25 Electric Potential

At more than 115 years old, the Eiffel Tower is arguably the world's most famous landmark. Among other things, the tower is an engineering feat, a work of art, a scenic lookout, and a radio tower. Less well known is the tower's ability to protect people, trees, and other buildings from being struck by lightning that might emanate from thunderheads behind it.

How can the Eiffel Tower protect people from lightning?

The answer is in this chapter.

25-1 Introduction

In the last few chapters we have explored the nature of interaction forces between charged particles. We have developed the concept of electric field as a way to represent the forces a point charge would experience at any point in the space surrounding a collection of charges.

In certain situations it is difficult to understand the motions of charges in terms of an electric field. This difficulty is analogous to problems encountered in describing the motion of an object in the presence of gravitational forces. We developed the concepts of work and energy in Chapters 9 and 10 to deal with these problems. We will now investigate the application of the concepts of work and energy to situations in which the forces involved are electrostatic forces. In this chapter we develop the concept of electric potential—commonly referred to as voltage. We then explore some of its properties, including how charges are distributed on a metal conductor placed in an electric field.

Since the concept of potential or voltage is essential to an understanding of electric circuits, we will use the concept of electric potential in the next chapter to help us understand the role that batteries play in maintaining currents.

25-2 Electric Potential Energy

Newton's law for the gravitational force and Coulomb's law for the electrostatic force are mathematically similar. In Section 14-2 we saw that the gravitational force between two particle-like masses depends directly on the product of the masses and inversely with the square of the distance between them (Eq. 14-2). In like manner, the electrostatic forces between two point charges depend directly on the product of the charges and inversely with the square of the distance between them (Eq. 22-4). This similarity gives us a starting point in our search for additional useful concepts related to the interactions between charged objects. In this chapter, we consider whether some of the general features we have established for the gravitational force apply to the electrostatic force as well.

For example, the gravitational force is a *conservative force*. The work done by it is independent of the path along which an object moves. In experimental tests the work done by the electrostatic force has also been found to be *path independent*. If a charged particle moves from point i to point f while an electrostatic force is acting on it, the work W done by the force is the same for all paths between points i and f. Hence, we can infer that the electrostatic force is a conservative force as well.

Definition of Electric Potential Energy

In Chapter 10 we defined potential energy as the energy associated with the configuration of a system of objects that interact and hence exert forces on each other. We then proceeded to define gravitational potential energy as the negative of the amount of gravitational work objects in the system do on each other when their positions relative to one another change. From Eq. 10-5, $\Delta U \equiv -W^{\text{cons}}$ or $\Delta U^{\text{grav}} = -W^{\text{grav}}$ (Eq. 10-6). This general definition of work can be applied to a system of charges that interact by means of electrostatic forces.

Since electrostatic forces, like gravitational forces, are conservative, then it makes sense to assign an **electric potential energy change** ΔU to a system of interacting charges in a similar manner. If we cause or allow a system to change its configuration from an initial potential energy state U_1 to a different final state U_2, the internal electrostatic forces do a total amount of work W^{elec} on the particles in the system. As in

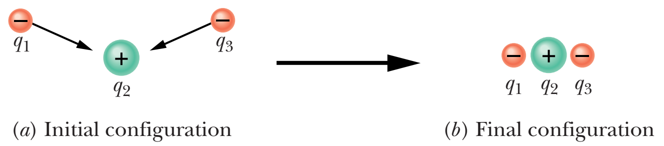


FIGURE 25-1 ■ (*a*) A system of three charges is in an initial configuration in which the charges are separated and have an electric potential energy U_1 associated with them. (*b*) Since both of the negative charges will be attracted to the positive charge, they will coalesce into a final configuration with potential energy U_2. The net electrostatic work the charges do on each other is positive so the system loses potential energy. Thus $U_2 < U_1$ so that $\Delta U < 0$.

Chapter 10, we define the potential energy change ΔU as the negative of the work the system does on itself when it undergoes the reconfiguration. This can be expressed symbolically as

$$\Delta U = U_2 - U_1 \equiv W^{\text{ext}} = -W^{\text{elec}}. \tag{25-1}$$

Figure 25-1 shows a system of charges losing electrostatic potential energy as a result of a natural reconfiguration. Figure 25-2 shows the same system gaining electrostatic potential energy as an external agent (doing positive work) causes the system to reconfigure. This results in the system doing negative work on itself.

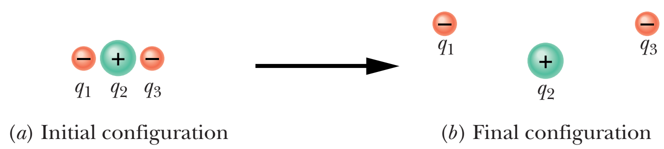


FIGURE 25-2 ■ (*a*) A system of three charges is in an initial configuration in which the charges are close together and have an electric potential energy U_1 associated with them. (*b*) Since q_1 and q_3 are both attracted to q_2, it will take positive external work, W^{ext}, to pull the charges apart. The net electrostatic work the charges do on each other is negative so the system gains potential energy. Thus $U_2 > U_1$ so that $\Delta U > 0$.

As you may recall, we determined that only differences in gravitational potential energy were physically significant. In Chapter 10 the system of masses we considered consisted of the Earth and a single object near its surface. We chose a convenient height at which to set the gravitational potential energy to zero. For example, we may have defined an Earth–object system as having zero potential energy when an object is at floor level or at the level of a tabletop. In doing so, we set the absolute scale for gravitational potential energy differently in different situations. This is legitimate since only potential energy *differences* are meaningful.

Potential energy difference is also of primary importance in keeping track of electric potential energy. Typically, we *define the electric potential energy of a system of charges to be zero when the particles are all infinitely separated from each other*, just as we did in Chapter 14 with the general form of gravitational potential energy. Using this zero of electric potential energy makes sense because the charges making up such a system have no interaction forces in that configuration. Using a standard reference potential (instead of moving it around as we typically do for Earth–object systems) allows us to find unique values of U_1 and U_2. For example suppose several charged particles come together from initially infinite separations (state 1) to form a system of nearby particles (state 2). Then using the conventional reference configuration, the initial potential energy U_1 is zero. If W^{elec} represents the internal work done by the electrostatic forces between particles during the move in from infinity, then from Eq. 25-1,

$$\Delta U = U_2 - U_1 = U_2 = -W^{\text{elec}}. \tag{25-2}$$

Since U_1 is zero, the final potential energy U_2 of the system can simply be denoted as U. Then, in terms of symbols,

$$U \equiv -W^{\text{elec}} \quad \text{(for initial potential energy = 0).} \tag{25-3}$$

As usual, the use of the symbol "≡" signifies that the expression is a definition.

External Forces and Energy Conservation

Since opposite charges attract, they will come together naturally if they are free to move. In these cases the charges "fall together" and the potential energy of the system of charges will be reduced. Similarly, like charges that are free will move apart and their potential energy will also be reduced. However, we can raise the potential energy of a system of charges by using energy from another system. Two common examples of external agents that can raise the potential energy of a system of charges are the Van de Graaff generator and the battery. Van de Graaff generators (see Fig. 25-3) use mechanical energy to force charges of like sign onto metal conductors. Batteries use chemical potential energy (which is actually a combination of electric and quantum effects) to force charges onto an electrode having the same sign charges.

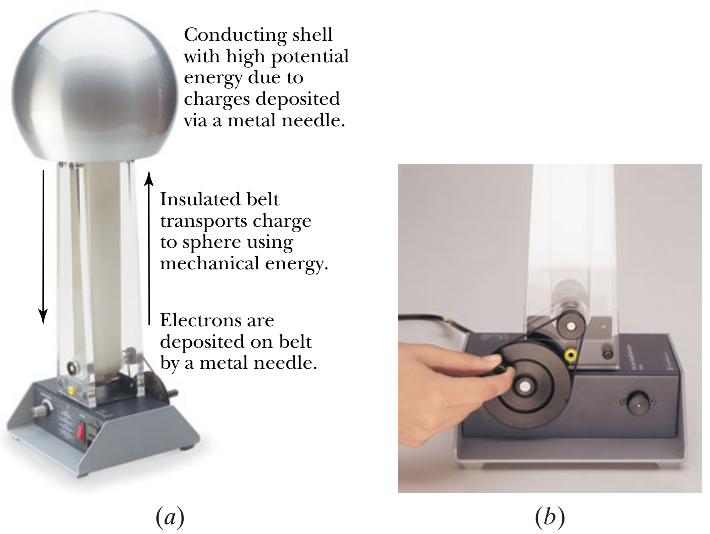


FIGURE 25-3 ■ A Van de Graaff generator uses mechanical energy from either (*a*) a motor or (*b*) a hand crank to transport charge to a conducting sphere, raising its potential energy. (Photo courtesy of PASCO scientific.)

Suppose an *external force* outside of the system under consideration causes a test particle of charge q to move from an initial location to a final location in the presence of an unchanging electric field generated by the source charges in the system. As the test charge moves, our outside force does work W^{ext} on the charge. At the same time, the electric field does work W^{elec} on it. By the work-kinetic energy theorem, the change ΔK in the kinetic energy of the particle is

$$\Delta K = K_2 - K_1 = W^{\text{ext}} + W^{\text{elec}}.$$

But since $W^{\text{elec}} = -\Delta U$,

$$\Delta K + \Delta U = W^{\text{ext}}. \tag{25-4}$$

Now suppose the particle is stationary before and after the move. Then K_2 and K_1 are both zero, and this reduces to

$$\Delta U = W^{\text{ext}} \quad \text{(for no kinetic energy change).} \tag{25-5}$$

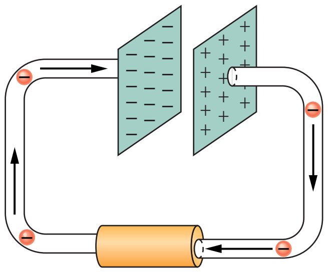


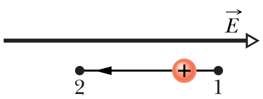
FIGURE 25-4 ■ A 1.5 V D-cell can act as an external agent that does the work needed to move electrons through a wire from a metal plate with excess positive charges to one with excess negative charges.

That is, the work, W^{ext}, done by our external force during the move is equal to the change in electric potential energy—provided there is no change in kinetic energy.

So in what direction will a positive or negative charge move if released? Will the charge move to raise or lower the potential energy of the system? The expression above can be used to determine this. For example, let the external force (perhaps the push or pull of your hand) do positive work. Recall from Section 9-4 the sign convention associated with work in general. If W^{ext} is positive, then ΔU must also be positive (by the equation above) and so we know that $U_2 > U_1$. In other words, the motion of a charge from a lower potential energy to a higher potential energy requires positive work to be done on the system. This motion would not happen if the particle were simply released. Spontaneous or naturally occuring motion is associated with reduced potential energy. This is very similar to the situations encountered in Section 9-8, where we considered an object under the influence of the Earth's attractive gravitational force. Often a battery does this work in a circuit, as in Fig. 25-4.

READING EXERCISE 25-1: Why is a configuration with charges separated by an infinite distance a good choice for our reference (zero) potential energy? Would a zero separation be equally good? Why or why not?

READING EXERCISE 25-2: In the figure, a proton moves from point 1 to point 2 in a uniform electric field directed as shown. (a) Does the electric field do positive or negative work on the proton? (b) Does the electric potential energy of the proton increase or decrease? (c) In this case we don't choose the potential energy to be zero at infinity. Why not?



25-3 Electric Potential

When considering gravitational potential energy we dealt primarily with a system consisting of the Earth and a single object much smaller than the Earth. If the object were to fall toward the Earth the interaction forces between them would be equal in magnitude, but as the object moves toward the Earth, the Earth's motion would be negligibly small. Thus the change in the system's gravitational potential energy would simply be the change in potential energy of the falling object. Similarly, as we did in Section 23-2, we can consider systems in which a small "test" charge moves in the presence of an electric field but does not change the electric field significantly. In these systems, the electric potential energy of the system can be calculated as the negative of work done by the electric field on a single test charge as we bring it to a location of interest from infinity.

> In the next several chapters we will focus primarily on systems in which the change in potential energy of a single test charge moving in an electric field is for all practical purposes the same as the change in potential energy of the entire system of charges.

This situation applies if the only charge that moves is our test charge. In this case, the electric field generated by the fixed source charges remains the same, so that the change in the system's potential energy will be proportional to the magnitude of the test charge. (This will not be true if other charges move.)

Defining Electric Potential

Recall that we defined and used the concept of electric field as the *electric force per unit charge* so we could easily analyze the forces experienced by a charge of any sign or magnitude. It is advantageous to develop an analogous concept for the determination of the electric potential energy of a system associated with the change in location of a test charge of any reasonable sign or magnitude. We will do that now, defining **electric potential** as a potential energy *per unit charge.* Once we have chosen a reference configuration with zero energy, our electric potential (potential energy per unit charge) has a unique value at any point in space. For example, suppose we move a test particle of positive charge $q_t = 1.60 \times 10^{-19}$ C from a location at infinity where the electric potential energy is defined as zero to a location in an electric field where the particle has an electric potential energy of 2.40×10^{-17} J. Then the change in electric potential, ΔV, of the system associated with the change in location of a test charge can be calculated as

$$\Delta V = V_2 - V_1 = \frac{2.40 \times 10^{-17}\text{ J}}{1.60 \times 10^{-19}\text{ C}} - 0 = 150\text{ J/C}.$$

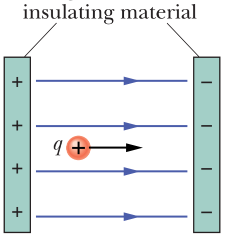


FIGURE 25-5 ■ A test charge moves in an electric field created by a stable configuration of source charges. If the test charge doesn't affect the electric field significantly as it changes location, the change in electric potential, ΔV, of the system (consisting of the source charges and the test charge) is due entirely to the work per unit charge done on the test charge by the electric field.

Next, suppose we replace that test particle with one having twice as much positive charge, 3.20×10^{-19} C. We would find that, at the same point, the second particle has an electric potential energy of 4.80×10^{-17} J, twice that of the first particle. However, the potential energy per unit charge or electric potential would be the same, still 150 J/C.

Thus, the system potential energy per unit charge, which can be symbolized as U/q_t, is independent of the charge q_t of the test particle we happen to be considering (Fig. 25-5). It is *characteristic only of the electric field* that is present. The potential energy per unit charge at a point in an electric field is defined as the electric potential V (or simply the **potential**) at that point. Thus V is defined as

$$V \equiv \frac{U}{q}. \tag{25-6}$$

Note that potential energy and charge are both scalar quantities, so the electric potential is also a scalar, not a vector.

The *electric potential difference,* ΔV, associated with moving a charge q between any two points 1 and 2 in an electric field is equal to the difference between the potential energy per unit charge at the two points:

$$\Delta V = V_2 - V_1 = \frac{U_2}{q} - \frac{U_1}{q} = \frac{\Delta U}{q}. \tag{25-7}$$

Using $\Delta U = U_2 - U_1 = -W^{\text{elec}}$ (Eq. 25-1) to substitute the work done by electrostatic forces $-W^{\text{elec}}$ for ΔU in the equation above, we can define the potential difference between points 1 and 2 as

$$\Delta V = V_2 - V_1 \equiv -\frac{W^{\text{elec}}}{q} \quad \text{(potential difference defined).} \tag{25-8}$$

That is, the potential difference between two points is the negative of the work done by the electrostatic force to move a unit charge from one point to the other. A potential difference can be positive, negative, or zero, depending on the signs and magnitudes of the charge q and the electrostatic work W^{elec}.

As we already mentioned, we have set $U_1 = 0$ infinitely far from any charges as our reference potential energy. So since $V \equiv U/q$ (Eq. 25-6), the electric potential must also be zero there. Then using Eq. 25-8, we can define the electric potential V (measured relative to infinity) at any point in an electric field to be

$$V = -\frac{W_\infty^{\text{elec}}}{q} \qquad \text{(potential defined relative to infinity)}, \tag{25-9}$$

where W_∞^{elec} is the work done by the electrostatic force on a charged particle as that particle moves in from infinity to point f. As was the case with potential difference ΔV, a potential V can be positive, negative, or zero, depending on the signs and magnitudes of q and W_∞^{elec}.

The SI unit for electric potential that follows from Eq. 25-9 is the joule per coulomb. This combination occurs so often that a special unit, the *volt* (abbreviated V) is used to represent it. Thus,

$$1 \text{ volt} \equiv 1 \text{ joule/coulomb}. \tag{25-10}$$

Although the terms electric potential energy and electric potential are very similar, they are not the same thing. This is probably one of the reasons why it is so common to refer to electric potential as **voltage** after its unit—the volt.

This new unit called the volt allows us to adopt a more conventional unit for the electric field $\vec{E}$, which we have measured up to now in newtons per coulomb. With two unit conversions, we obtain

$$1 \text{ N/C} = \left[1\frac{\text{N}}{\text{C}}\right]\left[\frac{1 \text{ V}}{1 \text{ J/C}}\right]\left[\frac{1 \text{ J}}{1 \text{ N}\cdot\text{m}}\right] = 1 \text{ V/m}. \tag{25-11}$$

The conversion factor in the second set of parentheses comes from Eq. 25-10, and that in the third set of parentheses is derived from the definition of the joule. From now on, we shall express values of the electric field in volts per meter rather than in newtons per coulomb.

The Electron Volt

Because we often have situations in which the charges involved are very small (a few times the charge of an electron), we define an energy unit that is a convenient one for energy measurements in the atomic and subatomic domain. One *electron-Volt* (eV) is the energy equal to the work required to move a single positive elementary charge e (the charge magnitude of the electron or the proton) through a potential difference of exactly one volt. Equation 25-8,

$$\Delta V = V_2 - V_1 = -\frac{W^{\text{ elec}}}{q} = \frac{-W^{\text{elec}}}{e},$$

tells us that the magnitude of this work is $e\,\Delta V$, so

$$\begin{aligned} 1 \text{ eV} &= e(1 \text{ V}) \\ &= (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.60 \times 10^{-19} \text{ J}, \end{aligned} \tag{25-12}$$

where the units for electron volt are joules because it is actually a unit of energy rather than electric potential.

READING EXERCISE 25-3: In the figure shown in Reading Exercise 25-2, we moved a proton from point i to point f in a uniform electric field directed as shown. (a) Does our external force do positive or negative work? (b) Does the proton move to a point of higher or lower potential?

TOUCHSTONE EXAMPLE 25-1: Electron Motion

An electron starts from rest at a point in space at which the electric potential is 9.0 V. If the only force acting on the electron is that associated with the electric potential, how fast will the electron be moving when it passes a second point in space where the electric potential is 10.0 V?

SOLUTION First we need to convince ourselves that this problem describes a physical situation that is even possible. Equation 25-4 tells us that $\Delta K + \Delta U = W^{\text{ext}}$. Since $W^{\text{ext}} = 0$ here and since ΔK must be positive if the electron speeds up, this means that ΔU must be negative. But is it? After all, ΔV is positive here since $V_2 - V_1 = +1.0$ V. However, the charge of the electron is negative, so Eq. 25-7 tells us that:

$$\Delta U = q\,\Delta V = (-e)\Delta V = (-e)(+1.0\text{ V}) = -1.0\text{ eV},$$

so that $$\Delta K = -\Delta U = -(-1.0\text{ eV}) = +1.0\text{ eV},$$

which is positive.

The **Key Idea** here is that a negative charge *loses* potential energy and *gains* kinetic energy when it moves from a region of *lower* potential to a region of *higher* potential. This is just the opposite of what would happen to a positive charge!

Now that we know the electron has 1.0 eV of kinetic energy, we need to determine how fast it is going. The **Key Idea** here is that 1 eV $= 1.60 \times 10^{-19}$ J (Eq. 25-12). Then

$$\Delta K = K_2 - K_1 = (\tfrac{1}{2})mv_2^2 - 0,$$

which gives us

$$\begin{aligned} v_2 &= \sqrt{2\,\Delta K/m} \\ &= \sqrt{2(1.0\text{ eV})(1.60 \times 10^{-19}\text{ J/eV}/(9.1 \times 10^{-31}\text{ kg})} \\ &= 5.9 \times 10^5\text{ m/s}. \end{aligned} \qquad \text{(Answer)}$$

25-4 Equipotential Surfaces

We are interested in what our knowledge of electric potential can tell us about how small test charges might move. We can infer from the discussion above that charged particles will not spontaneously move from one point to another point of equal potential. This is quite analogous to movement of mass in a gravitational field. A skier on a flat surface with no kinetic energy will not spontaneously move from one part of the surface to another. On the other hand, if the skier is on a slope and is free to move, the skier will spontaneously start moving down the slope, from higher to lower potential energy. Thus, it would be useful to know where all the points of equal potential energy are in a given region of space. That way, we can easily infer the directions of the forces on each of the charges. An **equipotential surface** is defined as a surface having the same potential at all points on it. Topographical maps show equipotential surfaces (lines on a two-dimensional map) in regard to gravitational potential energy.

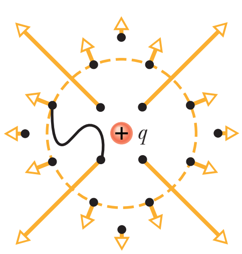


FIGURE 25-6 All of the electric field vectors created by the presence of a single charge point radially outward in three dimensions. If a test charge moves around on a sphere that is centered on the charge (where the dashed circle shows a cross section of the sphere), no work is done on it by the electric field since all the electric field vectors on surface elements of the sphere are normal to the sphere. If the charge is moved from one radius to the other (black squiggly line) it has to move parallel to the field vectors some of the time, and work is done on it.

Let's consider the electric field associated with a source consisting of a single fixed point charge we designate as the source charge. What happens if we place a test charge at a distance r from the source charge and move it around? If we move the charge anywhere on the surface of a sphere of radius r, no electrostatic work is done on the test charge as it is always moving perpendicular to the electric field vectors. However, we cannot move our test charge from one distance from the source charge to another distance without the electric field doing work on it. This is illustrated in Fig. 25-6. Thus, any sphere centered on the source charge is an equipotential surface. If our source charge is positive, then the potential decreases as the distance from the source charge increases. We know this because from $\Delta U = -W^{\text{elec}}$ (Eq. 25-1) a charge naturally moves from high potential energy to low. Thus the equipotential surfaces associated with a positive point charge consist of an infinite family of concentric spheres centered on the source charge. Each sphere has a different potential.

An equipotential surface can be either imaginary, such as a mathematical sphere, or a real, physical surface such as the outside of a wire. The set of all equipotential surfaces fills all of space, since every point in space has some value of electric potential associated with it. We could draw an equipotential surface through any one of these points, just like we can draw a field line through every point in space. However, in order to simplify illustrations and diagrams, we typically show just a few of the surfaces.

No work W^{elec} is done on a charged particle by an electric field when the particle moves between two points i and f on the same equipotential surface. This follows from Eq. 25-8,

$$\Delta V = V_2 - V_1 = -\frac{W^{\text{elec}}}{q},$$

which tells us W^{elec} must be zero if $V_2 = V_1$. Because of the path independence of work (and thus of potential energy and potential), $W^{\text{elec}} = 0$ for *any* path connecting points 1 and 2, regardless of whether that path lies entirely on the equipotential surface. In other words, if the charge moves away from the equipotential surface during the motion, the work done (positive or negative) is exactly canceled by the work done (negative or positive) in moving back onto the surface.

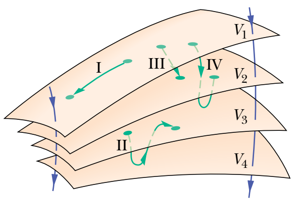


FIGURE 25-7 ■ Portions of four equipotential surfaces at electric potentials $V_1 = 100$ V, $V_2 = 80$ V, $V_3 = 60$ V, and $V_4 = 40$ V. Four paths along which a test charge may move are shown. Two electric field lines are also indicated.

Figure 25-7 shows a *family* of equipotential surfaces associated with the electric field due to some distribution of charges. The work done by the electrostatic force on a charged particle as the particle moves from one end to the other of paths I and II is zero because each of these paths begins and ends on the same equipotential surface. The work done as the charged particle moves from one end to the other of paths III and IV is not zero but has the same value for both these paths because the initial and final potentials are identical for the two paths; that is, paths III and IV connect the same pair of equipotential surfaces.

As we already noted, the equipotential surfaces produced by a point charge or a spherically symmetrical charge distribution are a family of concentric spheres. For a uniform electric field it is not difficult to see that the equipotential surfaces are a family of planes perpendicular to the field lines.

The fact that the value of the potential is constant along an equipotential surface implies that the electric field must always be perpendicular to the equipotential surfaces. Why? Because, if $\vec{E}$ were *not* perpendicular to an equipotential surface, it would have a component lying along that surface. This component would then do work on a

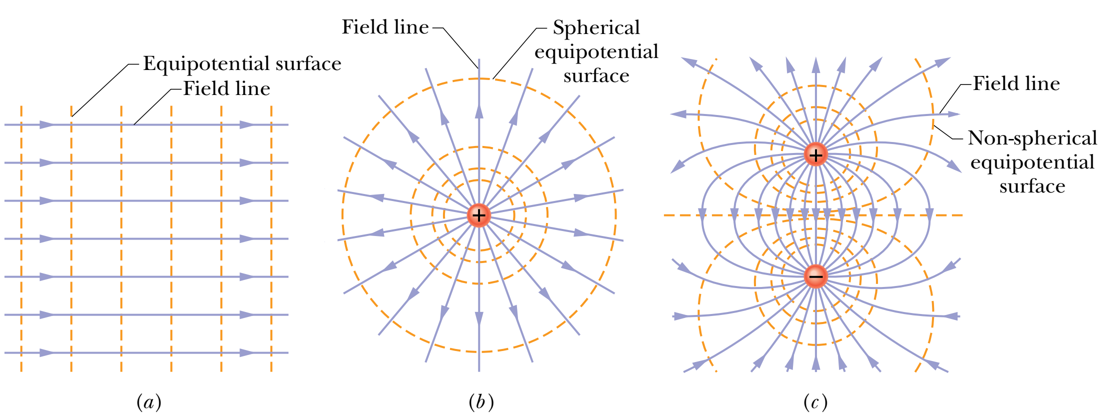


FIGURE 25-8 ■ Electric field lines (solid purple lines with arrows) and cross sections of equipotential surfaces (dashed gold lines) for (*a*) a uniform field with planar equipotential surfaces, (*b*) the field of a point charge with spherical equipotential surfaces, and (c) the field of an electric dipole with distorted equipotential surfaces that are not quite spherical.

charged particle as it moved along the surface. However, to prove that work cannot be done if the surface is truly an equipotential surface we use Eq. 25-8 once again,

$$\Delta V = V_2 - V_1 = -\frac{W^{\text{elec}}}{q}.$$

The only possible conclusion is that the electric field lines must be perpendicular to the surface everywhere along it.

If electric field lines are perpendicular to an equipotential surface, then conversely the equipotential surface must be perpendicular to the field lines. Thus, equipotential surfaces are always perpendicular to the direction of the electric field $\vec{E}$, which is tangent to the field lines. Figure 25-8 shows electric field lines and cross sections of the equipotential surfaces for a uniform electric field and for the field associated with a point charge and with an electric dipole.

25-5 Calculating Potential from an *E*-Field

Can we calculate the potential difference between any two points 1 and 2 in an electric field if we know the electric field vector $\vec{E}$ all along any path connecting those points? We can if we can find the work done on a charge by the field as the charge moves from 1 to 2, and then use Eq. 25-8 again,

$$\Delta V = V_2 - V_1 = -\frac{W^{\text{elec}}}{q}.$$

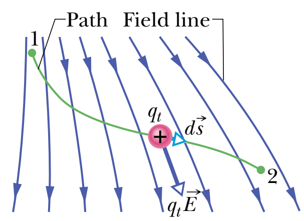


FIGURE 25-9 ■ A test charge q_t moves from point 1 to point 2 along the path shown in a nonuniform electric field represented by curved electric field lines. During a displacement $d\vec{s}$, an electrostatic force $q_t\vec{E}$ acts on the test charge. This force points in the direction of the field line at the location of the test charge.

For example, consider an arbitrary electric field, represented by the field lines in Fig. 25-9, and a positive test charge q_t moving along the path shown from point 1 to point 2. At any point on the path, an electrostatic force $q_t\vec{E}$ acts on the charge as it moves through an infinitesimally small differential displacement $d\vec{s}$. From Chapter 9, we know the differential work dW done on a particle by a force $\vec{F}$ during a displacement $d\vec{s}$ is

$$dW = \vec{F} \cdot d\vec{s}. \tag{25-13}$$

For the situation of Fig. 25-9, $\vec{F} = q_t\vec{E}$, and Eq. 25-13 becomes

$$dW^{\text{elec}} = q_t\vec{E} \cdot d\vec{s}. \tag{25-14}$$

To find the total work W^{elec} done on the particle by the field as the particle moves from point 1 to point 2, we sum—via integration—the differential work done on the charge as it moves through all the differential displacements $d\vec{s}$ along the path:

$$W^{\text{elec}} = q_t\int_1^2 \vec{E} \cdot d\vec{s}. \tag{25-15}$$

If we substitute the total electrical work W^{elec} from Eq. 25-15 into Eq. 25-8, $\Delta V = V_2 - V_1 = -W^{\text{elec}}/q$, we find

$$V_2 - V_1 = -\int_1^2 \vec{E} \cdot d\vec{s}. \tag{25-16}$$

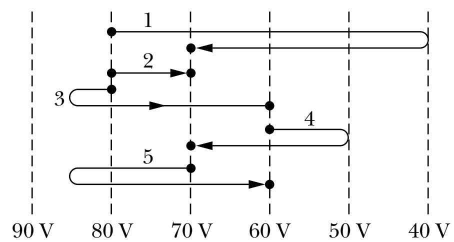
Thus, the potential difference $V_2 - V_1$ between any two points 1 and 2 in an electric field is equal to the negative of the *line integral* (meaning the integral along a particular path) of $\vec{E} \cdot d\vec{s}$ from 1 to 2. However, because the electrostatic force is conservative, all paths (whether easy or difficult to use) yield the same result. So, choose an easy-to-use path.

If the electric field is known throughout a certain region, Eq. 25-16 allows us to calculate the difference in potential between any two points in the field. If we choose the potential V_1 at point 1 to be zero, then Eq. 25-16 becomes

$$V = -\int_1^2 \vec{E} \cdot d\vec{s} \qquad \text{(for } V_1 = 0\text{)}, \tag{25-17}$$

where we have dropped the subscript 2 on V_2. Equation 25-17 gives us the potential V at any point 2 in the electric field *relative to the zero potential* at point 1. If we let point 1 be at infinity, then Eq. 25-17 gives us the potential V at any point 2 relative to the zero potential at infinity.

READING EXERCISE 25-4: The figure shows a family of parallel equipotential surfaces (in cross section) and five paths along which we shall move an electron from one surface to another. (a) What is the direction of the electric field associated with the surfaces? (b) For each path, is the work we do positive, negative, or zero? (c) Rank the paths according to the work we do, greatest first.



TOUCHSTONE EXAMPLE 25-2: Finding the Potential Difference

(a) Figure 25-10*a* shows two points 1 and 2 in a uniform electric field $\vec{E}$. The points lie on the same electric field line (not shown) and are separated by a distance d. Find the potential difference $V_2 - V_1$ by moving a positive test charge q_t from 1 to 2 along the path shown, which is parallel to the field direction.

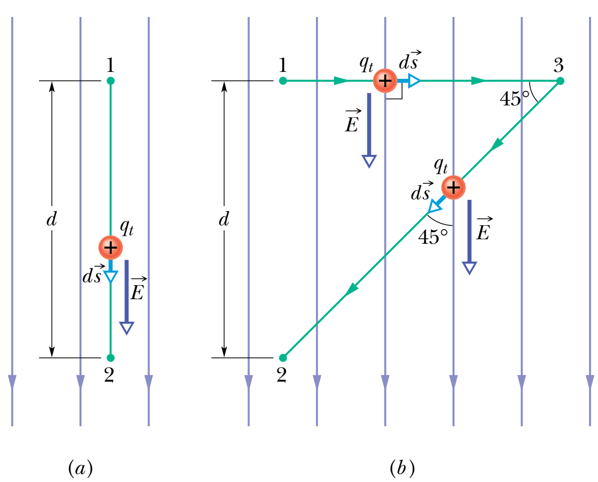


FIGURE 25-10 (*a*) A test charge q_t moves in a straight line from point 1 to point 2, along the direction of a uniform electric field. (*b*) Charge q_t moves along path 1-3-2 in the same electric field.

SOLUTION The **Key Idea** here is that we can find the potential difference between any two points in an electric field by integrating $\vec{E} \cdot d\vec{s}$ along a path connecting those two points according to Eq. 25-16. We do this by mentally moving a test charge q_t along that path, from initial point 1 to final point 2. As we move such a test charge along the path in Fig. 25-10*a*, its differential displacement $d\vec{s}$ always has the same direction as $\vec{E}$. Thus, the angle ϕ between $\vec{E}$ and $d\vec{s}$ is zero and the dot product in Eq. 25-16 is

$$\vec{E} \cdot d\vec{s} = |\vec{E}||d\vec{s}|\cos\phi = |\vec{E}||d\vec{s}|. \tag{25-18}$$

Equations 25-16 and 25-18 then give us

$$V_2 - V_1 = -\int_1^2 \vec{E} \cdot d\vec{s} = -\int_1^2 |\vec{E}||d\vec{s}|. \tag{25-19}$$

Since the field is uniform, E is constant over the path and can be moved outside the integral, giving us

$$V_2 - V_1 = -|\vec{E}|\int_1^2 |d\vec{s}| = -|\vec{E}||\vec{d}|,$$

in which the integral is simply the length d of the path. The minus sign in the result shows that the potential at point 2 in Fig. 25-10*a* is lower than the potential at point 1. This is a general result: The potential always decreases along a path that extends in the direction of the electric field lines.

(b) Now find the potential difference $V_2 - V_1$ by moving the positive test charge q_t from 1 to 2 along the path 1-3-2 shown in Fig. 25-10*b*.

SOLUTION ■ The **Key Idea** of (a) applies here too, except now we move the test charge along a path that consists of two lines: 1-3 and 3-2. At all points along line 1-3, the displacement $d\vec{s}$ of the test charge is perpendicular to $\vec{E}$. Thus, the angle ϕ between $\vec{E}$ and $d\vec{s}$ is 90°, and the dot product $\vec{E}\cdot d\vec{s}$ is 0. Equation 25-16 then tells us that points 1 and 3 are at the same potential: $V_3 - V_1 = 0$.

For line 3-2 we have $\phi = 45°$ and, from Eq. 25-16,

$$V_2 - V_1 = V_2 - V_3 = -\int_3^2 \vec{E}\cdot d\vec{s} = -\int_3^2 |\vec{E}|(\cos 45°)|d\vec{s}|$$

$$= -|\vec{E}|(\cos 45°)\int_3^2 |d\vec{s}|.$$

The integral in this equation is just the length of line 3-2; from Fig. 25-10*b*, that length is $d/\sin 45°$. Thus,

$$V_2 - V_1 = -|\vec{E}|(\cos 45°)\frac{|\vec{d}|}{\sin 45°} = -|\vec{E}||\vec{d}|. \qquad \text{(Answer)}$$

This is the same result we obtained in (a), as it must be; the potential difference between two points does not depend on the path connecting them. Moral: When you want to find the potential difference between two points by moving a test charge between them, you can save time and work by choosing a path that simplifies the use of Eq. 25-16.

25-6 Potential Due to a Point Charge

Imagine a single point charge in space. What would the value of the potential be at a distance of 3 m away from the charge? Consider a point P at a distance R from a fixed particle of positive charge q as in Fig. 25-11. To use Eq. 25-16,

$$V_2 - V_1 = -\int_1^2 \vec{E}\cdot d\vec{s},$$

we imagine that we move a positive test charge q_t from infinity to its final location at point P. We need to bring our test charge from infinity to a point P that is a distance R from the source charge. Because the path we choose will not change our final result, we are free to choose it. Mathematically, the simplest path between infinity and point P involves traveling along the same line that the electric field vectors lie along so no nonradial vector components of the electric field have to be considered.

We must then evaluate the dot product

$$\vec{E}\cdot d\vec{s} = |\vec{E}|\cos\phi|d\vec{s}| = (E)(ds)\cos\phi. \qquad (25\text{-}20)$$

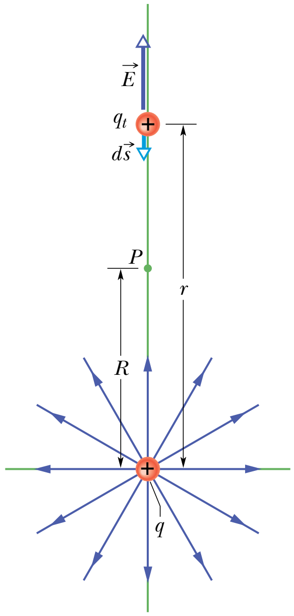


FIGURE 25-11 ■ The positive point charge q produces an electric field $\vec{E}$ and an electric potential ΔV at point P. We find the potential by moving a test charge q_t from its initial location at infinity to a point P. The test charge is shown at distance r from the point charge undergoing differential displacement $d\vec{s}$.

The electric field $\vec{E}$ in Fig. 25-11 is directed radially outward from the fixed particle. So the differential displacement $d\vec{s}$ of the test particle along our chosen path is radially inward and has the opposite direction as $\vec{E}$. That means that the angle $\phi = 180°$ and $\cos\phi = -1$. Because the path is radial, let us write ds as dr. Then, substituting the limits ∞ and R, we can write Eq. 25-16,

$$V_2 - V_1 = -\int_1^2 \vec{E}\cdot d\vec{s},$$

as

$$V_2 - V_1 = -\int_\infty^R E_r\,dr, \qquad (25\text{-}21)$$

where E_r is the component of the electric field in the radial direction. Next we set $V_1 = 0$ (at ∞) and $V_2 = V$ (at R). Then, for the magnitude of the electric field at the site of the test charge, we substitute from Chapter 23:

$$E = k\frac{|q|}{r^2}. \qquad (25\text{-}22)$$

With these changes, Eq. 25-21 then gives us

$$V - 0 = -k|q| \int_{\infty}^{R} \frac{1}{r^2}\, dr = k|q| \left[\frac{1}{r}\right]_{\infty}^{R}$$

$$= k\frac{|q|}{R} \qquad \text{(for positive } q\text{).} \qquad (25\text{-}23)$$

We want to generalize finding the potential relative to infinity for any distance, not just distance R. So, switching from R to r, we have an expression for potential at a distance r from a source charge of

$$V = k\frac{|q|}{r} \qquad \text{(for positive } q\text{).}$$

Although we have derived this expression above for a positively charged particle, the derivation also holds for a negatively charged particle as well. However, if q in Fig. 25-11 were a negative charge, the electric field vectors would point in the same direction as the path (radially inward). Thus, the differential displacement $d\vec{s}$ of the test particle along our chosen path has the same direction as $\vec{E}$. That means the angle $\phi = 0°$ and so $\cos\phi = +1$. This introduces a negative sign that remains throughout the derivation and results in a negative final result for the potential. So, we conclude that *the sign of V is the same as the sign of q*. This gives us

$$V = k\frac{q}{r} \qquad \text{(relative to infinity for either sign of charge),} \qquad (25\text{-}24)$$

as the electric potential V relative to infinity due to a particle of charge q at any radial distance r from the particle.

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Figure 25-12 shows a computer-generated plot of Eq. 25-24 for a positively charged particle; the magnitude of V is plotted vertically. Note that the magnitude increases as $r \to 0$. In fact, according to the expression above, V is infinite at $r = 0$, although Fig. 25-12 shows a finite, smoothed-off value there.

Equation 25-24 also gives the electric potential *outside or on the external surface of* a spherically symmetric charge distribution. We can prove this by using an

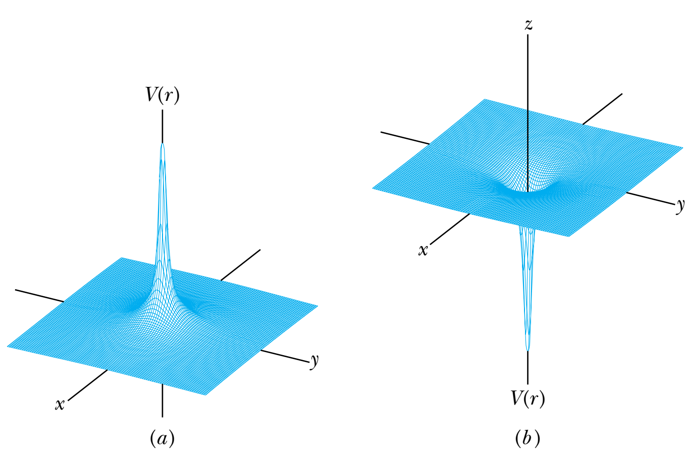


FIGURE 25-12 ■ (*a*) A computer-generated plot of the electric potential $V(r)$ due to a positive point charge located at the origin of an x-y plane. The potentials at points in that plane are plotted vertically. (Curved lines have been added to help you visualize the plot.) The infinite value of V predicted by Eq. 25-24 for $r = 0$ is not plotted. (*b*) The same plot of electric potential is shown for a negative charge.

electrostatic analogy to the shell theorem we found so useful in our study of gravitation (Section 14-2). This theorem allows us to replace the actual spherical charge distribution with an equal charge concentrated at its center. Then the derivation leading to Eq. 25-24 follows, provided we do not consider a point within the actual distribution.

TOUCHSTONE EXAMPLE 25-3: Near a Proton

The nucleus of a hydrogen atom consists of a single proton, which can be treated as a particle (or point charge).

(a) With the electric potential equal to zero at infinite distance, what is the electric potential V due to the proton at a radial distance $r = 2.12 \times 10^{-10}$ m from it?

SOLUTION ■ The **Key Idea** here is that, because we can treat the proton as a particle, the electric potential V it produces at distance r is given by Eq. 25-24,

$$V = k\frac{q}{r}.$$

Here charge q is $e(=1.6 \times 10^{-19}\text{ C})$. Substituting this and the given value for r, we find

$$V = \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})}{2.12 \times 10^{-10}\text{ m}}$$

$$= 6.78\text{ V}. \qquad \text{(Answer)}$$

(b) What is the electric potential energy U in electron-volts of an electron at the given distance from the nucleus? (The potential energy is actually that of the electron–proton system—the hydrogen atom.)

SOLUTION ■ The **Key Idea** here is that when a particle of charge q is located at a point where the electric potential due to other charges is V, the electric potential energy U is given by Eq. 25-6 ($V = U/q$). Using the electron's charge $-e$, we find

$$U = qV = (-1.60 \times 10^{-19}\text{ C})(6.78\text{ V})$$

$$= -1.0848 \times 10^{-18}\text{ J} = -6.78\text{ eV}. \qquad \text{(Answer)}$$

(c) If the electron moves closer to the proton, does the electric potential energy increase or decrease?

SOLUTION ■ The **Key Ideas** of parts (a) and (b) apply here also. As the electron moves closer to the proton, the electric potential V due to the proton at the electron's position increases because r decreases). Thus, the value of V in part (b) increases. Because the electron is negatively charged, this means that the value of U becomes more negative. Hence, the potential energy U of the electron (that is, of the system or atom) decreases.

25-7 Potential and Potential Energy Due to a Group of Point Charges

Now let's consider what happens when there are lots of charges. First we will look at the case where we only move a small test charge while all the other charges remain fixed. In this case, the changes in the system's potential energy as the test charge moves lead us to the same definition of electric potential, V, as we already developed. At the end of this section we consider the situation in which many charges move and find that the total potential energy of the system changes. In this case, even though the system has a potential energy associated with it, we cannot define an electric potential.

We found in Chapter 23 that the electric field arising from a group of point charges satisfies a superposition principle. That is, the total electric field is the sum of the individual electric fields arising from each individual point charge. Since the potential V is the line integral of the electric field and the integral of a sum of terms is the sum of the integrals, the superposition principle also holds for electrostatic potential.

Hence, we use the principle of superposition to find the electric potential at a particular location due to a group of point charges. We calculate the potential resulting from the influence of each charge in the system one at a time, using Eq. 25-24 with the sign of the charge included. Then we sum the potentials. For n charges, the net potential (measured relative to a zero at infinity) is

$$V = \sum_{i=1}^{n} V_i = k \sum_{i=1}^{n} \frac{q_i}{r_i} \quad \text{(potential due to } n \text{ point charges).} \tag{25-25}$$

Here q_i is the value of the ith charge, and r_i is the radial distance of the given point from the ith charge. The sum in Eq. 25-25 is an *algebraic sum,* not a vector sum like the sum used to calculate the electric field resulting from a group of point charges. Herein lies an important computational advantage of potential over electric field: it is a lot easier to sum several scalar quantities than to sum several vector quantities whose directions and components must be considered.

In Section 25-2, we discussed the electric potential energy of a charged particle as an electrostatic force does work on it. In that section, we assumed that the charges that produced the force were fixed in place, so that neither the force nor the corresponding electric field could be influenced by the presence of the test charge. If we consider a system with charges that move when a test charge moves around, there is no logical way to determine a charge-independent electric potential for it. The electric field will keep changing due to the presence of the test charge. But we can take a broader view and find the electric potential energy of the entire *system* of charges due to the electric field produced *by* those same charges.

We can start simply by pushing two bodies that have charges of like sign into the same vicinity. For example, imagine that we have one excess electron on the conducting shell of the Van de Graaff generator shown in Fig. 25-3 and we want to put a second electron in place. Our second electron is sprayed on the insulated belt and the generator motor does work as it forces the second electron toward the conducting shell in the presence of the first one. The first electron is no doubt relocating and acting on the second electron during the forcing process. Nonetheless, we can keep track of the work the motor does. This work is stored as electric potential energy in the two-body system (provided the kinetic energy of the bodies does not change). As we bring up a third electron we can measure the work we have to bring it up to the shell in the presence of the other two electrons, which are relocating as a result of the interactions of all three electrons. The work needed to bring the third electron to the conducting shell adds to the work needed to bring up the second electron. The total work is stored as the potential energy of the three-body system. This process of doing more work and causing the excess electrons on the shell to relocate goes on until there are billions and billions of electrons on the conducting shell. If you later release the charges but touch the shell with a conductor attached to the ground, you can recover this stored energy, in whole or in part, as the kinetic energy of the charged bodies as they rush away from each other.

We define the electric potential energy *of a system of point charges* in terms of the final locations of all the charges as follows:

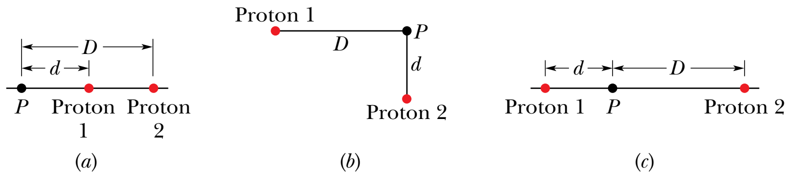
> The electric potential energy of a system of point charges that are not moving is equal to the work that must be done by an external agent to assemble the system one charge at a time.

We assume that the charges are stationary both in their initial infinitely distant positions and in their final assembled configuration. In equation form, the total electric potential energy of the system is given by the sum of the potential energies of all the possible pairs in the system so that

$$U = \sum_{\text{all pairs}} k \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} \quad \text{(system potential energy of } n \text{ point charges).} \tag{25-26}$$

READING EXERCISE 25-5: So far in this chapter, we have discussed two ways to calculate the electric potential V. Describe how one would calculate the electric potential given information about the charge distribution (the magnitudes of the charges and where they are located). Describe how one would calculate the electric potential given information regarding the electric field $\vec{E}$.

READING EXERCISE 25-6: The figure shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point P by the protons, greatest first.



TOUCHSTONE EXAMPLE 25-4: A Square of Charges

What is the electric potential at point P, located at the center of the square of point charges shown in Fig. 25-13*a*? The distance d is 1.3 m, and the charges are

$$q_1 = +12 \text{ nC}, \qquad q_3 = +31 \text{ nC},$$
$$q_2 = -24 \text{ nC}, \qquad q_4 = +17 \text{ nC}.$$

SOLUTION ■ The **Key Idea** here is that the electric potential V at P is the algebraic sum of the electric potentials contributed

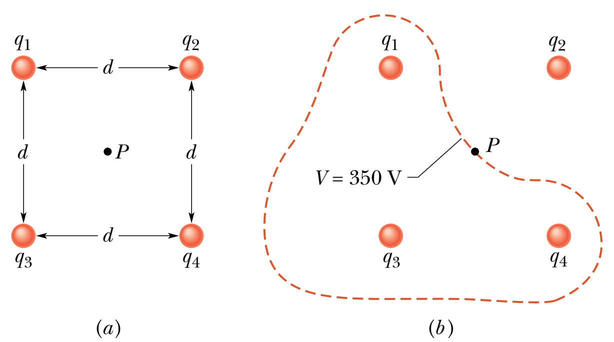


FIGURE 25-13 ■ (*a*) Four point charges are held fixed at the corners of a square. (*b*) The closed curve is a cross section, in the plane of the figure, of the equipotential surface that contains point P. (The curve is only roughly drawn.)

by the four point charges. (Because electric potential is a scalar, the orientations of the point charges do not matter.) Thus, from Eq. 25-25, we have

$$V = \sum_{i=1}^{4} V_i = k\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right).$$

The distance r is $d/\sqrt{2}$, which is 0.919 m, and the sum of the charges is

$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9} \text{ C}$$
$$= 36 \times 10^{-9} \text{ C}.$$

Thus,
$$V = \frac{(8.99 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-9} \text{ C})}{0.919 \text{ m}}$$
$$\approx 350 \text{ V}. \qquad \text{(Answer)}$$

Close to any of the three positive charges in Fig. 25-13*a*, the potential has very large positive values. Close to the single negative charge, the potential has very large negative values. Therefore, there must be points within the square that have the same intermediate potential as that at point P. The curve in Fig. 25-13*b* shows the intersection of the plane of the figure with the equipotential surface that contains point P. Any point along that curve has the same potential as point P.

TOUCHSTONE EXAMPLE 25-5: A Dozen Electrons

(a) In Fig. 25-14*a*, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?

SOLUTION ■ The **Key Idea** here is that the electric potential V at C is the algebraic sum of the electric potentials contributed by all the electrons. (Because electric potential is a scalar, the orientations of the electrons do not matter.) Because the electrons all have the same negative charge $-e$ and are all the same distance R from C, Eq. 25-25 gives us

$$\Delta V = -12k\frac{e}{R}. \qquad \text{(Answer)} \qquad (25\text{-}27)$$

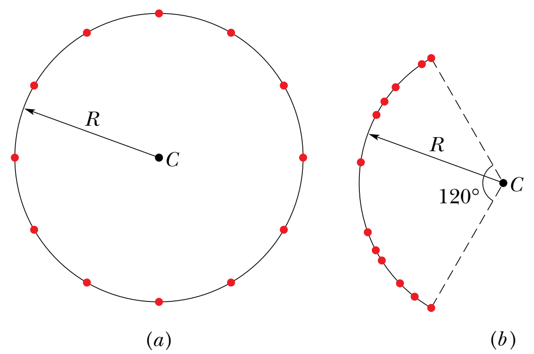


FIGURE 25-14 ■ (a) Twelve electrons uniformly spaced around a circle. (b) Those electrons are now nonuniformly spaced along an arc of the original circle.

For the electric field at C, the **Key Idea** is that electric field is a vector quantity and thus the orientation of the electrons *is* important. Because of the symmetry of the arrangement in Fig. 25-14a, the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at C,

$$\vec{E} = 0. \qquad \text{(Answer)}$$

(b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (Fig. 25-14b), what then is the potential at C? How does the electric field at C change (if at all)?

SOLUTION ■ The potential is still given by Eq. 25-27, because the distance between C and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, because the arrangement is no longer symmetric. There is now a net field that is directed toward the charge distribution.

25-8 Potential Due to an Electric Dipole

Electrically neutral matter is made of equal amounts of positive and negative charges. Electric forces pull in opposite directions on those charges. Thus, an electric field can cause a small separation of the positive and negative charges in matter (called polarization). In addition, many molecules distribute their electrons throughout their volume in a nonuniform way. This results in their having more positive charge on one end and one negative charge on the other end. For example, the water molecule shown in Fig. 23-22 has a nonuniform charge distribution.

A small separation produces an electric field very similar to that of a pair of equal and opposite charges separated by a small distance. If the charges were right on top of each other, their electric fields would cancel and they would appear neutral. But if they are a bit separated, their fields don't cancel perfectly, leaving a field pattern known as an electric dipole. The electric dipole fields produced by molecules play an essential role in a large number of processes in chemistry and biology, as well as in determining the electrical properties of matter such as color and transparency.

Now let us apply Eq. 25-25,

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i},$$

to an electric dipole to find the potential at an arbitrary point P in Fig. 25-15a. At P, the positive point charge (at distance $r_{(+)}$) sets up potential $V_{(+)}$ and the negative point charge (at distance $r_{(-)}$) sets up potential $V_{(-)}$. Then the net potential at P is given by Eq. 25-25 as

$$\begin{aligned} V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} &= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right) \\ &= \frac{q}{4\pi\varepsilon_0}\,\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}. \end{aligned} \qquad (25\text{-}28)$$

Naturally occurring dipoles—such as those possessed by many molecules—are quite small, so we are usually interested only in points that are relatively far from the

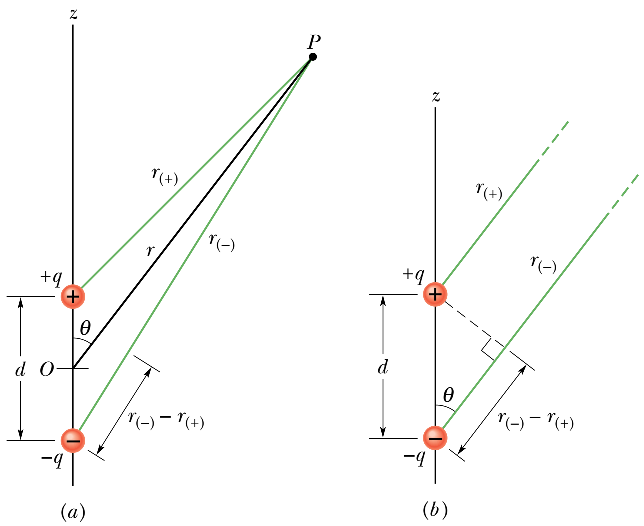


FIGURE 25-15 ■ (*a*) Point P is a distance r from the midpoint O of a dipole. The line OP makes an angle θ with the dipole axis. (*b*) If P is far from the dipole, the lines of lengths $r_{(+)}$ and $r_{(-)}$ are approximately parallel to the line of length r, and the dashed line is approximately perpendicular to the line of length $r_{(-)}$.

dipole, such that $r \gg d$, where d is the distance between the charges. Under those conditions, the approximations that follow from Fig. 25-15*b* are

$$r_{(-)} - r_{(+)} \approx d \cos \theta \quad \text{and} \quad r_{(-)} r_{(+)} \approx r^2.$$

If we substitute these quantities into Eq. 25-28, we can approximate V to be

$$V \approx \frac{q}{4\pi\varepsilon_0} \frac{d \cos \theta}{r^2} \qquad \text{(for } r >> d\text{)}.$$

Here θ is measured from the dipole axis as shown in Fig. 25-15*a*. We can now write V as

$$V \approx k \frac{p \cos \theta}{r^2} \qquad \text{(electric dipole for } r >> d\text{)}, \tag{25-29}$$

in which $p(=qd)$ is the magnitude of the electric dipole moment $\vec{p}$ defined in Section 23-7. The vector $\vec{p}$ is directed along the dipole axis, from the negative to the positive charge. (Thus, θ is measured from the direction of $\vec{p}$.)

Induced Dipole Moment

Many molecules such as water have *permanent* electric dipole moments. In other molecules (called nonpolar molecules) and in every isolated atom, the centers of the positive and negative charges coincide (Fig. 25-16*a*) and thus no dipole moment is set up. However, if we place an atom or a nonpolar molecule in an external electric field, the field affects the locations of the electrons relative to the nuclei and separates the

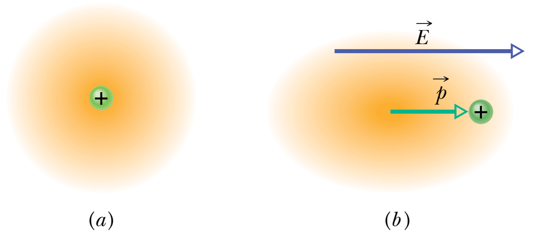


FIGURE 25-16 ■ (*a*) An atom, showing the positively charged nucleus (green) and a cloud of negatively charged electrons (gold shading). The centers of positive and negative charge coincide. (*b*) If the atom is placed in an external electric field $\vec{E}$, the electron orbits are distorted so that the centers of positive and negative charge no longer coincide. An induced dipole moment $\vec{p}$ appears. The distortion is exaggerated here by many orders of magnitude.

centers of positive and negative charge (Fig. 25-16*b*). Because the electrons are negatively charged, they tend to be shifted in a direction opposite the field. This shift sets up a dipole moment $\vec{p}$ pointing in the direction of the field. This dipole moment is said to be induced by the field, and the atom or molecule is then said to be polarized by the field (it has a positive side and a negative side). When the field is removed, the induced dipole moment and the polarization disappear.

READING EXERCISE 25-7: Suppose three points are set at equal (large) distances r from the center of the dipole in Fig. 25-15: Point a is on the dipole axis above the positive charge, point b is on the axis below the negative charge, and point c is on a perpendicular bisector through the line connecting the two charges. Rank the points according to the electric potential of the dipole there, greatest (most positive) first. ■

25-9 Potential Due to a Continuous Charge Distribution

When a charge distribution q is continuous (as on a uniformly charged thin rod or disk), we cannot use a summation to find the potential V at a point P. Instead, we must choose a differential element of charge dq. A differential element of charge is a very small bit of charge, small enough so we can treat it as if it were a point charge. We can then determine the potential dV at P due to dq, and then integrate over the entire charge distribution.

Let us again take the zero of potential to be at infinity. If we treat the element of charge dq as a point charge, then we can use Eq. 25-24,

$$V = k\frac{q}{r},$$

to express the potential dV at point P due to dq:

$$dV = k\frac{dq}{r} \qquad \text{(positive or negative } dq\text{).} \tag{25-30}$$

Here r is the distance between P and dq. To find the total potential V at P, we integrate to sum the potentials due to all the charge elements:

$$V = \int dV = k\int \frac{dq}{r}. \tag{25-31}$$

The integral must be taken over the entire charge distribution. Note that because the electric potential is a scalar, there are *no vector components* to consider in the equation above.

We now examine a continuous charge distribution, a line of charge.

Line of Charge

In Fig. 25-17*a*, a thin, nonconducting rod of length L has a positive charge of uniform linear density λ. Let us determine the electric potential V due to the rod at point P, a perpendicular distance d from the left end of the rod.

We consider a differential element dx of the rod as shown in Fig. 25-17*b*. This (or any other) element of the rod has a differential charge of

$$dq = \lambda\, dx. \tag{25-32}$$

This element produces a potential dV at point P, which is a distance $r = (x^2 + d^2)^{1/2}$ from the element. Treating the element as a point charge, we can use Eq. 25-30,

$$dV = k\frac{dq}{r},$$

to write the potential dV as

$$dV = k\frac{dq}{r} = k\frac{\lambda\, dx}{(x^2 + d^2)^{1/2}}. \tag{25-33}$$

Since the charge on the rod is positive and we have taken $V = 0$ at infinity, we know dV in this expression must be positive.

We now find the total potential V (measured relative to a zero at infinity) produced by the rod at point P by integrating along the length of the rod, from $x = 0$ to $x = L$. We evaluate the integral using an integral table or a symbolic manipulation program like Mathcad or Maple. We then find

$$\begin{aligned} V = \int dV &= \int_0^L k\frac{\lambda}{(x^2 + d^2)^{1/2}}\, dx \\ &= k\lambda \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}} \\ &= k\lambda[\ln(x + (x^2 + d^2)^{1/2})]_0^L \\ &= k\lambda[\ln(L + (L^2 + d^2)^{1/2}) - \ln d]. \end{aligned}$$

We can simplify this result by using the general relation $\ln A - \ln B = \ln(A/B)$. We then find

$$V = k\lambda \ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right]. \tag{25-34}$$

Because V is the sum of positive values of dV, it should be positive—but does this expression give a positive V? Since the argument of the logarithm is greater than one, the logarithm is a positive number and V is indeed positive.

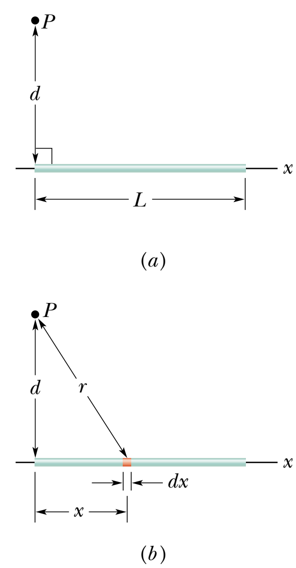


FIGURE 25-17 ■ (a) A thin, uniformly charged rod produces an electric potential V at point P. (b) A differential element of charge produces a differential potential dV at P.

25-10 Calculating the Electric Field from the Potential

In Section 25-5, you saw how to find the potential at a point f if you know the electric field along a path from a reference point to point f. In this section, we propose to go the other way—that is, to find the electric field when we know the potential. As Fig. 25-8 shows, graphically finding the direction of the field is easy: If we know the potential V at all points near an assembly of charges, we can draw in a family of equipotential surfaces. The electric field lines, sketched perpendicular to those surfaces, reveal the direction of $\vec{E}$. What we are seeking here is the mathematical equivalent of this graphical procedure.

Figure 25-18 shows cross sections of a family of closely spaced equipotential surfaces, the potential difference between each pair of adjacent surfaces being dV. As the figure suggests, the field $\vec{E}$ at any point P is perpendicular to the equipotential surface through P.

Suppose a positive test charge q_t moves through a displacement $d\vec{s}$ from one equipotential surface to the adjacent surface. From Eq. 25-8, we can relate the change

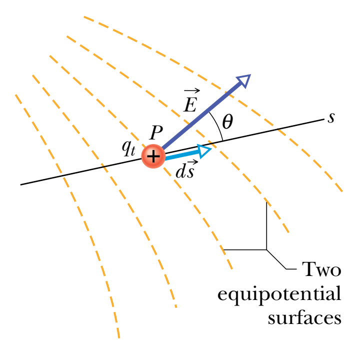


FIGURE 25-18 ■ A test charge q_t undergoes a displacement $d\vec{s}$ from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement $d\vec{s}$ makes an angle θ with the direction of the electric field $\vec{E}$.

in electric potential to the work done by the electric field on our test charge

$$\Delta V = V_2 - V_1 = -\frac{W^{\text{elec}}}{q_t}.$$

Let's consider the potential difference associated with an infinitesimally small displacement denoted by $d\vec{s}$. We see that the electric field does an infinitesimal amount of work on the test charge during the move. Using Eq. 25-8, we can denote this as $-q_t dV$. From Eq. 25-14, $dW = q_t\vec{E}\cdot d\vec{s}$, and Fig. 25-18, we see that the infinitesimal work done by the force may also be written as $(q_t\vec{E})\cdot d\vec{s}$ or $q_t|\vec{E}|(\cos\phi)|d\vec{s}|$, where ϕ is the angle between the electric field and displacement vectors as shown in Fig. 25-18. Equating these two expressions for the work yields

$$-q_t\, dV = q_t|\vec{E}|(\cos\phi)|d\vec{s}|, \tag{25-35}$$

or

$$\vec{E}(\cos\phi) = -\frac{dV}{ds}. \tag{25-36}$$

Since $E_s = |\vec{E}|\cos\phi$ is the component of $\vec{E}$ in the direction of $d\vec{s}$, the equation above becomes

$$E_s = -\frac{\partial V}{\partial s}. \tag{25-37}$$

We have added a subscript to the component of $\vec{E}$ and switched to the partial derivative symbols to emphasize that this expression involves only the variation of ΔV along a specified axis (here called the s axis) and only the component of $\vec{E}$ along that axis. In words, Eq. 25-37 is essentially the inverse of Eq. 25-16,

$$V_2 - V_1 = -\int_1^2 \vec{E}\cdot d\vec{s},$$

and states:

> The component of $\vec{E}$ in any direction is the negative of the rate of change of the electric potential with distance in that direction. Hence, $\vec{E}$ points in the direction of decreasing electric potential V.

If we take the s axis to be, in turn, the x, y, and z axes, we find that the x-, y-, and z-components of $\vec{E}$ at any point are

$$E_x = -\frac{\partial V}{\partial x}; \qquad E_y = -\frac{\partial V}{\partial y}; \qquad E_z = -\frac{\partial V}{\partial z}. \tag{25-38}$$

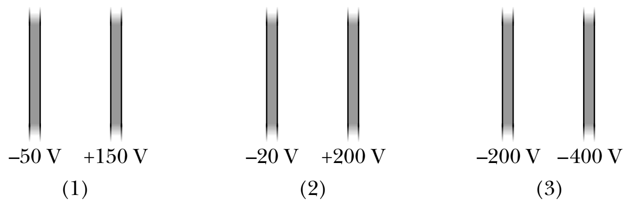
Thus, if we know V for all points in the region around a charge distribution—that is, if we know the function $V(x,y,z)$—we can find the components of $\vec{E}$, and thus $\vec{E}$ itself, at any point by taking partial derivatives. Each component of the electric field is simply the negative of the slope of the curve representing the electric potential vs. distance along each chosen axis.

For the simple situation in which the electric field $\vec{E}$ is uniform, the equipotential surfaces are a set of parallel planes that lie perpendicularly to the direction of the electric field. In addition, for a given potential difference, the distance between any two equipotential planes is the same. So, when the component of the electric field along the direction of $d\vec{s}$ is uniform, we can rewrite Eq. 25-37 ($E_s = -\partial V/\partial s$) in terms of the magnitude of the electric field $E = |\vec{E}|$ as

$$E = \left|\frac{\Delta V}{\Delta s}\right|, \tag{25-39}$$

where Δs is the component of displacement perpendicular to the equipotential surfaces. Equation 25-36 tells us that whenever the potential is constant along a surface so that $\Delta V = 0$, the electric field is zero. The component of the electric field is zero in any direction parallel to the equipotential surfaces. Thus, for a given potential difference ΔV, the magnitude of the electric field is given by the magnitude of the potential difference divided by the distance between any two equipotential surfaces.

READING EXERCISE 25-8: The figure shows three pairs of parallel plates with the same separation, and the electric potential of each plate. The electric field between the plates is uniform and perpendicular to the plates. (a) Rank the pairs according to the magnitude of the electric field between the plates, greatest first. (b) For which pair is the electric field pointing rightward? (c) If an electron is released midway between the third pair of plates, does it remain there, move rightward at constant speed, move leftward at constant speed, accelerate rightward, or accelerate leftward?



READING EXERCISE 25-9: In what ways is the superposition principle for energy discussed above the same as, and different from, the superposition principle for electric field? ■

TOUCHSTONE EXAMPLE 25-6: Obtaining $\vec{E}$ from V

The electric potential at any point on the axis of a uniformly charged disk is given by

$$V = \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2 + R^2} - z).$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

SOLUTION ■ We want the electric field $\vec{E}$ as a function of distance z along the axis of the disk. For any value of z, the direction of $\vec{E}$ must be along that axis because the disk has circular symmetry about that axis. Thus, we want the component E_z of $\vec{E}$ in the direction of z. Then the **Key Idea** is that this component is the negative of the rate of change of the electric potential with distance z. Thus, from the last of Eqs. 25-38, we can write

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}(\sqrt{z^2 + R^2} - z)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right). \quad \text{(Answer)}$$

25-11 Potential of a Charged Isolated Conductor

In Section 24-8, we concluded $\vec{E} = 0$ for all points inside an electrically isolated conductor. We then used Gauss' law to prove that an excess charge placed on an isolated conductor lies entirely on its surface. (This is true even if the conductor has an empty internal cavity.) Here we use the first of these facts to prove an extension of the second:

> An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.

This fact is rather obvious since any potential difference inside a conductor requires an electric field inside it. The nonzero electric field would, in turn, cause the free conduction electrons to redistribute themselves until the potential difference disappears.

The mathematical proof that an electrically isolated conductor is an equipotential region follows directly from Eq. 25-16,

$$V_2 - V_1 = -\int_1^2 \vec{E} \cdot d\vec{s}.$$

Since $\vec{E} = 0$ for all points within a conductor, it follows directly that $V_2 = V_1$ for all possible pairs of points i and f in the conductor.

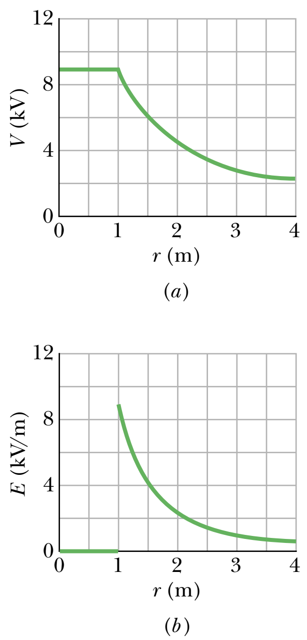


FIGURE 25-19 ■ (*a*) A plot of $V(r)$ both inside and outside a charged spherical shell of radius 1.0 m. (*b*) A plot of the electric field magnitude, $E(r)$, for the same shell.

A Spherical Shell with No External Electric Field

Figure 25-19*a* shows a plot of potential against radial distance r from the center for an isolated spherical conducting shell of 1.0 m radius, having a net excess charge of 1.0 μC. In the absence of an external field, we know by symmetry the surface charges will be uniformly distributed over the surface of the shell. For points outside the shell, we can calculate $V(r)$, the electric potential. Obviously this potential also has a spherical symmetry and can be given by Eq. 25-24,

$$V = k\frac{q}{r},$$

because the total charge on the shell, denoted as q, behaves for external points as if it were concentrated at the center of the shell. That equation holds right up to the surface of the shell. Now let us push a small test charge through the shell—assuming a small hole exists—to its center. No extra work is needed to do this because no net electric force acts on the test charge once it is inside the shell. Thus, the potential at all points inside the shell has the same value as on the surface, as shown in the Fig. 25-19*a* graph.

The Fig. 25-19*b* graph shows the variation of electric field with radial distance for the same shell. Note that $\vec{E} = 0$ everywhere inside the shell. The curves of Fig. 25-19*b* can be derived from the curve of Fig. 25-19*a* by differentiating with respect to r, using Eq. 25-37 (the derivative of a constant, recall, is zero). The curve of Fig. 25-19*a* can be derived from the curves of Fig. 25-19*b* by integrating

$$E_s = -\frac{\partial V}{\partial s}.$$

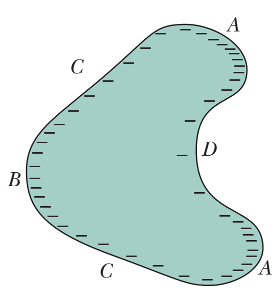


FIGURE 25-20 ■ The magnitude of the charge density on a conductor is greatest on a convex surface with a small radius of curvature (A) and least on a concave surface having small radius of curvature (D). The ranking of the magnitude of the charge density is $A > B > C > D$.

The Charge Distribution on a Nonspherical Conductor

Consider a nonspherical charged conductor. Assume the conductor is electrically isolated and there is no external electric field in its vicinity. It turns out its surface charges do not distribute themselves uniformly. When compared to the uniform density of excess charge on a spherical conductor, the charges redistribute themselves so there is a higher charge density when the radius of curvature is convex and small and a lower charge density where the radius of curvature is concave and small (Fig. 25-20). Why? We can use the characteristics of equipotential surfaces to develop a qualitative explanation for this phenomenon.

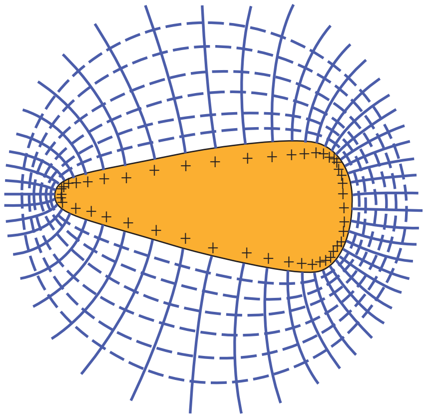


FIGURE 25-21 ■ The net positive charge on an odd-shaped isolated conductor distributes itself on the conductor's surface so the electric field generated by it is zero inside and normal to the surface elements of the conductor. This requires the equipotential surfaces (shown with dotted lines) to be closest together on the left where the conductor's convex radius of curvature is smallest. The electric field lines (shown with solid lines) and the excess charges also have the greatest density on the left where the curvature of the conductor's surface is smallest.

The explanation is as follows: There is no electric field inside the conductor, and the electric field at each point on the surface of the conductor must be normal (in other words perpendicular) to the surface. This requirement is obvious since any component of electric field parallel to the surface would cause free electrons to reconfigure themselves until all tangential components along the surface disappear. This also means the entire surface of our conductor is an equipotential surface no matter what its shape is. However, if we are far away from our charged conductor, the equipotential surfaces look more and more like those of a point charge. Thus, the family of equipotential surfaces that are each ΔV apart from the previous one become more and more spherical in shape. As the successive equipotential surfaces morph (change shape) slowly from that of our odd-shaped surface to that of a sphere, the parts of the equipotential surfaces near small-radius convex surface elements must be closer together than those elements having large radii of curvature. This is shown in Fig. 25-21. Now, equipotential surfaces more closely spaced occur where the electric field is the strongest and can do the most work on test charges, but the electric field is largest where the charge density that is its source is largest. The implication is that:

> On an isolated conductor the concentration of charges and hence the strength of the electric field is greater near sharp points where the curvature is large.

An Isolated Conductor in an External Electric Field

Suppose an *uncharged* isolated conductor is placed in an *external electric field*, as in Fig. 25-22. The electric field at the conductor's surface must have the same characteristics as it does when no external field is present. However, this doesn't mean its charges will be distributed in the same way as if no external electric field were present. All points of the conductor still come to a single potential regardless of whether the conductor is electrically neutral or has an excess charge. The free conduction electrons distribute themselves on the surface in such a way that the electric field they produce at interior points cancels the external electric field that would otherwise be there. Furthermore, the electron distribution causes the net electric field at all points on the surface to be normal to the surface. If the conductor in Fig. 25-22 could somehow be removed, leaving the surface charges frozen in place, the pattern of the electric field would remain absolutely unchanged for both exterior and interior points.

One common natural source of an external electric field that can affect isolated metal objects are excess negative charges at the bases of clouds contributing to the onset of thunderstorms. Such an external electric field can cause charge separation in conducting objects at the Earth's surface such as golf clubs and rock hammers. Since

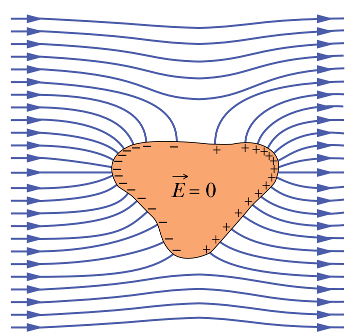


FIGURE 25-22 ■ An uncharged conductor is suspended in an external electric field. The free electrons in the conductor distribute themselves on the surface as shown, so as to reduce the net electric field inside the conductor to zero and make the net field outside normal to each surface element.

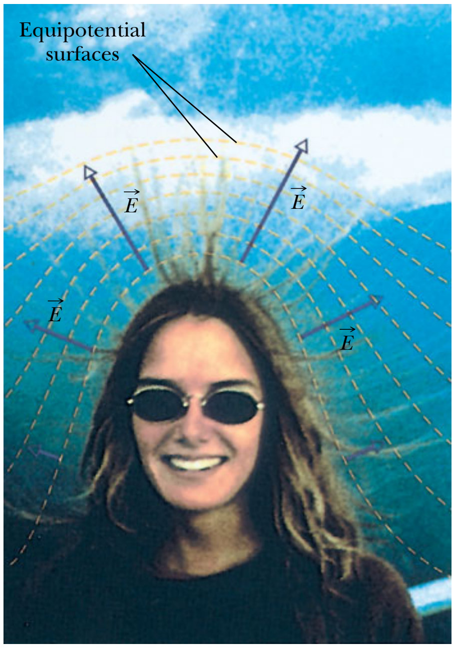


FIGURE 25-23 ■ This enhanced photograph shows the result of an overhead cloud system creating a strong electric field $\vec{E}$ near a woman's head. Many of the hair strands extended along the field, which was perpendicular to the equipotential surfaces and greatest where those surfaces were closest, near the top of her head.

these objects have points where the curvature is high, the surface charge density—and thus the external electric field, which is proportional to it—may reach very high values. The air around sharp points may become ionized, producing the corona discharge that golfers and mountaineers see on their tools when thunderstorms threaten. Such corona discharges, like hair that stands on end, are often the precursors of lightning strikes.

The cells and blood inside a human body contain salt water that acts as a conductor. The natural oil found on hair is also conductive. A person placed in a strong electric field can act like an uncharged conductor. For example, the woman shown in Fig. 25-23 was standing on a platform connected to the mountainside, and was at about the same potential as the mountainside. Overhead, a cloud system that had a high degree of charge separation with excess negative charges at its base moved in and created a strong electric field around her and the mountainside. Electrostatic forces due to this field drove some of the conduction electrons in the woman downward through her body, leaving her head and strands of her hair positively charged. The magnitude of this electric field was apparently large, but less than the value of about 3×10^6 V/m needed to cause electrical breakdown of the air molecules. (That value was exceeded when lightning struck the platform shortly after the picture was taken.)

As we just discussed, the surface charges on a nonspherical conductor concentrate in regions where the curvature is greatest. Thus, we expect the electric field to be greatest near the top of the woman's head—an equipotential surface. This suspicion is confirmed because the strands of her hair, containing excess positive charge, are pulled out most strongly where her head has the most curvature. Also, the strands of hair are extended along the direction of $\vec{E}$ perpendicular to her head. Since the magnitude of $\vec{E}$ was greatest just above her head, this is where the equipotential surfaces were most closely spaced. A sketch showing this close spacing is shown in Fig. 25-23.

The lesson here is simple. If an electric field causes the hairs on your head to stand up, you'd better run for shelter rather than pose for a snapshot.

What If Lightning Might Strike?

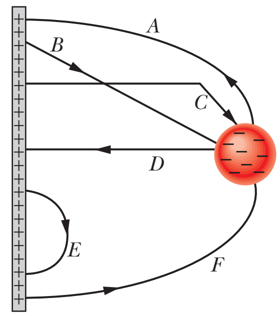
Speaking of lightning, what is the best way to protect yourself if lightning strikes? There are two ways to protect yourself using your knowledge of how conductors behave in electric fields. One is to enclose yourself in a relatively spherical conducting shell. The other is to use a lightning rod.

FIGURE 25-24 ■ A large spark jumps to a car's body and then exits by moving across the insulating left front tire, leaving the person inside unharmed because the electric potential difference remains zero inside the car.

Using a Spherical Shell: If you enclose yourself inside a more or less spherical cavity, the electric field inside the cavity is guaranteed to be zero. A car (unless it is a convertible) is almost ideal (Fig. 25-24) because it protects the passengers from the effects of lightning for the same reason that the Faraday cage shown in Chapter 24 protects the demonstrator from the high voltage caused by the transfer of charge to the cage by a Van de Graaff generator.

Using a Lightning Rod: If you live in an area where thunderstorms are common, you can embed the base of a tall metal lightning rod in the ground. Recall that the bottoms of thunderclouds have an excess of negative charge that creates strong electric fields at the Earth's surface. What happens if a conducting rod, like the Eiffel Tower, has a sharp point *and* is taller than its immediate surroundings? A couple of factors come into play. First, the distance $|\Delta s|$ between the cloud bases and the top of a lightning rod is smaller than the distance to the ground, even though the electric field strength near the top of a tall rod is not really uniform. Equation 25-39 ($E = |\Delta V/\Delta s|$) tells us that the magnitude of the electric field between the cloud bases and the top of the rod is greater than that between the clouds and the ground. Second, as we discussed earlier in this section the magnitude of the electric field near a conductor that has a sharp point is quite strong compared to that on level ground. This means that the free electrons at the top of a lightning rod (such as the Eiffel Tower) will move toward the ground leaving a large accumulation of positive metal ions at the sharp point at the top of the tower. The tip of the rod will attract electrons from the atmosphere to it and down to the ground in a corona discharge process that can serve to prevent a major discharge or lightning strike in the vicinity of the tower. Lightning is shown hitting the Eiffel Tower in Fig. 25-25.

READING EXERCISE 25-10: The figure below shows the region in the neighborhood of a negatively charged conducting sphere and a large positively charged conducting plate extending far beyond the region shown. Someone claims lines A through F are possible field lines describing the electric field lying in the region between the two conductors. (a) Examine each of the lines and indicate whether it is a correctly drawn field line. If a line is not correct, explain why. (b) Redraw the diagram with a pattern of field lines that is more nearly correct. (Based on Arnold Arons' *Homework and Test Questions,* Wiley, New York, 1994.)



■

READING EXERCISE 25-11: Why are the equipotential surfaces shown in Fig. 25-23 closer together just above the woman's head than they are at the side of her head? ■

FIGURE 25-25 ■ In this historic 1902 postcard photo, bolts of lightning are shown converging at the top of the Eiffel Tower. The tower is acting as a "lightning rod" protecting people, trees, and other buildings from being struck by lightning.

Problems

SEC. 25-3 ■ ELECTRIC POTENTIAL

1. Car Battery A particular 12 V car battery can send a total charge of 3.0×10^5 C through a circuit, from one terminal to the other. (a) How many coulombs of charge does this represent? (b) If this entire charge undergoes a potential difference of 12 V, how much energy is involved?

2. Ground and Cloud The electric potential difference between the ground and a cloud in a particular thunderstorm is 1.2×10^9 V. What is the magnitude of the change in the electric potential energy (in multiples of the electron-volt) of an electron that moves between the ground and the cloud?

3. Lightning Flash In a given lightning flash, the potential difference between a cloud and the ground is 1.0×10^9 V and the quantity of charge transferred is 30 C. (a) What is the decrease in energy of that transferred charge. (b) If all that energy could be used to accelerate a 1000 kg automobile from rest, what would be the automobile's final speed? (c) If the energy could be used to melt ice, how much ice would it melt at 0°C? The heat of fusion of ice is 3.33×10^5 J/kg.

SEC. 25-5 ■ CALCULATING THE POTENTIAL FROM AN *E*-FIELD

4. From *A* to *B* When an electron moves from A to B along an electric field line in Fig. 25-26, the electric field does 3.94×10^{-19} J of work on it. What are the electric potential differences (a) $V_B - V_A$, (b) $V_C - V_A$, and (c) $V_C - V_B$?

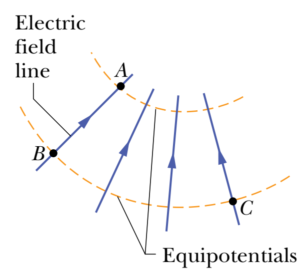


FIGURE 25-26 ■ Problem 4.

5. Infinite Sheet An infinite nonconducting sheet has a surface charge density $\sigma = 0.10\ \mu\text{C/m}^2$ on one side. How far apart are equipotential surfaces whose potentials differ by 50 V?

6. Parallel Plates Two large, parallel, conducting plates are 12 cm apart and have charges of equal magnitude and opposite sign on their facing surfaces. An electrostatic force of 3.9×10^{-15} N acts on an electron placed anywhere between the two plates. (Neglect fringing.) (a) Find the electric field at the position of the electron. (b) What is the potential difference between the plates?

7. Geiger Counter A Geiger counter has a metal cylinder 2.00 cm in diameter along whose axis is stretched a wire 1.30×10^{-4} cm in diameter. If the potential difference between the wire and the cylinder is 850 V, what is the electric field at the surface of (a) the wire and (b) the cylinder? (*Hint:* Use the result of Problem 30 of Chapter 24.)

8. Field Inside The electric field inside a nonconducting sphere of radius R, with charge spread uniformly throughout its volume, is radially directed and has magnitude

$$E(r) = |\vec{E}(r)| = \frac{|q|r}{4\pi\varepsilon_0 R^3}.$$

Here q (positive or negative) is the total charge within the sphere, and r is the distance from the sphere's center. (a) Taking $V = 0$ at the center of the sphere, find the electric potential $V(r)$ inside the sphere. (b) What is the difference in electric potential between a point on the surface and the sphere's center? (c) If q is positive, which of those two points is at the higher potential?

9. Uniformly Distributed A charge q is distributed uniformly throughout a spherical volume of radius R. (a) Setting $V = 0$ at infinity, show that the potential at a distance r from the center, where $r < R$, is given by

$$V = \frac{q(3R^2 - r^2)}{8\pi\varepsilon_0 R^3}.$$

(*Hint:* See Section 24-6.) (b) Why does this result differ from that in (a) of Problem 8? (c) What is the potential difference between a point on the surface and the sphere's center? (d) Why doesn't this result differ from that of (b) of Problem 8?

10. Infinite Sheet Two Figure 25-27 shows, edge-on, an infinite nonconducting sheet with positive surface charge density σ on one side. (a) Use Eq. 25-16 and Eq. 24-16 to show that the electric potential of an infinite sheet of charge can be written $V = V_0 - (\sigma/2\varepsilon_0)z$, where V_0 is the electric potential at the surface of the sheet and z is the perpendicular distance from the sheet. (b) How much work is done by the electric field of the sheet as a small positive test charge q_0 is moved from an initial position on the sheet to a final position located a distance z from the sheet?

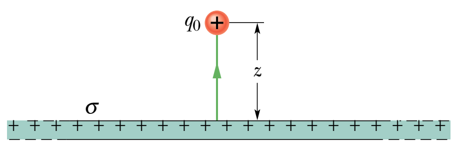


FIGURE 25-27 ■ Problem 10.

11. Thick Spherical Shell A thick spherical shell of charge Q and uniform volume charge density ρ is bounded by radii r_1 and r_2, where $r_2 > r_1$. With $V = 0$ at infinity, find the electric potential ΔV as a function of the distance r from the center of the distribution, considering the regions (a) $r > r_2$, (b) $r_2 > r > r_1$, and (c) $r < r_1$. (d) Do these solutions agree at $r = r_2$ and $r = r_1$? (*Hint:* See Section 24-6.)

SEC. 25-7 ■ POTENTIAL AND POTENTIAL ENERGY DUE TO A GROUP OF POINT CHARGES

12. Space Shuttle As a space shuttle moves through the dilute ionized gas of Earth's ionosphere, its potential is typically changed by -1.0 V during one revolution. By assuming that the shuttle is a sphere of radius 10 m, estimate the amount of charge it collects.

13. Diametrically Opposite Consider a point charge $q = 1.0\ \mu$C, point A at distance $d_1 = 2.0$ m from q, and point B at distance $d_2 = 1.0$ m. (a) If these points are diametrically opposite each other, as in Fig. 25-28a, what is the electric potential difference $V_A - V_B$? (b) What is that electric potential difference if points A and B are located as in Fig. 25-28b?

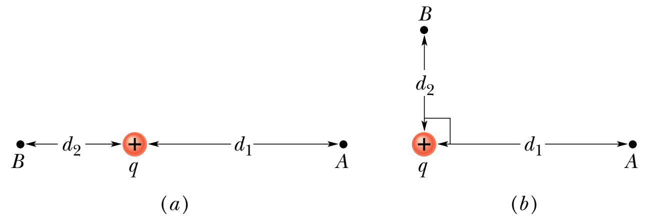


FIGURE 25-28 ■ Problem 13.

14. Field Lines and Equipotentials Figure 25-29 shows two charged particles on an axis. Sketch the electric field lines and the equipotential surfaces in the plane of the page for (a) $q_1 = +q$ and $q_2 = +2q$ and (b) $q_1 = +q$ and $q_2 = -3q$.

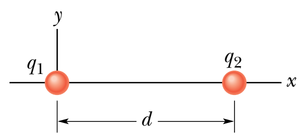


FIGURE 25-29 ■ Problems 14, 15, 16.

15. In Terms of d In Fig. 25-29, set $V = 0$ at infinity and let the particles have charges $q_1 = +q$ and $q_2 = -3q$. Then locate (in terms of the separation distance d) any point on the x axis (other than at infinity) at which the net potential due to the two particles is zero.

16. *E*-Field Is Zero Two particles, of charges q_1 and q_2, are separated by distance d in Fig. 25-29. The net electric field of the particles is zero at $x = d/4$. With $V = 0$ at infinity, locate (in terms of d) any point on the x axis (other than at infinity) at which the electric potential due to the two particles is zero.

17. Spherical Drop of Water A spherical drop of water carrying a charge of 30 pC has a potential of 500 V at its surface (with $V = 0$ at infinity). (a) What is the radius of the drop? (b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop?

18. Charge and Charge Density What are (a) the charge and (b) the charge density on the surface of a conducting sphere of radius 0.15 m whose potential is 200 V (with $V = 0$ at infinity)?

19. Field Near Earth An electric field of approximately 100 V/m is often observed near the surface of Earth. If this were the field over the entire surface, what would be the electric potential of a point on the surface? (Set $V = 0$ at infinity.)

20. Center of Rectangle In Fig. 25-30, point P is at the center of the rectangle. With $V = 0$ at infinity, what is the net electric potential at P due to the six charged particles?

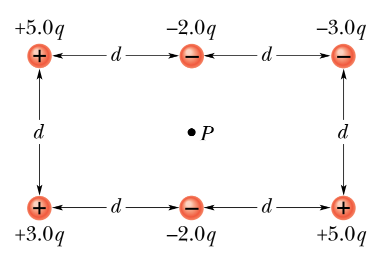


FIGURE 25-30 ■ Problem 20.

21. Potential at P In Fig. 25-31, what is the net potential at point P due to the four point charges, if $V = 0$ at infinity?

22. Potential Energy (a) What is the electric potential energy of two electrons separated by 2.00 nm? (b) If the separation increases, does the potential energy increase or decrease?

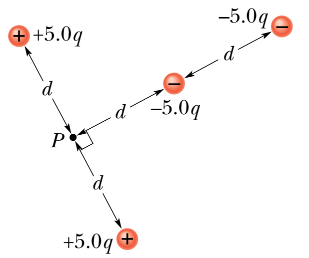


FIGURE 25-31 ■ Problem 21.

23. Work Required Derive an expression for the work required to set up the four-charge configuration of Fig. 25-32, assuming the charges are initially infinitely far apart.

24. Electric Potential Energy What is the electric potential energy of the charge configuration of Fig. 25-13*a*? Use the numerical values provided in Touchstone Example 25-4.

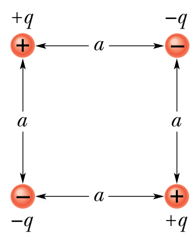


FIGURE 25-32 ■ Problem 23.

25. The Rectangle In the rectangle of Fig. 25-33, the sides have lengths 5.0 cm and 15 cm, $q_1 = -5.0\ \mu\text{C}$, and $q_2 = +2.0\ \mu\text{C}$. With $V = 0$ at infinity, what are the electric potentials (a) at corner A and (b) at corner B? (c) How much work is required to move a third charge $q_3 = +3.0\ \mu\text{C}$ from B to A along a diagonal of the rectangle? (d) Does this work increase or decrease the electric energy of the three-charge system? Is more, less, or the same work required if q_3 is moved along paths that are (e) inside the rectangle but not on a diagonal and (f) outside the rectangle?

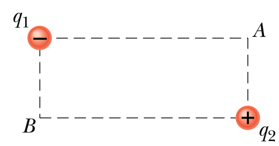


FIGURE 25-33 ■ Problem 25.

26. How Much Work In Fig. 25-34, how much work is required to bring the charge of $+5q$ in from infinity along the dashed line and place it as shown near the two fixed charges $+4q$ and $-2q$? Take distance $d = 1.40$ cm and charge $q = 1.6 \times 10^{-19}$ C.

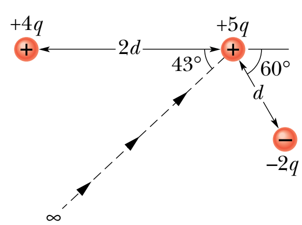


FIGURE 25-34 ■ Problem 26.

27. A Particle of Positive Charge A particle of positive charge Q is fixed at point P. A second particle of mass m and negative charge $-q$ moves at constant speed in a circle of radius r_1, centered at P. Derive an expression for the work W that must be done by an external agent on the second particle to increase the radius of the circle of motion to r_2.

28. How Much Energy Calculate (a) the electric potential established by the nucleus of a hydrogen atom at the average distance ($r = 5.29 \times 10^{-11}$ m) of the atom's electron (take $V = 0$ at infinite distance), (b) the electric potential energy of the atom when the electron is at this radius, and (c) the kinetic energy of the electron, assuming it to be moving in a circular orbit of this radius centered on the nucleus. (d) How much energy is required to ionize the hydrogen atom (that is, to remove the electron from the nucleus so that the separation is effectively infinite)? Express all energies in electron-volts.

29. Fixed at Point *P* A particle of charge q is fixed at point P, and a second particle of mass m and the same charge q is initially held a distance r_1 from P. The second particle is then released. Determine its speed when it is distance r_2 from P. Let $q = 3.1\ \mu\text{C}$, $m = 20$ mg, $r_1 = 0.90$ mm, and $r_2 = 2.5$ mm.

30. Thin Plastic Ring A charge of -9.0 nC is uniformly distributed around a thin plastic ring of radius 1.5 m that lies in the yz plane with its center at the origin. A point charge of -6.0 pC is located on the x axis at $x = 3.0$ m. Calculate the work done on the point charge by an external force to move the point charge to the origin.

31. Tiny Metal Spheres Two tiny metal spheres A and B of mass $m_A = 5.00$ g and $m_B = 10.0$ g have equal positive charges $q = 5.00\ \mu\text{C}$. The spheres are connected by a massless nonconducting string of length $d = 1.00$ m, which is much greater than the radii of the spheres. (a) What is the electric potential energy of the system? (b) Suppose you cut the string. At that instant, what is the acceleration of each sphere? (c) A long time after you cut the string, what is the speed of each sphere?

32. Conducting Shell on Support A thin, spherical, conducting shell of radius R is mounted on an isolating support and charged to a potential of $-V$. An electron is then fired from point P at distance r from the center of the shell ($r \gg R$) with initial speed v_1 and directly toward the shell's center. What value of v_1 is needed for the electron to just reach the shell before reversing direction?

33. Two Electrons Two electrons are fixed 2.0 cm apart. Another electron is shot from infinity and stops midway between the two. What is its initial speed?

34. Charged, Parallel Surfaces Two charged, parallel, flat conducting surfaces are spaced $d = 1.00$ cm apart and produce a potential difference $\Delta V = 625$ V between them. An electron is projected from one surface directly toward the second. What is the initial speed of the electron if it stops just at the second surface?

35. An Electron Is Projected An electron is projected with an initial speed of 3.2×10^5 m/s directly toward a proton that is fixed in place. If the electron is initially a great distance from the proton, at what distance from the proton is the speed of the electron instantaneously equal to twice the initial value?

SEC. 25-8 ■ POTENTIAL DUE TO AN ELECTRIC DIPOLE

36. Ammonia The ammonia molecule NH_3 has a permanent electric dipole moment equal to 1.47 D, where 1 D = 1 debye unit = 3.34×10^{-30} C·m. Calculate the electric potential due to an ammonia molecule at a point 52.0 nm away along the axis of the dipole. (Set $V = 0$ at infinity.)

37. Three Particles Figure 25-35 shows three charged particles located on a horizontal axis. For points (such as P) on the axis with $r \gg d$, show that the electric potential $V(r)$ is given by

$$V(r) = \frac{kq}{r}\left(1 + \frac{2d}{r}\right).$$

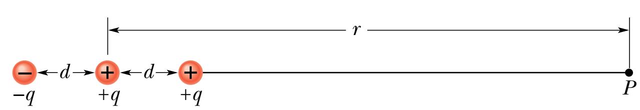


FIGURE 25-35 ■ Problem 37.

(*Hint:* The charge configuration can be viewed as the sum of an isolated charge and a dipole.)

SEC. 25-9 ■ POTENTIAL DUE TO A CONTINUOUS CHARGE DISTRIBUTION

38. Plastic Rod (*a*) Figure 25-36*a* shows a positively charged plastic rod of length L and uniform linear charge density λ. Setting $V = 0$ at infinity and considering Fig. 25-17 and Eq. 25-34, find the electric potential at point P without written calculation. (b) Figure 25-36*b* shows an identical rod, except that it is split in half and the right half is negatively charged; the left and right halves have the same magnitude λ of uniform linear charge density. With V still zero at infinity, what is the electric potential at point P in Fig. 25-36*b*?

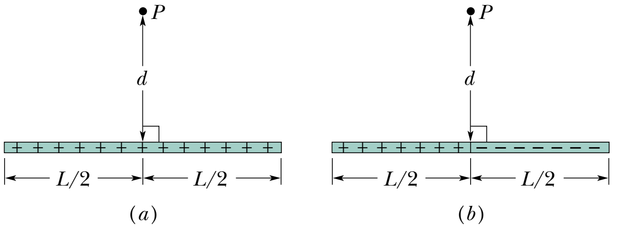


FIGURE 25-36 ■ Problem 38.

39. Nonlinear Charge Density The plastic rod shown in Fig. 25-37 has length L and a nonuniform linear charge density $\lambda = cx$, where c is a positive constant. With $V = 0$ at infinity, find the electric potential at point P_1 on the axis, at distance d from one end.

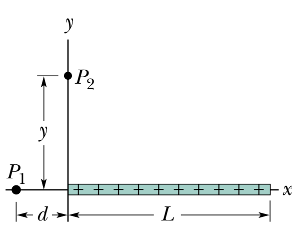


FIGURE 25-37 ■ Problems 39, 40, 44, 45.

40. Rod of Length *L* Figure 25-37 shows a plastic rod of length L and uniform positive charge Q lying on an x axis. With $V = 0$ at infinity, find the electric potential at point P_1 on the axis, at distance d from one end of the rod.

SEC. 25-10 ■ CALCULATING THE ELECTRIC FIELD FROM THE POTENTIAL

41. Points in the *xy* Plane The electric potential at points in an xy plane is given by $V = (2.0\ \text{V/m}^2)x^2 - (3.0\ \text{V/m}^2)y^2$. What are the magnitude and direction of the electric field at the point (3.0 m, 2.0 m)?

42. Parallel Metal Plates Two large parallel metal plates are 1.5 cm apart and have equal but opposite charges on their facing surfaces. Take the potential of the negative plate to be zero. If the potential halfway between the plates is then +5.0 V, what is the electric field in the region between the plates?

43. Show That (a) Using Eq. 25-31, show that the electric potential at a point on the central axis of a thin ring of charge of radius R and a distance z from the ring is

$$V = \frac{kq}{\sqrt{z^2 + R^2}}.$$

(b) From this result, derive an expression for the E-field magnitude $|\vec{E}| = E$ at points on the ring's axis; compare your result with the calculation of E in Section 23-7

44. Why Not The plastic rod of length L in Fig. 25-37 has the nonuniform linear charge density $\lambda = cx$, where c is a positive constant. (a) With $V = 0$ at infinity, find the electric potential at point P_2 on the y axis, a distance y from one end of the rod. (b) From that result, find the electric field component E_y at P_2. (c) Why cannot the field component E_x at P_2 be found using the result of (a)?

45. Find Component (a) Use the result of Problem 39 to find the electric field component E_x at point P_1 in Fig. 25-37 (*Hint:* First substitute the variable x for the distance d in the result.) (b) Use symmetry to determine the electric field component E_y at P_1.

SEC. 25-11 ■ POTENTIAL OF A CHARGED ISOLATED CONDUCTOR

46. Hollow Metal Sphere An empty hollow metal sphere has a potential of +400 V with respect to ground (defined to be at $V = 0$) and has a charge of 5.0×10^{-9} C. Find the electric potential at the center of the sphere.

47. Excess Charge What is the excess charge on a conducting sphere of radius $r = 0.15$ m if the potential of the sphere is 1500 V and $V = 0$ at infinity?

48. Widely Separated Consider two widely separated conducting spheres, 1 and 2, the second having twice the diameter of the first. The smaller sphere initially has a positive charge q, and the larger one is initially uncharged. You now connect the spheres with a long thin wire. (a) How are the final potentials V_1 and V_2 of the spheres related? (b)What are the final charges q_1 and q_2 on the spheres, in terms of q? (c) What is the ratio of the final surface charge density of sphere 1 to that of sphere 2?

49. Two Metal Spheres Two metal spheres, each of radius 3.0 cm, have a center-to-center separation of 2.0 m. One has a charge of $+1.0 \times 10^{-8}$ C; the other has a charge of -3.0×10^{-8} C. Assume that the separation is large enough relative to the size of the spheres to permit us to consider the charge on each to be uniformly distributed (the spheres do not affect each other). With $V = 0$ at infinity, calculate (a) the potential at the point halfway between their centers and (b) the potential of each sphere.

50. Charged Metal Sphere A charged metal sphere of radius 15 cm has a net charge of 3.0×10^{-8} C. (a) What is the electric field at the sphere's surface? (b) If $V = 0$ at infinity, what is the electric potential at the sphere's surface? (c) At what distance from the sphere's surface has the electric potential decreased by 500 V?

51. Surface Charge Density (a) If Earth had a net surface charge density of 1.0 electron per square meter (a very artificial assumption), what would its potential be? (Set $V = 0$ at infinity.) (b) What would be the electric field due to the Earth just outside its surface?

52. Concentric Spheres Two thin, isolated, concentric conducting spheres of radii R_1 and R_2 (with $R_1 < R_2$) have charges q_1 and q_2. With $V = 0$ at infinity, derive expressions for the electric field magnitude $E(r)$ and the electric potential $V(r)$, where r is the distance from the center of the spheres. Plot $E(r)$ and $V(r)$ from $r = 0$ to $r = 4.0$ m for $R_1 = 0.50$ m, $R_2 = 1.0$ m, $q_1 = +2.0\ \mu$C, and $q_2 = +1.0\ \mu$C.

Additional Problems

53. Work Done Consider a charge $q = -2.0\ \mu C$ that moves from A to B or C to D along the paths shown in Fig. 25-38. This charge is moving in the presence of a uniform electric field of magnitude $E = 100$ N/C.

(a) What is the total work done on the charge if the distance between A and B is 0.62 m?

(b) What is the total work done on the charge if the distance between C and D is 0.58 m?

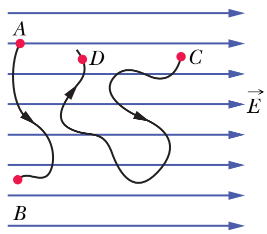


FIGURE 25-38 ■ Problem 53.

54. Orienteering an Electric Potential. (a) Figure 25-39 shows a contour plot of part of a range of hills in Virginia. The outer part of the figure is at sea level (marked 0). Each contour line from the region marked 0 shows a level 10 m higher than the previous line. The maximum height is 70 m and is shown by the number 70.

Answer the following questions by giving the pair of grid markers (a letter and a number) closest to the point being requested.

i. Where is there a steep cliff?
ii. Where is there a pass between two hills?
iii. Where is the easiest climb up the hill?

(b) Now suppose the figure represents a plot of the electric equipotentials for the surface of a glass plate, and the numbers now represent voltage. The maximum is 70 V and each contour line from the region marked 0 shows a level 10 V higher than the previous line.

i. Where would a test charge placed on the glass feel the strongest electric force? In what direction would the force point?
ii. Is there a place on the glass where a charge could be placed so it feels no electric force? Where?

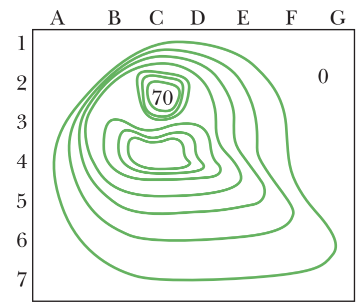


FIGURE 25-39 ■ Problem 54.