$$P(A|B) = P(A|B^c)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)}$$

$$P(A \cap B) \cdot P(B^c) = P(A \cap B^c) \cdot P(B)$$

if independent we can write as below:

$$P(A) \cdot P(B) \cdot P(B^c) = P(A) \cdot P(B^c) \cdot P(B)$$

so they're independent