Student Information

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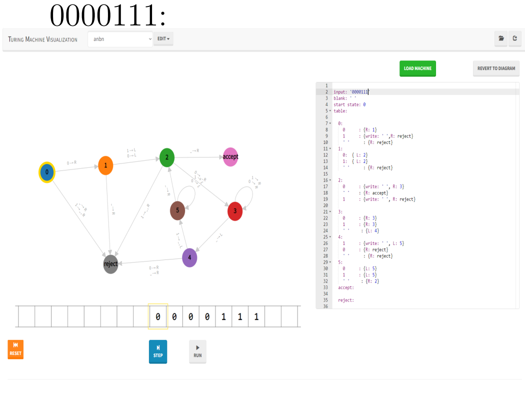
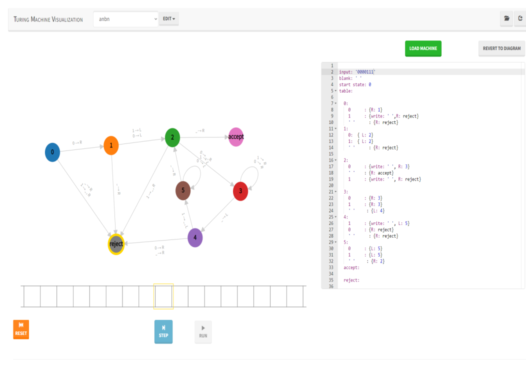
Answer 1)

The idea of this turing machine is the following: It starts with the state 0. If it reads a 1 then it immediately goes to the reject state and halts. Else, It goes to the state 1. If it reads a blank symbol than the word is "0" and it should be rejected. Otherwise in this state we are trying to find the leftmost character in the tape and then the machine goes to state 2. In state 2, we write blank symbol over the leftmost 0 and go to the state 3. In state 3, we go to the end of the input. Then, we go to the state 4 and write blank symbol over the rightmost 1. In state 5, we again go to the leftmost character in the tape and then go back to the state 2 and try to apply same step until the machine halts. Eventually, if the number of the 1's and 0's are not the same or are not in the correct order then the machine goes from state 4 to reject state and halts; or, it matches all 0's and 1's and changes them to the blank symbol and from state 2 it goes to the accepting state and halts.

Here are the machine and the example inputs. The initial and final states are given respectively:

0000111:

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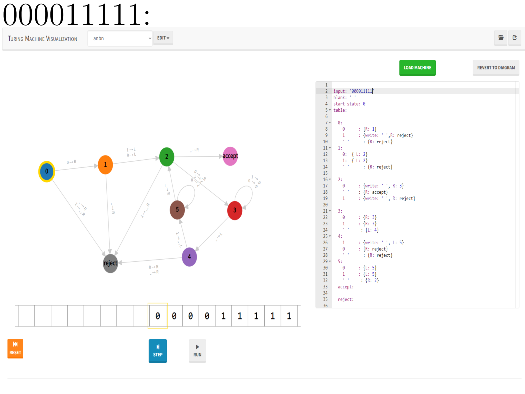 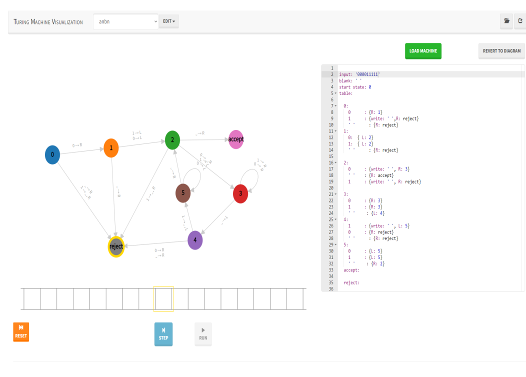

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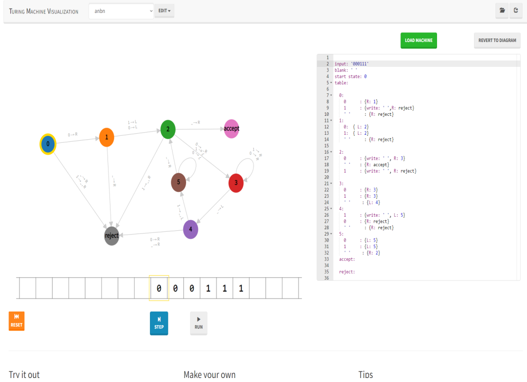

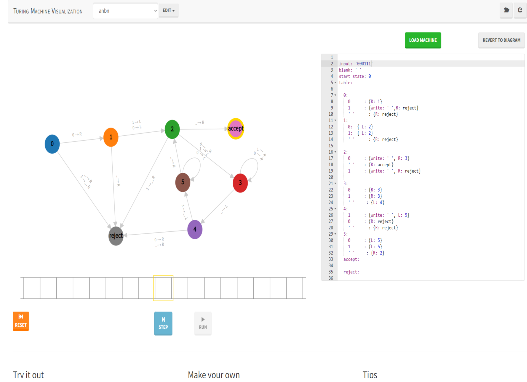

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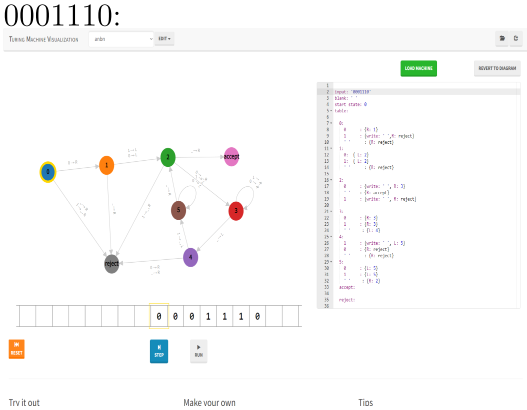

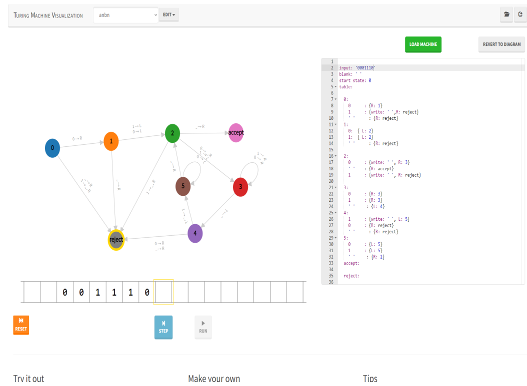

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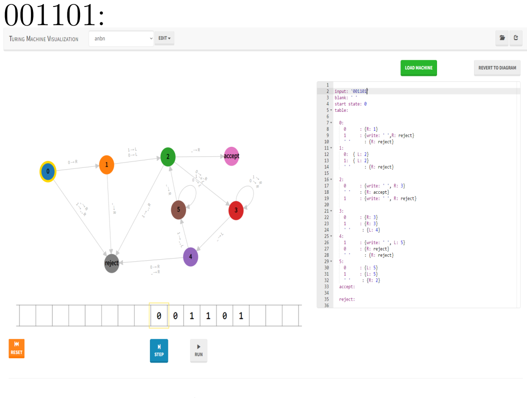

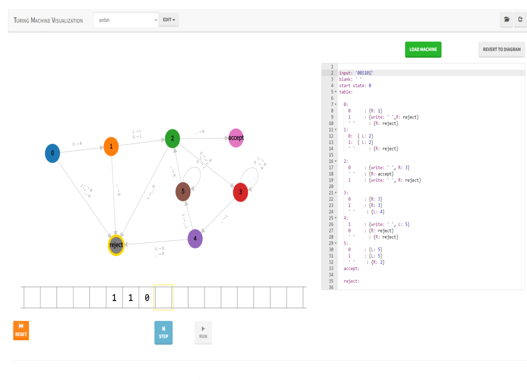

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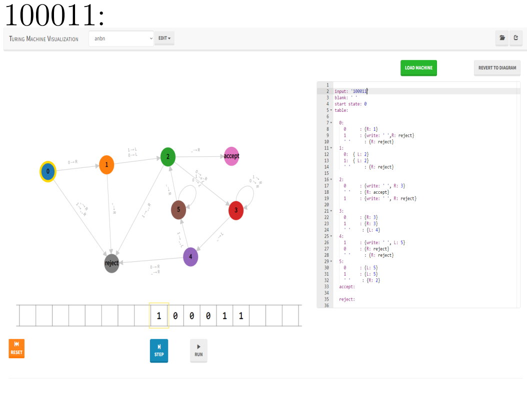
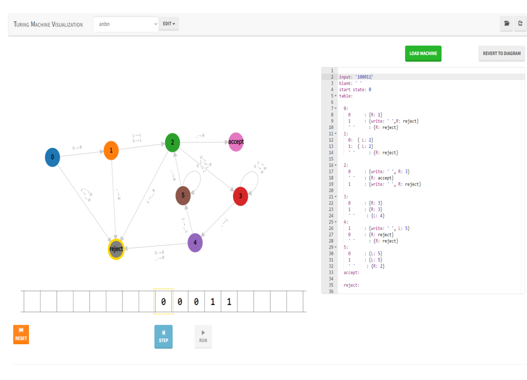


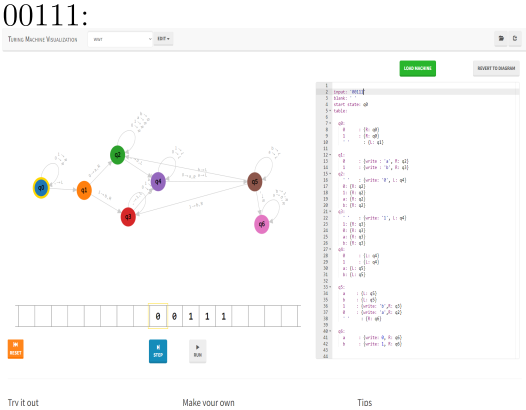


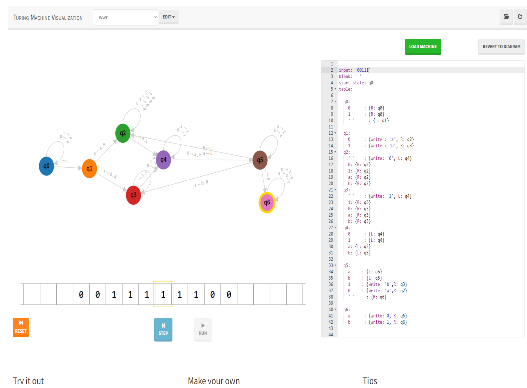
Answer 2)

We start with the state 0 and remain there until we find the rightmost symbol. Then we go to the state 1. If the symbol is "0" we change it to "a" and go to state 2; if it is "1" we change it to "b" and go to the state 3. Then in both state 2 and 3 we try to find the first blank symbol and write over "0" if we come from state 2, "1" if we come from state 3 and go to the state 4. Then, the function of states 4 and 5 is to scan left and find the first unchanged symbol. After that, according to the symbol, we go either state 2 or state 3 again and apply same steps until when we scan left there is no unchanged symbol left. When that happens, we go from state 5 to state 6. Finally, from the leftmost character, we change a's to "0" and b's to "1" back.

Here are machine and the example inputs. The initial and final configurations are given respectively:

00111:





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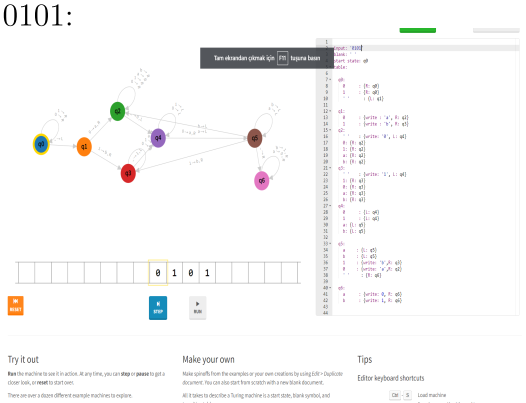

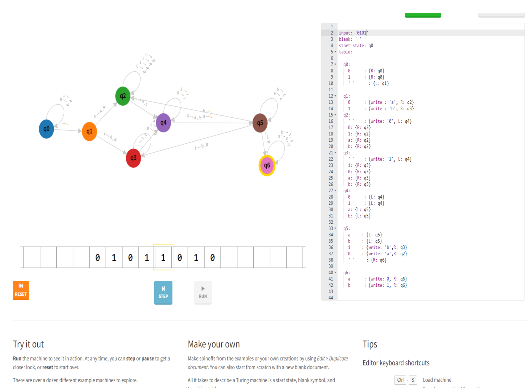

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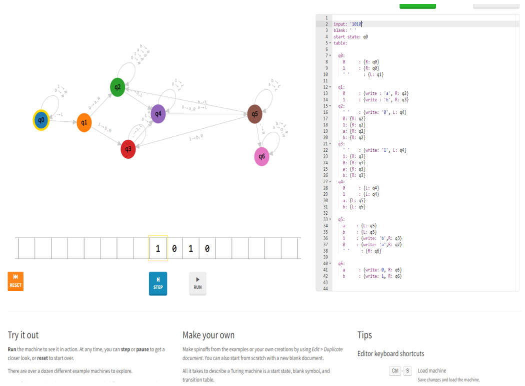

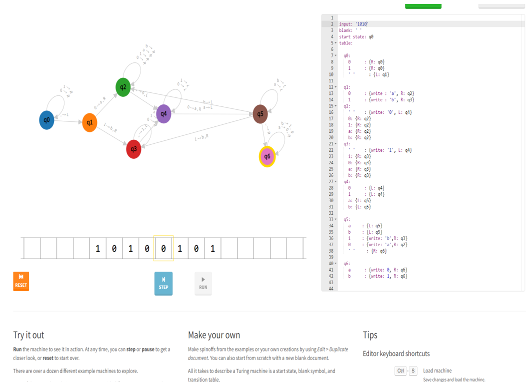

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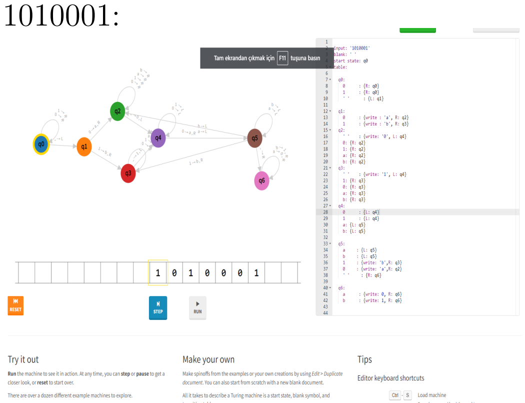

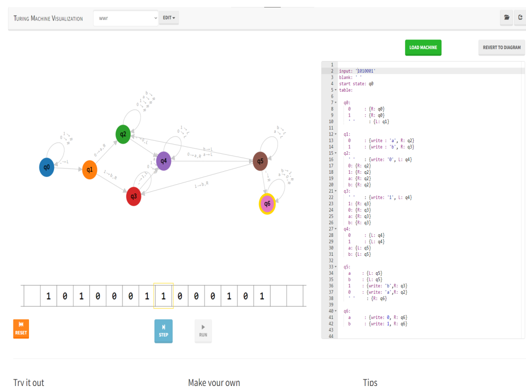

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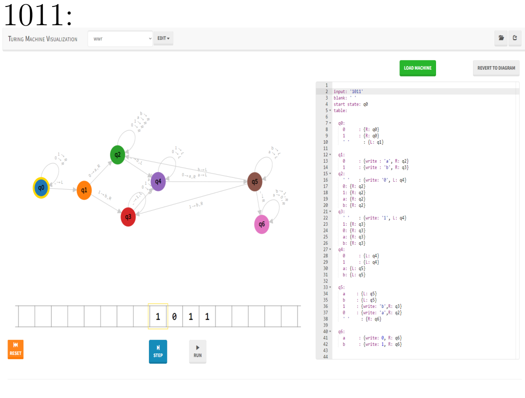

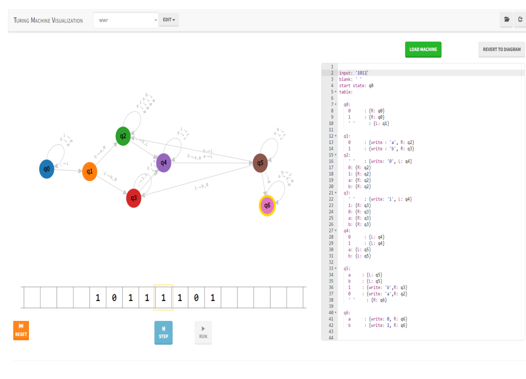

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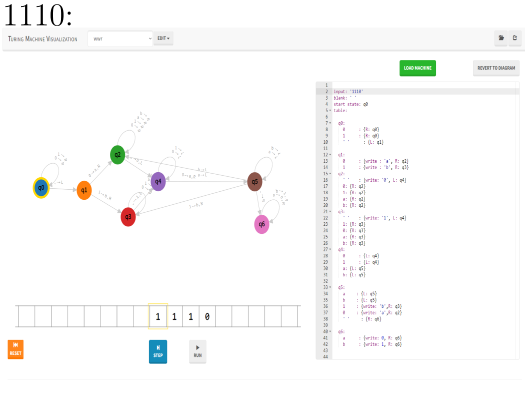

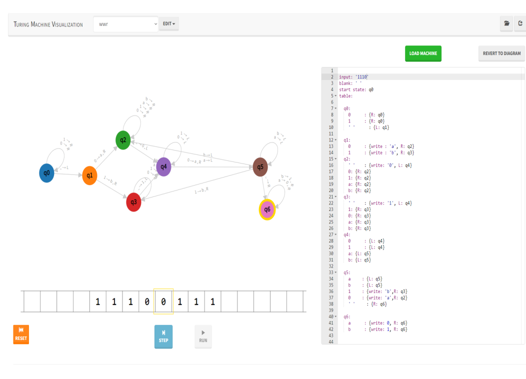

Answer 3)

$>$: input symbol
$\sqcup$: blank symbol
A 2 dimensional Turing machine M_{2D} can be defined as $M_{2D} = \{K, \Sigma, \delta, s, H\}$ such that K, s and H are as in the definition of standard Turing machine and Σ has one more symbol, which is bottom marker Δ. δ is the transition function from $(K - H) \times \Sigma$ to $K \times (\Sigma \cup \{\leftarrow, \rightarrow, \uparrow, \downarrow\})$ such that for all $q \in (K - H)$ if $\delta(q, >) = (p, b)$ then $b = \rightarrow$ and if $\delta(q, \Delta) = (p, b)$ then $b = \uparrow$, and for all $a \in \Sigma$ if $\delta(q, a) = (p, b)$ then $b \neq \rightarrow$ and $b \neq \uparrow$.

The configuration for the M_{2D}:

$$K \times (N \times N) \times \Sigma$$

That is, configurations shows the current state, the current coordinates of the head, and the head content.

Let $\Sigma_0 \subset \Sigma \backslash \{>, \Delta, \sqcup\}$ be the input alphabet of M. Assume there exists halting states $q_{yes}, q_{no} \in H$ such that $H = \{q_{yes}, qno\}$ for accepting and rejecting respectively. The machine M_{2D} decides a language L if for any input $w \in L$ the starting configuration $(s, 1, 1, a)$ yields either (q_{yes}, i, j, a') or (q_{no}, i, j, a') for $i, j \geq 1$, $a \in \Sigma$.

This machine can be simulated by standard Turing machine. We can concatenate the rows of the 2 dimensional tape contiguously and place a marker at the end of every row to know the endings. Also there is not significant computation power difference. The up-down transitions in the 2-D TM that is done in single step can be done in the standard TM by 1+k left-right transitions where k is the row the length of the 2-D machine. Therefore, the performance is still polynomial in time t and n.