4) $y = e^{\frac{x+1}{x}}$ eğrisini çiziniz.

Çözüm: 1. T.K. $= \mathbb{R} - \{0\}$

2. $x \neq 0$, $y \neq 0$

3. $\left.\begin{array}{l} \lim\limits_{x \to 0^+} e^{\frac{x+1}{x}} = +\infty \\ \lim\limits_{x \to 0^-} e^{\frac{x+1}{x}} = 0 \end{array}\right\} \Rightarrow x = 0$ D.A.

$\lim\limits_{x \to \mp\infty} e^{\frac{x+1}{x}} = e \Rightarrow y = e$ Y.A.

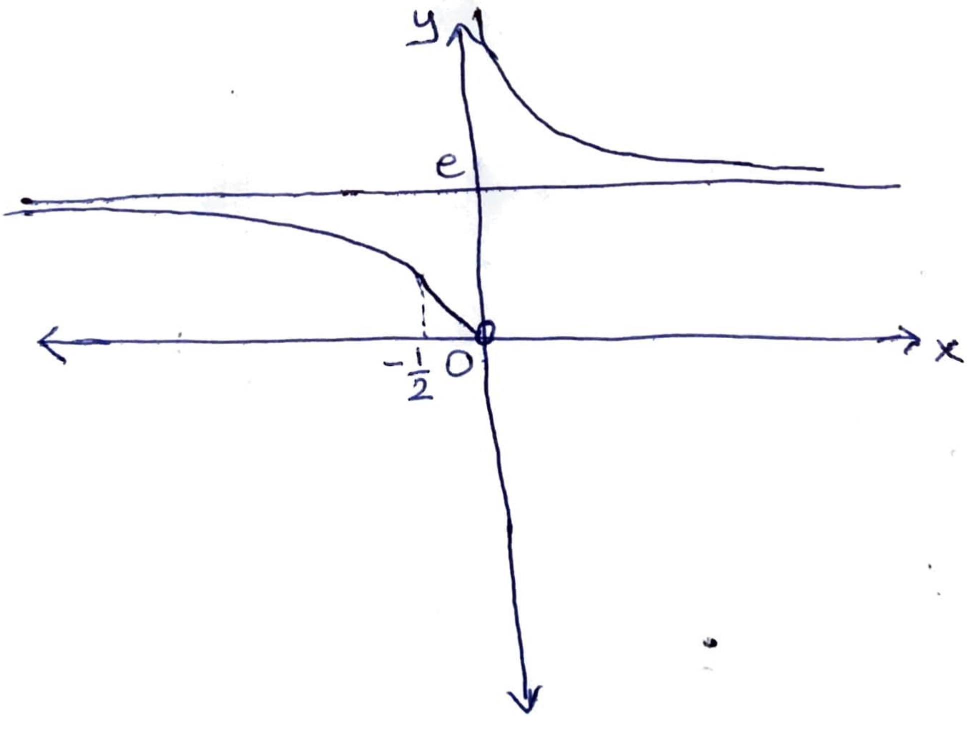
4. $y' = -\frac{1}{x^2} e^{\frac{x+1}{x}} < 0$ olduğundan azalandır.

5. $y'' = \frac{2}{x^3} e^{\frac{x+1}{x}} + \frac{1}{x^4} e^{\frac{x+1}{x}} = \frac{2x+1}{x^4} e^{\frac{x+1}{x}}$

		$-1/2$	
y''	$-$	0	$+$
y	$\cap$		$\cup$

6.

$-\infty$		$-\frac{1}{2}$		0		$+\infty$
	$\searrow$ $\cap$		$\searrow$ $\cup$	$0 \parallel +\infty$	$\searrow$ $\cup$	



5) $y=\sqrt{x^2-2x-3}$ eğrisini çiziniz.

<u>Çözüm:</u> 1. $T.K.=\{x\in\mathbb{R}: x^2-2x-3\geq 0\}=(-\infty,-1]\cup[3,+\infty)$

		-1		3	
x^2-2x-3	$+$		$-$		$+$

2. $x\neq 0$, $y=\sqrt{x^2-2x-3}=0 \Longrightarrow x=-1,\ x=3$

3. Düşey asimptot yoktur.

$$\lim_{x\to\mp\infty}\sqrt{x^2-2x-3}=\lim_{x\to\mp\infty}|x|\sqrt{1-\underbrace{\frac{2}{x}}_{\to 0}-\underbrace{\frac{3}{x^2}}_{\to 0}}=+\infty$$

olduğundan yatay asimptot yoktur.

Eğik asimptot $y=|x-1|$ dir. $x\to+\infty$ için eğik asimptot $y=x-1$ ve $x\to-\infty$ için eğik asimptot $y=-x+1$ dir.

4. $y'=\dfrac{x-1}{\sqrt{x^2-2x-3}}$

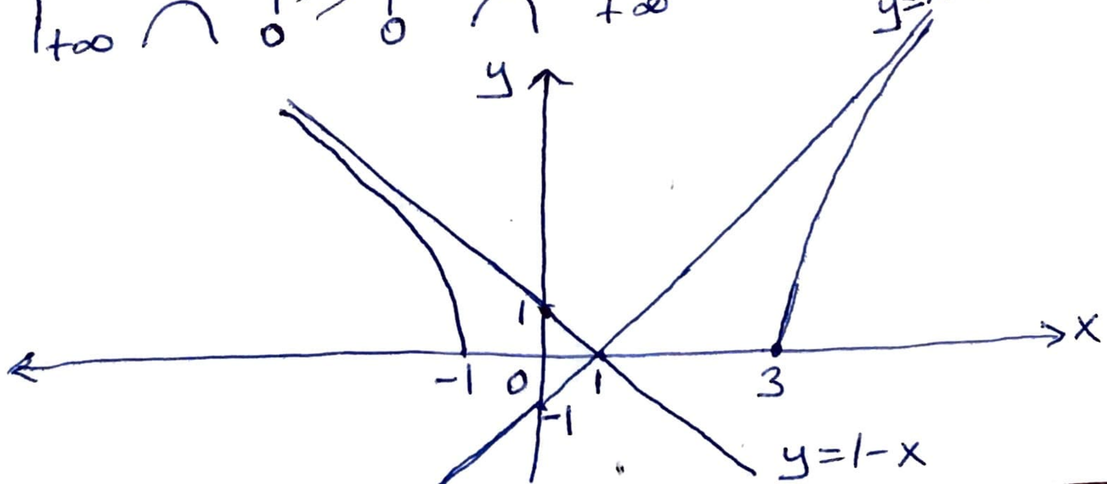
		1	
y'	$-$	0	$+$
y	$\searrow$		$\nearrow$

5. $$y''=\frac{\sqrt{x^2-2x-3}-\dfrac{(x-1)^2}{\sqrt{x^2-2x-3}}}{x^2-2x-3}=\frac{\cancel{x^2}-2x-3-(x^2-2x+1)}{(x^2-2x-3)^{3/2}}=\frac{-4}{(x^2-2x-3)^{3/2}}$$

$y''<0$ olduğundan konkavdır.

6.

$-\infty$		-1		3		$+\infty$
	$\searrow$				$\nearrow$	
$+\infty$	$\frown$	0		0	$\frown$	$+\infty$



6) $y = \ln\left(\frac{x-1}{x+1}\right)$ eğrisini çiziniz.

Çözüm: 1. $T.K. = \{x \in \mathbb{R} : \frac{x-1}{x+1} > 0,\ x+1 \neq 0\} = (-\infty, -1) \cup (1, +\infty)$

		-1		1	
$\frac{x-1}{x+1}$	$+$	|	$-$	0	$+$

2. $x \neq 0$, $y = \ln\left(\frac{x-1}{x+1}\right) = 0 \Rightarrow \frac{x-1}{x+1} = 1 \Rightarrow -1 = 1$ olamaz, o halde $y \neq 0$ dır.

3. $\lim\limits_{x \to -1^-} \ln\left(\frac{x-1}{x+1}\right) = +\infty \Rightarrow x = -1$ D.A.

$\lim\limits_{x \to 1^+} \ln\left(\frac{x-1}{x+1}\right) = -\infty \Rightarrow x = 1$ D.A.

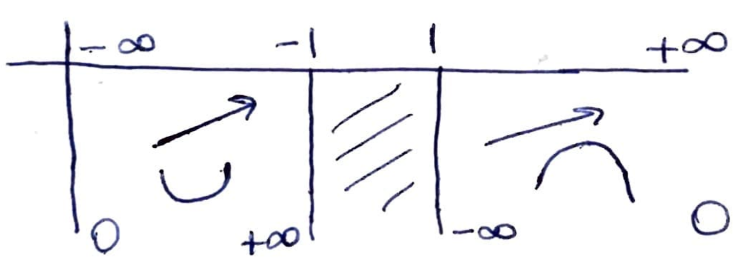
$\lim\limits_{x \to \mp\infty} \ln\left(\frac{x-1}{x+1}\right) = \ln 1 = 0 \Rightarrow y = 0$ Y.A.

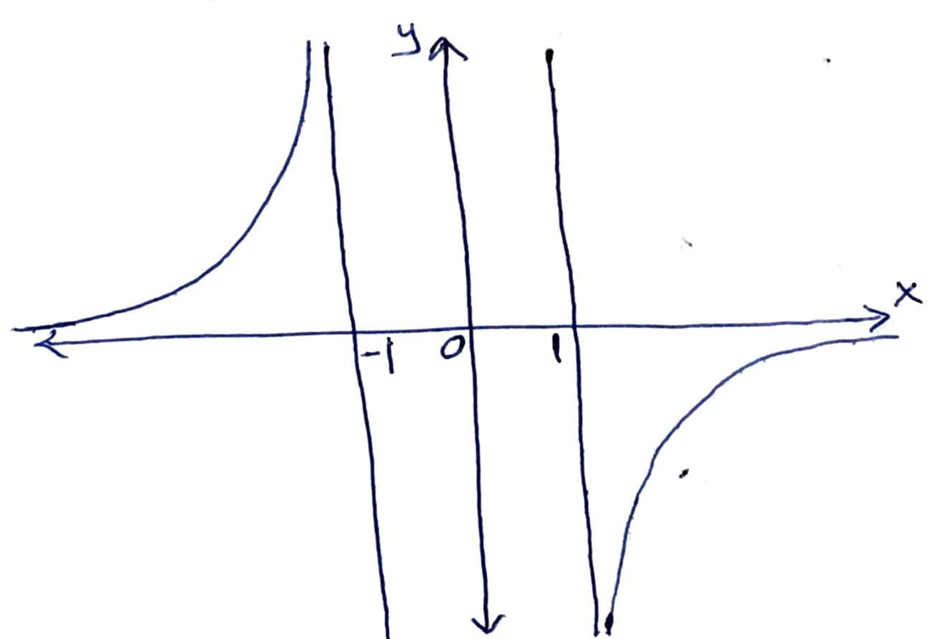
4. $y' = \dfrac{\frac{\cancel{x+1-(x-1)}}{(x+1)^{\cancel{2}}}}{\frac{x-1}{\cancel{x+1}}} = \dfrac{2}{x^2-1}$

		-1		1	
y'	$+$		$-$		$+$
y	↗		↘		↗

5. $y'' = \dfrac{-4x}{(x^2-1)^2}$

		0	
y''	$+$	0	$-$
y	$\cup$		$\cap$

6. 



7) $y=\left(\frac{x+2}{x+3}\right)^2$ eğrisini çiziniz.

Çözüm: 1. $T.K.=\mathbb{R}-\{-3\}$

2. $x=0 \Rightarrow y=\frac{4}{9}$, $y=0 \Rightarrow x=-2$

3. $\lim_{x\to -3}\left(\frac{x+2}{x+3}\right)^2=+\infty \Longrightarrow x=-3$ D.A.

$\lim_{x\to \mp\infty}\left(\frac{x+2}{x+3}\right)^2=1 \Rightarrow y=1$ Y.A.

4. $y'=2\left(\frac{x+2}{x+3}\right)\cdot\frac{x+3-(x+2)}{(x+3)^2}=\frac{2(x+2)}{(x+3)^3}$

		-3		-2	
y'	$+$		$-$		$+$
y	$\nearrow$		$\searrow$		$\nearrow$

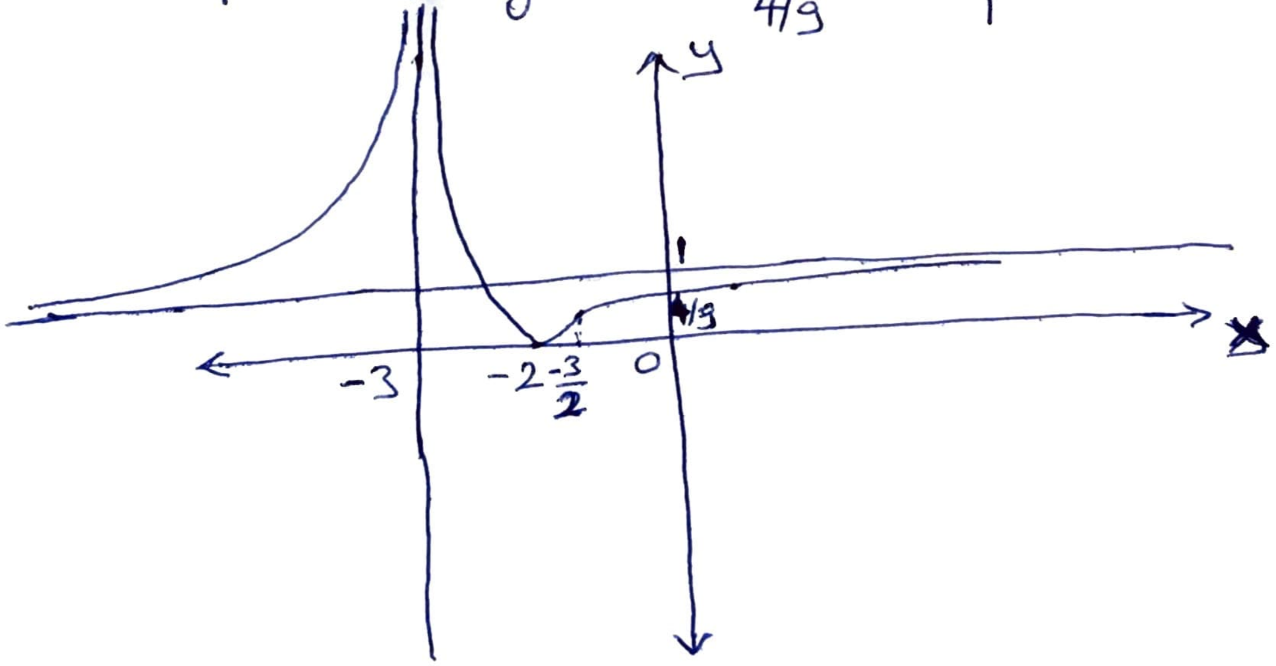
5. $y''=\frac{2(x+3)^3-3(x+3)^2\cdot 2\cdot(x+2)}{(x+3)^6}=\frac{2(x+3)^2(x+3-3x-6)}{(x+3)^6}$

$=\frac{2(-2x-3)}{(x+3)^4}$

		$-3/2$	
y''	$+$	0	$-$
y	$\cup$		$\cap$

6.

$-\infty$		-3		-2		$-\frac{3}{2}$		0		$+\infty$
	$\nearrow$ $\cup$		$\searrow$ $\cup$		$\nearrow$ $\cup$		$\nearrow$ $\cap$		$\nearrow$ $\cap$	
1		$+\infty$ $+\infty$		0				$4/9$		1



8) $y=\sqrt{\dfrac{x^3}{x-1}}$ eğrisini çiziniz.

Çözüm: 1. $T.K. = \left\{x\in\mathbb{R} : \dfrac{x^3}{x-1}\geq 0,\ x-1\neq 0\right\}=(-\infty,0]\cup(1,+\infty)$

		0		1	
$\frac{x^3}{x-1}$	+	0	−	|	+

2. $x=0 \Leftrightarrow y=0$

3. $\lim\limits_{x\to 1^+}\sqrt{\dfrac{x^3}{x-1}}=+\infty \Rightarrow x=1$ D.A.

$\lim\limits_{x\to\mp\infty}\sqrt{\dfrac{x^3}{x-1}}=+\infty$ olduğundan Y.A. yoktur.

$$m_1=\lim_{x\to\infty}\frac{\sqrt{\frac{x^3}{x-1}}}{x}=\lim_{x\to\infty}\frac{\cancel{x}\sqrt{\frac{x}{x-1}}}{\cancel{x}}=1$$

$$n_1=\lim_{x\to\infty}\left(\sqrt{\frac{x^3}{x-1}}-1.x\right)=\lim_{x\to\infty}\frac{\frac{x^3}{x-1}-x^2}{\sqrt{\frac{x^3}{x-1}}+x}=\lim_{x\to\infty}\frac{\frac{x^{\cancel{2}}}{x-1}}{\cancel{x}\left(\sqrt{\frac{x}{x-1}}+1\right)}=\frac{1}{2}$$

$x\to\infty$ için eğik asimptot $y=x+\frac{1}{2}$ olur.

$$m_2=\lim_{x\to-\infty}\frac{\sqrt{\frac{x^3}{x-1}}}{x}=\lim_{x\to-\infty}\frac{-\cancel{x}\sqrt{\frac{x}{x-1}}}{\cancel{x}}=-1$$

$$n_2=\lim_{x\to-\infty}\left(\sqrt{\frac{x^3}{x-1}}+x\right)=\lim_{x\to-\infty}\frac{\frac{x^3}{x-1}-x^2}{\sqrt{\frac{x^3}{x-1}}-x}=\lim_{x\to-\infty}\frac{\frac{x^{\cancel{2}}}{x-1}}{-\cancel{x}\left(\sqrt{\frac{x}{x-1}}+1\right)}=\frac{-1}{2}$$

$x\to-\infty$ için eğik asimptot $y=-x-\frac{1}{2}$ olur.

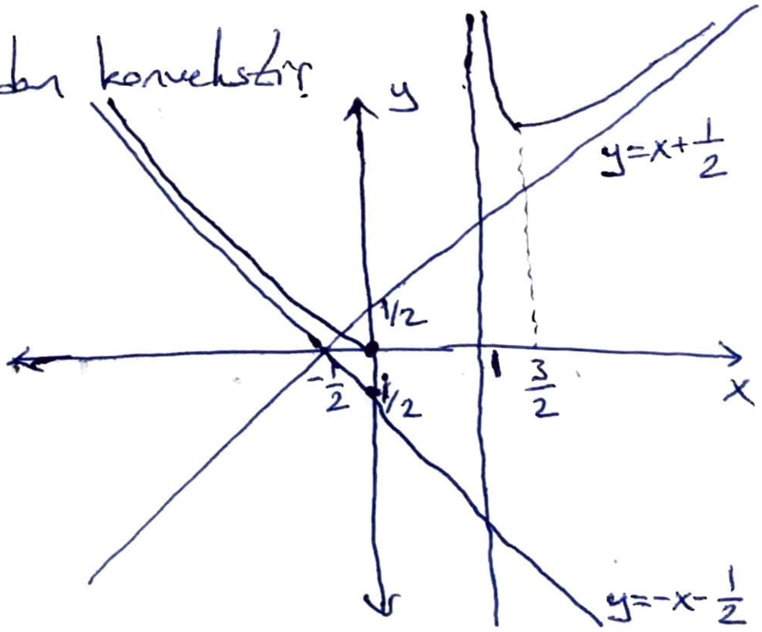
4. $$y'=\frac{\frac{3x^2(x-1)-x^3}{(x-1)^2}}{2\sqrt{\frac{x^3}{x-1}}}=\frac{x^2(2x-3)}{2(x-1)^2}\cdot\frac{(x-1)^{1/2}}{x^{3/2}}=\frac{x^{1/2}(2x-3)}{2(x-1)^{3/2}}$$

		3/2	
y'	−	0	+
y	↘		↗

5. $y''=\dfrac{3}{4\sqrt{x}(x-1)^{5/2}}>0$ olduğundan konvekstir.

6.

$-\infty$		0		1		3/2		$+\infty$
	↘ ∪	0	//////	$+\infty$	↘ ∪		↗ ∪	



9) $y=\ln\left(\frac{x^2-4}{1-x^2}\right)$ eğrisini çiziniz.

<u>Çözüm:</u> 1. $T.K.=\left\{x\in\mathbb{R}: \frac{x^2-4}{1-x^2}>0,\ 1-x^2\neq 0\right\}$

	-2		-1		1		2	
$\frac{x^2-4}{1-x^2}$	$-$	0	$+$	$\vert$	$-$	$\vert$	$+$	0 $-$

$T.K.=(-2,-1)\cup(1,2)$

2. $x\neq 0$, $y=\ln\left(\frac{x^2-4}{1-x^2}\right)=0\Rightarrow \frac{x^2-4}{1-x^2}=1\Rightarrow x^2-4=1-x^2\Rightarrow x=\mp\frac{\sqrt5}{2}$

3. $\lim_{x\to-2^+}\ln\left(\frac{x^2-4}{1-x^2}\right)=-\infty\Rightarrow x=-2$ D.A.

$\lim_{x\to-1^-}\ln\left(\frac{x^2-4}{1-x^2}\right)=+\infty\Rightarrow x=-1$ D.A.

$\lim_{x\to1^+}\ln\left(\frac{x^2-4}{1-x^2}\right)=+\infty\Rightarrow x=1$ D.A.

$\lim_{x\to2^-}\ln\left(\frac{x^2-4}{1-x^2}\right)=-\infty\Rightarrow x=2$ D.A.

$\lim_{x\to\mp\infty}\ln\left(\frac{x^2-4}{1-x^2}\right)$ yoktur, çünkü $x\to\infty$, $x\to-\infty$ gitmez

4. $y'=\dfrac{\frac{2x(1-x^2)+2x(x^2-4)}{(1-x^2)^2}}{\frac{x^2-4}{1-x^2}}=\dfrac{-6x}{(1-x^2)(x^2-4)}$

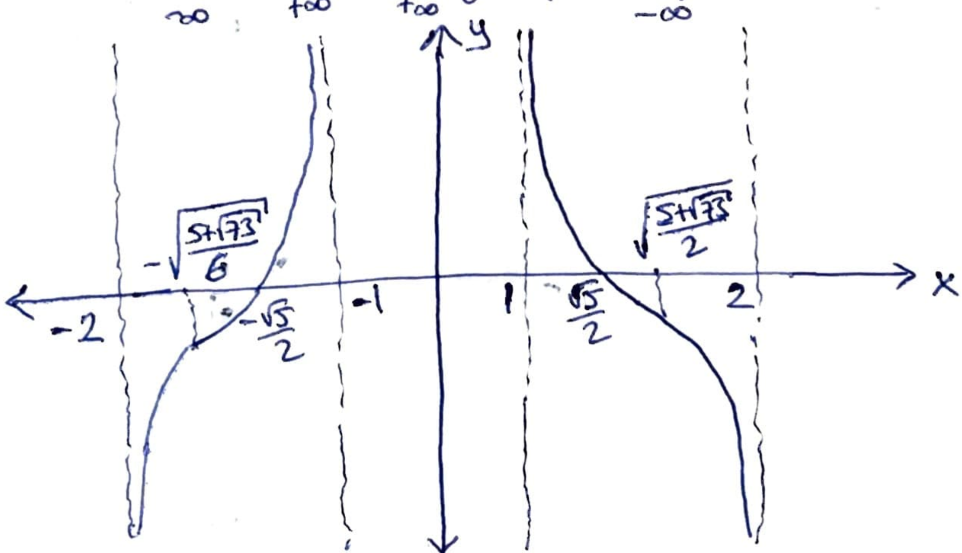
		0	
y'	$+$	0	$-$
y	$\nearrow$		$\searrow$

5. $y''=\dfrac{-6(3x^4-5x^2-4)}{(1-x^2)^2(x^2-4)^2}$

		$-\sqrt{\frac{5+\sqrt{73}}{6}}$		$\sqrt{\frac{5+\sqrt{73}}{6}}$	
y''	$-$		$+$		$-$
y	$\cap$		$\cup$		$\cap$

6.

$-\infty$	-2	$-\sqrt{\frac{5+\sqrt{73}}{6}}$	$-\sqrt5/2$	-1	1	$\frac{\sqrt5}{2}$	$\sqrt{\frac{5+\sqrt{73}}{6}}$	2	
	$\nearrow$ $\cap$	$\nearrow$ $\cup$	$\nearrow$ $\cup$		$\searrow$ $\cup$	$\searrow$ $\cup$	$\searrow$ $\cap$		
	$-\infty$		$+\infty$		$+\infty$	0		$-\infty$	



10) $y = x^2 e^{-x} + 1$ eğrisini çiziniz.

Çözüm: 1. T.K. $= \mathbb{R}$

2. $x = 0 \Rightarrow y = 1$, $y \neq 0$ olur, çünkü $x^2 e^{-x} \geq 0$ dır.

3. $\lim\limits_{x \to -\infty} (x^2 e^{-x} + 1) = +\infty$

$\lim\limits_{x \to \infty} (x^2 e^{-x} + 1) = 1 + \lim\limits_{x \to \infty} \dfrac{x^2}{e^x} = 1 + \lim\limits_{x \to \infty} \dfrac{2x}{e^x} = 1 + \lim\limits_{x \to \infty} \dfrac{2}{e^x} = 1$

olduğundan $y = 1$ Y.A. olur. Düşey asimptot yoktur.

4. $y' = 2x e^{-x} - x^2 e^{-x} = (2x - x^2) e^{-x}$

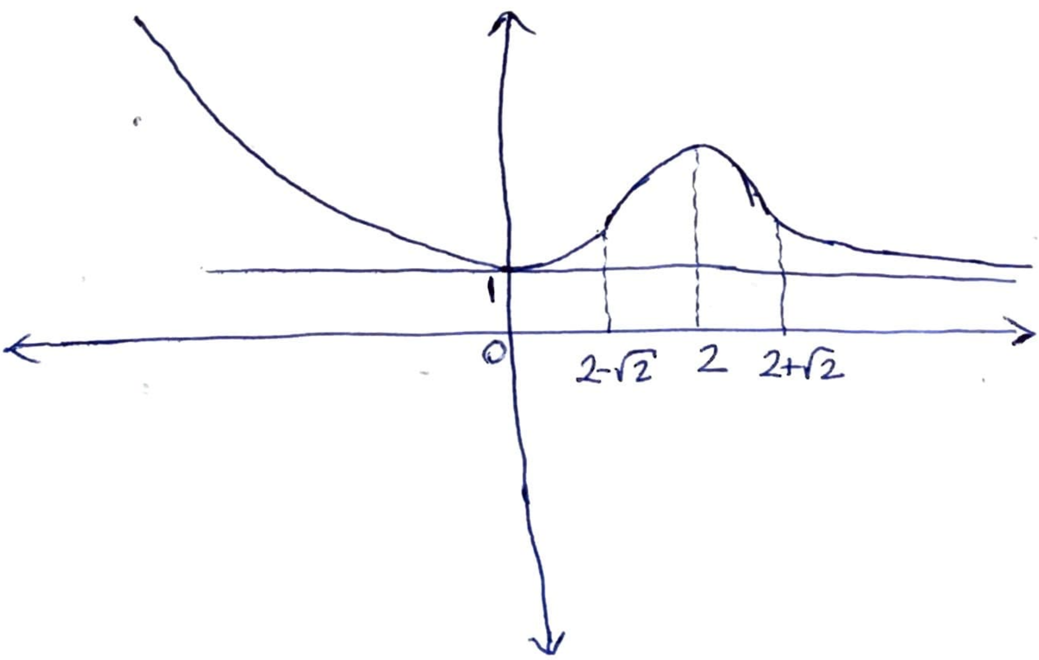
		0		2	
y'	$-$	0	$+$	0	$-$
y	$\searrow$		$\nearrow$		$\searrow$

5. $y'' = (2 - 2x) e^{-x} - (2x - x^2) e^{-x} = (x^2 - 4x + 2) e^{-x} = 0 \Rightarrow x = 2 \mp \sqrt{2}$

		$2-\sqrt{2}$		$2+\sqrt{2}$	
y''	$+$		$-$		$+$
y	$\cup$		$\cap$		$\cup$

6.

$-\infty$		0		$2-\sqrt{2}$		2		$2+\sqrt{2}$		$+\infty$
$+\infty$	$\searrow$ $\cup$	1	$\nearrow$ $\cup$		$\nearrow$ $\cap$		$\searrow$ $\cap$		$\searrow$ $\cup$	1



11) $y=\dfrac{x}{\ln(2x)-1}$ eğrisini çiziniz.

Çözüm: 1. $T.K.=\{x\in\mathbb{R}: 2x>0,\ \ln(2x)-1\neq 0\}=(0,\tfrac{e}{2})\cup(\tfrac{e}{2},+\infty)$

$\ln(2x)-1=0 \Rightarrow \ln(2x)=1 \Rightarrow 2x=e \Rightarrow x=\dfrac{e}{2}$

$\ln(2x)-1\neq 0 \Rightarrow x\neq \dfrac{e}{2}$

2. $x\neq 0$, $y\neq 0$

3. $\left.\begin{array}{l}\lim\limits_{x\to \frac{e}{2}^-}\dfrac{x}{\ln(2x)-1}=-\infty \\ \lim\limits_{x\to \frac{e}{2}^+}\dfrac{x}{\ln(2x)-1}=+\infty\end{array}\right\}\Rightarrow x=\dfrac{e}{2}$ D.A.

$\lim\limits_{x\to+\infty}\dfrac{x}{\ln(2x)-1}\overset{\frac{\infty}{\infty}}{=}\lim\limits_{x\to+\infty}\dfrac{1}{\frac{1}{x}}=+\infty$ old. Y.A. yoktur

$m=\lim\limits_{x\to\infty}\dfrac{\frac{x}{\ln(2x)-1}}{x}=\lim\limits_{x\to\infty}\dfrac{1}{\ln(2x)-1}=0$

$n=\lim\limits_{x\to\infty}\left(\dfrac{x}{\ln(2x)-1}-0\right)=+\infty$ olduğundan E.A. yoktur.

4. $y'=\dfrac{\ln(2x)-1-\frac{1}{\cancel{x}}\cdot\cancel{x}}{(\ln(2x)-1)^2}=\dfrac{\ln(2x)-2}{(\ln(2x)-1)^2}$

		$\frac{e^2}{2}$	
y'	$-$	0	$+$
y	↘		↗

$\ln(2x)-2=0 \Rightarrow 2x=e^2 \Rightarrow x=e^2/2$

5. $y''=\dfrac{\frac{1}{x}(\ln(2x)-1)^2-\frac{2}{x}(\ln(2x)-1)(\ln(2x)-2)}{(\ln(2x)-1)^4}=\dfrac{3-\ln(2x)}{x(\ln(2x)-1)^3}$

		0		$\frac{e}{2}$		$\frac{e^3}{2}$	
y''	$+$		$-$		$+$		$-$
y	∪		∩		∪		∩

6.

0		$\frac{e}{2}$		$\frac{e^2}{2}$		$\frac{e^3}{2}$		$+\infty$
0	↘ ∩	$-\infty$ ‖ $+\infty$	↘ ∪		↗ ∪		↗ ∩	$+\infty$