Case Study 4: Heat Transfer in Shallow Fluidized Bed

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Abstract

In this analysis we utilize data from a chemical engineering experiment studying heat transfer in a shallow fluidized bed to develop models for the heat transfer coefficient. Using a normal linear model we found that our crude estimation of velocity was the best predictor for the heat transfer coefficient. We also ran a binary logistic regression to predict the heat transfer coefficient exceeding 100 where, again, velocity was most informative. Finally, we used our linear regression model to attempt to model the thermal efficiency as well. However, results were quite poor with the model missing all modes in the original data.

Introduction

Fluidized bed experiments are useful for mixing solid particles with gases or liquids and have long been of interest to engineers and physicists (Tsimring, 2000). In this analysis we will utilize data from a chemical engineering shallow fluidized bed experiment to investigate the heat transfer coefficient. The results have the potential to influence multiple industrial application such as the design and construction of drying, cooling and combustion systems (Singh & Li, 2018).

Materials and Methods

Data

The data we will be using for this analysis was collected from a chemical engineering experiment designed to study heat transfer in a shallow fluidized bed. The data includes 20 observations for 6 variables: X_1 - fluidizing gas flow rate (lbs/hr), X_2 - supernatant gas flow rate (lbs/hr), X_3 - supernatant gas inlet nozzle opening (mm), X_4 - supernatant gas inlet temperature (F), Y_1 - heat transfer coefficient, and Y_2 - thermal efficiency. All data is positive and continuous save for the inlet nozzle opening size which is discrete.

Exploratory Data Analysis

In our initial data visualizations for the response variables, heat transfer coefficient and thermal efficiency, we noticed that there were potential outliers in the data. Using the simple IQR test we confirmed that the largest observations for both response variables qualified as outliers. Moving forward, we decided to test our models on both data including and excluding the potential outliers to see if they had a dramatic affect on the results. We then explored the potential for relationships between the response and predictor variables. Without manipulation there were no strong trends in the data. After research regarding the heat transfer coefficient in particular, we decided to create a crude velocity variable utilizing information from both the supernatant gas flow rate and the supernatant gas inlet nozzle opening size (Engineering ToolBox, 2004).

There seemed to be a weak linear relationship between velocity and the heat transfer coefficient as shown in Figure 1. Further plots and analysis are available in the Appendix.

$$V = \frac{\text{gas flow rate}}{\text{gas flow area}} = \frac{X_2}{\frac{1}{2}X_3\pi}$$

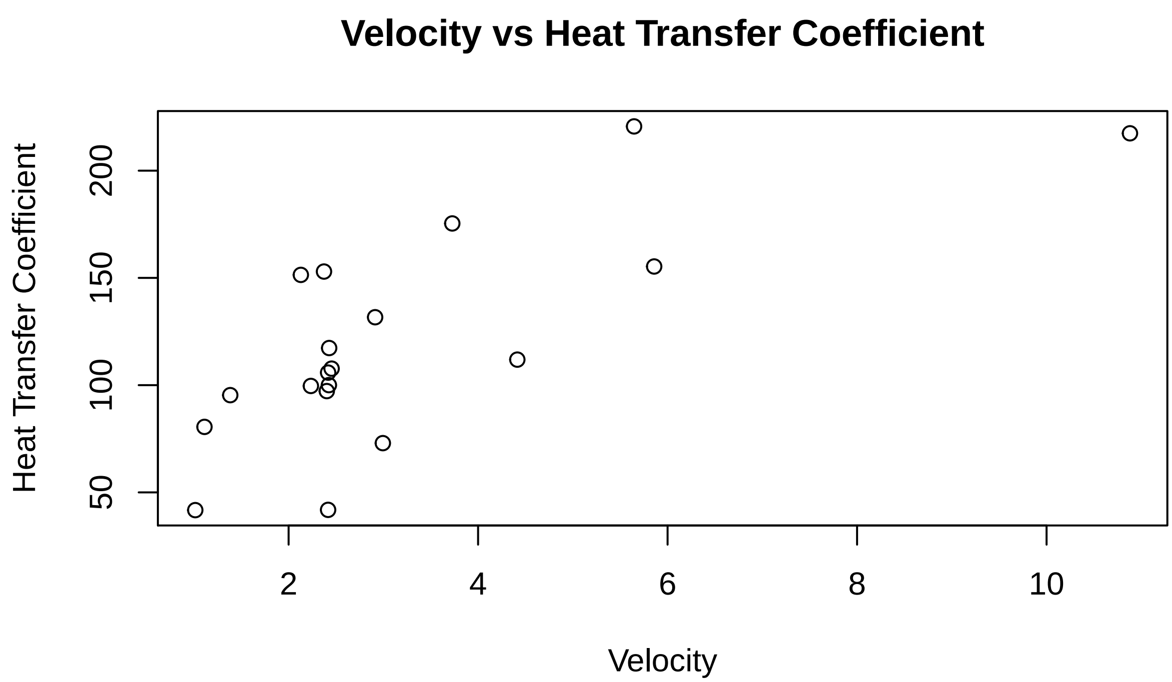


Figure 1: Weak linear relationship between velocity and the heat transfer coefficient

Model

Model 1

Our first task was to develop a model to satisfy questions 1 and 2 for which the main goal is predicting the heat transfer coefficient Y_1. As noted in the EDA, there seemed to be a relationship between velocity and Y_1 so we definitely wanted to include that in the model as a fixed effect. It seemed intuitive that there might be a relationship between velocity and the supernatant gas inlet temperature so we included an interaction term between V and X_4. From our initial investigation of the data it was unclear how the gas flow rates directly influenced the heat transfer coefficient; the supernatant gas flow rate had been incorporated into the velocity variable already so we decided to include both a fixed effect for the fluidizing gas flow rate as well as an interaction term between the fluidizing and supernatant gas flow rates.

$$
\begin{aligned}
(Y_{i1}) &\sim \text{Normal}(X_i^T \beta_i, \sigma^2) \\
X_i &= [1, V_i, X_{i1}, VX_{i4}, X_{i1}X_{i2}] \\
\beta &= [\alpha, \beta_1, \beta_2, \beta_3, \beta_4] \\
V &\sim Gamma(2, \frac{1}{2}) \\
X_{i1} &\sim Gamma(5, \frac{1}{2})
\end{aligned}
$$

Although the heat transfer coefficient does not extend below zero in our dataset we settled on a Normal linear regression using the `BRMS` package for ease of use and lack of an obvious alternate choice. Diffuse Gamma priors were chosen for both the velocity and the fluidizing gas flow rate while the intercepts were left with the default flat priors and the variance was set to the default diffuse Student-t prior. The weakly informative Gamma priors seemed fitting as they set a lower bound of zero on two variables which we know to be positive (in this scenario).

Model 2

Our next distinct task, presented in question 3, was to build a model to predict whether the heat transfer coefficient exceeded 100. To do this we created a new indicator variable for Y_1 and utilized our previous model and priors but changing the family to that of Bernoulli with a logit link function. Discussion about the lack of additional information added to this model can be found in Results and in the Appendix.

Results

Addressing question 1, the first table in the Appendix displays the coefficient estimates and 95% credible intervals for the fixed effects; we see that the velocity has the strongest relationship with the heat transfer coefficient. We also see that zero is not contained in the credible intervals for the intercept, velocity, and fluidized gas flow rate which indicates that these variables had non-zero slopes and thus provide significant information when predicting the heat transfer coefficient. However, notice that neither of the interaction terms seem to be informative with coefficients close to zero and credible intervals containing zero. This may be partially due to the fact that the velocity itself incorporates information about the supernatant gas flow rate. Interestingly, though, when the X_1X_2 interaction term is removed, model performance decreases. Ultimately, the posterior predictive checks seem to capture both the main peak as well as the secondary mode. Note that this model was run with and without the suspected outlier in Y_1 and results are reported using the data without the outlier. Discussion regarding model fit can be found in the Appendix.

To predict Y_1 using our Model 1 and address question 2, we included additional information for X_4 and V. We used the most common inlet nozzle size and the mean supernatant gas inlet temperature to hopefully provide an adequate estimate. We estimated the new value of Y_1 to be 71.73 with $\text{P}(Y_1 > 100) \approx 0.042$. Details regarding this calculation can be found in the Appendix.

Addressing question 2, the second table in the Apprendix displays the coefficient estimates and 95% credible intervals for the fixed effects; we notice that, similar to Model 1, velocity again is the strongest predictor for whether the heat transfer coefficient will exceed 100 while only velocity and fluidized gas flow rate have credible intervals that don't contain zero. We did find that the model correctly classified whether Y_1 exceeded 100 some 85% of the time which is a decent performance.

Using the original Model 1 to predict thermal efficiency and address question 4, we again see in the third table in the Appendix that the coefficient estimates for the fixed effects are similar to those found when predicting the heat transfer coefficient although both velocity and fluidized gas flow rate are less informative. However, when viewing the posterior predictive checks we see that this model does not capture the characteristics of the data at all; not only does the model underestimate the primary mode, but it fails to capture both of the secondary modes. Note that this model is run with all the data and removing the potential ourlier for Y_2 does not improve model performance. Further discussion regarding the model fit can be found in the Appendix.

Discussion

Results for the first model are relatively accurate although shortcomings exist in the decision to use a Normal linear regression. Since all of the data for our response variable is positive, it would have made more sense to choose a distribution that only allowed for positive values. Furthermore, the prior specifications are quite vague and model performance could be improved with the use of more informative choices. The prediction of Y_1 given new X_1 and X_2 values is certainly flawed as information about both X_3 and X_4 had to be assumed. Based on the data, it also seems that the estimation of $\mathrm{P}(Y_1 > 100)$ is lower than observed. Instead of using the same predictors as for Question 1, it would likely be useful to more extensively investigate the choice of prediction variables.

References

- Boyd, J. H. (1984). Heat Transfer Between a Supernatant Gas and a Flowing Shallow Gas-Fluidized Bed of Solids (Unpublished master's thesis). Virginia Polytechnic Institute and State University. doi:https://vtechworks.lib.vt.edu/bitstream/handle/10919/76414/LD5655.V855_1984.B694.pdf?sequence=1&isAllowed=y
- Engineering ToolBox, (2004). Orifice, Nozzle and Venturi Flow Rate Meters. [online] Retrieved April 02, 2021, from https://www.engineeringtoolbox.com/orifice-nozzle-venturi-d_590.html.
- Singh, R., &; Li, F. (2018, April 24). Industrial applications of Fluidized beds And Fluidization of fine particles (INVITED TALK). Retrieved April 02, 2021, from https://www.aiche.org/conferences/world-congress-on-particle-technology/2018/proceeding/session/industrial-applications-fluidized-beds-and-fluidization-fine-particles-invited-talk
- Smeets, L., & Van de Schoot, R. (2019, August 25). WAMBS BRMS tutorial: Popularity data. Retrieved April 02, 2021, from https://www.rensvandeschoot.com/brms-wambs/
- Tsimring, L. S. (2000, April 7). Dynamics of a shallow fluidized bed. Retrieved April 02, 2021, from https://biocircuits.ucsd.edu/lev/grain/fluidbed/fluidbed_slides_html/

Appendix

Table 1

	Estimate	CI
Intercept	-41.14	(-75.80,-13.98)
Velocity	11.41	(2.75,20.15)
X1	0.26	(0.11,0.46)
V:X4	0.01	(-0.03,0.05)
X1:X2	0	(0,0)

Table 2

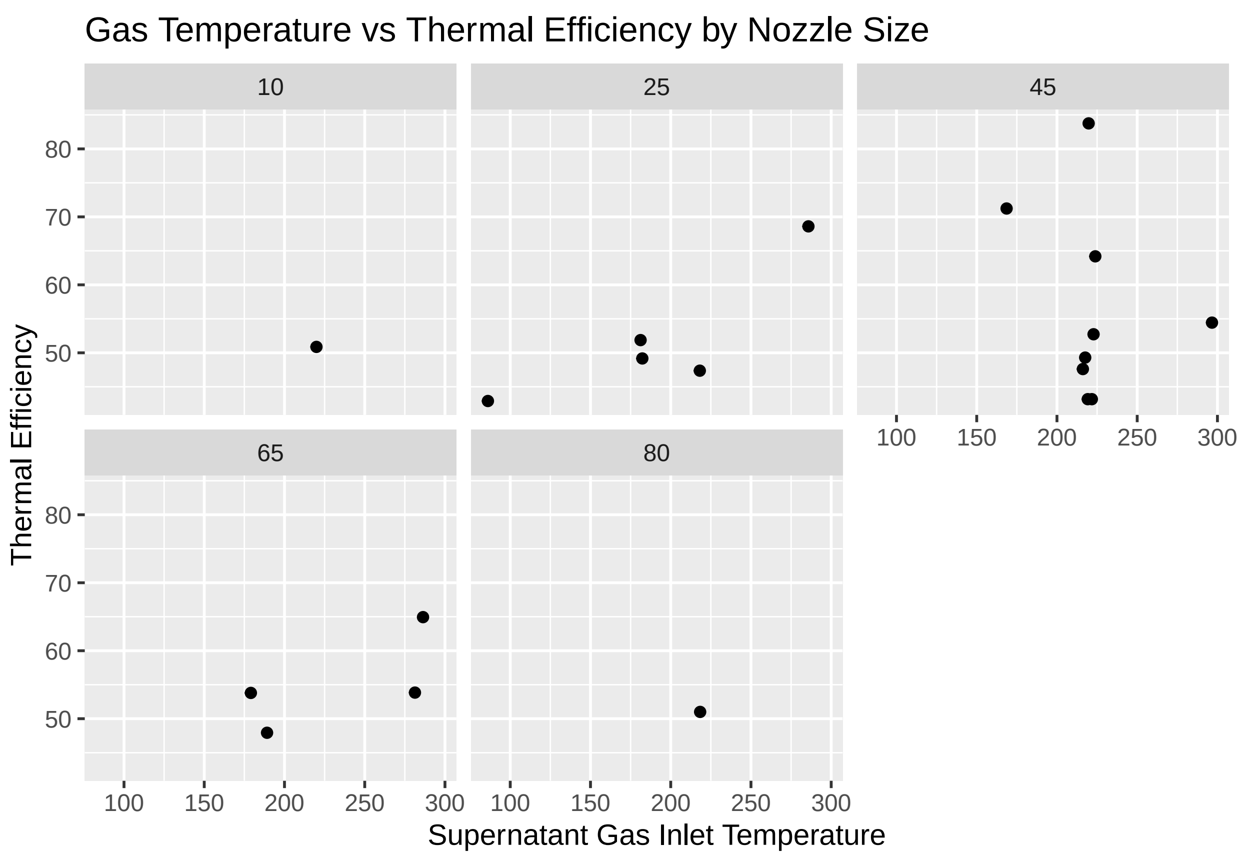
	Estimate	CI
Intercept	-18.91	(-33.81,-8.37)
Velocity	2.66	(0.39,6.51)
X1	0.07	(0.03,0.13)
V:X4	0	(-0.02,0.02)
X1:X2	0	(0,0)

Table 3

	Estimate	CI
Intercept	35.96	(11.30,56.26)
Velocity	2.58	(0.35,6.76)
X1	0.20	(0.08,0.36)
V:X4	-0.01	(-0.03,0)
X1:X2	0	(0,0)

EDA

In figure 2 we see that there are no clear relationships between either of the response variables and any of the explanatory variables. We see in figure 3 that, after accounting for inlet nozzle size, there is a bit of a relationship between supernatant gas temperature and thermal efficiency. This observation lead to the inclusion of the interaction term between velocity and supernatant gas temperature in both models. In figure 4 we see thee box and whisker plot for the heat transfer coefficient in which a potential outlier is present at about 550.



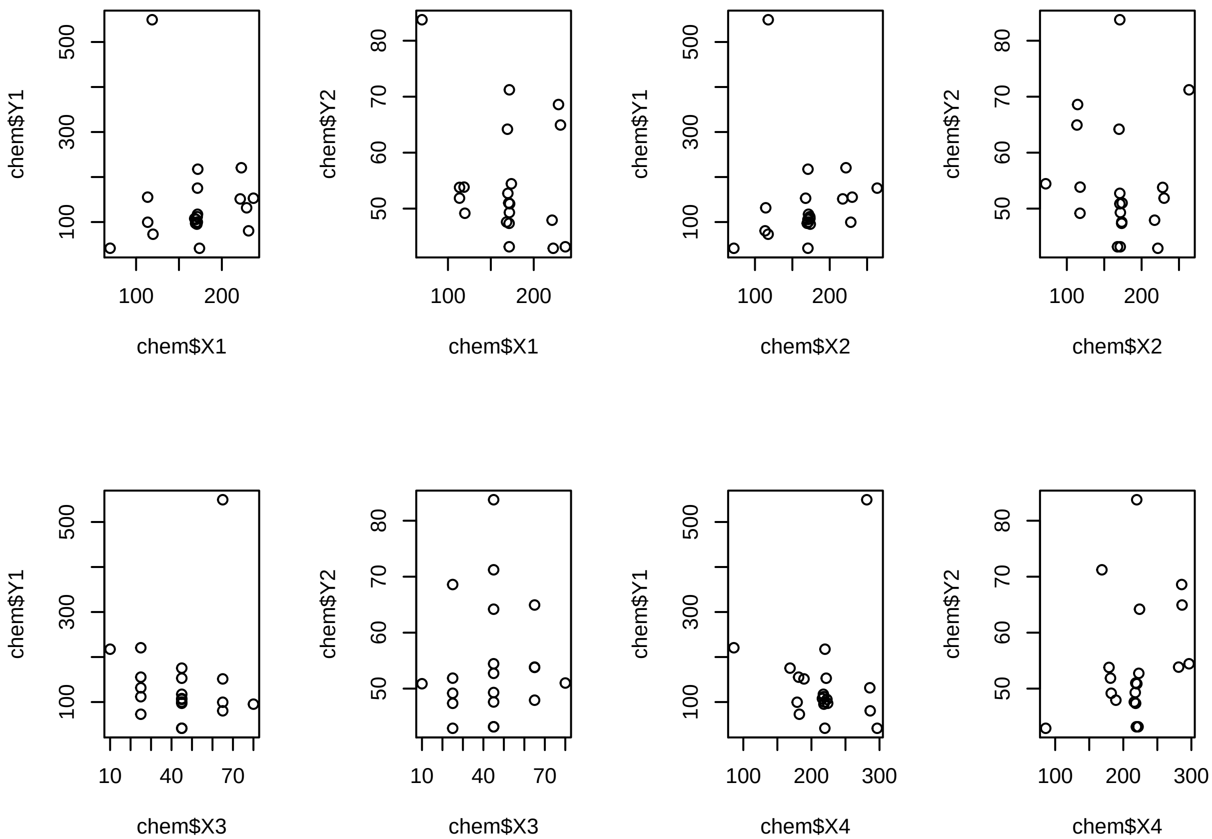
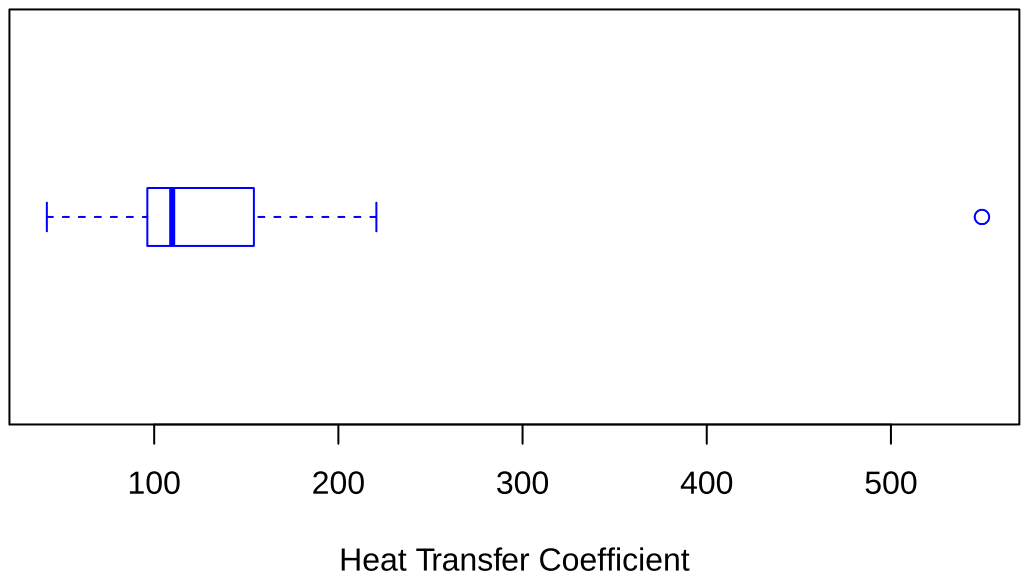


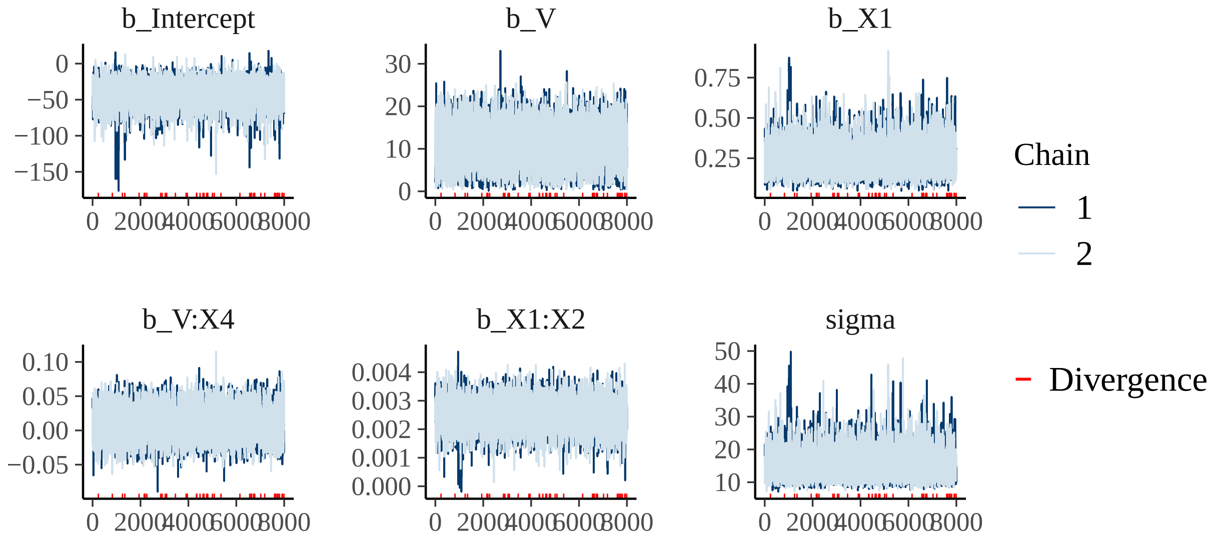
Figure 2: Exhaustive comparison of all predictor variables to both response variables

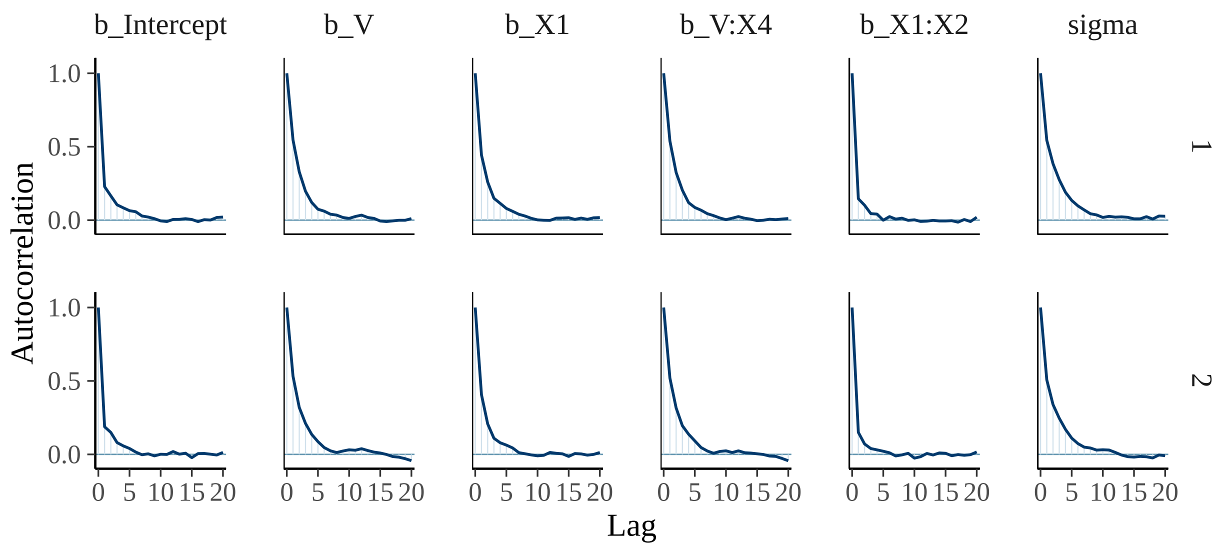


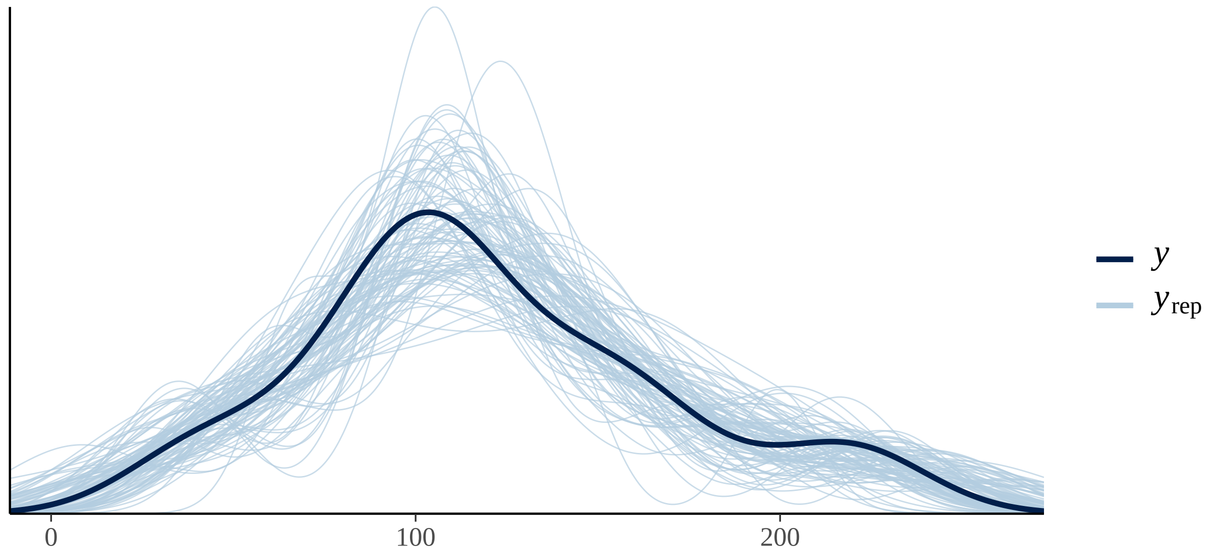
Model 1: Goodness of Fit

Predicting the Heat Transfer Coefficient

In the traceplot below we see that there is relatively good mixing for both chains although multiple divergent transitions are present. In the lag-1 autocorrelation plot we see that for most parameters the autocorrelation goes to zero although the correlation seems to be more persistent in the interaction terms and in the variance. As discussed previously, the posterior predictive checks fit the model well, capturing both the primary and secondary modes. These diagnostics are for the data that does not include the previously identified outlier; removing the potential outlier resulted in marginal improvements in the results.

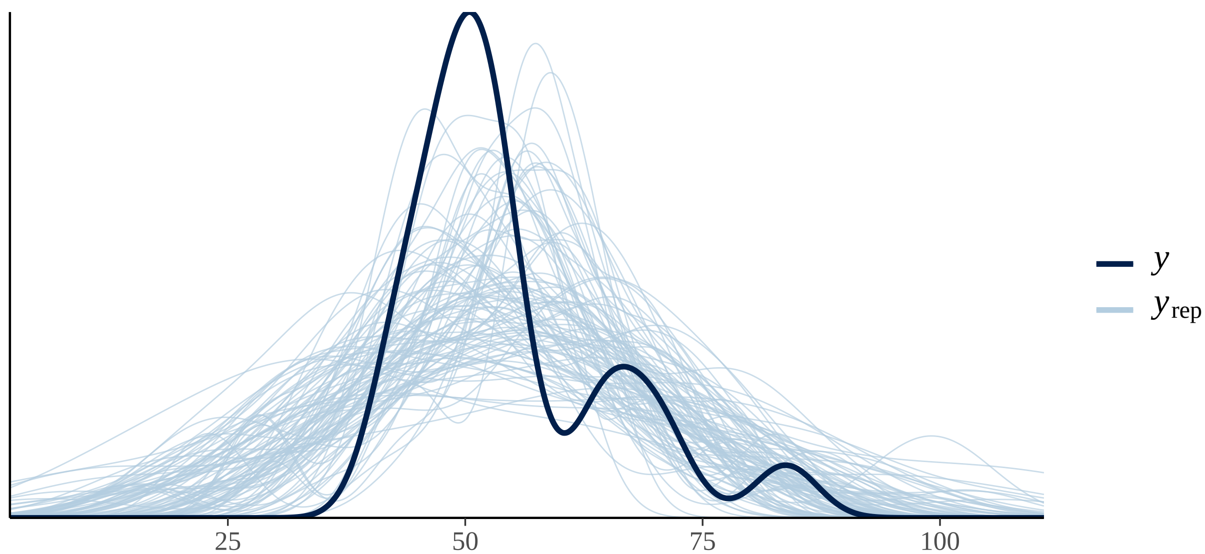

b_Intercept
b_V
b_X1
b_V:X4
b_X1:X2
sigma
Chain
1
2
Divergence


b_Intercept
b_V
b_X1
b_V:X4
b_X1:X2
sigma
Autocorrelation
Lag
1
2



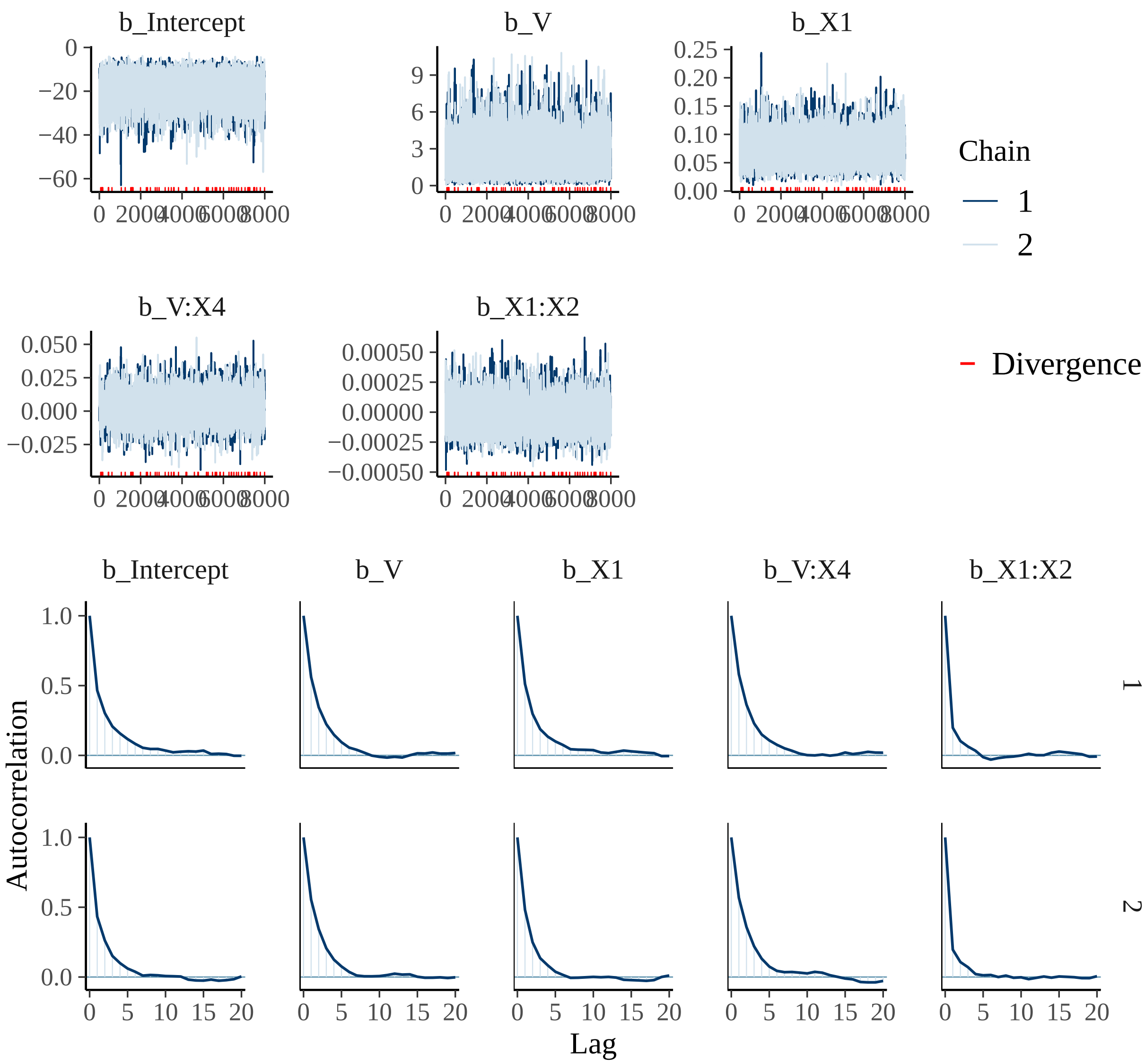
Predicting the Thermal Efficiency

We see in the posterior predictive plot below that the model for Y_1 did not fit well for Y_2 at all. The model severely underestimates the main mode and does not capture the secondary modes at all.



Model 2: Goodness of Fit

The model fit here is similar to that of model 1 although, in the autocorrelation plot below, we see that the variance is less problematic. The posterior predictions seem to capture the data relatively well although there seems to be significant overestimation between 0.75 and 1. The predictor variables are the same as in the previous model; multiple different formulas were tried for this model with none producing a better model. For example, a fixed effect for X_4 was added with no substantial shift in results.


b_Intercept
b_V
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Chain
1
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b_V:X4
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Divergence
b_Intercept
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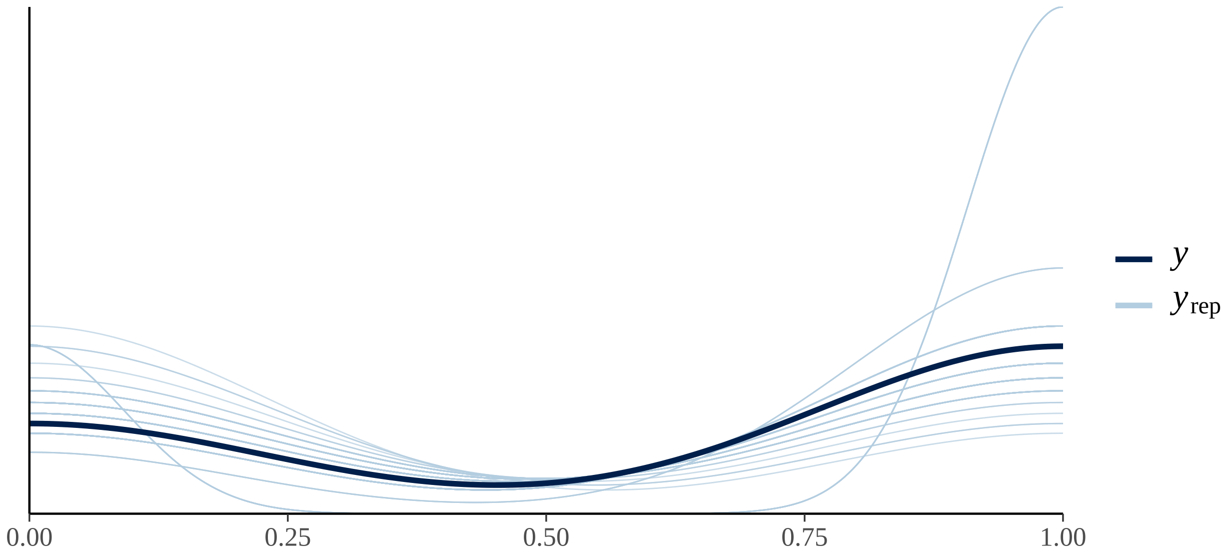
Autocorrelation
Lag



Further Discussion

Let's discuss the prediction results and process for question 2 a bit more thoroughly. As mentioned previously, estimated data for X_3 and X_4 had to be included in order to make a prediction. We took the most frequent nozzle size of 45 mm and the mean supernatant temperature which was about 213 degrees F. If we instead use the next two most common nozzle sizes, 25mm and 65mm, the results vary widely with $\mathrm{P}(Y_1 > 100) = 0.45$ and $\mathrm{P}(Y_1 > 100) = 0.0119$ respectively.