Probability HW

Note that this is only the written form of the HW. There are a second set of questions that are coding based. These are provided as separate files.

For the written component of the HW, solutions to each problem (1-5) should be on different sheets of paper.

1. It is estimated that on a typical night, 1 in 1000 drivers approaching a certain roadblock has too much alcohol in his or her blood. If a driver has consumed too much alcohol, the test will give a positive result with probability 0.99. If the driver has not consumed too much alcohol, the test will give a negative result with probability 0.999. If a driver's test comes out positive, what is the probability that the driver has indeed consumed too much alcohol?
   Let us suppose that on New Years Eve, the '1 in 1000' figure goes up to 20%. Calculate the probability that a driver who has tested positive at a roadblock is over the legal limit.

2. Let the probability space *S* be all the cards in a standard deck, with each card *c* having probability 1/52. Let *X* and *Y* be random variables with values in *{t, f}* so that

$$X(c) = \begin{cases} t & \text{if } c \text{ is a heart} \\ f & \text{if } c \text{ is not a heart} \end{cases}$$

$$Y(c) = \begin{cases} t & \text{if } c \text{ is a Jack} \\ f & \text{if } c \text{ is not a Jack} \end{cases}$$

$$Z(c) = \begin{cases} t & \text{if } c \text{ is a the Jack of hearts or the Queen of Spades} \\ f & \text{if } c \text{ is not the Jack of hearts and also not the Queen of Spades} \end{cases}$$

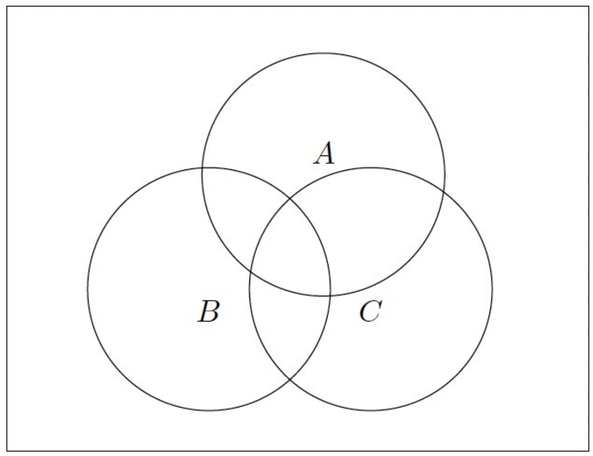
   a. Find the event $(X = t) \cap (Z = t)$. Describe what cards this represents in words.
   b. Find $Pr(X = t | Z = t)$.
   c. Find $Pr((X = t) \cap (Y = t) | Z = t)$.
   d. Show that the events $X = t$ and $Y = t$ are independent.

3. Suppose that A and B are events in a probability space, and suppose that $\Pr(A|B) = \Pr(A|B^c)$. Prove that A and B are independent.
   Hint: $A = (A \cap B) \cup (A \cap \overline{B})$.

4. Suppose two events A and B respectively have probability P(A) = 0.2, P(B) = 0.5.
    a. If A and B are mutually exclusive, what is the probability of the complement of A intersect B: $P((A \cap B)^c)$?
    b. If instead, A and B are independent, what is the probability of $P((A \cap B)^c)$?

Problem 5:

Here is a figure that you'll want to copy several times as you work this exercise:



(a) Sketch $\overline{A}$.

(b) Sketch (in a different figure) $\overline{B}$.

(c) Sketch $\overline{A} \cap \overline{B}$.

(d) Sketch $\overline{A \cup B}$.

(e) Your answers to (5c) and (5d) should be the same. Based on this, fill in the blank:

$$\overline{C} \cap \overline{B} \;=\; \underline{\hspace{3cm}}$$

(f) By following steps similar to what we saw in in (5a)–(5d), give a pictorial proof that $\overline{B} \cup \overline{C} = \overline{B \cap C}$.