Student Information

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Answer 1

$$B_i = \{ \text{ white balls in box } i \text{ } \}$$

a)

$P\{$at least one ball is white $\} = P\{$not all balls are black$\}$

$= \overline{P\{\text{all balls are black}\}} = 1 - P\{\text{all balls are black}\}$ (complement)

$= 1 - P\{\overline{B_1} \cap \overline{B_2} \cap \overline{B_3}\}$

$= 1 - (P\{\overline{B_1}\} * P\{\overline{B_2}\} * P\{\overline{B_3}\})$ (independence)

$= 1 - (\frac{8}{10} * \frac{11}{15} * \frac{9}{12}) = 0.56$

b)

$P\{$all balls are white$\} = P\{B_1 \cap B_2 \cap B_3\}$

$P\{B_1\} * P\{B_2\} * P\{B_3\})$ (independence)

$\frac{2}{10} * \frac{4}{15} * \frac{3}{12} = \frac{1}{75} = 0.0133$

c)

Here are the probabilities of three possible events:

$P\{$getting 2 white ball from first box$\}$
$= P\{B_1$ in first draw$\} * P\{B_1$ in second draw$\} = \frac{2}{10} * \frac{1}{9} = 0.0222$

$P\{$getting 2 white ball from second box$\}$
$= P\{B_2$ in first draw$\} * P\{B_2$ in second draw$\} = \frac{4}{15} * \frac{3}{14} = 0.0571$

$P\{$getting 2 white ball from third box$\} = P\{B_3$ in first draw$\} * P\{B_3$ in second draw$\} = \frac{3}{12} * \frac{2}{11} =$ 0.0455

As can be seen, Choosing the second box is rational since probability of getting 2 white balls from it is the highest.

d)

This time, we can pick 2 balls from the same box just like previous question but also we can pick from different boxes as well. There are 6 events, 3 of them are calculated in part c, so let us calculate the other ones as well.

$P\{\text{getting 1 white ball from first box and 1 white ball from second box}\}$
$= P\{B_1\} * P\{B_2\} = \frac{2}{10} * \frac{4}{15} = 0.0533$

$P\{\text{getting 1 white ball from first box and 1 white ball from third box}\}$
$= P\{B_1\} * P\{B_3\} = \frac{2}{10} * \frac{3}{12} = 0.050$

$P\{\text{getting 1 white ball from second box and 1 white ball from third box}\}$
$= P\{B_2\} * P\{B_3\} = \frac{4}{15} * \frac{3}{12} = 0.0667$

Taking these three and the ones we calculated in part c), the event that has highest probability is picking one from the second box and one from the third box. Notice that when picking balls from different boxes the individual probabilities are independent since one does not affect the other's probability and therefore in which order you pick balls does not matter.

e)

Let X be a random variable that represents the number of white balls at the end. It can take values from 0(all balls are black) to 3(all balls are white). Therefore we can calculate the expected value as:

$\mu = E(X) = \sum_x x * P(x) = 0 * P\{x = 0\} + 1 * P\{x = 1\} + 2 * P\{x = 2\} + 3 * P\{x = 3\}$
$= 1 * P\{x = 1\} + 2 * P\{x = 2\} + 3 * P\{x = 3\}$

We should calculate that probabilities:

$P\{x = 1\} = P\{\text{exactly 1 white ball}\}$
$= P\{B_1\} * P\{\overline{B_2}\} * P\{\overline{B_3}\} + P\{B_2\} * P\{\overline{B_1}\} * P\{\overline{B_3}\} + P\{B_3\} * P\{\overline{B_1}\} * P\{\overline{B_2}\}$
$= \frac{2}{10} * \frac{11}{15} * \frac{9}{12} + \frac{4}{15} * \frac{8}{10} * \frac{9}{12} + \frac{3}{12} * \frac{8}{10} * \frac{11}{15} = 0.417$

$P\{x = 2\} = P\{\text{exactly 2 white ball}\}$
$= P\{B_1\} * P\{B_2\} * P\{\overline{B_3}\} + P\{B_1\} * P\{B_3\} * P\{\overline{B_2}\} + P\{B_2\} * P\{B_3\} * P\{\overline{B_1}\}$
$= \frac{2}{10} * \frac{4}{15} * \frac{9}{12} + \frac{2}{10} * \frac{3}{12} * \frac{11}{15} + \frac{3}{12} * \frac{4}{15} * \frac{8}{10} = 0.130$

$P\{x=3\} = P\{\text{exactly 3 white ball}\}$
$= P\{B_1\} * P\{B_2\} * P\{B_3\}$
$= \frac{2}{10} * \frac{4}{15} * \frac{3}{12} = 0.0133$

$\mu = 1 * 0.417 + 2 * 0.130 + 3 * 0.0133 = 0.7169$

f)

Let I,II and III denotes the picking from the corresponding box numbers and W denotes the picking a white ball. We are asked to find $P\{I|W\}$. We can find it by the conditional probability formula:

$P\{I|W\} = \frac{P\{I \cap W\}}{P\{W\}}$

$P\{I \cap W\} = P\{W \cap I\} = P\{W|I\} * P\{I\} = \frac{2}{10} * \frac{1}{3} = 0.0667$

$P\{W\} = P\{W|I\} * P\{I\} + P\{W|II\} * P\{II\} + P\{W|III\} * P\{III\}$
$= \frac{2}{10} * \frac{1}{3} + \frac{4}{15} * \frac{1}{3} + \frac{3}{12} * \frac{1}{3} = 0.239$

$P\{I|W\} = \frac{0.0667}{0.239} = 0.279$

Answer 2

$D =$ the ring is destroyed $C_S =$ Sam is corrupted $C_F =$ Frodo is corrupted

a)

We are given the followings in the question:

$P\{D|\overline{C_S}\} = 0.9$
$P\{D|C_S\} = 0.5$
$P\{C_S\} = 0.1$
$P\{\overline{C_S}\} = 0.9$

And we are asked to find $P\{C_S|D\}$. We can find it using the Bayes rule for two events:

$P\{C_S|D\} = \frac{P\{D|C_S\}*P\{C_S\}}{P\{D|C_S\}*P\{C_S\}+P\{D|\overline{C_S}\}*P\{\overline{C_S}\}}$

Putting the values into formula:

$P\{C_S|D\} = \frac{0.5*0.1}{0.5*0.1+0.9*0.9} = 0.0581$

b)

We are given the followings:
$P\{C_F\} = 0.25$
$P\{D|C_F\} = 0.2$
$P\{D|C_S \cap C_F\} = 0.05$
$P\{D|\overline{C_S} \cap \overline{C_F}\} = 0.9$
Also we can calculate $P\{C_S \cap C_F\}$ using Sam's probability information from part a and knowing that their corruption probabilities are independent:

$P\{C_S \cap C_F\} = P\{C_S\} \cap P\{C_F\} = 0.1 * 0.25 = 0.025$

We are asked to find $P\{C_S \cap C_F|D\}$. From Bayes Rule:

$$P\{C_S \cap C_F|D\} = \frac{P\{D|C_S \cap C_F\} * P\{C_S \cap C_F\}}{P\{D\}} \tag{1}$$

We know the terms in the numerator. We should find $P\{D\}$. We can find it using the law of total probability. So let us calculate the followings first:

$P\{C_F \backslash C_S\} = P\{C_F \backslash (C_S \cap C_F)\} \rightarrow P\{C_F \backslash (C_S \cap C_F)\} = P\{C_F\} - P\{C_S \cap C_F\} = 0.25 - 0.025 = 0.225$ (since these events are disjoint.)

$P\{C_S \backslash C_F\} = P\{C_S \backslash (C_S \cap C_F)\} \rightarrow P\{C_S \backslash (C_S \cap C_F)\} = P\{C_S\} - P\{C_S \cap C_F\} = 0.1 - 0.025 = 0.075$ (since these events are disjoint.)

$P\{\overline{C_S} \cap \overline{C_F}\} = P\{\overline{C_S}\} * P\{\overline{C_P}\} = 0.9 * 0.75 = 0.675$

$P\{D|C_F \setminus C_S\} = P\{D|C_F\} - P\{D|C_S \cap C_F\} = 0.2 - 0.05 = 0.15$

$P\{D|C_S \setminus C_F\} = P\{D|C_S\} - P\{D|C_S \cap C_F\} = 0.5 - 0.05 = 0.45$

Now we can calculate $P\{D\}$ the law of total probability:

$P\{D\} = P\{D|C_F \setminus C_S\} * P\{C_F \setminus C_S\} + P\{D|C_S \setminus C_F\} * P\{C_S \setminus C_F\} + P\{D|C_S \cap C_F\} * P\{C_S \cap C_F\} + P\{D|\overline{C_S} \cap \overline{C_F}\} * P\{\overline{C_S} \cap \overline{C_F}\}$
$= 0.15 * 0.225 + 0.45 * 0.075 + 0.05 * 0.025 + 0.8 * 0.675 = 0.676$

Putting the values into (1):

$$P\{C_S \cap C_F|D\} = \frac{0.05 * 0.025}{0.676} = 0.00185 \tag{2}$$

Answer 3

a)

Let Z = A + I be the total number snowy days. We want Z to be 4. So we can calculate its probability:

$P_Z(4) = P\{A + I = 4\} = P\{A = 3 \cap I = 1\} + P\{A = 2 \cap I = 2\}$
$= P(3, 1) + P(2, 2) = 0.2 + 0.12 = 0.32$

b)

They are independent. For every possible event, it can be seen that $P_{(A,I)}(a, i) = P_A(a) * P_I(i)$:

$P_A(1) = P_{A,B}(1, 1) + P_{A,B}(1, 2) = 0.18 + 0.12 = 0.3$
$P_A(2) = P_{A,B}(2, 1) + P_{A,B}(2, 2) = 0.3 + 0.2 = 0.5$
$P_A(3) = P_{A,B}(3, 1) + P_{A,B}(3, 2) = 0.12 + 0.08 = 0.2$
$P_B(1) = P_{A,B}(1, 1) + P_{A,B}(2, 1) + P_{A,B}(3, 1) = 0.18 + 0.3 + 0.12 = 0.6$
$P_B(2) = P_{A,B}(1, 2) + P_{A,B}(2, 2) + P_{A,B}(3, 2) = 0.12 + 0.2 + 0.08 = 0.4$

$P_{(A,I)}(1, 1) = 0.18 = P_A(1) * P_I(1) = 0.3 * 0.6 = 0.18$
$P_{(A,I)}(1, 2) = 0.12 = P_A(1) * P_I(2) = 0.3 * 0.4 = 0.12$
$P_{(A,I)}(2, 1) = 0.3 = P_A(2) * P_I(1) = 0.5 * 0.6 = 0.3$
$P_{(A,I)}(2, 2) = 0.2 = P_A(2) * P_I(2) = 0.5 * 0.4 = 0.2$
$P_{(A,I)}(3, 1) = 0.12 = P_A(3) * P_I(1) = 0.2 * 0.6 = 0.12$
$P_{(A,I)}(3, 2) = 0.08 = P_A(3) * P_I(2) = 0.2 * 0.4 = 0.08$

Hence, the snowy days in Ankara and Istanbul are independent.