

Biostatistics Week VII

Ege Ülgen, M.D.

18 November 2021



ACIBADEM
MEHMET ALİ AYDINLAR
ÜNİVERSİTESİ

Hypothesis Testing - Steps

1. Check assumptions, determine H_0 and H_a , choose α

- Assumptions differ based on the test
- The null hypothesis always contains equality (=)

2. Calculate the appropriate test statistic

- z , t , χ^2 , ...

3. Calculate critical values/p value

- With the aid of precalculated tables/software

4. Decide whether to reject/fail to reject H_0

- Reject if the statistic is within the critical region/ $p \leq \alpha$

χ^2 Test of Association

- Used to assess the association between two categorical variables
- More generally, used to investigate the significance of the difference between expected and observed values
- Are the 2 categorical variables **independent**?

χ^2 Test – Test Statistic

$$\chi^2 = \sum \frac{(\textit{observed} - \textit{expected})^2}{\textit{expected}}$$

χ^2 Test – Example

TABLE III—Changes in frequency of physical exercise in patients with angina between baseline and review at two years

	No (%) of patients	
	Intervention group	Control group
Increased	108 (34)	63 (21)
No change	120 (38)	74 (25)
Decreased	89 (28)	163 (54)

χ^2 Test – Example

	Intervention Group	Control Group	Total
Increased	108	63	171
No change	120	74	194
Decreased	89	163	252
Total	317	300	617

$$expected_{1,1} = 317 \times \frac{171}{617} \quad expected_{1,2} = 300 \times \frac{171}{617}$$

$$expected_{2,1} = 317 \times \frac{194}{617} \quad expected_{2,2} = 300 \times \frac{194}{617}$$

$$expected_{3,1} = 317 \times \frac{252}{617} \quad expected_{3,2} = 300 \times \frac{252}{617}$$

χ^2 Test – Example

OBSERVED	Intervention Group	Control Group
Increased	108	63
No change	120	74
Decreased	89	163

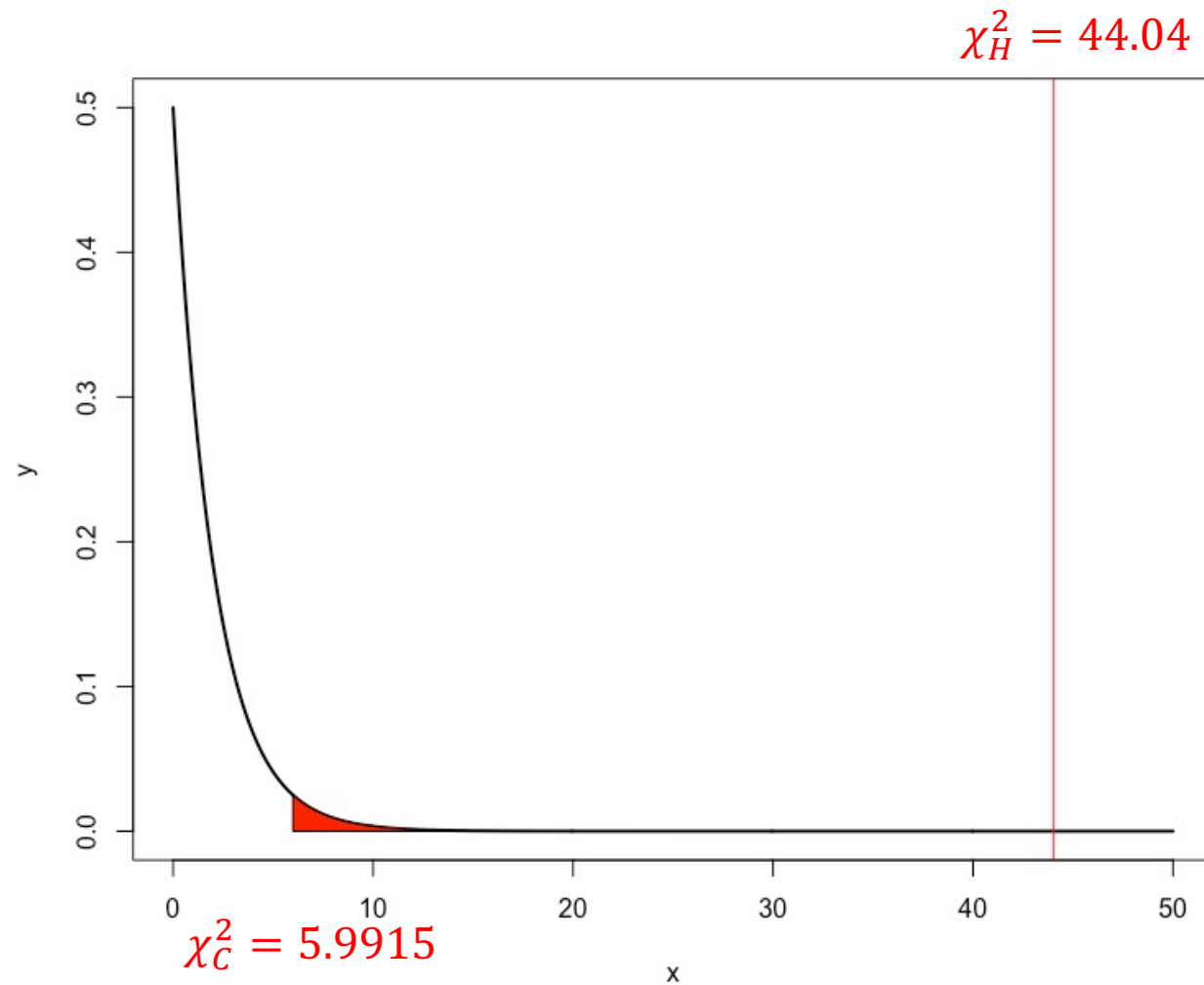
EXPECTED	Intervention Group	Control Group
Increased	87.86	83.14
No change	99.67	94.33
Decreased	139.47	122.53

χ^2 Test – Test Statistic

$$\chi^2 = \sum \frac{(\textit{observed} - \textit{expected})^2}{\textit{expected}}$$

$$\chi_H^2 = 44.04 \sim \chi_{(3-1)(2-1)=2}^2$$

χ^2 Test – Test Statistic



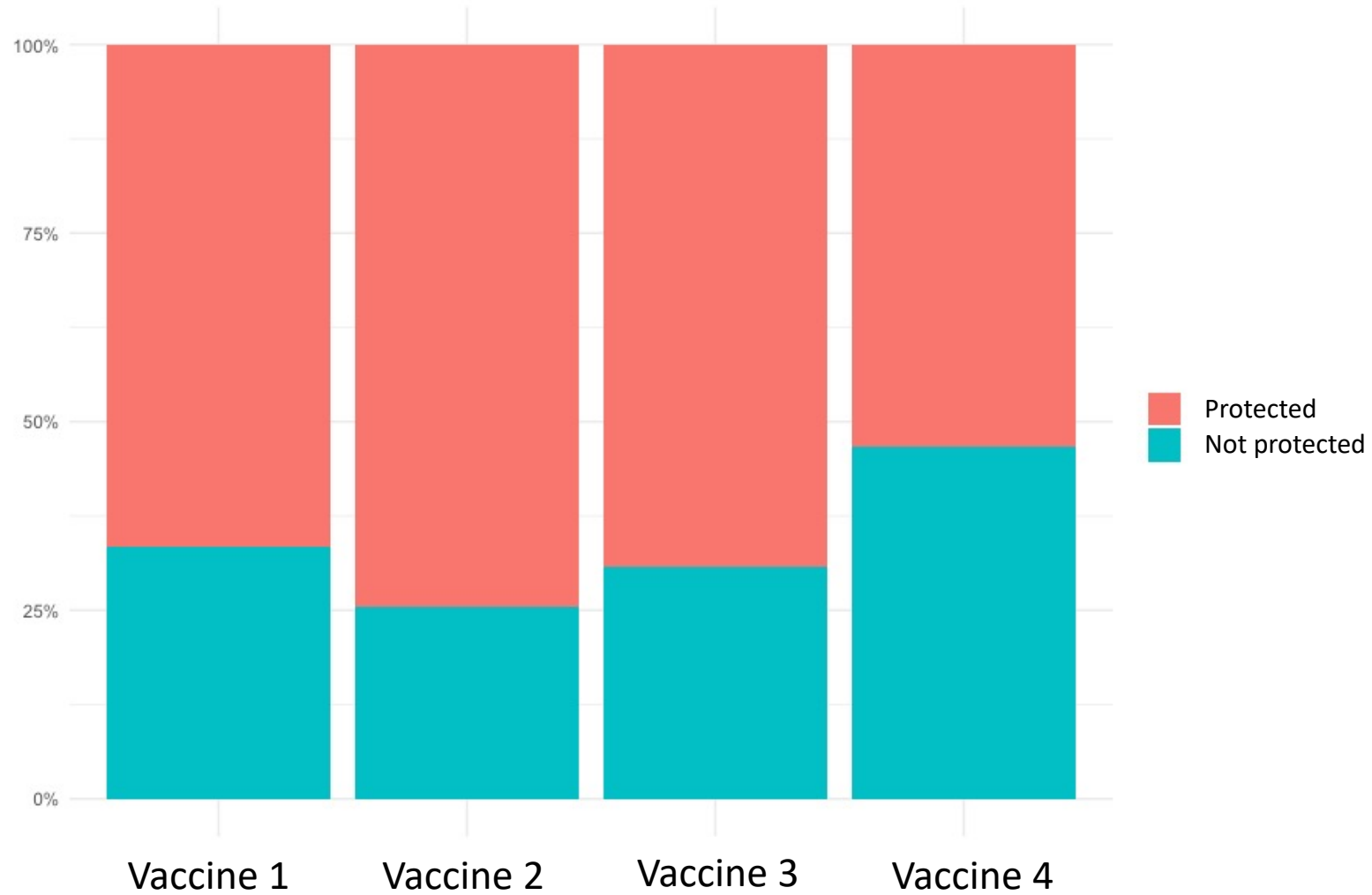
$p < 0.001$

χ^2 Test – Example

- Is there a significant difference between the efficacy of 4 different COVID vaccines?

	Protected	Not protected
Vaccine 1	82	41
Vaccine 2	70	24
Vaccine 3	45	20
Vaccine 4	48	42

χ^2 Test – Example



χ^2 Test – Example

1. Check assumptions, determine H_0 and H_a , choose α
 - H_0 : there is **no difference** in efficacy
 H_a : there is a difference in efficacy
 - $\alpha = 0.05$
2. Calculate the appropriate test statistic

$$\chi_H^2 = 9.297 \sim \chi_3^2$$

χ^2 Test – Example

	Protected	Not protected	Total
Vaccine 1	82	41	123
Vaccine 2	70	24	94
Vaccine 3	45	20	65
Vaccine 4	48	42	90
Total	245	127	372

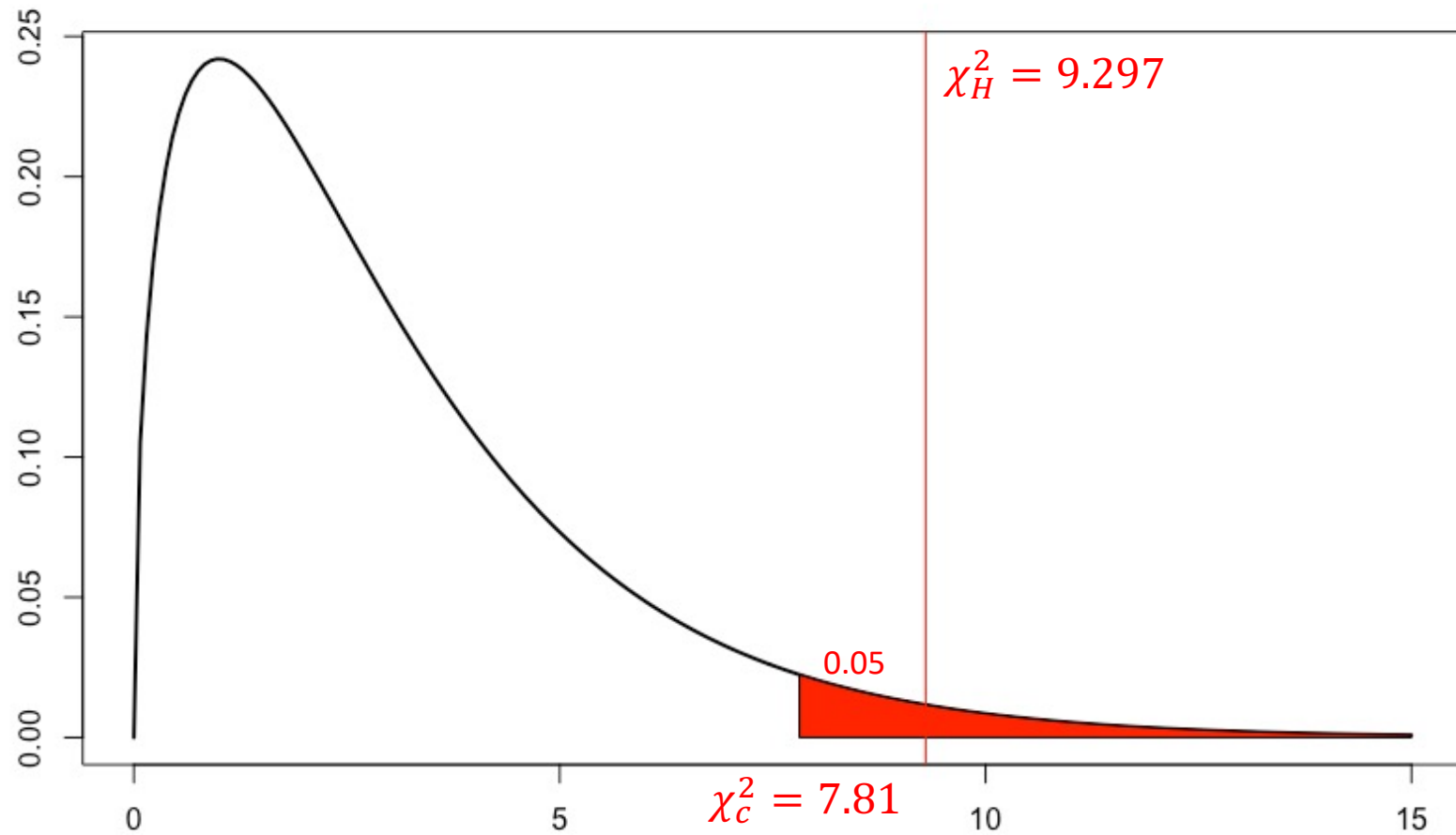
$$expected_{4,1} = 245 \times \frac{90}{372} = 59$$

$$expected_{4,2} = 127 \times \frac{90}{372} = 31$$

$$\chi_H^2 = \sum_{j=1}^m \sum_{i=1}^n \frac{(observed_{ij} - expected_{ij})^2}{expected_{ij}} \sim \chi_{(m-1)(n-1)}^2$$

$$\chi_H^2 = 9.297 \sim \chi_3^2$$

χ^2 Test – Example



$p = 0.025592$

