Student Information

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1)

a)

Let $G_a = (V_a, \Sigma, R_a, S_a)$ be the grammar of the language L_1 where S_a is the starting symbol and:

$$V_a = \{a, b, S_a\}$$

$$\Sigma = \{a, b\}$$

$$R_a = \{S_a \to S_a b S_a b S_a a S_a, S_a \to S_a b S_a a S_a b S_a, S_a \to S_a a S_a b S_a b S_a, S_a \to e\}$$

The reason of constructing those transitions is that we must derive 2 b's for every a at every step. But there is no restriction on the order so we should write all possible orderings of letters (bab,abb,bba) and we should place the nonterminal between every letter to derive all of the orderings.

b)

Let $G_b = (V, \Sigma, R, S_b)$ be the grammar of the language L_2 where S_b is the starting symbol and:

$$V_b = \{a, b, S_b\}$$
$$\Sigma = \{a, b\}$$
$$R_b = \{S_b \to a S_b b, S \to a S_b bb, S_b \to e\}$$

In the definition of L_2, $m \le n \le 2m$ means that for every b, we should have at least 1 , at most 2 a's. In addition, since empty string is in the language, we should use empty string to end derivations.

c)

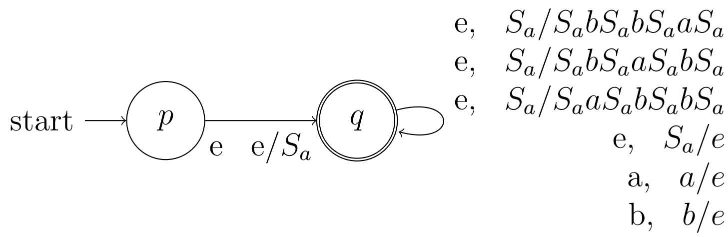
Let $M = \{K, \Sigma, V, \Delta, s, F\}$ be the pushdown automation of the language L_1 where:

$$K = \{p, q\}$$

$$s = p$$
$$F = \{q\}$$

$$
\begin{aligned}
\Delta = \{&((p,e,e),(q,S_a)),\\
&((q,e,S_a),(q,S_abS_abS_aaS_a)),\\
&((q,e,S_a),(q,S_abS_aaS_abS_a)),\\
&((q,e,S_a),(q,S_aaS_abS_abS_a)),\\
&((q,e,S_a),(q,e)),\\
&((q,a,a),(q,e)),\\
&((q,b,b),(q,e))\}
\end{aligned}
$$

Now let us draw the PDA:



d)

Let S_{new} be the new starting symbol and let $G_{union} = \{V_a \cup V_b \cup \{S_{new}\}, \Sigma, R_{new}, S_{new}\}$ where $R_{new} = R_a \cup R_b \cup \{S_{new} \to S_a, S_{new} \to S_b\}$. Then, $L(G) = L(G_a) \cup L(G_b)$ because if a word $w \in L(G_a)$ or $w \in L(G_b)$ then it can be produced by the new grammar G by going to the S_a or S_b (according to the which previous grammars accepts it) and applying same steps as before.

2)

a)

We can show that G_1 is ambiguous by showing 2 different leftmost derivation for the same string accepted by grammar. Choose the string as "00111":

$$D_1 = S \Longrightarrow_L AS \Longrightarrow_L 0A1S \Longrightarrow_L 0A11S \Longrightarrow_L 00111S \Longrightarrow_L 00111 \tag{1}$$

$$D_2 = S \Longrightarrow_L AS \Longrightarrow_L A1S \Longrightarrow_L 0A11S \Longrightarrow_L 00111S \Longrightarrow_L 00111 \tag{2}$$

Since D_1 and D_2 are different leftmost derivations, the grammar is ambiguous.

b)

The reason of the ambiguity is the fact that there is no precedence of matching 0's to 1's (0A1) or pumping 1's only (A1) . To remove the ambiguity, we can split these transitions into 2 different nonterminals and give one of them precedence (My grammar pumps the single 1's first.). The new V and R of G_1 after removing ambiguity is:

$$V = \{0, 1, S, A, B\}$$

$$R = \{S \to AS, S \to e, A \to A1, A \to B, B \to 0B1, B \to 01\}$$

c)

The derivation D for the given string "00111" after changing the grammar:

$$D = S \Longrightarrow_L AS \Longrightarrow_L A1S \Longrightarrow_L B1S \Longrightarrow_L 0B11S \Longrightarrow_L 00111S \Longrightarrow_L 00111$$