PHYS 301 Lecture 13 11.11.24

Central Force Motion

$$\mu\ddot{r} = \mu r\dot{\theta}^2 + f(r)$$

$$\mu(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0 \longrightarrow \vec{L} = \vec{\mu}\vec{r}\times\vec{v} = \ell\hat{k} \qquad \ell = \mu r^2\dot{\theta}$$

constant

$$\frac{1}{2}\mu\dot{r}^2 + \underbrace{\frac{\ell^2}{2\mu r^2} + U(r)}_{\equiv U_{eff}(r)} = E \;:\; \text{constant} \qquad \left\{-\frac{dU}{dr} = f\right\}$$

$$\mu \equiv \frac{mM}{M+m}$$

Kepler Problem

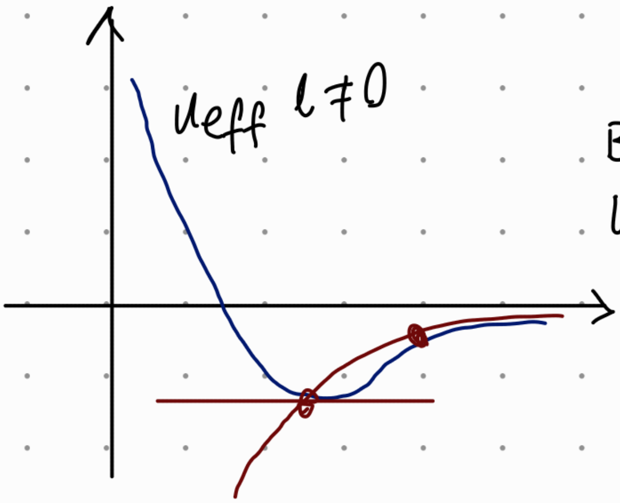
$$U(r) = \frac{-GMm}{r}$$

$$\left.\begin{aligned}\frac{d\theta}{dt} &= \frac{\ell}{\mu r^2}\\ \frac{dr}{dt} &= \sqrt{\frac{2E}{\mu} - \frac{\ell^2}{\mu^2 r^2} - \frac{2U}{\mu}}\end{aligned}\right\} \frac{dr}{d\theta} = g(r) \rightarrow \int\frac{dr}{g(r)} = \int d\theta$$

$$r = \frac{r_0}{1 - \epsilon\cos\theta} \qquad r_0 = \frac{\ell^2}{\mu C} \qquad \epsilon = \sqrt{1 + \frac{E\ell^2}{\mu C^2}}$$

$\downarrow$ eccentricity

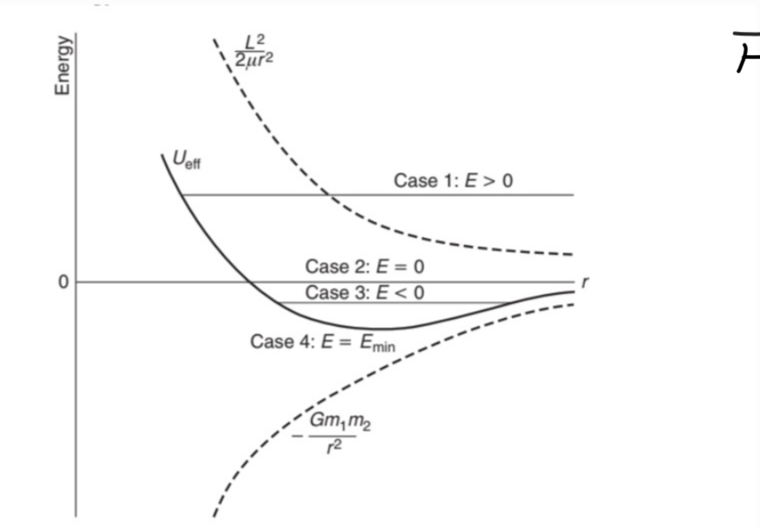
$C = GmM \qquad \epsilon > 0$



Bound orbits must have $E<0$

Unbound orbits have $E \geq 0$

From 2nd Ed. Pg. 382

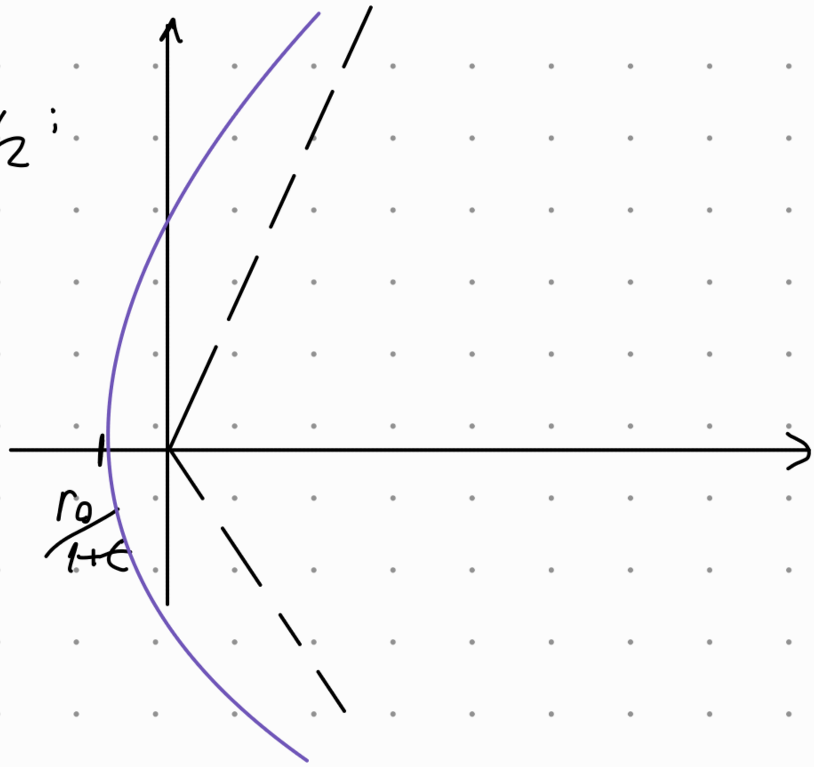


$\epsilon = 0 \Rightarrow r = r_0$

Hyperbola?

- $\epsilon = 2 \quad \cos\theta = 1/2$:

see there is a bound to θ when $\epsilon > 1$



- $\epsilon = 1/2$

$x = r\cos\theta \qquad y = r\sin\theta$

$$\sqrt{x^2+y^2} = \frac{r_0\sqrt{x^2+y^2}}{\sqrt{x^2+y^2} - \epsilon x}$$

$(\dots)$

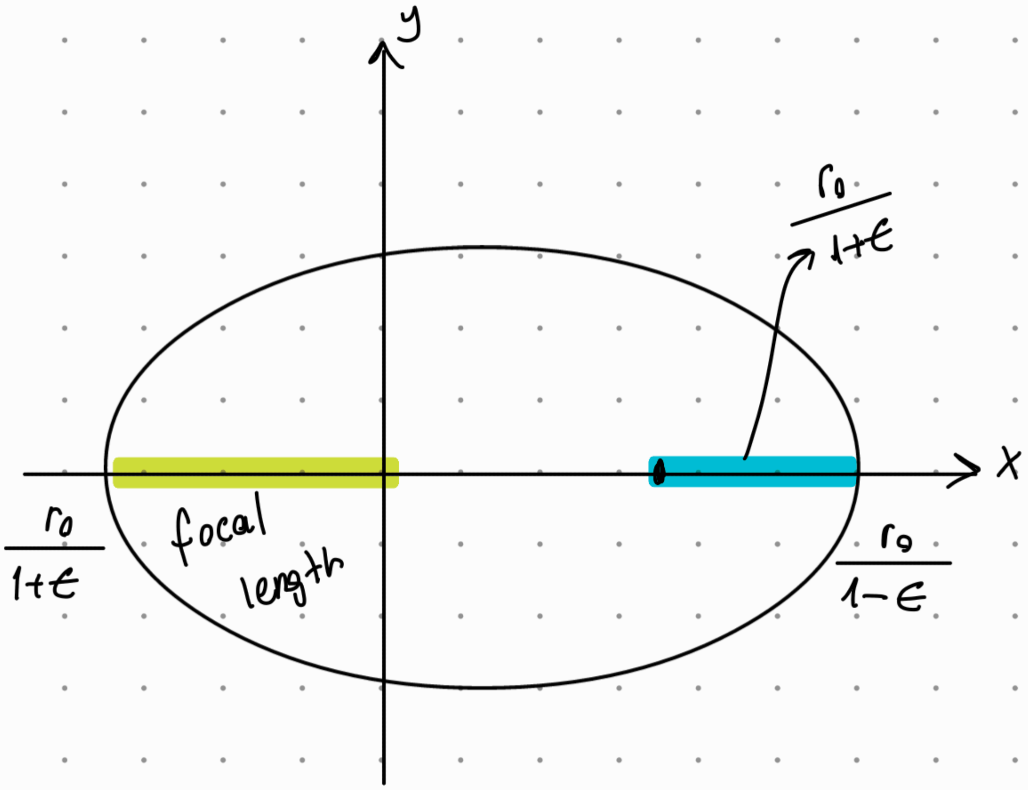
$$\boxed{(1-\epsilon^2)x^2 - 2r_0\epsilon x + y^2 = r_0^2}$$

quadratic curve

- hyperbola
- parabola
-

"Show this is a circle when $\epsilon = 0$."

"Scattering Solutions"



major axis

$$\frac{r_0}{1+\epsilon} + \frac{r_0}{1-\epsilon} = \frac{2 r_0}{1-\epsilon^2} = \frac{\frac{2\ell^2}{\mu C}}{-\frac{2E\ell^2}{\mu C^2}} = \frac{C}{-E}$$

$$1 - \left(1 + \frac{2E\ell^2}{\mu C^2}\right) = \frac{2E\ell^2}{\mu C^2}$$

- Say you want to change your circular orbit for another one.

→ What happens?

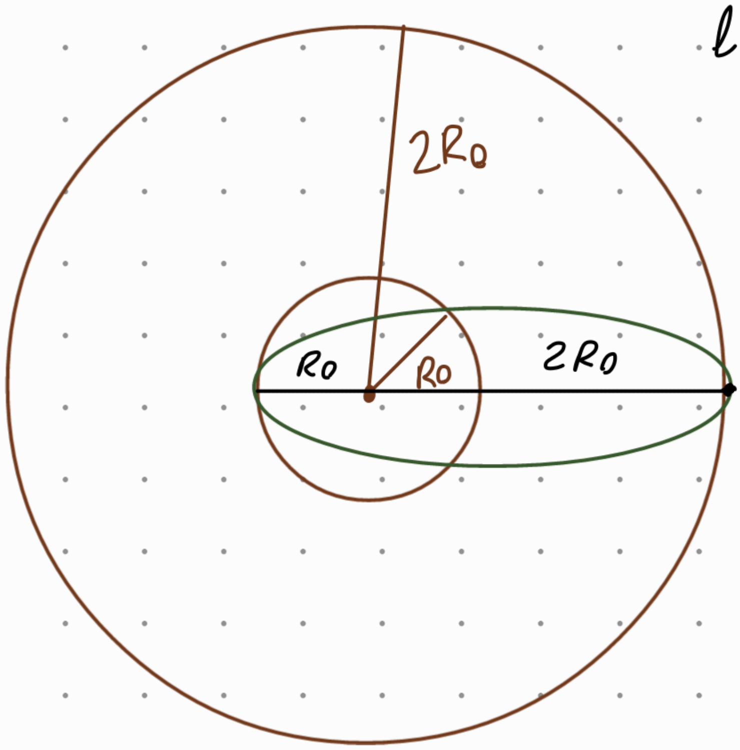
Dünyadan Kurtulmak

$$\frac{1}{2}\mu \dot{r}^2 - \frac{GMm}{R_e} = E \geq 0$$

$$(\dot{r}^2)_{min} = \frac{2GMm}{\mu R_e} = \frac{2G(M+m)}{R_e} = \frac{2GM_e}{R_e}$$

"How did Voyager get out of solar system?"

→ uranus, jupiter helped (Slingshot)



$$\ell = \mu R_0^2 \dot{\theta}$$

$$= -\frac{1}{2} \frac{\mu C^2}{\ell^2}$$

$$E = \frac{\ell^2}{2\mu R_0^2} - \frac{C}{R_0} = \frac{\mu C^2}{2\ell^2} - \frac{C^2 \mu}{\ell^2}$$

$$-\frac{\ell^2}{\mu R_0^2} + \frac{C}{R} = 0$$

$$R_0 = \frac{\ell^2}{\mu C}$$

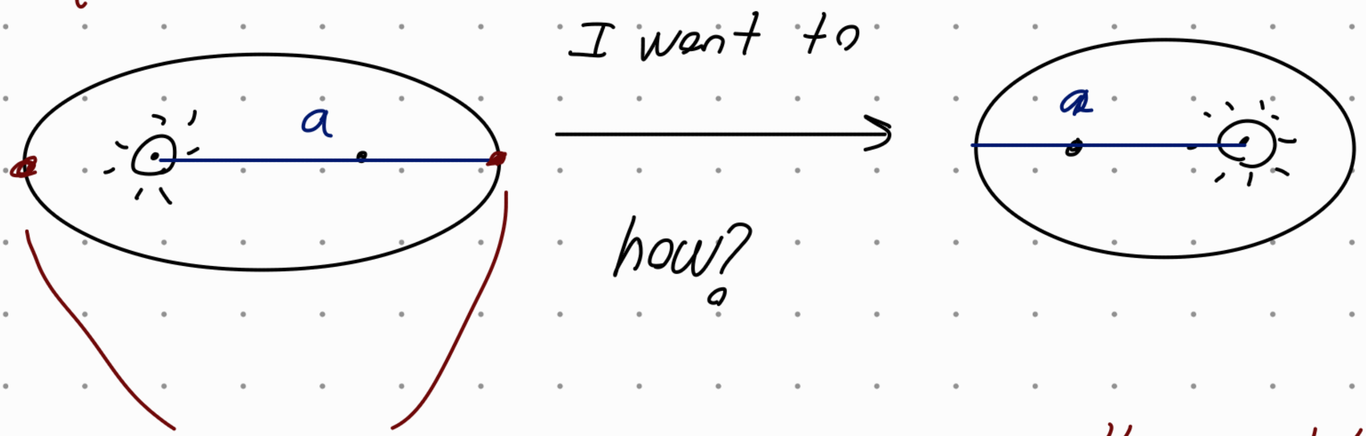
$$\boxed{= -\frac{1}{2} \frac{C}{R_0}}$$

"effective shock"

$$\frac{r_0}{1+\epsilon} = 2R_0 \qquad r_0 = (1-\epsilon)\,2R_0 \qquad \frac{1+\epsilon}{1-\epsilon} = 2$$

"What to do for low ϵ?"

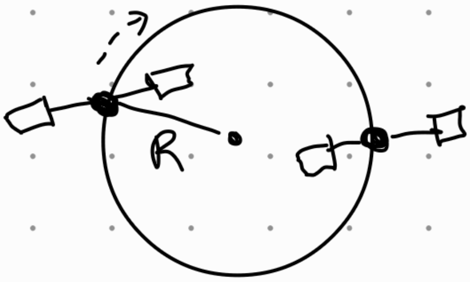
→ Question



need a circle passing from this points.

→ Question

"Can you send help to the other satellite in the same circular orbit"



→ Slow down and wait up for the other one to catch up to you

→ Question How to fall down to earth?

$\omega \swarrow$ $L \swarrow$ U_{eff} gets shallow, you fall down.