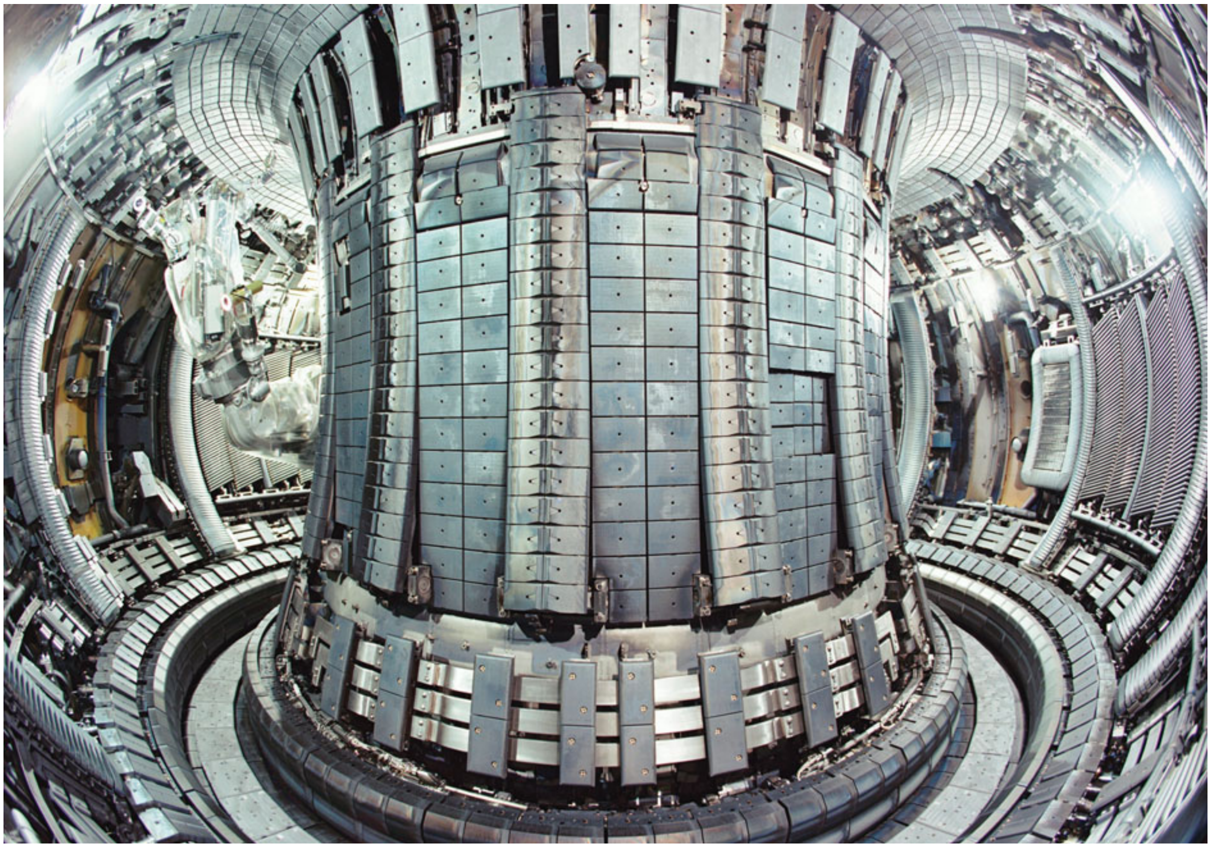
29 Magnetic Fields



Ocean water contains huge quantities of the light atomic nuclei found in "heavy water" needed to produce fusion power. If we could produce a cost-effective fusion reactor, the world's power problems could be solved. We have known this for over 50 years and still not produced fusion power. Why? A key problem is that it takes a temperature of at least 100 million degrees Celsius to force two light nuclei to fuse together. At this temperature, any material we tried to squeeze together to fuse would be so hot that it would vaporize any material it touches. The torus-shaped chamber of the large Tokomak reactor in this photo was built in an attempt to contain fusion reactions.

How can this Tokomak contain matter at 100 million degrees Celsius?

The answer is in this chapter.

FIGURE 29-1 ■ A large electromagnet is used to collect and transport scrap metal at a steel mill.

29-1 A New Kind of Force?

In Chapter 14 we studied gravitational interaction forces that we experience on an everyday basis. Gravitational forces are so weak that it takes a source the size of a planet or star to produce a noticeable effect. This made the study of the effects of gravity near the Earth's surface relatively simple. In most cases, we treat the gravitational force on an object as a constant.

Then, in Chapter 22, we studied the electrostatic force—a long-range force that is much stronger than the gravitational force. If you run a comb through your hair, a bit of paper near the comb hops up and sticks to the comb. The electrostatic force exerted on the paper by the comb is somewhat larger than the gravitational force that the whole Earth exerts on the paper.

Are there any other long-range (or, action-at-a-distance) forces, or are we done? If you think about your personal experiences, you probably have had the opportunity to play with small disk-shaped refrigerator magnets or pairs of bar magnets. On a larger scale, **electromagnets** are used for sorting scrap metal (Fig. 29-1) and many other things. Magnets are fun because they behave in such an unusual way. You can use one magnet to chase a second magnet around a table without even touching it. But if you come at the magnet from a slightly different direction, it will suddenly seem to change what it's doing and will be pulled toward the other. A refrigerator magnet will seem to leap to the door of the refrigerator, being drawn to it from a distance. Clearly a long-range force is at work here. But is it a new kind of force? Or is it merely a form of gravitational or electrical force?

29-2 Probing Magnetic Interactions

We know from our everyday experiences with small bar magnets that we can *feel* a force on one bar magnet as it interacts with another. This means we can use a bar magnet as a test object for investigating the nature of magnetic interactions. In order to answer the question of whether magnetic interactions are really gravitational or electrostatic forces, let's investigate what happens when a small bar magnet or disk-shaped refrigerator magnet experiences a significant force.

Is the Magnetic Force a Type of Gravitational Force?

The force on our test magnet near the Earth's surface is clearly *in addition* to the gravitational attraction of the Earth. The fact that a refrigerator magnet can stick to the refrigerator and not fall means that it is experiencing a force that is stronger than the gravitational force exerted on it by the entire Earth.

What happens if we replace our test magnet with another *nonmagnetic* object of equal mass and the same shape? We find that the magnetic force disappears. Hence, we must surmise that the force we detected with the bar magnet is not a gravitational force associated with the presence of another object. It is too strong and exists only for certain probe objects. Furthermore, we know from playing with magnets that the force can be attractive or repulsive. As we know, this is not true for the gravitational force.

Is the Magnetic Force a Type of Electrostatic Force?

Could the magnetic force be the electrostatic force we have learned about? After all, the magnetic force, like the electrostatic force, is sometimes attractive and sometimes repulsive. To test this idea, we replace our test magnet with a test charge (such as a tiny Styrofoam ball charged by a rubber rod) at the former location of our test magnet. Again, we find that our new probe (the charge) is only weakly attracted—as is any charged object to a neutral object. So, we must also surmise that *the force the bar magnet detects is not a type of electrostatic force.*

The Magnetic Force and a Moving Charge

We have just described observations that show that forces between magnets are fundamentally different from either electrostatic or gravitational forces. So it appears that we have a new action-at-a-distance force to learn about. This force can be either attractive or repulsive. We can detect this force with a magnet, and so we will refer to it as a **magnetic force.**

Having completed our investigations of electric force in earlier chapters, we now take the electric charge we had been using as a probe and move it rapidly away from a magnet. When we do this, we find something strange. When we *move* the charge, we do detect a force!

> **OBSERVATION:** A magnet exerts a force on a moving charged object, but not on a stationary charged object.

Furthermore, when we try moving the charge at different velocities, we find that the larger the magnitude of the velocity, the larger is the force exerted on the charge. Is the same true for *uncharged,* nonmagnetic masses? Experimentation shows the same is *not* true for uncharged masses. No magnetic force is detected when an uncharged, nonmagnetic mass is used as a probe—regardless of whether the probe is moving or stationary.

In the early 19th century, both Oersted and Ampère discovered that magnets interact with moving charges. In fact, these two scientists showed that current-carrying wires both exert forces on and feel forces from bar magnets. Their observations provide us with important information in our quest to understand the magnetic force. We have found that magnetic forces are not just exerted on other magnets. Magnetic forces are also exerted on a nonmagnetic small charged particle in rough proportion to the degree to which the particle is *both* charged and moving. What is the simplest relationship between magnetic force, charge, and velocity that is consistent with our observations? Mathematically stated, it is a proportional relationship given by

$$|\vec{F}^{\,\text{mag}}| \propto |q| v,$$

where $|q|$ represents the amount of electric charge on the particle and v is the particle's speed.

Is this relationship correct? Well, if it is, we should see a doubling of the force when we double the velocity of the charged particle we are using as a probe. Experimentally, this does turn out to be the case. Furthermore, we also find that doubling the charge on the probe doubles the force detected. Hence, the linear relationship expressed above is a good start toward a more precise mathematical description of the magnetic force on a moving charged particle. We will return to experimentation as a means for developing a precise expression for the magnetic force in just a moment.

29-3 Defining a Magnetic Field $\vec{B}$

When we play with two bar magnets, we quickly see that the magnetic force can be attractive or repulsive. Furthermore, if we observe more carefully, we find that the strength of the force decreases as the distance between the two magnets is increased. These observations are distinctly reminiscent of our observations of the electrostatic force between two charges. So our first guess in developing a model of the magnetic force might turn out to be somewhat similar to our model of the electrostatic force.

In order to develop a model of magnetism that parallels our model of electrostatics, we should have two different kinds of "magnetic charges." These conceptual objects are referred to as **magnetic monopoles.** We can model our bar magnet as containing a south and a north pole with at least some separation between them. If we assume that like poles repel and unlike poles attract, then this model allows us to correctly predict all our observations. Playing with bar magnets informs us that poles of the same kind repel one another and poles of different kinds attract one another. This is just as we found for electric charges. However, careful observation of the interaction between bar magnets shows that their behavior is similar to that of electric dipoles. Recall that an electric dipole consists of two charges of opposite sign with a small spacing between them. If two electric dipoles that are placed with all their charges lying on the same line are brought together, they will attract. Why? Because a negative charge from the end of one dipole will be closest to the positive charge of the other dipole. However, if we turn one of the electric dipoles around so the dipoles are anti-aligned, then the two like charges will be closest together. Now the dipoles will repel.

Two bar magnets when aligned and then anti-aligned will behave just like electric dipoles. For this reason, we often refer to magnets as *magnetic dipoles*. That is, one end appears to be one kind of magnetic charge and the other end appears to be the other kind of magnetic charge. By convention, we can assign names to the poles of a bar magnet as follows. If we suspend a bar magnet by a string placed halfway between its ends and take other magnetic sources away from its vicinity, one pole of the magnet will point more or less north and the other more or less south. We can call the north-pointing end the north pole of the magnet and the other end the south pole of the magnet.

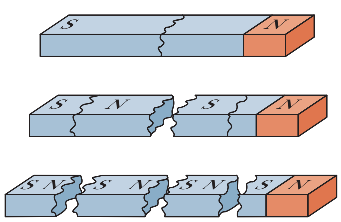


FIGURE 29-2 ■ Whenever a magnet is broken into pieces between its poles, the pieces behave like smaller, weaker magnets.

This idea that a bar magnet is a magnetic dipole with a north charge at one end and a south charge at the other end provides us with a start in describing magnetic interactions. However, to continue with the analogy between the magnetism and electricity, we would like to isolate a magnetic charge. After all, we can separate a negative charge from a positive charge. So we need to be able to separate the north pole of a bar magnet from the south pole of a bar magnet. To do this, we take our bar magnet and cut it in half. But, when we do this we find a surprising thing. The result of breaking the bar magnet in half is simply that we have two weaker half-sized bar magnets. Each one still behaves as a dipole with both a north and a south pole. If we again try to break the magnet in half, we find we have a still smaller magnet, but still with a north and south pole (Fig. 29-2). In fact, if we break the magnet down into subatomic parts, we find that even the electrons, protons, and neutrons within atoms behave as magnetic dipoles (that is, *very* little bar magnets).

As it turns out, the magnetic effect of a bar magnet arises from the combination of the effects of the little bar magnets in the electrons in iron, nickel, and cobalt aligning with each other and producing a strong effect. Each electron's magnet is small, but when you turn them in the same direction and add them all up, the total effect is strong—the full magnetic effect of the bar magnet. So, in short, although the existence of separate magnetic charges (or magnetic monopoles) have been predicted by some physicists, they have never actually been found.

Does the fact that we cannot find an isolated magnetic monopole mean that we must abandon our effort to find parallels between magnetic and electrostatic forces? Not at all. In Chapter 23, we found that the concept of an electric field was quite useful. With so many different possible sources of significant electrostatic forces, it was helpful to think about the force field associated with a given charge (the source of electrostatic force)—without having to decide on what object the force will be exerted on. That is, we wanted to separate the discussion of the source of the force from the discussion of the object the force is exerted on. So we defined the electric field $\vec{E}$ as

$$\vec{E} = \frac{\vec{F}^{\,\text{elec}}}{q}. \qquad \text{(Eq. 23-4)}$$

We determined the electric field $\vec{E}$ at a point by putting a test particle of charge q at rest at that point and measuring the electrostatic force $\vec{F}^{\,\text{elec}}$ acting on the particle. We saw that *electric charges* set up an electric field that can then affect other electric charges.

Perhaps the same idea could be useful to us in describing magnetic forces. If we could develop a parallel concept of a magnetic field, we could separate the issue of sources of magnetic forces from discussions of the objects that magnetic forces are exerted on. This would be helpful since the concept of a magnetic monopole is so problematic. If a magnetic monopole were available, we could define the magnetic field $\vec{B}$ in a way similar to that used for electric fields. However, because such particles have not been found, we must using another method to define a magnetic field $\vec{B}$.

For nonmagnetic particles, we have already observed that the magnetic force is proportional to the charge and the magnitude of the velocity of the particle being acted on (the probe). We can use this information and define the magnetic field in terms of the force $\vec{F}^{\,\text{mag}}$ exerted on a moving, electrically charged test particle. The magnitude of the force seems to depend on the direction of the particle's velocity $\vec{v}$ as well. We will examine this effect in more detail in the next section, but for now we define the magnetic field $\vec{B}$ in terms of the *maximum* force magnitude we measure after trying all different directions for $\vec{v}$. So we can express the *magnitude* of the magnetic field $\vec{B}$ in terms of this maximum force magnitude as:

$$B = \frac{F^{\text{mag}}_{\text{max}}}{|q|v}, \qquad (29\text{-}1)$$

where q is the particle's charge and v is its speed.

Having defined the magnitude of the magnetic field is a big step forward. It is a concept that will turn out to be extremely useful. Right now, it is helpful because we have not identified the source of the force exerted on our probe. But, having defined the magnitude of the magnetic field in this way, we can at least say that we know that there is a vector magnetic field in the region of space we have been probing. We make extensive use of the concept of a magnetic field in this chapter. Next we turn our attention to this issue of how to define the direction of the magnetic field.

29-4 Relating Magnetic Force and Field

In order to determine the direction of the magnetic field, we can fire a charged particle through a region of space where a magnetic field $\vec{B}$ is known to exist. If we shoot the charged particle in various directions, we find something surprising—the direction of $\vec{F}^{\,\text{mag}}$ is always perpendicular to the direction of $\vec{v}$ (Fig. 29-3). After many such trials we find that when the particle's velocity $\vec{v}$ is along a particular axis through the region of space, force $\vec{F}^{\,\text{mag}}$ is zero. Furthermore, we find that for all other directions of $\vec{v}$, the *magnitude* of $\vec{F}^{\,\text{mag}}$ depends on the direction of $\vec{v}$. In fact, it is proportional to $|\vec{v}|\sin\phi$ where ϕ is the angle between the zero-force axis and the direction of $\vec{v}$. Thinking back to our work on torque and angular momentum, these observations suggest that a cross product is involved. But a cross product of what two vectors?

Clearly, one of the two vectors involved in the cross product is the velocity vector. Our observation that the force is zero when the velocity is along a certain axis implies

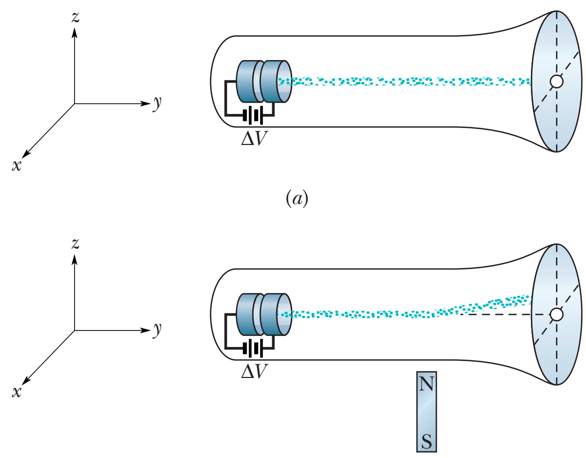


FIGURE 29-3 ■ (*a*) An electron beam is accelerated by a voltage source and travels through an evacuated glass tube to the center of a phosphorescent screen. (*b*) If a magnet is oriented vertically and placed just below the beam (along the $+z$ axis), the electrons are deflected horizontally along the $-x$ axis.

that the other vector must be aligned with this "zero magnetic force" axis. Referring back to our definition of the magnetic field magnitude, B, in Eq. 29-1, we note that the magnitude of the observed magnetic force is given by

$$F^{\text{mag}}_{\text{max}} = |q|vB,$$

where v is the particle speed and $|q|$ is the amount of charge the particle has. Suppose the direction of the magnetic field is taken to be along the "zero magnetic force" axis. We could then represent all of our observations with the following vector equation, known as the *magnetic force law* or ***Lorentz force law*:**

$$\vec{F}^{\text{mag}} = q\vec{v} \times \vec{B} \qquad \text{(magnetic force law).} \tag{29-2}$$

That is, the force $\vec{F}^{\text{mag}}$ on the particle is equal to the charge q times the cross product of its velocity $\vec{v}$ and the magnetic field $\vec{B}$. If this expression is correct, the force on a negatively charged particle should be opposite in direction from the force on a positively charged particle. This does in fact turn out to be the case.

Furthermore, expressing the magnetic force on a charged particle moving through a magnetic field as $\vec{F}^{\text{mag}} = q\vec{v} \times \vec{B}$ requires that we adopt a standard convention for the *direction* of the magnetic field. That is,

> The direction of a magnetic field is defined to be related to the direction of the force on and the velocity of a positively charged particle by $\vec{F}^{\text{mag}} = q\vec{v} \times \vec{B}$.

Although this is not a very intuitive statement of how one goes about finding the direction of a magnetic field, we are forced to use it if we want to use $\vec{F}^{\text{mag}} = q\vec{v} \times \vec{B}$ to determine the magnitude and direction of the magnetic force on a moving charged particle.

Using the mathematical definition of a cross product to evaluate this expression, we see that we can write the magnitude of the magnetic force as

$$F^{\text{mag}} = |q\vec{v}||\vec{B}|\sin\phi = |q|vB\sin\phi, \tag{29-3}$$

where ϕ is the smaller angle (the one whose value lies between 0° and 180°) between the directions of velocity $\vec{v}$ and magnetic field $\vec{B}$.

We have seen that magnetic force and electric force are not the same. However, a magnetic force *is* exerted on a moving charged particle as well as on bar magnets. This suggests that there is a profound connection between electricity and magnetism—even though they are *not* the same thing. As it turns out, the theory of relativity, treated in Chapter 38, reveals a deep underlying connection between $\vec{E}$ and $\vec{B}$. Furthermore, much of the technology that makes our lives more comfortable today results from an understanding of this relationship. In Chapter 30, we show how moving electrical charges can create magnetic fields and in Chapter 31 we show an even deeper and more surprising link between electricity and magnetism (called Faraday's law). What we find is that a magnetic field can, if it changes in time, create an electric field without any electric charge present!

Finding the Magnetic Force on a Moving Charged Particle

Equation 29-3 reveals that the magnitude of the force $\vec{F}^{\,\text{mag}}$ acting on a particle in a magnetic field is proportional to the amount of charge $|q|$ and speed v of the particle. Thus, the force is equal to zero if the charge is zero or if the particle is stationary. Equation 29-3 also tells us that the magnitude of the force is zero if $\vec{v}$ and $\vec{B}$ are either parallel ($\phi = 0°$) or antiparallel ($\phi = 180°$), and the force is a maximum when $\vec{v}$ and $\vec{B}$ are perpendicular to each other.

Equation 29-2 tells us all this and the direction of $\vec{F}^{\,\text{mag}}$. From Section 12-4, we know that the cross product $\vec{v} \times \vec{B}$ in Eq. 29-2 is a vector that is perpendicular to the two vectors $\vec{v}$ and $\vec{B}$. The right-hand rule (Fig. 29-4*a*) specifies that the thumb of the right hand points in the direction of $\vec{v} \times \vec{B}$ when the fingers sweep $\vec{v}$ into $\vec{B}$. If q is positive, then (by Eq. 29-2) the force $\vec{F}^{\,\text{mag}}$ has the same sign as $\vec{v} \times \vec{B}$ and thus must be in the same direction. That is, for positive q, $\vec{F}^{\,\text{mag}}$ is directed along the thumb as in Fig. 29-4*b*. If q is negative, then the force $\vec{F}^{\,\text{mag}}$ and the cross product $\vec{v} \times \vec{B}$ have opposite signs and thus must be in opposite directions. For negative q, $\vec{F}^{\,\text{mag}}$ is directed opposite the thumb as in Fig. 29-4*c*.

Regardless of the sign of the charge, however,

> The force $\vec{F}^{\,\text{mag}}$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

Thus, $\vec{F}^{\,\text{mag}}$ *never* has a component parallel to $\vec{v}$. This means that $\vec{F}^{\,\text{mag}}$ cannot change the particle's speed $v = |\vec{v}|$ (and thus it cannot change the particle's kinetic energy). The force can change only the direction of $\vec{v}$ (and thus the direction of travel); only in this sense can $\vec{F}^{\,\text{mag}}$ accelerate the particle. If there are no other forces acting on the charged particle and the velocity of the particle is perpendicular to the direction of the magnetic field, this means that the particle will move in a circle. If the particle has a component perpendicular to the magnetic field *and* a component of velocity parallel

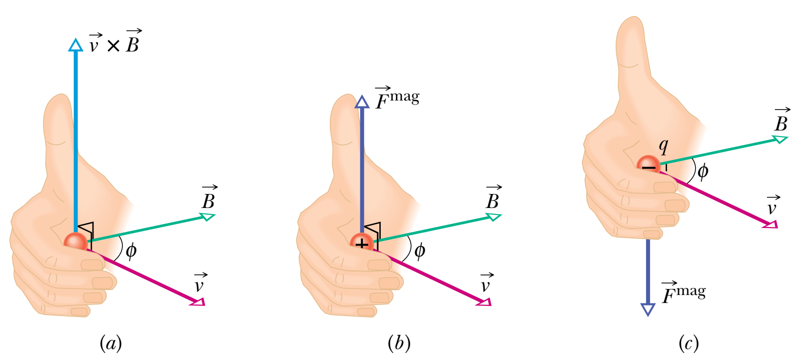


FIGURE 29-4 ■ (*a*) The right-hand rule (in which $\vec{v}$ is swept into $\vec{B}$ through the smaller angle ϕ between them) gives the direction of $\vec{v} \times \vec{B}$ as the direction of the thumb. (*b*) If q is positive, then the direction of $\vec{F}^{\,\text{mag}} = q\vec{v} \times \vec{B}$ is in the direction of $\vec{v} \times \vec{B}$. (*c*) If q is negative, then the direction of $\vec{F}^{\,\text{mag}}$ is opposite that of $\vec{v} \times \vec{B}$.

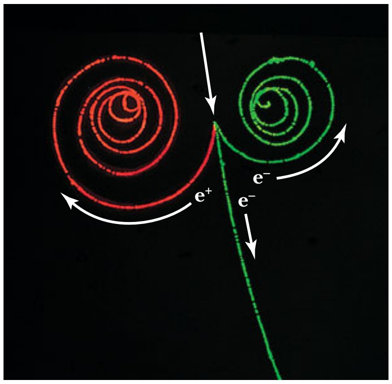


FIGURE 29-5 ■ Color enhanced tracks showing two electrons (e^-) and a positron (e^+) in a bubble chamber that is immersed in a uniform magnetic field that is directed out of the plane of the page.

to the magnetic field, the particle will move along a *helix* of constant radius. These paths are discussed in more detail in Section 29-5.

To develop a feeling for the relationship between the magnetic force on a moving charged particle and the magnetic field, $\vec{F}^{\,mag} = q\vec{v} \times \vec{B}$, consider Fig. 29-5. This figure shows some tracks left by charged particles moving rapidly through a *bubble chamber* at the Lawrence Berkeley Laboratory. The chamber, which is filled with liquid hydrogen, is immersed in a strong uniform magnetic field that is directed out of the plane of the figure. An incoming gamma ray particle—which leaves no track because it is uncharged—transforms into an electron (spiral track marked e^-) and a positron (track marked e^+) while it knocks an electron out of a hydrogen atom (long track marked e^-). At first these newly created charged particles are moving in the same direction as the gamma ray. As they move, they each experience a magnetic force of magnitude $F^{mag} = |q|vB$ and begin to move in a circular path given by $F^{mag} = mv^2/r$. Since $qvB = mv^2/r$, a particle has a path of radius $r = mv/|q|B$. You can use Eq. 29-2 and Fig. 29-4 to confirm that the three tracks made by these two negative particles and one positive particle curve in the proper directions. It is interesting to note that the electrons and positron do not move in a pure circle. Instead, they move in a shrinking spiral because they are slowed down through their interaction with the gas in the bubble chamber. This makes sense because $r = mv/|q|B$ and as each particle's speed, v, becomes smaller, so does its radius r. When this happens, the magnetic force, which is proportional to the particle's velocity, decreases and so the radius of the particle's path decreases.

What Produces a Magnetic Field?

We have discussed how a charged plastic rod produces a vector field—the electric field $\vec{E}$—at all points in the space around it. Similarly, a magnet produces a vector field—the **magnetic field** $\vec{B}$—at all points in the space around it. You get a hint of that magnetic field whenever you attach a note to a refrigerator door with a small magnet, or accidentally erase a computer disk by bringing it near a strong magnet. The magnet acts on the door or disk *by means of* its magnetic field.

In a common type of magnet, a wire coil is wound around an iron core and a current is sent through the coil; the strength of the magnetic field is determined by the size of the current. In industry, such **electromagnets** are used for sorting scrap metal (Fig. 29-1) among many other things. You are probably more familiar with **permanent magnets**—magnets, like the refrigerator-door type, that do not need current to have a magnetic field.

How then are magnetic fields set up? We know about two ways to create magnetic fields. (1) We observe that moving electrically charged particles, such as the current in a wire or charged beams of cosmic rays create magnetic fields. (2) We find that elementary particles such as protons, neutrons, and electrons have *intrinsic* magnetic moments that create magnetic fields. In Chapter 30 we discuss how moving charges create magnetic fields, and in Chapter 32 we consider the role of intrinsic magnetic moments in the creation of magnetic fields. In this chapter we stay focused on how to represent magnetic fields and how they influence charged particles that are moving.

The SI unit for $\vec{B}$ that follows from Eqs. 29-2 and 29-3 is the newton per coulomb-meter per second. For convenience, the SI unit for magnetic field is called the tesla (T):

$$1 \text{ tesla} = 1 \text{ T} = 1\frac{\text{newton}}{(\text{coulomb})(\text{meter/second})}.$$

Recalling that a coulomb per second is an ampere, we have

$$1 \text{ T} = 1\frac{\text{newton}}{(\text{coulomb/second})(\text{meter})} = 1\frac{\text{N}}{\text{A}\cdot\text{m}}. \tag{29-4}$$

TABLE 29-1
Some Approximate Magnetic Fields

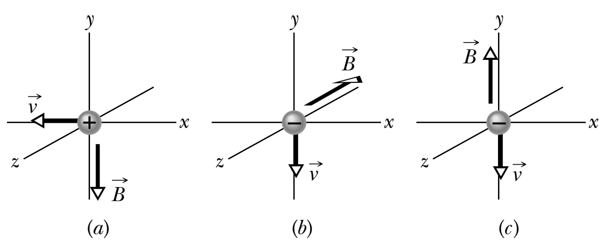
At the surface of a neutron star	10^8 T
Near a big electromagnet	1.5 T
Near a small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in a magnetically shielded room	10^{-14} T

An earlier (non-SI) unit for $\vec{B}$, that is still in common use is the *gauss* (G), and

$$1 \text{ tesla} = 10^4 \text{ gauss}. \qquad (29\text{-}5)$$

Table 29-1 lists the magnetic fields that occur in a few situations. Note that Earth's magnetic field near the planet's surface is about 10^{-4} T ($= 100\,\mu$T or 1 gauss).

READING EXERCISE 29-1: The figure shows three situations in which a charged particle with velocity $\vec{v}$ travels through a uniform magnetic field $\vec{B}$. In each situation, what is the direction of the magnetic force $\vec{F}^{\,\text{mag}}$ on the particle?



Magnetic Field Lines

We can represent magnetic fields with field lines, as we did for electric fields. Similar rules apply; that is, (1) the direction of the tangent to a magnetic field line at any point gives the direction of $\vec{B}$ at that point, and (2) the spacing of the lines represents the magnitude of $\vec{B}$—the magnetic field is stronger where the lines are closer together, and conversely.

Figure 29-6*a* shows how the magnetic field near a *bar magnet* (a permanent magnet in the shape of a bar) can be represented by magnetic field lines. The lines all pass through the magnet, and they all form closed and continuous loops (even those that are not shown closed in the figure). They don't start or end anywhere. Since electric field lines begin and end on electric charges, this is consistent with our assumption that there are no magnetic charges (monopoles). As shown with field lines, the external magnetic effects of a bar magnet are strongest near its ends, where the field lines are most closely spaced. Thus, the magnetic field of the bar magnet in Fig. 29-6*b* collects the iron filings mainly near the two ends of the magnet. Overall, outside of the bar magnet the field lines look just like they would for an electric dipole, but inside the magnet they point in the opposite direction.

The (closed) field lines enter one end of a magnet and exit the other end. The end of a magnet from which the field lines emerge is called the *north pole* of the magnet; the other end, where field lines enter the magnet, is called the *south pole*. (Remember that the direction of the field line is related to the direction of the force on a moving positively charged particle.) Some of the magnets we use to fix notes on refrigerators

(*a*) (*b*)

FIGURE 29-6 ■ (*a*) The magnetic field lines for a bar magnet. (*b*) A "cow magnet"—a bar magnet that is intended to be slipped down into the rumen (first stomach) of a cow to prevent accidentally ingested bits of scrap iron from reaching the cow's intestines. The iron filings at its ends reveal the directions of the magnetic field lines in the vicinity of the magnet.

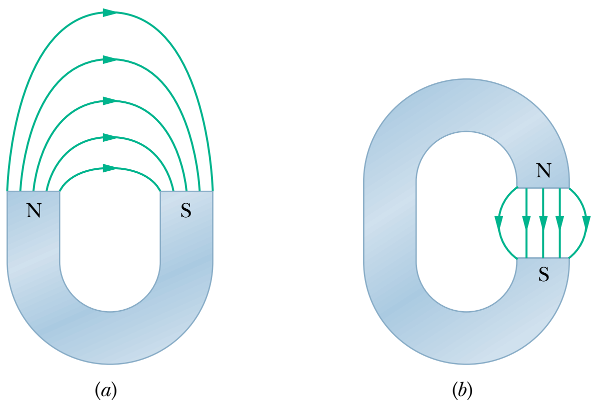


FIGURE 29-7 ■ (*a*) A horseshoe magnet and (*b*) a C-shaped magnet. (Only a few of the possible of the external field lines are shown.)

are short bar magnets. Figure 29-7 shows two other common shapes for magnets: a *horseshoe magnet* and a magnet that has been bent around into the shape of a C so that the *pole faces* are facing each other. (The magnetic field between the pole faces can then be approximately uniform.) Regardless of the shape of the magnets, if we place two of them near each other we find:

> Opposite magnetic poles attract each other, and like magnetic poles repel each other.

Earth has a magnetic field that is produced in its core. We discuss current theories about the nature and origin of the Earth's magnetic field in Section 32-9. On Earth's surface, we can detect this magnetic field with a compass, which is essentially a slender bar magnet on a low-friction pivot. This bar magnet, or this needle, turns because its north pole end is attracted toward the Arctic region, or North Pole, of Earth. Thus, the *south* pole of Earth's magnetic field must be located toward the North Pole. Logically, we then should call the pole there a south pole. However, because we call that direction north, we are trapped into the statement that Earth has a *geomagnetic north pole* in that direction.

With more careful measurement we would find that in the northern hemisphere, the magnetic field lines of Earth generally point down into Earth and toward the Arctic. In the southern hemisphere, they generally point up out of Earth and away from the Antarctic—that is, away from Earth's *geomagnetic south pole*.

TOUCHSTONE EXAMPLE 29-1: Proton in a Magnetic Field

A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What is the magnitude of the magnetic deflecting force acting on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg. (Neglect Earth's magnetic field.)

SOLUTION ■ Because the proton is charged and moving through a magnetic field, a magnetic force $\vec{F}^{\,\text{mag}}$ can act on it. The **Key Idea** here is that, because the initial direction of the proton's velocity is not along a magnetic field line, $\vec{F}^{\,\text{mag}}$ is not simply zero. To find the magnitude of $\vec{F}^{\,\text{mag}}$, we can use Eq. 29-3 provided we first find the proton's speed $|\vec{v}| = v$. We can find v from the given kinetic energy, since $K = \frac{1}{2}mv^2$. Solving for $|\vec{v}|$, we find

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3\text{ MeV})(1.60 \times 10^{-13}\text{ J/MeV})}{1.67 \times 10^{-27}\text{ kg}}}$$
$$= 3.2 \times 10^{7}\text{ m/s}.$$

Equation 29-3 then yields

$$F^{\text{mag}} = |q|vB_{\sin}\phi$$
$$= (1.60 \times 10^{-19}\text{ C})(3.2 \times 10^{7}\text{ m/s}) \times (1.2 \times 10^{-3}\text{ T})(\sin 90^\circ)$$
$$= 6.1 \times 10^{-15}\text{ N}. \qquad \text{(Answer)}$$

This may seem like a small force, but it acts on a particle of small mass, producing a large magnitude of acceleration; namely,

$$a = \frac{F^{\text{mag}}}{m} = \frac{6.1 \times 10^{-15}\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}} = 3.7 \times 10^{12}\ \text{m/s}^2.$$

To find the direction of $\vec{F}^{\text{mag}}$, we use the **Key Idea** that $\vec{F}^{\text{mag}}$ has the direction of the cross product $q\vec{v} \times \vec{B}$. Because the charge q is positive, $\vec{F}^{\text{mag}}$ must have the same direction as $\vec{v} \times \vec{B}$, which can be determined with the right-hand rule for cross products (as in Fig. 29-4*b*). We know that $\vec{v}$ is directed horizontally from south to north and $\vec{B}$ is directed vertically up. The right-hand rule shows us that the deflecting force $\vec{F}^{\text{mag}}$ must be directed horizontally from west to east, as Fig. 29-8 shows. (The array of dots in the figure represents a magnetic field directed out of the plane of the figure. An array of Xs would have represented a magnetic field directed into that plane.)

If the charge of the particle were negative, the magnetic deflecting force would be directed in the opposite direction—that is, horizontally from east to west. This is predicted automatically by Eq. 29-2, if we substitute a negative value for q.

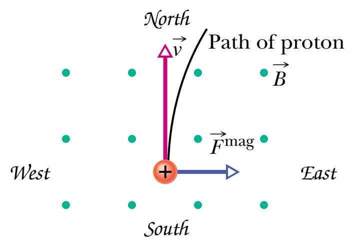


FIGURE 29-8 ■ An overhead view of a proton moving from south to north with velocity $\vec{v}$ in a chamber. A magnetic field is directed vertically upward in the chamber, as represented by the array of dots (which resemble the tips of arrows). The proton is deflected toward the east.

29-5 A Circulating Charged Particle

Remember that when we studied projectile motion we found that the (vertical) gravitational acceleration had no effect on the horizontal velocity of the projectile. Furthermore, when we studied uniform circular motion, we found that the (radial) centripetal acceleration only changed the direction of the object's velocity (keeping it moving in a circle), but did not speed it up or slow it down. This is a general relationship: The component of acceleration that is perpendicular to the direction of velocity only changes the direction of the velocity, not the magnitude.

We have a similar situation here. If we have a charged particle whose size is small enough to ignore, the magnetic force the particle feels is always perpendicular to its velocity and not its magnitude. As we established earlier, if the velocity and magnetic field are perpendicular (and there are no other forces on the particle), the particle will move in a circle.

If a particle moves in a circle at constant speed, we can be sure that the net force acting on the particle is constant in magnitude and is centripetal. That is, the force points toward the center of the circle, always perpendicular to the particle's velocity. Think of a stone tied to a string and whirled in a circle on a smooth horizontal surface, or of a satellite moving in a circular orbit around the Earth. In the first case, the tension in the string provides the necessary force and centripetal acceleration. In the second case, Earth's gravitational attraction provides the force and acceleration.

Figure 29-9 shows another example of a centripetal magnetic force: A beam of electrons is projected into a chamber by an *electron gun* G. The electrons enter in the plane of the page with speed v and move in a region of uniform magnetic field $\vec{B}$ directed out of the plane of the figure. As a result, a magnetic force $\vec{F}^{\text{mag}} = q\vec{v} \times \vec{B}$ continually deflects the electrons, and because the particle's velocity, $\vec{v}$, and the magnetic field it passes through, $\vec{B}$, are always perpendicular to each other, this deflection causes the electrons to follow a circular path. The path is visible in the photo because atoms of gas in the chamber emit light when some of the circulating electrons collide with them.

We would like to determine the parameters that characterize the circular motion of these electrons, or of any particle having an amount of charge $|q|$ and mass m moving perpendicular to a uniform magnetic field $\vec{B}$ at speed v. From Eq. 29-3, the force acting on the particle has a magnitude of $|q|vB$. From Newton's Second Law ($\vec{F} = m\vec{a}$) applied to uniform circular motion (Eq. 5-34),

$$F^{\text{mag}} = m\frac{v^2}{r}, \tag{29-6}$$

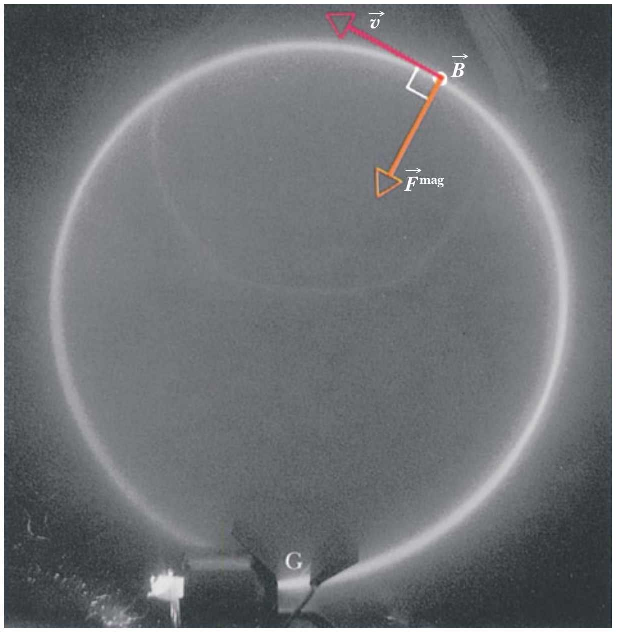


FIGURE 29-9 ■ Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field $\vec{B}$, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force $\vec{F}^{\,mag}$; for circular motion to occur, $\vec{F}^{\,mag}$ must point toward the center of the circle. Use the right-hand rule for cross products to confirm that $\vec{F}^{\,mag} = q\vec{v} \times \vec{B}$ gives $\vec{F}^{\,mag}$ the proper direction. (Don't forget to incude the sign of q.)

we have

$$|q|vB = \frac{mv^2}{r}. \tag{29-7}$$

Solving for r, we find the radius of the circular path as

$$r = \frac{mv}{|q|B} \quad \text{(radius of circular path).} \tag{29-8}$$

The period T (the time for one full revolution) is equal to the circumference divided by the speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\,\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \quad \text{(period).} \tag{29-9}$$

The frequency f (the number of revolutions per unit time) is

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \quad \text{(frequency).} \tag{29-10}$$

The angular frequency ω of the motion is then

$$\omega = 2\pi f = \frac{|q|B}{m} \quad \text{(angular or cyclotron frequency).} \tag{29-11}$$

The quantities T, f, and ω do not depend on the speed of the particle (provided that speed is much less than the speed of light). Fast particles move in large circles and slow ones in small circles, but all particles with the same charge-to-mass ratio q/m take the same time T (the period) to complete one round trip. A bigger velocity makes the particle travel in a larger circle. The increase in speed is exactly compensated by the increase in distance, so the time it takes to go around the circle is the same. We see later that this plays an important role in the construction of a charged particle accelerator known as a **cyclotron.** Using Eq. 29-2, you can show that if you are looking in the direction of $\vec{B}$, the direction of rotation for a positive particle is always counterclockwise; the direction for a negative particle is always clockwise.

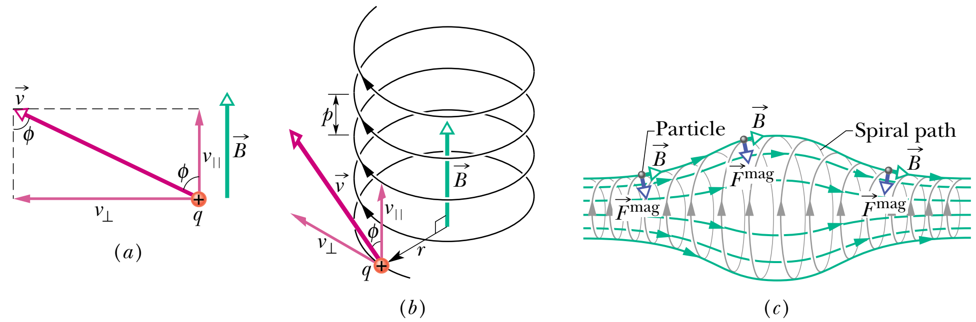


FIGURE 29-10 ■ (*a*) A charged particle moves in a uniform magnetic field $\vec{B}$, its velocity $\vec{v}$ making an angle ϕ with the field direction. (*b*) The particle follows a helical path, of radius *r* and pitch *p*. (*c*) A charged particle spiraling in a nonuniform magnetic field. (The particle can become trapped, spiraling back and forth between the strong field regions at either end.) Note that the magnetic force vectors at the left and right sides have a component pointing toward the center of the figure.

Helical Paths

As we discussed in regard to the electrons and positron in the bubble chamber of Figure 29-5, if the velocity of a charged particle moving through a magnetic field is changing, the particle will move in a shrinking spiral, rather than a circle. One way this can happen is for the particle to be slowed by frictional or other forces. Furthermore, if the velocity of a charged particle has a component parallel to the (uniform) magnetic field, the particle will move in a helical path about the direction of the field vector. Figure 29-10*a*, for example, shows the velocity vector $\vec{v}$ of such a particle resolved into two components, one parallel to $\vec{B}$ and one perpendicular to it:

$$v_\parallel = |\vec{v}|\cos\phi \quad \text{and} \quad v_\perp = |\vec{v}|\sin\phi. \tag{29-12}$$

The parallel component determines the *pitch p* of the helix—that is, the distance between adjacent turns (Fig. 29-10*b*). The perpendicular component determines the radius of the helix and is the quantity to be substituted for $|\vec{v}|$ in Eq. 29-8.

Figure 29-10*c* shows a charged particle spiraling in a nonuniform magnetic field. The more closely spaced field lines at the left and right sides indicate that the magnetic field is stronger there. When the field at an end is strong enough, the particle "reflects" from that end. If the particle reflects from both ends, it is said to be trapped in a *magnetic bottle*.

Confining Particles in a Tokomak Reactor

In the chapter opener we explained that in order to induce fusion reactions capable of releasing large amounts of energy, we must fuse light atoms together. To do this we need to confine ions having very high energy, and hence high temperature. Magnetic fields are ideal for containing the ions because both the ions and the electrons are charged and will spiral along magnetic field lines instead of hitting the walls of a containment vessel.

Scientists have not yet been able to confine charged particles at high enough temperatures to achieve controlled fusion. However, experiments reveal that one of the most effective configurations of magnetic field lines for containing the light atomic ions is shaped like a torus. A torus is basically a donut shape. The containment vessel of the Joint European Torus, commonly known as a tokomak, is shown at the beginning of this chapter. In a tokomak reactor, the magnetic field is produced by a series of magnetic coils that are evenly spaced around the torus-shaped containment vessel as shown in Fig. 29-11. The magnetic field lines form continuous loops inside the ring of the torus. In theory, when a tokomak is working properly, the high temperature ions and electrons should revolve in helical paths around the field lines. An ion can then travel in a continuous loop until it undergoes a fusion reaction with another ion.

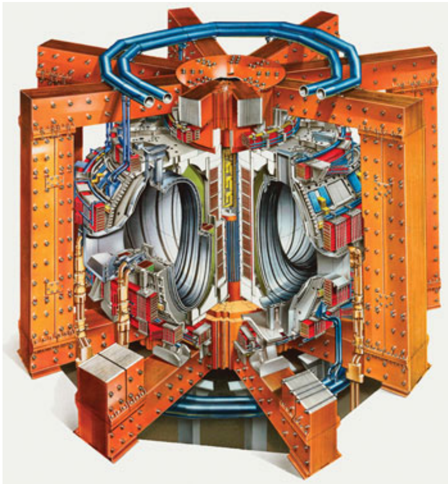


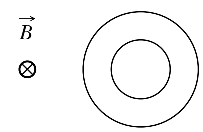
FIGURE 29-11 ■ A cutaway drawing of the JET tokomak showing the donut shaped containment vessel and surrounding magnetic coils.

Particles Trapped in the Earth's Magnetic Field

The terrestrial magnetic field acts as a magnetic bottle, trapping electrons and protons; the trapped particles form the *Van Allen radiation belts,* which loop well above the Earth's atmosphere between Earth's north and south geomagnetic poles. These particles bounce back and forth, from one end of this magnetic bottle to the other, within a few seconds.

When a large solar flare shoots additional energetic electrons and protons into the radiation belts, an electric field is produced in the region where electrons normally reflect. This field eliminates the reflection and instead drives electrons down into the atmosphere, where they collide with atoms and molecules of air, causing that air to emit light. This light forms the aurora—a curtain of light that hangs down to an altitude of about 100 km. Green light is emitted by oxygen atoms, and pink light is emitted by nitrogen molecules, but often the light is so dim that we perceive only white light.

READING EXERCISE 29-2: The figure shows the circular paths of two particles that travel at the same speed in a uniform magnetic field $\vec{B}$, which is directed into the page. One particle is a proton; the other is an electron (which is less massive). The relative sizes of the circles are not to scale. (a) Which particle follows the smaller circle, and (b) does that particle travel clockwise or counterclockwise?



TOUCHSTONE EXAMPLE 29-2: Mass Spectrometer

Figure 29-12 shows the essentials of a *mass spectrometer,* which can be used to measure the mass of an ion; an ion of mass m (to be measured) and charge q is produced in source S. The initially stationary ion is accelerated by the electric field due to a potential difference ΔV. The ion leaves S and enters a separator chamber in which a uniform magnetic field $\vec{B}$ is perpendicular to the path of the ion. The magnetic field causes the ion to move in a semicircle, striking (and thus altering) a photographic plate at distance x from the entry slit. Suppose that in a certain trial $B = 80.000$ mT and $\Delta V = 1000.0$ V, and ions of charge $q = +1.6022 \times 10^{-19}$ C strike the plate at $x = 1.6254$ m. What is the mass m of the individual ions, in unified atomic mass units ($1 \text{ u} = 1.6605 \times 10^{-27}$ kg)?

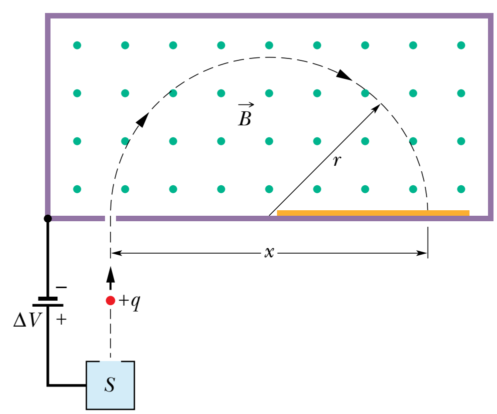


FIGURE 29-12 ■ Essentials of an early model of a mass spectrometer. A positive ion, after being accelerated from its source S by potential difference ΔV, enters a chamber of uniform magnetic field $\vec{B}$. There it travels through a semicircle of radius r and strikes a photographic plate at a distance x from where it entered the chamber.

SOLUTION ■ One **Key Idea** here is that, because the (uniform) magnetic field causes the (charged) ion to follow a circular path, we can relate the ion's mass m to the path's radius r with Eq. 29-8 ($r = m|\vec{v}|/|q\vec{B}|$). From Fig. 29-12 we see that $r = x/2$, and we are given the magnitude $|\vec{B}|$ of the magnetic field. However, we don't know the ion's speed v in the magnetic field, after it has been accelerated due to the potential difference ΔV.

To relate v and ΔV, we use the **Key Idea** that mechanical energy ($E^{\text{mec}} = K + U$) of the mass spectrometer system is conserved during the acceleration. When the ion emerges from the source, its kinetic energy is approximately zero. At the end of the acceleration, its kinetic energy is $\frac{1}{2}mv^2$. Also, during the acceleration, the positive ion moves through a change in potential of $-\Delta V$. Thus, because the ion has positive charge q, its potential energy changes by $-q\Delta V$. If we now write the conservation of the system's mechanical energy as

$$\Delta K + \Delta U = 0,$$

we get

$$\tfrac{1}{2}mv^2 - q\Delta V = 0$$

or

$$v = \sqrt{\frac{2|q\Delta V|}{m}}. \tag{29-13}$$

Substituting this into Eq. 29-8 gives us

$$r = \frac{mv}{|q\vec{B}|} = \frac{m}{|q|B}\sqrt{\frac{2|q\Delta V|}{m}} = \frac{1}{B}\sqrt{\frac{2m|\Delta V|}{|q|}}.$$

Thus,
$$x = 2r = \frac{2}{B}\sqrt{\frac{2m|\Delta V|}{|q|}}.$$

Solving this for m and substituting the given data yield

$$m = \frac{B^2|q|x^2}{8|\Delta V|}$$
$$= \frac{(0.080000\ \text{T})^2\,(1.6022 \times 10^{-19}\ \text{C})(1.6254\ \text{m})^2}{8(1000.0\ \text{V})}$$
$$= 3.3863 \times 10^{-25}\ \text{kg} = 203.93\ \text{u}. \qquad \text{(Answer)}$$

29-6 Crossed Fields: Discovery of the Electron

As we have seen, both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ can produce a force on a charged particle. When the two fields are perpendicular to each other, they are said to be *crossed fields*. Here we shall examine what happens to charged particles—namely, electrons—as they move through crossed fields. We use as our example the experiment that led to the discovery of the electron in 1897 by J. J. Thomson at Cambridge University.

Figure 29-13 shows a modern, simplified version of Thomson's experimental apparatus—a *cathode ray tube* (which is like the picture tube in a standard television set). Charged particles (which we now know as electrons) are emitted by a hot filament at the rear of the evacuated tube and are accelerated by an applied potential difference ΔV. After the electrons pass through a slit in screen C, they form a narrow beam. They then pass through a region of crossed $\vec{E}$ and $\vec{B}$ fields, headed toward a fluorescent screen S, where they produce a spot of light (on a television screen the spot is part of the picture). The forces on the charged particles in the crossed-fields region can deflect them from the center of the screen. By controlling the magnitudes and directions of the fields, Thomson could thus control where the spot of light appeared on the screen. Recall that the force on a negatively charged particle due to an electric field is directed opposite the field. Thus, for the particular field arrangement of Fig. 29-13, electrons are forced up the page by the electric field $\vec{E}$ and down the page by the magnetic field $\vec{B}$; that is, the forces are *in opposition*. Thomson's procedure was equivalent to the following series of steps:

1. Set $\vec{E} = 0$ N/C and $\vec{B} = 0$ T and note the position of the spot on screen S due to the undeflected beam.
2. Turn on $\vec{E}$ and measure the resulting beam deflection.
3. Maintaining $\vec{E}$, now turn on $\vec{B}$ and adjust its value until the beam returns to the undeflected position. (With the forces in opposition, they can be made to cancel.)

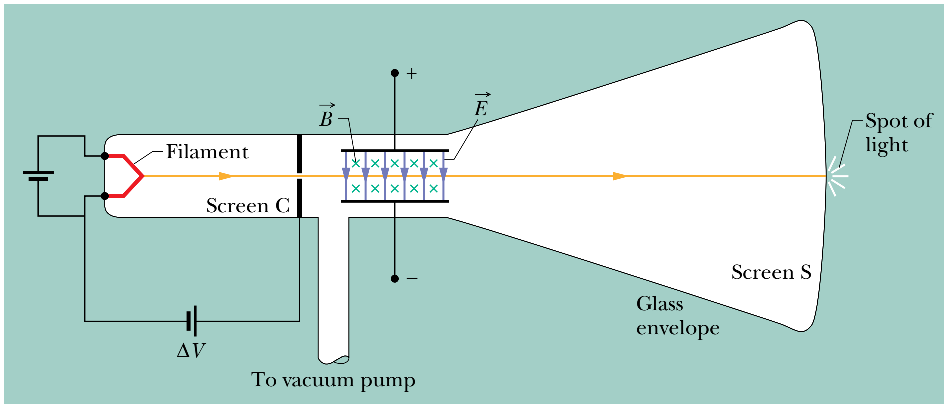


FIGURE 29-13 ■ A modern version of J. J. Thomson's apparatus for measuring the ratio of mass to the amount of charge of an electron. The electric field $\vec{E}$ is established by connecting a battery across the deflecting-plate terminals. The magnetic field $\vec{B}$ is set up by means of a current in a system of coils (not shown). The magnetic field shown is into the plane of the figure, as represented by the array of Xs (which resemble the feathered ends of arrows).

We discussed the deflection of a charged particle moving perpendicular to an electric field $\vec{E}$ between two plates (step 2 here) in Touchstone Example 23-4. We found that the magnitude of the deflection of the particle at the far end of the plates is

$$|\Delta y| = \frac{|q|EL^2}{2mv^2}, \tag{29-14}$$

where v is the particle's initial speed (which was v_x in Touchstone Example 23-4), m its mass, and q its charge, and L is the length of the plates. So long as the particle's deflection is small, we can apply this same equation to the beam of electrons in Fig. 29-13; if necessary, we can calculate the deflection by measuring the deflection of the beam on screen S and then working back to calculate the deflection y at the end of the plates. (Because the direction of the deflection is set by the sign of the particle's charge, Thomson was able to show that the particles lighting up his screen were negatively charged.)

When the two fields in Fig. 29-13 are adjusted so that the two deflecting forces cancel (step 3), we have from Eqs. 29-1 and 29-3,

$$|q|E = |q|vB\sin(90°) = |q|vB,$$

so the particle speed v is given by the ratio of the field magnitudes

$$v = \frac{E}{B}. \tag{29-15}$$

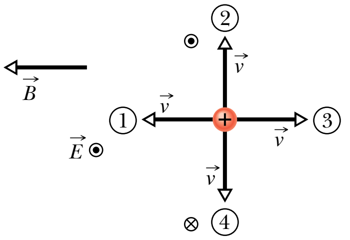
Thus, the crossed fields allow us to measure the speed of the charged particles passing through them. Substituting Eq. 29-15 for $|\vec{v}|$ in Eq. 29-14 and rearranging yield

$$\frac{m}{|q|} = \frac{B^2L^2}{2|\Delta y|E}, \tag{29-16}$$

in which all quantities on the right can be measured. Thus, the crossed fields allow us to measure the mass-charge amount ratio $m/|q|$ of the particles moving through Thomson's apparatus.

Thomson claimed that these particles are found in all matter. He also claimed that they are lighter than the lightest known atom (hydrogen) by a factor of more than 1000. (The exact ratio proved later to be 1836.15.) His $m/|q|$ measurement, coupled with the boldness of his two claims, is considered to be the moment of "discovery of the electron."

READING EXERCISE 29-3: The figure shows four directions for the velocity vector $\vec{v}$ of a positively charged particle moving through a uniform electric field $\vec{E}$ (directed out of the page and represented by an encircled dot) and a uniform magnetic field $\vec{B}$ (pointing to the left). (a) Rank directions 1, 2, 3, and 4 according to the magnitude of the net force on the particle, greatest first. (b) Of all four directions, which might result in a net force of zero?



29-7 The Hall Effect

In Chapters 22 and 26 we claimed that currents in solid conductors are due to moving electrons, and that the positive nuclei are at rest. What evidence do we have for this claim? In the late 1870s, Edwin H. Hall, a 24-year-old graduate student at the

Johns Hopkins University, investigated the deflection of electric current passing through copper wire when the wire is placed in a magnetic field. The result of his work, which is called the **Hall effect** after him, allows us to answer important questions about the nature of charge carriers. For example, Hall's findings allowed him to determine whether charge carriers in a conductor are positive or negative. In addition, Hall's measurements enabled him to deduce the number of charge carriers per unit volume contained in a given conductor.

What happens to a current-carrying metal wire in a magnetic field if the charge carriers are positive and negative charges are at rest? Figure 29-14*a* shows a copper strip of width d, carrying a current i that is assumed to be made up of positive charge carriers (the convention at the time) moving from the top of the figure to the bottom. The charge carriers drift (with an average speed $|\langle\vec{v}\rangle|$) in the direction of the current, from top to bottom. At the instant shown in Fig. 29-14*a*, an external magnetic field $\vec{B}$, pointing into the plane of the figure, has just been turned on. From Eq. 29-2 we see that a deflecting magnetic force $\vec{F}^{\,\text{mag}}$ will act on each drifting positive charge, pushing it toward the right edge of the strip.

As time goes on, positive charges pile up on the right edge of the strip, leaving uncompensated negative charges in fixed positions at the left edge. The separation of positive and negative charges produces a constant electric field $\vec{E}$ within the strip, pointing from right to left. This field exerts an average electrostatic force $\langle\vec{F}^{\,\text{elec}}\rangle$ on a typical positive charge, tending to push it back toward the left.

An equilibrium quickly develops in which the electric force on each positive charge (pushing left) builds up until it just cancels the magnetic force (pushing right). When this happens, as Fig. 29-14*b* shows, the force due to $\vec{B}$ and the force due to $\vec{E}$ are in balance. The drifting positive charges then move along the strip toward the bottom of the page at an average velocity $\langle\vec{v}\rangle$, with no further collection of positive charge on the right edge of the strip and thus no further increase in the electric field $\vec{E}$.

A *Hall potential difference* ΔV is associated with the electric field across strip width d. Because the field is constant, we use Eq. 25-39 to get

$$|\Delta V| = Ed. \tag{29-17}$$

By connecting a voltmeter across the width, we can measure the potential difference between the two edges of the strip. Moreover, the voltmeter can tell us which edge is at higher potential. This information, in turn, tells us whether our charge carriers are positive or negative.

So what do we find? For the situation of Fig. 29-14*a*, we find that the *left* edge is at *higher* potential, meaning we have a buildup of positive charge there. This result is inconsistent with our assumption that the charge carriers are positive.

Suppose we make the opposite assumption, that the charge carriers in current i are negative, as shown in Fig. 29-15. The negative charge carriers drift (with an average speed $|\langle\vec{v}\rangle|$) in the *opposite* direction of the conventional current, from bottom to top. You can use the magnetic force law (Eq. 29-2) to convince yourself that as these charge carriers move from bottom to top in the strip, they are pushed to the right edge by $\vec{F}^{\,\text{mag}}$ and thus that the *left* edge is at higher potential. Because that last statement is in fact what we actually observe with a voltmeter, we conclude that the charge carriers must be negative.

Now for the quantitative part. When the electric and magnetic forces are in balance (Fig. 29-14*b*), Eqs. 29-1 and 29-3 give us a relationship between the magnitudes of the electric and magnetic fields:

$$eE = e|\langle\vec{v}\rangle|B, \tag{29-18}$$

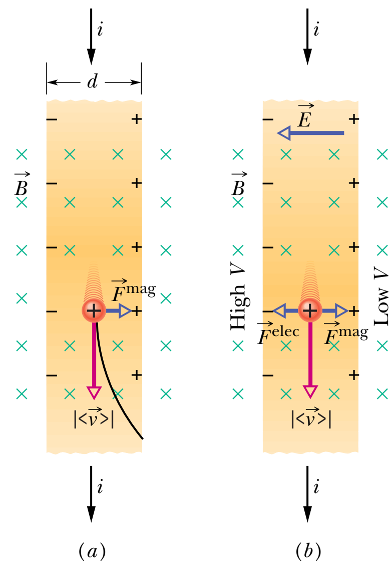


FIGURE 29-14 ■ What would happen if a positive current were to flow through a strip of copper immersed in a magnetic field $\vec{B}$? (*a*) As soon as the magnetic field is turned on, the positive charges follow a curved path as shown. (*b*) A short time later positive charges pile up on the right side of the strip. Thus, the right side of the strip has a higher potential than the left side. Since the higher potential is observed on the left not the right, we conclude that the *charge carriers are not positive.*

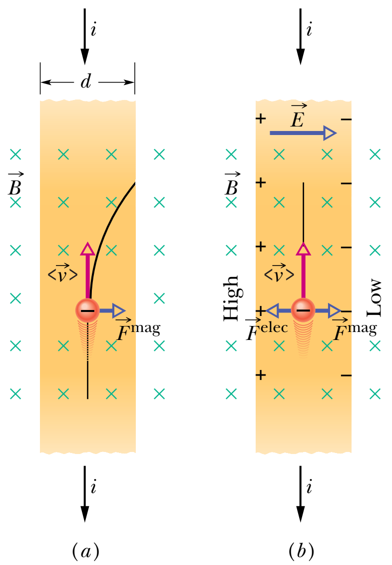


FIGURE 29-15 What should happen when the conventional current, i, actually consists of a negative electron current flowing in the opposite direction? (a) As soon as the magnetic field is turned on, electrons follow the curved path shown. (b) A short time later negative charges pile up on the right side of the strip so that a higher potential develops on the left. Since this prediction matches experimental findings we must conclude that the *charge carriers are negative*.

where e is the amount of the charge on the electron. From Eq. 26-21, the average or drift speed $|\langle\vec{v}\rangle|$ is

$$|\langle\vec{v}\rangle| = \frac{|\vec{J}|}{ne} = \frac{|i|}{neA}, \tag{29-19}$$

in which $|\vec{J}|(=|i|/A)$ is the current density in the strip, A is the cross-sectional area of the strip, and n is the *number density* of charge carriers (their number per unit volume).

In Eq. 29-18, substituting $|\Delta V|/d$ for E (Eq. 29-17) and substituting for $|\langle\vec{v}\rangle|$ with the rightmost term in Eq. 29-19, we obtain

$$n = \frac{|i|B}{e\ell|\Delta V|}, \tag{29-20}$$

in which $\ell = A/d$ is the thickness of the strip. With this equation we can find n from measurable quantities.

It is also possible to use the Hall effect to measure directly the average or drift speed $|\langle\vec{v}\rangle|$ of the charge carriers, which you may recall is of the order of centimeters per hour. In this clever experiment, the metal strip is moved mechanically through the magnetic field in a direction opposite that of the drift velocity of the charge carriers. The speed of the moving strip is then adjusted until the Hall potential difference vanishes. At this condition, with no Hall effect, the velocity of the charge carriers *with respect to the laboratory frame* must be zero, so the velocity of the strip must be equal in magnitude but opposite in direction to the velocity of the negative charge carriers.

TOUCHSTONE EXAMPLE 29-3: Motional Potential Difference

Figure 29-16 shows a solid metal cube, of edge length $d = 1.5$ cm, moving in the positive y direction at a constant velocity $\vec{v}$ of magnitude 4.0 m/s. The cube moves through a uniform magnetic field $\vec{B}$ of magnitude 0.050 T directed toward positive z.

(a) Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?

SOLUTION ■ One **Key Idea** here is that, because the cube is moving through a magnetic field $\vec{B}$, a magnetic force $\vec{F}^{\text{mag}}$ acts on its charged particles, including its conduction electrons. A second **Key Idea** is how $\vec{F}^{\text{mag}}$ causes an electric potential difference between certain faces of the cube. When the cube first begins to move through the magnetic field, its electrons do also. Because each electron has charge $q = -e$ and is moving through a magnetic field with velocity $\vec{v}$, the magnetic force $\vec{F}^{\text{mag}}$ acting on it is given by Eq. 29-2. Because q is negative, the direction of $\vec{F}^{\text{mag}}$ is opposite the cross product $\vec{v} \times \vec{B}$, which is in the positive direction of the x axis in Fig. 29-16. Thus, $\vec{F}^{\text{mag}}$ acts in the negative direction of the x axis, toward the left face of the cube (which is hidden from view in Fig. 29-16).

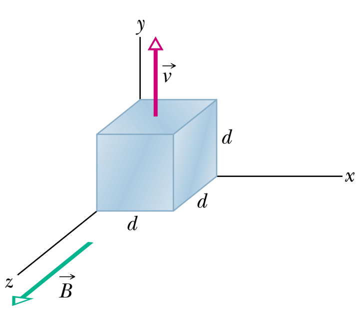


FIGURE 29-16 A solid metal cube of edge length d moves at constant velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$.

Most of the electrons are fixed in place in the molecules of the cube. However, because the cube is a metal, it contains conduction electrons that are free to move. Some of those conduction electrons are deflected by $\vec{F}^{\text{mag}}$ to the left cube face, making that face negatively charged and leaving the right face positively charged. This charge separation produces an electric field $\vec{E}$ directed from the positively charged right face to the negatively charged left face. Thus, the left face is at a lower electric potential, and the right face is at a higher electric potential.

(b) What is the potential difference between the faces of higher and lower electric potential?

SOLUTION ■ The **Key Ideas** here are these:

1. The electric field $\vec{E}$ created by the charge separation produces an electric force $\vec{F}^{\text{elec}} = q\vec{E}$ on each electron. Because q is

negative, this force is directed opposite the field $\vec{E}$—that is, toward the right. Thus on each electron, $\vec{F}^{\,\text{elec}}$ acts toward the right and $\vec{F}^{\,\text{mag}}$ acts toward the left.

2. When the cube had just begun to move through the magnetic field and the charge separation had just begun, the magnitude of $\vec{E}$ began to increase from zero. Thus, the magnitude of $\vec{F}^{\,\text{elec}}$ also began to increase from zero and was initially smaller than the magnitude $\vec{F}^{\,\text{mag}}$. During this early stage, the net force on any electron was dominated by $\vec{F}^{\,\text{mag}}$, which continuously moved additional electrons to the left cube face, increasing the charge separation.

3. However, as the charge separation increased, eventually magnitude $|\vec{F}^{\,\text{elec}}|$ became equal to magnitude $|\vec{F}^{\,\text{mag}}|$. The net force on any electron was then zero, and no additional electrons were moved to the left cube face. Thus, the magnitude of $\vec{F}^{\,\text{elec}}$ could not increase further, and the electrons were then in equilibrium.

We seek the potential difference ΔV between the left and right cube faces after equilibrium was reached (which occurred quickly). We can obtain the magnitude of ΔV with Eq. 29-17 ($|\Delta V| = Ed$) provided we first find the magnitude $|\vec{E}| = E$ of the electric field at equilibrium. We can do so with the equation for the balance of force magnitudes ($|\vec{F}^{\,\text{elec}}| = |\vec{F}^{\,\text{mag}}|$).

For F^{elec}, we substitute $|q|E$. For F^{mag}, we substitute $|q|vB\sin\phi$ from Eq. 29-3. From Fig. 29-16, we see that the angle ϕ between v and B is 90°; so $\sin\phi = 1$. We can now write ($F^{\text{elec}} = F^{\text{mag}}$) as

$$|q|E = |q|vB\sin 90° = |q|vB.$$

This gives us $E = vB$, so Eq. 29-17 ($|\Delta V| = Ed$) becomes

$$|\Delta V| = |V_{\text{left}} - V_{\text{right}}| = vBd. \tag{29-21}$$

Substituting known values gives us

$$|\Delta V| = (4.0 \text{ m/s})(0.050 \text{ T})(0.015 \text{ m})$$
$$= 0.0030 \text{ V} = 3.0 \text{ mV}.$$

Since the left face of the cube has excess negative charges, the right face is at a higher potential than the left face by 3.0 mV. (Answer)

29-8 Magnetic Force on a Current-Carrying Wire

We have just seen that a magnetic field exerts a sideways force on electrons moving in a wire. This force must then be transmitted to the wire itself, because the conduction electrons cannot escape sideways out of the wire.

In Fig. 29-17*a*, a vertical wire, carrying no current and fixed in place at both ends, extends through the gap between the vertical pole faces of a magnet represented by the shaded circle. The magnetic field between the faces is directed outward from the page. In Fig. 29-17*b*, a current is sent upward through the wire; the wire deflects to the right. In Fig. 29-17*c*, we reverse the direction of the current and the wire deflects to the left.

Figure 29-18 shows what happens inside the wire of Fig. 29-17. We see one of the conduction electrons, drifting downward with an assumed average (drift) speed $|\langle\vec{v}\rangle|$. Equation 29-3, in which we must put $\phi = 90°$, tells us that a force of magnitude $F^{\text{mag}} = e|\langle\vec{v}\rangle|B$ must act on a typical electron. From Eq. 29-2 we see that this force must be directed to the right. We expect then that the wire as a whole will experience a force to the right, in agreement with Fig. 29-17*b*.

If, in Fig. 29-18, we were to reverse *either* the direction of the magnetic field *or* the direction of the current, the force on the wire would reverse, being directed now to the left. Note too that it does not matter whether we consider negative charges drifting downward in the wire (the actual case) or positive charges drifting upward. The direction of the deflecting force on the wire is the same. We are safe then in dealing with a current of positive charge.

Consider a length L of the wire in Fig. 29-18. All the conduction electrons in this section of wire will drift past a plane that is parallel to xx' (shown in Fig. 29-18) in a time $\Delta t = L/|\langle\vec{v}\rangle|$. Thus, in that time the charge that will pass through the plane is given by

$$q = i\,\Delta t = i\frac{L}{|\langle\vec{v}\rangle|}.$$

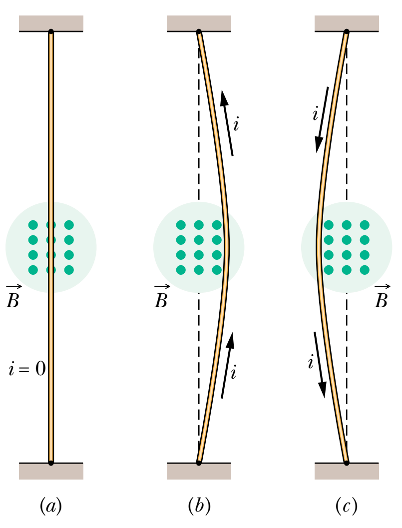


FIGURE 29-17 ■ A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (*a*) Without current in the wire, the wire is straight. (*b*) With upward current, the wire is deflected rightward. (*c*) With downward current, the deflection is leftward. Connections for getting the current into one end of the wire and out of the other are not shown.

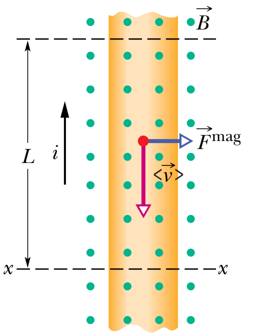


FIGURE 29-18 ■ A close-up view of a section of the wire of Fig. 29-17*b*. The current direction is upward, which means that electrons drift downward. A magnetic field that emerges from the plane of the page causes the electrons and the wire to be deflected to the right.

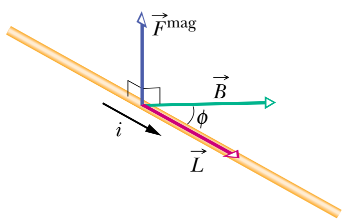


FIGURE 29-19 ■ A wire carrying current i makes an angle ϕ with magnetic field $\vec{B}$. The wire has length L in the field and length vector $\vec{L}$ (in the direction of the current). A magnetic force $\vec{F}^{\text{mag}} = i\vec{L} \times \vec{B}$ acts on the wire.

Substituting this into Eq. 29-3 yields the following expressions for the magnitude of the magnetic force

$$F^{\text{mag}} = |q||\langle\vec{v}\rangle|B\sin\phi = \frac{|i|L|\langle\vec{v}\rangle|B}{|\langle\vec{v}\rangle|}\sin 90^\circ$$

or

$$F^{\text{mag}} = |i|LB. \tag{29-22}$$

This equation gives the magnetic force that acts on a length L of straight wire carrying a current i and immersed in a magnetic field $\vec{B}$ that is perpendicular to the wire.

If the magnetic field is *not* perpendicular to the wire, as in Fig. 29-19, the magnetic force is given by a generalization of Eq. 29-22:

$$\vec{F}^{\text{mag}} = |i|\vec{L} \times \vec{B} \qquad \text{(force on a current).} \tag{29-23}$$

Here $\vec{L}$ is a *length vector* that has magnitude $|\vec{L}|$ and is directed along the wire segment in the direction of the (conventional) current. The magnitude of the magnetic field is

$$F^{\text{mag}} = |i|LB\sin\phi, \tag{29-24}$$

where ϕ is the smaller angle between the directions of $\vec{L}$ and $\vec{B}$. The direction of $\vec{F}^{\text{mag}}$ is that of the cross product $\vec{L} \times \vec{B}$, because we take current i to be a positive quantity. Equation 29-23 tells us that $\vec{F}^{\text{mag}}$ is always perpendicular to the plane defined by $\vec{L}$ and $\vec{B}$, as indicated in Fig. 29-19.

Equation 29-23 is equivalent to Eq. 29-2 in that either can be taken as the defining equation for $\vec{B}$. In practice, we define $\vec{B}$ from Eq. 29-23. It is much easier to measure the magnetic force acting on a wire than that on a single moving charge.

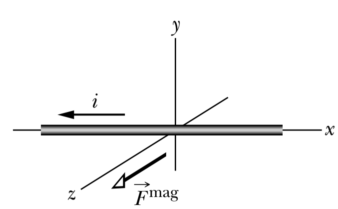
If a wire is not straight or the field is not uniform, we can imagine it broken up into small straight segments and apply Eq. 29-23 to each short segment $d\vec{L}$. The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write

$$d\vec{F}^{\text{mag}} = i\,d\vec{L} \times \vec{B}, \tag{29-25}$$

and we can find the resultant force on any given arrangement of currents by integrating Eq. 29-25 over that arrangement.

In using Eq. 29-25, bear in mind that there is no such thing as an isolated current-carrying wire segment of length $d\vec{L}$. There must always be a way to introduce the current into the segment at one end and take it out at the other end.

READING EXERCISE 29-4: The figure shows a current i through a wire in a uniform magnetic field $\vec{B}$, as well as the magnetic force $\vec{F}^{\text{mag}}$ acting on the wire. The field is oriented so that the magnitude force is a maximum. In what direction is the field?



TOUCHSTONE EXAMPLE 29-4: Levitating a Wire

A straight, horizontal length of copper wire has a current $i = 28$ A through it. If this current is directed out of the page as shown in Fig. 29-20, what are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) is 46.6 g/m.

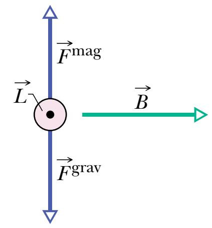


FIGURE 29-20 ■ A current-carrying wire (shown in cross section) can be made to "float" in a magnetic field. The current in the wire emerges from the plane of the page, and the magnetic field is directed to the right.

SOLUTION ■ One **Key Idea** is that, because the wire carries a current, a magnetic force $\vec{F}^{\,mag}$ can act on the wire if we place it in a magnetic field $\vec{B}$. To balance the downward gravitational force $\vec{F}^{\,grav}$ on the wire, we want $\vec{F}^{\,mag}$ to be directed upward (Fig. 29-20).

A second **Key Idea** is that the direction of $\vec{F}^{\,mag}$ is related to the directions of $\vec{B}$ and the wire's length vector $\vec{L}$ by Eq. 29-23. Because $\vec{L}$ is directed horizontally (and the current is taken to be positive), Eq. 29-23 and the right-hand rule for cross products tell us that $\vec{B}$ must be horizontal and rightward (in Fig. 29-20) to give the required upward $\vec{F}^{\,mag}$.

The magnitude of $\vec{F}^{\,mag}$ is given by Eq. 29-24 ($|\vec{F}^{\,mag}| = |i\vec{L}||\vec{B}|\sin\phi$). Because we want $\vec{F}^{\,mag}$ to balance $\vec{F}^{\,grav}$, we want

$$|i|LB\sin\phi = mg, \qquad (29\text{-}26)$$

where mg is the magnitude of $\vec{F}^{\,grav}$ and m is the mass of the wire. We also want the minimal field magnitude B for $\vec{F}^{\,mag}$ to balance $\vec{F}^{\,grav}$. Thus, we need to maximize $\sin\phi$ in Eq. 29-26. To do so, we set $\phi = 90°$, thereby arranging for $\vec{B}$ to be perpendicular to the wire. We then have $\sin\phi = 1$, so Eq. 29-26 yields a magnetic field magnitude of

$$|\vec{B}| = B = \frac{mg}{|i|L\sin\phi} = \frac{(m/L)g}{|i|}. \qquad (29\text{-}27)$$

We write the result this way because we know m/L, the linear density of the wire. Substituting known data then gives us a magnitude of

$$B = \frac{(46.6 \times 10^{-3}\text{ kg/m})(9.8\text{ m/s}^2)}{28\text{ A}}$$
$$= 1.6 \times 10^{-2}\text{ T}. \qquad \text{(Answer)}$$

This is about 160 times the strength of Earth's magnetic field. As stated in the second paragraph of this solution, the right-hand rule tells us that B must point to the right.

29-9 Torque on a Current Loop

Much of the world's work is done by electric motors. The forces that do this work are magnetic. In principle a direct current motor can be constructed from a single loop of current-carrying wire that is immersed in a magnetic field and is attached to a battery. If the current were to flow through the loop in the same direction all the time, the magnetic field would push on this loop in one direction at one instant of time, but would reverse the direction of the force when the loop was rotated halfway around. We would get a vibration that would quickly damp out. We can, however, get a continuous rotation if we use a connection, called a commutator, that reverses the current direction when the loop has gone halfway around (Fig. 29-21). Then, the force will continue to push the loop in the same direction and the motor will spin. Although many essential details have been omitted, the figure does suggest how the action of a magnetic field on a current loop produces rotary motion. To understand how the dc motor works in detail, we need to understand how a magnetic field can cause a current-carrying wire loop to rotate by exerting a torque on it.

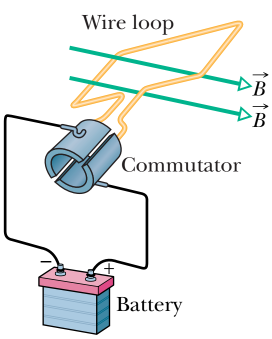


FIGURE 29-21 ■ The elements of an electric motor. A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator reverses the direction of the current every half-revolution so that the torque always acts in the same direction.

How a Current Loop Can Experience a Torque

Figure 29-22*a* shows a front view of a rectangular loop of sides a and b. The loop is carrying a current i and is immersed in a uniform magnetic field $\vec{B}$. We start our consideration of the torque on the loop with a special case in which the plane of the loop is parallel to the magnetic field as shown in Fig. 29-22*a*.

Let's use Eq. 29-24 to find the forces on each side of the loop for our special case. For sides 1 and 3 the vector $\vec{L}$ points in the direction of the current and has magnitude a. The angle between $\vec{L}$ and $\vec{B}$ for these is $\phi = 0°$. Thus, the magnitude of the forces acting on this side is

$$F_1^{mag} = F_3^{mag} = |i|aB\sin 0° = 0\text{ N}.$$

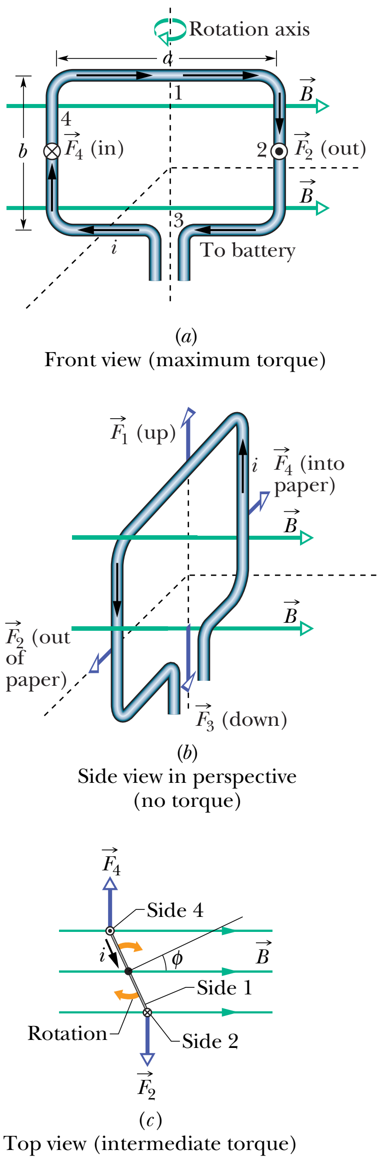


FIGURE 29-22 ■ A rectangular loop, of length a and width b and carrying a current i, is located in a uniform magnetic field. A torque $\vec{\tau}$ that is perpendicular to the magnetic field acts on the loop. The angle, ϕ, perpendicular or normal to the plane of the loop and the B-field varies. (a) The plane of the loop is aligned with the magnetic field so that $\phi = 90°$. (b) A perspective drawing of the loop after it has rotated to $\phi = 0°$ due to the torque exerted on it by the magnetic field. (c) A top view of the loop when it is part way between $\phi = 90°$ (part a) and $\phi = 0°$ (part b).

The situation is different for sides 2 and 4. For them, $\vec{L}$, which has magnitude b, is perpendicular to $\vec{B}$ so $\phi = 90°$. Thus, the forces $\vec{F}_2$ and $\vec{F}_4$ have the common magnitude given by

$$F_2^{\text{mag}} = F_4^{\text{mag}} = |i|bB\sin 90° = |i|bB. \tag{29-28}$$

However, since the direction of the current is different on each of these sides, the right-hand rule tells us that these two forces point in opposite directions. The vector $\vec{F}_2$ points out of the page while the vector $\vec{F}_4$ points into the page. However, as Fig. 29-22a shows, these two forces do *not* share the same line of action so they *do* produce a net torque. The torque tends to rotate the loop toward an orientation for which the plane of the loop is perpendicular to the direction of the magnetic field $\vec{B}$. At $\phi = 90°$ that torque has a moment arm of magnitude $a/2$ about the central axis of the loop. The magnitude of the torque due to forces $\vec{F}_2$ and $\vec{F}_4$ is then (see Fig. 29-22a),

$$\tau_{90} = \frac{a}{2}F_2 + \frac{a}{2}F_4 = |i|\frac{a}{2}(bB) + |i|\frac{a}{2}(bB) = |i|abB. \tag{29-29}$$

As the coil in Fig. 29-22a starts to rotate, the moment arm between sides 2 and 4 decreases, and it reaches zero when the loop is in the position shown in Fig. 29-22b. In general, the torque on the loop is given by

$$\tau' = |i|abB\sin\phi, \tag{29-30}$$

where ϕ is the smaller angle normal to the area subtended by the loop and the external magnetic field (Fig. 22-22c).

Suppose we replace the single loop of current with a *coil* of N loops, or *turns*. Further, suppose that the turns are wound tightly enough that they can be approximated as all having the same dimensions and lying in a plane. Then the turns form a *flat coil* and a torque $\vec{\tau}'$ with the magnitude found in Eq. 29-29 acts on each of the turns. The total torque on the coil then has magnitude

$$\tau = N\tau' = N|i|AB\sin\phi, \tag{29-31}$$

in which $A(= ab)$ is the area enclosed by the coil. Equation 29-31 holds for all flat coils, no matter what their shape, provided the magnetic field is uniform.

How a DC Motor Works

Consider the operation of a motor like that shown in Fig. 29-21. When the coil is at the point where the plane of the coil is perpendicular to the field direction so $\phi = 0°$, the polarity of the battery is suddenly reversed. Since the coil is accelerated by the initial torque on it, it sails past the point where $\phi = 0°$, and a new torque takes over and continues to rotate the coil in the same direction. This automatic reversal of the current occurs every half cycle and is accomplished with a commutator that electrically connects the rotating coil with the stationary contacts connected to the battery (or other power source).

29-10 The Magnetic Dipole Moment

We can describe the current-carrying coil of the preceding section with a single vector $\vec{\mu}$, its magnetic dipole moment. The direction of the magnetic dipole $\vec{\mu}$ is determined by another right hand rule similar to the one shown in Fig. 29-4. If you wrap your right hand around the coil in the direction of the positive current, your thumb points in the direction of the magnetic dipole $\vec{\mu}$.

We define the magnitude of $\mu = |\vec{\mu}|$ as

$$\mu = N|i|A \qquad \text{(magnetic moment magnitude)}, \tag{29-32}$$

in which N is the number of turns in the coil, $|i|$ is the magnitude current through the coil, and A is the area enclosed by each turn of the coil. (Equation 29-32 tells us that the unit of $\vec{\mu}$ is the ampere-square meter.) Using $\vec{\mu}$, we can rewrite Eq. 29-31 for the magnitude of the torque on the coil due to a magnetic field as

$$\tau = |\vec{\mu}||\vec{B}|\sin\phi = \mu B \sin\phi, \tag{29-33}$$

in which ϕ is the smallest angle between the vectors $\vec{\mu}$ and $\vec{B}$.

We can generalize this to the vector relation

$$\vec{\tau} = \vec{\mu} \times \vec{B}, \tag{29-34}$$

which reminds us very much of the corresponding equation for the torque exerted by an *electric* field on an *electric* dipole—namely, Eq. 23-37:

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

In each case the torque exerted by the external field—either magnetic or electric—is equal to the vector product of the corresponding dipole moment and the field vector.

A magnetic dipole in an external magnetic field has a **magnetic potential energy** that depends on the dipole's orientation in the field. For electric dipoles,

$$U(\theta) = -\vec{p}\cdot\vec{E}.$$

In strict analogy, we can write for the magnetic case

$$U(\theta) = -\vec{\mu}\cdot\vec{B}. \tag{29-35}$$

A magnetic dipole has its lowest energy ($-|\vec{\mu}||\vec{B}|\cos\theta = -\mu B$) when its dipole moment $\vec{\mu}$ is lined up with the magnetic field (Fig. 29-23). It has its highest energy ($-\mu B\cos 180° = +\mu B$) when the vector $\vec{\mu}$ is directed opposite the field.

When a magnetic dipole rotates in the presence of a magnetic field from an initial orientation θ_1 to another orientation θ_2, the work $W_{B\to\mu}$ done on the dipole by the magnetic field is

$$W_{B\to\mu} = -\Delta U = -(U_2 - U_1), \tag{29-36}$$

where U_2 and U_1 are calculated with Eq. 29-35. If an external torque acts on the dipole during the change in its orientation, then work $W_{\text{ext}\to\mu}$ is done on the dipole by the external torque. *If the dipole is stationary* before and after the change in its orientation, then work $W_{\text{ext}\to\mu}$ is the negative of the work done on the dipole by the field. Thus,

$$W_{\text{ext}\to\mu} = -W_{B\to\mu} = U_2 - U_1. \tag{29-37}$$

So far, we have identified only a current-carrying coil as a magnetic dipole. However, a simple bar magnet is also a magnetic dipole, as is a rotating sphere of charge. Earth itself is (approximately) a magnetic dipole. And, most subatomic particles, including the electron, the proton, and the neutron, have magnetic dipole moments. As you will see in Chapter 32, all these quantities can be viewed as current loops. For comparison, some approximate magnetic dipole moments are shown in Table 29-2.

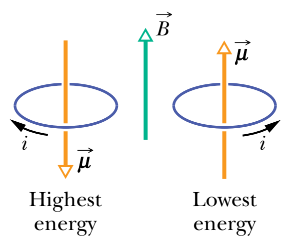
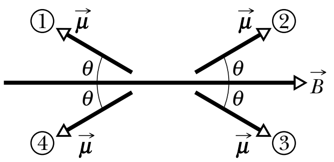


FIGURE 29-23 ■ The orientations of highest and lowest energy of a magnetic dipole in an external magnetic field $\vec{B}$. In each case, the direction of the current i determines the direction of the magnetic dipole moment $\vec{\mu}$ shown in Fig. 29-23 via the right-hand rule.

TABLE 29-2
Some Magnetic Dipole Moments

A small bar magnet	5 J/T
Earth	8.0×10^{22} J/T
A proton	1.4×10^{-26} J/T
An electron	9.3×10^{-24} J/T

READING EXERCISE 29-5: The figure shows four orientations, at angle θ, of a magnetic dipole moment $\vec{\mu}$ in a magnetic field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.



TOUCHSTONE EXAMPLE 29-5: Coil in an External Magnetic Field

Figure 29-24 shows a circular coil with 250 turns, an area A of $2.52 \times 10^{-4}\ \mathrm{m}^2$, and a current of 100 μA. The coil is at rest in a uniform magnetic field of magnitude $|\vec{B}| = 0.85$ T, with its magnetic dipole moment $\vec{\mu}$ initially aligned with $\vec{B}$.

(a) In Fig. 29-24, what is the direction of the current in the coil?

SOLUTION ■ The **Key Idea** here is to apply the right-hand rule to the coil by curling your fingers around the current in the coil so your right thumb points in the $\vec{\mu}$ direction. Thus, in the wires on the near side of the coil—those we see in Fig. 29-24—the current is from top to bottom.

(b) How much work would the torque applied by an external agent have to do on the coil to rotate it 90° from its initial orientation, so that $\vec{\mu}$ is perpendicular to $\vec{B}$ and the coil is again at rest?

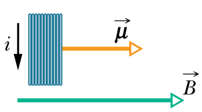


FIGURE 29-24 ■ A side view of a circular coil carrying a current and oriented so that its magnetic dipole moment $\vec{\mu}$ is aligned with magnetic field $\vec{B}$.

SOLUTION ■ The **Key Idea** here is that the work $W_{\text{ext}\to\mu}$ done by the applied torque would be equal to the change in the coil's potential energy due to its change in orientation. From Eq. 29-37 ($W_{\text{ext}\to\mu} = U_2 - U_1$), we find

$$\begin{aligned} W_{\text{ext}\to\mu} &= U(90^\circ) - U(0^\circ) \\ &= -\mu B\cos 90^\circ - (-\mu B\cos 0^\circ) = 0 + \mu B \\ &= \mu B. \end{aligned}$$

Substituting for $\vec{\mu}$ from Eq. 29-32 ($\mu = N|i|A$), we find that

$$\begin{aligned} W_{\text{ext}\to\mu} &= (N|i|AB) \\ &= (250)(100 \times 10^{-6}\ \mathrm{A})(2.52 \times 10^{-4}\ \mathrm{m}^2)(0.85\ \mathrm{T}) \\ &= 5.356 \times 10^{-6}\ \mathrm{J} \approx 5.4\ \mu\mathrm{J}. \qquad \text{(Answer)} \end{aligned}$$

29-11 The Cyclotron

Physicists have been able to use their understanding of how charged particles behave in magnetic fields to develop devices that can accelerate protons to high speeds. These high-energy protons are extremely useful to scientists for several reasons. Collisions between energetic protons and matter allow them to learn about the nature of atomic and subatomic particles. High-energy protons and ions can also be used to create new radioactive elements. In addition, physicians can use high-energy protons to destroy tumors in cancer patients. In 1939, E. O. Lawrence was awarded a Nobel Prize in physics for the development of the cyclotron—the first of many magnetic accelerators capable of accelerating protons, ions, and electrons.

The principles that govern the operation of the cyclotron are quite simple. Figure 29-9 showed experimental evidence that a charged particle projected into an evacuated chamber perpendicular to a uniform magnetic field moves in a circular orbit. We used the magnetic force law (Eq. 29-2) to derive the frequency of revolution of the orbit. In Eq. 29-10 we found that $f = |q|B/2\pi m$. This is known as the cyclotron frequency, and its derivation had a rather surprising outcome. The frequency, f, with which a charged particle moves in its circular orbit depends only on its charge, its mass, and the magnetic field strength. So f is independent of speed. This is because a particle with low speed moves in a small circle whereas one with a higher speed moves in a larger circle. The particle speeds and orbital sizes are related in such a way that all charged particles take the same amount of time to make a revolution in a uniform magnetic field. (At least this is true for all speeds that are well below the speed of light.) Lawrence used the fact that the orbital frequency of a charged particle does not depend on its speed in the design of the cyclotron.

The original cyclotron was first used to accelerate protons. It consisted of two hollow semicircular disks shaped more or less like a capital D as shown in Fig. 29-25. In early cyclotrons, the dees, as they are called, were made of copper sheeting. The diameter of a dee was only about one meter. The dees were then placed in a vacuum chamber and oriented perpendicular to a large uniform magnetic field having a strength of a few teslas. There was a small gap between them. The dees were connected to an electrical oscillator that can alternate the potential difference across the gap between them at exactly the same frequency as an orbiting proton would have in the magnetic field. This arrangement is shown in Fig. 29-26.

To begin the operation of the original cyclotron, an oscillator was set at the cyclotron frequency. Then hydrogen gas was leaked into the vacuum chamber. Next a beam of high-energy electrons was injected into the center of the chamber so that other electrons were knocked out of hydrogen atoms. This ionization process produced protons. At a time when the oscillator caused the left dee to be at a lower potential than the right dee, the proton received a kick in the direction of the right dee. It moved into the right dee where the electric field was zero. However, the magnetic field penetrated the dee and caused the proton to start into a small, low-speed orbit. Only half a cycle later the proton reached the gap again. Since the oscillator was tuned to the cyclotron frequency, the potential of the right dee was now lower than that of the left dee and the proton got another kick as it crossed the gap. The proton then proceeded into another circular orbit that involved a larger speed and radius, given by Eq. 29-8,

$$r = \frac{mv}{|q|B}.$$

When the proton reached the gap again it completed one full cycle but so had the alternating voltage oscillator. Thus the proton got another kick. This process continued, with the circulating proton always being in step with the oscillations of the dee potential. When the proton finally spiraled out to the edge of the dee system, a deflector plate sent it out through a portal. The path of such a proton is shown in Fig. 29-27.

Recall that the key to the operation of the cyclotron is that the frequency f at which the proton circulates in the field (and that does not depend on its speed) must be equal to the fixed frequency f_{osc} of the electrical oscillator, or

$$f = f_{\text{osc}} \qquad \text{(resonance condition).} \tag{29-38}$$

This *resonance condition* says that, if the energy of the circulating proton is to increase, energy must be fed to it at a frequency f_{osc} that is equal to the natural frequency f at which the proton circulates in the magnetic field.

Combining Eqs. 29-10 and 29-38 allows us to write the resonance condition as

$$|q|B = 2\pi m f_{\text{osc}}. \tag{29-39}$$

For the proton, q and m are fixed. The oscillator (we assume) is designed to work at a single fixed frequency f_{osc}. We can then either "tune" the cyclotron by varying either $\vec{B}$ or f_{osc} until Eq. 29-39 is satisfied. Then many protons can circulate through the magnetic field and emerge as a beam.

If the cyclotron is powerful enough to accelerate protons, electrons, or ions to speeds close to that of light, relativistic effects come into play. In such cases the simple resonance condition between orbital and oscillator frequencies no longer hold. More sophisticated magnetic field-based high-energy accelerators called synchrotrons and betatrons have been designed. We introduce relativistic effects in Chapter 38 where we discuss special relativity.

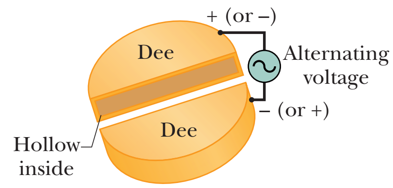


FIGURE 29-25 ■ Cyclotron dees are hollow semicircular metal containers that are open along their diameters.

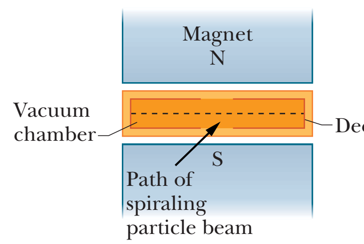


FIGURE 29-26 ■ Cutaway view of dees placed between the poles of a large electromagnet. The dotted line showns the plane in which the paths of the particles orbit.

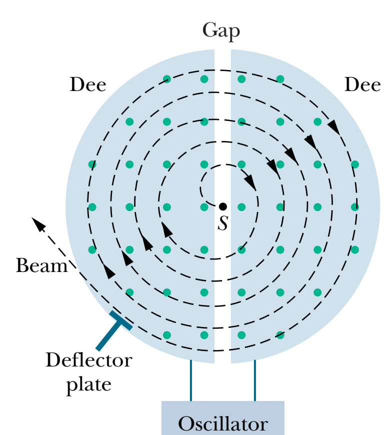


FIGURE 29-27 ■ Top view of dees showing the path of a charged particle beam in a cyclotron. Each time the particle passes through the gap three things happen: (1) the particle gets a kick and is accelerated to a higher speed, (2) the oscillator changes the sign of the gap's potential difference, and (3) the particle goes into a new semicircular orbit with a larger radius than before.

TOUCHSTONE EXAMPLE 29-6: Cyclotron

Suppose a cyclotron is operated at an oscillator frequency of 12 MHz and has a dee radius $R = 53$ cm.

(a) What is the magnitude of the magnetic field needed for deuterons to be accelerated in the cyclotron? A deuteron is the nucleus of deuterium, an isotope of hydrogen. It consists of a proton and a neutron and thus has the same charge as a proton. Its mass is $m = 3.34 \times 10^{-27}$ kg.

SOLUTION ■ The **Key Idea** here is that, for a given oscillator frequency f_{osc}, the magnetic field magnitude B required to accelerate any particle in a cyclotron depends on the ratio m/q of mass to charge for the particle, according to Eq. 29-39. For deuterons and the oscillator frequency $f_{osc} = 12$ MHz, we find

$$B = \frac{2\pi m f_{osc}}{q} = \frac{(2\pi)(3.34 \times 10^{-27}\ \text{kg})(12 \times 10^6\ \text{s}^{-1})}{1.60 \times 10^{-19}\ \text{C}}$$

$$= 1.57\ \text{T} \approx 1.6\ \text{T}. \qquad \text{(Answer)}$$

Note that, to accelerate protons, B would have to be reduced by a factor of 2, providing the oscillator frequency remained fixed at 12 MHz.

(b) What is the resulting kinetic energy of the deuterons?

SOLUTION ■ One **Key Idea** here is that the kinetic energy $\frac{1}{2}mv^2$ of a deuteron exiting the cyclotron is equal to the kinetic energy it had just before exiting, when it was traveling in a circular path with a radius approximately equal to the radius R of the cyclotron dees. A second **Key Idea** is that we can find the speed v of the deuteron in that circular path with Eq. 29-8 ($r = mv/|q|B$). Solving that equation for v, substituting R for r, and then substituting known data, we find

$$v = \frac{RqB}{m} = \frac{(0.53\ \text{m})(1.60 \times 10^{-19}\ \text{C})(1.57\ \text{T})}{3.34 \times 10^{-27}\ \text{kg}}$$

$$= 3.99 \times 10^7\ \text{m/s}.$$

This speed corresponds to a kinetic energy of

$$K = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}(3.34 \times 10^{-27}\ \text{kg})(3.99 \times 10^7\ \text{m/s})^2 \qquad \text{(Answer)}$$

$$= 2.7 \times 10^{-12}\ \text{J},$$

or about 17 MeV.

Problems

SEC. 29-3 ■ DEFINING A MAGNETIC FIELD $\vec{B}$

1. Alpha Particle An alpha particle travels at a velocity $\vec{v}$ of magnitude 550 m/s through a uniform magnetic field $\vec{B}$ of magnitude 0.045 T. (An alpha particle has a charge of $+3.2 \times 10^{-19}$ C and a mass of 6.6×10^{-27} kg.) The angle between $\vec{v}$ and $\vec{B}$ is 52°. What are the magnitudes of (a) the force $\vec{F}^{\,mag}$ acting on the particle due to the field and (b) the acceleration of the particle due to $\vec{F}^{\,mag}$? (c) Does the speed of the particle increase, decrease, or remain equal to 550 m/s?

2. TV Camera An electron in a TV camera tube is moving at 7.20×10^6 m/s in a magnetic field of strength 83.0 mT. (a) Without knowing the direction of the field, what can you say about the greatest and least magnitudes of the force acting on the electron due to the field? (b) At one point the electron has an acceleration of magnitude 4.90×10^{14} m/s^2. What is the angle between the electron's velocity and the magnetic field?

3. Proton Traveling A proton traveling at 23.0° with respect to the direction of a magnetic field of strength 2.60 mT experiences a magnetic force of 6.50×10^{-17} N. Calculate (a) the proton's speed and (b) its kinetic energy in electron-volts.

4. Force on Charges An electron that has velocity

$$\vec{v} = (2.0 \times 10^6\ \text{m/s})\hat{\text{i}} + (3.0 \times 10^6\ \text{m/s})\hat{\text{j}}$$

moves through the magnetic field $\vec{B} = (0.030\ \text{T})\hat{\text{i}} - (0.15\ \text{T})\hat{\text{j}}$. (a) Find the force on the electron. (b) Repeat your calculation for a proton having the same velocity.

5. Television Tube Each of the electrons in the beam of a television tube has a kinetic energy of 12.0 keV. The tube is oriented so that the electrons move horizontally from geomagnetic south to geomagnetic north. The vertical component of Earth's magnetic field points down and has a magnitude of 55.0 μT. (a) In what direction will the beam deflect? (b) What is the magnitude of the acceleration of a single electron due to the magnetic field? (c) How far will the beam deflect in moving 20.0 cm through the television tube?

SEC. 29-5 ■ A CIRCULATING CHARGED PARTICLE

6. Accelerated from Rest An electron is accelerated from rest by a potential difference of 350 V. It then enters a uniform magnetic field of magnitude 200 mT with its velocity perpendicular to the field. Calculate (a) the speed of the electron and (b) the radius of its path in the magnetic field.

7. Field Perpendicular to Beam A uniform magnetic field is applied perpendicular to a beam of electrons moving at 1.3×10^6 m/s. What is the magnitude of the field if the electrons travel in a circular arc of radius 0.35 m?

8. Heavy Ions Physicist S. A. Goudsmit devised a method for measuring the masses of heavy ions by timing their periods of revolution in a known magnetic field. A singly charged ion of iodine makes 7.00 rev in a field of 45.0 mT in 1.29 ms. Calculate its mass, in atomic mass units. (Actually, the method allows mass measurements to be carried out to much greater accuracy than these approximate data suggest.)

9. Kinetic Energy An electron with kinetic energy 1.20 keV circles in a plane perpendicular to a uniform magnetic field. The orbit radius is 25.0 cm. Find (a) the speed of the electron, (b) the magnetic field, (c) the frequency, and (d) the period of the motion.

10. Circular Path An alpha particle ($q = +2e$, $m = 4.00$ u) travels in a circular path of radius 4.50 cm in a uniform magnetic field with magnitude $B = 1.20$ T. Calculate (a) its speed, (b) its period of revolution, (c) its kinetic energy in electron-volts, and (d) the potential difference through which it would have to be accelerated to achieve this energy.

11. Frequency of Revolution (a) Find the frequency of revolution of an electron with an energy of 100 eV in a uniform magnetic field of magnitude 35.0 μT. (b) Calculate the radius of the path of this electron if its velocity is perpendicular to the magnetic field.

12. Source of Electrons A source injects an electron of speed $v = 1.5 \times 10^7$ m/s into a uniform magnetic field of magnitude $B = 1.0 \times 10^{-3}$ T. The velocity of the electron makes an angle $\theta = 10°$ with the direction of the magnetic field. Find the distance d from the point of injection at which the electron next crosses the field line that passes through the injection point.

13. Beam of Electrons A beam of electrons whose kinetic energy is K emerges from a thin-foil "window" at the end of an accelerator tube. There is a metal plate a distance d from this window and perpendicular to the direction of the emerging beam (Fig. 29-28). Show that we can prevent the beam from hitting the plate if we apply a uniform magnetic field $\vec{B}$ such that its magnitude is

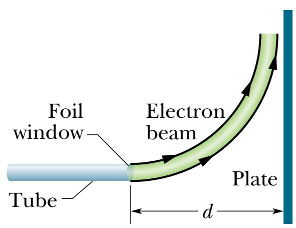


FIGURE 29-28 ■ Problem 13.

$$B \geq \sqrt{\frac{2mK}{e^2d^2}},$$

in which m and e are the electron mass and charge. How should $\vec{B}$ be oriented?

14. Proton, Deuteron, Alpha A proton, a deuteron ($q = +e$, $m = 2.0$ u), and an alpha particle ($q = +2e$, $m = 4.0$ u) with the same kinetic energies enter a region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. Compare the radii of their circular paths.

15. Nuclear Experiment In a nuclear experiment a proton with kinetic energy 1.0 MeV moves in a circular path in a uniform magnetic field. What energy must (a) an alpha particle ($q = +2e$, $m = 4.0$ u) and (b) a deuteron ($q = +e$, $m = 2.0$ u) have if they are to circulate in the same circular path?

16. Uniform Magnetic Field A proton of charge $+e$ and mass m enters a uniform magnetic field $\vec{B} = B\hat{\text{i}}$ with an initial velocity $\vec{v} = v_{1x}\hat{\text{i}} + v_{1y}\hat{\text{j}}$. Find an expression in unit-vector notation for its velocity $\vec{v}$ at any later time t.

17. Mass Spectrometer A certain commercial mass spectrometer (see Touchstone Example 29-2) is used to separate uranium ions of mass 3.92×10^{-25} kg and charge 3.20×10^{-19} C from related species. The ions are accelerated through a potential difference of 100 kV and then pass into a uniform magnetic field, where they are bent in a path of radius 1.00 m. After traveling through 180° and passing through a slit of width 1.00 mm and height 1.00 cm, they are collected in a cup. (a) What is the magnitude of the (perpendicular) magnetic field in the separator? If the machine is used to separate out 100 mg of material per hour, calculate (b) the current of the desired ions in the machine and (c) the thermal energy produced in the cup in 1.00 h.

18. Half Circle In Fig 29-29, a charged particle moves into a region of uniform magnetic field B, goes through half a circle, and then exits that region. The particle is either a proton or an electron (you must decide which). It spends 130 ns within the region. (a) What is the magnitude $|\vec{B}|$? (b) If the particle is sent back through the magnetic field (along the same initial path) but with 2.00 times its previous kinetic energy, how much time does it spend within the field?

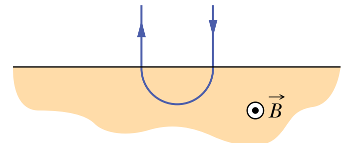


FIGURE 29-29 ■ Problem 18.

19. Positron A positron with kinetic energy 2.0 keV is projected into a uniform magnetic field $\vec{B}$ of magnitude 0.10 T, with its velocity vector making an angle of 89° with $\vec{B}$. Find (a) the period, (b) the pitch p, and (c) the radius r of its helical path.

20. Neutral Particle A neutral particle is at rest in a uniform magnetic field $\vec{B}$. At time $t = 0$ it decays into two charged particles, each of mass m. (a) If the charge of one of the particles is $+q$, what is the charge of the other? (b) The two particles move off in separate paths, both of which lie in the plane perpendicular to $\vec{B}$. At a later time the particles collide. Express the time from decay until collision in terms of m, $|\vec{B}|$, and $|q|$.

SEC. 29-6 ■ CROSSED FIELDS: DISCOVERY OF THE ELECTRON

21. Horizontal Motion An electron with kinetic energy 2.5 keV moves horizontally into a region of space in which there is a downward-directed uniform electric field of magnitude 10 kV/m. (a) What are the magnitude and direction of the (smallest) uniform magnetic field that will cause the electron to continue to move horizontally? Ignore the gravitational force, which is small. (b) Is it possible for a proton to pass through the combination of fields undeflected? If so, under what circumstances?

22. At One Instant A proton travels through uniform magnetic and electric fields. The magnetic field is $\vec{B} = (-2.5 \text{ mT})\hat{\text{i}}$. At one instant the velocity of the proton is $\vec{v} = (2000 \text{ m/s})\hat{\text{j}}$. At that instant, what is the magnitude of the net force acting on the proton if the electric field is (a) $(4.0 \text{ V/m})\hat{\text{k}}$ and (b) $(4.0 \text{ V/m})\hat{\text{i}}$?

23. Potential Difference An electron is accelerated through a potential difference of 1.0 kV and directed into a region between two parallel plates separated by 20 mm with a potential difference of 100 V between them. The electron is moving perpendicular to the electric field of the plates when it enters the region between the plates. What magnitude of uniform magnetic field, applied perpendicular to both the electron path and the electric field, will allow the electron to travel in a straight line?

24. Electric and Magnetic Field An electric field of magnitude 1.50 kV/m and a magnetic field of 0.400 T act on a moving electron to produce no net force. (a) Calculate the minimum speed $|\vec{v}|$ of the electron. (b) Draw a set of vectors $\vec{E}$, $\vec{B}$, and $\vec{v}$ that could yield the net force.

25. Ion Source An ion source is producing ions of ^{6}Li (mass = 6.0 u), each with a charge of $+e$. The ions are accelerated by a potential difference of 10 kV and pass horizontally into a region in which there is a uniform vertical magnetic field of magnitude $|\vec{B}| = 1.2$ T. Calculate the strength of the smallest electric field, to be set up over the same region, that will allow the ^{6}Li ions to pass through undeflected.

26. Initial Velocity An electron has an initial velocity of $(12.0 \text{ km/s})\hat{j} + (15.0 \text{ km/s})\hat{k}$ and a constant acceleration of $(2.00 \times 10^{12} \text{ m/s}^2)\hat{i}$ in a region in which uniform electric and magnetic fields are present. If $\vec{B} = (400\ \mu\text{T})\hat{i}$, find the electric field $\vec{E}$.

SEC. 29-7 ■ THE HALL EFFECT

27. Field Ratio (a) In Fig 29-14, show that the ratio of the magnitudes of the Hall electric field $\vec{E}$ to the electric field $\vec{E}^{\text{curr}}$ responsible for moving charge (the current) along the length of the strip is

$$\frac{E}{E^{\text{curr}}} = \frac{B}{ne\rho}$$

where ρ is the resistivity of the material and n is the number density of the charge carriers and e is the amount of charge on the electron. (b) Compute this ratio numerically for Problem 28. (See Table 26-2.)

28. Strip of Copper A strip of copper 150 μm wide is placed in a uniform magnetic field $\vec{B}$ of magnitude 0.65 T, with $\vec{B}$ perpendicular to the strip. A current $i = 23$ A is then sent through the strip such that a Hall potential difference ΔV appears across the width of the strip. Calculate ΔV. (The number of charge carries per unit volume for copper is 8.47×10^{28} electrons/m^3.)

29. Metal Strip A metal strip 6.50 cm long, 0.850 cm wide, and 0.760 mm thick moves with constant velocity $\vec{v}$ through a uniform magnetic field of magnitude $|\vec{B}| = 1.20$ mT directed perpendicular to the strip, as shown in Fig. 29-30. A potential difference of 3.90 μV is measured between points x and y across the strip. Calculate the speed $|\vec{v}|$.

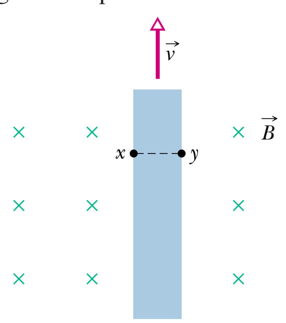


FIGURE 29-30 ■ Problem 29.

SEC. 29-8 ■ MAGNETIC FORCE ON A CURRENT-CARRYING WIRE

30. A Wire Carries a Current A wire 1.80 m long carries a current of 13.0 A and makes an angle of 35.0° with a uniform magnetic field of magnitude $B = 1.50$ T. Calculate the magnitude of the magnetic force on the wire.

31. Horizontal Conductor A horizontal conductor that is part of a power line carries a current of 5000 A from south to north. The magnitude of the Earth's magnetic field is 60.0 μT. The field is directed toward the north and is inclined downward at 70° to the horizontal. Find the magnitude and direction of the magnetic force on 100 m of the conductor due to Earth's field.

32. Along the x Axis A wire 50 cm long lying along the x axis carries a current of 0.50 A in the positive x direction. It passes through a magnetic field $\vec{B} = (0.0030 \text{ T})\hat{j} + (0.0100 \text{ T})\hat{k}$. Find the magnetic force on the wire.

33. A Wire of Length A wire of 62.0 cm length and 13.0 g mass is suspended by a pair of flexible leads in a uniform magnetic field of magnitude 0.440 T (Fig. 29-31). What are the magnitude and direction of the current required to remove the tension in the supporting leads?

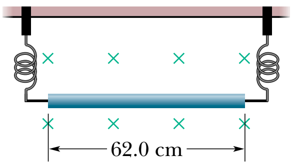


FIGURE 29-31 ■ Problem 33.

34. Electric Train Consider the possibility of a new design for an electric train. The engine is driven by the force on a conducting axle due to the vertical component of Earth's magnetic field. To produce the force, current is maintained down one rail, through a conducting wheel, through the axle, through another conducting wheel, and then back to the source via the other rail. (a) What amount of current is needed to provide a modest force of magnitude 10kN? Take the vertical component of Earth's field to be 10 μT and the length of the axle to be 3.0 m. (b) At what rate would electric energy be lost for each ohm of resistance in the rails? (c) Is such a train totally or just marginally unrealistic?

35. Copper Rod A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What is the magnitude of the smallest magnetic field (not necessarily vertical) that would cause the rod to slide?

SEC. 29-9 ■ TORQUE ON A CURRENT LOOP

36. Current Loop A single-turn current loop, carrying a current of 4.00 A, is in the shape of a right triangle with sides 50.0, 120, and 130 cm. The loop is in a uniform magnetic field of magnitude 75.0 mT whose direction is parallel to the current in the 130 cm side of the loop. (a) Find the magnitude of the magnetic force on each of the three sides of the loop. (b) Show that the total magnetic force on the loop is zero.

37. Rectangular Coil Figure 29-32 shows a rectangular 20-turn coil of wire, of dimensions 10 cm by 5.0 cm. It carries a current of 0.10 A and is hinged along one long side. It is mounted in the xy plane, at 30° to the direction of a uniform magnetic field of magnitude 0.50 T. Find the magnitude and direction of the torque acting on the coil about the hinge line.

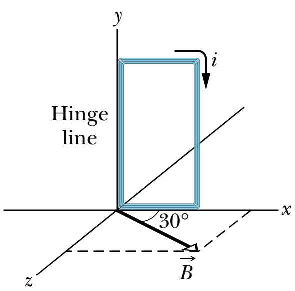


FIGURE 29-32 ■ Problem 37.

38. Arbitrarily Shaped Coil Prove that the relation $\tau = N|i|AB \sin\phi$ (Eq. 29-31) holds for closed loops of arbitary shape and not only for rectangular loops as in Fig. 29-22. (*Hint:* Replace the loop of arbitrary shape with an assembly of adjacent long, thin, approximately rectangular loops that are nearly equivalent to the loop of arbitrary shape as far as the distribution of current is concerned.)

39. Show That A length L of wire carries a current i. Show that if the wire is formed into a circular coil, then the magnitude of the maximum torque in a given magnetic field is developed when the coil has one turn only. Also show that maximum torque has the magnitude $\tau = L^2iB/4\pi$.

40. Zero Total Force A closed wire loop with current i is in a uniform magnetic field $\vec{B}$, with the plane of the loop at angle θ to the direction of $\vec{B}$. Show that the total magnetic force on the loop is zero. Does your proof also hold for a nonuniform magnetic field?

41. Wire Ring Figure 29-33 shows a wire ring of radius a that is perpendicular to the general direction of a radially symmetric, diverging magnetic field. The magnetic field at the ring is everywhere of the same magnitude $|\vec{B}|$, and its direction at the ring everywhere makes an angle θ with a normal to the plane of the ring. The twisted lead wires have no effect on the problem. Find the magnitude and direction of the force the field exerts on the ring if the ring carries a positive current i.

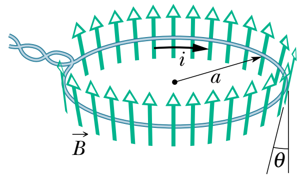


FIGURE 29-33 ■ Problem 41.

42. Maximum Torque A particle of charge q moves in a circular wire loop of radius a with speed $|\vec{v}|$. Find the maximum torque exerted on the loop by a uniform magnetic field of magnitude $|\vec{B}|$.

43. Wooden Cylinder Figure 29-34 shows a wooden cylinder with mass $m = 0.250$ kg and length $L = 0.100$ m, with $N = 10.0$ turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the axis of the cylinder. Also the plane of the coil is parallel to the inclined plane. There is a vertical, uniform magnetic field of magnitude 0.500 T. What is the least amount of current $|i|$ through the coil that will prevent the cylinder from rolling down a plane inclined at an angle θ to the horizontal?

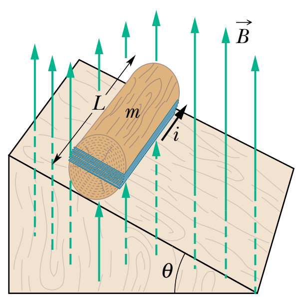


FIGURE 29-34 ■ Problem 43.

SEC. 29-10 ■ THE MAGNETIC DIPOLE MOMENT

44. Earth's Moment The magnitude of magnetic dipole moment of Earth is 8.00×10^{22} J/T. Assume that this is produced by charges flowing in Earth's molten outer core. If the radius of their circular path is 3500 km, calculate the amount of current associated with each moving charge.

45. Calculate the Current A circular coil of 160 turns has a radius of 1.90 cm. (a) Calculate the current that results in a magnetic dipole moment of $2.30\ \text{A} \cdot \text{m}^2$. (b) Find the maximum magnitude of torque that the coil, carrying this current, can experience in a uniform 35.0 mT magnetic field.

46. Moment and Torque A circular wire loop whose radius is 15.0 cm carries an amount of current of 2.60 A. It is placed so that the normal to its plane makes an angle of 41.0° with a uniform magnetic field of magnitude 12.0 T. (a) Calculate the magnitude of the magnetic dipole moment of the loop. (b) What is the magnitude of torque that acts on the loop?

47. Right Triangle A current loop, carrying an amount of current of 5.0 A, is in the shape of a right triangle with sides 30, 40, and 50 cm. The loop is in a uniform magnetic field of magnitude 80 mT whose direction is parallel to the current in the 50 cm side of the loop. Find the magnitude of (a) the magnetic dipole moment of the loop and (b) the torque on the loop.

48. Wall Clock A stationary circular wall clock has a face with a radius of 15 cm. Six turns of wire are wound around its perimeter; the wire carries a current of 2.0 A in the clockwise direction. The clock is located where there is a constant, uniform external magnetic field of magnitude 70 mT (but the clock still keeps perfect time). At exactly 1:00 P.M., the hour hand of the clock points in the direction of the external magnetic field. (a) After how many minutes will the minute hand point in the direction of the torque on the winding due to the magnetic field? (b) Find the torque magnitude.

49. Concentric Loops Two concentric, circular wire loops, of radii 20.0 and 30.0 cm, are located in the xy plane; each carries a clockwise current of 7.00 A (Fig. 29-35). (a) Find the magnitude of the net magnetic dipole moment of this system. (b) Repeat for reversed current in the inner loop.

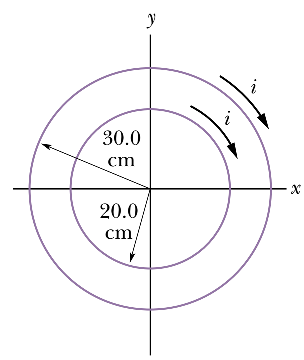


FIGURE 29-35 ■ Problem 49.

50. *ABCDEFA* Figure 29-36 shows a current loop $ABCDEFA$ carrying a current $i = 5.00$ A. The sides of the loop are parallel to the coordinate axes, with $AB = 20.0$ cm, $BC = 30.0$ cm, and $FA = 10.0$ cm. Calculate the magnitude and direction of the magnetic dipole moment of this loop. (*Hint:* Imagine equal and opposite currents i in the line segment AD; then treat the two rectangular loops $ABCDA$ and $ADEFA$.)

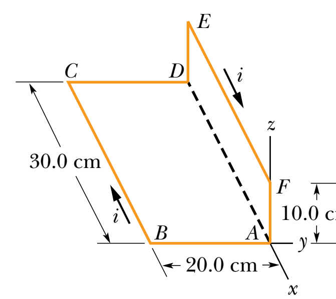


FIGURE 29-36 ■ Problem 50.

51. Circular Loop A circular loop of wire having a radius of 8.0 cm carries a current of 0.20 A. A vector of unit length and parallel to the dipole moment $\vec{\mu}$ of the loop is given by $0.60\hat{\text{i}} - 0.80\hat{\text{j}}$. If the loop is located in a uniform magnetic field given by $\vec{B} = (0.25\ \text{T})\hat{\text{i}} + (0.30\ \text{T})\hat{\text{k}}$, find (a) the torque on the loop (in unit-vector notation) and (b) the magnetic potential energy of the loop.

Additional Problems

52. Permanent Magnet You can observe that a permanent magnet can exert forces on moving charges or currents. (a) If a magnet exerts a force on a moving charge, would the magnet experience any forces? Explain. (b) In the case of the gravitational or electrostatic interaction between two objects, each object has a common property, such as mass in the case of gravitational interaction or excess charge in the case of the electrostatic interaction. A permanent magnet and a moving electron seem very different. Can you think of any way that they might have a common property? Explain.

53. U-Shaped Magnet An electron having a velocity of magnitude v enters a region between the poles of a U-shaped magnet. This region has a uniform magnetic field, $\vec{B}$, pointing out of the paper in the positive z direction as shown in Fig. 29-37.

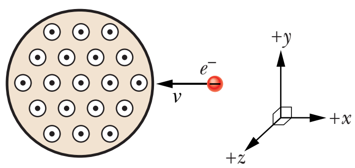


FIGURE 29-37 ■ Problem 53.

(a) If the magnetic field points out of the paper, where is the north pole of the magnet—in front of or behind the image shown? (b) Use the right-hand rule to find the direction of force on the electron as it passes into the region where the magnetic field is uniform. (c) Sketch the path of the electron, assuming that the magnetic field is relatively weak. (d) If the speed of the electron is 4.79×10^6 m/s and the magnitude of the magnetic field is 0.234 T, what is the magnitude of the force on the electron?

54. A Velocity Selector A group of physicists at Argonne National Laboratory in Illinois wants to bombard metals with monoenergetic beams of alpha particles to study radiation damage. (Alpha particles are helium nuclei, which consist of two neutrons and two protons and thus have a net charge of $+2e$ where e is the amount of the charge on the electron.) They have managed to create a beam of alpha particles from the decay of radioactive elements, but some of the alpha particles lose energy as they collide with other atoms in the source. As a new physicist assigned to the group you have been asked to use a velocity selector to select only the alpha particles in the beam that are close to one velocity and get rid of the others. The velocity selector consists of: (1) a power supply capable of delivering large potential differences between capacitor plates and (2) a large permanent magnet that has a uniform magnetic field perpendicular to the beam. The setup for the velocity selector is shown in Fig. 29-38. The direction of the B-field is out of the paper. Your magnet has a field of 0.22 T and the capacitor plates have a spacing of 2.5 cm. You are asked to figure out how the velocity selector works and then tell your group what voltage to put across the capacitor plates to select a velocity of 4.2×10^6 m/s.

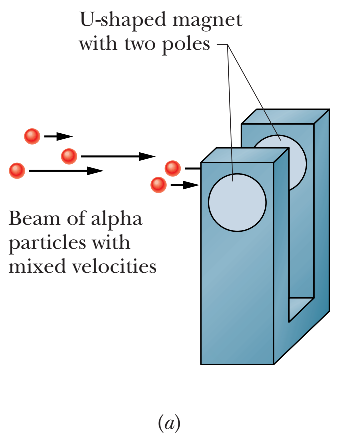


FIGURE 29-38a ■ Problem 54.

This is your first job and you feel overwhelmed by the assignment, but you calm down and begin to analyze the situation one step at a time. You come up with the following:

(a) The magnet is oriented so its magnetic field is out of the paper in the diagram you are given, so you use the right-hand rule to determine the direction of the magnetic force on an alpha particle passing from left to right into the magnetic field. What direction did you come up with for the force?

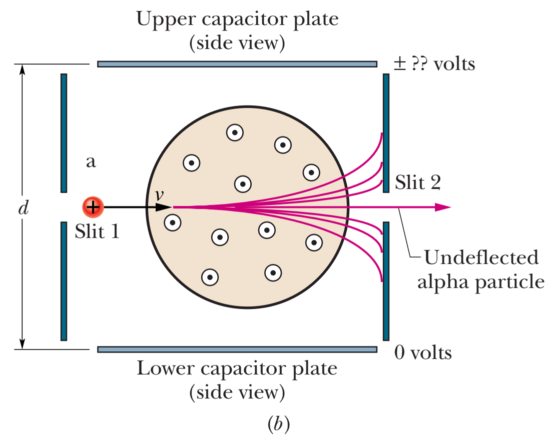


FIGURE 29-38b ■ Enlarged view of the region between the poles of the magnet showing a single alpha particle having a speed v entering the region of uniform magnetic field.

(b) You realize that by using $\vec{F}^{\,mag} = q\vec{v} \times \vec{B}$, you can calculate the magnitude of force on an alpha particle moving at speed v just as it enters the uniform magnetic field as a function of the charge on the alpha particle and the magnitude of the magnetic field B. What is the expression for the magnitude of the force in terms of e, v, and B?
(c) You realize that you might be able to put just the right voltage across the two capacitor plates so that the electrical force on a given alpha particle will be equal in magnitude and opposite in direction to the Lorentz magnetic force. Then any alpha particles with just the right velocity will pass straight through the poles of the magnet without being deflected. First you think about whether the voltage on the upper capacitor plate should be positive or negative to give a canceling force. What do you decide?
(d) Next you realize that if you know the electric field between the plates and the charge on the alpha particle then you can compute the electrical force on it. What is the relationship between the electrical force $\vec{F}^{\,elec}$, charge, q, and electric field $\vec{E}$?
(e) Finally, you use the fact that the magnitude of the electric field between capacitor plates is given by $E = |\Delta V|/d$ where d is the spacing between the plates. Show that the voltage needed to have the electrical force and the magnetic force be "equal and opposite" can be calculated using the equation $|\Delta V| = vBd$. Calculate the voltage needed.

55. Region A—Region B Figure 29-39 shows a charged particle that is moving in the positive x direction when it encounters region A with a uniform magnetic field. Its path is bent in a half-circle and then moves into region B also with a uniform magnetic field. The particle undergoes another half revolution. Finally it passes between two charged capacitor plates and is deflected downward in the negative y direction.

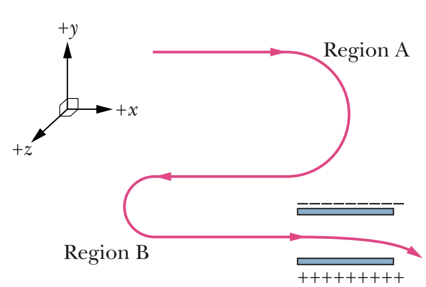


FIGURE 29-39 ■ Problem 55.

(a) Is the charge positive or negative? Explain.
(b) What is the direction of the magnetic field in region A? Explain.
(c) What is the direction of the magnetic field in region B? Explain.
(d) Which region has the larger magnetic field, A or B? Explain.

56. A Mass Spectrometer It is possible to accelerate ions to a known kinetic energy in an electric field. Sometimes chemists and physicists do this as part of a method to identify the chemical elements present in a beam of ions by determining the mass of each ion. This can be done by bending the ion beam in a uniform magnetic field and measuring the radius of the semicircular path each ion takes. A device that does this is called a mass spectrometer. A schematic of a mass spectrometer is shown in Fig. 29-40.

Boron is the fifth element in the periodic table so it always has 5 protons. However, different isotopes of boron have 3, 5, 6, 7, or 8 neutrons in addition to the 5 protons to make up boron-8, boron-10, boron-11 and so on. As a research chemist for the Borax Company you have been asked to use a mass spectrometer to determine the relative abundance of different isotopes of boron in a sample of boron obtained from a mine near Death Valley in California. You decide to accelerate a beam of singly charged boron ions (i.e., those that have lost one of their orbital electrons). You use an accelerating potential difference of -2.68×10^3 volts. The boron beam then enters a uniform magnetic field you set up to have a magnetic flux density of 0.182 T in a direction perpendicular to the direction of the boron beam. You observe two bright spots on your photographic plate with the spot corresponding to a radius of 13.0 cm having four times the intensity of the one corresponding to a radius of 13.6 cm. There are very faint spots at 11.6 cm, 14.2 cm, and 14.8 cm. Which isotope of boron has approximately 80% abundance? Which one has about 20% abundance? Which ones are present in only trace amounts? Please show all your reasoning and calculations. *Hints:* (1) An atomic mass unit is given by 1.66×10^{-27} kg, which is close to the mass of the proton and neutron. (2) Find the velocity of each isotope of boron in meters per second just after it has been accelerated by the potential difference of -2.68×10^3 volts. (3) It is helpful to do the calculations for each of the five isotopes on a spreadsheet.

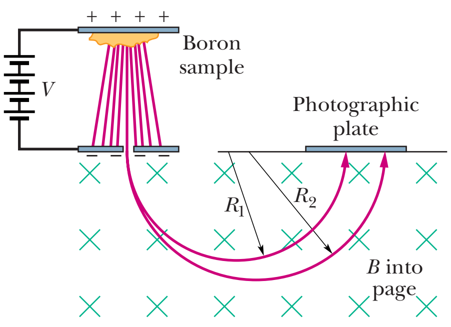


FIGURE 29-40 ■ Problem 56.

57. Bubble Chamber Tracks Energetic gamma rays like those coming from outer space can disappear near a heavy nucleus producing a rapidly moving pair of particles consisting of an electron and a positron. (A positron is a small positively charged particle that has the same mass and amount of charge as an electron). This process is called pair production. A device called a bubble chamber allows one to observe the path taken by electron–positron pairs produced by gamma rays. The study of bubble chamber tracks in the presence of magnetic fields has revealed a great deal about high-energy gamma rays, the processes of pair production, and the loss of energy by electrons and positrons. A sample bubble chamber track is shown in Fig. 29-41.

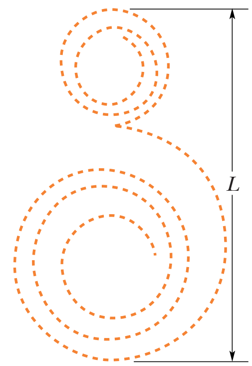


FIGURE 29-41 ■ Problem 57.

(a) If the magnetic field is uniform pointing into the paper, which trajectory (the upper one or the lower one) shows the motion of the positron? Explain your reasoning. (b) In which part of the spiral does the positron have the greatest energy—the large radius part or the small radius part? Explain the reasons for your answer. (c) Is the electron moving faster, slower, or at the same speed as the positron at the point in time when the two particles are created? Cite the evidence for your answer. (d) Suppose the bubble chamber photograph in Fig. 29-41 is an enlargement of the actual event so that the length L is actually only 2.4×10^{-3} m. Show that the radius of curvature of the electron path just after the electron is created is approximately 0.8×10^{-3} m. *Hint:* Measure L in picture units to find a scale factor and then measure the appropriate feature of the electron path in picture units and use the scale factor to find R in meters. (e) Use the Lorentz force law and the expression for centripetal force to find the equation relating the speed of the electron to B, R, e, and m. (f) Suppose the magnitude of the magnetic field in the bubble chamber is $B = 0.54$ T. Calculate the approximate speed of the electron when it is first created in the bubble chamber.

58. Three Force Fields We have studied three long-range forces: gravity, electricity, and magnetism. Compare and contrast these three forces giving at least one feature that all three forces have in common, and at least one feature that distinguishes each force.

59. Comparing $\vec{E}$ and $\vec{B}$ Fields We have studied two fields: electric and magnetic. Explain why we introduce the idea of field, and compare and contrast the electric and magnetic fields. In your comparison, be certain to discuss at least one similarity and one difference.

60. Anti-matter Ion Cosmic Rays An international consortium is presently building a device to look for anti-matter nuclei in cosmic rays to help us decide whether there are galaxies made of anti-matter. Anti-matter is just like ordinary matter except the basic particles (anti-protons and anti-electrons) have opposite charge from ordinary matter counterparts. Anti-protons are negative, and anti-electrons (positrons) are positive.

A schematic of the device is shown in Fig. 29-42. A cosmic ray—say, a carbon nucleus or an anti-carbon nucleus—enters the device at the left where its position and velocity are measured. It then passes through a (reasonably uniform) magnetic field. Its path is bent in one direction if its charge is positive and in the opposite direction it its charge is negative. Its deflection is measured as it goes out of the device.

(a) In Fig. 29-42, what is the direction of the magnetic field? How do you know?
(b) Which path is followed by each particle in the device? How do you know?
(c) If you were given the magnetic field, B, the size of the device, D, the amount of charge on the incoming particle, q, and the mass of the incoming particle, M, would this be enough to calculate the displacement of the charge, d? If so, describe briefly how you would

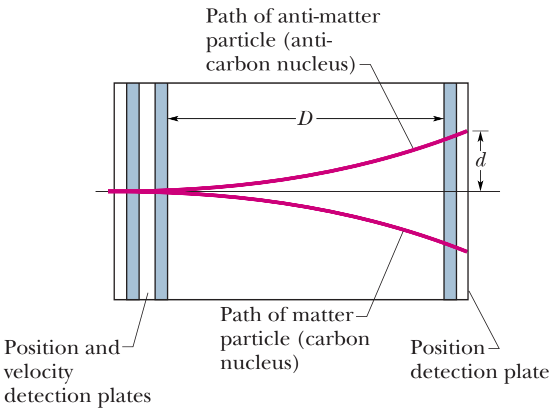


FIGURE 29-42 ■ Problem 60.

do it (but don't do it). If not, explain what additional information you would need (but don't estimate it).

61. Magnets and Charge A bar magnet is hung from a string through its center as shown in Fig. 29-43. A charged rod is brought up slowly into the position shown. In what direction will the magnet tend to rotate? Suppose the charged rod is replaced by a bar magnet with the north pole on top. In what direction will the magnet tend to rotate? Is there a difference between what happens to the hanging magnet in the two situations? Explain why you either do or do not think so.

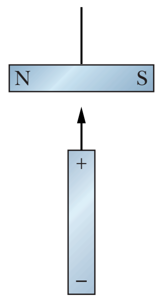


FIGURE 29-43 ■ Problem 61.

62. Buying a Mass Spectrometer* You are assigned the task of working with a desktop-sized magnetic spectrometer for the purpose of measuring the ratio of C^{12} to C^{14} atoms in a sample in order to determine the sample's age. For this problem, let's concentrate on the magnet that will perform the separation of masses. Suppose you have burned and vaporized the sample so that the carbon atoms are in a gas. You now pass this gas through an "ionizer" that on the average strips one electron from each atom. You then accelerate the ions by putting them through an electrostatic accelerator—two capacitor plates with small holes that permit the ions to enter and leave. (From the University of Washington Physics Education Group)

The two plates are charged so that they are at a voltage difference of ΔV volts. The electric field produced by charges on the capacitor plates accelerates the ions to an energy of $q\Delta V$. These are then introduced into a nearly constant, vertical magnetic field. If we ignore gravity, the magnetic field will cause the charged particles to follow a circular path in a horizontal plane. The radius of the circle will depend on the atom's mass. (Assume the whole device will be placed inside a vacuum chamber.)

Answer these three questions about how the device works.

(a) We want to keep the voltage at a moderate level. If ΔV is 1000 volts, how big of a magnetic field would we require to have a plausible tabletop-sized instrument? Is this a reasonable magnetic field to have with a tabletop-sized magnet?

(b) Do the C^{12} and C^{14} atoms hit the collection plate far enough apart? (If they are not separated by at least a few millimeters at the end of their path, we will have trouble collecting the atoms in separate bins.)

(c) Can we get away with ignoring gravity? (*Hint:* Calculate the time it would take the atom to travel its semicircle and calculate how far it would fall in that time.)

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* C14 is a radioactive isotope of carbon that behaves chemically almost identically to its more common but slightly lighter sibling, C12. The amount of C14 in the atmosphere stays about constant since it is being produced continually by cosmic rays. Once carbon from the air is bound into an organic substance, the C14 will decay with half of them vanishing every 5730 years. The ratio of C14 to C12 in an organic substance therefore tells how long ago it died.