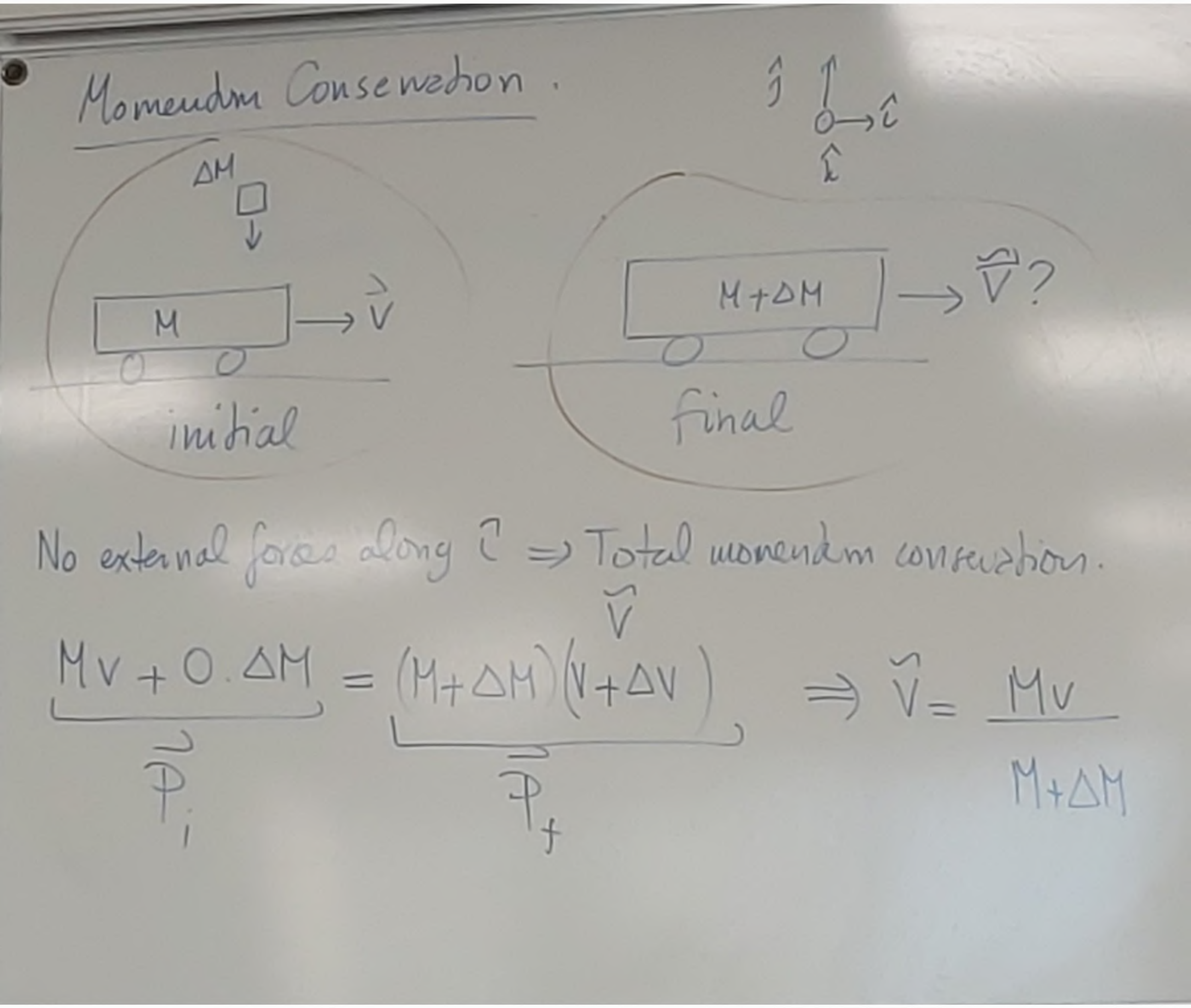
PHYS 301 Lecture 9

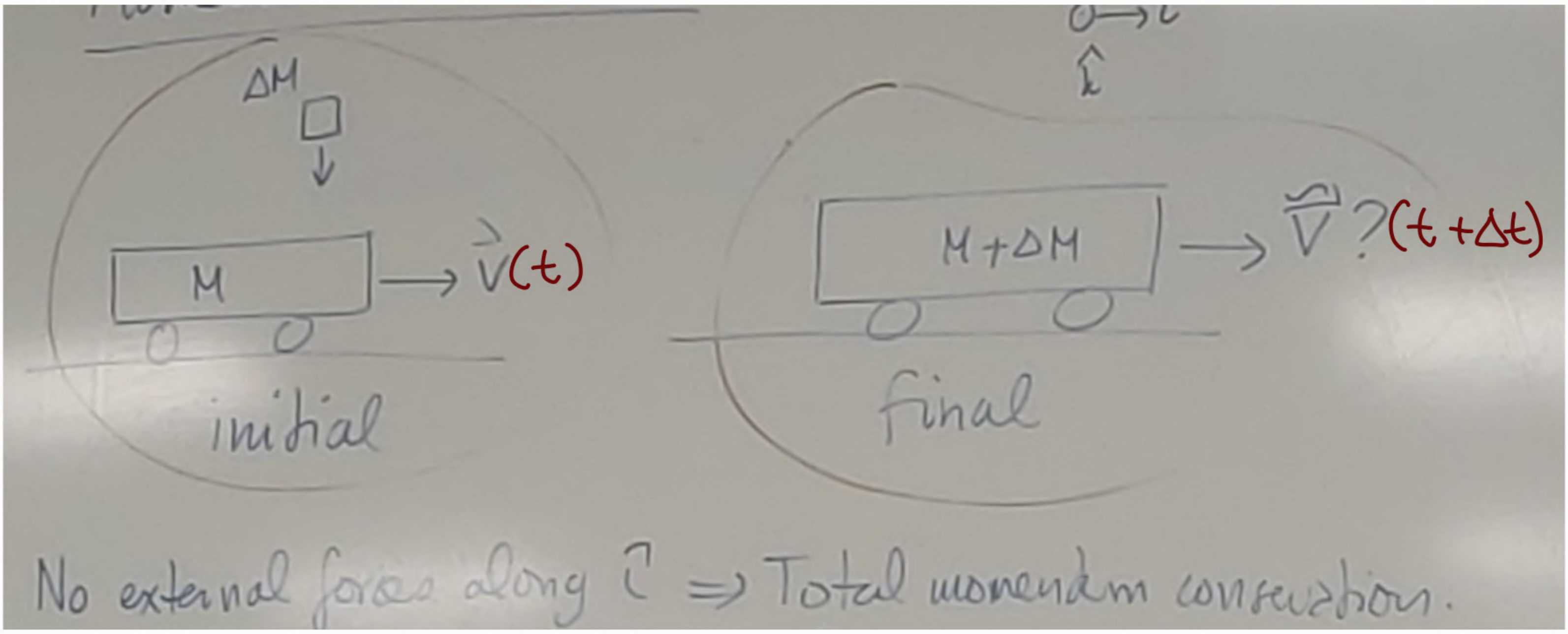
21.10.24

Momentum Conservation

You can, by throwing masses on a car, accelerate it, but $\vec{F}_{ext} = 0$.



We can go one step further:



$$\Delta \vec{P}_{part.} = \vec{F}_{part} \cdot \Delta t$$

$$(M+\Delta M)(V(t)+\Delta V) = M V(t)$$

$$\frac{\Delta M}{\Delta t} V + M \frac{\Delta V}{\Delta t} + \frac{\Delta M \Delta V}{\Delta t} = 0$$

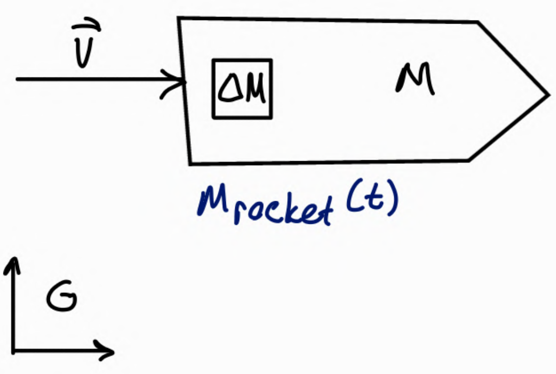
↳ quadratic. Washes out in the limiting process.

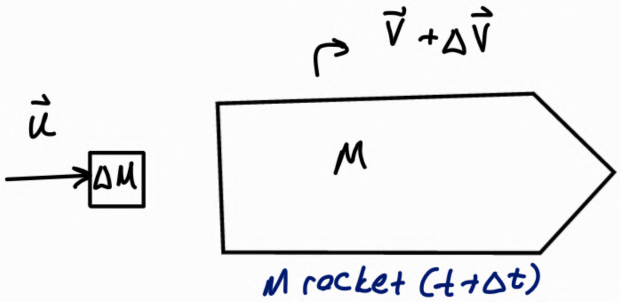
$$\frac{\Delta M}{\Delta t} V + M \frac{\Delta V}{\Delta t} = 0$$

⊛ Always consider the entire system without making major assumptions. (Write momentum conservation for the entire system)

$$\begin{aligned} \vec{r}_p' &= \vec{r}_p - \vec{v}_{rel}\, t \\ \vec{u}_p' &= \vec{u}_p - \vec{v}_{rel} \\ \vec{u}_e &= u - \vec{v} - \Delta\vec{v} \end{aligned}$$

Example: Rocket Equation





- If for the rocket (G'), the gas is ejected with $\vec{u}_e$ (engine parameter)

Momentum conservation for G:

$$\vec{P}_f - \vec{P}_i = \vec{u}\Delta M + M(\vec{V}+\Delta\vec{V}) - \Delta M\vec{V} - M\vec{V}$$

$$= (\vec{u}_e + \cancel{\vec{V}} + \Delta\vec{V})\Delta M + \cancel{M\vec{V}} + M\Delta\vec{V} - \cancel{\Delta M\vec{V}} - \cancel{M\vec{V}}$$

$$= \vec{u}_e\Delta M + \underbrace{\Delta M\Delta\vec{V}}_{\text{quadratic}\ldots} + M\Delta\vec{V}$$

$$= \vec{u}_e\Delta M + M\Delta\vec{V}.$$

$M = M_{rocket}$

$\Delta M = -\Delta M_{rocket}$

$\vec{V}_{rocket}$

$$= -\vec{u}_e\Delta M_r + M_r\Delta\vec{V}_r$$

$$= \vec{F}_{ext}^{\,sys}\Delta t$$

$$(\vec{F}_{ext}^{\,rocket} + \vec{F}_{ext}^{\,\Delta M})$$

(*) $\vec{F}_{ext}^{\,\Delta M}$ is there... But its very small (and because of Δt) it is washed out.

$$\boxed{\vec{F}_{ext}^{\,rocket} = -\vec{u}_e\frac{dM_{rocket}}{dt} + M_{rocket}\frac{d\vec{V}_{rocket}}{dt}}$$

The force caused by the ejected gas.

You can play around with this

→ Let's see if this eqn. makes any sense:

- If $\vec{F}_r^{\,ext} = 0$

$$M_r\frac{d\vec{V}_r}{dt} = \vec{u}_e\frac{dM_r}{dt} \Rightarrow d\vec{V}_r = \vec{u}_e\frac{dM_r}{M_r}$$

$$\vec{V}_r(t_f) - \vec{V}_r(t_i) = \vec{u}_e \ln\left(\frac{M_r(t_f)}{M_r(t_i)}\right) \text{ if } \vec{u}_e \text{ is constant.}$$

• $\vec{V}_r(t_i) = 0$

$$\vec{V}_f = \vec{u}_e \ln\left(\frac{M_f}{M_i}\right) = -\vec{u}_e \ln\left(\frac{M_i}{M_f}\right)$$

Lift off the ground: ($\vec{v}_r(t_i) = 0$)

$$M\vec{g} = -\vec{u}_e \frac{dM}{dt} + M\frac{d\vec{V}}{dt}$$

$$\vec{g}\,dt = -\vec{u}_e \frac{dM}{M} + d\vec{V}$$

$$\vec{g}\,\Delta t = -\vec{u}_e \ln\left(\frac{M_f}{M_i}\right) + \vec{V}_f - \cancelto{0}{\vec{V}_i}$$

$$\vec{V}_f = -\vec{u}_e \ln\left(\frac{M_i}{M_f}\right) + \vec{g}\,\Delta t$$

$\vec{V}_f = V_f\,\hat{j}$

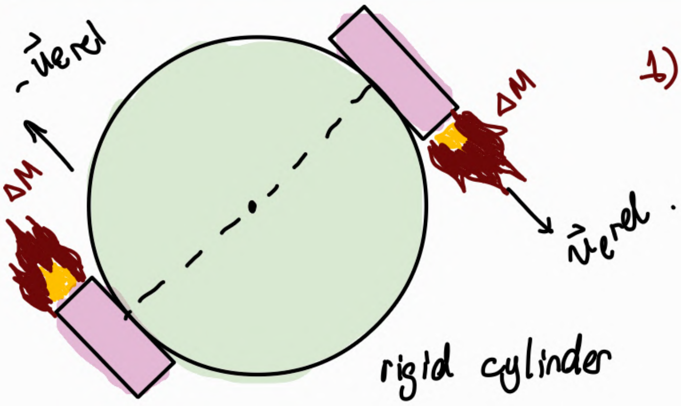
$\vec{u}_e = -u_e\,\hat{j}$

$\vec{g} = -g\,\hat{j}$

$$V_f = u_e \ln\left(\frac{M_i}{M_f}\right) \boxed{-g\,\Delta t}$$

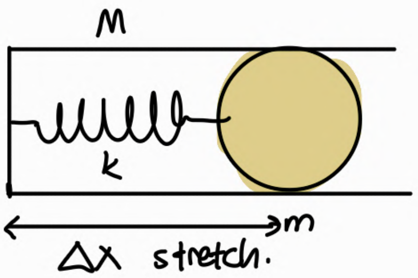
burn time!

PROBLEM IDEAS



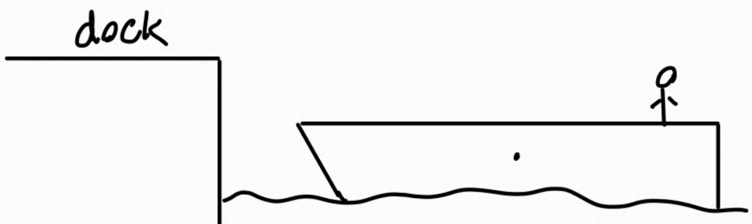
1) • Contact forces are always central. (May tap)
So, L (angular momentum) is conserved.
↳ You can solve using that.

2)



"Geri tepme fiziği"

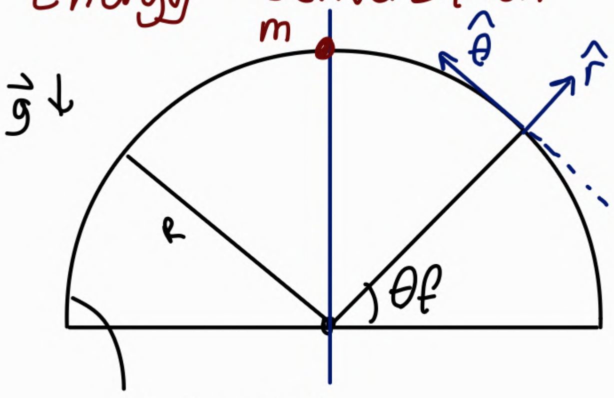
3)



Dock slips under you

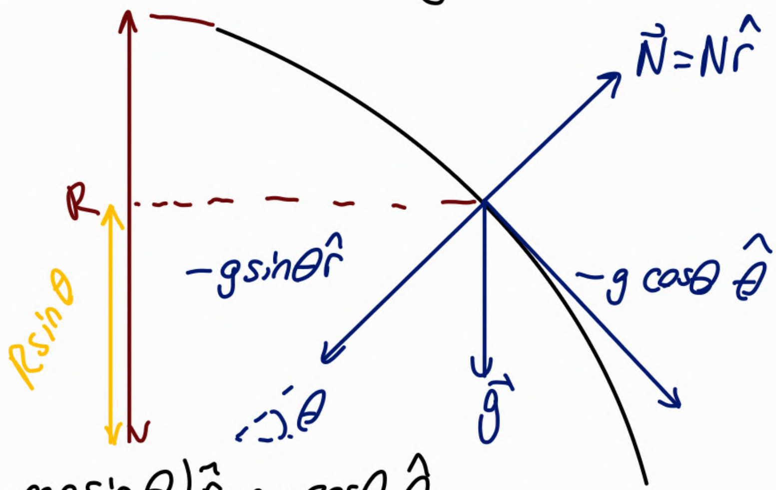
Is a "burnout process" better than throwing all gas at once?
You can calculate this (Assume you are throwing balls out the stationary car window. Start with mass $M_i = M+10m$)

Energy Conversation



⊛ Flies of when normal force vanishes.

⊛ Motion is not "düzgün dairesel"



$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$r = R$$

$$\ddot{\vec{r}} = -R\dot{\theta}^2\hat{r} + R\ddot{\theta}\hat{\theta}$$

$$\vec{F}_{net} = m(-R\dot{\theta}^2\hat{r} + R\ddot{\theta}\hat{\theta}) = (N - mg\sin\theta)\hat{r} - mg\cos\theta\,\hat{\theta}$$

To find N we use energy conservation.

$$\rightarrow \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$(\dot{\vec{r}}\cdot\dot{\vec{r}}) = \dot{r}^2 + r^2\dot{\theta}^2 \Rightarrow R^2\dot{\theta}^2$$

$$E_{Tot}^{initial} \stackrel{?}{=} E_{Tot}^{\theta}$$

$$E_{Tot}^{initial} = E_{Tot}^{\theta}$$

$$\cancel{m}g\cancel{R} = \cancel{m}g\cancel{R}\sin\theta + \frac{1}{2}\cancel{m}R^{\cancel{2}}\dot{\theta}^2$$

$$gR = gR\sin\theta + \frac{1}{2}R^2\dot{\theta}^2 \Rightarrow \boxed{R\dot{\theta}^2 = 2g(1-\sin\theta)}$$

$$N = -mR\dot{\theta}^2 + mg\sin\theta = -m\,2g(1-\sin\theta) + mg\sin\theta$$

$$N=0, \longrightarrow 2\cancel{mg}(1-\sin\theta) = \cancel{mg}\sin\theta$$

$$2 - 2\sin\theta = \sin\theta \qquad 3\sin\theta_f = 2$$

$$3\sin\theta_f = 2 \quad \text{or} \quad \sin\theta_f = \frac{2}{3}$$