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1)

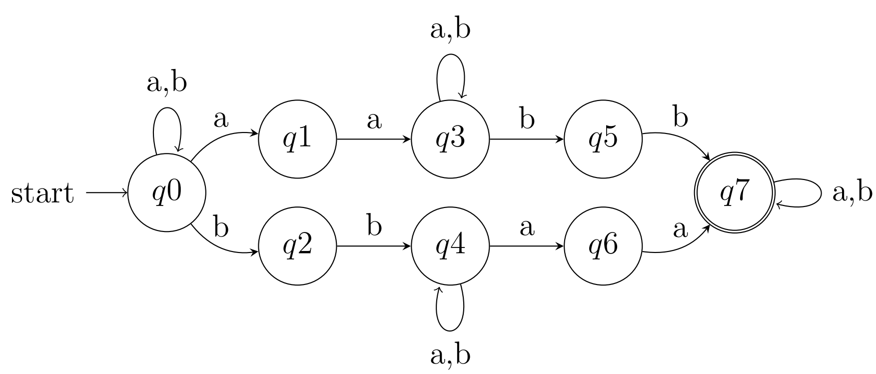
a)

The language must have at least 1 aa and 1 bb. So we can write a regular expression such that either aa comes first or bb comes first and the any word from Σ^* can come from the beginning or at the end or between the aa and bb:

$$(a \cup b)^*((aa(a \cup b)^*bb) \cup (bb(a \cup b)^*aa))(a \cup b)^*$$

b)

$K = \{q0, q1, q2, q3, q4, q5, q6, q7\}$
$\Sigma = \{a, b\}$
$s = q0$
$F = \{q7\}$
$\Delta = \{(q0, a, q0), (q0, b, q0), (q0, a, q1), (q0, b, q2), (q1, a, q3), (q2, b, q4), (q3, a, q3),$
$(q3, b, q3), (q3, b, q5), (q4, a, q4), (q4, b, q4), (q4, a, q6), (q5, b, q7), (q6, a, a7), (q7, a, q7), (q7, b, q7)\}$



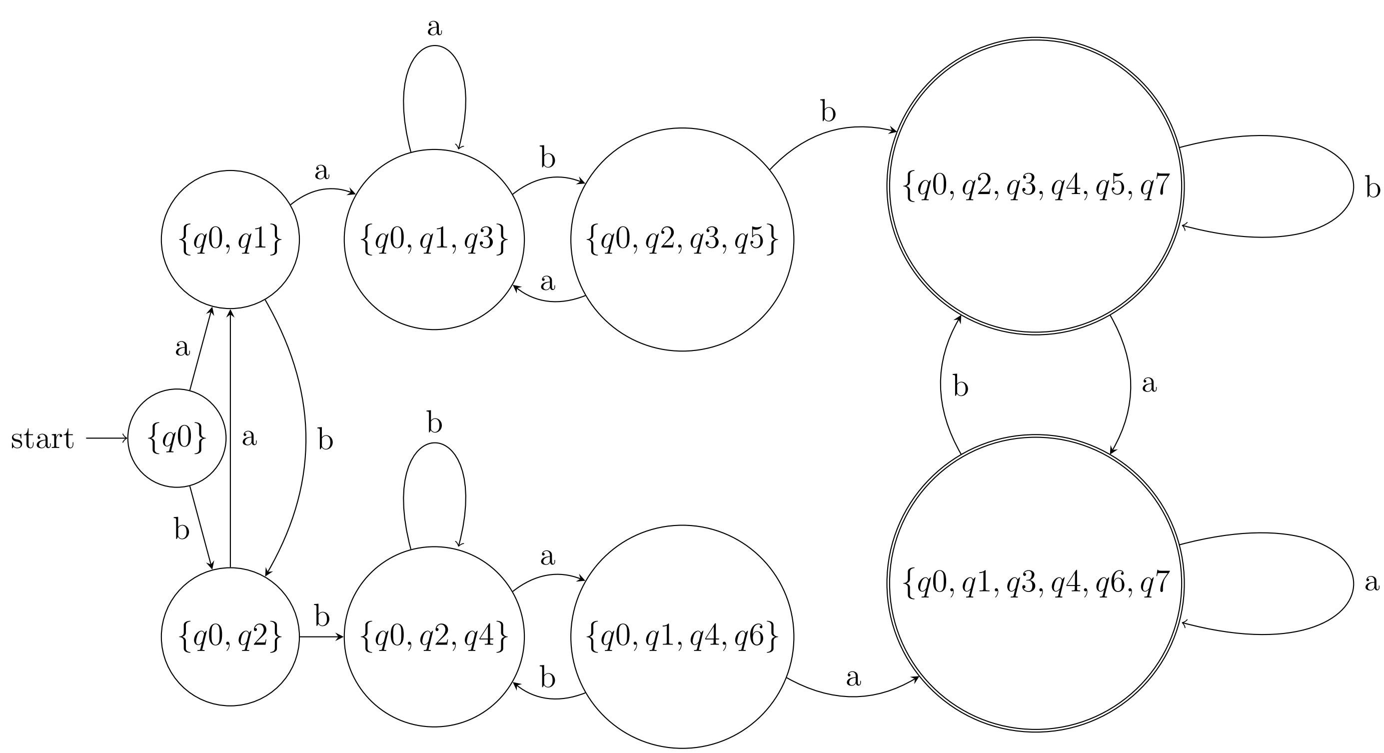
c)

Since the equivalent NFA have no empty transition, we do not have to check for E(q) closures. We have to use the δ' transition and if we find a new state we must add it to the DFA.
$\delta'(\{q0\}, a) = \{q0, q1\}$
$\delta'(\{q0, q1\}, a) = \{q0, q1, q3\}$
$\delta'(\{q0, q1, q3\}, a) = \{q0, q1, q3\}$
$\delta'(\{q0, q1, q3\}, b) = \{q0, q2, q3, q5\}$
$\delta'(\{q0, q2, q3, q5\}, a) = \{q0, q1, q3\}$

$\delta'(\{q0, q2, q3, q5\}, b) = \{q0, q2, q3, q4, q5, q7\}$
$\delta'(\{q0, q2, q3, q4, q5, q7\}, a) = \{q0, q1, q3, q4, q6, q7\}$
$\delta'(\{q0, q1, q3, q4, q6, q7\}, a) = \{q0, q1, q3, q4, q6, q7\}$
$\delta'(\{q0, q1, q3, q4, q6, q7\}, b) = \{q0, q2, q3, q4, q5, q7\}$
$\delta'(\{q0, q2, q3, q4, q5, q7\}, b) = \{q0, q2, q3, q4, q5, q7\}$
$\delta'(\{q0, q1\}, b) = \{q0, q2\}$
$\delta'(\{q0, q2\}, a) = \{q0, q1\}$
$\delta'(\{q0, q2\}, b) = \{q0, q2, q4\}$
$\delta'(\{q0, q2, q4\}, a) = \{q0, q1, q4, q6\}$
$\delta'(\{q0, q2, q4\}, b) = \{q0, q2, q4\}$
$\delta'(\{q0, q1, q4, q6\}, a) = \{q0, q1, q3, q4, q6, q7\}$
$\delta'(\{q0, q1, q4, q6\}, b) = \{q0, q2, q4\}$
$\delta'(\{q0\}, b) = \{q0, q2\}$

$K = \{\{q0\}, \{q0, q1\}, \{q0, q2\}, \{q0, q1, q3\}, \{q0, q2, q4\}, \{q0, q2, q3, q5\},$
$\{q0, q1, q4, q6\}, \{q0, q1, q3, q4, q6, q7\}, \{q0, q2, q3, q4, q5, q7\}\}$
$\Sigma = \{a, b\}$
$s = q0$
$F = \{\{q0, q1, q3, q4, q6, q7\}, \{q0, q2, q3, q4, q5, q7\}\}$(states including q7)

Now we can draw the DFA:



d)

It can be seen that the word bbabb does not belong to the language as it does not have an aa string. We can show it on the NFA by different ways since the machine is non-deterministic. Below is a one example:

$$
\begin{aligned}
(q0, bbabb) &\vdash_M (q2, babb) \\
(q2, babb) &\vdash_M (q4, abb) \\
(q4, abb) &\vdash_M (q4, bb) \\
(q4, bb) &\vdash_M (q4, b) \\
(q4, b) &\vdash_M (q4, e) \\
(q0, bbabb) &\vdash_M^* (q4, e)
\end{aligned}
$$

The reading process ended in state $q4$. However, this is not the final state. Therefore this word is not accepted.

Similarly, we can do the same thing for DFA. But this time, there is only one possible choice for every transition and final state is unique for the word bbabb:

$$
\begin{aligned}
(\{q0\}, bbabb) &\vdash_M (\{q0, q2\}, babb) \\
(\{q0, q2\}, babb) &\vdash_M (\{q0, q2, q4\}, abb) \\
(\{q0, q2, q4\}, abb) &\vdash_M (\{q0, q1, q4, q6\}, bb) \\
(\{q0, q1, q4, q6\}, bb) &\vdash_M (\{q0, q2, q4\}, b) \\
(\{q0, q2, q4\}, b) &\vdash_M (\{q0, q2, q4\}, e) \\
(\{q0\}, bbabb) &\vdash_M^* (\{q0, q2, q4\}, e)
\end{aligned}
$$

The reading process ended in the state $\{q0, q2, q4\}$, which is not one of the final states. Therefore we have proved for both NFA and DFA that bbabb is not accepted.

1)

In the following questions, I will use the Theorem 2.3.1 from Lecture Book which says that Regular Languages are closed under:
1)Union
2)Concatenation
3)Kleene star
4)Complementation
5)Intersection

a)

By the above information, If we want to find whether L_2 is regular or not, we can work with L_1 because if L_1 is regular, than its complement L_2 must be regular as well.

$$L_1 = \{a^m b^n | m > n \text{ and } m, n \in N\}$$

Let L_1 be a regular language and $k \geq 1$ be its pumping length. Choose a word w from language such that $|w| \geq k$:

$w = a^{k+1}b^k, |w| = 2k + 1$

w can be rewritten as $w = xyz$ such that $y \neq e$,$|xy| < k$ and $xy^iz \in L_1$ for each $i \geq 0$.

Let:

$x = a^\alpha, y = a^\beta$ and $z = a^{k+1-\alpha-\beta}b^k$ ($\beta \geq 1$ since $y \neq e$)

$xy^iz = a^{k+1+(i-1)\beta}b^k$

Choose i = 0:

$xy^0z = a^{k+1-\beta}b^k$

Since we assume that this word is in the language:

$k + 1 - \beta > k$
$1 - \beta > 0$
$1 > \beta$

However, as we stated before, $\beta >\geq$ must hold for w to be in language. Therefore there is a contradiction. Our assumption is wrong. L_1 is NOT regular. Hence L_2 is NOT REGULAR.

b)

$$L_4 = \{a^n b^n | n \in N^+\} \tag{1}$$

Let L_4 be a regular language and $k \geq 1$ be its pumping length. Choose a word w from language such that $|w| \geq k$:

$w = a^k b^k, |w| = 2k$

w can be rewritten as $w = xyz$ such that $y \neq e$,$|xy| < k$ and $xy^iz \in L_4$ for each $i \geq 0$.

Let:

$x = a^\alpha, y = a^\beta$ and $z = a^{k-\alpha-\beta}b^k$ ($\beta \geq 1$ since $y \neq e$)

$xy^iz = a^{k+(i-1)\beta}b^k$

Choose i = 0:

$xy^0z = a^{k-\beta}b^k \notin L_4$

Therefore, L_4 is NOT REGULAR.

$$L_5 = \{a^m b^n | n, m \in N\} \tag{2}$$

If a language can be shown with a regular expression, it is regular. The language L_5 is actually a REGULAR language and a^*b^* is the simple regular expression R for that language.

$$L_6 = b^*a(ab^*a)^* \tag{3}$$

In the above language, L_6 is already written as a regular expression. Since all regular expressions define regular languages, L_6 is REGULAR.

Observe that every word that is in L_4 is also in the L_5 because L4 denotes a subset of $L5$ such that the number of a's and the number of b's are equal and there are at least 1 of them. Therefore:

$$L_4 \cup L5 = L5$$

As we mentioned above, regular languages are closed under union so $L_5 \cup L_6$ is also builds a regular language. Hence:

$L_4 \cup L5 \cup L6$ is REGULAR.