27 Circuits

The electric eel (*Electrophorus*) lurks in rivers of South America, killing the fish on which it preys with pulses of current. It does so by producing a potential difference of several hundred volts along its length; the resulting current in the surrounding water, from near the eel's head to the tail region, can be as much as one ampere. If you were to brush up against this eel while swimming, you might wonder (after recovering from the very painful stun):

How can the electric eel manage to produce a current that large without shocking itself?

The answer is in this chapter.

27-1 Electric Currents and Circuits

Knowing how to analyze circuits by predicting the currents through their elements and the potential differences across them is a valuable skill. Such knowledge enables engineers and scientists to design electrical devices and helps them make productive use of existing devices. Our goal in this chapter is to understand the behavior of relatively simple electric circuits by applying concepts such as current, potential difference, and resistors developed in the previous chapter. We will start by considering very simple ideal circuits and then go on to consider circuits with multiple loops and batteries such as those shown in Fig. 27-1. Toward the end of the chapter we will introduce the concept of emf or electromotive force associated with batteries and other power sources. In particular, we will consider how to extend our analysis to the behavior of circuits powered by nonideal batteries that have internal resistance.

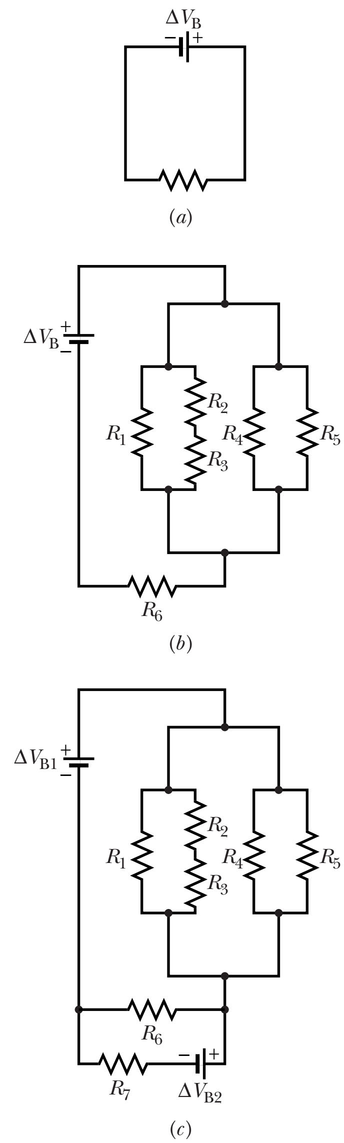


FIGURE 27-1 ■ Several types of ideal circuits we will learn to analyze in this chapter consist of ideal batteries, conducting wires with negligible resistance and ohmic resistors. (*a*) A single-loop circuit. (*b*) A single-battery, multiple-loop circuit. (*c*) A multiple-loop circuit with multiple batteries.

Ideal Circuits

As we so often do in developing physical ideas, we start by analyzing how a system behaves under ideal conditions. Only then do we introduce real-world complexities that require us to modify our methods of analysis. The ideal circuits we consider first have three characteristics:

1. **They are powered by ideal batteries.** As stated in Section 26-3, an ideal battery "maintains a constant potential difference across its terminals." This means there is a negligible amount of "electric friction" and the potential difference, ΔV_B, across the terminals of an ideal battery stays the same, regardless of the amount of charge flowing through it. But as the chemical potential energy of a real battery decreases, it develops some *internal resistance,* and the potential difference across its terminals decreases if its current increases.
2. **All circuit elements, other than the battery and connecting wires, are ohmic devices having a significant resistance.** As discussed in Section 26-5, an *ohmic device has a constant value of resistance, R, that is not a function of the amount of current passing through it.* Although lightbulbs and some other circuit elements are not ohmic, standard carbon resistors obey Ohm's law and have a constant resistance over a large current range. We make use of the fact that the potential difference across the terminals of an ohmic device is directly proportional to the current, i, flowing through it and is given by $\Delta V = iR$ (Eq. 26-7).
3. **Ideal conducting wires connect the battery to circuit elements.** Copper wiring is used in most circuits found in consumer devices, households, and industries. We can use Eq. 26-7 and data from Table 26-2 to determine that the resistance of a 30 cm length of common 22 gauge copper wire is about 0.1 Ω. If this wire was connected to a 10 Ω resistor, the additional resistance of the wire would add 1% to the overall resistance. In connecting larger resistors, the influence of the resistance of the wire is even smaller. Because the resistance in the wire is so small, the potential difference between the ends of even a relatively long continuous connecting wire is for all practical purposes negligible. In ideal circuits, we assume there is no potential drop across connecting wires.

READING EXERCISE 27-1: Show that the resistance of a 30 cm (≈12 inch) length of 22 gauge copper wire of diameter 0.024 cm has a resistance of about 0.1 Ω. *Hint:* You will need to use information from Table 26-2 along with Eq. 26-7. ■

27-2 Current and Potential Difference in Single-Loop Circuits

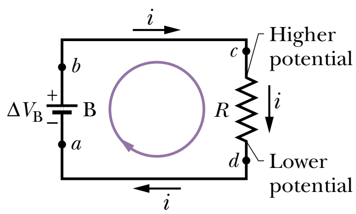


FIGURE 27-2 ■ A single-loop circuit in which a resistor R is connected across an ideal battery B with potential difference ΔV_B. The resulting current i is the same throughout the circuit.

Suppose we want to design or operate an electrical device such as a CD player or refrigerator. The operation of the given device will require a certain minimum current or potential difference. How would we calculate the amount of current in a circuit or the potential difference between two points within the device? That is the topic of this section.

We start out our discussion of current in circuits by focusing on the part of the circuit outside of the battery. That is, we will focus on current that passes from one battery terminal, through the circuit, and back to the other terminal. At the end of the chapter we will review and extend our previous discussions about what goes on inside devices like batteries and generators.

Consider the simple *single-loop* circuit of Fig. 27-2 consisting of an ideal battery, a resistor, R, and two ideal connecting wires. Unless otherwise indicated, we assume that wires in circuits have negligible resistance. Their function, then, is merely to provide pathways along which charge carriers can move. Through use of stored chemical energy (a form of internal potential energy), the battery keeps one of its terminals (called the positive terminal and often labeled +) at a higher electric potential than the other terminal (called the negative terminal and labeled −).

The mobile negative charge carriers in the circuit wires move preferentially toward the positive terminal and away from the negative terminal. As a result, for the circuit shown in Fig. 27-2, we have a net flow of negative charge in a counterclockwise direction. In Chapter 26, we discussed the fact that a flow of negative electrons in one direction is macroscopically indistinguishable from a flow of positive charges in the other direction. For historical reasons we continue the practice established in that chapter of working with current as if the charge carriers are positive.

The direction of the conventional current in the circuit shown in Fig. 27-2 is noted with arrows that are labeled i. Unless otherwise noted, we will continue the practice of using conventional (positive) current in our analysis of electric circuits. We will reach the same conclusions about the fundamental behavior of circuits as we would if we had used electron currents.

To begin learning how to calculate currents in circuits, let's start with the ideal circuit depicted in Fig. 27-2. We have marked the points just before and after each element with the letters a, b, c, and d. Let's start at point a and proceed around the circuit in either direction, adding any changes in potential we encounter. Once we return to our starting point, we must also have returned to our starting potential. In words, the potential energy change per unit of charge traveling through the battery plus the potential energy change of the charge traveling through the wires and the resistors must be zero. This can be denoted as

$$\Delta V_{a\to b} + \Delta V_{b\to c} + \Delta V_{c\to d} + \Delta V_{d\to a} = \Delta V_{a\to a} = 0\text{ V}.$$

For our simple circuit in Fig. 27-2 the charges gain potential while traveling from a to b due to the energy boost from the battery so that $\Delta V_{a\to b} = V_b - V_a = \Delta V_B$. The charges then flow freely from b to c through the first segment of the ideal conductor with no potential loss since the wire has a negligible resistance. Then the charges flow through the resistor, R. Finally, they flow back to point a, through another length of ideal wire.

$$\Delta V_B + \Delta V_{c\to d} = 0\text{ V},$$

where ΔV_B represents a positive change in potential per unit charge as charges proceed from point a to point b by moving through the battery. Recall that if our ohmic resistor has a fixed value R, then we noted in Eq. 26-7 that $\Delta V = iR$ where i is the current passing through the circuit. However, Eq. 26-7 didn't specify whether the ΔV refers to $\Delta V_{c\to d}$ or $\Delta V_{d\to c} = -\Delta V_{c\to d}$. It is clear from the context that if we proceed through the loop from c to d, $\Delta V_{c\to d}$ must be negative so it will cancel the ΔV_{B}, which we know is positive. This tells us the following about the mathematics of finding the potential difference across a resistor:

$$\Delta V_{d\to c} = iR \quad \text{and} \quad \Delta V_{c\to d} = -iR.$$

In other words, charges lose potential as they travel through a resistor. This makes sense physically because resistors give off energy in the form of heat and light. So our battery acts as a pump to increase the potential energy of a charge and the charge loses potential energy in passing through a resistive device.

This can be summarized as the loop rule.

Loop rule: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

This is often referred to as *Kirchhoff's loop rule* (or *Kirchhoff's voltage law*), after German physicist Gustav Robert Kirchhoff. This rule is analogous to what happens when you hike around a mountain. If you start from any point on a mountain and return to the same point after walking around it, the algebraic sum of the changes in elevation you encounter must be zero. Thus, you end up at the same gravitational potential as you had before you started. Although we developed this rule through consideration of a single-loop circuit, it also holds for any complete loop in a *multiloop* circuit, no matter how complicated.

In Fig. 27-2, we will start at point a, whose potential is V_a, and mentally walk clockwise around the circuit until we are back at a, keeping track of potential changes as we move. (Our starting point is at the low-potential terminal of the battery—the negative terminal.) The potential difference between the battery terminals is equal to ΔV_{B}. When we pass through the battery from the low to high-potential terminal, the change in potential is positive.

As we walk along the top wire to the top end of the resistor, there is no potential change because the wire has negligible resistance; it is at the same potential as the high-potential terminal of the battery. So too is the top end of the resistor. When we pass through the resistor in the direction of the current flow, the potential decreases by an amount equal to $-iR$. We know the potential decreases because we are moving from the higher potential terminal of the resistor to the lower potential terminal.

For a walk around a single-loop circuit of total resistance R in *the direction of the current* our loop rule gives us

$$\Delta V_{\mathrm{B}} - iR = 0\ \mathrm{V}.$$

Solving this equation for i gives us

$$i = \frac{\Delta V_{\mathrm{B}}}{R} \qquad \text{(single-loop circuit).} \tag{27-1}$$

If we apply the loop rule to a complete walk around a single-loop circuit of total resistance R *against the direction of current*, the rule gives us

$$-\Delta V_{\mathrm{B}} + iR = 0\ \mathrm{V},$$

and we again find that

$$i = \frac{\Delta V_{\mathrm{B}}}{R} \qquad \text{(single-loop circuit).}$$

Thus, you may mentally circle a loop in either direction to apply the loop rule.

To prepare for circuits more complex than Fig. 27-2, let us summarize two rules for finding potential differences as we move around a chosen loop:

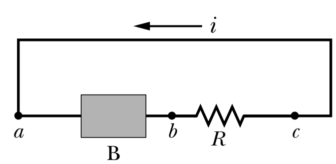
> **RESISTANCE RULE:** For a move through a resistor in the direction of the conventional current, the change in potential is $-iR$; in the opposite direction of current flow it is $+iR$.

> **POTENTIAL RULE:** For a move through a source of potential difference from low potential (for example, the negative terminal on a battery denoted a) to high potential (for example, the positive terminal on a battery denoted b) the change in potential is positive and given by $V_b - V_a = \Delta V_B$; in the opposite direction it is negative and given by $V_a - V_b = -\Delta V_B$.

What happens to the amount of current as it passes through a resistor? Is the current going into the resistor the same as the current coming out of the resistor? Or does a resistor (for example, a lightbulb) "use up" current? Recall that in Fig. 26-5 we depicted observations involving batteries and bulbs that clearly showed current is constant throughout a single loop circuit when resistors are connected in series. You can easily replicate these observations using fresh flashlight batteries, copper wires, and 1.5 V bulbs.

READING EXERCISE 27-2: It is asserted above that we can infer that the current flow into and out of a resistor is the same because three lightbulbs connected in series glow equally brightly. Suppose the resistors shown in Fig. 27-3*a* are lightbulbs. Describe the brightness of the third bulb relative to the first and second bulbs under the following assumptions: (a) All the current is used up by the first bulb; (b) most of the current is used up by the first bulb; (c) a small amount of the current was used up by the first bulb. ■

READING EXERCISE 27-3: The figure to the right shows the conventional current i in a single-loop circuit with a battery B and a resistor R (and wires of negligible resistance). At points a, b, and c, rank (a) the amount of the current and (b) the electric potential, greatest first.



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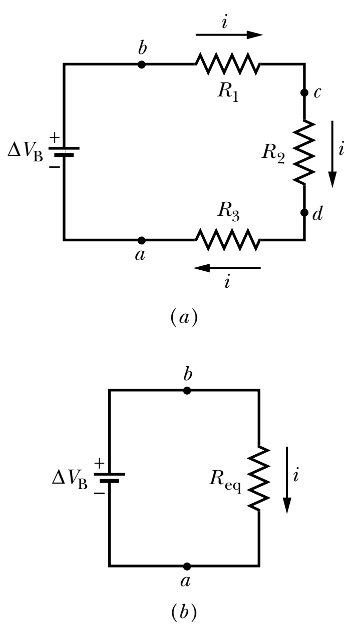

FIGURE 27-3 ■ (*a*) Three resistors are connected in series between points a and b. (*b*) An equivalent circuit, with the three resistors replaced with their equivalent resistance R_{eq}.

27-3 Series Resistance

We now turn our attention to more complicated single-loop circuits. Figure 27-3*a* shows three resistors connected in series to an ideal battery with potential difference ΔV_B between its terminals. Note that the three resistors are connected one after another between b and c, c and d, and d and a. Also an ideal battery maintains a potential difference across the series of resistors (between points a and b). If we apply the loop rule for charges moving in the direction of conventional current from point a at the negative terminal of the battery and proceeding through the loop until we encounter point a, again we get

$$\Delta V_{a\to b} + \Delta V_{b\to c} + \Delta V_{c\to d} + \Delta V_{d\to a} = 0\ V. \tag{27-2}$$

Because we know that current is not used up by a resistor, we know the current flowing through the loop is the same everywhere, and so the current through each resistor must be the same. We also assume there is no potential difference along any segment of wire. If we consider the three resistors separately, applying the loop rule in the same manner (starting at the positive terminal of the battery and proceeding through the loop in the direction of conventional current) gives

$$\Delta V_B + (-iR_1) + (-iR_2) + (-iR_3) = 0\ \text{V}.$$

By rearranging terms in the equation above we get

$$\Delta V_B - i(R_1 + R_2 + R_3) = 0\ \text{V}, \tag{27-3}$$

and defining an equivalent resistance as $R_{eq} = R_1 + R_2 + R_3$ we find that Eq. 27-3 reduces to the same form as Eq. 27-1 with the equivalent resistance playing the role of the resistance in a circuit that has only one resistance. This is illustrated in Fig. 27-3.

Equating these two expressions tells us two things. First, the potential difference across the whole series of resistors is equal to the sum of the potential differences across the three resistors. Second, the potential difference across the whole series of resistors is equal to the potential difference across our ideal battery. Figure 27-3*b* shows the equivalent resistance, with a new resistor R_{eq}, that can replace the three resistors of Fig. 27-3*a*.

The result $R_{eq} = R_1 + R_2 + R_3$ is not surprising because it is compatible with the experimental findings we presented in Section 26-6: the resistance of a length of wire is directly proportional to its length (Eq. 26-8). Imagine three different carbon resistors like those depicted in the previous chapter (Fig. 26-21). Suppose these resistors are connected by ideal conductors (with almost no resistivity) having the same graphite material in their centers each with the same cross-sectional area. Giving the resistors different values of resistance would involve having the centers of the resistors be three different lengths. We would then expect the total resistance to be proportional to the sum of the three lengths of the resistors' graphite centers.

Obviously, we can extend our method of finding the equivalent resistance from 3 to N resistors by expanding Eq. 27-3 into the equation

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N = \sum_{j=1}^{N} R_j \quad (N\text{ resistors in series}). \tag{27-4}$$

Note that when resistors are in series, their equivalent resistance is always *greater* than that of any of the individual resistors. Also, the current moving through resistors wired in series can move along only a single route. If there are additional routes so the currents in different resistors are different, the resistors are not connected in series.

In general:

> If N resistors in series were covered by a box, the resistors could be replaced by a single equivalent resistor with a value. $R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N$. Someone making measurements outside the box could not tell whether there is a single equivalent resistor or a series of individual resistors.

In short, we conclude that if we replace a series of resistors with a single equivalent resistor, the new circuit will have the same overall potential differences and currents as the original one (so long as we don't measure potential drops between the resistors wired in series).

More on Ammeters

Analog ammeters work by measuring the torque exerted by magnetic forces on a current-carrying wire. We discuss more about their operation in Chapter 29 on magnetic

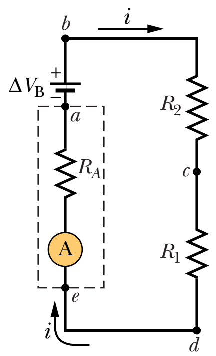

FIGURE 27-4 ■ This depicts how an ammeter can be inserted into a series circuit to measure the current. The third resistor represents the small resistance R_A of the ammeter itself.

fields. However, we continue our discussion of these devices from Chapter 26 and consider some important attributes the ammeter must have.

Recall from Chapter 26 that to measure the current in a wire, you are to break or cut the wire and insert the ammeter in series with an arm of the circuit so the current to be measured passes through the meter. (In Fig. 27-4, ammeter A is set up to measure current i).

When measuring the current in a circuit (or anything else for that matter) it is imperative that the measurement tool does not significantly change the quantity you are trying to measure. Hence, it is essential that the resistance R_A of the ammeter be very small compared to other resistances in the circuit. Otherwise, the presence of the meter will significantly change the current flow in the circuit, and measured current will be an inaccurate representation of the true current.

READING EXERCISE 27-4: In Fig. 27-3*a*, if $R_1 > R_2 > R_3$, rank the three resistances according to (a) the current through them and (b) the potential difference across them, greatest first. ■

READING EXERCISE 27-5: Consider an ammeter inserted into the circuit shown in Fig. 27-4. Compare the amount of current flowing through R_1 under the following three conditions: (a) without the ammeter inserted, (b) when the ammeter has a resistance much less than the equivalent resistance of $R_1 + R_2$, and (c) when the ammeter has a resistance equal to the equivalent resistance of $R_1 + R_2$. Explain your reasoning. Discuss the implications of your result on designing an ammeter. ■

27-4 Multiloop Circuits

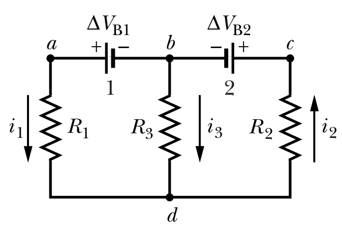

FIGURE 27-5 ■ A multiloop circuit consisting of three branches: left-hand branch *bad*, right-hand branch *bcd*, and central branch *bd*. The circuit has three loops we could choose to follow: left-hand loop *badb*, right-hand loop *bcdb*, and big loop *badcb*.

Figure 27-5 shows a circuit containing more than one loop. There are two points (*b* and *d*) at which the current branches split off or come together. We call such branching points **junctions.** For the circuit shown in Fig. 27-5, we would say there are two junctions, at *b* and *d*, and there are three *branches* connecting these junctions. The branches are the left branch (*bad*), the right branch (*bcd*), and the central branch (*bd*).

What are the currents in the three branches? We arbitrarily label the currents, using a different subscript for each branch. Because current is not used up and there are no additional branching points, current i_1 has the same value everywhere in branch *bad*, i_2 has the same value everywhere in branch *bcd*, and i_3 is the current through branch *bd*. The directions of the currents are assigned arbitrarily.

Consider junction *d* for a moment: charge comes into that junction via incoming currents i_1 and i_3, and it leaves via outgoing current i_2. Because charged particles neither accumulate nor disperse at the junction, the total incoming charge must be equal to the total outgoing charge. Hence, through conservation of charge arguments, we conclude that the total current coming into junction *d* must equal the total current leaving junction *d*,

$$i_{in} = i_{out},$$

or

$$i_1 + i_3 = i_2. \tag{27-5}$$

You can easily check that application of this condition to junction *b* leads to exactly the same equation. This expression for the current in branch 2 thus suggests a general principle:

> **JUNCTION RULE:** The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

This rule is often called *Kirchhoff's junction rule* (or *Kirchhoff's current law*). It is simply a statement of the conservation of charge for a steady flow of charge—there is neither a buildup nor a depletion of charge at a junction. Thus, our basic tools for solving complex circuits are the *loop rule* (based on the conservation of energy) and the *junction rule* (based on the conservation of charge).

The relationship between i_1, i_2, and i_3 above is a single equation involving three unknowns. To solve the circuit completely (that is, to find all three currents), we need two more equations involving those same unknowns. We obtain them by applying the loop rule twice. In the circuit of Fig. 27-5, we have three loops from which to choose: the left-hand loop (*badb*), the right-hand loop (*bcdb*), and the big loop (*badcb*). Which two loops we choose turns out not to matter so long as we manage to pass through all the circuit elements at least once. For now, let's choose the left-hand loop and the right-hand loop.

If we traverse the left-hand loop in a counterclockwise direction from point b, the loop rule gives us

$$\Delta V_{B1} - i_1R_1 + i_3R_3 = 0\text{ V}, \tag{27-6}$$

where ΔV_{B1} is the difference in potential between the terminals of battery 1. If we traverse the right-hand loop in a counterclockwise direction from point b, the loop rule gives us an equation involving battery 2,

$$-i_3R_3 - i_2R_2 - \Delta V_{B2} = 0\text{ V}. \tag{27-7}$$

We now have three equations (Eqs. 27-5, 27-6, and 27-7) containing the three unknown currents, and they can be solved by a variety of mathematical techniques.

If we had applied the loop rule to the big loop, we would have obtained (moving counterclockwise from b) the equation

$$\Delta V_{B1} - i_1R_1 - i_2R_2 - \Delta V_{B2} = 0\text{ V}.$$

This equation may look like fresh information, but in fact it is only the sum of Eqs. 27-6 and 27-7. (It would, however, yield the proper results when used with Eq. 27-5 and either 27-6 or 27-7.)

It is important to note that the assumed direction of the currents in a branch of the circuit do not have to be correct to get a correct solution. We must only keep track of the assumptions we have made. If in solving the resulting algebraic expressions we find that one of our currents turns out to have a negative value, then (because of the negative value) we know we made a wrong assumption about the direction of the current in that branch of the circuit.

In general, the total number of equations needed will be equal to the total number of independent loops in the circuit. The number of independent loops is simply the minimum number of loops needed to cover every branch in the circuit. Although some branches could be covered twice, every circuit element would be "covered" at least once. For example, we need at least two equations to cover all the loops in the circuit in Fig. 27-5 and at least three equations to cover all the loops in the more complex circuit in Fig. 27-6.

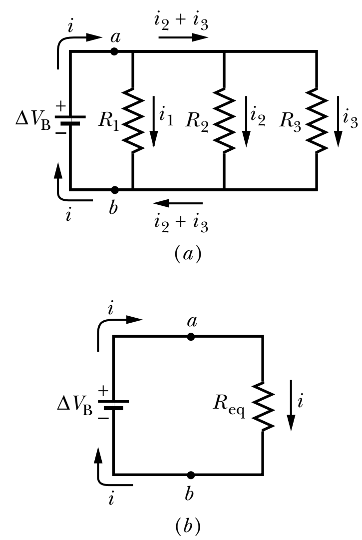

FIGURE 27-6 ■ (a) Three resistors connected in parallel across points a and b. (b) An equivalent circuit, with the three resistors replaced with their equivalent resistance R_{eq}.

27-5 Parallel Resistance

Figure 27-6*a* shows three resistances connected by branching junctions. Resistances that are parts of separate loops like those in Fig. 27-6*a* are said to be connected *in parallel* to the battery. Resistors connected "in parallel" are directly wired together on one side and directly wired together on the other side, and a potential difference ΔV

is applied across the pair of connected sides. Thus, the resistances have the same potential difference ΔV across them, producing a current through each. Because we are assuming ideal wires, there is no potential difference across the wires. Therefore, the potential across the top branch of the circuit is constant everywhere equal to the potential at the positive pole of the battery, and the potential across the bottom branch of the circuit is constant everywhere equal to the potential at the negative pole of the battery. In general,

> When a potential difference ΔV is applied across resistances connected in parallel, each resistor has the same potential difference ΔV across it.

Notice that we have again labeled the currents in each of the branches i_1, i_2, and i_3. We have discussed the way in which the current into a junction is equal to the current out of the junction. We have not yet discussed in what proportions currents divide when there is a branch (a choice of path) in a circuit. Are all three currents i_1, i_2, and i_3 equal? If not, which of these currents is largest? The answer to this question becomes clear when we write out the expressions for current through each of the resistors in Fig. 27-6 using the potential rule for loops. For the case pictured here, we have

$$i_1 = \frac{\Delta V}{R_1}, \quad i_2 = \frac{\Delta V}{R_2}, \quad \text{and} \quad i_3 = \frac{\Delta V}{R_3}. \tag{27-8}$$

Since each resistor is connected so it has the same potential difference across it, it is straightforward to see how the sizes of the currents compare to each other. If the resistances are all equal, the current through each is the same. However, if the three resistances are not equal, more current flows through the smaller resistances. This outcome is consistent with what we might predict based solely on an understanding that a resistor is just a device that resists the flow of current.

If we want to simplify how we think about a circuit that has resistors wired in parallel (like that shown in Fig. 27-6*a*), we can treat the three resistors in parallel as if they have been replaced by a single equivalent resistor R_{eq}. Figure 27-6*b* shows the three parallel resistances replaced with an equivalent resistance R_{eq}. The applied potential difference ΔV_B is maintained by a battery. We can see from this figure that the potential difference across the equivalent resistance would have to be the same as the potential difference applied across each of the original resistors. Furthermore, the equivalent resistor would have to have the same total current ($i_1 + i_2 + i_3$) through it as the original three resistors.

> Resistances connected in parallel can be replaced with an equivalent resistance R_{eq}. If the equivalent resistance has the same potential difference applied across it, then the current through it will equal the sum of currents flowing through the original resistors.

To derive an expression for R_{eq} in Fig. 27-6*b*, we first write the current in each of the resistors in Fig. 27-6*a* as

$$i_1 = \frac{\Delta V}{R_1}, \quad i_2 = \frac{\Delta V}{R_2}, \quad \text{and} \quad i_3 = \frac{\Delta V}{R_3}, \qquad \text{(Eq. 27-8)}$$

where ΔV is the potential difference between *a* and *b*. If we apply the junction rule at point *a* in Fig. 27-6*a* and then substitute these values, we find

$$i = i_1 + i_2 + i_3 = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right). \tag{27-9}$$

If we instead consider the parallel combination with the equivalent resistance R_{eq} (Fig. 27-6*b*), we have

$$i = \frac{\Delta V}{R_{eq}} = \Delta V\left(\frac{1}{R_{eq}}\right). \tag{27-10}$$

Comparing the two equations above leads to

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \tag{27-11}$$

The result $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$ is not surprising because it is compatible with the experimental findings we presented in Section 26-6: the resistance of a length of wire is inversely proportional to its cross-sectional area (Eq. 26-8). To see this connection, imagine three different carbon resistors like those depicted in the last chapter (Fig. 26-21) connected in parallel. Then giving them different values of resistance would involve having the centers of the resistors have three different cross-sectional areas. Because the resistors are connected in parallel, we would then expect the total cross-sectional area to be the sum of the three cross-sectional areas of the resistors' graphite centers so $A_{eq} = A_1 + A_2 + A_3$. Since the cross-sectional area and resistance are inversely proportional, we get $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$.

Extending Eq. 27-11 to the case of *n* resistors, we have

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} \quad (n \text{ resistors in parallel}). \tag{27-12}$$

Since we often deal with the case of two resistors in parallel, it is worth it for us to consider this case a bit more. For the case of two resistors, the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

With a bit of algebra, this becomes

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad (2 \text{ resistors in parallel}). \tag{27-13}$$

If you accidentally took the equivalent resistance to be the sum divided by the product, you would notice at once that this result would be dimensionally incorrect.

Note that when two or more resistors are connected in parallel, the equivalent resistance is smaller than any of the combining resistances.

More on the Voltmeter

Recall from our discussion in Chapter 26 that a meter used to measure potential differences is called a *voltmeter*. To measure the potential difference between any two points in the circuit, the voltmeter terminals are connected across those points, without breaking or cutting the wire. In Fig. 27-7, voltmeter V is set up to measure the potential difference across a resistor R_1. The voltmeter is inserted in parallel to R_1 by connecting its terminals to points *d* and *e* in the circuit.

To prevent the voltmeter from affecting a measurement, it is essential that the resistance R_V of a voltmeter be *very large* compared to the resistance of the circuit element across which the voltmeter is connected. Otherwise, the meter becomes an important circuit element by drawing a significant current through itself. This change

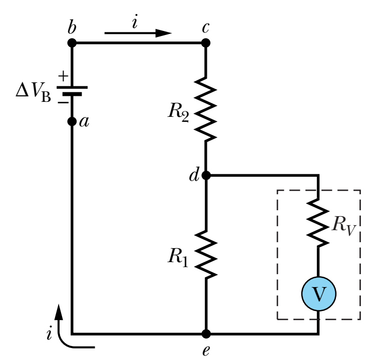

FIGURE 27-7 ■ A single-loop circuit, showing how to connect a voltmeter (V). The third resistor R_V represents the resistance of the voltmeter itself. We assume that R_V is very large compared to R_1 and R_2.

in current flow can alter the potential difference to be measured. On the other hand, even if the potential difference across the voltmeter is large, if a very small current flows through the voltmeter, the flow of current through R_1 will not change very much.

READING EXERCISE 27-6: A battery, with potential ΔV_B across it, is connected to a combination of two identical resistors and a current i flows through the battery. What is the potential difference across and the current through either resistor if the resistors are (a) in series, and (b) in parallel?

READING EXERCISE 27-7: Consider the voltmeter inserted into the circuit shown in Fig. 27-7. Describe what would happen if the voltmeter has a resistance $R_V \ll R_1$. How would this affect the potential difference measured across the resistor R_1? Describe what would happen if the voltmeter has a resistance $R_V \gg R_1$. How would this affect the potential difference measured across the resistor R_1? Which case would give the most "accurate" measure of the potential difference across the resistor when the voltmeter is not a part of the circuit?

READING EXERCISE 27-8: Suppose the resistors in Fig. 27-6*a* are all identical lightbulbs. Rank the brightness of the three bulbs. Compare the brightness of each of the bulbs to the brightness of one of the bulbs alone connected to the same battery.

TOUCHSTONE EXAMPLE 27-1: One Battery and Four Resistances

Figure 27-8*a* shows a multiloop circuit containing one ideal battery and four resistances with the following values:

$$R_1 = 20\ \Omega,\quad R_2 = 20\ \Omega,\quad \Delta V_B = 12\ \text{V},$$
$$R_3 = 30\ \Omega,\quad \text{and}\quad R_4 = 8.0\ \Omega.$$

(a) What is the current through the battery?

SOLUTION First note that the current through the battery must also be the current through R_1. Thus, one **Key Idea** here is that we might find that current by applying the loop rule to a loop that includes R_1 because the current would be included in the potential difference across R_1. Either the left-hand loop or the big loop will do. Noting that the potential difference arrow of the battery points upward so the current the battery supplies is clockwise, we might apply the loop rule to the left-hand loop, clockwise from point a. With i being the current through the battery, we would get

$$+\Delta V_B - iR_1 - iR_2 - iR_4 = 0\ \text{V}. \qquad \text{(incorrect)}$$

However, this equation is incorrect because it assumes that R_1, R_2, and R_4 all have the same current i. Resistances R_1 and R_4 do have the same current, because the current passing through R_4 must pass through the battery and then through R_1 with no change in value. However, that current splits at junction point b—only part passes through R_2, and the rest through R_3.

To distinguish the several currents in the circuit, we must label them individually as in Fig. 27-8*b*. Then, circling clockwise from a, we can write the loop rule for the left-hand loop as

$$+\Delta V_B - i_1R_1 - i_2R_2 - i_1R_4 = 0\ \text{V}.$$

Unfortunately, this equation contains two unknowns, i_1 and i_2; we need at least one more equation to find them.

A second **Key Idea** is that an easier option is to simplify the circuit of Fig. 27-8*b* by finding equivalent resistances. Note carefully that R_1 and R_2 are *not* in series and thus cannot be replaced with an equivalent resistance. However, R_2 and R_3 are in parallel, so we can use either Eq. 27-12 or Eq. 27-13 to find their equivalent resistance R_{23}. From the latter,

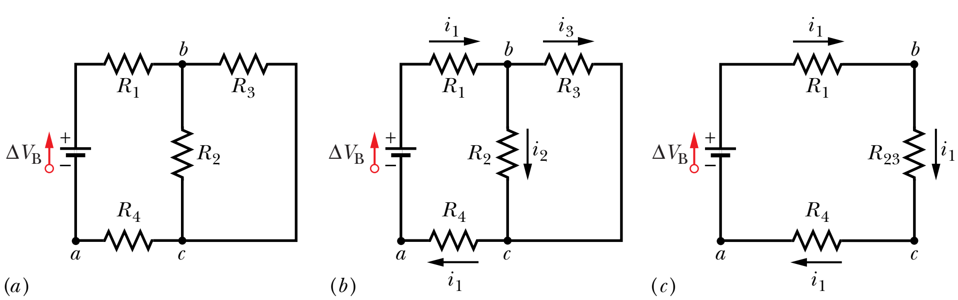

FIGURE 27-8 (*a*) A multiloop circuit with an ideal battery of potential difference ΔV_B and four resistances. (*b*) Assumed currents through the resistances. (*c*) A simplification of the circuit, with resistances R_2 and R_3 replaced with their equivalent resistance R_{23}. The current through R_{23} is equal to that through R_1 and R_4.

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{(20\ \Omega)(30\ \Omega)}{50\ \Omega} = 12\ \Omega.$$

We can now redraw the circuit as in Fig. 27-8*c*; note that the current through R_{23} must be i_1 because charge that moves through R_1 and R_4 must also move through R_{23}. For this simple one-loop circuit, the loop rule (applied clockwise from point *a*) yields

$$+\Delta V_B - i_1 R_1 - i_1 R_{23} - i_1 R_4 = 0\ \text{V}.$$

Substituting the given data, we find

$$12\ \text{V} - i_1(20\ \Omega) - i_1(12\ \Omega) - i_1(8.0\ \Omega) = 0\ \text{V},$$

which gives us

$$i_1 = \frac{12\ \text{V}}{40\ \Omega} = 0.30\ \text{A}. \qquad \text{(Answer)}$$

(b) What is the current i_2 through R_2?

SOLUTION ■ One **Key Idea** here is that we must work backward from the equivalent circuit of Fig. 27-8*c*, where R_{23} has replaced the parallel resistances R_2 and R_3. A second **Key Idea** is that, because R_2 and R_3 are in parallel, they both have the same potential difference across them as their equivalent R_{23}. We know the current through R_{23} is $i_1 = 0.30$ A. Thus, we can use Eq. 26-5 $R = \Delta V/i$ to find the potential difference ΔV_{23} across R_{23}:

$$\Delta V_{23} = i_1 R_{23} = (0.30\ \text{A})(12\ \Omega) = 3.6\ \text{V}.$$

The potential difference across R_2 is thus 3.6 V, so the current i_2 in R_2 must be, by Eq. 26-5,

$$i_2 = \frac{\Delta V_2}{R_2} = \frac{3.6\ \text{V}}{20\ \Omega} = 0.18\ \text{A}. \qquad \text{(Answer)}$$

(c) What is the current i_3 through R_3?

SOLUTION ■ We can answer by using the same technique as in (b), or we can use this **Key Idea**: The junction rule tells us that at point *b* in Fig. 27-8*b*, the incoming current i_1 and the outgoing currents i_2 and i_3 are related by

$$i_1 = i_2 + i_3.$$

This gives us

$$i_3 = i_1 - i_2 = 0.30\ \text{A} - 0.18\ \text{A} = 0.12\ \text{A}. \qquad \text{(Answer)}$$

TOUCHSTONE EXAMPLE 27-2: Three Batteries and Five Resistances

Figure 27-9 shows a circuit with three ideal batteries in it. Two of these batteries labeled ΔV_{B2} are identical. The circuit elements have the following values:

$$\Delta V_{B1} = 3.0\ \text{V}, \quad \Delta V_{B2} = 6.0\ \text{V}, \quad R_1 = 2.0\ \Omega, \quad R_2 = 4.0\ \Omega.$$

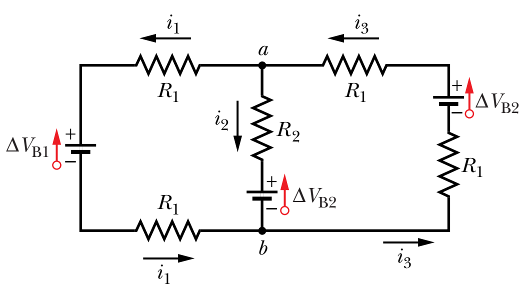

FIGURE 27-9 ■ A multiloop circuit with three ideal batteries and five resistances.

Find the amount and direction of the current in each of the three branches.

SOLUTION ■ It is not worthwhile to try to simplify this circuit, because no two resistors are in parallel, and the resistors that are in series (those in the right branch or those in the left branch) present no problem. So our **Key Idea** is to apply the junction and loop rules to this circuit.

Using arbitrarily chosen directions for the currents as shown in Fig. 27-9, we apply the junction rule at point *a* by writing

$$i_3 = i_1 + i_2. \qquad (27\text{-}14)$$

An application of the junction rule at junction *b* gives only the same equation, so we next apply the loop rule to any two of the three loops of the circuit. We first arbitrarily choose the left-hand loop, arbitrarily start at point *a*, and arbitrarily traverse the loop in the counterclockwise direction, obtaining

$$-i_1 R_1 - \Delta V_{B1} - i_1 R_1 + \Delta V_{B2} + i_2 R_2 = 0\ \text{V}.$$

Substituting the given data and simplifying yield

$$i_1(4.0\ \Omega) - i_2(4.0\ \Omega) = 3.0\ \text{V}. \qquad (27\text{-}15)$$

For our second application of the loop rule, we arbitrarily choose to traverse the right-hand loop clockwise from point *a*, finding

$$+i_3 R_1 - \Delta V_{B2} + i_3 R_1 + \Delta V_{B2} + i_2 R_2 = 0\ \text{V}.$$

Substituting the given data and simplifying yield

$$i_2(4.0\ \Omega) + i_3(4.0\ \Omega) = 0\ \text{V}. \qquad (27\text{-}16)$$

Using Eq. 27-14 to eliminate i_3 from Eq. 27-16 and simplifying give us

$$i_1(4.0\ \Omega) + i_2(8.0\ \Omega) = 0\ \text{V}. \qquad (27\text{-}17)$$

We now have a system of two equations (Eqs. 27-15 and 27-17) in two unknowns (i_1 and i_2) to solve either by hand (which is easy enough here) or with a math computer software package. (One solution technique is Cramer's rule, given in Appendix E.) We find

$$i_2 = -0.25\ \text{A}.$$

(The minus sign signals that our arbitrary choice of direction for i_2 in Fig. 27-9 is wrong; i_2 should point up through ΔV_{B2} and R_2.) Substituting $i_2 = -0.25$ A into Eq. 27-17 and solving for i_1 then give us

$$i_1 = 0.50\ \text{A}. \qquad \text{(Answer)}$$

With Eq. 27-14 we then find that

$$i_3 = i_1 + i_2 = 0.25\ \text{A}. \qquad \text{(Answer)}$$

The positive answers we obtained for i_1 and i_3 signal that our choices of directions for these currents are correct. We can now correct the direction for i_2 and write its amount as

$$i_2 = 0.25\ \text{A}. \qquad \text{(Answer)}$$

27-6 Batteries and Energy

So far we have discussed ideal batteries that can be characterized as maintaining a constant potential difference between their terminals no matter what current is flowing through them. Also, we have concentrated on analyzing what happens in the part of the circuit that lies outside the battery. In this section we consider more about what goes on inside batteries and how real, not so ideal, batteries behave.

The amazing thing about a battery is that positive charge carriers enter with a low potential energy and other carriers emerge from the battery at a higher potential. Energy transformations inside a battery enable charges to overcome the forces exerted on them by the electric field inside the battery. Positive carriers seem to move opposite to the battery's electric field, whereas negative charge carriers move with it. There must be some other force present inside an energy-providing device enabling charges to swim upstream against electrical forces. The outdated term given to this "force" is electromotive force. Its abbreviation, which we still use today, is emf. How is this "force" defined? Where does it come from in a typical battery?

We define the emf, $\mathscr{E}$, of a battery in terms of the work done per unit charge on charges flowing into it:

$$\mathscr{E} \equiv \frac{dW}{dq} \qquad \text{(definition of electromotive force).} \qquad (27\text{-}18)$$

In words, the battery emf is the work per unit charge it does to move charge from one terminal to the other. The SI unit for emf is the joule/coulomb. In Chapter 25 we defined one joule/coulomb as the *volt*. There must be some source of energy within a battery, enabling it to do work on the charges. The energy source may be chemical, as in a battery (or a fuel cell). Temperature differences may supply the energy, as in a thermopile; or the Sun may supply it, as in a solar cell. As you can see, the term electromotive force is very misleading since it is not a force at all, but has the same units as electrostatic potential (energy per unit charge). Furthermore emf is a scalar quantity and is not a vector quantity like a force is.

When a battery is connected to a circuit, it transfers energy to the charge carriers passing through it. Let's look at one example of how chemical action can do this. For this purpose we will consider the chemical reactions that take place inside one cell of a lead acid battery used in most automobiles. A lead acid battery consists of several cells wired together in series. Each cell has two metal plates surrounded by a liquid bath of chemicals. In a lead-acid cell, the negative plate is made of pure lead, and the positive plate is made of lead-oxide. These plates are immersed in sulfuric acid mixed with water. The acid dissociates in the water into hydronium ions (H_3O^+) and bisulfate ions ($HSO_4{}^-$). This is shown in Fig. 27-10. Both the lead and lead oxide can react

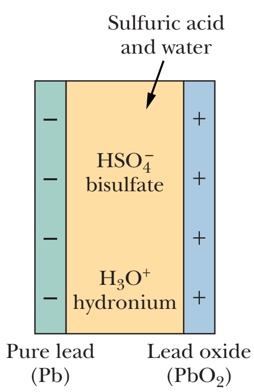

FIGURE 27-10 ■ The chemical constituents of the lead acid battery.

with the bisulfate ions as follows:

$$Pb + HSO_4^- + H_2O \rightarrow PbSO_4 + H_3O^+ + 2e^-$$
$$PbO_2 + HSO_4^- + 3H_3O^+ + 2e^- \rightarrow PbSO_4 + 4H_2O$$

The two electrons produced on the pure lead plate pile up on it. The second reaction removes the two electrons it needs from the lead oxide plate. Thus, each time the pair of reactions occur, electrons are added to the negative plate and removed from the positive plate. If the cell were not connected to a circuit, the reactions would stop when the charge difference gets so large that the energy needed to put more charges on the plates is greater than the energy released by the reactions. If the battery is connected to an external circuit, then as the charges flow through the circuit, they are removed from one plate and put back on the other; the process can keep going until all the sulfuric acid (HSO_4) is consumed.

Note that when we talk about a battery as a charge pump, this is somewhat misleading because the electrons removed by the chemical reaction at one battery terminal (plate) are not the same electrons released at the other terminal.

There are hundreds of different types of chemical batteries. The lead-acid battery action described here simply serves as an example of how chemical reactions can cause charge separation in a battery.

27-7 Internal Resistance and Power

In our evaluation of circuits up to this point, we have assumed the current passes through the battery (or other emf source) without encountering any resistance within it. We call such a battery or other emf device "ideal."

An **ideal emf device** is one that lacks any resistance to the movement of charge through it. The potential difference between the terminals of an *ideal* emf device is equal to the emf of the device. For example, an ideal battery with an emf of 12.0 V has a potential difference of 12.0 V between its terminals. Very fresh alkaline batteries are nearly ideal.

A **real emf device** has internal resistance to the movement of charge through it. For a real emf device (for example, a real battery), the only situation for which the potential difference between its terminals is equal to its emf is when the device is not connected to a circuit, and thus does not have current through it. However, when the device has current through it, the potential difference between its terminals differs from its emf.

Figure 27-11*a* shows circuit elements that describe the behavior of a real battery, with internal resistance r, wired to an external resistor of resistance R. The internal

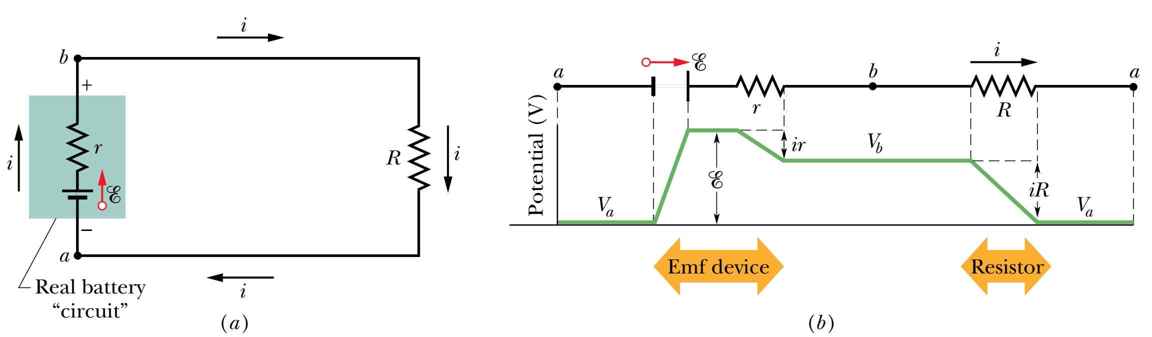

FIGURE 27-11 ■ (*a*) A single-loop circuit containing a real battery having internal resistance r and emf $\mathscr{E}$. (*b*) The same circuit, now spread out in a line. The potentials encountered in traversing the circuit clockwise from a are also shown. The potential V_a is arbitrarily assigned a value of zero, and other potentials in the circuit are graphed relative to V_a.

resistance of the battery is the electrical resistance of the conducting materials of the battery and thus is an unavoidable feature of any real battery. However, as an illustration, a real battery is depicted in Fig. 27-11*b* as if it could be separated into an ideal battery with potential difference $\mathscr{E}$ between its terminals and a resistor of resistance r. The order in which the symbols for these separated parts are drawn does not matter.

If we apply the potential (loop) rule, proceeding clockwise and beginning at point a, the *changes* in potential give us

$$\Delta V_{a \to b} + \Delta V_R = 0\,\mathrm{V},$$

or

$$\mathscr{E} + \Delta V_{\text{internal resistance}} + \Delta V_R = 0\,\mathrm{V}. \tag{27-19}$$

It is customary to keep track of potential differences as if the charge carriers are positive. Thus, we go through both resistances in the direction of the *conventional* current (defined in the previous chapter as the direction of flow we would find if the charge carriers were positive instead of negative):

$$\mathscr{E} - ir - iR = 0\,\mathrm{V}. \tag{27-20}$$

Solving for the current, we find

$$i = \frac{\mathscr{E}}{R + r}. \tag{27-21}$$

Note that this equation reduces to Eq. 27-1 if the battery is ideal so that $r = 0\,\Omega$.

Figure 27-11*b* shows graphically the changes in electric potential around the circuit. (To better link Fig. 27-11*b* with the *closed circuit* in Fig. 27-11*a*, imagine curling the graph into a cylinder with point a at the left overlapping point a at the right.) Note how traversing the circuit is like walking up and down a (potential) mountain and returning to your starting point—you also return to the starting elevation.

In this book, if a battery is not described as real or if no internal resistance is indicated, you can assume for simplicity that it is ideal.

Implications of Internal Resistance in Real EMF Devices

To understand the implications of internal resistance in emf devices for real circuits, let's try to make our understanding a bit more quantitative. To start with, let's see how $\Delta V_\mathrm{B} = \Delta V_{a \to b} = V_b - V_a$, the potential difference across the battery terminals in Fig. 27-11, is affected by the existence of an internal resistance in the battery. To calculate $V_b - V_a$, we start at point a and follow the shorter path around to b, which takes us clockwise through the battery. We then have

$$V_a + \mathscr{E} - ir = V_b,$$

or

$$V_b - V_a = \Delta V_\mathrm{B} = \mathscr{E} - ir, \tag{27-22}$$

where r is the internal resistance of the battery and $\mathscr{E}$ is the emf of the battery. This expression tells us the potential difference of the battery is equal to the emf minus the drop in potential associated with internal resistance.

Furthermore, if we refer back to Eq. 27-21,

$$i = \frac{\mathscr{E}}{R + r},$$

and substitute this expression for current (in the circuit shown in Fig. 27-11) into our expression for the potential difference across the battery terminals, we get

$$\Delta V_B = \mathscr{E} - \left(\frac{\mathscr{E}r}{R + r}\right).$$

With some algebra, we get the following generally applicable expression:

$$\Delta V_B = \mathscr{E}\frac{R}{R + r}. \tag{27-23}$$

For example, suppose that in Fig. 27-11, $\mathscr{E} = 12$ V, $R = 10\ \Omega$, and $r = 2.0\ \Omega$. Then the equation above tells us the potential across the battery's terminals is

$$\Delta V_B = (12\text{ V})\frac{10\ \Omega}{10\ \Omega + 2.0\ \Omega} = 10\text{ V}.$$

In "pumping" charge through itself, the battery (via electrochemical reactions) does work per unit charge of $\mathscr{E} = 12$ J/C, or 12 V. However, because of the internal resistance of the battery, it produces a potential difference of only 10 J/C, or 10 V, across its terminals.

If the internal resistance becomes large compared to the overall resistance in the circuit, the available potential difference of the battery, electrical generator, or other emf device will drop significantly. This drop in available potential difference results in a reduction in the amount of current in the circuit. This is especially important to consider when circuits are designed with a low resistance so they will carry a large current.

For example, consider the circuit shown in Fig. 27-6 (three resistors in parallel with a battery) and let $R = 3\ \Omega$ for each resistor. The equivalent resistance in the circuit is $R_{eq} = 1\ \Omega$. If the potential difference source is taken to be an ideal battery (internal resistance $r = 0$), the current in the circuit is

$$i = \frac{\Delta V_B}{R_{eq}} = \frac{12\text{ V}}{1\ \Omega} = 12\text{ A}.$$

The 12 amps are split evenly between each branch (because the resistances are all equal), so each resistor has 4 amps of current flowing through it.

However, if the potential difference source is a real battery with $\mathscr{E} = 12$ V and internal resistance $r = 2.0\ \Omega$, then the available potential difference from the battery is

$$\Delta V_B = (12\text{ V})\frac{1\,\Omega}{1\ \Omega + 2.0\ \Omega} = 4\text{ V}.$$

The total current in the circuit is then

$$i = \frac{\Delta V_B}{R_{eq}} = \frac{4\text{ V}}{1\ \Omega} = 4\text{ A}.$$

This current is still split between each of the branches of the circuit, so for the case of the real battery, the current flowing through each resistor is now only 4/3 amp. In comparison to the 4 amps produced by the ideal battery, one can see how the internal resistance of an emf device can play a significant role in the functioning of real circuits.

Power

When a battery or some other type of emf device does work on the charge carriers to establish a current i, it transfers energy from its source of energy (such as the chemical source in a battery) to the charge carriers. Because a real emf device has an internal resistance r, it also transfers energy to internal thermal energy via resistive dissipation, discussed in Chapter 26. Let us relate these transfers.

The net rate P of energy transfer from the emf device to the charge carriers is given by

$$P = i\,\Delta V, \tag{27-24}$$

where ΔV is the potential across the terminals of the emf device. (Note that this is the power associated with the transfer). If we apply this expression to the circuit shown in Fig. 27-11 (from Eq. 27-24 above), we can substitute $\Delta V_B = \mathscr{E} - ir$ into Eq. 27-24 to find

$$P = i(\mathscr{E} - ir) = i\mathscr{E} - i^2 r. \tag{27-25}$$

We see that the term $i^2 r$ in Eq. 27-25 is the rate P_r of energy transfer to thermal energy within the emf device:

$$P_r = i^2 r \qquad \text{(internal dissipation rate).} \tag{27-26}$$

Then the term $i\mathscr{E}$ in Eq. 27-25 must be the rate P_{emf} at which the emf device transfers energy to *both* the charge carriers and to internal thermal energy. Thus,

$$P_{\text{emf}} = i\mathscr{E} \qquad \text{(power of emf device).} \tag{27-27}$$

If a battery is being *recharged*, with a "wrong way" current through it, the energy transfer is then from the charge carriers to the battery—both to the battery's chemical energy and to the energy dissipated in the internal resistance r. The rate of change of the chemical energy is given by Eq. 27-27, the rate of dissipation is given by Eq. 27-26, and the rate at which the carriers supply energy is given by Eq. 27-24.

As is the case for mechanics, the accepted SI unit for electrical power is the watt. One watt is equal to one joule-sec.

TOUCHSTONE EXAMPLE 27-3: Two Real Batteries

Let's consider a circuit with two *nonideal* batteries that have internal resistances. Since the potential differences across the terminals of these batteries are not constant, we characterize each battery in terms of its emf ($\mathscr{E}_1$ or $\mathscr{E}_2$) and internal resistances (r_1 or r_2). The emfs and resistances in the circuit of Fig. 27-12*a* have the following values:

$$\mathscr{E}_1 = 4.4\text{ V},\quad \mathscr{E}_2 = 2.1\text{ V},\quad r_1 = 2.3\ \Omega,\quad r_2 = 1.8\ \Omega,\quad R = 5.5\ \Omega.$$

(a) What is the current i in the circuit?

SOLUTION ■ The **Key Idea** here is that we can get an expression involving the current i in this single-loop circuit by applying the loop rule. Although knowing the direction of i is not necessary, we can easily determine it from the emfs of the two batteries. Because $\mathscr{E}_1$ is greater than $\mathscr{E}_2$, battery 1 controls the direction of i, so that direction is clockwise. Let us then apply the loop rule by going counterclockwise—against the current—and starting at point a. We find

$$-\mathscr{E}_1 + ir_1 + iR + ir_2 + \mathscr{E}_2 = 0\text{ V}.$$

Check that this equation also results if we apply the loop rule clockwise or start at some point other than a. Also, take the time to compare this equation term by term with Fig. 27-12*b*, which shows the potential changes graphically (with the potential at point a arbitrarily taken to be zero).

Solving the above loop equation for the current i, we obtain

$$i = \frac{\mathscr{E}_1 - \mathscr{E}_2}{R + r_1 + r_2} = \frac{4.4\text{ V} - 2.1\text{ V}}{5.5\ \Omega + 2.3\ \Omega + 1.8\ \Omega}$$
$$= 0.2396\text{ A} \approx 240\text{ mA}. \qquad \text{(Answer)}$$

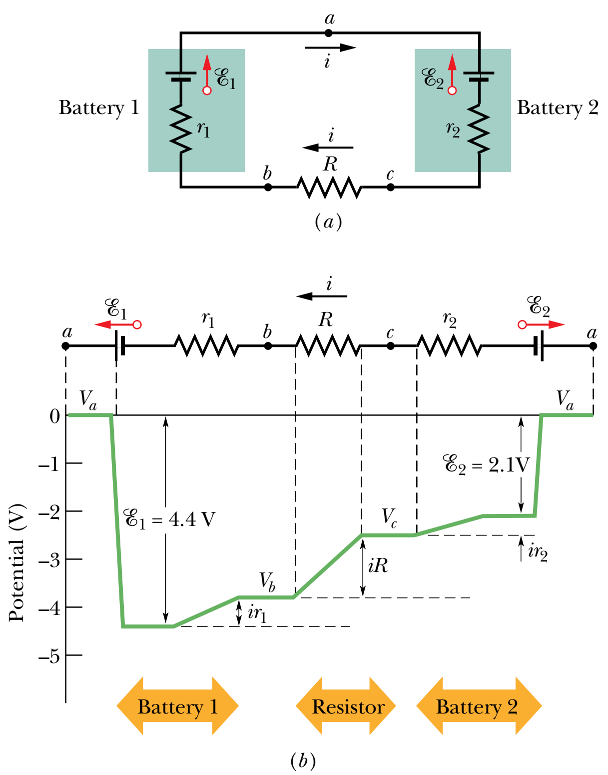

FIGURE 27-12 ■ (a) A single-loop circuit containing two real batteries and a resistor. The batteries oppose each other; that is, they tend to send current in opposite directions through the resistor. (b) A graph of the potentials encountered in traversing this circuit counterclockwise from point a, with the potential at a arbitrarily taken to be zero. (To better link the circuit with the graph, mentally cut the circuit at a and then unfold the left side of the circuit toward the left and the right side of the circuit toward the right.)

(b) What is the potential difference between the terminals of battery 1 in Fig. 27-12a?

SOLUTION ■ The **Key Idea** is to sum the potential differences between points a and b. Let us start at point b (effectively the negative terminal of battery 1) and travel clockwise through battery 1 to point a (effectively the positive terminal), keeping track of potential changes. We find that

$$V_b - ir_1 + \mathscr{E}_1 = V_a,$$

which gives us

$$\begin{aligned} V_a - V_b &= -ir_1 + \mathscr{E}_1 \\ &= -(0.2396\text{ A})(2.3\ \Omega) + 4.4\ V \\ &= +3.84\text{ V} \approx 3.8\text{ V}, \end{aligned}$$

which is less than the emf of the battery. You can verify this result by starting at point b in Fig. 27-12a and traversing the circuit counterclockwise to point a.

TOUCHSTONE EXAMPLE 27-4: Electric Eel

Electric fish generate current with biological cells called *electroplaques,* which are physiological emf devices. The electroplaques in the South American eel shown in the photograph that opens this chapter are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. The arrangement is suggested in Fig. 27-13a; each electroplaque has an emf $\mathscr{E}$ of 0.15 V and an internal resistance r of 0.25 Ω. The water surrounding the eel completes a circuit between the two ends of the electroplaque array, one end at the animal's head and the other near its tail.

(a) If the water surrounding the eel has resistance $R_{\text{water}} = 800\ \Omega$, how much current can the eel produce in the water?

SOLUTION ■ The **Key Idea** here is that we can simplify the circuit of Fig. 27-13a by replacing combinations of emfs and internal resistances with equivalent emfs and resistances. We first consider a single row. The total emf $\mathscr{E}_{\text{row}}$ along a row of 5000 electroplaques is the sum of the emfs:

$$\mathscr{E}_{\text{row}} = 5000\mathscr{E} = (5000)(0.15\text{ V}) = 750\text{ V}.$$

The total resistance R_{row} along a row is the sum of the internal resistances of the 5000 electroplaques:

$$R_{\text{row}} = 5000r = (5000)(0.25\ \Omega) = 1250\ \Omega.$$

We can now represent each of the 140 identical rows as having a single emf $\mathscr{E}_{\text{row}}$ and a single resistance R_{row}, as shown in Fig. 27-13b.

In Fig. 27-13b, the emf between point a and point b on any row is $\mathscr{E}_{\text{row}} = 750$ V. Because the rows are identical and because they are all connected together at the left in Fig. 27-13b, all points b in

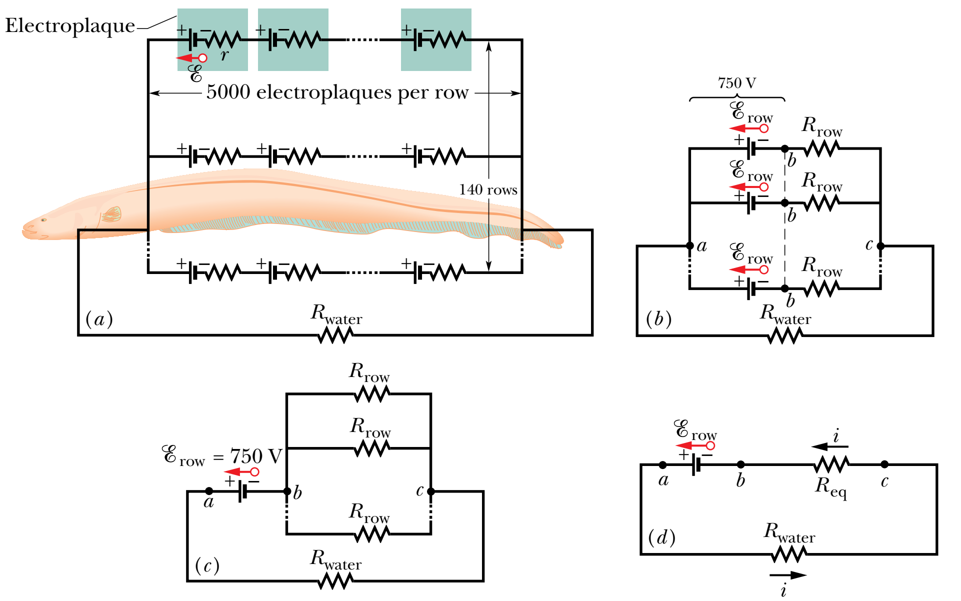

FIGURE 27-13 ■ (*a*) A model of the electric circuit of an eel in water. Each electroplaque of the eel has an emf $\mathscr{E}$ and internal resistance r. Along each of 140 rows extending from the head to the tail of the eel, there are 5000 electroplaques. The surrounding water has resistance R_{water}. (*b*) The emf $\mathscr{E}_{row}$ and resistance R_{row} of each row. (*c*) The emf between points a and b is $\mathscr{E}_{row}$. Between points b and c are 140 parallel resistances R_{row}. (*d*) The simplified circuit, with R_{eq} replacing the parallel combination.

that figure are at the same electric potential. Thus, we can consider them to be connected so that there is only a single point b. The emf between point a and this single point b is $\mathscr{E}_{row} = 750$ V, so we can draw the circuit as shown in Fig. 27-13*c*.

Between points b and c in Fig. 27-13*c* are 140 resistances $R_{row} = 1250\ \Omega$, all in parallel. The equivalent resistance R_{eq} of this combination is given by Eq. 27-12 as

$$\frac{1}{R_{eq}} = \sum_{j=1}^{140} \frac{1}{R_j} = 140 \frac{1}{R_{row}},$$

or

$$R_{eq} = \frac{R_{row}}{140} = \frac{1250\ \Omega}{140} = 8.93\ \Omega.$$

Replacing the parallel combination with R_{eq}, we obtain the simplified circuit of Fig. 27-13*d*. Applying the loop rule to this circuit counterclockwise from point b, we have

$$\mathscr{E}_{row} - iR_{water} - iR_{eq} = 0\ \text{V}.$$

Solving for i and substituting the known data, we find

$$i = \frac{\mathscr{E}_{row}}{R_{water} + R_{eq}} = \frac{750\ \text{V}}{800\ \Omega + 8.93\ \Omega}$$
$$= 0.927\ \text{A} \approx 0.93\ \text{A}. \qquad \text{(Answer)}$$

If the head or tail of the eel is near a fish, much of this current could pass along a narrow path through the fish, stunning or killing it.

(b) How much current i_{row} travels through each row of Fig. 27-13*a*?

SOLUTION ■ The **Key Idea** here is that since the rows are identical, the current into and out of the eel is evenly divided among them:

$$i_{row} = \frac{i}{140} = \frac{0.927\ \text{A}}{140} = 6.6 \times 10^{-3}\ \text{A}. \qquad \text{(Answer)}$$

Thus, the current through each row is small, about two orders of magnitude smaller than the current through the water. This tends to spread the current through the eel's body, so that it need not stun or kill itself when it stuns or kills a fish.

Problems

SECS. 27-2 AND 27-3 ■ CURRENT AND POTENTIAL DIFFERENCE IN SINGLE LOOP CIRCUITS, SERIES RESISTANCE

1. Three Resistors In Fig. 27-14, take $R_1 = R_2 = R_3 = 10\ \Omega$. If the potential difference across the ideal battery is $\Delta V_B = 12$ V, find: (a) the equivalent resistance of the circuit and (b) the direction the current flows in the circuit. (c) Which point, A or B, is at higher potential?

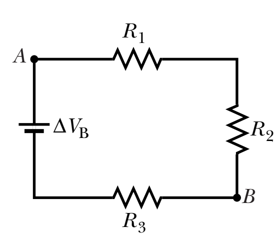

FIGURE 27-14 ■ Problems 1, 3, and 5.

2. Two Ideal Batteries Figure 27-15 shows two ideal batteries with $\Delta V_{B1} = 12$ V and $\Delta V_{B2} = 8$ V. (a) What is the direction of the current in the resistor? (b) Which battery is doing positive work? (c) Which point, A or B, is at the higher potential?

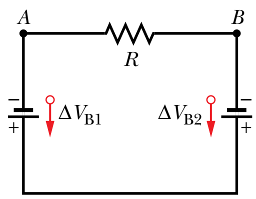

FIGURE 27-15 ■ Problem 2.

3. Total Current In Fig. 27-14, take $R_1 = 10\ \Omega$, $R_2 = 15\ \Omega$, and $R_3 = 20\ \Omega$. If the potential difference across the ideal battery is $\Delta V_B = 15$ V, find: (a) the equivalent resistance of the circuit, (b) the current through each of the resistors, and (c) the total current in the circuit.

4. If Potential at *P* Is In Fig. 27-16, if the potential at point P is 100 V, what is the potential at point Q?

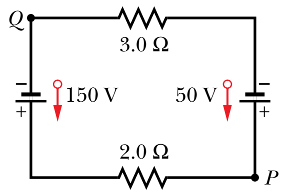

FIGURE 27-16 ■ Problem 4.

5. Voltages In Fig. 27-14, take $R_1 = 12\ \Omega$, $R_2 = 15\ \Omega$, and $R_3 = 25\ \Omega$. If the potential difference across the ideal battery is $\Delta V_B = 15$ V, find the potential differences across each of the resistors.

6. Neglecting Wires Figure 27-17 shows a 6.00 Ω resistor connected to a 12.0 V battery by means of two copper wires. The wires each have length 20.0 cm and radius 1.00 mm. In such circuits we generally neglect the potential differences along wires and the transfer of energy to thermal energy in them. Check the validity of this neglect for the circuit of Fig. 27-17: What are the potential differences across (a) the resistor and (b) each of the two sections of wire? At what rate is energy lost to thermal energy in (c) the resistor and (d) each of the two sections of wire?

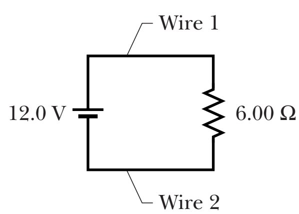

FIGURE 27-17 ■ Problem 6.

7. Single Loop The current in a single-loop circuit with one resistance R is 5.0 A. When an additional resistance of 2.0 Ω is inserted in series with R, the current drops to 4.0 A. What is R?

8. Ohmmeter A simple ohmmeter is made by connecting an ideal 1.50 V flashlight battery in series with a resistance R and an ammeter that reads from 0 to 1.00 mA, as shown in Fig. 27-18. Resistance R is adjusted so that when the clip leads are shorted together, the meter deflects to its full-scale value of 1.00 mA. What external resistance across the leads results in a deflection of (a) 10%, (b) 50%, and (c) 90% of full scale? (d) If the ammeter has a resistance of 20.0 Ω and the internal resistance of the battery is negligible, what is the value of R?

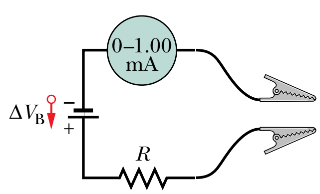

FIGURE 27-18 ■ Problem 8.

SECS. 27-4 AND 27-5 ■ MULTILOOP CIRCUITS AND PARALLEL RESISTANCE

9. Sizes and Directions What are the sizes and directions of the currents through resistors (a) R_2 and (b) R_3 in Fig. 27-19, where each of the three resistances is 4.0 Ω?

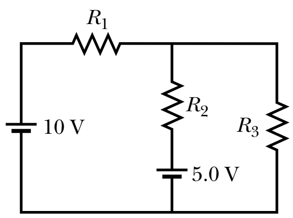

FIGURE 27-19 ■ Problem 9.

10. Changes The resistances in Figs. 27-20*a* and *b* are all 6.0 Ω, and the batteries are ideal 12 V batteries. (a) When switch S in Fig. 27-20*a* is closed, what is the change in the electric potential difference ΔV_{R_1} across resistor 1, or does ΔV_{R_1} remain the same? (b) When switch S in Fig. 27-20*b* is closed what is the change in the electric potential difference ΔV_{R_1} across resistor 1, or does ΔV_{R_1} remain the same?

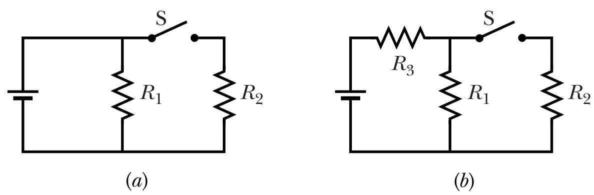

FIGURE 27-20 ■ Problem 10.

11. Equivalent (a) In Fig. 27-21, what is the equivalent resistance of the network shown? (b) What is the current in each resistor? Put $R_1 = 100\ \Omega$, $R_2 = R_3 = 50\ \Omega$, $R_4 = 75\ \Omega$, and $\Delta V_B = 6.0$ V; assume the battery is ideal.

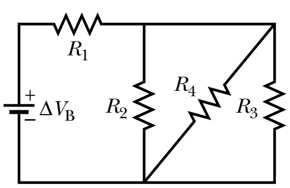

FIGURE 27-21 ■ Problem 11.

12. Plots Plot 1 in Fig. 27-22*a* gives the electric potential difference ΔV_{R_1} set up across R_1 versus the current i that can appear in resistor 1. Plots 2 and 3 are similar plots for resistors 2 and 3, respectively. Figure 27-22*b* shows a circuit with those three resistors and a 6.0 V battery. What is the current in resistor 2 in that circuit?

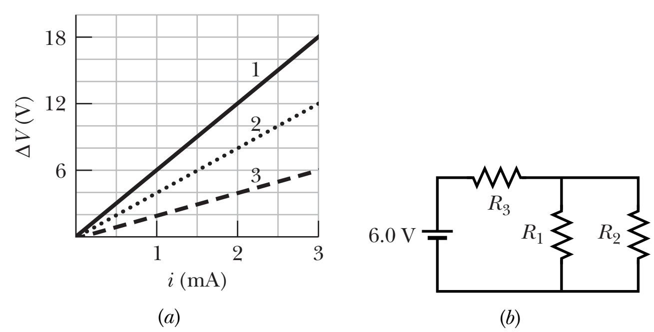

FIGURE 27-22 ■ Problem 12.

13. Equivalent Resistance Two In Fig. 27-23, $R = 10\ \Omega$. What is the equivalent resistance between points A and B? (*Hint:* This circuit section might look simpler if you first assume that points A and B are connected to a battery.)

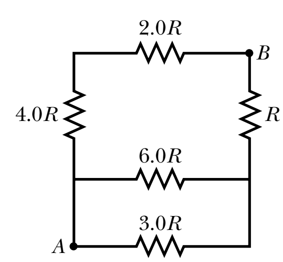

FIGURE 27-23 ■ Problem 13.

14. Three Switches Figure 27-24 shows a circuit containing three switches, labeled S_1, S_2, and S_3. Find the current at a for all possible combinations of switch settings. Put $\Delta V_B = 120$ V, $R_1 = 20.0\ \Omega$, and $R_2 = 10.0\ \Omega$. Assume that the battery has no resistance.

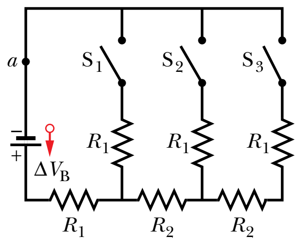

FIGURE 27-24 ■ Problem 14.

15. Two Lightbulbs Two lightbulbs, one of resistance R_1 and the other of resistance R_2, are connected to a battery (a) in parallel and (b) in series. Which bulb is brighter in each case if $R_1 = R_2$? How is your answer different if $R_1 > R_2$?

16. Calculate Potential In Fig. 27-5, calculate the potential difference between points c and d by as many paths as possible. Assume that $\Delta V_{B1} = 4.0$ V, $\Delta V_{B2} = 1.0$ V, $R_1 = R_2 = 10\ \Omega$, and $R_3 = 5.0\ \Omega$.

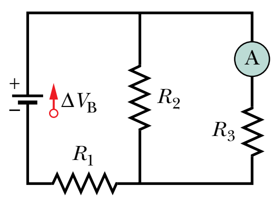

FIGURE 27-25 ■ Problem 17.

17. Ammeter (a) In Fig. 27-25, determine what the ammeter will read, assuming $\Delta V_B = 5.0$ V (for the ideal battery), $R_1 = 2.0\ \Omega$, $R_2 = 4.0\ \Omega$, and $R_3 = 6.0\ \Omega$. (b) The ammeter and the source of emf are now physically interchanged. Show that the ammeter reading remains unchanged.

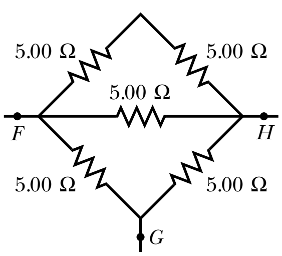

FIGURE 27-26 ■ Problem 18.

18. Equivalent Resistance In Fig. 27-26, find the equivalent resistance between points (a) F and H and (b) F and G. (*Hint*: for each pair of points, imagine that a battery is connected across the pair).

19. Current in Each In Fig. 27-27 find the current in each resistor and the potential difference between points a and b. Put $\Delta V_{B1} = 6.0$ V, $\Delta V_{B2} = 5.0$ V, $\Delta V_{B3} = 4.0$ V, $R_1 = 100\ \Omega$, and $R_2 = 50\ \Omega$.

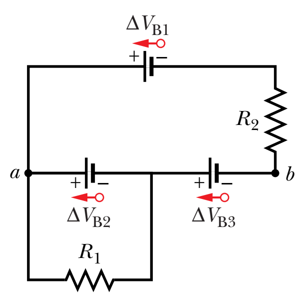

FIGURE 27-27 ■ Problem 19.

20. Two Resistors By using only two resistors—singly, in series, or in parallel—you are able to obtain resistances of 3.0, 4.0, 12, and 16 Ω. What are the two resistances?

21. Wire of Radius A copper wire of radius $a = 0.250$ mm has an aluminum jacket of outer radius $b = 0.380$ mm. (a) There is a current $i = 2.00$ A in the composite wire. Using Table 26-2, calculate the current in each material. (b) If a potential difference $V = 12.0$ V between the ends maintains the current, what is the length of the composite wire?

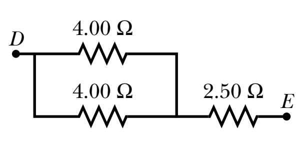

FIGURE 27-28 ■ Problem 22.

22. Between D and E In Fig. 27-28, find the equivalent resistance between points D and E. (*Hint*: Imagine that a battery is connected between points D and E.)

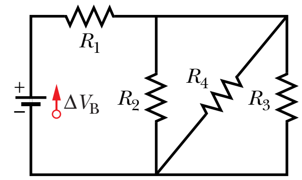

FIGURE 27-29 ■ Problem 24.

23. Four Resistors Four 18.0 Ω resistors are connected in parallel across a 25.0 V battery. What is the current through the battery?

24. Network Shown (a) In Fig. 27-29, what is the equivalent resistance of the network shown? (b) What is the current in each resistor? Put $R_1 = 100\ \Omega$, $R_2 = R_3 = 50\ \Omega$, $R_4 = 75\ \Omega$, and $\Delta V_B = 6.0$ V; assume the battery is ideal.

25. Nine Copper Wires Nine copper wires of length l and diameter d are connected in parallel to form a single composite conductor of resistance R. What must be the diameter D of a single copper wire of length l if it is to have the same resistance?

26. Voltmeter A voltmeter (of resistance $R_{\Delta V}$) and an ammeter (of resistance R_A) are connected to measure a resistance R, as in Fig. 27-30*a*. The resistance is given by $R = \Delta V/i$, where ΔV is the voltmeter reading and i is the current in the resistance R. Some of the current i' registered by the ammeter goes through the voltmeter, so that the ratio of the meter readings ($=\Delta V/i'$) gives only an *apparent* resistance reading R'. Show that R and R' are related by

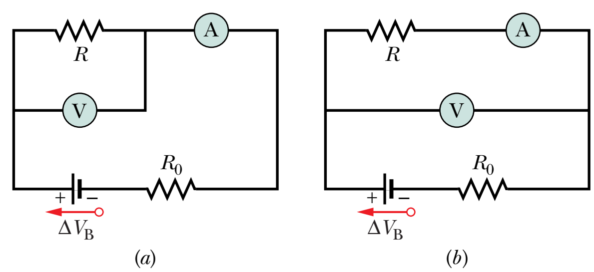

FIGURE 27-30 ■ Problems 26 to 28.

$$\frac{1}{R} = \frac{1}{R'} - \frac{1}{R_{\Delta V}}.$$

Note that as $R_{\Delta V} \to \infty$, $R' \to R$. Ignore R_0 for now.

27. Ammeter and Voltmeter (See Problem 26.) If an ammeter and a voltmeter are used to measure resistance, they may also be connected as in Fig. 27-30*b*. Again the ratio of the meter readings gives only an apparent resistance R'. Show that now R' is related to R by

$$R = R' - R_A,$$

in which R_A is the ammeter resistance. Note that as $R_A \to 0\ \Omega$, $R' \to R$. Ignore R_0 for now.

28. What Will the Meters Read (See Problems 26 and 27.) In Fig. 27-30, the ammeter and voltmeter resistances are 3.00 Ω and 3.00 Ω, respectively. Take $\Delta V_B = 12.0$ V for the ideal battery and $R_0 = 100\ \Omega$. If $R = 85.0\ \Omega$, (a) what will the meters read for the two different connections (Figs. 27-30*a* and *b*)? (b) What apparent resistance R' will be computed in each case?

29. Given a Number You are given a number of 10 Ω resistors, each capable of dissipating only 1.0 W without being destroyed. What is the minimum number of such resistors that you need to combine in series or in parallel to make a 10 Ω resistance that is capable of dissipating at least 5.0 W?

30. Asymptote In Fig. 27-31*a*, resistor 3 is a variable resistor and the battery is an ideal 12 V battery. Figure 27-31*b* gives the current i through the battery as a function of R_3. The curve has an asymptote of 2.0 mA as $R_3 \to \infty$. What are (a) resistance R_1 and (b) resistance R_2?

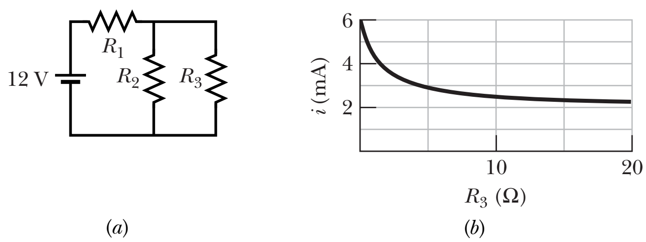

FIGURE 27-31 ■ Problem 30.

31. Box Figure 27-32 shows a section of a circuit. The electric potential difference between points A and B that connect the section to the rest of the circuit is $V_A - V_B = 78$ V, and the current through the 6.0 Ω resistor is 6.0 A. Is the device represented by "Box" absorbing or providing energy to the circuit and at what rate?

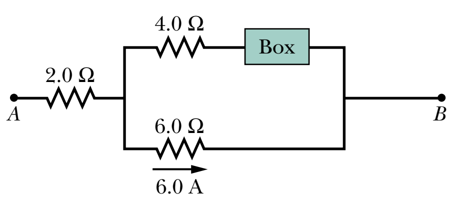

FIGURE 27-32 ■ Problem 31.

32. Arrangement of *N* Resistors In Fig. 27-33, a resistor and an arrangement of n resistors in parallel are connected in series with an ideal battery. All the resistors have the same resistance. If one more identical resistor were added in parallel to the n resistors already in parallel, the current through the battery would change by 1.25%. What is the value of n?

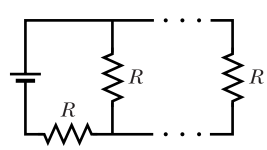

FIGURE 27-33 ■ Problem 32.

33. Rate of Energy Transfer In Fig. 27-34, where each resistance is 4.00 Ω, what are the sizes and directions of currents (a) i_1 and (b) i_2? At what rates is energy being transferred at (c) the 4.00 V battery and (d) the 12.0 V battery, and for each, is the battery supplying or absorbing energy?

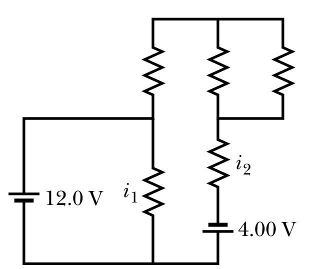

FIGURE 27-34 ■ Problem 33.

34. Both Batteries Are Ideal Both batteries in Fig. 27-35*a* are ideal. ΔV_{B1} of battery 1 has a fixed value but ΔV_{B2} of battery 2 can be varied between 1.0 V and 10 V. The plots in Fig. 27-35*b* give the currents through the two batteries as a function of ΔV_{B2}. You must decide which plot corresponds to which battery, but for both plots, a negative current occurs when the direction of the current through the battery is opposite the direction of that battery's potential difference. What are (a) ΔV_{B1} (b) resistance R_1, and (c) resistance R_2?

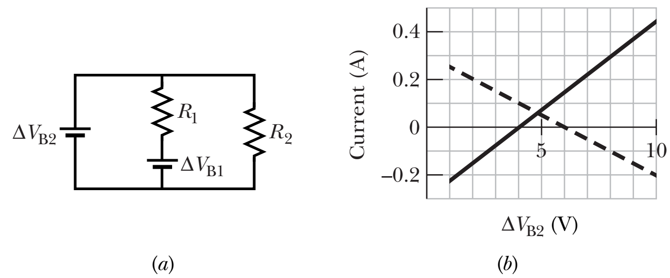

FIGURE 27-35 ■ Problem 34.

35. Work Done by Ideal Battery (a) How much work does an ideal battery with $\Delta V_B = 12.0$ V do on an electron that passes through the battery from the positive to the negative terminal? (b) If 3.4×10^{18} electrons pass through each second, what is the power of the battery?

36. Portion of a Circuit Figure 27-36 shows a portion of a circuit. The rest of the circuit draws current i at the connections A and B, as indicated. Take $\Delta V_{B1} = 10$ V, $\Delta V_{B2} = 15$ V, $R_1 = R_2 = 5.0$ Ω, $R_3 = R_4 = 8.0$ Ω, and $R_5 = 12$ Ω. For each of four values of i—0, 4.0, 8.0, and 12 A—find the current through each ideal battery and state whether the battery is charging or discharging. Also find the potential difference ΔV_{AB} between points A and B.

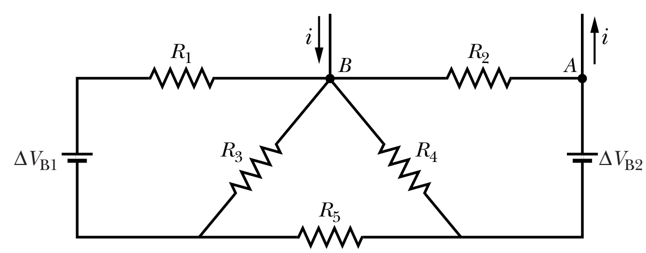

FIGURE 27-36 ■ Problem 36.

37. Adjusted Value In Fig. 27-37, R_s is to be adjusted in value by moving the sliding contact across it until points a and b are brought to the same potential. (One tests for this condition by momentarily connecting a sensitive ammeter between a and b; if these points are at the same potential, the ammeter will not deflect.) Show that when this adjustment is made, the following relation holds:

$$R_x = R_s\left(\frac{R_2}{R_1}\right).$$

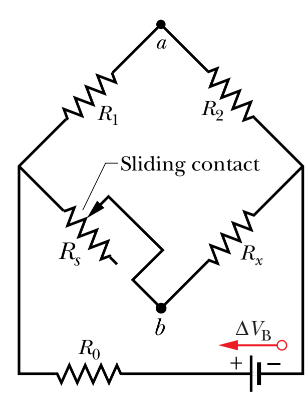

FIGURE 27-37 ■ Problem 37.

An unknown resistance (R_x) can be measured in terms of a standard (R_s) using this device, which is called a Wheatstone bridge.

38. What Are the Currents In Fig. 27-38, what are currents (a) i_2, (b) i_4, (c) i_1, (d) i_3, and (e) i_5?

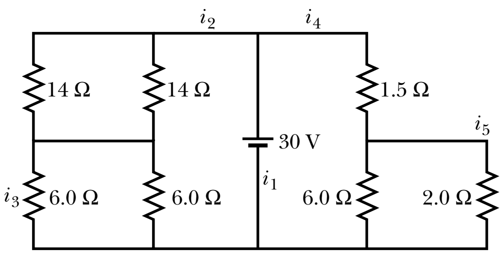

FIGURE 27-38 ■ Problem 38.

39. Sizes and Directions Two What are the sizes and directions of (a) current i_1 and (b) current i_2 in Fig. 27-39, where each resistance is 2.00 Ω? (Can you answer this making only mental calculations?) (c) At what rate is energy being transferred in the 5.00 V battery at the left, and is the energy being supplied or absorbed by the battery?

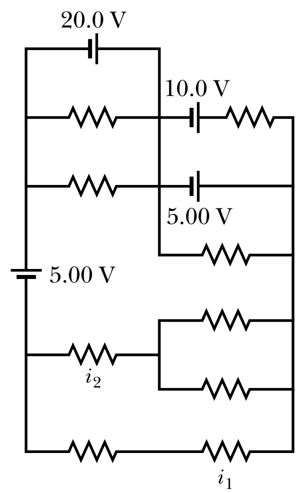

FIGURE 27-39 ■ Problem 39.

40. Size and Direction Three (a) What are the size and direction of current i_1 in Fig. 27-40, where each resistance is 2.0 Ω? What are the powers of (b) the 20 V battery, (c) the 10 V battery, and (d) the 5.0 V battery, and for each, is energy being supplied or absorbed?

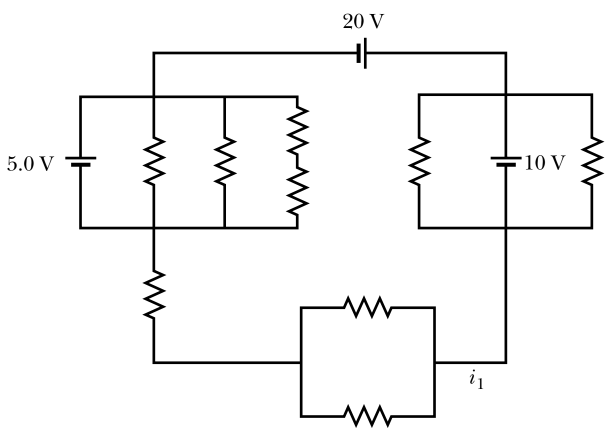

FIGURE 27-40 ■ Problem 40.

41. Size and Direction Four (a) What are the size and direction of current i_1 in Fig. 27-41? (b) How much energy is dissipated by all four resistors in 1.0 min?

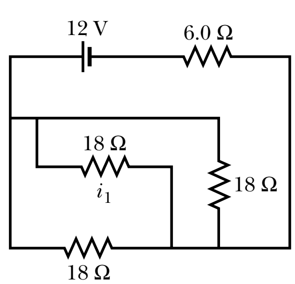

FIGURE 27-41 ■ Problem 41.

SEC. 27-7 ■ INTERNAL RESISTANCE AND POWER

42. Chemical Energy A 5.0 A current is set up in a circuit for 6.0 min by a rechargeable battery with a 6.0 V emf. By how much is the chemical energy of the battery reduced?

43. Flashlight Battery A standard flashlight battery can deliver about 2.0 W·h of energy before it runs down. (a) If a battery costs 80¢, what is the cost of operating a 100 W lamp for 8.0 h using batteries? (b) What is the cost if energy is provided at 12¢ per kilowatt-hour?

44. Power Supplied Power is supplied by a device of emf $\mathscr{E}$ to a transmission line with resistance R. Find the ratio of the power dissipated in the line for $\mathscr{E} = 110\ 000$ V to that dissipated for $\mathscr{E} = 110$ V, assuming the power supplied is the same for the two cases.

45. Car Battery A certain car battery with a 12 V emf has an initial charge of 120 A·h. Assuming that the potential across the terminals stays constant until the battery is completely discharged, for how long can it deliver energy at the rate of 100 W?

46. Energy Transferred A wire of resistance 5.0 Ω is connected to a battery whose emf $\mathscr{E}$ is 2.0 V and whose internal resistance is 1.0 Ω. In 2.0 min, (a) how much energy is transferred from chemical to electrical form? (b) How much energy appears in the wire as thermal energy? (c) Account for the difference between (a) and (b).

47. Assume the Batteries Assume that the batteries in Fig. 27-42 have negligible internal resistance. Find (a) the current in the circuit, (b) the power dissipated in each resistor, and (c) the power of each battery, stating whether energy is supplied by or absorbed by it.

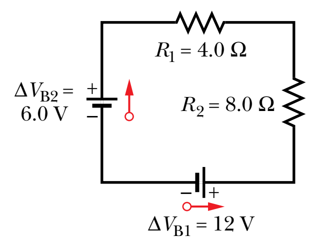

FIGURE 27-42 ■ Problem 47.

48. Both Batteries In Fig. 27-43*a*, both batteries have emf $\mathscr{E} = 1.20$ V and the external resistance R is a variable resistor. Figure 27-43*b* gives the electric potentials ΔV_T between the terminals of each battery as functions of R: Curve 1 corresponds to battery 1 and curve 2 corresponds to battery 2. What are the internal resistances of (a) battery 1 and (b) battery 2?

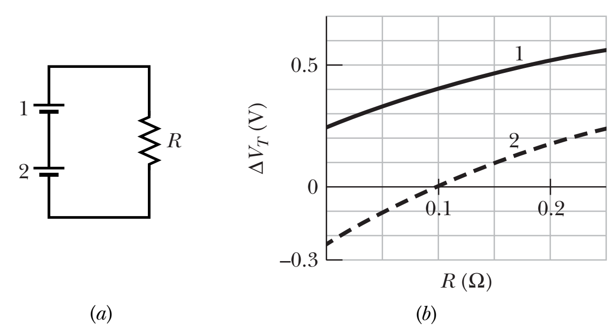

FIGURE 27-43 ■ Problem 48.

49. Find Internal Resistance The following table gives the electric potential difference ΔV_T across the terminals of a battery as a function of current i being drawn from the battery. (a) Write an equation that represents the relationship between the terminal potential difference ΔV_T and the current i. Enter the data into your graphing calculator and perform a linear regression fit of ΔV_T versus i. From the parameters of the fit, find (b) the battery's emf and (c) its internal resistance.

i (A):	50	75	100	125	150	175	200
ΔV_T (V):	10.7	9.0	7.7	6.0	4.8	3.0	1.7

50. Make Plots In Fig. 27-11a, put $\mathscr{E} = 2.0$ V and $r = 100\ \Omega$. Plot (a) the current and (b) the potential difference across R, as functions of R over the range 0 to 500 Ω. Make both plots on the same graph. (c) Make a third plot by multiplying together, for various values of R, the corresponding values on the two plotted curves. What is the physical significance of this third plot?

51. Energy Converted A car battery with a 12 V emf and an internal resistance of 0.040 Ω is being charged with a current of 50 A. (a) What is the potential difference across its terminals? (b) At what rate is energy being dissipated as thermal energy in the battery? (c) At what rate is electric energy being converted to chemical energy? (d) What are the answers to (a) and (b) when the battery is used to supply 50 A to the starter motor?

52. What Value of *R* (a) In Fig. 27-44, what value must R have if the current in the circuit is to be 1.0 mA? Take $\mathscr{E}_1 = 2.0$ V, $\mathscr{E}_2 = 3.0$ V, and $r_1 = r_2 = 3.0\ \Omega$. (b) What is the rate at which thermal energy appears in R?

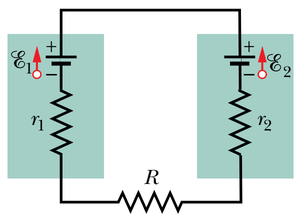

FIGURE 27-44 ■ Problem 52.

53. Circuit Section In Fig. 27-45, circuit section AB absorbs energy at a rate of 50 W when a current $i = 1.0$ A passes through it in the indicated direction. (a) What is the potential difference between A and B? (b) emf device X does not have internal resistance. What is its emf? (c) What is its *polarity* (the orientation of its positive and negative terminals)?

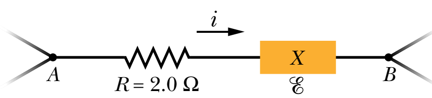

FIGURE 27-45 ■ Problem 53.

54. Lights of an Auto When the lights of an automobile are switched on, an ammeter in series with them reads 10 A and a voltmeter connected across them reads 12 V. See Fig. 27-46. When the electric starting motor is turned on, the ammeter reading drops to 8.0 A and the lights dim somewhat. If the internal resistance of the battery is 0.050 Ω and that of the ammeter is negligible, what are (a) the emf of the battery and (b) the current through the starting motor when the lights are on?

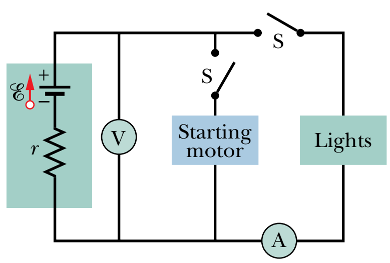

FIGURE 27-46 ■ Problem 54.

55. Same EMF Two batteries having the same emf $\mathscr{E}$ but different internal resistances r_1 and r_2 ($r_1 > r_2$) are connected in series to an external resistance R. (a) Find the value of R that makes the potential difference zero between the terminals of one battery. (b) Which battery is it?

56. Starting Motor The starting motor of an automobile is turning too slowly, and the mechanic has to decide whether to replace the motor, the cable, or the battery. The manufacturer's manual says that the 12 V battery should have no more than 0.020 Ω internal resistance, the motor no more than 0.200 Ω resistance, and the cable no more than 0.040 Ω resistance. The mechanic turns on the motor and measures 11.4 V across the battery, 3.0 V across the cable, and a current of 50 A. Which part is defective?

57. Maximum Power (a) In Fig. 27-11a, show that the rate at which energy is dissipated in R as thermal energy is a maximum when $R = r$. (b) Show that this maximum power is $P = \mathscr{E}^2/4r$.

58. Solar Cell A solar cell generates a potential difference of 0.10 V when a 500 Ω resistor is connected across it, and a potential difference of 0.15 V when a 1000 Ω resistor is substituted. What are (a) the internal resistance and (b) the emf of the solar cell? (c) The area of the cell is 5.0 cm^2, and the rate per unit area at which it receives energy from light is 2.0 mW/cm^2. What is the efficiency of the cell for converting light energy to thermal energy in the 1000 Ω external resistor?

59. Maximum Energy Two batteries of emf $\mathscr{E}$ and internal resistance r are connected in parallel across a resistor R, as in Fig. 27-47a. (a) For what value of R is the rate of electrical energy dissipation by the resistor a maximum? (b) What is the maximum energy dissipation rate?

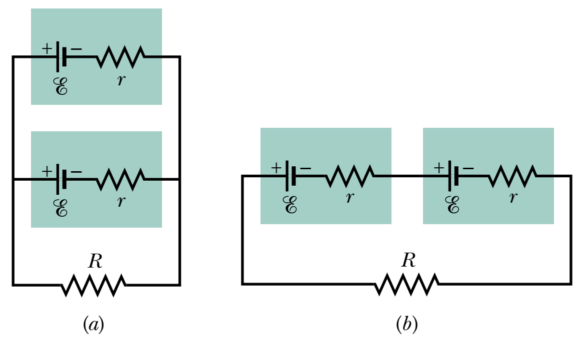

FIGURE 27-47 ■ Problems 59 and 60.

60. Either Parallel or Series You are given two batteries of emf $\mathscr{E}$ and internal resistance r. They may be connected either in parallel (Fig. 27-47a) or in series (Fig. 27-47b) and are to be used to establish a current in a resistor R. (a) Derive expressions for the current in R for both arrangements. Which will yield the larger current (b) when $R > r$ and (c) when $R < r$?

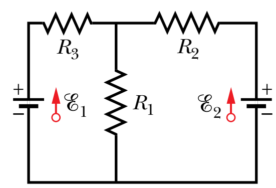

FIGURE 27-48 ■ Problem 61.

61. Batteries Are Ideal In Fig. 27-48, $\mathscr{E}_1 = 3.00$ V, $\mathscr{E}_2 = 1.00$ V, $R_1 = 5.00\ \Omega$, $R_2 = 2.00\ \Omega$, $R_3 = 4.00\ \Omega$, and both batteries are ideal. What is the rate at which energy is dissipated in (a) R_1, (b) R_2, and (c) R_3? What is the power of (d) battery 1 and (e) battery 2?

62. For What Value of *R* In the circuit of Fig. 27-49, for what value of R will the ideal battery transfer energy to the resistors (a) at a rate of 60.0 W, (b) at the maximum possible rate, and (c) at the minimum possible rate? (d) What are those rates?

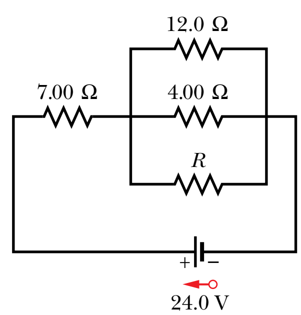

FIGURE 27-49 ■ Problem 62.

63. Calculate Current (a) Calculate the current through each ideal battery in Fig. 27-50. Since the batteries are ideal $\mathcal{E} = \Delta V_B$ in each case. Assume that $R_1 = 1.0\ \Omega$, $R_2 = 2.0\ \Omega$, $\mathcal{E}_1 = 2.0$ V and $\mathcal{E}_2 = \mathcal{E}_3 = 4.0$ V. (b) Calculate $V_a - V_b$.

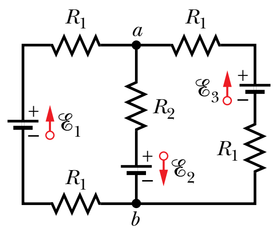

FIGURE 27-50 ■ Problem 63.

64. Constant Value In the circuit of Fig. 27-51, $\mathcal{E}$ has a constant value but R can be varied. Find the value of R that results in the maximum heating in that resistor. The battery is ideal.

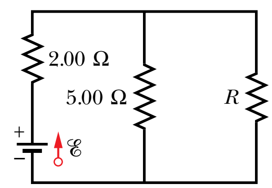

FIGURE 27-51 ■ Problem 64.

Additional Problems

65. True or False For the circuit in Fig. 27-52, indicate whether the statements are true or false. If a statement is false, give a correct statement.

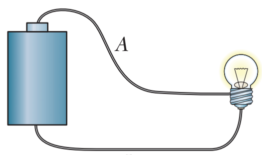

FIGURE 27-52 ■ Problem 65.

(a) Some of the current is used up when the bulb is lit; the current in wire B is smaller than the current in wire A.
(b) A current probe will have the same readings if connected to read the current in wire A or wire B. The current flows from the battery, through wire A, through the bulb, and then back to the battery through wire B.
(c) The current flows toward the bulb in both wires A and B.
(d) The (positive) current flows from the battery, through wire A, and then back to the battery through wire B.
(e) If wire A is left connected but wire B is disconnected, the bulb will still light.

66. Use the Model (a) Use our model for electric current to rank the networks shown in Fig. 27-53 in order by resistance. Explain your reasoning. (b) If a battery were connected to each of the circuits, in which case would the current through the battery be the largest? The smallest? Explain your reasoning.

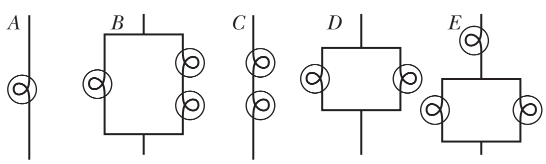

FIGURE 27-53 ■ Problem 66.

67. Examine the Circuits Examine the circuits shown in Fig. 27-54 and indicate whether you think each of the following two statements are true or false. Please explain your reasoning.

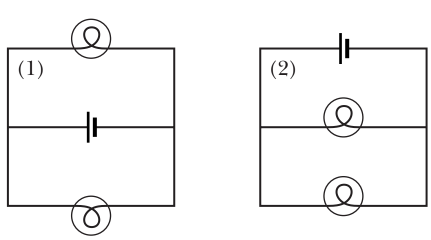

FIGURE 27-54 ■ Problem 67.

(a) Circuits 1 and 2 are different. The brightness of the two bulbs in circuit 1 are the same, but in circuit 2 the bulb closest to the battery in brighter than the bulb that is further away.
(b) Circuit diagrams only show electrical connections, so the drawings in circuits 1 and 2 are electrically equivalent and the brightness of the two bulbs is the same in both circuits 1 and 2.

68. Which Diagram (a) Identify which of the nice, neat circuit diagrams (A, B, C, or D) in Fig. 27-55*c* corresponds to the messy circuit drawing in Fig. 27-55*a*. Explain the reasons for your answer.

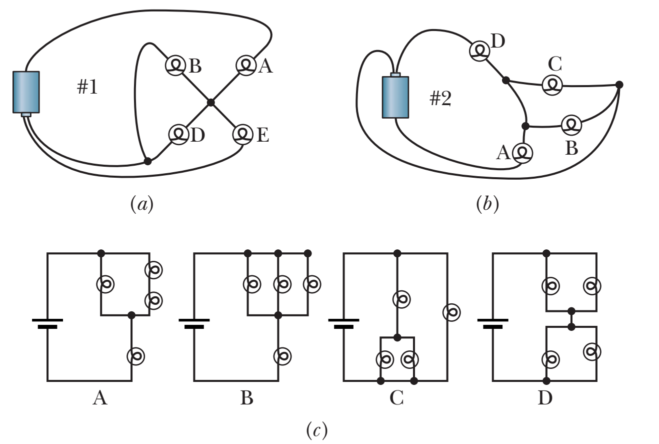

FIGURE 27-55 ■ Problem 68.

(b) Which neat circuit diagram corresponds to the messy circuit drawing in Fig. 27-55*b*. Explain the reasons for your answer.

69. At Which Point (a) For the circuit in Fig. 27-56, at which point A, B, C, D or E is the voltage the lowest? Explain. (b) At which point is the potential energy of a positive charge the highest? Explain. (c) At which point is the current the largest? Explain.

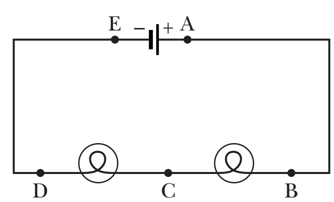

FIGURE 27-56 ■ Problem 69.

70. Bulbs 1 Through 6 (a) For the circuit shown in Fig. 27-57, rank bulbs 1 through 6 in order of descending brightness. Explain the reasoning for your ranking. (b) Now assume that the filament of lightbulb 6 breaks. Again rank the bulbs in order of descending brightness. Explain the reasoning for your ranking.

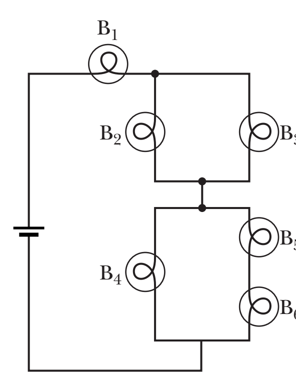

FIGURE 27-57 ■ Problem 70.

71. The Circuit Diagram The circuit diagram in Fig. 27-58 shows two unlabeled resistors attached to identical bulbs. Explain how you would interpret the brightness of bulbs A and B to decide which resistor is larger.

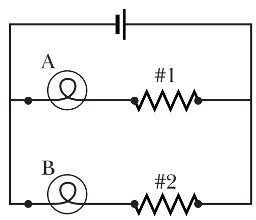

FIGURE 27-58 ■ Problem 71.

72. Three Circuits Which of the three circuits shown in Fig. 27-59, if any, are electrically identical? Which are different? Explain your answers.

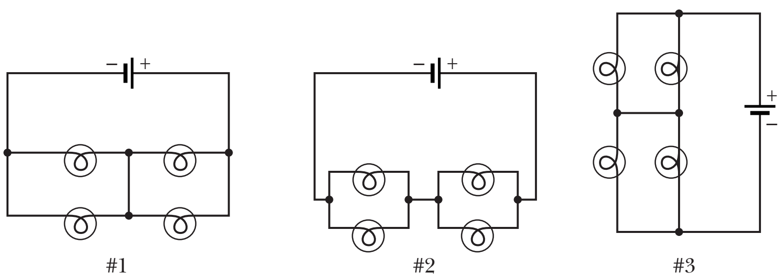

FIGURE 27-59 ■ Problem 72.

73. An Unscrewed Bulb Examine the circuit shown in Fig. 27-60. (a) Rank the bulbs according to brightness and explain your reasoning. (b) How will the brightness of bulbs 1 and 3 change if bulb 4 is unscrewed? Explain. (c) How will the brightness of bulbs 1, 3, 5, and 6 change if a conducting wire is connected between points A and F? Explain.

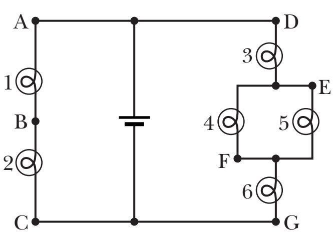

FIGURE 27-60 ■ Problem 73.

74. Examine the Circuit Examine the circuit shown in Fig. 27-61. (a) Assume that the switch is *open*. State which bulbs or combination of bulbs are in series, and in parallel. (b) Assume that the switch is *closed*. State whether the bulbs in the circuit are arranged in series or parallel.

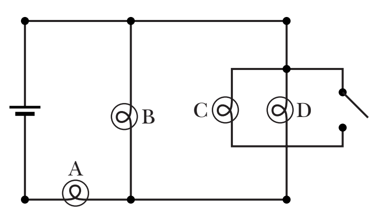

FIGURE 27-61 ■ Problem 74.

75. Examine the Circuit Two Examine the circuit shown in Fig. 27-62. (a) Assume that the switch is *open*. Rank the bulbs according to brightness and explain your reasoning. (b) Assume that the switch is *closed*. Rank the bulbs according to brightness and explain your reasoning.

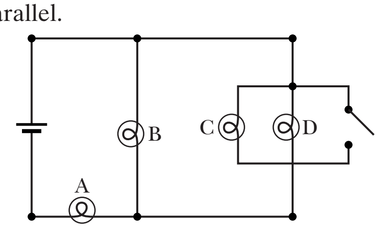

FIGURE 27-62 ■ Problem 75.

76. More Current If the batteries in Fig. 27-63 are identical, which circuit draws more current? Circuit A? Circuit B? Neither? Show your calculations and reasoning.

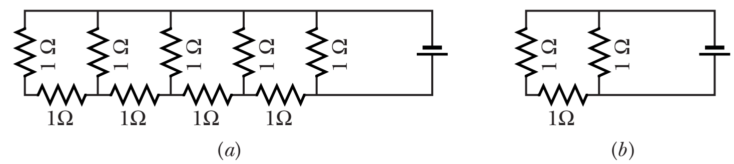

FIGURE 27-63 ■ Problem 76.

77. Which Are Connected In the circuits shown in Fig. 27-64, state which resistors are connected in series with which other resistors, which are connected in parallel with which other resistors, and which are neither in series nor parallel.

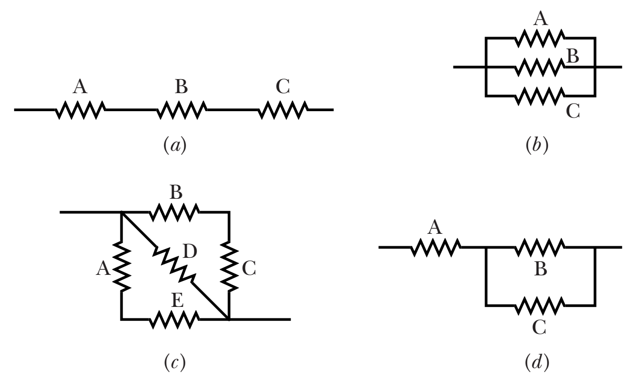

FIGURE 27-64 ■ Problem 77.

78. Lots of Batteries and a Bulb Figure 27-65 shows identical batteries connected in different arrangements to the same lightbulb. Assume the batteries have negligible internal resistances. The positive terminal of each battery is marked with a plus. Rank these arrangements on the basis of bulb brightness from the highest to the lowest. Please explain your reasoning.

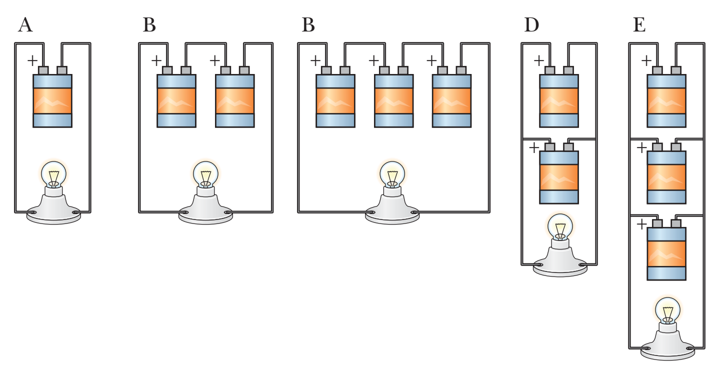

FIGURE 27-65 ■ Problem 78.

79. Constant Current Source We have studied batteries that provide a fixed voltage across their terminals. In that case, we had to examine our circuit and use our physical principles in order to calculate the current through the battery. In neuroscience, it is sometimes useful to use a constant current source (CCS), which instead provides a fixed amount of current through itself. In this case, we have to use our physical principles in order to calculate the voltage drop across the source.

Suppose we have a constant current source (denoted CSS) that always provides a current of $i_c = 10^{-6}$ amps. For the three circuits shown in Fig. 27-66, find the voltage drop across the current source. Each resistor has a resistance $R = 2000\ \Omega$. (If you prefer, you may leave your answer in terms of the symbols i_c and R.)

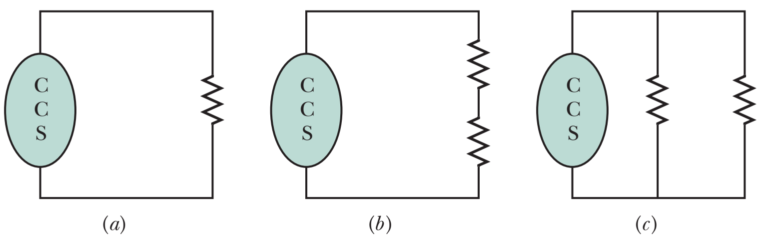

FIGURE 27-66 ■ Problem 79.

80. Tracking Around a Circuit The circuit shown in Fig. 27-67 contains an ideal battery and three resistors. The battery has an emf of 1.5 V, $R_1 = 2\ \Omega$, $R_2 = 3\ \Omega$, and $R_3 = 5\ \Omega$. Also shown in Fig. 27-67 is a graph tracking some quantity around the circuit. Make three

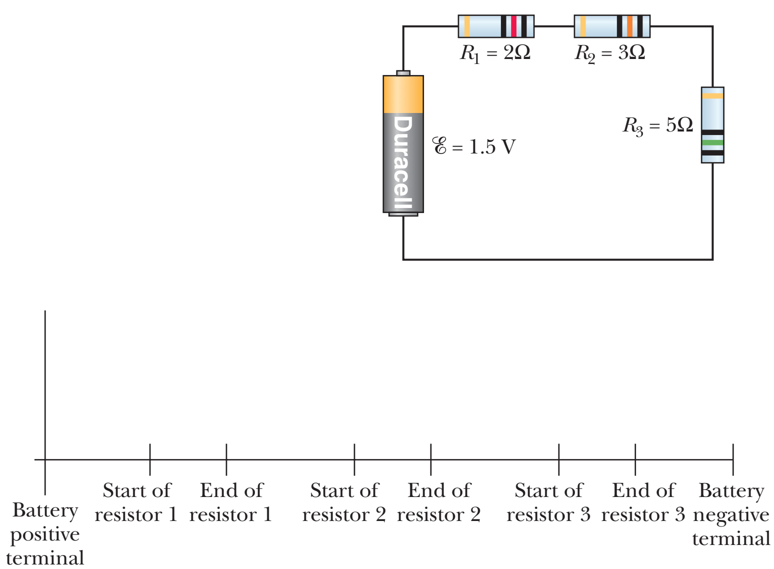

FIGURE 27-67 ■ Problem 80.

copies of this graph. On the first, plot the voltage a test charge would experience as it moved through the circuit. On the second, plot the electric field a test charge would experience as it moved through the circuit. On the third, plot the current one would measure crossing a plane perpendicular to the wire of the circuit as one goes through the circuit.

81. Modeling a Nerve Membrane (From a homework set in a graduate course in synaptic physiology) As a result of a complex set of biochemical reactions, the cell membrane of a nerve cell pumps ions (Na^+ and K^+) back and forth across itself, thereby maintaining an electrostatic potential difference from the inside to the outside of the membrane. Modifications on the conditions can result in changes in those potentials.

Part of the process can be modeled by treating the membrane as if it were a simple electric circuit consisting of batteries, resistors, and a switch. A simple model of the membrane of a nerve cell is shown in Fig. 27-68. It consists of two batteries (ion pumps) with voltages $\Delta V_1 =$ 100 mV and $V_2 = 50$ mV. The resistance to flow across the membrane is represented by two resistors with resistances $R_1 = 10$ K Ω and $R_2 = 90$ K Ω. The variability is represented by a switch, S_1.

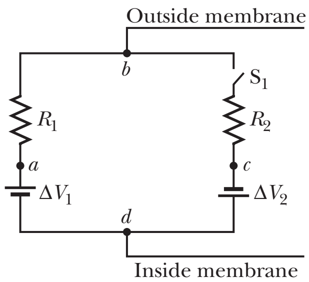

FIGURE 27-68 ■ Problem 81.

Four points on the circuit are labeled by the letters a–d. The point b represents the outside of the membrane and the point d the inside of the membrane.

(a) What is the voltage difference across the membrane (i.e., between d and b) when the switch is open?
(b) What is the current flowing around the loop when the switch is closed?
(c) What is the voltage drop across the resistor R_1 when the switch is open? Closed?
(d) What is the voltage drop across the resistor R_2 when the switch is open? Closed?
(e) What is the potential difference across the membrane (i.e., between d and b) when the switch is closed?
(f) If the locations of resistances R_1 and R_2 were reversed, would the voltages across the cell membrane be different?

82. Find the Five Currents Consider the circuit in Fig. 27-69. (a) Apply the junction rule to junctions d and a and the loop rule to the three loops to produce five simultaneous, linearly independent equations. (b) Represent the five linear equations by the matrix equation $[A][B] = [C]$, where

$$[B] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}.$$

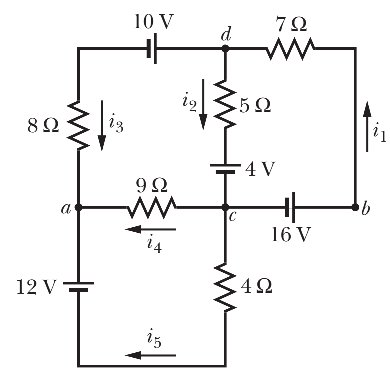

FIGURE 27-69 ■ Problem 82 and 83.

What are the matrices $[A]$ and $[C]$? (c) Have the calculator perform $[A]^{-1}[C]$ to find the values of i_1, i_2, i_3, i_4, and i_5.

83. Knowing the Currents For the same situation as in Problem 82 and having already solved for the five unknown currents, do the following. (a) Find the electric potential difference across the 9 Ω resistor. (b) Find the rate at which work is being done on the 7 Ω resistor. (c) Find the rate at which the 12 V battery is doing work on the circuit. (d) Find the rate at which the 4 V battery is doing work on the circuit. (e) Of the points in the circuit labeled a and c, which is at the higher electric potential?