

Basics of GNSS

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Contents

- Coordinates System
 - Satellite Position
- }
- 1st period
-
- Measurements Errors
 - Calculating Position and DOP
 - Improved Position
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- 2nd period
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- Basics of GNSS receiver
 - Future GNSS
- }
- 3rd period

Lecture

- Any comments and questions are welcome.
- Simple problem (15min.) is assigned for each period. After summer school, please submit it to the staff by the end of this school.
- My lecture is mainly for smooth transition to SDR and RTKLIB in the following lectures.
- **GPS** is mainly used in this lecture.

References

My presentation is mainly based on ...

- *Compendium of GPS*

<http://www.u-blox.com/>

- *Global Positioning System: Signals, Measurements, and Performance Second Edition (2006)*

By Pratap Misra and Per Enge

<http://www.gpstextbook.com/>

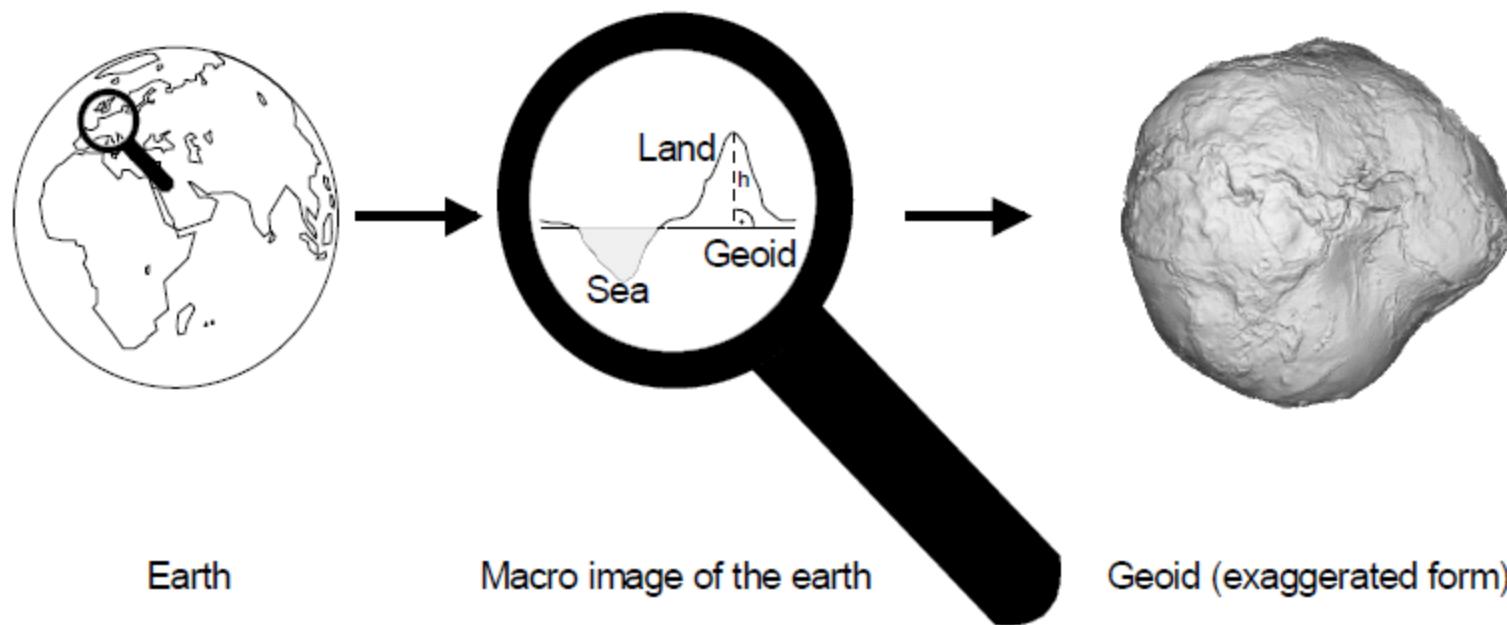
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Coordinate systems

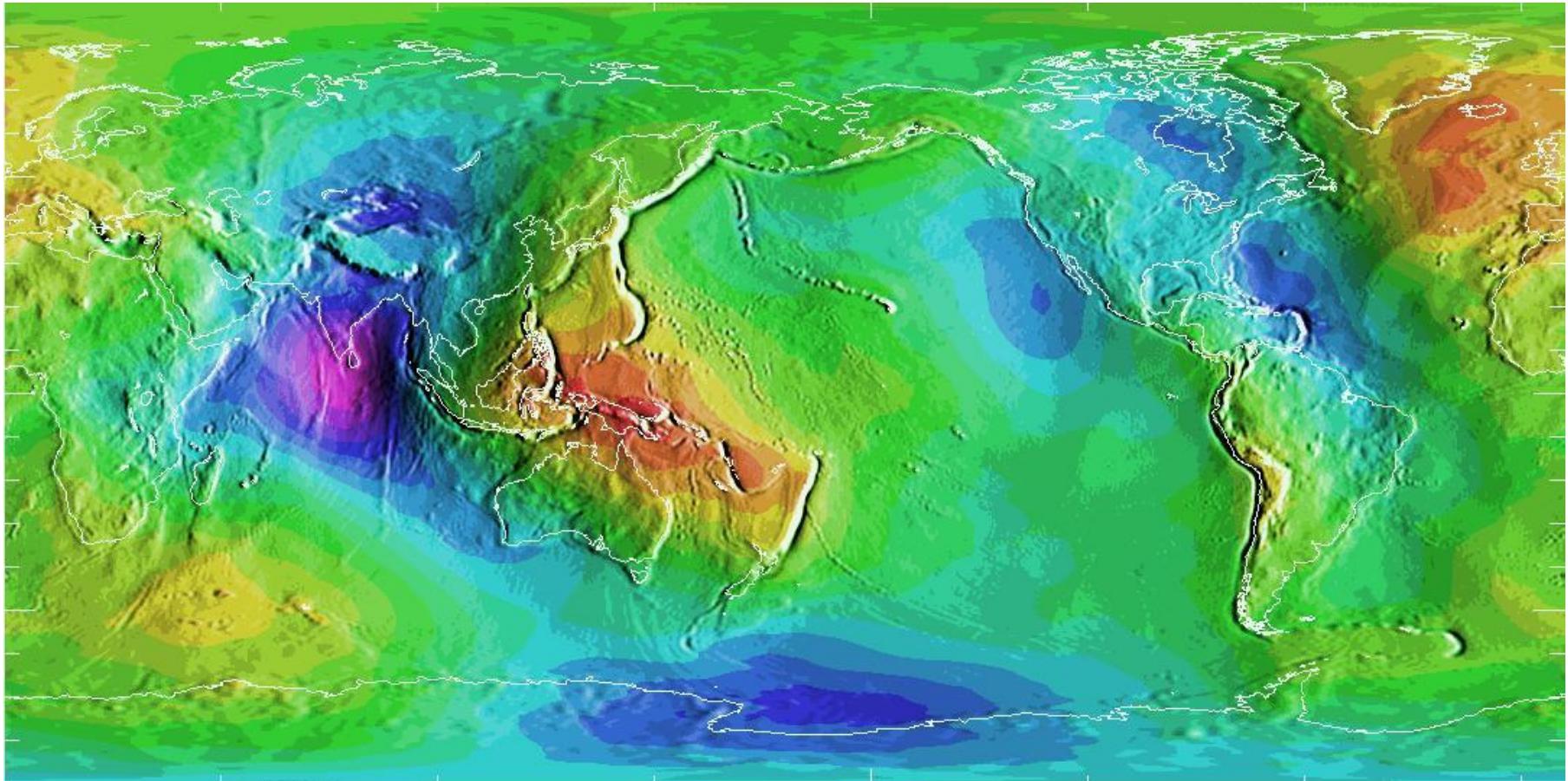
- A significant problem to overcome when using a GNSS system is the fact that **there are a great number of different coordinate systems worldwide.**
- As a result, the position measured and calculated does not always correspond with one's supposed position.
- In order to understand how GNSS systems function, it is necessary to examine some of the basics of **geodesy**.

What is Geoid ?



- The Geoid represents the true shape of the earth; defined as the surface, where **the mean sea level is zero**. However, a Geoid is a difficult shape to manipulate when conducting calculations.

World Geoid



http://principles.ou.edu/earth_figure_gravity/geoid/

Color Scale, Upper (Red) : 85.4 meters and higher;
Color Scale, Lower (Magenta) :-107.0 meters and lower

Geoid Height in Japan

The screenshot shows a Windows application window titled "ジオイド高計算". The interface is divided into two main sections: "入力値" (Input Value) on the left and "計算結果" (Calculation Result) on the right.

Input Value (Left):

- Selection: 1点毎の計算 (Point by Point Calculation) is selected.
- Method: 数値入力 (Numerical Input) is selected.
- Coordinates:
 - Latitude: 36.103774792
 - Longitude: 140.087855042
- Unit Selection: 度分秒 (Degree, Minute, Second) is selected.

Calculation Result (Right):

- Used Parameters: 日本のジオイド2011+2000 Ver1.0
- Coordinates:

緯度	36.103774792
経度	140.087855042
- Output Value:

ジオイド高	40.118 m
-------	----------

Buttons:

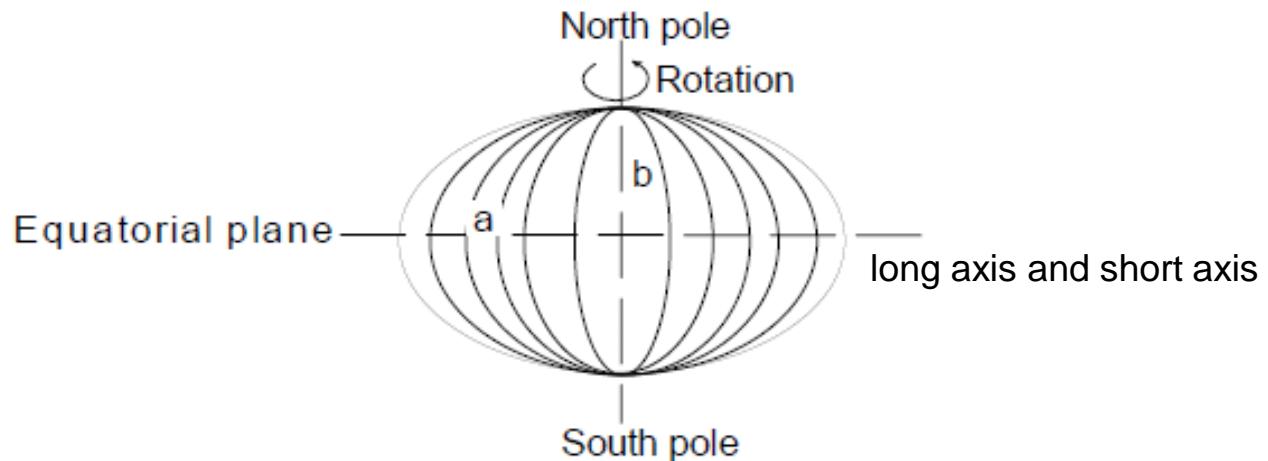
- 計算実行 (Calculate) button at the bottom left.
- 印刷 (Print) button at the bottom center.

<http://surveycalc.gsi.go.jp/sokuchi/geoid/calcgh/calcframe.html>

- TUMSAT
36.41 m
- Narita
35.24 m
- Mt. Fuji
42.50 m
- Osaka
37.45 m

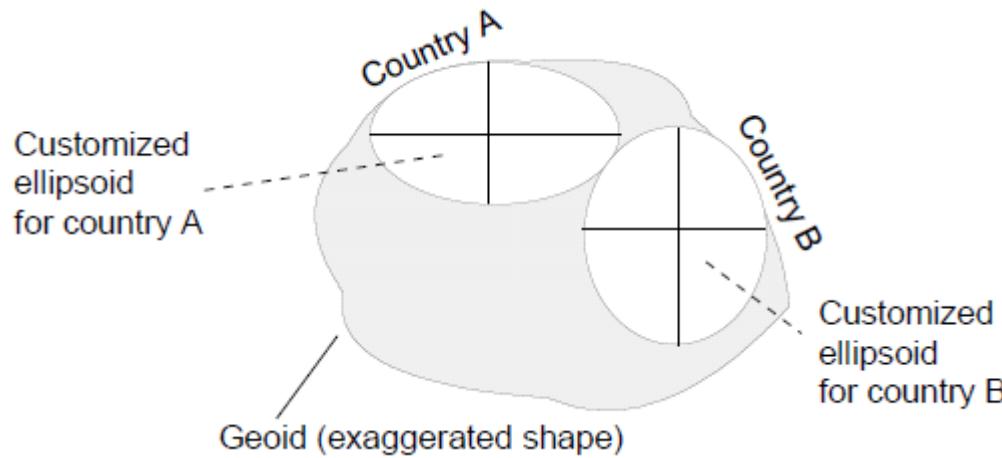
What is Ellipsoid ?

$$f = \frac{a-b}{a}$$

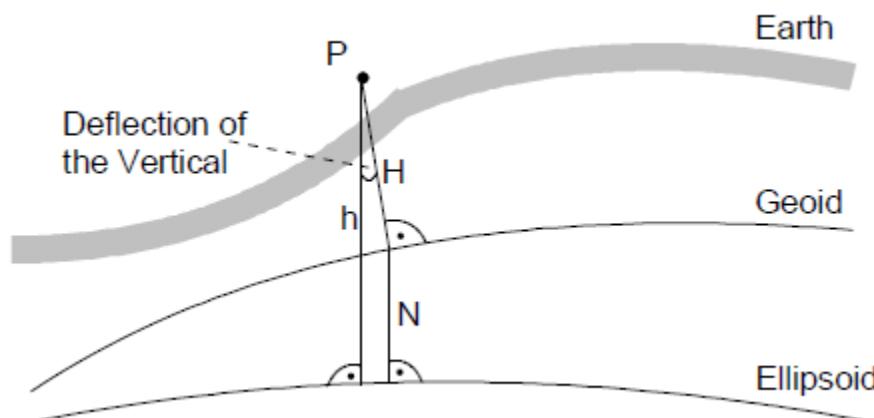


- A simpler, more definable shape is needed when carrying out daily surveying operations. Such a substitute surface is known as an **ellipsoid**. A spheroid is obtained like the above figure.

Datum, map reference system



Each country has developed its **own customized non-geocentric ellipsoid** as a reference surface for carrying out surveying operations.

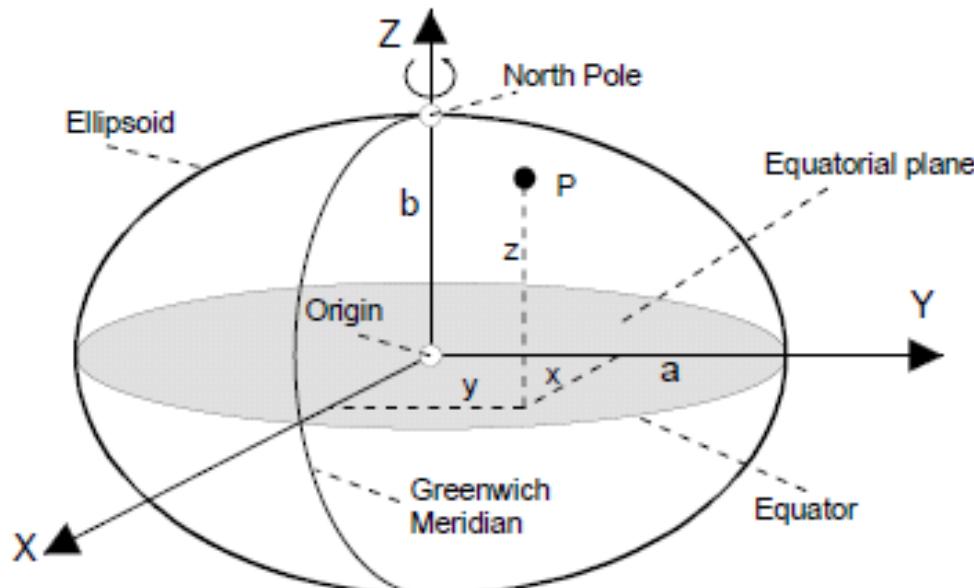


An ellipsoid is well suited for describing the positional coordinates of a point in degrees of longitude and latitude.

Ellipsoidal Height = Undulation N + Geoid Height

Worldwide reference ellipsoid WGS-84

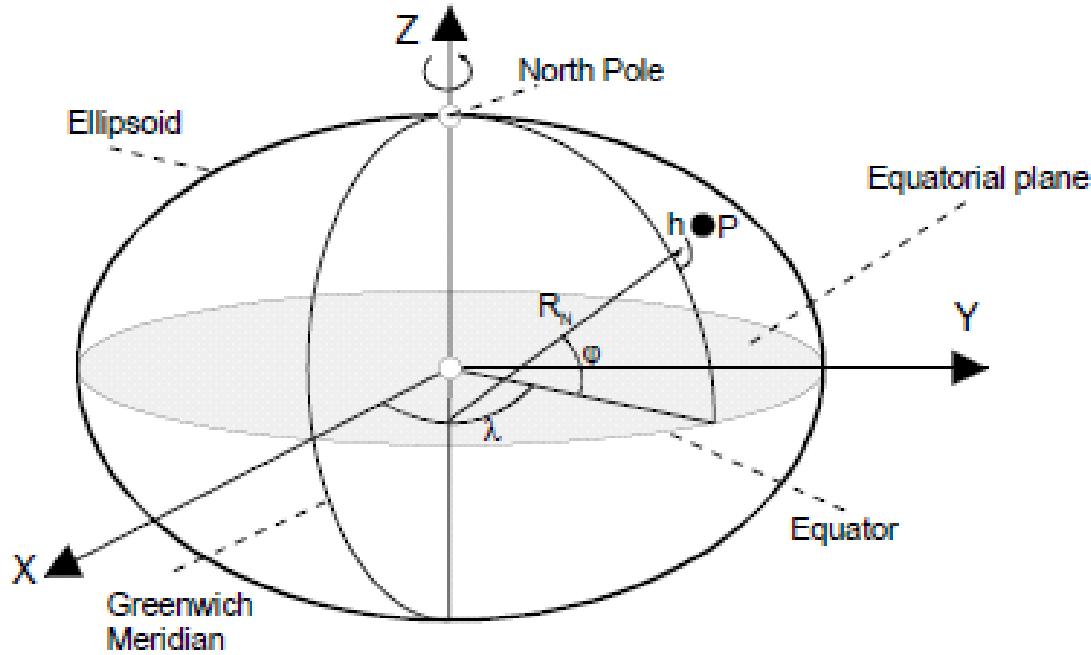
(World Geodetic System 1984)



Parameter of WGS-84 Reference Ellipsoids		
Semi major axis a (m)	Semi minor axis b (m)	Flattening (1:)
6,378,137.00	6,356,752.31	298,257223563

- The WGS-84 coordinate system is geocentrically positioned with respect to the center of the Earth. Such a system is called **ECEF** (Earth Centered, Earth Fixed)
- The WGS-84 is a three-dimensional, right-handed, Cartesian coordinate system with its origin at the center of mass of an ellipsoid.

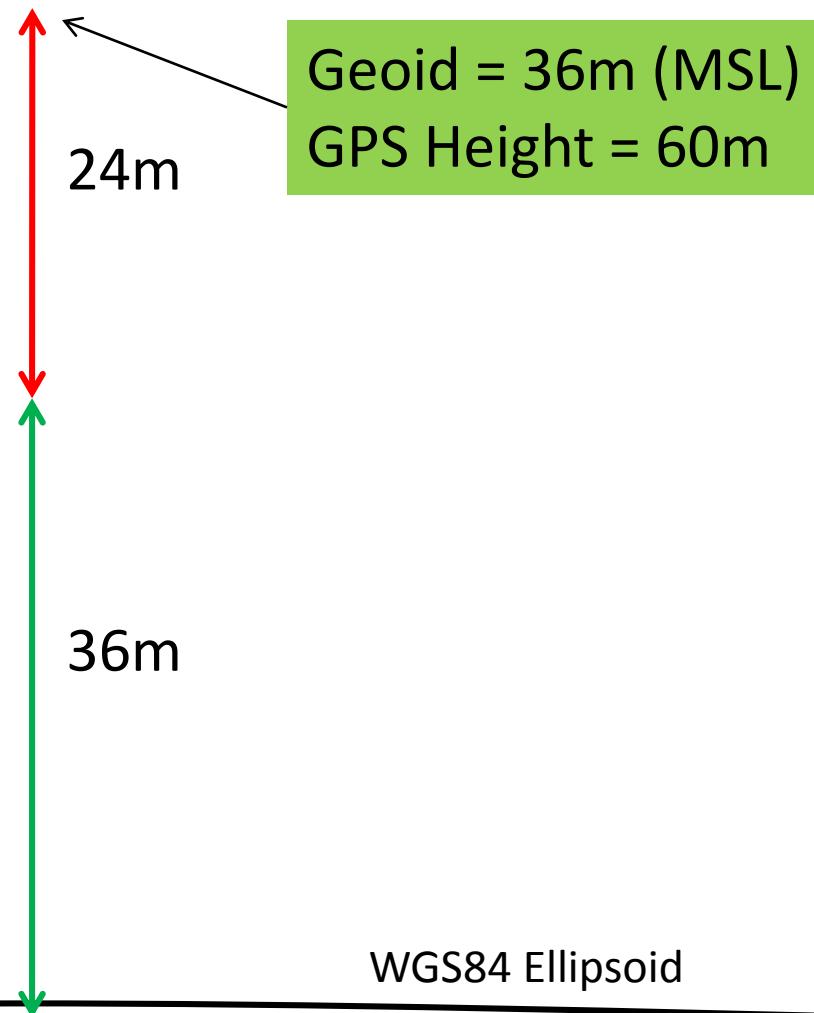
Ellipsoidal Coordinates



Ellipsoidal coordinates (Φ, λ, h) , rather than Cartesian coordinates (X, Y, Z) are generally used for further processing. Φ corresponds to **latitude**, λ corresponds to **longitude** and h to the **Ellipsoidal height**.

Ellipsoidal Height (GPS)

= Geoid Height + Orthometric Height



Tokyo Datum

- Japan has used Tokyo Datum based on Vessel ellipsoidal for many years. We have just started WGS84 since 2002.
- Orthometric height is still based on the height above mean sea level in Tokyo.
- In horizontal plane, there was about 400 m deviation in Tokyo only due to the difference between WGS84 and Tokyo Datum.

How about GLO, GAL, BeiDou ?

- Each navigation system uses the different coordinates system, but the coordinates for Galileo and BeiDou are quite similar to WGS84.
- GLONASS adopts PZ-90.02. We need to consider the difference if we combine GPS and GLONASS.

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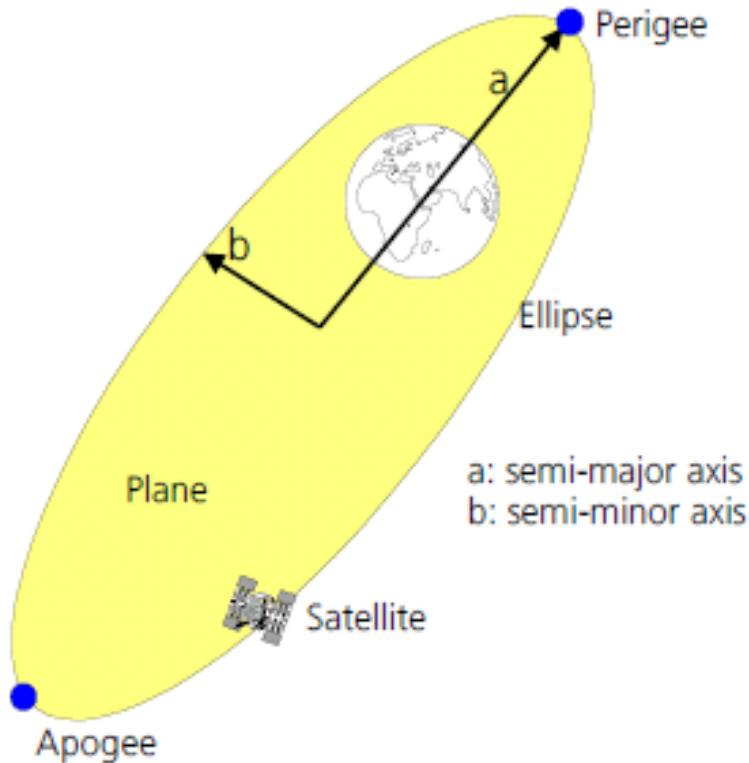
Satellite Position Calculation

- Calculating satellite position is mainly based on two methods.
- One is based on **almanac** data. The another one is based on **ephemeris** data.
- After Kepler's law introduction, brief explanation about almanac and ephemeris are introduced here.

Keplerian Elements

- Epoch (time)
- Semi-major Axis (km)
- Eccentricity
- Inclination (radian)
- RAAN (Right Ascension of Ascending Node) (radian)
- Argument of Perigee (radian)
- Mean Anomaly (radian)

Kepler's first law



- The **Apogee** expresses the furthest point of an elliptical orbit from the center of the Earth.
- The **Perigee** is the closest point of the orbital ellipse to the Earth.

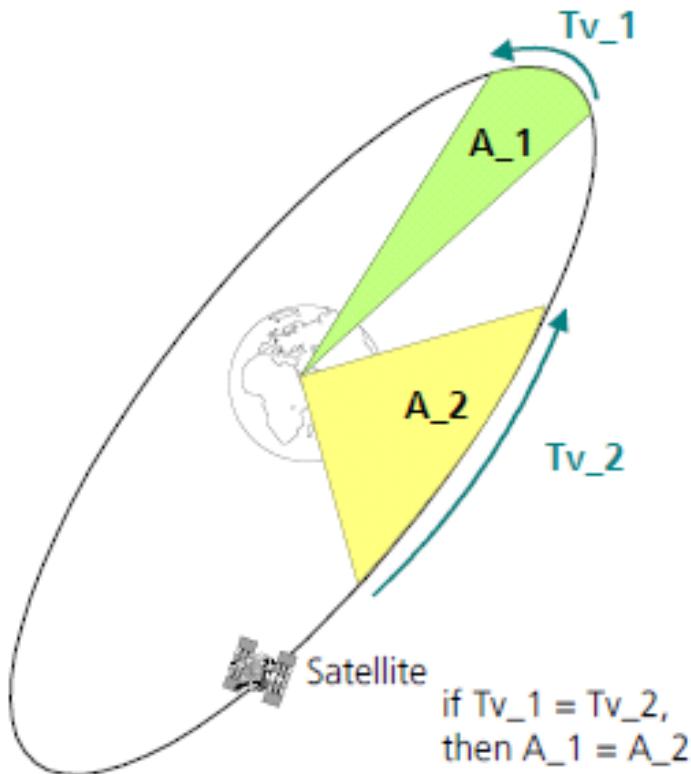
Semi-major axis and Eccentricity

①

②

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Kepler's second law



- The second law states that: **“A line joining a planet and the sun sweeps out equal areas during equal intervals of time”**
- For satellites this means left figure.

Kepler's third law

$\frac{P^2}{a^3}$ is constant for all planets.

P = orbital Period, a = semi-major axis of the orbital ellipse

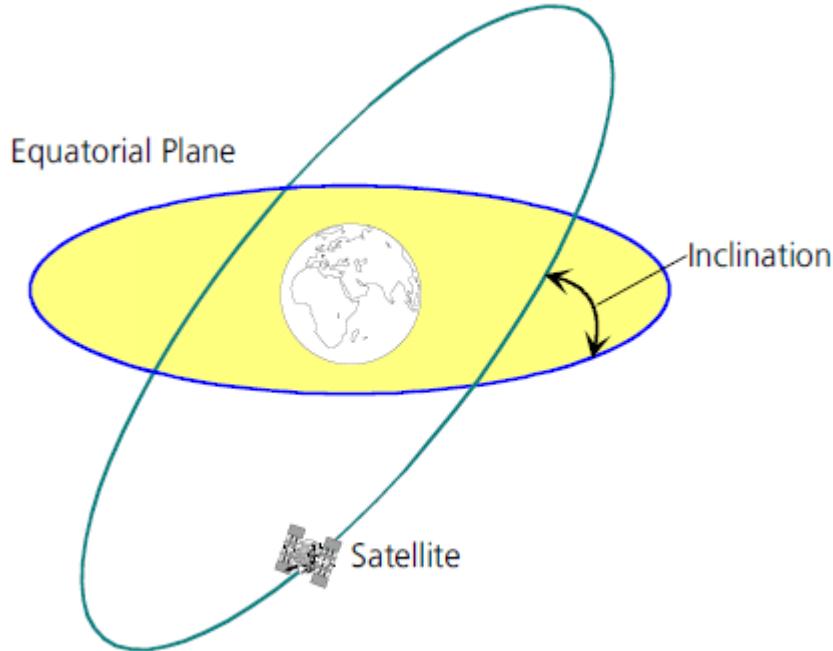
$$h = \sqrt[3]{3,9860042 \cdot 10^{14} \frac{\text{m}^3}{\text{s}^2} \cdot \left(\frac{P}{2\pi}\right)^2} - R_e \quad [\text{m}]$$

R_e: Radius of the Earth (6378.137km)

P: orbital period of the satellite around the Earth

- This law states that the **squares of the orbital periods** of planets are directly proportional to the **cubes of the semi-major axis of the orbits**.

Satellite orbits



The spatial orientation:
Orbital inclination, eccentricity,
length, altitude

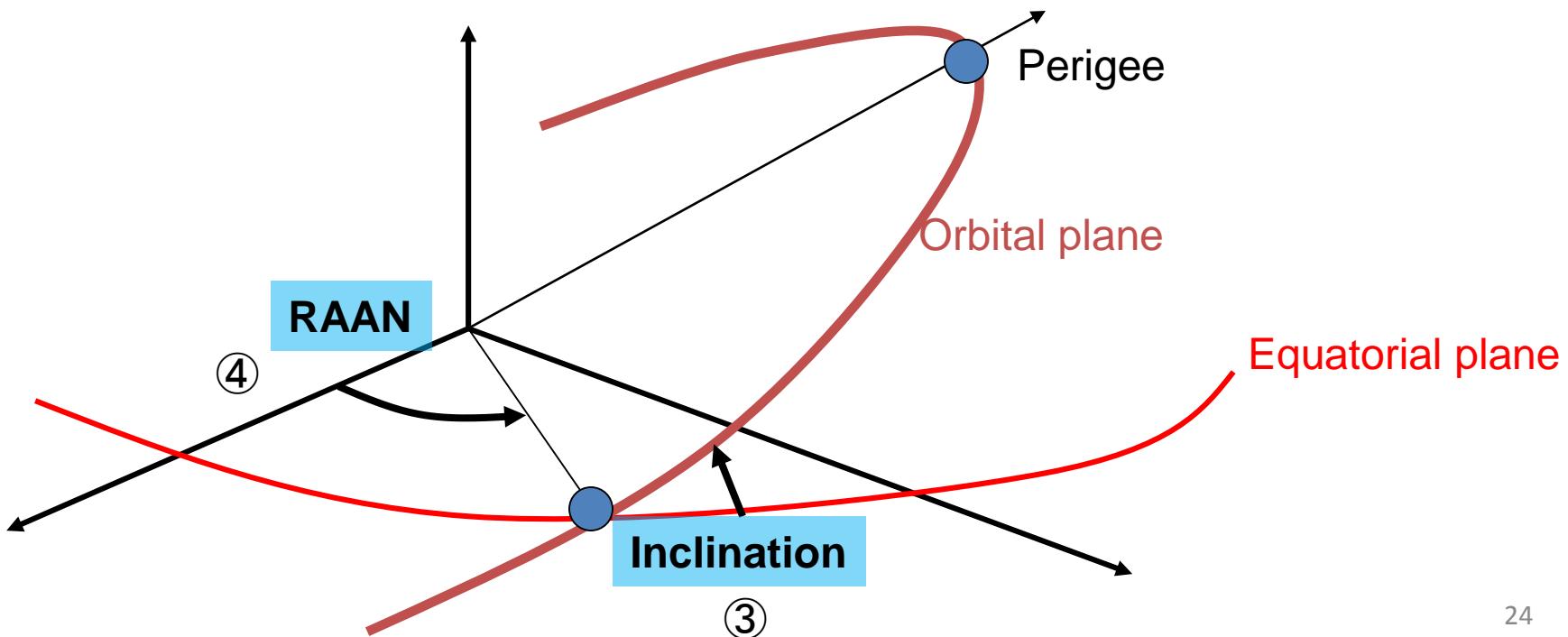
The parameters of motion:
Orbital period

The **Ephemeris** of a satellite is a mathematical description of its orbit. The high precision satellite orbital data is necessary for a receiver to calculate the satellite's exact position in space at any given time.

Orbital data with reduced exactness is referred to as an **Almanac**,
23

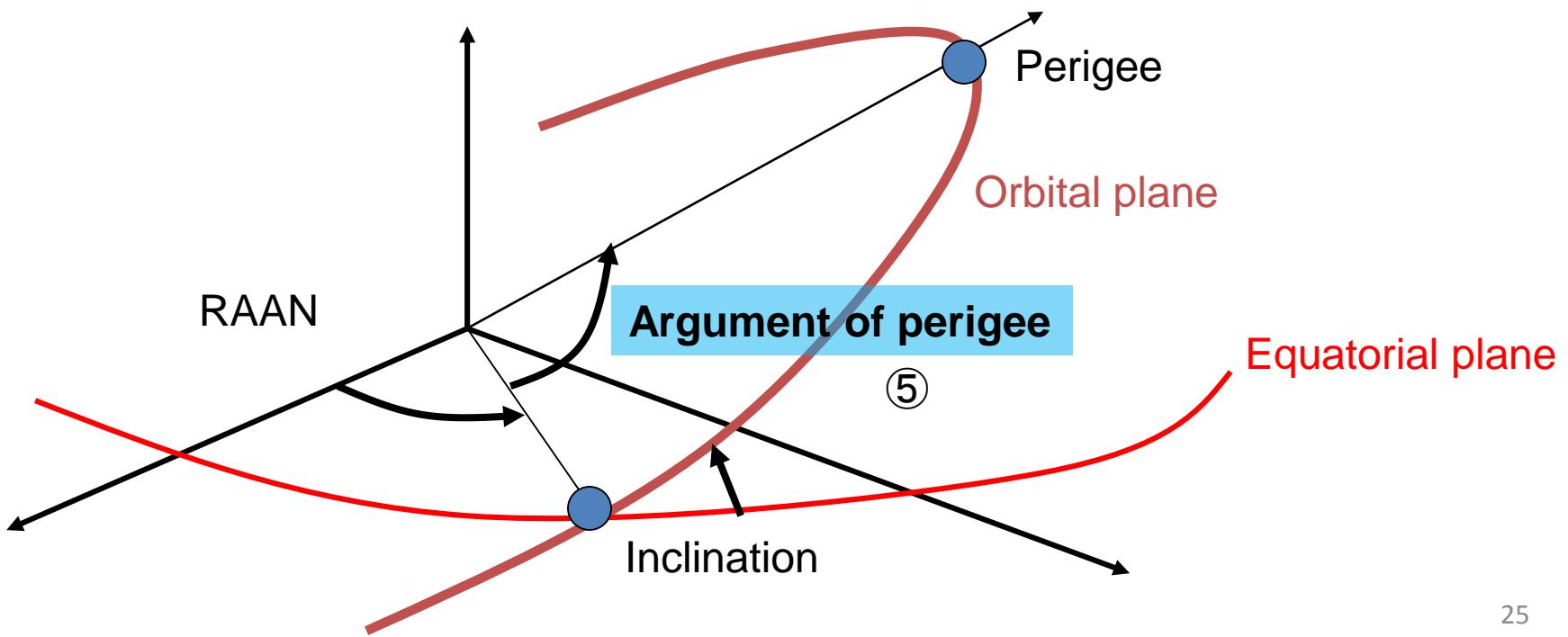
Orbital Plane

- **Inclination**: the angle between orbital plane and equatorial plane
- **Right Ascension of Ascending Node** : the geocentric R.A. of a satellite as it intersects the Earth's equatorial plane traveling northward (ascending)



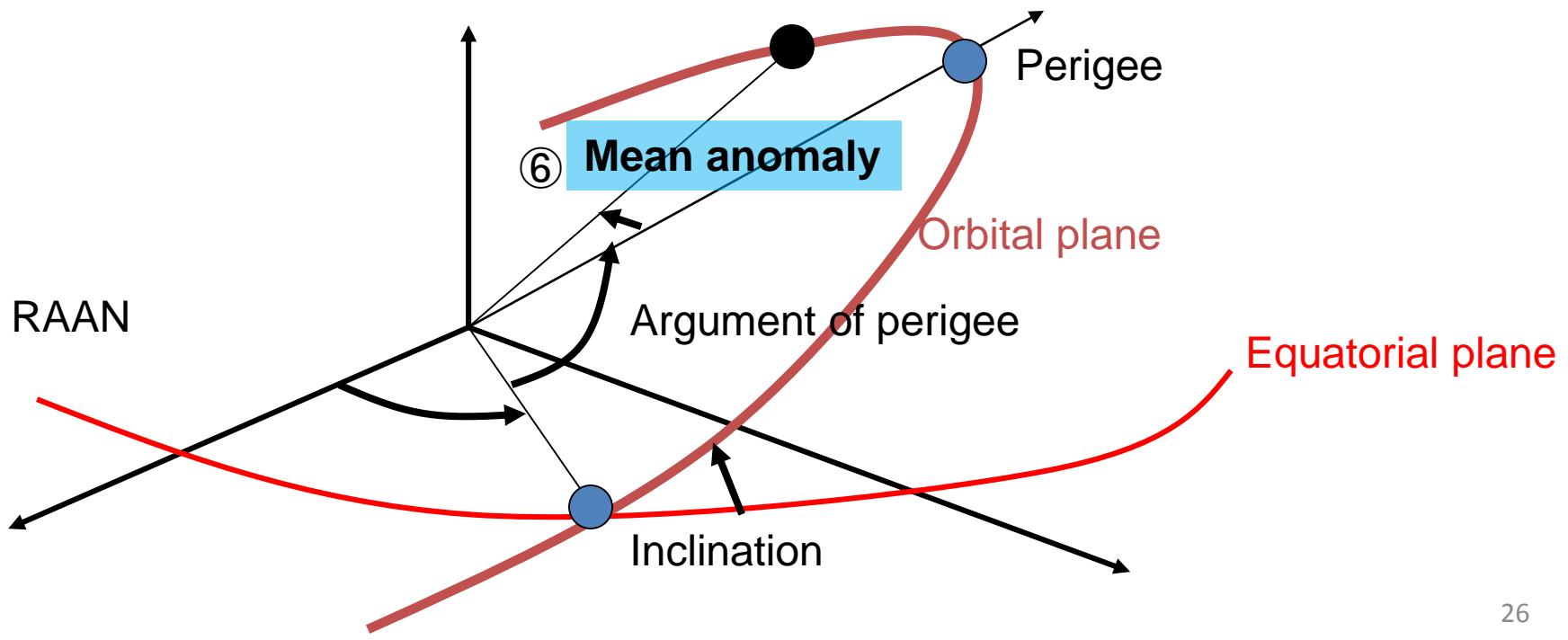
Direction of a semi-major axis

- **Argument of Perigee**: the angle between the perigee and the orbit's RAAN



Satellite position on orbital plane

- Mean anomaly : relating position and time for a body moving in a orbital plane



Almanac

```
***** Week 424 almanac for PRN-01 *****
ID:          01
Health:      000
😊 Eccentricity:      0.6912231445E-002
😊 Time of Applicability(s): 405504.0000
😊 Orbital Inclination(rad): 0.9911766052
😊 Rate of Right Ascen(r/s): -0.7417838788E-008
😊 SQRT(A)  (m 1/2): 5153.549316
😊 Right Ascen at Week(rad): -0.1640348434E+000
😊 Argument of Perigee(rad): -1.812852621
😊 Mean Anom(rad): -0.1197433472E+000
😊 Af0(s): 0.1583099365E-003
😊 Af1(s/s): 0.3637978807E-011
week:        424
```

- The current Almanac Data can be viewed over the internet.
- Accuracy

Almanac: 100-1000m 1week

Ephemeris: 1-2m 2hours

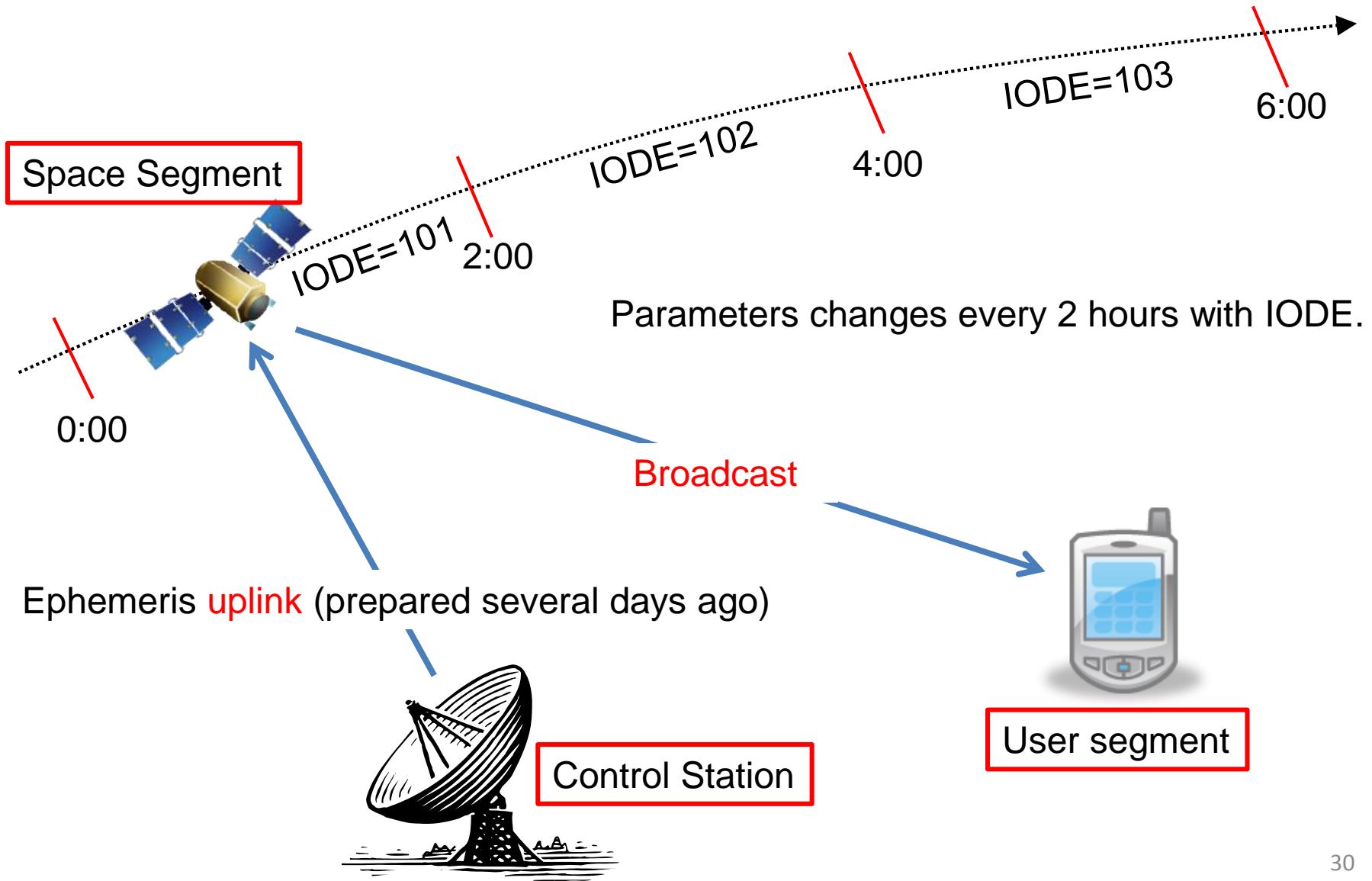
Ephemeris

- Almanac + Perturbation
- 16 coefficients
- Good thing is that ephemeris parameters are similar for each GNSS.
- Calculating satellite position based on several equations shown in ICD is very simple.
- Accuracy : 1-2m, 2 hours life for GPS

Perturbation

- **Perturbation** is the complex motion of a massive body subject to forces other than the gravitational attraction of a single other massive body.
1. Non-spherical gravitational potential of earth
 2. Resistance from atmosphere
 3. Attraction from sun and moon
 4. Solar radiation pressure

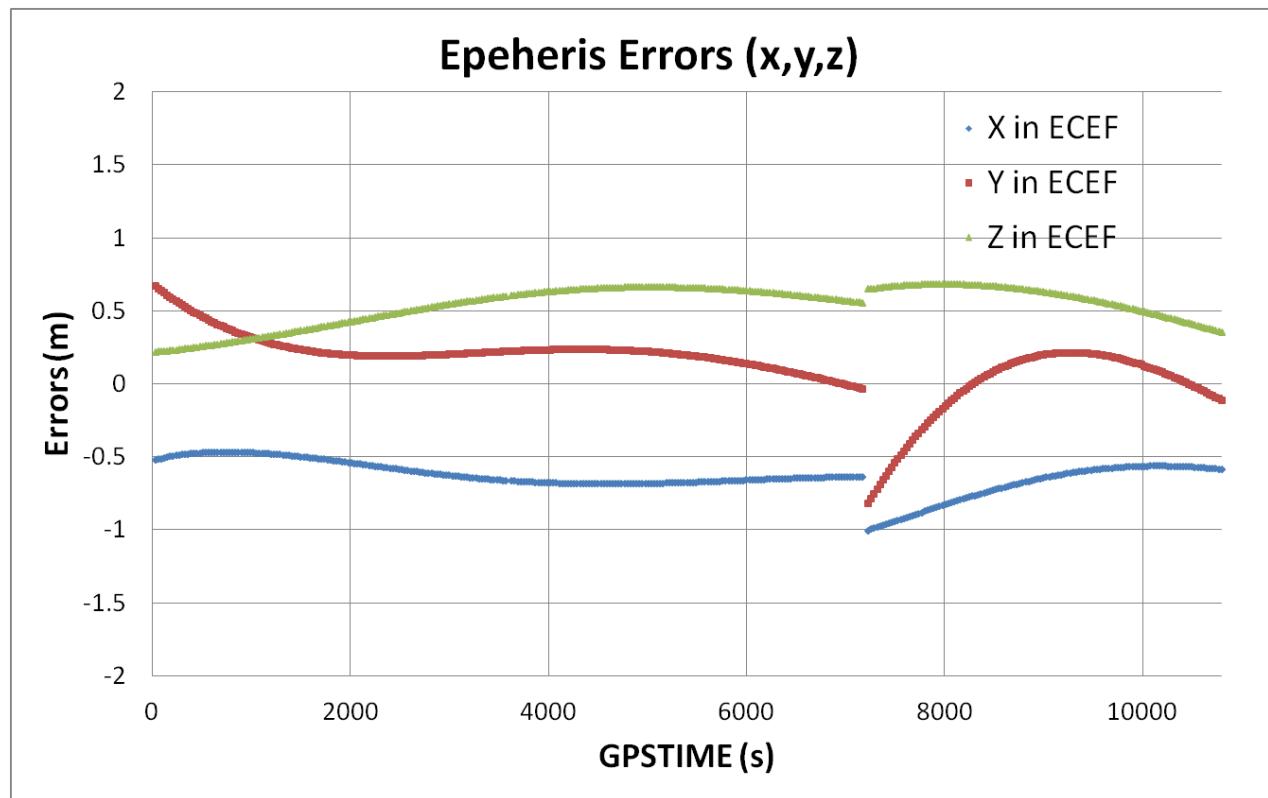
Image of using Ephemeris



Real Ephemeris Errors

(based on precise orbit data)

- Precise orbit data (-1cm) also can be obtained over the internet (sp3 file).



How about GLO, GAL, BeiDou ?

- **GLONASS** adopts the different method to estimate satellite position.
- **Galileo** and **BeiDou** broadcast same ephemeris parameters as GPS/QZS. You can use same source code for Galileo and BeiDou.
- The only thing should be careful is system time and GEO (geostationary earth orbit) for BeiDou.

Update Cycle of Ephemeris

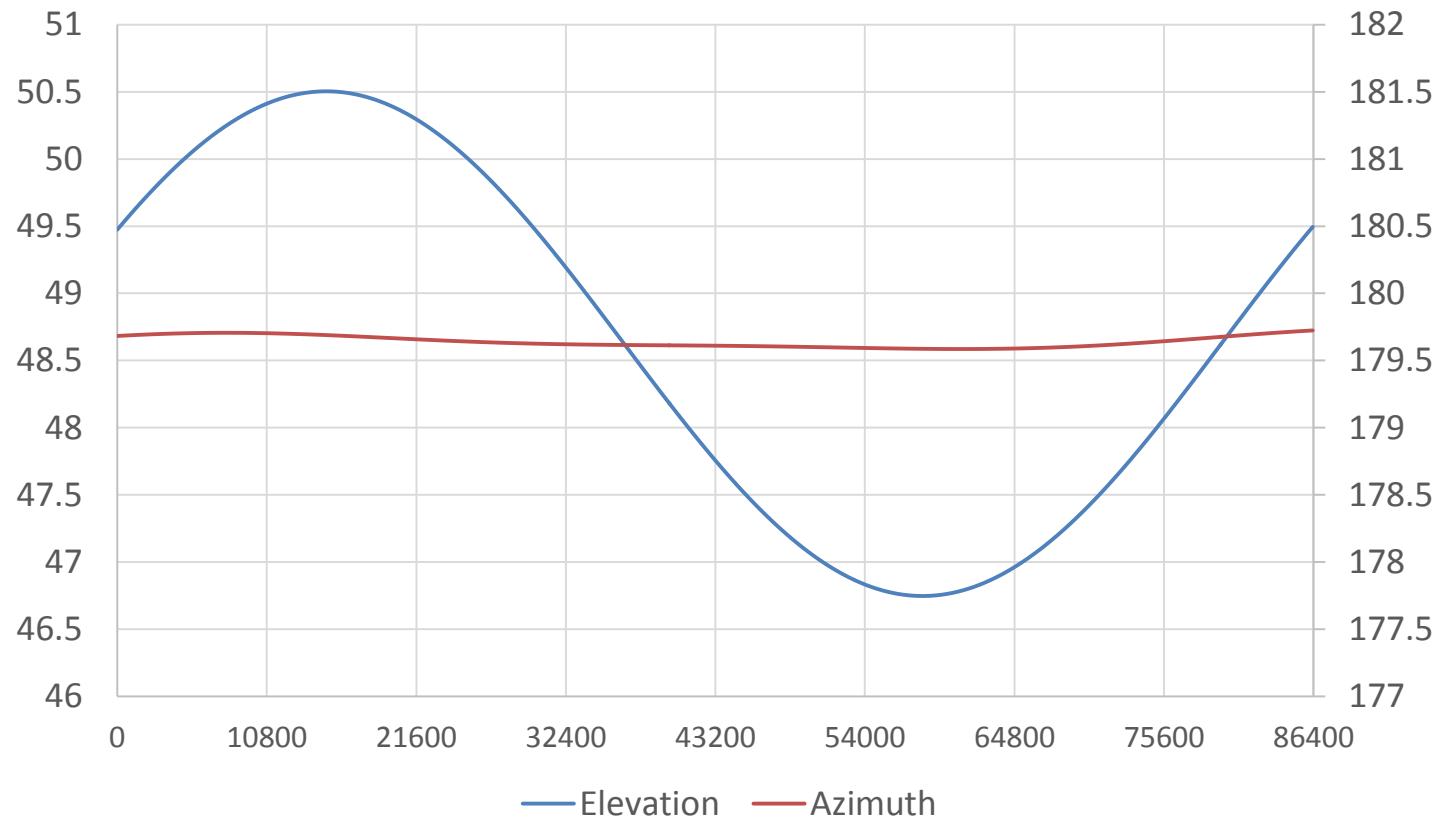
- GPS : 2hour
- GLONASS : 30 minute (interpolation use)
- BeiDou : 1hour
- Galileo : 10 minute
- QZSS : 15 minute

It is not formal information.

I just got these information based on the ephemeris data of NetR9.

BeiDou GEO1

Elevation and Azimuth of 24 hour from Tokyo



System Time Difference

Trimble - 2014-07-19T0 x 700 S Florida Ave - Goo x

192.244.150.156

アプリ おすすめサイト IE ブックマーク Trimble NetR9 K... Citibank® online citi Citibank-Japan Yahoo! JAPAN LIVE REAL TIME ... 直流電動機のトル...

Receiver Status - Position

Receiver Status

- Activity
- Position
- Position (Graph)
- Vector
- Google Map
- Google Earth
- Identity
- Receiver Options

Satellites

- Web Services
- Data Logging
- Receiver Configuration
- I/O Configuration
- Bluetooth
- OmniSTAR
- Network Configuration
- Security
- Firmware
- Programmatic Interface
- Help

Position:

Lat: 35° 39' 58.50429" N
Lon: 139° 47' 32.28375" E
Hgt: 61.444 [m]
Type: Autonomous
Datum: WGS-84

Satellites Used:33

GPS(11): 1, 3, 4, 8, 11, 17, 19, 20, 28, 30, 32
GLONASS(9): 1, 7, 8, 13, 14, 15, 22, 23, 24
Galileo(2): 11, 12
BeiDou(10): 1, 2, 3, 4, 6, 7, 8, 10, 11, 12
QZSS(1): 193

Velocity:

East : 0.00 [m/s]
North : 0.00 [m/s]
Up : -0.01 [m/s]

Error Estimates(1σ):

East: 0.441 [m]
North: 0.494 [m]
Up: 1.023 [m]
Semi Major Axis: 0.502 [m]
Semi Minor Axis: 0.432 [m]
Orientation: 20.306°

Position Solution Detail:

Position Dimension:	3D
Position Type:	Autonomous
Motion Info:	N/A
Augmentation:	GPS+GLN+GAL+BDS+QZSS
RTK Solution:	N/A
RTK Init:	N/A
RTK Mode:	N/A
RTK Network Mode:	N/A
Age of Corrections:	N/A
SBAS PRN:	N/A
Height Mode:	Normal
Correction Controls:	Off

Satellites Tracked:33

GPS (11): 1, 3, 4, 8, 11, 17, 19, 20, 28, 30, 32
GLONASS (9): 1, 7, 8, 13, 14, 15, 22, 23, 24
Galileo (2): 11, 12
BeiDou (10): 1, 2, 3, 4, 6, 7, 8, 10, 11, 12
QZSS (1): 193

Receiver Clock:

GPS Week: 1801
GPS Seconds: 548472
Offset: 0.00000 [msec]
Drift: -0.00001 [ppm]

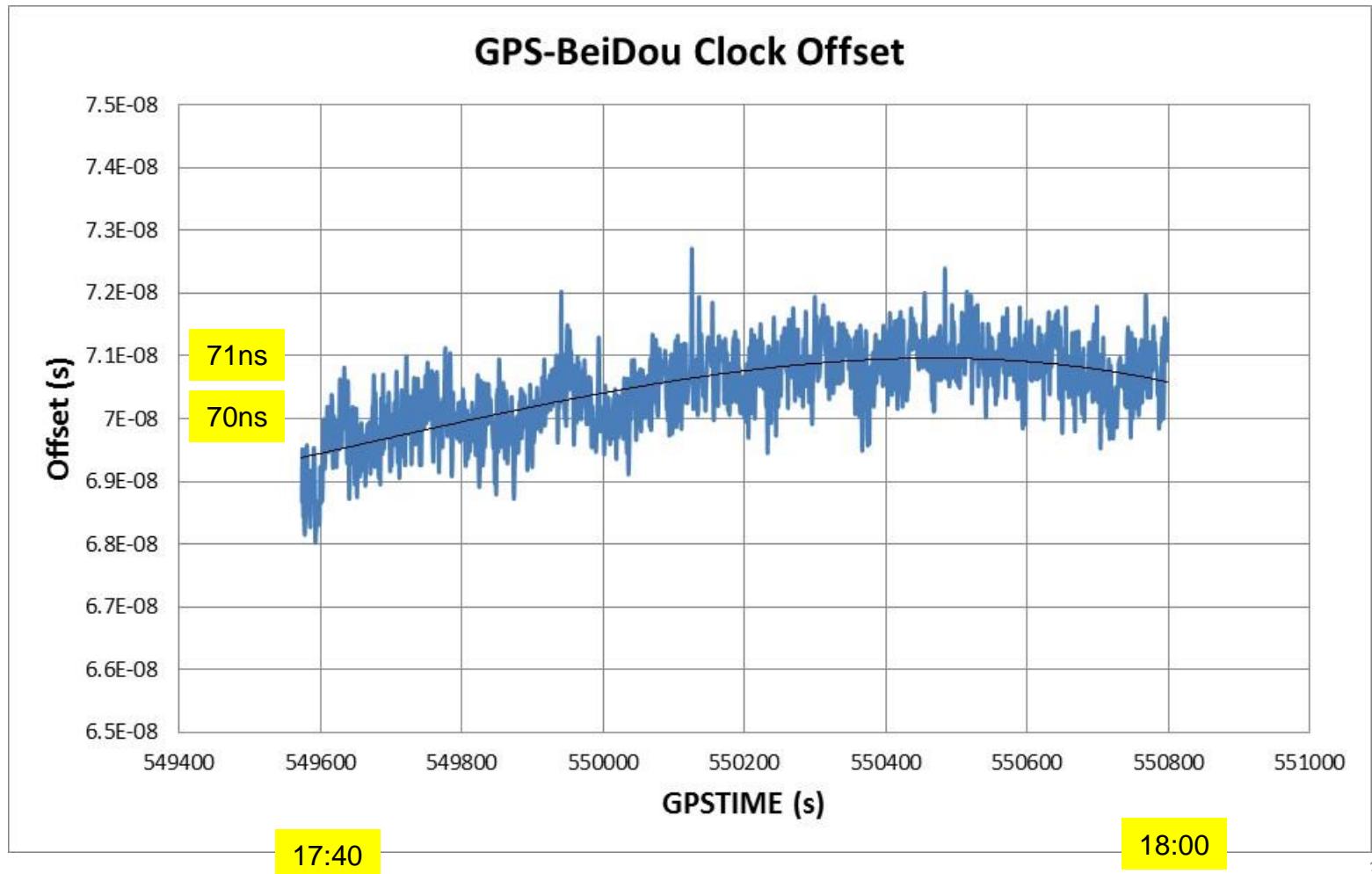
Multi-System Clock Offsets:

Master Clock System: GPS
GLONASS Offset: 385.9 [ns]
Galileo Offset: 27.8 [ns]
BeiDou Offset: -71.3 [ns]
GLONASS Drift: 0.020 [ns/s]
Galileo Drift: -0.046 [ns/s]
BeiDou Drift: 0.021 [ns/s]

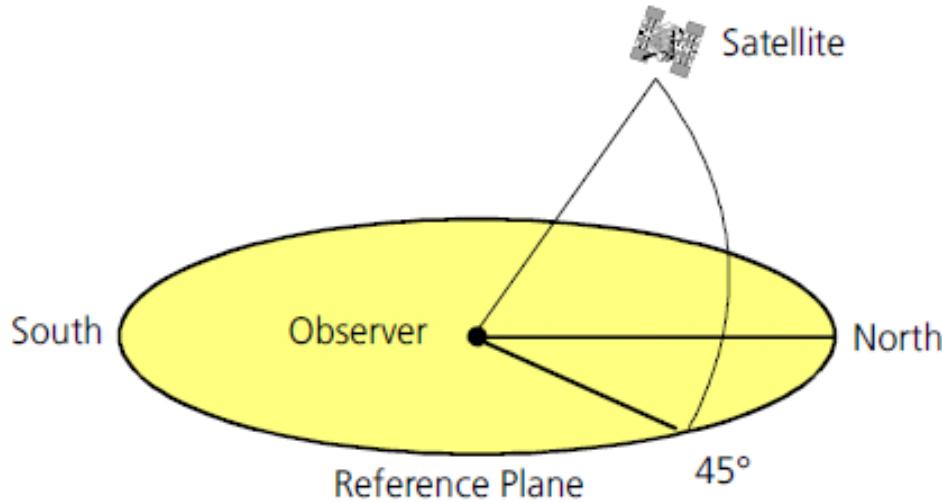
2014-07-19T08:20:56Z (UTC)

GPS-BeiDou System Time Difference

-based on observation data (7/19)-



Elevation, Azimuth



- The Elevation describes the angle of a satellite relative to the horizontal plane.
- The Azimuth is the angle between the satellite and true North.

Quiz

- Calculate the distance in millimeter between the following two surveyed positions.

#1 35.6662474, 139.7923025

#2 35.6662474, 139.7923026 seventh decimal place

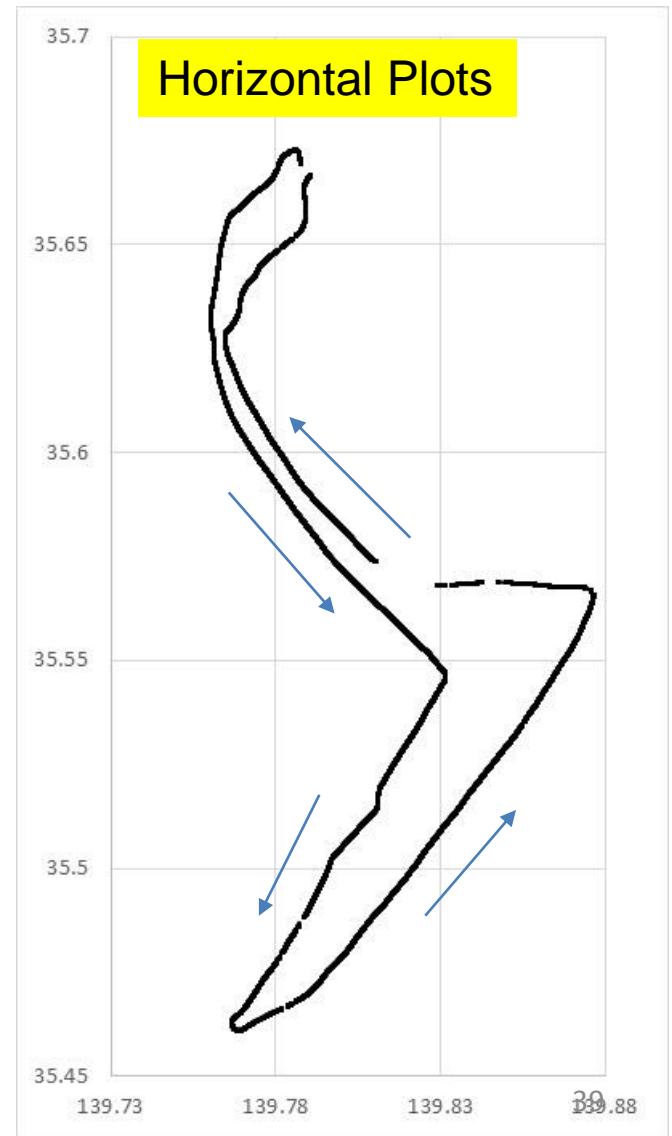
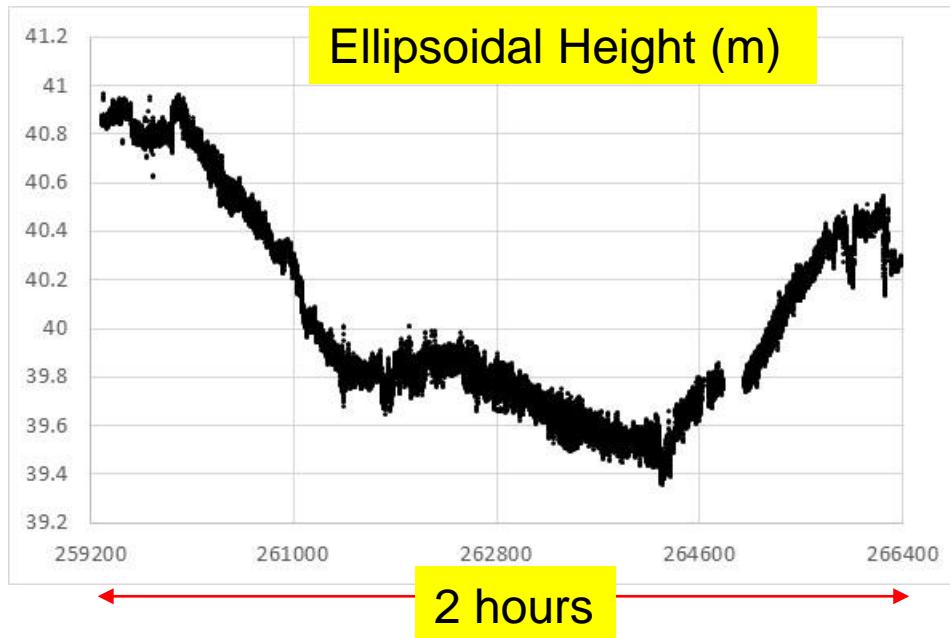
#1 35.6662474, 139.7923025

#2 35.6662475, 139.7923025

Problem 1

Two figures are RTK results of small ship (Yayoi).
You can see the height variation.
Please tell me why there was the variation
in altitude direction.

2013/12/4 9:00-11:00



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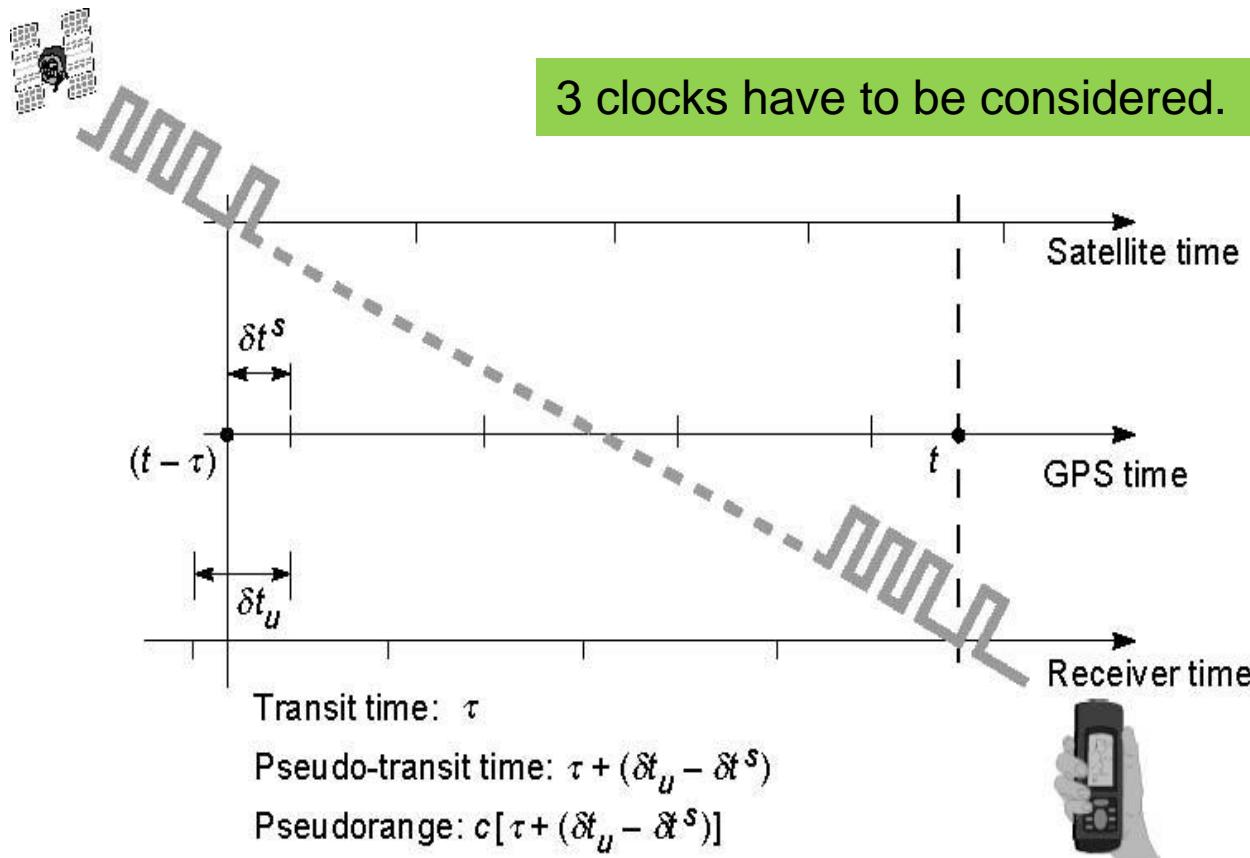
What are measurements and errors ?

- Measurement Models
- Control Segment Errors
- Signal Propagation Modeling Errors
- Measurement Errors
- User Range Error
- Empirical Data
- Combining Code and Carrier Measurement

Why we learn measurements and errors ?

- Needless to say, “position, velocity and time” are important for users.
- The ability to improve final performance of the above outputs **strongly depends on how can we estimate or possibly mitigate measurements errors.**
- Measurements errors strongly depends on the **environment and receiver performance.**

Code Phase Measurement

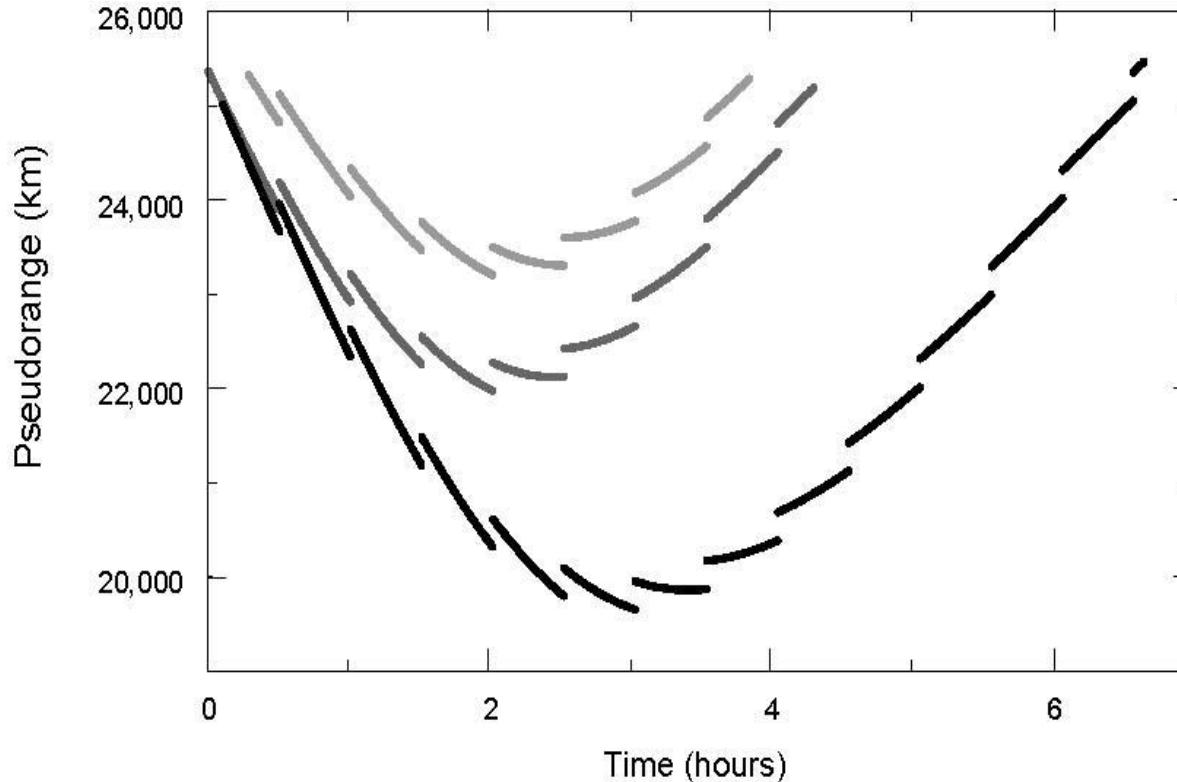


3 clocks are not synchronized.

Satellite clock error can be corrected using navigation message.

User clock error can be estimated as an unknown parameter in the positioning.

Real Pseudo-range Measurements



The variations of pseudo-range are mainly due to the satellite motion and earth rotation. Several gaps in all satellites are due to receiver clock offset. Receiver usually offset their own clock because the receiver clock error continues to increase.

Carrier Phase Measurement

$$\phi(t) = \phi_u(t) - \phi^s(t - \tau) + N$$

$$\phi(t) = f \cdot \tau + N$$

$$= \frac{r(t, t - \tau)}{\lambda} + N$$

$\phi_u(t)$ carrier phase in the receiver

$\phi^s(t - \tau)$ carrier phase in the satellite

τ transit time

N integer ambiguity

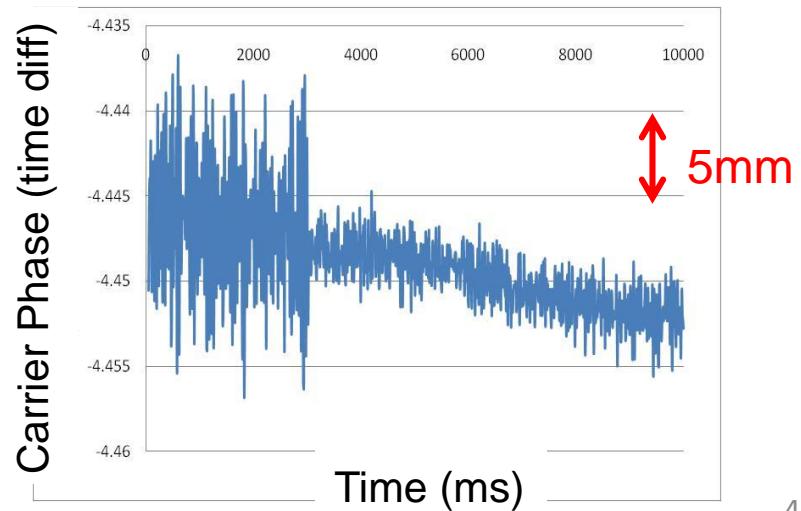
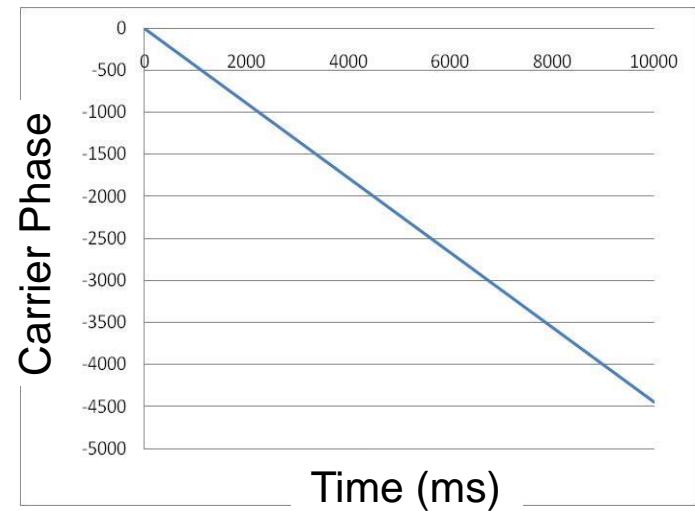
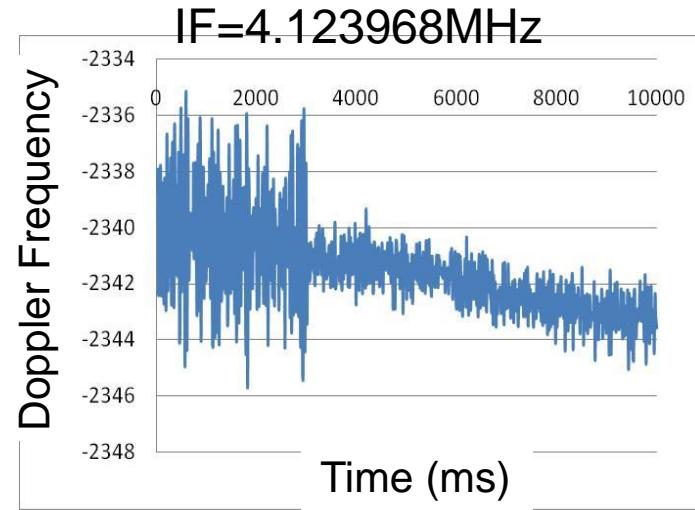
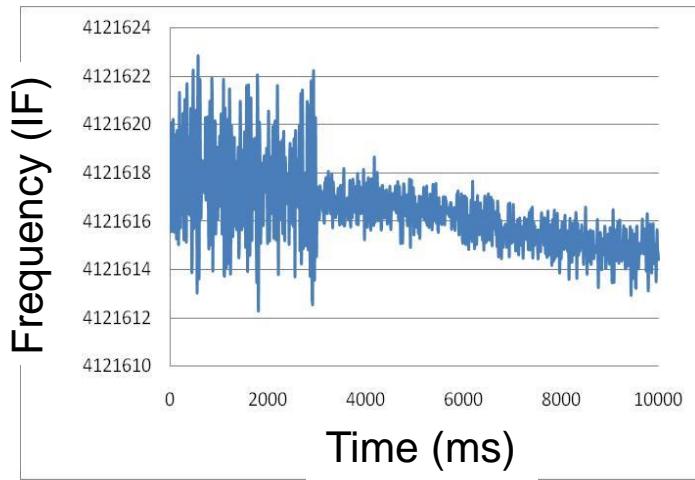
f, λ Doppler frequency and wavelength

r(t, t - τ) geometrical range

Clock error and measurements errors are assumed zero.

Carrier phase measurement is accumulated Doppler frequency. Be careful about "f". In the receiver, carrier frequency is basically converted to "IF".

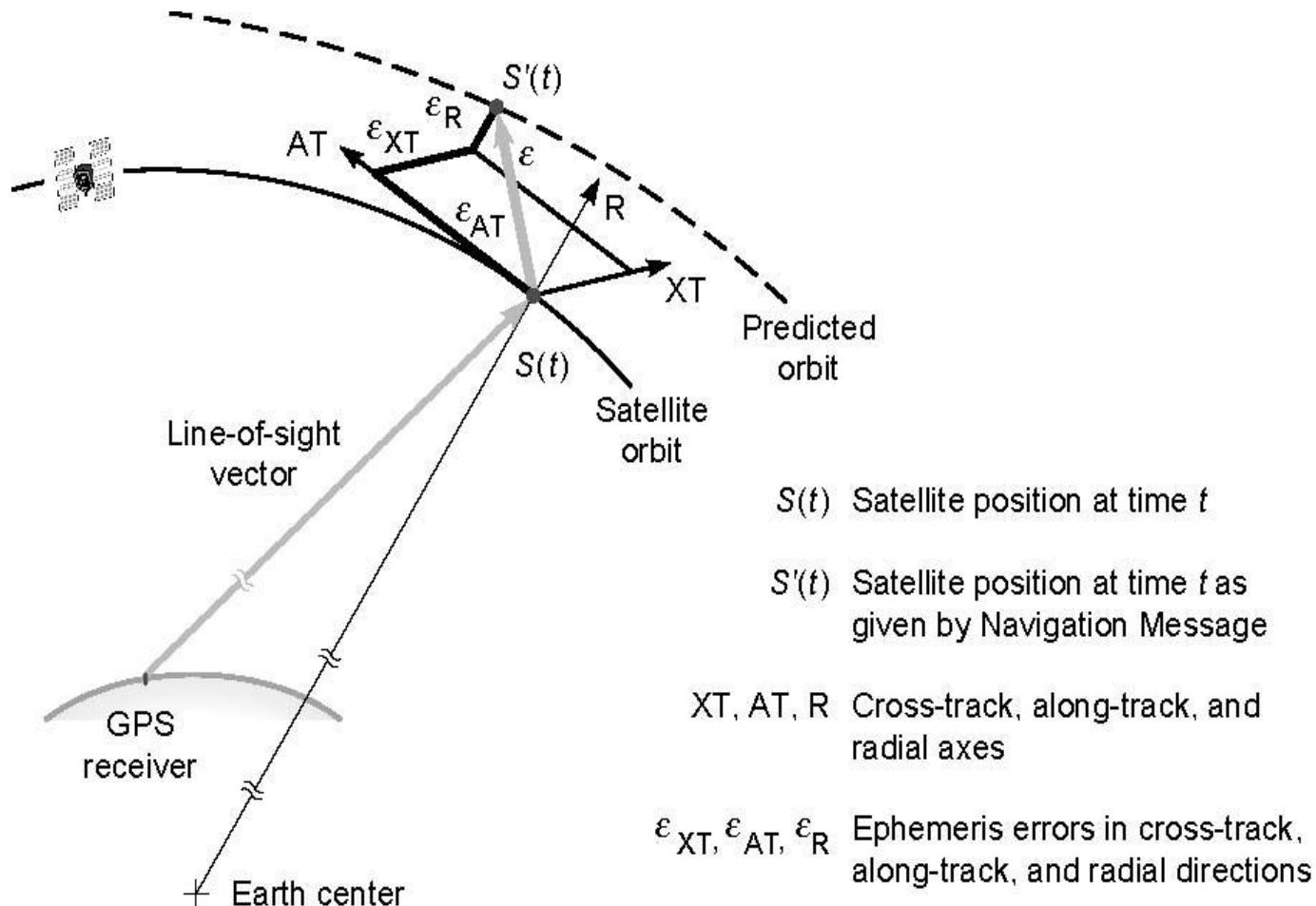
Real Carrier Phase Measurements



Noise and Bias

- Measurement errors are often categorized as noise and bias.
- #1 Errors in the parameter values broadcast by a satellite in its navigation message for which the **Control Segment** is responsible
- #2 Uncertainties associated with the **propagation medium** which affect the travel time of the signal from a satellite to the receiver
- #3 **Receiver noise** which affects the precision of a measurement, and **interference** from signals reflected from surfaces in the vicinity of the antenna

Control Segment Errors

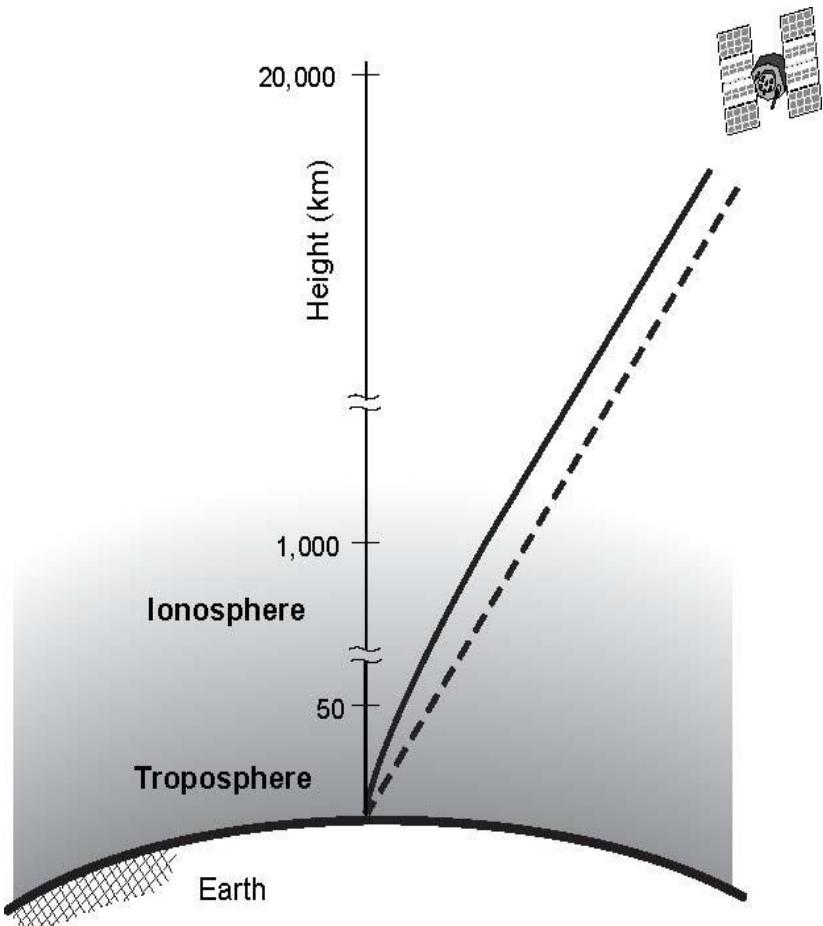


Satellite clock and ephemeris errors

Ephemeris/ Clock	Accuracy (RMS)	Real-time	Update	Sample
Navigation	1m/5ns	○	2hour	
Ultra-Rapid (predicted half)	0.05m/3ns	○	4/day	15 min
Ultra-Rapid (observed half)	0.03m/150ps	3-9 hours	4/day	15 min
Rapid	0.025m/75ps	17-41 hours	1/day	15/5 min
Final	0.025m/75ps	12-18 days	1/week	15/5 min

IGS site (2009)

Signal refraction, Wave propagation, and Dispersive media

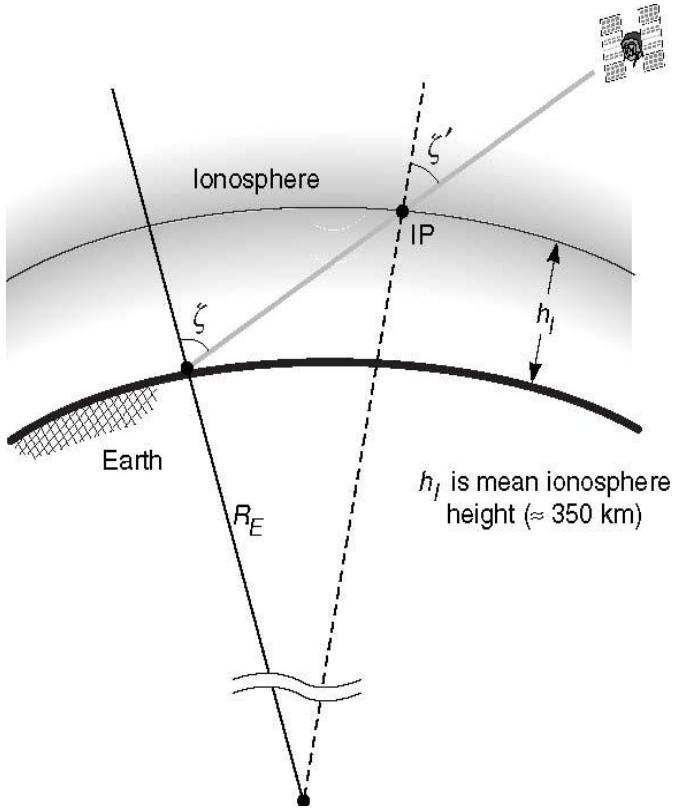


Refraction of GPS signals in the earth's atmosphere results in changes to both speed and direction.

Increase in path length due to bending of the signal ray is generally insignificant.

The effect of the change in speed of propagation, however, can result in pseudo-range measurement error of several meters or more.

Ionospheric delay

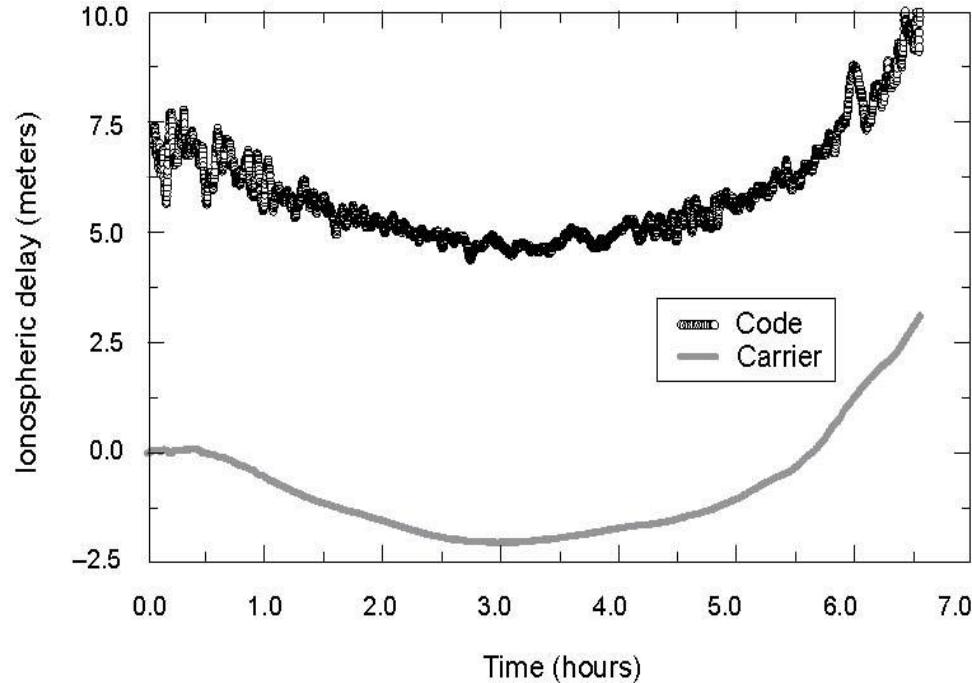


The ionosphere is a region of ionized gases. The state of the ionosphere is determined primarily by the intensity of the solar activity.

The speed of propagation of radio signals in the ionosphere depends on the number of free electrons in the path of a signal, defined as the total electron content (TEC): the number of electrons in a tube of 1 m^2 cross section extending from the receiver to the satellite.

The increased path length is accounted for in terms of a multiplier of the zenith delay. The multiplier is called **Obliquity Factor**.

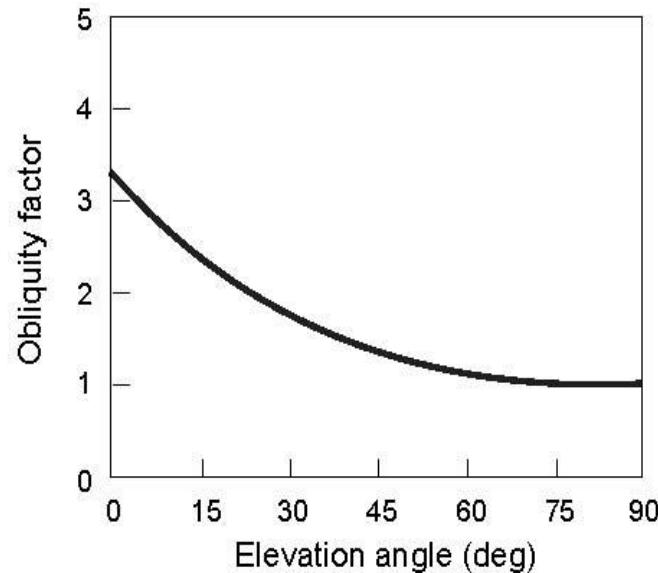
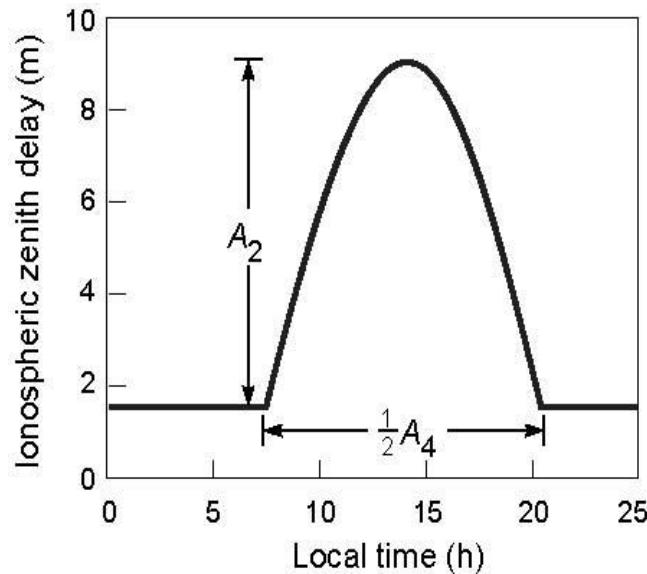
Ionospheric delay estimation



Ionospheric delay (L1) estimates obtained from code and carrier phase measurements at both L1 and L2. The Code-based estimates are noisy, the carrier-based estimates are precise and ambiguous.

Be careful the satellite side bias in the code measurements when you use these estimates for standalone positioning.

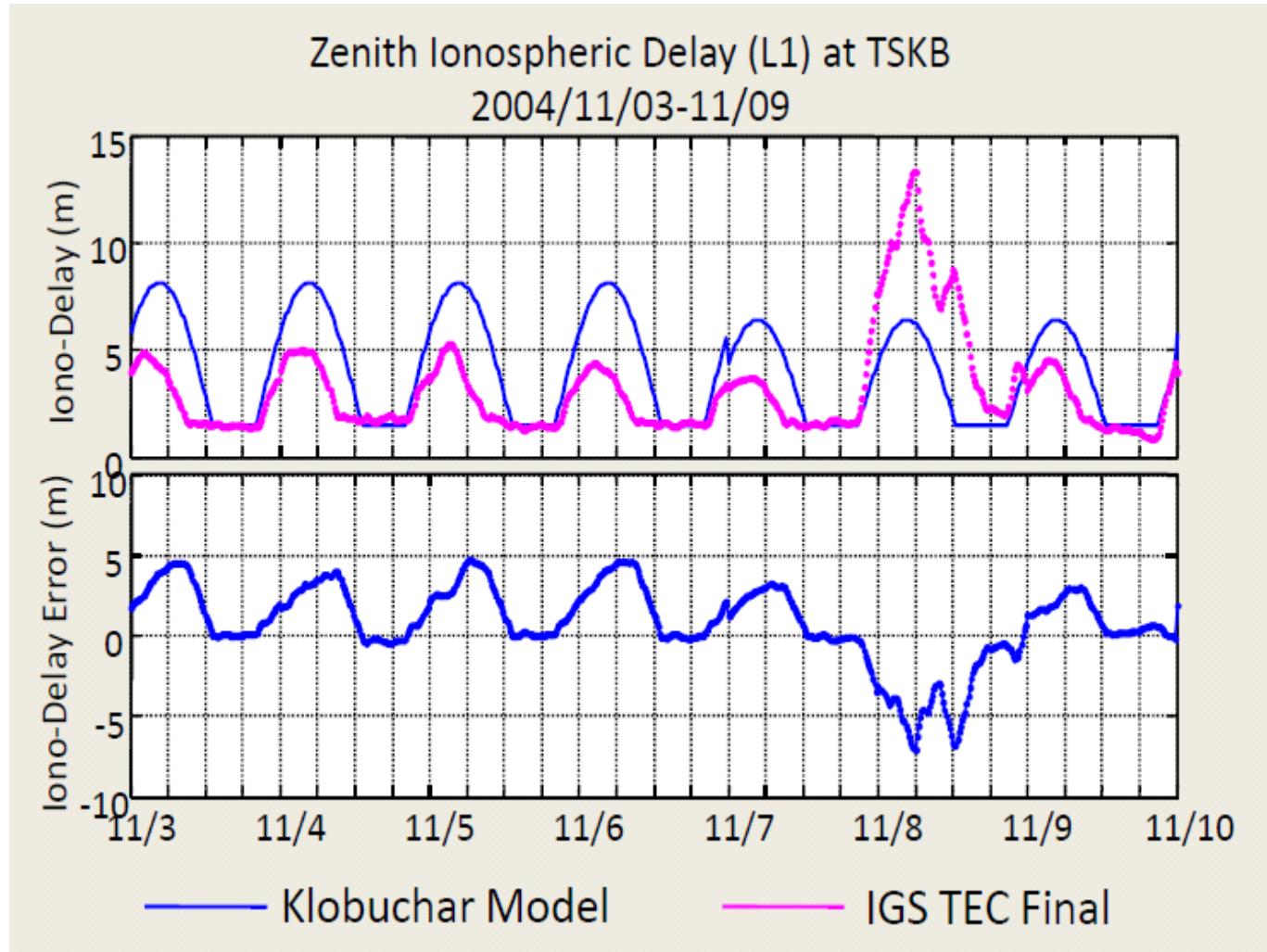
Broadcast Model



The Klobuchar ionospheric model. Parameter values A_2 and A_4 are selected by the Control Segment to reflect the prevailing ionospheric conditions and are broadcast by the satellites.

For Galileo, **NeQuick model** will be used to estimate ionospheric errors.

Accuracy evaluation of Klobuchar model based estimates

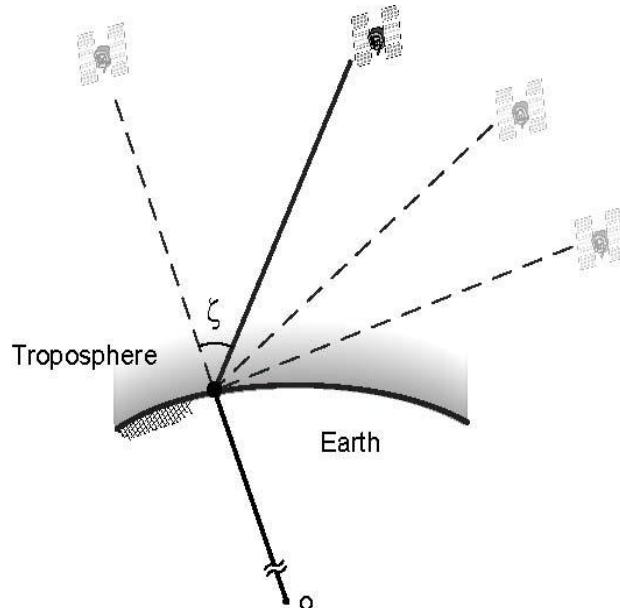


Tropospheric delay

- The GPS signals are also reflected by the lower part of the earth's atmosphere composed of **gases and water vapor.**
- The speed of propagation of GPS signals in the troposphere is lower than that in free space and, therefore, the apparent range to a satellite appears longer, typically by **2.5-25 m** depending on the satellite elevation angle.
- Water vapor density carries with the local weather and can change quickly. Fortunately, **most of the tropospheric delay is due to the more predictable dry atmosphere.**

Tropospheric models

- **Saastamoinen model** was derived using gas laws and simplifying assumptions regarding changes in temperature and water vapor with altitude.
- **Hopfield model** is based on a relationship between dry refractivity at height h to that at the surface. It was derived empirically on the basis of extensive measurements.



Obliquity factor is defined same as ionosphere, but the value is different because the height is different.

30 degrees : 2
15 degrees : 4
10 degrees : 6
5 degrees : 10

Measurement Errors

(**Receiver Noise** and Multipath)

- The code and carrier measurements are affected by random measurement noise, called **receiver noise**. It depends on the signal strength.

code : deci-meter level

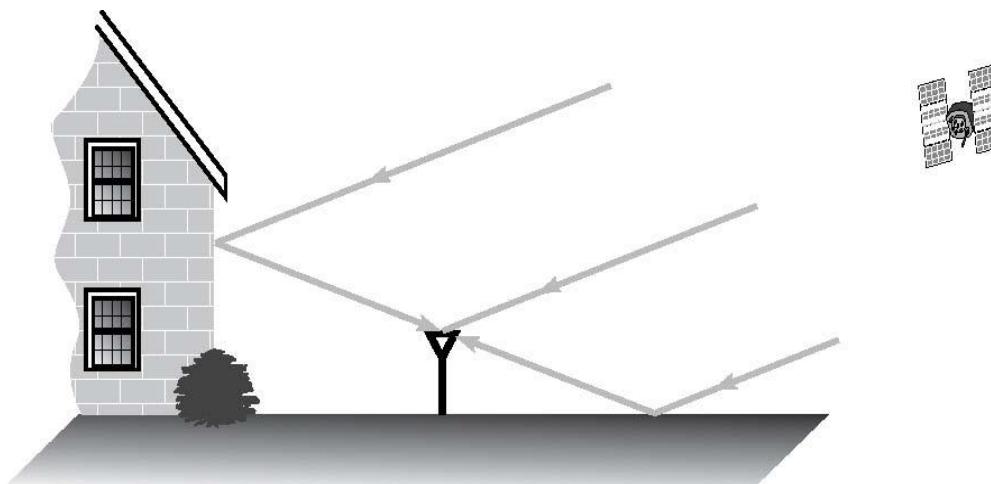
carrier : mm level

Figure ?

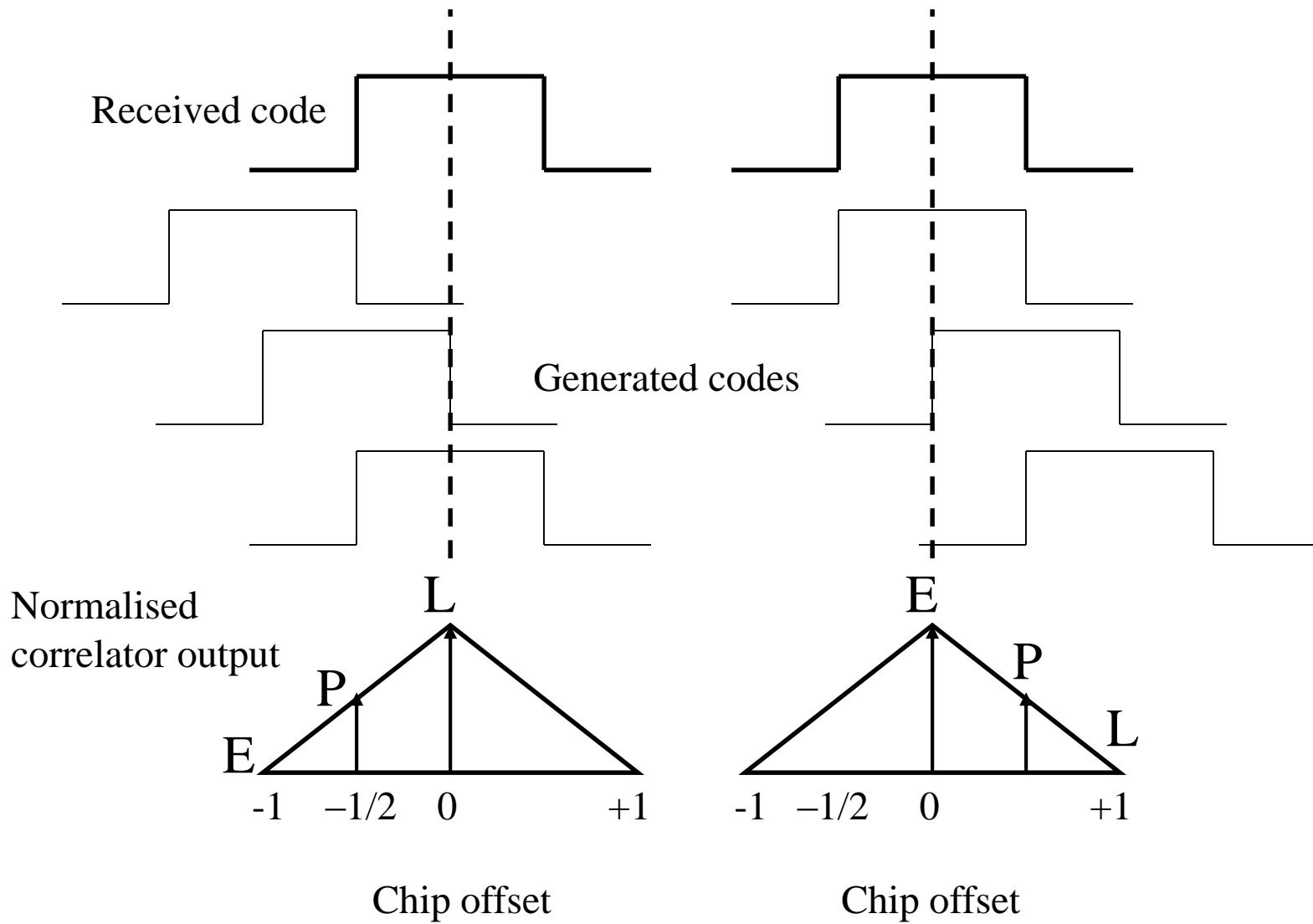
Measurement Errors

(Receiver Noise and **Multipath**)

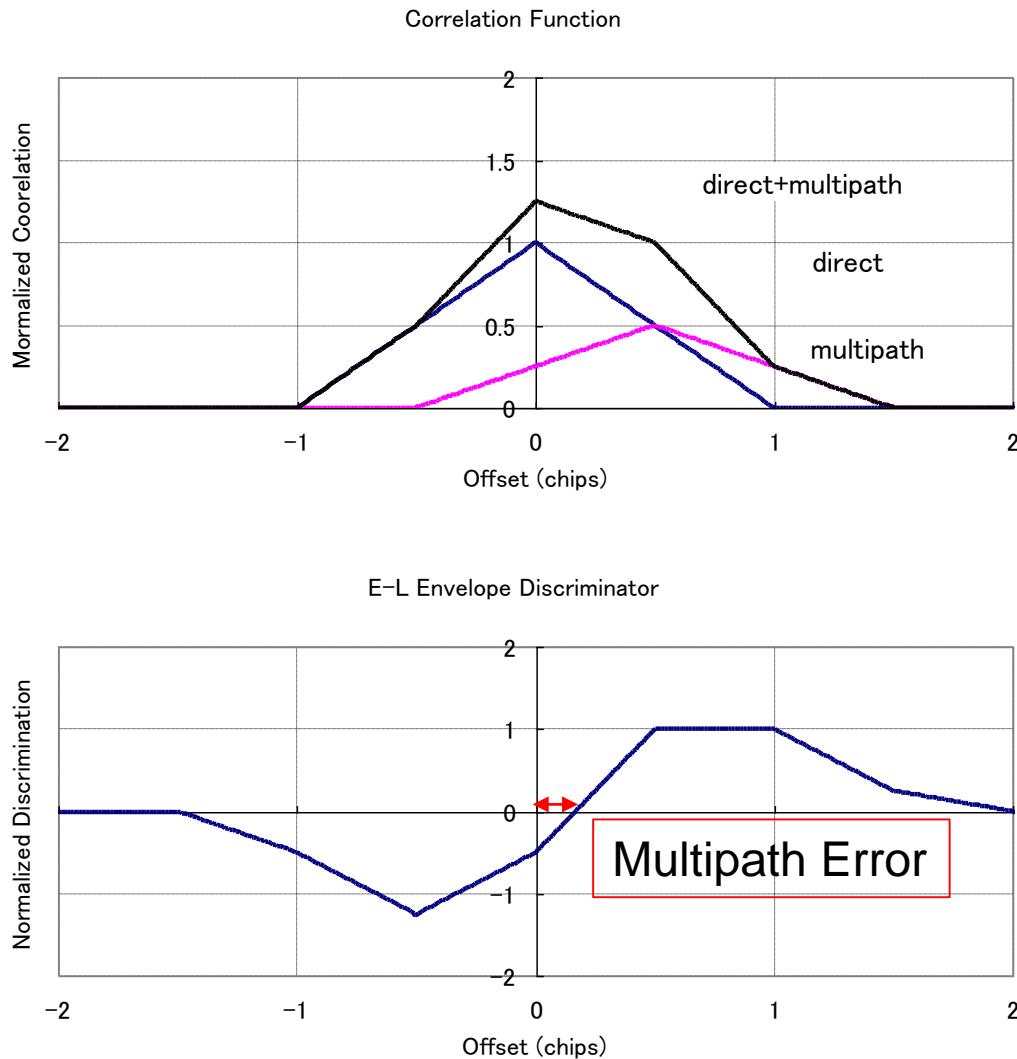
- Multipath refers to the phenomenon of a signal reaching an antenna via two or more paths.
- The range measurement error due to multipath depends on the **strength** of the reflected signal and the **delay** between direct and reflected signals.
- Mitigation of multipath errors : **Antenna or Receiver**



What is Correlation Function ?



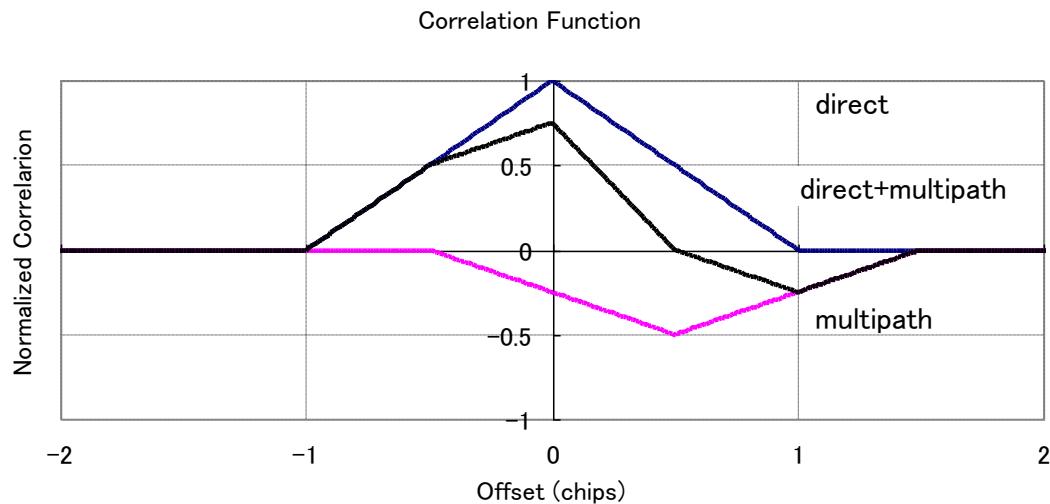
In-phase Multipath Example



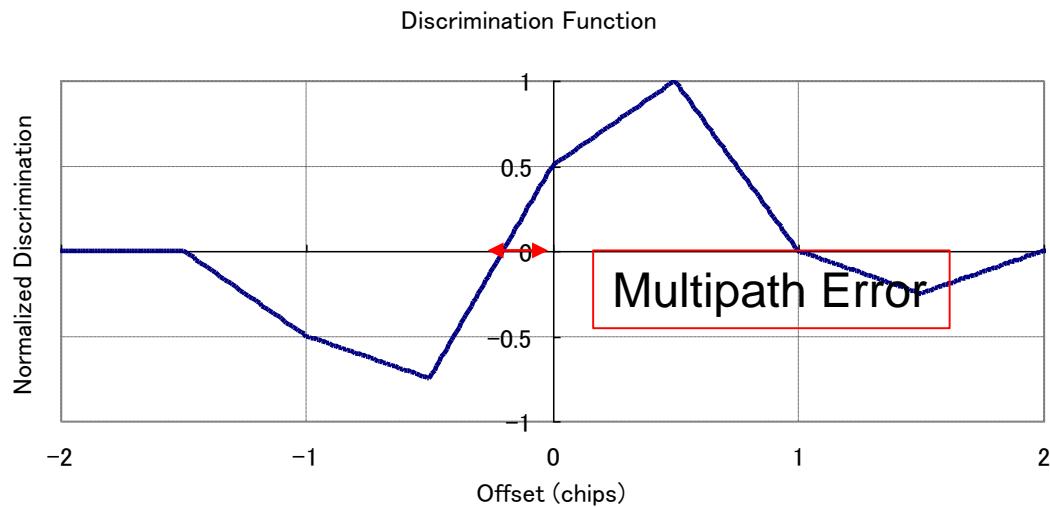
Standard 1chip correlator
D/M ratio = 0.5
Bandwidth = No limit
Multipath delay = 0.5 chip (150m)

Early-late discriminator
= 1.0 (0.5+0.5) chip

Out-of-phase Multipath Example

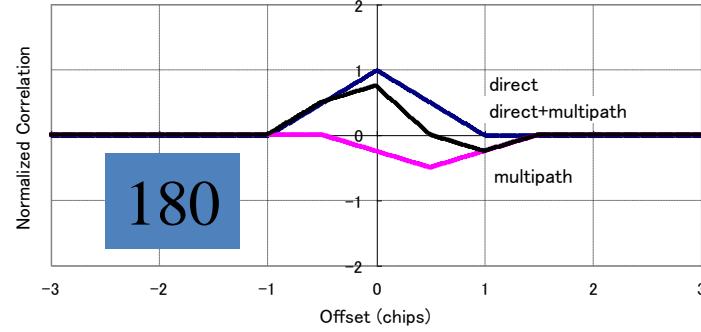
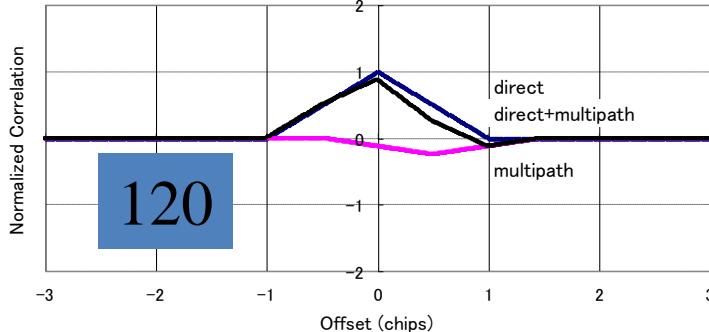
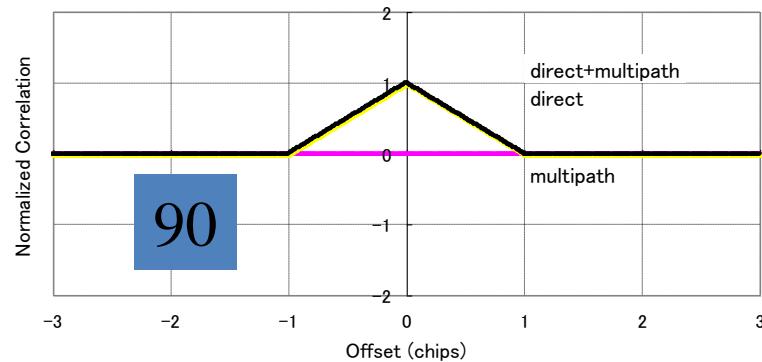
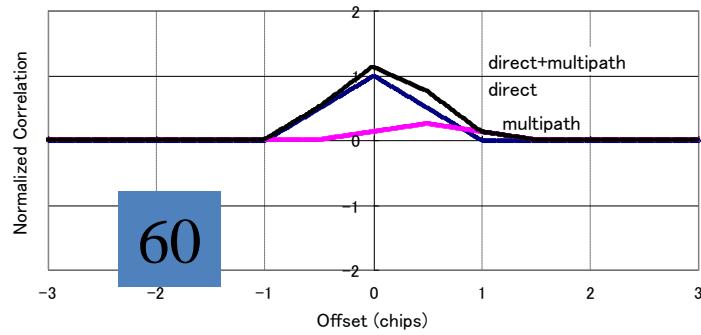
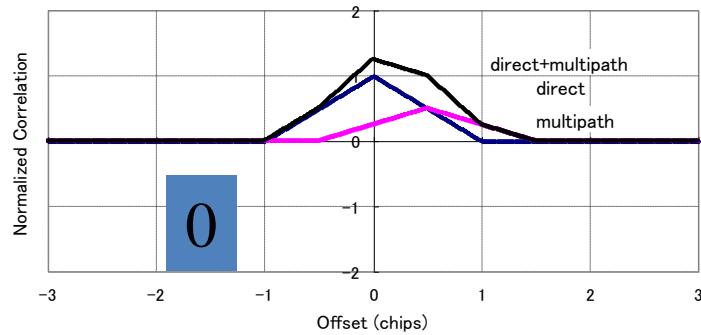


Standard 1chip correlator
D/M ratio = 0.5
Bandwidth = No limit
Multipath delay = 0.5 chip (150m)

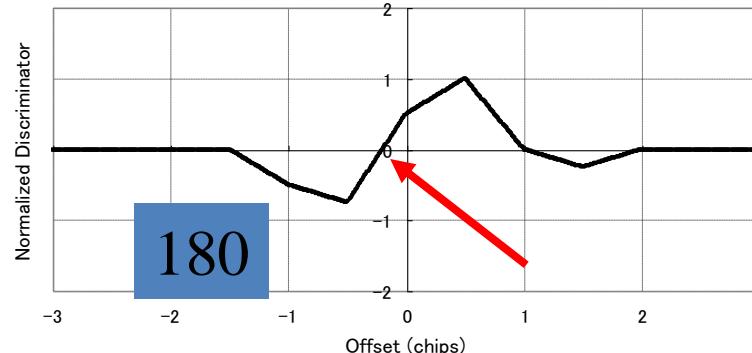
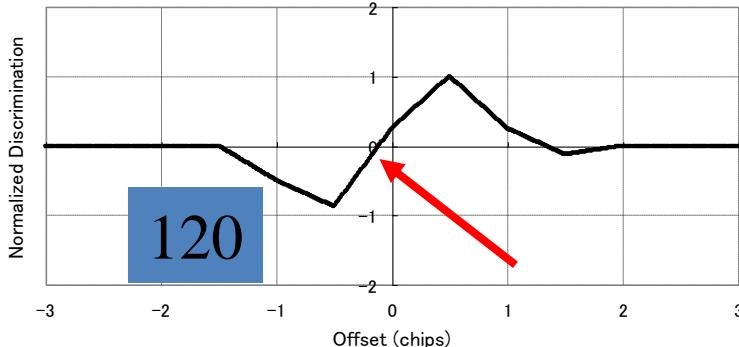
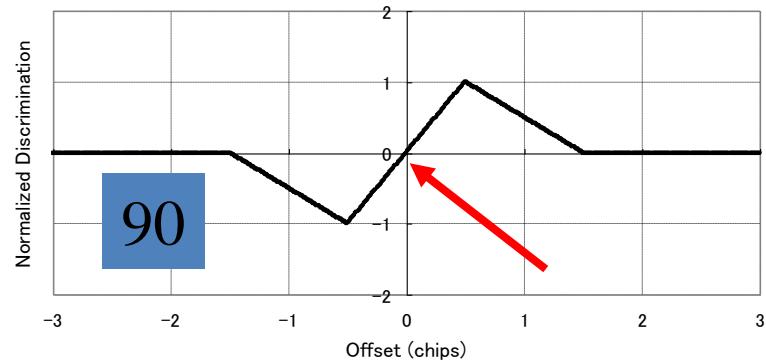
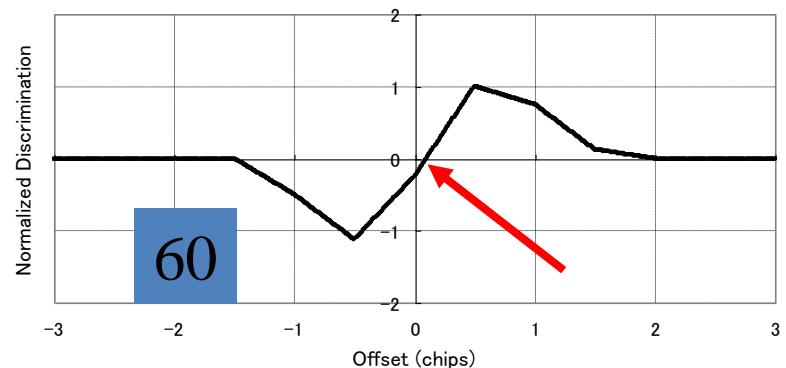
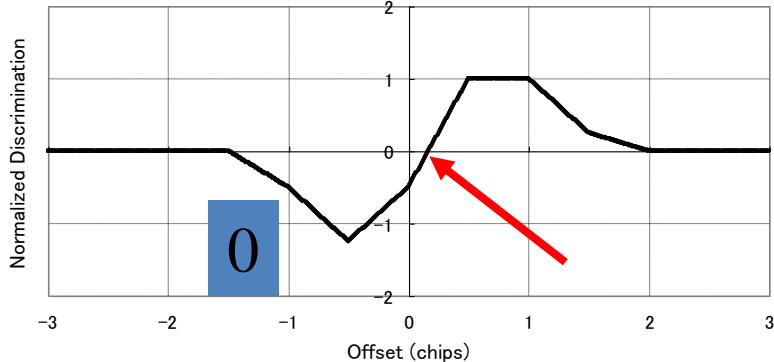


Early-late discriminator
= 1.0 (0.5+0.5) chip

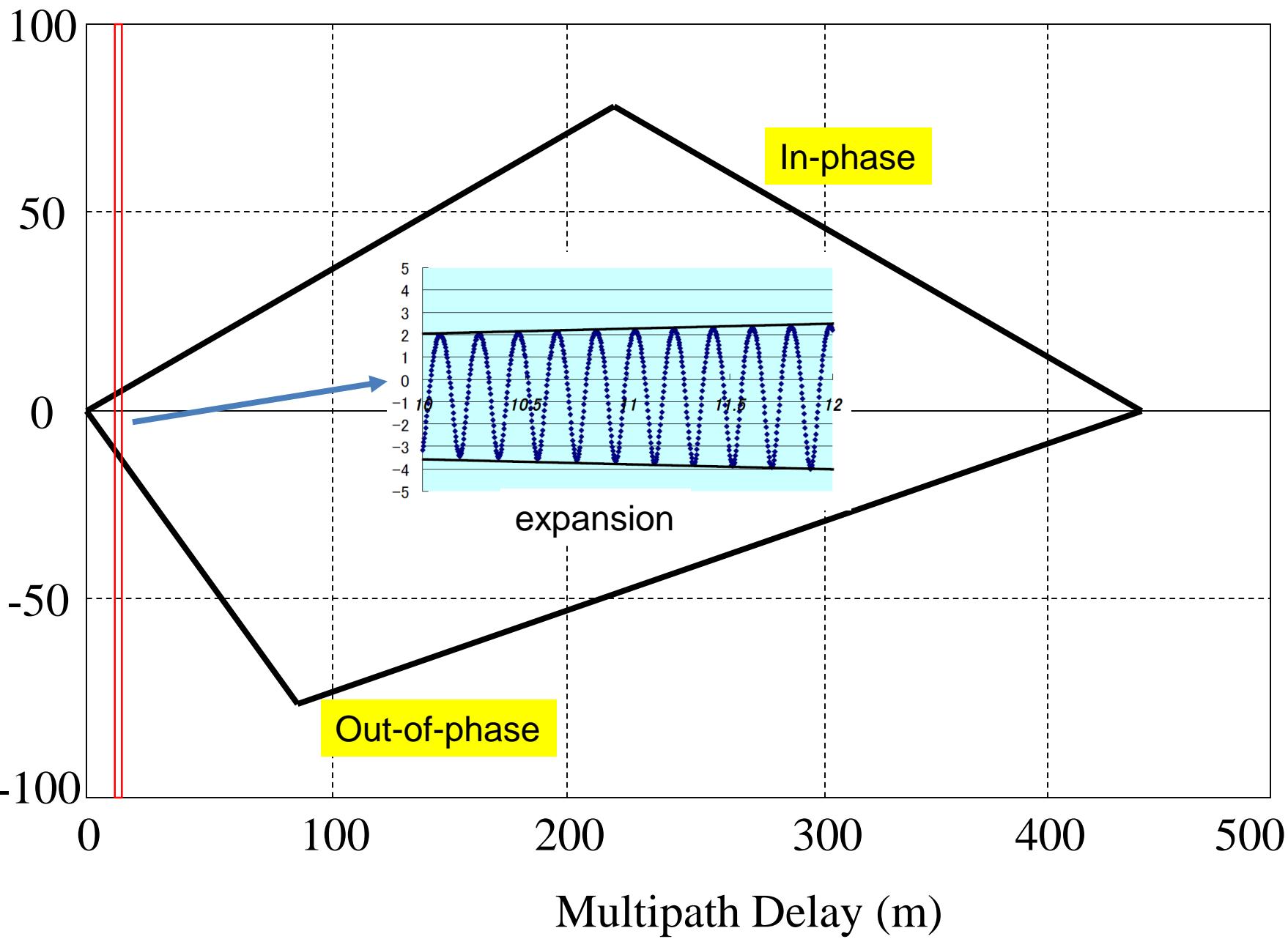
Phase of multipath to direct-path changes...



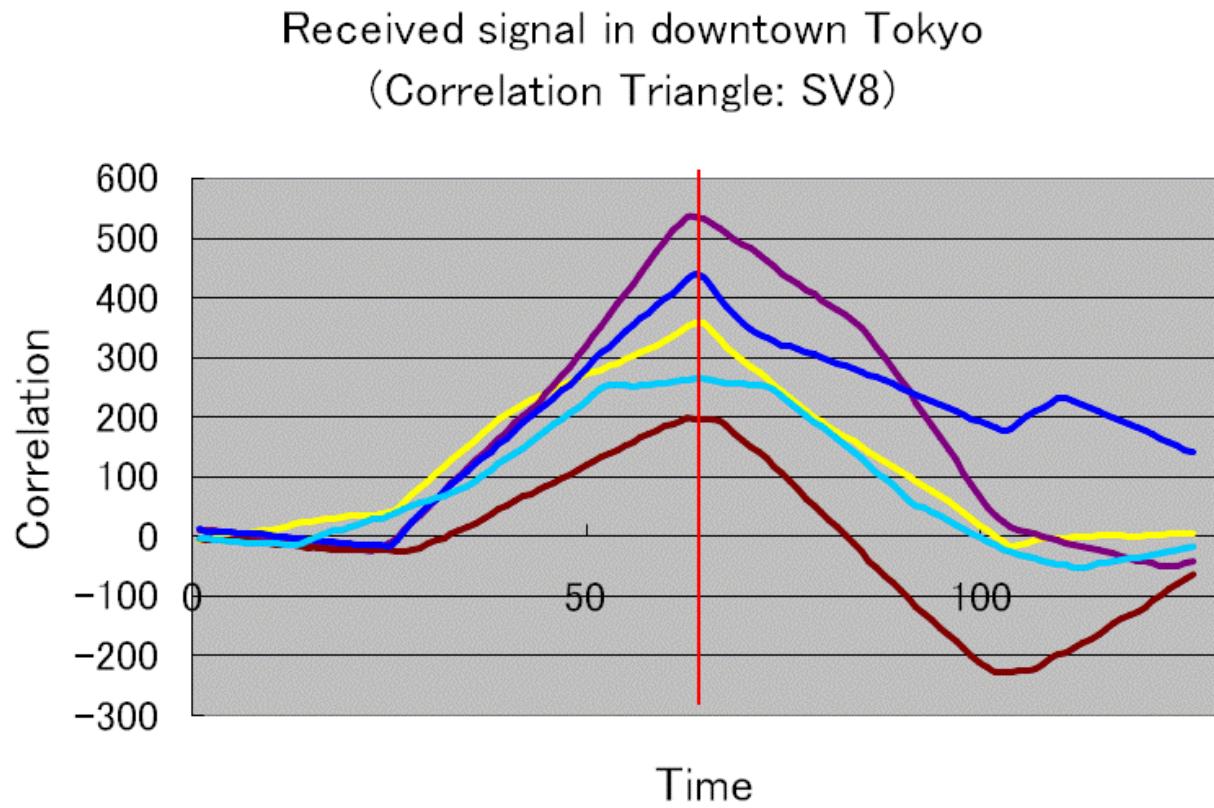
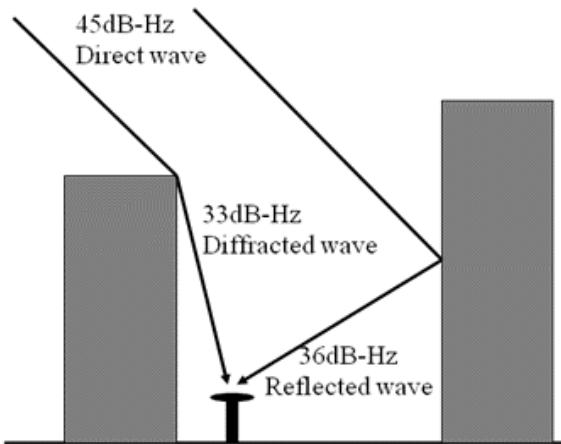
Output of early-late discriminator changes...



Multipath Error (m)



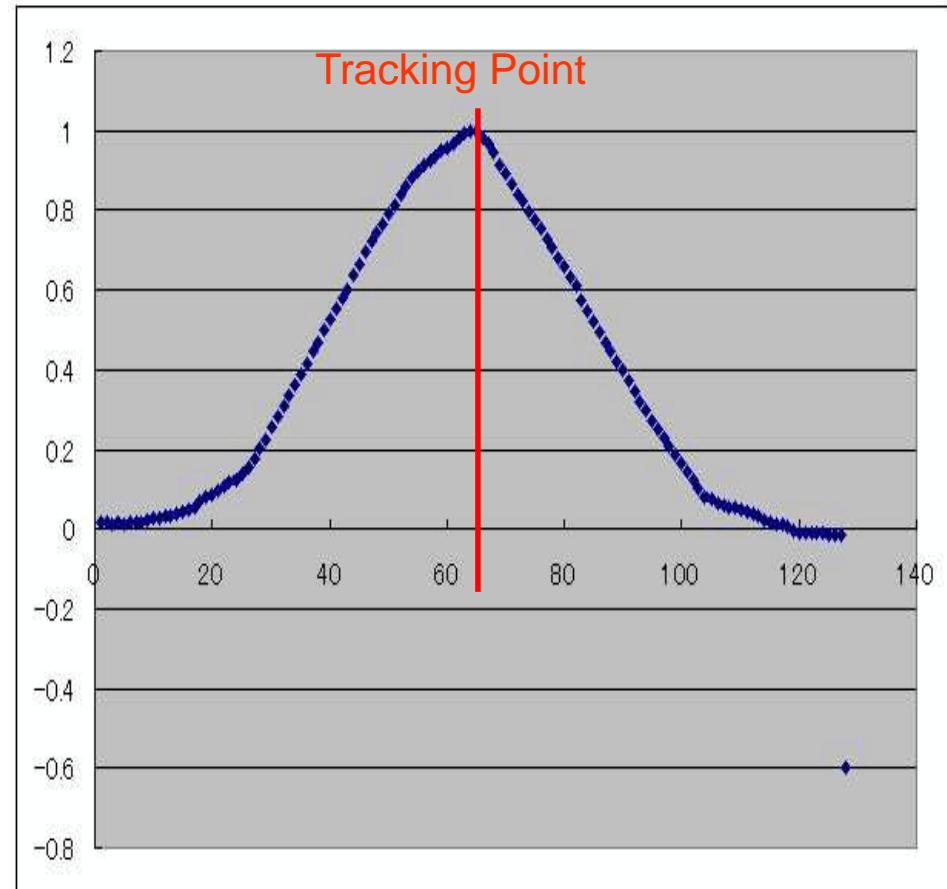
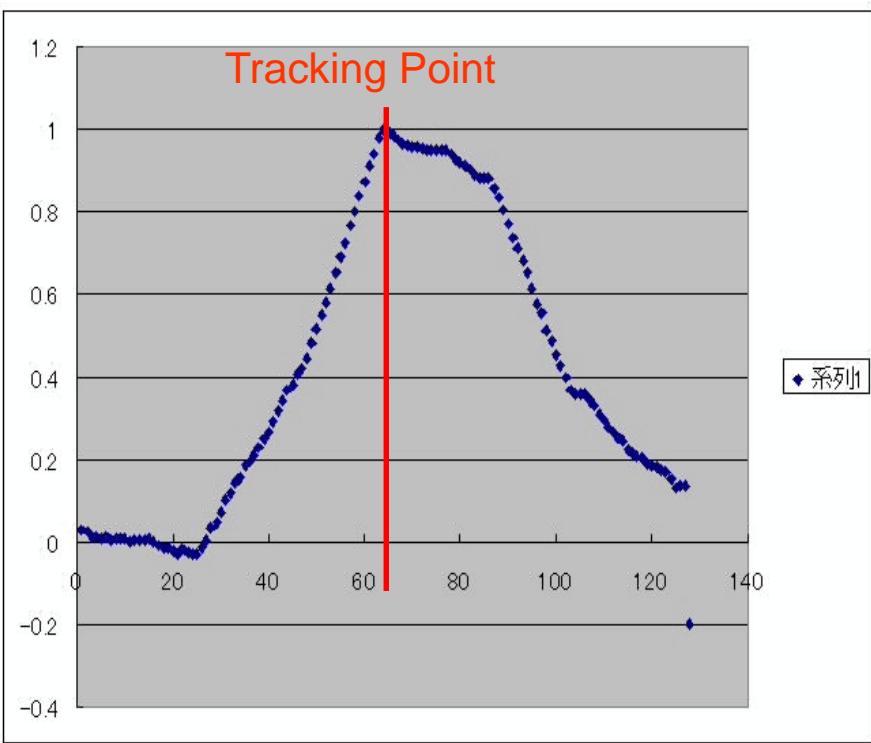
Typical Multipath Case in Urban Canyon



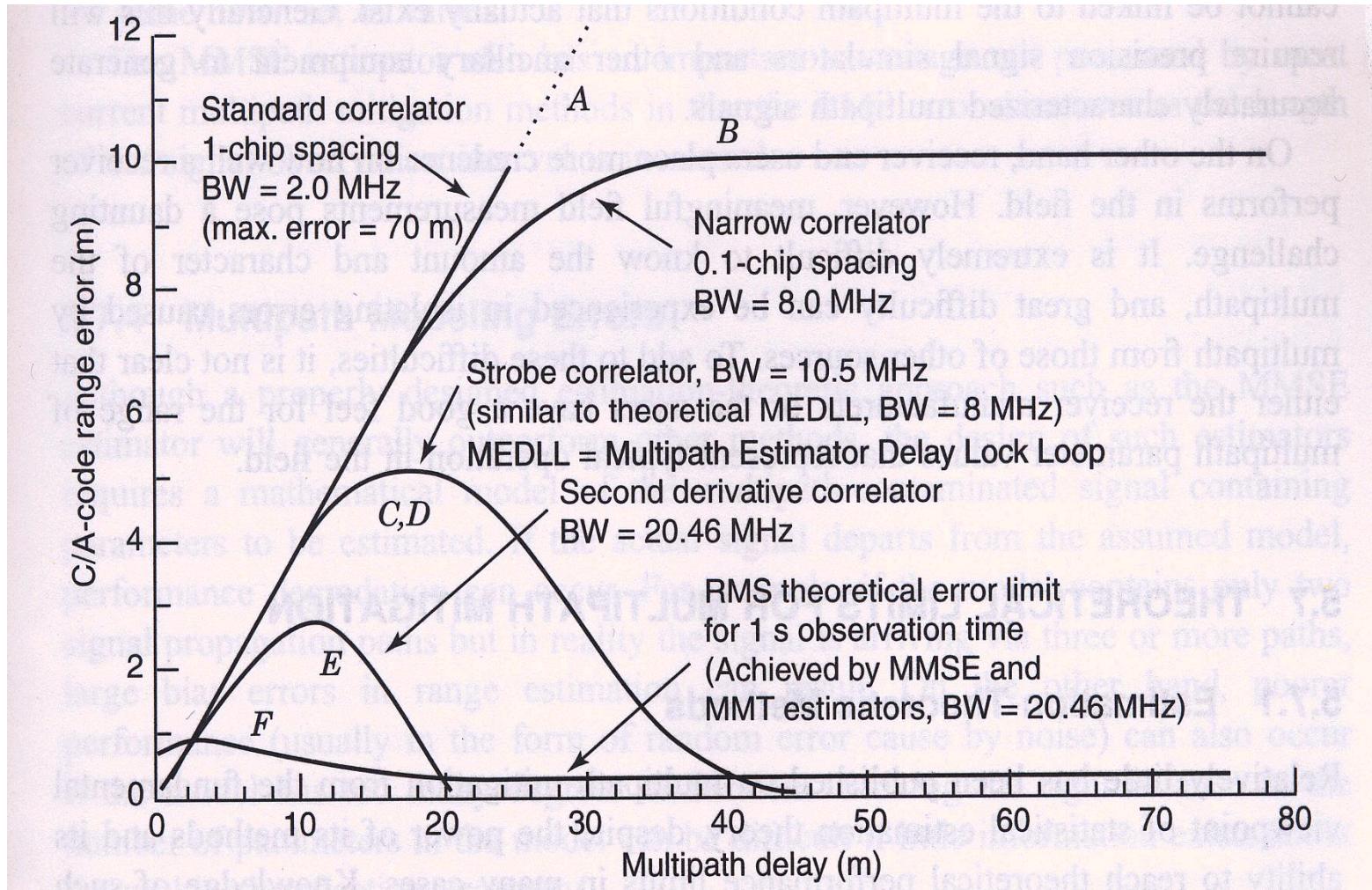
Sampling frequency = 40 MHz
Furuno SQM Receiver
Data : Over 10 years ago

Real Correlation Variation at Marunouchi

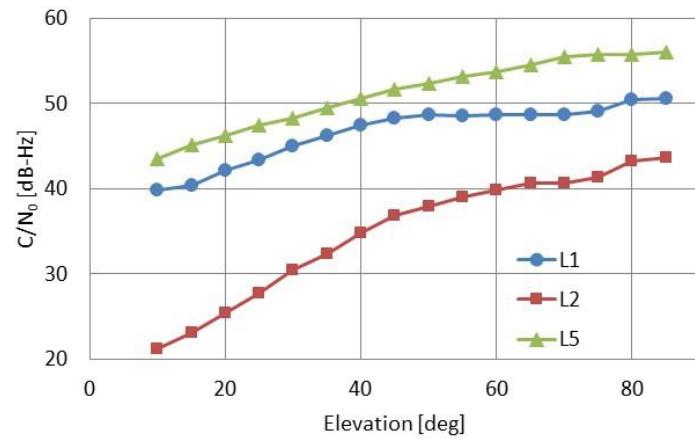
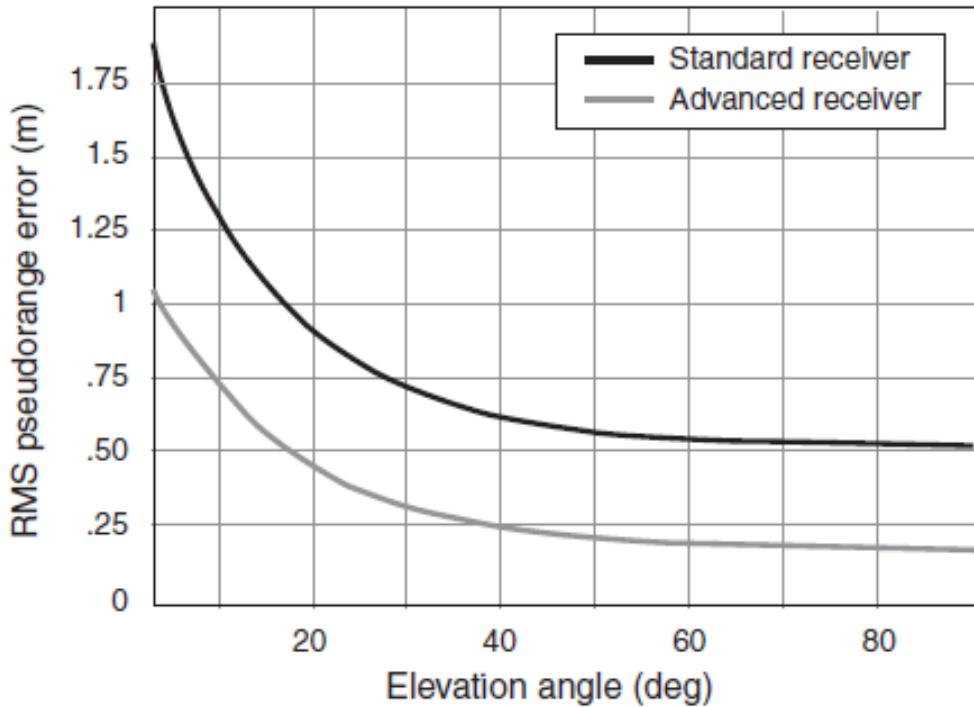
Severe multipath reception case (car stopped at traffic signal...)



Multipath Mitigation Technique (Receiver inside)



Measurement error for two types of receiver



C/N₀ and Elevation

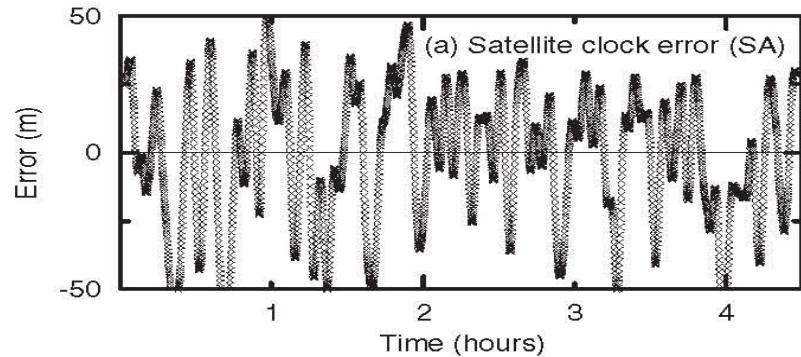
You can change Elevation angle to signal strength

Typical pseudo-range measurement errors for L1 receiver

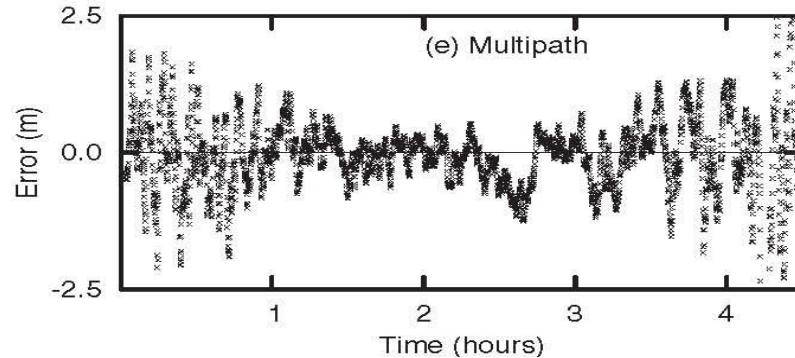
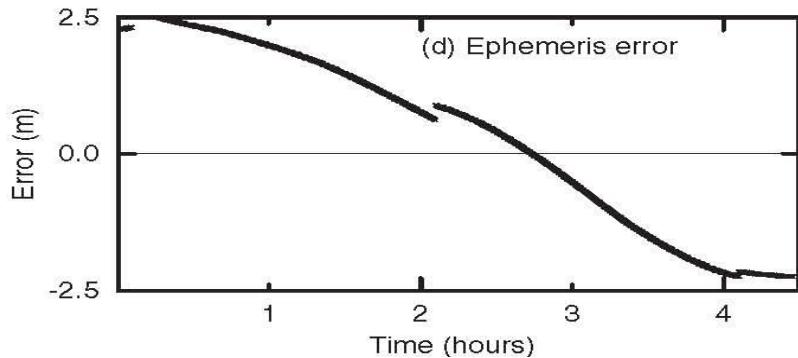
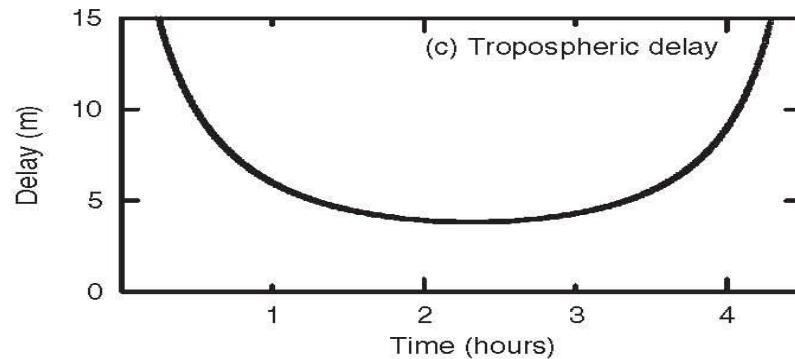
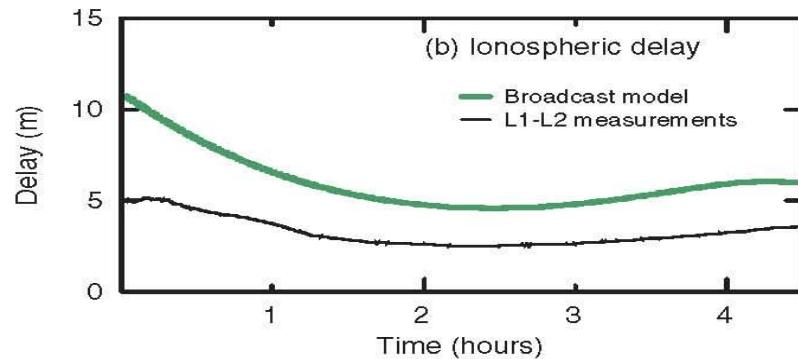
Error Source	RMS Range Error
Satellite clock and ephemeris parameters	3 m (SIS URE)
Atmospheric propagation modeling	5 m
Receiver noise and multipath	1 m
User range error (URE)	6 m

URE : User Range Error
SIS : Signal-in-Space

Measurement Error : Empirical Data



1997 : SA was activated.

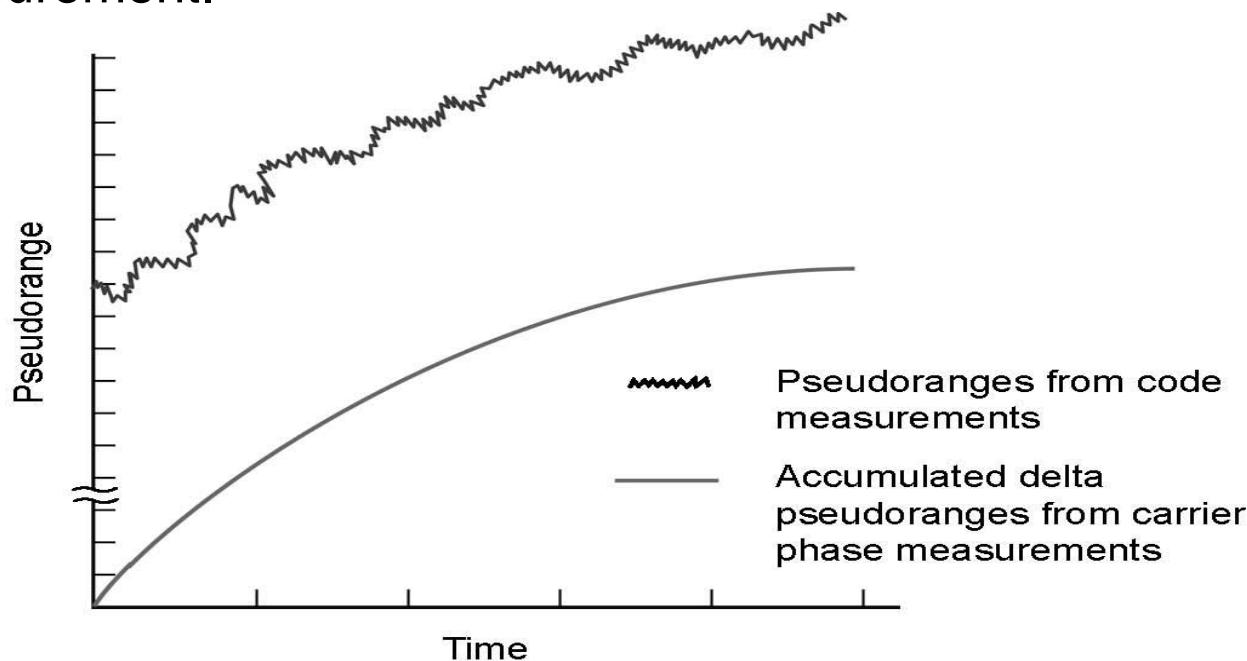


Why we discuss about measurement errors ?

- Back to bias and noise errors discussion, noise errors of pseudo-range can be mitigated to some degree using **carrier phase smoothing technique.**
- On the other hand, you have to estimate **bias errors** as accurate as possible **by yourself** to improve positioning performance.
- All kinds of improved techniques are essentially same in terms of estimating or eliminating bias or noise errors.

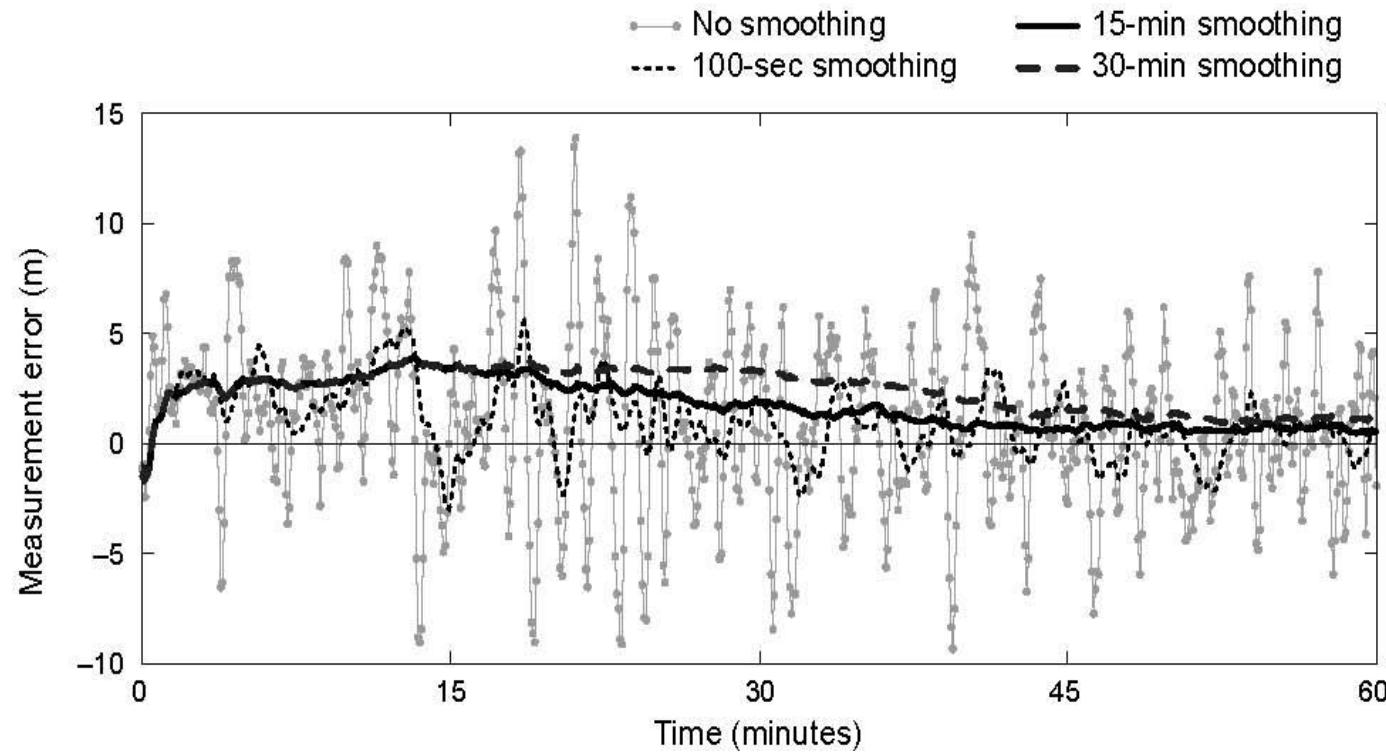
Combining Code and Carrier Measurements

Carrier phase measurement can be used to smooth pseudo-range Measurement.



The code based measurements are noisy. The carrier-based estimates are precise but ambiguous, and the plot starts arbitrarily at zero value.

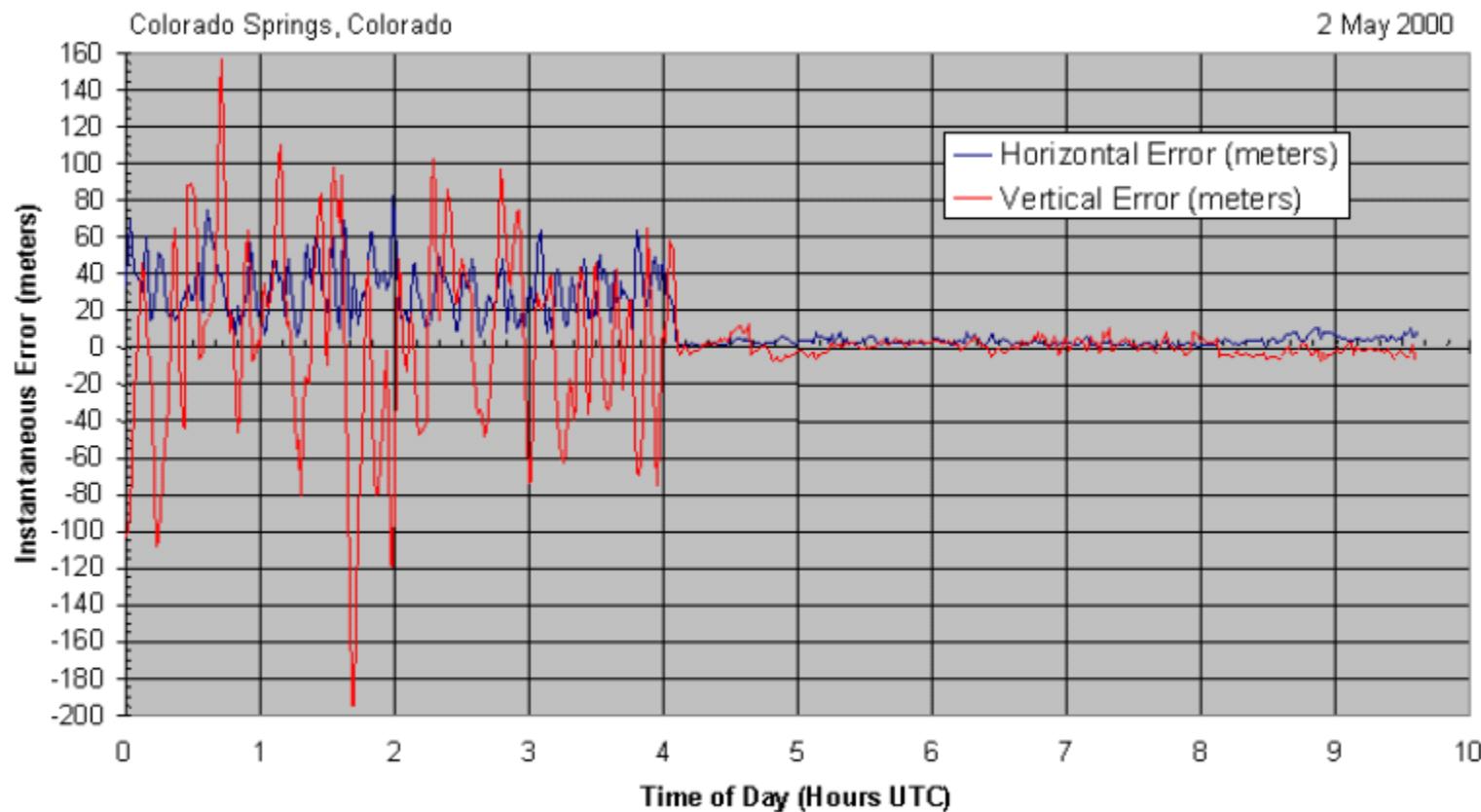
Carrier-smoothed pseudo-ranges with different filter lengths



$$\bar{\rho}(t_i) = \frac{1}{M} \rho(t_i) + \frac{(M-1)}{M} [\bar{\rho}(t_{i-1}) + (\Phi(t_i) - \Phi(t_{i-1}))],$$

$$\bar{\rho}(t_1) = \rho(t_1).$$

Deactivation the artificial distortion of the signal



On September 18, 2007, the US DoD reported that with the next generation of GPS satellites (GPS III), satellite navigation signals can no longer be artificially distorted

GPS Measurement Errors

Source	Potential error size	Error mitigation using single point positioning
Satellite clock model	2 m (rms)	→
Satellite ephemeris prediction	2 m (rms) along the LOS	→
Ionospheric delay	2-10 m (zenith) Obliquity factor 3 at 5°	1-5 m (single-freq.) within 1m (dual-freq.)
Tropospheric delay	2.3-2.5m (zenith) Obliquity factor 10 at 5°	0.1-1 m
Multipath (open sky)	Code : 0.5-1 m Carrier : 0.5-1 cm	→
Receiver Noise	Code : 0.25-0.5 m (rms) Carrier : 1-2 mm (rms)	→

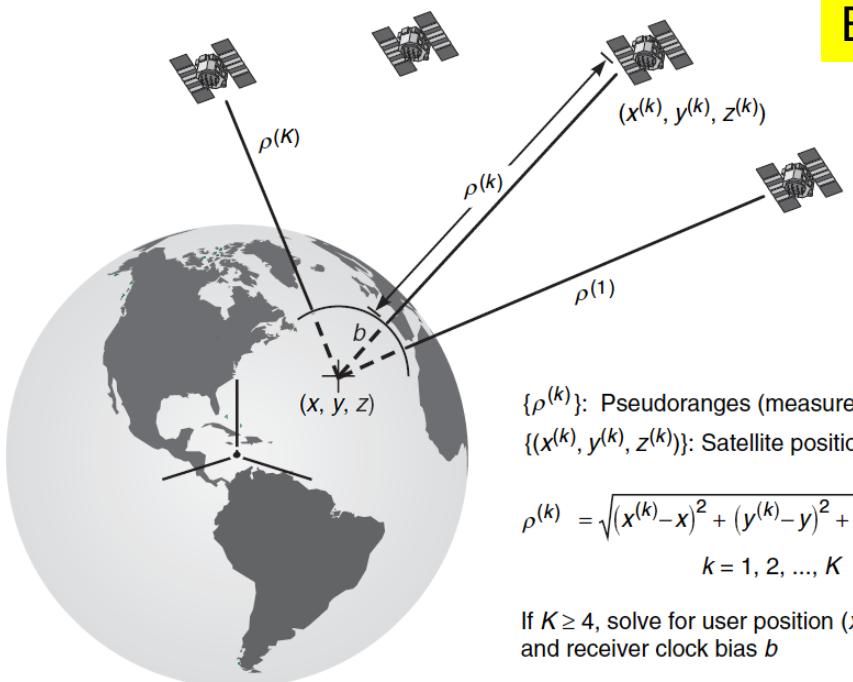
Contents

- Coordinates System
 - Satellite Position
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 - Future GNSS
- }
- 3rd period

Positioning Performance of GNSS

Positioning Performance =

Measurements Accuracy × DOP



Ephemeris errors should be considered...

Horizontal accuracy =
Measurements accuracy × HDOP

$\{\rho^{(k)}\}$: Pseudoranges (measurements)
 $\{(x^{(k)}, y^{(k)}, z^{(k)})\}$: Satellite positions (known)

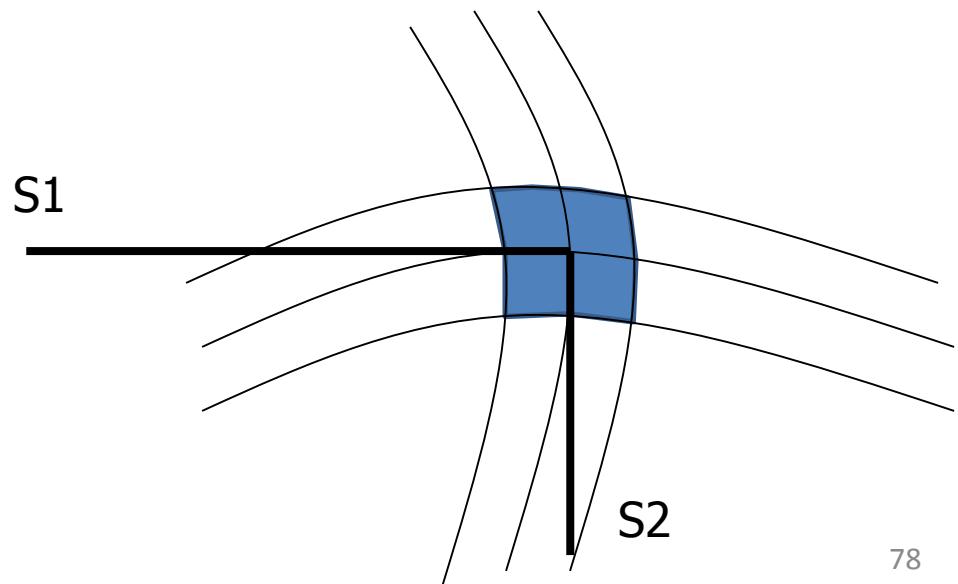
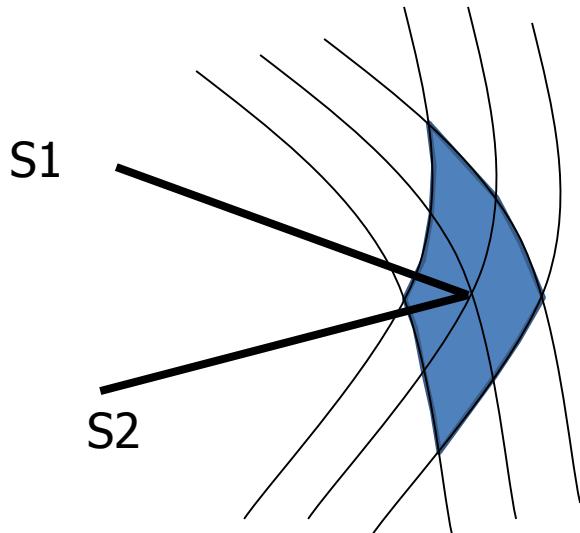
$$\rho^{(k)} = \sqrt{(x^{(k)} - x)^2 + (y^{(k)} - y)^2 + (z^{(k)} - z)^2} - b$$
$$k = 1, 2, \dots, K$$

If $K \geq 4$, solve for user position (x, y, z) ,
and receiver clock bias b

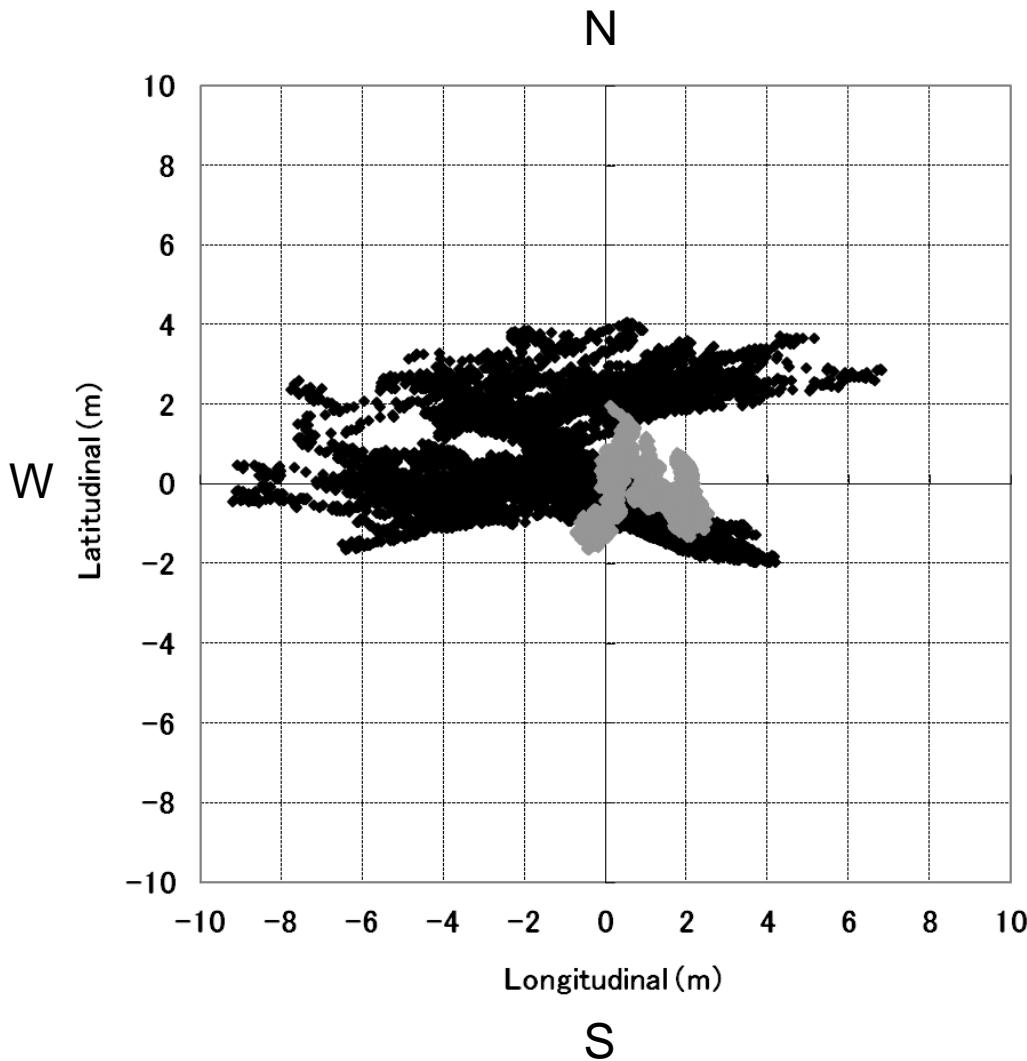
What is DOP ?

(dilution of precision : DOP)

- If the measurements errors are zero, the calculated user position is true.
- However, if the measurements include some errors, the accuracy depends on measurement errors as well as the geometry of satellites (=DOP).



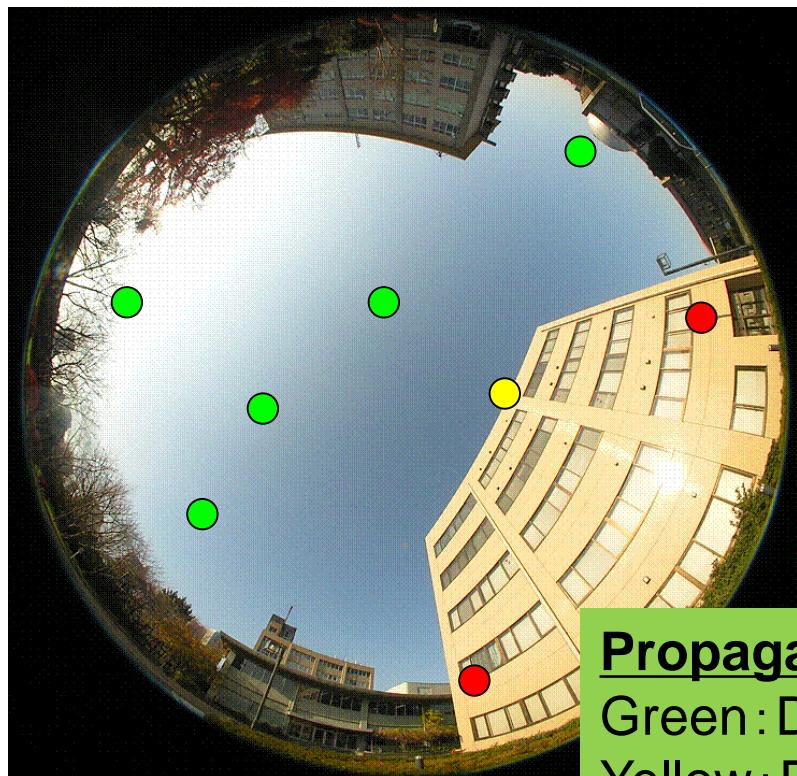
All Satellites VS. East Visible Satellites



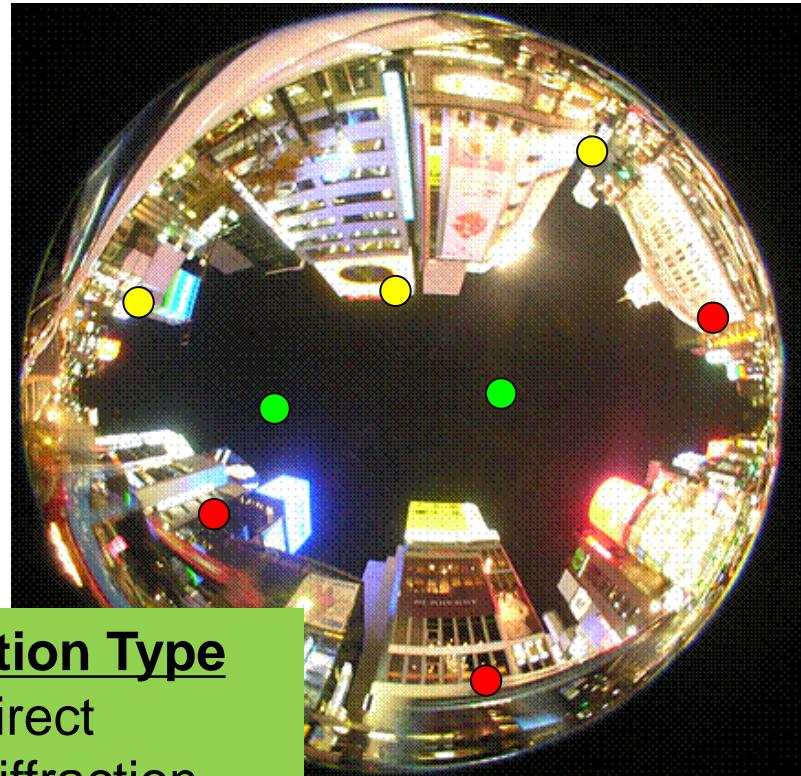
- Only east side satellites are used in the dark color plots.
(average=4.6)
- All satellites are used in the light color plots.
(average=8.7)

Sky Views in two different places

(same constellation but different performance)



Kaiyodai



Ginza

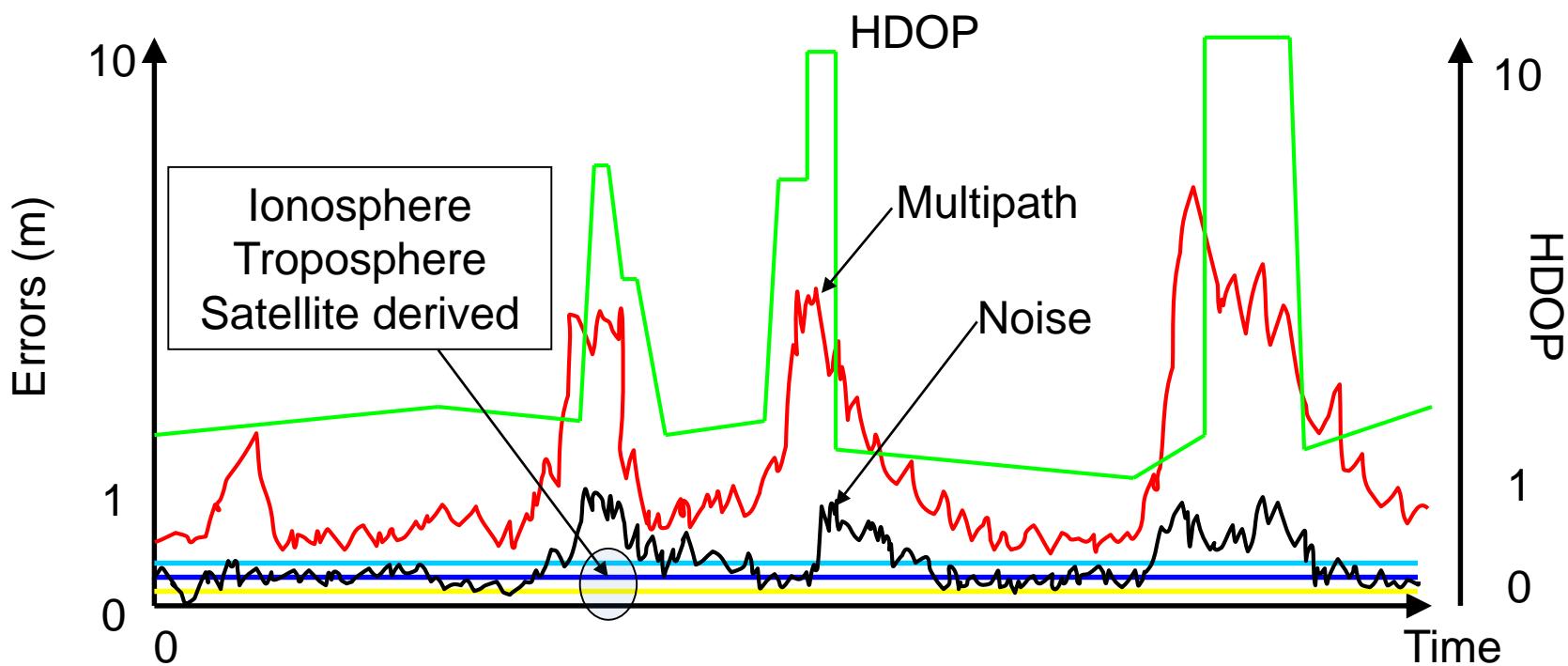
Propagation Type

Green: Direct

Yellow: Diffraction

Red: Masking
+Reflection

Temporal Measurements Errors and DOP Variation (sub-urban)



Position Estimation

- Satellite position in the transmitted time “ $t - \tau$ ”.
- Pseudo-range between satellite and user in the received time “ $t”$

$$\rho^{(k)}(t) = r^{(k)}(t, t - \tau) + c[\delta t_u(t) - \delta t^{(k)}(t - \tau)] + I^{(k)}(t) + T^{(k)}(t) + \varepsilon_{\rho}^{(k)}(t)$$



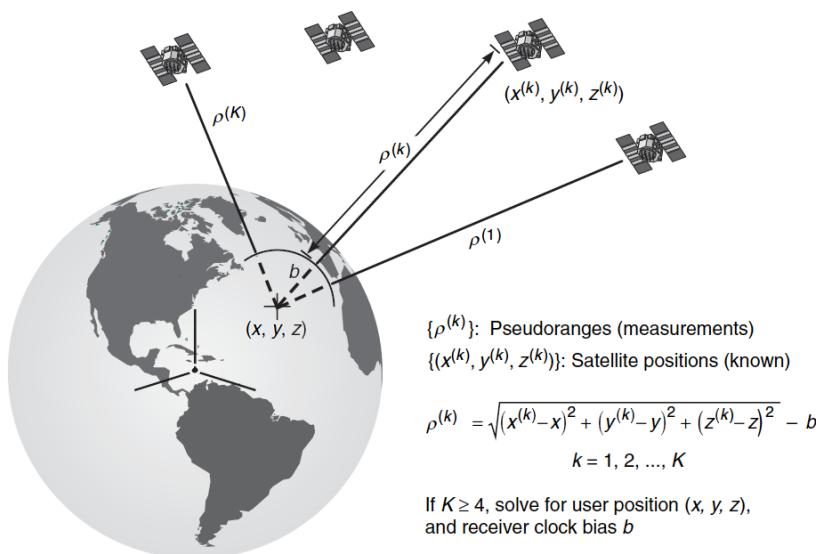
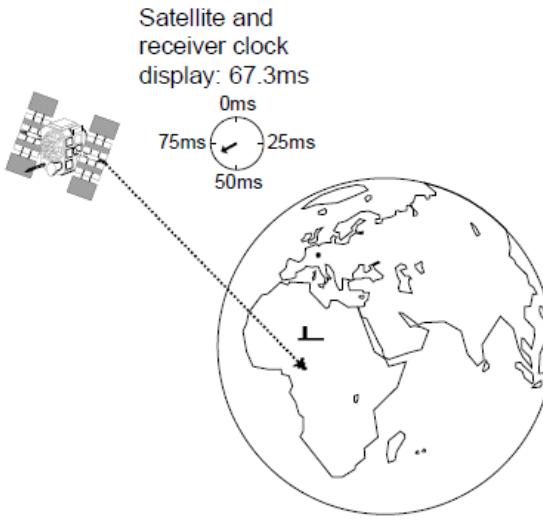
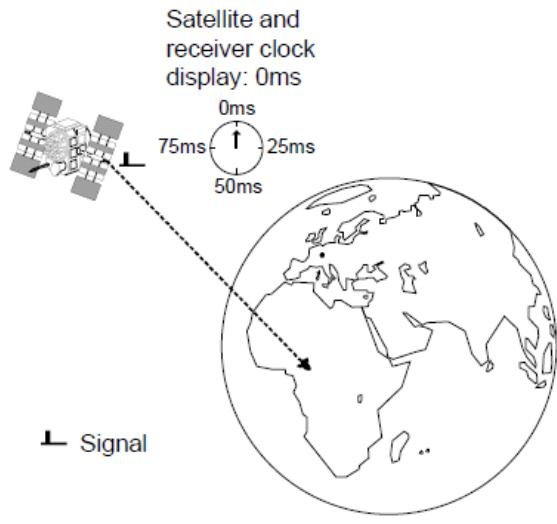
Clock Errors

The reason why we call “pseudo-range” is from second term.

Satellite clock and Receiver clock are not synchronized.

How many unknown parameter do we have ?

x, y, z, receiver clock offset



- Satellite clock is corrected using navigation data.
- Fortunately, receiver clock offset is **same** for all satellites.
- Therefore, unknown variables should be solved are **x, y, z and receiver clock offset**.

Least Square Method

Core Component of Positioning in LS method

```
for(i=0;i<SATn;i++){  
    prn = SVn[i];  
    r2[i] = sqrt((SVx[prn]-init[0])*(SVx[prn]-init[0])  
                +(SVy[prn]-init[1])*(SVy[prn]-init[1])  
                +(SVz[prn]-init[2])*(SVz[prn]-init[2]));  
    r3[i] = Pr1[prn] + SV_corrtme[prn] - Iono[prn] - Tropo[prn] - r2[i];  
    .....  
}
```

Init[0],init[1],init[2] are respectively X, Y, Z position.

After several iterations, Init[0],init[1],init[2] become final solution of positioning.

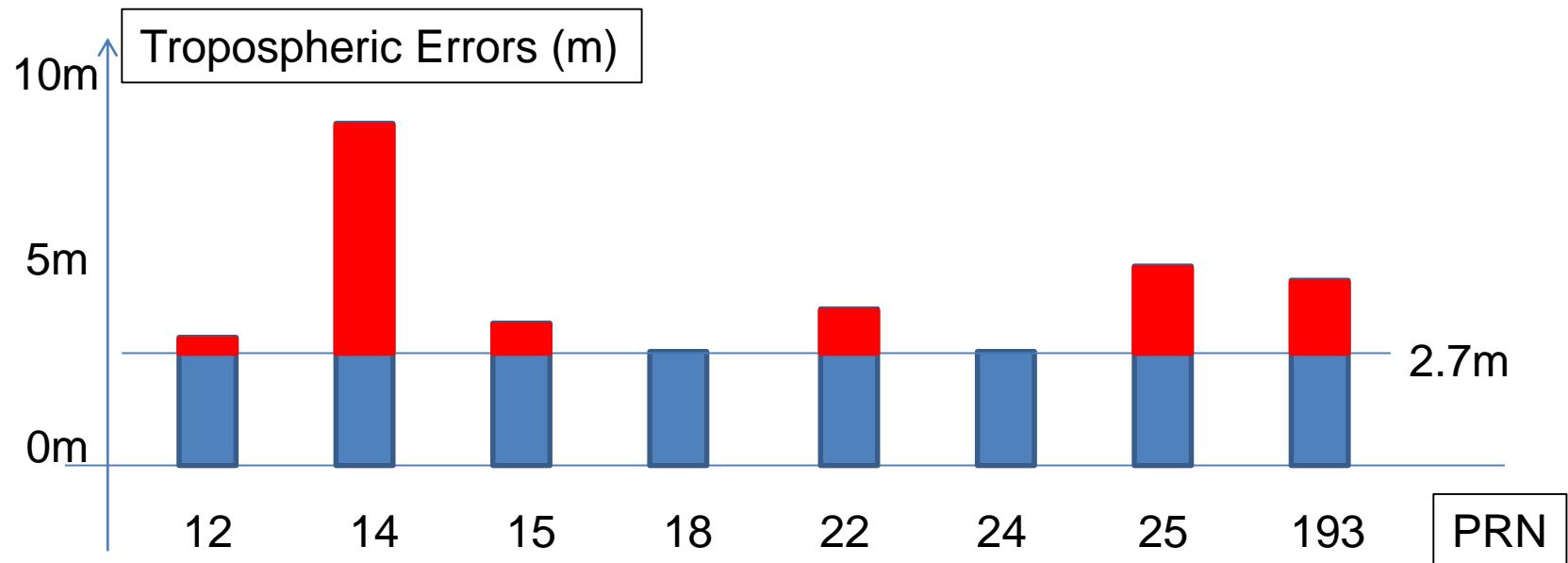
Clearly, the accuracy depends on the accuracy of **several terms in red color**.

Multi-path/Noise terms **can not be estimated**. That's why they are not included.

The more accurate input data we have, the more accurate position we can get.

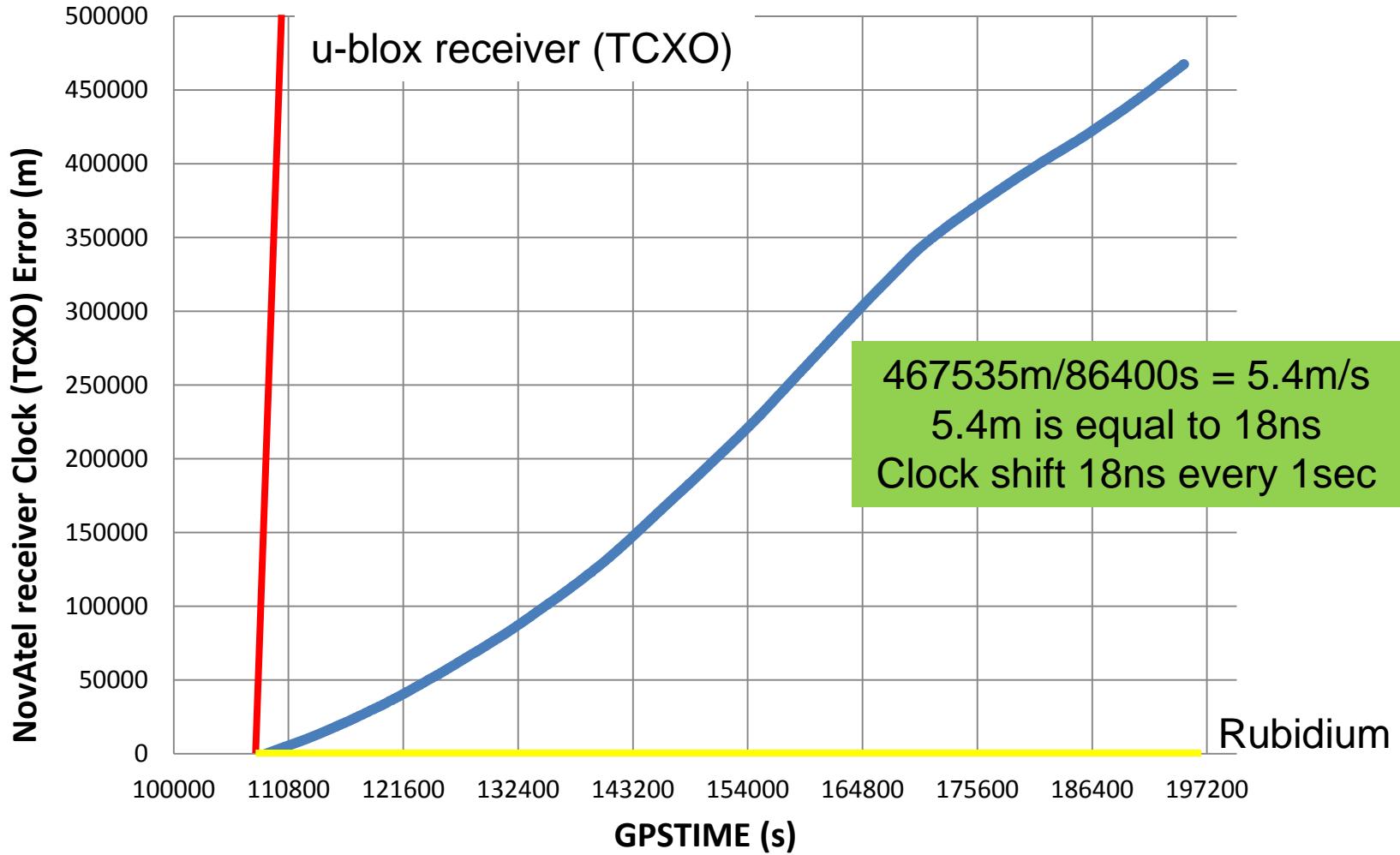
Common Biases are negligible

- Please remember that the common biases to all satellites are negligible in LS method. **They are absorbed into clock offset term.**



What is receiver clock offset ?

Receiver clock offset is coproduct of single positioning



Single Point Positioning

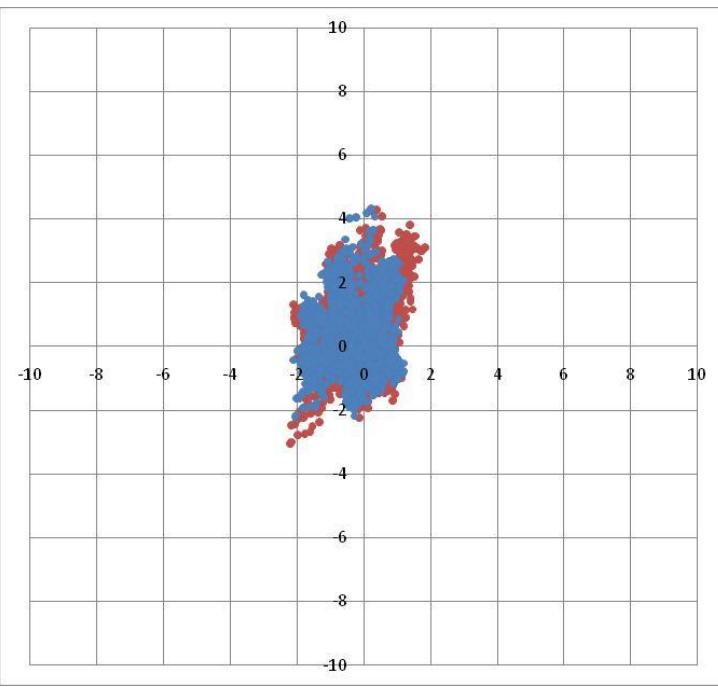
- 4 unknown variables (x, y, z, clock) are present.
- At least 4 visible satellites are required.
- DOP value has to be checked if it is small.
- With true satellite positions and true range between satellites and user antenna, the calculated position is true (only one solution).
- It is impossible in a practical sense.
- Least-Square method (LS method) is mainly used for the estimation of user antenna position.

Example of Iterations in LS method

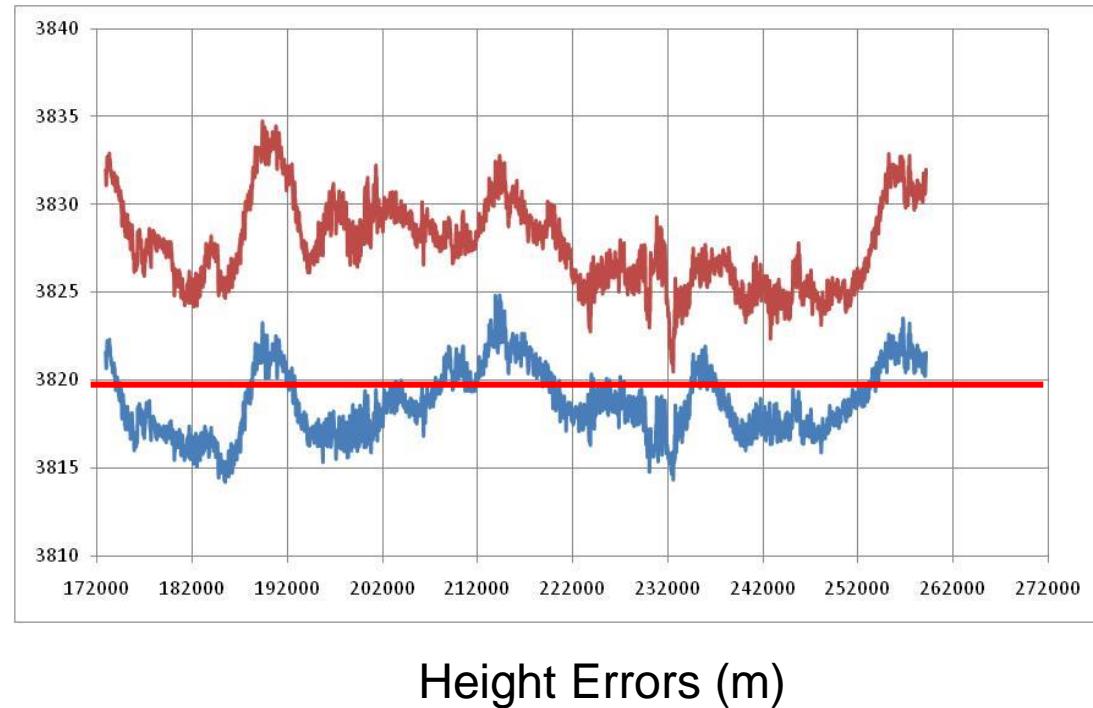
- The user antenna was located in Etchujima campus.
- If we set $(0, 0, 0)$ as a initial x, y, z positions,
- After the first iteration, the estimated position was **35.156, 139.191, 1252955m**. (on the sea close to Yugawara-machi in Kanagawa pref.)
- Secondly, it was **35.624, 139.727, 42298m** (close to Gotanda-station)
- Thirdly, it was **35.666166, 139.792192, 116m** (about 30m away from antenna)
- Fourth, it was **35.666246, 139.792322, 63m** (within 2m from antenna)

Reference Station at Mt. Fuji

(6/1/2010)



Horizontal Errors (m)



Height Errors (m)

Blue : Stand Alone Positioning

Red : Stand Alone Positioning without Iono and Tropo Estimation

Doppler Effect



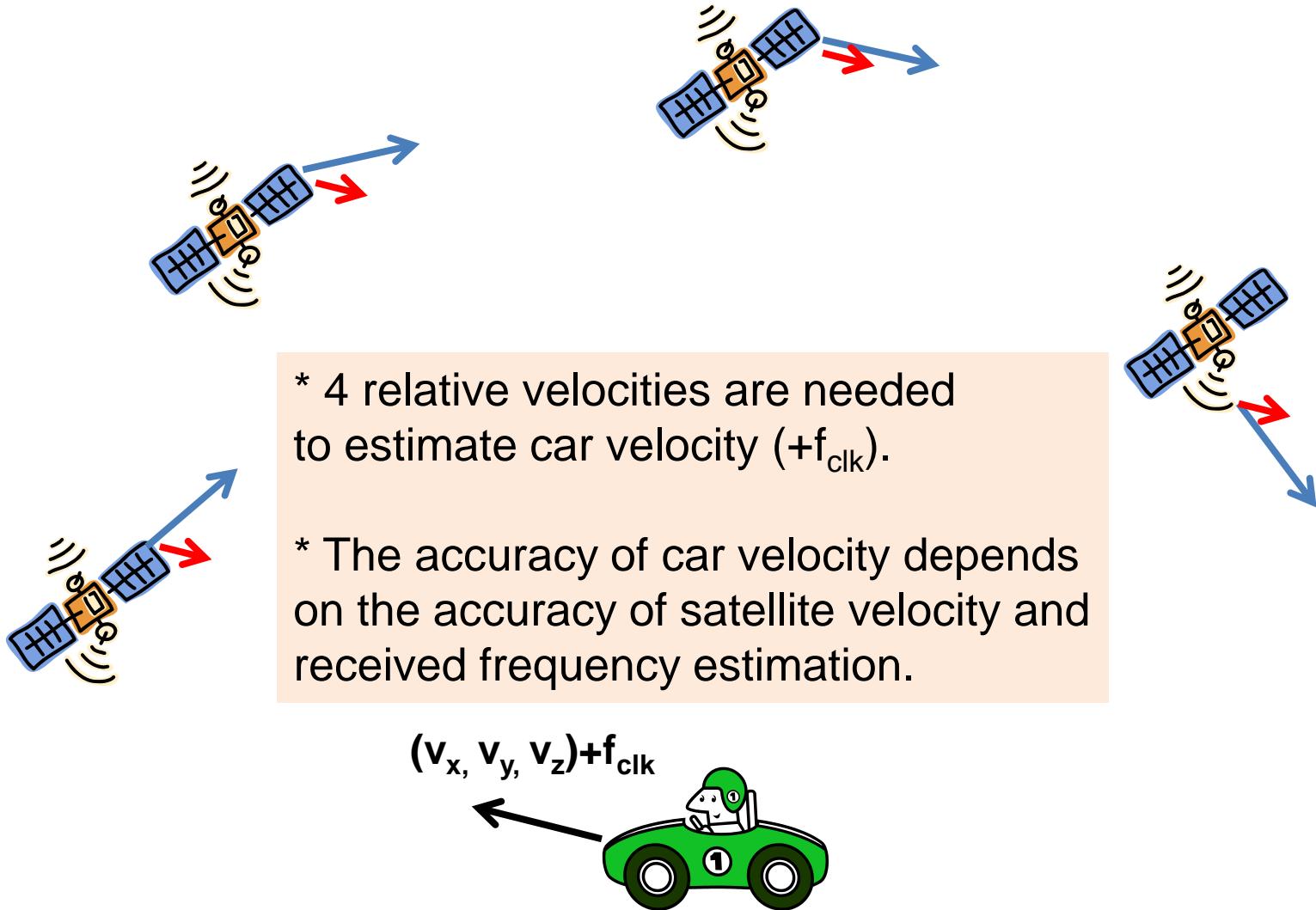
- Receiver is set in the car.
- Received frequency is
- “cs” is speed of light.
- Doppler frequency “ f_D ” is equal to “ $f_{obs} - f_{source}$ ”
- FLL (frequency lock loop) tries to estimate “ f_D ”.
- Once we can estimate “ f_D ”, “ v_o ” can be resolved.

$$f_{obs} = f_s \frac{cs - v_o}{cs - v_s}$$

Velocity Estimation

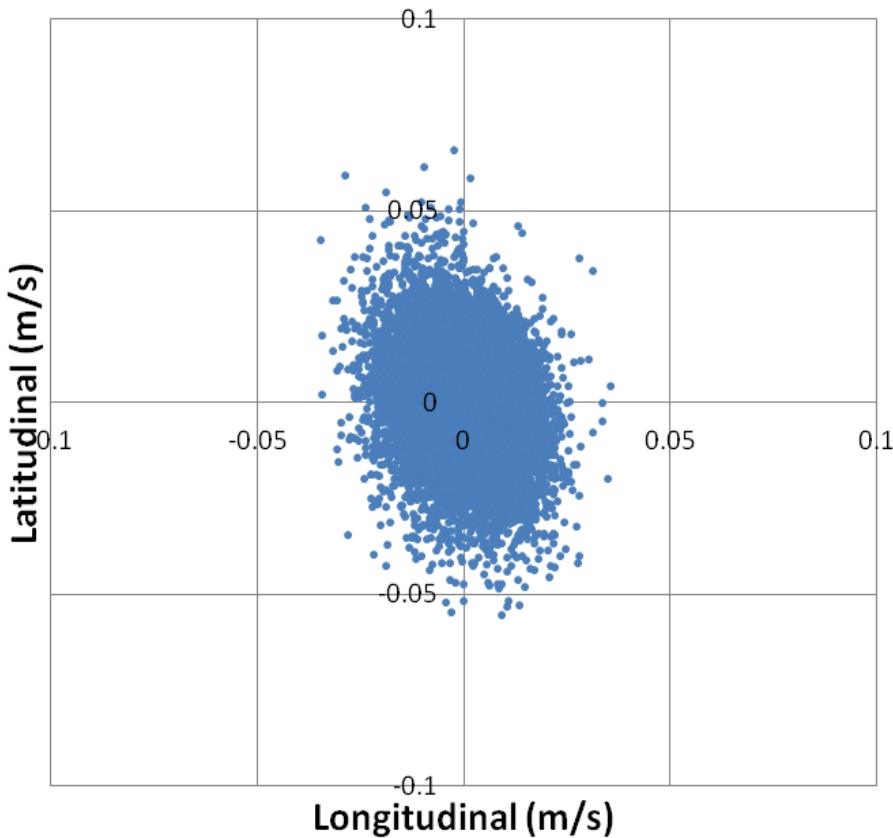
- Velocity estimation in GPS is just same as shown in the previous slide.
- The differences are as follows.
 - * **3 dimension velocity (v_x, v_y, v_z) have to be estimated.**
 - * **Frequency in the receiver is based on on-board clock.**
 - 4 unknown variables (v_x, v_y, v_z, f_{clk}) have to be estimated using at least 4 visible satellites. DOP is also important.
 - Velocity estimation is same as position estimation.

Image of Velocity Estimation



Performance of GPS based Velocity

Velocity in static condition



std = 1.6 cm/s

Accuracy in terms of frequency
GPS L1 wavelength = 19cm
1Hz : 19cm
0.1Hz : 1.9cm

Accuracy in terms of satellite velocity
 $sv_vel[t] = (sv_vel[t+1] - sv_vel[t-1]) / 2$
based on ephemeris parameters
Accuracy is quite good.

Moving Platform

(Koto-ku Ariake)



- Origination : 0,0
- Velocity was accumulated.
- Data Rate : 5Hz
- Period : 650 sec
- Receiver : NovAtel OEM6
- Left and right rounds : 6 times
- End point : 36.76m,-62.91m
- RTK : 35.75m,-65.18m

Deviation after 11 minutes velocity accumulation was about 2-3 m.

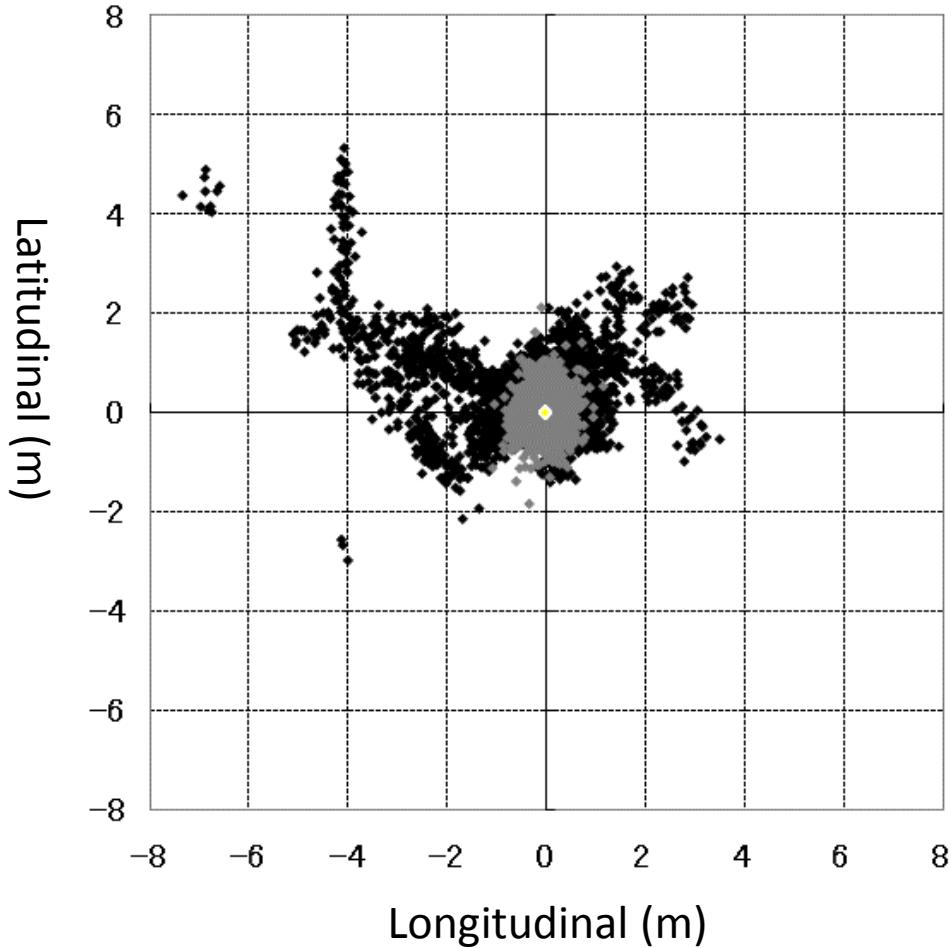
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Improved GPS

- DGPS and RTK are powerful method for error mitigation.
- DGPS uses the fact that the most of error sources change slowly in the time domain if the distance between reference and user is approx. within 100km.

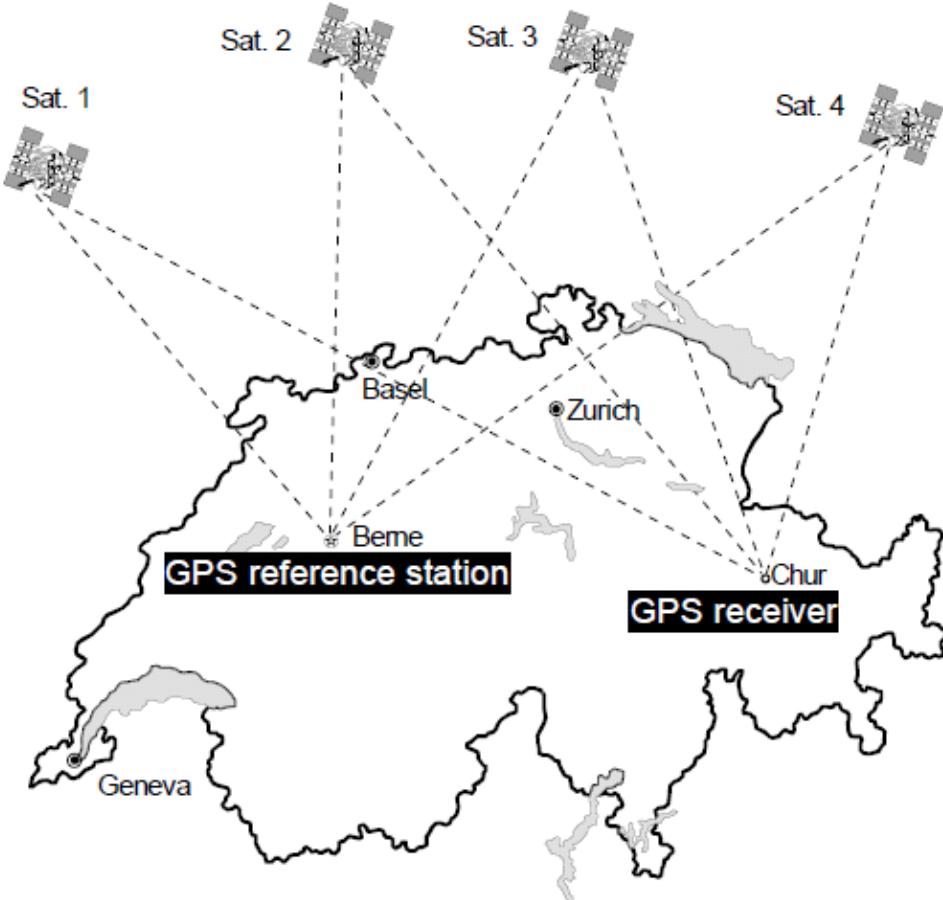
DGPS and RTK Performance



Single Positioning
DGPS
RTK

Rooftop (Lab.)
15 s interval
24 hours
Reference : Ichikawa

Image of DGPS

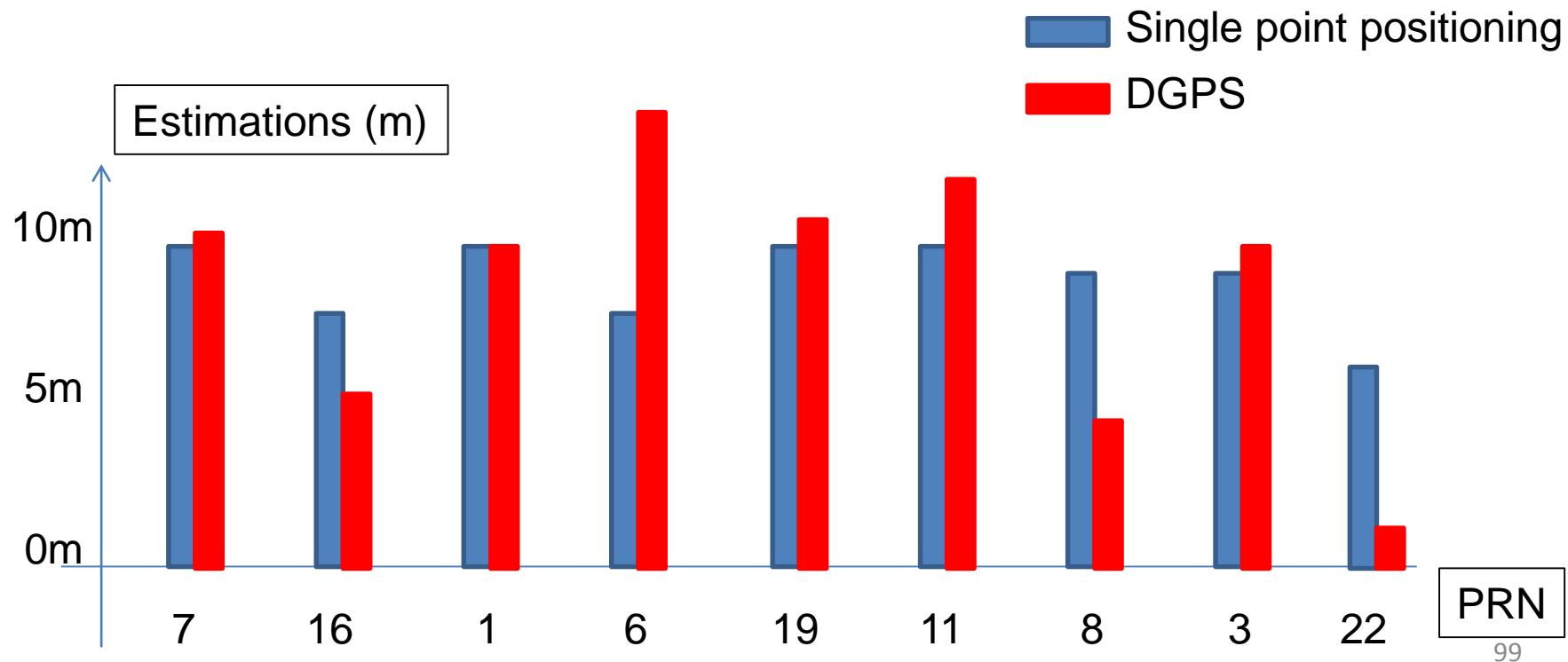


- Determination of the correction values at the reference station
- **Transmission of the correction values** from the reference station to the GPS user
- **Compensation** for the determined pseudo-ranges to correct the calculated position of the GPS user

$$\text{Correction [prn]} = \text{Pseudo-range[prn]} - \text{True-range [prn]}$$

Real Correction Data

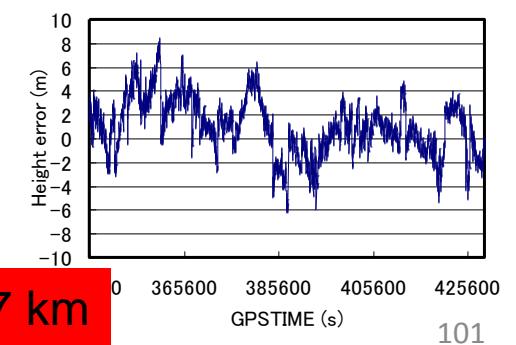
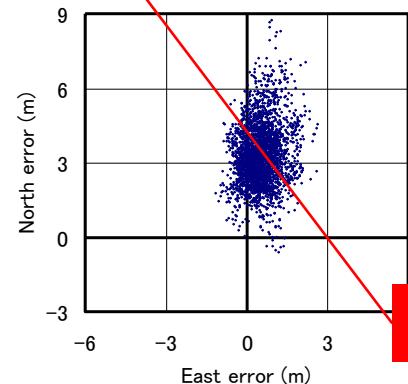
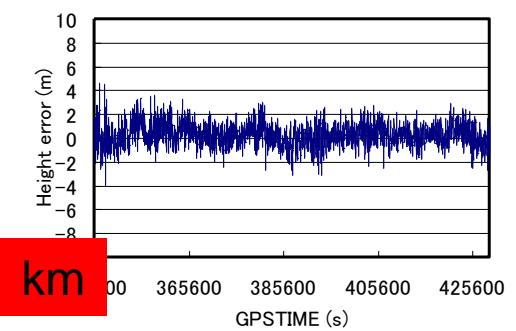
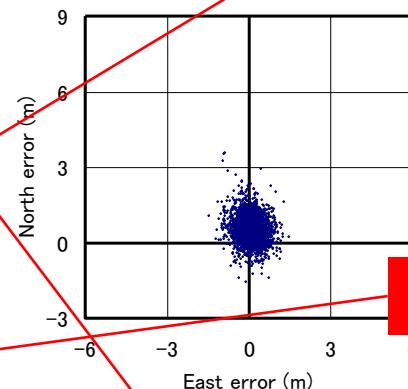
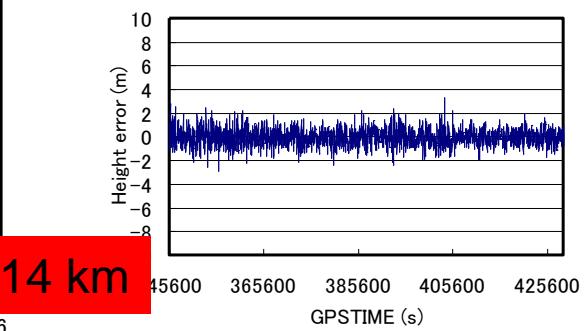
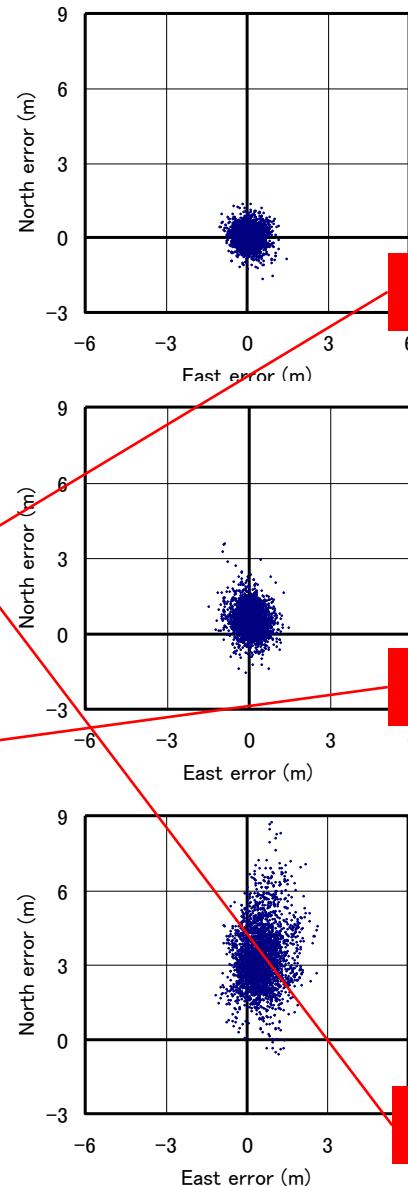
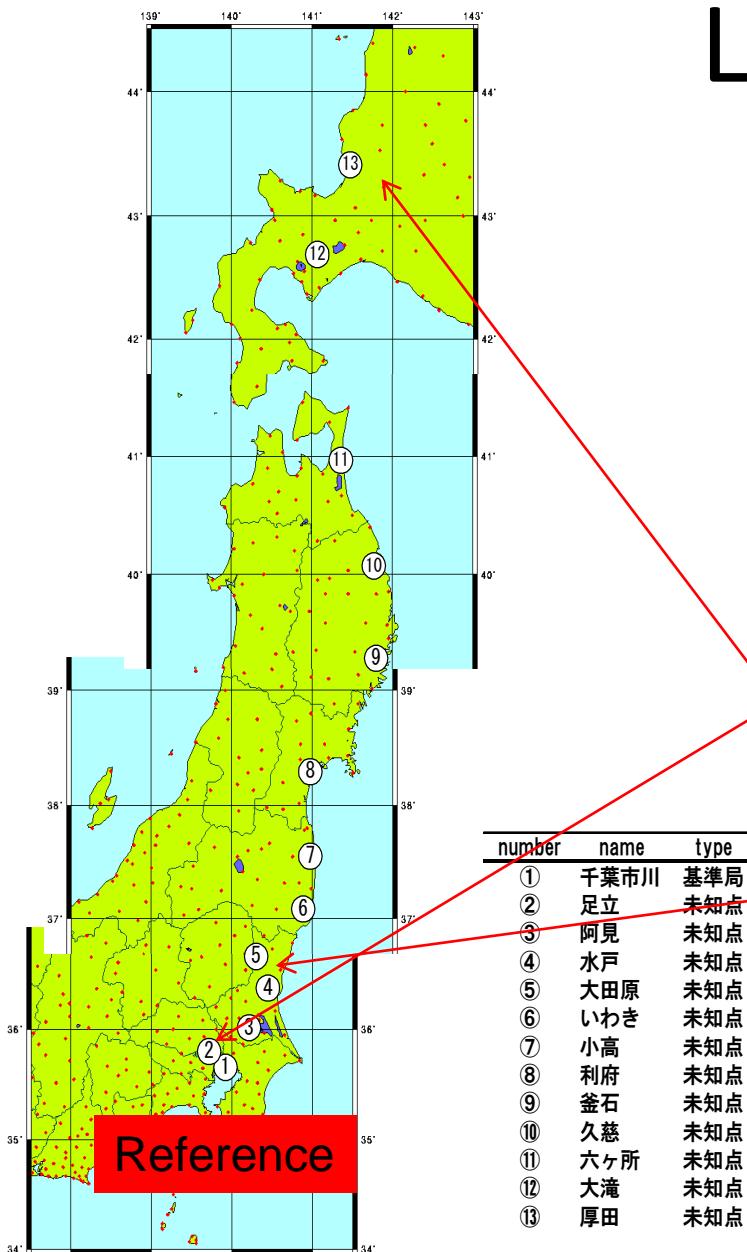
- Correction [prn] =
Pseudo-range[prn] – True-range [prn]
- Correction data provides the **better estimations in each satellite in LS method.**



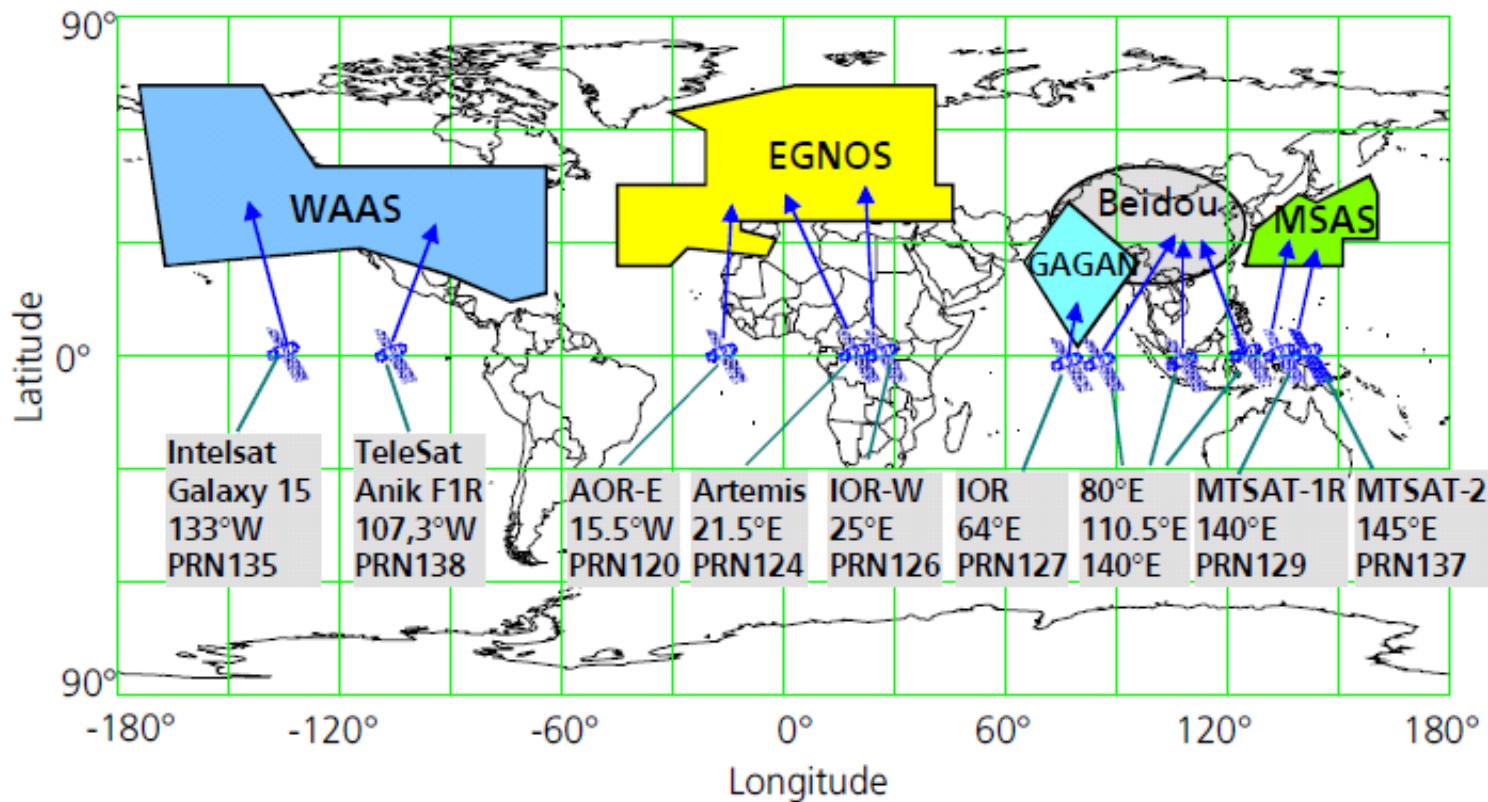
DGPS mitigates ...

Source	Potential error size	Error mitigation using DGPS
Satellite clock model	2 m (rms)	0.0 m
Satellite ephemeris prediction	2 m (rms) along the LOS	0.1 m (rms)
Ionospheric delay	2-10 m (zenith) Obliquity factor 3 at 5°	0.2 m (rms)
Tropospheric delay	2.3-2.5m (zenith) Obliquity factor 10 at 5°	0.2 m (rms) + altitude effect
Multipath (open sky)	Code : 0.5-1 m Carrier : 0.5-1 cm	→
Receiver Noise	Code : 0.25-0.5 m (rms) Carrier : 1-2 mm (rms)	→

Limitation of DGPS



SBAS

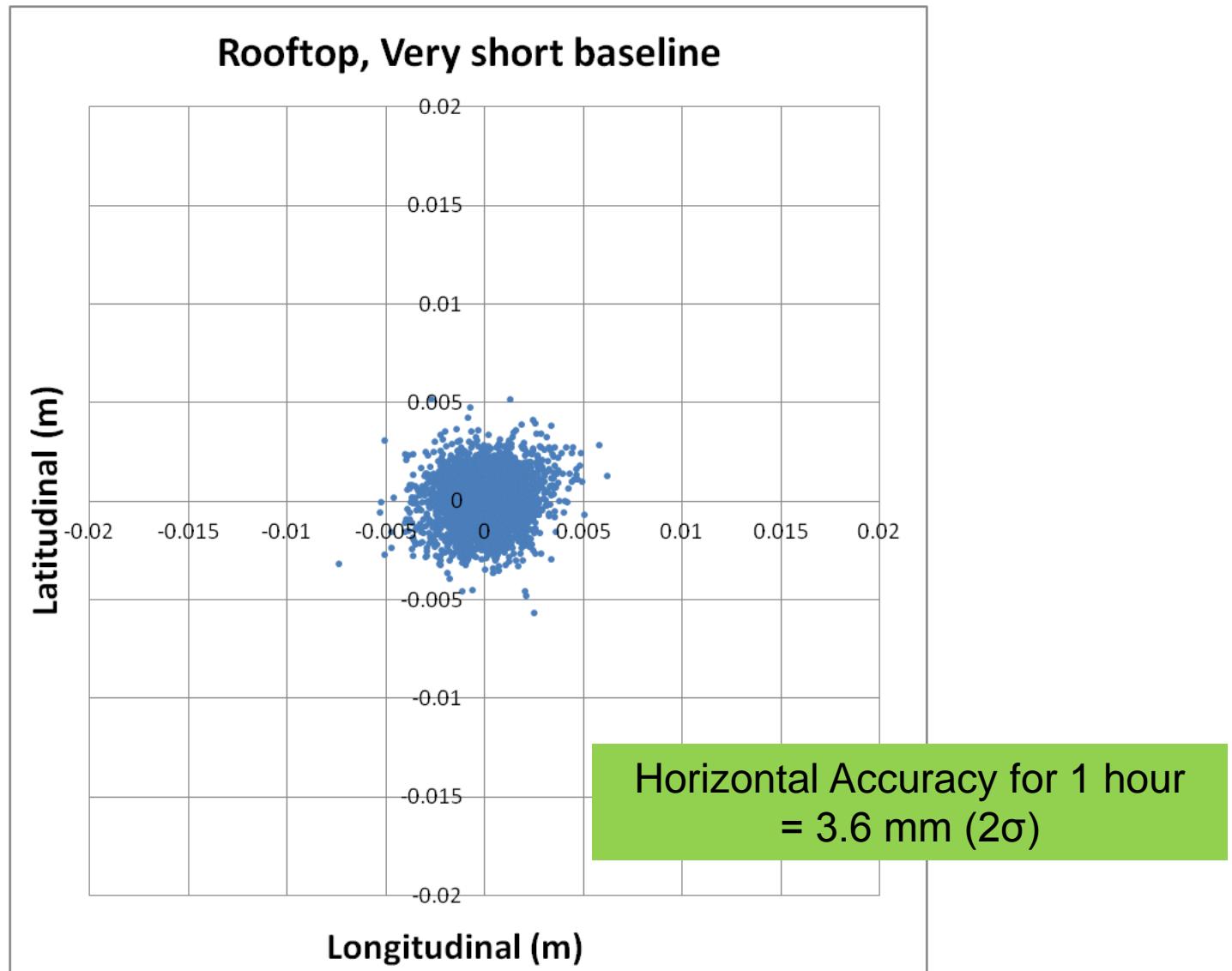


Without the installation of the reference stations, you can use correction data through the SBAS satellite such as MTSAT in Japan. Under quiet ionospheric condition, the performance is generally good within 1-2 m .
(Small robot car demo)

RTK (Real Time Kinematic)

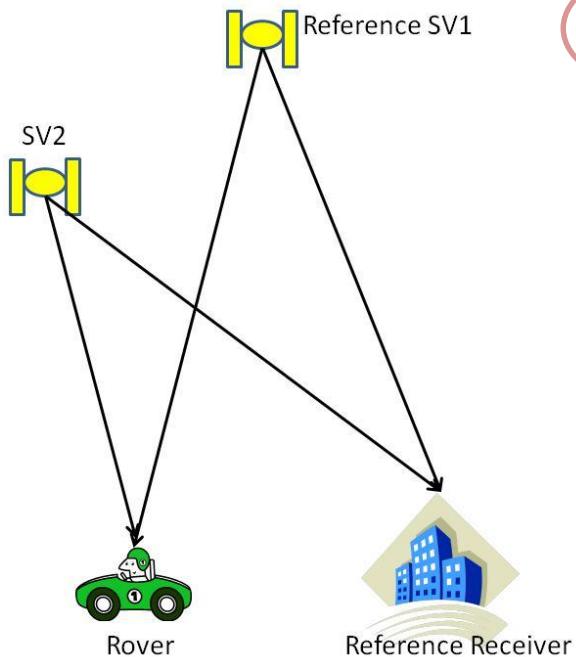
- The concept of RTK is same as DGPS.
- RTK uses carrier phase measurements. DGPS uses pseudo-range measurements.
- GPS receiver is able to measure 1/100 of wavelength of L1 frequency (19 cm).
- If you have high-end receiver, you know your position within 1-2cm accuracy as long as you have 5 or more LOS satellites.

RTK performance



Key Concept of RTK

(double difference technique)



$$\begin{aligned}
 P_{rov_ref}^{sv1-sv2} &= (P_{rov}^{sv1} - P_{ref}^{sv1}) - (P_{rov}^{sv2} - P_{ref}^{sv2}) \\
 &= \rho_{rov}^{sv1} + c(dt_{sv1} - dT_{rov}) + ion_{rov}^{sv1} + tropo_{rov}^{sv1} + mp_{rov}^{sv1} + noise_{rov}^{sv1} \\
 &\quad - [\rho_{ref}^{sv1} + c(dt_{sv1} - dT_{ref}) + ion_{ref}^{sv1} + tropo_{ref}^{sv1} + mp_{ref}^{sv1} + noise_{ref}^{sv1}] \\
 &\quad - [\rho_{rov}^{sv2} + c(dt_{sv2} - dT_{rov}) + ion_{rov}^{sv2} + tropo_{rov}^{sv2} + mp_{rov}^{sv2} + noise_{rov}^{sv2}] \\
 &\quad + [\rho_{ref}^{sv2} + c(dt_{sv2} - dT_{ref}) + ion_{ref}^{sv2} + tropo_{ref}^{sv2} + mp_{ref}^{sv2} + noise_{ref}^{sv2}] \\
 &= \rho_{rov}^{sv1} - \rho_{ref}^{sv1} + \rho_{rov}^{sv2} - \rho_{ref}^{sv2} \quad \text{Completely zero assumed zero within 10 km} \\
 &\quad + (mp_{rov}^{sv1} + noise_{rov}^{sv1}) - (mp_{ref}^{sv1} + noise_{ref}^{sv1}) \\
 &\quad - (mp_{rov}^{sv2} + noise_{rov}^{sv2}) + (mp_{ref}^{sv2} + noise_{ref}^{sv2})
 \end{aligned}$$

Generating new observation data !!!

Ambiguity Resolution

$$P_{rov_ref}^{sv1_sv2} = r_{rov_ref}^{sv1_sv2} + \mathcal{E}_{p,rov_ref}^{sv1_sv2}$$

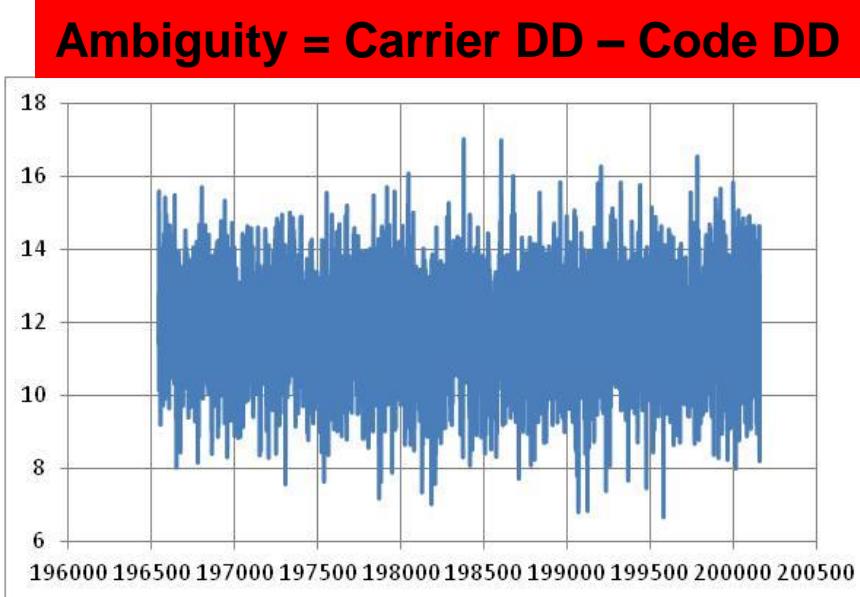
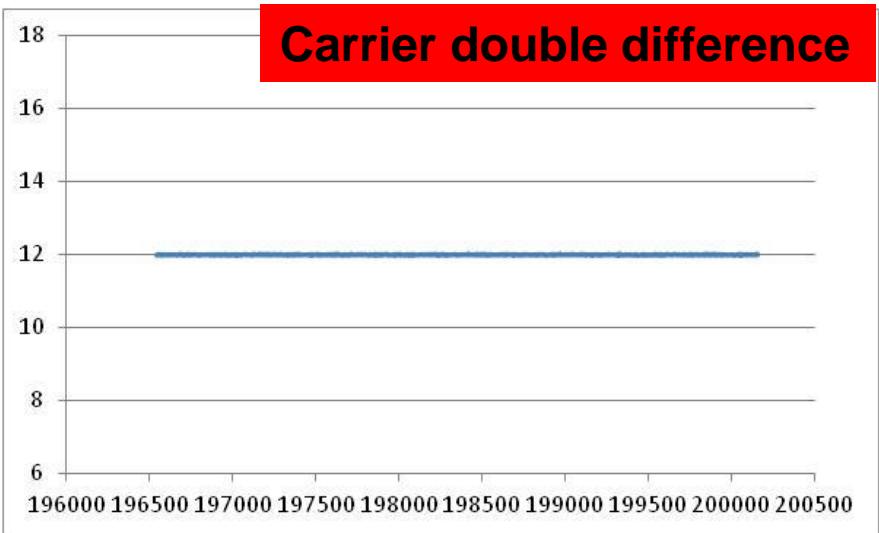
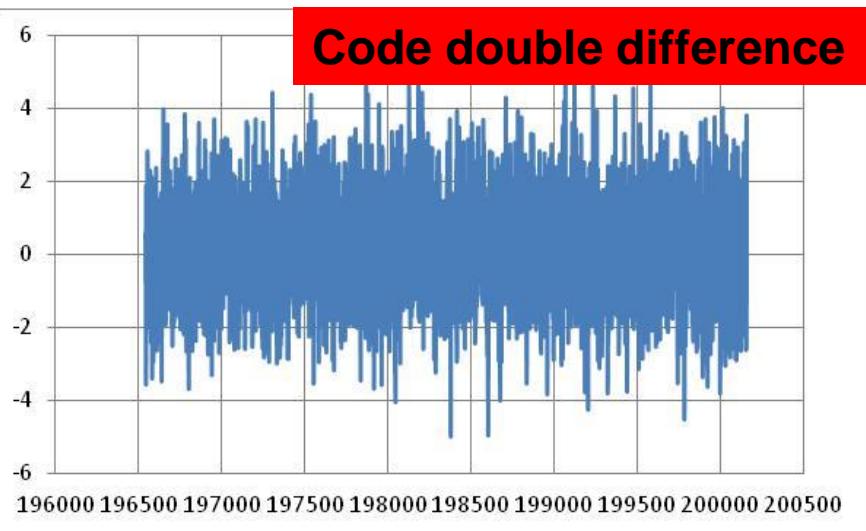
$$\phi_{rov_ref}^{sv1_sv2} = r_{rov_ref}^{sv1_sv2} + N_{rov_ref}^{sv1_sv2} + \mathcal{E}_{\phi,rov_ref}^{sv1_sv2}$$

- Once you can resolve **integer N** in carrier phase double difference, you can get accurate position within 1-2cm.
- It can be imagine that the **pseudo-range accuracy** is important. Also **combinations of frequency** is vital.

Combinations	Wavelength
L1	0.19 cm
L1-L2	0.86 m
L2-L5	5.86 m

Double Differenced Observation

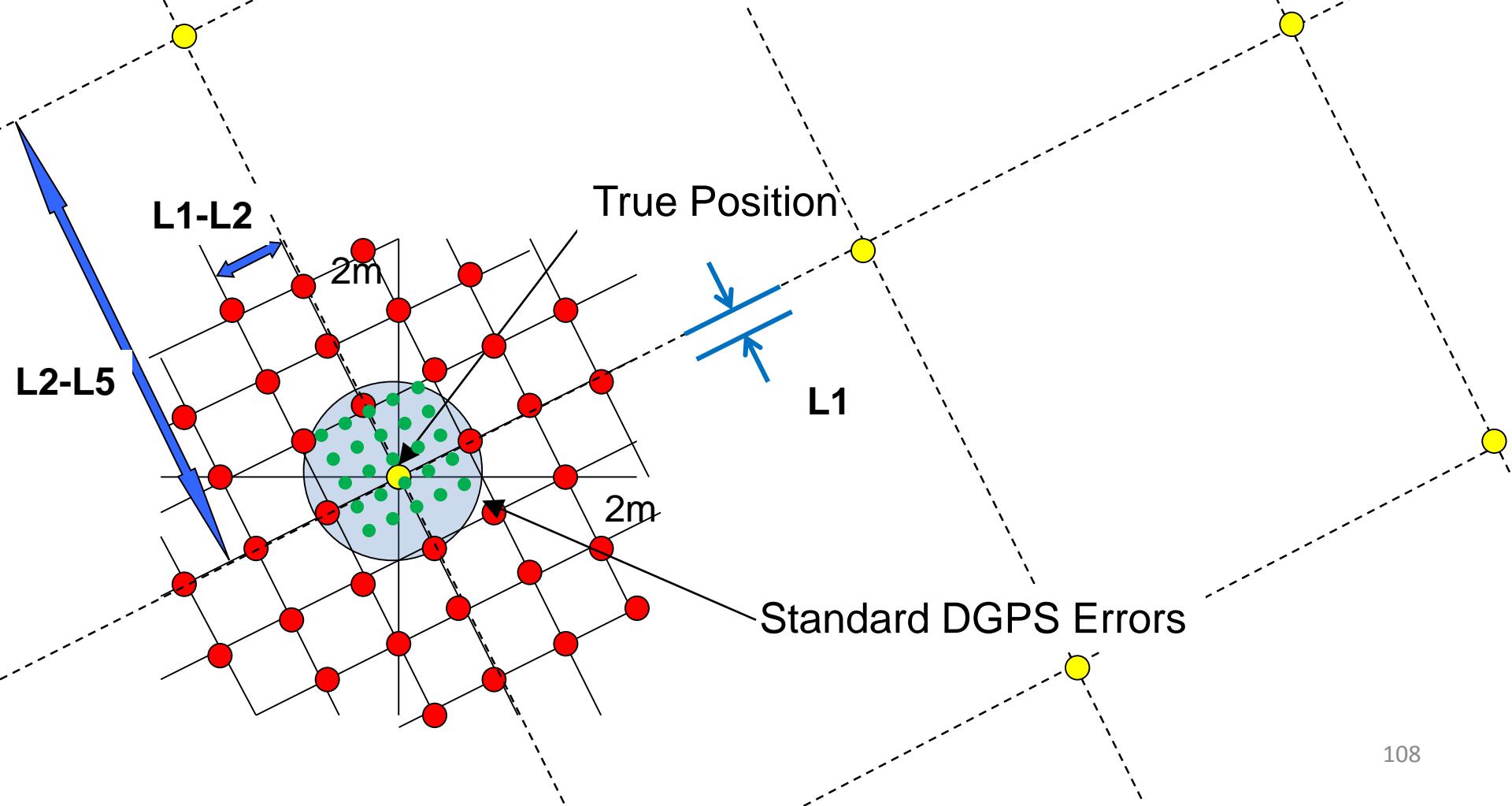
(open sky condition : prn19->prn3 : 1 hour)



**Average = 11.8
Std = 1.4**

DGPS Errors and the effect of frequency combinations

Ideal 2 dimension is assumed.

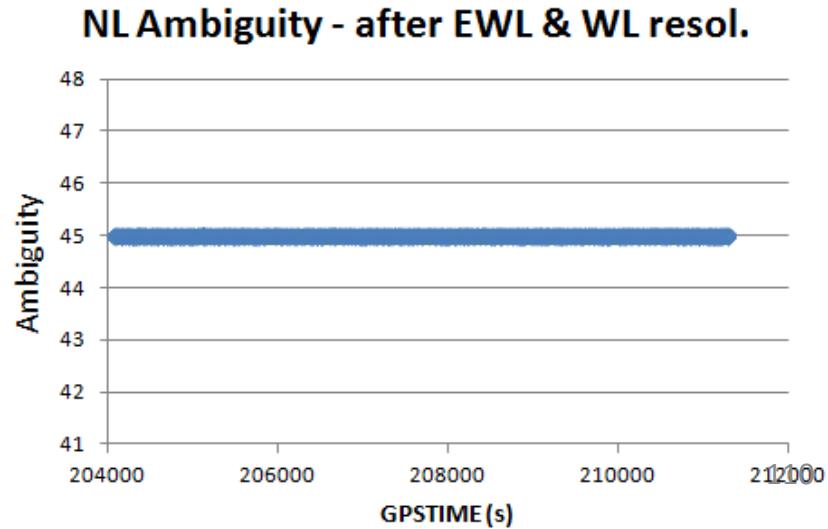
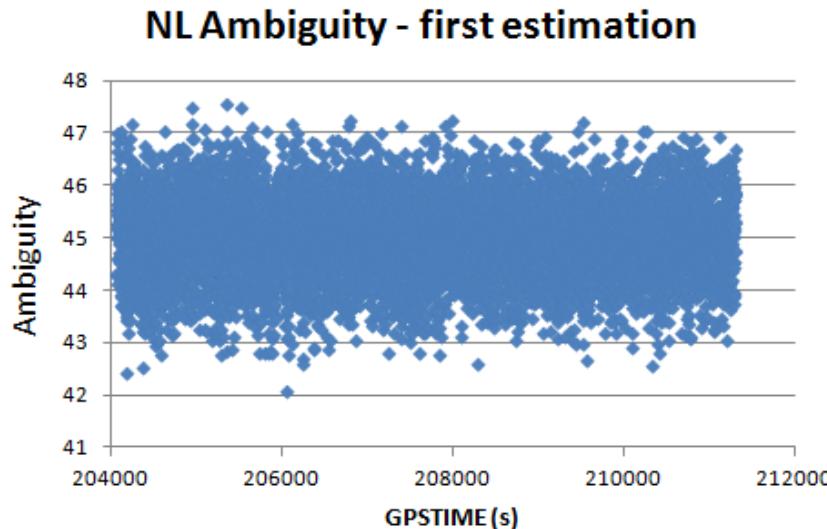
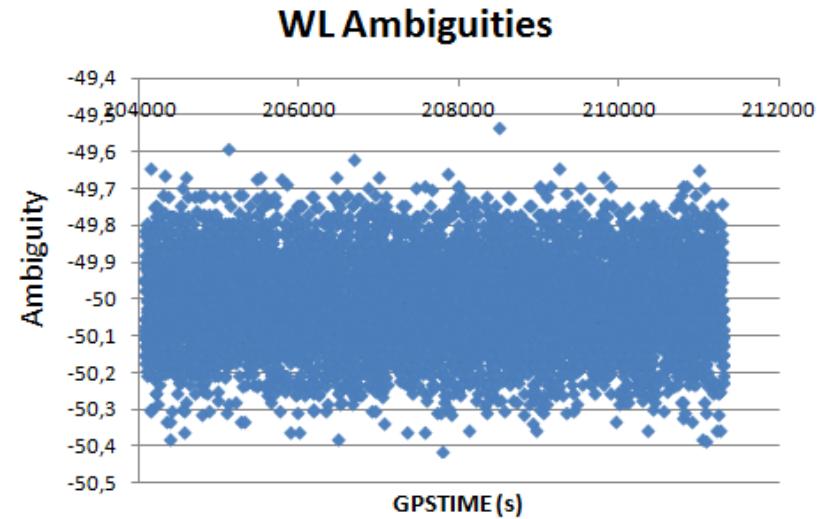
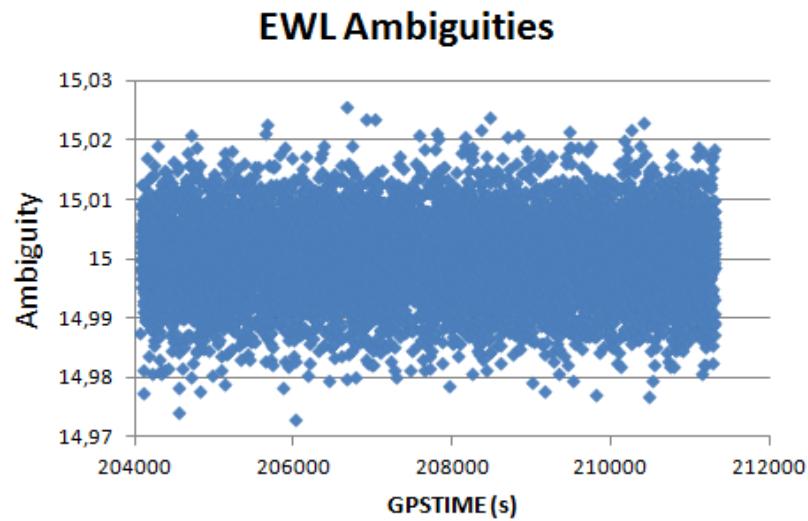


Galileo RTK & Ambiguity Resolution

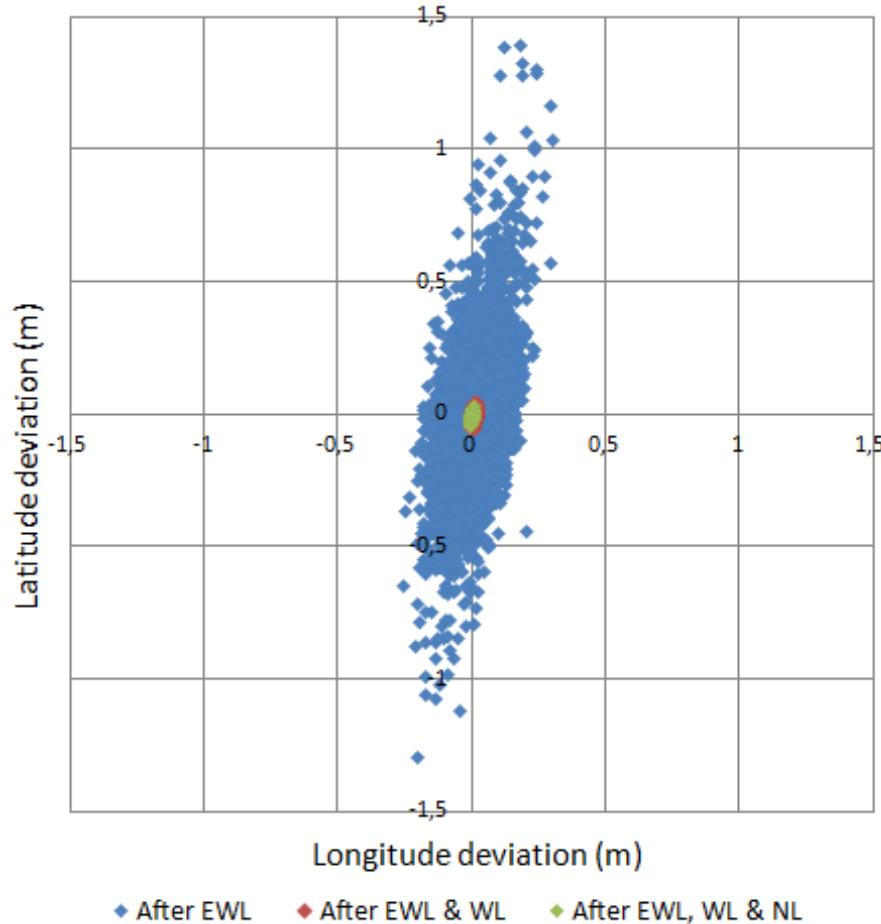
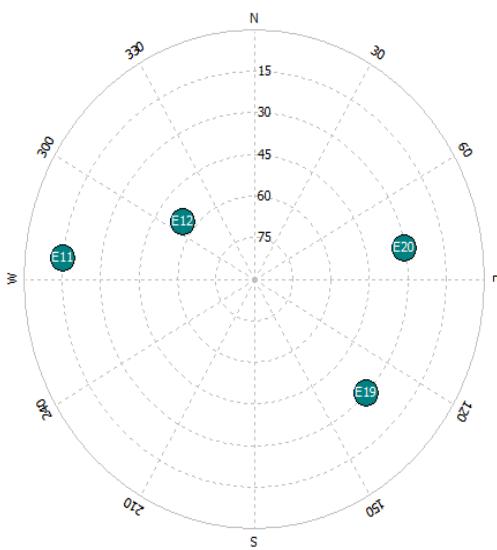
- Linear combinations
 - Enable to create a new frequency, with a longer wavelength, useful for ambiguity resolution
 - Galileo frequencies:
 - E1: 1575.42 MHz
 - E2: 1191.795 MHz (original E5altboc frequency)
 - E5: 1176.45 MHz (original E5a frequency)
 - Galileo linear combinations:

New carrier name	Frequencies	Wavelength (λ)	Accuracy
Extra-wide lane (EWL)	E2 – E5	19.55 m	43 cm
Wide lane (WL)	E1 – E2	78 cm	2 cm
Narrow lane (NL)	E1	19 cm	0.3 cm <small>109</small>

Ambiguities for Galileo



Horizontal Test Results

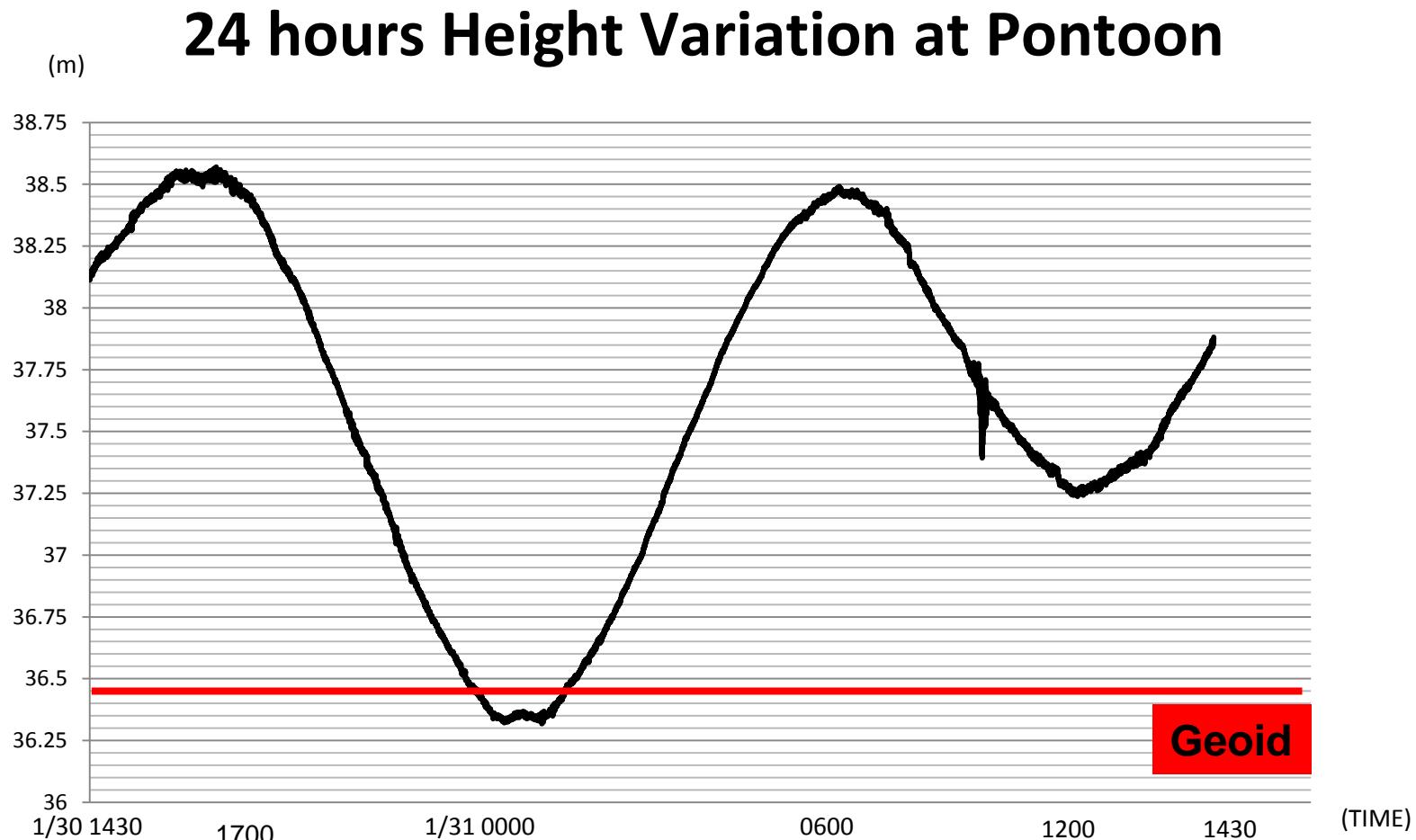


*Rover coordinates deviations from the reference station coordinates
Combining the 3 positioning results*

Tide Observation



1 Epoch RTK at Pontoon



Problem 2

- Please download the DOP calculation program (VS2010) and almanac for multi-GNSS.

<http://www.gnss-learning.org/index.php?id=70>

- Please answer which satellite is ID=111.
- Using the above program, please plot the position of satellite (ID=111) above the Earth's surface (you just need to add the source).

Hint

- Firstly, how can we estimate the position of satellite above the Earth's surface ?
- Above the Earth's surface means latitude and longitude description here.
- http://surveycalc.gsi.go.jp/sokuchi/surveycalc/trans_alg/trans_alg.html

Contents

- Coordinates System
 - Satellite Position
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- Measurements Errors
 - Calculating Position and DOP
 - Improved Position
- }
- 2nd period
-
- Basics of GNSS receiver
 - Multi-GNSS
- }
- 3rd period

Why we use Software Receiver ?

- If you want to improve signal processing in the GNSS receiver.
- ...develop receiver by yourself. All observation data from GNSS receiver is a kind of borrowed data
- ...integrate with other sensors.
- ...evaluate unique system like IMES or early phase new satellite.
- Good educational tool like this summer school.

Data Acquisition

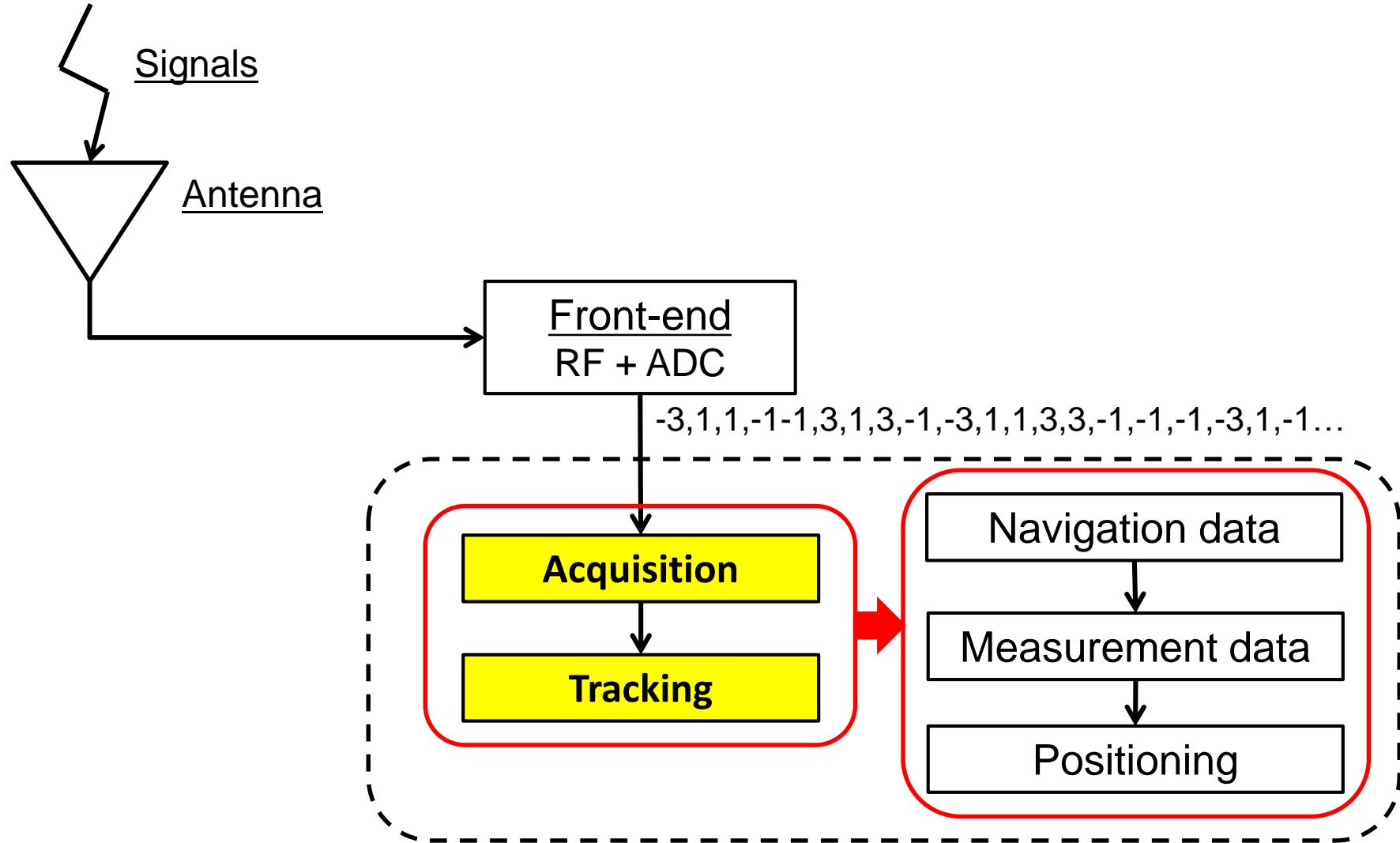
- Front-end is needed.
- The front-end manufactured by “IP-solutions” was used in this demonstration.
- You just bring **notebook** and **front-end** where you want to obtain the raw data for SDR (software defined radio).



Important parameters for SDR

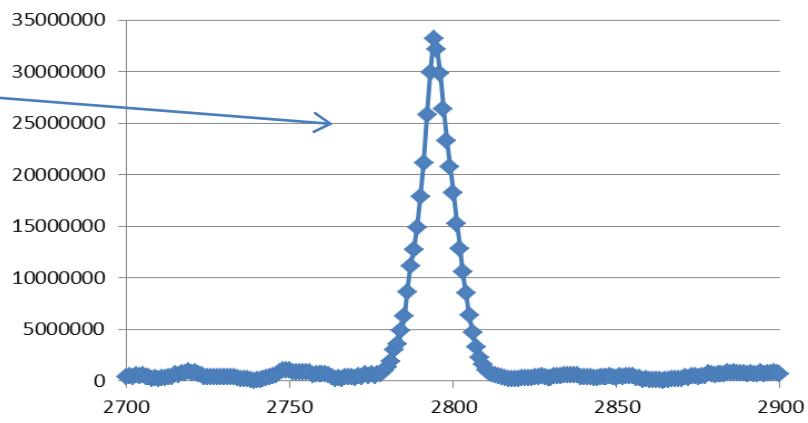
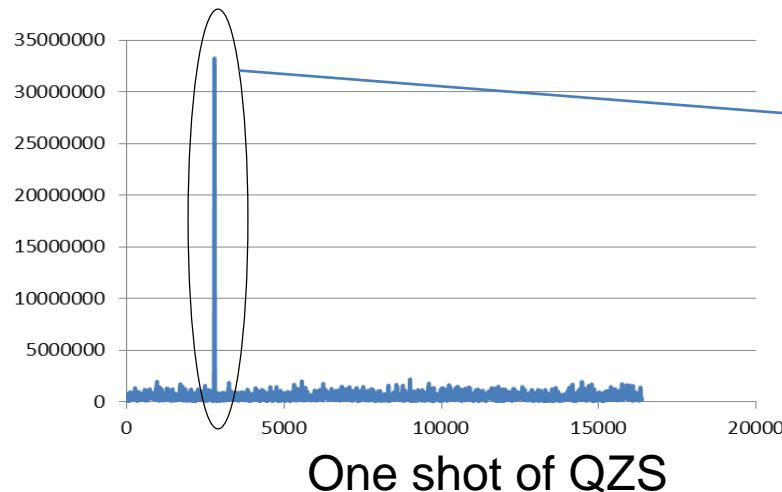
settings.IF = 4.123968MHz
settings.samplingFreq = 16.367667MHz
2 bit (-3,-1,1,3)

Brief Structure of SDR



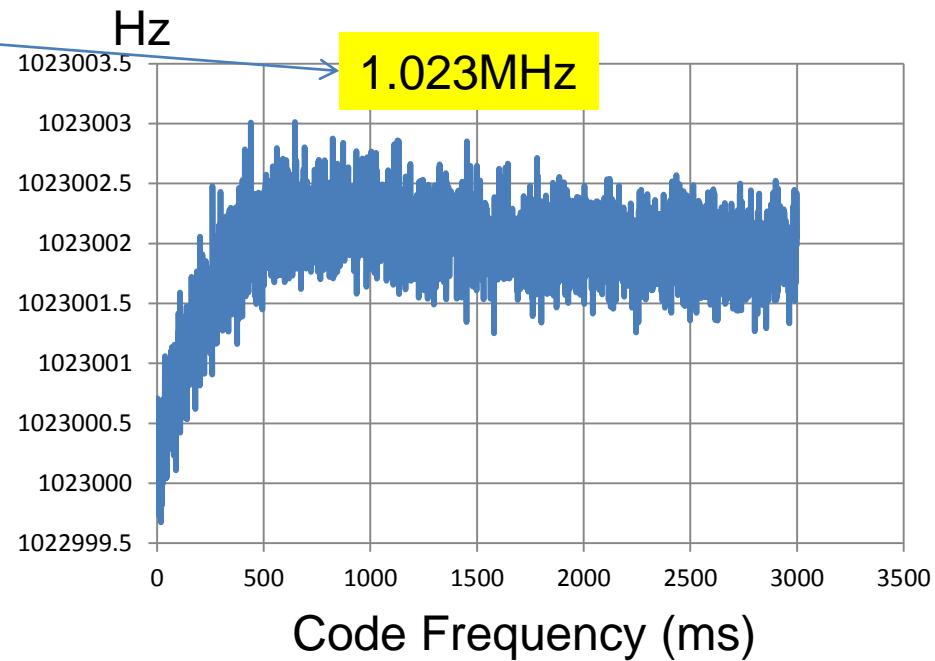
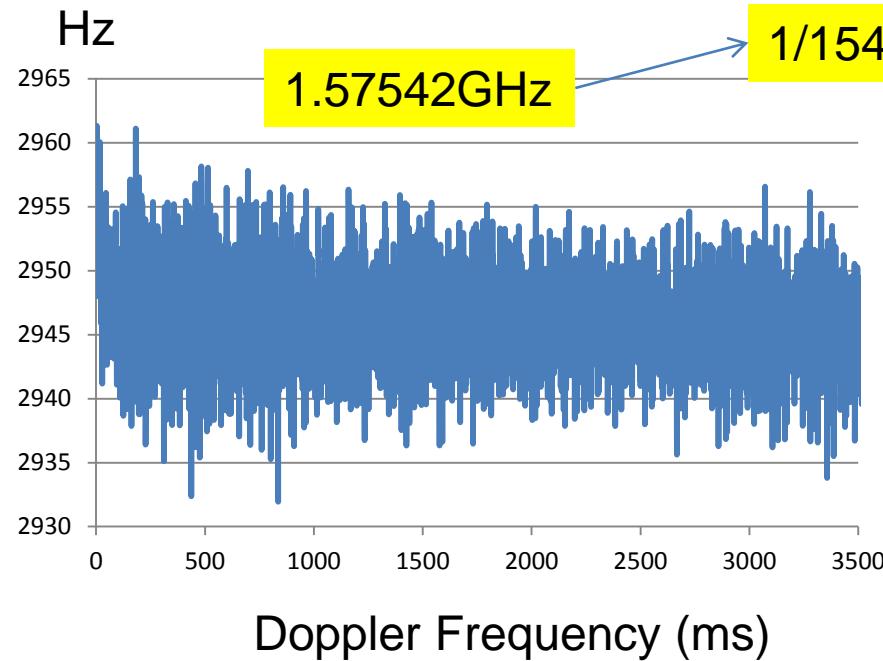
Acquisition (FFT based)

- **Acquisition** is to acquire the approximate code-phase and Doppler frequency of GNSS signals. **Tracking** is difficult without acquisition information.



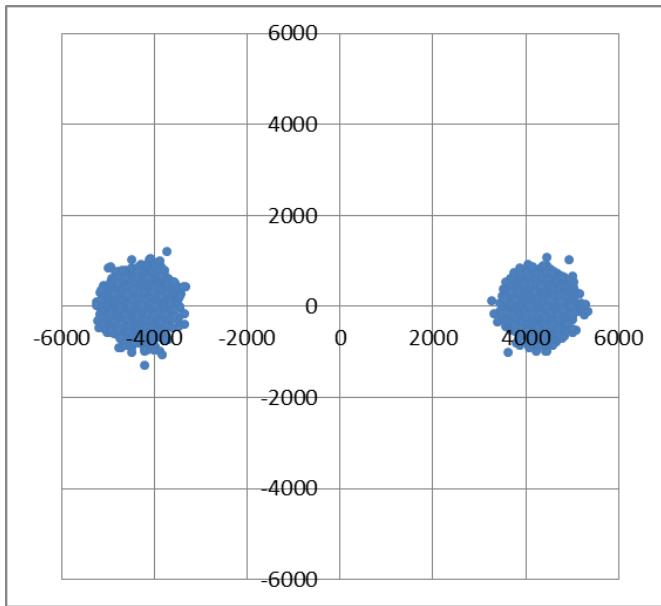
Tracking

- **Tracking** is to continuously track the code-phase and Doppler frequency of GNSS signals.
Loop filter is used in the tracking loop.

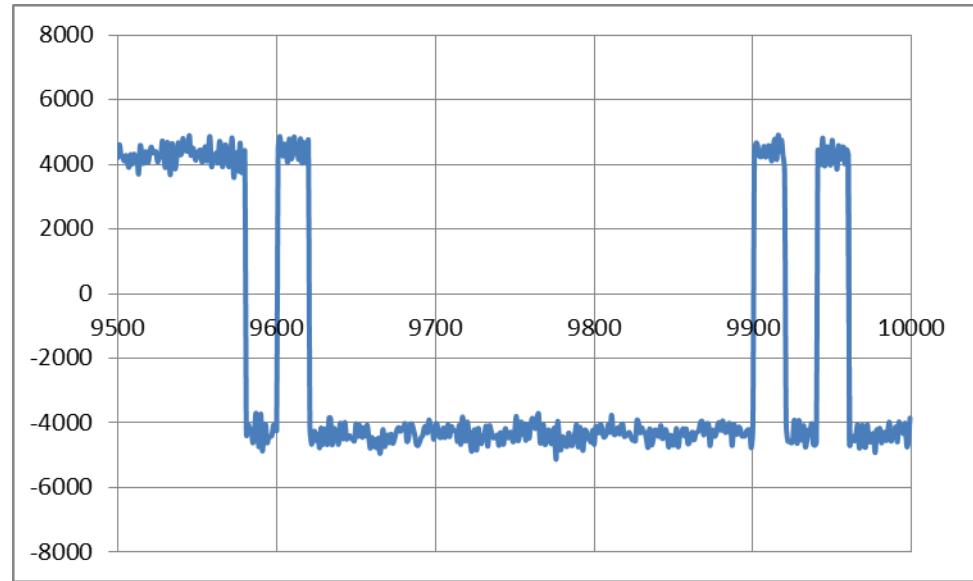


I and Q, Navigation decode

(outputs of tracking)

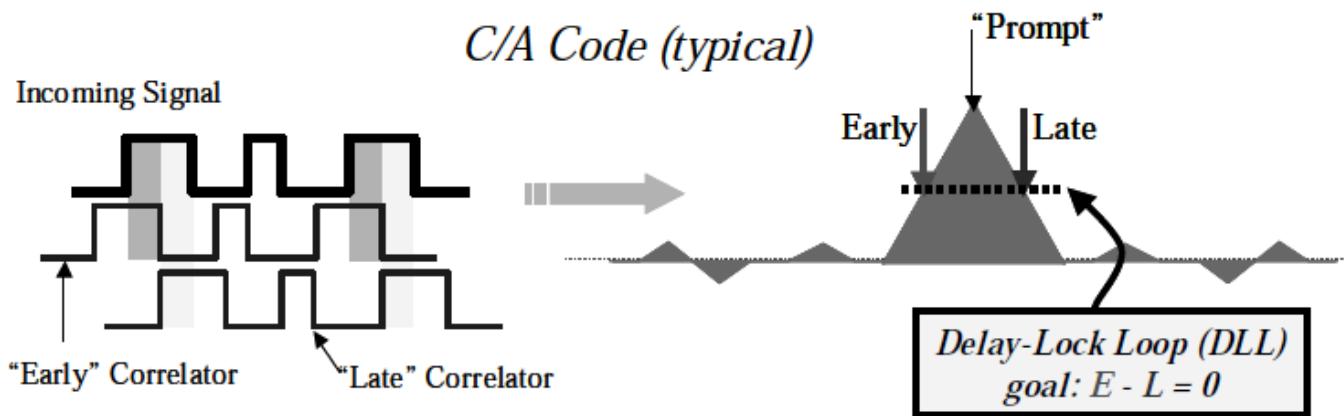
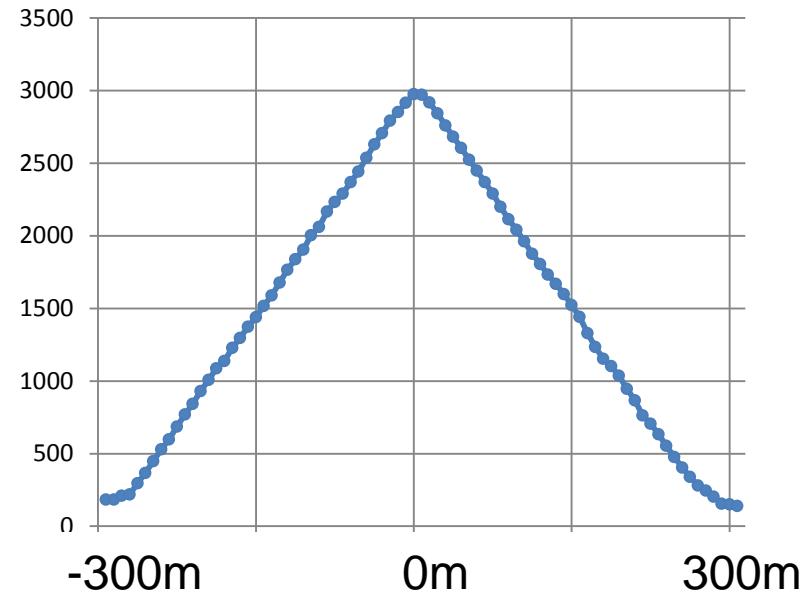
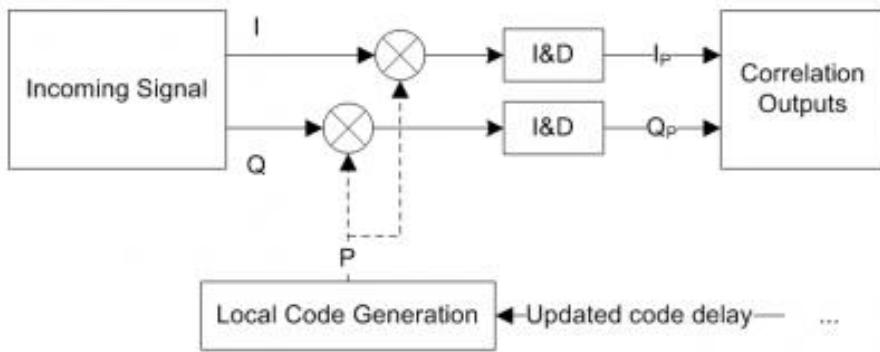


I and Q
(BPSK in GPS L1 C/A)



In phase correlation value from 9.5s to 10s

Correlation



Demonstration data was obtained at...

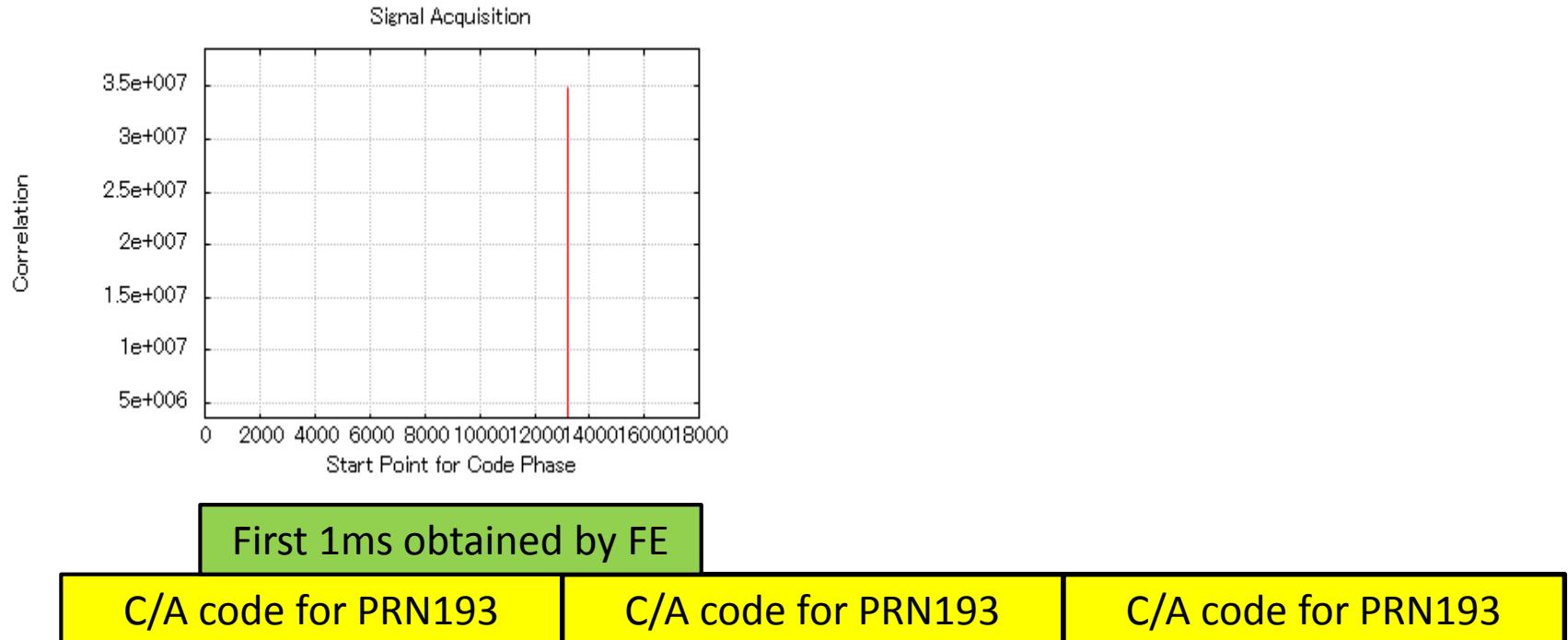
- Rooftop of our building
- Under high-rise building in Toyosu
- Inside laboratory

Signal Acquisition

PRN	Initial Code Phase	Doppler Frequency (Hz)
1	8800	1525
8	29	34
11	3781	494
19	10236	-302
20	2023	4974
28	9612	4248
32	11444	3593
193	13226	1220

These results are very important for signal tracking.
Without these results, signal tracking is impossible.

Image of Initial Code Phase



Sampling Frequency is 16.36776MHz

It means that the number of sampling is 16367 in every 1ms.

GPS L1 C/A code repeats every 1ms.

Finally, we can synchronize the code timing in tracking.

Navigation Decode

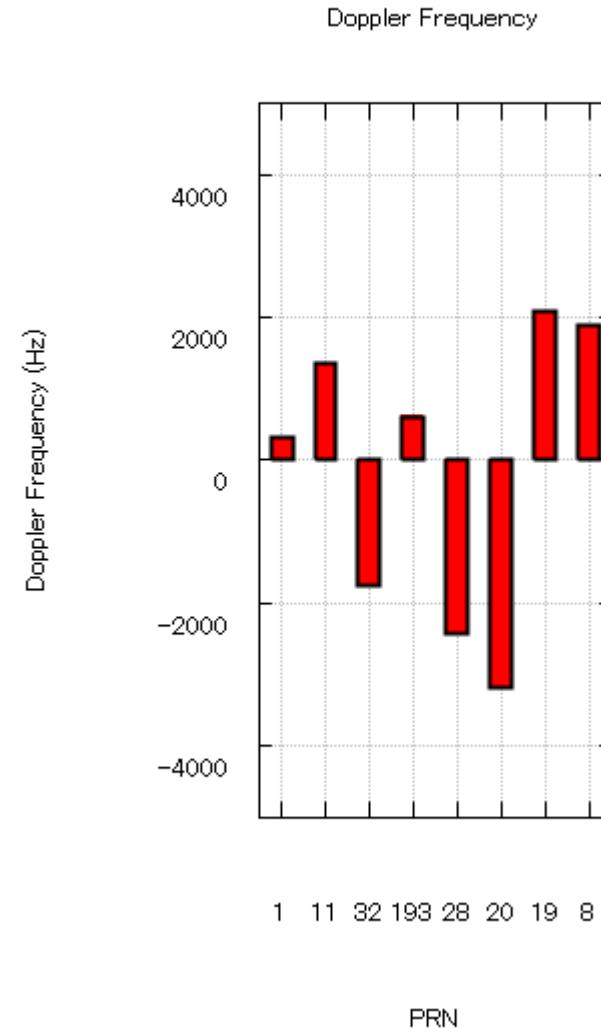
- At least about 30-40 seconds are needed to decode ephemeris.
- If decoding was OK, you can use ephemeris data of each satellite for navigation.
- **Input_ephemeris.txt** is deduced from SDR. It is similar to RINEX navigation file.

Signal Tracking

-demonstration-

- You can see several outputs.
 - Navigation Decode
 - Doppler frequency
 - Signal Strength
 - Correlation value

Doppler Frequency Check

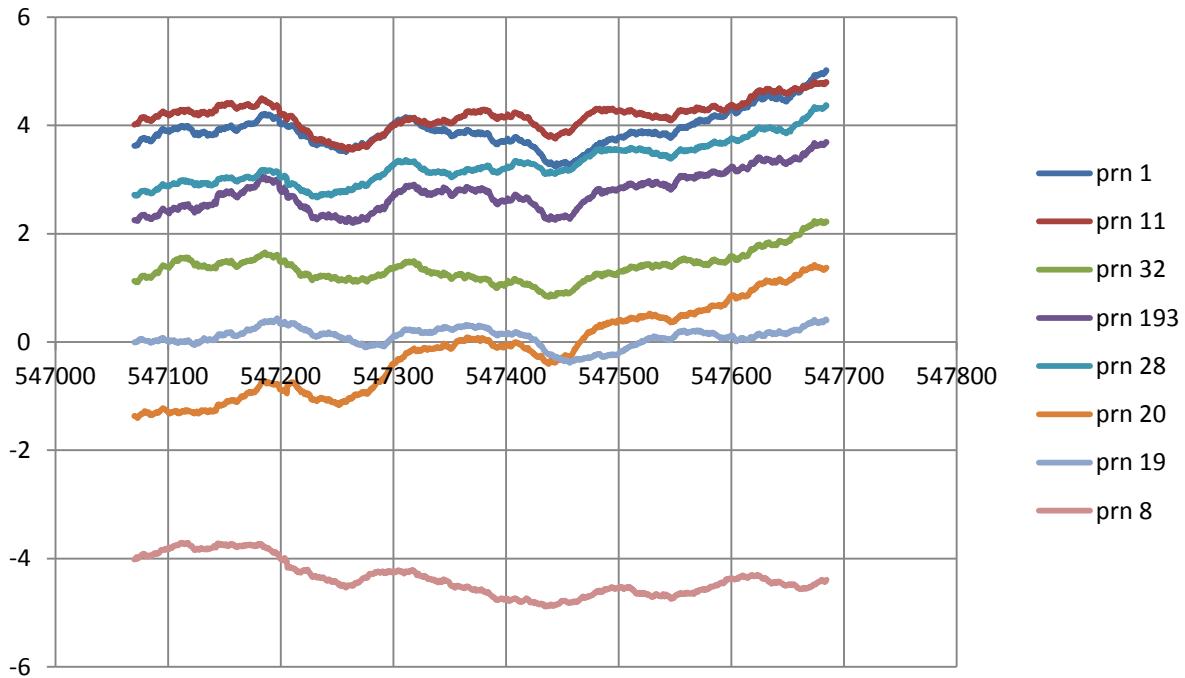


Positioning and Velocity

-demonstration-

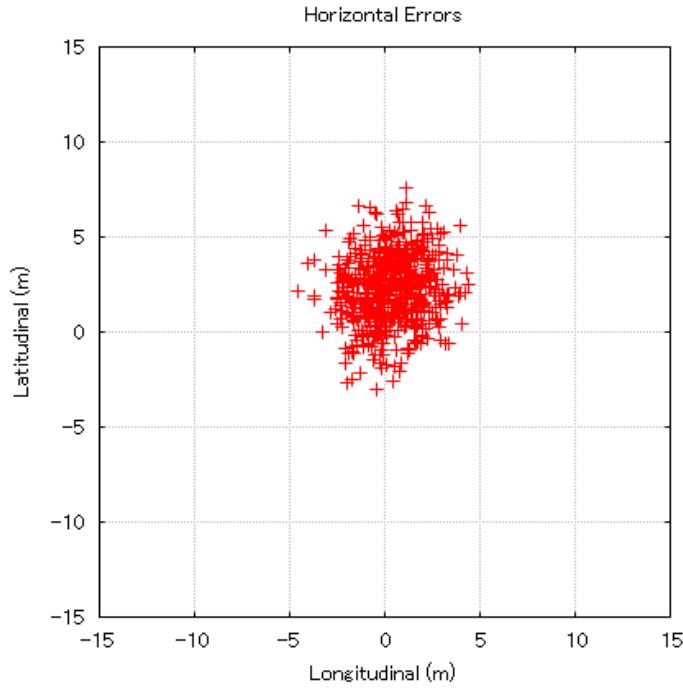
- Single point positioning
- Velocity information
- DGPS

Actual Correction Data for DGPS

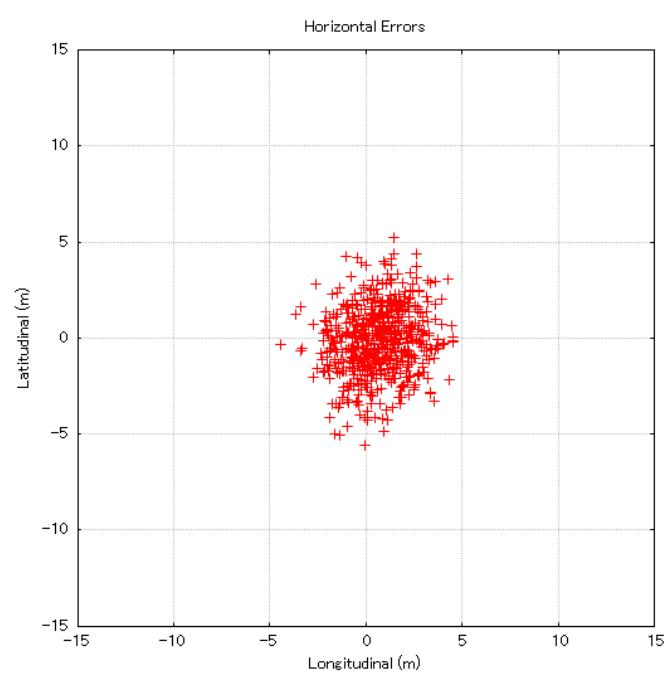


The reference data was obtained near the rover antenna using commercial receiver.

How accuracy can be improved ?



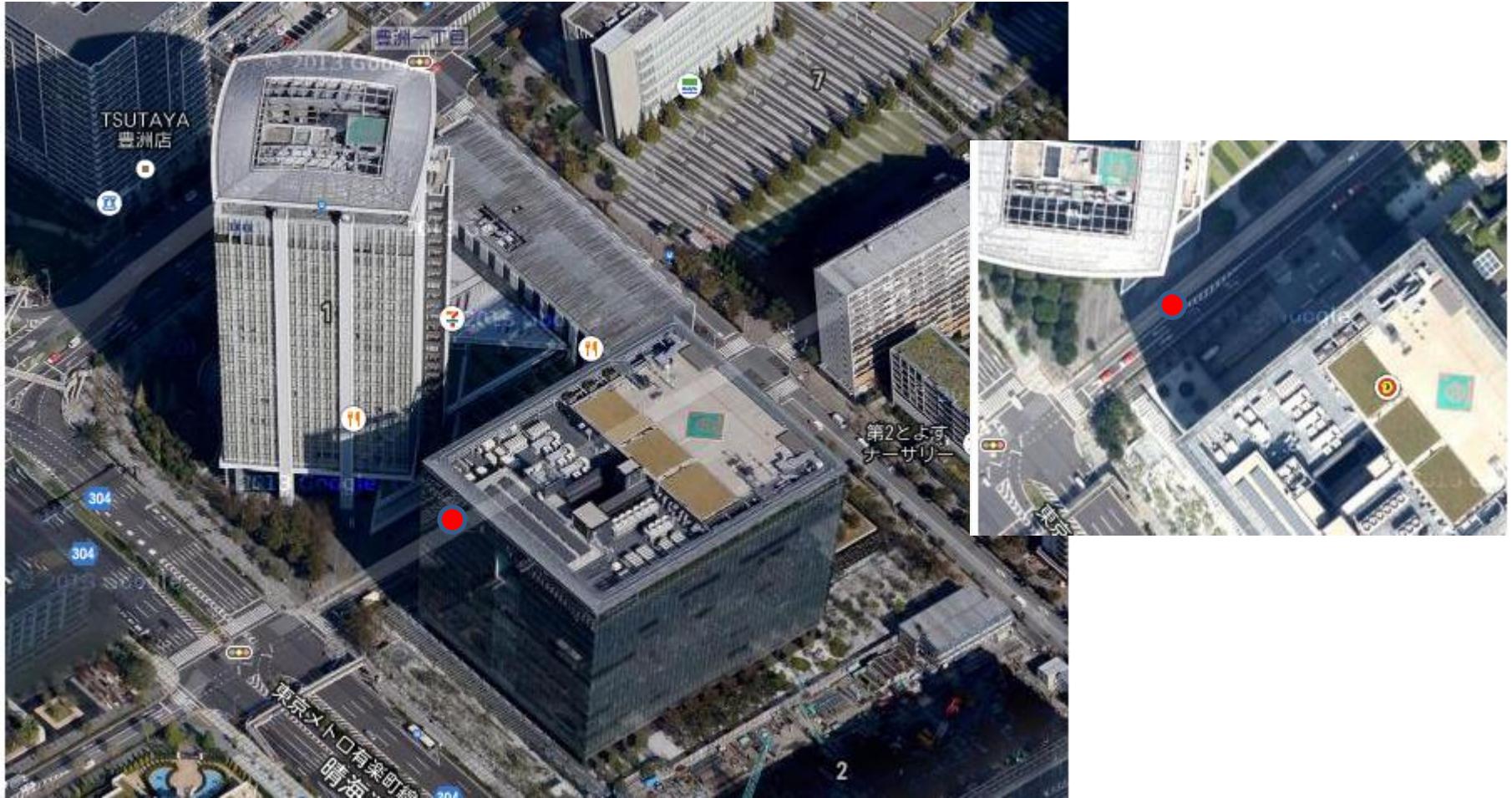
Single Point Positioning



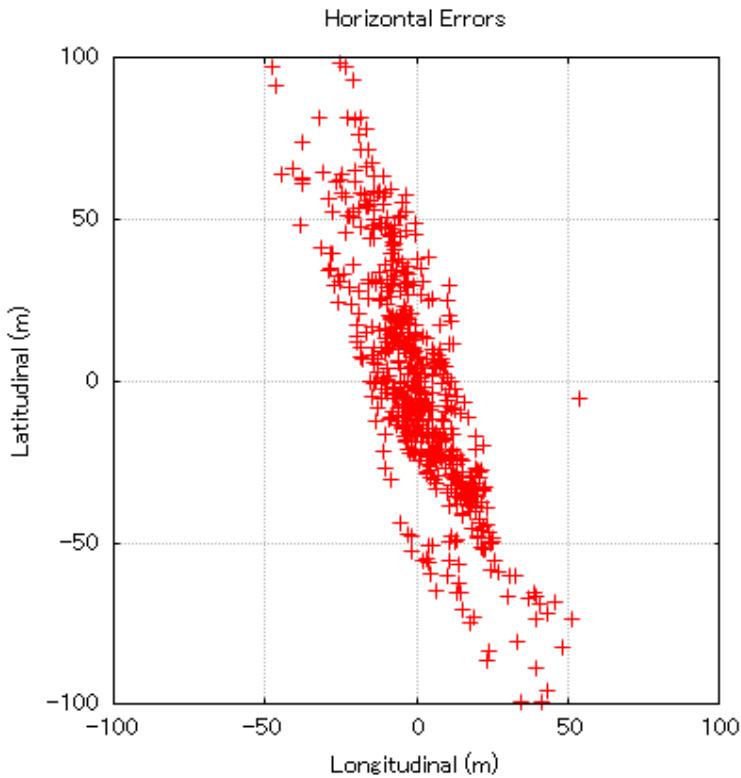
DGPS

If you use carrier phase to smooth pseudo-range, accuracy can be improved dramatically. The power of DGPS is to reduce bias.

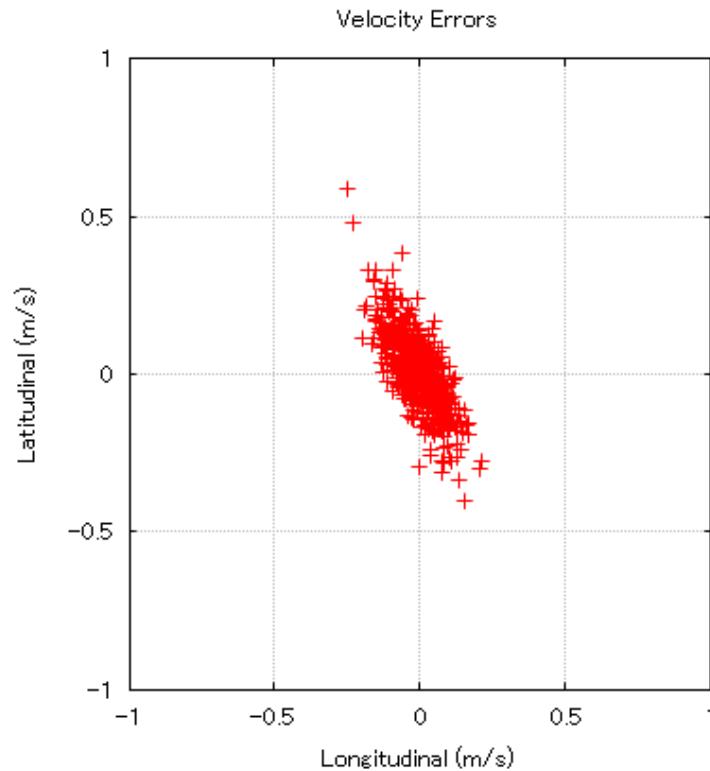
Strong Multipath Condition in Toyosu



How degree multipath and DOP affect GPS ?

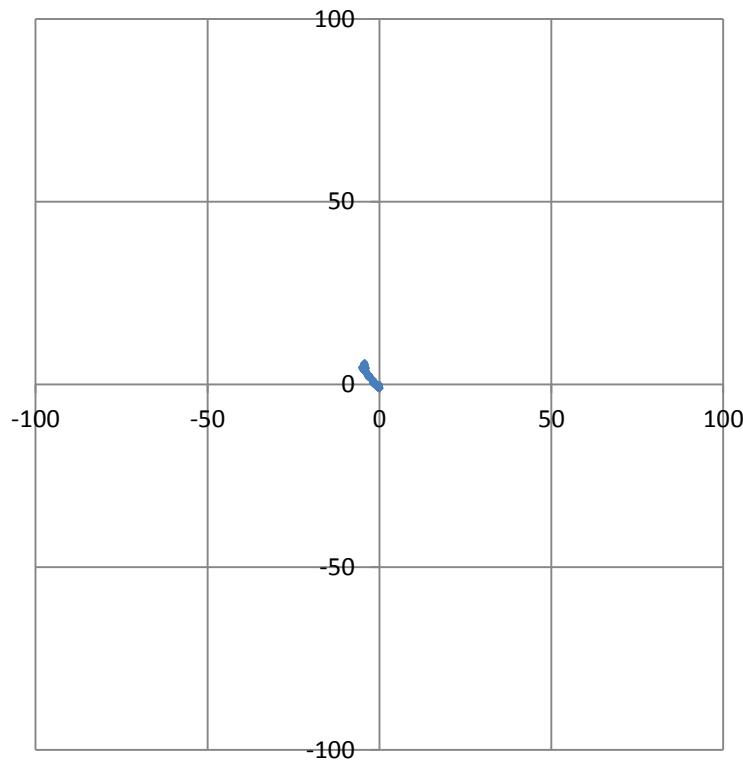


Positioning

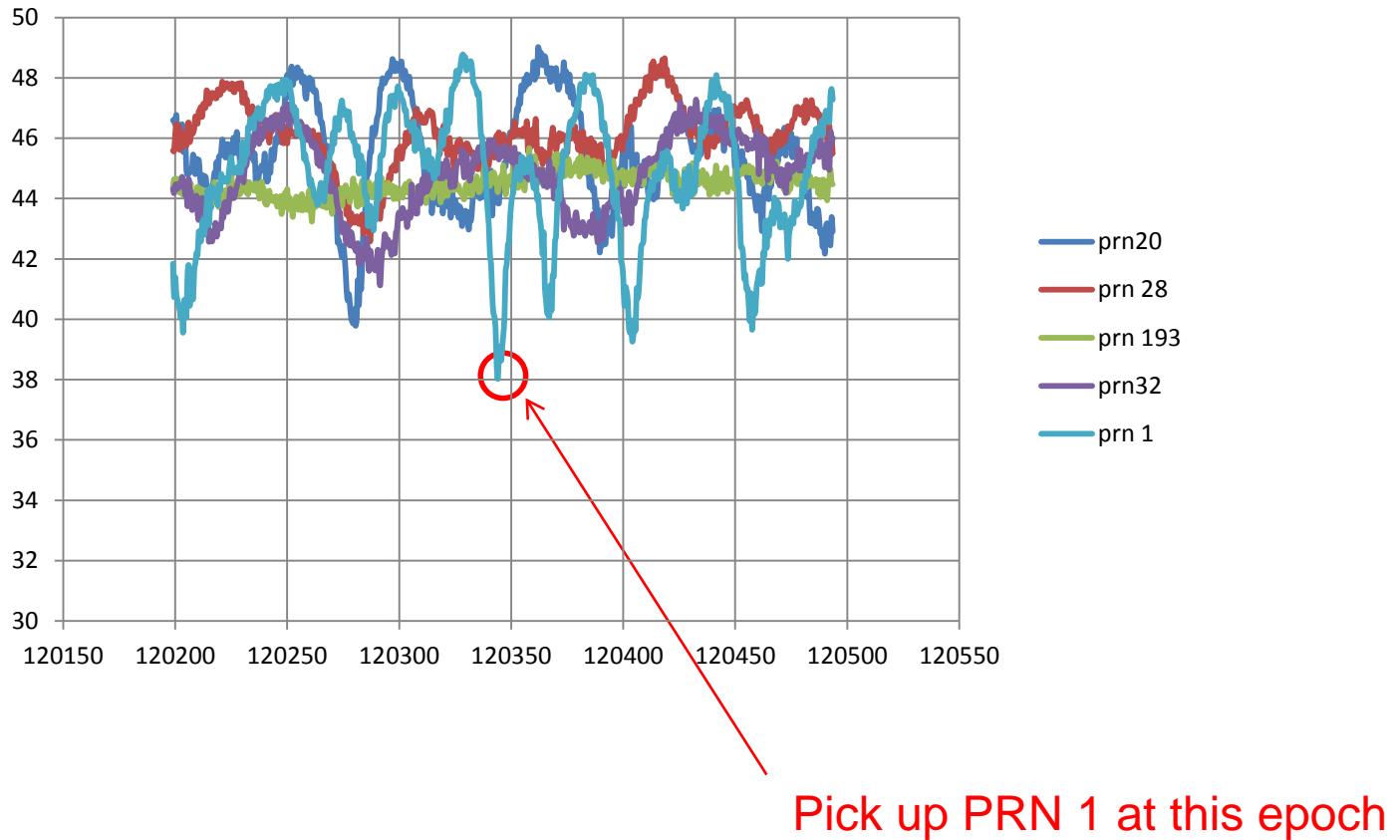


Velocity

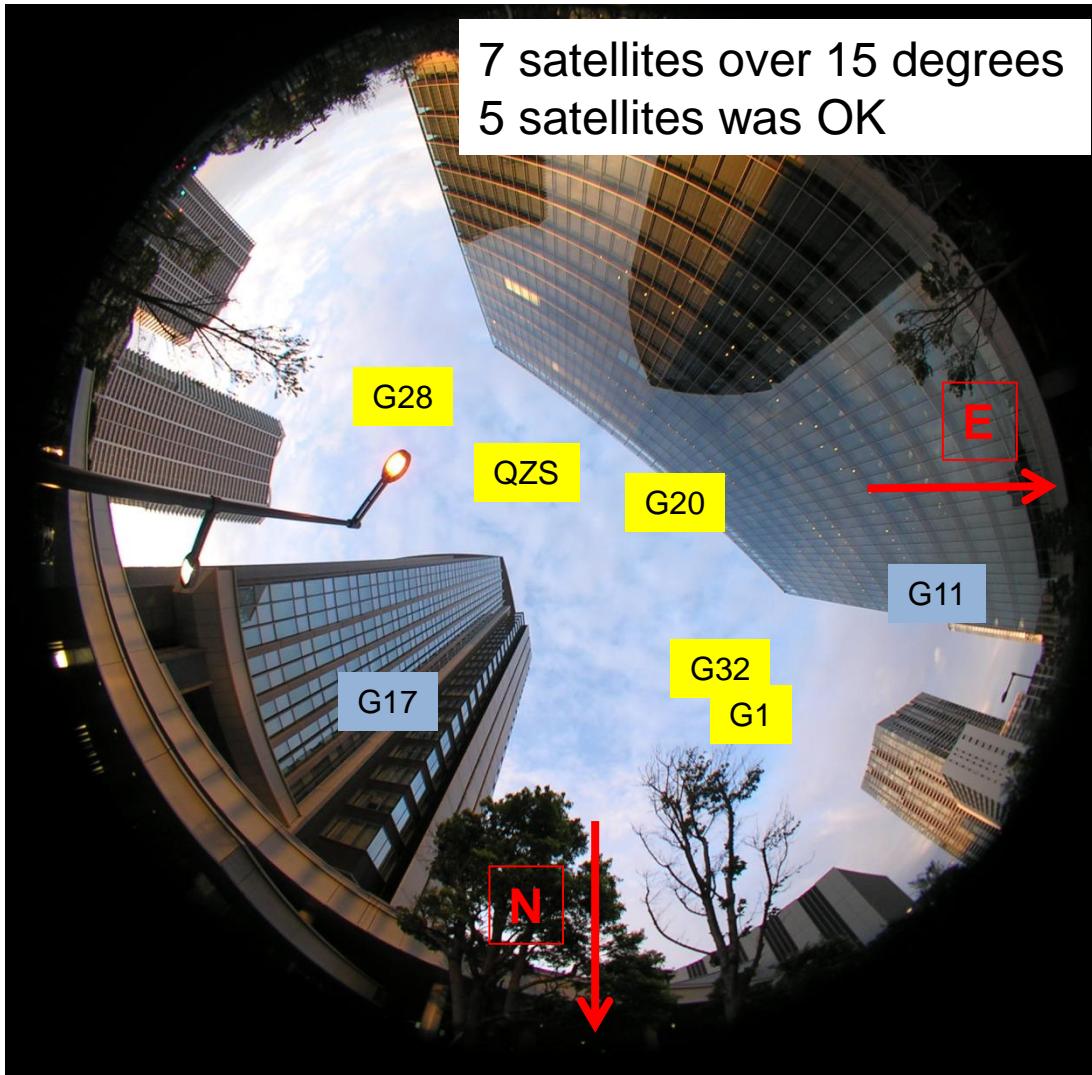
Velocity Accumulation



Multipath affects Signal Strength

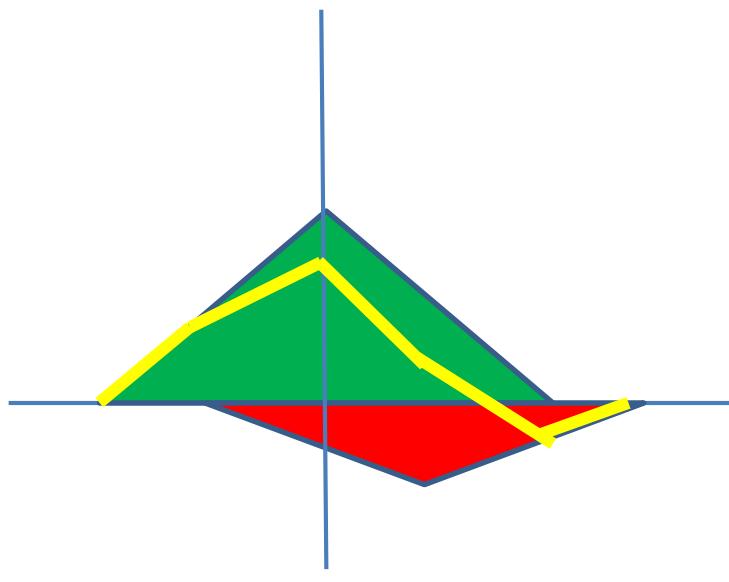
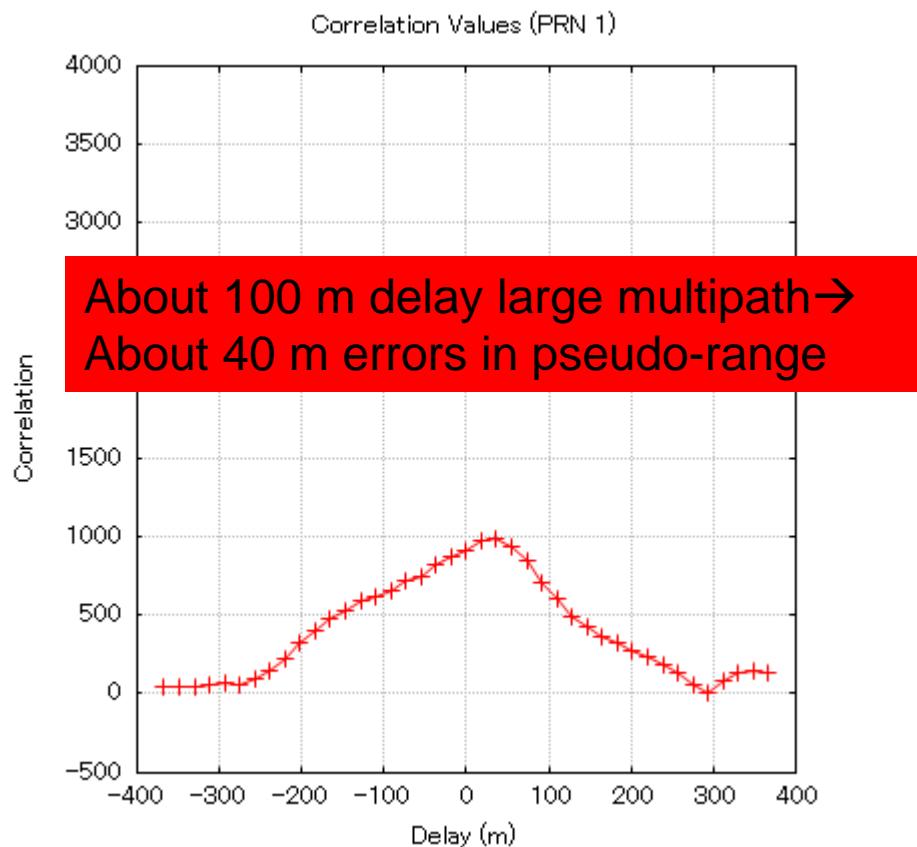


Fish Eye View



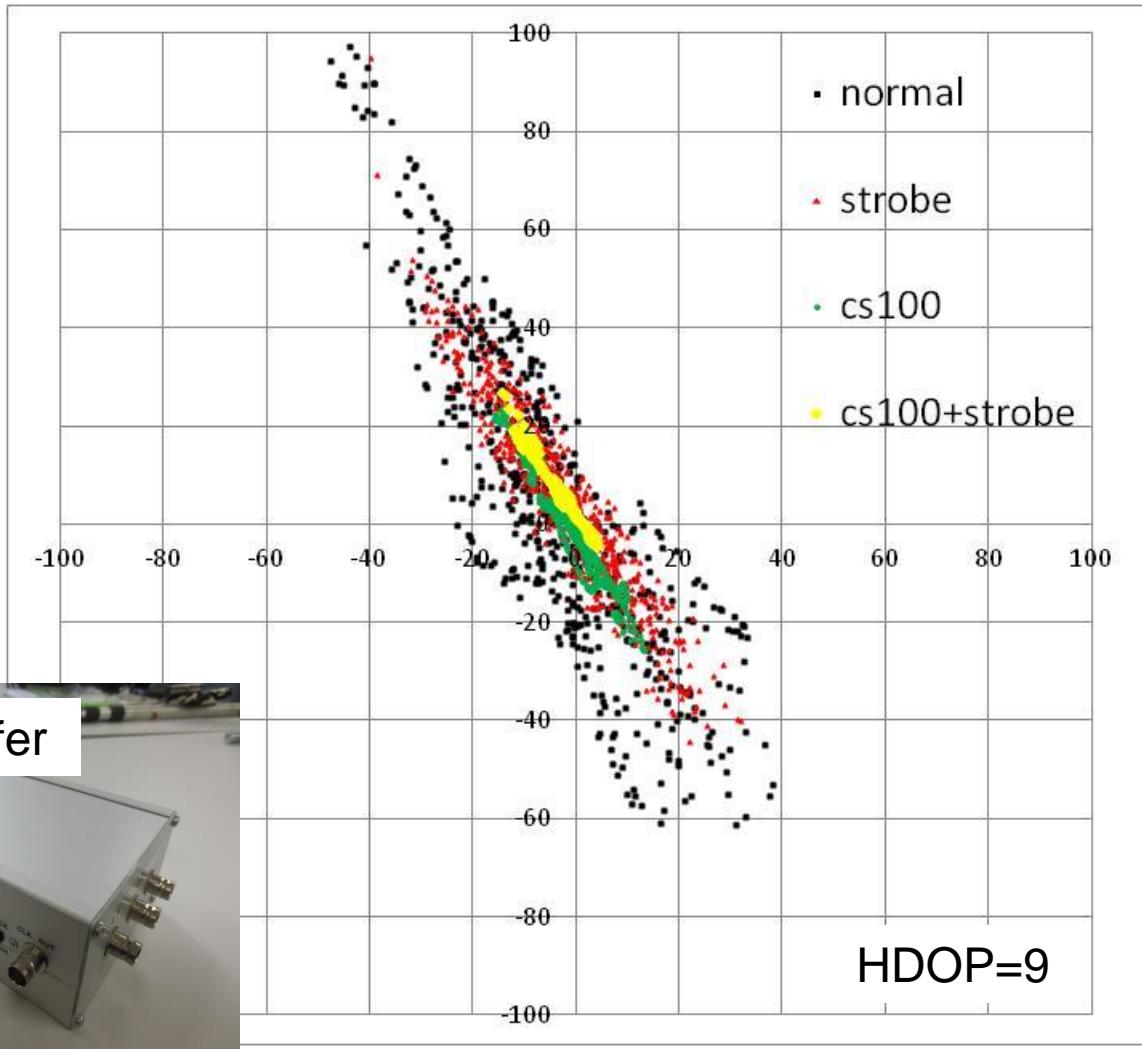
Multipath Contaminated Data

- One shot of correlation of PRN 1 in the case of large multipath.



Same period data with different FE

(BW=13MHz, SF=40.96MHz)



STD=35.8m

STD=23.8m

STD=14.1m

STD=4.3m

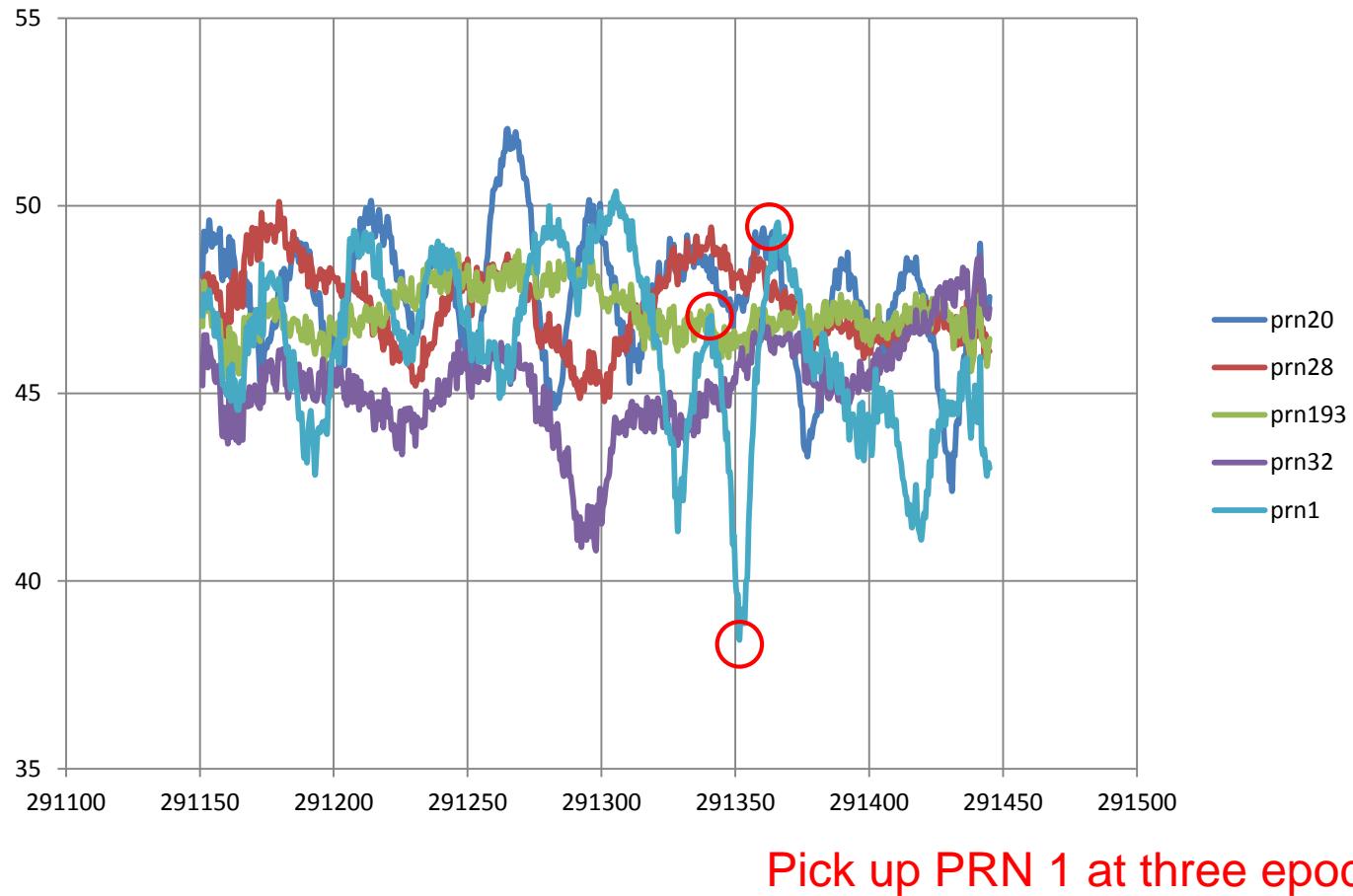
JAVAD receiver
=1.0 m

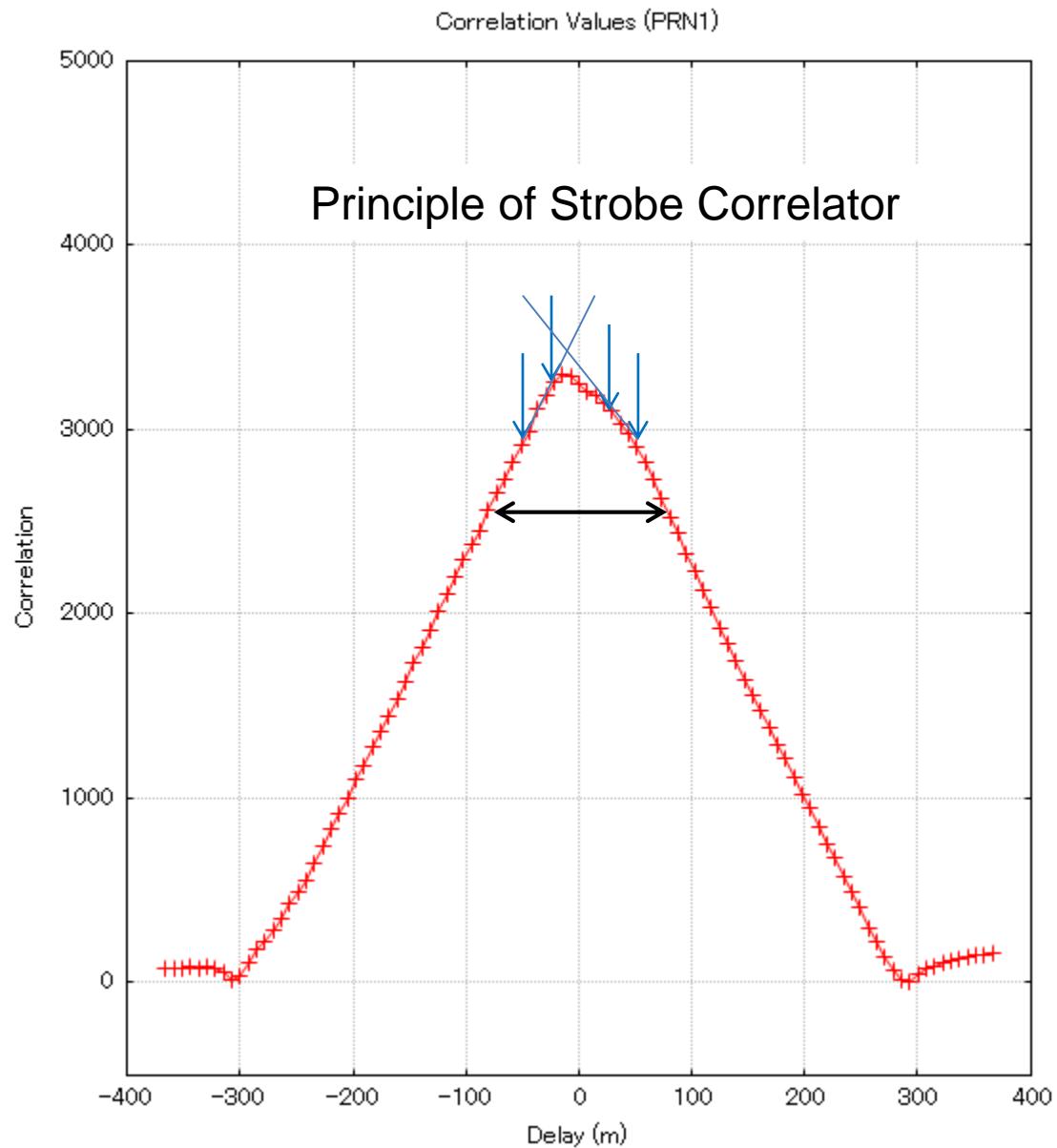


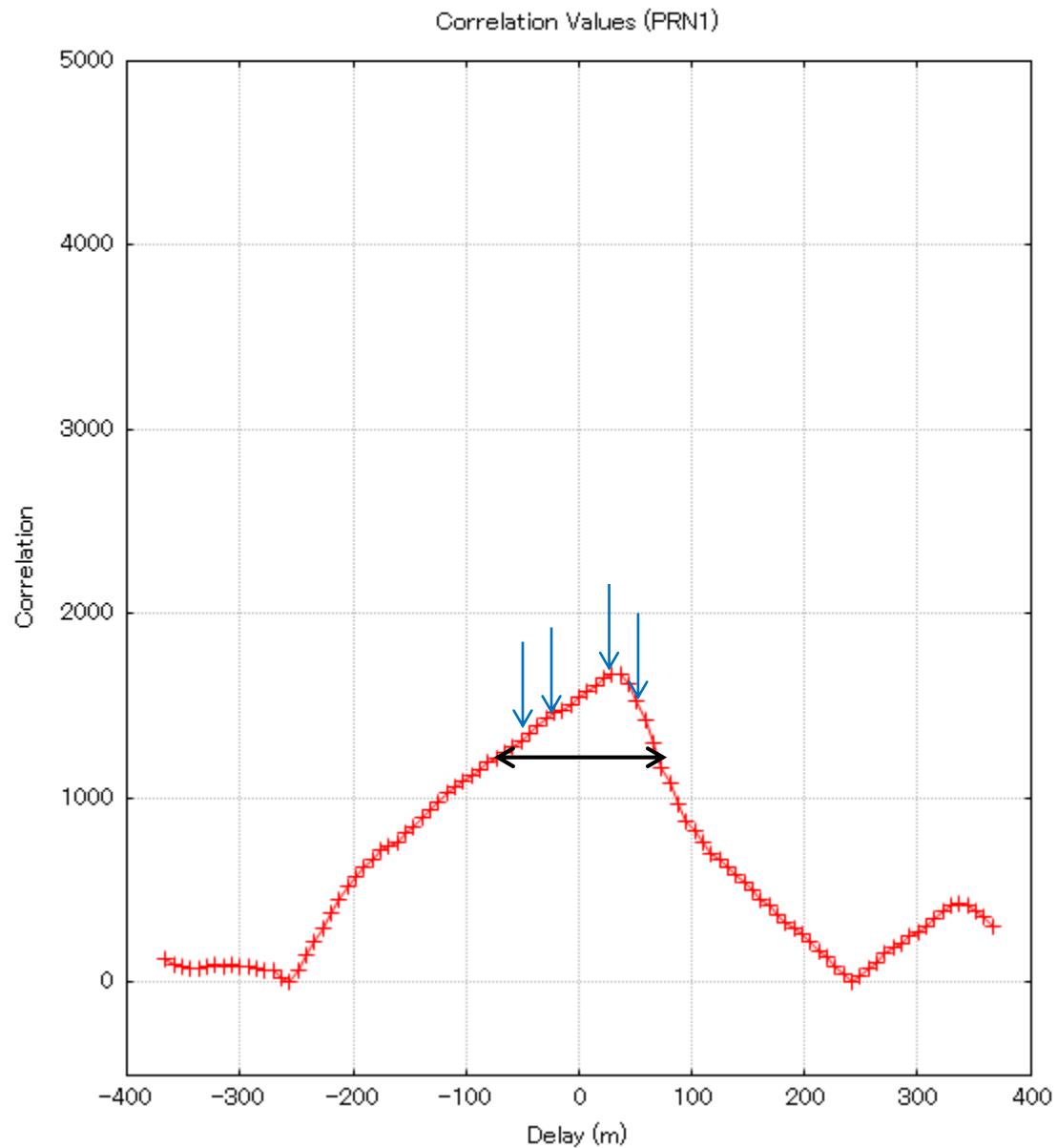
Why can we mitigate large errors ?

- As you already learned, **carrier smoothing technique** is quite effective to mitigate pseudo-range noise under static condition.
- **Strobe correlator** is still a kind of best correlator to mitigate multipath errors.

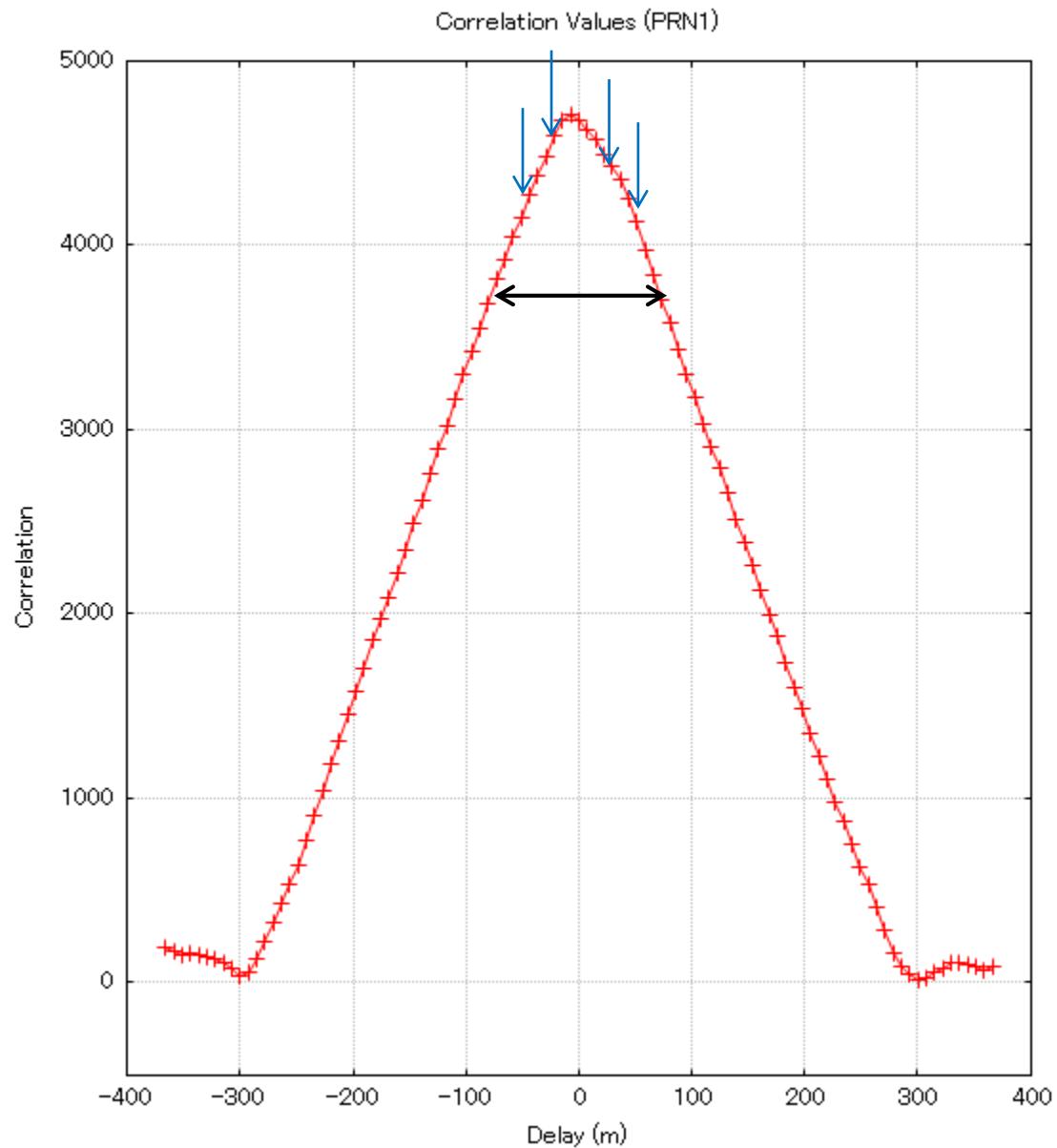
Temporal Signal Strength





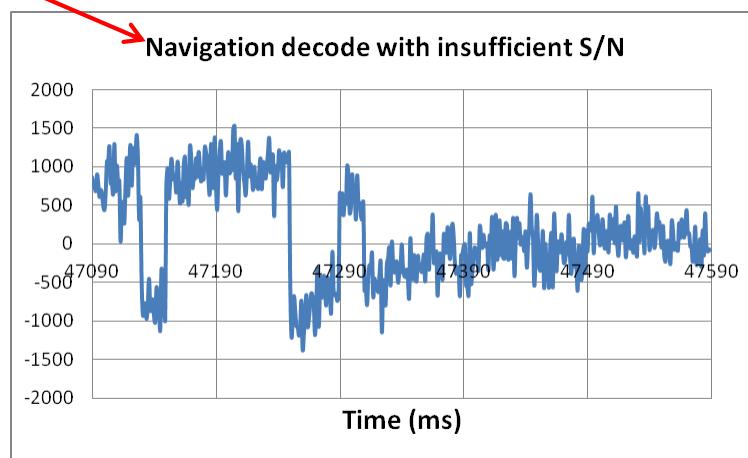
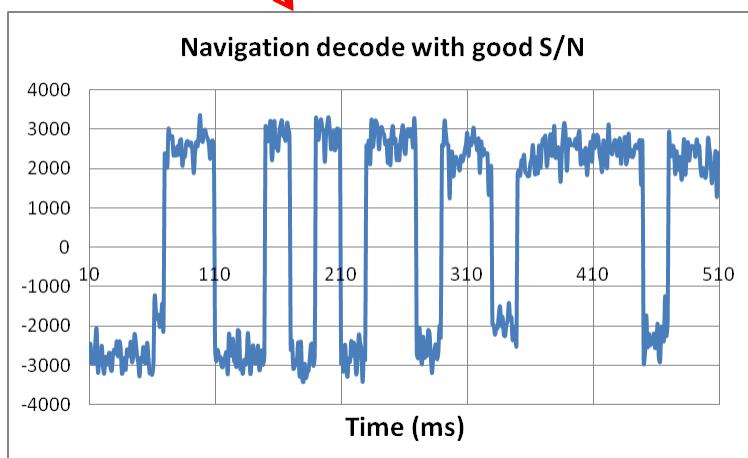
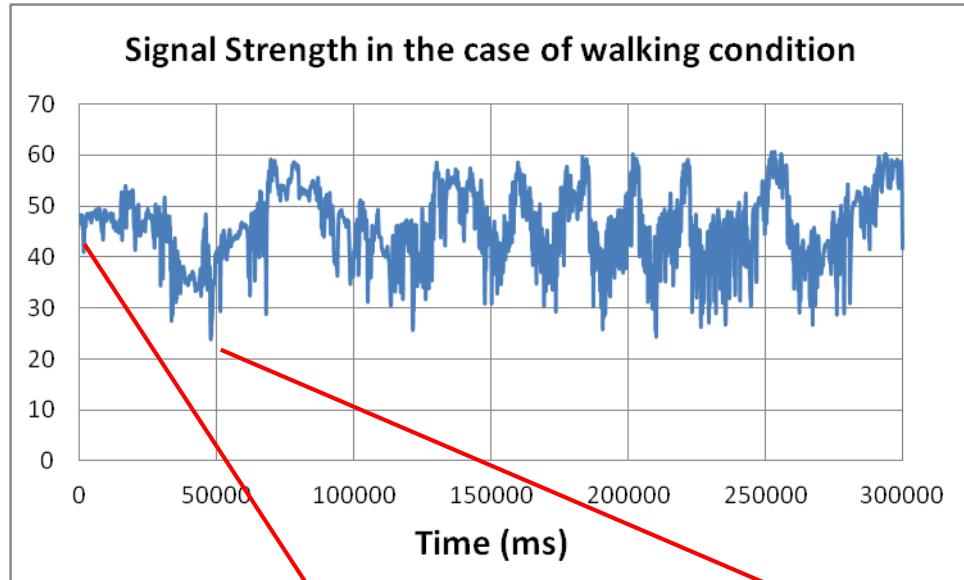


GPSTIME
=291351.5



GPSTIME
=291366.0

5 min IMES tracking in Lab.



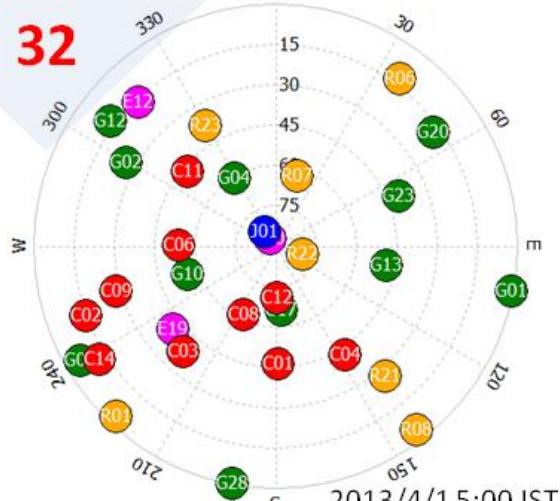
Front-end

Contents

- Coordinates System
 - Satellite Position
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- 1st period
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- Measurements Errors
 - Calculating Position and DOP
 - Improved Position
- }
- 2nd period
-
- Basics of GNSS receiver
 - Multi-GNSS
- }
- 3rd period

Satellites position at TOKYO

Green:GPS, Blue:QZSS



Yellow : GLONASS

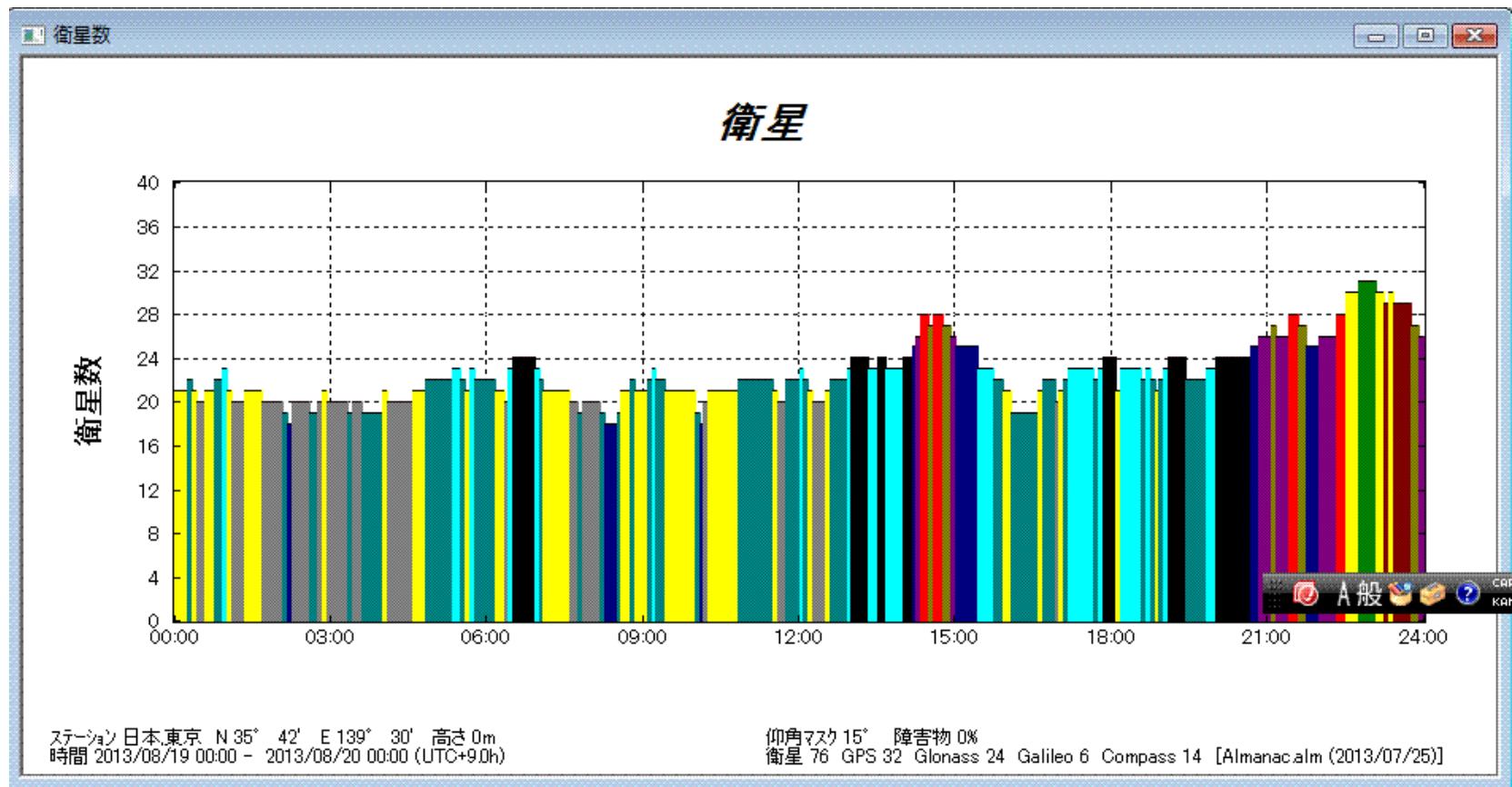
Red : BeiDou

Pink : Galileo

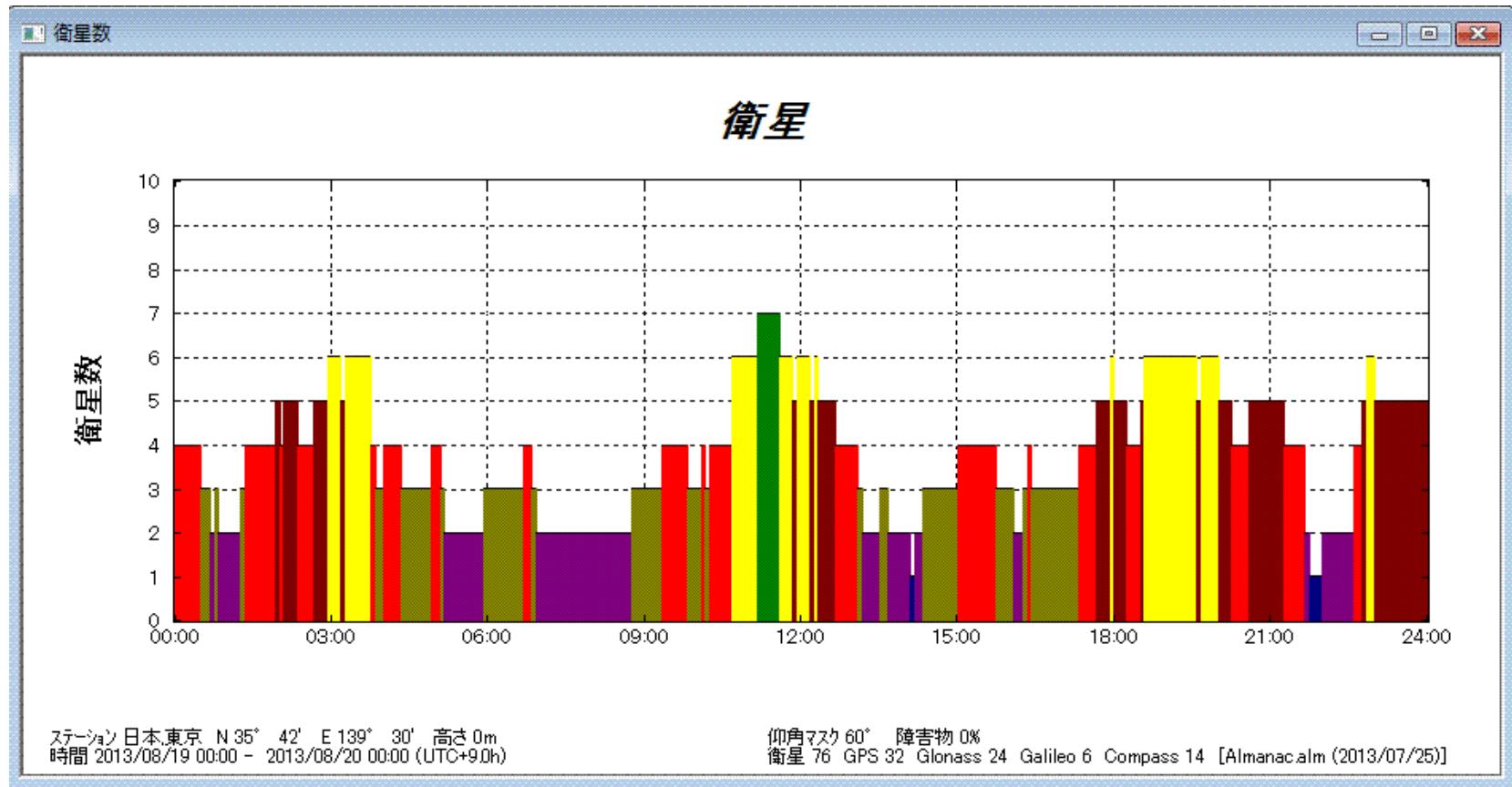
Current Constellation

- GLOANSS has been operated for long time like GPS, however it is a little hard to use due to FDMA.
- BEIDOU and GALILEO have been just available now !
- Number of visible satellite increases from 12 to 30 if we use all navigation satellites.

24-hour number of visible satellites (GPS/GLO/GAL/BeiJing mask=15°)

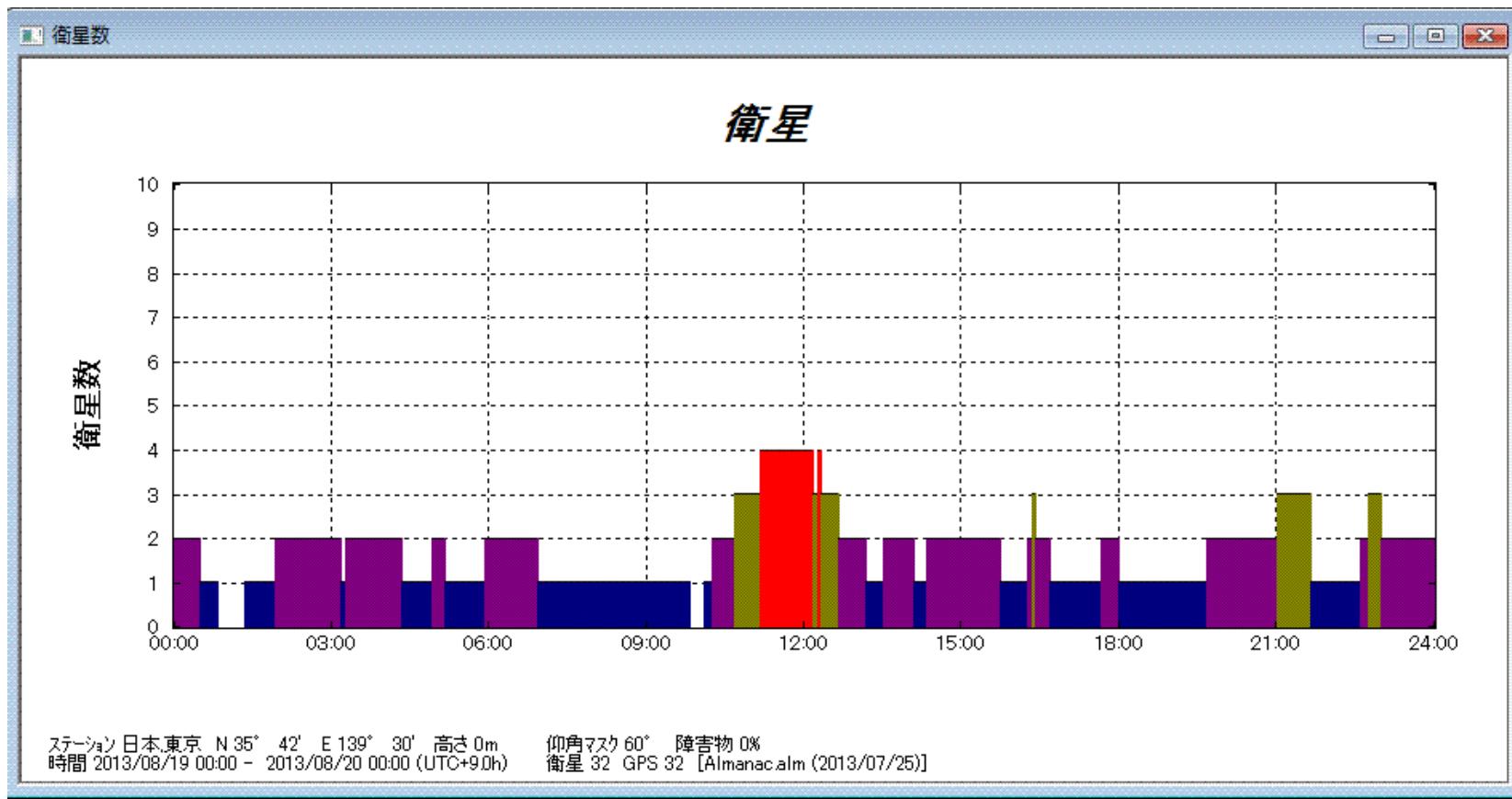


24-hour number of visible satellites (GPS/GLO/GAL/BeiJing mask=60°)

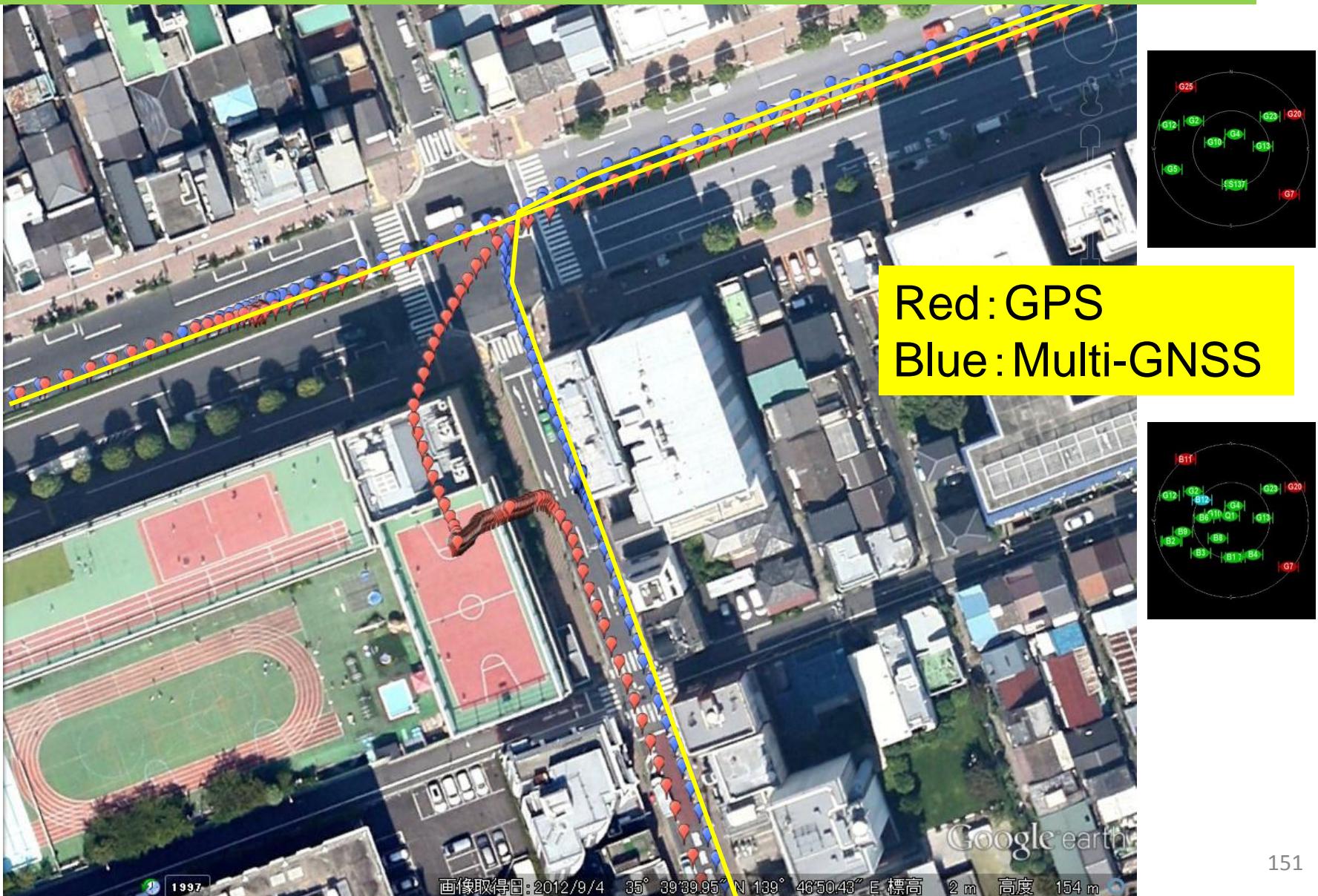


What if we have 4 QZS satellites ?

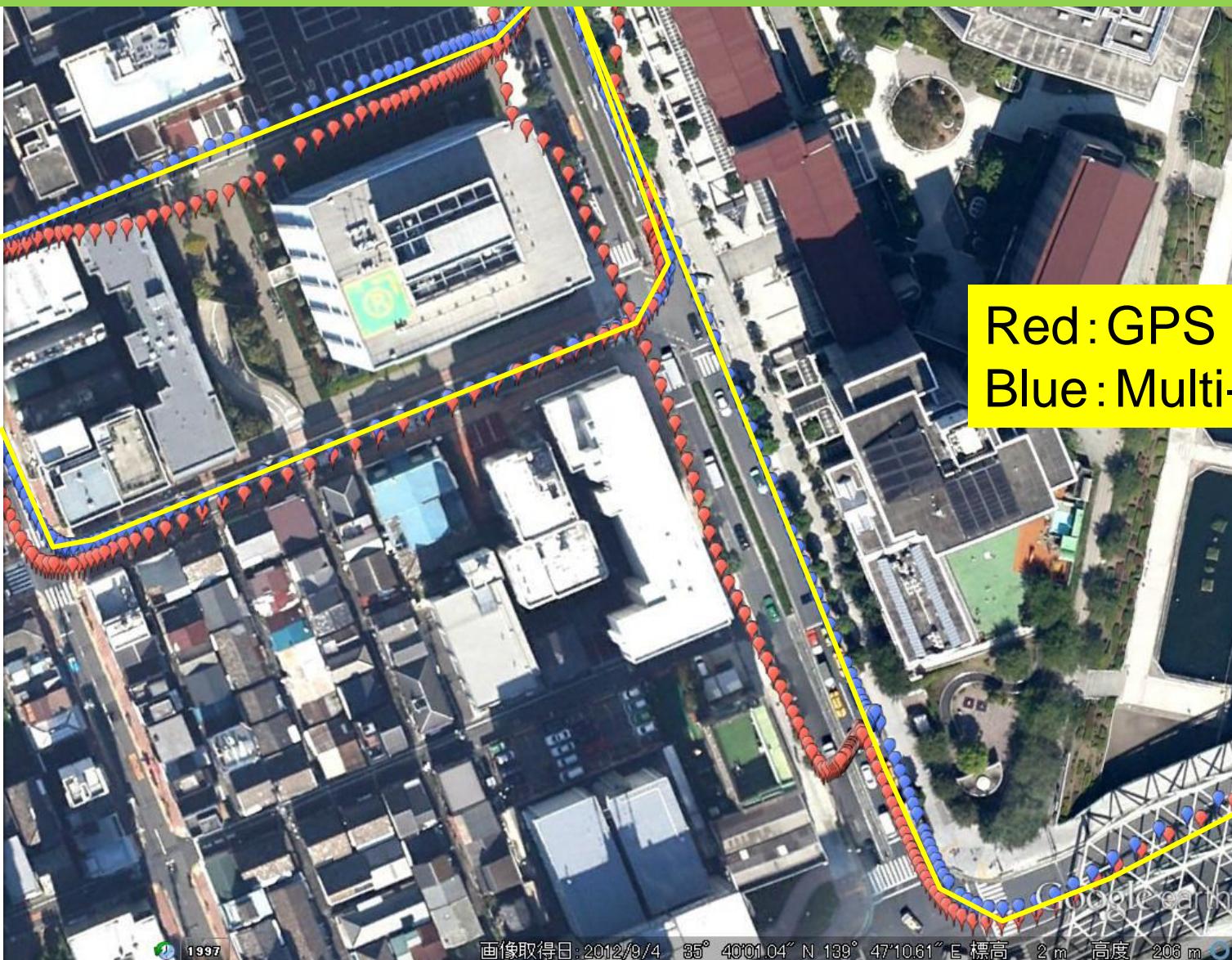
24-hour number of visible satellites (GPS mask=60°)



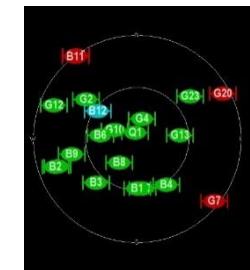
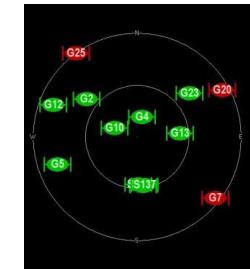
Low-cost commercial receiver comparison (ublox 5 and 8) Semi-urban with narrowed road



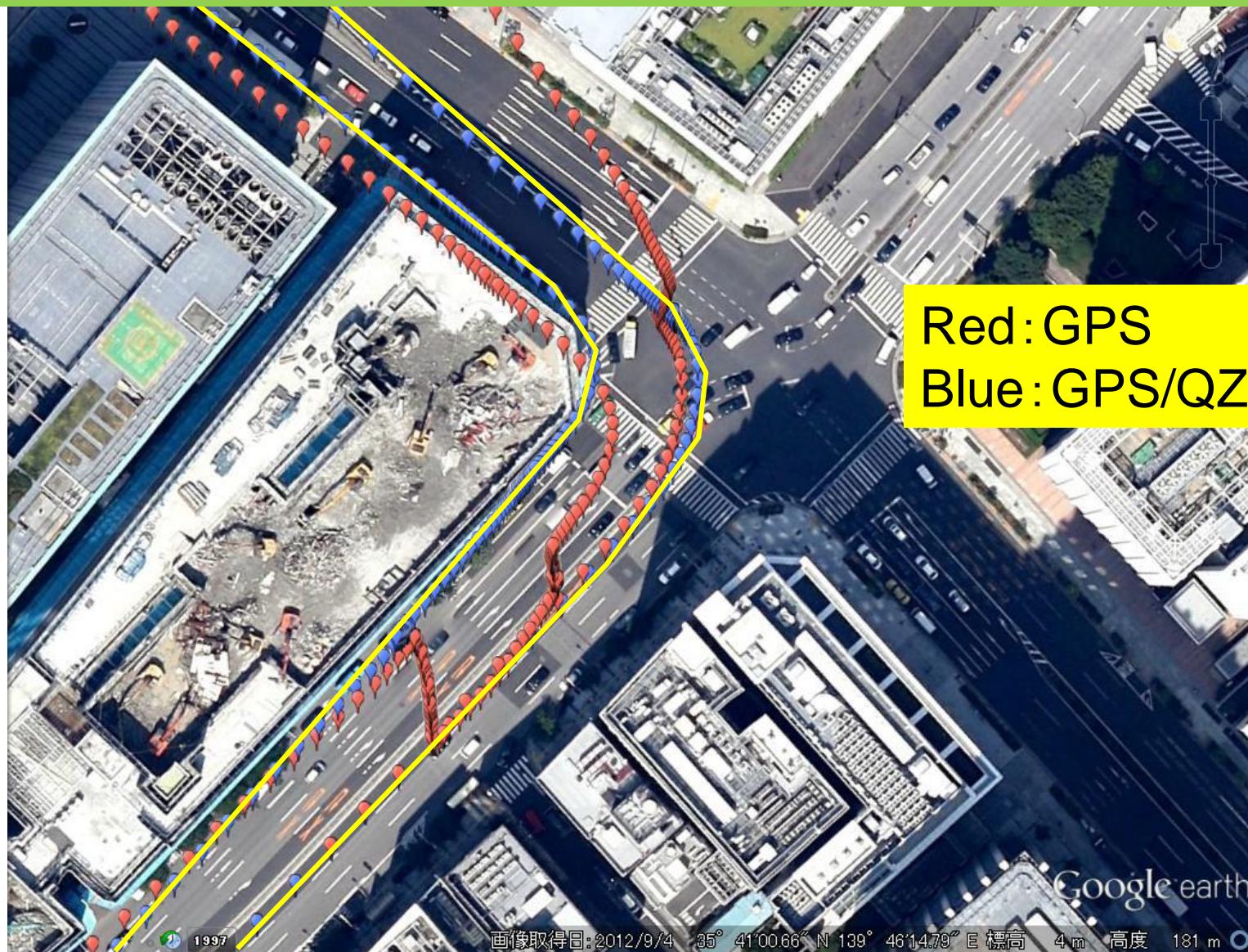
Low-cost commercial receiver comparison (ublox 5 and 8) Dense-urban with narrowed road



Red : GPS
Blue : Multi-GNSS



Low-cost commercial receiver comparison (ublox 5 and 8) Dense-urban with wide road

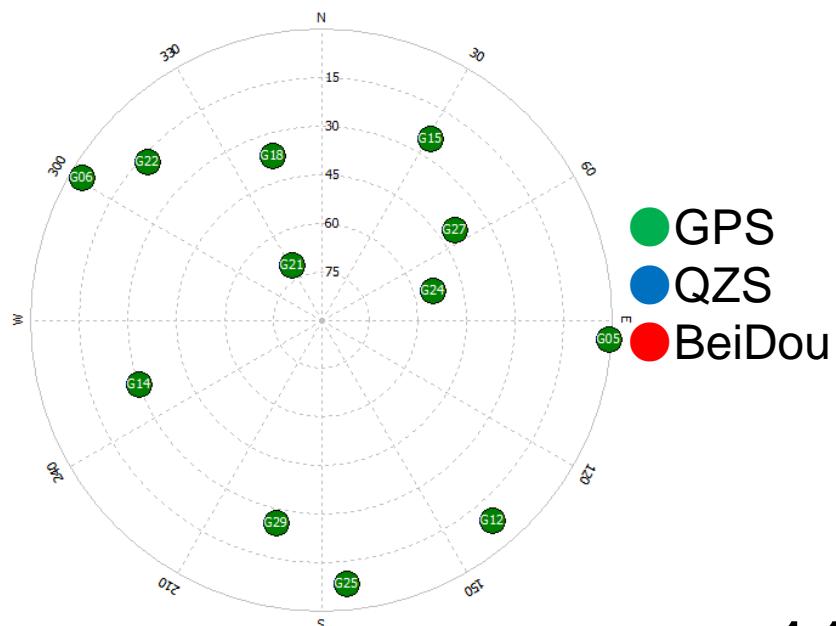


Bangkok Downtown

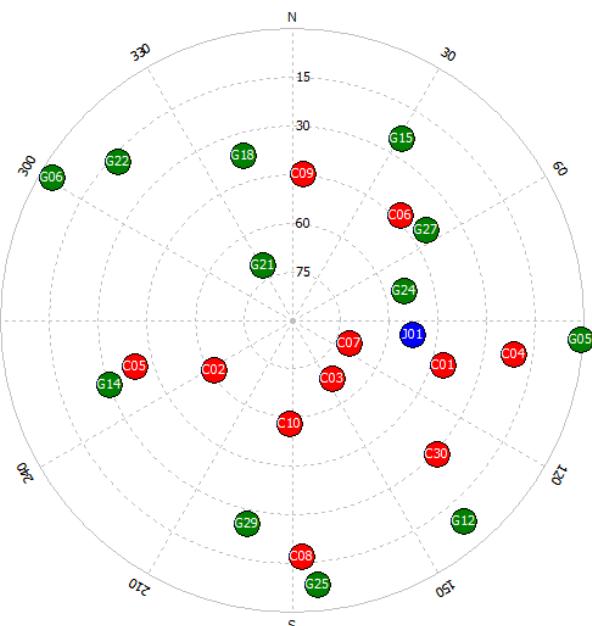


Constellation

GPS

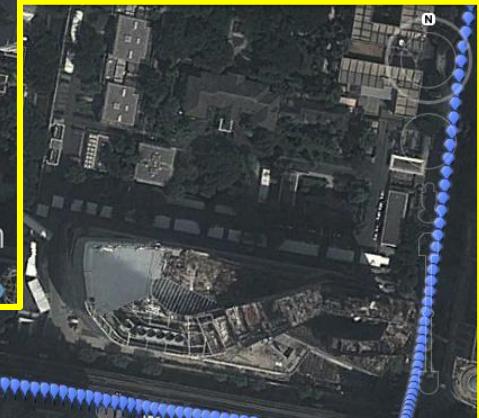
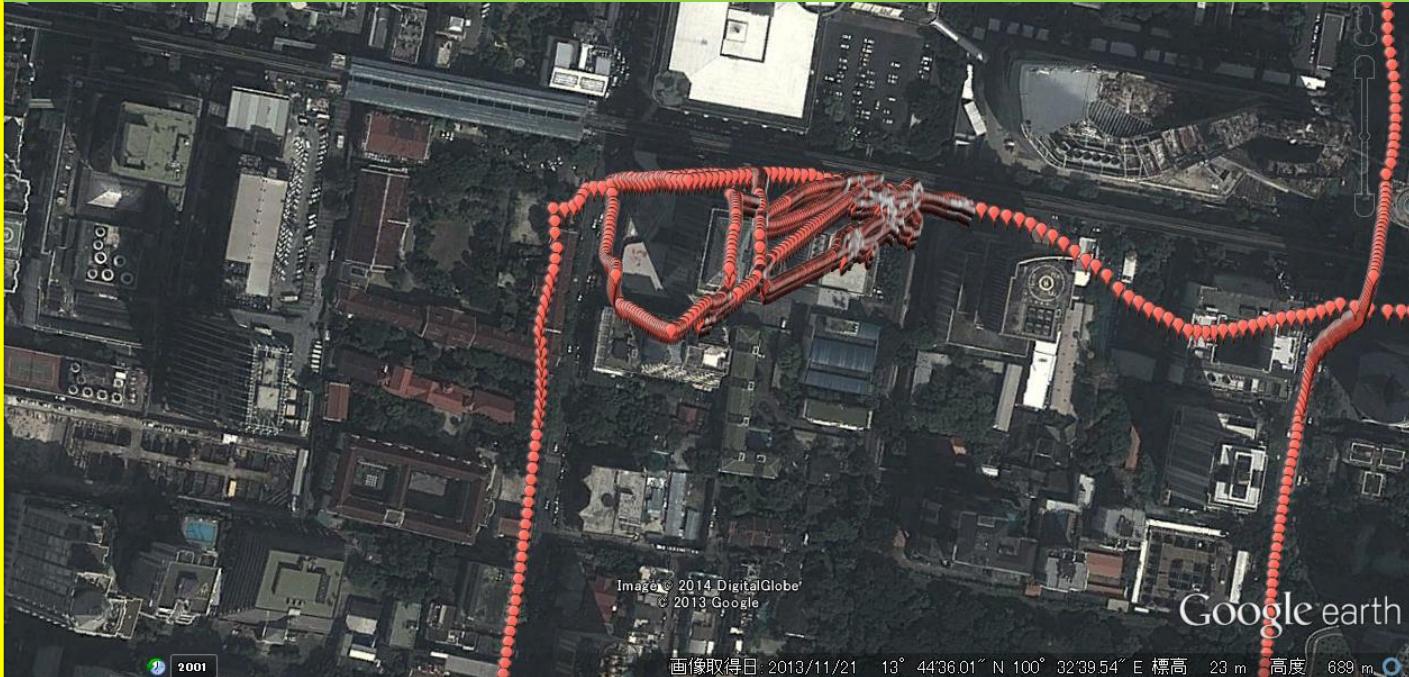


GPS/QZS/BeiDou



12 → +11 → 23

Low-cost commercial receiver comparison (ublox 5 and 8) Dense-urban with narrowed road and under an overpass



● GPS

● GPS/QZS/BeiDou



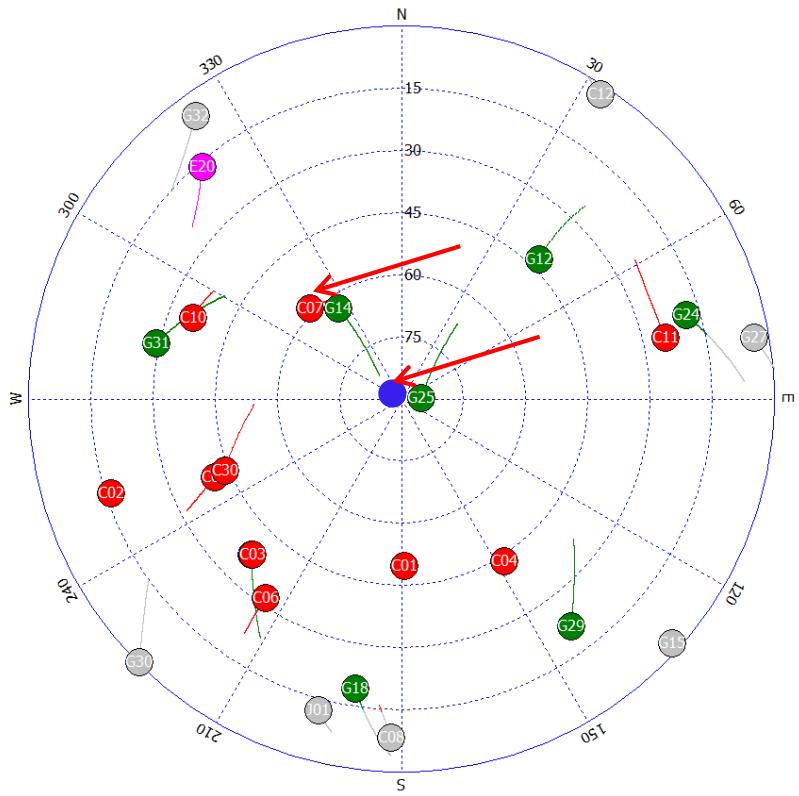
Low-cost commercial receiver comparison (ublox 5 and 8) Dense-urban with narrowed road



RTK TEST using GPS/QZS/BeiDou

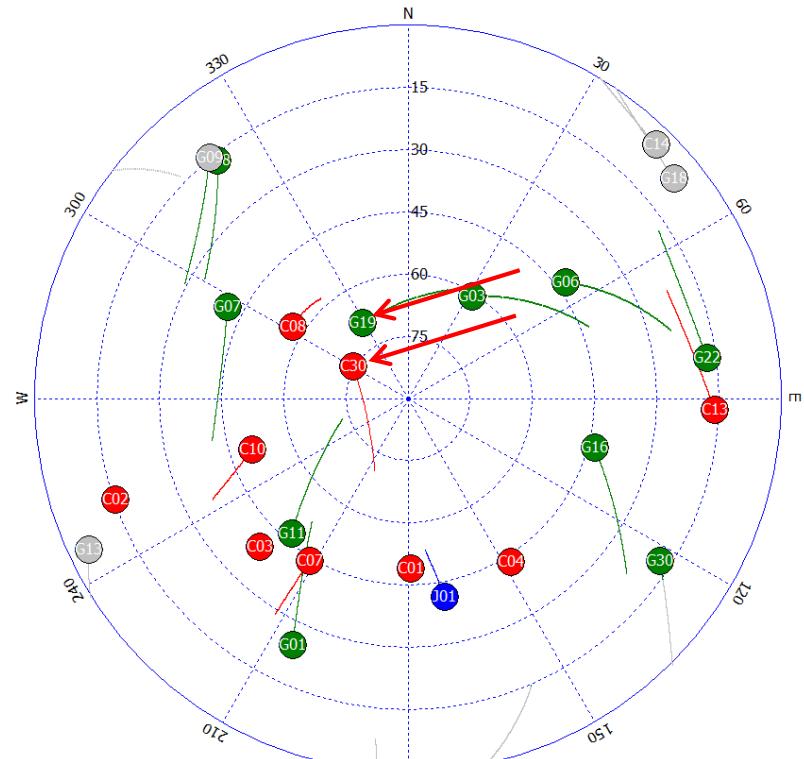
- The data was obtained in 10/2/2013 at two places.
- **Tsukishima (sub-urban), Yaesu (dense-urban)**
- The rover antenna was installed on the rooftop of the car. Reference antenna was installed at Kaiyodai.
- Receivers : Trimble NetR9 (Ref/Rover)
- No smoothed pseudo-range and no filter
- Single-epoch RTK
- Mask angle and SNR threshold were set.
- POS/LV was used as a reference system

Satellite Constellation



Kaiyodai-Tsukushima (40min.)

15:00-15:40



Kaiyodai-Yaesu (60min.)

10:00-11:00

Reference satellite

Test Course

Yaesu

Start (Kaiyodai)

Tsukishima



Results in semi-urban area

	FIX rate	Maximum Interval without fix	Percentage below 0.5m (Horizontal)
GPS	21.7 %	195 s	99.96 %
GPS/QZS	39.8 %	176 s	99.73 %
GPS/QZS/BeiDou	71.6 %	60 s	99.85 %

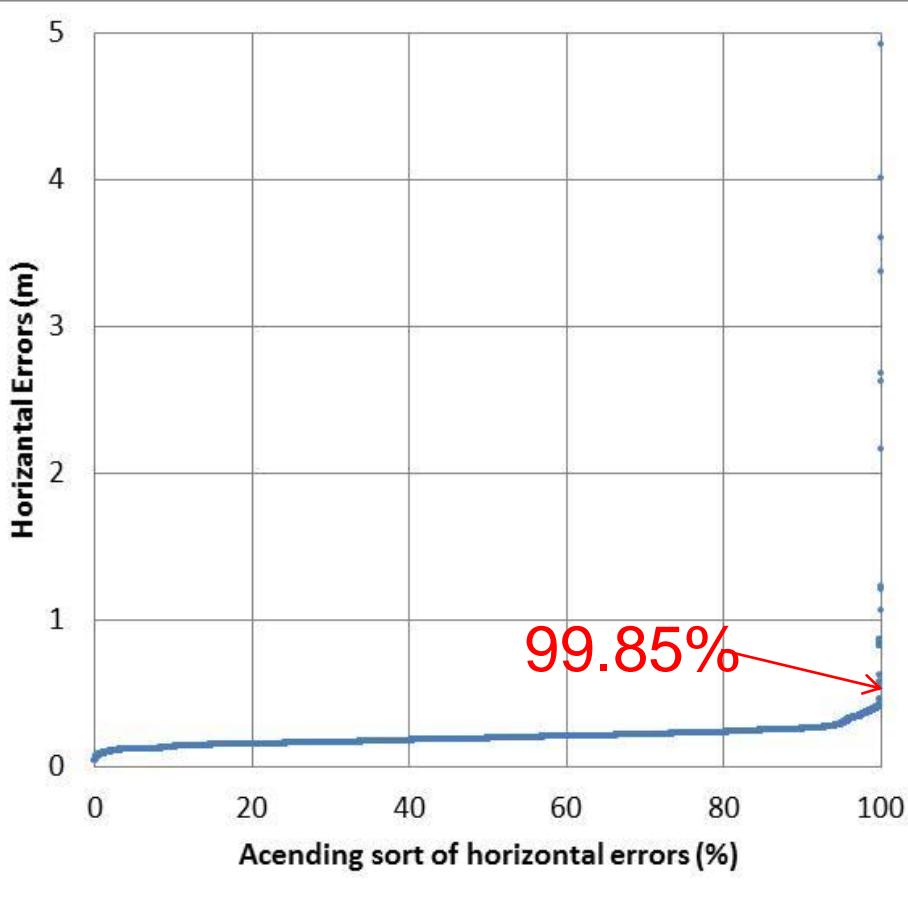
“POS/LV” assures 20-30 cm errors under this route condition
“60 s” interval happened under the elevated road

Results in dense-urban area

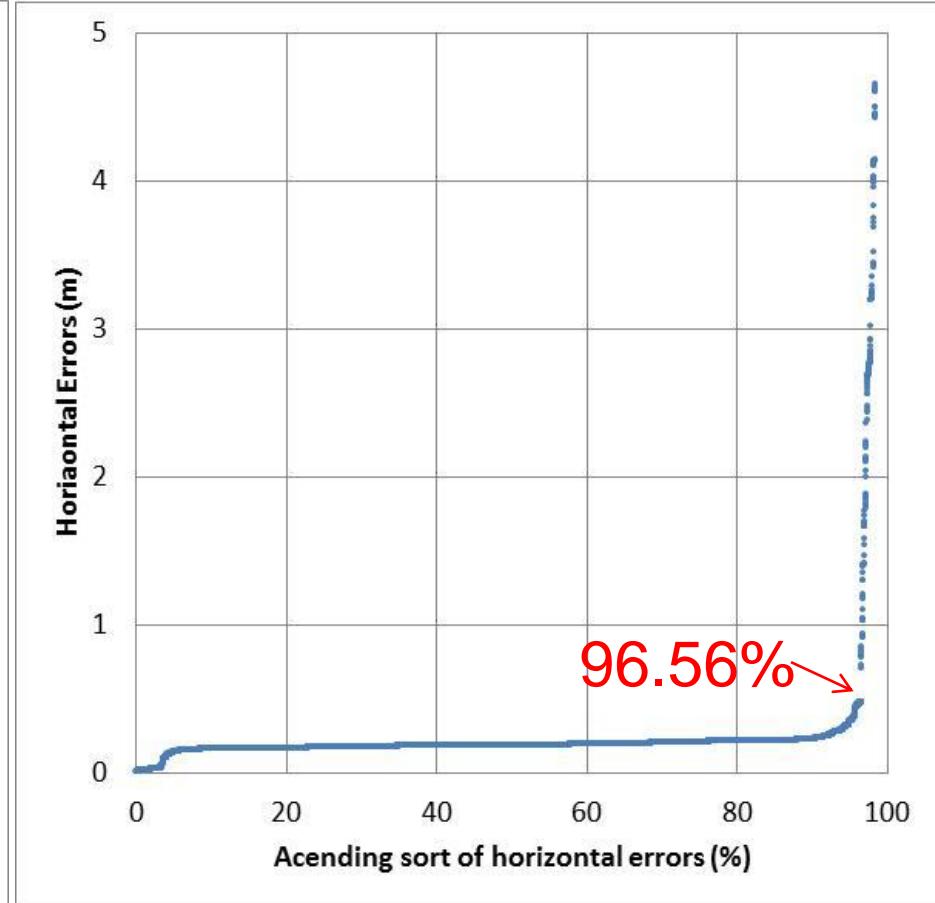
	FIX rate	Maximum Interval without fix	Percentage below 0.5m (Horizontal)
GPS	22.0 %	416 s	99.74 %
GPS/QZS	27.1 %	415 s	99.80 %
GPS/QZS/BeiDou	33.1 %	128 s	96.56 %

“POS/LV” assures 20-30 cm errors under this route condition

All horizontal errors of fix solutions (ascending sort)



Tsukishima (semi-urban)

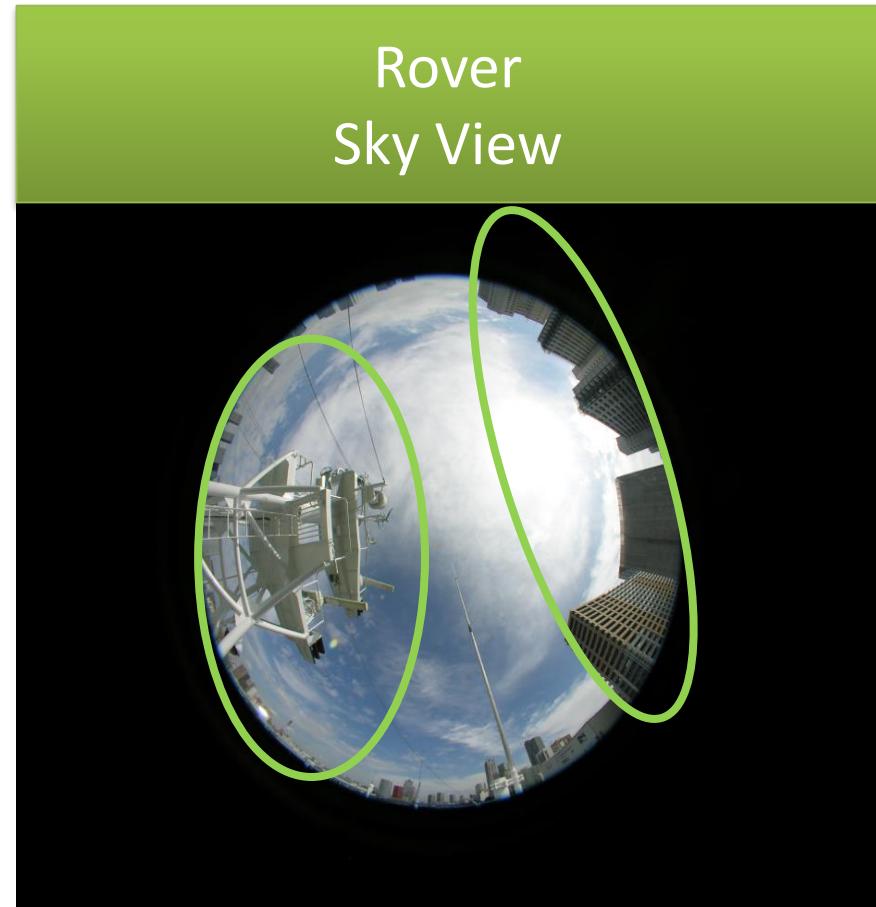


Yaesu (dense-urban)

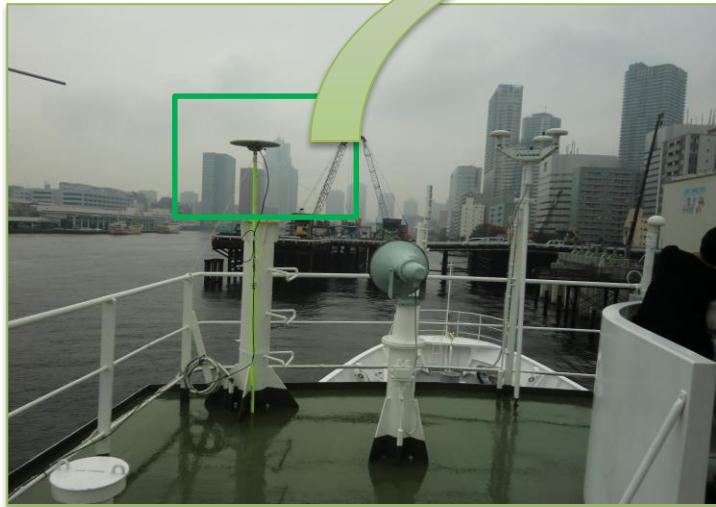
RTK Test at Ship

- The data was obtained in summer 2013
- Reference was installed at Kaiyodai
- Rover was installed at ship
- RTK-GPS/QZS/BeiDou

The condition at rover side was not perfect. There are some buildings and steel tower of ship itself.



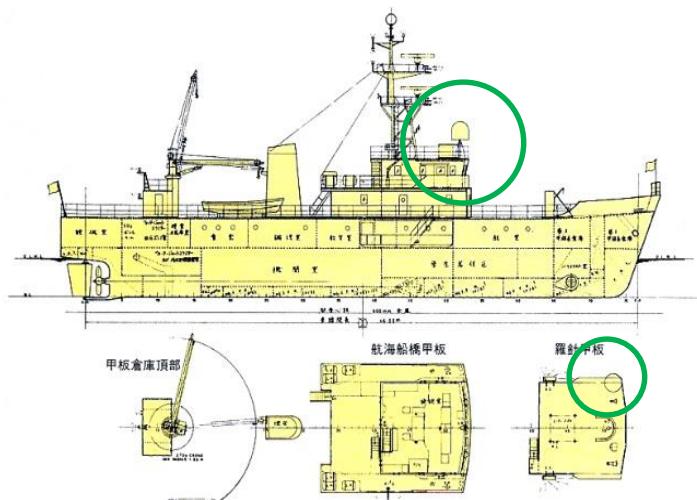
Antenna Installation



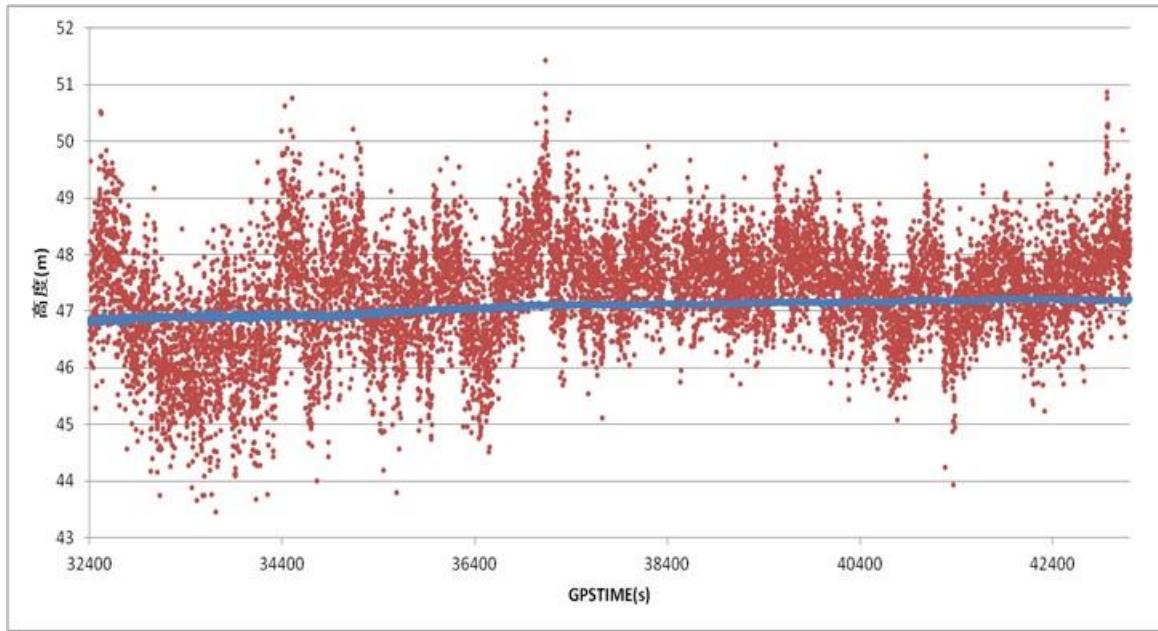
Trimble Zephyr Geodetic 2



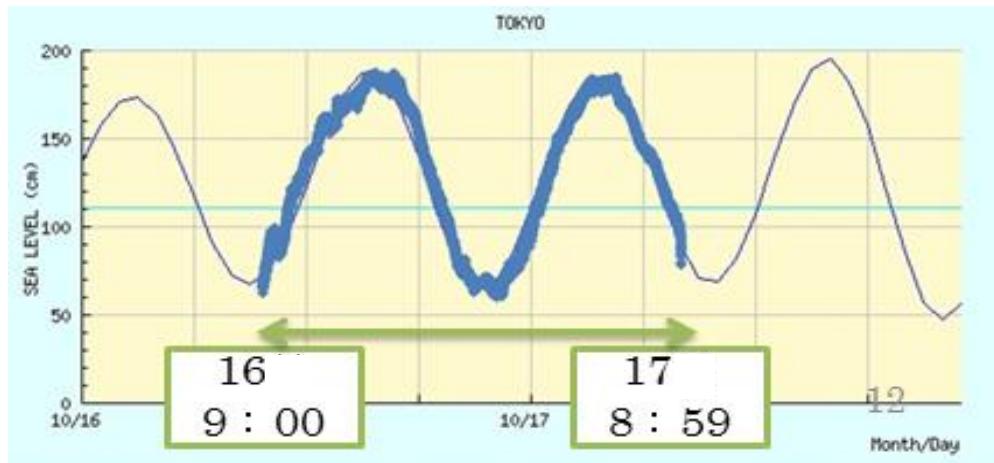
Trimble NetR9



DGNSS vs. RTK-GNSS in altitude



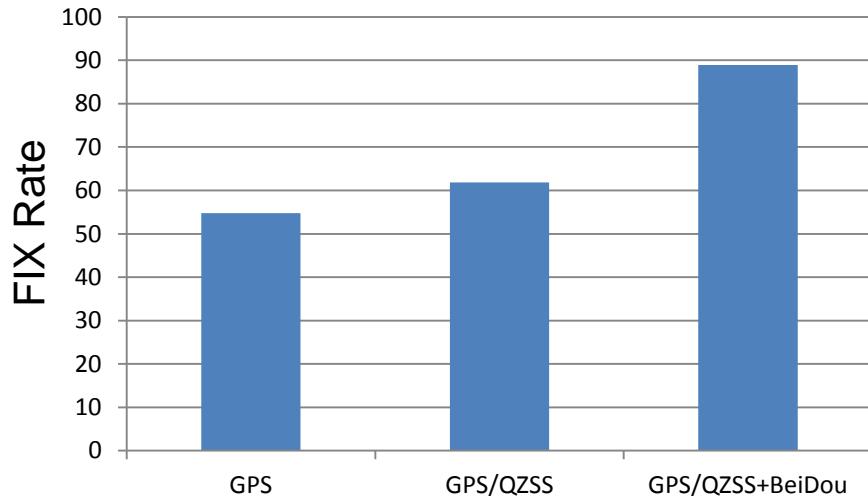
3 hours data
Blue : RTK
Red : DGNSS



Accordance with sea tide data
for 24 hours

Summary of RTK-GNSS FIX Rate

	RTK FIX Rate(%)		
	GPS	GPS/QZSS	GPS/QZSS+BeiDou
7/26	40.9	49.1	90.3
7/27	50.3	60.8	87.6
7/28	62.2	68.5	91.7
7/29	64.7	70.3	92.5
7/30	55.7	60.7	82.7

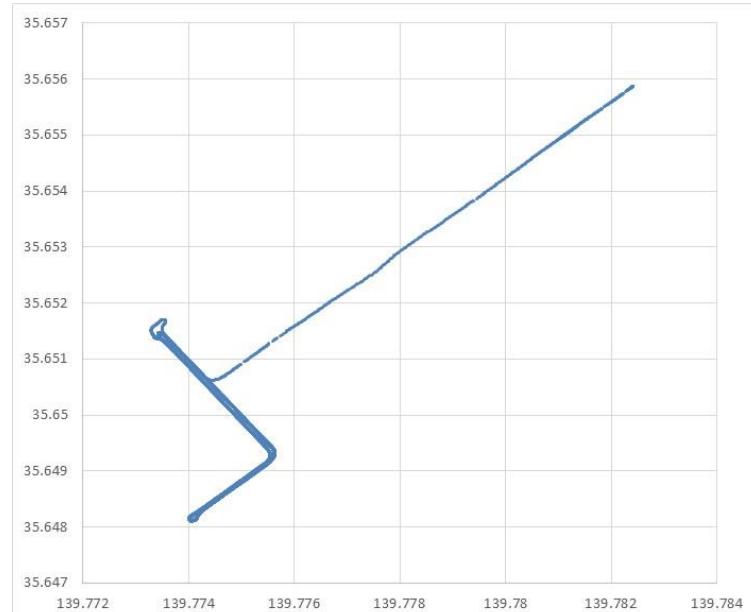


These results clearly shows the improvement by adding QZS and BeiDou.

+ QZS : 5-10%
+ BeiDou : more than 20%

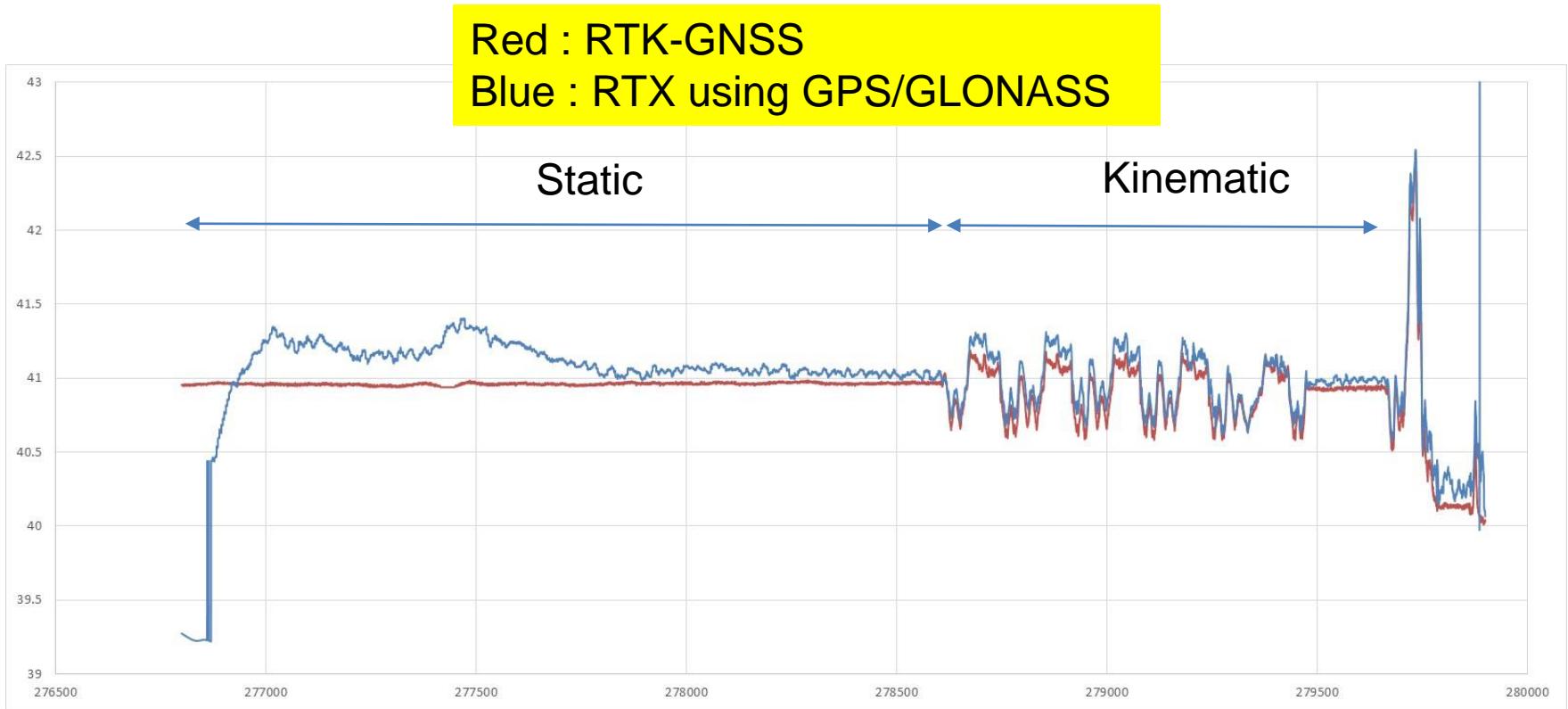
Precise Point Positioning Test using commercial service

- 30 minutes static and 15 minutes kinematic
- Trimble SPS855+RTX (PPP) option
- Comparison with RTK results
- Omni-star was used
- Open Sky



Horizontal plots at Harumi Area

Altitude Comparison between RTK and RTX (PPP)



The accuracy was maintained within several centi-meters after 15 minutes of power on. Small bias (about 10cm) was deduced from other reason.

Problem 3

- It will be announced on site.

Any comments and questions ?

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