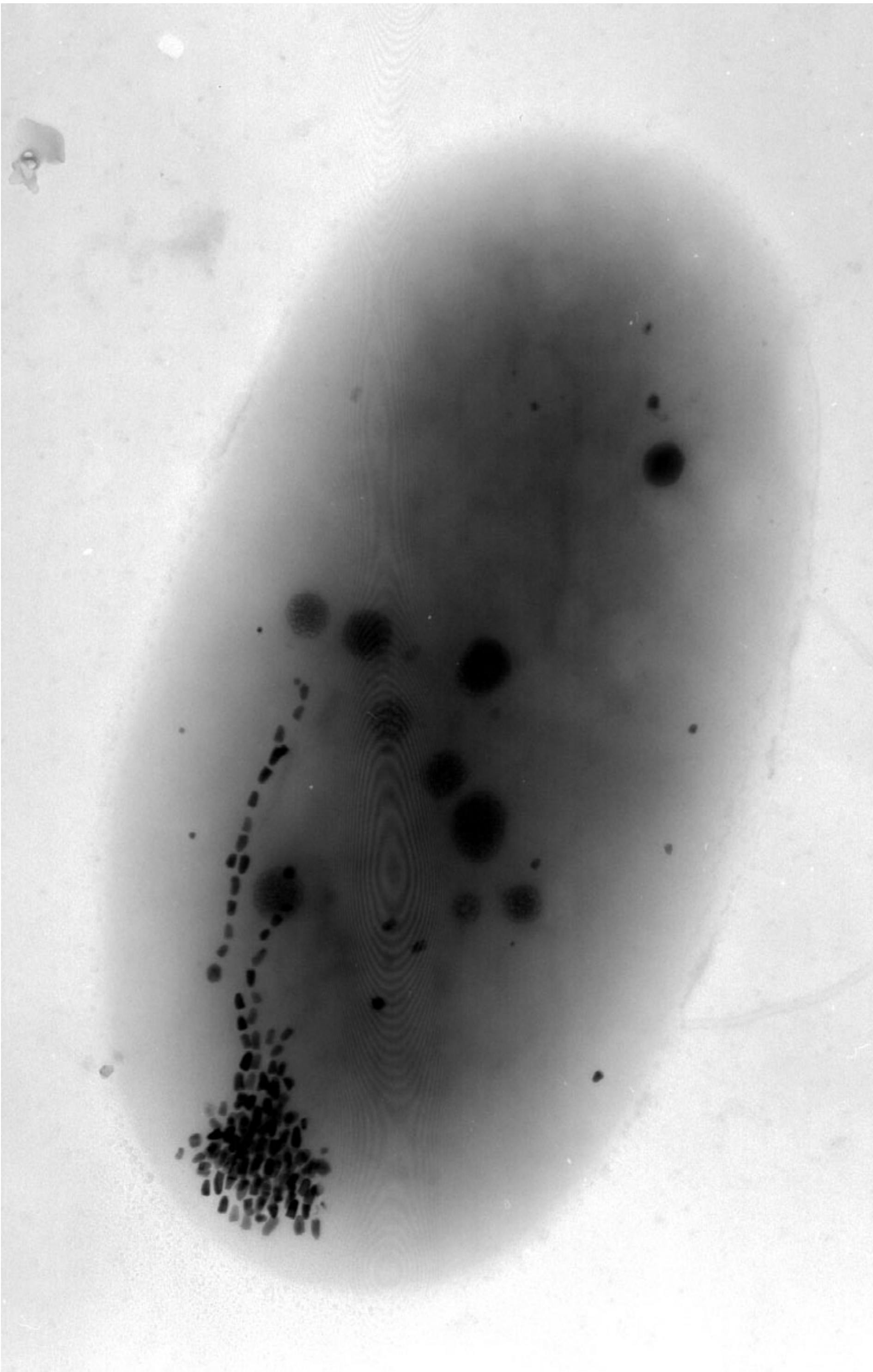
32 Inductors and Magnetic Materials



This is a microscopic view of a bacterium found in Australia that will swim to the muddy bottom of a pond to escape oxygen in its environment and find the nutrients it needs to survive. But if this bacterium were transported to a pond in the United States, it would swim to the top of the pond and die.

How does this bacterium know how to swim down in Australia but not in the U.S.?

The answer is in this chapter.

32-1 Introduction

In the previous chapter we described how an electric car or toothbrush could be charged without electrical contacts. Likewise the guitar pickup described in Section 31-5 amplifies sound. These devices make practical use of inductance. In this chapter we consider some additional practical uses of inductance phenomena in common electric circuit elements known as inductors and transformers. You will consider the basic behaviors of these elements in circuits where the voltage changes in time. Then you will move on to what appears to be an unrelated topic—the behavior of magnetic materials.

The simplest magnetic structure contained in magnetic materials is a magnetic dipole. We will trace the origin of magnetic dipoles, and the associated magnetic properties of materials back to atoms and electrons. You will then reconsider inductors and transformers and learn how magnetic materials can be used to enhance their performance. Some of the first inductors are pictured in Fig. 32-1.

Finally, we will discuss recent theories that enable us explain why the Earth behaves like a huge magnetic dipole, and we will consider the possible role induction plays in explaining the characteristics and changing nature of the Earth's magnetic field.

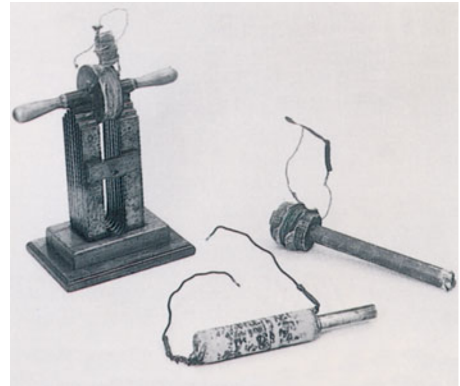


FIGURE 32-1 ■ The crude inductors with which Michael Faraday discovered the law of induction. In those days amenities such as insulated wire were not commercially available. It is said that Faraday insulated his wires by wrapping them with strips cut from one of his wife's petticoats.

32-2 Self-Inductance

Let's explore how the phenomenon of inductance introduced in the previous chapter can be useful in the design of electric circuits with changing currents. In Section 31-3 we saw that when two coils are near each other, a changing current in one of the coils can induce an emf in the other according to Faraday's law ($\mathscr{E} = -N d\,\Phi^{\text{mag}}/dt$). But if the second coil is part of an electric circuit, the current induced in it can also induce an emf in the first coil. This phenomenon, known as **mutual induction,** is used in the design of *inductive chargers*—noncontact charging systems like those used for electric toothbrushes and other devices. In multiple-loop coils the emfs produced by mutual induction are proportional to the number of loops in the coil. For this reason mutual induction is also used in the design of **transformers**—devices that can transform time-varying voltages to larger or smaller time-varying voltages.

In addition, when current in a single coil with one or more loops changes, this induces an emf in the *same* coil. This emf is produced as the result to the changing flux the coil produces in the area it encloses. This process is known as **self induction.** In general,

> A self-induced emf $\mathscr{E}_L$ appears in any coil whenever its current is changing.

According to Lenz's law the self-induced emf acts to oppose the change of current in the coil. For this reason, coils of wire called **inductors** (sometimes called "chokes") are useful in circuits whenever it is desirable to stabilize currents. In a circuit diagram, an inductor is denoted by a symbol that looks like helical loops of wire ( ). See Figs. 32-2 and 32-3. In addition, inductors can be combined with resistors and capacitors to modify the characteristics in circuits driven by oscillating voltage sources. In this chapter we consider the role of inductors in stabilizing currents. In the next chapter we will study the behavior of inductors in circuits with oscillating voltages.

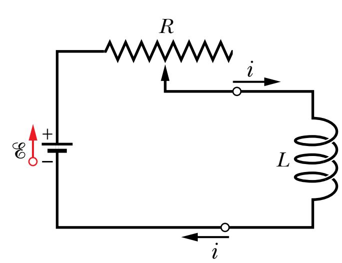


FIGURE 32-2 ■ If the current in a coil is changed by varying the contact position on a variable resistor, a self-induced emf $\mathscr{E}_L$ will appear in the coil while the current is changing.

A typical **inductor** consists of a wire that is coiled into a very large number of loops wrapped around a piece of hollow cardboard or perhaps a magnetic rod. Inductors come in many shapes. A common shape is a **solenoid,** which consists of a tightly

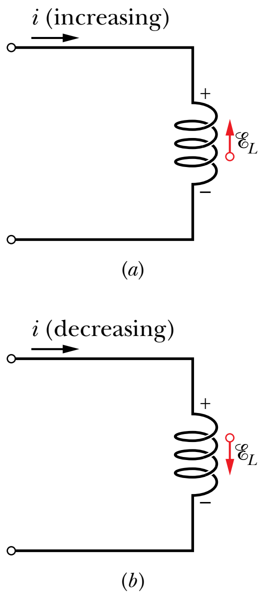


FIGURE 32-3 ■ The arrow and the + and − signs on either side of the inductor indicate the direction the emf $\mathscr{E}_L$ acts in relative to the direction of the current in the circuit alongside the coil. (*a*) The current i is increasing and the self-induced emf $\mathscr{E}_L$ appears along the coil in a direction such that it opposes the increase. (*b*) The current i is decreasing and the self-induced emf appears in a direction such that it opposes the decrease.

wound helical coil of wire—like the one Faraday wound shown in the lower part of Fig. 32-1. Because the magnetic field inside a solenoid is very uniform, it is not difficult to calculate the emfs created by current changes in solenoids. For this reason, we shall consider a solenoid as our basic type of inductor. Also, at first we assume that all inductors are air-core inductors that have no magnetic materials such as iron in their vicinity to distort their magnetic fields.

The Mathematics of Self-Inductance

We start our mathematical treatment of self-inductance with a solenoid-shaped inductor of length l and total number of loops N. When a charge flows through an inductor, the coil produces a magnetic field inside its coils whose strength is *directly proportional* to the current. For an ideal solenoid, the magnitude of the magnetic field is given by Eq. 30-25,

$$B = \mu_0 n|i| \quad \text{(inside an ideal solenoid)}, \qquad \text{(Eq. 30-25)}$$

where μ_0 is the magnetic constant and n the number of turns per unit length.

This magnetic field yields an amount of flux over the area A enclosed by the coil of $|\Phi^{\text{mag}}| = BA = n(\mu_0 A|i|)$. Now, if we try to change the current by changing the resistance in the circuit shown in Fig. 32-2, then the magnetic field and hence the flux at the center of the coil changes. According to Faraday's law this change in flux will produce an emf in the coil given by Eq. 31-7 ($\mathscr{E} = -Nd\,\Phi^{\text{mag}}/dt$). According to Lenz's law this emf will act to oppose the change in the current. Thus, if you close a switch that connects a voltage source to an inductor, the induced "back" emf will retard the rise in current through the circuit. An emf that acts to oppose a change in current is known as a **back emf.** Alternately, if a current already exists in a circuit then opening a switch will slow the rate of reduction of the current (Fig. 32-3). Applying Faraday's law and noting that the total number of turns N is the product of the turns per unit length, n, and the length, l, of the solenoid gives us

$$\mathscr{E}_L = -N\frac{d\Phi^{\text{mag}}}{dt} = -nl\frac{d\Phi^{\text{mag}}}{dt} = -\mu_0 A n^2 l \frac{di}{dt} \quad \text{(solenoidal air-core inductor)}, \qquad (32\text{-}1)$$

where $\mathscr{E}_L$ is the self-induced emf in the solenoid.

If the solenoid is very much longer than its radius, then Eq. 32-1 expresses its inductance to a good approximation. However, we have neglected the spreading of the magnetic field lines near the ends of the solenoid, just as the parallel-plate capacitor formula $C = \varepsilon_0 A/d$ neglects the fringing of the electric field lines near the edges of the capacitor plates.

Equation 32-1 tells us that the amount of self-induced back emf is directly proportional to the rate of change of the current through the coil. The minus sign tells us that $\mathscr{E}_L$ is a back emf. It is customary to combine the product of constants (which for a solenoid is $\mu_0 A n^2 l$) and write this proportionality between the self-induced emf and the rate of current change as

$$\mathscr{E}_L = -L\frac{di}{dt}, \qquad (32\text{-}2)$$

where L is known as the self-inductance of the coil. As we learned in Chapter 31, the minus sign in the equation indicates that the emf acts to oppose the change in current. From Eq. 32-2 we see that when the inductance L is large, a large emf will be produced for a given rate of current change.

This combination of terms (such as area, length, and so on) that makes up the constant of proportionality, L, is only valid for a long solenoid. The terms will be different if the coil has a flat shape or if the inductor wire is wrapped around an iron core. In addition, since any electric circuit is basically a loop of some sort, all circuits have a certain amount of self-inductance even when no inductor is present. Self-inductance is usually negligible, but it can be significant when high-voltage circuits are switched on or off or when the circuit current oscillates at high frequencies. If we have a complicated geometry and cannot calculate inductance simply, the inductance L can be determined experimentally by measuring both the emf and the rate of change of current and taking the ratio of these quantities. Thus, for any geometry the **self-inductance** of an inductor or a circuit can be defined as the ratio of the induced emf to the rate of current change or

$$L \equiv -\frac{\mathscr{E}_L}{di/dt} \quad \text{(self-inductance defined).} \tag{32-3}$$

For any inductor having a self-inductance L, Eqs. 32-1 and 32-2 tell us that $N(d\phi^{\text{mag}}/dt) = L\, di/dt$. Thus we conclude that $\mathscr{E}_L = -Nd\,\Phi^{\text{mag}}/dt = -\,L\,di/dt$, so $Li = N\Phi^{\text{mag}}$, where N is the number of turns in the coil producing flux and i is the current in the coil producing the flux. The windings of the inductor are said to be *linked* by the shared flux, and the product $N\Phi^{\text{mag}}$ is called the *magnetic flux linkage*. This leads us to an alternate definition of inductance (which is equivalent to that given in Eq. 32-3):

$$L \equiv \frac{N\Phi^{\text{mag}}}{i} \quad \text{(alternative definition of self-inductance).} \tag{32-4}$$

The inductance L is thus a measure of the flux linkage produced by the inductor per unit of current.

Because the SI unit of magnetic flux is the tesla-square meter, the SI unit of inductance is the tesla square-meter per ampere ($\text{T} \cdot \text{m}^2/\text{A}$). We call this the **henry** (H), after American physicist Joseph Henry, the co-discoverer, with Faraday, of the law of induction. Thus,

$$1 \text{ henry} = 1 \text{ H} = 1 \text{T} \cdot \text{m}^2/\text{A}. \tag{32-5}$$

In any inductor (such as a flat coil, a solenoid, or a toroid) a self-induced emf appears whenever the current changes with time. The amount of the current has no influence on the amount of induced emf. Only the rate of change of the current matters.

You can find the *direction* of a self-induced emf from Lenz's law. The minus signs in Eqs. 32-2 and 32-3 indicate that—as the law states and Fig. 32-2 shows—the self-induced emf $\mathscr{E}_L$ has an orientation such that it opposes the change in current i.

Ideal Inductors

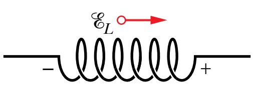
In Section 31-7 we saw that we cannot define an electric potential for an emf that is induced by a changing magnetic flux. This means that when a self-induced emf is produced, we cannot define an electric potential within the inductor itself. However, electric potentials can still be defined at points in a circuit that are not within the inductor—points where the electric fields are due to charge distributions.

Moreover, we can define a self-induced potential difference ΔV_L *across an inductor* (between its terminals, which we assume to be outside the region of changing flux). If the inductor is ideal so that its wire has negligible resistance, the amount of the measured voltage change ΔV_L is equal to the amount of the self-induced emf $\mathscr{E}_L$.

If, instead, the wire in the inductor has resistance R_L, we mentally separate the inductor into a resistance R_L (which we take to be outside the region of changing flux) and an ideal inductor of self-induced emf $\mathscr{E}_L$. As with a real battery of emf $\mathscr{E}$ and internal resistance R, the potential difference across the terminals of a real inductor then differs from the emf. Unless otherwise indicated, we assume here that inductors are ideal.

READING EXERCISE 32-1: (a) What happens to the inductance of a solenoid if: (a) the number of turns per unit length doubles, (b) the cross-sectional area enclosed by the windings doubles?

READING EXERCISE 32-2: The figure shows an emf $\mathscr{E}_L$ induced in a coil. Which of the following can describe the current through the coil: (a) constant and rightward, (b) constant and leftward, (c) increasing and rightward, (d) decreasing and rightward, (e) increasing and leftward, (f) decreasing and leftward?



32-3 Mutual Induction

In this section we return to the case of two interacting coils, which we started discussing in the previous section. We saw earlier that if two coils are close together as in Fig. 32-4 (or Fig. 31-10), a steady current i in one coil will set up a magnetic flux Φ^{mag} at the other coil (*linking* the other coil). If we change the current, i, in the first coil with time, an emf $\mathscr{E}$ given by Faraday's law ($\mathscr{E} = -N d\,\Phi^{\text{mag}}/dt$) will be induced in the second coil. We called this process **mutual induction,** to suggest the mutual interaction of the two coils and to distinguish it from *self-induction,* in which only one coil is involved.

Let us look at mutual induction quantitatively. For any inductor having a self-inductance L, Eq. 32-3 tells us that

$$L \equiv -\frac{\mathscr{E}_L}{di/dt}$$

where i is the current in the coil producing the flux. Figure 32-4a shows two circular coils near each other that share a common central axis. Assume there is a steady current i_1 in coil 1, produced by the battery in the external circuit. This current creates a magnetic field represented by the lines of $\vec{B}_1$ in the figure. Coil 2 is connected to a

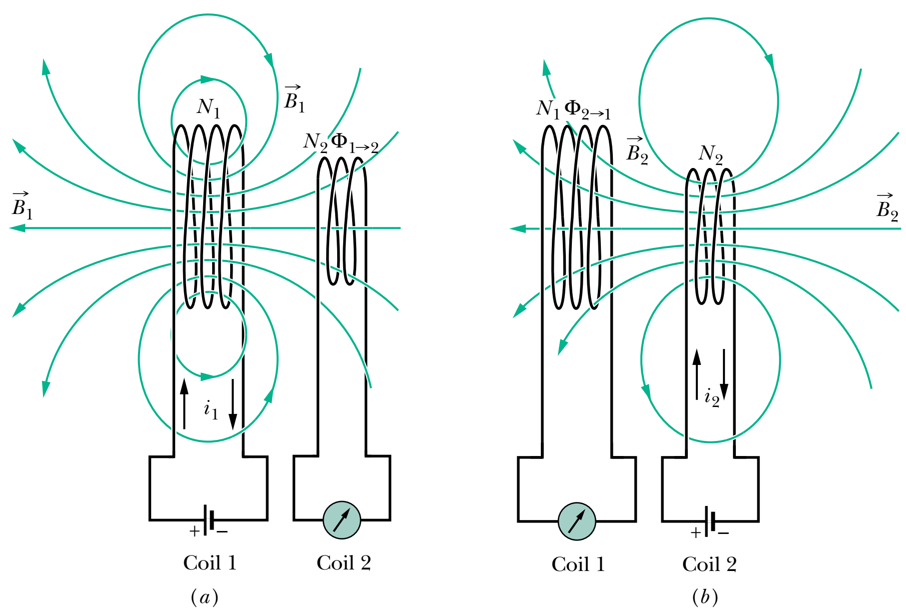


FIGURE 32-4 ■ Mutual induction. (a) If the current in coil 1 changes, an emf will be induced in coil 2. (b) If the current in coil 2 changes, an emf will be induced in coil 1.

sensitive meter but contains no battery. A magnetic flux $\Phi_{1\to2}$ (the flux associated with the current in coil 1 that passes through coil 2) links the N_2 turns of coil 2.

Suppose that by external means we cause i_1 to vary with time. Then by analogy to the definition of self-inductance, we can write a mutual induction equation that is analogous to Eq. 32-2,

$$\mathscr{E}_2 = -M_{1\to2}\frac{di_1}{dt}.$$

This leads us to define the mutual inductance $M_{1\to2}$ of coil 2 due to coil 1 as

$$M_{1\to2} \equiv -\frac{\mathscr{E}_2}{di_1/dt} \qquad \text{(mutual inductance defined).} \qquad (32\text{-}6)$$

Once again we can formulate an alternate definition of mutual induction using the relationship between flux linkage in coil 2 and the current in coil 1, which is $M_{1\to2}i_1 = N_2\Phi_{1\to2}$. The factor N_2 is the number of turns in coil 2 and the factor $\Phi_{1\to2}$ is the magnetic flux present inside coil 2 due to coil 1. This allows us to define mutual inductance as

$$M_{1\to2} \equiv \frac{N_2\Phi_{1\to2}}{i_1} \qquad \text{(alternate definition of mutual inductance).} \qquad (32\text{-}7)$$

If we take the time derivative of all terms in the expression $M_{1\to2}i_1 = N_2\Phi_{1\to2}$ we can write

$$\mathscr{E}_2 = -M_{1\to2}\frac{di_1}{dt} = -N_2\frac{d\Phi_{1\to2}}{dt}. \qquad (32\text{-}8)$$

According to Faraday's law, the right side of this equation is just the amount of the emf $\mathscr{E}_2$ appearing in coil 2 due to the changing current in coil 1. As usual, the minus sign reminds us that induced emf acts to oppose the change in current.

Let us now interchange the roles of coils 1 and 2, as in Fig. 32-4*b*; that is, we set up a current i_2 in coil 2 by means of a battery, and this produces a magnetic flux $\Phi_{2\to1}$ that links coil 1. If we change i_2 with time, we have, by the arguments given above,

$$\mathscr{E}_1 = -M_{2\to1}\frac{di_2}{dt} = -N_1\frac{d\Phi_{2\to1}}{dt}. \qquad (32\text{-}9)$$

Thus, we see that the emf induced in either coil is proportional to the rate of change of current in the other coil. The proportionality constants $M_{1\to2}$ and $M_{2\to1}$ seem to be different. We assert, without proof, that they are in fact the same so that no subscripts are needed. (This conclusion is true but is not obvious.) Thus, we have

$$M_{1\to2} = M_{2\to1} = M, \qquad (32\text{-}10)$$

and we can rewrite Eqs. 32-9 and 32-10 as

$$\mathscr{E}_2 = -M\frac{di_1}{dt} \qquad (32\text{-}11)$$

and

$$\mathscr{E}_1 = -M\frac{di_2}{dt}. \qquad (32\text{-}12)$$

The induction is indeed mutual. The SI unit for M (as for L) is the henry.

TOUCHSTONE EXAMPLE 32-1: Two Coupled Coils

Figure 32-5 shows two circular close-packed coils, the smaller (radius R_2, with N_2 turns) being coaxial with the larger (radius R_1, with N_1 turns) and in the same plane.

(a) Derive an expression for the mutual inductance M for this arrangement of these two coils, assuming that $R_1 \gg R_2$.

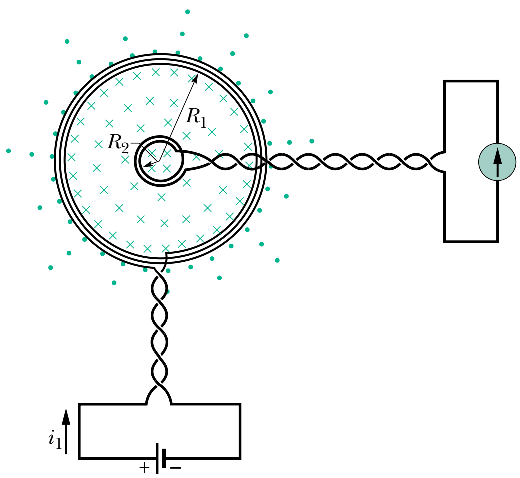


FIGURE 32-5 ■ A small coil is located at the center of a large coil. The mutual inductance of the coils can be determined by sending current i_1 through the large coil.

SOLUTION ■ The **Key Idea** here is that the mutual inductance M for these coils is the ratio of the flux linkage ($N\Phi$) through one coil to the current i in the other coil, which produces that flux linkage. Thus, we need to assume that currents exist in the coils; then we need to calculate the flux linkage in one of the coils.

The magnetic field through the larger coil due to the smaller coil is nonuniform in both magnitude and direction, so the flux in the larger coil due to the smaller coil is nonuniform and difficult to calculate. However, the smaller coil is small enough for us to assume that the magnetic field through it due to the larger coil is approximately uniform. Thus, the flux in it due to the larger coil is also approximately uniform. Hence, to find M we shall assume a current i_1 in the larger coil and calculate the flux linkage $N_2\Phi_{1\to 2}$ in the smaller coil:

$$M_{1\to 2} = \frac{N_2\Phi_{1\to 2}}{i_1}. \tag{32-13}$$

A second **Key Idea** is that the flux $\Phi_{1\to 2}$ through each turn of the smaller coil is, from Eq. 31-1,

$$\Phi_{1\to 2} = B_1 A_2,$$

where B_1 is the magnitude of the magnetic field at points within the small coil due to the larger coil, and $A_2 (= \pi R_2^2)$ is the area enclosed by the coil. Thus, the flux linkage in the smaller coil (with its N_2 turns) is

$$N_2\Phi_{1\to 2} = N_2 B_1 A_2. \tag{32-14}$$

A third **Key Idea** is that to find B_1 at points within the smaller coil, we can use Eq. 30-28, with z set to 0 because the smaller coil is in the plane of the larger coil. That equation tells us that each turn of the larger coil produces a magnetic field of magnitude $\mu_0 i_1/2R_1$ at points within the smaller coil. Thus, the larger coil (with its N_1 turns) produces a total magnetic field of magnitude

$$B_1 = N_1\frac{\mu_0 i_1}{2R_1} \tag{32-15}$$

at points within the smaller coil.

Substituting Eq. 32-15 for B_1 and πR_2^2 for A_2 in Eq. 32-14 yields

$$N_2\Phi_{1\to 2} = \frac{\pi\mu_0 N_1 N_2 R_2^2 i_1}{2R_1}.$$

Substituting this result into Eq. 32-7, and using Eq. 32-10, we find

$$M = M_{1\to 2} = \frac{N_2\Phi_{1\to 2}}{i_1} = \frac{\pi\mu_0 N_1 N_2 R_2^2}{2R_1}. \quad \text{(Answer)} \tag{32-16}$$

Just as capacitance does not depend on the amount of charge on capacitor plates, mutual inductance, M, does not depend on the current in the coils.

(b) What is the value of M for $N_1 = N_2 = 1200$ turns, $R_2 = 1.1$ cm, and $R_1 = 15$ cm?

SOLUTION ■ Equation 32-16 yields

$$M = \frac{(\pi)(4\pi \times 10^{-7}\ \text{H/m})(1200)(1200)(0.011\ \text{m})^2}{(2)(0.15\ \text{m})}$$

$$= 2.29 \times 10^{-3}\ \text{H} \approx 2.3\ \text{mH}. \quad \text{(Answer)}$$

Consider the situation if we reverse the roles of the two coils—that is, if we produce a current i_2 in the smaller coil and try to calculate M from Eq. 32-7 in the form

$$M_{2\to 1} = \frac{N_2\Phi_{2\to 1}}{i_2}.$$

The calculation of $\Phi_{2\to 1}$ (the nonuniform flux of the smaller coil's magnetic field encompassed by the larger coil) is not simple. If we were to do the calculation numerically using a computer, we would find M to be 2.3 mH, as above! This emphasizes that Eq. 32-10 ($M_{1\to 2} = M_{2\to 1} = M$) is not obvious.

32-4 *RL* Circuits (With Ideal Inductors)

In Section 28-9 we saw that if we suddenly switch an emf $\mathscr{E}$ on in a series circuit containing a resistor R and a capacitor C, the charge on the capacitor q does not build up immediately to its final equilibrium value $C\mathscr{E}$ but approaches it in an exponential fashion:

$$q = C\mathscr{E}(1 - e^{t/\tau_C}). \tag{32-17}$$

The rate at which the charge builds up is determined by the capacitive time constant τ_C, defined in Eq. 28-42 as

$$\tau_C = RC. \tag{32-18}$$

If we suddenly remove the emf from this same circuit, the charge does not immediately fall to zero but approaches zero in an exponential fashion:

$$q = q_0 e^{-t/\tau_C}. \tag{32-19}$$

The time constant τ_C describes the fall of the charge as well as its rise and q_0 is the initial charge on the capacitor.

An analogous slowing of the rise (or fall) of the current occurs if we introduce an emf $\mathscr{E}$ into (or remove it from) a single-loop circuit containing a resistor R and an inductor L. We assume the inductor is ideal and has a resistance R_L that is much less than R. When the switch S in Fig. 32-6 is closed on a, for example, the current in the resistor starts to rise. If the inductor were not present, the current would rise rapidly to a steady value $\mathscr{E}/R$. Because of the inductor, however, a self-induced emf $\mathscr{E}_L$ appears in the circuit. As predicted from Lenz's law, this emf opposes the rise of the current. This means that it opposes the battery emf $\mathscr{E}$ in polarity. Thus the current in the resistor responds to the *difference* between two emfs, a constant one $\mathscr{E}$ due to the battery, and a variable one $\mathscr{E}_L(= -L\, di/dt)$ due to self-induction. As long as $\mathscr{E}_L$ is present, the current in the resistor will be less than $\mathscr{E}/R$. As time goes on, the rate at which the current increases becomes less rapid and the amount of the self-induced emf, which is proportional to di/dt, becomes smaller. Thus, the current in the circuit approaches $\mathscr{E}/R$ asymptotically.

We can generalize these results as follows: When a switch is opened or closed in a dc circuit, an inductor initially acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire that has some resistance R_L.

Now let us analyze the situation quantitatively. With the switch S in Fig. 32-6 thrown to a, the circuit is equivalent to that of Fig. 32-7. Let us apply the loop rule, starting at point x in this figure and moving clockwise around the loop along with current i.

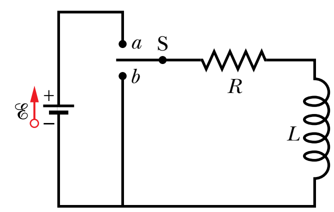


FIGURE 32-6 ■ An *RL* circuit. When switch S is closed on a, the current rises and approaches a limiting value of $\mathscr{E}/R$.

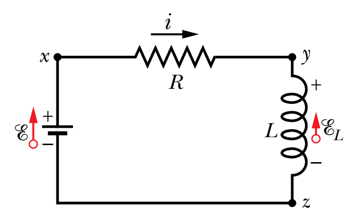


FIGURE 32-7 ■ The circuit of Fig. 32-6 with the switch closed on a. We apply the loop rule for circuits clockwise, starting at x.

1. *Resistor*. Because we move through the resistor in the direction of current i, the electric potential decreases by iR. Thus, as we move from point x to point y where these points lie *outside* the inductor, we encounter a potential change of $-iR$.
2. *Inductor*. Because current i is changing, there is a self-induced emf $\mathscr{E}_L$ in the inductor. The amount of $\mathscr{E}_L$ is given by Eq. 32-2 as $L\, di/dt$. The direction of $\mathscr{E}_L$ is upward in Fig. 32-7 because current i is downward through the inductor and increasing. Thus, as we move from point y to point z, opposite the direction of $\mathscr{E}_L$, we encounter a potential change of $-L\, di/dt$.
3. *Battery*. As we move from point z back to starting point x, we encounter a potential change of $+\mathscr{E}$ due to the battery's emf.

Thus, the loop rule gives us

$$-iR - L\frac{di}{dt} + \mathscr{E} = 0$$

or

$$L\frac{di}{dt} + Ri = \mathscr{E} \quad (RL \text{ circuit}). \tag{32-20}$$

Equation 32-20 is a differential equation involving the variable i and its first derivative di/dt. To solve it, we seek the function $i(t)$ such that when $i(t)$ and its first derivative are substituted in Eq. 32-20, the equation is satisfied and the initial condition $i(0) = 0$ A is satisfied.

Equation 32-20 and its initial condition are of exactly the form of Eq. 28-38 for an RC circuit, with i replacing q, L replacing R, and R replacing $1/C$. The solution of Eq. 32-20 must then be of exactly the form of Eq. 28-39 with the same replacements. That solution is

$$i = \frac{\mathscr{E}}{R}(1 - e^{-(R/L)t}), \tag{32-21}$$

which we can rewrite as

$$i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) \quad \text{(rise of current)}. \tag{32-22}$$

Here τ_L, the inductive time constant, is given by

$$\tau_L = \frac{L}{R} \quad \text{(time constant)}. \tag{32-23}$$

What happens to the current described in Eq. 32-22 between the time the switch is closed (at time $t = 0$ s) and a later time ($t \to \infty$)? If we substitute $t = 0$ s into Eq. 32-22, the exponential becomes $e^{-0} = 1$. Thus, Eq. 32-22 tells us that the current is initially $i = 0$ A, as expected. Next, if we let t go to infinity, then the exponential goes to $e^{-\infty} = 0$. Thus, Eq. 32-22 tells us that the current goes to its equilibrium value of $\mathscr{E}/R$.

We can also examine the potential differences in the circuit. The graphs of Fig. 32-8 show experimental data describing how the potential differences $|\Delta V_R| = iR$ across a resistor and $|\Delta V_L| = L\,di/dt$ across an inductor vary with time for particular

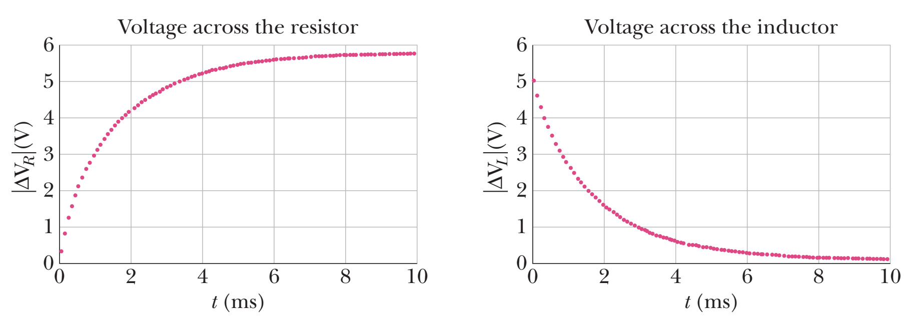


FIGURE 32-8 ■ A computer data acquisition system is used to record the time variation of potential differences (*a*) ΔV_R across the resistor in Fig. 32-7 and (*b*) ΔV_L across the inductor in that circuit. The data were obtained at 10 000 samples per second for $R = 9830\ \Omega$, $L \approx 20$ H, and $\mathscr{E} = 5.88$ V. The inductor has a direct current resistance of 167 Ω, so it is not ideal. The data show some different characteristics than those predicted by Eqs. 32-22 and 32-23.

values of $\mathscr{E}$, L, and R. Compare this figure carefully with the corresponding figure for an *RC* circuit (Fig. 28-23).

To show that the quantity $\tau_L(=L/R)$ has the dimension of time, we convert from henries per ohm as follows:

$$1\frac{\mathrm{H}}{\Omega} = 1\frac{\mathrm{H}}{\Omega}\left(\frac{1\,\mathrm{V\cdot s}}{1\,\mathrm{H\cdot A}}\right)\left(\frac{1\,\Omega\cdot\mathrm{A}}{1\,\mathrm{V}}\right) = 1\,\mathrm{s}.$$

The first quantity in parentheses is a conversion factor based on Eq. 32-20, and the second one is a conversion factor based on the relation $\Delta V = iR$.

The physical significance of the time constant follows from Eq. 32-21. If we put $t = \tau_L = L/R$ in this equation, it reduces to

$$i = \frac{\mathscr{E}}{R}(1 - e^{-1}) = 0.63\frac{\mathscr{E}}{R}. \tag{32-24}$$

Thus, the time constant τ_L is the time it takes the current in the circuit to reach about 63% of its final equilibrium value $\mathscr{E}/R$. Since the potential difference ΔV_R across the resistor is proportional to the current i, a graph of the increasing current versus time has the same shape as that of ΔV_R in Fig. 32-8*a*.

If the switch S in Fig. 32-6 is closed on *a* long enough for the equilibrium current $\mathscr{E}/R$ to be established and then is thrown to *b*, the effect will be to remove the battery from the circuit. (The connection to *b* must actually be made an instant before the connection to *a* is broken. A switch that does this is called a *make-before-break* switch.)

With the battery gone, the current through the resistor will decrease. However, because of the inductor it cannot drop immediately to zero but must decay to zero over time. The differential equation that governs the decay can be found by putting $\mathscr{E} = 0$ in the *RL* circuit voltage loop equation (Eq. 32-20):

$$L\frac{di}{dt} + iR = 0. \tag{32-25}$$

By analogy with Eqs. 28-44 and 28-45, the solution of this differential equation that satisfies the initial condition $i(0) = i_0 = \mathscr{E}/R$ is

$$i = \frac{\mathscr{E}}{R}e^{-t/\tau_L} = i_0e^{-t/\tau_L} \quad \text{(decay of current).} \tag{32-26}$$

We see that both current rise (Eq. 32-21) and current decay (Eq. 32-26) in an *RL* circuit are governed by the same inductive time constant, τ_L.

We have used i_0 in Eq. 32-26 to represent the current at time $t = 0$. In our case that happened to be $\mathscr{E}/R$, but it could be any other initial value.

TOUCHSTONE EXAMPLE 32-2: Two Inductors and Three Resistors

Figure 32-9*a* shows a circuit that contains three identical resistors with resistance $R = 9.0\ \Omega$, two identical ideal inductors with inductance $L = 2.0$ mH, and an ideal battery with emf $\mathscr{E} = 18$ V.

(a) What is the current i through the battery just after the switch is closed?

SOLUTION ■ The **Key Idea** here is that just after the switch is closed, the inductor acts to oppose a change in the current through it. Because the current through each inductor is zero before the switch is closed, it will also be zero just afterward. Thus, immediately after the switch is closed, the inductors act as broken wires, as indicated in Fig. 32-9*b*. We then have a single-loop circuit

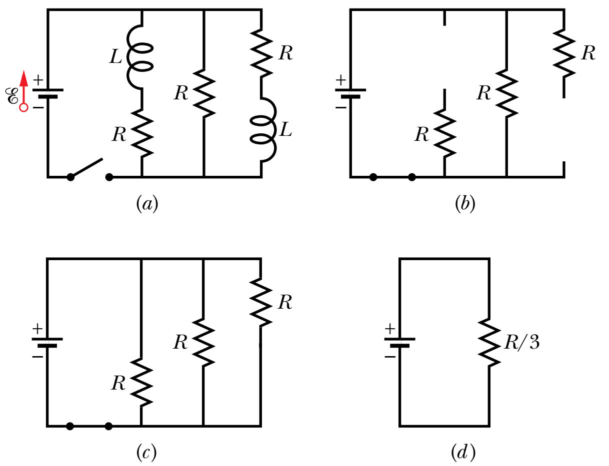


FIGURE 32-9 ■ (a) A multiloop RL circuit with an open switch. (b) The equivalent circuit just after the switch has been closed. (c) The equivalent circuit a long time later. (d) The single-loop circuit that is equivalent to circuit (c).

for which the loop rule gives us

$$\mathscr{E} - iR = 0.$$

Substituting given data, we find that

$$i = \frac{\mathscr{E}}{R} = \frac{18\text{ V}}{9.0\ \Omega} = 2.0\text{ A}. \qquad \text{(Answer)}$$

(b) What is the current i through the battery long after the switch has been closed?

SOLUTION ■ The **Key Idea** here is that long after the switch has been closed, the currents in the circuit have reached their equilibrium values, and the inductors act as simple connecting wires, as indicated in Fig. 32-9c. We then have a circuit with three identical resistors in parallel; from Eq. 27-12, their equivalent resistance is $R^{\text{eq}} = R/3 = (9.0\ \Omega)/3 = 3.0\ \Omega$. The equivalent circuit shown in Fig. 32-9d then yields the loop equation $\mathscr{E} - iR^{\text{eq}} = 0.0$ V, or

$$i = \frac{\mathscr{E}}{R^{\text{eq}}} = \frac{18\text{ V}}{3.0\ \Omega} = 6.0\text{ A}. \qquad \text{(Answer)}$$

32-5 Inductors, Transformers, and Electric Power

In most countries the electrical power used in homes and industries involves voltages and currents that change over time periodically, often sinusoidally. Such power is usually referred to as alternating current or ac electricity. Alternating electrical power is usually generated using induction. An **ac generator** simply consists of a magnet or electromagnet rotating inside an inductor coil or, alternatively, an inductor coil rotating in a magnetic field like that shown in Fig. 32-10.

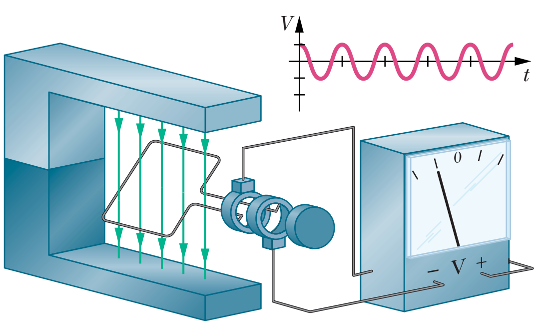


FIGURE 32-10 ■ A simplified diagram of an electric generator showing how a crank can be used to rotate a pickup coil in a magnetic field such that the flux through the coil is changing periodically. Most large generators have a geometry in which the coil rotates outside of an electromagnet.

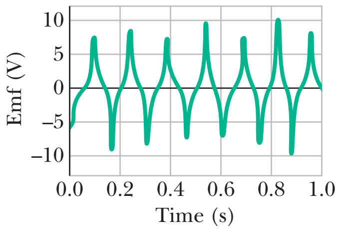


FIGURE 32-11 ■ A periodic emf is induced in a coil that is being turned by a hand crank in the presence of a magnet. The generator is similar to that shown in Fig. 32-10. If the coil were less bulky and the $\vec{B}$-field were more uniform, the emf would vary sinusoidally when the crank is turned steadily.

Generators don't care what form of energy is used to cause the rotation (Fig. 32-11). The shaft can turn when steam produced by a coal-fired or nuclear power plant pushes on propeller-like blades. In hydroelectric plants, falling water can provide the rotational energy. Since the potential difference and current in a generator vary sinusoidally, the voltages and currents are reported as root mean square (or rms) values. The use of rms values is explained in the next chapter, where we deal with alternating-current circuits in more detail.

The Role of Transformers

Generators typically produce power at low voltage, but it is important to transmit this power from generation stations to consumers with minimum energy loss. It turns out that the losses are minimized when ac power is transmitted at high voltages. The reason has to do with how power loss is related to current and voltage. The total power generated is given by Eq. 26-10 as $P^{\text{gen}} = i^{\text{gen}}\Delta V^{\text{gen}}$. If this power is transmitted to consumers with an rms current i^{gen} flowing over long distances, then the power lost in heating transmission lines is given by

$$P^{\text{lost}} = (i^{\text{gen}})^2 R \qquad \text{(power lost in transmission)}, \tag{32-27}$$

where R is the total resistance of the wires that make up the transmission lines. The power available to consumers is then $P^{\text{gen}} - P^{\text{lost}}$. Although we can't get something for nothing, it is obvious from Eq. 32-27 that reorganizing the generated power so that it is transmitted at high voltage and low current would greatly reduce the transmission losses. In other words, we would like to achieve

$$P^{\text{gen}} = i^{\text{gen}}\Delta V^{\text{gen}} = i^{\text{trans}}\Delta V^{\text{trans}}, \tag{32-28}$$

where $\Delta V^{\text{trans}} \gg \Delta V^{\text{gen}}$ so that $i^{\text{trans}} \ll i^{\text{gen}}$.

As an example, consider the 735 kV line used to transmit electric energy from the La Grande 2 hydroelectric plant in Quebec to Montreal, 1000 km away. Suppose that the current is 500 A. Then from Eq. 32-28, energy is supplied at the average rate

$$P^{\text{gen}} = i^{\text{gen}}\Delta V^{\text{gen}} = (7.35 \times 10^5\,\text{V})(500\text{ A}) = 368\text{ MW}.$$

The resistance of the transmission line is about 0.220 Ω/km. Thus, there is a total resistance of about 220 Ω for the 1000 km stretch. Energy is dissipated due to that resistance at a rate of about

$$P^{\text{lost}} = (i^{\text{gen}})^2 R = (500\text{ A})^2(220\ \Omega) = 55.0\text{ MW},$$

which is nearly 15% of the supply rate.

Imagine what would happen if we could halve the current and double the voltage. Energy would be supplied by the plant at the same average rate of 368 MW as before, but now energy would be dissipated at the much lower rate of about

$$P^{\text{lost}} = (i^{\text{gen}})^2 R = (250\text{ A})^2(220\ \Omega) = 13.8\text{ MW},$$

This rate of energy loss is *only 4% of the supply rate*. Hence the general energy transmission rule: Transmit at the highest possible voltage and the lowest possible current. There is an upper limit to the voltage that can be used. If the voltage gets too high, the power line insulation and the surrounding air will not be able to prevent the current from passing through them and leaking to the ground.

The Ideal Transformer

The *ideal transformer* in Fig. 32-12 consists of two coils, a *primary* and a *secondary*. These coils have different numbers of turns and are wound around the same iron core. The coils experience mutual induction. The iron core concentrates the flux so that it is the same in both coils. (We will discuss the role iron plays in Section 32-7 on ferromagnetism.) In use, the **primary coil,** of N_p turns, is connected to an alternating-current generator whose emf $\mathscr{E}$ at any time t is given by

$$\mathscr{E} = \mathscr{E}^{\max}\sin\omega t. \tag{32-29}$$

The **secondary coil,** of N_s turns, is connected to load resistance R, but its circuit is an open circuit as long as switch S is open (which we assume for the present). Thus, there

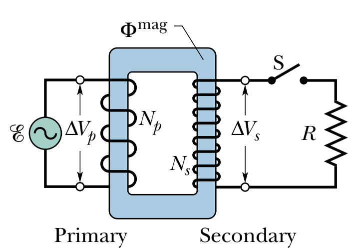


FIGURE 32-12 ■ An ideal transformer (two coils wound on an iron core) in a basic transformer circuit. An ac generator produces current in the coil at the left (the *primary*). The coil at the right (the *secondary*) is connected to the resistive load R when switch S is closed.

can be no current through the secondary coil. We assume further for this ideal transformer that the resistances of the primary and secondary coils (or **windings**) are negligible as are energy losses in the iron core. Well-designed, high-capacity transformers can have energy losses as low as 1%, so our assumptions are reasonable.

For the assumed conditions, the primary winding (or *primary*) is a pure inductance that carries a small alternating primary current i_p. This current induces an alternating magnetic flux Φ^{mag} in the iron core. Because the core extends through the secondary winding (or *secondary*), this induced flux also extends through the turns of the secondary. At any given time the flux in the primary and secondary coils are the same. Therefore, Faraday's law of induction (Eq. 31-7) tells us that the amount of the induced emf per turn, denoted as $\mathscr{E}_{\text{turn}}$ is the same for both the primary and the secondary coils. Also, the voltage ΔV_p across the primary is equal to the emf induced in the primary, and the voltage ΔV_s across the secondary is equal to the emf induced in the secondary. Thus, we can write

$$\mathscr{E}_{\text{turn}} = \frac{d\Phi^{\text{mag}}}{dt} = \frac{\Delta V_p}{N_p} = \frac{\Delta V_s}{N_s},$$

and thus,

$$\Delta V_s = \Delta V_p \frac{N_s}{N_p} \quad \text{(transformation of voltage).} \tag{32-30}$$

If $N_s > N_p$, the transformer is called a **step-up transformer** because it steps the primary's voltage ΔV_p up to a higher voltage ΔV_s. Alternatively, if $N_s < N_p$, the device is a **step-down transformer.**

So far, with switch S open, no energy is transferred from the generator to the rest of the circuit. Now let us close S to connect the secondary to the resistive load R. (In general, the load would also contain inductive and capacitive elements, but here we neglect the capacitance.) We find that now energy is transferred from the generator. To see why, let's explore what happens when we close switch S.

1. An alternating current i_s appears in the secondary circuit, with corresponding energy dissipation rate $i_s^2 R = (\Delta V_s^2)/R$ in the resistive load. Since the emf produced in the secondary coil is a back emf that opposes the direction of the change in current in the primary, the secondary current is out of phase with the primary current.
2. This current produces its own alternating magnetic flux in the iron core, and this flux induces (from Faraday's law and Lenz's law) an opposing emf in the primary windings.
3. The voltage ΔV_p of the primary, however, cannot change in response to this opposing emf because it must always be equal to the emf $\mathscr{E}$ that is provided by the generator; closing switch S cannot change this fact.

In order to relate i_s to i_p, we can apply the principle of conservation of energy. For the ideal transformer without losses in the magnetic core, the power drawn from the primary is equal to the power transferred to the secondary (via the alternating magnetic field linking the two coils). Conservation of energy requires that

$$i_p \Delta V_p = i_s \Delta V_s. \tag{32-31}$$

Substituting for ΔV_s from Eq. 32-30, we find that

$$i_s = i_p \frac{N_p}{N_s} \quad \text{(transformation of currents).} \tag{32-32}$$

This equation tells us that the amount of the current i_s in the secondary can be greater than, less than, or the same as the amount of current i_p in the primary, depending on the *ratio of turns (or loops) in the coils given by* N_p/N_s.

Current i_p appears in the primary circuit because of the resistive load R in the secondary circuit. To find i_p, we substitute $i_s = \Delta V_s/R$ into Eq. 32-32 and then we substitute for ΔV_s from Eq. 32-30. We find

$$i_p = \frac{1}{R}\left(\frac{N_s}{N_p}\right)^2 \Delta V_p. \tag{32-33}$$

This equation has the form $i_p = \Delta V_p/R_{\text{eq}}$, where equivalent resistance R_{eq} is

$$R_{\text{eq}} = \left(\frac{N_p}{N_s}\right)^2 R. \tag{32-34}$$

Here R is the actual resistance in the secondary circuit and R_{eq} is the value of the load resistance as "seen" by the generator. The generator produces the current i_p and voltage ΔV_p as if it were connected to a resistance R_{eq}.

Impedance Matching

Equation 32-34 suggests still another function for the transformer. For maximum transfer of energy from an emf device to a resistive load, the resistance of the emf device and the resistance of the load must be equal. The same relation holds for ac circuits (discussed in Chapter 33) except that the *impedance* (rather than just the resistance) of the generator must be matched to that of the load. Often this condition is not met. For example, in a music-playing system, the amplifier can have high impedance and the speaker set have low impedance. We can match the impedances of the two devices by coupling them through a transformer with a suitable turns ratio N_p/N_s.

READING EXERCISE 32-3: An alternating-current emf device has a smaller resistance than that of the resistive load; to increase the transfer of energy from the device to the load, a transformer will be connected between the two. (a) Should N_s be greater than or less than N_p? (b) Will that make it a step-up or step-down transformer? ■

32-6 Magnetic Materials—An Introduction

Today, magnets and magnetic materials are ubiquitous. In addition to naturally magnetic lodestones, magnets are also in VCRs, audiocassettes, credit cards, electronic speakers, audio headsets, and even the inks in paper money. In fact, some breakfast cereals that are "iron fortified" contain small bits of magnetic materials (you can collect them from a slurry of cereal and water with a magnet). In this section we are interested in understanding more about why so-called bulk matter, made of billions upon billions of individual atoms, has magnetic properties.

Characteristics of Magnetic Materials

When we speak of magnetism in everyday conversation, we usually have a mental picture of a bar magnet, a disk magnet (probably clinging to a refrigerator door), or even a tiny compass needle. That is, we picture a *ferromagnetic* material made of iron having strong, permanent magnetism. Although most bulk matter does not behave like the familiar iron bar magnets, it turns out that almost all bulk materials have some magnetic behaviors. There are three general types of magnetism: ferromagnetism, paramagnetism, and diamagnetism.

1. ***Ferromagnetism*** is present if a material produces a strong magnetic field of its own in the presence of an external field, and if its magnetic field partially persists after the external field is removed. We usually use the term *ferromagnetic material,* and also the common term *magnetic material*, to refer to materials that exhibit primarily ferromagnetism. Iron, nickel, and cobalt (and compounds and alloys of these elements) are ferromagnetic.
2. ***Paramagnetism*** is present if a material that is placed in an external magnetic field is attracted to the region of greater magnetic field and produces a magnetic field of its own—but only while it is in the presence of the external field. The term *paramagnetic material* usually refers to materials that exhibit primarily paramagnetism. This type of magnetism is exhibited by materials such as liquid oxygen and aluminum as well as transition elements, rare earth elements, and actinide elements (see Appendix G).
3. ***Diamagnetism*** is present if a material that is placed near a magnet is repelled from the region of greater magnetic field. This is opposite to the behavior of the other two types of magnetism. Diamagnetism is exhibited by all common materials, but it is so weak that it is masked if the material exhibits magnetism of either of the other two types. Thus, the term *diamagnetic material* refers to materials that only exhibit diamagnetism. Metals such as bismuth, copper, gold, silver, and lead, as well as many nonmetals such as water and most organic compounds, are diamagnetic. Because people and other animals are made largely of water and organic compounds, they are diamagnetic too.

FIGURE 32-13 ■ A bar magnet is a magnetic dipole. The orientations of the iron filings suggest the direction of magnetic field lines.

What causes magnetism? Why are there three types of magnetism? We now believe that magnetism is caused by tiny magnetic dipoles that are intrinsic to the atoms contained in all materials. For this reason, understanding the characteristics of magnetic dipoles is essential to understanding the behavior of magnetic materials. We will conclude this section with a discussion of magnetic dipoles, and then in the next two sections we will explore how the characteristics of atomic magnetic dipoles help us understand the three types of magnetism.

Characteristics of Magnetic Dipoles

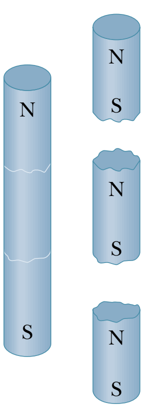


FIGURE 32-14 ■ If you break a bar magnet, each fragment becomes a smaller magnet, with its own north and south poles. It is impossible to break a fragment into separate north and south poles.

Both the bar magnets with which we are familiar and small coils of wire carrying current are magnetic dipoles. Let's review some of the characteristics of magnetic dipoles that we have already discussed. A magnetic dipole:

- Has a magnetic field pattern associated with it similar to that of an electric dipole (like that shown in Fig. 32-13 or that described by the equations derived in Sections 23-6 and 30-7).
- *Always has two poles,* which we have chosen to call north (seeking) and south (seeking) because of the way they behave when placed in the Earth's magnetic field, as shown in Fig. 32-14 (see Section 29-3 for a review).
- Can be described by a magnetic dipole moment $\vec{\mu}$, which is a vector quantity whose *magnitude* tells us how *strong* the magnetic field associated with the dipole is and whose direction tells how the field pattern is oriented. The orientation is along the axis of a bar magnet pointing from its south pole and to its north pole or perpendicular to the plane of a current-carrying coil with a direction determined by the right-hand rule (Section 29-10). *Note:* Our use of conventional notation is unfortunate here. The magnetic moment $\vec{\mu}$ should not be

confused with the permeability constant μ_0 or μ that sometimes appears in the same equation.

- Will attempt to align its magnetic dipole moment with an external magnetic field, $\vec{B}$, because the dipole experiences a torque, $\vec{\tau}$, given by $\vec{\tau} = \vec{\mu} \times \vec{B}$ (Section 30-7).
- Has a potential energy U in an external magnetic field given by $U = -\vec{\mu} \cdot \vec{B}$, so that the dipole's potential energy is a minimum when its dipole moment is aligned with an external magnetic field.

Magnetism in Atoms

We believe that the combined effect of tiny magnetic dipole moments in atoms are responsible for all magnetic interactions in bulk matter. Before we attempt to explain why different materials exhibit certain types of magnetism, we need to discuss what is known about atomic magnetism.

The focus of this book is on classical physics. However, understanding atomic phenomena requires some familiarity with quantum physics, which is in general beyond the scope of this book. So, we will present some basic ideas of quantum physics that apply to atomic magnetism without discussing the existing body of experimental evidence.

So far in our classical treatment of magnetism we have already identified two sources of magnetic dipole fields: (1) electric charges that create a current if they move in a loop and (2) magnetic dipoles consisting of a bar or rod of magnetized iron. Also, some effects of atomic magnetism can be explained using a classical model that identifies two types of atomic magnetic dipoles—orbital and spin. First, if we think of electrons as "orbiting" around a nucleus, then an orbit is a current loop with an orbital magnetic moment. Second, we think of the electron as having an intrinsic magnetic dipole moment that we call spin. This model is quite comfortable because it is rather like the familiar picture of the Earth spinning about its own axis as it orbits the Sun. But when we try to predict the magnetic behavior of various types of materials using this classical model, its usefulness is limited and it is completely wrong in many ways.

The bad classical predictions are not surprising since quantum physics, devised to explain atomic behavior, tells us that: (1) We cannot think of electrons as having distinct orbits. Instead we visualize them as swarming about in the vicinity of a nucleus without having distinct paths. So all we can know is something about the probability of finding the electron at various locations in the vicinity of the nucleus and that these probabilities are different for each type of atom or molecule. (2) The spin magnetic moments are a fundamental property of electrons and should not be thought of as being produced by an electron spinning about an internal axis. (3) The spin and orbital magnetic moments associated with atomic electrons are quantized. This means they can only have certain values.

Next let's examine the characteristics of these two types of atomic magnetic moments in more detail.

Spin Magnetic Dipole Moment

An electron has an intrinsic **spin magnetic dipole moment** $\vec{\mu}^{\,\text{spin}}$. (By *intrinsic,* we mean that $\vec{\mu}^{\,\text{spin}}$ is a basic characteristic of an electron, like its mass and electric charge.) According to quantum theory,

1. $\vec{\mu}^{\,\text{spin}}$ itself cannot be measured directly. Only its component along a single axis can be well-defined (and therefore measured) at any one time.

2. A measured component of $\vec{\mu}^{\,\text{spin}}$ is *quantized*, which is a general term that means it is restricted to certain values.

Let us assume that the component of the spin magnetic moment $\vec{\mu}^{\,\text{spin}}$ is measured along the z axis of a coordinate system you have chosen. Then the measured component μ_z^{spin} can have only the two values given by

$$\mu_z^{\text{spin}} = +\frac{eh}{4\pi m} \quad \text{or} \quad \mu_z^{\text{spin}} = -\frac{eh}{4\pi m}, \tag{32-35}$$

where $h = 6.63 \times 10^{-34}\,\text{J}\cdot\text{s}$ and is the well-known Planck constant used often in quantum physics. The constants e and m represent the charge and mass of the electron, respectively. The plus and minus signs given in Eq. 32-35 describe the direction of μ_z^{spin} along the chosen z axis. The plus sign indicates that μ_z^{spin} is parallel to the z axis, and the electron is said to be "spin up." When μ_z^{spin} is antiparallel to the z axis, the minus sign is used and the electron is said to be "spin down."

The combination of constants in Eq. 32-35 is called the *Bohr magneton* μ_{B}, which can be calculated from the known values of Planck's constant and the electron charge and mass:

$$\mu_{\text{B}} = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \text{J/T} \qquad \text{(Bohr magneton value for an electron).} \tag{32-36}$$

Spin magnetic dipole moments of electrons and other elementary particles can be expressed in terms of μ_{B}. In terms of the Bohr magneton, we can substitute in to Eq. 32-35 to rewrite the expression for the two possible values of μ_z^{spin} as

$$\mu_z^{\text{spin}} = +\mu_{\text{B}} \quad \text{or} \quad \mu_z^{\text{spin}} = -\,\mu_{\text{B}}. \tag{32-37}$$

When an electron is placed in an external magnetic field $\vec{B}^{\,\text{ext}}$, a potential energy U can be associated with the orientation of the electron's spin magnetic dipole moment $\vec{\mu}^{\,\text{spin}}$ just as a potential energy can be associated with the orientation of the magnetic dipole moment $\vec{\mu}$ of a current loop placed in an external magnetic field $\vec{B}^{\text{ext}}$. From Eq. 29-35, the potential energy for the electron due to its spin orientation has only two possible values

$$U^{\,\text{spin}} = -\vec{\mu}^{\,\text{spin}} \cdot \vec{B}^{\,\text{ext}} = -\mu_z^{\text{spin}} B^{\,\text{ext}} = \pm\ \mu_{\text{B}} B^{\text{ext}}, \tag{32-38}$$

where the z axis is taken to be in the direction of $\vec{B}^{\,\text{ext}}$.

Again, although we use the word "spin" here, according to quantum theory the fact that electrons have intrinsic magnetic moments does not mean that they spin like tops.

Protons and neutrons also have intrinsic magnetic dipole moments. In fact, these nuclear magnetic moments are a critical element in the development of magnetic resonance imaging—a valuable diagnostic tool in medicine. The masses of protons and neutrons are almost 2000 times that of the electron, so the magnetic moment for these particles is much smaller than that of the electron. For this reason, the contributions of nuclear dipole moments to the magnetic fields of atoms are negligible.

Orbital Magnetic Dipole Moment

An electron that is part of an atom has an additional dipole magnetic moment. This is called its **orbital magnetic moment** $\vec{\mu}^{\,\text{orb}}$. Again, although we use the "orbital" here, electrons do not orbit the nucleus of an atom like planets orbiting the Sun. According to quantum physics, an "orbit" roughly defines a region in space where the electron is most likely to be found. The orientation of this region specifies the direction of the electron's orbital angular momentum. The so-called "outer electrons" in an atom with many electrons will tend to be found further from its nucleus. An outer electron has a larger orbital angular momentum and, hence, a larger magnetic moment. It turns out that in any given atom there are typically more than two possible quantized values for the z-components of orbital magnetic moments. We can express these possible components along a chosen z axis in terms of the Bohr magneton as

$$\mu_z^{\text{orb}} = -m_l^{\max}\mu_B, \ldots, -3\mu_B, -2\mu_B, -1\mu_B, 0\mu_B, 1\mu_B, 2\mu_B, 3\mu_B, \ldots, m_l^{\max}\mu_B, \quad (32\text{-}39)$$

where $m_l^{\max}$ is an integer that designates the magnitude of the orbital magnetic moment component an electron can have.

When an atom is placed in an external magnetic field $\vec{B}^{\text{ext}}$, an orbital potential energy U^{orb} can be associated with the orientation of the orbital magnetic dipole moment of each electron in the atom. Its value is

$$U^{\text{orb}} = -\vec{\mu}^{\text{orb}} \cdot \vec{B}^{\text{ext}} = -\mu_z^{\text{orb}} B^{\text{ext}}, \quad (32\text{-}40)$$

where the z axis is taken in the direction of $\vec{B}^{\text{ext}}$ so that $\vec{B}^{\text{ext}} = B^{\text{ext}}\,\hat{k}$.

The Magnetic Dipole Moment of an Atom

Each electron in an atom has an orbital magnetic dipole moment and a spin magnetic dipole moment that combine vectorially. The resultant of these two vector quantities combines vectorially with similar resultants for all other electrons in the atom, and the resultant for each atom combines with those for all the other atoms in a bulk sample of matter. If the combination of all these magnetic dipole moments produces a magnetic field, then the material is magnetic.

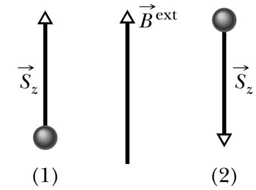
We have one more step in preparing to explain the magnetic behavior of bulk material on the basis of the magnetic behavior of its atoms. We need to consider how the spin and orbital magnetic moments associated with all the electrons in a single atom of a certain element (such as iron, arsenic, and so on) could combine to determine its total magnetic moment.

If the spin and orbital magnetic moments of all the electrons in a given atom lined up with each other and then if all the individually aligned atoms lined up with each other in a solid or liquid, we would have an incredibly strong magnet. Most materials are not strongly magnetic because whenever possible it is natural for the electron magnetic moments in an atom to cancel out. Atomic electrons are located in regions around the nucleus called shells. The number of electrons in a shell is governed by a quantum mechanical rule known as the *Pauli exclusion principle*. This exclusion principle requires that no two electrons in the same shell can have both the same components of orbital magnetic moment and the same components of spin magnetic moment. When all different combinations of orbital and spin states have occurred in a shell it is full. Once a shell is full the pairing of electrons in their locations with respect to the nucleus cancel each other. Each successive shell has more possible orbital magnetic moment components than its electrons can have. In addition, the electrons in a given shell have more energy than the ones in the previous shell. If the atom is in its lowest possible energy state, the shells usually fill in order.

In an atom with many electrons, the number of electrons in a full shell is $4n + 2$, where n is 0, 1, 2, 3, and so on. So the first shell can have 2 electrons, the second 6 electrons, the third 10, the fourth 14, and so on. Typically it is unpaired outermost electrons in an atom that determine the magnetic behavior of bulk matter. Magnetism depends critically on how atoms combine with each other as a result of the sharing and interaction of the outermost electrons.

READING EXERCISE 32-4: In this section, we discuss three types of magnetism in materials. Which one is associated with a refrigerator magnet? A standard paper clip? A piece of silver wire? Explain your reasoning.

READING EXERCISE 32-5: The figure shows the spin orientations of two particles in an external magnetic field $\vec{B}^{\text{ext}}$. (a) If the particles are electrons, which spin orientation is at lower potential energy? (b) If, instead, the particles are protons, which spin orientation is at lower potential energy? Explain your reasoning.



32-7 Ferromagnetism

Iron, cobalt, nickel, gadolinium, dysprosium, and alloys of these become strongly magnetized in the presence of an external magnetic field. Because they retain this magnetism when the external field is removed, we call them ferromagnetic.

Atomic Magnetic Moments in Ferromagnetism

Although most heavy elements are not ferromagnetic, all ferromagnetic materials are relatively heavy elements with complex electronic structures. The lightest of these is iron, which has 26 electrons. The best explanation to date for iron's ferromagnetism involves the complex behavior of its electrons. The 20 innermost electrons are paired in such a way that their spin and orbital magnetic moments cancel each other. The other 6 electrons behave in a manner that is unusual for most materials. Instead of piling into the third shell that has plenty of room for them, 2 of the 6 electrons move out into the fourth shell. These 2 electrons form outer conduction electrons. The key to the ferromagnetism of iron is that the third shell is unfilled, which allows 4 of 14 electrons in that shell to have spin magnetic moments that end up being aligned. However, these aligned electrons do not participate in chemical bonding with other atoms. The detailed quantum mechanical calculations reveal that this unusual arrangement of electrons gives an individual iron *atom* both a net magnetic moment and a lower energy—a situation that is similar for cobalt and nickel.

Even though individual iron atoms have permanent magnetic dipole moments, we might assume that their orientations relative to each other are random, leaving a bulk sample of iron with no net magnetic moment. We know this is not the case. Although various explanations have been put forth to explain why individual atoms line up in ferromagnetic materials, the situation is not well understood. It is currently believed that the spin-aligned electrons in the third shell, which causes the magnetism, influence the outermost conduction electrons, which are wandering through the material. Because of the Pauli exclusion principle, the spin magnetic moment of a conduction electron will have a tendency to be aligned in a direction opposite to that of the third-shell electrons. This anti-aligned conduction electron could, in turn, influence the alignment of the third shell electrons in a neighboring atom. This interaction could align the third-shell spin magnetic moments in the two neighboring atoms, and so on. The jargon for this quantum physical effect, in which spins of the electrons in one atom interact with those of neighboring atoms via conduction electrons, is called **exchange coupling.** The result is an alignment of the magnetic dipole moments of the atoms, in spite of the randomizing effects of thermal energy that causes atomic collisions. We currently believe that this type of coupling is what gives ferromagnetic materials their permanent magnetism.

Magnetic Domains

Exchange coupling in which spins of the electrons in one atom interact with those of neighboring atoms via conduction electrons, produces strong alignment of adjacent atomic dipoles in a ferromagnetic *material.* So we might expect that all the atoms in a sample of iron would align themselves into a permanent magnet even in the absence of an external magnetic field. This doesn't happen. Instead, a piece of iron, nickel, or cobalt is always made up of a number of *magnetic domains*. Each domain is a region in which the alignment of the atomic dipoles is essentially perfect. The domains, however, are not all aligned. For the sample as a whole, the domains are so oriented that they largely cancel each other as far as their external magnetic effects are concerned.

Two reasons are often given for the existence of domains in ferromagnetic materials. First, calculations reveal that a pure sample with perfectly aligned atoms (known as a *single crystal*) has a lower energy state when there are distinct domains with boundaries between them. Second, most real samples have impurities that can cause even more boundaries between domains to form.

A photograph of a single crystal of nickel is shown in Fig. 32-15. A suspension of powdered iron oxide was sprinkled on the crystal surface. The domain boundaries, which are thin regions in which the alignment of the elementary dipoles changes from a certain orientation in one domain to a different orientation in the other, are the sites of intense, but highly localized and nonuniform, magnetic fields. The suspended iron oxide particles are attracted to some of the more prominent boundaries and show up as the white lines. Although the atomic dipoles in each domain are completely aligned as shown by the arrows, the crystal as a whole has a very small resultant magnetic moment.

Actually, a piece of iron as we ordinarily find it is not a single crystal but an assembly of many tiny crystals, randomly arranged; we call it a polycrystalline solid. Each tiny crystal, however, has its array of variously oriented domains, just as in Fig. 32-15. We can magnetize such a specimen by placing it in an external magnetic field B_z^{ext} of gradually increasing strength, and measuring the magnetization B_z^{M} of the iron. (The measurement process is explained in the next subsection on Bulk Properties.) A common way to display the results is to plot a magnetization curve. If the piece of iron had all of its magnetic dipoles aligned perfectly with the external field, its magnetization would be a maximum represented by $B_{\text{max}}^{\text{M}}$. The magnetization curve consists of a plot of the ratio $B_z^{\text{M}}/B_{\text{max}}^{\text{M}}$ as a function of the external field (shown in Fig. 32-16). Note that $B^{\text{M}}/B_{\text{max}}^{\text{M}}$ is always less than one, so the iron does not become perfectly magnetized.

By photographing domain patterns as in Fig. 32-15, we see two microscopic effects that serve to explain the shape of the magnetization curve: One effect is a growth in size of the domains that are oriented along the external field at the expense of those that are not. The second effect is a shift of the orientation of the dipoles within a domain, as a unit, to become closer to the field direction.

Exchange coupling and domain shifting give us the following result:

> A ferromagnetic material placed in an external magnetic field $\vec{B}^{\text{ext}}$ develops a strong magnetic dipole moment in the direction of $\vec{B}^{\text{ext}}$. If the field is nonuniform, the ferromagnetic material is attracted toward a region of greater magnetic field from a region of lesser field.

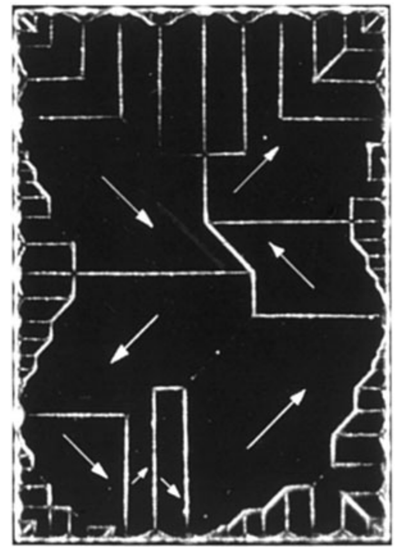


FIGURE 32-15 ■ A photograph of domain patterns within a single crystal of nickel; white lines reveal the boundaries of the domains. The white arrows superimposed on the photograph show the orientations of the magnetic dipoles within the domains and thus the orientations of the net magnetic dipoles of the domains. The crystal as a whole is unmagnetized if the net magnetic field (the vector sum over all the domains) is zero.

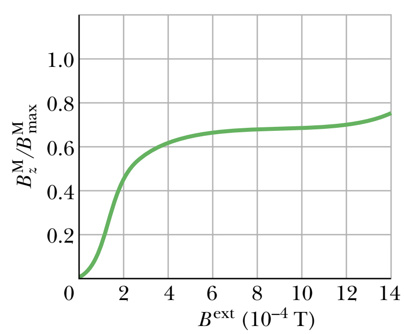


FIGURE 32-16 ■ A magnetization curve for a ferromagnetic core material in the Rowland ring of Fig. 32-17. On the vertical axis, 1.0 corresponds to complete alignment (saturation) of the atomic dipoles within the material.

Bulk Properties of Ferromagnetic Materials

If the temperature of a ferromagnetic material is raised above a certain critical value, called the **Curie temperature,** the exchange coupling ceases to be effective. Most such materials then become simply paramagnetic. That is, the dipoles still tend to align with an external field but much more weakly, and thermal agitation can now more easily disrupt the alignment. The Curie temperature for iron is 1043 K (= 770°C).

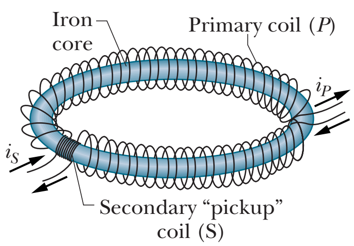


FIGURE 32-17 ■ A toroidal Rowland ring coil in which a current i_P is sent through a primary coil P. This current is used to study the behavior of the ferromagnetic material of the iron core inside the windings. The extent of magnetization of the core determines the total magnetic field $\vec{B}$ within coil P. Field $\vec{B}$ can be measured by means of a secondary or "pickup" coil.

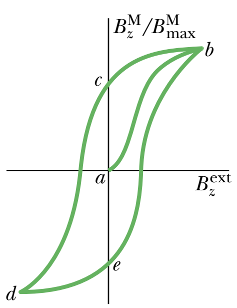


FIGURE 32-18 ■ A magnetization curve (ab) for a ferromagnetic specimen and an associated hysteresis loop ($bcdeb$).

We can express the extent to which a given paramagnetic sample is magnetized by finding the ratio of its magnetic dipole moment to its volume V. This vector quantity, the magnetic dipole moment per unit volume, is called the **magnetization** $\vec{M}$ of the sample, and its magnitude is

$$\vec{M} = \frac{\text{measured magnetic moment}}{V}. \quad (32\text{-}41)$$

The unit of $\vec{M}$ is the ampere-square meter per cubic meter, or ampere per meter (A/m). Complete alignment of the atomic dipole moments, called **saturation** of the sample, corresponds to the maximum magnetization of magnitude $M^{\text{max}} = N\mu/V$ where N is the number of atoms in the volume V.

The magnetization of a ferromagnetic material such as iron can be studied using a toroidal coil called a *Rowland ring* (Fig. 32-17). A Rowland ring is basically a long solenoid with an iron cylinder at its core, except the whole thing is bent into the shape of a donut. Assume that the ring's primary coil P has n turns per unit length and carries current i_P. If the iron core were not present, the magnitude of the magnetic field inside the coil caused by the "external" solenoid windings (as distinct from the magnetization of a core material inside the windings) would be given by Eq. 30-25,

$$B^{\text{ext}} = n\mu_0|i_P| \quad \text{(no iron core).} \quad (32\text{-}42)$$

Here μ_0 represents the magnetic constant (or permeability) of air (and is not a magnetic moment).

If an iron core is present, the magnitude of the magnetic field B inside the coil is proportional to B^{ext} but is on the order of 1000 to 10 000 times greater due to the magnetization of the iron core. This magnetization results from the alignment of the atomic dipole moments within the iron. The field $\vec{B}$ inside the coil should be the vector sum of the field $\vec{B}^{\text{ext}}$ contributed by the coil without the core and the field $\vec{B}^{\text{M}}$ contributed by the magnetization of the core. Since the magnitude of $\vec{B}^{\text{ext}}$ field is much smaller than that produced by the core magnetization

$$\vec{B} = \vec{B}^{ext} + \vec{B}^{\text{M}} \approx \vec{B}^{\text{M}}. \quad (32\text{-}43)$$

To determine $\vec{B}^{\text{M}}$ we use a secondary coil S to measure $\vec{B}$ and hence $\vec{B}^{\text{M}}$. If needed, we compute $\vec{B}^{\text{ext}}$ using Eq. 32-42.

Figure 32-18 shows a magnetization curve for a ferromagnetic material in a Rowland ring: the ratio of magnitudes $B^{\text{M}}/B^{\text{M}}_{\text{max}}$ is plotted as a function of B^{ext} (where $B^{\text{M}}_{\text{max}}$ is the maximum possible value of B^{M}, corresponding to saturation). The curve is similar to that for the magnetization curve for a paramagnetic substance shown in Fig. 32-19. Both curves are measures of the extent to which an applied magnetic field can align the atomic dipole moments of a material.

For the ferromagnetic core described by the graph in Fig. 32-16, the alignment of the dipole moments is about 70% complete for $B^{\text{ext}} \approx 1 \times 10^{-3}$ T. If B^{ext} were increased to 1 T, the alignment would be almost complete (but $B^{\text{ext}} = 1$ T, and thus almost complete saturation, is quite difficult to achieve).

Hysteresis

Magnetization curves for ferromagnetic materials are not retraced as we increase and then decrease and then reverse the external magnetic field $\vec{B}^{\text{ext}}$. Let's assume that we choose the z axis to be along the direction of the external magnetic field. Figure 32-18 is a plot of the z-component of the magnetization field B_z^{M} versus the z-component of the external field B_z^{ext} during the following operations with a Rowland ring: (1) Starting with the iron unmagnetized (point a), increase the current in the toroid until $B_z^{\text{ext}} = n\mu_0|i|$ has the value corresponding to point b; (2) reduce the current in the toroid winding (and thus B^{ext}) back to zero (point c); (3) reverse the toroid current and increase it in amount until B^{ext} has the value corresponding to point d; (4) reduce the current to zero again (point e); (5) reverse the current once more until point b is reached again.

The lack of retraceability shown in Fig. 32-18 is called **hysteresis,** and the curve *bcdeb* is called a *hysteresis loop*. Note that at points c and e the iron core is magnetized, even though there is no current in the toroid windings; this is the familiar phenomenon of permanent magnetism. In fact when engineers are designing permanent magnets, they look for materials that have a high degree of hysteresis.

Hysteresis can be understood through the concept of magnetic domains. When the magnetic field in the coil due to the current in the solenoid windings, $\vec{B}^{\text{ext}}$, is increased and then decreased back to its initial value, the domains do not return completely to their original configuration but retain some "memory" of their alignment after the initial increase. This memory of magnetic materials is essential for the magnetic storage of information, as on cassette tapes and computer disks.

This memory of the alignment of domains can also occur naturally. When lightning sends currents along multiple tortuous paths through the ground, the currents produce intense magnetic fields that can suddenly magnetize any ferromagnetic material in nearby rock. Because of hysteresis, such rock material retains some of that magnetization after the lightning strike (after the currents disappear) then becomes lodestones.

Inductors and Transformers with Iron Cores

Based on our discussion above of the Rowland ring, it is clear that the use of iron and iron alloys in inductors and transformers can literally increase the performance of these devices by a thousandfold or more.

A great deal of engineering has gone into the design of cores for large inductors and high-performance transformers. For example, these cores should not behave like permanent magnets with large hysteresis. Instead, they should have small hysteresis so that the magnetization of the core can change rapidly in the presence of alternating currents. In addition, transformer cores are not single hunks of iron. Rather, they are built up in layers to prevent eddy currents from being induced in the cores that could reduce the efficiency of the power transfer from the primary to secondary coils in a transformer.

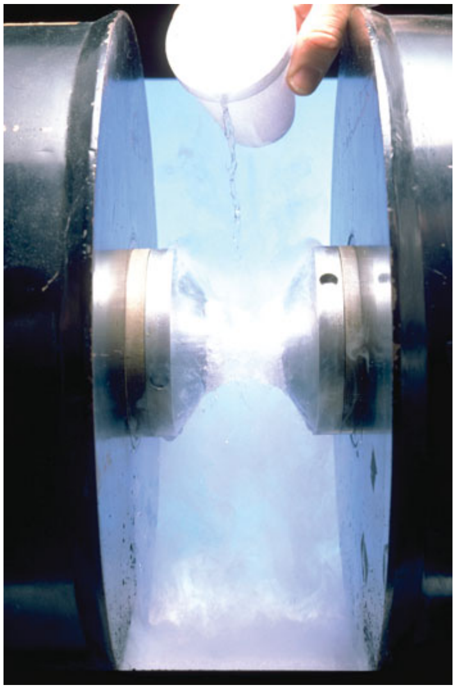
READING EXERCISE 32-6: Iron is a ferromagnetic material. Why then isn't every piece of iron—for example, an iron nail—a naturally strong magnet? ■

READING EXERCISE 32-7: What is hysteresis and why does it occur? ■

32-8 Other Magnetic Materials

Paramagnetism

In paramagnetic materials, the spin and orbital magnetic dipole moments of the electrons in individual atoms do not cancel but add vectorially to give each *atom* a net (and permanent) magnetic dipole moment $\vec{\mu}$. In the absence of an external magnetic field, these atomic dipole moments are randomly oriented, and the net magnetic dipole moment of the *material* is zero. However, if a sample of the material is placed in an external magnetic field $\vec{B}^{\text{ext}}$, the magnetic dipole moments tend to line up with the field, which gives the sample a net magnetic dipole moment not unlike that found in a ferromagnetic sample. However, paramagnetic materials lack the exchange coupling needed to set up permanent magnetic domains. Paramagnetism is fairly weak compared to ferromagnetism because the forces of alignment from external magnetic


Liquid oxygen is suspended between the two pole faces of a magnet because the liquid is paramagnetic and is magnetically attracted to the magnet.

fields are smaller than the randomizing forces due to thermal motions. Also, paramagnetic materials do not retain their magnetism once an external magnetic field is turned off.

A paramagnetic material placed in an external magnetic field $\vec{B}^{\text{ext}}$ develops a magnetic dipole moment in the direction of $\vec{B}^{\text{ext}}$. If the field is not uniform, the paramagnetic material is attracted toward a region of greater magnetic field from a region of lesser field.

As is the case for ferromagnetism, we can express the extent to which a given paramagnetic sample is magnetized by measuring the magnetization $\vec{M}$ (defined in Eq. 32-41). In 1895, Pierre Curie discovered that the magnitude of the magnetization of a paramagnetic sample is directly proportional to the external magnetic field magnitude B^{ext} and inversely proportional to the temperature T in kelvins; that is,

$$M = C\frac{B^{\text{ext}}}{T}. \tag{32-44}$$

Equation 32-44 is known as **Curie's law,** and C is called the **Curie constant.** Curie's law is reasonable in that increasing $\vec{B}^{\text{ext}}$ tends to align the atomic dipole moments in a sample and thus to increase $\vec{M}$, whereas increasing T tends to disrupt the alignment via thermal agitation and thus to decrease $\vec{M}$. However, the law is actually an approximation that is valid only when the ratio B^{ext}/T is not too large.

Figure 32-19 shows the ratio M/M^{max} as a function of B^{ext}/T for a sample of the salt potassium chromium sulfate, in which chromium ions are the paramagnetic substance. The plot is called a **magnetization curve.** The straight line for Curie's law fits the experimental data at the left, for B^{ext}/T below about 0.5 T/K. The curve that fits all the data points is based on quantum physics. The data on the right side, near saturation, are very difficult to obtain because they require very strong magnetic fields (about 100 000 times Earth's field), even at the very low temperatures noted in Fig. 32-19.

Diamagnetism

The *atoms* in diamagnetic materials have no net magnetic dipole moments. However, diamagnetic *materials* do undergo a very weak nonpermanent alignment in the presence of an external magnetic field. The strength of the alignments is still proportional to the strength of the external magnetic field (as is the case for both ferro- and paramagnetism). However, the behavior of diamagnetic materials is not very temperature dependent.

The most interesting characteristic of diamagnetism is that in the presence of an external magnetic field that is nonuniform, each atom experiences a net force that is directed *away* from the region of greater magnetic field. Thus, in diamagnetism the

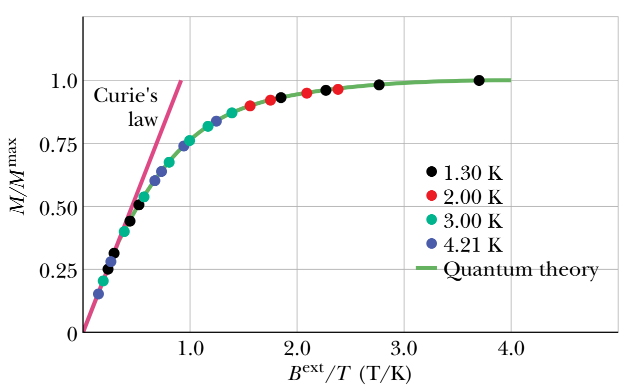


FIGURE 32-19 ■ A *magnetization curve* for potassium chromium sulfate, a paramagnetic salt. The ratio of the magnitudes of the salt magnetization $\vec{M}$ to the maximum possible magnetization $\vec{M}^{\text{max}}$ is plotted versus the ratio of the magnitude of the applied magnetic field B^{ext} to the temperature T. Curie's law fits the data at the left; quantum theory fits all the data. (Based on research by Warren E. Henry, 1909–2001.)

alignment of atomic magnetic moments with an external magnetic field is opposite to that associated with ferromagnetic and paramagnetic materials. In general,

> A diamagnetic material placed in an external magnetic field $\vec{B}^{\text{ext}}$ develops a magnetic dipole moment directed opposite $\vec{B}^{\text{ext}}$. If the field is nonuniform, the diamagnetic material is repelled from a region of greater magnetic field toward a region of lesser field.

Animals like the frog shown in Fig. 32-20 are diamagnetic. This frog has been placed in the diverging magnetic field near the top end of a vertical current-carrying solenoid; every atom in the frog was repelled upward, away from the region of stronger magnetic field at that end of the solenoid. The frog moved upward into weaker and weaker magnetic field until the upward magnetic force balanced the gravitational force on it, and there it hung in midair. People are also diamagnetic, so if we built a large enough solenoid, we could also suspend a person in midair.

FIGURE 32-20 ■ An overhead view of a diamagnetic frog that is being levitated in a magnetic field. The $\vec{B}$-field is produced by current in a vertical solenoid below the frog. The solenoid's upward magnetic force on the frog balances the downward gravitational force on the frog. (The frog is not in discomfort; the sensation is like floating in water, which frogs don't seem to mind.)

READING EXERCISE 32-8: The figure here shows two paramagnetic spheres located near the south pole of a bar magnet. Are (a) the magnetic forces on the spheres and (b) the magnetic dipole moments of the spheres directed toward or away from the bar magnet? (c) Is the magnetic force on sphere 1 greater than, less than, or equal to that on sphere 2?

READING EXERCISE 32-9: The figure shows two diamagnetic spheres located near the south pole of a bar magnet. Are (a) the magnetic forces on the spheres and (b) the magnetic dipole moments of the spheres directed toward or away from the bar magnet? (c) Is the magnetic force on sphere 1 greater than, less than, or equal to that on sphere 2?

32-9 The Earth's Magnetism

The Earth has a magnetic field associated with it that behaves approximately like that of a magnetic dipole. In other words, the Earth's magnetic field can be thought of as being produced by a bar magnet that straddles the center of the planet with its axis more or less aligned with the Earth's rotation axis. Figure 32-21 is an idealized depiction of the Earth's dipole field that ignores the distortion of field lines caused by charged particles streaming out of the Sun and other factors.

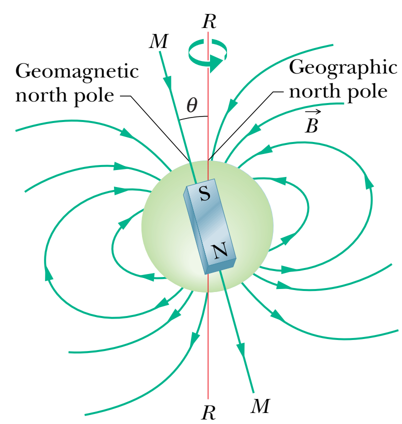


FIGURE 32-21 ■ An idealized view of the Earth's magnetic field as a dipole field. At present, dipole axis *MM* makes an angle of 8.7° with Earth's rotational axis *RR*. The "south pole" of the dipole is in Earth's northern hemisphere.

Characteristics of the Earth's Magnetic Field

For the idealized magnetic field shown in Fig. 32-21, the Earth's magnetic dipole moment $\vec{\mu}$ has a magnitude of 8.0×10^{22} J/T. The point where the Earth's rotation axis intersects the surface is known at the *geographic north pole*. In 2001, the geological survey of Canada placed the direction of the Earth's dipole moment at an angle of $\theta = 8.7°$ from the rotation axis (*RR*) of the Earth.* The *dipole axis* (*MM* in Fig. 32-21) lies along $\vec{\mu}$ and intersects the Earth's surface at the *geomagnetic north and south poles*, These days the magnetic north pole is estimated to be somewhere in the Arctic Ocean north of Canada and the south pole is in the Antarctic Ocean. Since the poles are currently moving at about 40 km/yr, the possibility exists that the magnetic north pole could pass north of Alaska and in about fifty years end up in Siberia, although this outcome is not certain.

*http://www.geolab.nrcan.gc.ca/geomag/northpole_e.shtml

The lines of the magnetic field $\vec{B}$ generally emerge in the southern hemisphere and reenter Earth in the northern hemisphere. Thus, the magnetic pole that is in the Earth's northern hemisphere and known as a "north magnetic pole" *is really the south pole of the Earth's magnetic dipole*. This means that the north pole of a compass is attracted to the Earth's geographic north pole.

The direction of the magnetic field varies from location to location on the Earth. The field direction at any location on the Earth's surface is commonly specified in terms of two angles. The **field declination** is the angle (left or right) between geographic north (which is toward 90° latitude) and the horizontal component of the field. The **field inclination** is the angle (up or down) between a horizontal plane and the field's direction.

The field's inclination and declination at a given location can be measured with a *compass* and a *dip meter*. A **compass** is simply a needle-shaped magnet that is mounted so it can rotate freely about a vertical axis. When it is held in a horizontal plane, the north-pole end of the needle points, generally, toward the geomagnetic north pole (really a south magnetic pole). The angle between the compass needle and geographic north is the field declination.* A dip meter, used to measure inclination, is simply another needle-shaped magnet mounted so it can rotate freely about a *horizontal* axis. If the plane of the dip meter is aligned with the direction of the compass needle used to measure the declination, then the angle the dip meter needle makes with the horizontal is defined as the inclination angle. The magnetic north pole is defined as the location in the northern hemisphere for which the dip angle is 90°.

Causes of the Earth's Magnetism

The mechanisms that produce the Earth's magnetic field are not completely understood. However, it is helpful to begin our discussion of the latest models with a consideration of what is known about the Earth's formation and structure.

The Earth's Structure: Measurements of the spread of seismic waves tell us that the structure of the Earth is rather like that of a chocolate-covered cherry with gooey liquid between the cherry and the chocolate. This structure makes sense when we consider the currently accepted theory that the Earth was formed five billion years ago as a conglomeration of colliding meteorites and comets. Iron and other dense elements from meteorites were pulled by gravitational forces toward the center of the Earth. Compounds made of lighter elements, as well as the water contained in comets, migrated toward the surface. In between the solid core at the center of the Earth and the solid crust at the Earth's surface there is the gooey liquid consisting of molten lava (Fig. 32-22).

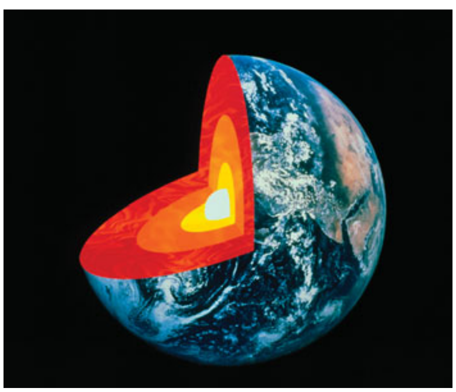


FIGURE 32-22 ■ Seismic data reveal that the Earth has an **inner core** (white) of solid iron with a radius of about 1200 km, an **outer core** (yellow) of iron rich molten lava about 2200 km thick, a more or less solid **mantle** (orange and red—not to scale) of less dense matter about 2600 km thick, and a very thin **crust** of rocks and soils at the surface with an average thickness of 20 km.

Continuous Molten Lava Currents: Many scientists believe that most of the Earth's magnetic field is produced by electromagnetic interactions that depend on the molten lava acting like a moving electrical conductor. We know from our study of Faraday's law that if even a small magnetic field is present in a region of the core, the electrical currents can be induced in the conducting fluid that travels through it. These induced currents can, in turn, produce magnetic fields that can act on other parts of the liquid core that are also moving. Thus a continuous cycle of induction and magnetic field production can take place as long as the material in the liquid core keeps flowing. In principle, this process is rather like that described for the generator shown in Fig. 32-10.

Two mechanisms have been proposed that explain the flow of molten lava in the liquid core. One possible mechanism is thermal convection produced by the temperature

*Inclination is the angle that a magnetic needle makes with the plane of the horizon. It is also called the angle of dip. Declination is the angle between magnetic north and geographic north.

difference between the hot solid inner core and the much cooler mantle. A second proposed mechanism for the flow of lava involves condensation of the heavier elements onto a growing inner core. This causes lighter, less dense, elements to flow toward the Earth's surface. In either case liquid convection currents are produced that are not unlike those in a pot of boiling water.

The Earth's magnetic field depends critically on the existence of *continuous* convection currents that requires the solid core to remain very hot for billions of years. Some scientists believe that nuclear energy in the core is being transformed to thermal energy through the decay of heavy radioactive elements. Other scientists have suggested that thermal energy can be released if the inner core expands by condensing material from the liquid core.

Changes in the Earth's Magnetic Field Over Time: Some mysterious characteristics of the Earth's magnetic field have been gleaned from fossil records and other geomagnetic measurements. The strength of the field and the location of the magnetic poles are constantly changing. For example, in recent years the geographic location of the magnetic poles has changed by an average of about 100 meters a day. These relatively small day-to-day changes are not obvious to someone a long distance from a magnetic pole who uses a compass and dip meter to measure a local field direction. It's another story when longer time scales are involved. We can use simple instruments to detect changes over a time period of a year or more. When even longer time periods are considered, the changes have been dramatic. In fact, the orientations of magnetized minerals imbedded in ancient rocks indicate that the Earth's magnetic field has completely reversed itself many times in the Earth's five billion year history, though reversals seem to take 1000 years or more.

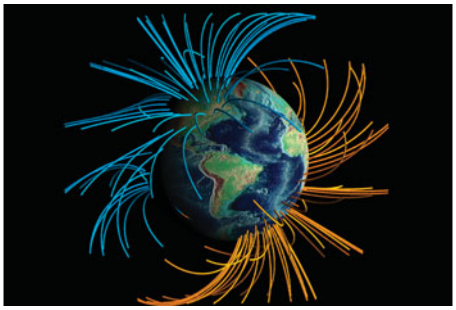


Figure 32-23 ■ This image shows one of many configurations of the Earth's magnetic field lines created by the model developed by Glatzmaier and Roberts.

The Glatzmaier/Roberts Model: A few years ago, two scientists, Gary Glatzmaier and Paul Roberts, developed a comprehensive numerical model of the electromagnetic and fluid dynamic processes in the Earth's interior. When this model was run on a CRAY supercomputer for thousands of hours these investigators were able to simulate over 300,000 years of magnetic field conditions. Their results showed many of the key features revealed by geological data, including the existence of a dipole field outside the Earth, a preference for approximate alignment between the Earth's dipole moment and its rotation axis, field strength variations, migration of the magnetic poles over the Earth's surface, and several field reversals. One such configuration of magnetic field lines is seen in Fig. 32-23.

There is still a great deal to be learned about the actual mechanisms responsible for the continual changes in the Earth's magnetic field, but scientists expect to resolve many of their uncertainties within the next few decades.

Magnetic Bacteria

The survival of many organisms depends on their ability to sense the Earth's magnetic field. For example, it is believed that the Earth's dipole field is critical to the navigation of migrating birds and fish as well as certain types of bacteria.

Magnetotactic bacteria are one-celled organisms that can be found almost anywhere in the world where there are ponds, marshes, or muddy lake bottoms. Many species of these bacteria are anaerobic or microanaerobic and must burrow in mud both to get away from oxygen and to feed on nutrients. Notice that on the lower left side of the bacterium shown in the photo at the beginning of this chapter there is a string of tiny 100-nanometer-long particles. These particles, known as magnetosomes, are oriented along the bacterium's long axis. An enlarged view of a set of magnetosomes is shown in Fig. 32-24.

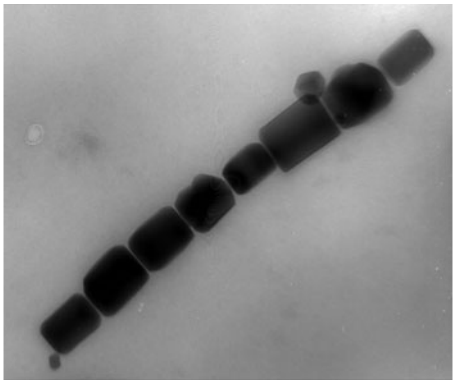


Figure 32-24 ■ The type of bacterium shown in the puzzler at the beginning of the chapter is magnetotactic because it contains a chain of dense iron-rich magnetosomes each having a length of about 100 nm. The chain shown in this transmission electron micrograph has a net magnetic moment and tends to align itself with the Earth's magnetic field.

Magnetotactic bacteria synthesize these magnetic particles out of iron-oxygen or iron sulfur compounds. Each magnetosome is just big enough to have a permanent magnetic dipole moment and just small enough to be a single ferromagnetic domain. When strung together like a set of microscopic refrigerator magnets, the array has a net dipole moment. So instead of bumbling around randomly, these bacteria align with the Earth's magnetic field. This allows them to swim naturally along field lines.

Through natural selection, the bacteria that have their magnetosome strings oriented so they swim down along magnetic field lines to the mud at the bottom of a pond or lake will survive and multiply. Those that don't will swim up and die. An examination of the pattern of the Earth's magnetic field lines shown in Fig. 32-21 reveals that "down" is opposite to the direction of the field lines in the southern hemisphere and in the same direction as the field lines in the northern hemisphere. Thus, an Australian bacterium evolved to swim down in its normal habitat would swim up if transported to the United States. Alternatively, a healthy bacterium that evolves in the United States would be preset by evolutionary processes to have its magnetosomes oriented in the opposite direction, so it will swim down.

It is interesting to note that the orientations of bacterial magnetosome strings in fossils have helped scientists piece together evidence for past changes in the Earth's magnetic field.

READING EXERCISE 32-10: Describe the ways in which the Earth's magnetic field varies over its surface. Does the Earth's magnetic field vary in time as well?

Problems

SEC. 32-2 ■ SELF-INDUCTANCE

1. Close-Packed Coil The inductance of a close-packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5.0 mA.

2. Circular Coils and Flux A circular coil has a 10.0 cm radius and consists of 30.0 closely wound turns of wire. An externally produced magnetic field of magnitude 2.60 mT is perpendicular to the coil. (a) If no current is in the coil, what is the magnitude of the magnetic flux that links its turns? (b) When the current in the coil is 3.80 A in a certain direction, the net flux through the coil is found to vanish. What is the inductance of the coil?

3. Equal Currents, Opposite Directions Two long parallel wires, both of radius a and whose centers are a distance d apart, carry equal currents in opposite directions. Show that, neglecting the flux within the wires, the inductance of a length l of such a pair of wires is given by

$$L = \frac{\mu_0 l}{\pi} \ln \frac{d - a}{a}$$

(*Hint:* Calculate the flux through a rectangle of which the wires form two opposite sides.)

4. Wide Copper Strip A wide copper strip of width W is bent to form a tube of radius R with two parallel planar extensions, as shown in Fig. 32-25. There is a current i through the strip, distributed uniformly over its width. In this way a "one-turn solenoid" is formed. (a) Derive an expression for the magnitude of the magnetic field $\vec{B}$ in the tubular part (far away from the edges). (*Hint:* Assume that the magnetic field outside this one-turn solenoid is negligibly small.) (b) Find the inductance of this one-turn solenoid, neglecting the two planar extensions.

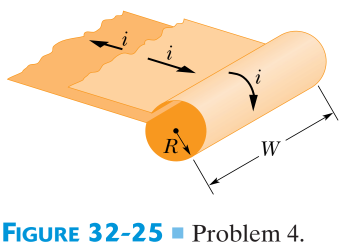


FIGURE 32-25 ■ Problem 4.

5. Inductor Carries Steady Current A 12 H inductor carries a steady current of 2.0 A. How can a 60 V self-induced emf be made to appear in the inductor?

6. At a Given Instant At a given instant the current and self-induced emf in an inductor are directed as indicated in Fig. 32-26. (a) Is the current increasing or decreasing? (b) The induced emf is 17 V and the rate of change of the current is 25 kA/s; find the inductance.

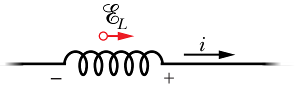


FIGURE 32-26 ■ Problem 6.

7. Inductors in Series Two inductors L_1 and L_2 are connected in series and are separated by a large distance. (a) Show that the equivalent inductance is given by

$$L_{eq} = L_1 + L_2.$$

(*Hint:* Review the derivations for resistors in series and capacitors in series. Which is similar here?) (b) Why must their separation be large for this relationship to hold? (c) What is the generalization of (a) for N inductors in series?

8. Current Varies with Time The current i through a 4.6 H inductor varies with time t as shown by the graph of Fig. 32-27. The inductor has a resistance of 12 Ω. Find the magnitude of the induced emf $\mathscr{E}$ during the time intervals (a) $t_1 = 0$ to $t_2 = 2$ ms, (b) $t_2 = 2$ ms to $t_3 = 5$ ms, (c) $t_3 = 5$ ms to $t_4 = 6$ ms. (Ignore the behavior at the ends of the intervals.)

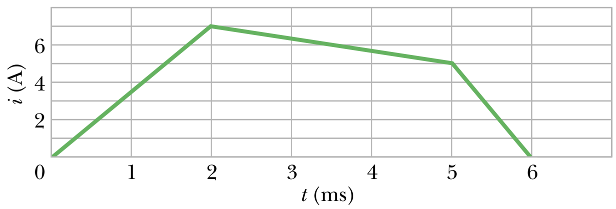


FIGURE 32-27 ■ Problem 8.

9. At What Rate At time $t = 0$ ms, a 45 V potential difference is suddenly applied to the leads of a coil with inductance $L = 50$ mH and resistance $R = 180\ \Omega$. At what rate is the current through the coil increasing at $t = 1.2$ ms?

10. Inductors in Parallel Two inductors L_1 and L_2 are connected in parallel and separated by a large distance. (a) Show that the equivalent inductance is given by

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

(*Hint:* Review the derivations for resistors in parallel and capacitors in parallel. Which is similar here?) (b) Why must their separation be large for this relationship to hold? (c) What is the generalization of (a) for N inductors in parallel?

11. What Is L The inductance of a closely wound coil is such that an emf of 3.0 mV is induced when the current changes at the rate of 5.0 A/s. A steady current of 8.0 A produces a magnetic flux of 40 μ Wb through each turn. (a) Calculate the inductance of the coil. (b) How many turns does the coil have?

SEC. 32-3 ■ MUTUAL INDUCTION

12. Coil 1, Coil 2 Coil 1 in Fig. 32-4 has $L_1 = 25$ mH and $N_1 = 100$ turns. Coil 2 has $L_2 = 40$ mH and $N_2 = 200$ turns. The coils are rigidly positioned with respect to each other; their mutual inductance M is 3.0 mH. A 6.0 mA current in coil 1 is changing at the rate of 4.0 A/s. (a) What magnetic flux $\Phi_{1\to 2}$ links coil 2, and what self-induced emf appears there? (b) What magnetic flux $\Phi_{2\to 1}$ links coil 1, and what mutually induced emf appears there?

13. Two Coils at Fixed Locations Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate 15.0 A/s, the emf in coil 1 is 25.0 mV. (a) What is their mutual inductance? (b) When coil 2 has no current and coil 1 has a current of 3.60 A, what is the flux linkage in coil 2?

14. Two Solenoids Two solenoids are part of the spark coil of an automobile. When the current in one solenoid falls from 6.0 A to zero in 2.5 ms, an emf of 30 kV is induced in the other solenoid. What is the mutual inductance M of the solenoids?

15. Two Connected Coils Two coils, connected as shown in Fig. 32-28, separately have inductances L_1 and L_2. Their mutual inductance is M. (a) Show that this combination can be replaced by a single coil of equivalent inductance given by

$$L_{\text{eq}} = L_1 + L_2 + 2M.$$

(b) How could the coils in Fig. 32-28 be reconnected to yield an equivalent inductance of

$$L_{\text{eq}} = L_1 + L_2 - 2M?$$

(This problem is an extension of Problem 7, but the requirement that the coils be far apart has been removed.)

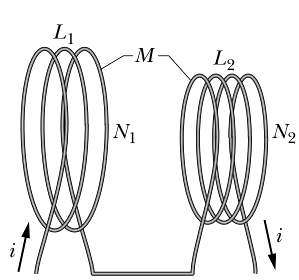


FIGURE 32-28 ■ Problem 15.

16. Coil Around Solenoid A coil C of N turns is placed around a long solenoid S of radius R and n turns per unit length as in Fig. 32-29. Show that the mutual inductance for the coil–solenoid combination is given by $M = \mu_0 \pi R^2 n N$. Explain why M does not depend on the shape, size, or possible lack of close-packing of the coil.

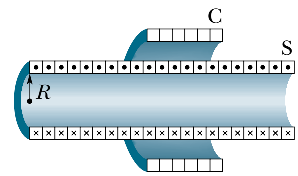


FIGURE 32-29 ■ Problem 16.

17. Coaxial Solenoid Figure 32-30 shows, in cross section, two coaxial solenoids. Show that the mutual inductance M for a length l of this solenoid–solenoid combination is given by $M = \pi R_1^2 l \mu_0 n_1 n_2$, in which n_1 and n_2 are the respective numbers of turns per unit length and R_1 is the radius of the inner solenoid. Why does M depend on R_1 and not on R_2?

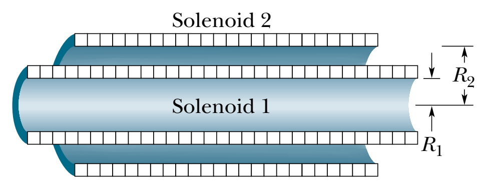


FIGURE 32-30 ■ Problem 17.

18. Coils Over a Toroid Figure 32-31 shows a coil of N_2 turns wound as shown around part of a toroid of N_1 turns. The toroid's inner radius is a, its outer radius is b, and its height is h. Show that the mutual inductance M for the toroid–coil combination is

$$M = \frac{\mu_0 N_1 N_2 h}{2\pi} \ln \frac{b}{a}.$$

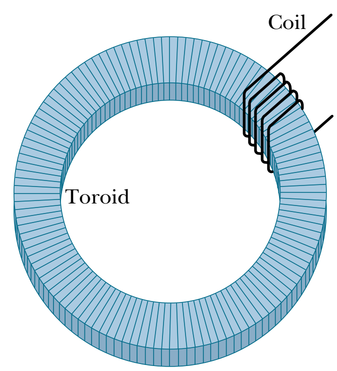


FIGURE 32-31 ■ Problem 18.

19. Rectangular Loop A rectangular loop of N close-packed turns is positioned near a long straight wire as shown in Fig. 32-32. (a) What is the mutual inductance M for the loop–wire combination? (b) Evaluate M for $N = 100$, $a = 1.0$ cm, $b = 8.0$ cm, and $l = 30$ cm.

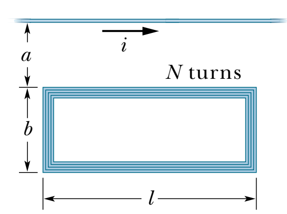


FIGURE 32-32 ■ Problem 19.

SEC. 32-4 ■ *RL* CIRCUITS (WITH IDEAL INDUCTORS)

20. Inductive Time Constant The current in an RL circuit builds up to one third of its steady-state value in 5.00 s. Find the inductive time constant.

21. How Long Must We Wait In terms of τ_L, how long must we wait for the current in an RL circuit to build up to within 0.100% of its equilibrium value?

22. In Terms of the emf Consider the RL circuit of Fig. 32-6. In terms of the battery emf $\mathscr{E}$, (a) what is the self-induced emf ΔV_2 when the switch has just been closed on a, and (b) what is ΔV_2 when $t = 2.0\tau_L$? (c) In terms of τ_L, when will ΔV_2 be just one-half the battery emf $\mathscr{E}$?

23. First Second The current in an RL circuit drops from 1.0 A to 10 mA in the first second following removal of the battery from the circuit. If L is 10 H, find the resistance R in the circuit.

24. emf Varies with Time Suppose the emf of the battery in the circuit of Fig. 32-7 varies with time t so that the current is given by $i(t) = 3.0\text{ A} + (5.0\text{ A/s})t$, where i is in amperes and t is in seconds. Take $R = 4.0\ \Omega$ and $L = 6.0$ H, and find an expression for the battery emf as function of time. (*Hint:* Apply the loop rule.)

25. Solenoid A solenoid having an inductance of 6.30 μH is connected in series with a 1.20 kΩ resistor. (a) If a 14.0 V battery is inserted into the circuit, how long will it take for the current through the resistor to reach 80.0% of its final value? (b) What is the current through the resistor at time $t = 1.0\tau_L$?

26. Wooden Toroidal Core A wooden torodial core with a square cross section has an inner radius of 10 cm and an outer radius of 12 cm. It is wound with one layer of wire (of diameter 1.0 mm and resistance per meter 0.020 Ω/m). What are (a) the inductance and (b) the inductive time constant of the resulting toroid? Ignore the thickness of the insulation on the wire.

27. Suddenly Applied At time $t = 0$ ms, a 45.0 V potential difference is suddenly applied to a coil with $L = 50.0$ mH and $R = 180\ \Omega$. At what rate is the current increasing at $t = 1.20$ ms?

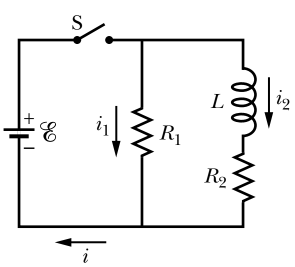


FIGURE 32-33 ■ Problem 28.

28. In the Circuit In the circuit of Fig. 32-33, $\mathscr{E} = 10$ V, $R_1 = 5.0\ \Omega$, $R_2 = 10\ \Omega$, and $L = 5.0$ H. For the two separate conditions (I) switch S just closed and (II) switch S closed for a long time, calculate (a) the current i_1 through R_1, (b) the current i_2 through R_2, (c) the current i through the switch, (d) the potential difference across R_2, (e) the potential difference across L, and (f) the rate of change di_2/dt.

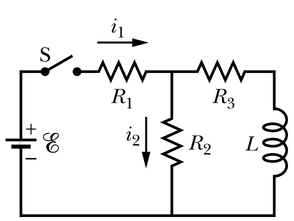


FIGURE 32-34 ■ Problem 29.

29. In the Figure In Fig. 32-34, $\mathscr{E} = 100$ V, $R_1 = 10.0\ \Omega$, $R_2 = 20.0\ \Omega$, $R_3 = 30.0\ \Omega$, and $L = 2.00$ H. Find the values of i_1 and i_2 (a) immediately after closing of switch S, (b) a long time later, (c) immediately after the reopening of switch S, and (d) a long time after the reopening.

30. What Is the Constant Figure 32-35a shows a circuit consisting of an ideal battery with emf $\mathscr{E} = 6.00\ \mu$V, a resistance R, and a small wire loop of area 5.0 cm^2. For the time interval $t_1 = 10$ to $t_2 = 20$ s, an external magnetic field is set up throughout the loop. The field is uniform, its direction is into the page in Fig. 32-35a, and the field magnitude is given by $B = at$, where B is in teslas, a is a constant with units of teslas per second, and t is in seconds. Figure 32-35b gives the current i in the circuit before, during, and after the external field is set up. Find a.

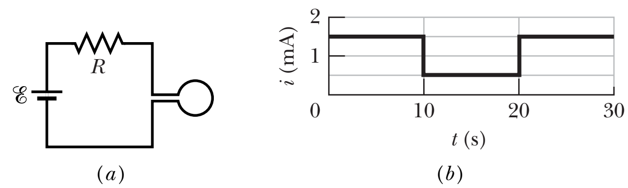


FIGURE 32-35 ■ Problem 30.

31. Once the Switch Is Closed Once the switch S is closed in Fig. 32-36 the time required for the current to reach any obtainable value depends, in part, on the value of resistance R. Suppose the emf $\mathscr{E}$ of the ideal battery is 12 V and the inductance of the ideal (resistanceless) inductor is 18 mH. How much time is needed for the current to reach 2.00 A if R is (a) 1.00 Ω, (b) 5.00 Ω, and (c) 6.00 Ω? (d) Why is there a huge jump between the answers to (b) and (c)? (e) For what value of R is the time required for the current to reach 2.00 A least? (f)What is that least time? (*Hint:* Rethink Eq. 32-21.)

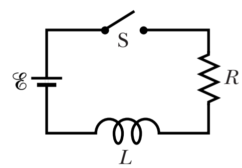


FIGURE 32-36 ■ Problems 31, 34, and 61.

32. Circuit Shown In the circuit shown in Fig. 32-37, switch S is closed at time $t = 0$. Thereafter, the constant current source, by varying its emf, maintains a constant current i out of its upper terminal. (a) Derive an expression for the current through the inductor as a function of time. (b) Show that the current through the resistor equals the current through the inductor at time $t = (L/R)\ln 2$.

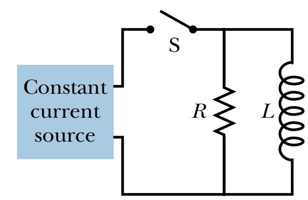


FIGURE 32-37 ■ Problem 32.

33. When Is the Flux Equal In Fig. 32-38a, switch S has been closed on A long enough to establish a steady current in the inductor of inductance $L_1 = 5.00$ mH and the resistor of resistance $R_1 = 25\ \Omega$. Similarly, in Fig. 32-38b, switch S has been closed on A long enough to establish a steady current in the inductor of inductance $L_2 = 3.00$ mH and the resistor of resistance $R_2 = 30\ \Omega$. The ratio Φ_{02}/Φ_{01} of the magnetic flux through a turn in inductor 2 to that in inductor 1 is 1.5. At time $t = 0$, the two switches are closed on B. At what time t is the flux through a turn in the two inductors equal?

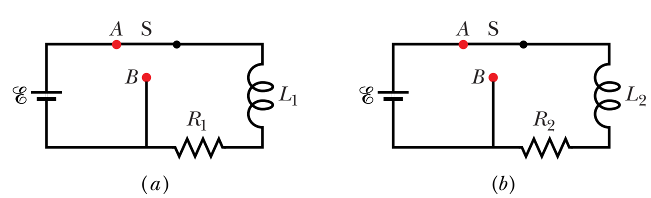


FIGURE 32-38 ■ Problem 33.

34. When Is emf Equal Switch S in Fig. 32-36 is closed at time $t = 0$, initiating the buildup of current in the 15.0 mH inductor and the 20.0 Ω resistor. At what time is the emf across the inductor equal to the potential difference across the resistor?

SEC. 32-5 ■ INDUCTORS, TRANSFORMERS, AND ELECTRIC POWER

35. A Transformer A transformer has 500 primary turns and 10 secondary turns. (a) If ΔV_p is 120 V (rms), what is ΔV_s with an open circuit? (b) If the secondary now has a resistive load of 15 Ω, what are the currents in the primary and secondary?

36. A Generator A generator supplies 100 V to the primary coil of a transformer of 50 turns. If the secondary coil has 500 turns, what is the secondary voltage?

37. Audio Amplifier In Fig. 32-39 let the rectangular box on the left represent the (high-impedance) output of an audio amplifier, with $r = 1000\ \Omega$. Let $R = 10\ \Omega$ represent the (low-impedance) coil of a loudspeaker. For maximum transfer of energy to the load R we must have $R = r$, and that is not true in this case. However, a transformer can be used to "transform" resistances, making them behave electrically as if they were larger or smaller than they actually are. Sketch the primary and secondary coils of a transformer that can be introduced between the amplifier and the speaker in Fig. 32-39 to match the impedances. What must be the turns ratio?

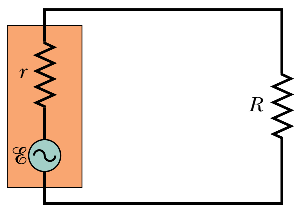


FIGURE 32-39 ■ Problem 37.

38. Autotransformer Figure 32-40 shows an "autotransformer." It consists of a single coil (with an iron core). Three taps T_N are provided. Between taps T_1 and T_2 there are 200 turns, and between taps T_2 and T_3 there are 800 turns. Any two taps can be considered the "primary terminals" and any two taps can be considered the "secondary terminals." List all the ratios by which the primary voltage may be changed to a secondary voltage.

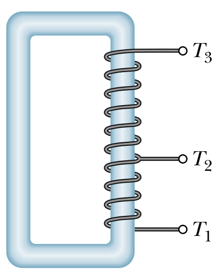


FIGURE 32-40 ■ Problem 38.

SEC. 32-6 ■ MAGNETIC MATERIALS—AN INTRODUCTION

39. Orbital Magnetic Dipole What is the measured component of the orbital magnetic dipole moment of an electron with (a) $m_l = 1$ and (b) $m_l = -2$?

40. Energy Difference What is the energy difference between parallel and antiparallel alignment of the z-component of an electron's spin magnetic dipole moment with an external magnetic field of magnitude 0.25 T, directed parallel to the z axis?

41. Electron in an Atom If an electron in an atom has an orbital angular momentum with $m_l = 0$, (a) what is the component μ_z^{orb}? If the atom is in an external magnetic field $\vec{B}$ of magnitude 35 mT and directed along z axis, what are the potential energies associated with the orientations of (b) the electron's orbital magnetic dipole moment and (c) the electron's spin magnetic dipole moment? (d) Repeat (a) through (c) for $m_l = -3$.

42. Spin Magnetic Moment An electron is placed in a magnetic field $\vec{B}$ that is directed along a z axis. The energy difference between parallel and antiparallel alignments of the z-component of the electron's spin magnetic moment with $\vec{B}$ is 6.00×10^{-25} J. What is the magnitude of $\vec{B}$?

43. How Many Suppose that ±4 are the limits to the values of m_l for an electron in an atom. (a) How many different values of the z-component μ_z^{orb} of the electron's orbital magnetic dipole moment are possible? (b) What is the greatest magnitude of those possible values? Next, suppose that the atom is in a magnetic field of magnitude 0.250 T, in the positive direction of the z axis. What are (c) the maximum potential energy and (d) the minimum potential energy associated with those possible values of μ_z^{orb}?

44. NMR and MRI Nuclear Magnetic Resonance (NMR) and Magnetic Resonance Imaging (MRI) exploit the interactions between charged particles and very strong magnetic fields in order to produce images (including images of soft tissue). The magnetic field in a certain MRI machine is 0.5 Tesla. What is the maximum difference in energy that one might measure for a single electron placed in this field?

SEC. 32-7 ■ FERROMAGNETISM

45. Saturation Magnetization The saturation magnetization M^{max} of the ferromagnetic metal nickel is 4.70×10^5 A/m. Calculate the magnetic moment of a single nickel atom. (The density of nickel is 8.90 g/cm^3 and its molar mass is 58.71 g/mol.)

46. Iron The dipole moment associated with an atom of iron in an iron bar has magnitude 2.1×10^{-23} J/T. Assume that all the atoms in the bar, which is 5.0 cm long and has a cross-sectional area of 1.0 cm^2, have their dipole moments aligned. (a) What is the magnitude of the dipole moment of the bar? (b) What is the magnitude of the torque that must be exerted to hold this magnet perpendicular to an external field of 1.5 T? (The density of iron is 7.9 g/cm^3.)

47. Earth's Magnetic Moment The magnetic dipole moment of Earth has magnitude 8.0×10^{22} J/T. (a) If the origin of this magnetism were a magnetized iron sphere at the center of the Earth, what would be its radius? (b) What fraction of the volume of the Earth would such a sphere occupy? Assume complete alignment of the dipoles. The density of the Earth's inner core is 14 g/cm^3. The magnetic dipole moment of an iron atom is 2.1×10^{-23} J/T. (*Note:* The Earth's inner core is in fact thought to be in both liquid and solid forms and partly iron, but a permanent magnet as the source of the Earth's magnetism has been ruled out by several considerations. For one, the temperature is certainly above the Curie point.)

48. Mines and Boreholes Measurements in mines and boreholes indicate that the Earth's interior temperature increases with depth at the average rate of 30 C°/km. Assuming a surface temperature of 10°C, at what depth does iron cease to be ferromagnetic? (The Curie temperature of iron varies very little with pressure.)

SEC. 32-8 ■ OTHER MAGNETIC MATERIALS

49. Electron Assume that an electron of mass m and charge magnitude e moves in a circular orbit of radius r about a nucleus. A uniform magnetic field $\vec{B}$ is then established perpendicular to the plane of the orbit. Assuming also that the radius of the orbit does not change and that the change in the speed of the electron due to field $\vec{B}$ is small, find an expression for the change in the orbital magnetic dipole moment of the electron due to the field.

50. Loop Model Figure 32-41 shows a loop model (loop L) for a diamagnetic material. (a) Sketch the magnetic field lines through and about the material due to the bar magnet. (b) What are the directions of the loop's net magnetic dipole moment $\vec{\mu}$ and the conventional current i in the loop? (c) What is the direction of the magnetic force on the loop?

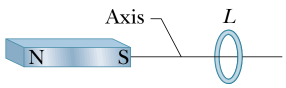


FIGURE 32-41 ■ Problems 50 and 54.

51. Cylindrical Magnet A magnet in the form of a cylindrical rod has a length of 5.00 cm and a diameter of 1.00 cm. It has a uniform magnetization of 5.30×10^3 A/m. What is the magnitude of its magnetic dipole moment?

52. Paramagnetic Gas A magnetic field of magnitude 0.50 T is applied to a paramagnetic gas whose atoms have an intrinsic magnetic dipole moment of magnitude 1.0×10^{-23} J/T. At what temperature will the mean kinetic energy of translation of the gas atoms be equal to the energy required to reverse such a dipole end for end in this magnetic field?

53. Paramagnetic Salt A sample of the paramagnetic salt to which the magnetization curve of Fig. 32-19 applies is to be tested to see whether it obeys Curie's law. The sample is placed in a uniform 0.50 T magnetic field that remains constant throughout the experiment. The magnetization M is then measured at temperatures ranging from 10 to 300 K. Will Curie's law be valid under these conditions?

54. Paramagnetic Material Repeat Problem 50 for the case in which loop L is the model for a paramagnetic material.

55. Electron's Kinetic Energy An electron with kinetic energy K travels in a circular path that is perpendicular to a uniform magnetic field, the electron's motion is subject only to the force due to the field. (a) Show that the magnetic dipole moment of the electron due to its orbital motion has magnitude $\mu = K/|\vec{B}|$ and that it is in the direction opposite that of $\vec{B}$. (b) What are the magnitude and direction of the magnetic dipole moment of a positive ion with kinetic energy K_{ion} under the same circumstances? (c) An ionized gas consists of 5.3×10^{21} electrons/m^3 and the same number density of ions. Take the average electron kinetic energy to be 6.2×10^{-20} J and the average ion kinetic energy to be 7.6×10^{-21} J. Calculate the magnetization of the gas when it is in a magnetic field of 1.2 T.

56. Magnetization Curve A sample of the paramagnetic salt to which the magnetization curve of Fig. 32-17 applies is held at room temperature (300 K). At what applied magnetic field will the degree of magnetic saturation of the sample be (a) 50% and (b) 90%? (c) Are these fields attainable in the laboratory?

SEC. 32-9 ■ THE EARTH'S MAGNETISM

57. New Hampshire In New Hampshire the average horizontal component of the Earth's magnetic field in 1912 was 16 μT and the average inclination or "dip" was 73°. What was the corresponding magnitude of the Earth's magnetic field?

58. Earth's Field Assume the average value of the vertical component of the Earth's magnetic field is 43 μT (downward) for all of Arizona, which has an area of 2.95×10^5 km^2, and calculate the net magnetic flux through the rest of the Earth's surface (the entire surface excluding Arizona). Is that net magnetic flux outward or inward?

59. Earth's Field Two Use the results of Problem 60 to predict the Earth's magnetic field (both magnitude and inclination) at (a) the geomagnetic equator, (b) a point at geomagnetic latitude 60°, and (c) the north geomagnetic pole.

60. Magnetic Field of Earth The magnetic field of the Earth can be approximated as the magnetic field of a dipole, with horizontal and vertical components, at a point a distance r from the Earth's center, given by

$$B_h = \frac{\mu_0 \mu}{4\pi r^3} \cos \lambda_m, \qquad B_v = \frac{\mu_0 \mu}{2\pi r^3} \sin \lambda_m,$$

where λ_m is the *magnetic latitude* (this type of latitude is measured from the geomagnetic equator toward the north or south geomagnetic pole). Assume that the Earth's magnetic dipole moment is $\mu = 8.00 \times 10^{22}$ A · m^2. (a) Show that the magnitude of the Earth's field at latitude λ_m is given by

$$B = \frac{\mu_0 \mu}{4\pi r^3} \sqrt{1 + 3 \sin^2 \lambda_m}\,.$$

(b) Show that the inclination ϕ_i of the magnetic field is related to the magnetic latitude λ_m by

$$\tan \phi_i = 2 \tan \lambda_m.$$

Additional Problems

61. Rate of Energy Transfer In Fig. 32-36, a 12.0 V ideal battery, a 20 Ω resistor, and an ideal inductor are connected by a switch at time $t = 0$ s. At what rate is the battery transferring energy to the inductor's field at $t = 1.61\tau_L$?

62. Compass Needle You place a magnetic compass on a horizontal surface, allow the needle to settle into equilibrium position, and then give the compass a gentle wiggle to cause the needle to oscillate about that equilibrium position. The frequency of oscillation is 0.312 Hz. The Earth's magnetic field at the location of the compass has a horizontal component of 18.0 μT. The needle has a magnetic moment of 0.680 mJ/T. What is the needle's rotational inertia about its (vertical) axis of rotation?

63. Induced Current in a Coil A long narrow coil is surrounded by a short wide coil as shown in Fig. 32-42. Both coils have negligible resistance. The short wide coil has a diameter d_S, n_S turns per unit length, and a length S. Its ends are connected through a resistor of resistance R. The long narrow inner coil has a diameter d_L, n_L turns per unit length, and a length L. Its ends are connected across a variable power source.

For each of the partial sentences below, indicate whether they are correctly completed by the phrase greater than (>), less than (<), or the same as (=). If you cannot determine which is the case from the information given, indicate not sufficient information (NSI).

The current through an inner coil is increased from 0.0 amps to 0.1 amps over a period of 10 seconds in a smooth fashion according to the rule

$$i_L(t) = (0.01\ \text{A/s})\ t.$$

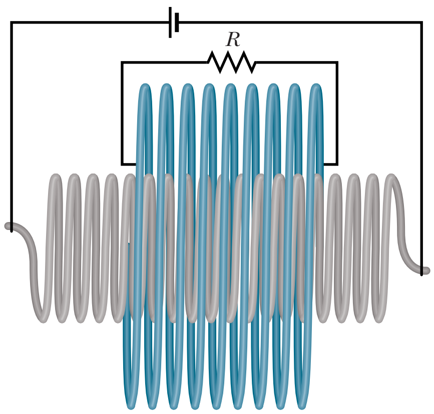


FIGURE 32-42 ■ Problem 63.

(a) The magnitude of the current in the long narrow coil at time $t = 1$ s is ________ the current in that coil at time $t = 5$ s.
(b) The magnitude of the current in the short wide coil at time $t = 1$ s is ________ the current in that coil at time $t = 5$ s.
(c) The magnitude of the current in the long narrow coil at time $t = 1$ s is ________ the current in the short wide coil at that same time.
(d) If the long narrow coil was compressed to half its length (without changing its diameter) before the current was turned on, the current in the short wide coil would be ________ it was without the compression.

64. Inducing Current Figure 32-43 shows a solenoid and two hoops. When the switch is closed, the solenoid carries a current in the direction indicated. The planes of the small loops are parallel to the planes of the hoops of the solenoid.

Hoop 1 consists of a single turn of resistive wire that has a resistance per unit length of λ. Hoop 2 consists of N turns of the same wire. Each hoop is a circle of radius r.

(a) The switch is closed and remains closed for a few seconds. Hoop 1 is then moved to the right. Is there a current flow induced? If there is a current, indicate the direction and explain how you figured it out.
(b) The hoops are now returned to their original locations and held fixed. The switch is opened. For a short time, the magnetic field at the hoops decreases like

$$B_x(t) = B_x(0) - \gamma t,$$

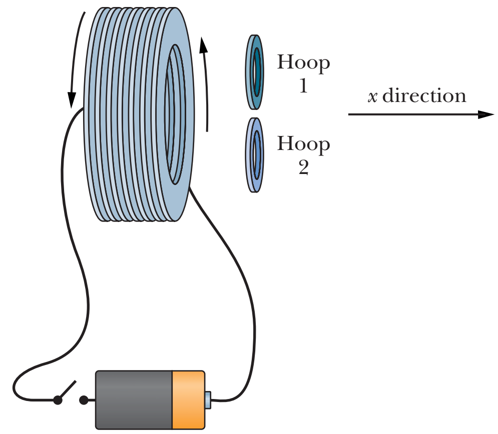


FIGURE 32-43 ■ Problem 64.

where γ is a constant with units of gauss per second. Is there a current flow induced in the hoops? If there is a current, indicate the direction.
(c) Calculate the current flow in each hoop for situation (b).
(d) If $B_x(0) = 10$ gauss, $\gamma = 2.5$ gauss/s, the resistivity of the wire is 1 $\mu\Omega$/m, and the hoops have a radius of 2 cm, calculate the current induced in hoop 1 as the B-field from the solenoid begins to fall.