# Biostatistics
# Week IV

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# Hypothesis Testing - Steps

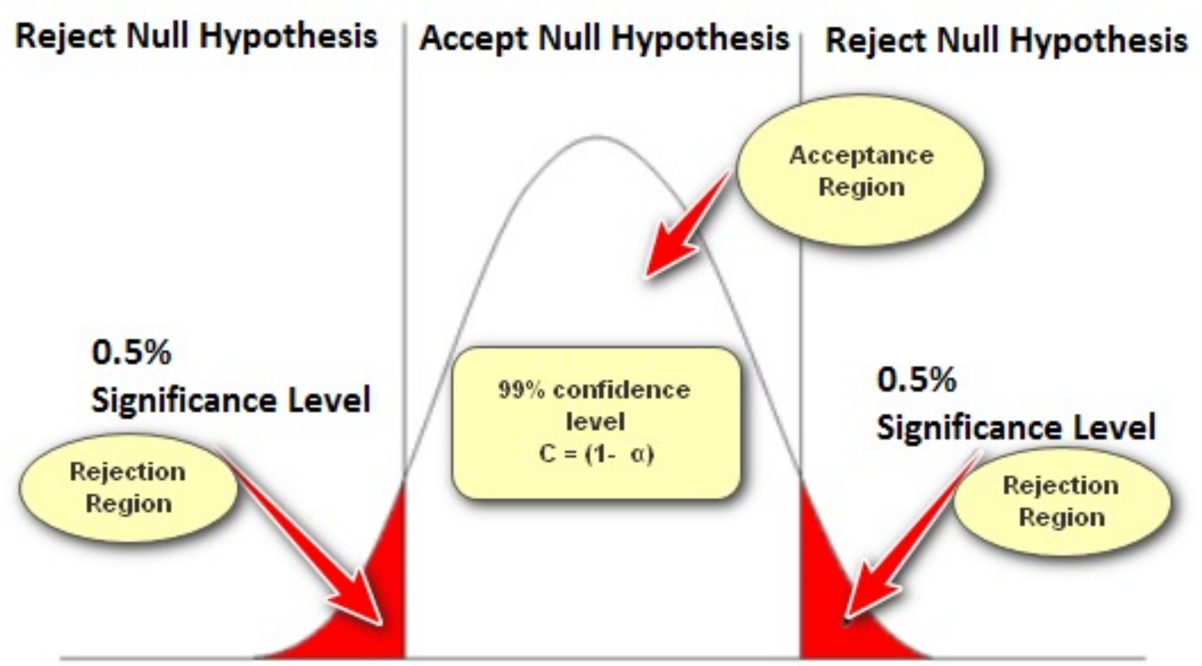
1. **Check assumptions, determine $H_0$ and $H_a$, choose $\alpha$**
    - Assumptions differ based on the test
    - The null hypothesis always contains equality (=)
2. **Calculate the appropriate test statistic**
    - z, t, $\chi^2$, …
3. **Calculate critical values/p value**
    - With the aid of precalculated tables/software
4. **Decide whether to reject/fail to reject $H_0$**
    - Reject if the statistic is within the critical region/$p \leq \alpha$

<table>
<tr><td rowspan="2">$H_0$</td><td colspan="2">Decision</td></tr>
<tr><td>Accept $H_0$</td><td>Reject $H_0$</td></tr>
<tr><td>$H_0$ is True</td><td>Correct decision</td><td>Type I Error<br>$\alpha$</td></tr>
<tr><td>$H_0$ is False</td><td>Type II Error<br>$\beta$</td><td>Correct decision</td></tr>
</table>

*Barai A. Hypothesis Testing | All a beginner needs to know [Internet]. Medium. 2020 [cited 2021 Oct 4]. Available from: https://medium.com/analytics-vidhya/hypothesis-testing-all-a-beginner-needs-to-know-dd1bdeebdbf1*

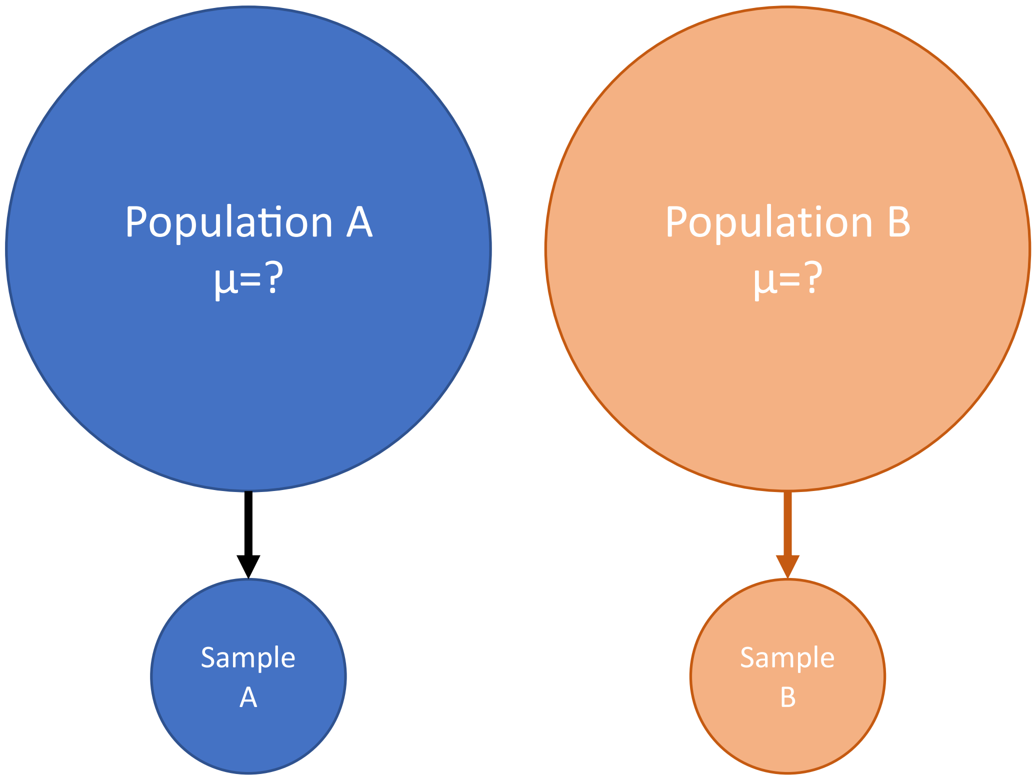
# Reminder

$$\text{test statistic} = \frac{\text{estimator} - \text{null value}}{\text{standard error of estimator}}$$



# Two-Sample t-Test

- The **two-sample t-test** (also known as the **independent samples t-test**) is a method used to test whether the unknown population means of two groups are equal or not



# Two-sample t-Test

$H_0: \mu_X = \mu_Y$

$H_a: \mu_X \neq \mu_Y$

or

$\mathbf{H_0: \mu_X - \mu_Y = 0}$

$\mathbf{H_a: \mu_X - \mu_Y \neq 0}$

# Two-sample t-Test

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}} \sim \mathrm{t}(\mathrm{m}),$$

$$m = \frac{(w_X + w_Y)^2}{\left(\frac{w_X^2}{n_X - 1} + \frac{w_Y^2}{n_Y - 1}\right)}$$

$$w_X = s_X^2 / n_X, \quad w_Y = s_Y^2 / n_Y$$

# Two-sample t-Test – Example I

| id | treatment | perc_benefit |
|---|---|---|
| 158 | trt1 | 37.2549020 |
| 392 | trt1 | -4.3864459 |
| 457 | trt1 | -5.1075269 |
| 487 | trt1 | 36.7043369 |
| 723 | trt1 | 5.1303099 |
| 832 | trt1 | 3.1806616 |
| 894 | trt1 | -3.9062500 |
| 1104 | trt1 | 5.9443608 |
| 1283 | trt1 | -0.8601855 |
| 1288 | trt1 | -3.1674208 |

| id | treatment | perc_benefit |
|---|---|---|
| 15 | trt2 | 10.0978368 |
| 143 | trt2 | 0.5048635 |
| 470 | trt2 | -0.8156940 |
| 536 | trt2 | 50.0000000 |
| 549 | trt2 | -3.0303030 |
| 750 | trt2 | -2.8977108 |
| 891 | trt2 | 26.3872135 |
| 997 | trt2 | 4.3651179 |
| 1000 | trt2 | 2.3582125 |
| 1209 | trt2 | 8.9702189 |

- Mean percentage benefit is 7.078674 for group 1, and 9.593976 for group 2
- Is the difference a significant one?

# Two-sample t-Test – Example I (cont.)

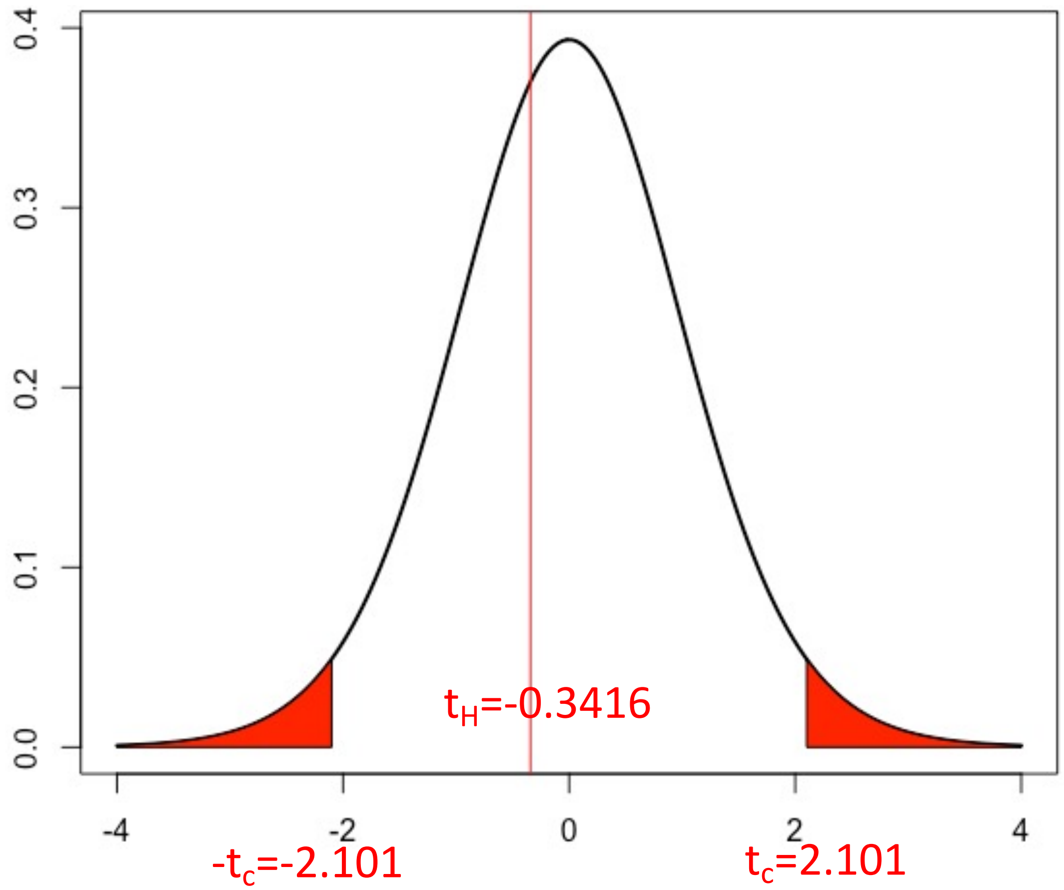
1. Check assumptions, determine $H_0$ and Ha, choose $\alpha$
   - We check that the variables are normally distributed
   - $H_0$: $\mu_1 = \mu_2$ $H_a$: $\mu_1 \neq \mu_2$
   - $\alpha = 0.05$

2. Calculate the appropriate test statistic

$$t = -0.3416 (\sim t_{17.98834})$$

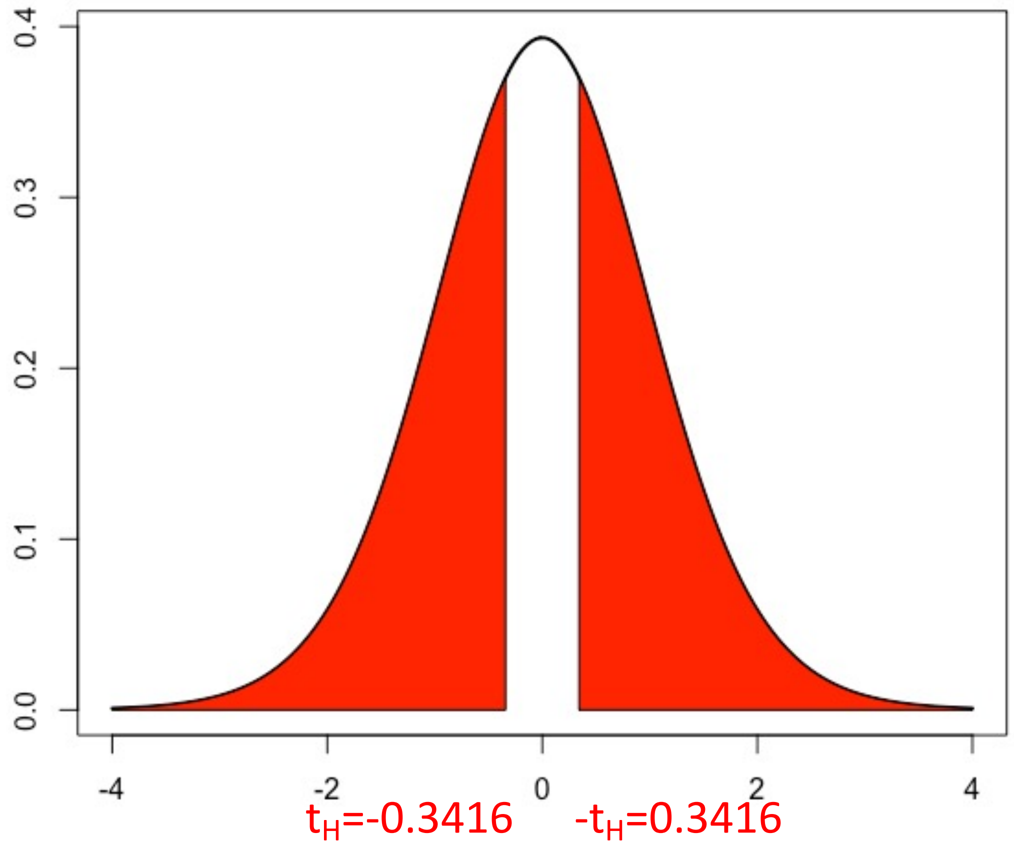
# Two-sample t-Test – Example I (cont.)

3. Calculate **critical values**/p value
4. Decide whether to reject/fail to reject $H_0$

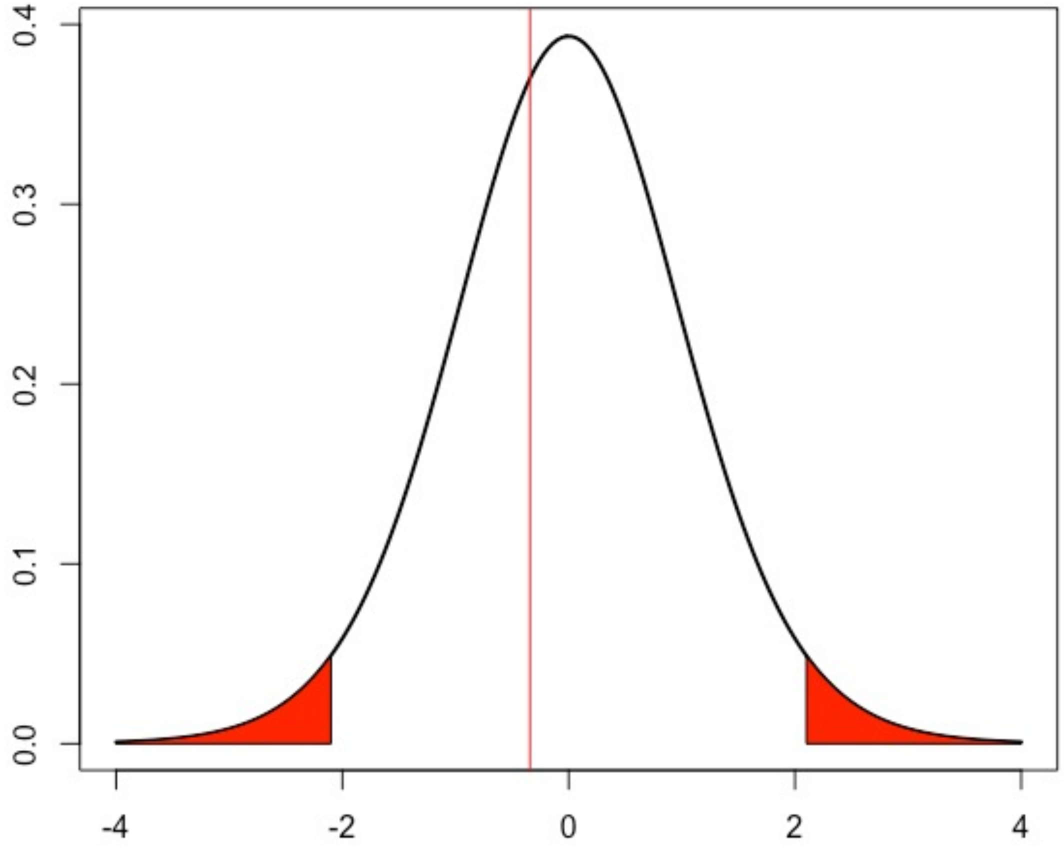


# Two-sample t-Test – Example I (cont.)

3. Calculate critical values/**p value**
4. Decide whether to reject/fail to reject $H_0$



# Two-sample t-Test – Example I (cont.)




95% confidence interval for $\mu_1 - \mu_2$ = [-17.98 , 12.95]

# Two-sample t-Test – Example I (cont.)

- there is not enough evidence to say mean percentage benefit for treatment 1 and treatment 2 are significantly different

# Two-sample t-Test – Example II

- In a study,
  - The sedimentation rate of 12 arthritis patients was measured:

    $\bar{X}_1 = 82.79$ mm and $s_1 = 18.4$ mm
  - The sedimentation rate of 15 healthy controls was measured

    $\bar{X}_2 = 69.03$ mm and $s_2 = 21.4$ mm
- Is there a difference between the mean sedimentation rates of the two groups?

# Two-sample t-Test – Example II (cont.)

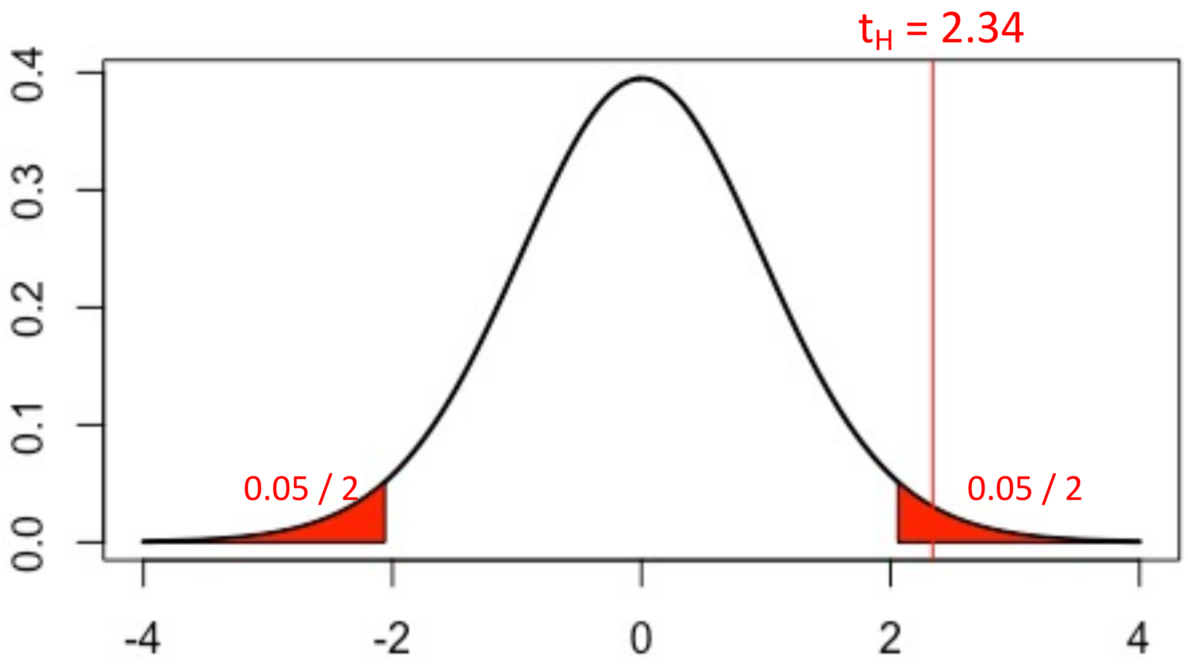
1. Check assumptions, determine $H_0$ and Ha, choose $\alpha$
   - We check that the variables are normally distributed
   - $H_0$: $\mu_1 = \mu_2$     $H_a$: $\mu_1 \neq \mu_2$
   - $\alpha = 0.05$

2. Calculate the appropriate test statistic

$$t = 2.34 \qquad (\sim t_{25})$$

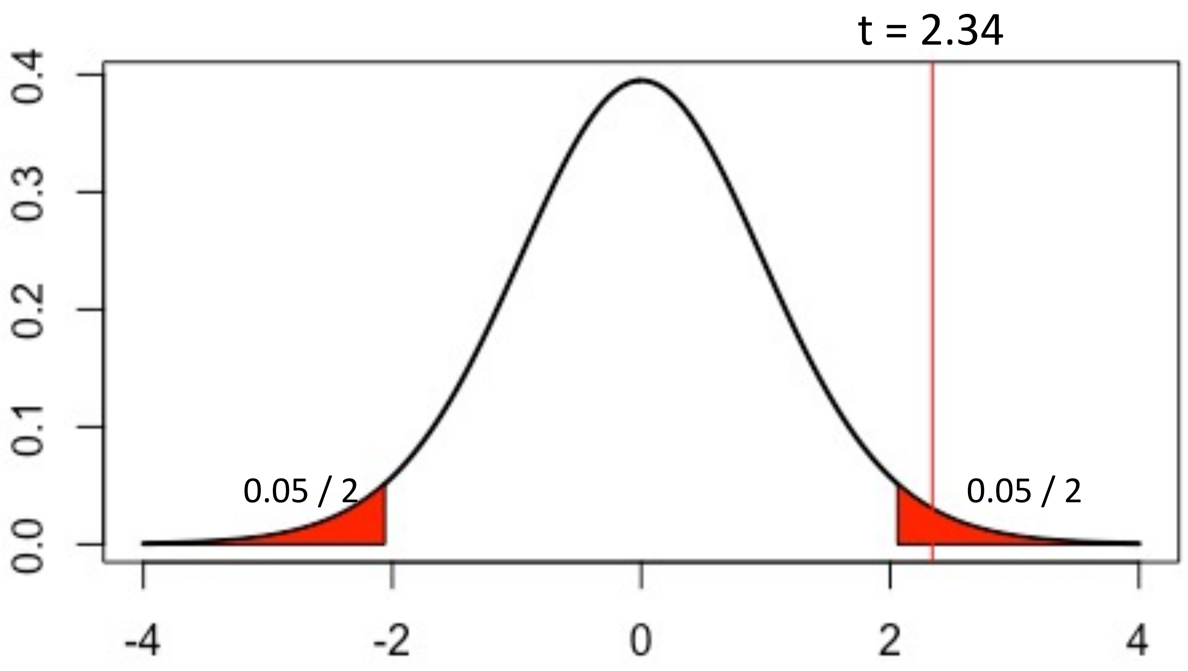
# Two-sample t-Test – Example II (cont.)

3. Calculate critical values/p value
4. Decide whether to reject/fail to reject $H_0$




p = 0.027568

# Two-sample t-Test – Example II (cont.)




p = 0.027568

95% confidence interval for $\mu_1 - \mu_2$ = [3.52, 33]

# Two-sample t-Test – Example II (cont.)

- With 95% confidence, there is enough evidence to say that there is a difference between the mean sedimentation rates of the two groups

*Wong DT, Dallaire A, Singh KP, Madhusudan P, Jackson T, Singh M, et al. High-flow nasal oxygen improves safe apnea time in morbidly obese patients undergoing general anesthesia: a randomized controlled trial. Anesth Analg. 2019 Oct;129(4):1130–6.*

# Two-sample t-Test – Example III

- "Morbidly obese patients undergoing general anesthesia are at risk of hypoxemia during anesthesia induction"
- A randomized controlled trial investigating:
- Does high-flow nasal oxygenation provide longer safe apnea time compared to conventional facemask oxygenation during anesthesia induction in morbidly obese surgical patients?

*Wong DT, Dallaire A, Singh KP, Madhusudan P, Jackson T, Singh M, et al. High-flow nasal oxygen improves safe apnea time in morbidly obese patients undergoing general anesthesia: a randomized controlled trial. Anesth Analg. 2019 Oct;129(4):1130–6.*

# Two-sample t-Test – Example III (cont.)

- Safe Apnea time in Control Group (n = 20)
  - $\overline{X_c} = 185.5$
  - $s_c = 53$
- Safe Apnea time in High-Flow Nasal Oxygenation Group (n = 20)
  - $\overline{X_T} = 261.4$
  - $s_T = 77.7$

# Two-sample t-Test – Example III (cont.)

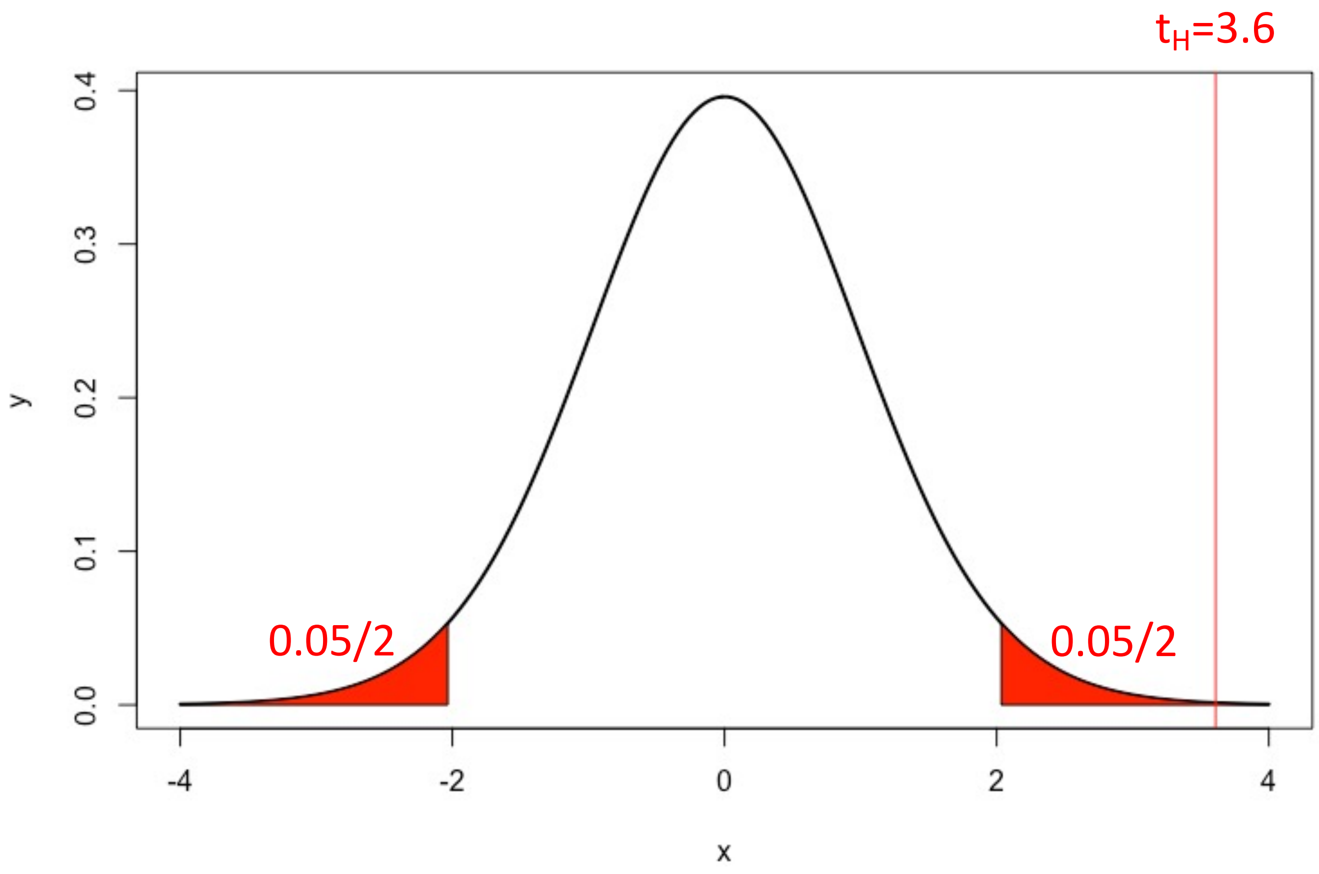
1. Check assumptions, determine $H_0$ and Ha, choose $\alpha$
   - We check that the variables are normally distributed
   - $H_0$: $\mu_c = \mu_T$ $H_a$: $\mu_c \neq \mu_T$
   - $\alpha = 0.05$

2. Calculate the appropriate test statistic

$$t = 3.6 \qquad (\sim t_{33.53})$$

# Two-sample t-Test – Example III (cont.)

3. Calculate **critical values**/p value
4. Decide whether to reject/fail to reject $H_0$



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# Two-sample t-Test – Example III (cont.)

**Table 2. Study Outcomes: Safe Apnea Time, Minimum $Spo_2$, Plateau $ETco_2$, and Time to Regain Baseline $Spo_2$**

| | Control Group (n = 20) | High-Flow Nasal Oxygenation Group (n = 20) | Mean Difference (95% CI) | *P* Value |
|---|---|---|---|---|
| Safe apnea time (s) | 185.5 ± 53.0 | 261.4 ± 77.7 | 75.9 (33.3–118.5) | .001 |
| Minimum $Spo_2$ (%) | 87.9 ± 4.7 | 90.9 ± 3.5 | 3.1 (0.4–5.7) | .026 |
| Plateau $ETco_2$ (mm Hg) | 38.8 ± 2.5 | 37.9 ± 3.0 | −0.8 (−2.6 to 0.9) | .33 |
| Time to regain baseline $Spo_2$ (s) | 49.6 ± 20.8 | 37.3 ± 6.8 | −12.3 (−22.2 to −2.4) | .016 |

Values represent mean ± SD.
Control group: facemask oxygenation.
Abbreviations: CI, confidence interval; $ETco_2$, end-tidal carbon dioxide; $Spo_2$, oxygen saturation measured by pulse oximetry.

"Safe apnea time was significantly longer (261.4 ± 77.7 vs 185.5 ± 52.9 seconds; mean difference [95% CI], 75.9 [33.3–118.5]; *P* = .001)…"

Barai A. Hypothesis Testing | All a beginner needs to know [Internet]. Medium. 2020 [cited 2021 Oct 4]. Available from: https://medium.com/analytics-vidhya/hypothesis-testing-all-a-beginner-needs-to-know-dd1bdeebdbf1

# Brief Summary