

$$\phi - \phi_0 = \int_{r_0}^r \frac{L}{r^2 \sqrt{2m[-U(r)] - \frac{L^2}{r^2}}} dr$$

$$\text{For } \mu = 0 : U(r) = -\frac{v_1 r^2}{2r^2}$$

$$L \in L \quad u = \frac{1}{r} \quad U(u) = -\frac{v_1 r^2 u^2}{2}$$

$$du = -r^{-2}$$

$$\phi - \phi_0 = \int_{u_0}^0 \frac{L}{\sqrt{\frac{4\pi m v_1 r^2 u^2}{2} - L^2 u^2}} du$$

$$\phi - \phi_0 = \int_{u_0}^0 \frac{L}{u \sqrt{M v_1 r^2 - L^2}} du$$

$$\phi - \phi_0 = \frac{L}{\sqrt{M v_1 r^2 - L^2}} \int_{u_0}^0 \frac{1}{u} du$$

$$\phi - \phi_0 = \frac{L}{\sqrt{M v_1 r^2 - L^2}} \left[\ln\left(\frac{1}{u}\right) - \ln\left(\frac{1}{u_0}\right) \right]$$

~~$$\text{Let } \text{def } u = \frac{1}{r}$$~~

~~comes:~~
 ~~$\frac{1}{r} = A e^{k \theta + \epsilon}$~~
 ~~$\frac{1}{r_0} = B e^{k \theta + \epsilon}$~~
 ~~$\frac{1}{r} - \frac{1}{r_0} = C e^{k \theta + \epsilon}$~~
 ~~$\ln\left(\frac{1}{r} - \frac{1}{r_0}\right) = k \theta + \epsilon$~~
 ~~$\ln\left(\frac{1}{r} - \frac{1}{r_0}\right) = k \theta + \epsilon$~~

$$\phi - \phi_0 = \frac{L}{\sqrt{M v_1 r^2 - L^2}} \ln\left(\frac{r}{r_0}\right)$$

~~$\ln\left(\frac{1}{r} - \frac{1}{r_0}\right) = k \theta + \epsilon$~~

~~$\ln\left(\frac{1}{r} - \frac{1}{r_0}\right) = k \theta + \epsilon$~~

$$\text{Let } \phi_0 = 0 : \phi = \frac{L}{\sqrt{M v_1 r^2 - L^2}} \ln\left(\frac{r}{r_0}\right)$$

~~$\phi = \frac{1}{r}$~~