Student Information

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Answer 1

The sample mean is calculated as $\overline{X} = 16.96$. The formula for the confidence interval for the population mean can be used since the sample has normal distribution:

$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

a)

$n = 10, \overline{X} = 16.96, \sigma = 3$

%90 Confidence Interval:

$$1 - \alpha = 0.9$$
$$\alpha = 0.01$$
$$\frac{\alpha}{2} = 0.05$$
$$z_{\alpha/2} = 1.645$$

So, the %90 confidence interval for the mean is:

$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$16.96 \pm 1.645\frac{3}{\sqrt{10}} = 16.96 \pm 1.56$$
$$= [15.40, 18.52]$$

%99 Confidence Interval:

$$1 - \alpha = 0.99$$
$$\alpha = 0.001$$
$$\frac{\alpha}{2} = 0.005$$
$$z_{\alpha/2} = 2.576$$

So, the %95 confidence interval for the mean is:

$$16.96 \pm 2.576 \frac{3}{\sqrt{10}} = 16.96 \pm 2.44$$

$$= [14.52, 19.40]$$

b)

The margin of error Δ is equal to:

$$\Delta = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Hence:

$$n \geq (z_{\alpha/2} \frac{\sigma}{\Delta})^2$$

should be satisfied.

For the %98 confidence level:

$$1 - \alpha = 0.98$$

$$\alpha = 0.02$$

$$\alpha/2 = 0.01$$

$$z_{\alpha/2} = z_{0.01} = 2.326$$

$$n \geq (2.326 * \frac{3}{1.55})^2 = 20.27$$

Therefore, the smallest required sample size must be 21, which is closest to our decimal number result.

Answer 2

a)

They are not enough. How the distrubiton type of ratings should be known because if the sample size is small and the ratings are not normally distrubuted, we can not assume it is normally distributed therefore we can not construct proper confidence intervals when needed. Secondly, knowing the standard deviation is beneficial since if it is large, than the food quality of restaurant is not consistent and we might get very bad food.

b)

Let us construct a confidence interval for the population mean, rating for this question.Since the sample is very large we can assume it is normally distributed and construct confidence interval. From the part a), the confidence interval for the population mean is the following:

$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

We know $\overline{X} = 7.5$ and $n = 256$. To attain a confidence level of $1 - \alpha = 0.95$ (with a significance level of %5):

$$\alpha = 0.05$$

$$\alpha/2 = 0.025$$

$$z_{\alpha/2} = 1.960$$

Finally, we need the population standard deviation. Since the sample size is very large, we can replace the true standard error with its estimator. So we are allowed to use the sample standard deviation $s = 0.8$. Hence:

$$I = 7.5 \pm 1.960 * \frac{0.8}{\sqrt{256}} = [7.402, 7.598]$$

is our confidence interval. As can be seen, $7.4 \notin I$. Therefore, restaurant A can not be in our list of candidate restaurants.

c)

If we substitute the new value of s to the interval I from part b), the new interval is:

$$I_{new} = 7.5 \pm 1.960 * \frac{1}{\sqrt{256}} = [7.4, 7.6]$$

This time, $7.4 \in I_{new}$. Therefore, we can now include the restaurant A to our list.

d)

We don't need to construct any confidence interval because actually the upper bound is not necessary for us. If the rating is greater than 7.5 then it can be as greater as it can be. It does not matter for us.

Answer 3

$n_A = 20$, $\overline{X_A} = 211$ min, $s_A = 5.2$ min, $n_B = 32$, $\overline{X_B} = 133$ min, $s_A = 22.8$ min
Null hypothesis = B indeed provides at least 90 minutes better performance.
Alternative hypothesis = B can't provide at least 90 minutes better performance.
$H_0 : \mu_A - \mu_B \geq 90$
$H_A : \mu_A - \mu_B < 90$(Left tail alternative)
$\alpha = 0.01$ (%1 Level of Significance)

a)

It is stated that we can assume the population variances are equal so we can use the pooled sample variance to estimate common variance:

$$s_p^2 = \frac{(n_A - 1) * s_A^2 + (n_B - 1) * s_B^2}{n_A + n_B - 2}$$

$$s_p = \sqrt{\frac{19 * 5.2^2 + 31 * 22.8^2}{50}} = 18.237$$

Then we can use this pooled sample variance to apply Z-test:

$$Z = \frac{\overline{X}_A - \overline{X}_B - (\mu_A - \mu_B)}{s_p * \sqrt{\frac{1}{n_A} + \frac{1}{n_B}}}$$

$$Z = \frac{211 - 133 - 90}{18.237 * \sqrt{\frac{1}{20} + \frac{1}{32}}} = -2.308$$

Since the H_A is left-tail alternative, the test is one sided and we need z_α:

$$z_\alpha = z_{0.01} = 2.326$$

For left tail alternative we should:

$$\begin{cases} \text{reject } H_0 & \text{if } Z \leq -z_\alpha \\ \text{accept } H_A & \text{if } Z > -z_\alpha \end{cases}$$

According to our calculations, $Z = -2.308 > -z_\alpha = -2.326$. Therefore, we accept the null hypothesis. Computer B provide at least 90 minutes better performance.

b)

This time we can't assume the population variances are equal. Therefore we should use T-test this time. For this, let us first calculate the degrees of freedom:

$$v = \frac{(\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B})^2}{\frac{s_A^4}{n_A^2(n_A-1)} + \frac{s_B^4}{n_B^2(n_B-1)}}$$

$$v = \frac{(\frac{5.2^2}{20} + \frac{22.8^2}{32})^2}{\frac{5.2^4}{20^2(19)} + \frac{22.8^4}{32^2(31)}} = 35.9$$

We can round it up to 36 to use Table A5 from book. T-test statistic, on the other hand, is equal to:

$$t = \frac{\overline{X}_A - \overline{X}_B - (\mu_A - \mu_B)}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

$$t = \frac{211 - 133 - 90}{\sqrt{\frac{5.2^2}{20} + \frac{22.8^2}{32}}} = -2.86$$

Finally, from the Table-A5 with $\alpha = 0.01$ and $v = 36$:

$$t_\alpha = t_{0.01} = 2.434$$

For a left tail alternative,

$$\begin{cases} \text{reject } H_0 & \text{if } t \leq -t_\alpha \\ \text{accept } H_A & \text{if } t > -t_\alpha \end{cases}$$

Because $t = -2.86 \leq -t_\alpha = -2.434$, we should Reject the null hypothesis. Thus, researcher can not claim computer B provides at least 90 minutes or better performance.