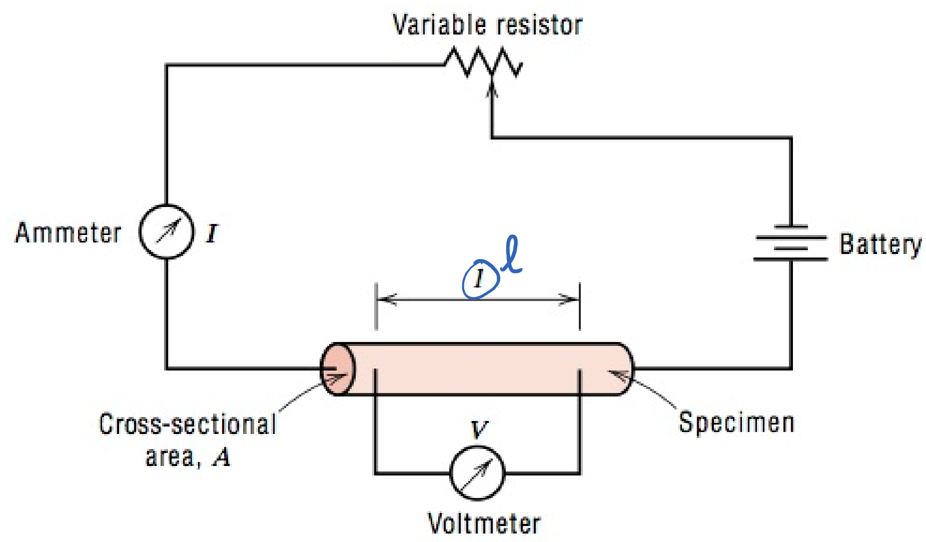
Unit 5 - Electrical properties



Ohm's law

$$V = IR$$

Resistivity

$$\rho = \frac{RA}{l}$$

Conductivity

$$\sigma = \frac{1}{\rho}$$

Current density

$$J = \frac{I}{A} = \sigma \mathcal{E}$$

$$\mathcal{E} = E = \frac{V}{L}$$

TABLE 18–1 Resistivity and Temperature Coefficients (at 20°C)

Material	Resistivity, ρ ($\Omega \cdot$ m)	Temperature Coefficient, α (C°)$^{-1}$
Conductors		
Silver	1.59×10^{-8}	0.0061
Copper	1.68×10^{-8}	0.0068
Gold	2.44×10^{-8}	0.0034
Aluminum	2.65×10^{-8}	0.00429
Tungsten	5.6×10^{-8}	0.0045
Iron	9.71×10^{-8}	0.00651
Platinum	10.6×10^{-8}	0.003927
Mercury	98×10^{-8}	0.0009
Nichrome (Ni, Fe, Cr alloy)	100×10^{-8}	0.0004
Semiconductors†		
Carbon (graphite)	$(3–60) \times 10^{-5}$	−0.0005
Germanium	$(1–500) \times 10^{-3}$	−0.05
Silicon	0 .1–60	−0.07
Insulators		
Glass	$10^{9} – 10^{12}$	
Hard rubber	$10^{13} – 10^{15}$	

† Values depend strongly on the presence of even slight amounts of impurities.

2. Classical (Drude) model

$$\bar{v} = \mu \mathcal{E}$$

$$\mu = \frac{e\tau}{m_e}$$

Group exercise

(a) Using the data in Table ~~12.1~~ 4.1, compute the resistance of a copper wire 3 mm (0.12 in.) in diameter and 2 m (78.7 in.) long. **(b)** What would be the current flow if the potential drop across the ends of the wire is 0.05 V? **(c)** What is the current density? **(d)** What is the magnitude of the electric field across the ends of the wire?

$$R = \rho \frac{l}{A} = \frac{(1.55\times10^{-8}\,\Omega\cdot\cancel{m})(2\,\cancel{m})}{\pi(1.5\times10^{-3}\,\cancel{m})^2}$$

$$= 4.4\times10^{-3}\,\Omega$$

$V = IR \qquad El = \Delta V$

3. Bloch theory.

- perfect crystal has no resistance

$$\rho_{total} = \rho_T + \rho_i + \rho_d$$

thermal   impurities   deformation

4. Temp. dependence of resistivity.

$$\rho_T = \rho_0(1+\alpha(T-T_0)) \qquad (25\text{-}5) \text{ Giáncoli}$$

Group exercise

P25.28 (sort of)

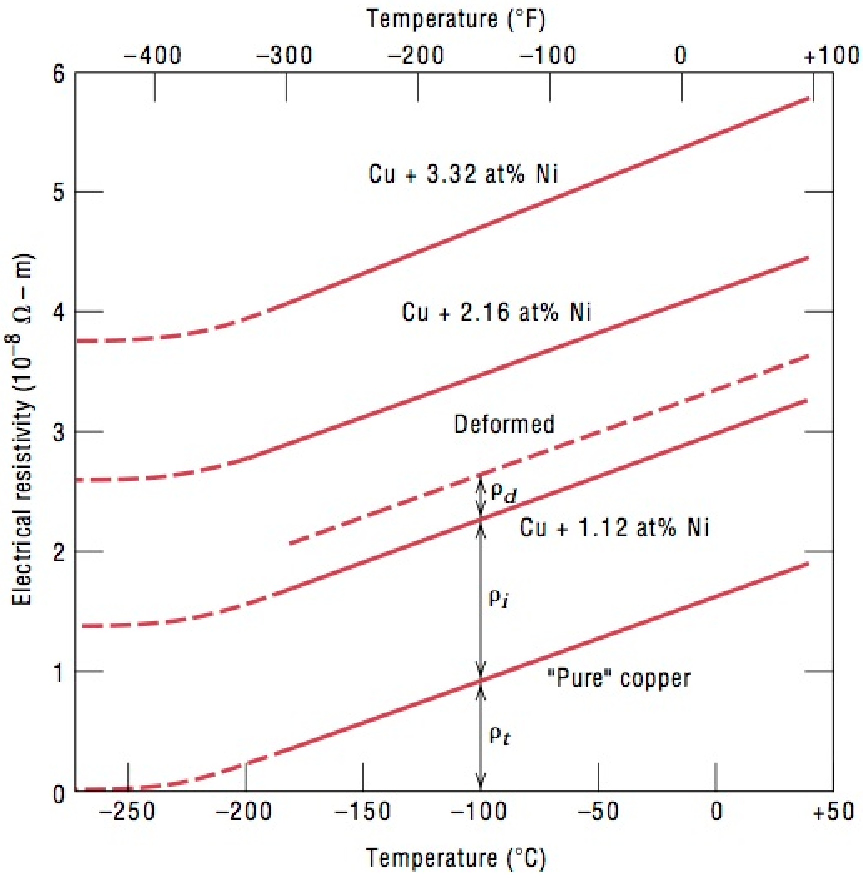
The tungsten filament of a lightbulb has a resistance of 12 ohms at 20°C and 140 ohms when hot. (a) Estimate the temperature of filament when hot neglecting any change in length and area. (b) Estimate the temperature taking in to account the change of length and area of the filament. The coefficient of thermal expansion for tungsten is $5.5\times10^{-6}\,{}^{\circ}\mathrm{C}^{-1}$.

5. Impurity dependence of resistivity.

$$\rho_i = A\,c_i(1-c_i)$$

Group exercise

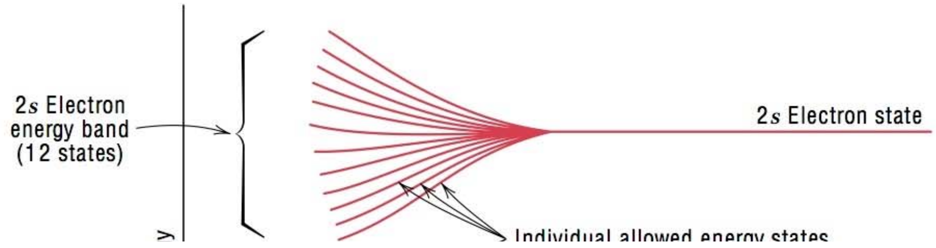
(a) Using the data in Figure 12.8, determine the values of ρ_0 and a from Equation 12.10 for pure copper. Take the temperature T to be in degrees Celsius. **(b)** Determine the value of A in Equation 12.11 for nickel as an impurity in copper, using the data in Figure 12.8. **(c)** Using the results of parts a and b, estimate the electrical resistivity of copper containing 1.75 at% Ni at 100°C.



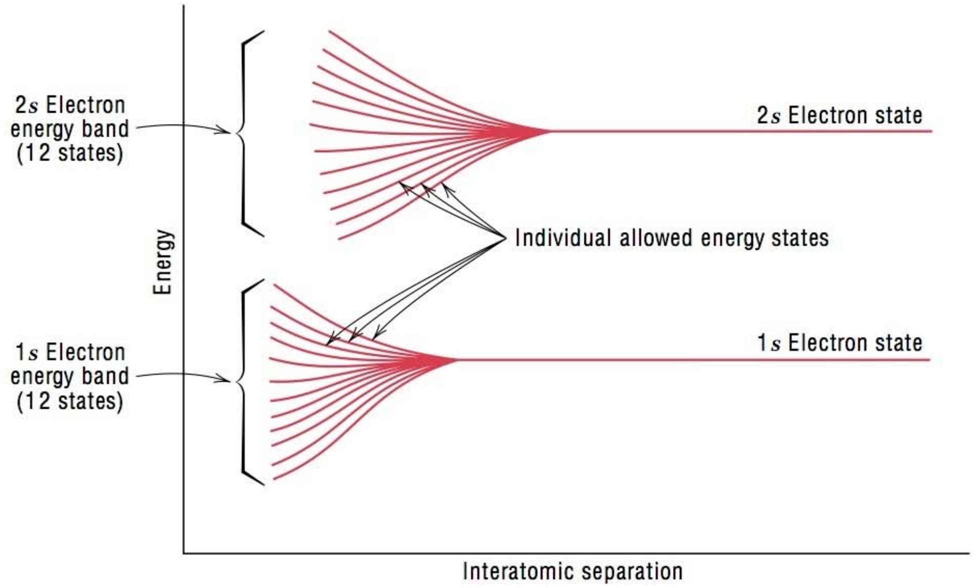
Band Theory. - two explanations

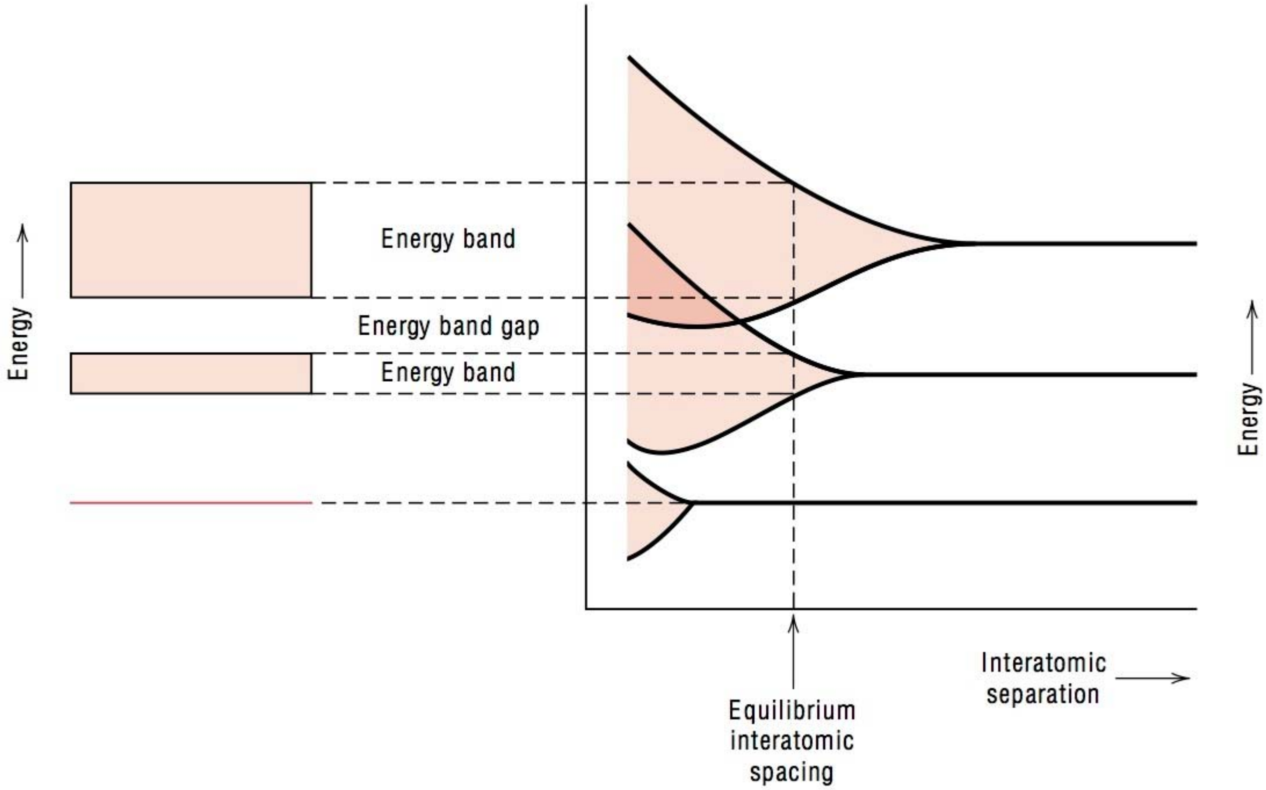
① Atomic explanation

Schematic plot of electron energy versus interatomic separation for an aggregate of 12 atoms (N = 12). Upon close approach, each of



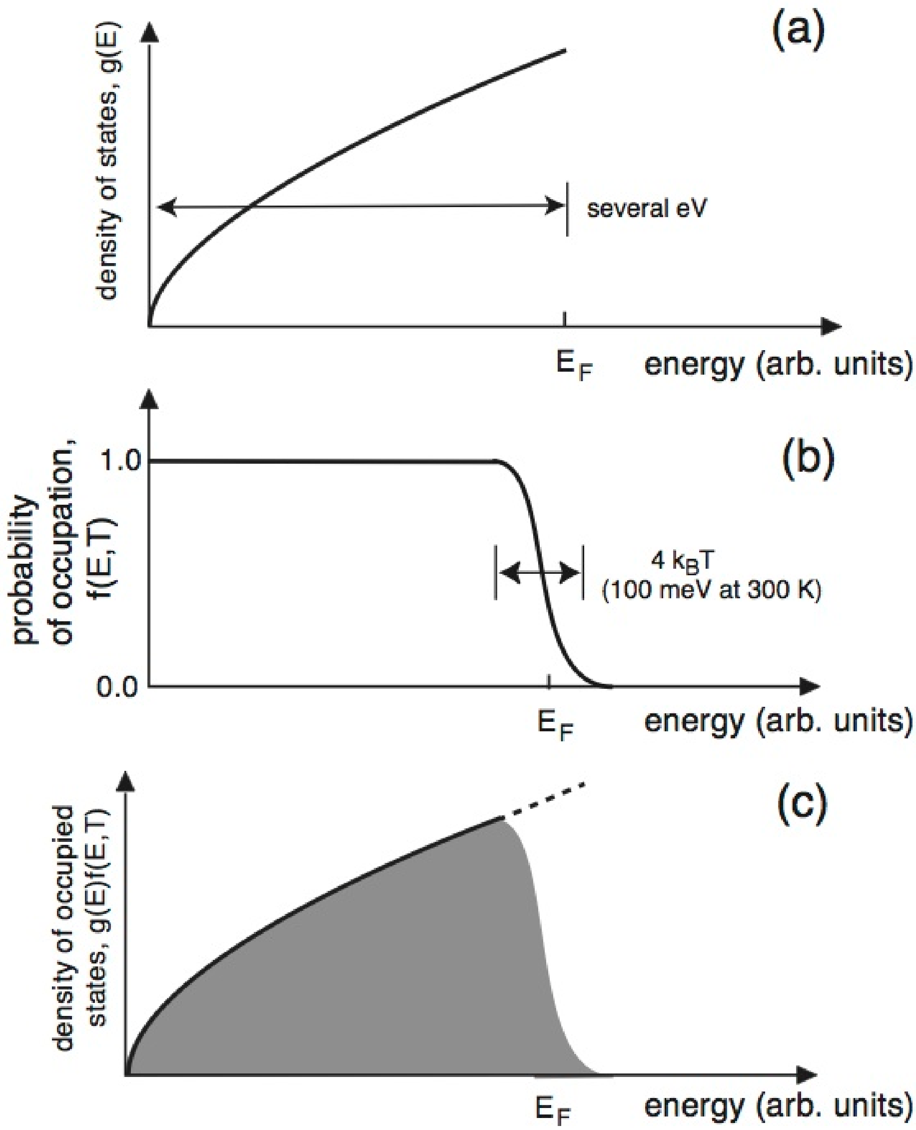
Schematic plot of electron energy versus interatomic separation for an aggregate of 12 atoms ($N = 12$). Upon close approach, each of the $1s$ and $2s$ atomic states splits to form an electron energy band consisting of 12 states.



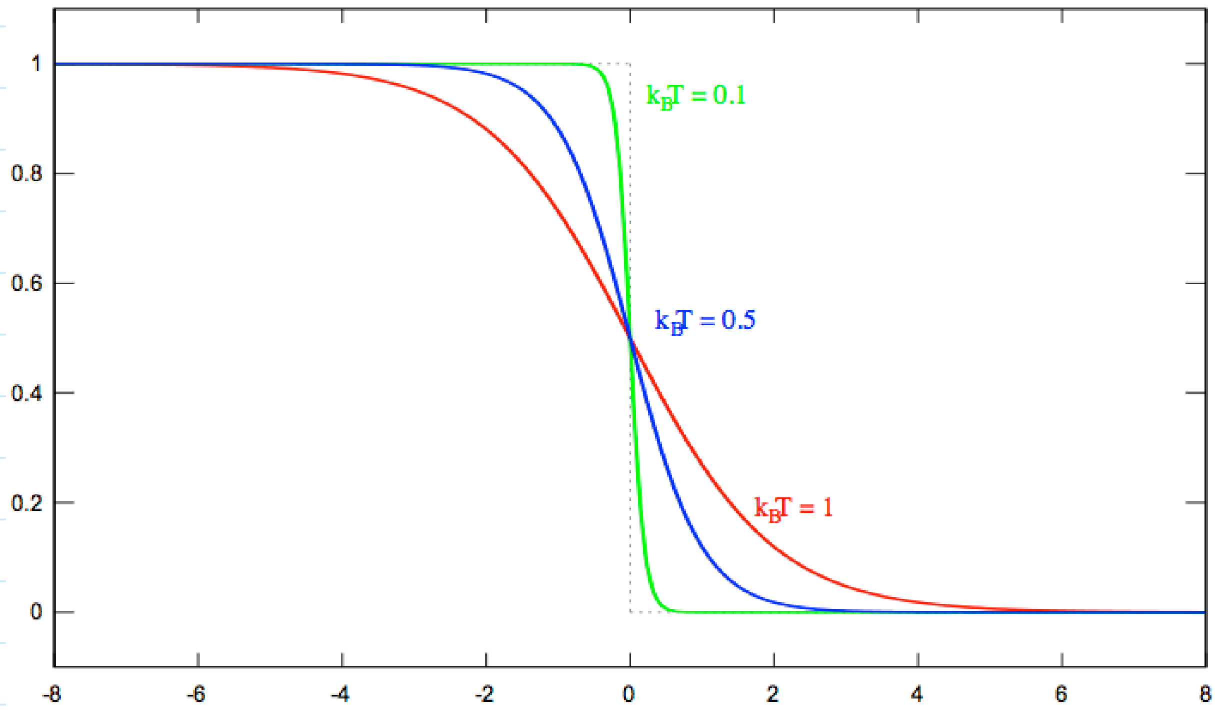


The conventional representation of the electron energy band structure for a solid material at the equilibrium interatomic separation.

② Free electron model



2. Fermi-Dirac distribution

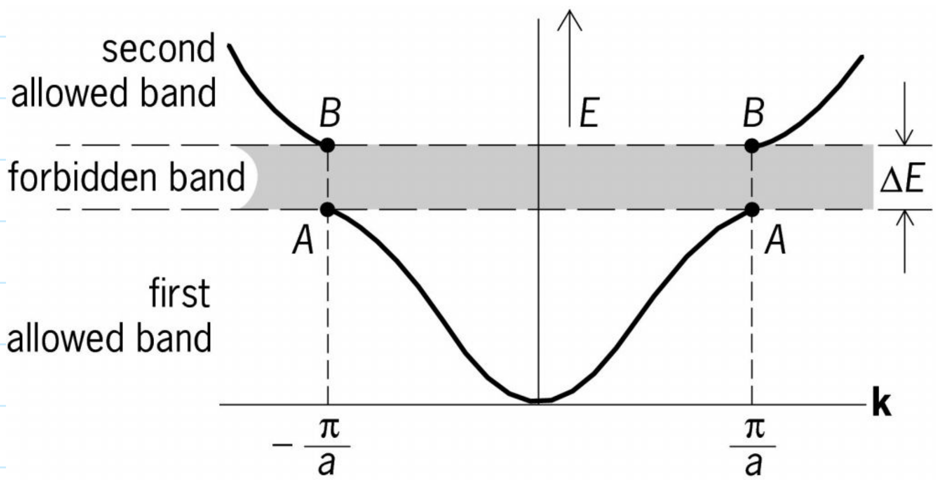


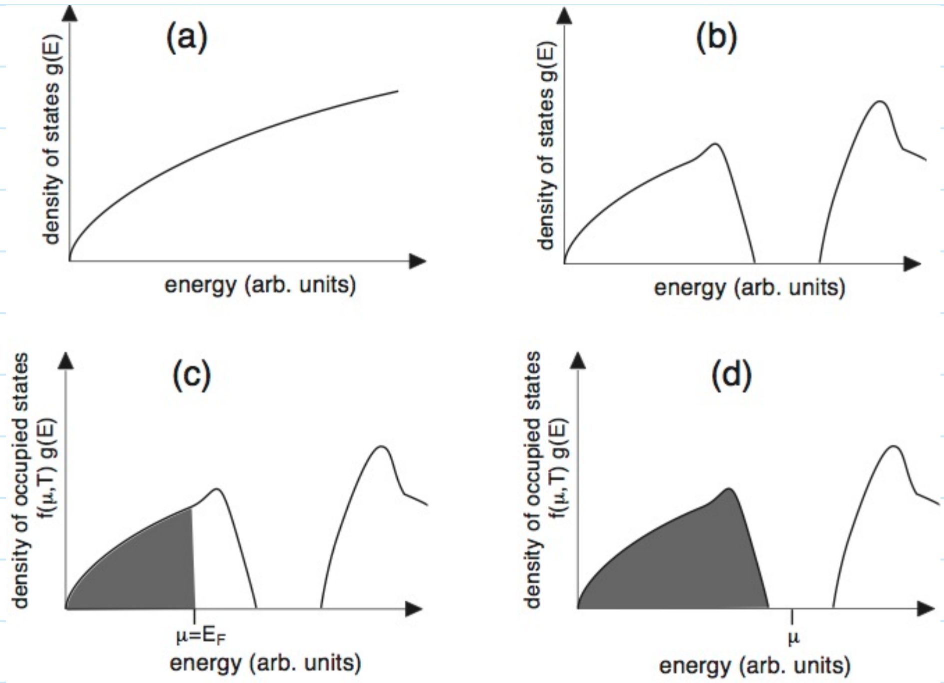
$$E_f = \frac{\hbar^2}{2m_e}\left(\frac{3\pi^2 N}{V}\right)^{2/3} \qquad \hbar = \frac{h}{2\pi}$$

Group exercise

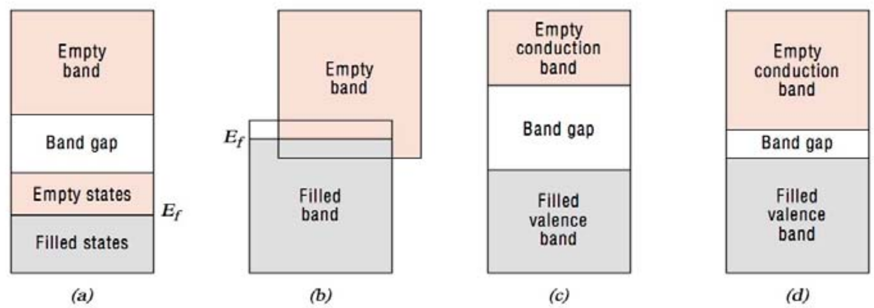
Calculate the Fermi energy for copper assuming there is one conduction electron for each copper atom.

Almost free (Bloch model)





Conductors, insulators & semiconductors.



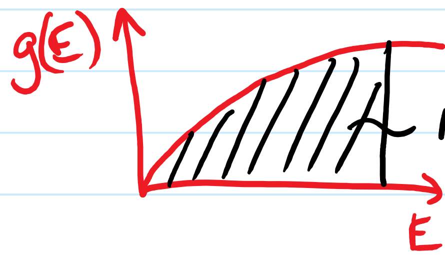
(*a*) The electron band structure found in metals such as copper, in which there are available electron states above and adjacent to filled states, in the same band. (*b*) The electron band structure of metals such as magnesium, wherein there is an overlap of filled and empty outer bands. (*c*) The electron band structure characteristic of insulators; the filled valence band is separated from the empty conduction band by a relatively large band gap (>2 eV). (*d*) The electron band structure found in the semiconductors, which is the same as for insulators except that the band gap is relatively narrow (<2 eV).

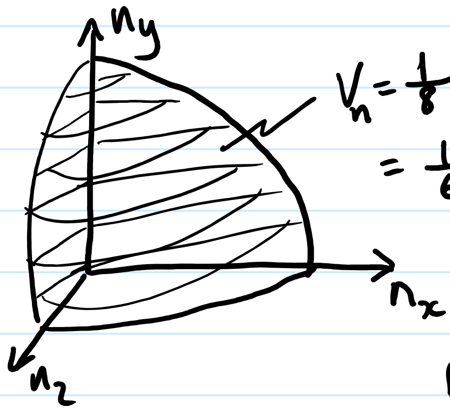
Group exercise

(a) Calculate the Fermi temperature for copper ($kT_F = E_F$). (b) To estimate how sharp the cutoff is at the Fermi energy E_F, calculate how far the energy of a state must be above the Fermi energy so that its probability of being filled is 1% at room temperature. (c) What fraction of the Fermi energy is that for copper?

$g(E) \propto E^{1/2}$

Q. Why does $g(E) \propto E^{1/2}$?





$$V_n = \frac{1}{8}\left(\frac{4}{3}\pi r^3\right) \qquad r^2 = n_{max}^2 = n_x^2 + n_y^2 + n_z^2$$

$$= \frac{1}{6}\pi n_{max}^3$$

$$V_n = \frac{N}{2} = \frac{\pi n_{max}^3}{6} \Rightarrow n_{max} = \left(\frac{3N}{\pi}\right)^{\frac{1}{3}}$$

$$E_f = \frac{\hbar^2}{2m_e}\left(\frac{\pi}{L}\right)^2 n_{max}^2 \qquad (4.16)$$

$$= \frac{\hbar^2}{2m_e}\left(\frac{\pi}{L}\right)^2\left(\frac{3N}{\pi}\right)^{2/3}$$

$$= \frac{\hbar^2}{2m_e}\left(\frac{\pi^3}{L^3}\right)^{2/3}\left(\frac{3N}{\pi}\right)^{2/3}$$

$$E_f = \frac{\hbar^2}{2m_e}\left(\frac{3\pi^2 N}{V}\right)^{2/3} \quad (4.17) \quad (\text{where } V = L^3)$$

Area under Fig 4.23 $= N = \int_0^{E_f} g(E)\,dE$

$$\frac{d}{dE} \Rightarrow \frac{dN}{dE} = g(E)$$

$$N^{2/3} = \frac{2m_e}{\hbar^2}\left(\frac{V}{3\pi^2}\right)^{2/3} E \qquad (\quad)^{3/2}$$

$$N = \left(\frac{2m_e}{\hbar^2}\right)^{3/2} \frac{V}{3\pi^2} E^{3/2}$$

$$\frac{dN}{dE} = \left(\frac{2m_e}{\hbar^2}\right)^{3/2} \frac{V}{\cancel{3}\pi^2}\frac{\cancel{3}}{2} E^{\frac{1}{2}}$$

$$\boxed{g(E) = \frac{dN}{dE} = \frac{V}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} E^{1/2}} \qquad (4.19).$$