Research question, hypothesis and refined design

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Domain and scope of research - ACM 2012

- Domain:
  A. Mathematics of Computing -> Mathematical Analysis -> Calculus -> Lambda Calculus. Relevant Articles: Greco and Palmigiano (2023), Arrial et al. (2023), Accattoli et al. (2022b), Valiappan (2022)
  B. Mathematics of Computing -> Mathematical Analysis -> Calculus -> Differential Calculus. Relevant Articles: Bangaru et al. (2021), Chen et al. (2017)
  C. Mathematics of Computing -> Mathematical Analysis -> Calculus -> Integral Calculus.
  D. Mathematics of computing -> Continuous mathematics -> Calculus -> Lambda calculus. Relevant Articles: Bordg and Doña Mateo (2023), Accattoli et al. (2022a), Blot (2022)
  E. Mathematics of computing -> Continuous mathematics -> Topology -> Algebraic topology. Relevant Articles: Mavrogiannis & Knepper (2017), Rajsbaum & Raventós-Pujol (2022), Xiang (2021)
- Scope: Quantum computational logic and calculi to express it will be studied with call by value method in this research.
- Assumptions: The provable assumption of expressions that are semantically the same would result into results that are semantically the same for all languages and calculi that fit into notion of computation, suggests that there will be inconsistencies in the results for languages and calculi that is not proved to satisfy computation universality.
- Limitations: Correctness of the invented calculi itself based on notion of computation is a limitation for testing the correctness caused by evaluation method.
- Delimitations: Quantum computational logic will be the only subject of quantum computation branch rather than algorithms or hardware, as only call by value will be the chosen evaluation method to measure the correctness of new approaches of representation of the logic as clear as possible.

Gaps in the literature and research question

- Gaps: There are studies that focus on different calculi, some focus on different types of logic, some research about expressions and recursion in the expressions were made but there is not enough research about studying the evaluation of the expressions generated by calculus. In the domain of mathematics of computation, there are many studies of algorithms and certain problems that are known to be difficult ranging from computer graphics to networks. Examples of these can be seen in Rajsbaum & Raventós-Pujol (2022), Xiang (2021), Mavrogiannis & Knepper (2017), Bangaru et al. (2021), Chen et al. (2017), Accattoli et al. (2022b). An example study takes subject call by value evaluation method with lambda calculus however this study was focused on time and space efficiency of algorithms run instead focusing on quantum logic system Accattoli et al. (2022a). Another research works on the display of linear logic using calculi instead of quantum logic. Greco & Palmigiano (2023). In one the studies type theory was chosen as a subject, but logic can be a perspective to study separately Bordg & Doña Mateo (2023). different algorithms were studied to solve a certain problem, using call by value name and call by value strategies on different lambda-calculi Arrial et al. (2023). Moreover, studies of normalization with different calculi was done, however logic or evaluation strategies were not factored in Valliappan et al. (2022). Therefore, study of call-by-value evaluation method in calculi that represents quantum logic is a research topic that has not been covered widely yet.

- Research Question: Which calculus approach for representing quantum computational logic, used to generate semantically same expressions would have more correctness compared to quantum lambda calculus if these expressions were evaluated with call-by-value method?

hypothesis + research methods

Null Hypothesis:

The correctness of evaluations of expressions from different calculus approaches that does not satisfy computational universality gathered by using same evaluation methods is not significant to favor one of the approaches used to describe quantum computational logic.

Alternate Hypothesis:

IF Vectorial, Lambda-S, Lambda-S1, and Connective calculi methods, that describe quantum computational logic, would be used to generate any set of expressions that can be generated with the given calculi while semantically sharing the same meaning, and are evaluated with the call-by-value method

THEN the output (simplified expressions) will have different correctness levels when compared to quantum lambda calculus' output.

Research Methods:

- Type: Primary Research. A field research needs to be conducted to experiment and collect data that does not already exist to be able to draw a conclusion; sample expressions need to be generated, evaluated and compared to verify the hypothesis.
- Objective: Quantitative Research. An empirical observation is required to investigate the fundamental connection of quantitative properties and phenomena along with their relationship; the resulting expressions will be observed and compared to the trusted logic system's output and the relationship between their properties will lead to conclusion.
- Form: Empirical Research. A hypothesis will be tested by the knowledge gained from direct experimentation; the evaluation of results from the experiment with expressions will be used to test the hypothesis.
- Reasoning: Deductive Reasoning. The facts that these calculi that are specified do not fulfill the computation universality and that the expressions that share the same semantics should result into simplified expressions that share the same semantics when they are evaluated with the same methods regardless of their language, results into this specific hypothesis where the question is asked if these calculi are off from the precision of universal computation definition, how far are they?

General and specific research objectives for experimental purposes towards hypothesis testing using statistical tools

O1: Generate Data

- Fifteen expressions will be generated using quantum lambda calculus over the course of a week.
- The generated expressions will be translated to other four calculi mentioned in the research while conserving the semantics over the course of a week, making it a total of seventy-five expressions.
- These expressions will be evaluated using call-by-value method over the course of three weeks.

O2: Evaluate Data

- The resulting expressions from the four calculi will be verified against the resulting expressions from quantum lambda calculus and a conclusion will be drawn from the correctness values over the course of two weeks.

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Performance metrics of your experiment

- Correctness – That is the only performance metric for this experiment because the focus is on the capability of calculi to evaluate expressions correctly. When the resulting expressions from semantically same expressions are compared with a system's results that is considered correct, quantum lambda calculus, the resulting expressions will be decided whether they are correct or not. Overall, the analysis of these results will suggest the correctness of calculi approach based on computational universality.