Sabancı University
Faculty of Engineering and Natural Sciences

CS301 – Algorithms

Homework 4

Due: May 30, 2023 @ 23.55
(Upload to SUCourse)

PLEASE NOTE:

- Provide only the requested information and nothing more. Unreadable, unintelligible and irrelevant answers will not be considered.
- You can collaborate with your `TA/INSTRUCTOR ONLY` and discuss the solutions of the problems. However you have to write down the solutions on your own.
- Plagiarism will not be tolerated.

Late Submission Policy:

- Your homework grade will be decided by multiplying what you normally get from your answers by a "submission time factor (STF)".
- If you submit on time (i.e. before the deadline), your STF is 1. So, you don't lose anything.
- If you submit late, you will lose 0.01 of your STF for every 5 mins of delay.
- We will not accept any homework later than 500 mins after the deadline.
- SUCourse+'s timestamp will be used for STF computation.
- If you submit multiple times, the last submission time will be used.

Question	Points	Score
1	30	
2	40	
3	30	
Total:	100	

Question 1 [30 points]

Design a flow network $G = (V, E, s, t, c)$ with $|V| \leq 4$ and $c : V \times V \to \{0, 1\}$ such that max–flow function for G is not unique. On the flow network you design, show at least two different max–flow functions and state the value of the max–flow.

Question 2 [40 points]

We know that the value of the maximum flow is unique in flow networks. However, there can be more than one max–flow function achieving this maximum value.

Now, consider a flow network $G = (V, E, s, t, c)$ where we have the following property:

$\forall u_1, u_2, v_1, v_2 \in V$:

$$[\, c(u_1, u_2) \neq 0 \;\wedge\; c(v_1, v_2) \neq 0 \;\wedge\; (u_1, u_2) \neq (v_1, v_2)\,] \implies [\, c(u_1, u_2) \neq c(v_1, v_2)\,]$$

Claim A: For such flow networks, there is exactly one max–flow function.

Is Claim A true or false?

If true, prove it (no partial points if no proof is given).

If false, give a counter example by using a flow network of at most 4 nodes on which you need to provide two different max–flow functions (no partial points if no counter example is given or if the counter example uses 5 or more nodes).

Question 3 [30 points]

Let $G = (V, E, s, t, c)$ be a flow network, f_1 and f_2 be two flow functions on G. Let $F : V \times V \to R$ be defined as $\forall u, v \in V : F(u, v) = f_1(u, v) + f_2(u, v)$.

Is F guaranteed to be a flow on G?

If yes, prove it (no partial points if no proof is given).

If no, for only one of the constraints of flow functions, show that it does not necessarily hold for F, by giving a counter example on a flow network of at most 3 nodes (no partial points if no counter example is provided or the counter example uses 4 or more nodes).