State Space Exploration of Analog Circuits by Visualized Multi-Parallel Particle Simulation

Sebastian Steinhorst Markus Peter Lars Hedrich
Electronic Design Methodology Group
Department of Computer Science
University of Frankfurt/Main, Germany
Email: {steinhorst, peter, hedrich}@em.cs.uni-frankfurt.de

***Abstract*— In this contribution, a novel approach to exploring analog circuit behavior in the circuit's state space using visualization and multi-parallel particle simulation techniques is presented. Insights not acquirable by conventional circuit characterization methods can be obtained by combining different visualization aspects. The introduction of a visualized multi-parallel particle simulation allows to observe the system's dynamics in a new, up to now unavailable form, revealing possible design errors typically not detected by transient circuit simulation. The acquired results are compared to those of transient simulation and formal verification using an analog model checking approach.**

I. Introduction

Analog circuit design still suffers from a lack of formalism, making the success of a design approach highly dependent on the designer's experience and expertise. In the area of analog circuit characterization, finding the right test benches and drawing the right conclusions from simulation results poses serious challenges to designers; challenges that can be approached with innovative tools.

When considering dynamic circuit behavior, the undisputed industry standard is transient simulation with its results commonly investigated in a 2-dimensional s(t)-plot with signal value axis and time axis. However, this established method may be assisted by more specialized approaches as advanced concepts for state space-based transient signal inspection and circuit verification are emerging.

While vector field visualization techniques have contributed significantly to understanding complex data in physics and chemistry [1], visualization in electronic circuit design is still in its infancy with transient signal plots representing the state of the art. A visualization of the n-dimensional vector field representing the analog circuit's behavior has not yet been considered as a possible approach to support conventional transient simulation. As dynamic structures can be difficult to identify in such high-dimensional vector fields originating from state space representation of analog circuits, application of visual aids is mandatory. Approaches such as line integral convolution (LIC) [2] or anaglyph stereo vision [3] facilitate the understanding of 2-D and 3-D visualization but are not covering dynamic transient behavior. Therefore, a novel approach to consider analog circuits in a state space representation using visualized multi-parallel vector field particle simulation will be presented in this paper, and the application to example circuits will prove the practicability for real-world problems.

This paper is organized as follows. In section II the state space representation of analog circuits and the vector field sampling strategy is described. On the obtained vector field, the visualized multi-parallel particle simulation approach is introduced in section III. A case study applies the new particle simulation approach to a modified ring oscillator circuit in section IV, identifying a hidden design error. A Schmitt trigger circuit is analyzed in section V and conclusions are drawn in section VI.

II. State Space Representation

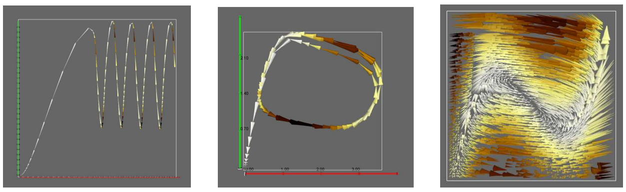
For our approach, a vector field representation of the analog circuit's dynamics in state space has to be obtained. An analog circuit can be described by a nonlinear first order differential algebraic equation (DAE) system

$$f(\dot{x}(t), x(t), u(t)) = 0 \tag{1}$$

representing the input vector $u(t)$ and the vector of the system variables $x(t)$, set up by applying modified nodal analysis (MNA) to the circuit netlist. The state space of the resulting DAE system, spanned by the system variables of the energy-storing elements $x_e(t)$ and the input variables, is now sampled using numerical integration steps with a defined short integration time constant, representing the behavior of the system variables' derivatives $\dot{x}_e(t)$.

Starting at equally distributed random sample points over the state space of the system, the sampling process is guided by the divergence of the length and angle of neighboring vectors. Sampling density is increased in areas of non-homogeneous vector field orientation, resulting in a discrete vector field V_D with the set of sample points $Q = \{q_1, ..., q_m\}$. This described modeling process of the vector field is performed by the analog modeling tool *amcheck* [4].

For a better understanding of the state space representation, the transient response for I_L of a tunnel diode oscillator circuit, shown in Figure 2, is plotted over time in Figure 1(a). For both system variables I_L (y-axis) and V_C (x-axis) of this circuit, the transient response is plotted in Figure 1(b) in state space representation with transition time implicitly shown with a color mapping. Figure 1(c) shows the complete discrete vector field of the tunnel diode oscillator's state space.



(a) (b) (c)

Fig. 1. Transient response of the oscillator's system variable I_L plotted over time (a). Transient response of the two system variables I_L and V_C in state space representation (b). Vector field representation of complete state space over I_L and V_C (c).

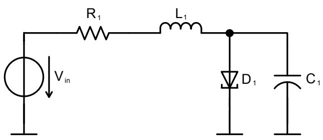


Fig. 2. Schematic of tunnel diode oscillator circuit.

III. Multi-Parallel State Space Particle Simulation

By visualizing the vector field resulting from the state space sampling described in the previous section, the dynamic behavior of the system under investigation can be analyzed. Vector field visualization is only possible with a restriction to 3 dimensions, while state space dimensions of common analog circuits can vary between 2 and more than 4. Therefore, a selection of the main dimensions has to be made to project to the 3-dimensional view.

Particle simulation is a common approach for vector field visualization and mature algorithms have been developed [5]. In contrast to a static approach such as line integral convolution, time-dependent motion of the particles visualizes the transient behavior of the circuit in an animation sequence.

We consider a vector field $V : \mathbb{R}^n \to \mathbb{R}^n$ on which for the discrete set $Q = \{q_1, ..., q_m\}$ of m sample points q_i in the state space, generated from equation 1, the discrete vector field $V_D : Q \to V_D$ is defined:

$$V_D(q_i) = \{v_i | \exists q_i \in Q : v_i = \frac{\partial q_i}{\partial t}\} \quad (2)$$

In other words, the discrete vector field V_D is represented by position vectors q_i determining the sample points in the

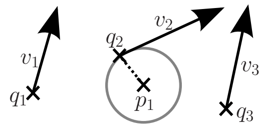


Fig. 3. Determining nearest sample point q_2 in state space for particle p_1 within discrete vector field V_D.

state space and the direction vectors $v_i = V_D(q_i)$ giving the motion direction and speed within V at position q_i.

For the injected particles $p_i \in \mathbb{R}^n$, their tangent vector $V(p_i) = \frac{\partial p_i}{\partial t}$ has to be approximated with respect to the discrete vector field V_D. Hence, a mapping is necessary which assigns a nearest sample point $q_j \in Q$ to each particle $p_i \in P$ from the set of particles P, as illustrated in Figure 3:

$$M(p_i) = \{arg\ min_{q_j \in Q} ||p_i - q_j||_2\} \quad (3)$$

Therewith, for each particle p_i, its next position can be calculated according to a time step Δt and the nearest direction vector $v_j = V_D(M(p_i))$:

$$p_i(t + \Delta t) = p_i(t) + v_j \cdot \Delta t \quad (4)$$

When starting the particle simulation, an equally distributed amount of particles is inserted into the vector field of the state space and for each particle, the nearest vector regarding euclidean distance determines its direction and speed of movement as stated above.

Algorithm 1 recapitulates the introduced particle simulation algorithm.

Algorithm 1: Particle Simulation Algorithm

```
while animation running do
    foreach each particle p_i in state space do
        detect nearest sample point q_j = M(p_i) with
        respect to euclidean distance;
        get direction vector v_j = V_D(q_j);
        p_i.position = p_i.position + v_j · Δt
    end
end
```

Each of the particles represents an independent simulation run with the starting position indicating its initial condition. While the visualization is projected to a 3-dimensional representation, the motion vector of the particles is calculated with full dimensionality. Thus, the motion is determined by all dimensions of the state space, revealing additional information exceeding the 3-dimensional plot.

For the tunnel diode oscillator circuit's state space with two state space dimensions, the particle animation is illustrated in four steps in Figures 4(a) to 4(d).

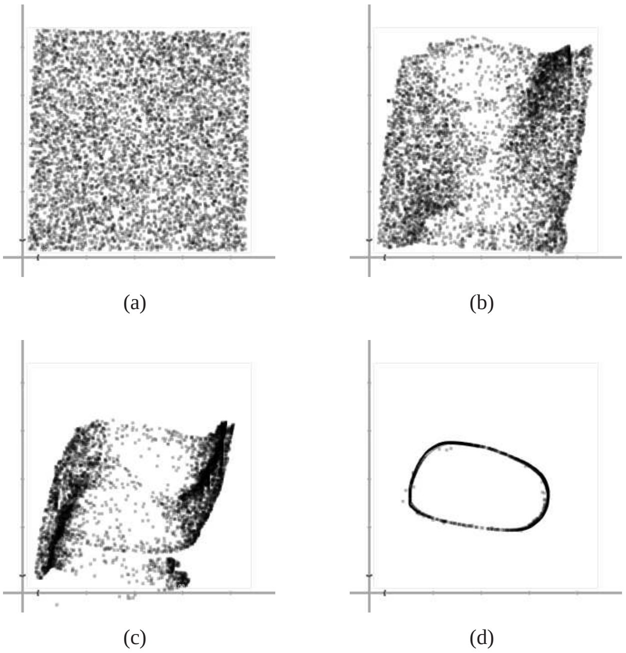


Fig. 4. Particle simulation for tunnel diode oscillator circuit with increasing time from (a) to (d).

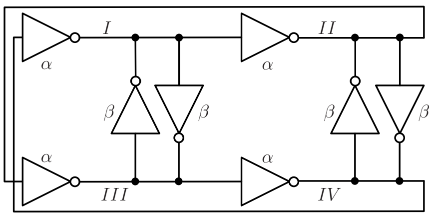


Fig. 5. Modified ring oscillator with an even number of inverter stages and cross-coupling.

IV. Case study: Ring Oscillator

The example circuit illustrated in Figure 5 is a modified ring oscillator with an even number of inverter stages and cross-coupling [6]. Due to the bridges β, this circuit will oscillate if there is a ratio α/β of the transistor sizes in the feedback chain to those of the bridges within the interval [0.4, 2.0]. The crucial point of this circuit is its proneness to initial conditions, avoiding it to oscillate when the α/β ratio reaches or exceeds the interval boundaries. While several simulation runs for this critical transistor ratios with random initial conditions can show perfect oscillation, particular initial conditions exist which are preventing this circuit from oscillating. Figure 6(a) shows the vector field of the oscillation area with an α/β ratio of 1.0 projected to the state space variables V_I, V_{II}, and V_{III}. The particle simulation converges into an oscillation circle without aggregation areas where particles are not oscillating, as illustrated in Figure 6(b).

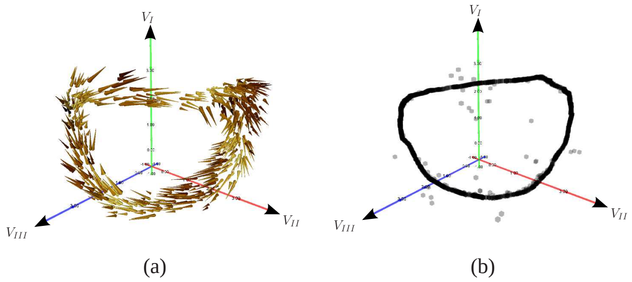


Fig. 6. Vector field representation of the oscillation area (a) and the particle simulation of the oscillator circuit's dynamic behavior (b) for transistor ratio $\alpha/\beta = 1.0$.

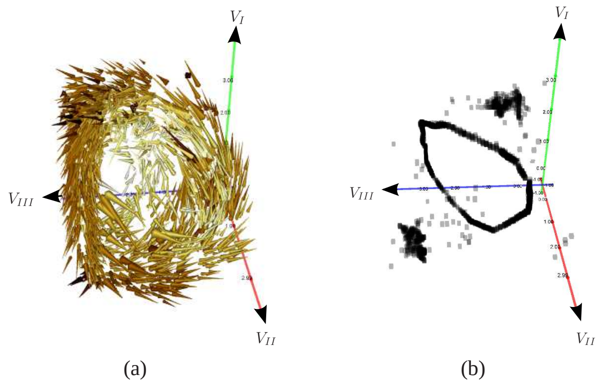


Fig. 7. Vector field representation of the oscillation area (a) and the particle simulation of the oscillator circuit's dynamic behavior (b) for transistor ratio $\alpha/\beta = 2.1$.

In terms of runtime, the costly part is the initial vector field sampling, taking 54 seconds on a Pentium 4 with 2.8 GHz clock frequency and 2 GB of RAM. The particle simulation for 5000 particles runs in real-time, allowing user-interactive zooming and rotation of the visualization during the simulation.

In contrast to the perfect oscillation with transistor ratio 1.0, Figure 7(a) shows a larger area of vectors belonging to possible oscillations for transistor ratio 2.1. The particle simulation clearly reveals two aggregation areas where the system reaches a steady state and will not oscillate when initial conditions lead into this area as illustrated in Figure 7(b).

The results detected by the particle simulation can be reproduced with transient simulations from the detected initial conditions. In order to compare the results with another state space-based approach, a formal verification of the circuit using a property specification with the Analog Specification Language (ASL) and the ASL-MCT model checking tool [7] was conducted. Similar to the vector field modeling, this approach is based on a state space sampling strategy using the homogeneity criterion of the angle and length of the vectors for partitioning the state space in hypercubes of homogeneous behavior. Each of these hyper-

```
# Get the oscillation set
osci_set = select oscillation;

# Check if oscillation exists (bool. result to %has_osci)
$set_is_not_empty(osci_set, %has_osci);

# Check if circuit has steady states (bool. result to %has_steady)
$set_is_not_empty(steadystates,%has_steady);

# Assertion: %has_osci shall be true
for %has_osci assert true;

# Assertion: %has_steady shall be false
for %has_steady assert false;

# Which states lead into steady states?
into_steady = reach steadystates;

# Which states can reach osci_set?
into_osci = reach osci_set;

# Which states run into steady states and not into the oscillation?
# These states are initial conditions that will never run into an
# oscillation, while states that are in the set into_steady and in
# into_osci can possibly run into steady states even if they can
# run into the oscillation area
bad_initial_conditions
  = into_steady and not into_osci;
```

Listing 1. ASL specification for oscillator verification.

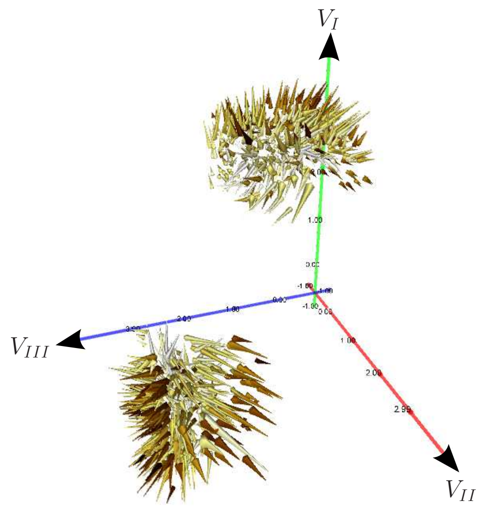


Fig. 8. Vector field of initial conditions leading into the non-oscillating steady states for transistor ratio $\alpha/\beta = 2.1$.

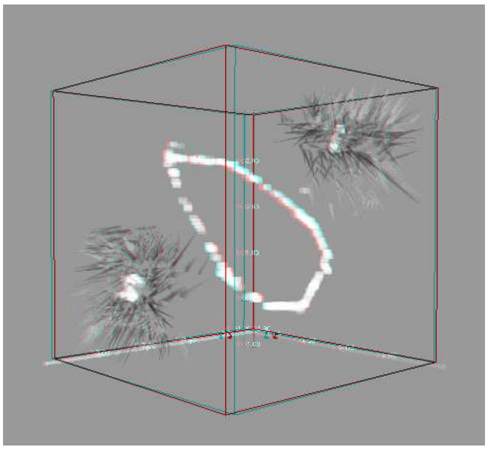


Fig. 9. Anaglyph stereoscopic visualization of the particle simulation and vector field of bad initial conditions for transistor ratio $\alpha/\beta = 2.1$.

cubes represents a state of the discrete state space and can be transferred into a graph structure, where transitions are determined by the vectors between consecutive hypercubes. Subsequently, the model checking algorithms are applied to the generated graph structure.

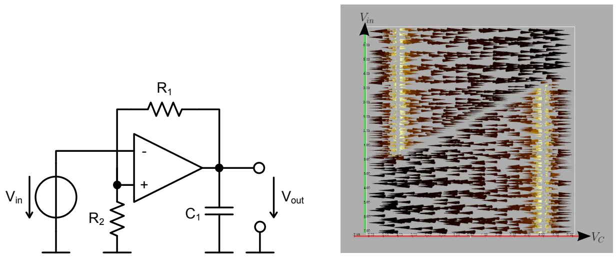
The ASL specification of the oscillation properties is given in Listing 1 and when applied to the circuit model, for transistor ratio 1.0 no initial conditions violating the oscillation behavior are existent. In contrast, for transistor ratio 2.1, initial conditions are detected for which the circuit will not run into an oscillation. This area of bad initial conditions is illustrated in Figure 8, complementing the aggregation areas of the particle simulation in Figure 7(b). A combined anaglyph 3D-visualization of the bad initial conditions detected by model checking and the aggregation areas resulting from particle simulation is shown in Figure 9, indicating congruency of the results acquired from both methods. The anaglyph plot can be viewed with standard red/cyan anaglyph glasses, giving a stereoscopic impression of the visualized structures.

While both particle simulation and model checking detect the bad initial conditions for the oscillator circuit, using the particle simulation approach, no effort for writing a specification of the properties is needed. Instead, it is embedded in a visualization system that is highly interactive. However, the model checking approach was developed under the objective of a precise and fully automated verification without user interaction, offering specification possibilities beyond the complexity of what can be visually identified.

V. Application to other circuit types: Schmitt Trigger

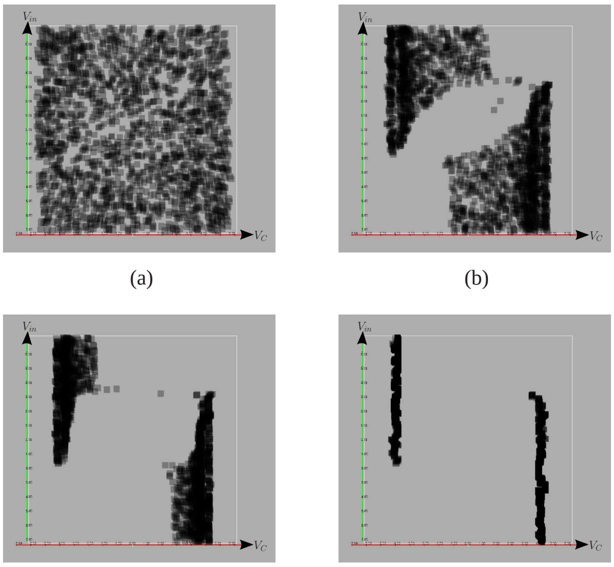
While the newly introduced particle simulation is especially useful for exploration of the behavior of oscillating circuits, the application to other types of circuits can deliver important insights just as well. In the following example, an inverting Schmitt trigger circuit, as illustrated in Figure 10(a), is analyzed ($R_1 = R_2 = 100\,k\Omega,\ C_1 = 100\,pF$).

The vector field representation of the state space dimensions V_{in} and V_{C_1} depicted in Figure 10(b) shows the typical hysteresis behavior of the Schmitt trigger. All



(a) (b)

Fig. 10. Circuit schematic of inverting Schmitt trigger (a) and vector field representation of state space plotted for V_{in} and V_{C_1} (b).



(a) (b) (c) (d)

Fig. 11. Particle simulation for Schmitt trigger circuit with increasing time from (a) to (d).

vectors are pointing to either of the two aggregation areas that represent the attractors of the DC transfer function of the circuit.

The particle simulation is illustrated in Figures 11(a) to 11(d), visualizing the circuit behavior from any initial condition with any input voltage. Depending on their initial position, the particles aggregate either in the areas of high or low output voltages during the animation sequence. Hence, the dynamic behavior of the Schmitt trigger circuit can be examined by conducting one parallel particle simulation.

The vector field sampling took 12 seconds on a Pentium 4 with 2.8 GHz clock frequency and 2 GB of RAM, enabling the interactive real-time particle simulation.

VI. Conclusion

While transient simulation is the general purpose tool for circuit characterization, this paper demonstrated how shifting the perspective from a signal-based time domain representation to a state space representation with its visualization and particle simulation possibilities can provide additional insights into analog circuit behavior. Due to the initial conditions of the multi-parallel simulation approach being distributed over the complete state space, much more certainty on the coverage of the performed simulation is given.

As has been demonstrated with the given example, the results obtained by state space particle simulation can compare to those of formal verification and hence did not depend on the verification engineer to detect the critical initial conditions for simulation runs. Future work will include extending the particle simulation with input stimuli. This will offer the possibility not only to simulate the inherent state space dynamics, but to consider the circuit behavior under the existence of input signals. In addition, by trading in real-time interactivity for higher accuracy, the calculation of the particle trajectories could be precomputed using a commercial circuit simulator.

Providing deeper insights into circuit dynamics, state space representation already proved its value in the area of formal verification. The visualized state space particle simulation may bring the benefits of this approach to another new area of application.

Acknowledgment

The authors would like to thank Georg Denk, Carsten Hammer (Titan Simulator Development, Qimonda AG, Munich, Germany), Peter Jores (AE/EIM3, Robert Bosch GmbH, Reutlingen, Germany), and Victor Konrad for very fruitful discussions concerning this work.

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