1

Systems of equations involving only two unknowns can be analyzed graphically. For example, consider the system

$$2x + 3y = 1, \tag{1}$$
$$x - y = 2. \tag{2}$$

Use *linspace* to discretized x into 200 equally-spaced points. Then, use "plot" command to plot these two lines in the interval $x \in [-10, 10]$.

Note: To plot another line on top of the already existing line use "hold on" after the first plot command:
$>> plot(x1, y1); hold\, on;$
$>> plot(x2, y2);$
This will first plot y1 vs x1, and then plot y2 vs x2 on top of the first line.

See if you can find the intersection through the graphic user interface. Next, form the matrix and the right-hand-side array that correspond to this linear system i.e. find $\mathbf{A}$ and $\mathbf{B}$ in $\mathbf{Ax} = \mathbf{B}$, and solve for $\mathbf{x} = [x; y]$. You should find $x = 1.40$, $y = -0.60$.

Also use "doc" to lookup *logspace* command. Use *logspace* to divide $x \in [10^{-5}, 10]$ into 200 equally logarithmically spaced points and then Plot $\sin(x)/x$.

2

Form the following matrices

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -3 & 1 & 1 \\ 1 & 4 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 0.50 & 0.35 & 0.15 \\ 0.35 & 0.6 & 0.05 \\ 0.15 & 0.05 & 0.80 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ -3 & 2 \end{bmatrix}$$

Recall : The multiplication operator in MATLAB is $*$. If A and B are two matrices that have the right number of rows and columns to be multiplied then $A * B$ is the matrix product. Note that if b is any scalar then $b * A$ is a matrix of the same shape as A with entries that are just b times the corresponding entry in A (as we would expect from the definition of scalar multiplication given in class). Using the matrix A, B and C above type the following :

$$
\begin{aligned}
&A * A \\
&A * B \\
&B * A \\
&A * C \\
&C * C' \\
&C' * C \\
&(A + A')/2
\end{aligned}
$$

- Did Matlab refuse to do any of these?Why?
- Does $A * B = B * A$?
- Is $(A + A')/2$ always a symmetric matrix is $(A - A')/2$ always anti-symmetric?
- Confirm $(A * B)' = B' * A'$.
- Confirm $inv(A * B) = inv(A) * inv(B)$

3

form a 4×4 matrix with random entries, and evaluate the following:

$$
\begin{aligned}
&A^2 - A * A \\
&A^{-1} * A^2 * A^{-1} \\
&inv(A)^2 * A^2 \\
&inv(A^2) * A^2 \\
&A^3 * A^{-3} \\
&A * A' - A' * A \\
&A * A - A. * A \\
&A. \wedge 3 - A^3 \\
&A.\hat{}(-1) - A^{-1}
\end{aligned}
$$

4

Form two 5×5 matrix with random entries, and evaluate the following:

$$
\begin{aligned}
& det(A) \\
& det(3 * A) \\
& 3 * det(A) \\
& det(A') - det(A) \\
& det(A) + det(B) \\
& det(A + B) \\
& det(A * B) \\
& det(B * A) \\
& det(A' * B') \\
& det(B' * A') \\
& det(A^{-1}) \\
& 1/det(A) \\
& det(A^{-1} * B) \\
& det(A)/det(B) \\
& det(B)/det(A) \\
& det(B^{-1} * A * B) \\
& det(B^{-1} * B)
\end{aligned}
$$

5

Determine the eigenvalues and eigenvectors of a random 5×5 matrix. To check for correctness evaluate

$$
\begin{aligned}
E_1^i &= |\det(\mathbf{A} - \lambda_i \mathbf{I})|, \quad i = 1, 2, .., 5 \\
E_2^i &= \text{mean}\left(|\mathbf{A}\mathbf{v}_i - \lambda_i \mathbf{v}_i|\right),
\end{aligned}
$$

where $\mathbf{v}_i$ is the i^{th} eigenvector. Both of these quantities (for $i = 1, .., 5$) should be zero within machine precision.

Repeat this for $N = 15$ different random matrices within a loop this time with 10×10 random matrices and plot $E_1^T = \sum_{i=1}^{10} E_1^i$ and $E_2^T = \sum_{i=1}^{10} E_2^i$ as a function of N.

6

Using Taylor expansion we can express $\exp(x)$ as

$$
e^x = \sum_{n=1}^{N} \frac{x^n}{n!}
$$

Use "for loops" to analyze the error for evaluating exp(1), $error = \exp(1) - \sum_{n=1}^{N} \frac{1^n}{n!}$, as the number of terms in Taylor expansion is increased. Plot the error as a function of N, using "plot", "loglog", and "semilogy".

7 More fun

Want something a bit more challenging? Try to reproduce Table 1.1 and Fig. 1.2 in the main text I gave you the week before. Then extend the range of h by using 100 logarithmically-spaced points for $h \in [10^{-10}, 10^{-1}]$. Plot the same variables as in Fig. 1.2. Do you see anything unusual? Can you explain it?