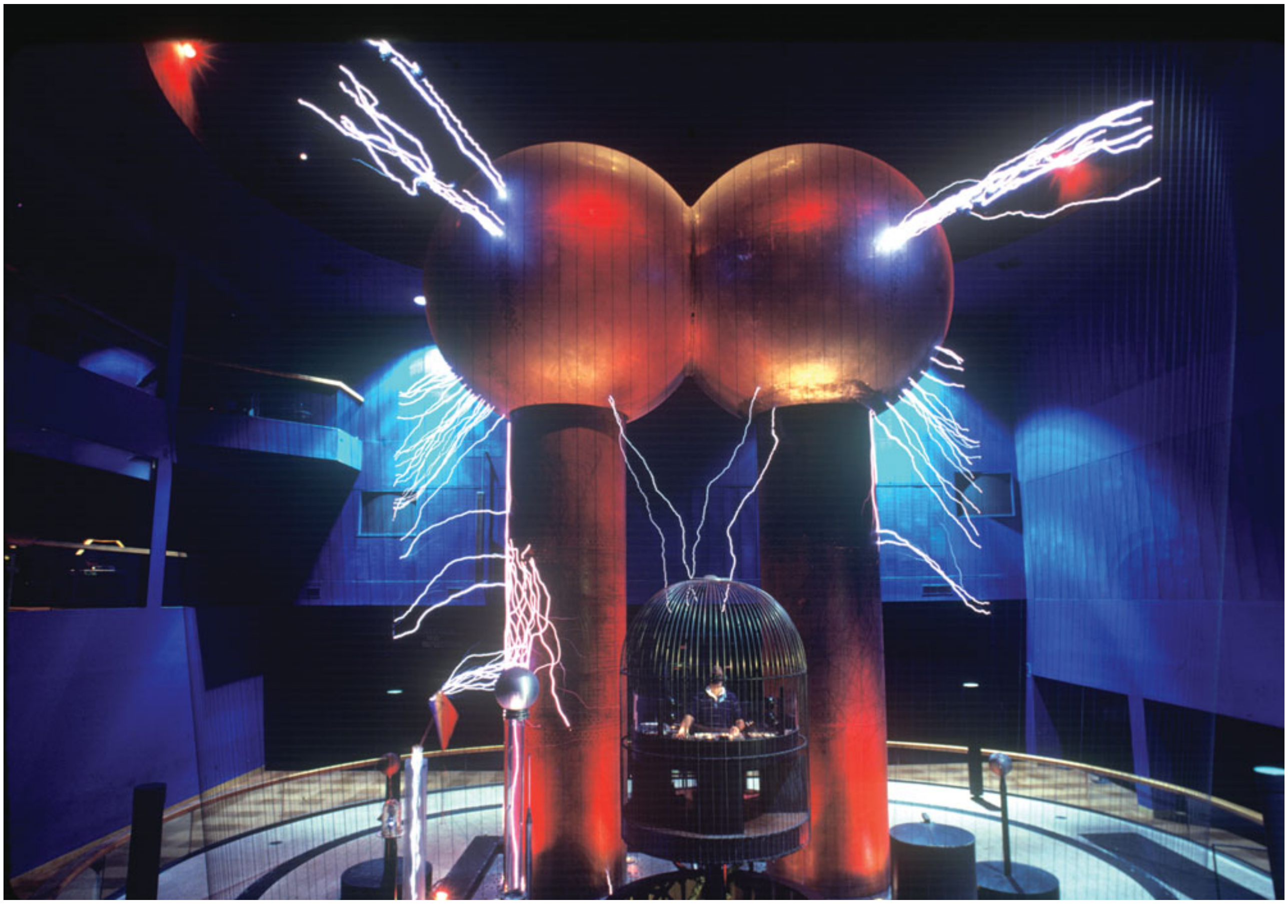
24 Gauss' Law



A demonstrator at the Boston Museum of Science is enclosed in a large conducting cage made of wire mesh. An electrical discharge from a giant Van de Graaff generator, like the one discussed in Chapter 25, is charging the metal cage to a dangerously high voltage. Yet the demonstrator cannot detect the fact that the cage is electrically charged even while touching the inside of the cage.

How can a closed conducting surface such as this metal cage or an automobile prevent someone from being harmed by lightning or other high-voltage sources?

The answer is in this chapter.

24-1 An Alternative to Coulomb's Law

We associate a vector electric field with a distribution of charges. The electric field has a vector at every location in space telling us what force a test charge q_t will experience at that location. In Sections 23-5 through 23-10 in the last chapter, we used Coulomb's law and the principle of superposition to calculate the electric field vectors at various points in space due to charges that were distributed in different ways. Although Coulomb's law can be used to calculate the electric force (and hence electric field) exerted on a test charge by any possible arrangement of charges we could imagine, this is usually a very difficult task. For example, even calculating the electric field outside the surface of a hollow, charged, conducting sphere would require us to do a triple integration.

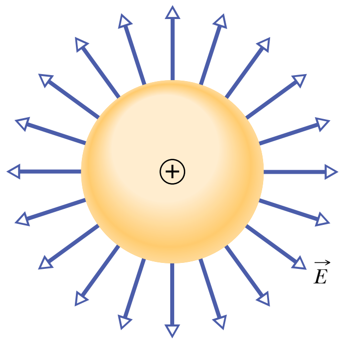


FIGURE 24-1 ■ If a single charge is located at the center of an imaginary sphere, Coulomb's law tells us the magnitudes of the electric field vectors are the same at all points on the surface of the sphere and the direction of each electric field vector is normal (perpendicular) to the surface. Only the field vectors that lie in the plane of the page are shown in this drawing.

In Chapter 23 we used Coulomb's law to find electric fields from charge distributions, but what if we want to turn our calculation around and determine a distribution of charges from an electric field pattern? Unless our distribution of charges is very simple, this reverse calculation is also difficult to perform using Coulomb's law. Thus Coulomb's law appears to be valid but difficult to use in many circumstances. In this chapter we introduce Gauss' law as another method for relating a known electric field to the charge distribution generating it and, conversely, for relating a known charge distribution to its associated electric field. Gauss' law in the integral form discussed in this chapter allows us to find electric fields easily for very symmetrical charge distributions.

To explore how we might find a general relationship between a collection of charges and their electric field, let's consider the electric field associated with the simplest possible charge distribution—a point charge (see Fig. 24-1). By applying Coulomb's law we have already found that the magnitude of the charge's electric field *decreases* as the inverse square of the distance r, as expressed in Eq. 23-8,

$$E = |\vec{E}| = k\frac{|q|}{r^2}.$$

However, if we construct an imaginary spherical surface around our source charge we find that the surface area of the sphere *increases* as the square of the distance of the spherical surface from the source charge. The equation for the surface area is given by $A = 4\pi r^2$. Thus, we see that the product of the electric field magnitude and the surface area of any imaginary spherical boundary is constant no matter how large or small the distance from the charge is, as shown in Eq. 24-1,

$$EA = k\frac{|q|}{r^2}(4\pi r^2) = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}(4\pi r^2) = \frac{|q|}{\varepsilon_0}. \tag{24-1}$$

Here we use Eq. 22-7 to replace the electrostatic constant k with $1/4\pi\varepsilon_0$ where ε_0 is the electric (or permittivity) constant.

Equation 24-1 is remarkable for two reasons. First, as the electric field magnitude gets smaller, the area over which it can act gets larger by exactly the same factor. Second, the product of the electric field magnitude anywhere on a spherical surface and the area of the spherical surface is *proportional* to the amount of charge $|q|$ enclosed by that surface. Does this proportionality still exist when the closed surface takes on other shapes? These questions were addressed by German mathematician and physicist Carl Friedrich Gauss (1777–1855). We begin our study of Gauss' approach to relating charge distributions, electric fields, and closed surfaces to each other by defining a new quantity called electric flux.

24-2 Electric Flux

For the case of a single point charge at the center of an imaginary sphere, Eq. 24-1 tells us that the product of the electric field magnitude (at the surface of a sphere) and the surface area of the sphere are proportional to the charge. This product EA is known as the **electric flux** through the sphere. In our simple situation the directions of electric field vectors created by the point charge happen to be normal (that is, perpendicular) to the surface of our imaginary sphere at all points along its surface. What if we have a complex array of charges or decide to surround our charge with an imaginary enclosure with a different shape? In that case we need to break our surface into little elements of area and find the component of the electric field vector that is normal to each area element as depicted in Fig. 24-2. We took a similar approach in Section 15-10 in defining *volume flux* for fluids flowing in pipes and streams. If the definitions of volume flux and normal vector for an area are not familiar to you, we suggest you read this earlier section.

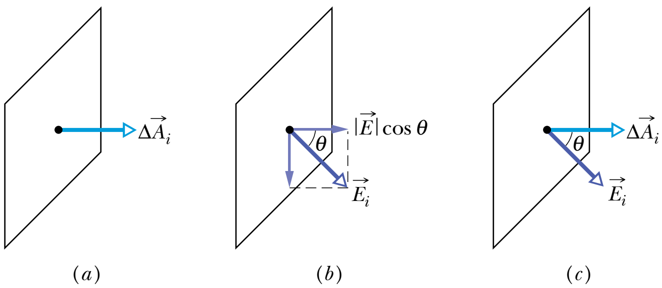


FIGURE 24-2 ■ (*a*) A small area vector element $\Delta\vec{A}$ is perpendicular to the plane of a square loop of area A with a magnitude of A. (*b*) The component of $\vec{E}$ perpendicular to the plane of the loop is $|\vec{E}|\cos\theta$, where θ is the angle between $\vec{E}$ and a normal to the plane. (*c*) The area vector $\Delta\vec{A}$ makes an angle θ with $\vec{E}$.

If we know the nature of the velocity vector field, $\vec{v}$, characterizing the motion of the fluid, we can use the definition of volume flux presented in Chapter 15 to calculate the amount of fluid flowing through any very small element, $\Delta\vec{A}_i$, of a larger surface area*. If we look at the ith element of a larger area, the *volume flux* element, Φ_i, for that small area is defined as the scalar or dot product of the normal vector representing an area element and the velocity vector at the location of the area element as shown in Eq. 15-33,

$$\Phi_i \equiv \vec{v}_i \cdot \Delta\vec{A}_i \qquad \text{(volume flux definition for a small area element).}$$

What is a normal vector? Recall that we defined the normal vector to a small flat area to allow us to represent both the magnitude and the orientation of an element of area. If the element of area is part of a closed surface completely surrounding a space, we define the normal vector to be pointing *out* of the surface (Fig. 24-3). The normal vector points at right angles, or normal, to the plane of the area and has a magnitude equal to the area (Fig. 24-4).

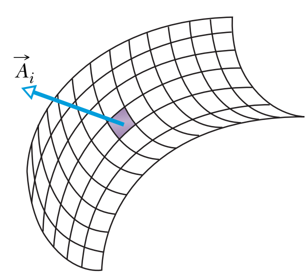


FIGURE 24-3 ■ In order to make net flux calculations, a curved surface area must be divided into N small area elements. Each element must be small enough so it is essentially flat and has electric field vectors that have the same magnitude and direction at every location on a given surface element. The ith area element and its normal vector are shown assuming that an outside piece of a closed surface is being shown here.

Although *electric flux* does not involve the flow of anything, we define it in a way mathematically analogous to volume flux introduced in Chapter 15. An **electric flux element** is defined as the dot product of the normal vector representing an area element and the electric field vector at the location of the area element as shown in Fig. 24-2 and in Eq. 24-2,

$$\Phi_i \equiv (E_i)(\Delta A_i)\cos\theta = \vec{E}_i \cdot \Delta\vec{A}_i \qquad \text{(electric flux definition for a small area),} \qquad (24\text{-}2)$$

where E_i and ΔA_i are magnitudes while θ is the angle between the two vectors. If a curved surface like the one in Fig. 24-3 is broken into small area elements, each of the $\Delta\vec{A}_i$ vectors can point in different directions.

*Our use of the symbol ΔA_i instead of just A_i is to signify that the areas are very small. In this context, the delta does not signify change.

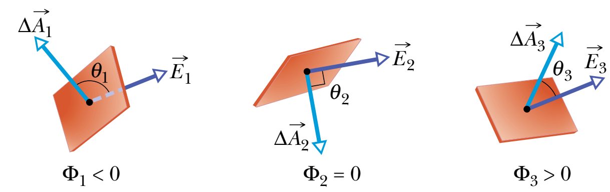


FIGURE 24-4 ■ Three small areas that subtend different angles with respect to various electric field vectors. The first flux element is negative, the second zero, and the third positive. Note that nothing is "flowing" in the case of electric flux to exist.

As is the case for volume flux, if our area is not small enough to be considered as flat or if the electric field vectors are not uniform over the area we choose, then we must break the area into smaller elements that are essentially flat (Fig. 24-4). We can then determine the net electric flux as the sum of individual flux elements. For N flux elements, this is given by

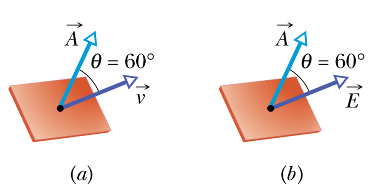
$$\begin{aligned}\Phi^{\text{net}} &= \Phi_1 + \Phi_2 + \cdots + \Phi_N \\ &= \vec{E}_1 \cdot \Delta\vec{A}_1 + \vec{E}_2 \cdot \Delta\vec{A}_2 + \cdots + \vec{E}_N \cdot \Delta\vec{A}_N \qquad \text{(net electric flux)}, \qquad (24\text{-}3) \\ &= \sum_{n=1}^{N} \vec{E}_n \cdot \Delta\vec{A}_n\end{aligned}$$

where $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$, and so on represent the electric field vectors at the location of each of the N area elements. The flux associated with an electric field is a scalar, and its SI unit is the newton-meter-squared per coulomb or $[\text{N} \cdot \text{m}^2/\text{C}]$.

Some possible orientations for area elements and electric field vectors needed to calculate electric flux elements are shown in Fig. 24-4.

In everyday language the term flux is often used to represent flow or change. This is suggested by expressions such as "an influx of population" or "the economy is in a state of flux." These popular uses of the word flux can be deceptive when applied to electrostatic phenomena that we are dealing with in Chapters 22 through 25. Electric flux can be defined whenever an electric field exists, even when an electric field is static and not changing. Furthermore, even if a redistribution of charges causes an electric field to change over time, the changing flux associated with electric field is not related to the flow of anything.

> Instead of representing change or flow, **electric flux** at an area represents the summation over a surface of flux elements. Each flux element represents the product of an essentially flat area element on the surface and the component of the electric field vector that lies along the normal to that area element.



READING EXERCISE 24-1: The figure shows two situations in which the angle between a field vector and the normal vector representing the orientation of the area is $\theta = 60°$. Assume the magnitude of the area in each case is $\Delta A = 2 \times 10^{-4}\ \text{m}^2$. (a) If the imaginary area element is placed at a location in a stream where the magnitude of the stream velocity is $v = 3$ m/s, what is the volume flux through the area? Is anything flowing through the area element? If so, what? (b) Suppose the imaginary area element is placed in an electric field where the magnitude of the field vector is $E = 3$ N/C. What is the electric flux through the area element? Is anything flowing through the area element? If so, what? ■

24-3 Net Flux at a Closed Surface

In the introductory section we posed the question of whether there is a proportionality between an enclosed charge distribution and the flux at a surface that encloses it. To answer this question we need to examine carefully the procedures for determining net electric flux at an imaginary surface that encloses charges. The word "enclose" is important here. In the discussion that follows, we will not be discussing calculations of electric flux at any arbitrary surface. We will limit our discussion to the electric flux at closed surfaces that are continuous and connected. That is, a **closed** surface must be without cuts or edges. Nothing can get into or out of such surfaces without passing through the surface itself.

In order to define the net electric flux at any closed surface, consider Fig. 24-5, which shows an arbitrary (irregularly shaped) imaginary surface immersed in a *nonuniform* electric field. For historical reasons, any imaginary closed surface used in the calculation of a net electric flux is called a **Gaussian surface.** Since the electric field vector might be different at each location on our Gaussian surface, we must divide the entire surface into small area elements and take the sum as shown in Eq. 24-3.

Let's consider the arbitrary closed surface shown in Fig. 24-5. The vectors $\Delta\vec{A}_i$ and $\vec{E}_i$ for each square have some angle θ_i between them. Figure 24-5 shows an enlarged view of three small squares (1, 2, and 3) on the Gaussian surface, and the angle θ_i between $\vec{E}_i$ and $\Delta\vec{A}_i$. Our net flux equation (Eq. 24-3) instructs us to visit each square on the Gaussian surface, to evaluate the scalar product $\vec{E}_i \cdot \Delta\vec{A}_i$ at the location of each, and to sum the results algebraically (that is, with signs included) for all the squares that make up the surface. The sign or a zero resulting from each scalar product determines whether the flux at a square is positive, negative, or zero. Squares like 1, in which $\vec{E}_1$ points inward, make a negative contribution to the sum. Squares like 2, in which $\vec{E}_2$ lies in the surface, make zero contribution. Squares like 3, in which $\vec{E}_3$ points outward, make a positive contribution. (Note that the particular signs for the flux elements discussed above are a consequence of the convention adopted on the previous page; the area vectors point outward for closed surfaces.)

The exact definition of the flux of the electric field at a surface is found by allowing the area of the squares shown in Fig. 24-5 to become smaller and smaller, approaching a differential limit dA. The normal vectors for each tiny surface area then approach a differential limit $d\vec{A}$. Thus, the electric flux at a closed surface is given by the integral of the electric field components parallel to the normal of each surface area element over the magnitude of each surface area element. In mathematical notation the equation for electric flux becomes

$$\Phi^{\text{net}} \equiv \lim_{\Delta\vec{A}\to 0} \sum_{i=1}^{N} \vec{E}_i \cdot \Delta\vec{A}_i$$

$$= \oint \vec{E} \cdot d\vec{A} \qquad \text{(net electric flux at a Gaussian surface).} \qquad (24\text{-}4)$$

The circle on the integral sign indicates that the integration is to be taken over the entire closed surface (Gaussian surface).

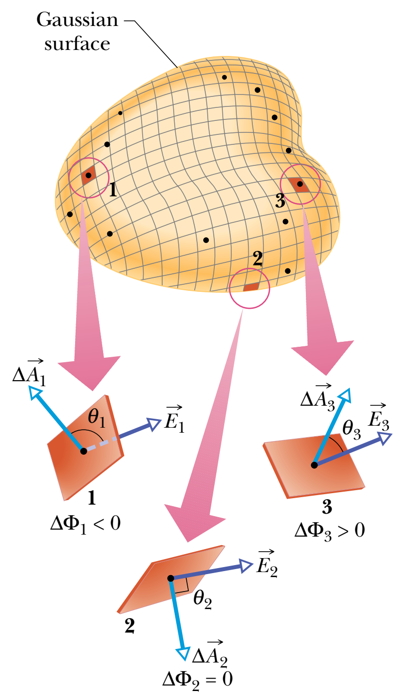


FIGURE 24-5 ■ A Gaussian surface of arbitrary shape is immersed in an electric field. The surface is divided into small area elements. The electric field vectors and the area vectors are shown for three representative area elements marked 1, 2, and 3. The other electric field vectors are not shown.

TOUCHSTONE EXAMPLE 24-1: Net Flux for a Uniform Field

Figure 24-6 shows a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field $\vec{E}$, with the cylinder axis parallel to the field. What is the flux Φ^{net} of the electric field through this closed surface?

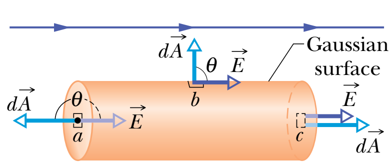


FIGURE 24-6 ■ A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

SOLUTION ■ The **Key Idea** here is that we can find the flux Φ through the surface by integrating the scalar product $\vec{E}\cdot d\vec{A}$ over the Gaussian surface. We can do this by writing the flux as the sum of three terms: integrals over the left disk cap a, the cylinder surface b, and the right disk cap c. Thus, from Eq. 24-4,

$$\Phi^{\text{net}} = \oint \vec{E}\cdot d\vec{A}$$

$$= \int_a \vec{E}\cdot d\vec{A} + \int_b \vec{E}\cdot d\vec{A} + \int_c \vec{E}\cdot d\vec{A}. \qquad (24\text{-}5)$$

For all points on the left cap, the angle θ between $\vec{E}$ and $d\vec{A}$ is 180°, and the magnitude E of the field is constant. Thus,

$$\int_a \vec{E}\cdot d\vec{A} = \int E(\cos 180°)\, dA = -E\int dA = -EA,$$

where $\int dA$ gives the cap's area, $A(=\pi R^2)$. Similarly, for the right cap, where $\theta = 0$ for all points,

$$\int_c \vec{E}\cdot d\vec{A} = \int E(\cos 0°)\, dA = E\int dA = +EA.$$

Finally, for the cylindrical surface, where the angle θ is 90° at all points,

$$\int_b \vec{E}\cdot d\vec{A} = \int E(\cos 90°)\, dA = 0.$$

Substituting these results into Eq. 24-5 leads us to

$$\Phi = -EA + EA = 0. \qquad \text{(Answer)}$$

This result is perhaps not surprising because the field lines that represent the electric field all pass entirely through the Gaussian surface, entering through the left end cap, leaving through the right end cap, and giving a net flux of zero.

TOUCHSTONE EXAMPLE 24-2: Flux for a Nonuniform Field

A *nonuniform* electric field given by $\vec{E} = (3.0\ \text{N/C}\cdot\text{m})x\hat{\text{i}} + (4.0\ \text{N/C})\hat{\text{j}}$ pierces the Gaussian cube shown in Fig. 24-7. What is the electric flux through the right face, the left face, and the top face?

SOLUTION ■ The **Key Idea** here is that we can find the flux Φ through the surface by integrating the scalar product $\vec{E}\cdot d\vec{A}$ over each face.

Right face: An area vector $\vec{A}$ is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector $d\vec{A}$ for the right face of the cube must point in the positive x direction. In unit vector notation, then,

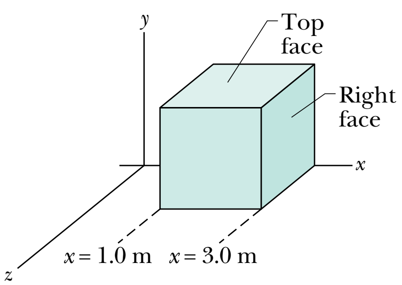


FIGURE 24-7 ■ A Gaussian cube with one edge on the x axis lies within a nonuniform electric field.

$$d\vec{A} = dA\,\hat{\text{i}}.$$

From Eq. 24-4, the flux Φ_r through the right face is then

$$\begin{aligned}\Phi_r &= \int \vec{E}\cdot d\vec{A} = \int [(3.0\ \text{N/C}\cdot\text{m})x\hat{\text{i}} + (4.0\ \text{N/C})\hat{\text{j}}]\cdot(dA\hat{\text{i}}) \\ &= \int [(3.0\ \text{N/C}\cdot\text{m})(x)(dA)\hat{\text{i}}\cdot\hat{\text{i}} + (4.0\ \text{N/C})(dA)\hat{\text{j}}\cdot\hat{\text{i}}] \\ &= \int (3.0\ \text{N/C}\cdot\text{m})x\,dA + (0.0\ \text{N}\cdot\text{m}^2/\text{C}) = (3.0\ \text{N/C}\cdot\text{m})\int x\,dA.\end{aligned}$$

We are about to integrate over the right face, but we note that x has the same value everywhere on that face—namely, $x = 3.0$ m. This means we can substitute that constant value for x. Then

$$\Phi_r = (3.0\ \text{N/C}\cdot\text{m})\int (3.0\ \text{m})\,dA = (9.0\ \text{N/C})\int dA.$$

Now the integral merely gives us the area $A = 4.0\ \text{m}^2$ of the right face, so

$$\Phi_r = (9.0\ \text{N/C})(4.0\ \text{m}^2) = 36\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

Left face: The procedure for finding the flux through the left face is the same as that for the right face. However, two factors change. (1) The differential area vector $d\vec{A}$ points in the negative x direction and thus $d\vec{A} = -dA\hat{\text{i}}$. (2) The term x again appears in our integration, and it is again constant over the face being considered. However, on the left face, $x = 1.0$ m. With these two changes, we find that the flux Φ_l through the left face is

$$\Phi_l = -12\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

Top face: The differential area vector $d\vec{A}$ points in the positive y direction and thus $d\vec{A} = dA\hat{\text{j}}$. The flux Φ_t through the top face is then

$$\begin{aligned}\Phi_t &= \int [(3.0\ \text{N/C}\cdot\text{m})x\hat{\text{i}} + (4.0\ \text{N/C})\hat{\text{j}}]\cdot(dA\hat{\text{j}}) \\ &= \int [(3.0\ \text{N/C}\cdot\text{m})(x\,dA)\hat{\text{i}}\cdot\hat{\text{j}} + (4.0\ \text{N/C})(dA)\hat{\text{j}}\cdot\hat{\text{j}}] \\ &= (0.0\ \text{N}\cdot\text{m}^2/\text{C}) + \int (4.0\ \text{N/C})\,dA) = (4.0\ \text{N/C})\int dA \\ &= 16\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}\end{aligned}$$

24-4 Gauss' Law

Let's return for a moment to the consequence of Coulomb's law we presented in the first section, where we surrounded a single charge with a spherical Gaussian surface. We found that a flux-like quantity (namely, the product of the magnitude of the electric field at the sphere's surface multiplied by the area of the sphere's surface) is equal to a constant times the enclosed charge. The surprising thing is this is true no matter what the radius of the sphere is, because the amount by which the surface area of the sphere increases just compensates for the amount by which the electric field magnitude decreases. This suggests that the net flux through a Gaussian surface of any shape enclosing a single charge will be proportional to the amount of charge enclosed.

Visualizing Flux through a Gaussian Surface

Since the relationship between flux and charge enclosed by a Gaussian surface is hard to visualize in three dimensions, let's consider the special case of an infinitely long rod that has a uniform charge density. While infinitely long rods do not exist, our result will be valid providing the Gaussian surfaces are far from the ends of the rod. Imagine a Gaussian surface that has the shape of a coin and surrounds a small segment of the rod. This is shown in Fig. 24-8.

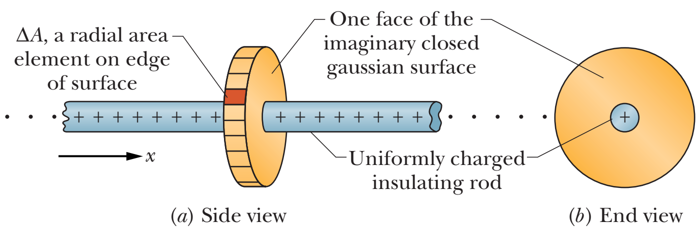


FIGURE 24-8 ■ (*a*)An infinitely long uniformly charged rod has a Gaussian surface that looks like a coin with front and back faces that are perpendicular to the rod and enclose a small charge. (*b*) An end view of the rod and Gaussian surface face can help us visualize flux at the surface's edges.

Because the charged rod is infinitely long it is symmetric about any point on it. As we showed in Section 23-5 (see Fig. 23-12), it turns out the electric field vectors created by a symmetric pair of charges point outward in a radial direction and have no components parallel to the line that the charges lie on (in this case, the line determined by the rod). We can also show that for a thin rod the field magnitude falls off as $1/r$ where r is the radial distance from the center of the rod. (Likewise, a similar negatively charged rod has electric field vectors pointing radially inward). The key factor in surrounding a piece of long rod with a coin-shaped closed surface is that all the flux at the surface will be at the edges and there will be no flux at the faces of the surface. For this reason, we can calculate and depict the "amount" of flux at elements of area on the edges of the surface by looking at an end view of the rod. This is true not only for coin-like closed surfaces that have circular faces but also for any shaped faces so long as the two faces are parallel to each other and perpendicular to the rod. End views depicting flux amounts as green rectangles are shown in Fig. 24-9 for three different imaginary Gaussian surfaces outlined in red.

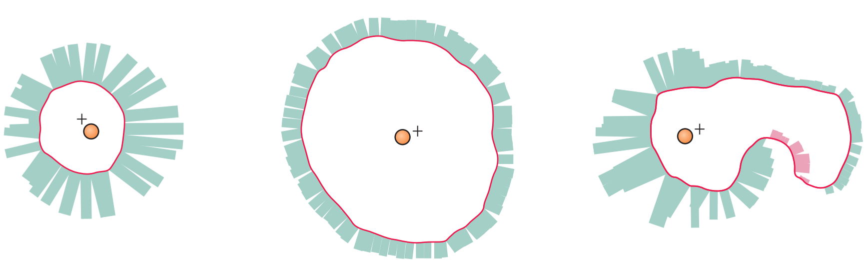


FIGURE 24-9 ■ Three imaginary Gaussian surfaces surround the same point charge. Here the red lines show only a two-dimensional cross section of three-dimensional surfaces. The contribution of electric flux at a series of small area elements is calculated and represented by rectangles. Note that whenever part of a surface is close to the charge, the flux elements are bigger but there are fewer of them. We can see visually that the net flux (which is proportional to the area occupied by all the outgoing flux (shown as green) minus the incoming flux (shown as pink) is approximately the same in the three cases.

A small bundle of enclosed charge yields the same net electric flux at a Gaussian surface no matter what the shape of the surface. By superposition, if there are two charges enclosed by a Gaussian surface, each charge contributes its proportional share to the net flux no matter where each of the charges is located, provided both are *inside* the Gaussian surface. This leads us to a statement of Gauss' law that describes a plausible general relationship between the net flux through a Gaussian surface of any shape and the total enclosed charge no matter how it is distributed.

GAUSS' LAW: The net flux through any imaginary closed surface is directly proportional to the net charge enclosed by that surface.

Based on consideration of SI units, the constant of proportionality must be $1/\varepsilon_0$ where ε_0 is the permittivity constant, so that the mathematical expression of Gauss' law is

$$\Phi^{\text{net}} = \frac{q^{\text{enc}}}{\varepsilon_0} \qquad \text{(Gauss' law).} \tag{24-6}$$

By substituting the definition of electric flux at a Gaussian surface, $\Phi^{\text{net}} \equiv \oint \vec{E} \cdot d\vec{A}$, we can also write Gauss' law as

$$\Phi^{\text{net}} = \oint \vec{E} \cdot d\vec{A} = \frac{q^{\text{enc}}}{\varepsilon_0} \qquad \text{(Gauss' law).} \tag{24-7}$$

Here, the circle on the integral sign indicates that the surface over which we integrate must be "closed." The use of the permittivity constant for a vacuum, ε_0, in Eqs. 24-6 and 24-7 indicates this form of Gauss' law only holds when the net charge is located in air or some other medium that doesn't polarize easily. In Section 28-6, we modify Gauss' law to include situations in which so-called dielectric materials that can polarize, such as paper, oil, or water, are present. In Fig. 24-10 we show how the net flux can have the same value for two different charge distributions involving the same amount of enclosed charge.

Gauss' law is useful for finding both charge and flux. That is, if we can calculate the net flux through a closed surface, we can deduce the amount of charge enclosed. On the other hand, if we know the amount of charge enclosed, we can use Gauss' law to deduce the net flux through any surface that encloses the charge.

Interpreting Gauss' Law

One use of Gauss' law is to calculate how much net charge is contained inside any closed surface. To make the calculation, you need know only the net electric flux at the surface enclosing the collection of charges. This net flux is related to the strength of the normal components of the electric field at all locations on the surface.

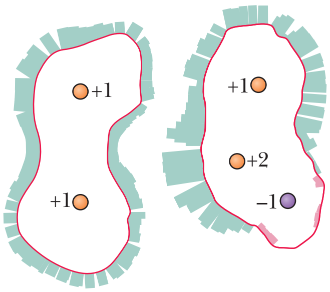


FIGURE 24-10 ■ Each Gaussian surface encloses a different charge distribution but encloses the same net charge. The electric flux calculated at the edges of the surface is represented by green rectangles (outward flux) or pink rectangles (inward flux). The total space covered by all of the green rectangles minus that occupied by the pink rectangles turns out to be the same for the two situations, which is compatible with the predictions of Gauss' law.

In Eqs. 24-6 and 24-7, the net charge q^{enc} is the algebraic sum of all the *enclosed* positive and negative charges, and it can be positive, negative, or zero. We include the sign, rather than just use the amount of enclosed charge, because the sign tells us something about the net flux at the Gaussian surface. Here we continue to use our convention that the normal area vectors representing the area elements of a closed surface point *outward*. If the net charge enclosed, q^{enc}, is positive, its electric field vectors point mostly outward too. This leads to a net flux that is *outward* and positive as shown in Fig. 24-11*a*. If q^{enc} is negative, the area vector still points outward but the electric field vector points inward. This leads to a net flux that is *inward* and negative, as shown in Fig. 24-11. Figure 24-11*c* shows how positive and negative charges inside a Gaussian surface can lead to zero net flux.

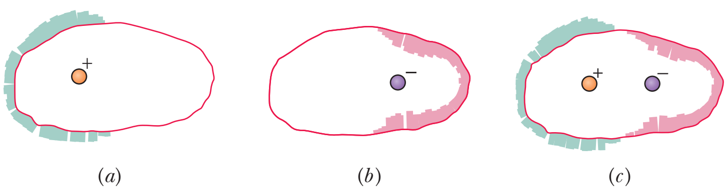


FIGURE 24-11 ■ Each of these Gaussian surfaces has the same shape. (*a*) One unit of enclosed positive charge causes a positive net outward flux shown in green. (*b*) One unit of enclosed negative charge causes a negative net inward flux shown in pink. Note that the amount of negative flux is the same as the amount of positive flux shown in the previous diagram. (*c*) If both the positive and negative charges are enclosed the net charge is zero and so is the net flux.

Charge outside a Gaussian surface, no matter how large or how close it may be, is not included in the term q^{enc} in Gauss' law. We expect this since there is no source of electric field inside the surface, and negative and positive flux elements will cancel each other, as shown in Fig. 24-12. The exact form or location of the charges inside the Gaussian surface is also of no concern; the only things that matter are the amount of the net charge enclosed and its sign. The quantity $\vec{E}$ on the left side of Eq. 24-7, however, is the electric field resulting from *all* charges, both those inside and those outside the Gaussian surface. This may seem to be inconsistent, but keep in mind the electric field due to a charge outside the Gaussian surface contributes zero net flux on the surface (as shown in Fig. 24-12). This is the case even though a charge outside the surface does contribute to the actual values of the electric field at each point on the surface.

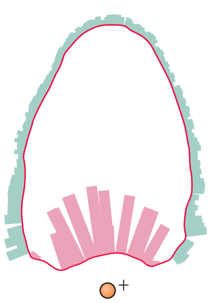


FIGURE 24-12 ■ A charge element along a rod is located *outside* a Gaussian surface. When the electric flux is calculated at each area element using Coulomb's law, its outward values are represented by green rectangles and the inward flux by pink rectangles. The net flux is zero because the negative inward flux at the portion of the surface near the charge just cancels the positive outward flux at the location of the portions of the surface far away from the charge.

Let us apply these ideas to Fig. 24-13, which shows the electric field lines surrounding two point charges, equal in amount but opposite in sign. Four Gaussian surfaces are also shown, in cross section. Let us consider each in turn.

Surface S_1 (encloses only the positive charge): The electric field is dominated by the nearby positive charge and so points outward for the majority of the points on this surface. Thus, the flux of the electric field at this surface is positive, and so is the net charge within the surface, as Gauss' law requires. (That is, if Φ is positive, q^{enc} must be also.)

Surface S_2 (encloses only the negative charge): The electric field is dominated by the nearby negative charge and so points inward for the majority of the points on this surface. Thus, the flux of the electric field is negative and so is the enclosed charge, as Gauss' law requires.

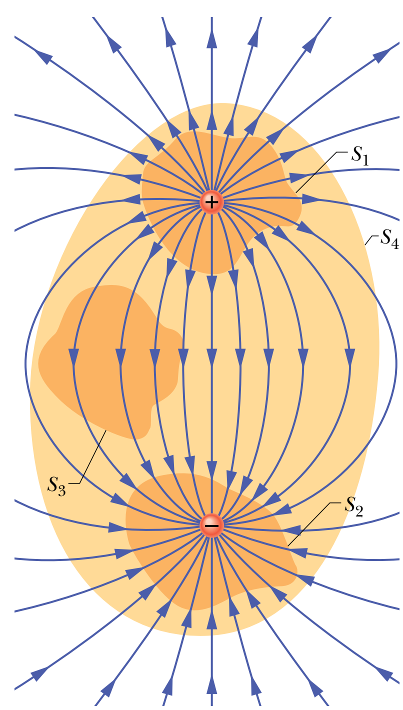


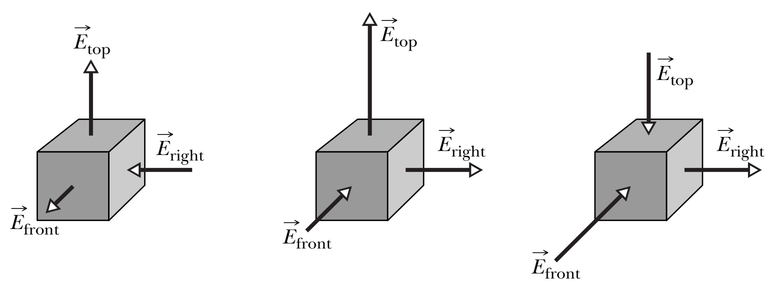
FIGURE 24-13 ■ An idealization showing two point charges of equal amount and opposite sign are shown with the field lines that depict their net electric field as if all lines lie in a plane. The cross sections of four Gaussian surfaces are shown. Surface S_1 encloses the positive charge, S_2 encloses the negative charge, and S_3 encloses no charge. Since S_4 surrounds both charges, it encloses no net charge.

Surface S_3 (encloses no charges): Since $q^{\text{enc}} = 0$ and there are comparable contributions to the electric field at points on the surface from both charges, the field on some parts of the surface will point out and on other parts it will point in. Gauss' law (Eq. 24-7) requires the net electric flux through this surface to be zero. That is reasonable because in calculating the net flux, the inward and outward flux elements cancel each other.

Surface S_4 (encloses both charges): This surface encloses no *net* charge, because equal amounts of positive and negative charge are enclosed. Gauss' law requires the net flux of the electric field at this surface be zero. That is reasonable because in this case the field vectors point outward for the portion of the surface nearest to the positive charge (yielding positive flux) and inward for the portion of the surface near the negative charge (yielding negative flux). In calculating the net flux, the positive and negative flux elements cancel each other, even though the field is nonzero along most of the surface.

What would happen if we were to bring an enormous charge Q up close to (but still outside of) surface S_4 in Fig. 24-13? The pattern of the electric field would certainly change, but the net flux for the four Gaussian surfaces would not change. We can understand this because the inward and outward flux elements associated with the added Q at any of the four surfaces would cancel each other, making no contribution to the net flux at any of them. The value of Q would not enter Gauss' law in any way, because Q lies outside all four of the Gaussian surfaces that we are considering.

READING EXERCISE 24-2: The figure shows three situations in which a Gaussian cube sits in an electric field. The arrows indicate the directions of the electric field vectors for the top, front, and right faces of each cube. The flux at the six sides of each cube is listed in the table below. In which situations do the cubes enclose (a) a positive net charge, (b) a negative net charge, and (c) zero net charge?



Flux [N · m²/C]

Face	Cube 1	Cube 2	Cube 3
Front	+2	−4	−7
Back	−3	−3	+8
Left	+7	+3	+2
Right	−4	+5	+5
Top	+5	+10	−6
Bottom	−7	−6	−5

24-5 Symmetry in Charge Distributions

Why go through all this trouble to develop a method of calculating electric fields that is equivalent to Coulomb's law? We suggested in the introduction to this chapter that it is because Gauss' law makes it possible to calculate the field for highly symmetric charge distributions. What we mean by symmetric charge distributions are arrangements of charges that can be rotated about an axis or reflected in a mirror and still look the same. Figure 24-14 shows several examples of symmetric objects.

Why do charge distributions need to be symmetric in order for Gauss' law to be helpful in finding an electric field? Because we can use symmetry arguments to find the direction of the electric field and surfaces along which it is constant. This allows us to choose an imaginary Gaussian surface over which the electric field is constant. Then we can take the dot product and turn the vectors into scalar magnitudes. Finally, we know the electric field magnitude is constant at the surface we are integrating over, so we can pull the electric field vector outside of the integral sign. By following the steps we outlined, in some cases Gauss' law can be reduced to

$$\varepsilon_0 \oint E \cos\theta \, dA = \varepsilon_0 E \oint \cos\theta \; dA = |q^{\text{enc}}|.$$

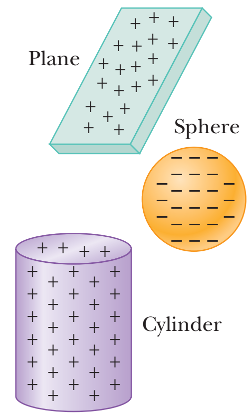


FIGURE 24-14 ■ Some symmetrically charged objects—a plane, a sphere, and a cylinder.

Better still is to be able to find a Gaussian surface over which both the electric field and the angle between the field and area vectors, θ, are constant over the entire area. In that case, both the electric field and the cosine functions can be moved outside the integral and Gauss' law reduces to:

$$(\varepsilon_0 E \cos\theta) \oint dA = |q^{\text{enc}}|.$$

This expression is very easy to evaluate because the integral of dA is simply the magnitude of the total area of the Gaussian surface, which we will denote as A. Hence, if we can find a Gaussian surface over which the field and angle θ are constant, Gauss' law allows us to calculate the electric field of an extended charge distribution without doing an integral. In those cases, Gauss' law tells us that the electric field magnitude is

$$E = \frac{|q^{\text{enc}}|}{\varepsilon_0 A \cos\theta} \qquad \text{(constant } E \text{ and } \theta\text{)}, \qquad (24\text{-}8)$$

where A is the area of the Gaussian surface, θ is the angle between the field and each area vector, and q^{enc} is the net charge *enclosed by the Gaussian surface*. In some cases where the angle, θ, has one value for some parts of a surface and another value for other parts of a surface, we can handle the calculation by breaking the surface integral into parts.

A word of caution: There are only a few charge distributions with sufficient symmetry for Gauss' law to be useful. These include single point charges and spherically symmetric ones. Charge distributions that work with Gauss' law also include the infinitely long cylinder, with cylindrical symmetry, and that of a uniformly charged slab with infinitely long sides with planar symmetry. Fortunately, there are many physical situations for which these geometries are important. Hence, Gauss' law is an extraordinarily useful tool.

However, for many charge distributions, we cannot use Gauss' law to find the field because the flux integral on the left-hand side of the expression

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q^{\text{enc}}$$

is too complicated to evaluate. In these cases, Gauss' law is still valid but not useful.

24-6 Application of Gauss' Law to Symmetric Charge Distributions

As we determined in the last section, Gauss' law is useful if we already know what the general shape of the vector electric field plot looks like. In some cases we can derive this knowledge from symmetry of the charge distribution without using equations or doing calculations. Only then can we choose an imaginary closed surface and use the mathematical form of Gauss' law to calculate the magnitude of the electric field at points on the surface. In this section we take this approach to determining the electric field for three highly symmetric charge distributions.

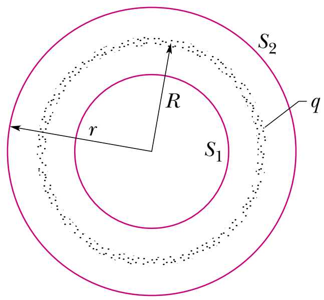


FIGURE 24-15 ■ A thin, charged, spherical shell with total charge q, in cross section. Two Gaussian surfaces S_1 and S_2 are also shown in cross section. Surface S_2 encloses the shell, and S_1 encloses only the empty interior of the shell.

Spherical Symmetry for a Shell of Charge

Figure 24-15 shows a charged spherical shell of total positive charge q and radius R and two concentric spherical Gaussian surfaces, S_1 and S_2. (Note that we chose the shape of the Gaussian surface to mirror the symmetry of the charge distribution.) Because the charge distribution is spherically symmetric no matter how we rotate the spherical shell around its center, the shell looks the same. This means that the electric field must have a spherical symmetry too. Thus, it must have the same magnitude at every point on the spherical Gaussian surface S_2 and it must point in a radial direction. Further, since the area vector points radially outward at all points on S_2, the angle between the electric field $\vec{E}$ and the area $\vec{A}$ is constant. As a result of the spherical symmetry of the distributed charge, we know the electric field also points in a radial direction at all points on S_2. Hence, the angle θ is not only constant but it is also 0° at all points on the surface. Applying Gauss' law to surface S_2 then comes down to evaluating the expression for the electric field magnitude that we derived in Eq. 24-8 for constant E and θ,

$$E = \frac{|q^{\text{enc}}|}{\varepsilon_0 A \cos\theta}.$$

Note that $\cos\theta = \cos 0 = 1$ and the area of a sphere (the Gaussian sphere) of radius r is $4\pi r^2$. Hence, for any $r \geq R$, we find that

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q^{\text{enc}}|}{r^2} \quad \text{(spherical shell, field at } r \geq R\text{)}. \tag{24-9}$$

What is surprising is that outside the shell the electric field is the same as if the shell of charge were replaced by a single point-like charge, q, provided that the single charge is placed where the center of the shell of charge was. Thus, if the charge on a shell is evenly distributed, a shell of total charge q would produce the same force on a small test charge placed anywhere *outside* the shell as a single point-like charge q would.

> A shell with a uniform charge distribution attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.

This shell theorem is identical to the one developed by Isaac Newton for gravitation in Section 14-2.

What happens to the electric field inside the shell of charge? Applying Gauss' law to surface S_1, for which $r < R$, leads directly to

$$\vec{E} = 0\ [N/C] \quad \text{(spherical shell, field at } r < R\text{)}, \tag{24-10}$$

because this Gaussian surface encloses no charge. Thus, when a small test charge is enclosed by a shell of uniform charge distribution, the shell exerts no net electrostatic force on it.

> A shell of uniform charge exerts no electrostatic force on a charged particle that is located inside the shell.

A Spherically Symmetric Charge Distribution

Any spherically symmetric charge distribution, such as that of Fig. 24-16, can be constructed with a nest of concentric spherical shells. This is a good starting point for treating a wide variety of charged objects with nearly spherical distribution of charge such as nuclei and atoms. For purposes of applying the two shell theorems stated above, the volume charge density ρ, defined as the charge per unit volume, should have a single value for each shell but need not be the same from shell to shell. Thus, for the charge distribution as a whole, ρ, can vary only with r, the radial distance from the center of the sphere and not with direction. We can then examine the effect of the charge distribution "shell by shell."

In Fig. 24-16*a* the entire charge lies within a Gaussian surface with $r > R$. The charge produces an electric field on the Gaussian surface as if the charge were a point charge located at the center, and Eq. 24-9 holds.

Figure 24-16*b* shows a Gaussian surface with $r < R$. To find the electric field at points on this Gaussian surface, we consider two sets of charged shells—one set inside the Gaussian surface and one set outside. The charge lying *outside* the Gaussian surface does not set up a net electric field on the Gaussian surface. Gauss' law tells us that the charge *enclosed* by the surface sets up an electric field as if that enclosed charge were concentrated at the center. Letting q' represent that enclosed charge, we can then write the electric field magnitude as

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{|q^{\text{enc}}|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q'(r)|}{r^2} \quad \text{(spherical distribution, field at } r < R\text{)}, \tag{24-11}$$

where the term $q'(r)$ signifies that q' depends on r. (It is not the product of q' and r.)

Equation 24-11 is valid for any spherically symmetric charge distribution, even one that is not uniform. For example, Fig. 24-16 shows a situation in which the volume charge density is spherically symmetric but larger near the center of the sphere than further out. In other words, Eq. 24-11 is valid whenever $\rho = \rho(r)$ or ρ is a constant. But the equation is not useful unless we know how to use a knowledge of the volume charge density to determine the charge q' enclosed by a sphere of radius r.

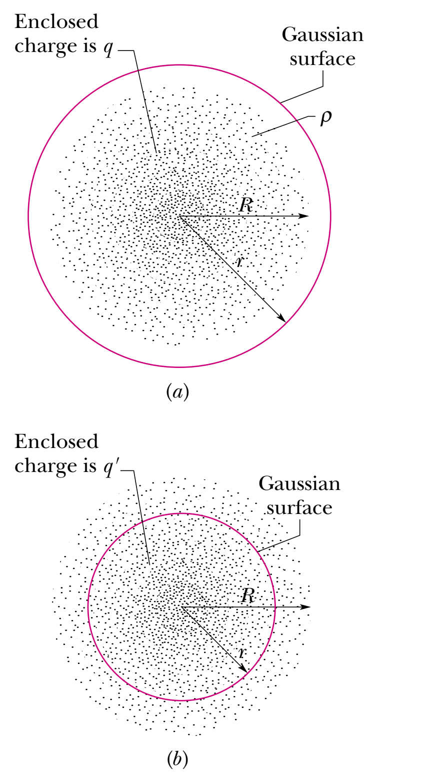


FIGURE 24-16 ■ Spherically symmetric distributions of charge of radius R, whose volume charge density ρ is a function only of distance from the center. The charged object is not a conductor, so the charge is assumed to be fixed in position. A cross-section of concentric spherical Gaussian surface with $r > R$ is shown in (a). A similar Gaussian surface with $r < R$ is shown in (b).

Spherical Symmetry for a Uniform Volume Charge Distribution

Consider the simple case where the charge is distributed uniformly through the volume of a sphere of radius R containing an excess charge q. In this case it is possible to find the magnitude of the electric field at any location inside the sphere in terms of the total charge in the sphere.

Whenever the total charge q enclosed within a sphere of radius R is distributed uniformly, we can use the definition of volume charge density (presented in Table 23-2) and the knowledge that the volume of a sphere of radius R is given by $\frac{4}{3}\pi R^3$ to write

$$\rho \equiv \frac{q}{V} = \frac{q}{\frac{4}{3}\pi R^3}. \tag{24-12}$$

Since the charge density is a constant the amount of charge in a smaller sphere of radius r is proportional to its volume. Since its volume is $V' = \frac{4}{3}\pi r^3$ then $q' = \rho V' = \rho\,(\frac{4}{3}\pi r^3)$. Substituting Eq. 24-12 for ρ gives

$$q' = q\frac{r^3}{R^3}. \tag{24-13}$$

Substituting this into Eq. 24-11 gives us the electric field magnitude in terms of the total charge on the sphere.

$$E = \left(\frac{|q|}{4\pi\varepsilon_0 R^3}\right)r \qquad \text{(uniform volume charge density for } r \leq R). \qquad (24\text{-}14)$$

Cylindrical Symmetry for a Uniform Line Charge Distribution

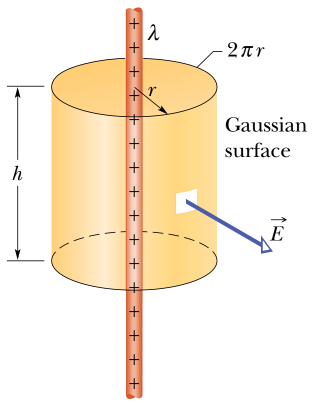


FIGURE 24-17 ■ A Gaussian surface in the form of a closed cylinder surrounds a section of a very long, uniformly charged, cylindrical plastic rod.

Figure 24-17 shows a section of a very long thin cylindrical plastic rod with a uniform distribution of positive charge, so that linear charge density λ (as defined in Table 23-2) is constant. Let us find an expression for the magnitude of the electric field $\vec{E}$ outside of the rod at a distance r from its axis in terms of the linear charge density of the rod. In doing so, we assume that r is small compared to the length of the rod so that we can ignore the effect of the rod's ends.

We start by choosing a Gaussian surface that matches the cylindrical symmetry of the rod. So our imaginary surface is a circular cylinder of radius r and length h, coaxial with the rod. The Gaussian surface must be closed, so we include two end caps as part of the surface. We pick the end caps of the Gaussian surface so they are far from the end of the rod.

Imagine that, while you are not watching, someone rotates the plastic rod around its longitudinal axis or moves it a finite distance along the axis. When you look again at the rod, you will not be able to detect any change in either the appearance of the rod or the behavior of the electric field that surrounds it. Furthermore, when we experiment, we find that if the rod is flipped end for end we still detect no change in the rod's electric field. What does this tell us about the nature of the electric field? If the electric field has only a component that points radially inward or outward from the rod, then the field should be unaffected by the changes in orientation that we have discussed. If however, the field had any component tangent to the rod's surface, pointed toward or away from the rod, we would detect a change in the electric field as we rotated or flipped the rod. Hence, we conclude from these symmetry arguments that at every point on the cylindrical part of the Gaussian surface, the electric field must have the same magnitude $E = |\vec{E}|$ and must be directed radially outward (for a positively charged rod).

Since $2\pi r$ is the circumference of the cylinder and h is its height, the area A of the cylindrical surface is $2\pi rh$. The flux of $\vec{E}$ at this cylindrical surface is then

$$\Phi = EA\cos\theta = E(2\pi rh)\cos 0 = E(2\pi rh).$$

There is no flux at the end caps because $\vec{E}$, being radially directed, is parallel to the end caps at every point, so $\vec{E}$ is perpendicular to the normal and the dot product vanishes. Thus the flux through the cylindrical surface is equal to the net flux ($\Phi^{\text{net}} = \Phi$).

According to Gauss' law, shown in Eq. 24-6,

$$\Phi^{\text{net}} = \frac{q^{\text{enc}}}{\varepsilon_0}.$$

We can find the enclosed charge in terms of the linear charge density, defined as the charge per unit length. If the charge enclosed by the surface that encompasses a length h of the rod has a uniform density λ, then $q^{\text{enc}} = \lambda h$. Thus, the previous two equations reduce to $E(2\pi rh) = |\lambda|h/\varepsilon_0$, so that

$$E = \frac{|\lambda|}{2\pi\varepsilon_0 r} \qquad \text{(long line of uniformly distributed charge).} \qquad (24\text{-}15)$$

This is the expression for the electric field magnitude due to a very long, straight line of uniformly distributed charge, at a point that is a radial distance r from the line. The direction of $\vec{E}$ is radially outward from the line of charge if the charge is positive, and radially inward if it is negative. Equation 24-15 also approximates the field of a *finite* line of charge, at points that are not too near the ends (compared with the distance from the line).

A Sheet of Uniform Charge

Figure 24-18 shows a portion of a thin, very large, sheet with a uniform (positive) surface charge density σ (as defined in Table 23-2). A large sheet of thin plastic wrap, uniformly charged on one side, can serve as a simple example of a nonconducting sheet. A large sheet of aluminum foil serves as an example of a conducting sheet. Let us find the electric field $\vec{E}$ a distance r from the uniformly charged sheet. Here we assume that we are far from the edges of the sheet and that the thickness of the sheet is much less than r.

Even though it doesn't have the same shape as a charged sheet, something called a Gaussian pillbox turns out to make a useful imaginary surface in this case. The pillbox is a closed cylinder with end caps of area A, arranged so that it is perpendicular to the sheet with each end cap located at the same distance from the sheet. This Gaussian pillbox is shown in Fig. 24-18*a*. Using symmetry (considerations like those used earlier in this section or those depicted in Fig. 23-12 and Fig. 23-13 in the previous chapter), $\vec{E}$ must be perpendicular to the sheet and hence to the end caps. Furthermore, since the charge is positive, $\vec{E}$ is directed *away* from the sheet, and thus the electric field vectors point in an outward direction from the two Gaussian end caps. Because the electric field vectors are perpendicular to the normal vectors on the curved surface, there is no flux at this portion of the Gaussian surface. Thus $\vec{E}\cdot d\vec{A}$ is simply EdA—the product of the magnitudes of $\vec{E}$ and $d\vec{A}$. In this case Gauss' law (Eq. 24-7) gives us

$$\oint \vec{E}\cdot d\vec{A} = q^{\text{enc}}/\varepsilon_0.$$

Since there are two caps on our pillbox we need to break the integral into two parts so in terms of the area and electric field magnitudes,

$$EA + EA = \int_{\substack{\text{end}\\\text{caps}}} EdA = |q^{\text{enc}}|/\varepsilon_0.$$

Next we can find the amount of charge on the sheet enclosed by our Gaussian pillbox in terms of the surface charge density, σ, on the sheet. Since the surface charge is uniform and the surface charge density is defined as the ratio of the charge on a given surface to its area, we know that $\sigma = q^{\text{enc}}/A$. If we replace q^{enc} in the equation above with σA and solve it for the electric field magnitude we get

$$E = \frac{|\sigma|}{2\varepsilon_0} \quad \text{(sheet of uniformly distributed charge).} \tag{24-16}$$

The equation holds whether the sheet is conducting or nonconducting as long as the layer of charge on the sheet is thin.

Equation 24-16 tells us that *the electric field has the same value for all locations outside a large uniformly charged sheet and points in a direction that is perpendicular to the sheet*. This result is quite surprising! The fact that the net field is perpendicular to the sheet can be explained using symmetry arguments. But how can it be that as you get farther away from the charged sheet the electric field doesn't decrease? The answer lies in considering the influences of the charges as we move away from the sheet. When a test charge is placed very close to the sheet, the

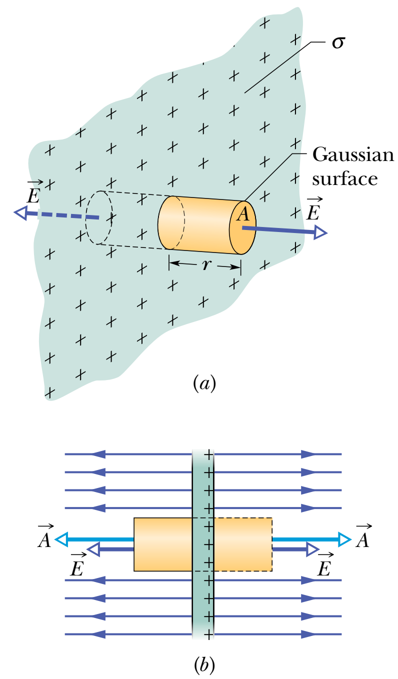
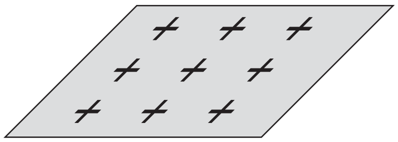


FIGURE 24-18 ■ Perspective view (*a*) and side view (*b*) of a portion of a very large, thin plastic sheet, uniformly charged with surface charge density σ. A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.

influence on it by the charge closest to it dominates. If the test charge is moved farther from the sheet the influence of the nearest sheet charge gets weaker, but the normal components of the electric field vectors from neighboring sheet charges start to contribute and compensate for the loss of influence of the nearest sheet charge. If the test charge is moved even farther the influence of the nearest and nearby charges diminish but the components of additional surrounding charges come into play and so on.

Equation 24-16 agrees with what we would have found by integration of the electric field components that are produced by individual charges. That would be a very time-consuming and challenging integration, and note how much more easily we obtain the result using Gauss' law. This is one reason for devoting a whole chapter to Gauss' law. For certain symmetric arrangements of charge, it is much easier to use it than to integrate field components.

READING EXERCISE 24-3: Consider an array of 9 charges evenly distributed on a square insulating sheet as shown in the diagram. Use symmetry arguments to explain why the electric field vector anywhere on a line normal to the central charge and passing through it has no component that is parallel to the sheet.



TOUCHSTONE EXAMPLE 24-3: $\vec{E}$ for Two Sheets of Charge

Figure 24-19*a* shows portions of two large, parallel, nonconducting sheets, each with a fixed uniform charge on one side. The amounts of the surface charge densities are $\sigma_{(+)} = 6.8\,\mu\text{C/m}^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3\,\mu\text{C/m}^2$ for the negatively charged sheet.

Find the electric field $\vec{E}$ (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.

SOLUTION The **Key Idea** here is that with the charges fixed in place, we can find the electric field of the sheets in Fig. 24-19*a* by (1) finding the field of each sheet as if that sheet were isolated and (2) adding the vector fields of the isolated sheets via the superposition principle. (The vector addition is simple here since the fields lie along the same axis. We can add the fields algebraically because they are parallel to each other.) From Eq. 24-16, the magnitude $E_{(+)}$ of the electric field due to the positive sheet at any point is

$$|\vec{E}_{(+)}| = \frac{|\sigma_{(+)}|}{2\varepsilon_0} = \frac{6.8 \times 10^{-6}\ \text{C/m}^2}{(2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}$$
$$= 3.84 \times 10^5\ \text{N/C}.$$

Similarly, the magnitude $|\vec{E}_{(-)}|$ of the electric field at any point due to the negative sheet is

$$|\vec{E}_{(-)}| = \frac{|\sigma_{(-)}|}{2\varepsilon_0} = \frac{4.3 \times 10^{-6}\ \text{C/m}^2}{(2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}$$
$$= 2.43 \times 10^5\ \text{N/C}.$$

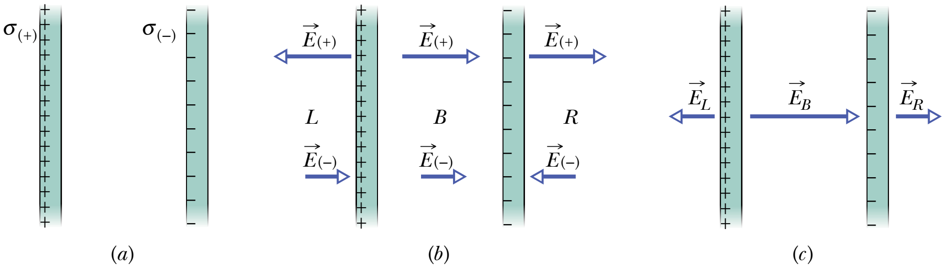


FIGURE 24-19 (*a*) Two large, parallel insulating sheets, uniformly charged on one side. (*b*) The individual electric fields resulting from the two charged sheets. (*c*) The net field due to both charged sheets, found by superposition.

Figure 24-19b shows the fields set up by the sheets to the left of the sheets (L), between them (B), and to their right (R).

The resultant fields in these three regions follow from the superposition principle. To the left of the sheets, the field magnitude is

$$|\vec{E}_L| = |\vec{E}_{(+)}| - |\vec{E}_{(-)}|$$
$$= 3.84 \times 10^5 \text{ N/C} - 2.43 \times 10^5 \text{ N/C}$$
$$= 1.4 \times 10^5 \text{ N/C.} \qquad \text{(Answer)}$$

Because $|E_{(+)}|$ is larger than $|E_{(-)}|$, the net electric field $\vec{E}_L$ in this region points to the left, as Fig. 24-19c shows. To the right of the sheets, the electric field $\vec{E}_R$ has the same magnitude but points to the right, as Fig. 24-19c shows.

Between the sheets, the two fields add and we have

$$|\vec{E}_B| = |\vec{E}_{(+)}| + |\vec{E}_{(-)}|$$
$$= 3.84 \times 10^5 \text{ N/C} + 2.43 \times 10^5 \text{ N/C}$$
$$= 6.3 \times 10^5 \text{ N/C.} \qquad \text{(Answer)}$$

The electric field $\vec{E}_B$ points to the right.

24-7 Gauss' Law and Coulomb's Law

If Gauss' law and Coulomb's law are equivalent, we should be able to derive each from the other. Here we derive Coulomb's law from Gauss' law and some symmetry considerations.

Figure 24-20 shows a positive point charge q, around which we have drawn a concentric spherical Gaussian surface of radius r. Let us divide this surface into differential areas $d\vec{A}$. By definition, the area vector $d\vec{A}$ at any point is perpendicular to the surface and directed outward from the interior. From the symmetry of the situation, we know at any point the electric field $\vec{E}$ is also perpendicular to the surface and directed outward from the interior. Thus, since the angle θ between $\vec{E}$ and $d\vec{A}$ is zero, we can rewrite Gauss' law expressed in Eq. 24-7 as

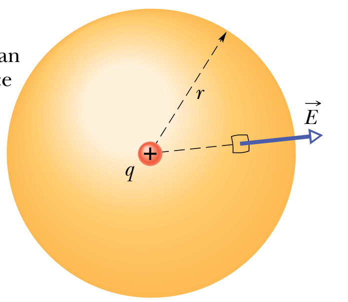


FIGURE 24-20 ■ A spherical Gaussian surface centered on a point charge q.

$$\oint \vec{E} \cdot d\vec{A} = \oint E\,dA = q^{\text{enc}}/\varepsilon_0. \qquad (24\text{-}17)$$

Here $q^{\text{enc}} = q$. Although the magnitude of the vector $\vec{E}$ varies radially with the distance from q, it has the same value everywhere on the spherical surface. Since the integral in this equation is taken over that surface, the electric field magnitude ($E = |\vec{E}|$) is a constant in the integration and can be brought out in front of the integral sign. That gives us

$$\varepsilon_0 E \oint dA = |q^{\text{enc}}|. \qquad (24\text{-}18)$$

The integral is now merely the sum of the magnitudes of all the differential area elements $d\vec{A}$ on the sphere and thus is just the surface area, $4\pi r^2$. Substituting this, we have

$$\varepsilon_0 E(4\pi r^2) = |q^{\text{enc}}|,$$

or since $q = q^{\text{enc}}$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q^{\text{enc}}|}{r^2} = k\frac{|q|}{r^2}. \qquad (24\text{-}19)$$

This is exactly the electric field due to a point charge (Eq. 23-8), which we found using Coulomb's law. Thus, we have shown that Gauss' law and Coulomb's law give us the same result for the electric field due to a single point-like charge. However, Gauss' law is also valid for complex arrays of charges. It can be shown using the principle of superposition that the information about electric fields obtained by using either Gauss' or Coulomb's law will yield the same results even for charge arrays. The difference between the two laws is this: It is easier to use Coulomb's law if we have an array of a few point-like charges, and it is easier to use Gauss' law if we have certain kinds of highly symmetric charge distributions like those discussed in Section 24-6. In still other situations, it is quite difficult to use either law.

READING EXERCISE 24-4: There is a certain net flux Φ^{net} at a Gaussian sphere of radius r enclosing an isolated charged particle. Suppose the enclosing Gaussian surface is changed to (a) a larger Gaussian sphere, (b) a Gaussian cube with edge length equal to r, and (c) a Gaussian cube with edge length equal to $2r$. In each case, is the net flux at the new Gaussian surface greater than, less than, or equal to Φ^{net}? ■

24-8 A Charged Isolated Conductor

Gauss' law permits us to prove an important theorem about isolated conductors:

> If excess charges are placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. Once the charges stop moving, none of the excess charge will be found within the body of the conductor.

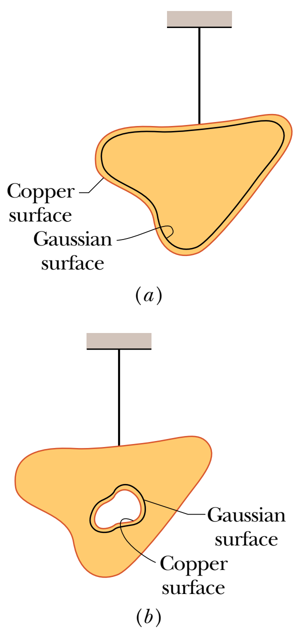


FIGURE 24-21 ■ (a) A lump of copper with a charge q hangs from an insulating thread. A Gaussian surface is placed within the metal, just inside the actual surface. (b) The lump of copper now has a cavity within it. A Gaussian surface lies within the metal, close to the cavity surface.

This might seem reasonable, considering charges with the same sign repel each other. You might imagine that by moving to the surface, the added charges are getting as far away from each other as they can. We turn to Gauss' law for verification of this speculation.

Figure 24-21a shows, in cross section, an isolated lump of copper hanging from an insulating thread and having an excess charge q. We place a Gaussian surface just inside the actual surface of the conductor.

Once the excess charges stop moving, the electric field inside this conductor must be zero. If this were not so, the field would exert forces on the conduction (free) electrons, which are always present in a conductor such as copper, and thus current would always exist within a conductor. (That is, charge would flow from place to place within the conductor.) Of course, there are no such perpetual currents in an isolated conductor, and so we know that the internal electric field is zero.

An internal electric field *does* appear as a conductor is being charged. However, the added charge quickly distributes itself in such a way that the net internal electric field—the vector sum of the electric fields due to all the charges, both inside and outside—is zero. The movement of charge then ceases because there are drag forces known as *resistance* in conductors that dissipate the charges' kinetic energies and eventually bring them to rest. Since the net field is zero, the net force on each charge is zero. So, once the charges are stopped by resistance in the conductor, they remain at rest. Some special materials can be "superconductors" at very low temperatures and allow charges to move without resistance. Therefore, these materials can support long-lasting currents.

If $\vec{E}$ is zero everywhere inside our copper conductor, it must be zero for all points on the Gaussian surface because that surface, though close to the surface of the conductor, is definitely inside the conductor. This means the flux at the Gaussian surface must be zero. Gauss' law then tells us the net charge inside the Gaussian surface must also be zero. Then because the excess charge is not inside the Gaussian surface, it must be outside that surface, which means it must lie on the actual surface of the conductor.

An Isolated Conductor with a Cavity

Figure 24-21b shows the same hanging conductor, but now with a cavity totally within the conductor. It is perhaps reasonable to suppose that when we scoop out the electrically neutral material to form the cavity we do not change the distribution of charge or the pattern of the electric field that exists in Fig. 24-21*a*. Again, we can turn to Gauss' law for a quantitative proof.

We draw a Gaussian surface surrounding the cavity, close to its surface but inside the conducting body. Because $\vec{E} = 0$ inside the conductor, there can be no flux at this new Gaussian surface. Therefore, from Gauss' law, that surface can enclose no net charge. We conclude there is no net charge on the cavity walls; all the excess charge remains on the outer surface of the conductor, as in Fig. 24-21*a*.

The Conductor Removed

Consider now an object that has the same shaped surface, but consists of only a conducting shell of charge. This is equivalent to enlarging the cavity of Fig. 24-21*b* until it consumes the entire conductor, leaving only the charges. The electric field would not change at all; it would remain zero inside the thin shell of charge and would remain unchanged for all external points. This reminds us that the electric field is set up by the charges and not by the conductor. The conductor simply provides an initial pathway for the charges to take up their positions.

The External Electric Field

You have seen that the excess charge on an isolated conductor moves entirely to the conductor's surface. However, unless the conductor is spherical, the charge does not distribute itself uniformly. Put another way, the surface charge density σ (charge per unit area) varies over the surface of any nonspherical conductor. Generally, this variation makes the determination of the electric field set up by the surface charges very difficult.

Suppose we know the surface charge density, σ, on a region of a conductor. Then it is easy to use Gauss' law to calculate the electric field just outside the surface of a conductor. To do this, we consider a section of the surface small enough to permit us to neglect any curvature and thus to take the section to be flat. We then imagine a tiny cylindrical Gaussian surface to be embedded in the section as in Fig. 24-22: One end cap is fully inside the conductor, the other is fully outside, and the cylinder is perpendicular to the conductor's surface.

The electric field $\vec{E}$ at and just outside the conductor's surface must also be perpendicular to that surface. If it were not, then it would have a component along the conductor's surface exerting forces on the surface charges, causing them to move. However, such motion would violate our implicit assumption that we are dealing with electrostatic equilibrium. Therefore, $\vec{E}$ is perpendicular to the conductor's surface.

We now sum the flux at the Gaussian surface. There is no flux at the internal end cap, because the electric field within the conductor is zero. There is no flux at the curved surface of the cylinder, because internally (in the conductor) there is no electric field and externally the electric field is parallel to the curved portion of the Gaussian surface. The only flux at the Gaussian surface is at the external end cap, where $\vec{E}$ is perpendicular to the plane of the cap. We assume the cap area A is small enough that the field magnitude $|\vec{E}|$ is constant over the cap. Then the amount of the flux at the cap is $|\vec{E}|A$, and that is the net amount of flux $|\Phi^{\text{net}}|$ at the Gaussian surface.

The charge q^{enc} enclosed by the Gaussian surface lies on the conductor's surface in an area A. If σ is the charge per unit area, then q^{enc} is equal to σA. When we substitute σA for q^{enc} and $|\vec{E}|A$ for $|\Phi^{\text{net}}|$, Gauss' law, $\varepsilon_0|\Phi^{\text{net}}| = |q^{\text{enc}}|$, becomes $\varepsilon_0|\vec{E}|A = |\sigma|A$, from which we find

$$|\vec{E}| = \frac{|\sigma|}{\varepsilon_0} \quad \text{(conducting surface).} \tag{24-20}$$

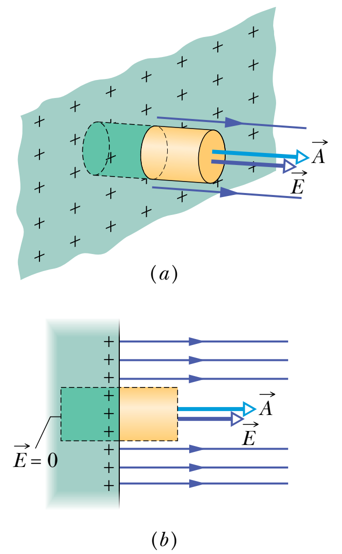


FIGURE 24-22 ■ Perspective view (*a*) and side view (*b*) of a tiny portion of a large, isolated conductor with excess positive charge on its surface. A (closed) cylindrical Gaussian surface, embedded perpendicularly in the conductor, encloses some of the charge. Electric field lines pierce the external end cap of the cylinder, but not the internal end cap. The external end cap has area A and area vector $\vec{A}$.

Thus, the magnitude of the electric field at a location just outside a conductor is proportional to the surface charge density at that location on the conductor. If the charge on the conductor is positive, the electric field is directed away from the conductor as in Fig. 24-22. It is directed toward the conductor if the charge is negative.

The difference between Eq. 24-20 and Eq. 24-16 ($|\vec{E}| = |\sigma|/2\varepsilon_0$) results from the fact that our conductor is no longer thin so that one of our Gaussian pillbox endcaps lies inside the conductor where the electric field is zero. Although the situation in Figs. 24-18 and 24-22 look similar, there is an important difference. There must be other charges in Fig. 24-22 that contribute to making the field zero inside the conductor. Even though these charges are outside the Gaussian surface and therefore do not contribute to the total flux, they change the values of the E field on the surface and therefore change the value we extract.

The field vectors in Fig. 24-22 point toward negative charges somewhere in the environment. If we bring those charges near the conductor, the charge density at any given location on the conductor's surface changes, and so does the magnitude of the electric field. However, the relation between the amount of the surface charge per unit area and the electric field magnitude is still given by Eq. 24-20,

$$E = \frac{|\sigma|}{\varepsilon_0}.$$

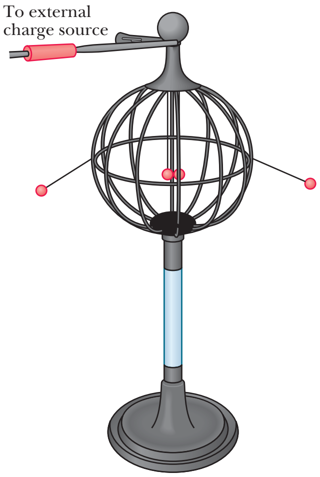


FIGURE 24-23 ■ A charged Faraday cage consisting of a sphere made of curved brass rods. Charges on the outside of the cage travel along conducting strings to the small balls causing them to be repelled from the cage. There is no charge inside the cage so the balls in the cage do not repel.

The Faraday Cage

The fact that an isolated conductor with a cavity has no electric field inside of it has led to the construction of a very valuable electrical device. Many research environments today involve the measurement of very low power electrical signals. This might occur when measuring the electrical signals from the neuron of a live mouse running a maze or while trying to measure the electrical properties of a microscopic device meant as part of a micro-miniaturized computer chip. In our modern world there are numerous electrical signals traveling through space, arising from everything from the 60 Hz power running in our walls to the radio signals from TV stations and cellular phones. These signals can interfere with sensitive electrical measurements.

To prevent these stray electric fields from ruining sensitive measurements, researchers often conduct their experiments inside a thin-walled metal cage known as a Faraday cage. Examples of Faraday cages are shown in the photo on the first page of this chapter as well as in Fig. 24-23. The Faraday cage in Fig. 24-23 is like the object shown in Fig. 24-21*b* except that now the "cavity" takes up almost the whole volume of the material. In addition, the thin metal shell in a Faraday cage is typically made of wire mesh. As long as the mesh is fairly fine, charge can spread out evenly on its surface. This type of cage can prevent even strong electrical signals from producing electric fields inside the cage. How? The external electric field induces charges on the surface of the Faraday cage to move so that the field they produce will precisely cancel the external field at points inside the surface. This rearrangement occurs naturally and is predictable by Gauss' law. This is why a demonstrator in a Faraday cage that is highly charged by a Van de Graaff generator can touch the inside of the cage and survive as shown in the opening photograph. The principle of the Faraday cage is also what makes it safe to be inside an automobile in a lightning storm. Even if lightning strikes your car, the effects inside the conductor are substantially reduced. This would not be the case if you were in a wooden crate, because the lightning could pass right through it. The crate could also catch on fire.

READING EXERCISE 24-5: Suppose a single positive charge is suddenly placed in the cavity shown in Fig. 24-21*b*. What has to happen in the conductor at the cavity walls to ensure that the electric field everywhere inside the conductor remains at zero? ■

TOUCHSTONE EXAMPLE 24-4: Spherical Metal Shell

Figure 24-24*a* shows a cross section of a spherical metal shell of inner radius R. A point charge of $-5.0\,\mu C$ is located at a distance $R/2$ from the center of the shell. If the shell is electrically neutral, what are the (induced) charges on its inner and outer surfaces? Are those charges uniformly distributed? What is the field pattern inside and outside the shell?

SOLUTION ■ Figure 24-24*b* shows a cross section of a spherical Gaussian surface within the metal, just outside the inner wall of the shell. One **Key Idea** here is that the electric field must be zero inside the metal (and thus on the Gaussian surface inside the metal). This means that the electric flux through the Gaussian surface must also be zero. Gauss' law then tells us that the *net* charge enclosed by the Gaussian surface must be zero. With a point charge of $-5.0\,\mu C$ within the shell, a charge of $+5.0\,\mu C$ must lie on the inner wall of the shell.

If the point charge were centered, this positive charge would be uniformly distributed along the inner wall. However, since the point charge is off-center, the distribution of positive charge is skewed, as suggested by Fig. 24-24*b*, because the positive charge tends to collect on the section of the inner wall nearest the (negative) point charge.

A second **Key Idea** is that because the shell is electrically neutral, its inner wall can have a charge of $+5.0\,\mu C$ only if electrons, with a total charge of $-5.0\,\mu C$, leave the inner wall and move to the outer wall. There they spread out uniformly, as is also suggested by Fig. 24-24*b*. This distribution of negative charge is uniform because the shell is spherical and because the skewed distribution of positive charge on the inner wall cannot produce an electric field in the shell to affect the distribution of charge on the outer wall.

The field lines inside and outside the shell are shown approximately in Fig. 24-24*b*. All the field lines intersect the shell and the point charge perpendicularly. Inside the shell the pattern of field lines is skewed owing to the skew of the positive charge distribution. Outside the shell the pattern is the same as if the point charge were centered and the shell were missing. In fact, this would be true no matter where inside the shell the point charge happened to be located.

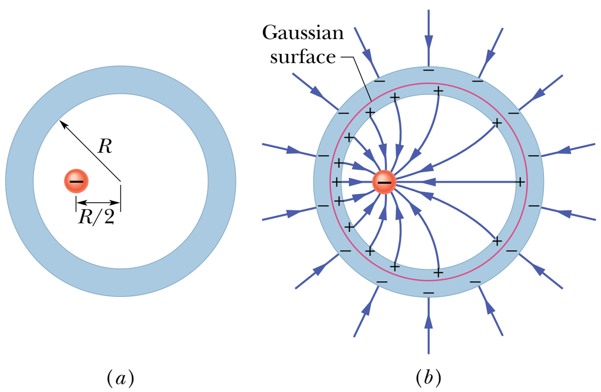


FIGURE 24-24 ■ (*a*) A negative point charge is located within a spherical metal shell that is electrically neutral. (*b*) As a result, positive charge is nonuniformly distributed on the inner wall of the shell, and an equal amount of negative charge is uniformly distributed on the outer wall. The electric field lines are shown.

Problems

SEC. 24-3 ■ NET FLUX AT A CLOSED SURFACE

1. Cube The cube in Fig. 24-25 has edge lengths of 1.40 m and is oriented as shown with its bottom face in the x-y plane at $z = 0.00$ m. Find the electric flux through the right face if the uniform electric field, in newtons per coulomb, is given by (a) $6.00\hat{i}$, (b) $-2.00\hat{j}$, and (c) $-3.00\hat{i} + 4.00\hat{k}$. (d) What is the total flux through the cube for each of these fields?

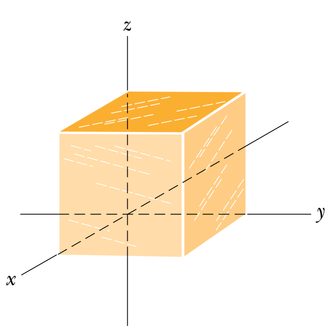


FIGURE 24-25 ■ Problems 1, 5, and 10.

2. Square Surface The square surface shown in Fig. 24-26 measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude $|\vec{E}| = 1800$ N/C. The field lines make an angle of 35° with a normal to the surface, as shown. Take that normal to be directed "outward," as though the surface were one face of a box. Calculate the electric flux through the surface.

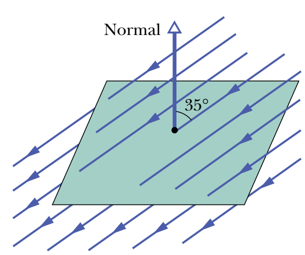


FIGURE 24-26 ■ Problem 2.

SEC. 24-4 ■ GAUSS' LAW

3. Charge at Center of Cube A point charge of 1.8 μC is at the center of a cubical Gaussian surface 55 cm on edge. What is the net electric flux through the surface?

4. Four Charges You have four point charges, $2q$, q, $-q$, and $-2q$. If possible describe how you would place a closed surface that encloses at least the charge $2q$ (and perhaps other charges) and through which the net electric flux is (a) 0 (b) $+3q/\varepsilon_0$, and (c) $-2q/\varepsilon_0$.

5. Flux Through Cube Find the net flux through the cube of Problem 1 and Fig. 24-25 if the electric field is given by (a) $\vec{E} = (3.00\,y\,[\mathrm{N/(C\cdot m)}])\hat{j}$

and (b) $\vec{E} = -(4.00\text{ N/C})\hat{\text{i}} + (6.00\text{ N/C} + 3.00\,y\,[\text{N/(C}\cdot\text{m)}])\hat{\text{j}}$. (c) In each case, how much charge is enclosed by the cube?

6. Butterfly Net In Fig. 24-27, a butterfly net is in a uniform electric field of magnitude $\vec{E}$. The rim, a circle of radius a, is aligned perpendicular to the field. Find the electric flux through the netting.

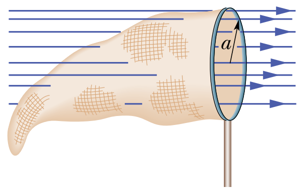


FIGURE 24-27 ■ Problem 6.

7. Earth's Atmosphere It is found experimentally that the electric field in a certain region of Earth's atmosphere is directed vertically down. At an altitude of 300 m the field has magnitude 60.0 N/C; at an altitude of 200 m, the magnitude is 100 N/C. Find the net amount of charge contained in a cube 100 m on edge, with horizontal faces at altitudes of 200 and 300 m. Neglect the curvature of Earth.

8. Shower When a shower is turned on in a closed bathroom, the splashing of the water on the bare tub can fill the room's air with negatively charged ions and produce an electric field in the air as great as 1000 N/C. Consider a bathroom with dimensions of 2.5 m × 3.0 m × 2.0 m. Along the ceiling, floor, and four walls, approximate the electric field in the air as being directed perpendicular to the surface and as having a uniform magnitude of 600 N/C. Also, treat those surfaces as forming a closed Gaussian surface around the room's air. What are (a) the volume charge density ρ and (b) the number of excess elementary charges e per cubic meter in the room's air?

9. Point Charge A point charge q is placed at one corner of a cube of edge a. What is the flux through each of the cube faces? (*Hint*: Use Gauss' law and symmetry arguments.)

10. Surface of Cube At each point on the surface of the cube shown in Fig 24-25, the electric field is along the y-axis. The length of each edge of the cube is 3.0 m. On the right surface of the cube, $\vec{E} = (-34\text{ N/C})\hat{\text{j}}$, and on the left face of the cube $\vec{E} = (+20\text{ N/C})\hat{\text{j}}$. Determine the net charge contained within the cube.

SEC. 24-6 ■ APPLICATION OF GAUSS' LAW TO SYMMETRIC CHARGE DISTRIBUTIONS

11. Conducting Sphere A conducting sphere of radius 10 cm has an unknown charge. If the electric field 15 cm from the center of the sphere has the magnitude 3.0×10^3 N/C and is directed radially inward, what is the net charge on the sphere?

12. Charge Causes Flux A point charge causes an electric flux of $-750\text{ N}\cdot\text{m}^2/\text{C}$ to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

13. Rutherford In a 1911 paper, Ernest Rutherford said: "In order to form some idea of the forces required to deflect an α particle through a large angle, consider an atom [as] containing a point positive charge Ze at its center and surrounded by a distribution of negative electricity $-Ze$ uniformly distributed within a sphere of radius R. The electric field E . . . at a distance r from the center for a point inside the atom [is]

$$E = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)\text{''}.$$

Verify this equation.

14. Concentric Spheres Two charged concentric spheres have radii of 10.0 cm and 15.0 cm. The charge on the inner sphere is 4.00×10^{-8} C, and that on the outer sphere is 2.00×10^{-8} C. Find the electric field (a) at $r = 12.0$ cm and (b) at $r = 20.0$ cm.

15. Proton A proton with speed $v = 3.00 \times 10^5$ m/s orbits just outside a charged sphere of radius $r = 1.00$ cm. What is the charge on the sphere?

16. Charge at Center of Shell A point charge $+q$ is placed at the center of an electrically neutral, spherical conducting shell with inner radius a and outer radius b. What charge appears on (a) the inner surface of the shell and (b) the outer surface? What is the net electric field at a distance r from the center of the shell if (c) $r < a$, (d) $b > r > a$, and (e) $r > b$? Sketch field lines for those three regions. For $r > b$, what is the net electric field due to (f) the central point charge plus the inner surface charge and (g) the outer surface charge? A point charge $-q$ is now placed outside the shell. Does this point charge change the charge distribution on (h) the outer surface and (i) the inner surface? Sketch the field lines now. (j) Is there an electrostatic force on the second point charge? (k) Is there a net electrostatic force on the first point charge? (l) Does this situation violate Newton's Third Law?

17. Solid Nonconducting Sphere A solid nonconducting sphere of radius R has a nonuniform charge distribution of volume charge density $\rho = \rho_s r/R$, where ρ_s is a constant and r is the distance from the center of the sphere. Show (a) that the total charge on the sphere is $Q = \pi\rho_s R^3$ and (b) that

$$|\vec{E}| = k\frac{|Q|}{R^4}r^2$$

gives the magnitude of the electric field inside the sphere.

18. Hydrogen Atom A hydrogen atom can be considered as having a central point-like proton of positive charge $+e$ and an electron of negative charge $-e$ that is distributed about the proton according to the volume charge density $\rho = A\exp(-2r/a_1)$. Here A is a constant, $a_1 = 0.53 \times 10^{-10}$ m is the *Bohr radius*, and r is the distance from the center of the atom. (a) Using the fact that hydrogen is electrically neutral, find A. (b) Then find the electric field produced by the atom at the Bohr radius.

19. Sphere of Radius *a* In Fig 24-28 an insulating sphere, of radius a and charge $+q$ uniformly distributed throughout its volume, is concentric with a spherical conducting shell of inner radius b and outer radius c. This shell has a net charge of $-q$. Find expressions for the electric field, as a function of the radius r, (a) within the sphere ($r < a$), (b) between the sphere and the shell ($a < r < b$), (c) inside the shell ($b < r < c$), and (d) outside the shell ($r > c$). (e) What are the charges on the inner and outer surfaces of the shell?

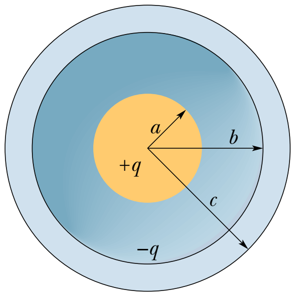


FIGURE 24-28 ■ Problem 19.

20. Uniform Volume Charge Density Figure 24-29a shows a spherical shell of charge with uniform volume charge density ρ. Plot E due to the shell for distances r from the center of the shell ranging from zero to 30 cm. Assume that $\rho = 1.0 \times 10^{-6}\text{ C/m}^3$, $a = 10$ cm, and $b = 20$ cm.

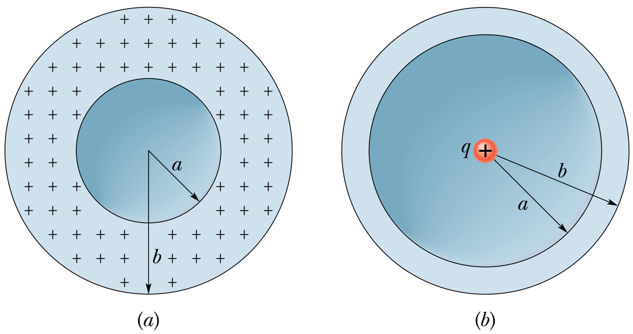


FIGURE 24-29 ■ Problems 20 and 21.

21. Nonconducting Spherical Shell In Fig. 24-29*b*, a nonconducting spherical shell, of inner radius a and outer radius b, has a positive volume charge density $\rho = A/r$ (within its thickness), where A is a constant and r is the distance from the center of the shell. In addition, a positive point charge q is located at that center. What value should A have if the electric field in the shell ($a \le r \le b$) is to be uniform? (*Hint:* The constant A depends on a but not on b.)

22. Show That A nonconducting sphere has a uniform volume charge density ρ. Let $\vec{r}$ be the vector from the center of the sphere to a general point P within the sphere. (a) Show that the electric field at P is given by $\vec{E} = \rho\vec{r}/3\varepsilon_0$. (Note that the result is independent of the radius of the sphere.) (b) A spherical cavity is hollowed out of the sphere, as shown in Fig. 24-30. Using superposition concepts, show that the electric field at all points within the cavity is uniform and equal to $\vec{E} = \rho\vec{a}/3\varepsilon_0$, where $\vec{a}$ is the position vector from the center of the sphere to the center of the cavity. (Note that this result is independent of the radius of the sphere and the radius of the cavity.)

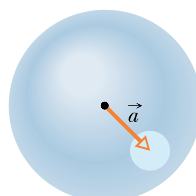


FIGURE 24-30 ■ Problem 22.

23. Spherically Symmetrical A spherically symmetrical but nonuniform volume distribution of charge produces an electric field of magnitude $|\vec{E}| = Kr^4$, directed radially outward from the center of the sphere. Here r is the radial distance from that center, and K is a positive constant. What is the volume density ρ of the charge distribution as a function of r?

24. Long Metal Tube Figure 24-31 shows a section of a long, thin-walled metal tube of radius R, with a positive charge per unit length λ on its surface. Derive expressions for $|\vec{E}|$ in terms of the distance r from the tube axis, considering both (a) $r > R$ and (b) $r < R$. Plot your results for the range $r = 0$ to $r = 5.0$ cm, assuming that $\lambda = 2.0 \times 10^{-8}$ C/m and $R = 3.0$ cm. (*Hint:* Use cylinderical Gaussian surfaces, coaxial with the metal tube.)

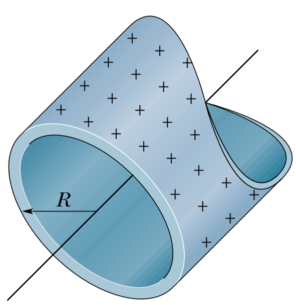


FIGURE 24-31 ■ Problem 24.

25. Infinite Line of Charge An infinite line of charge produces a field magnitude of 4.5×10^4 N/C at a distance of 2.0 m. Calculate the amount of linear charge density $|\lambda|$.

26. Long Straight Wire A long, straight wire has fixed negative charge with a linear charge density of -3.6 nC/m. The wire is to be enclosed by a thin, nonconducting cylinder of outside radius 1.5 cm, coaxial with the wire. The cylinder is to have positive charge on its outside surface with a surface charge density σ such that the net external electric field is zero. Calculate the required σ.

27. Cylindrical Rod A very long conducting cylindrical rod of length L with a total charge $+q$ is surrounded by a conducting cylindrical shell (also of length L) with total charge $-2q$, as shown in Fig. 24-32. Use Gauss' law to find (a) the electric field at points outside the conducting shell, (b) the distribution of charge on the shell, and (c) the electric field in the region between the shell and rod. Neglect end effects.

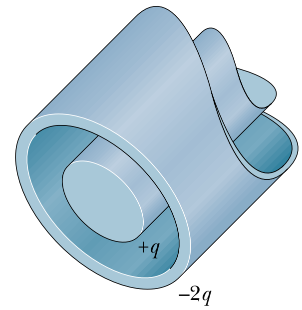


FIGURE 24-32 ■ Problem 27.

28. Solid Cylinder A long, nonconducting, solid cylinder of radius 4.0 cm has a nonuniform volume charge density ρ that is a function of the radial distance r from the axis of the cylinder, as given by $\rho = Ar^2$ with $A = 2.5\ \mu\text{C/m}^5$. What is the magnitude of the electric field at a radial distance of (a) 3.0 cm and (b) 5.0 cm from the axis of the cylinder?

29. Two Concentric Cylinders Two long, charged, concentric cylinders have radii of 3.0 and 6.0 cm. Assume the outer cylinder is hollow. The charge per unit length is 5.0×10^{-6} C/m on the inner cylinder and -7.0×10^{-6} C/m on the outer cylinder. Find the electric field at (a) $r = 4.0$ cm and (b) $r = 8.0$ cm, where r is the radial distance from the common central axis.

30. Geiger Counter Figure 24-33 shows a Geiger counter, a device used to detect ionizing radiation (radiation that causes ionization of atoms). The counter consists of a thin, positively charged central wire surrounded by a concentric, circular, conducting cylinder with an equal negative charge. Thus, a strong radial electric field is set up inside the cylinder. The cylinder contains a low-pressure inert gas. When a particle of radiation enters the device through the cylinder wall, it ionizes a few of the gas atoms. The resulting free electrons (labelled e) are drawn to the positive wire. However, the electric field is so intense that, between collisions with other gas atoms, the free electrons gain energy sufficient to ionize these atoms also. More free electrons are thereby created, and the process is repeated until the electrons reach the wire. The resulting "avalanche" of electrons is collected by the wire generating a signal that is used to record the passage of the original particle of radiation. Suppose that the radius of the central wire is 25 μm, the radius of the cylinder 1.4 cm, and the length of the tube 16 cm. If the electric field component E_r at the cylinder's inner wall is $+2.9 \times 10^4$ N/C, what is the total positive charge on the central wire?

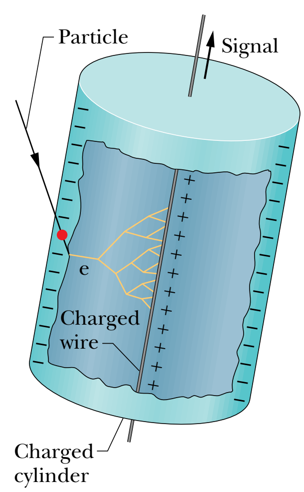


FIGURE 24-33 ■ Problem 30.

31. Charge Is Distributed Uniformly Charge is distributed uniformly throughout the volume of an infinitely long cylinder of radius R. (a) Show that, at a distance r from the cylinder axis (for $r < R$),

$$|\vec{E}| = \frac{|\rho| r}{2\varepsilon_0},$$

where $|\rho|$ is the amount of volume charge density. (b) Write an expression for $|\vec{E}|$ when $r > R$.

32. Parallel Sheets Figure 24-34 shows cross sections through two large, parallel, nonconducting sheets with identical distributions of positive charge with area charge density σ. What is $\vec{E}$ at points (a) above the sheets, (b) between them, and (c) below them?

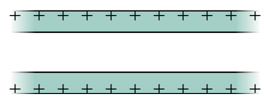


FIGURE 24-34 ■ Problem 32.

33. Square Metal Plate A square metal plate of edge length 8.0 cm and negligible thickness has a total charge of 6.0×10^{-6} C. (a) Estimate the magnitude E of the electric field just off the center of the plate (at, say, a distance of 0.50 mm) by assuming that the charge is spread uniformly over the two faces of the plate. (b) Estimate E at a distance of 30 m (large relative to the plate size) by assuming that the plate is a point charge.

34. Thin Metal Plates Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have excess surface charge of opposite signs. The amount of charge per unit area is given by $|\sigma| = 7.0 \times 10^{-22}$ C/m^2, with the negatively charged plate on the left. What are the magnitude and direction of the electric field $\vec{E}$ (a) to the left of the plates, (b) to the right of the plates, and (c) between the plates?

35. Ball on Thread In Fig. 24-35, a small, nonconducting ball of mass $m = 1.0$ mg and charge $q = 2.0 \times 10^{-8}$ C (distributed uniformly through its volume) hangs from an insulating thread that makes an angle $\theta = 30°$ with a vertical, uniformly charged nonconducting sheet (shown in cross section). Considering the gravitational force on the ball and assuming that the sheet extends far vertically and into and out of the page, calculate the surface charge density σ of the sheet.

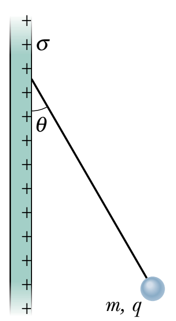


FIGURE 24-35 ■ Problem 35.

36. Large Metal Plates Two large metal plates of area 1.0 m^2 face each other. They are 5.0 cm apart and have equal but opposite charges on their inner surfaces. If the magnitude $|\vec{E}|$ of the electric field between the plates is 55 N/C, what is the amount of charge on each plate? Neglect edge effects.

37. An Electron Is Shot An electron is shot directly toward the center of a large metal plate that has excess negative charge with surface charge density -2.0×10^{-6} C/m^2. If the initial kinetic energy of the electron is 1.60×10^{-17} J and if the electron is to stop (owing to electrostatic repulsion from the plate) just as it reaches the plate, how far from the plate must it be shot?

38. Planar Slab A planar slab of thickness d has a uniform volume charge density ρ. Find the magnitude of the electric field at all points in space both (a) within and (b) outside the slab, in terms of x, the distance measured from the central plane of the slab.

SEC. 24-8 ■ A CHARGED ISOLATED CONDUCTOR

39. Photocopying Machine The electric field just above the surface of the charged drum of a photocopying machine has a magnitude $|\vec{E}|$ of 2.3×10^5 N/C. What is the surface charge density on the drum, assuming that the drum is a conductor?

40. Space Vehicles Space vehicles traveling through Earth's radiation belts can intercept a significant number of electrons. The resulting charge buildup can damage electronic components and disrupt operations. Suppose a spherical metallic satellite 1.3 m in diameter accumulates -2.4 μC of charge in one orbital revolution. (a) Find the resulting surface charge density. (b) Calculate the magnitude of the electric field just outside the surface of the satellite due to the surface charge.

41. Charged Sphere A uniformly charged conducting sphere of 1.2 m diameter has a surface charge density of 8.1 μC/m^2. (a) Find the net charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

42. Arbitrary Shape Conductor An isolated conductor of arbitrary shape has a net charge of $+10 \times 10^{-6}$ C. Inside the conductor is a cavity within which is a point charge $q = +3.0 \times 10^{-6}$ C. What is the charge (a) on the cavity wall and (b) on the outer surface of the conductor?

Additional Problems

43. If/Can If the electric field in a region of space is zero, can you conclude there are no electric charges in that region? Explain.

44. If/Than If there are fewer electric field lines leaving a Gaussian surface than there are entering the surface, what can you conclude about the net charge enclosed by that surface?

45. Net Flux What is the net electric flux through each of the closed surfaces in Fig. 24-36 if the value of q is $+1.6 \times 10^{-19}$ C?

46. Net Flux Two What is the net electric flux through each of the closed surfaces in Fig. 24-37 if the value of q is 8.85×10^{-12} C? Explain the reasons for your answers.

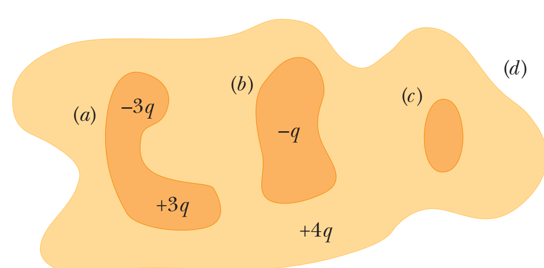


FIGURE 24-36 ■ Problem 45.

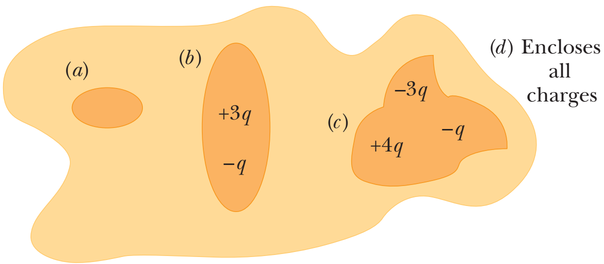


Figure 24-37 ■ Problem 46.

47. Fair Weather During fair weather, an electric field of about 100 N/C points vertically downward into Earth's atmosphere. Assuming that this field arises from charge distributed in a spherically symmetric manner over the surface of Earth, determine the *net* charge of Earth and its atmosphere if the radius of Earth and its atmosphere is 6.37×10^6 m.

48. Hollow Sphere Suppose a charge is located at the center of a hollow sphere as shown in Fig. 24-38.

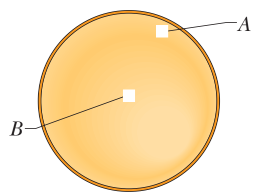


Figure 24-38 ■ Problem 48.

(a) Are the intersections of the field lines with the surface of the sphere uniformly distributed throughout? In other words, is the density of lines passing through the surface of the sphere uniform? Explain why or why not.
(b) Consider surface elements A and B, which have exactly the same area. Is the number of field lines passing through surface element A greater than, less than, or equal to the number of field lines through surface element B? Explain.
(c) Is the flux through surface element A greater than, less than, or equal to the flux through surface element B? Explain.

49. Center of Cube Suppose a charge is located at the center of the cube shown in Fig. 24-39.

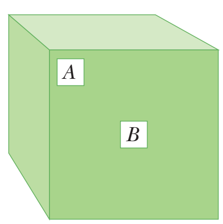


Figure 24-39 ■ Problem 49.

(a) Are the intersections of the field lines with a side of the cube uniformly distributed across the side? In other words, is the density of lines passing through the box uniform? Explain why or why not.
(b) Is the number of field lines through surface element A greater than, less than, or equal to the number of field lines through surface element B? Explain.
(c) Is the flux through surface element A greater than, less than, or equal to the flux through surface element B? Explain.

50. Using Gauss' Law Gauss' law is usually written as an equation in the form

$$\oint \vec{E} \cdot d\vec{A} = q^{\text{enc}}/\varepsilon_0.$$

(a) For this equation, specify what each term in this equation means and how it is to be calculated when doing some specific (but arbitrary—not a special case) calculation.

A long thin cylindrical shell like that shown in Fig. 24-40 has length L and radius R with $L >> R$ and is uniformly covered with a charge Q. If we look for the field near the cylinder somewhere about the middle, we can treat the cylinder as if it were an infinitely long cylinder. Using this assumption, we can calculate the magnitude and direction of the field at a point a distance d from the *axis* of the cylinder (outside the cylindrical shell; i.e., $L >> d > R$ but d not very close to R) using Gauss's law. Do so by explicitly following the steps below.

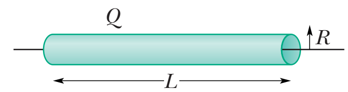


Figure 24-40 ■ Problem 50.

(b) Select an appropriate Gaussian surface. Explain why you chose it.
(c) Carry out the integral on the left side of the equation, expressing it in terms of the unknown value of the magnitude of the E field.
(d) What is the relevant value of q for your surface?
(e) Use your results in (c) and (d) in the equation and solve for the magnitude of E.

51. Interpreting Gauss Gauss' law states

$$\oint_A \vec{E} \cdot d\vec{A} = q_A/\varepsilon_0,$$

where A is a surface and q_A is a charge.

(a) Which of the following statements are true about the surface A appearing in Gauss' law for the equation to hold? You may list any number of these statements including all or none.

i. The surface A must be a closed surface (must cover a volume).
ii. The surface A must contain all the charges in the problem.
iii. The surface A must be a highly symmetrical surface like a sphere or a cylinder.
iv. The surface A must be a conductor.
v. The surface A is purely imaginary.
vi. The normals to the surface A must all be in the same direction as the electric field on the surface.

(b) Which of the following statements are true about the charge q_A appearing in Gauss' law? You may list any number of these statements including all or none.

i. The charge q_A must be all the charge lying on the Gaussian surface.
ii. The charge q_A must be the charge lying within the Gaussian surface.
iii. The charge q_A must be all the charge in the problem.
iv. The charge q_A flows onto the Gaussian surface once the surface is established.
v. The electric field E in the integral on the left of Gauss' law is due only to the charge q_A.
vi. The electric field E in the integral on the left on Gauss' law is due to all charges in the problem.