Q1 Dice Entropy

5 Points

Consider a six sided dice. The probability of each side is given by (p1,p2,p3,p4,p5,p6). In each of the cases below, calculate the entropy of this dice. You may use a computer or calculate for this. In all cases, report your answer in the form x.xx. That is, two decimal places with the leading 0. So if your answer is 0.12, report 0.12, not just .12. If your answer is 0, report 0.00. There should not be an issue of ambiguous rounding on this, if you use a computer and only round at the end.

Q1.1 a

1 Point

(1,0,0,0,0,0)

0.00

Q1.2 b

1 Point

(0.5,0.5,0,0,0,0)

0.69

Q1.3 c

1 Point

(0.5,0.25,0.25,0,0,0

1.04

Q1.4 d
1 Point

(1/6, 1/6, 1/6, 1/6, 1/6, 1/6)

1.79

Q1.5 e
1 Point

(1/2, ¼, 1/8, 1/16, 1/32,0.03125)

1.34

Q2 Conditional Entropy proof
4 Points

Assume that a combined system is comprised of two discrete random variables X and Y. The conditional entropy is defined as the amount of information needed to describe Y if you already know the value of the random variable X. In math terms, it is

$$H(Y|X) = -\sum_{x\in X, y\in Y} p(x,y)\log\frac{p(x,y)}{p(x)}$$

where p(x,y) is the probability of (x and y). Prove that H(Y│X)=H(X,Y)-H(X)

This is referred to as the chain rule for entropy and it can be derived by appropriately manipulating the sum above. Note that here, H(X,Y) is the joint entropy (as opposed to cross entropy which uses the same notation).

Further prove that H(Y│X)=H(X│Y)-H(X)+H(Y), which is referred to as Bayes rule for conditional entropy. This can be prooved using the above relation without manipulating sums.

For these, you will upload a handwritten (or latex or similar) solution to both problems on a single sheet.

▾ **Screenshot 2024-02-12 at 12.19.35 PM.png** Download

Chain Rule for Entropy

We aim to prove that:

$$H(Y|X) = H(X,Y) - H(X)$$

Given:

$$H(Y|X) = -\sum_x \sum_y p(x,y) \log p(y|x)$$

$$H(X,Y) = -\sum_x \sum_y p(x,y) \log p(x,y)$$

$$H(X) = -\sum_x p(x) \log p(x)$$

By definition:

$$p(y|x) = \frac{p(x,y)}{p(x)}$$

Substitute $p(y|x)$ into $H(Y|X)$ based on Bayes rule:

$$H(Y|X) = -\sum_x \sum_y p(x,y) \log \left(\frac{p(x,y)}{p(x)}\right)$$

$$= -\sum_x \sum_y p(x,y) \log p(x,y) + \sum_x \sum_y p(x,y) \log p(x)$$

We know $\sum_y p(x,y) = p(x)$ based on above, thus:

$$= H(X,Y) + \sum_x p(x) \log p(x)$$

$$= H(X,Y) - H(X)$$

Hence proved the Chain Rule for Entropy.

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Bayes Rule for Conditional Entropy

We aim to prove that:

$$H(Y|X) = H(X|Y) - H(X) + H(Y)$$

Using the chain rule for entropy, we know:

$$H(X, Y) = H(Y|X) + H(X)$$

$$H(X, Y) = H(X|Y) + H(Y)$$

Thus, by equating the two expressions for $H(X, Y)$:

$$H(Y|X) + H(X) = H(X|Y) + H(Y)$$

Rearranging the terms gives us:

$$H(Y|X) = H(X|Y) - H(X) + H(Y)$$

given X.pdf Download

Q3
2 Points

In Phoenix, it is sunny 300 days per year, while in Bloomington, it is sunny 184 days per year. Calculate the entropy of the weather in both cities. Round your answers to two decimals.

Q3.1 Phoenix
1 Point

Phoenix

0.47

Q3.2 Bloomington
1 Point

Bloomington

0.69

Q4 Entropies from table

4 Points

Let p(x,y) be given by the following table. Calculate the following quantities. In all cases, round to two digits. You will insert your answers into the boxes below. For example, if your answer is 0.12, report 0.12, not 0.1 or .12.

To be clear, in this table, the columns are the values of X at a given Y value and the rows are the values of Y at a given X value.

$X \backslash Y$	0	1
0	$\frac{1}{3}$	$\frac{1}{3}$
1	0	$\frac{1}{3}$

Q4.1

1 Point

H(X)

0.64

Q4.2
1 Point

H(X|Y)

0.31

Q4.3
1 Point

H(X,Y) - Joint entropy

1.10

Q4.4
1 Point

I(X,Y)

0.33

Information Theory Written

● Ungraded

Student

Ege Otenen

Total Points

- / 15 pts

Question 1

Dice Entropy 5 pts

1.1	a	1 pt
1.2	b	1 pt
1.3	c	1 pt
1.4	d	1 pt
1.5	e	1 pt

Question 2

Conditional Entropy proof 4 pts

Question 3

(no title) 2 pts

3.1	Phoenix	1 pt
3.2	Bloomington	1 pt

Question 4

Entropies from table 4 pts

4.1	(no title)	1 pt
4.2	(no title)	1 pt
4.3	(no title)	1 pt
4.4	(no title)	1 pt