PHYS 311 Lecture 13

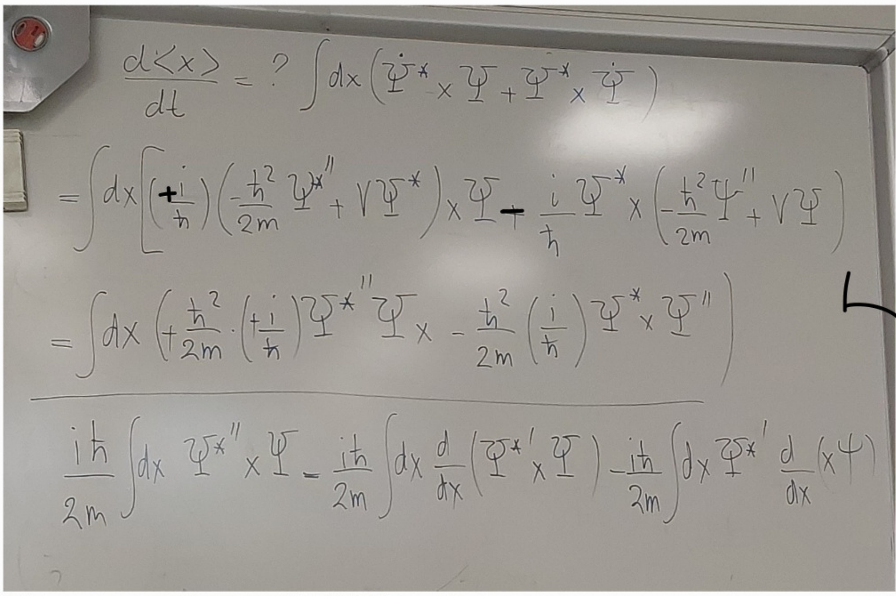
11.11.2024

Schrödingers Equation implies that expectation values of observables obey classical equations of motion.

$$\langle x \rangle = \int_{-\infty}^{\infty} dx\, \Psi^*(x,t)\, x\, \Psi(x,t)$$

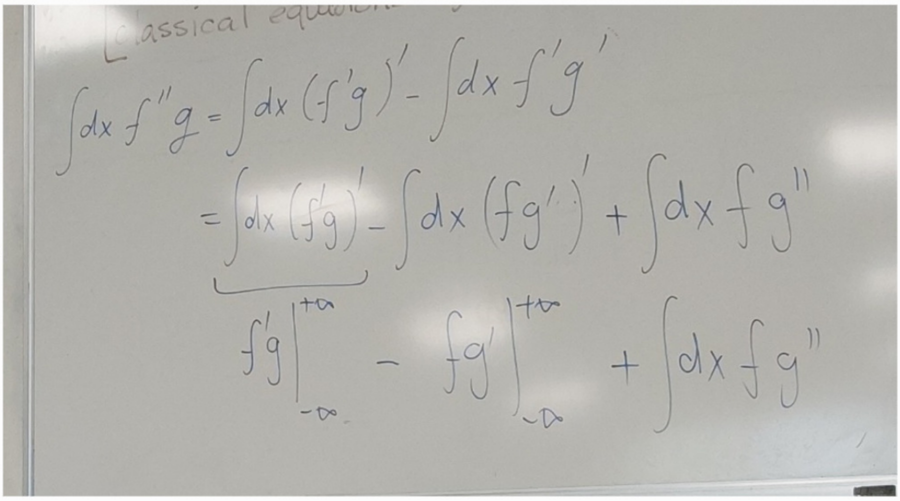
$$\langle p \rangle = \int_{-\infty}^{\infty} dx\, \Psi^*(x,t) \left[-i\hbar \frac{d\Psi}{dx}\right]$$

$$\boxed{\begin{aligned} \frac{\partial \psi}{\partial t} &= \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right]\Psi\left(\frac{-i}{\hbar}\right) \\ \frac{\partial \Psi^*}{\partial t} &= \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right]\Psi^*\left(\frac{i}{\hbar}\right) \end{aligned}}$$



$$\boxed{\begin{aligned} (fg)'' &= (f'g + fg')' \\ &= f''g + 2f'g' + g'' \end{aligned}}$$

This pic might still have + - problems!



$$\frac{d\langle x \rangle}{dt} = ?\int dx \left(\dot{\Psi}^* x \Psi + \Psi^* x \dot{\Psi}\right) = -\frac{i\hbar}{m}\int dx\, \Psi^* \frac{\partial \Psi}{\partial z}$$

$$= \frac{\langle p \rangle}{m}$$

$$\frac{d\langle p\rangle}{dt} = \langle F(x)\rangle \neq F(\langle x\rangle) \qquad \langle p\rangle = \int dx\, \Psi^* \left(-i\hbar \frac{\partial \Psi}{\partial x}\right)$$

if not linear.

- For harmonic oscillator:

$$\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m} \qquad \frac{d\langle p\rangle}{dt} = -k\langle x\rangle$$

- $$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi\right]$$

- $\vec{r}, t$ $\qquad |\Psi_p(t)|^2$

$$\boxed{\begin{aligned} \vec{r}' &= \vec{r} - \vec{R}(t) \\ t' &= t \end{aligned}}$$

Particle in a Box

$$\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) e^{-iE_n t/\hbar} = \Phi_n(x,t)$$

$$\langle x\rangle_n = L/2 \qquad \langle p\rangle = 0$$

$$\Phi_n^*(x,t)\Phi_n(x,t) = \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right)$$

Easiest example:

$$\Psi = \frac{\Phi_1(x,t) + \Phi_2(x,t)}{\sqrt{2}}$$

$$\overbrace{\Phi_n = e^{-i\frac{E_n t}{\hbar}} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}^{\text{energy eigenstates}}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$|\Psi|^2 = \frac{1}{2}(\phi_1^* + \phi_2^*)(\phi_1 + \phi_2)$$

$$= \frac{1}{2}\left(|\phi_1|^2 + |\phi_2|^2 + \phi_1^*\phi_2 + \phi_2^*\phi_1\right)$$

$$= \frac{1}{2}\,\frac{2}{L}\left(\sin^2\left(\frac{\pi x}{L}\right) + \sin^2\left(\frac{2\pi x}{L}\right) + 2\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{\overbrace{(E_2 - E_1)}^{\omega} t}{\hbar}\right)\right)$$

$$\langle x \rangle (1) = a + b\cos(\omega t)$$

$$\langle E \rangle \,?$$

$$i\hbar\frac{\partial\Psi}{\partial t} = \underbrace{\left[\frac{\hat{p}^2}{2m} + V(\hat{x})\right]}_{H}\Psi = \underbrace{\left[\frac{p^2}{2m} + V(x)\right]}_{H}\Psi$$

$$\hat{p}\Psi = -i\hbar\frac{\partial\Psi}{\partial x}$$

$$\hat{x}\Psi = x\Psi$$

$$\langle E \rangle = \int dx\, \Psi^* H \Psi$$

$$\frac{d\langle E \rangle}{dt} = 0 \qquad \frac{d\langle E^2 \rangle}{dt} = 0$$

- Schrodinger Thrm. lineerliği önemli.
- H'nin hermitsel olması önemli.

$$\Psi(x,0) \longrightarrow \Psi(x,t) = \mathcal{U}(t)\,\Psi(x,0)$$

$$\int \Psi^*(x,0)\,\Psi(x,0) = \int \Psi^*(x,t)\,\Psi(x,t)$$

$\mathcal{U}(t)$ must be a unitary operator.

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = (\dot{\mathcal{U}}\, i\hbar)\,\Psi(x,0) = i\hbar\, \dot{U}\, U^{-1}\, \Psi(x,t)$$

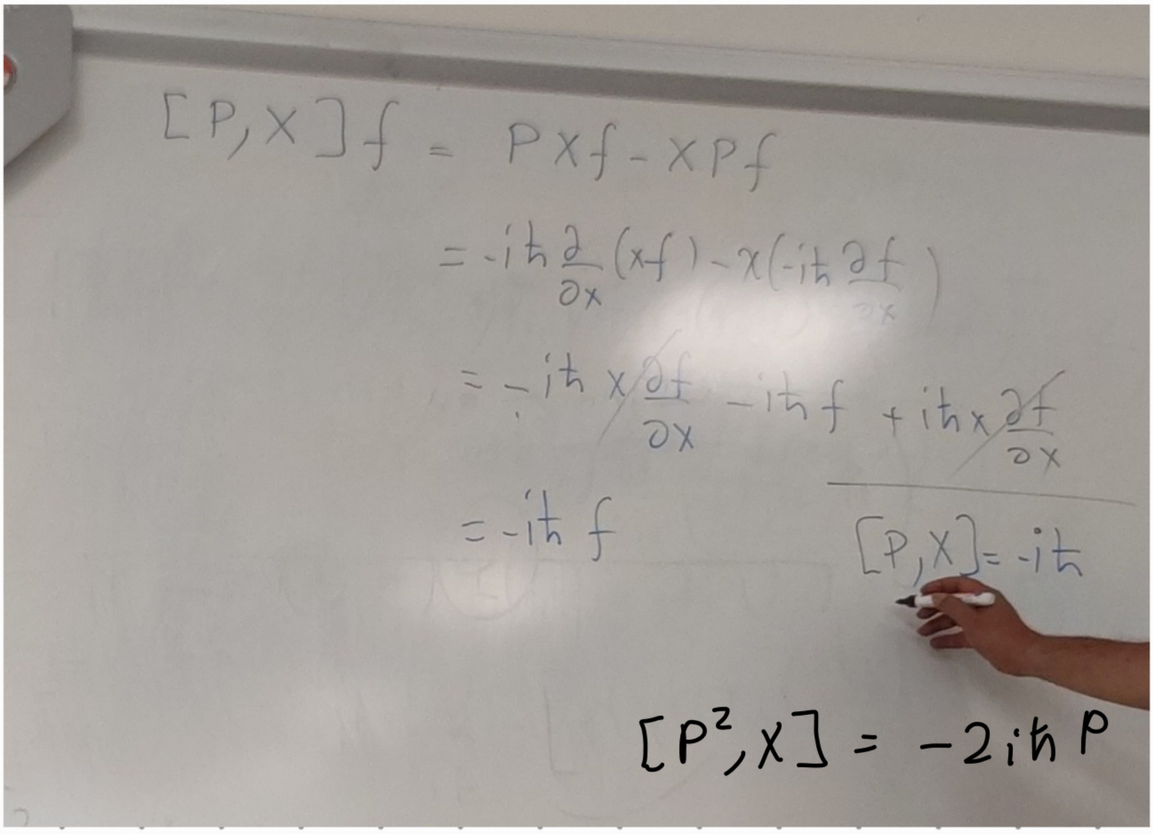
$$i\hbar \frac{\partial \Psi}{\partial t} = [i\hbar\, \dot{\mathcal{U}}\,\mathcal{U}^{-1}]\,\Psi$$

$$\langle A \rangle = \int dx\, \Psi^* A \Psi \qquad \left(\frac{\partial A}{\partial t} = 0\right)$$

$$i\hbar \frac{d}{dt}\langle A \rangle = \int dx \left[\underbrace{i\hbar \frac{\partial \Psi^*}{\partial t}}_{-\Psi^* H} A \Psi + \Psi^* A \underbrace{(i\hbar)\frac{\partial \Psi}{\partial t}}_{H\Psi} \right]$$

$$= \int dx\, \Psi^* [A, H]\, \Psi$$

$[A,B] = AB - BA$ | $[AB, C] = A[B,C] + [A,C]B$



Commutator: if not 0, they can't be observed together.

- Between P & X there'll always be uncertainty.

$$[A,B] = C$$

$$\Delta A \Delta B \geq \frac{|\langle C \rangle|}{2}$$ (Schwartz inequality)