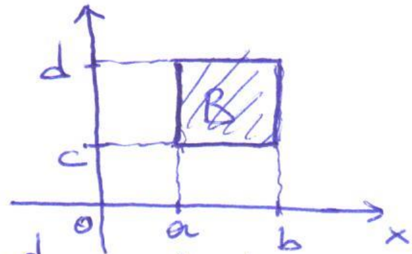
İki Katlı İntegraller

— $B=\{(x,y): a\le x\le b,\ c\le y\le d\}$ ve



$f: B\to\mathbb{R}$ fonksiyonu sürekli ise,

$$\iint_B f(x,y)\,dx\,dy=\int_c^d\left(\int_a^b f(x,y)\,dx\right)dy=\int_a^b\left(\int_c^d f(x,y)\,dy\right)dx$$

olur.

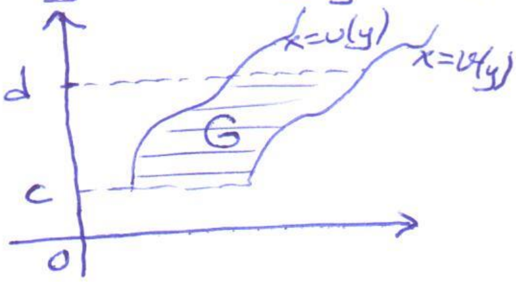
— $g,h:[a,b]\to\mathbb{R}$ fonksiyonları sürekli, $\forall x\in[a,b]$ için $g(x)\le h(x)$ ve $D=\{(x,y): a\le x\le b,\ g(x)\le y\le h(x)\}$ olsun.

$f: D\to\mathbb{R}$ fonksiyonu sürekli ise,

$$\iint_D f(x,y)\,dx\,dy=\int_a^b\left(\int_{g(x)}^{h(x)} f(x,y)\,dy\right)dx$$

olur.

— $u,v:[c,d]\to\mathbb{R}$ fonksiyonları sürekli, $\forall y\in[c,d]$ için $u(y)\le v(y)$ ve $G=\{(x,y): c\le y\le d,\ u(y)\le x\le v(y)\}$ olsun.



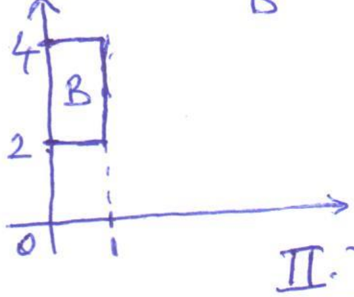
$f: G\to\mathbb{R}$ fonksiyonu sürekli ise,

$$\iint_G f(x,y)\,dx\,dy=\int_c^d\left(\int_{u(y)}^{v(y)} f(x,y)\,dx\right)dy$$

olur.

Örnek: $B=\{(x,y)\in\mathbb{R}^2: 0\le x\le 1,\ 2\le y\le 4\}$ ise $\iint_B x^2y\,dx\,dy=?$

Çözüm: $$\iint_B x^2y\,dx\,dy=\int_2^4\left(\int_0^1 x^2y\,dx\right)dy=\int_2^4\left[\frac{x^3y}{3}\Big|_0^1\right]dy$$

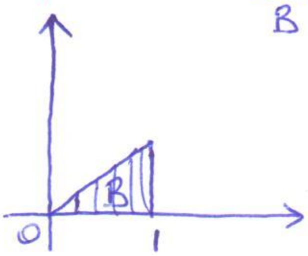


$$=\int_2^4\frac{y}{3}\,dy=\frac{y^2}{6}\Big|_2^4=\frac{16-4}{6}=2$$

II. Yol: $$\iint_B x^2y\,dx\,dy=\int_0^1\left(\int_2^4 x^2y\,dy\right)dx=2$$

Örnek: $B=\{(x,y)\in\mathbb{R}^2 : 0\le x\le 1,\ 0\le y\le x\}$ bölgesi üzerinde $f(x,y)=3-x-y$ fonksiyonunun integralini hesaplayınız.

Çözüm:

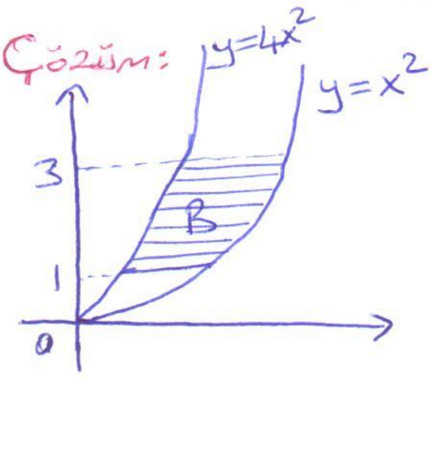


$$\iint_B (3-x-y)\,dx\,dy=\int_0^1\left(\int_0^x(3-x-y)\,dy\right)dx$$
$$=\int_0^1\left[3y-xy-\frac{y^2}{2}\Big|_0^x\right]dx$$
$$=\int_0^1\left(3x-\frac{3x^2}{2}\right)dx$$
$$=\frac{3x^2}{2}-\frac{\cancel{3}}{2}\cdot\frac{x^3}{\cancel{3}}\Big|_0^1$$
$$=1$$

<u>II. Yol</u>: $\displaystyle\iint_B (3-x-y)\,dx\,dy=\int_0^1\left(\int_y^1(3-x-y)\,dx\right)dy=1$

Örnek: Birinci bölgede $y=x^2$, $y=4x^2$ parabolleri ile $y=1$, $y=3$ doğruları tarafından sınırlanan bölge üzerinde $f(x,y)=xy$ fonksiyonunun integralini bulunuz.

Çözüm:

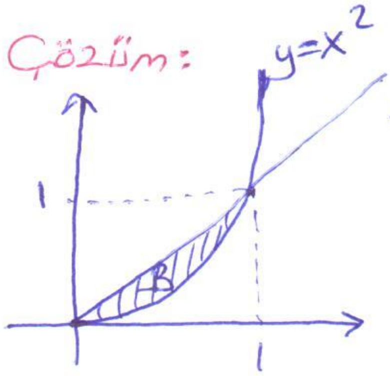


$$\iint_B xy\,dx\,dy=\int_1^3\left(\int_{\sqrt{y}/2}^{\sqrt{y}}xy\,dx\right)dy$$
$$=\int_1^3\left[\frac{x^2y}{2}\Big|_{\sqrt{y}/2}^{\sqrt{y}}\right]dy$$
$$=\int_1^3\left(\frac{y^2}{2}-\frac{y^2}{8}\right)dy=\int_1^3\frac{3y^2}{8}\,dy$$
$$=\frac{y^3}{8}\Big|_1^3$$
$$=\frac{13}{4}$$

<u>II. Yol</u>: $\displaystyle\iint_B xy\,dx\,dy=\int_{1/2}^{\sqrt{3}/2}\left(\int_1^{4x^2}xy\,dy\right)dx+\int_{\sqrt{3}/2}^{1}\left(\int_1^3xy\,dy\right)dx$
$\displaystyle+\int_1^{\sqrt{3}}\left(\int_{x^2}^3xy\,dy\right)dx=\frac{13}{4}$

Örnek: B bölgesi $y=x$ doğrusuyla $y=x^2$ parabolü arasında kalan kapalı bölge olduğuna göre $\iint_B (x+y+1)\,dx\,dy$ integralini hesaplayınız.

Çözüm:



$$\iint_B (x+y+1)\,dx\,dy = \int_0^1 \left(\int_{x^2}^{x} (x+y+1)\,dy \right) dx$$

$$= \int_0^1 \left[xy + \frac{y^2}{2} + y \Big|_{x^2}^{x} \right] dx$$

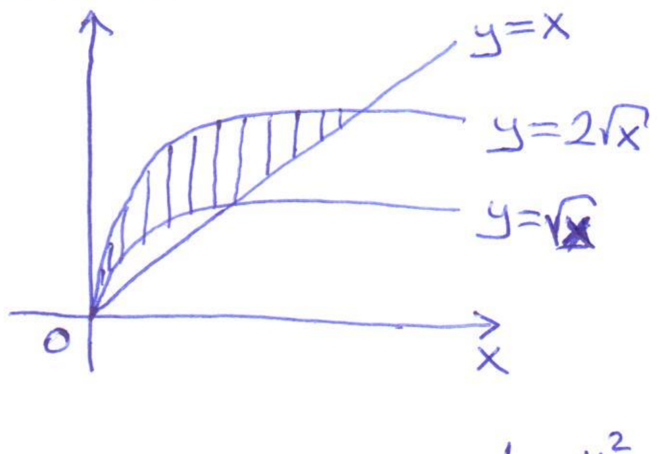
$$= \int_0^1 \left(-\frac{x^4}{2} - x^3 + \frac{x^2}{2} + x \right) dx$$

$$= \frac{19}{60}$$

II. Yol: $\iint_B (x+y+1)\,dx\,dy = \int_0^1 \left(\int_y^{\sqrt{y}} (x+y+1)\,dx \right) dy = \frac{19}{60}$

Örnek: $y=2\sqrt{x}$, $y=\sqrt{x}$ parabolleri ile $y=x$ doğrusu tarafından sınırlanan B bölgesi üzerinde $\iint_B dx\,dy$ integralini hesaplayınız

Çözüm:



$$\iint_B dx\,dy = \int_0^1 \left(\int_{\sqrt{x}}^{2\sqrt{x}} dy \right) dx + \int_1^4 \left(\int_x^{2\sqrt{x}} dy \right) dx$$

$$= \int_0^1 \sqrt{x}\,dx + \int_1^4 (2\sqrt{x} - x)\,dx$$

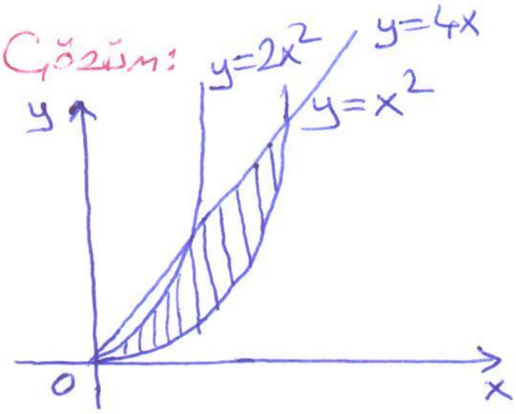
$$= \frac{2}{3} x^{3/2} \Big|_0^1 + \left[\frac{4}{3} x^{3/2} - \frac{x^2}{2} \Big|_1^4 \right]$$

$$= \frac{5}{2}$$

II. Yol: $\iint_B dx\,dy = \int_0^1 \left(\int_{y^2/4}^{y^2} dx \right) dy + \int_1^4 \left(\int_{y^2/4}^{y} dx \right) dy$

Örnek: B bölgesi $y=x^2$, $y=2x^2$ parabolleri ile $y=4x$ doğrusu tarafından sınırlanan bölge olsun. $f(x,y)=x+y+1$ fonksiyonunun B üzerindeki integralini bulunuz.

Çözüm:



$$\iint_B (x+y+1)\,dx\,dy = \int_0^2 \left(\int_{x^2}^{2x^2} (x+y+1)\,dy \right) dx + \int_2^4 \left(\int_{x^2}^{4x} (x+y+1)\,dy \right) dx$$

$$= \int_0^2 \left[xy + \frac{y^2}{2} + y \Big|_{x^2}^{2x^2} \right] dx + \int_2^4 \left[xy + \frac{y^2}{2} + y \Big|_{x^2}^{4x} \right] dx$$

$$= \int_0^2 \left(x^3 - \frac{3x^4}{2} + x^2 \right) dx + \int_2^4 \left(3x^2 - x^3 + 4x^2 - \frac{x^4}{2} + 4x \right) dx$$

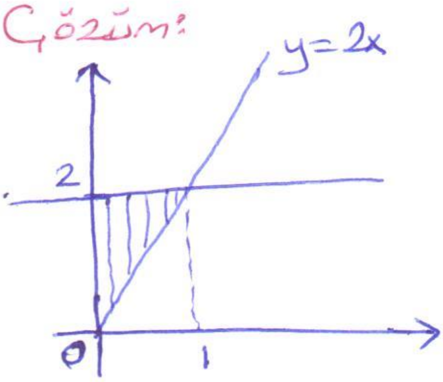
$$= \frac{432}{5}$$

II. Yol $$\iint_B (x+y+1)\,dx\,dy = \int_0^8 \left(\int_{\sqrt{\frac{y}{2}}}^{\sqrt{y}} (x+y+1)\,dx \right) dy + \int_8^{16} \left(\int_{y/4}^{\sqrt{y}} (x+y+1)\,dx \right) dy$$

$$= \frac{432}{5}$$

Örnek: $\int_0^1 \left(\int_{2x}^2 e^{y^2}\,dy \right) dx = ?$

Çözüm:



$$\int_0^1 \left(\int_{2x}^2 e^{y^2}\,dy \right) dx = \int_0^2 \left(\int_0^{y/2} e^{y^2}\,dx \right) dy$$

$$= \int_0^2 \left[x e^{y^2} \Big|_0^{y/2} \right] dy$$

$$= \int_0^2 \frac{y}{2} e^{y^2}\,dy$$

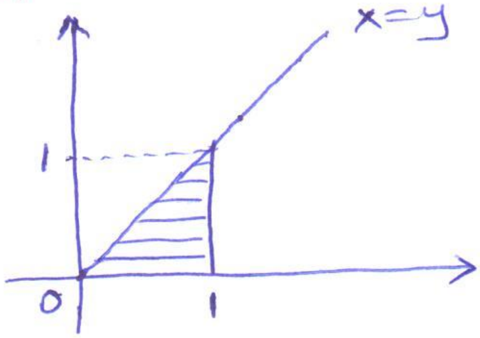
$$= \frac{1}{4} \int_0^2 e^{y^2} \cdot 2y\,dy$$

$$= \frac{1}{4} e^{y^2} \Big|_0^2$$

$$= \frac{1}{4}\left(e^4 - 1\right)$$

Örnek: $\displaystyle\int_0^1 \left(\int_y^1 \tan x^2 \, dx \right) dy = ?$

Çözüm:



$$\int_0^1 \left(\int_y^1 \tan x^2 \, dx \right) dy = \int_0^1 \left(\int_0^x \tan x^2 \, dy \right) dx$$

$$= \int_0^1 x \cdot \tan x^2 \, dx$$

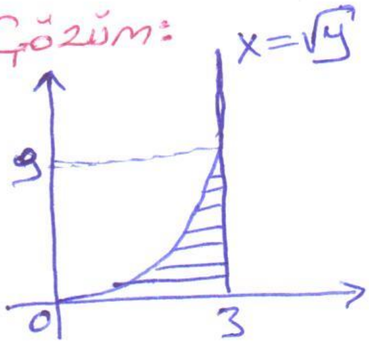
$$= \frac{1}{2} \int_0^1 \tan(x^2) \cdot 2x \, dx$$

$$= -\frac{1}{2} \cdot \ln(\cos x^2) \Big|_0^1$$

$$= -\frac{1}{2} \cdot \ln(\cos 1)$$

Örnek: $\displaystyle\int_0^9 \left(\int_{\sqrt{y}}^3 \sin x^3 \, dx \right) dy = ?$

Çözüm:



$$\int_0^9 \left(\int_{\sqrt{y}}^3 \sin x^3 \, dx \right) dy = \int_0^3 \left(\int_0^{x^2} \sin x^3 \, dy \right) dx$$

$$= \int_0^3 \left[y \sin(x^3) \Big|_0^{x^2} \right] dx$$

$$= \int_0^3 x^2 \cdot \sin(x^3) \cdot dx$$

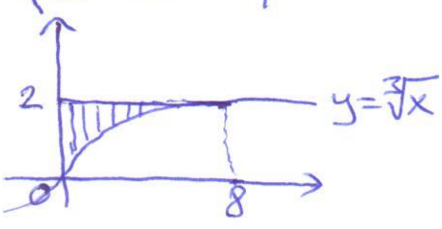
$$= \frac{1}{3} \int_0^3 \sin(x^3) \cdot 3x^2 \, dx$$

$$= \frac{-1}{3} \cdot \cos(x^3) \Big|_0^3$$

$$= \frac{-1}{3} \left(\cos(27) - 1 \right)$$

Örnek: $\displaystyle\int_0^8 \left(\int_{\sqrt[3]{x}}^2 \frac{dy}{y^4+1} \right) dx = ?$

Çözüm:



$$\int_0^8 \left(\int_{\sqrt[3]{x}}^2 \frac{dy}{y^4+1} \right) dx = \int_0^2 \left(\int_0^{y^3} \frac{dx}{y^4+1} \right) dy$$

$$= \int_0^2 \left[x \cdot \frac{1}{y^4+1} \Big|_0^{y^3} \right] dy$$

$$= \int_0^2 \frac{y^3}{y^4+1}\, dy = \frac{1}{4} \int_0^2 \frac{4y^3\, dy}{y^4+1}$$

$$= \frac{1}{4} \cdot \ln(y^4+1) \Big|_0^2$$

$$= \frac{1}{4} \cdot \ln(17)$$

İki Katlı İntegrallerde Bölge Dönüşümleri

uv-düzlemindeki bir D bölgesi

$$\begin{cases} x = g(u,v) \\ y = h(u,v) \end{cases}$$

dönüşümü yardımıyla xy-düzlemindeki bir B bölgesine dönüştürülsün. Jakobiyen

$$j = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

olmak üzere,

$$\iint_B f(x,y)\, dx\, dy = \iint_D f\big(g(u,v), h(u,v)\big) \cdot |j|\, du\, dv$$

olur. Özel olarak,

$$x = r\cos\theta$$
$$y = r\sin\theta$$

dönüşümü yardımıyla kutupsal koordinatlara geçilirse,

$$\iint_B f(x,y)\,dx\,dy = \iint_D f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

olur, çünkü

$$j = \frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r$$

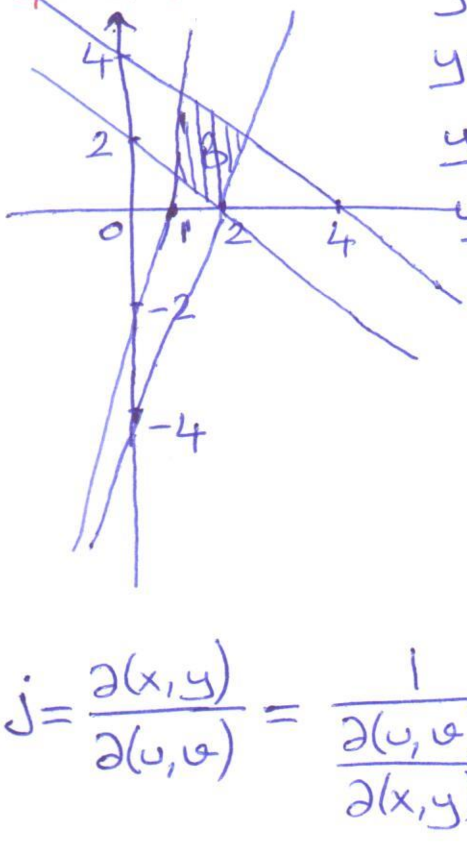
olur. $j = \dfrac{\partial(x,y)}{\partial(u,v)} = \dfrac{1}{\dfrac{\partial(u,v)}{\partial(x,y)}}$ yazılabilir.

Örnek: B bölgesi $y=2x-2$, $y=2x-4$, $y=-x+2$, $y=-x+4$ doğruları tarafından sınırlanan bölge olsun. Bu bölge

$$\begin{cases} u = x+y \\ v = 2x-y \end{cases}$$

dönüşümü yardımıyla uv-düzlemindeki D bölgesine dönüştürülüyor. $\iint_B (2x-y)^2\,dx\,dy$ integralini bölge dönüşümü ile hesaplayınız.

Çözüm:

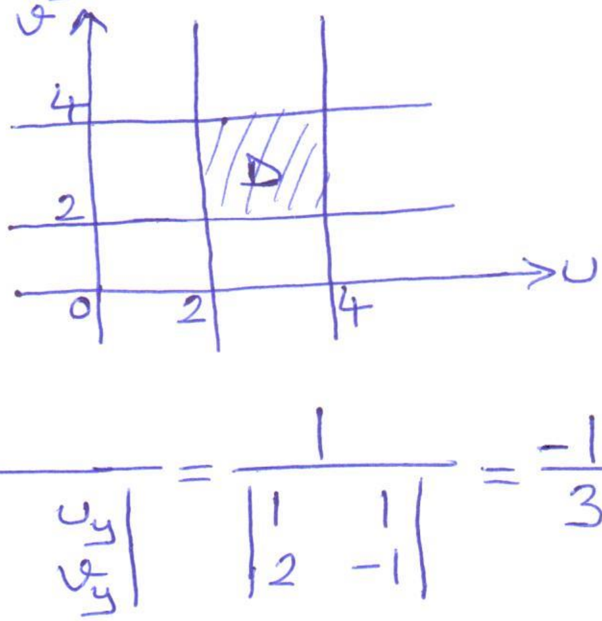


$y=2x-2 \Rightarrow 2x-y=2 \Rightarrow v=2$

$y=2x-4 \Rightarrow 2x-y=4 \Rightarrow v=4$

$y=-x+2 \Rightarrow x+y=2 \Rightarrow u=2$

$y=-x+4 \Rightarrow x+y=4 \Rightarrow u=4$



$$j = \frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\dfrac{\partial(u,v)}{\partial(x,y)}} = \frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{-1}{3}$$

$$\iint_B (2x-y)^2\,dxdy = \iint_D v^2 \cdot |J| \cdot du\,dv$$

$$= \int_2^4 \left(\int_2^4 v^2 \cdot \frac{1}{3}\, dv \right) du$$

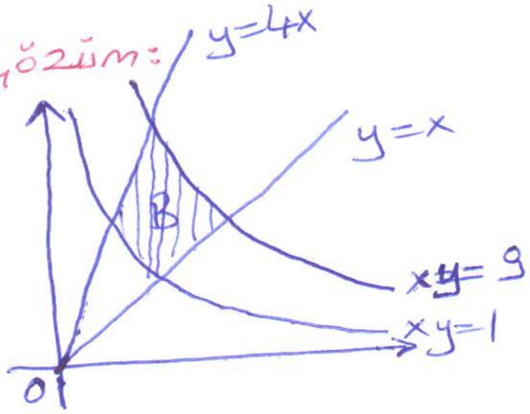
$$= \int_2^4 \left(\frac{v^3}{9} \Big|_2^4 \right) du$$

$$= \int_2^4 \frac{56}{9}\, du$$

$$= \frac{112}{9}$$

Örnek: B, xy-düzleminin birinci dörtte birlik bölgesinde $xy=1$, $xy=9$ hiperbolleri ve $y=x$, $y=4x$ doğruları ile sınırlı bölge olsun. $\iint_B \left(\sqrt{\frac{x}{y}} + \sqrt{xy}\right) dxdy = ?$

Çözüm:



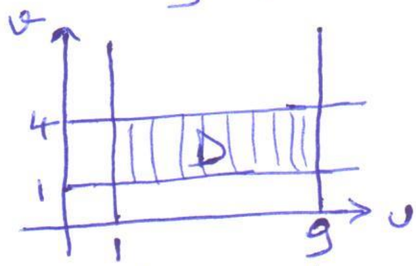
$$\begin{cases} u = xy \\ v = \frac{y}{x} \end{cases}$$

$xy=1 \Rightarrow u=1$

$xy=9 \Rightarrow u=9$

$y=x \Rightarrow v=1$

$y=4x \Rightarrow v=4$



$$J = \frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\frac{\partial(u,v)}{\partial(x,y)}} = \frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}} = \frac{1}{\begin{vmatrix} y & x \\ \frac{-y}{x^2} & \frac{1}{x} \end{vmatrix}} = \frac{1}{\frac{2y}{x}} = \frac{1}{2v}$$

$$\iint_B \left(\sqrt{\frac{x}{y}} + \sqrt{xy}\right) dx\,dy = \iint_D \left(\frac{1}{\sqrt{v}} + \sqrt{u}\right) \cdot \frac{1}{2v}\, du\,dv$$

$$= \int_1^4 \left(\int_1^9 \left(\frac{1}{\sqrt{v}} + \sqrt{u}\right) \frac{1}{2v}\, du \right) dv = \int_1^4 \left[\left(\frac{u}{\sqrt{v}} + \frac{2}{3} u^{3/2}\right) \frac{1}{2v} \Big|_1^9 \right] dv$$

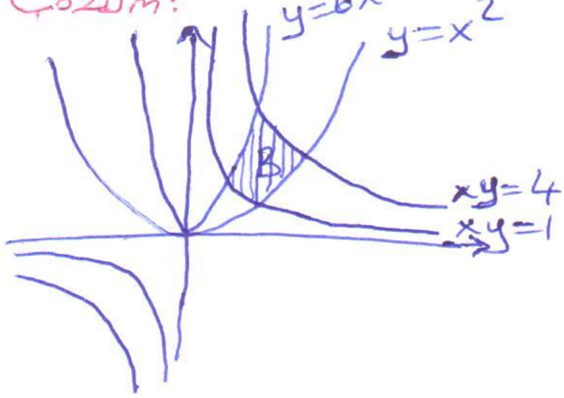
$$= \int_1^4 \left[\left(\frac{8}{\sqrt{v}} + \frac{2}{3} \cdot 26\right) \frac{1}{2v} \right] dv = \int_1^4 \left(4 v^{-3/2} + \frac{26}{3} \cdot \frac{1}{v}\right) dv$$

$$= 4 \cdot (-2) \cdot v^{-1/2} + \frac{26}{3} \cdot \ln|v| \Big|_1^4 = \left(-\frac{8}{2} + 8\right) + \frac{26}{3} \ln(4) = 4 + \frac{52}{3} \ln 2$$

Örnek: $y=x^2$, $y=6x^2$ parabolleri ile $xy=1$, $xy=4$ hiperbolleri tarafından sınırlanan B bölgesi için

$$\iint_B \frac{y}{x^2}\,dx\,dy = ?$$

Çözüm:



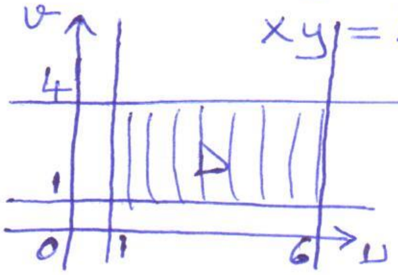
$$\begin{cases} u=\dfrac{y}{x^2} \\ v=xy \end{cases}$$

$y=x^2 \Longrightarrow u=1$

$y=6x^2 \Longrightarrow u=6$

$xy=1 \Longrightarrow v=1$

$xy=4 \Longrightarrow v=4$



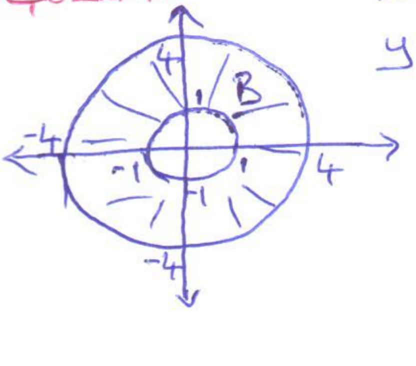
$$j=\frac{\partial(x,y)}{\partial(u,v)}=\frac{1}{\dfrac{\partial(u,v)}{\partial(x,y)}}=\frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}}=\frac{1}{\begin{vmatrix} -\dfrac{2y}{x^3} & \dfrac{1}{x^2} \\ y & x \end{vmatrix}}=\frac{1}{-\dfrac{3y}{x^2}}=\frac{-1}{3u}$$

$$\iint_B \frac{y}{x^2}\,dx\,dy=\iint_D u\cdot|j|\,du\,dv=\int_1^4\left(\int_1^6 \cancel{u}\cdot\frac{1}{3\cancel{u}}\,du\right)dv$$

$$=\int_1^4 \frac{5}{3}\,dv=5$$

Örnek: B bölgesi $x^2+y^2=1$ ve $x^2+y^2=16$ çemberleri arasında kalan bölge olduğuna göre $\iint_B (x^2+y^2)^{\frac{1}{4}}\,dx\,dy=?$

Çözüm:



$$\left.\begin{aligned} x&=r\cos\theta \\ y&=r\sin\theta \end{aligned}\right\} \Longrightarrow j=r$$

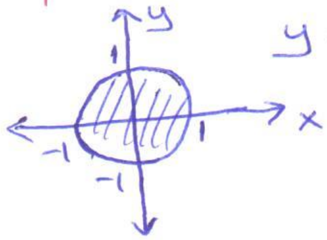
$$\iint_B (x^2+y^2)^{\frac{1}{4}}\,dx\,dy=\int_0^{2\pi}\left(\int_1^4 (r^2)^{\frac{1}{4}}\,r\,dr\right)d\theta$$

$$=\int_0^{2\pi}\left(\frac{2}{5}\,r^{5/2}\Big|_1^4\right)d\theta$$

$$=\int_0^{2\pi}\frac{62}{5}\,d\theta=\frac{124\pi}{5}$$

Örnek: B integrasyon bölgesi $x^2+y^2\leq 1$ dairesi olduğuna göre $\iint_B (1-x^2-y^2)\,dx\,dy = ?$

Çözüm: $\left.\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}\right\} \Rightarrow J = r$

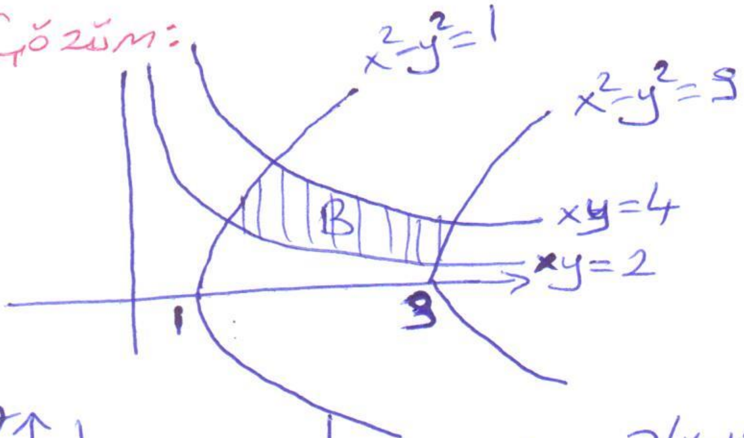


$$\iint_B (1-x^2-y^2)\,dx\,dy = \int_0^{2\pi}\left(\int_0^1 (1-r^2)\,r\,dr\right)d\theta$$
$$= \int_0^{2\pi}\left(\frac{r^2}{2}-\frac{r^4}{4}\Big|_0^1\right)d\theta$$
$$= \int_0^{2\pi}\frac{1}{4}\,d\theta = \frac{\pi}{2}$$

Örnek: B integrasyon bölgesi; birinci bölgede $x^2-y^2=1$, $x^2-y^2=9$, $xy=2$, $xy=4$ hiperbolleri tarafından sınırlanan bölge olduğuna göre

$$\iint_B (x^2+y^2)\,dx\,dy = ?$$

Çözüm:



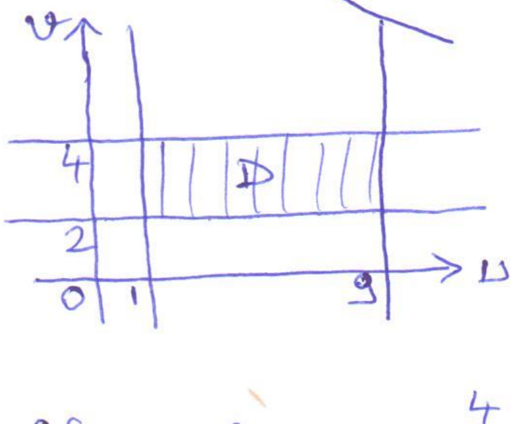
$\left\{\begin{array}{l} u = x^2-y^2 \\ v = xy \end{array}\right.$

$x^2-y^2=1 \Rightarrow u=1$

$x^2-y^2=9 \Rightarrow u=9$

$xy=2 \Rightarrow v=2$

$xy=4 \Rightarrow v=4$



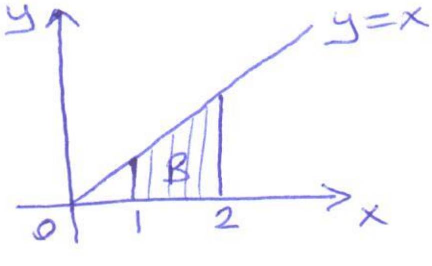
$$J = \frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\dfrac{\partial(u,v)}{\partial(x,y)}} = \frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}}$$

$$= \frac{1}{\begin{vmatrix} 2x & -2y \\ y & x \end{vmatrix}} = \frac{1}{2(x^2+y^2)}$$

$$\iint_B (x^2+y^2)\,dx\,dy = \int_2^4\left(\int_1^9 \cancel{(x^2+y^2)}\cdot\frac{1}{2\cancel{(x^2+y^2)}}\,du\right)dv = \int_2^4 4\,dv = 8$$

Örnek: $B=\{(x,y)\in\mathbb{R}^2 : 1\le x\le 2,\ 0\le y\le x\}$ olduğuna göre $\displaystyle\iint_B \frac{y\sqrt{x^2+y^2}}{x}\,dx\,dy = ?$

Çözüm:



$$1\le x\le 2 \Rightarrow 1\le r\cos\theta\le 2 \Rightarrow \frac{1}{\cos\theta}\le r\le \frac{2}{\cos\theta}$$

$$0\le y\le x \Rightarrow 0\le \frac{y}{x}\le 1 \Rightarrow 0\le \tan\theta\le 1$$

$$\Rightarrow 0\le\theta\le\frac{\pi}{4}$$

$$\left.\begin{array}{l} x=r\cos\theta \\ y=r\sin\theta \end{array}\right\} \Rightarrow J=r$$

$$\iint_B \frac{y}{x}\cdot\sqrt{x^2+y^2}\,dx\,dy = \int_0^{\pi/4}\left(\int_{1/\cos\theta}^{2/\cos\theta}(\tan\theta).r.r\,dr\right)d\theta$$

$$=\int_0^{\pi/4}\left[\frac{r^3}{3}\tan\theta\Big|_{\frac{1}{\cos\theta}}^{\frac{2}{\cos\theta}}\right]d\theta = \int_0^{\pi/4}\left(\frac{7}{3\cos^3\theta}\cdot\frac{\sin\theta}{\cos\theta}\right)d\theta$$

$$=\frac{7}{3}\int_0^{\pi/4}\frac{\sin\theta}{\cos^4\theta}\,d\theta = \frac{7}{9}\cdot\frac{1}{\cos^3\theta}\Big|_0^{\pi/4} = \frac{7}{9}\left(2\sqrt{2}-1\right)$$

Örnek: B bölgesi birinci bölgede $x+y=1$, $x+y=2$ doğrularıyla koordinat eksenleri arasında kalan yamuk olduğuna göre $\displaystyle\iint_B \frac{(x-y)^2}{1+x+y}\,dx\,dy$ integralini $\begin{cases} u=x+y+1 \\ v=x-y \end{cases}$ dönüşümü yardımıyla hesaplayın.

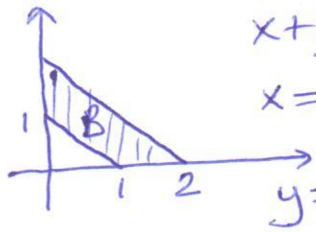
Çözüm:

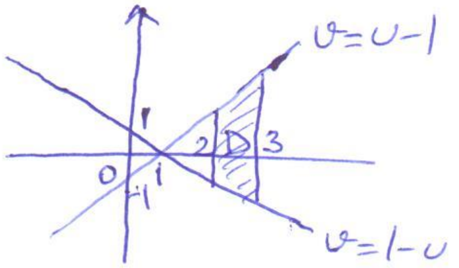
$x+y=1 \Rightarrow u=2$

$x+y=2 \Rightarrow u=3$

$x=0 \Rightarrow \left.\begin{array}{l} u=y+1 \\ v=-y \end{array}\right\} \Rightarrow v=1-u$

$y=0 \Rightarrow \left.\begin{array}{l} u=x+1 \\ v=x \end{array}\right\} \Rightarrow v=u-1$





$$J=\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\frac{\partial(u,v)}{\partial(x,y)}} = \frac{1}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-1}{2}$$

$$\iint_B \frac{(x-y)^2}{1+x+y}\,dx\,dy = \iint_D \frac{v^2}{u}\cdot|J|\,du\,dv = \int_2^3\left(\int_{1-u}^{u-1}\frac{v^2}{2u}\,dv\right)du = \int_2^3\left(\frac{v^3}{6u}\Big|_{1-u}^{u-1}\right)du$$

$$=\int_2^3\left[\frac{(u-1)^3}{6u}-\frac{(1-u)^3}{6u}\right]du = \int_2^3\frac{(u-1)^3}{3u}\,du = \int_2^3\frac{1}{3}\left(u^2-3u+3-\frac{1}{u}\right)du$$

$$=\frac{1}{3}\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln|u|\right)\Big|_2^3 = \frac{1}{3}\left(\frac{11}{6}+\ln\left(\frac{2}{3}\right)\right)$$