# Limits of Computation

14 - Robustness of **P**

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# The story so far

- We have discussed "computability",
- Church-Turing Thesis: it does not matter which notion of computation we use.
- in Complexity: does this matter if we restrict to class **P**?

THIS TIME

# Robustness of **P**

- we discuss *theses* similar to Church-Turing Thesis but now with *added time complexity.*
- "*robust*" means *resilient, hard-wearing,* so
- for a complexity class this means *resilient under compilation into other languages.*
- we focus on class **P**,as it turns out it is "*robust*" compared to other classes.

robust

not as robust



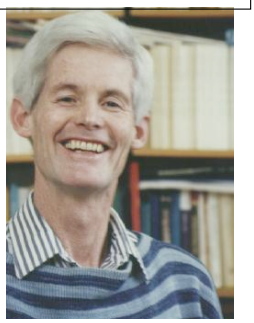
# Cook's (Invariance) Thesis

**Definition 14.2** (*Invariance Thesis*) All "reasonable" *sequential* notions of computation can simulate each other up to a polynomial factor.

in other words

$\mathbf{P}^{\mathtt{L}}$ is the same class of problems for all reasonable sequential (that is, nonparallel) computational models $\mathtt{L}$.

- Like CTT, this is a (widely believed) *thesis ("reasonable computational models" is not a formal notion).*
- We will give some evidence now.



Stephen Arthur Cook
Turing Award Winner 1982

# Evidence for Cook's thesis

**Lemma 14.1**

$$TM \preceq^{lintime-pg-ind} GOTO \preceq^{lintime-pg-ind} SRAM \preceq^{ptime} TM$$

proof by careful analysis of compilation results

**Lemma 14.2**

$$TM \preceq^{lintime} CA \preceq^{ptime} TM$$

proof by careful analysis of compilation results

Now use lifting lemma(s) from Lecture 13, slide 14

**Theorem 14.1** *It holds that*

$$\boldsymbol{P}^{CA} = \boldsymbol{P}^{TM} = \boldsymbol{P}^{GOTO} = \boldsymbol{P}^{SRAM}$$



# **LIN** is not so robust

**Lemma 14.3**

$$GOTO \preceq^{lintime-pg-ind} WHILE \preceq^{lintime} WH^1LE$$
$$WH^1LE \preceq^{lintime-pg-ind} WHILE \preceq^{lintime} GOTO$$

**Theorem 14.2**

$$\boldsymbol{LIN}^{GOTO} = \boldsymbol{LIN}^{WHILE} = \boldsymbol{LIN}^{WH^1LE}$$

- Linear time only robust for "similar" languages
- Too restrictive for all notions of computation.

# Is P the bee's knees then?

Can we even go as far a this:

**Definition 14.3** (*Cobham-Edmonds Thesis*) The tractable (feasible) problems are exactly those in **P**.

also often called Cook-Karp thesis

- only a thesis (what is a "tractable/feasible problem")?
- not widely believed. Why not? (next slide)



# Why we do not believe Cobham-Edmonds/Cook-Karp

- Is every polynomial time bound really a good time bound indicating feasibility?
- Is every time bound beyond polynomial really a bad time bound indicating intractability?

# END



Next time:
Next time:
Can we solve more problems given more time?