y' = family of curves
$-\frac{1}{y'}$ = orthoganol family of curves

SEPERATION OF VARIABLES

$\frac{dN}{dt} = -\lambda N$
$\frac{1}{N}\frac{dN}{dt} = -\lambda$
$\frac{dN}{N} = -\lambda dt$
ln N = -lambda t + c
N = exp(-lambda t + c)
N = exp(c)exp(-lambda t)
N(t) = N_0 exp(-lambda t)

ORTHOGANOL CURVES

$y' = \frac{y+1}{x}$
$\frac{dy}{dx} = \frac{-x}{y+1}$
-x-x-x-x-x–x-x-x-x-x-x
GENERAL SOLUTION FOR $y' + Py = Q$
$y(x) = e^{-I}\int Q(x)e^{I}dx + c_0 e^{-I}$
$I = \int P dx$
-x-x-x-x-x–x-x-x-x-x-x

THERMO EXACT DIFFERENTIAL

$\frac{dy}{dx} = \frac{P(x,y)}{Q(x,y)}$
$Qdy + Pdx = 0$
$dU + \vec{\nabla}U * d\vec{r}$
$dU = \frac{dU}{dx}dx + \frac{dU}{dy}dy$
$\vec{F} = \vec{\nabla}U$
$F_x = \frac{\partial U}{\partial x} \quad F_y = \frac{\partial U}{\partial y}$
$\frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x} = 0$
$\vec{\nabla} \times \vec{F} = 0$

SECOND ORDER HOMOGENOUS DIFFERENTIALS

$y'' + A_1 y' + A_0 y = 0$
Operator
$\frac{d}{dx}(operand) \to Df$
$D^2 y + A_1 Dy + A_0 y = 0$
$(D^2 + A_1 D + A_0)y = 0$
auxillary equation
$(D+a)(D+b)y = 0$
$(D+a)(Dy+by) = 0$
$D^2 y + aDy + bDy + aby = 0$
$A_1 = a+b \quad A_0 = ab$
$(D+a)y = 0$
$(D+b)y = 0 \to D_y = -ay \qquad c_1 e^{-ax}$
$D_x = -by \qquad c_2 e^{-bx}$
$y(x) = c_1 e^{-ax} + c_2 e^{-bx}$
$\frac{-A_1 +- \sqrt{A_1^2 - 4A_0}}{2}$
imaginary $\to$ sinusoidal; under damping
real $\to$ exponential decay; over damping
0 $\to$ sqrt drops out and exponential decay; critical damped
WHAT IF a=b
$(D+a)(D+a)y = 0$
$(D+a)u = 0$
$u = c_1 e^{-ax}$
$(D+a)y = u = c_1 e^{-ax}$
$y' + ay = c_1 e^{-ax}$
$I = \int a dx = ax$
$y = e^{-ax}\int c_1 e^{-ax} a^{ax} dx + c_2 e^{-ax}$
$= c_1 x e^{-ax} + c_2 e^{-ax}$
$y(x) = Re^{A_1 x/2}\sin(x\sqrt{A_0 - A_1^2/4} + q)$
$\sqrt{A_0} = \omega_0$ "natural frequency"
homogenous
$y'' + A_1 y' + A_0 y = 0$
$y(x) = (c_1 x + c_2)e^{-ax}$
inhomogeneous
$\omega_0 = \sqrt{A_0} \quad \omega' = \sqrt{\omega_0^2 - \frac{A_1^2}{4}}$
$y(x) = e^{(}-\frac{A_1 x}{2}(c_1 e^{i\omega' x} + c_2 e^{-i\omega' x})$
Any solution goes away as x $\to$ infinity
$y'' + A_1 y' + A_0 y = f(x)$
z(x) solves homogeneous equation (=0)
complementary solution
w(x) solves the inhomogeneous equation (=f(x))
particular solution
q(x) = z(x) + w(x)
total solution
———GENERAL SOLUTION COMPLEMENTARY SOLUTION—————
$y'' + A_1 y' + A_0 y = 0$
$y(x) = c_1 e^{-ax} + c_2 e^{-bx}$
a,b=$\frac{-A_1 +- \sqrt{A_1^2 - 4A_0}}{2}$

No general solution for w(x) like there is for z(x) because it depends on what f(x) is

$y'' + y'A_1 + yA_0 = B$
$w(x) = \frac{B}{A_0}$
$f(x) = \delta e^{\gamma x}$
Cases:

1 $\gamma = a \quad \& \quad \gamma \neq b \quad \& \quad a \neq b$

2 $\gamma = -a \quad OR \quad \gamma = -b \quad \& \quad a \neq b$

3 $\gamma = -a = -b$

$(D+a)(D+b)y = \delta e^{\gamma x}$ *remember operators....*
$(D+a)y = \delta e^{\gamma x}$
$u' + au = \delta e^{\gamma x}$
$u = e^{-ax}\int \delta e^{\gamma x} e^{ax} dx + ce^{-ax}$
CASE 1
$u = \frac{e^{-ax}\delta}{\gamma + a} e^{(\gamma+a)x} + ce^{-ax}$
$u(x) = \frac{\delta}{\gamma+a} e^{\gamma x} + ce^{-ax}$
$y' + by = \frac{\delta}{\gamma+a} e^{\gamma x} + ce^{-ax}$
$y = e^{-bx}\int [\frac{\delta}{\gamma+a} e^{\gamma x} + ce^{-ax}] e^{bx} dx + c_2 e^{-bx}$
$y = e^{-bx}\int \frac{\delta}{\gamma+a} e^{(\gamma+b)x} + c_1 e^{(b-n)x}\ dx + c_2 e^{-bx}$
$y = \frac{\delta e^{\gamma x}}{(\gamma+a)(\gamma+b)} + \frac{c_1 e^{-ax}}{b-a} + c_2 e^{-bx}$
CASE 2
$y'' + (a+b)y' + aby = f(x) = \delta e^{-ax}$
$y = Cxe^{-ax}$
$y' = Ce^{-ax} - aCxe^{-ax}$
$y'' = -aCe^{-ax} + a^2 Cxe^{-ax} - aCe^{-ax}$
$\delta e^{-ax} = y'' + (a+b)y' + aby$
particular: $C = \frac{\delta}{b-a}$
$y(x) = \frac{x}{b-a}\delta e^{-ax} + c_1 e^{-ax} + c_2 e^{-bx}$
CASE 3
$y_p = Cx^2 e^{\gamma x}$
$y(x) = Cx^2 e^{\gamma x} + c_1 e^{-ax} + c_2 e^{-bx}$

PARTIAL DIFFY Qs

f(x,y,z,t)
$\frac{\partial f}{\partial x} \quad \frac{\partial^2 f}{\partial x^2}$

LAPLANCE EQUATION

$\nabla^2 f = 0$
$\vec{\nabla} * \vec{\nabla} f$
$\vec{F} = -\vec{\nabla} U$
$\vec{\nabla} * \vec{F} = \rho$

POISSON'S EQUATION

$\nabla^2 f = \rho(x,y,z)$

DIFFUSION EQUATOIN

$\nabla^2 f = \frac{1}{\alpha^2}\frac{\partial f}{\partial t}$

WAVE EQUATION

$\nabla^2 f = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$

SCHRODINGER'S EQUATION

$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(\vec{r})\Psi = i\hbar\frac{\partial\Psi}{\partial t}$

SEPERATION OF VARIABLES

$f(x,y,z,t) \to$ look for solutions that are made up of mini solutions that each depend on one of the variables
$f(x,y,z,t) = X(x)Y(y)Z(z)P(t)$

Boundaries are important

Let's start with LaPlance!
$\nabla^2 T = 0 \to$ because its a static situation
$T(x=0,y) = 100$
$T(x,y=0) = T(x.y=a) = 0$
$T(x \to \infty, y) = 0$
$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$
$= \frac{\partial^2}{\partial x^2}(X(x)Y(y)) + \frac{\partial^2}{\partial y^2}(X(x)Y(y))$
$= Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} = 0$
$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}$
$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = k^2 \qquad \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k^2$

PLATE

T(x=0,y) = sin(pi*y/a)
T(x=b,y) = sin(2*pi*y/a)
T(x,y) = e^(kx)sin(ky)
e^(-ky)cos(ky)
Can rule out cosine; we also know k has to be quantize in npi/a
T(x,y) = sum((A_ne^(k_nx) + B_ne^(-k_nx))sin(k_ny))
T(x=0,y) = sin(pi*y/a)=sum((A_n+B_n)sin(npi/a))
A_1 + B_1 =1
A_n + B_n = 0
T(x=b,y) = sin(2piy/a) = sum((A_ne^(k_nb) + B_n^(-k_nb))*sin(npiy/a))
A_2e^(2pib/a) + B_2e^(-2pib/a) = 1
A_1 + B_1 = 1
A_2e^(2pib/a) + B_2e^(-2pib/a) = 1
A_1e^(2pib/a) + B_1e^(-2pib/a) = 0
A_2 + B_2 = 0
Four equations four unknowns
T(x,y) =
$T(x,y) = (\frac{-e^{-\pi b/a}}{2\sinh(\frac{\pi b}{a})}e^{\pi x/a} + \frac{e^{\pi b/a}}{2\sinh(\frac{\pi b}{a})}e^{-\pi x/a})\sin(\frac{\pi y}{a}) + (\frac{1}{2\sinh(\frac{2\pi b}{a})}e^{2\pi x/a} - \frac{1}{2\sinh(\frac{2\pi b}{a})}e^{-2\pi x/a})\sin(\frac{2\pi y}{a})$

Wave Equation

$\nabla^2 f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0$
$f(x,t) = X(x)P(t)$
$\frac{1}{X}\frac{d^2 X}{dx^2} - \frac{1}{c^2 P}\frac{d^2 P}{dt^2} = 0$
$\frac{d^2 X}{dx^2} = -k^2 X \quad \frac{d^2 P}{dt^2} = -c^2 k^2 P$
f(x,t) = sin(kx) sin(ckt)
cos(kx) cos(ckt)
f(x=0,t) = 0 f(x=l, t) = 0
k = n pi/l
cos(kx) and sin(ckt) drop out
f(x,t) = sum(A_nsin(npix/l)*cos(npit/l*c)
propogating wave
f(x, t=0) = sin(8pi/l x) x < l/8
0 x> l/8
f(x,t=0) = sum(A_n sin(npi/l x)
A_n = 2/l int 0 to l f(x,t=0)sin(n pi/l x)dx
A_n = 2/l int 0 to 1/8 sin(8pix/l)sin(npix/l)sx = 1/l int 0 to l/8 -cos(pix/l (8+n)) + cos(pix/l(8-n)))dx