Homework 5

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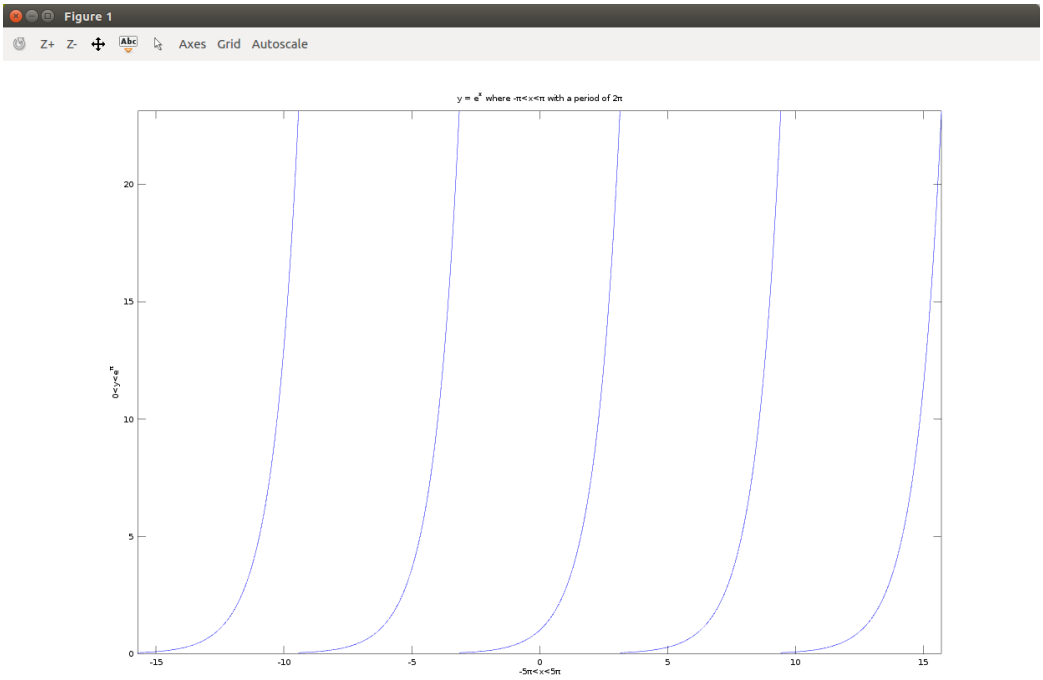
1 Introduction

I am very sorry for the quality of this homework set. I wish I had another five hours to simplify, write the transforms out, finish #3, and explain things properly. If you ever decide that you want one homework set to be rewritten Jen and I would very much like it to be this one (she's sitting next to me). Please meditate or take an aspirin before reading.

2 Boas 7.8.12

2.1 (a)

$$y = e^x \quad -\pi < x < \pi$$

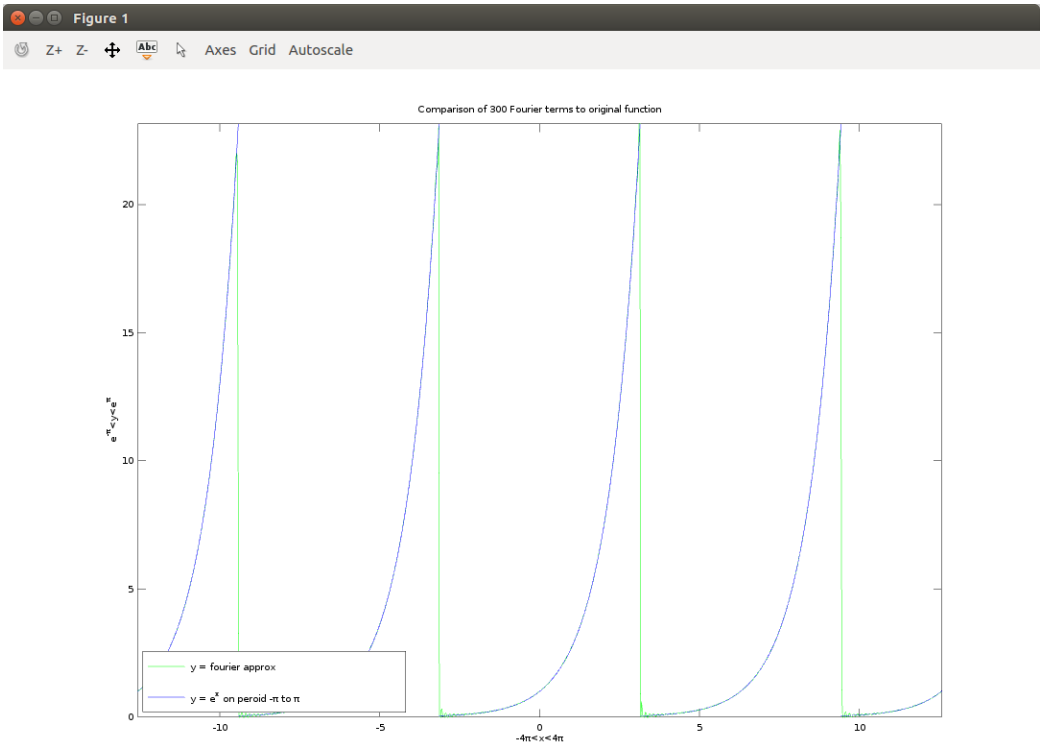


This is our function! Time to approximate!

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^x * e^{-in\pi x/\pi}dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{x-in\pi x/\pi}dx = \frac{1}{2\pi}(\frac{e^{x-inx}}{1-in})\Big|_{-\pi}^{\pi} = \frac{1}{2\pi}(\frac{e^{\pi(1-in)} - e^{\pi(in-1)}}{1-in}) =$$

$$\frac{e^{\pi}(-1)^n - e^{-\pi}(-1)^n}{2\pi(1-in)} = \frac{(-1)^n(e^{\pi} - e^{-\pi})}{2\pi(1-in)}$$

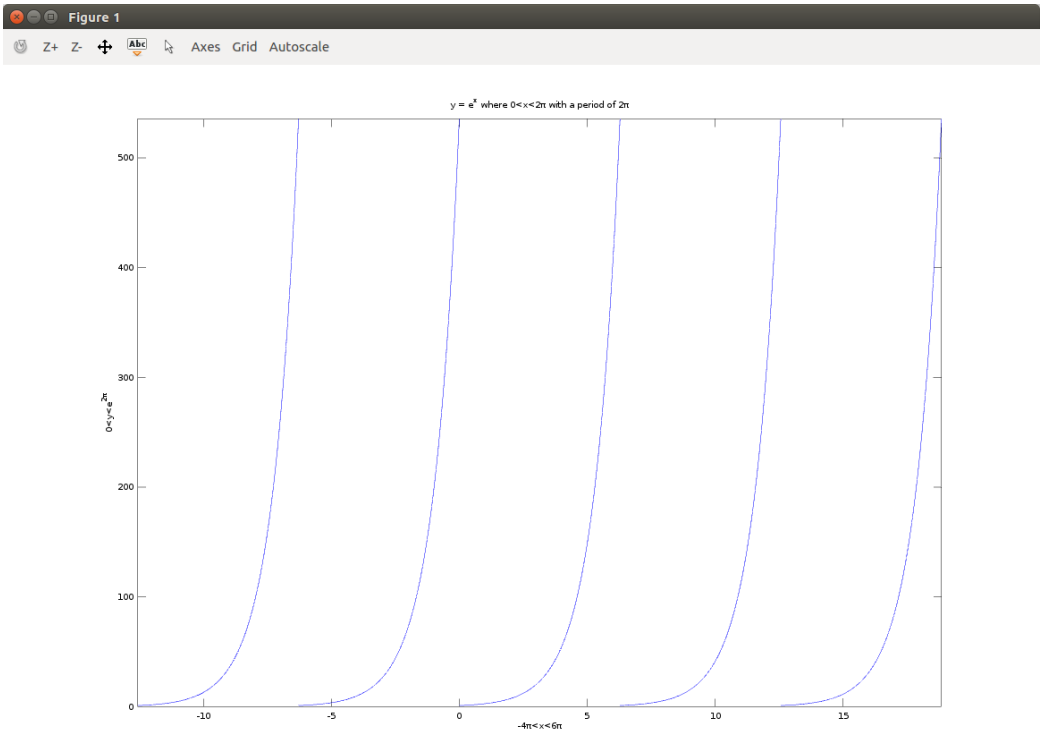
$$c_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^x dx = \frac{e^x}{2\pi}\Big|_{-\pi}^{\pi} = \frac{e^{\pi} - e^{-\pi}}{2\pi}$$



This may not look like much but you'd be surprised at how much time it took to write a program which could actually plot the function. It looks fantastic, but I lost so much time. Sometimes I think about Jordan and how much she gets done because she never sleeps.

2.2 (b)

$$y = e^x \quad 0 < x < 2\pi$$

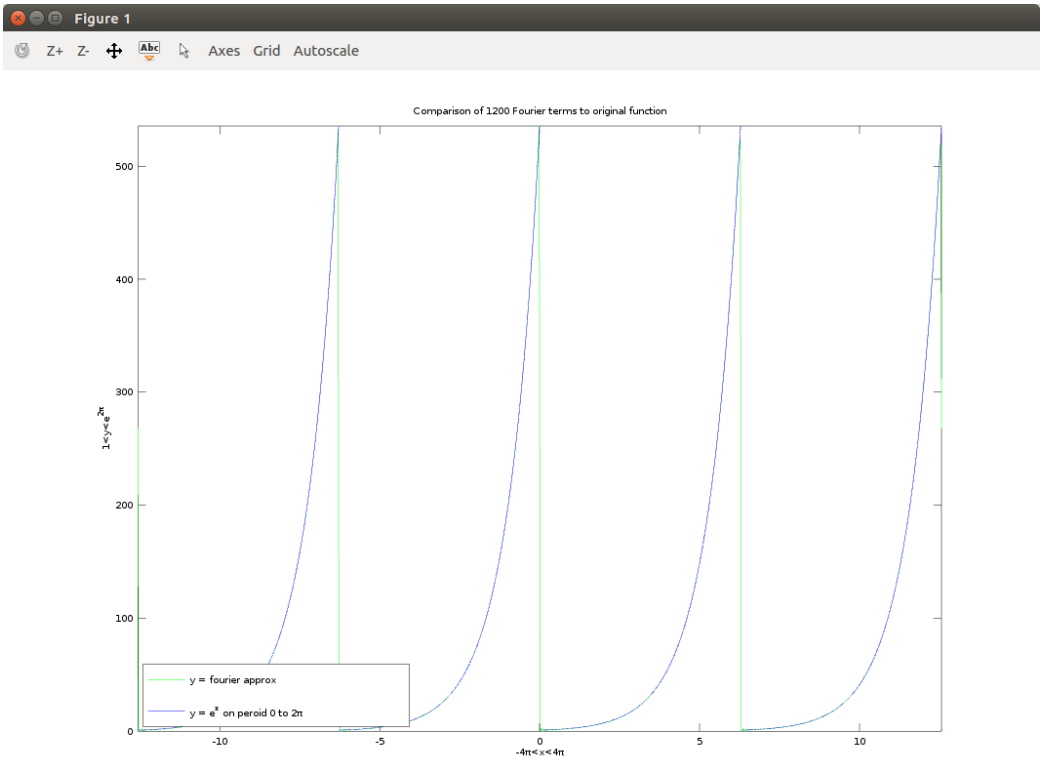


Same function, different interval.

$$c_n = \frac{1}{2\pi}\int_0^{2\pi} e^x * e^{-in\pi x/\pi}dx = \frac{1}{2\pi}\int_0^{2\pi} e^{x-in\pi x/\pi}dx = \frac{1}{2\pi}(\frac{e^{x-inx}}{1-in})\Big|_0^{2\pi} = \frac{1}{2\pi}(\frac{e^{2\pi(1-in)}-1}{1-in}) =$$

$$\frac{e^{2\pi}(\cos(2\pi n) - i\sin(2\pi n) - 1}{2\pi(1-in)} = \frac{e^{2\pi}-1}{2\pi(1-in)}$$

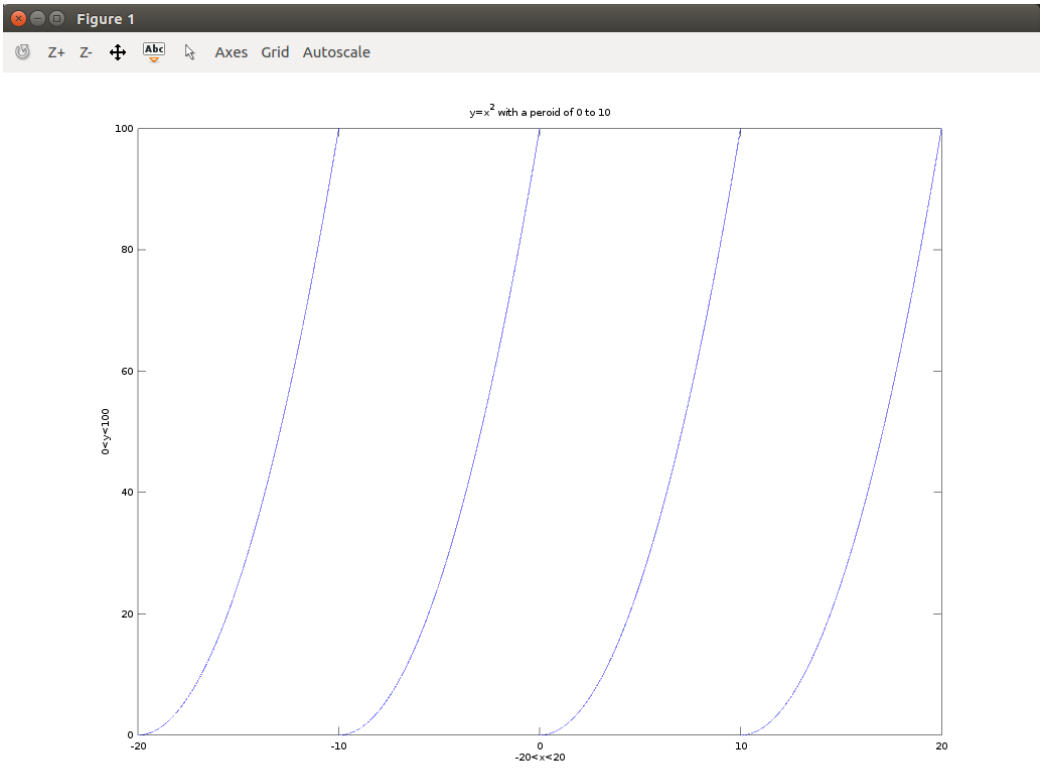
$$c_0 = \frac{1}{2\pi}\int_0^{2\pi} e^x dx = \frac{e^x}{2\pi}\Big|_0^{2\pi} = \frac{e^{2\pi}-1}{2\pi}$$



What a nice graph! Our series clearly approximates our function. Now its the fun part! Making the Fourier transforms....

3 Boas 7.8.18

$$y = x^2 \quad 0 < x < 10$$



Our function

$$c_n = \frac{1}{10}\int_0^{10} x^2 * e^{-in\pi x/5} dx$$

Integration by parts

$$u = \frac{x^2}{10} \qquad\qquad dv = e^{-in\pi x/5}dx$$

$$du = \frac{x}{5}dx \qquad\qquad v = \int e^{-in\pi x/5}dx = \frac{e^{-in\pi x/5}}{-in\pi/5}$$

$$\int_0^{10} udv = v*u - \int_0^{10} vdu = \left(\frac{x^2 e^{-in\pi x/5}}{-2in\pi}\right)\Big|_0^{10} - \int_0^{10} \frac{xe^{-in\pi x/5}dx}{-in\pi} = \frac{50e^{-2in\pi}}{-in\pi} - \int_0^{10} \frac{xe^{-in\pi x/5}dx}{-in\pi} =$$

Integration by parts: II - attack of the clone

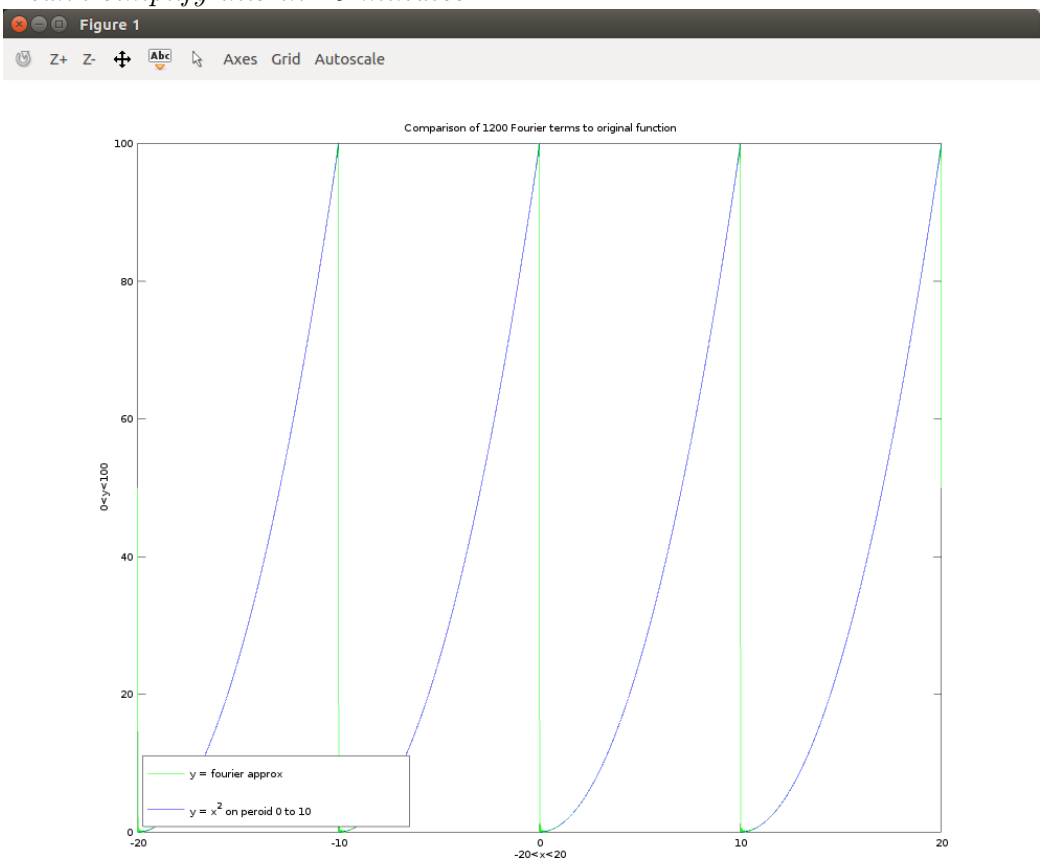
$$u = x \qquad dv = \frac{e^{-in\pi x/5}dx}{-in\pi}$$

$$du = dx \qquad v = \int \frac{e^{-in\pi x/5}dx}{-in\pi} = \frac{e^{-in\pi x/5}}{(-in\pi)^2/5}$$

$$\frac{50e^{-2in\pi}}{-in\pi} - (v*u - \int_0^{10} vdu) = \frac{50e^{-2in\pi}}{-in\pi} - (\frac{xe^{-in\pi x/5}}{(-in\pi)^2/5})\Big|_0^{10} + \int_0^{10} \frac{e^{-in\pi x/5}dx}{(-in\pi)^2/5} =$$

$$\frac{50e^{-2in\pi}}{-in\pi} + \frac{50e^{-2in\pi}}{n^2\pi^2} + \frac{e^{-in\pi x/5}}{(-in\pi)^3/25}\Big|_0^{10} = \frac{50e^{-2in\pi}}{-in\pi} + \frac{50e^{-2in\pi}}{n^2\pi^2} + \frac{25(e^{-2in\pi} - 1)}{in^3\pi^3}$$

I can't simplify this in 15 minutes



For what its worth it looks great.

4 Boas 7.8.24

Congratulations! You have one less problem to grade! Spend this time however you please!

5 Matlab Code

5.1 plotExp.m

```
function [retval] = plotExp (c,n,c0,delx,a,b)
%% Author: eggoeke <eggoeke@nephele>
%% Created: 2017-02-28
hold on;
var = @(x)c0;
for p = 1:(n-1)
  var = @(x)var(x) + (c(p).*exp((i*p*pi.*x)/delx))+(
     conj(c(p)).*exp((-i*p*pi.*x)/delx));
end
x1= linspace(a,b,10000);
plot(x1,var(x1),'g')
hold off

endfunction
```