31 Induction and Maxwell's Equations

The General Motors EV1 electric car was marketed in the southwest with two generations of vehicles in 1997 and 1999. Production was completed in 1999 and all leases have been assigned. The EV1 had no engine, tailpipe, valves, pistons, timing belts, or crankshaft. The EV1 came with an inductive charging system in which there was no metal-to-metal connection. The charger, which plugged into a 220-volt outlet, had a paddle which, when inserted into the charge port at the front of the car, provided the electricity to re-charge the batteries.

How can electric car batteries be charged without making electrical contact with the power source?

The answer is in this chapter.

31-1 Introduction

In the previous chapter we discovered that the moving charges that make up electric currents create magnetic fields. We also learned that both permanent magnets and moving charges can exert forces on each other. These discoveries have powerful practical consequences. They allow us to build electromagnets to create large magnetic fields. More significantly, they enable us to harness the forces these large magnetic fields can exert on moving charges to create electric motors capable of moving massive objects.

In 1820, when Oersted observed that electric currents create magnetic fields, a number of prominent scientists began to look for ways to use magnetic fields to create currents. For more than a decade, scientists searched for current induced by static magnetic fields and failed to find it. By 1831, both Michael Faraday (Fig 31-1) and an American physicist, Joseph Henry, had discovered that a *changing* magnetic field is required to induce electric current. This phenomenon is called **electromagnetic induction.**

The discovery of electromagnetic induction, usually credited to Faraday, was of tremendous technological importance. Induction made it possible to create electric power from motion. Indeed, by the end of the 19th century, systems had been developed for the generation and transmission of electric power. Applications of Faraday's and Henry's discoveries are found in the design of thousands of electrical devices including transformers, high-speed trains, inductive battery chargers, and electric guitar pickups.

Although the practical benefits of the discovery of induction are tremendous, so is its impact on science. Many scientists view Faraday's law of induction as one of the most profound laws in all of classical physics because it "closed the loop" between magnetism and electricity. By combining Faraday's law with Ampère's law, we can understand how electricity and magnetism can be treated as complimentary aspects of the same phenomenon. By the middle of the 19th century, James Clerk Maxwell incorporated the ideas of Faraday and others into a famous set of four equations describing electromagnetic phenomena. In this chapter you will learn about the characteristics of electromagnetic induction and about Maxwell's synthesis of electromagnetic interactions.

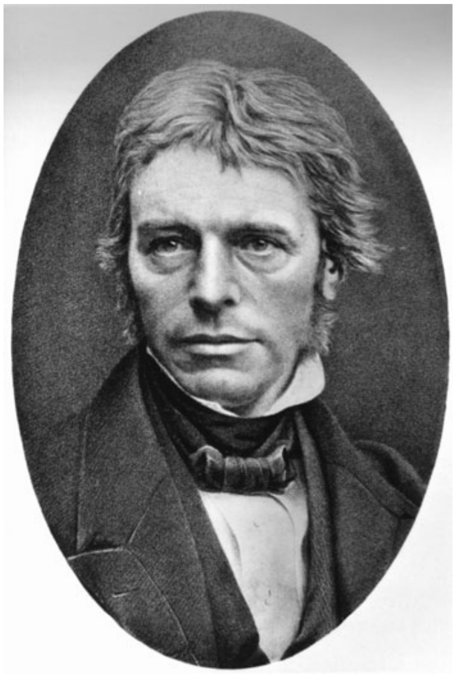


FIGURE 31-1 ■ Michael Faraday, a famous English scientist, is credited with the discovery of electromagnetic induction.

READING EXERCISE 31-1: Why did it take so long for scientists working in the early 19th century to actually observe magnetic induction? ■

31-2 Induction by Motion in a Magnetic Field

Let us start our treatment of electromagnetic induction by considering what happens if we move a coil of conducting wire at a constant velocity through a uniform magnetic field and then out of the field as shown in Fig. 31-2. This is not the observation made by Faraday. We will describe that later. Notice that the diagram shows the plane of the coil is always perpendicular to the direction of the magnetic field. Under what conditions can a current be induced? We will consider this situation from both an experimental and a theoretical perspective.

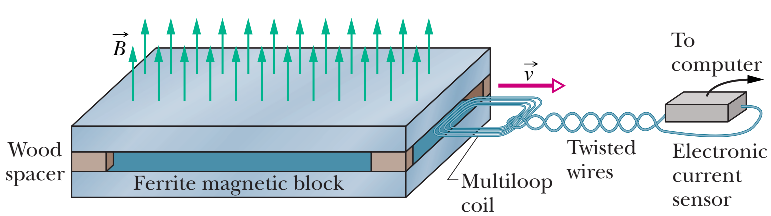


FIGURE 31-2 ■ It is not difficult to measure the current induced in a coil of wire while it is being pulled out of the gap between a pair of ferrite blocks separated by wooden spacers. The magnetic field in the central area between the magnetic blocks is essentially uniform. The ends of a multiloop coil are connected to an electronic current sensor.

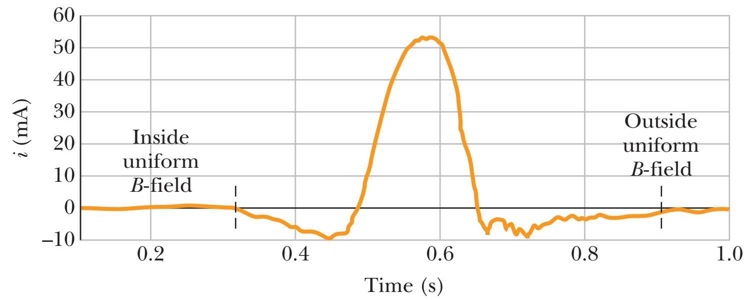


FIGURE 31-3 ■ A computer data acquisition system is used to measure the induced current 200 times a second as the coil shown in Fig. 31-2 is pulled steadily out of the uniform magnetic field in the central part of the gap between the two magnetic ferrite blocks. From 0.0 s to 0.3 s the entire coil is in the uniform magnetic field. After 0.9 s the coil is entirely outside the magnetic field. Between 0.3 s and 0.9 s part of the coil is in the B-field and part is outside of it.

A Conductor Moving Through a Magnetic Field—Observation

If we connect the ends of the coil to an ammeter, we see the needle jump back and forth a bit erratically during the time that the coil is passing out of the gap between the magnets. When the whole area of the coil is still in the central part of the gap between the magnets, the ammeter needle points to zero. When the coil has completely emerged from the region of space influenced by the magnets, the ammeter needle points to zero once again. This current jump can be seen in more detail using an electronic current sensor as shown in Fig. 31-3.

Both casual observation with a sensitive ammeter and the data gathered using an electronic current sensor show that for this situation:

- When the coil is not moving, there is no induced current no matter what the steady magnetic field is like at its location.
- When the coil is moving through a region where the magnetic field is entirely uniform or zero, there is no induced current.
- When the coil is moving through a region where the steady magnetic field is not uniform, a current is induced.

We can draw the following conclusion from these observations:

OBSERVATION: When a conducting loop moves perpendicular to a magnetic field, a current will be induced whenever the coil experiences a *changing* magnetic field.

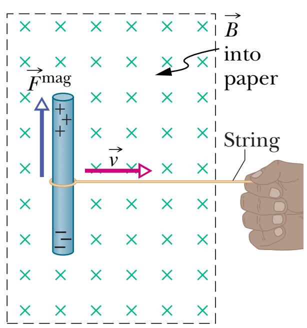


FIGURE 31-4 ■ A piece of wire is pulled through a uniform magnetic field at a constant velocity and becomes polarized.

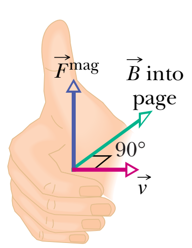


FIGURE 31-5 ■ The right-hand rule for the magnetic force law provides an "upward" force on positive changes and a "downward" force on mobile negative charges present in the wire shown in Fig. 31-4.

A Conductor Moving Through a Magnetic Field—Theory

Although it's not obvious without reflection, you are capable of predicting that a *changing* magnetic field is required to induce an electric current in a moving coil. This induction is a natural consequence of the magnetic force laws described in Eqs. 29-2 and 29-23.

Straight Conductor Moving in a Uniform $\vec{B}$-Field: Let's start by using the force law to predict what happens to a straight piece of conducting wire if we pull it at a constant velocity in a direction perpendicular to a uniform magnetic field (Fig. 31-4). Each of the charges in the conductor experiences a force given by the magnetic force law, $\vec{F}^{mag} = q\vec{v} \times \vec{B}$ (Eq. 29-2). The direction of the force is given by the right-hand rule for cross products as shown in Fig. 31-5. Since there are mobile electrons in metals, these electrons will move toward the bottom of the wire, exposing fixed excess positive charge at the top of the wire. Thus, a current will flow in the wire for an instant until the electric field created by the charge separation opposes any further electron flow.

Loop Moving in a Uniform $\vec{B}$-Field: Perhaps we can induce a current by forming a closed loop instead of a single length of wire. This doesn't help because both the left and right segments (a and c in Fig. 31-6) are perpendicular to the motion, so they become polarized in the same manner. This merely results in excess charge piling up on the top and bottom segments (b and d as shown in Fig. 31-6).

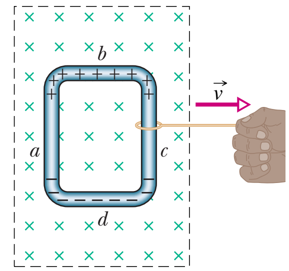


FIGURE 31-6 ■ A wire loop is pulled by a string through a uniform magnetic field at a constant velocity. Although excess charge accumulates on the top and bottom segments (b and d), no current is induced in the loop.

Loop Moving from a Uniform $\vec{B}$-Field to No Field: The easiest way to create a current using the polarization caused by the magnetic force law is to pull our loop in such a way that segment a is inside the magnetic field and segment c is not. Now what happens? We can see from Fig. 31-7 that the electrons in segment a, which is the trailing loop segment, continue to have magnetic forces exerted on them. But there are no forces on electrons in segment c of the loop because these electrons are not in the magnetic field. The only force that contributes to the current flow in the loop is the force on the left segment of wire, so the electrons in this segment of wire are pushed downward. The result is a net flow of electrons in a counterclockwise direction. Since "conventional current" as defined in Chapter 26 represents the flow of positive charge carriers, conventional current flow would be clockwise as shown in Fig. 31-7.

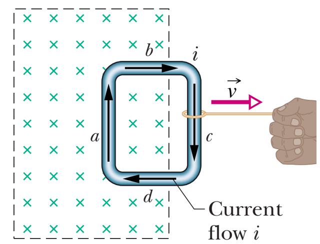


FIGURE 31-7 ■ A wire loop is pulled by a string through a region where the magnetic field is uniform on one side and zero on the other. Electrons from segment a are allowed to flow counterclockwise around the loop. The conventional current flow is clockwise.

Nonuniform $\vec{B}$-Field: Theoretically we still expect to be able to induce a current in our loop in any nonuniform magnetic field. For example, suppose the magnetic field in Fig. 31-7 is weaker (but not necessarily zero) on the right side of the loop (near segment c) than it is at the left (near segment a). In this case, the magnetic forces on electrons in the left and right segments of the loop will no longer be equal and the forces on the charges in one of the segments will overpower those on the charges in the other segments. This will cause a net current to be induced.

Our theoretical considerations enable us to conclude that by applying the magnetic force law, we can predict the results of the observations presented in the first part of this section: When a conducting loop moves perpendicular to a magnetic field, then a current will be induced whenever the coil experiences a *changing* magnetic field through it.

READING EXERCISE 31-2: In the discussion above, we determined that the forces on the electrons in the top and bottom segments of the wire loop shown in Fig. 31-6 did not contribute to the current flow. Why is this the case? ■

READING EXERCISE 31-3: Suppose the magnetic field shown in Fig. 31-6 varies continuously in such a way that it is always stronger on the right than it is on the left. What will be the direction of the resulting (conventional) current in the loop? Explain. ■

31-3 Induction by a Changing Magnetic Field

Michael Faraday made a significant contribution to physics when he asked: What happens if instead of moving the wire loop in a magnetic field we keep the wire loop *stationary* and move a magnet toward or away from the loop to create a "moving" or changing magnetic field? One might argue that since the electrons in the wire are not moving in this case, the velocity of the loop segments used in the magnetic force law expression is zero and so there should be no force on the electrons and therefore no current. On the other hand, in many ways these two situations are the same. In order to answer his question, Faraday made observations similar to those discussed below.

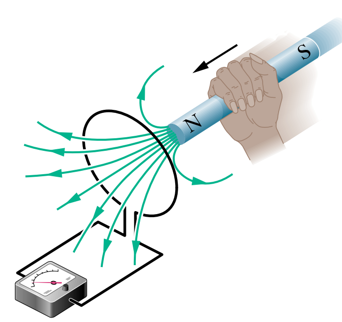


FIGURE 31-8 ■ An idealized setup showing a current meter registering nonzero currents in a stationary wire loop when a magnet is moving near the loop. (Typically a multiturn loop is needed to generate a detectable current.)

Observation 1, with a magnet: Figure 31-8 shows a conducting loop connected to a sensitive ammeter. Since there is no battery or other source of emf included, there is no current in the circuit. What happens if we move a bar magnet toward the loop? We observe that a current suddenly appears in the circuit! But the current disappears as soon as the magnet stops moving. If we then move the magnet away, a current again

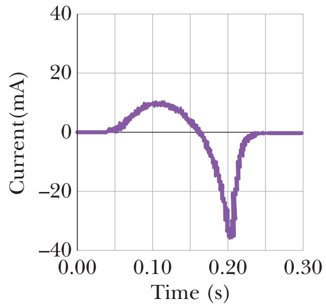


FIGURE 31-9 ■ The current induced as a magnet is dropped through a stationary multiturn coil (like that shown in Fig. 31-8). A computer data acquisition system is used to record current data at 2000 points/second.

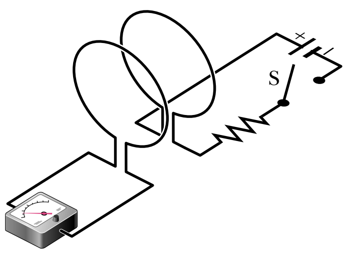


FIGURE 31-10 ■ An idealized setup showing an ammeter registering a current in the left-hand wire loop while switch S is being closed or opened (to turn the current in the right-hand loop on and off). No motion of the coils is involved. Faraday made essentially the same observation using multi-loop coils.

suddenly appears, but now in the opposite direction. If we experimented for a while, we would observe the following:

1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other, but it doesn't matter which one); the current disappears when the relative motion between them ceases. See Fig. 31-9 for a graph of the current induced by a magnet dropped through a stationary coil.
2. Faster motion produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current. Moving the south pole toward or away from the loop also causes current, but in the reversed direction.

We call the current produced in the loop an **induced current;** the work done per unit charge to produce that current (to move the conduction electrons that constitute the current) is called an **induced emf,** and the process of producing the current and emf is called induction. Currents that are caused by batteries in a circuit and those caused by induction in a wire loop are the same—mobile electrons are flowing through wires.

Observation 2, replacing the magnet with a current-carrying coil: Let us now perform a second observation. For this observation we use the apparatus of Fig. 31-10, with the two conducting loops close to each other but not touching. If we close switch S to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—an induced current—in the left-hand loop. If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction. We get an induced current (and thus an induced emf) only when the current in the right-hand loop is changing (either when turning on or off) and not when it is constant (even if the current is large). The outcome of this second observation is not surprising. We know from Ampère's law (Chapter 30) that the magnitude of the magnetic field surrounding a current-carrying wire increases as the current increases and its direction changes when the direction of current changes.

Faraday also noticed that the actual amount of magnetic field present at the area enclosed by the loop does not matter. Instead, the values of the induced emf and induced current are determined by the *rate* at which the amount changes.

When we pull all of these observations together, the way Faraday did, we conclude that

> Induced emf and current are present whenever the magnetic field present in the area subtended by the conducting loop *changes* for any reason.
>
> The amount of induced emf and current increases as a function of the rate of change of the magnetic field present at the area subtended by the loop.

Charging an Electric Car by Induction

Suppose we replace the switch in Fig. 31-10 with a source of current in the right-hand loop that varies over time sinusoidally. Then we create a magnetic field at the location of the left-hand loop that is also changing sinusoidally in time. This time-varying magnetic field then induces current in the left-hand loop that varies with time as well. By using some circuitry to filter out the negative current, we can use this induced current to charge a battery even though there is *no electrical contact* between the right- and left-hand loops. This type of noncontact charging is also used for charging familiar devices such as electric toothbrushes. Although an actual charger for an electric toothbrush or car like that discussed in the chapter-opening puzzler has more loops of wire and electrical circuits in it, it works on the same principle of electromagnetic induction discovered by Henry and Faraday in the early 19th century.

One drawback of inductive charging is that it is slower than direct charging. This is not a problem for electric toothbrush charging, but it is for electric car charging. This is probably one reason why the inductively charged General Motors EV1 cars like the one shown in the chapter puzzler have been taken off the market. You will learn more about the practical applications of induction in Chapter 32.

READING EXERCISE 31-4: Can the magnetic force law be used to explain why a current appears in a stationary loop when a bar magnet is brought close to it? If so, use your understanding of this force law to explain how this happens. If not, justify why not. ■

READING EXERCISE 31-5: Consider the induced current data shown in Fig. 31-9. The magnet is accelerating as it falls through the stationary coil. The magnet is dropped in free fall. The extrema of currents are about +8mA and −35mA. Why is the negative extremum larger? ■

31-4 Faraday's Law

We can enhance the predictive power of Faraday's qualitative observations by developing a mathematical formulation of electromagnetic induction. The mathematical expression that describes electromagnetic induction is commonly known as **Faraday's law.** Although we derive Faraday's law for a simplified situation using concepts and laws that we have already introduced, it can be applied to virtually any situation.

Magnetic Flux

To begin we use the concept of magnetic flux to quantify the amount of magnetic field at the area enclosed by a loop. In Chapter 24, in a similar situation, we needed to calculate the amount of an electric field present on a surface. There we determined electric flux for a small element of essentially flat area in Eq. 24-2 as $\Phi^{\text{elec}} = \vec{E} \cdot \Delta\vec{A}$ (the dot product of the normal vector representing a small area and the electric field vector at the location of the area). By analogy, the *magnetic flux* at the surface of a small area element $\Delta\vec{A}$ that is located in a magnetic field $\vec{B}$ is defined as

$$\Phi^{\text{mag}} = \vec{B} \cdot \Delta\vec{A} \qquad \text{(magnetic flux at an area } \Delta\vec{A}\text{)}. \tag{31-1}$$

Simply put, the flux of magnetic field Φ^{mag} at an area element A is the product of the area element and the component of the field *perpendicular* to it for a uniform magnetic field. The validity of this basic definition depends on the assumption that the magnetic field $\vec{B}$ is uniform over the surface element $\Delta\vec{A}$. If the field varies over the area, we must break the area up into little pieces in such a way that the field will be about constant for each piece. We then calculate the flux in each little piece and perform an integration to add up all the little contributions in analogy to the more general definition of electric flux.

From Eq. 31-1, we see that the SI unit for magnetic flux is the tesla-square meter, which is called the weber (abbreviated Wb):

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T}\cdot\text{m}^2. \tag{31-2}$$

Using our simplified formulation of magnetic flux, we are now ready to derive Faraday's law.

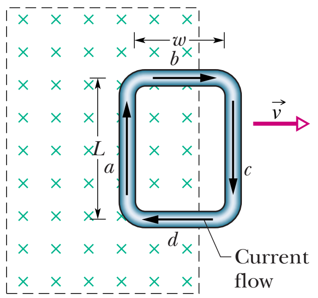


FIGURE 31-11 ■ A wire loop is moving at a constant velocity through a region where the magnetic field is uniform on one side and zero on the other. While this is happening, the magnetic flux at the area subtended by the coil is decreasing at a constant rate.

A Simplified Derivation of Faraday's Law

Consider the simple situation depicted in Fig. 31-7 in which a wire loop is being pulled out of a uniform magnetic field at a constant velocity. Next we derive the relationship between the emf induced in the loop and the rate of change of the magnetic flux enclosed by the loop. To help us with the derivation we have redrawn the situation and introduced symbols for the dimensions of the loop and the axis along which it moves in Fig. 31-11.

According to the magnetic force law, each charge in the left part of the loop (segment a) will experience a force of magnitude $F^{\text{mag}} = qvB$. As the positive and negative charges separate, an electric field of magnitude

$$E = \frac{F^{\text{mag}}}{|q|} = vB \tag{31-3}$$

will be generated. If segment a has a length L, then the potential difference of induced emf across it is given by

$$\mathscr{E} = EL = vBL. \tag{31-4}$$

Next we need to relate the right side of Eq. 31-4 to the rate at which the magnetic flux at the area subtended by the loop is decreasing as it moves out of the uniform B-field. If we designate the loop as being pulled in the x direction, then its velocity component can be expressed as $v_x = dx/dt$. Note that the area of the moving loop is decreasing at a rate given by $dA/dt = -L\,dx/dt = -Lv_x$. Since the magnetic field that subtends the left part of the area enclosed by the loop is constant, the rate of change of the magnetic flux at the loop can be expressed as

$$\frac{d\Phi^{\text{mag}}}{dt} = \frac{d(BA)}{dt} = B\frac{dA}{dt} = -v_x BL. \tag{31-5}$$

Combining Eqs. 31-4 with 31-5, we get an expression for Faraday's law for a single loop or coil,

$$\mathscr{E} = -\frac{d\Phi^{\text{mag}}}{dt} \quad \text{(Faraday's law for a single-turn coil).} \tag{31-6}$$

As you will see in the next section, the induced emf $\mathscr{E}$ tends to oppose the flux change, and the minus sign indicates that opposition. Faraday's law can also be expressed in words:

> The amount of the emf $\mathscr{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ^{mag} at the area enclosed by the loop changes with time.

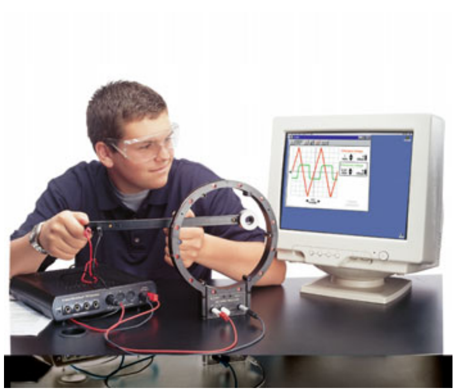


FIGURE 31-12 ■ It is quite easy to verify Faraday's law with modern apparatus and computer data acquisition systems. Here a student holds a small multiturn pickup coil inside a larger field coil that is generating a "sawtooth" magnetic field that increases and then decreases continuously. The B-field is shown on the jagged dark red trace on the computer screen. The induced current in the pickup coil is shown by the squarish lighter green trace. (Photo courtesy of PASCO scientific.)

If we change the magnetic flux at a coil of N turns, an induced emf appears in every turn and the total emf induced in the coil is the sum of these individual induced emfs. If the coil is tightly wound (closely packed), so that the same magnetic flux Φ^{mag} is present in each turn, the total emf induced in the coil is

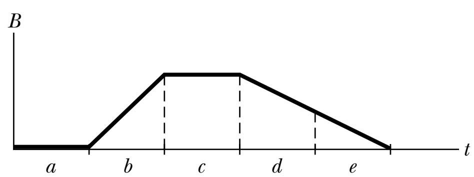
$$\mathscr{E} = -N\frac{d\Phi^{\text{mag}}}{dt} \quad \text{(Faraday's law for an } N\text{-turn coil).} \tag{31-7}$$

Although we have used simple geometry to derive Faraday's law (Eq. 31-7), experiments (such as the one shown in Fig. 31-12) have verified that the mathematical expression we have derived is true for any situation where the flux enclosed by a set of conducting loops or coils is changing. In fact, there are many ways to change the magnetic flux at a coil and thus induce emfs and currents:

1. Change the magnitude B of the magnetic field within the coil.
2. Change the area of the coil, or the portion of that area that happens to lie within the magnetic field (for example, by expanding the coil or sliding it out of the field).
3. Change the angle between the direction of the magnetic field $\vec{B}$ and the area of the coil (for example, by rotating the coil so that $\vec{B}$ is first perpendicular to the plane of the coil and then is along that plane).

Later on in the chapter we will derive a more general form of Faraday's law that relates flux change to electric field induction even when no charges or conducting loops are present.

READING EXERCISE 31-6: The graph gives the magnitude $B(t)$ of a magnetic field that exists throughout the area subtended by a conducting loop, perpendicular to the plane of the loop. Although it changes with time, at any particular instant the magnetic field is uniform over the area of the loop. (a) Rank the five time intervals (a, b, c, d, and e) shown on the graph according to the amount of the emf $|\mathscr{E}|$ induced in the loop, greatest first. (b) Explain your reasoning.



TOUCHSTONE EXAMPLE 31-1: Coil in a Long Solenoid

The long solenoid S shown (in cross section) in Fig. 31-13 has 220 turns/cm and carries a current $i = 1.5$ A; its diameter D is 3.2 cm. At its center we place a 130-turn, closely packed coil C of diameter $d = 2.1$ cm. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the size of the emf $|\mathscr{E}|$ that is induced in coil C while the current in the solenoid is changing?

SOLUTION The **Key Ideas** here are these:

1. Because coil C is located in the interior of the solenoid, it lies within the magnetic field produced by current i in the solenoid; thus, there is a magnetic flux Φ^{mag} present in coil C.
2. Because current i decreases, flux Φ^{mag} also decreases.
3. As Φ^{mag} decreases, emf $\mathscr{E}$ is induced in coil C, according to Faraday's law.

Because coil C consists of more than one turn, we apply Faraday's law in the form of Eq. 31-7 ($\mathscr{E} = -N\,d\Phi^{\text{mag}}/dt$), where the number of turns N is 130 and $d\Phi^{\text{mag}}/dt$ is the rate at which the flux in each turn changes.

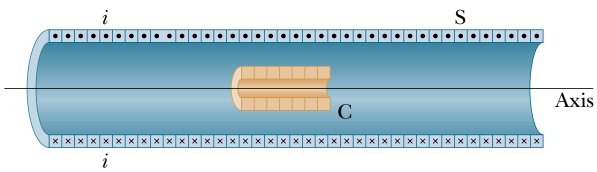


FIGURE 31-13 A coil C is located inside solenoid S, which carries current i.

Because the current in the solenoid decreases at a steady rate, flux Φ^{mag} also decreases at a steady rate and we can write $d\Phi^{\text{mag}}/dt$ as $\Delta\Phi^{\text{mag}}/\Delta t$. Then, to evaluate $\Delta\Phi^{\text{mag}}$, we need the final and initial flux. The final flux Φ_f^{mag} is zero because the final current in the solenoid is zero. To find the initial flux Φ_i^{mag}, we need two more **Key Ideas:**

4. The flux at the area enclosed by each turn of coil C depends on the area A and orientation of that turn in the solenoid's magnetic field $\vec{B}$. Because $\vec{B}$ is uniform and directed perpendicular to area A, the flux is given by Eq. 31-1 ($\Phi^{\text{mag}} = BA$).
5. The magnitude B of the magnetic field in the interior of a solenoid depends on the solenoid's current i and its number n of turns per unit length, according to Eq. 30-25 ($B = n\mu_0|i|$).

For the situation of Fig. 31-13, A is $\frac{1}{4}\pi d^2 (= 3.46 \times 10^{-4}\ \text{m}^2)$ and n is 220 turns/cm, or 22 000 turns/m. Substituting Eq. 30-25 into Eq. 31-1 then leads to

$$\begin{aligned}\Phi_i^{\text{mag}} &= BA = (n\mu_0|i|)A \\ &= (22\,000\ \text{turns/m})(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.5\ \text{A})(3.46 \times 10^{-4}\ \text{m}^2) \\ &= 1.44 \times 10^{-5}\ \text{Wb}.\end{aligned}$$

Now we can write

$$\frac{d\Phi^{\text{mag}}}{dt} = \frac{\Delta\Phi^{\text{mag}}}{\Delta t} = \frac{\Phi_f^{\text{mag}} - \Phi_i^{\text{mag}}}{\Delta t}$$

$$= \frac{(0 - 1.44 \times 10^{-5}\ \text{Wb})}{25 \times 10^{-3}\ \text{s}}$$

$$= -5.76 \times 10^{-4}\ \text{Wb/s} = -5.76 \times 10^{-4}\ \text{V}.$$

We are interested only in the size of the emf, so we ignore the minus signs here and in Eq. 31-7, writing

$$|\mathscr{E}| = \left|N\frac{d\Phi^{\text{mag}}}{dt}\right| = (130\ \text{turns})(5.76 \times 10^{-4}\ \text{V})$$

$$= 7.5 \times 10^{-2}\ \text{V} = 75\ \text{mV}. \qquad \text{(Answer)}$$

31-5 Lenz's Law

Soon after Faraday proposed his law of induction, Heinrich Friedrich Lenz devised a rule—now known as Lenz's law—for determining the direction of an induced current in a loop:

> An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that has induced the current.

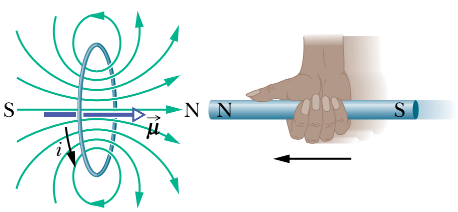


FIGURE 31-14 ■ Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment $\vec{\mu}$ oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

It is important to notice that it is the *change* in the flux that determines the direction of the induced current rather than the direction of the magnetic field or motion. Furthermore, the direction of an induced emf is that of the induced current. To get a feel for Lenz's law, let us apply it in two different but equivalent ways to Fig. 31-14, where the north pole of a magnet is being moved toward a conducting loop.

1. Opposition to Flux Change. In Fig. 31-14, with the magnet initially distant, there is no magnetic flux at the area encircled by the loop. As the north pole of the magnet then nears the loop with its magnetic field $\vec{B}$ directed *toward the left,* the flux at the loop increases. To oppose this increase in flux, the induced current i must set up its own field $\vec{B}_i$ *directed toward the right* inside the loop, as shown in Fig. 31-15*a*; then the rightward flux of field $\vec{B}_i$ opposes the increasing leftward flux of field $\vec{B}$. The right-hand rule of Fig. 30-19 then tells us that i must be counterclockwise in Fig. 31-15*a*.

2. Opposition to Pole Movement. The approach of the magnet's north pole in Fig. 31-14 increases the magnetic flux in the loop and thereby induces a current in the loop. From Fig. 30-22, we know that the loop then acts as a magnetic dipole with a south pole and a north pole, and that its magnetic dipole moment $\vec{\mu}$ is directed from south to north. To oppose the magnetic flux increase being caused by the approaching magnet, the loop's north pole (and thus $\vec{\mu}$) must face *toward* the approaching north pole so as

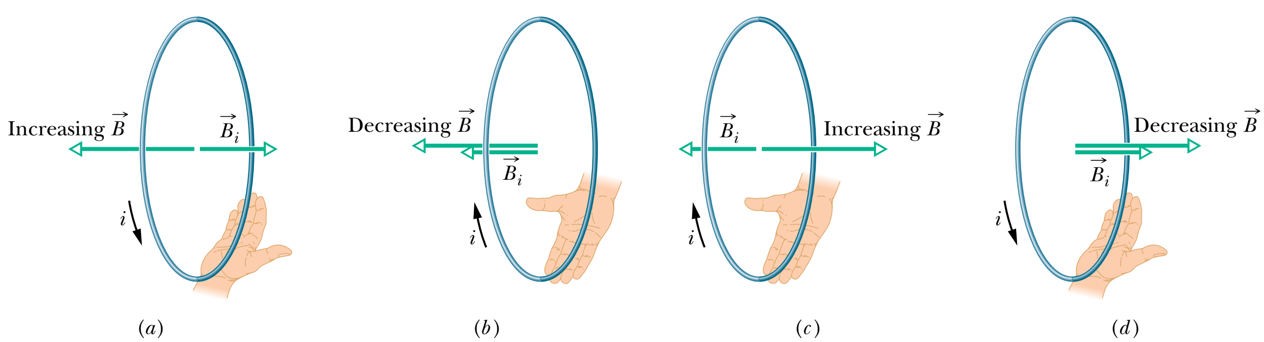


FIGURE 31-15 ■ The current i induced in a loop has the direction such that the current's magnetic field $\vec{B}_i$ opposes the change in the magnetic field $\vec{B}$ inducing i. The field $\vec{B}_i$ is always directed opposite an increasing field $\vec{B}$ shown in (*a*) and (*c*) and in the same direction as a decreasing field $\vec{B}$ shown in (*b*) and (*d*). The curled-straight right-hand rule gives the direction of the induced current based on the direction of the induced field.

to repel it (Fig. 31-14). Then the curled-straight right-hand rule for $\vec{\mu}$ (Fig. 30-22) tells us that the current induced in the loop must be counterclockwise in Fig. 31-14.

If we next pull the magnet away from the loop, a current will again be induced. Now, however, the loop will have a south pole facing the retreating north pole of the magnet, so as to oppose the retreat. Thus, the induced current will be clockwise.

As we noted above, be careful to remember that the flux of $\vec{B}_i$ always opposes the *change* in the flux of $\vec{B}$, but that does not always mean that $\vec{B}_i$ points opposite $\vec{B}$. For example, if we pull the magnet away from the loop in Fig. 31-14, the flux Φ^{mag} from the magnet is still directed to the left at the area subtended by the loop, but it is now decreasing. The flux of $\vec{B}_i$ must now be to the left inside the loop, to oppose the *decrease* in Φ^{mag}, as shown in Fig. 31-15*b*. Thus, $\vec{B}_i$ and $\vec{B}$ are now in the same direction.

Figures 31-15*c* and *d* show the situations in which the south pole of the magnet approaches and retreats from the loop, respectively. Figure 31-16 is a photo of a demonstration of Lenz's law in action.

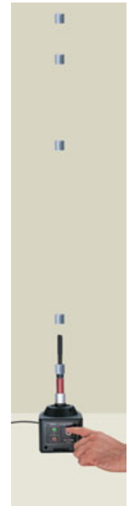


FIGURE 31-16 ■ This demonstration of Lenz's law occurs when an electromagnet is switched on suddenly. The current induced in a metal ring opposes the electromagnet's current. The repulsive forces between the magnet and the ring cause the ring to jump more than a meter. (Photo courtesy of PASCO scientific.)

Electric Guitars

Soon after rock began in the mid-1950s, guitarists switched from acoustic guitars to electric guitars—but it was Jimi Hendrix who first used the electric guitar as an electronic instrument. He was able to create new sounds that continue to influence rock music today. What is it about an electric guitar that enabled Hendrix to make different sounds?

Whereas an acoustic guitar depends for its sound on the acoustic resonance produced in the hollow body of the instrument by the oscillations of the strings, an electric guitar like that being played by Hendrix in Fig. 31-17 is a solid instrument, so there is no body resonance. Instead, the oscillations of the metal strings are sensed by electric "pickups" that send signals to an amplifier and a set of speakers.

The basic construction of a pickup is shown in Fig. 31-18. Wire connecting the instrument to the amplifier is coiled around a small magnet. The magnetic field of the magnet produces a north and south pole in the section of the metal string just above the magnet. That section of string then has its own magnetic field. When the string is plucked and thus made to oscillate, its motion relative to the coil changes the flux of its magnetic field at the area encircled by the coil, inducing a current in the coil. As the string oscillates toward and away from the coil, the induced current changes direction at the same frequency as the string's oscillations, thus relaying the frequency of oscillation to the amplifier and speaker.

On a Stratocaster©, there are three groups of pickups, placed near the bridge at the end of the wide part of the guitar body. The group closest to the bridge better detects the high-frequency oscillations of the strings; the group farthest from the near end better detects the low-frequency oscillations. By throwing a toggle switch on the guitar, the musician can select which group or which pair of groups will send signals to the amplifier and speakers.

To gain further control over his music, Hendrix sometimes rewrapped the wire in the pickup coils of his guitar to change the number of turns. In this way, he altered the amount of emf induced in the coils and thus their relative sensitivity to string oscillations. Even without this additional measure, you can see that the electric guitar offers far more control over the sound that is produced than can be obtained with an acoustic guitar.

FIGURE 31-17 ■ Jimi Hendrix playing his Fender Stratocaster©. This guitar has three groups of six electric pickups each (within the wide part of the body). A toggle switch (at the bottom of the guitar) allows the musician to determine which group of pickups sends signals to an amplifier and thus to a speaker system.

READING EXERCISE 31-7: Lenz's law states: "An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current." (a) Suppose there is a magnetic field directed into the plane of this page and that the strength of the field is decreasing. Would a magnetic field that opposes this change in magnetic flux be directed into the page, out of the page, or in some other direction? Explain your reasoning. (b) Suppose that there is a magnetic field directed into the plane of this page that is increasing in strength. Would a magnetic field that opposes this change in magnetic flux be directed into the page, out of the page, or in some other direction? Explain your reasoning. ■

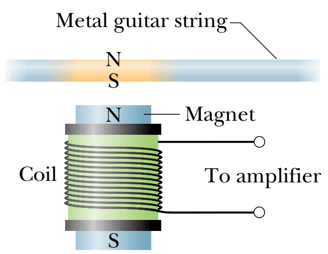
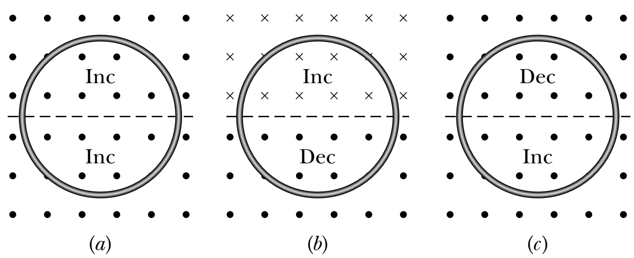


FIGURE 31-18 ■ A side view of an electric guitar pickup. When the metal string (which acts like a magnet) oscillates, it causes a variation in magnetic flux that induces a current in the coil.

READING EXERCISE 31-8: The figure shows three situations in which identical circular conducting loops are in uniform magnetic fields that are either increasing (Inc) or decreasing (Dec) in magnitude at identical rates. In each, the dashed line coincides with a diameter. (a) Rank the situations according to the amount of the current induced in the loops, greatest first. (b) Explain your reasoning.



TOUCHSTONE EXAMPLE 31-2: Induced Emf

Figure 31-19 shows a conducting loop consisting of a half-circle of radius $r = 0.20$ m and three straight sections. The half-circle lies in a uniform magnetic field $\vec{B}$ that is directed out of the page; the field magnitude is given by $B = (4.0\ \text{T/s}^2)t^2 + (2.0\ \text{T/s})t + 3.0$ T. An ideal battery with $\mathscr{E}_{\text{bat}} = 2.0$ V is connected to the loop. The resistance of the loop is 2.0 Ω.

(a) What are the amount and direction of the emf $\mathscr{E}^{\text{ind}}$ induced around the loop by field $\vec{B}$ at $t = 10$ s?

SOLUTION ■ One **Key Idea** here is that, according to Faraday's law, $\mathscr{E}^{\text{ind}}$ is equal to the negative rate $d\Phi^{\text{mag}}/dt$ at which the magnetic flux at the area encircled by the loop changes. A second **Key Idea** is that the flux at the loop depends on the loop's area A and its orientation in the magnetic field $\vec{B}$. Because $\vec{B}$ is uniform and is perpendicular to the plane of the loop, the flux is given by Eq. 31-1 ($\Phi^{\text{mag}} = BA$). Using this equation and realizing that only the field magnitude B changes in time (not the area A), we rewrite Faraday's law, Eq. 31-6, as

$$|\mathscr{E}^{\text{ind}}| = \left|\frac{d\Phi^{\text{mag}}}{dt}\right| = \left|\frac{d(BA)}{dt}\right| = A\left|\frac{dB}{dt}\right|.$$

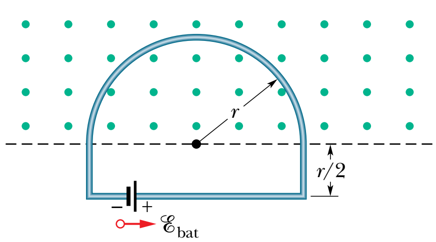


FIGURE 31-19 ■ A battery is connected to a conducting loop consisting of a half-circle of radius r that lies in a uniform magnetic field. The field is directed out of the page; its magnitude is changing.

A third **Key Idea** is that, because the flux penetrates the loop only within the half-circle, the area A in this equation is $\frac{1}{2}\pi r^2$. Substituting this and the given expression for B yields

$$|\mathscr{E}^{\text{ind}}| = A\left|\frac{dB}{dt}\right| = \frac{\pi r^2}{2}\frac{d}{dt}[(4.0\ \text{T/s})t^2 + (2.0\ \text{T/s})t + 3.0\ \text{T}]$$

$$= \frac{\pi r^2}{2}[(8.0\ \text{T/s}^2)t + (2.0\ \text{T/s})].$$

At $t = 10$ s, then,

$$|\mathscr{E}^{\text{ind}}| = \frac{\pi(0.20\ \text{m})^2}{2}[(8.0\ \text{T/s})(10\ \text{s}) + (2.0\ \text{T/s})]$$

$$= 5.152\ \text{V} \approx 5.2\ \text{V}. \qquad \text{(Answer)}$$

To find the direction of $\mathscr{E}^{\text{ind}}$, we first note that in Fig. 31-19 the flux at the loop is out of the page and increasing. Then the **Key Idea** here is that the induced field B^{ind} (due to the induced current) must oppose that increase, and thus be into the page. Using the curled-straight right-hand rule (Fig. 30-8*c*), we find that the induced current *contribution* must be clockwise around the loop. The induced emf $\mathscr{E}^{\text{ind}}$ must then also be clockwise.

(b) What is the current in the loop at $t = 10$ s?

SOLUTION ■ The **Key Idea** here is that two emfs tend to move charges around the loop. The induced $\mathscr{E}^{\text{ind}}$ tends to drive a current clockwise around the loop; the battery's $\mathscr{E}_{\text{bat}}$ tends to drive a current counterclockwise. Because $\mathscr{E}^{\text{ind}}$ is greater than $\mathscr{E}_{\text{bat}}$, the net emf $\mathscr{E}^{\text{net}}$ is clockwise, and thus so is the current. To find the current at $t = 10$ s, we use $i = \mathscr{E}/R$:

$$i = \frac{\mathscr{E}^{\text{net}}}{R} = \frac{\mathscr{E}^{\text{ind}} - \mathscr{E}_{\text{bat}}}{R}$$

$$= \frac{5.152\ \text{V} - 2.0\ \text{V}}{2.0\,\Omega} = 1.58\ \text{A} \approx 1.6\ \text{A}. \qquad \text{(Answer)}$$

31-6 Induction and Energy Transfers

Let us return to the simple situation we considered in Fig. 31-7. What are the consequences of the fact that a clockwise current is induced when the loop is pulled to the right and a counterclockwise current is induced when the loop is pushed to the left? If one pushes the loop back and forth (right and left), the result is an alternating current in the loop. This is current just like the current in our household electric system. It is a current that could run a motor, light a bulb, or provide heating through the resistive dissipation. If it took no effort on our part to push the loop back and forth, we could solve the energy crisis. Of course, it does take effort (work) on our part to push and pull the loop back and forth.

If you want to drag a metal loop out of a magnetic field at a constant velocity, you have to exert a force on the loop to balance the magnetic force associated with the charges moving in the magnetic field. This requires you to do work on the loop, but doing work adds energy to a system. We certainly cannot violate the principle of conservation of energy. So, where does this energy go? One place the energy could go is into an increase in the internal energy of the loop's wires. Since we observe a temperature rise in the wires, we conclude that the work done has been transformed into thermal energy—one form of internal energy. This makes sense. There is a current i in the loop that has some resistance R, and we learned in Section 26-7 that the electric power dissipation (or rate of thermal energy increase in the wires) is given by

$$P = i^2R \qquad \text{(resistive dissipation).} \qquad \text{(Eq. 26-11)}$$

How does this rate of energy loss compare to the rate we are doing work? Perhaps they are the same. In that case, we might conclude that the work we do in moving the loop is transformed into thermal energy in the loop. Let's work out the details.

Figure 31-11 shows a situation involving induced current. A rectangular loop of wire of width L has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. This field may be produced, for example, by a large electromagnet. The dashed lines in the figure show the assumed limits of the magnetic field; the fringing of the field at its edges is neglected. You are asked to pull this loop to the right at a constant velocity $\vec{v}$.

In the situation of Fig. 31-11, the flux of the field at the loop is changing with time. Let us now calculate the rate at which you do mechanical work as you pull steadily on the loop. The amount of work done by a force $\vec{F}$ in moving a loop a small distance $d\vec{x}$ in a time dt is

$$dW = \vec{F}\cdot d\vec{x}.$$

For simplicity, let us consider a force $\vec{F}$, which is completely in the direction of the displacement $d\vec{x}$. Then

$$dW = \vec{F}\cdot d\vec{x} = F dx.$$

The rate of doing work (which is called the *power* P) is

$$P = \frac{dW}{dt} = F\frac{dx}{dt}.$$

So

$$P = Fv, \qquad \text{(31-8)}$$

where v is the speed at which we move the loop.

Suppose that we wish to find an expression for the power, P, in terms of the magnitude B of the magnetic field and the characteristics of the loop—namely, its resistance

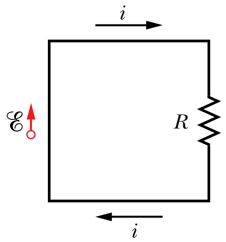


FIGURE 31-20 ■ A circuit diagram for the loop of Fig. 31-7 while it is moving.

R to current and its dimension L. As you move the loop to the right in Fig. 31-11, the portion of its area within the magnetic field decreases. Thus, the flux at the loop also decreases and, according to Lenz's law, a current is produced in the loop. It is the presence of this current that causes the force that opposes your pull.

To find the amount of the current, we first apply Faraday's law for a single loop in conjunction with Eq. 31-4. We can write the amount of this emf as

$$|\mathscr{E}| = \left|\frac{d\Phi^{\text{mag}}}{dt}\right| = BvL. \tag{31-9}$$

Figure 31-20 shows the loop depicted in Fig. 31-7, as a circuit. The induced emf, $\mathscr{E}$, is represented on the left, and the collective resistance R of the loop is represented on the right. The direction of the induced current i is shown as in Fig. 31-7, and we have already established that $\mathscr{E}$ must have the same direction as the conventional current, i.

To find the amount of the induced current, we cannot apply the loop rule for potential differences in a circuit because, as you will see in Section 31-7, we cannot define a potential difference for an induced emf. However, we can apply the equation $i = \mathscr{E}/R$. With Eq. 31-9, the current amount becomes

$$|i| = \frac{BvL}{R}. \tag{31-10}$$

Because three segments of the loop in Fig. 31-7 carry this current through the magnetic field, sideways deflecting forces act on those segments. From Chapter 29, we know that the magnitude of such a deflecting force is given in general notation by

$$F_d = |i\vec{L} \times \vec{B}|. \tag{31-11}$$

The deflecting forces acting on segments a, b, and d of the loop shown in Fig. 31-7 can be denoted as $\vec{F}_a$, $\vec{F}_b$, and $\vec{F}_d$. Application of the right-hand rule to each of these segments shows that the forces are perpendicular to each segment and point outward from the loop. Note, however, that from the symmetry, $\vec{F}_b$ and $\vec{F}_d$ are oppositely directed and equal in magnitude, so they cancel. This leaves only $\vec{F}_a$, which is directed opposite the force $\vec{F}$ you apply to the loop. Therefore, $\vec{F} = -\vec{F}_a$.

Using Eq. 31-11 to obtain the magnitude of $\vec{F}_a$ and noting that the angle between $\vec{B}$ and the length vector $\vec{L}$ for the left segment is 90°, we can write

$$F = F_a = |i|BL \sin 90° = |i|BL. \tag{31-12}$$

Substituting Eq. 31-10 for i in Eq. 31-12 then gives us

$$F = \frac{B^2vL^2}{R}. \tag{31-13}$$

Since B, L, and R are constants, the speed v at which you move the loop is constant if the magnitude F of the force you apply to the loop is also constant.

By substituting Eq. 31-13 into Eq. 31-8, we find the rate at which you do work on the loop as you pull it out of the magnetic field:

$$P = Fv = \frac{B^2v^2L^2}{R} \quad \text{(rate of doing work).} \tag{31-14}$$

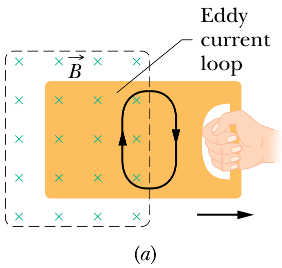
To complete our analysis, let us find the rate at which internal energy appears in the loop as you pull it along at constant speed. We calculate it from Eq. 26-11,

$$P = i^2R. \tag{31-15}$$

Substituting for i from Eq. 31-10, we find

$$P = \left(\frac{BvL}{R}\right)^2 R = \frac{B^2v^2L^2}{R} \qquad \text{(rate of internal energy gain),} \tag{31-16}$$

which is exactly equal to the rate at which you are doing work on the loop (Eq. 31-14). Thus, the work that you do in pulling the loop through the magnetic field is transferred to thermal energy in the loop, manifesting itself as a small increase in the loop's temperature.



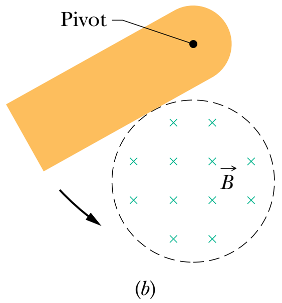


FIGURE 31-21 ■ (*a*) As you pull a solid conducting plate out of a magnetic field, *eddy currents* are induced in the plate. A typical loop of eddy current is shown; it has the same clockwise sense of circulation as the current in the conducting loop of Fig. 31-7. (*b*) A conducting plate is allowed to swing like a pendulum about a pivot and into a region of magnetic field. As it enters and leaves the field, eddy currents are induced in the plate.

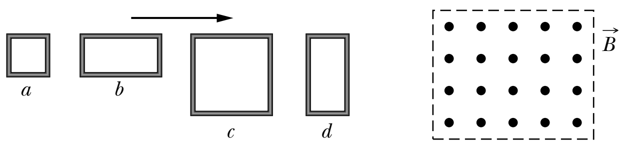
Eddy Currents

Suppose we replace the conducting loop of Fig. 31-7 with a solid conducting plate as shown in Fig. 31-21*a*. If we then move the plate out of the magnetic field, the relative motion of the field and the conductor again induces a current in the conductor. Thus, we again encounter an opposing force and must do work because of the induced current. With the plate, however, the conduction electrons making up the induced current do not follow one path as they do with the loop. Instead, the electrons swirl about within the plate as if they were caught in an eddy (or whirlpool) of water. Such a current is called an *eddy current* and can be represented as in Fig. 31-21*a as if* it followed a single path.

Eddy currents are used to cook food on an induction stove. To do this an oscillating current is sent through a conducting coil that lies just below the cooking surface. The magnetic field produced by that current oscillates and induces an oscillating current in the conducting cooking pan. Because the pan has some resistance to that current, the electrical energy of the current is continuously transformed to the pan's energy, resulting in a temperature increase of the pan and the food in it. What's amazing is that the stove itself might not get hot at all—only the pan.

As with the conducting loop of Fig. 31-7, the current induced in the plate results in mechanical energy being dissipated as it increases the pan's thermal energy. The dissipation is more apparent in the arrangement of Fig. 31-21*b*; a conducting plate, free to rotate about a pivot, is allowed to swing down through a magnetic field like a pendulum. Each time the plate enters and leaves the field, a portion of its mechanical energy is transferred to its thermal energy. After several swings, no mechanical energy remains and the warmed-up plate just hangs from its pivot.

READING EXERCISE 31-9: The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field $\vec{B}$ (directed out of the page) at the same constant velocity. (a) Rank the four loops according to the maximum amount of the emf induced as they move through the field, greatest first. (b) Explain your reasoning.



31-7 Induced Electric Fields

Let us place a copper ring of radius r in a uniform external magnetic field, as in Fig. 31-22*a*. The field—neglecting fringing—fills a cylindrical volume of radius R. Suppose that we increase the strength of this field at a steady rate, perhaps by increasing—in an appropriate way—the current in the windings of the electromagnet that produces the field. The magnetic flux at the ring will then change at a steady rate and—by Faraday's law—an induced emf and thus an induced current will appear in the ring. From Lenz's law we can deduce that the direction of the induced current is counterclockwise in Fig. 31-22*a*.

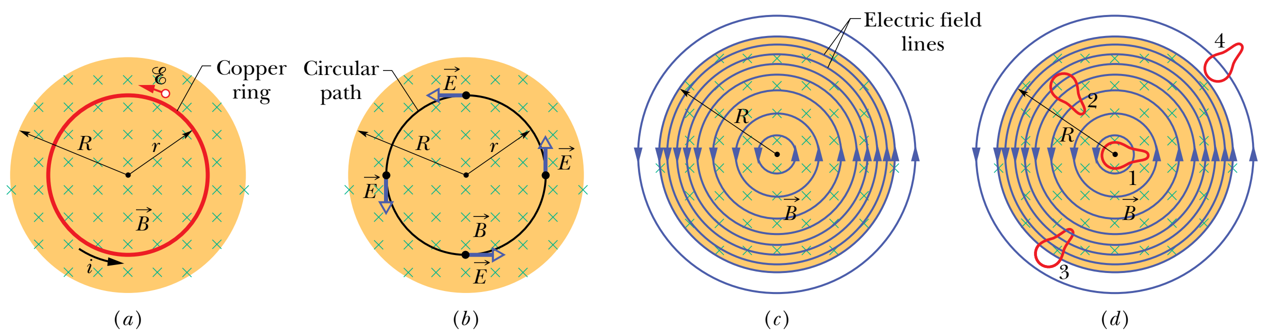


FIGURE 31-22 ■ (a) If the magnetic field increases at a steady rate, a constant induced current appears, as shown, in the copper ring of radius r. (b) An induced electric field exists even when the ring is removed; the electric field is shown at four points. (c) The complete picture of the induced electric field, displayed as field lines. (d) Four similar closed paths that enclose identical areas. Equal emfs are induced around paths 1 and 2, which lie entirely within the region of the changing magnetic field. A smaller emf is induced around path 3, which only partially lies in that region. No emf is induced around path 4, which lies entirely outside the magnetic field.

If there is a current in the copper ring, an electric field must be present along the ring; an electric field is needed to do the work of moving the conduction electrons. Moreover, the electric field must have been produced by the changing magnetic flux. This **induced electric field** $\vec{E}$ is just as real as an electric field produced by static charges; either field will exert a force $q\vec{E}$ on a particle of charge q.

By this line of reasoning, we are led to a more general and informative restatement of Faraday's law of induction:

> A changing magnetic field produces an electric field.

The striking feature of this statement is that the electric field is induced even if there is no copper ring.

To fix these ideas, consider Fig. 31-22b, which is just like Fig. 31-22a except the copper ring has been replaced by a hypothetical circular path of radius r. We assume, as previously, that the magnetic field $\vec{B}$ is increasing in magnitude at a constant rate dB/dt. The electric field induced at various points around the circular path must—from the symmetry—be tangent to the circle, as Fig. 31-22b shows.* Hence, the circular path is an electric field line. There is nothing special about the circle of radius r, so the electric field lines produced by the changing magnetic field must be a set of concentric circles, as in Fig. 31-22c.

As long as the magnetic field is increasing with time, the electric field represented by the circular field lines in Fig. 31-22c will be present. If the magnetic field remains constant with time, there will be no induced electric field and thus no electric field lines. If the magnetic field is decreasing with time (at a constant rate), the electric field lines will still be concentric circles as in Fig. 31-22c, but they will now have the opposite direction. All this is what we have in mind when we say: A changing magnetic field produces an electric field.

A Reformulation of Faraday's Law

Consider a particle of charge q moving around the circular path of Fig. 31-22b. The work W done on it in one revolution by the induced electric field is $q\mathscr{E}$, where $\mathscr{E}$ is the

* Arguments of symmetry would also permit the lines of $\vec{E}$ around the circular path to be radial, rather than tangential. However, such radial lines would imply that there are free charges, distributed symmetrically about the axis of symmetry, on which the electric field lines could begin or end; there are no such charges.

induced emf—that is, the work done per unit charge in moving the test charge around the path. From another point of view, the work is

$$\int \vec{F} \cdot d\vec{s} = qE(2\pi r), \tag{31-17}$$

where $|q|E$ is the magnitude of the force acting on the test charge and $2\pi r$ is the distance over which that force acts. Setting these two expressions for W equal to each other and canceling q, we find that

$$|\mathscr{E}| = 2\pi r E. \tag{31-18}$$

More generally, we can rewrite Eq. 31-17 to give the work done on a particle of charge q moving along any closed path:

$$W = \oint \vec{F} \cdot d\vec{s} = q\oint \vec{E} \cdot d\vec{s}. \tag{31-19}$$

(The circle indicates that the integral is to be taken around the closed path.) Substituting $q\mathscr{E}$ for W, we find that

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s}. \tag{31-20}$$

This integral reduces at once to Eq. 31-18 if we evaluate it for the special case of Fig. 31-22*b*.

With Eq. 31-20, we can expand the meaning of induced emf. Previously, induced emf meant the work per unit charge done in maintaining current due to a changing magnetic flux, or it meant the work done per unit charge on a charged particle that moves around a closed path in a changing magnetic flux. However, we can see in Fig. 31-22*b* and Eq. 31-20 that an induced emf can exist without the need of a current or particle: An induced emf is the sum—via integration—of quantities $\vec{E} \cdot d\vec{s}$ around a closed path, where $\vec{E}$ is the electric field induced by a changing magnetic flux and $d\vec{s}$ is a differential length vector along the closed path.

If we combine Eq. 31-20 with Faraday's law in Eq. 31-6 ($\mathscr{E} = -d\Phi^{\text{mag}}/dt$), we can rewrite Faraday's law as

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi^{\text{mag}}}{dt} \quad \text{(Faraday's law, general formula).} \tag{31-21}$$

This equation says simply that a changing magnetic field induces an electric field. The changing magnetic field appears on the right side of this equation, the electric field on the left.

Faraday's law in the form of Eq. 31-21 can be applied to *any* closed path that can be drawn in a changing magnetic field. But $\oint \vec{E} \cdot d\vec{s}$ can only be evaluated for symmetrical situations. Figure 31-22*d*, for example, shows four such paths, all having the same shape and area but located in different positions in the changing field. For paths 1 and 2, the induced emfs $\mathscr{E}(=\oint \vec{E} \cdot d\vec{s})$ are equal because these paths lie entirely in the magnetic field and thus have the same value of $d\Phi^{\text{mag}}/dt$. This is true even though the electric field vectors at points along these paths are different, as indicated by the patterns of electric field lines in the figure. For path 3 the induced emf is smaller because the enclosed flux Φ^{mag} (hence, $d\Phi^{\text{mag}}/dt$) is smaller, and for path 4 the induced emf is zero, even though the electric field is not zero at any point on the path.

A New Look at Electric Potential

Induced electric fields are produced not by static charges but by a changing magnetic flux. Although electric fields produced in either way exert forces on charged particles, there is an important difference between them. The difference is not in the way they affect charges at a given point (the electric force on a charge q in this field is still qE), but in their global properties. Their field lines behave differently and there is a problem defining the electric potential associated with induced electric fields. The simplest evidence of this difference is that the field lines of induced electric fields form closed loops, as in Fig. 31-22*c*. Field lines produced by static charges never do so but rather must start on positive charges and end on negative charges. Since the induced fields are not caused by charges, there is no place for the field lines to start or end. Instead, they form closed loops, similar to those of magnetic fields. (But these are still electric fields! They act on stationary charges whereas magnetic fields don't.)

So, a varying *magnetic field* is accompanied by circular *electric field lines*. An electric current is known to be accompanied by circular magnetic field lines. But is an electric *current* the only source of circular magnetic field lines? Might it be possible that a varying *electric field* is accompanied by a circulating *magnetic field?* This is a question we will consider in the next chapter.

What we are immediately concerned with is that the electric field lines make closed loops, which has a powerful implication for trying to define an electrostatic potential. Since the potential difference equals the work per unit charge, if we carry a charge around a loop of electric field line, the $\vec{E}$ field always acts in the direction of motion, so every small step we make makes a positive contribution to the work. But since the field follows a loop, we can come back to our starting point after having only done positive work! The implication is:

> Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

You can understand this statement quantitatively by considering what happens to a charged particle that makes a single journey around the circular path in Fig. 31-22*b*. It starts at a certain point and, on its return to that same point, has experienced an emf $\mathscr{E}$ of, let us say, 5 V; that is, work of 5 J/C has been done on the particle, and thus the particle should then be at a point that is 5 V greater in potential. However, that is impossible because the particle is back at the same point, which cannot have two different values of potential. We must conclude that potential has no meaning for electric fields that are set up by changing magnetic fields.

We can take a more formal look by recalling Eq. 25-16, which defines the potential difference between two points 1 and 2 in an electric field $\vec{E}$:

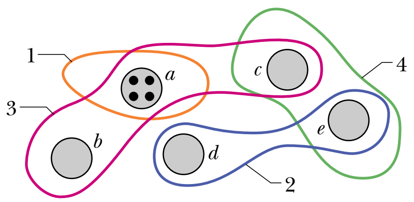
$$V_2 - V_1 = -\int_1^2 \vec{E} \cdot d\vec{s}. \tag{31-22}$$

In Chapter 25 we had not yet encountered Faraday's law of induction, so the electric fields involved in the derivation of Eq. 25-16 were those due to static charges. If 1 and 2 in Eq. 31-22 are the same point, the path connecting them is a closed loop, V_1 and V_2 are identical, and Eq. 31-22 reduces to

$$\oint \vec{E} \cdot d\vec{s} = 0 \text{ V}. \tag{31-23}$$

However, when a changing magnetic flux is present, this integral is *not* zero but is $-d\Phi^{\text{mag}}/dt$, as Eq. 31-21 asserts. Thus, assigning electric potential to an induced electric field leads us to a contradiction. We must conclude that electric potential difference is path dependent for the electric fields associated with induction.

READING EXERCISE 31-10: The figure shows five lettered regions in which a uniform magnetic field extends either directly out of the page (as in region a) or into the page. The field is increasing in magnitude at the same steady rate in all five regions; the regions are identical in area. Also shown are four numbered paths along which $\oint \vec{E} \cdot d\vec{s}$ has the magnitudes given below in terms of a unit "mag." Determine whether the magnetic fields in regions b through e are directed into or out of the page.



Path:	1	2	3	4
$\oint \vec{E} \cdot d\vec{s}$:	1(mag)	2(mag)	3(mag)	0(mag)

TOUCHSTONE EXAMPLE 31-3: Inducing an Electric Field

In Fig. 31-22b, take $R = 8.5$ cm and $dB/dt = 0.13$ T/s.

(a) Find an expression for the magnitude E of the induced electric field at points within the magnetic field, at radius r from the center of the magnetic field. Evaluate the expression for $r = 5.2$ cm.

SOLUTION The **Key Idea** here is that an electric field is induced by the changing magnetic field, according to Faraday's law. To calculate the field magnitude E, we apply Faraday's law in the form of Eq. 31-21. We use a circular path of integration with radius $r \le R$ because we want E for points within the magnetic field. We assume from the symmetry that $\vec{E}$ in Fig. 31-22b is tangent to the circular path at all points. The path vector $d\vec{s}$ is also always tangent to the circular path, so the dot product $\vec{E} \cdot d\vec{s}$ in Eq. 31-21 must have the magnitude $E\,ds$ at all points on the path. We can also assume from the symmetry that E has the same value at all points along the circular path. Then the left side of Eq. 31-21 becomes

$$\oint \vec{E} \cdot d\vec{s} = \oint E\,ds = E \oint ds = E(2\pi r). \qquad (31\text{-}24)$$

(The integral $\oint ds$ is the circumference $2\pi r$ of the circular path.)

Next, we need to evaluate the right side of Eq. 31-21. Because $\vec{B}$ is uniform over the area A encircled by the path of integration and is directed perpendicular to that area, the magnetic flux is given by Eq. 31-1:

$$\Phi^{\text{mag}} = BA = B(\pi r^2). \qquad (31\text{-}25)$$

Substituting this and Eq. 31-24 into Eq. 31-21 and dropping the minus sign, we find that the magnitude of the electric field is

$$E(2\pi r) = (\pi r^2)\frac{dB}{dt}$$

or

$$E = \frac{r}{2}\frac{dB}{dt}. \qquad \text{(Answer)}\ (31\text{-}26)$$

Equation 31-26 gives the magnitude of the electric field at any point for which $r \le R$ (that is, within the magnetic field). Substituting given values yields, for the magnitude of $\vec{E}$ at $r = 5.2$ cm,

$$E = \frac{(5.2 \times 10^{-2}\ \text{m})}{2}(0.13\ \text{T/s})$$

$$= 0.0034\ \text{V/m} = 3.4\ \text{mV/m}. \qquad \text{(Answer)}$$

(b) Find an expression for the magnitude E of the induced electric field at points that are outside the magnetic field, at radius r. Evaluate the expression for $r = 12.5$ cm.

SOLUTION The **Key Idea** of part (a) applies here also, except that we use a circular path of integration with radius $r = R$, because we want to evaluate E for points outside the magnetic field. Proceeding as in (a), we again obtain Eq. 31-24. However, we do not then obtain Eq. 31-25, because the new path of integration is now outside the magnetic field, and we need this **Key Idea**: The magnetic flux encircled by the new path is only that in the area πR^2 of the magnetic field region. Therefore,

$$\Phi^{\text{mag}} = BA = B(\pi R^2). \qquad (31\text{-}27)$$

Substituting this and Eq. 31-24 into Eq. 31-21 (without the minus sign) and solving for the magnitude of $\vec{E}$ yield

$$E = \frac{R^2}{2r}\frac{dB}{dt}. \qquad \text{(Answer)}\ (31\text{-}28)$$

Since E is not zero here, we know that an electric field is induced even at points that are outside the changing magnetic field, an

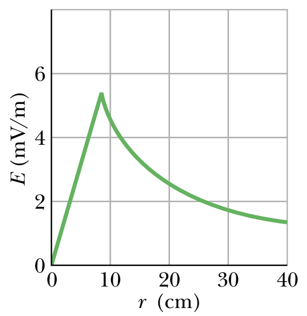


FIGURE 31-23 ■ A plot of the induced electric field $E(r)$ for the conditions given in Touchstone Example 31-3.

important result that (as you shall see in Section 32-5) makes transformers possible. With the given data, Eq. 31-28 yields the magnitude of $\vec{E}$ at $r = 12.5$ cm:

$$E = \frac{(8.5 \times 10^{-2}\ \text{m})^2}{(2)(12.5 \times 10^{-2}\ \text{m})}(0.13\ \text{T/s})$$

$$= 3.8 \times 10^{-3}\ \text{V/m} = 3.8\ \text{mV/m}. \qquad \text{(Answer)}$$

Equations 31-26 and 31-28 give the same result, as they must, for $r = R$. Figure 31-23 shows a plot of $E(r)$ based on these two equations.

31-8 Induced Magnetic Fields

Let's consider a region in space where no electric currents are present. As we have seen, a changing magnetic flux induces an electric field, and we end up with Faraday's law of induction in the form

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi^{\text{mag}}}{dt} \quad \text{(Faraday's law of induction).} \qquad (31\text{-}29)$$

Here $\vec{E}$ is the electric field induced along a closed loop by the changing magnetic flux Φ^{mag} encircled by that loop. Because symmetry is often so powerful in physics, we should be tempted to ask whether induction can occur in the opposite sense; that is, can a changing electric flux induce a magnetic field?

The answer is that it can; furthermore, the equation governing the induction of a magnetic field is almost symmetric with Eq. 31-21. We often call it Maxwell's law of induction after James Clerk Maxwell, and we write it as

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi^{\text{elec}}}{dt} \quad \text{(Maxwell's law of induction—no currents).} \qquad (31\text{-}30)$$

Here $\vec{B}$ is the magnetic field induced along a closed loop by the changing electric flux Φ^{elec} in the region encircled by that loop.

As an example of this sort of induction, we consider the charging of a parallel-plate capacitor with circular plates, as shown in Fig. 31-24*a*. (Although we shall focus on this particular arrangement, a changing electric flux will always induce a magnetic field whenever it occurs.) We assume that the charge on the capacitor is being increased at a steady rate by a constant current i in the connecting wires. Then the amount of the electric field between the plates must also be increasing at a steady rate.

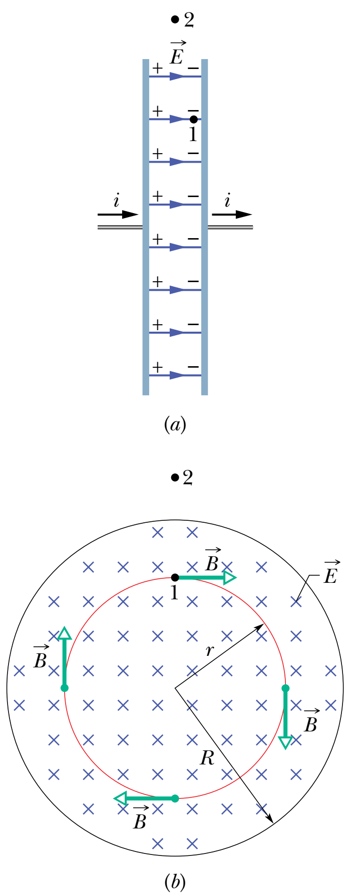


FIGURE 31-24 ■ (*a*) A circular parallel-plate capacitor, shown in side view, is being charged by a constant current i. (*b*) A view from within the capacitor, toward the plate at the right. The electric field $\vec{E}$ is uniform, is directed into the page (toward the plate), and grows in magnitude as the charge on the capacitor increases. The magnetic field $\vec{B}$ induced by this changing electric field is shown at four points on a circle with a radius r less than the plate radius R.

Figure 31-24*b* is a view of the right-hand plate of Fig. 31-24*a* from between the plates. The electric field is directed into the page. Let us consider a circular loop through point 1 in Figs. 31-24*a* and *b*, concentric with the capacitor plates and with a radius smaller than that of the plates. Because the electric field at the area subtended by the loop is changing, the electric flux at the loop must also be changing. According to Eq. 31-22, this changing electric flux induces a magnetic field around the loop.

Experiment proves that a magnetic field $\vec{B}$ *is* indeed induced around such a loop, directed as shown. This magnetic field has the same magnitude at every point around the loop and thus has circular symmetry about the central axis of the capacitor plates.

If we now consider a larger loop—say, through point 2 outside the plates in Figs. 31-24*a* and *b*—we find that a magnetic field is induced around that loop as well. Thus, while the electric field is changing, magnetic fields are induced between the plates, both inside and outside the gap. When the electric field stops changing, these induced magnetic fields disappear.

Although Eq. 31-30 is similar to Eq. 31-29, the equations differ in two ways. First, Eq. 31-30 has the two extra symbols, μ_0 and ε_0, but they appear only because we employ SI units. Second, Eq. 31-30 lacks the minus sign of Eq. 31-29. That difference in sign means that the induced electric field $\vec{E}$ and the induced magnetic field $\vec{B}$ have opposite directions when they are produced in otherwise similar situations.

To see this opposition of directions, examine Fig. 31-25, in which an increasing magnetic field $\vec{B}$, directed into the page, induces an electric field $\vec{E}$. The induced field $\vec{E}$ is counterclockwise, whereas the induced magnetic field $\vec{B}$ in Fig. 31-24*b* is clockwise.

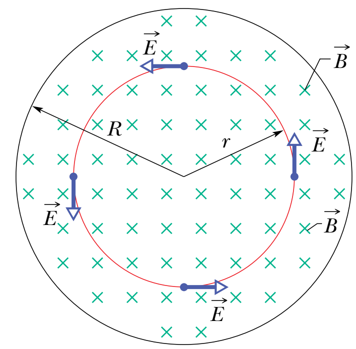


FIGURE 31-25 ■ A uniform magnetic field $\vec{B}$ in a circular region. The field, directed into the page, is increasing in magnitude. The electric field $\vec{E}$ induced by the changing magnetic field is shown at four points on a circle concentric with the circular region. Compare this situation with that of Fig. 31-24*b*.

Ampère–Maxwell Law

Now recall that the left side of Eq. 31-30, the integral of the dot product $\vec{B}\cdot d\vec{s}$ around a closed loop, appears in another equation—namely, Ampère's law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i^{\text{enc}} \qquad \text{(Ampère's law)}, \tag{31-31}$$

where i^{enc} is the current encircled by the closed loop. Thus, our two equations that specify the magnetic field $\vec{B}$ produced by means other than a magnetic material (that is, by a current and by a changing electric field) give the field in exactly the same form. We can combine the two equations into the single equation

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi^{\text{elec}}}{dt} + \mu_0 i^{\text{enc}} \qquad \text{(Ampère–Maxwell law)}. \tag{31-32}$$

When there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of Eq. 31-32 is zero, and Eq. 31-32 reduces to Eq. 31-31, Ampère's law. When there is a change in electric flux but no current (such as inside or outside the gap of a charging capacitor), the second term on the right side of Eq. 31-32 is zero, and Eq. 31-32 reduces to Eq. 31-30, Maxwell's law of induction.

READING EXERCISE 31-11: Referring back to Chapter 30, where we first studied Ampère's law, describe how we found the direction of the magnetic field produced by a current. What did the magnetic field lines look like for a long, straight, current-carrying wire? Discuss any connections or similarities between the case of the current-carrying wire and the case shown in Fig. 31-24.

TOUCHSTONE EXAMPLE 31-4: Inducing a Magnetic Field

A parallel-plate capacitor with circular plates of radius R is being charged as in Fig. 31-24*a*.

(a) Derive an expression for the magnitude of the magnetic field at radii r for the case $r \le R$.

SOLUTION ■ The **Key Idea** here is that a magnetic field can be set up by a current and by induction due to a changing electric flux; both effects are included in Eq. 31-32. There is no current between the capacitor plates of Fig. 31-24, but the electric flux there is changing. Thus, Eq. 31-32 reduces to

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi^{\text{elec}}}{dt}. \tag{31-33}$$

We shall separately evaluate the left and right sides of this equation.

Left side of Eq. 31-33: We choose a circular Ampèrian loop with a radius $r \le R$ as shown in Fig. 31-24, because we want to evaluate the magnetic field for $r \le R$—that is, inside the capacitor. The magnetic field $\vec{B}$ at all points along the loop is tangent to the loop, as is the path element $d\vec{s}$. Thus, $\vec{B}$ and $d\vec{s}$ are either parallel or antiparallel at each point of the loop. For simplicity, assume they are parallel (the choice does not alter our outcome here). Then

$$\oint \vec{B}\cdot d\vec{s} = \oint B \cdot ds \cos 0° = \oint B\, ds.$$

Due to the circular symmetry of the plates, we can also assume that $\vec{B}$ has the same magnitude at every point around the loop. Thus, B can be taken outside the integral on the right side of the above equation. The integral that remains is $\oint ds$, which simply gives the circumference $2\pi r$ of the loop. The left side of Eq. 31-33 is then $(B)(2\pi r)$.

Right side of Eq. 31-33: We assume that the electric field $\vec{E}$ is uniform between the capacitor plates and directed perpendicular to the plates. Then the electric flux Φ^{elec} encircled by the Ampèrian loop is EA, where A is the area encircled by the loop within the electric field. Thus, the right side of Eq. 31-33 is $\mu_0\varepsilon_0\, d(EA)/dt$.

Substituting our results for the left and right sides into Eq. 31-33, we get

$$B(2\pi r) = \mu_0\varepsilon_0 \frac{d(EA)}{dt}.$$

Because A is a constant, we write $d(EA)$ as $A\, dE$, so we have

$$B(2\pi r) = \mu_0\varepsilon_0 A \frac{dE}{dt}. \qquad (31\text{-}34)$$

We next use this **Key Idea:** The area A that is encircled by the Ampèrian loop within the electric field is the full area πr^2 of the loop, because the loop's radius r is less than (or equal to) the plate radius R. Substituting πr^2 for A in Eq. 31-34 and solving the result for B give us, for $r \le R$,

$$B = \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt}. \qquad \text{(Answer) (31-35)}$$

This equation tells us that, inside the capacitor, B increases linearly with increased radial distance r, from zero at the center of the plates to a maximum value at the plate edges (where $r = R$).

(b) Evaluate the field magnitude B for $r = R/5 = 11.0$ mm and $dE/dt = 1.50 \times 10^{12}$ V/m·s.

SOLUTION ■ From the answer to (a), we have

$$\begin{aligned} B &= \tfrac{1}{2}\mu_0\varepsilon_0 r \frac{dE}{dt} \\ &= \tfrac{1}{2}(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) \\ &\quad \times (11.0 \times 10^{-3}\ \text{m})(1.50 \times 10^{12}\ \text{V/m}\cdot\text{s}) \\ &= 9.18 \times 10^{-8}\ \text{T}. \qquad \text{(Answer)} \end{aligned}$$

(c) Derive an expression for the induced magnetic field for the case $r \ge R$.

SOLUTION ■ Our procedure is the same as in (a) except we now use an Ampèrian loop with a radius r that is greater than the plate radius R, to evaluate B outside the capacitor. Evaluating the left and right sides of Eq. 31-33 again leads to Eq. 31-34. However, we then need this subtle **Key Idea:** The electric field exists only between the plates, *not* outside the plates. Thus, the area A that is encircled by the Ampèrian loop in the electric field is *not* the full area πr^2 of the loop. Rather, A is only the plate area πR^2.

Substituting πR^2 for A in Eq. 31-34 and solving the result for B give us, for $r \ge R$,

$$B = \frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}. \qquad \text{(Answer) (31-36)}$$

This equation tells us that, outside the capacitor, B decreases with increased radial distance r, from a maximum value at the plate edges (where $r = R$). By substituting $r = R$ into Eqs. 31-35 and 31-36, you can show that these equations are consistent; that is, they give the same maximum value of B at the plate radius.

The magnitude of the induced magnetic field calculated in (b) is so small that it can scarcely be measured with simple apparatus. This is in sharp contrast to the magnitudes of induced electric fields (Faraday's law), which can be measured easily. This experimental difference exists partly because induced emfs can easily be multiplied by using a coil of many turns. No technique of comparable simplicity exists for multiplying induced magnetic fields. In any case, the experiment suggested by this sample problem has been done, and the presence of the induced magnetic fields has been verified quantitatively.

31-9 Displacement Current

If you compare the two terms on the right side of Eq. 31-32, you will see that the product $\varepsilon_0(d\Phi^{\text{elec}}/dt)$ in the first term must have the units associated with a current. Since no charge actually flows, historically, that product has been treated as being a fictitious current called the **displacement current** i^{dis}:

$$i^{\text{dis}} = \varepsilon_0 \frac{d\Phi^{\text{elec}}}{dt} \qquad \text{(displacement current).} \qquad (31\text{-}37)$$

"Displacement" is a poorly chosen term in that nothing is being displaced, but we are stuck with the word. Nevertheless, we can now rewrite Eq. 31-32 as

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{dis}^{enc} + \mu_0 i^{enc} \quad \text{(Ampère–Maxwell law)}, \tag{31-38}$$

in which i_{dis}^{enc} is the displacement current that is encircled by the integration loop.

Let us again focus on a charging capacitor with circular plates, as in Fig. 31-26*a*. The real current i that is charging the plates changes the electric field $\vec{E}$ between the plates. The fictitious displacement current i^{dis} between the plates is associated with that changing field $\vec{E}$. Let us relate these two currents.

The amount of excess charge $|q|$ on each of the plates at any time is related to the magnitude $|\vec{E}| = E$ of the field between the plates at that time by Eq. 28-4:

$$|q| = \varepsilon_0 AE, \tag{31-39}$$

in which A is the plate area. To get the real current i, we differentiate Eq. 31-39 with respect to time, finding

$$\frac{d|q|}{dt} = |i| = \varepsilon_0 A \frac{dE}{dt}. \tag{31-40}$$

To get the displacement current i^{dis}, we can use Eq. 31-37. Assuming that the electric field $\vec{E}$ between the two plates is uniform (we neglect any fringing), we can replace the electric flux Φ^{elec} in that equation with EA. Then Eq. 31-37 becomes

$$|i^{dis}| = \varepsilon_0 \left|\frac{d\Phi^{elec}}{dt}\right| = \varepsilon_0 \left|\frac{d(EA)}{dt}\right| = \varepsilon_0 A \left|\frac{dE}{dt}\right|. \tag{31-41}$$

Comparing Eqs. 31-40 and 31-41, we see that the real current i charging the capacitor and the fictitious displacement current i^{dis} between the plates have the same value:

$$i^{dis} = i \quad \text{(displacement current in a capacitor)}. \tag{31-42}$$

Thus, we can consider the fictitious displacement current i^{dis} to be simply a continuation of the real current i from one plate, across the capacitor gap, to the other plate. Because the electric field is uniformly spread over the plates, the same is true of this fictitious displacement current i^{dis}, as suggested by the spread of current arrows in Fig. 31-26*a*. Although no charge actually moves across the gap between the plates, the idea of the fictitious current i^{dis} can help us to quickly find the direction and magnitude of an induced magnetic field, as follows.

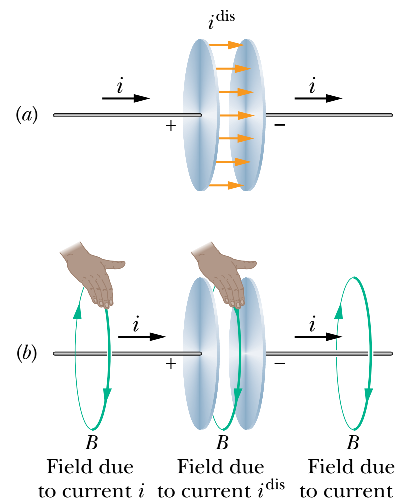


FIGURE 31-26 ■ (*a*) The displacement current i^{dis} between the plates of a capacitor that is being charged by a current i. (*b*) The right-hand rule for finding the direction of the magnetic field around a wire with a real current (as at the left) also gives the magnetic field direction around a displacement current (as in the center).

Finding the Induced Magnetic Field

In Chapter 30 we found the direction of the magnetic field produced by a real current i by using the right-hand rule of Fig. 30-4. We can apply the same rule to find the direction of an induced magnetic field produced by a fictitious displacement current i^{dis}, as shown in the center of Fig. 31-26*b* for a capacitor.

We can also use i^{dis} to find the magnitude of the magnetic field induced by a charging capacitor with parallel circular plates of radius R. We simply consider the space between the plates to be an imaginary circular wire of radius R carrying the imaginary current i^{dis}. Then, from Eq. 30-22, the magnitude of the magnetic field at a point inside the capacitor at radius r from the center is

$$B = \left(\frac{\mu_0 |i^{dis}|}{2\pi R^2}\right) r \quad \text{(inside a circular capacitor)}. \tag{31-43}$$

Similarly, from Eq. 30-19, the magnitude of the magnetic field at a point outside the capacitor at radius r is

$$B = \frac{\mu_0 |i^{\text{dis}}|}{2\pi r} \quad \text{(outside a circular capacitor).} \tag{31-44}$$

READING EXERCISE 31-12: Discuss the ways in which it is useful for us to think of the quantity $\varepsilon_0\, d\Phi^{\text{elec}}/dt$ as a current. ■

TOUCHSTONE EXAMPLE 31-5: Displacement Current

The circular parallel-plate capacitor in Touchstone Example 31-4 is being charged with a current i.

(a) Between the plates, what is the magnitude of $\oint \vec{B} \cdot d\vec{s}$, in terms of μ_0 and i, at a radius $r = R/5$ from their center?

SOLUTION ■ The first **Key Idea** of Touchstone Example 31-4a holds here too. However, now we can replace the product $\varepsilon_0 d\Phi^{\text{elec}}/dt$ in Eq. 31-32 with a fictitious displacement current i^{dis}. Then integral $\oint \vec{B} \cdot d\vec{s}$ is given by Eq. 31-38, but because there is no real current i between the capacitor plates, the equation reduces to

$$\oint \vec{B} \cdot d\vec{s} = \mu_0\, i_{\text{dis}}^{\text{enc}}. \tag{31-45}$$

Because we want to evaluate $\oint \vec{B} \cdot d\vec{s}$ at radius $r = R/5$ (within the capacitor), the integration loop encircles only a portion $i_{\text{dis}}^{\text{enc}}$ of the total displacement current i^{dis}. A second **Key Idea** is to assume that i^{dis} is uniformly spread over the full plate area. Then the portion of the displacement current encircled by the loop is proportional to the area encircled by the loop:

$$\frac{(\text{encircled displacement current } i_{\text{dis}}^{\text{enc}})}{(\text{total displacement current } i^{\text{dis}})} = \frac{\text{encircled area } \pi r^2}{\text{full plate area } \pi R^2}.$$

This gives us a current magnitude of

$$i_{\text{dis}}^{\text{enc}} = i^{\text{dis}} \frac{\pi r^2}{\pi R^2}.$$

Substituting this into Eq. 31-45, we obtain

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i^{\text{dis}} \frac{\pi r^2}{\pi R^2}. \tag{31-46}$$

Now substituting $i^{\text{dis}} = i$ (from Eq. 31-42) and $r = R/5$ into Eq. 31-46 leads to

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i \frac{(R/5)^2}{R^2} = \frac{\mu_0 i}{25}. \quad \text{(Answer)}$$

(b) In terms of the maximum induced magnetic field, what is the magnitude of the magnetic field induced at $r = R/5$, inside the capacitor?

SOLUTION ■ The **Key Idea** here is that, because the capacitor has parallel circular plates, we can treat the space between the plates as an imaginary wire of radius R carrying the imaginary current i^{dis}. Then we can use Eq. 31-43 to find the induced magnetic field magnitude B at any point inside the capacitor. At $r = R/5$, that equation yields

$$B = \left(\frac{\mu_0 |i^{\text{dis}}|}{2\pi R^2}\right) r = \frac{\mu_0 |i^{\text{dis}}|(R/5)}{2\pi R^2} = \frac{\mu_0 |i^{\text{dis}}|}{10\pi R}. \tag{31-47}$$

The maximum field magnitude B^{max} within the capacitor occurs at $r = R$. It is

$$B^{\text{max}} = \left(\frac{\mu_0 |i^{\text{dis}}|}{2\pi R^2}\right) R = \frac{\mu_0 |i^{\text{dis}}|}{2\pi R}. \tag{31-48}$$

Dividing Eq. 31-47 by Eq. 31-48 and rearranging the result, we find

$$B = \frac{B^{\text{max}}}{5}, \quad \text{(Answer)}$$

We should be able to obtain this result with a little reasoning and less work. Equation 31-43 tells us that inside the capacitor, B increases linearly with r. Therefore, a point $\frac{1}{5}$ the distance out to the full radius R of the plates, where B^{max} occurs, should have a field B that is $\frac{1}{5}B^{\text{max}}$.

31-10 Gauss' Law for Magnetic Fields

In this chapter and the two that precede it, we have investigated several fundamental aspects of electricity and magnetism. Furthermore, we have seen many ways in which magnetism and electricity are connected. When combined as a set of laws, these ideas provide a framework from which we can understand all of the electromagnetic phenomena that fill our world, much like Newton's laws do in regard to forces and motion.

However, there remains one last idea that we must discuss before our view of electromagnetism is complete. This idea is contained in an idea known as *Gauss' law for magnetic fields*. Gauss' law for magnetic fields is a formal way of saying that magnetic monopoles do not exist. The law asserts that the net magnetic flux Φ^{mag} at any closed Gaussian surface is zero:

$$\Phi^{\text{mag}} = \oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss' law for magnetic fields).} \qquad (31\text{-}49)$$

Contrast this with Gauss' law for electric fields,

$$\Phi^{\text{elec}} = \oint \vec{E} \cdot d\vec{A} = \frac{q^{\text{enc}}}{\varepsilon_0} \qquad \text{(Gauss' law for electric fields).}$$

In both equations, the integral is taken over a *closed* Gaussian surface. Gauss' law for electric fields says that this integral (the net electric flux at the surface) is proportional to the net electric charge q^{enc} enclosed by the surface. Gauss' law for magnetic fields says that there can be no net magnetic flux at the surface because there can be no net "magnetic charge" (individual magnetic poles) enclosed by the surface. The simplest magnetic structure that can exist and thus be enclosed by a Gaussian surface is a dipole, which consists of both a source and a sink for the field lines. Thus, there must always be as much magnetic flux into the surface as out of it, and the net magnetic flux must always be zero.

Gauss' law for magnetic fields holds for more complicated structures than a magnetic dipole, and it holds even if the Gaussian surface does not enclose the entire structure. Gaussian surface II near the bar magnet of Fig. 31-27 encloses no poles, and we can easily conclude that the net magnetic flux at it is zero. Gaussian surface I is more difficult to understand. It may seem to enclose only the north pole of the magnet because it encloses the label N and not the label S. However, a south pole must be associated with the lower boundary of the surface, because magnetic field lines enter the surface there. (The enclosed section is like one piece of the broken cylindrical magnet in Fig. 31-28.) Thus, Gaussian surface I encloses a magnetic dipole and the net flux at the surface is zero.

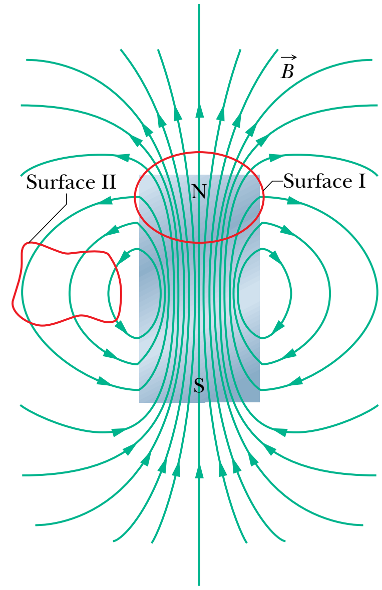


FIGURE 31-27 ■ The field lines for the magnetic field $\vec{B}$ of a short bar magnet. The red curves represent cross sections of closed, three-dimensional Gaussian surfaces.

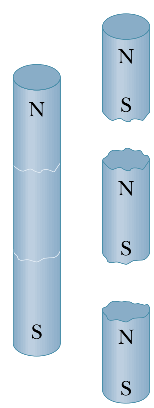
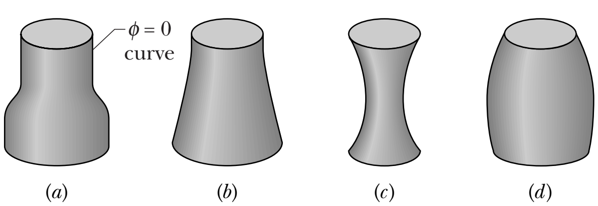


FIGURE 31-28 ■ If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles.

READING EXERCISE 31-13: The figure below shows four closed surfaces with flat top and bottom faces and curved sides. The table gives the areas A of the faces and the magnitudes B of the uniform and perpendicular magnetic fields at those faces; the units of A and B are arbitrary but consistent. (a) Rank the surfaces according to the magnitudes of the magnetic flux at their curved sides, greatest first. (b) Explain your reasoning.



Surface	A_{top}	B_{top}, direction	A_{bot}	B_{bot}, direction
a	2	6, outward	4	3, inward
b	2	1, inward	4	2, inward
c	2	6, inward	2	8, outward
d	2	3, outward	3	2, outward

31-11 Maxwell's Equations in a Vacuum

Many 18th and 19th century scientists contributed to our understanding of electricity and magnetism including Franklin, Coulomb, Gauss, Oersted, Biot, Savart, Lorentz, Ampère, Henry, Faraday, and Maxwell. But it was James Clerk Maxwell who reformulated many of the basic equations describing electric and magnetic effects we have already presented. A special case of Maxwell's equations are shown in Table 31-1 for situations in which no dielectric or magnetic materials are present.

It is amazing that these four rather compact equations can be used to *derive a complete description of all electromagnetic interactions that were understood by the end of the 19th century*. Taken together they describe a diverse range of phenomena, from how a compass needle points north to how a car starts when you turn the ignition key. They have been used to design electric motors, cyclotrons, television transmitters and receivers, telephones, fax machines, radar, and microwave ovens.

In addition, many of the equations you have seen since Chapter 22 can be derived from Maxwell's equations. Perhaps the most exciting intellectual outcome of Maxwell's equations is their prediction of electromagnetic waves and our eventual understanding of the self-propagating nature of these waves that will be introduced in Chapter 34. Maxwell's picture of electromagnetic wave propagation was not fully appreciated until scientists abandoned the idea that all waves had to propagate through an elastic medium and accepted Einstein's theory of special relativity formulated in the early part of the 20th century.

Because we now know that visible light is a form of electromagnetic radiation, these equations provide the basis for many of the equations you will see in Chapters 34 through 37, which introduce you to optics and optical devices such as telescopes and eyeglasses.

The significance of Maxwell's equations should not be underestimated. Richard Feynman, a leading famous 20th-century physicist, recognized this when he stated:

> Now we realize that the phenomena of chemical interaction and ultimately of life itself are to be understood in terms of electromagnetism . . . . The electrical forces, enormous as they are, can also be very tiny, and we can control them and use them in many ways . . . From a long view of the history of mankind—seen from, say, ten thousand years from now—there can be little doubt that the most significant event of the nineteenth century will be judged as Maxwell's discovery of the laws of electrodynamics.

TABLE 31-1
Maxwell's Equations for Vacuum[a]

Name	Equation	
Gauss' law for electricity (Eq. 24-7)	$\oint \vec{E}\cdot d\vec{A} = q/\varepsilon_0$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism (Eq. 31-49)	$\oint \vec{B}\cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law (Eq. 31-7)	$\oint \vec{E}\cdot d\vec{s} = -\dfrac{d\Phi^{\text{mag}}}{dt}$	Relates induced electric field to changing magnetic flux
Ampère–Maxwell law (Eq. 31-32)	$\oint \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0\dfrac{d\Phi^{\text{elec}}}{dt} + \mu_0 i$	Relates induced magnetic field to changing electric flux and to current

[a]Written on the assumption that no dielectric or magnetic materials are present.

READING EXERCISE 31-14: Discuss several ways in which Gauss' law for electricity and Gauss' law for magnetism are similar. Discuss several ways in which they are different. ■

READING EXERCISE 31-15: Discuss several ways in which Faraday's law and the Ampère–Maxwell law are similar. Discuss several ways in which they are different. ■

Problems

Sec. 31-4 ■ Faraday's Law

1. UHF Antenna A UHF television loop antenna has a diameter of 11 cm. The magnetic field of a TV signal is normal to the plane of the loop and, at one instant of time, its magnitude is changing at the rate 0.16 T/s. The magnetic field is uniform. What emf is induced in the antenna?

2. Small Loop A small loop of area A is inside of, and has its axis in the same direction as, a long solenoid of n turns per unit length and current i. If $i = I^{\text{max}} \sin \omega t$, find the magnitude of the emf induced in the loop.

3. Magnetic Flux The magnetic flux encircled by the loop shown in Fig. 31-29 increases according to the relation $\Phi^{\text{mag}} = (6.0\ \text{mWb/s}^2)t^2 + (3.7\ \text{mWb/s})t$. (a) What is the magnitude of the emf induced in the loop when $t = 2.0$ s? (b) What is the direction of the current through R?

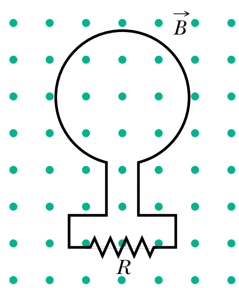


Figure 31-29 ■ Problems 3 and 13.

4. Calculate emf The magnitude of the magnetic field encircled by a single loop of wire, 12 cm in radius and of 8.5 Ω resistance, changes with time as shown in Fig. 31-30. Calculate the magnitude of the emf in the loop as a function of time. Consider the time intervals (a) $t_1 = 0.0$ s to $t_2 = 2.0$ s, (b) $t_2 = 2.0$ s to $t_3 = 4.0$ s, (c) $t_3 = 4.0$ s to $t_4 = 6.0$ s. The (uniform) magnetic field is perpendicular to the plane of the loop.

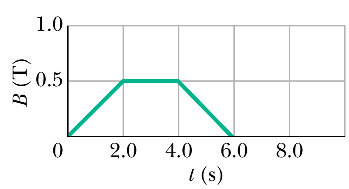


Figure 31-30 ■ Problem 4.

5. Uniform Magnetic Field A uniform magnetic field is normal to the plane of a circular loop 10 cm in diameter and made of copper wire (of diameter 2.5 mm). (a) Calculate the resistance of the wire. (See Table 26-2.) (b) At what rate must the magnetic field change with time if an induced current of 10 A is to appear in the loop?

6. Current in Solenoid The current in the solenoid of Touchstone Example 31-1 changes, not as stated there, but according to $i = (3.0\ \text{A}/s)t + (1.0\text{A}/s^2)t^2$. (a) Plot the induced emf in the coil from $t_1 = 0.0$ s to $t_2 = 4.0$ s. (b) The resistance of the coil is 0.15 Ω. What is the current in the coil at $t = 2.0$ s?

7. Coil Outside Solenoid In Fig. 31-31 a 120-turn coil of radius 1.8 cm and resistance 5.3 Ω is placed *outside* a solenoid like that of Touchstone Example 31-1. If the current in the solenoid is changed as in that sample problem, what current appears in the coil while the solenoid current is being changed?

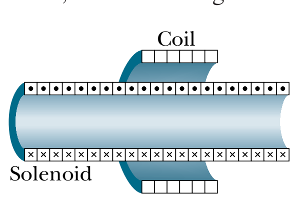


Figure 31-31 ■ Problem 7.

8. Elastic Conducting Material An elastic conducting material is stretched into a circular loop of 12.0 cm radius. It is placed with its plane perpendicular to a uniform 0.800 T magnetic field. When released, the radius of the loop starts to shrink at an instantaneous rate of 75.0 cm/s. What magnitude of emf is induced in the loop at that instant?

9. Square Loop A square loop of wire is held in a uniform, magnetic field 0.24 T directed perpendicularly to the plane of the loop. The length of each side of the square is decreasing at a constant rate of 5.0 cm/s. What emf is induced in the loop when the length is 12 cm?

10. Rectangular Loop A rectangular loop (area = 0.15 m^2) turns in a uniform magnetic field, $B = 0.20$ T. When the angle between the field and the normal to the plane of the loop is $\pi/2$ rad and increasing at 0.60 rad/s, what emf is induced in the loop?

Sec. 31-5 ■ Lenz's Law

11. Two Parallel Loops Though not to scale, Fig. 31-32 shows two parallel loops of wire with a common axis. The smaller loop (radius r) is above the larger loop (radius R) by a distance $x \gg R$. Consequently, the magnetic field due to the current i in the larger loop is nearly constant throughout the smaller loop. Suppose that x is increasing at the constant rate of $dx/dt = v$. (a) Determine the magnetic flux at the area bounded by the smaller loop as a function of x. (*Hint:* See Eq. 30-29.) In the smaller loop, find (b) the induced emf and (c) the direction of the induced current.

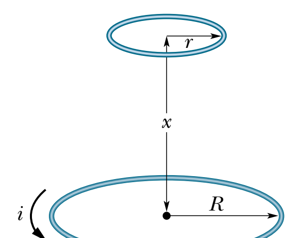


Figure 31-32 ■ Problem 11.

12. Circular Loop In Fig. 31-33, a circular loop of wire 10 cm in diameter (seen edge-on) is placed with its normal at an angle $\theta = 30°$ with the direction of a uniform magnetic field $\vec{B}$ of magnitude 0.50 T. The loop is then rotated such that the normal rotates in a cone about the field direction at the constant rate of 100 rev/min; the angle θ remains unchanged during the process. What is the emf induced in the loop?

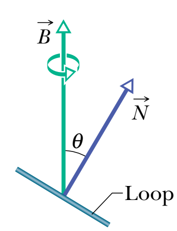


Figure 31-33 ■ Problem 12.

13. Flux At Loop In Fig. 31-29 let the flux encircled by the loop be $\Phi^{\text{mag}}(0)$ at time $t_1 = 0$. Then let the magnetic field $\vec{B}$ vary in a continuous but unspecified way, in both magnitude and direction, so that at time t_2 the flux is represented by $\Phi^{\text{mag}}(t_2)$. (a) Show that the net charge $q(t_2)$ that has passed through resistor R in time t_2 is

$$q(t_2) = \frac{1}{R}\left[\Phi^{\text{mag}}(0) - \Phi^{\text{mag}}(t_2)\right]$$

and is independent of the way $\vec{B}$ has changed. (b) If $\Phi^{\text{mag}}(t_2) = \Phi^{\text{mag}}(0)$ in a particular case, we have $q(t_2) = 0$. Is the induced current necessarily zero throughout the interval from 0 to t_2?

14. Big Loop, Little Loop A small circular loop of area 2.00 cm^2 is placed in the plane of, and concentric with, a large circular loop of radius 1.00 m. The current in the large loop is changed uniformly from 200 A to −200 A (a change in direction) in a time of 1.00 s, beginning at $t_1 = 0$. (a) What is the magnitude of the magnetic field at the center of the small circular loop due to the current in the large loop at $t_1 = 0$ s, $t_2 = 0.500$ s, and $t_3 = 1.00$ s? (b) What is the magnitude of the emf induced in the small loop at $t_2 = 0.500$ s? (Since the inner loop is small, assume the field $\vec{B}$ due to the outer loop is uniform over the area of the smaller loop.)

15. Copper Wire on Wooden Core One hundred turns of insulated copper wire are wrapped around a wooden cylindrical core of cross-sectional area 1.20×10^{-3} m^2. The two ends of the wire are connected to a resistor. The total resistance in the circuit is 13.0 Ω. If an externally applied uniform longitudinal magnetic field in the core changes from 1.60 T in one direction to 1.60 T in the opposite direction, how much charge flows through the circuit? (*Hint:* See Problem 13.)

16. Earth's Field At a certain place, Earth's magnetic field has magnitude $|\vec{B}| = 0.590$ gauss and is inclined downward at an angle of 70.0° to the horizontal. A flat horizontal circular coil of wire with a radius of 10.0 cm has 1000 turns and a total resistance of 85.0 Ω. It is connected to a meter with 140 Ω resistance. The coil is flipped through a half-revolution about a diameter, so that it is again horizontal. How much charge flows through the meter during the flip? (*Hint:* See Problem 13.)

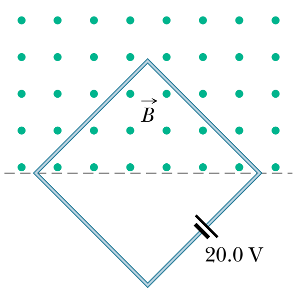


FIGURE 31-34 ■ Problem 17.

17. Square Loop A square wire loop with 2.00 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in Fig. 31-34. The loop contains a 20.0 V battery with negligible internal resistance. If the magnitude of the field varies with time according to $B = (0.0420\ \text{T}) - (0.870\ \text{T/s})t$, what are (a) the magnitude of the net emf in the circuit and (b) the direction of the current through the battery?

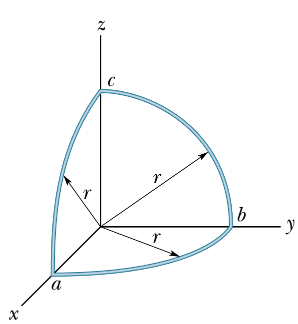


FIGURE 31-35 ■ Problem 18.

18. Three Circular Segments A wire is bent into three circular segments, each of radius $r = 10$ cm, as shown in Fig. 31-35. Each segment is a quadrant of a circle, ab lying in the xy plane, bc lying in the yz plane, and ca lying in the zx plane. (a) If a uniform magnetic field $\vec{B}$ points in the positive x direction, what is the magnitude of the emf developed in the wire when $\vec{B}$ increases at the rate of 3.0 mT/s in the x direction? (b) What is the direction of the current in segment bc?

19. Rectangular Coil A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field $\vec{B}$, as indicated in Fig. 31-36. The coil is connected to co-rotating cylinders, against which metal brushes slide to make contact. If we arbitrarily define emf as being positive during the first quarter-turn, (a) show that the emf induced in the coil is given (as a function of time t) by

$$\mathscr{E} = 2\pi f NabB \sin(2\pi f t) = \mathscr{E}_0 \sin(2\pi f t).$$

This is the principle of the commercial alternating-current generator. (b) Design a loop that will produce an emf with $\mathscr{E}_0 = 150$ V when rotated at 60.0 rev/s in a uniform magnetic field of 0.500 T.

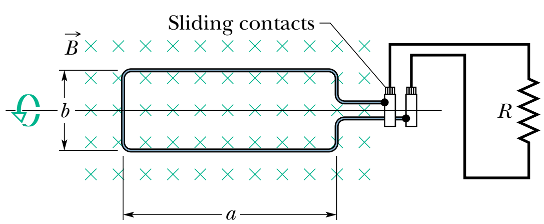


FIGURE 31-36 ■ Problem 19.

20. Semicircle A stiff wire bent into a semicircle of radius a is rotated with frequency f in a uniform magnetic field, as suggested in Fig. 31-37. What are (a) the frequency and (b) the amplitude of the varying emf induced in the loop?

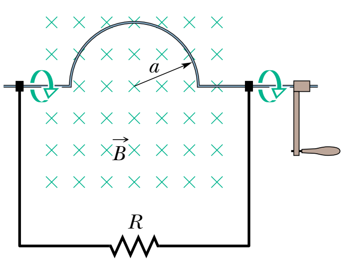


FIGURE 31-37 ■ Problem 20.

21. Electric Generator An electric generator consists of 100 turns of wire formed into a rectangular loop 50.0 cm by 30.0 cm, placed entirely in a uniform magnetic field with magnitude $B = 3.50$ T. What is the maximum value of the emf produced when the loop is spun at 1000 rev/min about an axis perpendicular to $\vec{B}$?

22. Closed Circular Loop In Fig. 31-38, a wire forms a closed circular loop, with radius $R = 2.0$ m and resistance 4.0 Ω. The circle is centered on a long straight wire; at time $t = 0$, the current in the long straight wire is 5.0 A rightward. Thereafter, the current changes according to $i = 5.0\ \text{A} - (2.0\ \text{A/s}^2)t^2$. (The straight wire is insulated, so there is no electrical contact between it and the wire of the loop.) What are the magnitude and direction of the current induced in the loop at times $t > 0$?

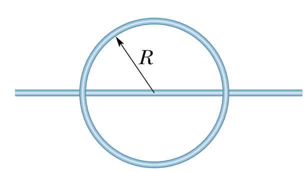


FIGURE 31-38 ■ Problem 22.

23. Square Loop Two In Fig. 31-39, the square loop of wire has sides of length 2.0 cm. A magnetic field is directed out of the page; its magnitude is given by $B = (4.0\ \text{T/m}\cdot\text{s}^2)\ t^2y$, where B is in teslas, t is in seconds, and y is in meters. Determine the emf around the square at $t = 2.5\,\text{s}$ and indicate whether its direction is clockwise or counterclockwise.

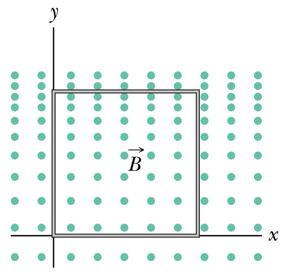


FIGURE 31-39 ■ Problem 23.

24. Square Loop Three For the situation shown in Fig. 31-40, $a = 12.0$ cm and $b = 16.0$ cm. The current in the long straight wire is given by $i = (4.50\ \text{A/s}^2)t^2 - (10.0\ \text{A/s})t$, where i is in amperes and t is in seconds. (a) Find the magnitude of the emf in the square loop at $t = 3.00$ s. (b) Indicate whether the direction of the induced current in the loop is clockwise or counterclockwise at $t = 3.00$ s.

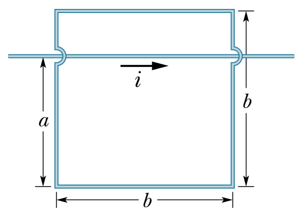


FIGURE 31-40 ■ Problem 24.

25. Parallel Copper Wires Two long, parallel copper wires of diameter 2.5 mm carry currents of 10 A in opposite directions. (a) Assuming that their central axes are 20 mm apart, calculate the magnetic flux per meter of wire that exists in the space between those axes. (b) What fraction of this flux lies inside the wires? (c) Repeat part (a) for parallel currents.

26. Rectangular Wire Loop A rectangular loop of wire with length a, width b, and resistance R is placed near an infinitely long wire carrying current i, as shown in Fig. 31-41. The distance from the long wire to the center of the loop is r. Find (a) the magnitude of the magnetic flux encircled by the loop and (b) the amount of induced current in the loop $|i^{\text{ind}}|$ as it moves away from the long wire with velocity $\vec{v}$. (c) Indicate whether the induced current is clockwise or counterclockwise.

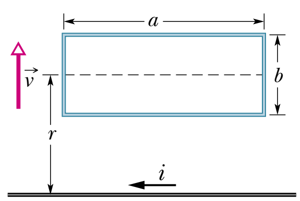


FIGURE 31-41 ■ Problem 26.

SEC 31-6 ■ INDUCTION AND ENERGY TRANSFERS

27. Internal Energy If 50.0 cm of copper wire (diameter = 1.00 mm) is formed into a circular loop and placed perpendicular to a uniform magnetic field that is increasing at the constant rate of 10.0 mT/s, at what rate does internal energy increase in the loop?

28. Loop Antenna A loop antenna of area A and resistance R is perpendicular to a uniform magnetic field $\vec{B}$. The field drops linearly to zero in a time interval Δt. Find an expression for the total internal energy added to the loop.

29. Rod on Rails A metal rod is forced to move with constant velocity $\vec{v}$ along two parallel metal rails, connected with a strip of metal at one end, as shown in Fig. 31-42. A magnetic field of magnitude $|\vec{B}| = 0.350$ T points out of the page. (a) If the rails are separated by 25.0 cm and the speed of the rod is 55.0 cm/s, what emf is generated? (b) If the rod has a resistance of 18.0 Ω and the rails and connector have negligible resistance, what is the current in the rod? (c) At what rate is mechanical energy being transformed to thermal energy?

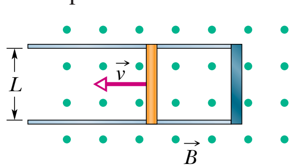


FIGURE 31-42 ■ Problem 29 and Problem 31.

30. Find Terminal Speed In Fig. 31-43, a long rectangular conducting loop, of width L, resistance R, and mass m, is hung in a horizontal, uniform magnetic field $\vec{B}$ that is directed into the page and that exists only above line aa. The loop is then dropped; during its fall, it accelerates until it reaches a certain terminal speed v_t. Ignoring air drag, find that terminal speed.

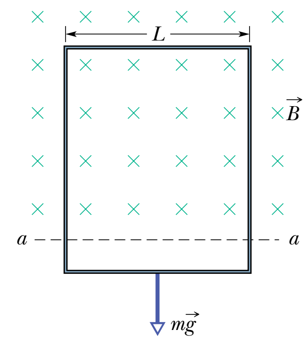


FIGURE 31-43 ■ Problem 30.

31. Rod on Rails Two The conducting rod shown in Fig. 31-42 has length L and is being pulled along horizontal, frictionless conducting rails at a constant velocity $\vec{v}$. The rails are connected at one end with a metal strip. A uniform magnetic field $\vec{B}$, directed out of the page, fills the region in which the rod moves. Assume that $L = 10$ cm, $v = 5.0$ m/s, and $B = 1.2$ T. (a) What is the magnitude of the emf induced in the rod? (b) What is the magnitude and direction (clockwise or counterclockwise) of the current in the conducting loop? Assume that the resistance of the rod is 0.40 Ω and that the resistance of the rails and metal strip is negligibly small. (c) At what rate is thermal energy added to the rod? (d) What magnitude of force must be applied to the rod by an external agent to maintain its motion? (e) At what rate does this external agent do work on the rod? Compare this answer with the answer to (c).

32. Rods Bent into V Two straight conducting rails form a right angle where their ends are joined. A conducting bar in contact with the rails starts at the vertex at time $t = 0$ and moves with a constant velocity of magnitude 5.20 m/s along them, as shown in Fig. 31-44. A magnetic field of magnitude $B = 0.350$ T is directed out of the page. Calculate (a) the flux through the triangle formed by the rails and bar at $t = 3.00$ s and (b) the magnitude of emf around the triangle at that time. (c) If we write the emf as $\mathscr{E} = at^n$, where a and n are constants, what is the value of n?

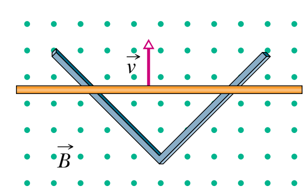


FIGURE 31-44 ■ Problem 32.

33. Rod on Conducting Rails Two Figure 31-45 shows a rod of length L caused to move at constant speed v along horizontal conducting rails. The magnetic field in which the rod moves is *not uniform* but is provided by a current i in a long wire parallel to the rails. Assume that $v = 5.00$ m/s, $a = 10.0$ mm, $L = 10.0$ cm, and $i = 100$ A. (a) Calculate the magnitude of the emf induced in the rod. (b) What is the magnitude of the current in the conducting loop?

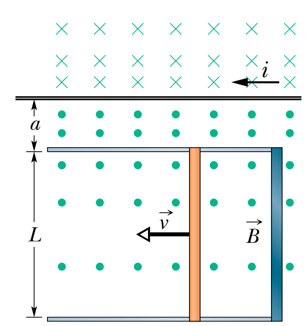


FIGURE 31-45 ■ Problem 33.

Assume that the resistance of the rod is 0.400 Ω and that the resistance of the rails and the strip that connects them at the right is negligible. (c) At what rate is internal energy added to the rod? (d) What magnitude of force must be applied to the rod by an external agent to maintain its motion? (e) At what rate does this external agent do work on the rod? Compare this answer to that for (c).

SEC. 31-7 ■ INDUCED ELECTRIC FIELDS

34. Two Circular Regions Figure 31-46 shows two circular regions R_1 and R_2 with radii $r_1 = 20.0$ cm and $r_2 = 30.0$ cm. In R_1 there is a uniform magnetic field of magnitude $B_1 = 50.0$ mT into the page, and in R_2 there is a uniform magnetic field of magnitude $B_2 = 75.0$ mT out of the page (ignore any fringing of these fields). Both fields are decreasing at the rate of 8.50 mT/s. Calculate the integral $\oint \vec{E} \cdot d\vec{s}$ for each of the three dashed paths.

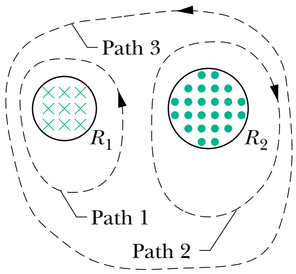


FIGURE 31-46 ■ Problem 34.

35. Long Solenoid A long solenoid has a diameter of 12.0 cm. When a current i exists in its windings, a uniform magnetic field of magnitude $B = 30.0$ mT is produced in its interior. By decreasing i, the field is caused to decrease at the rate of 6.50 mT/s. Calculate the magnitude of the induced electric field (a) 2.20 cm and (b) 8.20 cm from the axis of the solenoid.

36. Magnet Lab Early in 1981 the Francis Bitter National Magnet Laboratory at M.I.T. commenced operation of a 3.3-cm-diameter cylindrical magnet that produces a 30 T field, then the world's largest steady-state field. The field magnitude can be varied sinusoidally between the limits of 29.6 and 30.9 T at a frequency of 15 Hz. When this is done, what is the maximum value of the magnitude of the induced electric field at a radial distance of 1.6 cm from the axis? (*Hint:* See Touchstone Example 31-3.)

37. Drop to Zero Prove that the electric field $\vec{E}$ in a charged parallel-plate capacitor cannot drop abruptly to zero (as is suggested at point a in Fig. 31-47), as one moves perpendicular to the field, say, along the horizontal arrow in the figure. Fringing of the field lines always occurs in actual capacitors, which means that $\vec{E}$ approaches zero in a continuous and gradual way (see Problem 35 in Chapter 30). (*Hint:* Apply Faraday's law to the rectangular path shown by the dashed lines).

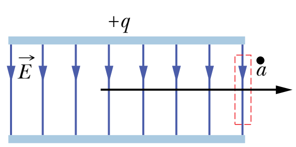


FIGURE 31-47 ■ Problem 37.

SEC 31-8 ■ INDUCED MAGNETIC FIELDS

38. Charging Capacitor Touchstone Example 31-4 describes the charging of a parallel-plate capacitor with circular plates of radius 55.0 mm. At what two radii r from the central axis of the capacitor is the magnitude of the induced magnetic field equal to 50% of its maximum value?

39. Induced Magnetic Field The induced magnetic field 6.0 mm from the central axis of a circular parallel-plate capacitor and between the plates has magnitude of 2.0×10^{-7} T. The plates have radius 3.0 mm. At what rate $|d\vec{E}/dt|$ is the electric field magnitude between the plates changing?

40. Parallel-Plate Capacitor Suppose that a parallel-plate capacitor has circular plates with radius $R = 30$ mm and a plate separation of 5.0 mm. Suppose also that a sinusoidal potential difference with a maximum value of 150 V and a frequency of 60 Hz is applied across the plates. That is,

$$\Delta V = (150\text{ V}) \sin[2\pi(60\text{ Hz})t].$$

(a) Find $B^{\max}(R)$, the maximum value of the magnitude of the induced magnetic field that occurs at $r = R$. (b) Plot $B^{\max}(r)$ for $0 < r < 10$ cm.

41. Uniform Electric Flux Figure 31-48 shows a circular region of radius $R = 3.00$ cm in which a uniform electric flux is directed out of the page. The total electric flux enclosed by the region is given by $\Phi^{\text{elec}} = (3.00\text{ mV} \cdot \text{m/s})t$, where t is time. What is the magnitude of the magnetic field that is induced at radial distances (a) 2.00 cm and (b) 5.00 cm?

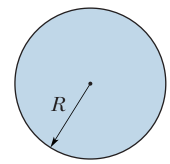


FIGURE 31-48 ■ Problems 41 through 44, and 57, 59, and 60.

42. Nonuniform Electric Flux Figure 31-48 shows a circular region of radius $R = 3.00$ cm in which an electric flux is directed out of the page. The flux encircled by a concentric circle of radius r is given by $\Phi^{\text{elec}} = (0.600\text{ V} \cdot \text{m/s})(r/R)t$, where $r \leq R$ and t is time. What is the magnitude of the induced magnetic field at radial distances (a) 2.00 cm and (b) 5.00 cm?

43. Uniform Electric Field In Fig. 31-48, a uniform electric field is directed out of the page within a circular region of radius $R = 3.00$ cm. The magnitude of the electric field is given by $E = (4.5 \times 10^{-3}\text{ V/m} \cdot \text{s})t$, where t is time. What is the magnitude of the induced magnetic field at radial distances (a) 2.00 cm and (b) 5.00 cm?

44. Nonuniform Electric Field In Fig. 31-48, an electric field is directed out of the page within a circular region of radius $R = 3.00$ cm. The magnitude of the electric field is given by $E = (0.500\text{ V/m} \cdot \text{s})(1 - r/R)t$, where t is the time and r is the radial distance ($r \leq R$). What is the magnitude of the induced magnetic field at radial distances (a) 2.00 cm and (b) 5.00 cm?

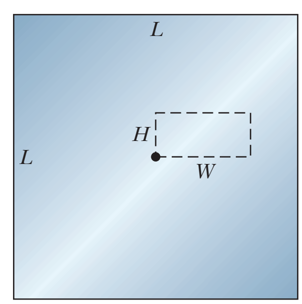


FIGURE 31-49 ■ Problem 45.

45. Discharging Capacitor A capacitor with square plates of edge length L is being discharged by a current of 0.75 A. Figure 31-49 is a head-on view of one of the plates from inside the capacitor. A dashed rectangular path is shown. If $L = 12$ cm, $W = 4.0$ cm, and $H = 2.0$ cm, what is the value of $\oint \vec{B} \cdot d\vec{s}$ around the dashed path?

46. Charging Capacitor The circuit in Fig. 31-50 consists of switch S, a 12.0 V ideal battery, a 20.0 MΩ resistor, and an air-filled capacitor. The capacitor has parallel circular plates of radius 5.00 cm,

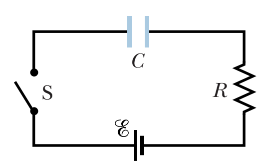


FIGURE 31-50 ■ Problem 46.

separated by 3.00 mm. At time $t = 0$ s, switch S is closed to begin charging the capacitor. The electric field between the plates is uniform. At $t = 250\ \mu s$, what is the magnitude of the magnetic field within the capacitor, at radial distance 3.00 cm?

SEC. 31-9 ■ DISPLACEMENT CURRENT

47. Prove That Displacement Prove that the displacement current in a parallel-plate capacitor of capacitance C can be written as $i^{\text{dis}} = C(d\Delta V/dt)$, where ΔV is the potential difference between the plates.

48. At What Rate At what rate must the potential difference between the plates of a parallel-plate capacitor with a 2.0 μF capacitance be changed to produce a displacement current of 1.5 A?

49. Current Density For the situation of Touchstone Example 31-4, show that the magnitude of the current density of the displacement current is $J^{\text{dis}} = \varepsilon_0\,(dE/dt)$ for $r \leq R$.

50. Being Discharged A parallel-plate capacitor with circular plates of radius 0.10 m is being discharged. A circular loop of radius 0.20 m is concentric with the capacitor and halfway between the plates. The displacement current through the loop is 2.0 A. At what rate is the magnitude of the electric field between the plates changing?

51. Displacement Current As a parallel-plate capacitor with circular plates 20 cm in diameter is being charged, the current density of the displacement current in the region between the plates is uniform and has a magnitude of 20 A/m^2. (a) Calculate the magnitude B of the magnetic field at a distance $r = 50$ mm from the axis of symmetry of this region. (b) Calculate dE/dt in this region.

52. Electric Field The magnitude of the electric field between the two circular parallel plates in Fig. 31-51 is $E = (4.0 \times 10^5\ \text{V}\cdot\text{m}) - (6.0 \times 10^4\ \text{V}\cdot\text{m/s})t$, with E in volts per meter and t in seconds. At $t = 0$ s, the field is upward as shown. The plate area is 4.0×10^{-2} m^2. For $t \geq 0$ s, (a) what are the magnitude and direction of the displacement current between the plates and (b) is the direction of the induced magnetic field clockwise or counterclockwise around the plates?

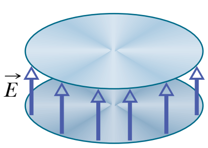


FIGURE 31-51 ■ Problem 52.

53. Magnitude of Electric Field The magnitude of a uniform electric field collapses to zero from an initial strength of 6.0×10^5 N/C in a time of 15 μs in the manner shown in Fig. 31-52. Calculate the amount of displacement current, $|i|$, through a 1.6 m^2 area perpendicular to the field, during each of the time intervals, a, b, and c shown on the graph. (Ignore the behavior at the ends of the intervals.)

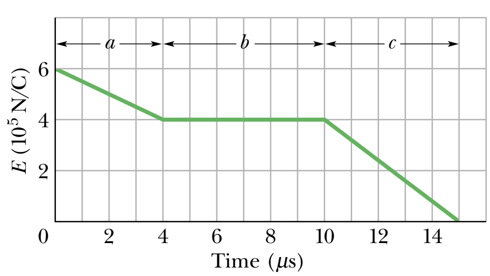


FIGURE 31-52 ■ Problem 53.

54. Displacement Current Two A parallel-plate capacitor with circular plates is being charged. Consider a circular loop centered on the central axis between the plates. The loop radius is 0.20 m, the plate radius is 0.10 m, and the displacement current through the loop is 2.0 A. What is the rate at which the magnitude of the electric field between the plates is changing?

55. Square Plates A parallel-plate capacitor has square plates 1.0 m on a side as shown in Fig. 31-53. A current of 2.0 A charges the capacitor, producing a uniform electric field $\vec{E}$ between the plates, with $\vec{E}$ perpendicular to the plates. (a) What is the displacement current i^{dis} through the region between the plates? (b) What is dE/dt in this region? (c) What is the displacement current through the square dashed path between the plates? (d) What is $\oint \vec{B} \cdot d\vec{s}$ around this square dashed path?

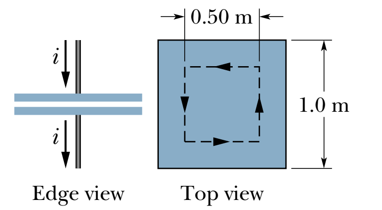


FIGURE 31-53 ■ Problem 55.

56. Consider a Loop A capacitor with parallel circular plates of radius R is discharging via a current of 12.0 A. Consider a loop of radius $R/3$ that is centered on the central axis between the plates. (a) How much displacement current is encircled by the loop? The maximum induced magnetic field has a magnitude of 12.0 mT. (b) At what radial distance from the central axis of the plate is the magnitude of the induced magnetic field 3.00 mT?

57. Uniform Displacement-Current Density. Figure 31-48 shows a circular region of radius $R = 3.00$ cm in which a displacement current is directed out of the page. The magnitude of the displacement current has a uniform density $J^{\text{dis}} = 6.00$ A/m^2. What is the magnitude of the magnetic field due to the displacement current at radial distances (a) 2.00 cm and (b) 5.00 cm?

58. Actual and Displacement Figure 31-54a shows current i that is produced in a wire of resistivity $1.62 \times 10^{-8}\ \Omega\cdot\text{m}$ in the direction indicated. The magnitude of the current versus time t is shown in Fig. 31-54b. Point P is at radius 9.00 mm from the wire's center. Determine the magnitude of the magnetic field at point P due to the real current i in the wire at (a) $t_1 = 20$ ms, (b) $t_2 = 40$ ms, (c) $t_3 = 60$ ms, and (d) $t_4 = 70$ ms. Next, assume that the electric field driving the current is confined to the wire. Then determine the magnitude of the magnetic field at point P due to the displacement current i^{dis} in the wire at (e) $t_1 = 20$ ms, (f) $t_2 = 40$ ms, (g) $t_3 = 60$ ms, and (h) $t_4 = 70$ ms. (i) When both magnetic fields are present at point P, what are their directions in Fig. 31-54a?

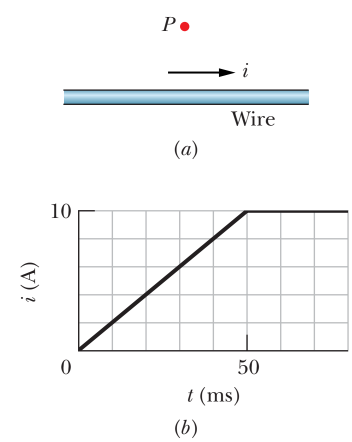


FIGURE 31-54 ■ Problem 58.

59. Nonuniform Displacement-Current Density. Figure 31-48 shows a circular region of radius $R = 3.00$ cm in which a displacement current is directed out of the page. The displacement current has a density of magnitude $J^{\text{dis}} = (4.00\ \text{A/m}^2)(1 - r/R)$, where r is the radial distance $r \leq R$. What is the magnitude of the magnetic field due to the displacement current at radial distances (a) 2.00 cm and (b) 5.00 cm?

60. Uniform Displacement Current. Figure 35-48 shows a circular region of radius $R = 3.00$ cm in which a uniform displacement current $i^{\text{dis}} = 0.500$ A is directed out of the page. What is the magnitude of the magnetic field due to the displacement current at radial distances (a) 2.00 cm and (b) 5.00 cm?

SEC. 31-10 ■ GAUSS' LAW FOR MAGNETIC FIELDS

61. Rolling a Sheet of Paper Imagine rolling a sheet of paper into a cylinder and placing a bar magnet near its end as shown in Fig. 31-55. (a) Sketch the magnetic field lines that pass through the surface of the cylinder. (b) What can you say about the sign of $\vec{B} \cdot d\vec{A}$ for every area $d\vec{A}$ on the surface? (c) Does this result contradict Gauss' law for magnetism? Explain.

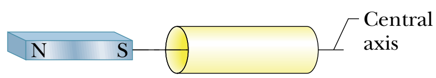


FIGURE 31-55 ■ Problem 61.

62. Die Suppose the magnetic flux at each of five faces of a die (singular of "dice") is given by $\Phi^{\text{mag}} = \pm N$ Wb, where $N(= 1$ to $5)$ is the number of spots on the face. The flux is positive (outward) for N even and negative (inward) for N odd. What is the flux at the sixth face of the die? Is it directed in or out?

63. Right Circular Cylinder A Gaussian surface in the shape of a right circular cylinder with end caps has a radius of 12.0 cm and a length of 80.0 cm. One end encircles an inward magnetic flux of 25.0 μWb. At the other end there is a uniform magnetic field of 1.60 mT, normal to the surface and directed outward. What is the net magnetic flux at the curved surface?

64. Weird Shape Figure 31-56 shows a closed surface. Along the flat top face, which has a radius of 2.0 cm, a magnetic field $\vec{B}$ of magnitude 0.30 T is directed outward. Along the flat bottom face, a magnetic flux of 0.70 mWb is directed outward. What are (a) the magnitude and (b) the net magnetic flux at the curved part of the surface?

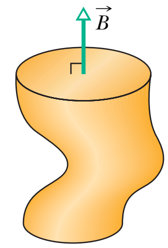


FIGURE 31-56 ■ Problem 64.

Additional Problems

65. Power from a Tether A few years ago, the space shuttle *Columbia* tried an experiment with a tethered satellite. The satellite was released from the shuttle and slowly reeled out on a long conducting cable as shown in Fig. 31-57 (not to scale). For this problem we will make the following approximations:

> The shuttle is moving at a constant velocity.
>
> The Earth's magnetic field is constant and uniform.
>
> The line of the tether, the velocity of the system, and the magnetic field are all perpendicular to each other.

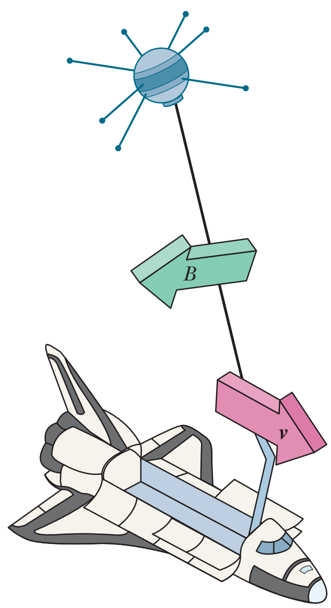


FIGURE 31-57 ■ Problem 65.

The Earth's field produces an emf from one end of the cable to the other. The idea is to use a system like this to generate electric power in space more efficiently than with solar panels.

(a) Explain why a voltage difference is produced.
(b) If the Earth's magnetic field is given by a magnitude $\vec{B}$, the shuttle–satellite system is moving with a velocity $\vec{v}$, and the tether has a length L, calculate the magnitude of the emf $\mathscr{E}$ from one end of the tether to the other.
(c) At the shuttle's altitude, the Earth's field is about 0.3 gauss and the shuttle's speed is about 7.5 km/s. The tether is 20 km long (!). What is the expected potential difference in volts?
(d) At the altitude of the shuttle, the thin atmosphere is lightly ionized, allowing a current of about 0.5 amps to flow from the satellite back to the shuttle through the thin air. What is the resistance of the 20 km of ionized air?

66. Building a Generator The apparatus shown in Fig. 31-58 can be used to build a motor. This device can also be used to build a generator that will produce a voltage. (a) Explain the setup that one would use to make a motor and explain how it works. Do the same for the generator. (b) Estimate the maximum voltage that would be produced if you cranked the generator by hand. (*Hint:* As a comparison for estimating the strength of the bar magnet, the Earth's magnetic field at our location is about 0.4 gauss.)

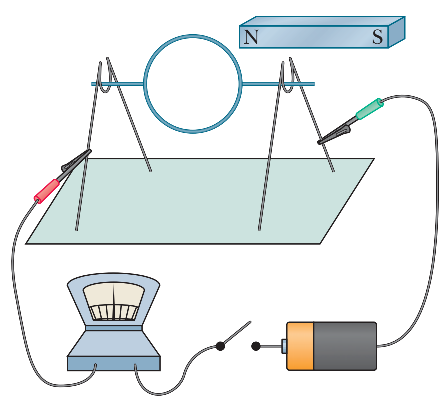


FIGURE 31-58 ■ Problem 66.

67. Faraday's Law Faraday's law describes the emf produced by magnetic fields in a variety of circumstances. State and discuss Faraday's law, being careful to include a discussion of different physical situations that may be described by the statement of the law.

68. Magnetic Field, Force, and Torque Figure 31-59 shows two long, current-carrying wires and a bar magnet. At the right is shown a compass specifying set of direction labels. For each of the vectors (a)–(e) below, select the direction label that best gives the direction of the item. If the magnitude of the item is zero, write 0. If none of the directions are correct, write N.

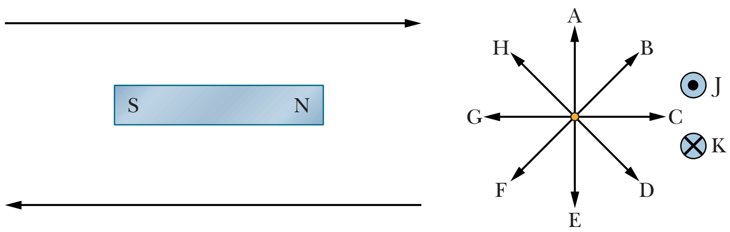


FIGURE 31-59 ■ Problem 68.

(a) The magnetic field due to the lower wire at the center of the upper wire
(b) The force on the lower wire due to the magnetic field from the upper wire
(c) The net torque acting on the upper wire
(d) The magnetic field due to the currents at the center of the magnet
(e) The net force acting on the lower wire due to the bar magnet

69. ***B* Increases in Time** In Fig. 31-60*a*, a uniform magnetic field $\vec{B}$ increases in magnitude with time t as given by Fig. 31-60*b*. A circular conducting loop of area 8.0×10^{-4} m^2 lies in the field, in the plane of the page. The amount of charge q that has passed point A on the loop is given in Fig. 31-60*c* as a function of t. What is the loop's resistance?

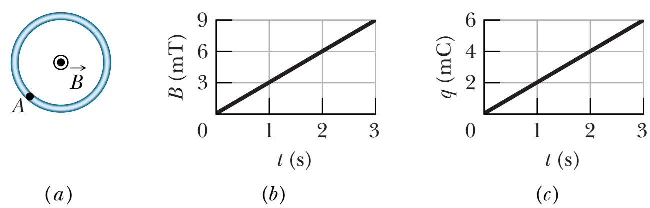


FIGURE 31-60 ■ Problem 69.

70. Circular Loop Around a Solenoid In Fig. 31-61*a*, a circular loop of wire is concentric with a solenoid and lies in a plane that is perpendicular to the solenoid's central axis. The loop has radius 6.00 cm. The solenoid has radius 2.00 cm, consists of 8000 turns per meter, and has a current i_{sol} that varies with time t as given in Fig. 31-61*b*. Figure 31-61*c* shows, as a function of time, the energy $E^{thermal}$ that is transformed to thermal energy in the loop. What is the loop's resistance?

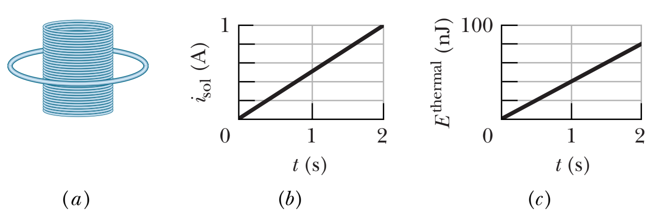


FIGURE 31-61 ■ Problem 70.

71. Magnitudes and Direction Figure 31-62*a* shows a wire that forms a rectangle and has a resistance of 5.0 mΩ. Its interior is split into three equal areas with different magnetic fields $\vec{B}_1$, $\vec{B}_2$, and $\vec{B}_3$ that are either directly out of or into the page, as indicated. The fields are uniform within each region. Figure 31-62*b* gives the change in the z components B_z of the three fields with time t. What are the magnitude and direction of the current induced in the wire?

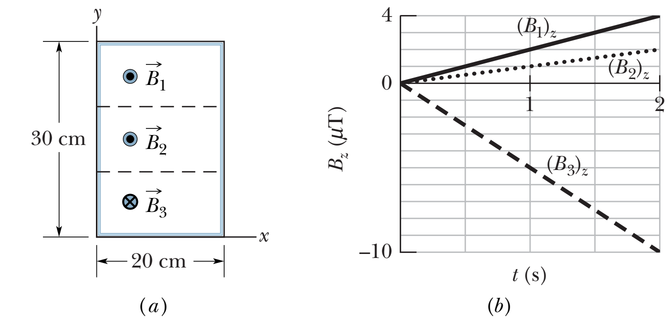


FIGURE 31-62 ■ Problem 71.

72. Two Concentric Regions Figure 31-63*a* shows two concentric circular regions in which uniform magnetic fields can change. Region 1, with radius $r_1 = 1.0$ cm, has an outward magnetic field $\vec{B}_1$ that is increasing in magnitude. Region 2, with radius $r_2 = 2.0$ cm, has an outward magnetic field $\vec{B}_2$ that may also be changing. Imagine that a conducting ring of radius R is centered on the two regions and then the emf $\mathscr{E}$ around the ring is determined. Figure 31-63*b* gives emf $\mathscr{E}$ as a function of the square of the ring's radius, R^2, to the outer edge of region 2. What are the rates of *B*-field magnitude change (a) dB_1/dt and (b) dB_2/dt? (c) Is the magnitude of $\vec{B}_2$ increasing, decreasing, or remaining constant?

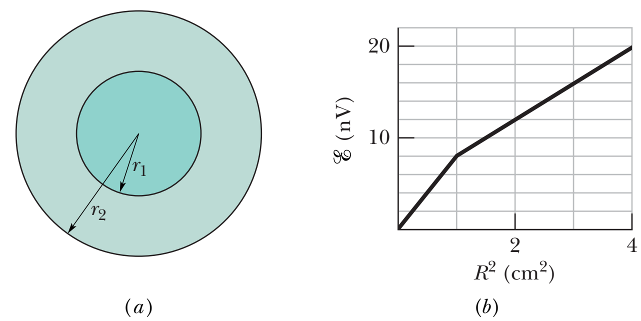


FIGURE 31-63 ■ Problem 72.

73. Pulled at Constant Speed Figure 31-64*a* shows a rectangular conducting loop of resistance $R = 0.020\ \Omega$, height $H = 1.5$ cm, and length $D = 2.5$ cm being pulled at constant speed $v = 40$ cm/s through two regions of uniform magnetic field. Figure 31-64*b* gives the current i induced in the loop as a function of the position x of the right side of the loop. For example, a current of 3.0 μA is induced clockwise as the loop enters region 1. What are the magnitudes and directions of the magnetic field in (a) region 1 and (b) region 2?

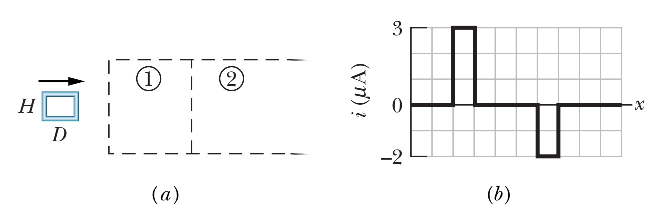


FIGURE 31-64 ■ Problem 73.

74. Plane Loop A plane loop of wire consisting of a single turn of area 8.0 cm^2 is perpendicular to a magnetic field that increases uniformly in magnitude from 0.50 T to 2.5 T in a time of 1.0 s. What is the resulting induced current if the coil has a total resistance of 2.0 Ω?

75. At What Rate Must *B* Change The plane of a rectangular coil of dimensions 5.0 cm by 8.0 cm is perpendicular to the direction of magnetic field B. If the coil has 75 turns and a total resistance of 8.0 Ω, at what rate must the magnitude of B change in order to induce a current of 0.10 A in the windings of the coil?

76. Rod on Rails 3 In the arrangement shown in Fig. 31-65, a conducting rod rolls to the right along parallel conducting rails connected on one end by a 6.0 Ω resistor. A 2.5 T magnetic field is directed *into* the paper. Let $L = 1.2$ m. Neglect the mass of the bar and friction. (a) Calculate the applied force required to move the bar to the right at a *constant* speed of 2.0 m/s. (b) At what rate is energy dissipated in the resistor?

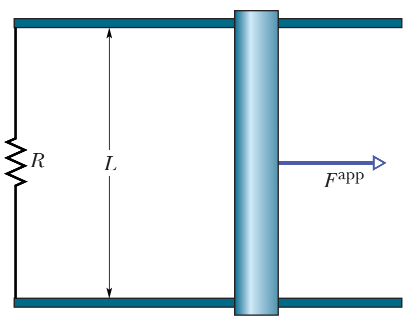


FIGURE 31-65 ■ Problem 76.

77. An Engineer An engineer has designed a setup with a small pickup coil placed in the center of a large field coil as shown in Fig. 31-66. Both coils have many turns of conducting wire. The field coil produces a magnetic field that is proportional in magnitude to the amount of current flowing through its wires. The pickup coil is smaller and its many turns can sense or "pick up" the changing magnetic field in the field coil. The pickup coil produces an emf that is proportional in magnitude to the rate of change of the magnetic field and the angle ϕ. Here ϕ is the angle between the normal to the field coil and the normal to the pickup coil. You have been hired as a consultant to check on the reliability of the engineer's work. You figure out how to use Faraday's law along with proportional reasoning to check on the validity of the results that have been reported without doing any formal calculations or measurements. Sketches from the engineer's notebook are shown in Fig. 31-66*b*. (a) Look at the graph pair in Fig. 31-66*b*. Sketch the measured emf induced in the pickup coil if the engineer has adjusted the scope so the maximum emf is the first positive grid line and the minimum emf is on the first negative grid line. Assume that the normal to each of the coils is pointing in the same direction. (b) According to the engineer's notebook, she fed exactly the same pattern of current to the field coil but she turned the pickup coil so its normal makes an angle of +45° with respect to the normal to the plane of the field coil. Carefully sketch the pattern of emf observed in the pickup coil. What is the maximum and minimum amplitude of the emf in "grid" units? (c) What happens when she flips the pickup coil over around so its normal is 180° from the normal to the field coil? Sketch the emf and use the correct signs for the values of the induced emf for this situation. Explain the reasons for the shape and magnitude of your sketch in each case.

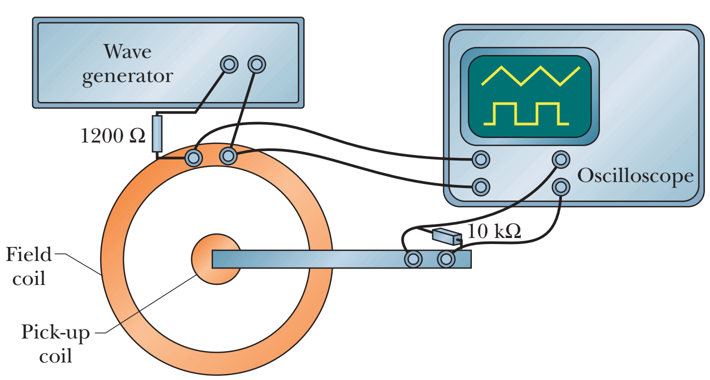


FIGURE 31-66*a* ■ Problem 77.

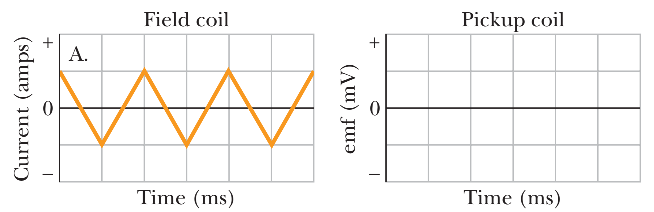


FIGURE 31-66*b* ■ Problem 77.

78. Engineer Task 2 You are still double-checking the work of the engineer from Problem 77. Consider the graph shown in Fig. 31-67. (a) What should our honest and competent engineer have reported for the pattern of emf values as a function of time? Assume that once again the normal to the pickup coil is in the same direction as the normal to the field coil. Please take care to sketch not only the shape of the emf graph but also its proper magnitude using the same gain setting on the oscilloscope as you did in Problem 77. Use a solid line for your sketch. (b) Suppose the engineer reduced the number of turns in the pickup coil by a factor of 2 and redid the measurements. Sketch a new graph showing the shape and proper magnitudes for the expected pickup coil emf using a dashed line. Explain the reasons for the shape and magnitude of your sketch in each case.

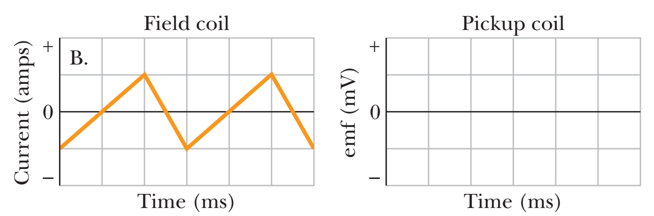


FIGURE 31-67 ■ Problem 78.

79. Engineer Task 3 You are still double-checking the work of the engineer from Problem 31-77. Assume that the number of turns in both the field and pickup coils is the same as in that problem, as is the oscilloscope setting. Consider the graph shown in Fig. 31-68. (a) What should our honest and competent engineer have reported for the pattern of current fed into the field coil as a function of time? Assume that once again the normal to the pickup coil is in the same

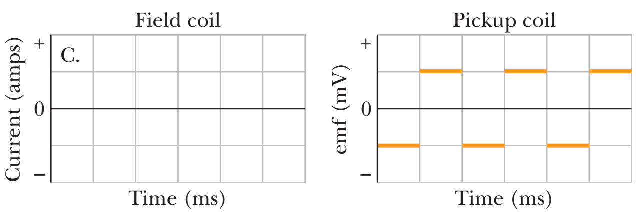


FIGURE 31-68 ■ Problem 79.

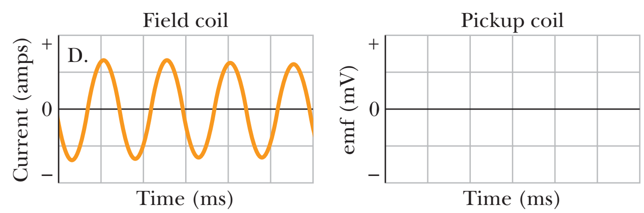


FIGURE 31-69 ■ Problem 80.

direction as the normal to the field coil. Please take care to sketch not only the shape of the emf graph but also its proper magnitude using the same gain setting on the oscilloscope as you did in Problem 31-77. Use a solid line for your sketch. (b) Suppose the engineer reduced the number of turns in the pickup coil by a factor of 2 and redid the measurements. Sketch a new graph showing the shape and proper magnitudes for emf in the field coil using a dashed line. Explain the reasons for the shape and magnitude of your sketch in each case.

80. Engineer Task 4 You are still double-checking the work of the engineer from Problem 31-77. Assume that the number of turns in both the field and pickup coils is the same as in that problem. Consider the graph shown in Fig. 31-69. What should our honest and competent engineer have reported for the pattern of emf induced in the pickup coil if the oscilloscope gain is adjusted to give a maximum value of emf of +2 oscilloscope grid units and a minimum value of −2 oscilloscope units? (*Hint:* What is the derivative of the sine function?) Explain the reason for the shape and magnitude of your sketch in each case.

81. Ring of Copper Figure 31-70 shows a ring of copper with its plane perpendicular to the axis of the nearby rod-shaped magnet. In which of the following situations will a current be induced in the ring? Choose all correct answers.

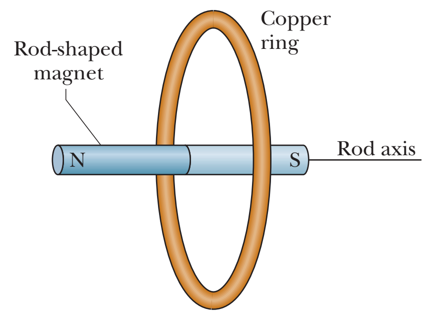


FIGURE 31-70 ■ Problem 81.

(a) The magnet is moved horizontally toward the left.
(b) The ring is moved away from the magnet.
(c) The ring is rotated around any of its diameters.
(d) The magnet is moved up or down.
(e) The ring is rotated around its center in the plane in which it lies.