Homework 5

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1 Decision version of this optimization problem

1.a Inputs

T: A set of projects where each project $t \in T$.

e_t: Positive integer that denotes the amount of effort required to complete the project t.

p_t: Positive integer that denotes the profit gained by completing the project t.

c: Positive integer that denotes the maximum amount of effort that can be spent on the projects.

1.b Outputs

The resulting output will be either “Yes” or “No”.

1.c Decision Problem

”Yes” if there exists a subset of projects S from T such that the total amount of effort spent to complete the projects in S is less than or equal to c and the total profit gained by completing the projects in S is greater or equal to p. In the remaining cases output is ”No”.

$$Output = \begin{cases} YES, & \exists S \subseteq T \text{ such that } \sum_{t \in S} e_t \leq c \text{ and } \sum_{t \in S} p_t \geq p. \\ NO, & \text{otherwise.} \end{cases}$$

2 Proof of the decision version is in NP.

To prove that this decision version is in NP, we must show that the problem can be verified in polynomial time. As I explained to verify the solution we need to check if the given S (the subset of the T) provides the necessary requirements. These requirements are the total amount of effort spent to complete the projects in S is less than or equal to c and the total profit gained by completing the projects in S is greater or equal to p. Logically total amount of effort spent to complete the projects in S can be found by summing the efforts of the projects in S. This operation would take O($|S|$), where $|S|$ is the size of the S. With the same logic, the total amount of profit made by completing the projects in S can be found by summing the profits made by the given projects in S. This operation would take O($|S|$), where $|S|$ is the size of the S. Therefore, verifying the solution takes O($|S|$) time. Moreover, since the size of S is polynomial, the solution can be verified in polynomial time. Which concludes that the Decision version of the problem is in NP.