**Sabancı University**
**Faculty of Engineering and Natural Sciences**

# CS301 – Algorithms

## Homework 1

Due: April 2, 2023 @ 23.55
(Upload to SUCourse - **no late submission**)

**PLEASE NOTE**:

- Provide only the requested information and nothing more. Unreadable, unintelligible and irrelevant answers will not be considered.
- You can collaborate with your `TA/INSTRUCTOR ONLY` and discuss the solutions of the problems. However you have to write down the solutions on your own.
- Plagiarism will not be tolerated.

**Late Submission Policy**:

- Your homework grade will be decided by multiplying what you normally get from your answers by a "submission time factor (STF)".
- If you submit on time (i.e. before the deadline), your STF is 1. So, you don't lose anything.
- If you submit late, you will lose 0.01 of your STF for every 5 mins of delay.
- We will not accept any homework later than 500 mins after the deadline.
- SUCourse+'s timestamp will be used for STF computation.
- If you submit multiple times, the last submission time will be used.

| Question | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 50 | |
| 4 | 10 | |
| Total: | 100 | |

**Question 1** [20 points]

(a) [5 points] What is the form of the input array that triggers the worst case of the insertion sort?

(b) [5 points] What is the complexity of this worst–case behavior in $\Theta$ notation?

(c) [10 points] Explain how this particular form of the array results in this complexity.

**Question 2** [20 points]

(a) [5 points] What is the form of the input array that triggers the best case of the insertion sort?

(b) [5 points] What is the complexity of this best–case behavior in $\Theta$ notation?

(c) [10 points] Explain how this particular form of the array results in this complexity.

**Question 3** [50 points]

Suppose that you are trying to prove $(5n+4)^2 = O(n^2)$ by using the formal definition of $O$–notation, where $n \geq 0$.

In order to show that $(5n+4)^2 = O(n^2)$ by using the formal definition of $O$–notation, we need to pick constants $c$ and $n_0$ such that for any $n \geq n_0$ we have

$$(5n+4)^2 \leq cn^2 \tag{1}$$

(a) [25 points] If you use $n_0 = 2$, what is the smallest $c$ value that makes the proof go through?

(b) [25 points] If you use $c = 36$, what is the smallest $n_0$ value that makes the proof go through?

**Question 4** [10 points]

Rank the following functions in descending order with respect to their growth rates.

$$\lg(n!) \qquad n! \qquad n2^n \qquad 2^{2^n} \qquad \lg^2 n \qquad (\lg n)!$$