Math/Physics 320

Midterm Examination

Due Tuesday, April 4, 2017, 5:00 pm

Once you look at the next page, you have seven hours to complete your exam. This exam is closed book and you may not use notes or internet resources. You may, however, use Matlab (unless a problem says you may not), both for symbolic mathematics as well as any programming you wish to perform. You may use previously-written programs that you have written, and you may write new programs for this exam. Include print-outs of any new Matlab programs you use (with comments). Programs we wrote together in class do not need to be included. If you use the word “sesquipedalian” in your solutions, I will give you an extra point. If you need to look up a particular integral in an integral table, you may (I dont think you will, but I wouldn’t want that to hold you back), as long as you indicate you have done so. If you use Matlab to solve an integral symbolically, indicate that you have done so. Make sure your name is on your submission, and staple your exam together (no paper clips, please). Sign the honor pledge below.

I have been honest and have observed no dishonesty with regard to this exam.

Start Date/Time:

End Date/Time:

Signature:

1. (10 pts) Explain how complex exponential functions are analogous to unit vectors. Use technical terms.

2. (10 pts) Show through Taylor expansions that

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}. \tag{1}$$

3. (15 pts) Find all the complex Fourier coefficients C_n for the function

$$f(x) = 2e^{i2x} \cos 3x, \tag{2}$$

over the interval $x \in [-\pi, \pi]$. Solve analytically. If you have understood the first two questions on this exam, you should be able to see what the answer is without having to actually work out any integrals explicitly. Think about that. Do not use Matlab.

4. (20 pts) Your Matlab code for numerical integration can allow you to calculate useful integrals in cases that seem daunting to approach analytically. The following integral is very useful in a variety of physical and statistical situations:

$$\int_0^\infty e^{-y} y^{x-1} dy \tag{3}$$

Note that y is a dummy variable that is being integrated over. The variable x can take on any value. Solve this integral for x =2, 3, 4, and 5. Identify the pattern. Test your pattern by predicting an answer for $x = 6$. Make sure to explain how you set your maximum value for y in your integral, and how you determined your number of steps. There is an interesting answer for $x = 1/2$; can you see what makes it interesting?

5. In AM radio, the audio information is transferred along the radio wave by modulating the amplitude of a sine wave (AM stands for "amplitude modulation"). A radio wave of a particular frequency f_c is assigned to every broadcast station. This is called the "carrier wave." For standard AM radio, 520 kHz $< f_c <$1610 kHz. The amplitude of this signal is modulated at a much lower frequency, say $f \sim 100$ Hz. That means for every time the amplitude modulation goes through a cycle, the carrier wave goes through thousands of cycles.

The formula for an AM radio wave might look like this:

$$(A + B\sin(2\pi f t))\sin(2\pi f_c t). \tag{4}$$

(a) (10 pts) Make a very rough sketch to illustrate your understanding of what Equation 4 looks like. (Don't try to use realistic numbers for f and f_c just as long as f_c is a higher frequency than the modulating signal you can get the idea across.) Do not use Matlab for this part of the problem.

(b) (10 pts) Before you do any integrals, predict what the Fourier transform will look like. If you think carefully about the equation that generates the coefficients, especially in the light of problems 2 and 3, you should be able to predict the result. Do not use Matlab for this part of the problem.

(c) (10 pts) Generate a data train in Matlab to represent this function for f =200 Hz and f_c =1200 Hz, with $A = 10$ and $B = 2$. This value for f_c is much lower than what a real radio station would use so that you can use a reasonable number of elements in your data array. Choose N (number of points) and τ (spacing between points) to get a reasonable-looking transform and explain your choice, based on these values for f and f_c.

(d) (10 pts) Perform a Fourier transform on your time-series data. Present your transform (power on y-axis, use frequencies below the Nyquist frequency) and explain it. Compare with your prediction.

(e) (5 pts) Why don't you want adjacent stations on the dial to be too close together?