarrow \underbrace{\alpha \equiv \frac{e^2}{4\pi\epsilon_0\hbar c}}_{(-13.6\text{ eV})\frac{1}{n^2}}$$

make sure it's coulomb

Schrödinger

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\psi = 0 \qquad \text{say } \psi = e^{i\omega t}\psi$$

$$\left(-\frac{\omega^2}{c^2} - \nabla^2\right)\psi = 0 \Rightarrow (\nabla^2 + k^2)\psi = 0 \qquad k = \frac{2\pi}{\lambda}$$

- Stationary waves obey this.

$$\left(\nabla^2 + \frac{4\pi^2}{h^2}p^2\right)\psi = 0 \rightarrow (\hbar^2\nabla^2 + p^2)\psi = 0$$

+ de Broglie !!!

"Then π is the momentum of the particle"

Definite energy as Monofreq... "match"

- $E = \frac{p^2}{2m} + V$ (nonrelativistic particle) $\quad p^2 = 2mE - 2mV$

$$(\hbar^2 \nabla^2 + 2mE - 2mV)\psi = 0$$

$$\boxed{\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi}$$

Time independent Schrödinger Equation

matrix · vector = eigenvalue · vector

"Is there a real solution for every E?"

No! Only certain discrete levels of E are allowed.

- ψ is now called the state function.

Particle in a (1D) box $\qquad \psi(x=0) = \psi(x=L) = 0$

$$\left.-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \right\} \quad \psi = A\sin(kx) + \overset{0}{\cancel{B}}\cos(kx)$$

$$(*)\ E = \frac{\hbar^2 k^2}{2m}$$

Boundary conditions make it non trivial.

When $\psi(x=0)=0$, B is forced to be 0.

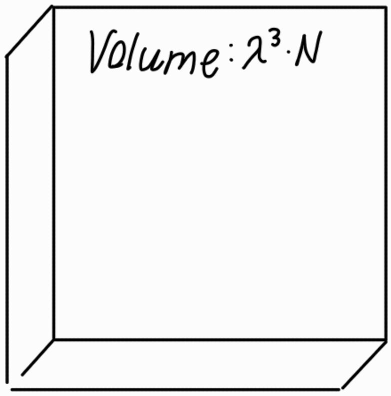
When $\psi(x=L)=0$, $\sin(kL)=0 \Rightarrow k = \frac{n\pi}{L}$, $k = \frac{2\pi}{\lambda}$

$$\boxed{\lambda = \frac{2L}{n}}$$

de Broglie wavelengths.

$$(*)\ E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2mL}$$

"point particles" is no longer in the language of quantum mechanics.



- Lowest E,
  Largest λ,
  Lowest n. more "quantum"

"Thermalizing a motion"

Thermal wavelength: $\bar{E} = \frac{3}{2}kT = \frac{\overline{p^2}}{2m} = \frac{h^2}{2m\lambda^2}$ $\quad \lambda = \sqrt{\frac{4h^2 m}{3kT}}$

"limiting temperature for quantum interactiveness"