1. Consider a two dimensional vector space. Suppose *w1, w2, w3* are vectors in this space. Prove that they cannot possibly be linearly independent.

Hint: Choose any basis *v1, v2*. Suppose *w1, w2* are linearly independent (a necessary pre-requisite for all 3 to be LI). By appropriately writing *w’s* in terms of *v’s*, prove that *w3* must be in the span of *w1, w2*.

1. Consider the two dimensional vector space with the standard basis i=[1,0], j=[0,1]. Suppose T is a coordinate transform where T(i) = v1 = [2,3] and T(j) = v2 = [-1,1]. Consider the vector v=[2,1]=2i+1j. These are the coordinates of v in the i,j basis. Find the coordinates [a,b] of this vector in terms of v1, v2. That is v = a\*v1 + b\*v2.
2. Let *f, g* be two linear functions. Define *h* to the composition of *f,g* so that *h(v) = f(g(v))*. Prove that *h* is also a linear function.
3. Consider a linear function on *R2* with the standard basis *i,j .* Suppose *f(i) = w* and *f(j) = cw.* An example of this would be *f(i) = [2,3], f(j)=[-4,-6].* This is a scenario where the function *f* compresses the 2D plan into a single line. That is, it maps the two linearly independent basis vectors into two vectors that are no longer linearly independent.

In class, we mentioned that in this case, there is the problem that such a linear function is no longer invertible since it is not a 1-1 function. Here you will prove this. The proof goes something like this.

Step 1: Suppose there is a where *f(v)=0*. If this is the case, then we can take any other vector *h* where *f(h) = l*. If we have such a *v*, it follows that *f(h +cv) = f(h)* for all values of *c.* That is, there are infinitely many things that all map to the same place. This is linearity and you do not need to show this. Your job is to show step 2.

Step 2: Find such a vector *v*.Note that we have fully described how *f* operates above by defining what it maps *i* and *j* to. Find a single where *f(v)=0* and prove this. It only takes a few lines. I want to see your *v* and your proof. Do this for the general function *f*, not the specific one that I attached numeric vectors to. For this general function *f(i) = w* and *f(j) = cw*.

1. Consider a 2x2 matrix A with the form shown below. Fill in the missing values so that this matrix has an eigen-value eigen-vector pair of . Here this should be interpreted as a right eigenvector and *v* should be a column vector.

