3. A and B are independent if $\boxed{P(A \cap B) = P(A).P(B)}$

First $P(A|B) = P(A)$ $\quad P(A) = \frac{P(A \cap B)}{P(B)}$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

chain rule

$\frac{P(B|A).P(A)}{P(B)} = \frac{P(A \cap B)}{P(B)} \longrightarrow Pr(A).Pr(B|A)$

Second

$Pr(A) = P(A \cap B) + P(A \cap B^c)$ $\quad$ 4

$Pr(A) = P(A|B).P(B) + \underbrace{P(A|B^c)}_{\text{given}}.P(B^c)$

$= P(A|B).P(B) + P(A|B).P(B^c)$ $\quad \cancel{Pr(A)}$

$= P(A|B)\,(\underbrace{P(B) + P(B^c)}_{1})$

$\boxed{Pr(A) = P(A|B)}$

$$P(A|B) = P(A|B^c)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)}$$

$$P(A \cap B) \cdot P(B^c) = P(A \cap B^c) \cdot P(B)$$

if independent we can write as below:

$$P(A) \cdot P(B) \cdot P(B^c) = P(A) \cdot P(B^c) \cdot P(B)$$

so they'r independent