TÜREV

$A \subset \mathbb{R}$, $a \in A$ ve f, A da tanımlı bir fonksiyon olsun $\lim\limits_{x \to a} \frac{f(x)-f(a)}{x-a}$ limiti veya $x=a+h$ alınarak $\lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$ limiti var ise, f fonksiyonu a noktasında türevlenebilir veya diferansiyellenebilir denir. $f'(a)$, $\frac{df}{dx}(a)$ veya $Df(a)$ sembollerinden biri ile gösterilir.

$f'(a^-) = \lim\limits_{x \to a^-} \frac{f(x)-f(a)}{x-a}$ limiti var ise, f fonksiyonu $x=a$ noktasında soldan türevlidir denir.

$f'(a^+) = \lim\limits_{x \to a^+} \frac{f(x)-f(a)}{x-a}$ limiti var ise, f fonksiyonu $x=a$ noktasında sağdan türevlidir denir.

$f'(a^+) = f'(a^-)$ ise, f fonksiyonu a noktasında türevlenebilirdir. Aksi halde, türevlenemezdir.
Ayrıca, f fonksiyonu türevli ise süreklidir, sürekli değil ise türevli değildir.

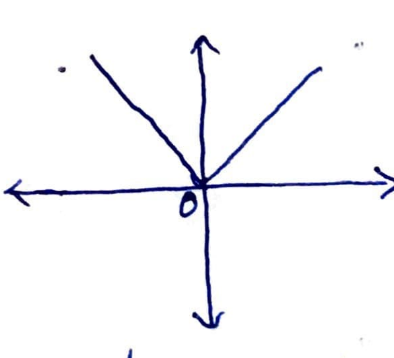
Örnekler:

1) $f(x)=|x|$ ise $f'(0)=?$

Çözüm: $f'(0) = \lim\limits_{x \to 0} \frac{f(x)-f(0)}{x-0} = \lim\limits_{x \to 0} \frac{|x|}{x}$

$f'(0^+) = \lim\limits_{x \to 0^+} \frac{|x|}{x} = \lim\limits_{x \to 0^+} \frac{x}{x} = \lim\limits_{x \to 0^+} 1 = 1$

$f'(0^-) = \lim\limits_{x \to 0^-} \frac{|x|}{x} = \lim\limits_{x \to 0^-} \frac{-x}{x} = \lim\limits_{x \to 0^-} (-1) = -1$



$f'(0^+) \neq f'(0^-)$ olduğundan $x=0$ da türevlenemezdir.

2) $f(x)=[\![x]\!]$ fonksiyonunun $x=3$ ve $x=\frac{7}{2}$ noktalarındaki türevlerini bulunuz.

Çözüm: $f'(3)=\lim\limits_{x\to 3}\frac{f(x)-f(3)}{x-3}=\lim\limits_{x\to 3}\frac{[\![x]\!]-3}{x-3}$

$$f'(3^+)=\lim_{x\to 3^+}\frac{[\![x]\!]-3}{x-3}=\lim_{x\to 3^+}\frac{3-3}{x-3}=\lim_{x\to 3^+}0=0$$

$$f'(3^-)=\lim_{x\to 3^-}\frac{[\![x]\!]-3}{x-3}=\lim_{x\to 3^-}\frac{2-3}{x-3}=\lim_{x\to 3^-}\frac{-1}{x-3}=+\infty$$

olduğundan $x=3$ de türevlenemezdir.

II. Yol: $x=3$ de fonksiyon sürekli olmadığından $x=3$ de türevlenemezdir.

$$f'\left(\frac{7}{2}\right)=\lim_{x\to\frac{7}{2}}\frac{f(x)-f\left(\frac{7}{2}\right)}{x-\frac{7}{2}}=\lim_{x\to\frac{7}{2}}\frac{[\![x]\!]-3}{x-\frac{7}{2}}=\lim_{x\to\frac{7}{2}}\frac{3-3}{x-\frac{7}{2}}=0$$

3) $f(x)=x\sqrt{x^2-2x+1}$ ise $f'(1)=?$

Çözüm: $f(x)=x\sqrt{x^2-2x+1}=x\sqrt{(x-1)^2}=x\cdot|x-1|$

$$f'(1)=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1}\frac{x\cdot|x-1|}{x-1}$$

$$f'(1^+)=\lim_{x\to 1^+}\frac{x\cdot|x-1|}{x-1}=\lim_{x\to 1^+}\frac{x\cancel{(x-1)}}{\cancel{x-1}}=\lim_{x\to 1^+}x=1$$

$$f'(1^-)=\lim_{x\to 1^-}\frac{x\cdot|x-1|}{x-1}=\lim_{x\to 1^-}\frac{-x(x-1)}{x-1}=\lim_{x\to 1^-}(-x)=-1$$

olduğundan $x=1$ de türevlenemezdir.

4) $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$ ise $f'(0)=?$

Çözüm: $f'(0)=\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim\limits_{x\to 0}\frac{x^2\sin\left(\frac{1}{x}\right)}{x}=\lim\limits_{x\to 0}x\cdot\sin\left(\frac{1}{x}\right)=0$

5) $f(x)=\begin{cases} 2x^2-3x+4\sin x+x^2\cos\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$ ise $f'(0)=?$

Çözüm: $f'(0)=\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim\limits_{x\to 0}\frac{2x^2-3x+4\sin x+x^2\cos\left(\frac{1}{x}\right)}{x}$

$$=\lim_{x\to 0}\left(2x-3+4\cdot\frac{\sin x}{x}+x\cdot\cos\frac{1}{x}\right)$$

$$=0-3+4+0$$

$$=1$$

6) g, $g(0)=g'(0)=0$ özelliğine sahip bir fonksiyon ve

$f(x)=\begin{cases} g(x)\cos\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$ ise $f'(0)=?$

Çözüm: $g'(0)=\lim\limits_{x\to 0}\frac{g(x)-g(0)}{x-0}=\lim\limits_{x\to 0}\frac{g(x)}{x}\Rightarrow\lim\limits_{x\to 0}\frac{g(x)}{x}=0$

$f'(0)=\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim\limits_{x\to 0}\frac{g(x)\cdot\cos\frac{1}{x}}{x}=\lim\limits_{x\to 0}\frac{g(x)}{x}\cdot\cos\left(\frac{1}{x}\right)=0$

7) $f'(1)=3$ olduğuna göre $\lim\limits_{h\to 0}\frac{f(1+h)-f(1-h)}{h}=?$

Çözüm: $\lim\limits_{h\to 0}\frac{f(1+h)-f(1-h)}{h}=\lim\limits_{h\to 0}\left(\frac{f(1+h)-f(1)}{h}-\frac{f(1-h)-f(1)}{h}\right)$

$=\lim\limits_{h\to 0}\frac{f(1+h)-f(1)}{h}-\lim\limits_{h\to 0}\frac{f(1-h)-f(1)}{h}$ $\left(\begin{array}{c} h=-k \\ h\to 0\Rightarrow k\to 0\end{array}\right)$

$=\lim\limits_{h\to 0}\frac{f(1+h)-f(1)}{h}-\lim\limits_{k\to 0}\frac{f(1+k)-f(1)}{-k}=2\cdot f'(1)=2\cdot 3=6$

8) $f'(3)=2$ olduğuna göre $\lim\limits_{h\to 0} \dfrac{f(3+4h)-f(3+2h)}{h}=?$

Çözüm: $\lim\limits_{h\to 0} \dfrac{f(3+4h)-f(3+2h)}{h} = \lim\limits_{h\to 0}\left(\dfrac{f(3+4h)-f(3)}{h} - \dfrac{f(3+2h)-f(3)}{h}\right)$

$= \lim\limits_{h\to 0} \dfrac{f(3+4h)-f(3)}{h} - \lim\limits_{h\to 0} \dfrac{f(3+2h)-f(3)}{h}$ $\boxed{\begin{array}{c|c} k=4h & l=2h \\ h\to 0 \Rightarrow k\to 0 & h\to 0 \Rightarrow l\to 0 \end{array}}$

$= \lim\limits_{k\to 0} \dfrac{f(3+k)-f(3)}{k/4} - \lim\limits_{l\to 0} \dfrac{f(3+l)-f(3)}{l/2}$

$= 4 \lim\limits_{k\to 0} \dfrac{f(3+k)-f(3)}{k} - 2 \lim\limits_{l\to 0} \dfrac{f(3+l)-f(3)}{l}$

$= 4 \cdot f'(3) - 2f'(3)$

$= 2 \cdot f'(3)$

$= 4$

9) $f(x) = x(x-1)(x-2)\cdots(x-1000)$ ise, $f'(0)=?$

Çözüm: $f'(0) = \lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x-0} = \lim\limits_{x\to 0} \dfrac{x(x-1)(x-2)\cdots(x-1000)}{x}$

$= \lim\limits_{x\to 0} (x-1)(x-2)\cdots(x-1000)$

$= (-1)\cdot(-2)\cdots(-1000)$

$= 1000!$

Türev Almada Genel Kurallar:

1) $f(x)=c \Rightarrow f'(x)=0$

2) $\left(c_1 f(x)+c_2 g(x)\right)'=c_1 f'(x)+c_2 g'(x)$

3) $\left(f(x).g(x)\right)'=f'(x).g(x)+g'(x).f(x)$

4) $g(x)\neq 0$ o.ü. $\left(\dfrac{f(x)}{g(x)}\right)'=\dfrac{f'(x).g(x)-g'(x).f(x)}{(g(x))^2}$ olur.

5) $(g\circ f)'(x)=\left(g(f(x))\right)'=f'(x).g'(f(x))$

6) $f(x)=x^k$ $(k\in\mathbb{R})$ ise $f'(x)=k.x^{k-1}$ olur.

$\left(f(x)\right)^k=k.f'(x).\left(f(x)\right)^{k-1}$

7) $f(x)=\sin x \Rightarrow f'(x)=\cos x$

$\left(\sin(u(x))\right)'=u'(x).\cos(u(x))$

8) $(\cos x)'=-\sin x$, $\left(\cos(u(x))\right)'=-u'(x).\sin x$

9) $(\tan x)'=1+\tan^2 x=\dfrac{1}{\cos^2 x}$

$\left(\tan(u(x))\right)'=u'(x).\left(1+\tan^2(u(x))\right)=\dfrac{u'(x)}{\cos^2(u(x))}$

10) $(\cot x)'=-(1+\cot^2 x)=\dfrac{-1}{\sin^2 x}$

$\left(\cot(u(x))\right)'=-u'(x).\left(1+\cot^2(u(x))\right)=\dfrac{-u'(x)}{\sin^2(u(x))}$

11) $(\arcsin x)'=\dfrac{1}{\sqrt{1-x^2}}$, $\left(\arcsin(u(x))\right)'=\dfrac{+u'(x)}{\sqrt{1-(u(x))^2}}$

12) $(\arccos x)'=\dfrac{-1}{\sqrt{1-x^2}}$, $\left(\arccos(u(x))\right)'=\dfrac{-u'(x)}{\sqrt{1-(u(x))^2}}$

13) $(\arctan x)'=\dfrac{1}{1+x^2}$, $\left(\arctan(u(x))\right)'=\dfrac{u'(x)}{1+(u(x))^2}$

14) $(\text{arccot}\, x)'=\dfrac{-1}{1+x^2}$, $\left(\text{arccot}(u(x))\right)'=\dfrac{-u'(x)}{1+(u(x))^2}$

15) $(\log_a x)' = \frac{1}{x} \cdot \log_a e$, $\quad (\log_a(u(x)))' = \frac{u'(x)}{u(x)} \cdot \log_a e$

16) $(\ln x)' = \frac{1}{x}$, $\quad (\ln(u(x)))' = \frac{u'(x)}{u(x)}$

17) $(a^x)' = a^x . \ln a$, $\quad (a^{u(x)})' = u'(x) . a^{u(x)} . \ln a$

18) $(e^x)' = e^x$, $\quad (e^{u(x)})' = u'(x) . e^{u(x)}$

19) $(\cosh x)' = \sinh x$, $\quad (\cosh(u(x)))' = u'(x) . \sinh(u(x))$

20) $(\sinh x)' = \cosh x$, $\quad (\sinh(u(x)))' = u'(x) . \cosh(u(x))$

21) $(\tanh x)' = \frac{1}{\cosh^2 x}$, $\quad (\tanh(u(x)))' = \frac{u'(x)}{\cosh^2(u(x))}$

22) $(\coth x)' = \frac{-1}{\sinh^2 x}$, $\quad (\coth(u(x)))' = \frac{-u'(x)}{\sinh^2(u(x))}$

<u>Örnekler:</u>

1) $f(x) = e^{2x^2+2x-1} + \ln(\cos(\sqrt{x})) \Longrightarrow f'(x) = ?$

<u>Çözüm:</u> $f'(x) = (4x+2)e^{2x^2+2x-1} + \frac{(\cos(\sqrt{x}))'}{\cos(\sqrt{x})}$

$$= (4x+2)e^{2x^2+2x-1} - \frac{1}{2\sqrt{x}} \cdot \tan(\sqrt{x})$$

2) $f(x) = \left(\frac{x}{2x+3}\right)^{2/3} + \sin^5(2x+3)$ ise $f'(x) = ?$

<u>Çözüm:</u> $f'(x) = \frac{2}{3} \cdot \frac{2x+3-2x}{(2x+3)^2} \cdot \left(\frac{x}{2x+3}\right)^{-1/3} + 5.2.\cos(2x+3).\sin^4(2x+3)$

$$\Longrightarrow f'(x) = \frac{2}{(2x+3)^2} \cdot \left(\frac{x}{2x+3}\right)^{-1/3} + 10.\cos(2x+3).\sin^4(2x+3)$$

3) $f(x) = \arccos(\sqrt{x}) . \arcsin(3e^{2x}+x)$ ise $f'(x) = ?$

<u>Çözüm:</u> $f'(x) = (\arccos(\sqrt{x}))' \arcsin(3e^{2x}+x) + (\arcsin(3e^{2x}+x))' . \arccos(\sqrt{x})$

$$f'(x) = -\frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-(\sqrt{x})^2}} \cdot \arcsin(3e^{2x}+x) + \frac{6e^{2x}+1}{\sqrt{1-(3e^{2x}+x)^2}} \cdot \arccos(\sqrt{x})$$

Örnekler:

1) $f(x)=g(x^2-x)$, $g'(6)=5 \Rightarrow f'(3)=?$

Çözüm: $f'(x)=(2x-1)g'(x^2-x)$

$\Rightarrow f'(3)=(2\cdot3-1)g'(3^2-3)=5\cdot g'(6)=25$

2) $f(3x+4)=\sin(ax)$, $f'(4)=2 \Rightarrow a=?$

Çözüm: $f(3x+4)=\sin(ax) \Rightarrow 3.f'(3x+4)=a.\cos(ax)$

$x=0$ alınırsa,

$3.f'(4)=a.\underbrace{\cos 0}_{1} \Rightarrow a=3.f'(4)=6$

3) $f(x)=g(x^3+2)$, $g'(10)=6 \Rightarrow f'(2)=?$

Çözüm: $f'(x)=3x^2.g'(x^3+2) \Rightarrow f'(2)=12.g'(10)=72$

4) $f(2x+5)=x^3-5x+1 \Rightarrow f(3)+f'(3)=?$

Çözüm: $f(2x+5)=x^3-5x+1 \xrightarrow{x=-1} f(3)=-1+5+1=5$

$2.f'(2x+5)=3x^2-5 \xrightarrow{x=-1} 2.f'(3)=3.1-5 \Rightarrow f'(3)=-1$

$f(3)+f'(3)=5-1=4$

5) Tek fonksiyonun türevinin çift fonksiyon ve çift fonksiyonun türevinin tek fonksiyon olduğunu gösteriniz

Çözüm: f tek fonksiyon ise $f(-x)=-f(x)$ olur. Her iki tarafın türevine geçilirse, $-f'(-x)=-f'(x) \Rightarrow f'(-x)=f'(x)$ olduğundan f' çift fonksiyondur.

$f(-x)=f(x) \Rightarrow -f'(-x)=f'(x) \Rightarrow f'(-x)=-f'(x) \Rightarrow f'$ tek fonk

6) f fonksiyonu 3. dereceden kökleri a, b, c olan bir fonksiyon ise, $\frac{a}{f'(a)} + \frac{b}{f'(b)} + \frac{c}{f'(c)} = ?$

Çözüm: $f(x) = A(x-a)(x-b)(x-c)$ olduğundan

$$\frac{a}{f'(a)} + \frac{b}{f'(b)} + \frac{c}{f'(c)} = \frac{a}{\underset{(b-c)}{A(a-b)(a-c)}} + \frac{b}{\underset{(c-a)}{A(b-a)(b-c)}} + \frac{c}{\underset{(a-b)}{A(c-a)(c-b)}}$$

$$= \frac{\cancel{ab} - \cancel{ac} + \cancel{bc} - \cancel{ab} + \cancel{ac} - \cancel{bc}}{A(a-b)(a-c)(b-c)} = 0$$

7) $y = 2^{\sin^3 x} \Rightarrow y' = ?$

Çözüm: $y' = (\sin^3 x)' \cdot 2^{\sin^3 x} \cdot \ln 2$

$\Rightarrow y' = 3 \cdot \sin^2 x \cdot \cos x \cdot 2^{\sin^3 x} \cdot \ln 2$

8) $y = 3^{\ln x^2} \Rightarrow y' = ?$

Çözüm: $y' = (\ln x^2)' \cdot 3^{\ln x^2} \cdot \ln 3$

$\Rightarrow y' = \frac{2}{x} \cdot 3^{\ln x^2} \cdot \ln 3$

9) $y = (5)^{3^x} \Rightarrow y' = ?$

Çözüm: $y' = (3^x)' \cdot (5)^{3^x} \cdot \ln 5 = 3^x \cdot \ln 3 \cdot (5)^{3^x} \cdot \ln 5$

10) $y = (2)^{2^{x^2}} \Rightarrow y' = ?$

Çözüm: $y' = (2^{x^2})' \cdot (2)^{2^{x^2}} \cdot \ln 2$

$\Rightarrow y' = 2x \cdot 2^{x^2} \cdot \ln 2 \cdot (2)^{2^{x^2}} \cdot \ln 2$

$\Rightarrow y' = 2x \cdot 2^{x^2} \cdot (2)^{2^{x^2}} \cdot \ln^2 2$

Logaritmik Türev Alma:

$$y=(f(x))^{g(x)} \Rightarrow \ln y=\ln (f(x))^{g(x)} \Rightarrow \ln y = g(x).\ln(f(x))$$

$$\Rightarrow \frac{y'}{y}=g'(x).\ln(f(x))+\frac{f'(x)}{f(x)}\cdot g(x)$$

$$\Rightarrow y'=(f(x))^{g(x)}\left[g'(x).\ln(f(x))+\frac{f'(x)}{f(x)}\cdot g(x)\right]$$

Örnekler:

1) $y=(1+x^2)^x \Rightarrow y'=?$

Çözüm: $\ln y = x.\ln(1+x^2)$

$$\frac{y'}{y}=\ln(1+x^2)+\frac{2x^2}{1+x^2}$$

$$y'=(1+x^2)^x\left[\ln(1+x^2)+\frac{2x^2}{1+x^2}\right]$$

2) $f(x)=x^{\sin x} \Rightarrow f'\left(\frac{\pi}{2}\right)=?$

Çözüm: $f(x)=x^{\sin x} \Rightarrow \ln(f(x))=\sin x.\ln x$

$$\Rightarrow \frac{f'(x)}{f(x)}=\cos x.\ln x+\frac{1}{x}\cdot\sin x$$

$$\Rightarrow f'(x)=x^{\sin x}.\left[\cos x.\ln x+\frac{\sin x}{x}\right]$$

$$\Rightarrow f'\left(\frac{\pi}{2}\right)=\frac{\pi}{2}\left(0+\frac{1}{\pi/2}\right)=1$$

3) $y=(2x^2-1)^{3x+1} \Rightarrow y'(1)=?$

Çözüm: $\ln y=(3x+1).\ln(2x^2-1)$

$$\frac{y'}{y}=3.\ln(2x^2-1)+\frac{4x}{2x^2-1}\cdot(3x+1)$$

$$y'=(2x^2-1)^{3x+1}\left[3.\ln(2x^2-1)+\frac{4x}{2x^2-1}\cdot(3x+1)\right] \Rightarrow y'(1)=16$$

4) $f(x)=(x^2+1)^{\arctan x} \Rightarrow f'(1)=?$

<u>Çözüm:</u> $\ln(f(x))=\arctan x \cdot \ln(x^2+1)$

$$\Rightarrow \frac{f'(x)}{f(x)}=\frac{1}{1+x^2}\cdot \ln(x^2+1)+\frac{2x}{x^2+1}\cdot \arctan x$$

$$\Rightarrow f'(x)=(x^2+1)^{\arctan x}\left[\frac{\ln(x^2+1)}{x^2+1}+\frac{2x}{x^2+1}\cdot \arctan x\right]$$

$$\Rightarrow f'(1)=2^{\frac{\pi}{4}}\left(\frac{\ln 2}{2}+\frac{\pi}{4}\right)$$

5) $y=\sin(2x^2+1)^{3x-1} \Rightarrow y'=?$

<u>Çözüm:</u>

$$f(x)=(2x^2+1)^{3x-1}$$

$$\ln(f(x))=(3x-1)\cdot \ln(2x^2+1)$$

$$\frac{f'(x)}{f(x)}=3\cdot \ln(2x^2+1)+\frac{4x}{2x^2+1}\cdot(3x-1)$$

$$f'(x)=(2x^2+1)^{3x-1}\left[3\cdot \ln(2x^2+1)+\frac{4x}{2x^2+1}\cdot(3x-1)\right]$$

$$y'=(2x^2+1)^{3x-1}\left[3\cdot \ln(2x^2+1)+\frac{4x}{2x^2+1}\cdot(3x-1)\right]\cdot \cos(2x^2+1)^{3x-1}$$

6) $y=\dfrac{(\ln x)^x}{x^{\ln x}} \Rightarrow y'=?$

<u>Çözüm:</u> $u=(\ln x)^x \Rightarrow \ln u = x\cdot \ln(\ln x) \Rightarrow u'=(\ln x)^x\left[\ln(\ln x)+\frac{1}{\ln x}\right]$

$$v=x^{\ln x} \Rightarrow \ln v=(\ln x)^2 \Rightarrow \frac{v'}{v}=2\cdot\frac{1}{x}\cdot \ln x \Rightarrow v'=2\cdot x^{\ln x}\cdot\frac{\ln x}{x}$$

$$y'=\frac{(\ln x)^x\left(\ln(\ln x)+\frac{1}{\ln x}\right)x^{\ln x}-2x^{\ln x}\cdot\frac{\ln x}{x}\cdot(\ln x)^x}{x^{2\ln x}}$$

7) $y=x^{(x^x)} \Rightarrow y'=?$

<u>Çözüm:</u> $f(x)=x^x \Rightarrow \ln(f(x))=x\cdot \ln x \Rightarrow \frac{f'(x)}{f(x)}=\ln x+1 \Rightarrow f'(x)=x^x\cdot(1+\ln x)$

$$y=x^{(x^x)} \Rightarrow \ln y = x^x\cdot \ln x \Rightarrow \frac{y'}{y}=x^x\cdot(1+\ln x)\cdot \ln x+\frac{1}{x}\cdot x^x$$

$$\Rightarrow y'=x^{(x^x)}\left[x^x(1+\ln x)\cdot \ln x+\frac{x^x}{x}\right]$$

Parametrik Denklemi Verilen Fonksiyonların Türevleri

$\left.\begin{array}{l} x=u(t) \\ y=v(t) \end{array}\right\}$ parametrik denklemi verilen fonksiyonun türevi

$$y' = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

ile hesaplanır.

Örnekler:

1) $\left.\begin{array}{l} x = a\cos^2 t \\ y = b\sin^2 t \end{array}\right\} \Rightarrow \frac{dy}{dx} = ?$

Çözüm: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2b\sin t\cos t}{-2a\cos t\sin t} = -\frac{b}{a}$

2) $\left.\begin{array}{l} x = 2(t-\sin t) \\ y = 2(1-\cos t) \end{array}\right\}$ parametrik denklemi ile verilen sikloid eğrisinin $t = \frac{\pi}{2}$ noktasındaki türevini bulunuz.

Çözüm: $y' = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\sin t}{2(1-\cos t)} = \frac{\sin t}{1-\cos t}$

$$y'\left(\frac{\pi}{2}\right) = \frac{1}{1-0} = 1$$

3) $\left.\begin{array}{l} x = \frac{1}{1+t} \\ y = \left(\frac{t}{t+1}\right)^2 \end{array}\right\} \Rightarrow \frac{dy}{dx}(1) = ?$

Çözüm: $\frac{dy}{dx} = \frac{2 \cdot \frac{t}{t+1} \cdot \frac{t+1-\cancel{t}}{(t+1)^2}}{-\frac{1}{(t+1)^2}} = \frac{-2t}{t+1}$

$$\frac{dy}{dx}(1) = \frac{-2}{1+1} = -1$$

4) $\left.\begin{array}{l} x = e^t \cos t \\ y = e^t \sin t \end{array}\right\} \Rightarrow \frac{dy}{dx}(0) = ?$

<u>Çözüm:</u> $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t \sin t + e^t \cos t}{e^t \cos t - e^t \sin t} = \frac{\sin t + \cos t}{\cos t - \sin t}$

$\frac{dy}{dx}(0) = \frac{0+1}{1-0} = 1$

5) $\left.\begin{array}{l} x = \cos^3 t \sin t \\ y = \sin^3 t \cos t \end{array}\right\} \Rightarrow \frac{dy}{dx} = ?$

<u>Çözüm:</u> $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3\cos^2 t \sin^2 t - \sin^4 t}{-3\cos^2 t \sin^2 t + \cos^4 t}$

6) $\left.\begin{array}{l} x = 2t^3 + t^2 \\ y = 2t + 3t^2 \end{array}\right\} \Rightarrow y'(4) = ?$

<u>Çözüm:</u> $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2+6t}{6t^2+2t} = \frac{1}{t}$

$\left.\frac{dy}{dx}\right|_{t=4} = \frac{1}{4}$