Trigonometrik integraller:

1. $\int \sin(ax).\sin(bx)dx$, $\int \sin(ax)\cos(bx)dx$, $\int \cos(ax)\cos(bx)dx$ tipindeki integraller:

$$\sin(ax).\sin(bx) = \frac{1}{2}\left[\cos(a-b)x - \cos(a+b)x\right]$$
$$\sin(ax).\cos(bx) = \frac{1}{2}\left[\sin(a+b)x + \sin(a-b)x\right]$$
$$\cos(ax).\cos(bx) = \frac{1}{2}\left[\cos(a+b)x + \cos(a-b)x\right]$$

Örnek: $\int \sin(2x).\sin(8x)dx = \frac{1}{2}\int(\cos 6x - \cos 10x)dx$
$$= \frac{1}{12}\sin 6x - \frac{1}{20}\sin 10x + c$$

Örnek: $\int \sin(3x).\cos(6x)dx = \frac{1}{2}\int(\sin 9x + \sin(-3x))dx$
$$= \frac{-1}{18}\cos 9x + \frac{1}{6}\cos(3x) + c$$

2. $\int \sin^m x.\cos^n x\, dx$ tipindeki integraller:

a) m veya n nin tek olma durumu:

m tek ise $\sin^m x = \sin^{m-1}x.\sin x$ yazılarak $\sin^{m-1}x$, $\cos x$ cinsinden yazılır ve $t=\cos x$ değişken değiştirmesi yapılır.

n tek ise $\cos^n x = \cos^{n-1}x\cos x$ yazılarak $\cos^{n-1}x$, $\sin x$ cinsinden yazılır ve $t=\sin x$ değişken değiştirmesi yapılır.

Örnek: $\int \sin^5 x.\cos^2 x\,dx = \int \sin^4 x.\sin x.\cos^2 x\,dx$

$t=\cos x$
$dt=-\sin x\,dx$

$$= \int (1-\cos^2 x)^2 \cos^2 x.\sin x\,dx$$
$$= -\int (1-t^2)^2.t^2 dt$$
$$= -\int (t^4-2t^2+1)t^2 dt$$
$$= -\int (t^6-2t^4+t^2)dt = -\frac{t^7}{7} + 2.\frac{t^5}{5} - \frac{t^3}{3} + c$$
$$= -\frac{1}{7}\cos^7 x + \frac{2}{5}\cos^5 x - \frac{1}{3}\cos^3 x + c$$

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Örnek: $\int \sin^3 x \cdot \cos^3 x \, dx = \int \sin^3 x \cdot \cos^2 x \cdot \cos x \, dx$

$$= \int \sin^3 x \cdot (1-\sin^2 x) \cos x \, dx \qquad \begin{array}{l} t = \sin x \\ dt = \cos x \, dx \end{array}$$

$$= \int t^3 (1-t^2) dt = \int (t^3 - t^5) dt$$

$$= \frac{t^4}{4} - \frac{t^6}{6} + c$$

$$= \frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + c$$

Örnek: $\int \sin^5 x \, dx = \int \sin^4 x \cdot \sin x \, dx = \int (1-\cos^2 x)^2 \sin x \, dx$

$$\begin{array}{l|} t = \cos x \\ dt = -\sin x \, dx \end{array} \qquad = -\int (1-t^2)^2 dt = -\int (t^4 - 2t^2 + 1) dt$$

$$= -\left(\frac{t^5}{5} - 2\frac{t^3}{3} + t\right) + c$$

$$= -\frac{1}{5}\cos^5 x + \frac{2}{3}\cos^3 x - \cos x + c$$

b) m ve n nin çift olma durumu:

$$\cos^2 x = \frac{1}{2}(1+\cos 2x) \quad \text{ve} \quad \sin^2 x = \frac{1}{2}(1-\cos 2x)$$

bağıntıları yardımıyla çözülür.

Örnek: $\int \sin^4 x \, dx = \int (\sin^2 x)^2 dx = \int \left[\frac{1}{2}(1-\cos 2x)\right]^2 dx$

$$= \frac{1}{4}\int (\cos^2 2x - 2\cos 2x + 1) dx$$

$$= \frac{1}{4}\int \left[\frac{1}{2}(1+\cos 4x) - 2\cos 2x + 1\right] dx$$

$$= \frac{1}{4}\int \left(\frac{1}{2}\cos 4x - 2\cos 2x + \frac{3}{2}\right) dx$$

$$= \frac{1}{4}\left(\frac{1}{8}\sin 4x - \sin 2x + \frac{3x}{2}\right) + c$$

Örnek: $\int \sin^2 x \cdot \cos^4 x \, dx = \int \frac{1}{2}(1-\cos 2x) \cdot \frac{1}{4}(1+\cos 2x)^2 dx$

$$= \frac{1}{8}\int (1-\cos^2 2x)(1+\cos 2x) dx = \frac{1}{8}\int \sin^2 2x \cdot (1+\cos 2x) dx = \frac{1}{8}\int \frac{1}{2}(1-\cos 4x)(1+\cos 2x) dx$$

$$= \frac{1}{16}\int (1 - \cos 4x + \cos 2x - \cos 4x \cdot \cos 2x) dx = \frac{1}{16}\int \left(1 - \cos 4x + \cos 2x - \frac{1}{2}(\cos 6x + \cos 2x)\right) dx$$

$$= \frac{1}{16}\left(x - \frac{1}{4}\sin 4x + \frac{1}{4}\sin 2x - \frac{1}{12}\sin 6x\right) + c$$

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3. $\int R(\sin x, \cos x)dx$ tipindeki integraller:

a) $R(-\sin x,\cos x)=-R(\sin x,\cos x)$ ve $R(\sin x,-\cos x)=-R(\sin x,\cos x)$ durumu

$R(-\sin x,\cos x)=-R(\sin x,\cos x)$ ise fonksiyon, $\cos x$'e bağlı rasyonel fonksiyon ile $\sin x$'in çarpımı şekline getirilerek $t=\cos x$ değişken değiştirmesi yapılır.

$R(\sin x,-\cos x)=-R(\sin x,\cos x)$ ise fonksiyon, $\sin x$'e bağlı rasyonel fonksiyon ile $\cos x$'in çarpımı şekline getirilerek $t=\sin x$ değişken değiştirmesi yapılır.

Örnek: $\int \frac{\cos^5 x}{\sin^2 x}dx=\int \frac{\cos^4 x}{\sin^2 x}\cdot\cos x\,dx=\int \frac{(1-\sin^2 x)^2}{\sin^2 x}\cos x\,dx$

$\boxed{\begin{aligned} t&=\sin x\\ dt&=\cos x\,dx\end{aligned}}$

$$=\int \frac{(1-t^2)^2}{t^2}dt=\int \frac{t^4-2t^2+1}{t^2}dt$$

$$=\int\left(t^2-2+\frac{1}{t^2}\right)dt=\frac{t^3}{3}-2t-\frac{1}{t}+c$$

$$=\frac{1}{3}\sin^3 x-2\sin x-\frac{1}{\sin x}+c$$

Örnek: $\int \cos x\cdot\tan^3 x\,dx=\int\cos x\cdot\frac{\sin^3 x}{\cos^3 x}dx=\int\frac{\sin^2 x}{\cos^2 x}\cdot\sin x\,dx$

$\boxed{\begin{aligned} t&=\cos x\\ dt&=-\sin x\,dx\end{aligned}}$

$$=\int\frac{1-\cos^2 x}{\cos^2 x}\cdot\sin x\,dx=\int\frac{1-t^2}{t^2}\cdot(-1)\cdot dt$$

$$=\int\left(1-\frac{1}{t^2}\right)dt=t+\frac{1}{t}+c$$

$$=\cos x+\frac{1}{\cos x}+c$$

Örnek: $\int\frac{\sin^2 x}{\cos^3 x}dx=\int\frac{\sin^2 x}{\cos^4 x}\cdot\cos x\,dx=\int\frac{\sin^2 x}{(1-\sin^2 x)^2}\cdot\cos x\,dx$

$\boxed{\begin{aligned} t&=\sin x\\ dt&=\cos x\,dx\end{aligned}}$

$$=\int\frac{t^2}{(1-t^2)^2}dt=\int\frac{t^2\,dt}{(1-t)^2(1+t)^2}$$

$$\frac{t^2}{(1-t)^2(1+t)^2}=\frac{A}{\underset{(1-t)(1+t)^2}{1-t}}+\frac{B}{\underset{(1+t)^2}{(1-t)^2}}+\frac{C}{\underset{(1-t)^2(1+t)}{1+t}}+\frac{D}{\underset{(1-t)^2}{(1+t)^2}}$$

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$$t^2=(-A+C)t^3+(-A+B-C+D)t^2+(A+2B-C-2D)t+A+B+C+D$$

$$\left.\begin{aligned} -A+C&=0\\ -A+B-C+D&=1\\ A+2B-C-2D&=0\\ A+B+C+D&=0 \end{aligned}\right\} \Rightarrow A=C=\frac{-1}{4}\ ,\quad B=D=\frac{1}{4}$$

$$\int \frac{\sin^2 x}{\cos^3 x}dx=\int\left(\frac{-\frac{1}{4}}{1-t}+\frac{\frac{1}{4}}{(1-t)^2}-\frac{\frac{1}{4}}{1+t}+\frac{\frac{1}{4}}{(1+t)^2}\right)dt$$

$$=\frac{1}{4}\left(\ln|t-1|+\frac{1}{1-t}-\ln|1+t|-\frac{1}{1+t}\right)+c$$

$$=\frac{1}{4}\left(\ln\left|\frac{1-t}{1+t}\right|+\frac{1}{1-t}-\frac{1}{1+t}\right)+c$$

$$=\frac{1}{4}\left(\ln\left|\frac{1-\sin x}{1+\sin x}\right|+\frac{1}{1-\sin x}-\frac{1}{1+\sin x}\right)+c$$

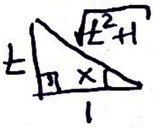
b) $R(-\sin x,-\cos x)=R(\sin x,\cos x)$ durumu:

$R(-\sin x,-\cos x)=R(\sin x,\cos x)$ ise, $\tan x=t$ değişken değiştirmesi yapılır.

Örnek: $\displaystyle\int\frac{dx}{\cos^2 x\cdot\sin^4 x}=\int\frac{\cancel{\frac{dt}{1+t^2}}}{\frac{1}{\cancel{t^2+1}}\cdot\frac{t^4}{(t^2+1)^2}}=\int\frac{(t^2+1)^2dt}{t^4}$

$\tan x=t\Rightarrow x=\arctan t$

$dx=\dfrac{dt}{1+t^2}$



$\sin x=\dfrac{t}{\sqrt{t^2+1}}$

$\cos x=\dfrac{1}{\sqrt{t^2+1}}$

$$=\int\frac{t^4+2t^2+1}{t^4}dt$$

$$=\int\left(1+\frac{2}{t^2}+\frac{1}{t^4}\right)dt$$

$$=t-\frac{2}{t}-\frac{1}{3t^3}+c$$

$$=\tan x-\frac{2}{\tan x}-\frac{1}{3\tan^3 x}+c$$

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$\sqrt{ax^2+bx+c}$ ifadesini bulunduran integraller:

1. $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ tipindeki integraller $(b^2-4ac>0)$:

$a<0$ ise $\int \frac{du}{\sqrt{k^2-u^2}} = \arcsin\left(\frac{u}{k}\right)+c$ şekline getirilir.

$a>0$ ise $\int \frac{du}{\sqrt{u^2\mp p}} = \ln\left|u+\sqrt{u^2\mp p}\right|+c$ şekline getirilir.

Örnek: $\int \frac{dx}{\sqrt{x^2-4x-5}} = \int \frac{dx}{\sqrt{(x-2)^2-9}} = \int \frac{dt}{\sqrt{t^2-9}}$

$\boxed{\begin{aligned} t&=x-2 \\ dt&=dx \end{aligned}}$

$$= \ln\left|t+\sqrt{t^2-9}\right|+c$$
$$= \ln\left|x-2+\sqrt{x^2-4x-5}\right|+c$$

Örnek: $\int \frac{dx}{\sqrt{-x^2+2x+3}} = \int \frac{dx}{\sqrt{4-(x-1)^2}} = \int \frac{dt}{\sqrt{4-t^2}}$

$-x^2+2x+3 = -(x^2-2x-3)$
$= 4-(x-1)^2$

$\boxed{\begin{aligned} t&=x-1 \\ dt&=dx \end{aligned}}$

$$= \arcsin\left(\frac{t}{2}\right)+c$$
$$= \arcsin\left(\frac{x-1}{2}\right)+c$$

Örnek: $\int \frac{dx}{\sqrt{x^2-2x-8}} = \int \frac{dx}{\sqrt{(x-1)^2-9}} = \int \frac{dt}{\sqrt{t^2-9}}$

$t=x-1$
$dt=dx$

$$= \ln\left|t+\sqrt{t^2-9}\right|+c$$
$$= \ln\left|x-1+\sqrt{x^2-2x-8}\right|+c$$

Örnek: $\int \frac{dx}{\sqrt{4x^2+4x+3}} = \int \frac{dx}{\sqrt{(2x+1)^2+2}} = \int \frac{\frac{1}{2}dt}{\sqrt{t^2+2}}$

$t=2x+1$
$dt=2dx$

$$= \frac{1}{2}\cdot\ln\left|t+\sqrt{t^2+2}\right|+c$$
$$= \frac{1}{2}\cdot\ln\left|2x+1+\sqrt{4x^2+4x+3}\right|+c$$

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2. $\int \frac{mx+n}{\sqrt{ax^2+bx+c}}\,dx$ tipindeki integraller:

$$\int \frac{mx+n}{\sqrt{ax^2+bx+c}}\,dx = \int \frac{\frac{m}{2a}(2ax+b) - \frac{mb}{2a} + n}{\sqrt{ax^2+bx+c}}\,dx$$

$$= \frac{m}{2a}\int \frac{2ax+b}{\sqrt{ax^2+bx+c}}\,dx + \left(n - \frac{mb}{2a}\right)\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

Örnek: $\int \frac{x\,dx}{\sqrt{x^2-2x+5}} = \int \frac{\frac{1}{2}(2x-2)+1}{\sqrt{x^2-2x+5}}\,dx$

$u = x^2-2x+5$

$du = (2x-2)dx$

$t = x-1$

$dt = dx$

$$= \frac{1}{2}\int \frac{2x-2}{\sqrt{x^2-2x+5}}\,dx + \int \frac{dx}{\sqrt{(x-1)^2+4}}$$

$$= \frac{1}{2}\int \frac{du}{\sqrt{u}} + \int \frac{dt}{\sqrt{t^2+4}}$$

$$= \sqrt{u} + \ln\left|t + \sqrt{t^2+4}\right| + c$$

$$= \sqrt{x^2-2x+5} + \ln\left|x-1+\sqrt{x^2-2x+5}\right| + c$$

Örnek: $\int \frac{3x+2}{\sqrt{x^2+4x+1}}\,dx = \int \frac{\frac{3}{2}(2x+4)-4}{\sqrt{x^2+4x+1}}\,dx$

$u = x^2+4x+1$

$du = (2x+4)dx$

$t = x+2$

$dt = dx$

$$= \frac{3}{2}\int \frac{2x+4}{\sqrt{x^2+4x+1}}\,dx - 4\int \frac{dx}{\sqrt{(x+2)^2-3}}$$

$$= \frac{3}{2}\int \frac{du}{\sqrt{u}} - 4\int \frac{dt}{\sqrt{t^2-3}}$$

$$= \frac{3}{2}\cdot 2\sqrt{u} - 4\cdot\ln\left|t+\sqrt{t^2-3}\right| + c$$

$$= 3\sqrt{x^2+4x+1} - 4\ln\left|x+2+\sqrt{x^2+4x+1}\right| + c$$

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3. $\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$ tipindeki integraller:

$t=\frac{1}{px+q}$ değişken değiştirmesi yapılarak, integral 1. tipteki integrale dönüşür.

Örnek: $\int \frac{dx}{(x-1)\sqrt{x^2+3}} = \int \frac{-\frac{1}{t^2}\,dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t}+1\right)^2+3}} = \int \frac{-dt}{t\sqrt{\frac{t^2+2t+1}{t^2}+3}}$

$t=\frac{1}{x-1}$

$x-1=\frac{1}{t}$

$dx=\frac{-1}{t^2}dt$

$u=2t+\frac{1}{2}$

$du=2dt$

$$= -\int \frac{dt}{\sqrt{4t^2+2t+1}}$$

$$= -\int \frac{dt}{\sqrt{\left(2t+\frac{1}{2}\right)^2+\frac{3}{4}}}$$

$$= -\int \frac{\frac{1}{2}du}{\sqrt{u^2+\frac{3}{4}}}$$

$$= -\frac{1}{2}\cdot \ln\left|u+\sqrt{u^2+\frac{3}{4}}\right|+c$$

$$= -\frac{1}{2}\ln\left|2t+\frac{1}{2}+\sqrt{4t^2+2t+1}\right|+c$$

$$= -\frac{1}{2}\ln\left|\frac{2}{x-1}+\frac{1}{2}+\sqrt{\frac{4}{(x-1)^2}+\frac{2}{x-1}+1}\right|+c$$

Örnek: $\int \frac{dx}{x\sqrt{2x-x^2}} = \int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\frac{2}{t}-\frac{1}{t^2}}} = \int \frac{-dt}{t\sqrt{\frac{2t-1}{t^2}}}$

$t=\frac{1}{x} \Rightarrow x=\frac{1}{t}$

$dx=\frac{-1}{t^2}dt$

$u=2t-1$

$du=2dt$

$$= -\int \frac{dt}{\sqrt{2t-1}} = -\int \frac{\frac{1}{2}du}{\sqrt{u}}$$

$$= -\frac{1}{2}\cdot 2\sqrt{u}+c$$

$$= -\sqrt{2t-1}+c$$

$$= -\sqrt{\frac{2}{x}-1}+c$$

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4. $\int \frac{P_n(x)\,dx}{\sqrt{ax^2+bx+c}}$ tipindeki integraller:

$P_n(x)$ n. dereceden bir polinom, $Q_{n-1}(x)$ $(n-1)$. dereceden bilinmeyen bir polinom ve k bilinmeyen bir sabit olmak üzere

$$\int \frac{P_n(x)\,dx}{\sqrt{ax^2+bx+c}} = Q_{n-1}(x)\sqrt{ax^2+bx+c} + k\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

ifadesinde eşitliğin her iki tarafının türevi alınarak polinom eşitliğinden $Q_{n-1}(x)$ polinomu ve k sabiti bulunur.

Örnek: $\int \frac{x^2\,dx}{\sqrt{1-x^2}} = ?$

$$\left(\int \frac{x^2\,dx}{\sqrt{1-x^2}}\right)' = \left((ax+b)\sqrt{1-x^2} + k\int \frac{dx}{\sqrt{1-x^2}}\right)'$$

$$\frac{x^2}{\sqrt{1-x^2}} = a\sqrt{1-x^2} - \frac{x}{\sqrt{1-x^2}}(ax+b) + \frac{k}{\sqrt{1-x^2}}$$

$$\frac{x^2}{\sqrt{1-x^2}} = \frac{a(1-x^2)-ax^2-bx+k}{\sqrt{1-x^2}}$$

$$x^2 = -2ax^2 - bx + a + k$$

$$-2a = 1 \Rightarrow a = \frac{-1}{2}$$

$$-b = 0 \Rightarrow b = 0$$

$$a + k = 0 \Rightarrow k = \frac{1}{2}$$

$$\int \frac{x^2\,dx}{\sqrt{1-x^2}} = -\frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\int \frac{dx}{\sqrt{1-x^2}}$$

$$= \frac{-x}{2}\sqrt{1-x^2} + \frac{1}{2}\arcsin x + c$$

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5. $\displaystyle\int \frac{dx}{(x-p)^n\sqrt{ax^2+bx+c}}$ tipindeki integraller:

$t=\frac{1}{x-p}$ değişken değiştirmesi yapılarak 4. tip integrale dönüştürülür.

Örnek: $$\int \frac{dx}{(x-1)^2\sqrt{x^2+2x-2}} = \int \frac{-\cancel{\frac{1}{t^2}}\,dt}{\cancel{\frac{1}{t^2}}\sqrt{\left(\frac{t+1}{t}\right)^2+2\frac{t+1}{t}-2}}$$

$t=\frac{1}{x-1} \Rightarrow x-1=\frac{1}{t}$

$dx=\frac{-1}{t^2}dt$

$$= \int \frac{-dt}{\sqrt{\frac{t^2+4t+1}{t^2}}}$$

$$= \int \frac{-t\,dt}{\sqrt{t^2+4t+1}}$$

$$= -\int \frac{\frac{1}{2}(2t+4)-2}{\sqrt{t^2+4t+1}}dt$$

$$= -\frac{1}{2}\int \frac{2t+4}{\sqrt{t^2+4t+4}}dt + 2\int \frac{dt}{\sqrt{(t+2)^2-3}}$$

$$= -\frac{1}{2}\cdot 2\sqrt{t^2+4t+4} + 2\ln\left|t+2+\sqrt{t^2+4t+1}\right| + c$$

$$= -\sqrt{\frac{1}{(x-1)^2}+\frac{4}{x-1}+4} + 2\ln\left|\frac{1}{x-1}+2+\sqrt{\frac{1}{(x-1)^2}+\frac{4}{x-1}+1}\right| + c$$

Örnek: $$\int \frac{dx}{x^3\sqrt{1+x^2}} = \int \frac{-\frac{1}{t^2}dt}{\frac{1}{t^3}\sqrt{1+\frac{1}{t^2}}} = \int \frac{-t\,dt}{\frac{\sqrt{t^2+1}}{t}} = \int \frac{-t^2dt}{\sqrt{t^2+1}}$$

$t=\frac{1}{x} \Rightarrow x=\frac{1}{t}$

$dx=\frac{-1}{t^2}dt$

$$\left(\int \frac{-t^2dt}{\sqrt{t^2+1}}\right)' = \left((at+b)\sqrt{t^2+1}+k\int\frac{dt}{\sqrt{t^2+1}}\right)'$$

$$\frac{-t^2}{\sqrt{t^2+1}} = a\sqrt{t^2+1}+\frac{t}{\sqrt{t^2+1}}(at+b)+\frac{k}{\sqrt{t^2+1}}$$

$$-t^2 = 2at^2+bt+a+k \Rightarrow a=\frac{-1}{2},\ b=0,\ k=\frac{1}{2}$$

$$\int \frac{dx}{x^3\sqrt{1+x^2}} = \frac{-t}{2}\sqrt{t^2+1}+\frac{1}{2}\int\frac{dt}{\sqrt{t^2+1}} = \frac{-t}{2}\sqrt{t^2+1}+\frac{1}{2}\ln\left|t+\sqrt{t^2+1}\right|+c$$

$$= \frac{-1}{2x}\sqrt{\frac{1}{x^2}+1}+\frac{1}{2}\ln\left|\frac{1}{x}+\sqrt{\frac{1}{x^2}+1}\right|+c$$

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Örnek: $\int \frac{x^4-2}{\sqrt{x^2+1}}dx = ?$

Çözüm: $\left(\int \frac{x^4-2}{\sqrt{x^2+1}}dx\right)' = \left((ax^3+bx^2+cx+d)\sqrt{x^2+1} + k\int \frac{dx}{\sqrt{x^2+1}}\right)'$

$$\frac{x^4-2}{\sqrt{x^2+1}} = (3ax^2+2bx+c)\sqrt{x^2+1} + \frac{x}{\sqrt{x^2+1}}(ax^3+bx^2+cx+d) + \frac{k}{\sqrt{x^2+1}}$$

$$\frac{x^4-2}{\sqrt{x^2+1}} = \frac{(3ax^2+2bx+c)(x^2+1)+ax^4+bx^3+cx^2+dx+k}{\sqrt{x^2+1}}$$

$$x^4-2 = 4ax^4+3bx^3+(3a+2c)x^2+(2b+d)x+c+k$$

$4a=1 \Rightarrow a=\frac{1}{4}$

$3b=0 \Rightarrow b=0$

$3a+2c=0 \Rightarrow c=\frac{-3}{8}$

$2b+d=0 \Rightarrow d=0$

$c+k=-2 \Rightarrow k=\frac{-13}{8}$

$$\int \frac{x^4-2}{\sqrt{x^2+1}}dx = \left(\frac{x^3}{4}-\frac{3x}{8}\right)\sqrt{x^2+1} - \frac{13}{8}\int \frac{dx}{\sqrt{x^2+1}}$$

$$= \frac{2x^3-3x}{8}\cdot\sqrt{x^2+1} - \frac{13}{8}\cdot\ln\left|x+\sqrt{x^2+1}\right| + c$$

Örnek: $\int \frac{x^2dx}{\sqrt{x^2-x+1}} = ?$

Çözüm: $\left(\int \frac{x^2dx}{\sqrt{x^2-x+1}}\right)' = \left((ax+b)\sqrt{x^2-x+1} + k\int \frac{dx}{\sqrt{x^2-x+1}}\right)'$

$$\frac{x^2}{\sqrt{x^2-x+1}} = a\sqrt{x^2-x+1} + \frac{2x-1}{2\sqrt{x^2-x+1}}(ax+b) + \frac{k}{\sqrt{x^2-x+1}}$$

$$\frac{2x^2}{2\sqrt{x^2-x+1}} = \frac{2a(x^2-x+1)+(2x-1)(ax+b)+2k}{2\sqrt{x^2-x+1}}$$

$$2x^2 = 4ax^2+(-3a+2b)x+(2a-b+2k)$$

$a=\frac{1}{2}$, $b=\frac{3}{4}$, $k=\frac{-1}{8}$

$$\int \frac{x^2dx}{\sqrt{x^2-x+1}} = \left(\frac{x}{2}+\frac{3}{4}\right)\sqrt{x^2-x+1} - \frac{1}{8}\int \frac{dx}{\sqrt{(x-\frac{1}{2})^2+\frac{3}{4}}}$$

$$= \left(\frac{x}{2}+\frac{3}{4}\right)\sqrt{x^2-x+1} - \frac{1}{8}\ln\left|x-\frac{1}{2}+\sqrt{x^2-x+1}\right| + c$$

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