Math 541: Written Homework 8
Due: 04/17

Theory : Show that for any integer k

$$\sum_{j=1}^{2^{k-1}-1} f\left(a+\frac{j}{2}h_{k-1}\right)=\sum_{j=1}^{2^{k-2}} f\left(a+\left(j-\frac{1}{2}\right)h_{k-1}\right)+\sum_{j=1}^{2^{k-2}-1} f\left(a+jh_{k-1}\right),$$

where $h_{k-1}=(b-a)/2^{k-1}$.

Coding : Using Romberg integration, compute

$$\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt,$$

erf(1) to 10^{-7}. Turn in a table of the values used in Romberg integration used to get to this result.

Coding : The study of light diffraction at a rectangular aperature involves the Fresnel integrals

$$c(t)=\int_0^t \cos\left(\frac{\pi}{2}\omega^2\right)d\omega,\ \ s(t)=\int_0^t \sin\left(\frac{\pi}{2}\omega^2\right)d\omega$$

Using adaptive quadrature, generate approximations to these integrals at $t=.1,.2,\cdots,1$ with an accuracy of 10^{-4}. Plot these values and turn in the plot. Running with this, what is the limit of these integrals as $t\to\infty$? Generate plots which establish you answer. How large of a value of t do you need to get a well defined limit?