Useful formula

- first kind Stirling

  the number of ways to arrange n objects into k nonempty cycles

  - $s(n,k) = s(n-1,k-1) + (n-1) * s(n-1,k)$

- second kind Stirling

  the number of ways to arrange n object into k nonempty subsets

  - $S(n,k) = \frac{1}{k!}\sum_{j=1}^{k}(-1)^{k-j}C(k,j)j^n$
  - $S(n,k) = S(n-1,k-1) + k * S(n-1,k)$

- catalan number

  - $C_0 = 1, C_{n+1} = \sum_{i=0}^{n} C_i C_{n-1}$
  - $C_0 = 1, C_{n+1} = \frac{2(2n+1)}{n+2} C_n$
  - $C_n = \binom{2n}{n} - \binom{2n}{n+1}$
  - $C_n = \frac{1}{n+1}\binom{2n}{n}$

Combinatorics: Box-Ball

N ball, M box

- ball same, box same, no empty
  $P_m(N)$

- ball same, box same, allow empty
  $P_m(N+M)$

- ball same, box unique, no empty
  $C(N-1, M-1)$

- ball same, box unique, allow empty
  $C(N+M-1, M-1)$

- ball unique, box same, no empty
  $S(N, M)$

- ball unique, box same, allow empty

$\sum_{i=1}^{M} S(N, i)$

- ball unique, box unique, no empty
  $M! * S(N, M)$

- ball unique, box unique, allow empty
  M^N