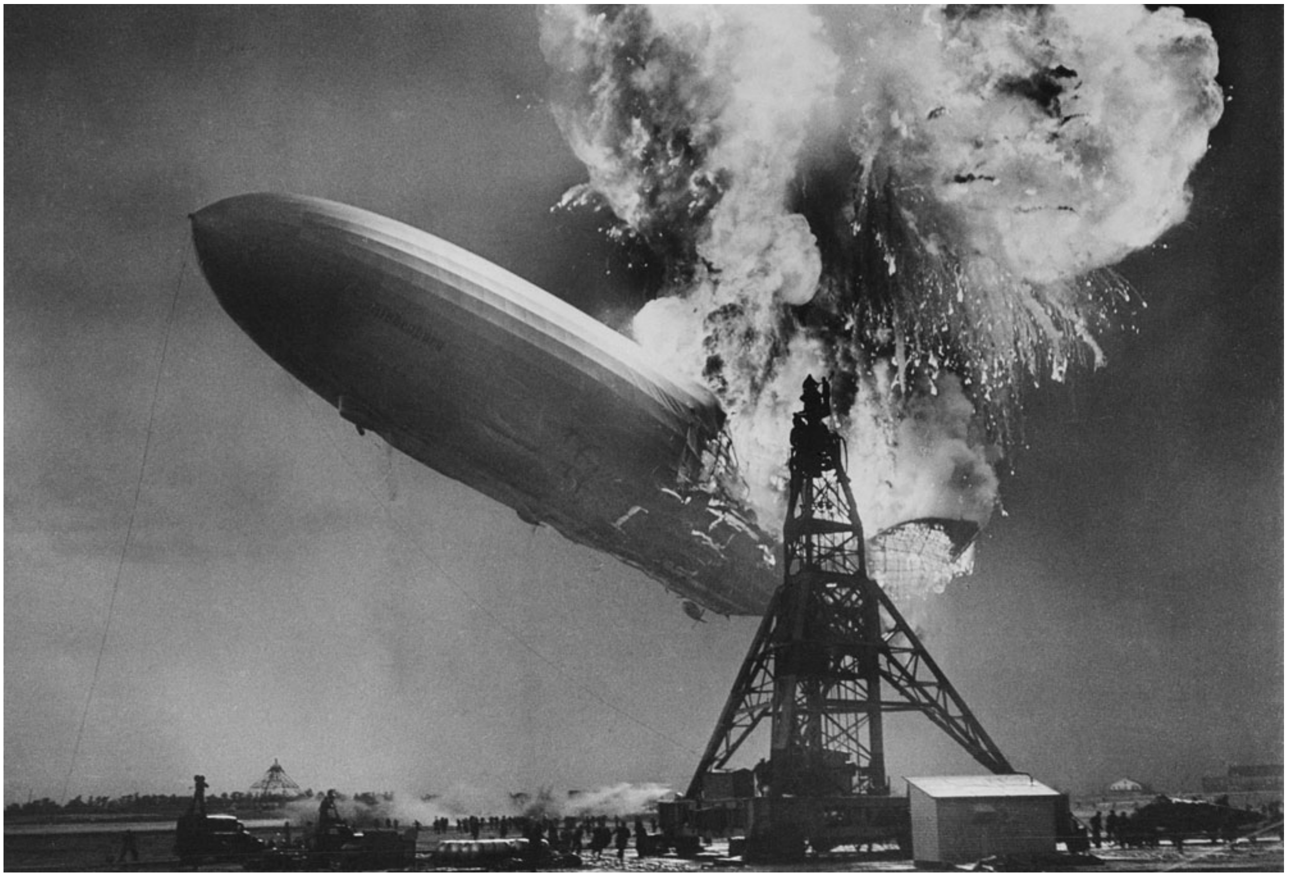
26 Current and Resistance



When the zeppelin *Hindenburg* was built, it was the pride of Germany. Almost three football fields long, it was the largest flying machine ever built. Although the zeppelin was kept aloft by 16 cells of highly flammable hydrogen gas, it made many uneventful trans-Atlantic trips. However, on May 6, 1937, the *Hindenburg* burst into flames while landing at a U.S. naval air station in New Jersey during a rainstorm. While its handling ropes were being let down to a ground crew, ripples were sighted on the outer fabric near the rear of the ship. Seconds later, flames erupted from that region and 32 seconds after that the Hindenburg fell to the ground.

After so many successful flights of hydrogen zeppelins, why did this one burst into flames?

The answer is in this chapter.

26-1 Introduction

The interpretation of electrostatics experiments (described in Chapters 22 through 25) is that matter consists of two kinds of electrical charges, positive and negative. At least some negative charge can be moved from one object to another, leaving the first positively charged (with a deficit of negative charge) and the second negatively charged (with an excess of negative charge). Once the charges stopped moving we explored the electrostatic forces between them.

It turns out that the electrical devices we encounter most often in modern life such as computers, lights, and telephones are not purely electrostatic but involve moving charges which we will come to call *electric currents*. In addition, natural phenomena such as lightning, the flow of protons between the Earth's magnetic poles, and cosmic ray currents involve electric currents.

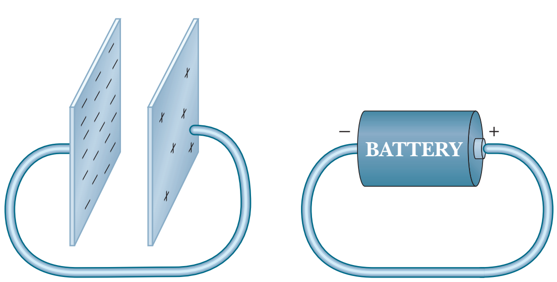
In this chapter we explore electric currents, or charge flow, with a primary focus on how current passes through conductors in electric circuits. We will see that the critical idea is to understand that a potential difference across a conductor causes a flow of charge (a current) through that conductor.

26-2 Batteries and Charge Flow

By the end of the 18th century, Alessandro Volta had discovered that when two metal plates were placed in contact with a moist piece of metal, they seemed to have electrical properties like those of rubbed amber and glass. To magnify this effect, Volta piled up pairs of unlike metals. When he grasped the plates (terminals) at each end of the pile with his hands, he claimed to feel electric charges move through his body on a continuous basis. Volta had invented the battery, and his experience with early batteries is an indication that there is a connection between electric charges, as discussed in Chapters 22–25, and the continuous flow of electricity created by batteries and other power sources. However, it is not obvious without further investigation that there is actually a connection between the sensation of electric flow that Volta experienced and the electric charges we believe exist based on the electrostatic observations discussed in Chapter 22.

In order to investigate this further, let's examine the results of several experiments involving a metal wire connected to oppositely charged conducting plates as shown in Fig. 26-1*a* and the same wire connected to a battery instead as shown in Fig. 26-1*b*.

Experiment 1 (Electrostatic Discharge): Suppose we use glass and amber rods that have been rubbed to transfer electrons to or from conducting plates. We can use a hanging amber or glass rod shown in Fig. 22-2 to verify that we have excess electric charge on each plate. Since it is easier to add excess electrons to a conductor than remove electrons from a conductor, the negatively charged plate will tend to have a



(*a*) (*b*)

FIGURE 26-1 ■ (*a*) When a conducting wire connects two oppositely charged plates, charge flows from the negatively charged plate to the positively charged plate until both plates have the same number of excess electrons. As a result the wire becomes hot. (*b*) When a battery is placed between the ends of the wire instead, the wire also becomes hot, indicating that charge is also flowing.

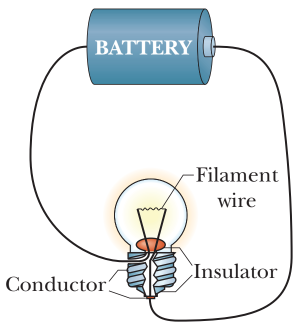


FIGURE 26-2 ■ If there is a complete conducting loop between the two terminals of a battery, a bulb will stay lit until the battery runs down.

greater magnitude of charge. Initially the negative electrons on the left plate repel each other and spread out but cannot leave the plate. Since a positive test charge placed between the plates will be repelled from the positive plate (on the right) and attracted to the negative plate (on the left), we know there is an electric field between the plates. So, Eq. 25-17 tells us there will be a potential difference between the plates.

If we connect the two plates with a piece of thread nothing happens. But when a conducting wire is connected across the two plates, (Fig. 26-1*a*) we observe that excess electrons on the left-hand plate will flow to the right-hand plate until both plates have the same number of excess electrons on them. This is not surprising since we expect the repulsive forces between the electrons on the left plate to push the charges through the wire while the attractive forces on the right plate pull on the charges. If we have enough excess charge on the plates, the wire will feel hot just after the discharge and then cool down again. If the wire has a properly connected small bulb in the middle of it, the bulb will light up briefly and then go out. We conclude from these observations that charge is flowing through the wire for a short time.

Experiment 2 (Battery Current): As we mentioned in Chapter 25, a battery is capable of doing work on electric charges and increasing their potential energy. So there must be a potential difference across its terminals. If we connect a piece of thread between the terminals of a battery nothing happens. On the other hand, if we connect a wire between the terminals of a battery, we observe that the wire gets very hot and stays that way for a long time as shown in Fig. 26-1*b*. If we also properly connect a bulb to the middle of the wire, as shown in Fig. 26-2, the bulb stays continuously lit until the battery eventually runs down. (In the next section we discuss how to connect a bulb to a battery properly, so that it lights.)

Because, at first, the electrostatic charging in Experiment 1 has the same result as the battery in Experiment 2, we infer that the underlying electric effects are the same in both cases. The hot wires and the lighting of bulbs lead us to conclude that charge is flowing through the wires. We call this flow of charge **electric current.**

READING EXERCISE 26-1: Although you were not provided with any details, what sensations might Volta have felt that led him to believe that electric charge was flowing through his body? ■

26-3 Batteries and Electric Current

There are some additional observations that help us understand the nature of electric current. Suppose we want to use a battery and perhaps a wire to light a flashlight bulb. By fiddling around we discover that many of the possible arrangements for lighting a bulb *do not work.* For example, none of the arrangements shown in Fig. 26-3 work.

To understand why these arrangements do not work, we need to examine a flashlight bulb much more carefully. The flashlight bulb consists of a piece of thin conducting "filament" wire encapsulated in glass that has no air inside. This wire glows and so gives off light when electric current passes through it. One end of the filament wire is in contact with a conductor that surrounds the bottom part of the bulb. The other end is connected to another conductor at the bulb's base. These conductors are separated by an insulator. A cutaway diagram of the bulb is shown in Fig. 26-4.

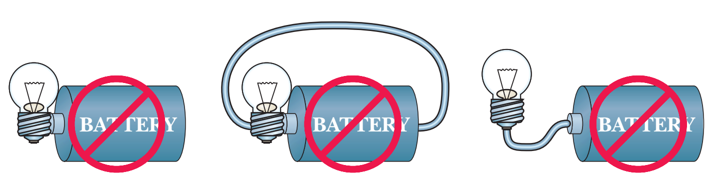


FIGURE 26-3 ■ Three of many arrangements of a battery and bulb and wire that do not cause the bulb to light.

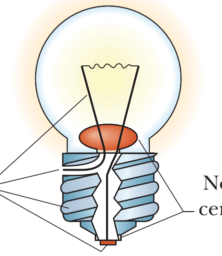


FIGURE 26-4 ■ A cutaway diagram of a flashlight bulb.

After some more fiddling we discover that all of the arrangements of wires, bulb, and battery that cause the bulb to light have one thing in common. They all have a continuous, complete loop or **circuit** for current to pass from one terminal of a battery through conductors back to other terminal of the battery. In addition to the arrangement shown in Fig. 26-2, another of the many arrangements that forms a complete loop and causes a bulb to light is shown in Fig. 26-5.

When bulb filaments get old they sometimes break. In this case the circuit is incomplete and our "burned out" bulb does not light. Another requirement is that the battery must have a potential difference between its terminals. When a battery loses its potential difference after much use we refer to it as a "dead battery."

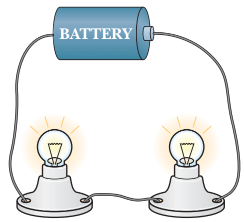


FIGURE 26-5 ■ When two identical bulbs in holders are connected in a row to a battery, they have the same brightness as each other. We conclude that the same current is passing through both bulbs. This indicates that the battery is not a source of excess charge used up by the bulbs.

What Is Stored in a Battery?

It is commonly (and wrongly) believed that batteries store excess charge that can be "used up" in a circuit, and that a battery is "dead" when this excess charge is used up. The fact that people often refer to "charging" and "discharging" batteries is evidence of this belief. Careful observation tells us that this idea is wrong. The excess charge a fresh alkaline flashlight battery would have to store to keep a flashlight bulb lit as long as it does is more than 20 000 coulombs. This is a hundred million times the amount of charge we can typically place on a light metal-coated ball on a string. Yet, we observe no forces between such a charged ball and a fresh battery. There are also no forces between a charged ball and the wires carrying current in a circuit.

> Observations indicate that both batteries and any current-carrying wires connected to them are *electrically neutral.*

We conclude from these observations that batteries do not store charge. Batteries store energy. The energy in the battery is transformed to mechanical energy, light energy, and thermal energy as it pushes charges through wires and bulbs. Thus our observations support the idea that a battery acts as a pump that absorbs electrons at the negative terminal and releases higher potential energy electrons from the positive terminal. We discuss how chemical reactions can create a charge pump in more detail in Section 27-6.

If we connect two identical bulbs to the same battery as shown in Fig. 26-5, they shine with the same brightness. Based on this observation we conclude that the same current is passing through both bulbs.

> When wires and other conducting elements such as bulbs are placed between the battery terminals to make a continuous loop or circuit, the battery acts as a pump that pushes charge carriers already available in the wires around the loop. The battery is not a source of charge and electrical elements like bulbs do not use up charge.

Figure 26-6*a* shows a very simplified representation of a small segment of wire made up of electrically neutral atoms. In Fig. 26-6*b* the ends of the wire segment are

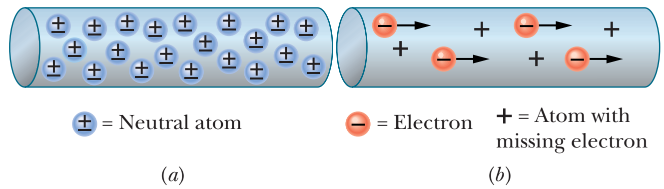


FIGURE 26-6 ■ (*a*) A representation of many electrically neutral atoms in a wire. (*b*) A diagram that shows a potential difference across the ends of the wire so a very small fraction of the electrons surrounding atoms start moving and a few ions with missing conduction electrons are present. These ions have excess positive charge. *Note:* The neutral atoms are still present but are not shown.

connected to a battery (not shown). A few of the conduction electrons in the metal start moving, but the stationary charges, consisting of neutral atoms and ions (with missing conduction electrons) still exist in the wire. The stationary ions neutralize the moving conduction electrons. To reduce clutter, Fig. 26-6*b* shows the stationary ions but not the neutral atoms. In other figures in this chapter we just show moving electrons and not the stationary ions. This type of depiction can give the false impression that there is excess charge in the wire. This is not so. Conducting wires are electrically neutral.

Defining Current Mathematically

Figure 26-2 shows a complete circuit with a battery (or other power source) that maintains a constant potential difference across its terminals. In this case, charge pushed through the circuit by the battery flows through a conducting wire, and then through the filament of a bulb, which is usually a very thin wire. In order to think more carefully about the current, we need to develop a mathematical definition for current.

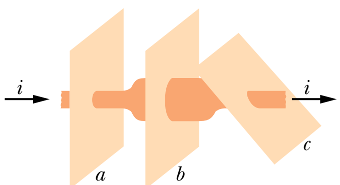


FIGURE 26-7 ■ The current i or charge per unit time through the conductor has the same value at imaginary planes a, b, and c as long as the planes cut through the entire conductor at the points of intersection.

Figure 26-7 shows a section of a conducting loop with different cross-sectional areas in which a current has been established. If net charge dq passes through a hypothetical plane (such as a) in time dt, then the current through that plane is defined as

$$i \equiv \frac{dq}{dt} \quad \text{(definition of current).} \tag{26-1}$$

Regardless of the details of the geometry of the charge flow, we can find the net charge passing through any plane in a time interval extending from 0 to t by integration:

$$q = \int dq = \int_0^t i\,dt. \tag{26-2}$$

Measurements of current through various locations in a single loop circuit show that the current is the same in all parts of a circuit where there are no junctions or alternate paths for the current to take. The current or rate of charge flow is the same passing through the imaginary planes a, b, and c shown in Fig. 26-7. Indeed, the current is the same for any plane that passes completely through the conducting elements in a continuous circuit with no branches, no matter what their locations or orientations. That is, a charge carrier must pass through plane a for every charge carrier that passes through plane c.

The unit for current is called the *ampere* (A), and it can be related to the coulomb by the expression

$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb/second} = 1 \text{C/s}.$$

The Directions of Currents

How can we tell whether there are positive or negative charges moving when a current is established in electrically neutral conductors? When we place a conducting wire between the plates shown in Fig. 26-8, charge carriers flow until the plates are neutralized.

It is not possible for us to design an experiment based on macroscopic observations that will allow us to tell whether the charge carriers are positive or negative because the end result (neutralized plates) will be the same in either case. Early experimenters with electricity had no knowledge of atomic structure and could only use macroscopic observations of electrical effects to guide them. They assumed that charge carriers were positive. Even though we now know that negatively charged electrons are the charge carriers in conductors, for historical reasons we will stick with the assumption that the charge carriers are positive. This historical assumption makes it easier to use traditional references on electricity, and all the characteristics of circuits we will study on a macroscopic level will be exactly the same. Furthermore, this early assumption would have been correct if Benjamin Franklin had decided to designate the excess charges on rubber rods as positive and those on glass as negative instead of the other way around!

Although the charge carriers in conductors are negative, other currents, for example, protons streaming out of our Sun, create positive currents. Also charge carriers in fluids can be either positive ions (atoms with missing electrons) or negative electrons or ions (atoms with extra electrons). In fact, the movement of charge within most batteries is due to the migration of positive ions that undergo chemical reactions. Also, currents in biological systems are carried by sodium and potassium ions, which are positive charge carriers.

Current arrows show only a direction (or sense) of flow of charge carriers along the connected conductors as they bend and turn between battery terminals, not a fixed direction in space. Since current is actually a flux, which is a scalar quantity, *these current arrows do not represent vectors with magnitude and direction.*

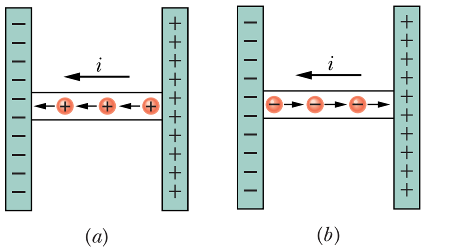


FIGURE 26-8 ■ No macroscopically-oriented experiment will allow us to detect whether the charge carriers in a conducting wire are (*a*) positive or (*b*) negative. So we define the current flow to be from right to left in both cases. Although stationary charges are not depicted, the conducting wires are neutral.

> A current arrow, although not a mathematical vector, is drawn in the direction in which positive charge carriers would move through wires and circuit elements from a higher potential to a lower (more negative) potential, even though the actual charge carriers are usually negative and move in the opposite direction.

Charge Conservation at Junctions

So far we have only considered circuits like the one shown in Fig. 26-9*a*, ones for which there is only one path for charge carriers to follow. Such circuits are called **series** circuits. However, it is also common to find circuits or portions of circuits in which charge carriers encounter a junction where they can take either of two (or more) paths as shown in Fig. 26-9*b*. We call this type of circuit a **parallel** circuit. Although we introduce the terms "series" and "parallel" here, we will focus on the quantitative evaluation of series and parallel circuits in Chapter 27.

Figure 26-9*b* shows the moving charge carriers splitting up at a junction and then moving in parallel. If the bulbs are *identical,* how do the currents split at junction 1?

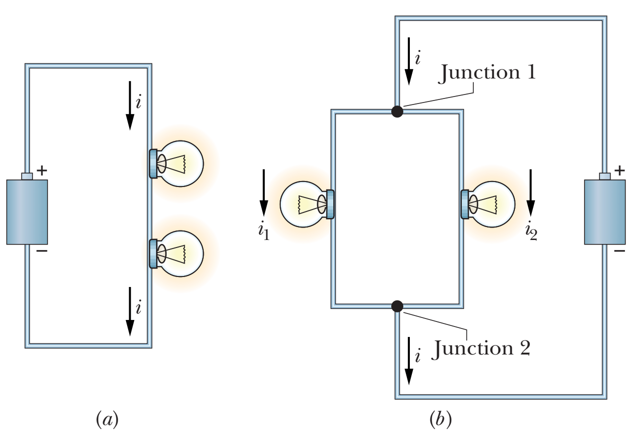


FIGURE 26-9 ■ We use a lightbulb as an example of a circuit element. (*a*) A series connection involves two or more circuit elements that are connected together so that the same current that passes through one element must pass through the other element. The potential differences across the elements is the sum of the drops across each element. (*b*) A parallel connection requires that one terminal of each two or more elements are connected together at one point and then the other terminal of each of the elements is connected together at another point. These points of connection are called junctions. Because of the connections at the junctions the potential difference across each element is the same when the parallel network is placed in a circuit.

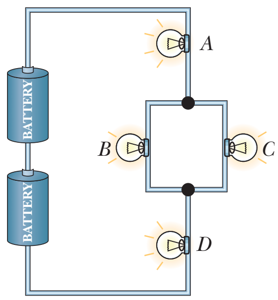


FIGURE 26-10 ■ To verify what we think would happen to current at the junctions, four identical lightbulbs and a battery are connected in a circuit having both series and parallel elements. Observations tell us that the brightness of bulb A is the same as that in bulb D. Bulbs B and C have the same brightness as each other but share the battery's current, so they are much dimmer than A and D. Note that even though they are not adjacent, bulbs A and D are in series.

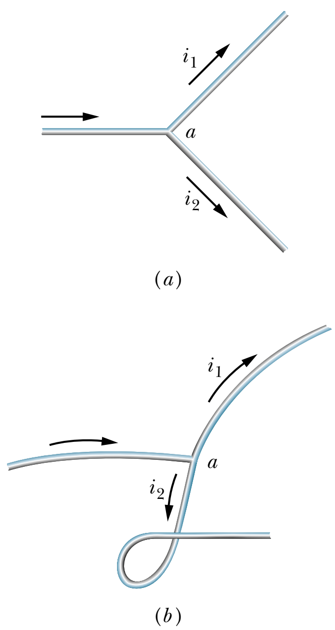


FIGURE 26-11 ■ The relation $i = i_1 + i_2$ is true at junction a no matter what the orientation of the three wire segments is.

What happens when they come back together at junction 2? Since the bulbs are identical, we expect that the current coming into junction 1 will divide equally so half takes the left path and half the right. If charge is conserved, when the currents combine at junction 2, they should add up to the original current. We can verify this by making a very simple observation with a couple of flashlight batteries in series and four bulbs as shown in Fig. 26-10. Bulbs A and D have the same brightness as each other. This fact indicates that the amount of current through bulb A is the same amount of current passing through bulb D and back through the battery. The fact that bulbs B and C have the same brightness as each other but are dimmer than A and D suggests that the current is splitting in half at the junction. We conclude

$$i = i_1 + i_2 = \frac{i}{2} + \frac{i}{2} \quad \text{(special case with identical parallel elements),} \tag{26-3}$$

where i is the total current and i_1 and i_2 are the currents in the two branches.

If charge is conserved, the magnitudes of the currents in two parallel branches *must* add to yield the magnitude of the current in the original conductor even when the branches have different circuit elements. This statement, called Kirchhoff's current law, states that in general

$$i = i_1 + i_2 \quad \text{(three-way junction of Fig. 26-11}a\text{).} \tag{26-4}$$

We treat Kirchhoff's circuit laws in more detail in Chapter 27.

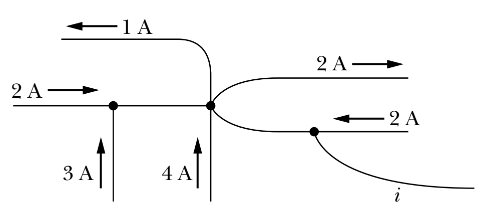
Experiments indicate that bending or reorienting the wires in space does not change the validity of Eq. 26-4. The fact that current is not affected by wire orientation can be explained by the accumulation of static surface charges. These charges keep the electric field associated with a potential difference pointing along a wire, regardless of how the wire twists. The lack of influence of bending is depicted in Fig. 26-11.

READING EXERCISE 26-2: Explain why the word "circuit," as used in everyday speech, is an appropriate term for application to electrical situations.

READING EXERCISE 26-3: Suppose a battery sets up a flow of charges through wires and a bulb. (a) Will the overall circuit, consisting of the battery, bulb, and wires, remain electrically neutral, become positive, or become negative? Explain. (b) Will the wires remain electrically neutral, become positive, or become negative? (c) What ordinary observations would support your answers?

READING EXERCISE 26-4: Apply your understanding of the concept of flux, the nature of electrical charge, and the definition of current presented to explain why a flow of equal amounts of opposite charge in the same direction would not be considered a current. That is, explain why there must be a net flow of charge through a surface for there to be a current.

READING EXERCISE 26-5: The figure below shows a portion of a circuit. What are the magnitude and direction of the current i in the lower right-hand wire?



TOUCHSTONE EXAMPLE 26-1: Charged Fuel

If you've ever gone to a gas station to fill a gas can with fuel for your lawn mower, you may have noticed the sign that tells you to take the gas can out of your car and place it on the ground before you fill it with gasoline. Why is this important? As fuel is pumped from its underground storage tank, it can acquire a net electrical charge. If so, as you pump fuel into a container, the can will build up a net electrical charge if it is electrically isolated from its surroundings. If this charge builds up to a sufficient level, it can create a spark, igniting the fumes around your container with very unfortunate consequences.

Suppose the maximum safe charge that can be deposited on your 5.0 gal gas can is 1.0 μC.

(a) What is the maximum safe charge per liter that the fuel you are pumping can have?

SOLUTION ■ The **Key Idea** here is simply that the maximum safe "charge density" is

$$(1.0\ \mu\text{C})/(5.0\ \text{gal}) = (0.20\ \mu\text{C/gal})(264\ \text{gal/m}^3)(1\ \text{m}^3/1000\ \text{L})$$
$$= 0.0528\ \mu\text{C/L}. \qquad \text{(Answer)}$$

(b) If the pump delivers fuel at a rate of 8.0 gallons per minute, what is the maximum safe electrical current associated with the flow of the fuel into the can?

SOLUTION ■ The **Key Idea** here is that the fuel delivery rate is

$$(8.0\ \text{gal/min})(1\ \text{L}/0.264\ \text{gal})(1\ \text{min}/60\ \text{s}) = 0.50505\ \text{L/s}.$$

Since each liter of fuel can deliver no more than 0.0528 μC safely, the maximum safe electrical current is just (0.0528 μC/L)(0.50505 L/s) = 0.027 μC/s = 27 nA.

26-4 Circuit Diagrams and Meters

As we move into the remaining sections in this chapter and the next, we will be drawing electric circuits with elements such as batteries, bulbs, wires, and switches. We will also be introducing new elements such as resistors and meters for measuring current and voltage.

Symbols for Basic Circuit Elements

Before proceeding with our study of current and resistance, we pause and introduce a few of the symbols scientists and engineers have created to represent circuit elements. Figure 26-12 shows the common symbols used to make the circuits we discuss in this chapter easier to draw.

Using these symbols, the circuit shown in Fig. 26-2 with a switch added can be represented as shown in Fig. 26-13.

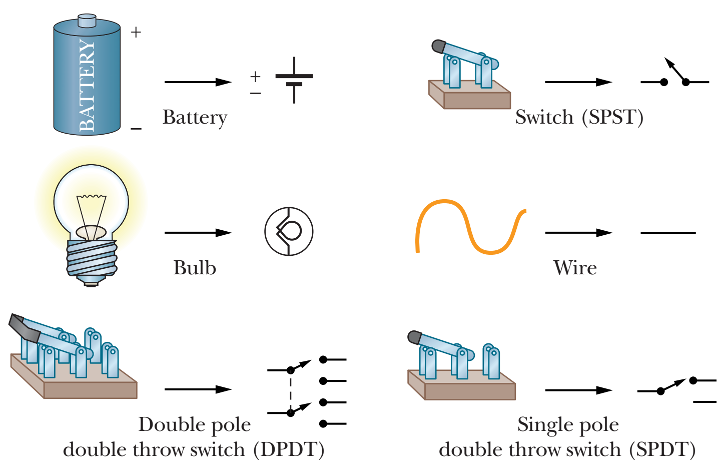


FIGURE 26-12 ■ Some circuit symbols.

Meters

Current and potential differences are very important properties of electrical circuits, and we have well-established convenient

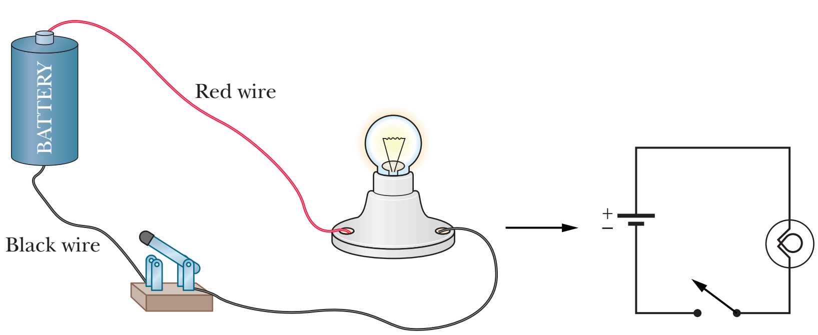


FIGURE 26-13 ■ A circuit sketch and corresponding diagram.

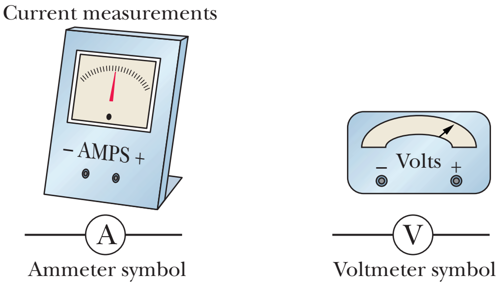


FIGURE 26-14 ■ An analog ammeter for measuring current and an analog voltmeter for measuring potential difference (or "voltage"), along with their circuit symbols.

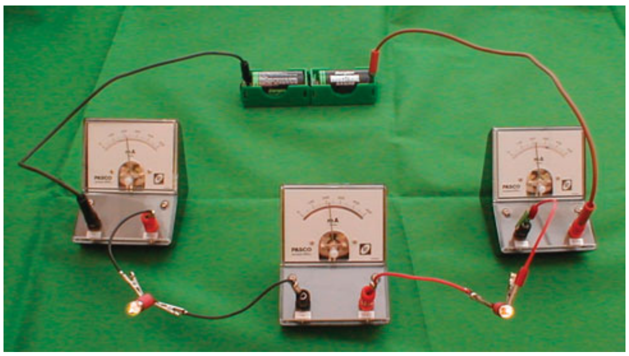


FIGURE 26-15 ■ Three analog ammeters measure the same current flowing through three locations in a series circuit consisting of two #14 flashlight bulbs.

ways to measure these quantities using meters. The device with which one measures current is called an **ammeter.** Potential difference is measured with a device called a **voltmeter.** An ammeter and voltmeter along with their circuit symbols are depicted in Fig. 26-14.

Since an ammeter measures current *through* a circuit (or a branch of a more complex circuit), it is placed in *series* with circuit elements. A voltmeter measures the potential difference between two locations (or points) in a circuit, so a voltmeter is placed across or in *parallel* with the two points of interest. This is shown in Fig. 26-15.

Often ammeters and voltmeters are combined in a device used to measure either potential difference or current. When the two or more meters are combined, the meter is typically called a **multimeter.** A digital multimeter is shown in Fig. 26-16. Many modern digital multimeters are also capable of measuring other quantities we will discuss, such as resistance and capacitance.

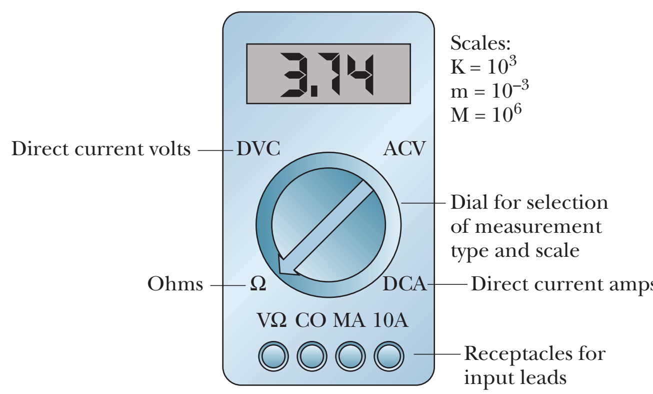


FIGURE 26-16 ■ The digital multimeter pictured can be configured to act as an ammeter to measure current *through* a given part of a circuit, a voltmeter to measure potential difference *across* any two points in a circuit, or the resistance of any circuit element.

READING EXERCISE 26-6: In Fig. 26-17, the voltmeter is attached across the bulb and the ammeter is inserted into the circuit. Why are these devices connected this way? How would the ammeter reading change if it were inserted in the circuit before the bulb instead of after it? ■

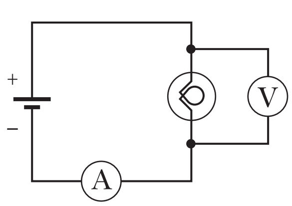


FIGURE 26-17 ■ A basic circuit for measuring the current flowing *through* a circuit element as a function of potential difference *across* it.

26-5 Resistance and Ohm's Law

In professional applications of physics like designing electronic devices, we often need to know what effect adding more circuit elements will have on the flow of current. Given devices like ammeters and voltmeters, with which we can measure current and potential difference, we can do quantitative studies of the relationship between current and potential difference. For example, what will happen to the current in a circuit element, such as a bulb, that is part of a circuit if we add more batteries in series with our original battery? What will happen to the current in a conducting wire as voltage increases? The experimental setup for this investigation is shown in Fig. 26-17. The results are presented in Fig. 26-18 as graphs of applied potential difference ΔV and the resulting current i in two different circuit elements.

We can draw several interesting conclusions from looking at the two graphs in Fig. 26-18. First, we see in both graphs that as the potential difference increases, the amount of current through a given device increases. Second, it is not possible to tell how much current exists just by knowing the potential difference across a circuit element. For instance, when 1.0 V is placed across the lightbulb, the current through it is greater than the current in the Nichrome wire with the same potential difference

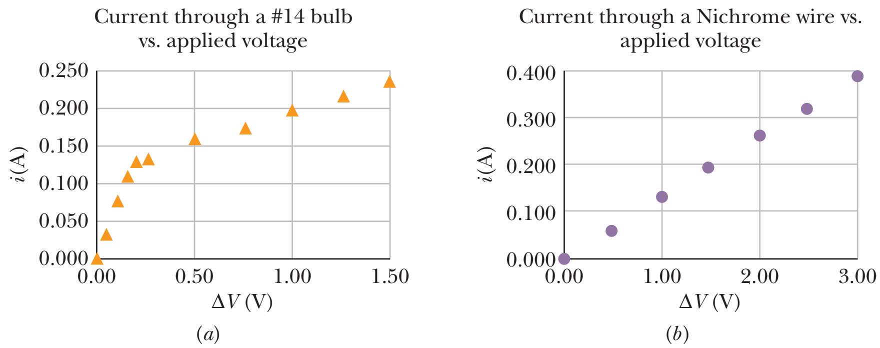


FIGURE 26-18 ■ Graph (a) shows ammeter data for current passing through a #14 lightbulb as a function of potential difference between the terminals of the lightbulb. Graph (b) shows ammeter data for the current through a length of cylindrical Nichrome wire as a function of potential difference between the ends of the wire.

across it. Third, for the length of Nichrome wire, the current is directly proportional to the potential difference, ΔV, across it. Thus, if we know the slope of the line, we can predict the current associated with any value of ΔV. Because of this direct proportionality, we refer to the Nichrome wire as a **linear** device. For the lightbulb, there is no convenient direct proportionality, so it is called a **nonlinear** device.

Definition of Resistance

In both the small bulb and the Nichrome wire, once we measure a specific potential difference, ΔV, across a circuit element and the corresponding current through it we have a measure of the *resistance* of the element to current but only at that ΔV. The **resistance** of a given circuit element is defined as the ratio of the potential difference across the element to the current through the element. When a small potential difference causes a relatively large current, the circuit element has a small resistance to flow of charge. Conversely, when the same potential difference produces a current that is small, we say the resistance is large. For example, in the data presented in Fig. 26-18, a potential difference of 1V across the bulb causes a current of 0.19A to flow, while the same potential difference across that Nichrome wire causes only 0.13A to flow. So we say that at the specific potential difference of 0.25V, the Nichrome wire has more resistance than the bulb.

We define resistance as the ratio of potential difference applied to the current that results:

$$R \equiv \frac{\Delta V}{i} \quad \text{(definition of } R\text{).} \tag{26-5}$$

Here we use the notation ΔV to emphasize we are dealing with the *difference* in potential between two locations in a circuit, which changes the potential energy of the charges as they flow. When discussing circuits, potential difference is often referred to by an alternate name of **voltage.**

The SI unit for resistance that follows from Eq. 26-5 is the volt per ampere. This combination occurs so often that we give it a special name, the **ohm** (symbol Ω); that is,

$$\begin{aligned} 1\,\text{ohm} &= 1\ \Omega = 1\ \text{volt/ampere} \\ &= 1\text{V/A}. \end{aligned} \tag{26-6}$$

If we rewrite Eq. 26-5 as

$$i = \frac{\Delta V}{R},$$

it emphasizes the fact that the potential difference across a device with resistance R *produces* an electric current. The most common way to express the definition of resistance in Eq. 26-5 is

$$\Delta V = iR. \tag{26-7}$$

For a linear device like Nichrome wire we will get the same value for R no matter what potential difference we impress across the device. However, we must be *careful* in the case of a nonlinear device like a light bulb to specify at what potential difference we are measuring the current, i, in order to determine its resistance.

Ohm's Law

As we just pointed out, our Nichrome wire has the same resistance no matter what the value of the applied potential difference (as shown in Fig. 26-18*b*). Other conducting devices, such as lightbulbs, have resistances that change with the applied potential difference (as shown in Fig. 26-18*a*). Although both the Nichrome wire and the bulb contain metallic conductors, the wire in the bulb is so thin that its temperature rises noticeably as the potential difference increases, and the bulb's resistance increases.

In 1827, George Simm Ohm, a Bavarian, reported that he had observed a linear relationship between current and potential difference for metallic conductors kept at a fairly constant temperature. Because of this, linear devices such as the length of Nichrome wire are sometimes referred to as **ohmic.**

> A device is said to obey Ohm's law whenever the current through it is *always* directly proportional to the potential difference applied. That is, the device's resistance is constant in the $\Delta V = iR$ relation.

Many elements used in electric circuits, whether they are conductors like copper or semiconductors like pure silicon or silicon containing special impurities, obey Ohm's law within some range of values of potential difference. If the current in a resistive device is large enough to cause significant temperature changes in it, then Ohm's law often breaks down.

It is sometimes contended that $R = \Delta V/i$ (or $\Delta V = iR$) is a statement of Ohm's law. That is not true! This equation is the defining equation for resistance, and it applies to all conducting devices, whether they obey Ohm's law or not. If we measure the potential difference ΔV across and the current i through any device, even a bulb or other non-ohmic device, we can find its resistance *at that value of* ΔV as $R \equiv \Delta V/i$. The essence of Ohm's law, however, is a plot of i versus ΔV that is a straight line, so that the value of R is independent of the value of ΔV.

Resistors

A conductor whose function in a circuit is to obey Ohm's law so that it provides a specified resistance to the flow of charge independent of the potential difference impressed across it is called a **resistor** (see Fig. 26-19). Carbon resistors are the most standard sources of ohmic resistance used in electrical circuits for several reasons. Unlike a lightbulb, a resistor has a resistance that remains constant as current changes. Carbon resistors are inexpensive to manufacture, and they can be produced with a large range of resistances. The circuit diagram symbol for a resistor is shown in Fig. 26-20.

A typical carbon resistor contains graphite, a form of carbon, suspended in a hard glue binder. It usually is surrounded by a plastic case with a color code painted on it as shown in Fig. 26-21.

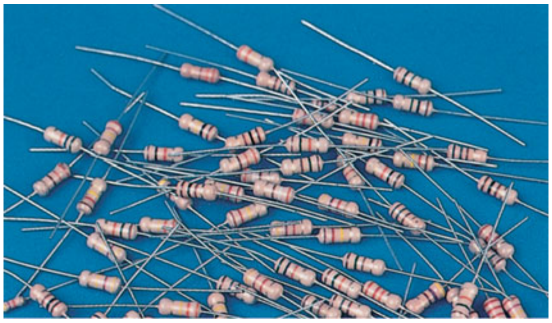


FIGURE 26-19 ■ An assortment of carbon resistors. The circular bands are color-coding marks that identify the value of the resistance.

READING EXERCISE 26-7: The following table gives the current i (in amperes) through three devices for several values of potential difference ΔV (in volts). From these data, determine which devices, if any, obey Ohm's law.

Device 1		Device 2		Device 3	
ΔV	i	ΔV	i	ΔV	i
2.00	4.50	2.00	1.50	2.00	6.50
3.00	6.75	3.00	2.50	3.00	8.75
4.00	9.00	4.00	3.00	4.00	11.00

FIGURE 26-20 ■ Circuit diagram symbol for an ohmic resistor.

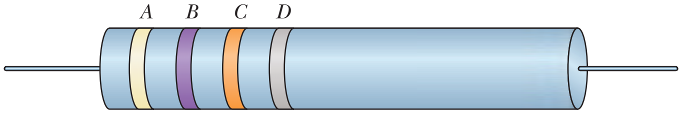


FIGURE 26-21 ■ Depiction of the four color bands on a color-coded resistor with $R = 47\ \text{K}\Omega \pm 10\%$. See Table 26-1 for details

26-6 Resistance and Resistivity

Next we consider how the resistance of ohmic circuit elements such as metal wires or carbon resistors depends on their geometries. That is, how does the resistance of a short, broad object change if we stretch it so it is long and thin? To determine this, we fix our investigation on a single material. For example, we might experiment with copper wire. Relatively thick copper wire is commonly used in electric circuits because it has a very low resistance compared to other circuit elements. Thus, it can be used to connect circuit elements without adding much resistance to a circuit.

Observations

Consider a conducting wire with a potential difference across its ends as shown in Fig. 26-22. To start with, we will keep the thickness of the wire fixed and just decrease its length. If we apply a potential difference across the ends of the wire and use current and potential difference measurements, we can determine its resistance as a function of length. We find that its resistance is proportional to its length L. Thus, we can write

$$R = kL.$$

If instead we fix the length of the wire and decrease its thickness or cross-sectional area A, then the measured resistance of the wire increases as its cross-sectional area

TABLE 26-1
The Resistor Code[a]

Black = 0	Blue = 6
Brown = 1	Violet = 7
Red = 2	Gray = 8
Orange = 3	White = 9
Yellow = 4	Silver = ±10%
Green = 5	Gold = ±5%

[a]The value in ohms $= AB \times 10^C \pm D$. (AB means the A band digit placed beside the B band digit, not A times B). The colors on bands A, B, and C represent the digits shown in Table 26-1. The D band represents the "tolerance" of the resistor. No band denotes ±20%, a silver band denotes ±10%, and a gold band denotes ±5%. For example, a resistor with bands of Blue-Gray-Red-Silver has a value: $AB \times 10^C \pm D = 68 \times 10^2\,\Omega \pm 10\%$ or $(6800 \pm 680)\,\Omega$, since $A = 6$, $B = 8$, $C = 2$, $D =$ silver, (±10%).

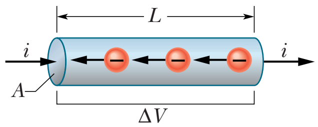


FIGURE 26-22 ■ A potential difference ΔV is applied between the ends of a conducting wire of length L and cross section A, establishing a current i. Although the stationary ions that neutralize the conduction electrons that make up the current are not shown, the wire is, as always, essentially neutral electrically.

decreases. In fact we get an inverse relationship so that

$$R = k'\frac{1}{A}.$$

To combine these two results, we write that R is proportional to L and inversely proportional to A with a new proportionality constant, ρ, which we define as the **resistivity** of the wire. Thus,

$$R = \rho\frac{L}{A}. \tag{26-8}$$

The results of these resistivity observations are important for two reasons. First, the fact that resistance varies inversely with cross-sectional area implies that current passes through the volume of the conductor, and not just along the surface. This knowledge will be useful as we continue to think about how charge moves through wires and other circuit elements.

Second, we know that every conducting material has a resistivity ρ. Is it the same for all materials? The answer is no. Is it the same if the length (or area) of a wire is changed? The answer is yes. What we observe is that if we apply the same potential difference between the ends of geometrically similar (same L and same A) rods of copper and of glass, very different currents result. This investigation reveals that resistivity varies with material. That is, it is a property of the *material* from which the object is fashioned.

We have just made an important distinction:

> Resistance is a property of an object. Resistivity is a property of a material.

It is important to note that resistivity is analogous in many ways to the concept of density. Density depends only on the kind of material being used (such as lead or Styrofoam). The density can be used to calculate the mass of a certain volume of a substance. Similarly, resistivity depends only on the material being used in the wire and not on the length or cross-sectional area of the wire. If you know the resistivity of a material then the resistance of a given wire can be calculated using Eq. 26-8 once its length and cross-sectional area are known.

Variation of Resistivity with Temperature

The values of most physical properties vary with temperature, and resistivity is no exception. Figure 26-23, for example, shows the variation of this property for copper over a wide temperature range. The relation between temperature and resistivity for copper—and for metals in general—is fairly linear over the temperature range commonly found in circuits. For such linear relations we can write an approximation based on the results of measurements as

$$\rho - \rho_0 \approx \rho_0\alpha(T - T_0) \quad \text{(approx. temperature dependence of } \rho\text{).} \tag{26-9}$$

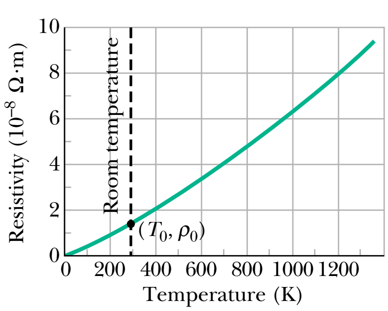


FIGURE 26-23 ■ The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point (T_0 = 293 K and $\rho_0 = 1.69 \times 10^{-8}\ \Omega\cdot\text{m}$).

Here T_0 is a selected reference temperature and ρ_0 is the resistivity at that temperature. Usually T_0 = 293 K (room temperature), for which $\rho_0 = 1.69 \times 10^{-8}\,\Omega \cdot \text{m}$ for copper. This approximate relationship is good enough for most engineering purposes.

Because temperature enters into this expression only as a difference, it does not matter whether you use the Celsius or Kelvin scale in that equation because the sizes of degrees on these scales are identical. The quantity α, called the **temperature coefficient of resistivity,** is chosen so that the equation gives good agreement with

TABLE 26-2
Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot$ m)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
Manganin[a]	48.2×10^{-8}	0.002×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

[a]An alloy specifically designed to have a small value of α.
[b]Pure silicon doped with phosphorus impurities to a charge carrier density of 10^{23} m^{-3}.
[c]Pure silicon doped with aluminum impurities to a charge carrier density of 10^{23} m^{-3}.

experimental values for temperatures in the chosen range. Some values of α for metals are listed in Table 26-2.

The *Hindenburg*

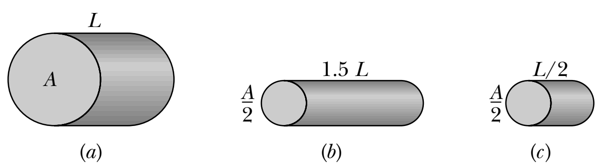
When the zeppelin *Hindenburg* was preparing to land on May 6th, 1937, the handling ropes were let down to the ground crew. Exposed to the rain, the ropes became wet (and thus were able to conduct a current). In this condition, the ropes "grounded" the metal framework of the zeppelin to which they were attached; that is, the wet ropes formed a conducting path between the framework and the ground, making the electric potential of the framework the same as the ground's. This should have also grounded the outer fabric of the zeppelin. The *Hindenburg,* however, was the first zeppelin to have its outer fabric painted with a sealant of large electrical resistivity. The fabric remained at the electric potential of the atmosphere at the zeppelin's altitude of about 43 m. Due to the rainstorm, that potential was large relative to the potential at ground level.

The handling of the ropes apparently ruptured one of the hydrogen cells and released hydrogen between that cell and the zeppelin's outer fabric, causing the reported rippling of the fabric. There was then a dangerous situation: the fabric was wet with conducting rainwater and was at a potential much different from the framework of the zeppelin. Apparently, charge flowed along the wet fabric and then sparked through the released hydrogen to reach the metal framework of the zeppelin, igniting the hydrogen in the process. The burning rapidly ignited the cells of hydrogen in the zeppelin and brought the ship down. If the sealant on the outer fabric of the *Hindenburg* had been of less resistivity (like that of other zeppelins), the *Hindenburg* disaster probably would not have occurred.

READING EXERCISE 26-8: Sketch a graph of i vs ΔV for a Nichrome wire like that in Fig. 26-22 but with the diameter of the wire cut in half.

READING EXERCISE 26-9: In the section above, we cited the fact the resistance of a wire to current was inversely proportional to the cross-sectional area of the wire as evidence that the current passes through the volume of the wire rather than along the surface of the wire. (a) Justify this assertion. (b) What expression would you expect to replace Eq. 26-8 if the current was along the surface of the wire instead?

READING EXERCISE 26-10: The figure shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference ΔV is placed across their lengths.



26-7 Power in Electric Circuits

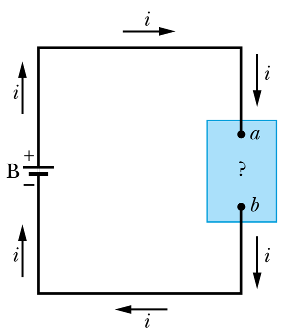
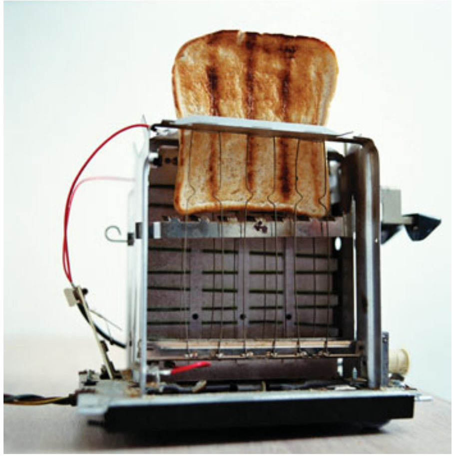


FIGURE 26-24 A battery B sets up a current i in a circuit containing an unspecified conducting device.



The wire coils within a toaster have appreciable resistance. When there is a current through them, electrical energy is transferred to thermal energy of the coils, increasing their temperature. The coils then emit infrared radiation and visible light that can toast bread.

Batteries store a certain amount of chemical energy. This chemical energy is transformed to electrical and other forms of energy as current flows through various circuit elements. At times we are interested in the rate at which a battery's energy is used up by a circuit. Just as we did in Section 9-10 where power is defined as the rate at which work is done by a force, we also use the term power to describe the rate at which electrical energy is delivered to a circuit.

We start our consideration of power by examining the energy delivered to an electrical device that is connected to a battery by ideal wires. Figure 26-24 shows a circuit consisting of a battery B that is connected by wires to an unspecified conducting device. The device might be a resistor, a storage battery (a rechargeable battery), a motor, or some other electrical device. If the wires in the circuit are thick enough they are ideal because they have essentially no resistance. When current is present in a wire with no resistance the entire wire is at the same potential. In other words, there is no potential difference between one end of an ideal wire and the other end. In this case, a battery maintains a potential difference of magnitude ΔV across its own terminals, and thus across the terminals of the unspecified device, with a greater potential at terminal a of the device than at terminal b.

Since there is an external conducting path between the two terminals of the battery, and since the battery maintains a fixed potential difference, the battery produces a steady current i in the circuit. This current is directed from terminal a to terminal b. The amount of charge dq moving between those terminals in time interval dt is equal to $i\,dt$. This charge dq moves through a decrease in potential difference across the terminals of the device of magnitude ΔV, and thus its electric potential energy U decreases in magnitude by the amount

$$dU = -dq\,\Delta V = -i\,dt(\Delta V).$$

The principle of conservation of energy tells us that the decrease in electric potential energy from a to b is accompanied by a transfer of energy to some other form. Since $P = dW/dt$ (Eq. 9-48), the power P associated with that transfer is the rate at which the battery does work. Since $dW = -dU = idt(\Delta V)$, we get

$$P = i\,\Delta V \qquad \text{(rate of electric energy transfer).} \tag{26-10}$$

Moreover, this power P is also the rate at which energy is transferred from the battery to the unspecified device. If that device is a motor connected to a mechanical load, the energy is transferred as work done on the load. If the device is a storage battery being charged, the energy is transferred to stored chemical energy in the storage battery. We know from observations that if the device is a resistor, the energy is transferred to internal thermal energy, tending to increase the resistor's temperature.

The unit of power following from the equation above is the volt-ampere (V · A). We can write it as

$$1\ \mathrm{V \cdot A} = \left(1\frac{\mathrm{J}}{\mathrm{C}}\right)\left(1\frac{\mathrm{C}}{\mathrm{s}}\right) = 1\frac{\mathrm{J}}{\mathrm{s}} = 1\ \mathrm{W}.$$

The course of an electron moving through a resistor at constant speed is much like that of a stone falling through syrup at constant terminal speed. The average kinetic energy of the electron remains constant, and its lost electric potential energy appears as thermal energy in the resistor and its surroundings. On a microscopic scale this energy transfer is due to collisions between the electron and the molecules of the resistor, which leads to an increase in the temperature of the resistor lattice. The mechanical energy thus transferred to thermal energy is *lost* because the transfer cannot be reversed. This energy transfer due to atomic collisions is discussed in more detail in Sections 26-10 and 26-11.

For a resistor or some other device with resistance R, we can combine Eqs. 26-5 ($R = \Delta V/i$) and 26-10 to obtain, for the rate of electric energy loss (or dissipation) due to a resistance, either

$$P = i^2R \quad \text{(resistive dissipation)} \tag{26-11}$$

or

$$P = \frac{(\Delta V)^2}{R} \quad \text{(resistive dissipation).} \tag{26-12}$$

Caution: We must be careful to distinguish these two new equations from Eq. 26-10: $P = i\,\Delta V$ applies to electric energy transfers of all kinds; $P = i^2R$ and $P = (\Delta V)^2/R$ apply only to the transfer of electric potential energy to thermal energy in a device with resistance.

READING EXERCISE 26-11: A potential difference ΔV is connected across a device with resistance R, causing current i through the device. Rank the following variations according to the change in the rate at which electrical energy is converted to thermal energy due to the resistance, greatest change first: (a) ΔV is doubled with R unchanged, (b) i is doubled with R unchanged, (c) R is doubled with ΔV unchanged, (d) R is doubled with i unchanged. ■

TOUCHSTONE EXAMPLE 26-2: Heating Wire

You are given a length of uniform heating wire made of a nickel-chromium-iron alloy called Nichrome; it has a resistance R of 72 Ω. At what rate is energy dissipated in each of the following situations? (1) A potential difference of 120 V is applied across the full length of the wire. (2) The wire is cut in half, and a potential difference of 120 V is applied across the length of each half.

SOLUTION ■ The **Key Idea** is that a current in a resistive material produces a transfer of electrical energy to thermal energy; the rate of transfer (dissipation) is given by Eqs. 26-10 to 26-12. Because we know the potential ΔV and resistance R, we use Eq. 26-12, which yields, for situation 1,

$$P = \frac{(\Delta V)^2}{R} = \frac{(120\ \mathrm{V})^2}{72\ \Omega} = 200\ \mathrm{W}. \quad \text{(Answer)}$$

In situation 2, the resistance of each half of the wire is (72 Ω)/2, or 36 Ω. Thus, the dissipation rate for each half is

$$P' = \frac{(120\ \mathrm{V})^2}{36\ \Omega} = 400\ \mathrm{W},$$

and that for the two halves is

$$P = 2P' = 800 \text{ W}. \qquad \text{(Answer)}$$

This is four times the dissipation rate of the full length of wire. Thus, you might conclude that you could buy a heating coil, cut it in half, and reconnect it to obtain four times the heat output. Why is this unwise? (What would happen to the amount of current in the coil?)

26-8 Current Density in a Conductor

We defined current so that it was a scalar—basically a "count" of the amount of charge crossing a surface per second with a sign to tell us in which direction the charge is crossing the surface—in the direction we choose as positive or opposite to it. Since in a current, charges are actually moving and have a velocity associated with them, there is a vector "hidden" in the concept of current. We can make it explicit by defining a new concept, the **current density.** If we have a volume that contains a set of moving charged particles, let the charge on each particle be e, let the density of the charges be n (number per unit volume), and let their average velocity be $\langle\vec{v}\rangle$. We then define the current density (or current percentage of cross-sectional area) as

$$\vec{J} \equiv ne\langle\vec{v}\rangle \qquad \text{(definition of current density).} \qquad (26\text{-}13)$$

As is the case for the volume flux of water described in Eq. 15-33, the total amount of charge flowing through a given element of area can be defined as the dot product of the current density and an area element. If the area element is infinitesimal we can write the amount of current through it as $\vec{J} \cdot d\vec{A}$, where $d\vec{A}$ is the area vector of the element, perpendicular to the plane of the area element. The total *conventional current* through the surface of a cross section of wire is then

$$i = \int \vec{J} \cdot d\vec{A}. \qquad (26\text{-}14)$$

In most electrical conductors the charge carriers are negative. As we mentioned earlier, the term "conventional current" refers to the direction of flow of positive charge carriers. For a typical conductor such as copper, the electrons are moving in the opposite direction to the direction of the conventional current.

In Section 26-6 we concluded that in steady current through a conductor the charges must be flowing throughout the volume of the conductor. The key evidence for this is the inverse proportionality between resistance and the cross-sectional area of a conductor.* If we further assume that the direction of the current is parallel to $d\vec{A}$, then $\vec{J}$ is also uniform and parallel to $d\vec{A}$. In this case Eq. 26-14 can be rewritten in terms of the magnitudes of the current density and area.

$$|i| = \int J dA = J \int dA = JA,$$

so

$$J = \frac{|i|}{A}, \qquad (26\text{-}15)$$

*However, steady charge flow throughout a conductor is not true for the high-frequency alternating currents we treat in Chapter 33.

where A is the total area of the surface. From these equations, we see that the SI unit for current density is the ampere per square meter (A/m^2).

In Chapter 23 we represented an electric field with electric field lines. Figure 26-25 shows how current density can be represented with a similar set of lines, which we can call *streamlines*. The current, which is toward the right in Fig. 26-25, makes a transition from the wider conductor at the left to the narrower conductor at the right. Because charge is conserved during the transition, the current or rate at which the charges flow through the wire cannot change. However, the current density (or rate of charge flow per unit of cross-sectional area) does change—it is greater in the narrower conductor. The spacing of the streamlines suggests this increase in current density; streamlines that are closer together imply greater current density.

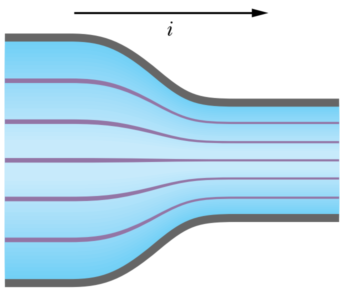
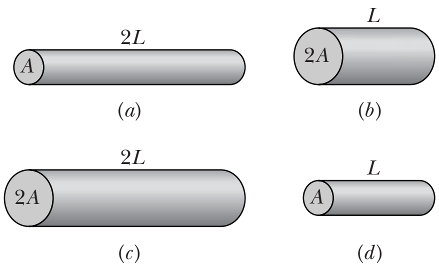


FIGURE 26-25 ■ Streamlines representing current density in the flow of charge through a constricted conductor.

READING EXERCISE 26-12: The sketches below show several copper wires with the same potential difference across them. Rank the current density magnitude from largest to smallest.



26-9 Resistivity and Current Density

Although i, ΔV, and R are the quantities that are directly measurable in electrical circuits, if we want to think more explicitly about what is happening in terms of the motion of charges it makes sense to reframe our Ohm's law relation in terms of the forces (or the electric field) and the current density. This gives us a generic relation that describes how forces affect the motion of charges without relying in any way on the properties of specific circuit elements in the way that Ohm's law does.

Recall that, for materials that obey Ohm's law, the resistance of a segment of a conductor R is related to the potential difference ΔV across it as well as the conventional current i passing through it. This relationship is given by

$$\Delta V = iR. \qquad \text{(Eq. 26-7)}$$

We can write this expression in an alternate form if we replace the potential difference ΔV with an expression involving the electric field $\vec{E}$. From Chapter 25, we know that the relationship between the electric field and the potential difference between two locations a and b is

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{s}.$$

For a wire of length L with one end at location a and the other at location b, (Fig. 26-26), the electric field $\vec{E}$ set up within the wire is constant. As a result, the expression above can be expressed in terms of the electric field magnitude E and the length of the wire L as

$$V_a - V_b = \pm EL,$$

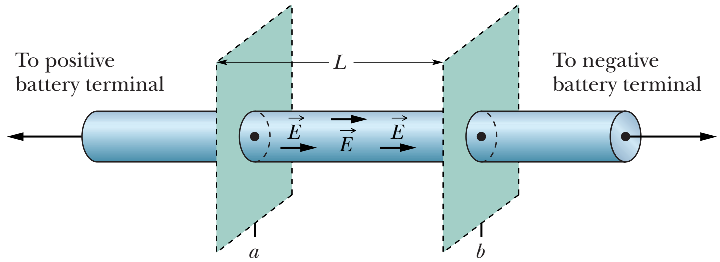


FIGURE 26-26 ■ A length L between points a and b along a current-carrying conductor.

where we use the plus sign if $\vec{E}$ and $d\vec{s}$ are in the same direction and the minus sign if $\vec{E}$ and $d\vec{s}$ point in opposite directions. Combining the expression above with

$$R = \frac{V_a - V_b}{i}$$

and ignoring signs gives us
$$R = \frac{EL}{i} = \frac{EL}{JA}.$$

The substitution for i comes from the relationship between current i, current density $\vec{J}$, and the cross section of the wire A. We compare this relation with that presented earlier when we introduced ρ as the resistivity of the material in Eq. 26-8:

$$R = \rho\frac{L}{A}.$$

By combining the previous two equations, we see that resistivity can be defined in terms of the magnitudes of the microscopic quantities $\vec{E}$ and $\vec{J}$ as

$$\rho \equiv \frac{E}{J} \quad \text{(definition of } \rho\text{)}. \tag{26-16}$$

If we combine the SI units of $\vec{E}$ and $\vec{J}$ we get, for the unit of ρ, the ohm-meter ($\Omega \cdot$ m):

$$\text{units of } \rho = \frac{\text{units of } E}{\text{units of } J} = \frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}}\,\text{m} = \Omega \cdot \text{m}.$$

(Do not confuse the *ohm-meter*, the unit of resistivity, with the *ohmmeter*, which is an instrument that measures resistance.)

Since $\vec{E}$ and $\vec{J}$ always point in the same direction, we can rewrite this expression in vector form as

$$\vec{E} = \rho\vec{J}. \tag{26-17}$$

However, be aware that these two relations hold only for *isotropic* materials—materials whose electrical properties are the same in all directions (like the metals used to make wires).

26-10 A Microscopic View of Current and Resistance

Our macroscopic studies tell us that there is a current in a conductor whenever there is a potential difference across it. Whenever Ohm's law holds, the current is directly proportional to the potential difference that causes it. Let's consider a length L of thin conducting wire with a potential difference of ΔV between its ends. What happens microscopically to the charge carriers in this situation?

We already know that the conduction electrons in a metal serve as charge carriers, and that when there is a steady current, we can represent the density of electrons as n and the charge on each electron as e. What does Ohm's law tell us about the average velocity $\langle\vec{v}\rangle$ of these electrons? When Ohm's law holds so that $\Delta V = iR$ (Eq. 26-7), then according to Eq. 26-13, the current density is proportional to the average velocity of the charge carriers by definition,

$$\vec{J} \equiv ne\langle\vec{v}\rangle. \qquad \text{(Eq. 26-13)}$$

Since $\vec{E} = \rho\vec{J}$ (Eq. 26-17), we find that the electric field $\vec{E}$ across the wire (associated with potential difference ΔV across the wire) is also proportional to the average velocity of the charge carriers,

$$\vec{E} = \rho ne\langle\vec{v}\rangle. \qquad (26\text{-}18)$$

However, the electrostatic force on a charge carrier is given by $\vec{F}^{\text{elec}} = e\vec{E}$ (Eq. 23-4), so that

$$\vec{F}^{\text{elec}} = e\vec{E} = \rho ne^2\langle\vec{v}\rangle. \qquad (26\text{-}19)$$

This is a dramatic and interesting result. It tells us that the average velocity, $\langle\vec{v}\rangle$, of a charge carrier is proportional to the electrostatic force on it! However, if the electrostatic force is the only force acting on the electron, then Newton's Second Law tells us that the electron should accelerate and not maintain a constant average velocity. To maintain a constant velocity, the *net force* on the charge carrier must be zero. Thus, there must be a second force. This situation is very similar to that associated with air drag where an object falling in the presence of a gravitational force reaches a terminal velocity as a result of an air drag acting in the opposite direction. Using Eq. 6-24 we see that

$$\vec{F}^{\text{net}} = \vec{F}^{\text{elec}} + \vec{D} = \rho ne^2\langle\vec{v}\rangle + \vec{D} = 0$$

so that

$$\vec{D} = -e\vec{E} = -\rho ne^2\langle\vec{v}\rangle. \qquad (26\text{-}20)$$

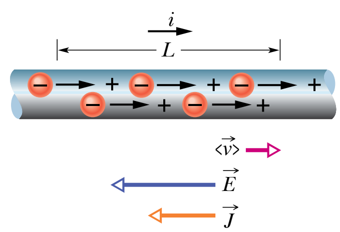


FIGURE 26-27 ■ Conduction electrons which are negative charge carriers drift at an average velocity $\langle\vec{v}\rangle$ in the opposite direction of the applied electric field $\vec{E}$. Their size is greatly exaggerated. By convention, the direction of the current density $\vec{J}$ and the sense of the arrow representing the flow of conventional current are drawn in that same direction.

This leads us to conclude that there must be a drag force that is proportional to the average velocity of the charge carriers. The air drag force on a falling object is attributed to the action of many small air molecules hitting the falling object as it moves. Similarly, we can imagine that a charge carrier is being slowed down by hitting many stationary atoms and ions as it passes through the conductor. The interactions between charge carriers and the atoms in a conductor can only be described properly using quantum mechanics. Nonetheless, we attempt to picture the flow of charge past positive ions in Fig. 26-27.

What Is a Typical Average Charge Carrier Speed?

Solving $\vec{J} \equiv ne\langle\vec{v}\rangle$ (Eq. 26-13) for the average velocity and recalling Eq. 26-15 ($J = |i|/A$), we obtain the following expression for the average speed of the charge carrier,

$$|\langle\vec{v}\rangle| = \frac{|i|}{nAe} = \frac{J}{ne}. \qquad (26\text{-}21)$$

The product ne, whose SI unit is the coulomb per cubic meter (C/m^3), is the *carrier charge density*.

At this point we can use Eq. 26-21 to find a typical value for the average speed for electrons flowing in a copper wire. Since copper has one conduction electron per atom we can use measurements for the density of copper atoms of $n = 8.5 \times 10^{28}$ atoms/m^3. Assume that our wire carries a current of 1.0 A and has a diameter of 2 mm so its cross-sectional area is 3×10^{-6} m^2. Then, according to Eq. 26-21, the average speed of the electrons is about

$$|\langle \vec{v} \rangle| = \frac{|i|}{nAe} = \frac{1 \text{ C/s}}{(8.5 \times 10^{28} \text{ atoms/m}^3)(3 \times 10^{-6} \text{ m}^2)(1.6 \times 10^{-19} \text{ C/atom})}$$
$$\approx 2.5 \times 10^{-5} \text{ m/s}.$$

This typical average speed is extremely small compared to very high speed random thermal motion of the electrons. It would take an electron about 11 hours to move across a 10 cm stretch of wire. Although the conduction electrons move along a wire very slowly like tired snails, there are so many of them that the current can actually be relatively large.

A Microscopic View of Resistivity

We can carry our microscopic analysis further, by relating the resistivity of a conductor to the properties of its charge carriers and the average time between electron collisions. If an electron of mass m is placed in an electric field of magnitude E, the electron will experience an acceleration given by Newton's Second Law:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{e\vec{E}}{m}. \tag{26-22}$$

The nature of the collisions experienced by conduction electrons is such that, after a typical collision, each electron will—so to speak—completely lose its memory of its previous average velocity. Between collisions a conduction electron will have a mean free path λ like that derived in Section 20-5 for molecules traveling in a gas. However, it moves with a typical random speed $v^{\text{eff}} = \lambda/\tau$ where τ is the average time between collisions. Each electron will then start off fresh after every encounter, moving off in a random direction. In the average time τ between collisions, a typical electron will undergo an acceleration $\vec{a}$ in a direction opposite to that of the electric field as shown in Fig. 26-27. Thus, the average speed (often called the drift speed), the electron acquires in that direction is given by $|\langle \vec{v} \rangle| = a\tau$. Using Eq. 26-22 we get

$$|\langle \vec{v} \rangle| = a\tau = \frac{eE\tau}{m}. \tag{26-23}$$

Combining this result with $\vec{J} = ne\langle \vec{v} \rangle$ yields the average velocity of

$$\langle \vec{v} \rangle = \pm\frac{\vec{J}}{ne} = \pm\frac{e\vec{E}\tau}{m},$$

where we use the plus (+) sign for positive charge carriers and the minus (−) sign for negative charge carriers. We can combine the last two terms in the previous equation and solve for $\vec{E}$ to get

$$\vec{E} = \left(\frac{m}{e^2 n\tau}\right)\vec{J}.$$

This equation shows a proportionality between the electric field in a wire and the amount of current. Note that the magnitude of the electric field in a wire is in turn proportional to the potential difference across the wire. Thus, our microscopic picture of resistivity for metallic conductors is consistent with our macroscopic measurements, and it predicts a proportionality between potential difference and current.

Comparing the equation above with Eq. 26-17 ($\vec{E} = \rho\vec{J}$) leads to an expression for the resistivity in terms of the mass and charge of the carriers, the charge density n, and the average time between collisions

$$\rho = \frac{m}{e^2 n\tau}. \tag{26-24}$$

Conductivity

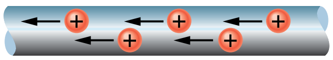
As well as referring to the resistivity of a material, we often speak of the conductivity σ of a material. This is simply the reciprocal of its resistivity, so

$$\sigma \equiv \frac{1}{\rho} \quad \text{(definition of } \sigma\text{)}. \tag{26-25}$$

The SI unit of conductivity is the reciprocal ohm-meter $(\Omega \cdot \text{m})^{-1}$. The unit name mhos per meter is sometimes used (mho is ohm backward). The definition of conductivity, σ, allows us to write Eq. 26-17($\vec{E} = \rho\vec{J}$) in the alternative form

$$\vec{J} = \sigma\vec{E}. \tag{26-26}$$

READING EXERCISE 26-13: The figure shows positive charge carriers moving leftward through a wire. Are the following leftward or rightward: (*a*) the conventional current i, (b) the current density $\vec{J}$, (c) the electric field $\vec{E}$ in the wire? *Hint*: You may want to review the discussion of conventional current in Section 26-8.



TOUCHSTONE EXAMPLE 26-3: Mean Free Time

What is the mean free time τ between collisions for the conduction electrons in copper?

SOLUTION ■ The **Key Idea** here is that the mean free time τ of copper is approximately constant, and in particular does not depend on any electric field that might be applied to a sample of the copper. Thus, we need not consider any particular value of applied electric field. However, because the resistivity ρ displayed by copper under an electric field depends on τ, we can find τ from Eq. 26-24 ($\rho = m/e^2 n\tau$). That equation gives us

$$\tau = \frac{m}{ne^2\rho}.$$

Taking the value of n, the number of conduction electrons per unit volume in copper, to be $8.5 \times 10^{28}\ \text{m}^{-3}$, and taking the value of ρ from Table 26-2, the denominator then becomes

$$(8.5 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})^2(1.69 \times 10^{-8}\ \Omega\cdot\text{m})$$
$$= 3.67 \times 10^{-17}\ \text{C}^2\cdot\Omega/\text{m}^2$$
$$= 3.67 \times 10^{-17}\ \text{kg/s},$$

where we converted units as

$$\frac{\text{C}^2\cdot\Omega}{\text{m}^2} = \frac{\text{C}^2\cdot\text{V}}{\text{m}^2\cdot\text{A}} = \frac{\text{C}^2\cdot\text{J/C}}{\text{m}^2\cdot\text{C/s}} = \frac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{m}^2/\text{s}} = \frac{\text{kg}}{\text{s}}.$$

Using these results and substituting for the electron mass m, we then have

$$\tau = \frac{9.1 \times 10^{-31}\ \text{kg}}{3.67 \times 10^{-17}\ \text{kg/s}} = 2.5 \times 10^{-14}\ \text{s}. \qquad \text{(Answer)}$$

(b) The mean free path λ of the conduction electrons in a conductor is the average distance traveled by an electron between collisions. (This definition parallels that in Section 20-5 for the mean free path of molecules in a gas.) What is λ for the conduction electrons in copper?

SOLUTION ■ The **Key Idea** here is that the distance d any particle travels in a certain time t at a constant speed v is $d = vt$. To estimate v^{eff}, the speed at which the electrons typically move between collisions, we can think of the electrons as a "gas" of particles in thermal equilibrium with their surroundings inside the metal wire. Equation 20-21 then tells us that a typical electron has a kinetic energy related to the Kelvin temperature of its environment by $(\frac{1}{2})m\langle v^2\rangle = (\frac{3}{2})k_B T$ (where k_B is the Boltzmann constant). Taking the electron's effective speed in a room temperature (300 K) environment to be $v^{\text{rms}} = \sqrt{\langle v^2\rangle}$ gives

$$v^{\text{eff}} = \sqrt{3k_B T/m} = \sqrt{(3(1.38\times10^{-23}\text{ J/K})(300\text{ K})/(9.11\times10^{-31}\text{ kg}))}$$
$$= 1.168\times10^5\text{ m/s}$$

and

$$\lambda = v^{\text{eff}}\tau = (1.168\times10^5\text{ m/s})(2.5\times10^{-14}\text{ s})$$
$$= 2.9\times10^{-9}\text{ m} = 2.9\text{ nm}. \qquad \text{(Answer)}$$

This is about 10 times the distance between nearest-neighbor atoms in a copper lattice. While this is a reasonable sounding result, it turns out that the actual value of λ is about 10 times larger than this due to quantum effects.

26-11 Other Types of Conductors

In the last few chapters we have assumed that the conductors under consideration are metallic like copper or nichrome. As you can see from Table 26-2, one of the distinctive properties of metallic conductors is that they have positive temperature coefficients indicating that their resistivities *increase* with temperature. This property seems reasonable since the thermal energy in the metal lattice causes the atoms in the metal to vibrate more, which further impedes the flow of conduction electrons. In addition, Eq. 26-9 indicates that this increase of resistivity with temperature is approximately *linear*.

There are other types of conductors with resistivities that do not simply increase linearly with temperature. The most important of these are semiconductors, which lie at the heart of the microelectronic revolution. The resistivity of semiconductors decreases more or less linearly with temperature. Superconductors are another class of conductors that do not have the same temperature behavior as conductors. Although the resistivity of superconductors increases with temperature, it does so in a very nonlinear fashion.

Because of the importance of semiconductors and superconductors we describe some of their properties here. Both of these nonmetallic conductors have some amazing properties that we describe briefly in this section. However, in the next few chapters we return to the study—within the framework of classical physics—of *steady* currents of *conduction electrons* moving through *metallic conductors*.

Semiconductors

The basic element found in virtually all semiconductors is either silicon or germanium. Table 26-3 compares the properties of silicon—a typical semiconductor—and copper—a typical metallic conductor. We see that silicon has significantly fewer charge carriers, a much higher resistivity, and a temperature coefficient of resistivity that is both large and negative. Thus, although the resistivity of copper increases with temperature, that of pure silicon decreases.

Pure silicon has such a high resistivity that it is effectively an insulator and not of much direct use in microelectronic circuits. However, its resistivity can be greatly

TABLE 26-3
Some Electrical Properties of Copper and Silicon[a]

Property	Copper	Silicon
Type of material	Metal	Semiconductor
Charge carrier density, m^{-3}	9×10^{28}	1×10^{16}
Resistivity, $\Omega\cdot m$	2×10^{-8}	3×10^{3}
Temperature coefficient of resistivity, K^{-1}	$+4\times10^{-3}$	-70×10^{-3}

[a]Rounded to one significant figure for easy comparison.

reduced in a controlled way by adding minute amounts of specific "impurity" atoms in a process called *doping*. Table 26-2 gives typical values of resistivity for silicon before and after doping with two different impurities, phosphorus and aluminum. Most semiconducting devices, such as transistors and junction diodes, are fabricated by the selective doping of different regions of the silicon with impurity atoms of different kinds.

A full explanation of the difference in resistivity between semiconductors and metallic conductors requires an understanding of quantum theory developed to explain atomic behavior. However, the difference has to do with the probability that electrons in a material can be made mobile. As we discuss in Section 22-6, in a metallic conductor some of the outermost electrons associated with an atom can move from one atom to the next without any additional energy. Thus, the electric field set up in the wire when a potential difference is applied drives current through a conductor.

In an insulator, considerable energy is required to free electrons so they can move through the material. Thermal energy cannot supply enough energy, and neither can any reasonable electric field applied to the insulator. Thus, no electrons are available to move through the insulator, and hence no current occurs even with an applied electric field. A semiconductor is like an insulator *except* that the energy required to free some electrons can be adjusted through doping. Doping can supply either electrons or positive charge carriers held very loosely within the material that are easy to get moving.*

In a semiconductor, the density of charge carriers is small but increases very rapidly with temperature as the increased thermal agitation makes more charge carriers available. This causes a *decrease* of resistivity with increasing temperature, as indicated by the negative temperature coefficient of resistivity for silicon in Table 26-3. The same increase in collision rate we noted for metals also occurs for semiconductors, but its effect is swamped by the rapid increase in the number of charge carriers.

Superconductors

In 1911, Dutch physicist Kamerlingh Onnes discovered that the resistivity of mercury absolutely disappears at temperatures below about 4 K (Fig. 26-28). This phenomenon of **superconductivity** is of vast potential importance in technology because it means charge can flow through a superconducting conductor without producing thermal energy losses. Currents created in a superconducting ring, for example, have persisted for several years without any measurable decrease; the electrons making up the current require a force and a source of energy at start-up time, but not thereafter.

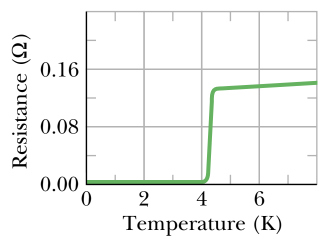
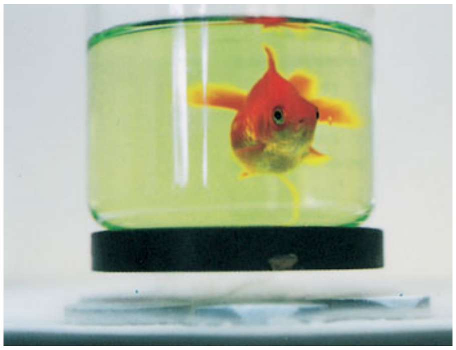


FIGURE 26-28 ■ The resistance of mercury drops to zero at a temperature of about 4 K.

Prior to 1986, the technological development of superconductivity was throttled by the cost of producing the extremely low temperatures that were required to achieve the effect. In 1986, however, new ceramic materials were discovered that become superconducting at considerably higher (and thus cheaper to produce) temperatures. Practical application of superconducting devices at room temperature may eventually become feasible.

Superconductivity is a much different phenomenon from conductivity. In fact, the best of the normal conductors, such as silver and copper, cannot become superconducting at any temperature, and the new ceramic superconductors are actually insulators when they are not at low enough temperatures to be in a superconducting state.

One explanation for superconductivity is that the electrons making up the current move in coordinated pairs. One of the electrons in a pair may electrically distort the molecular structure of the superconducting material as it moves through, creating a short-lived concentration of positive charge nearby. The other electron in the pair may then be attracted toward this positive charge. According to the theory, such coor-



A disk-shaped magnet is levitated above a superconducting material that has been cooled by liquid nitrogen. The goldfish is along for the ride.

*Explaining what positive charge carriers are and how they move is complex. For now just consider the charge carriers as negative (that is, electrons).

dination between electrons would prevent them from colliding with the molecules of the material and thus would eliminate electrical resistance. The theory worked well to explain the pre-1986, lower temperature superconductors, but new theories appear to be needed for the newer, higher temperature superconductors.

Problems

SEC. 26-3 ■ BATTERIES AND ELECTRIC CURRENT

1. Coulombs and Electrons A current of 5.0 A exists in a 10 Ω resistor for 4.0 min. How many (a) coulombs and (b) electrons pass through any cross section of the resistor in this time?

2. Charged Belt A charged belt, 50 cm wide, travels at 30 m/s between a source of charge and a sphere. The belt carries charge into the sphere at a rate corresponding to 100 μA. Compute the surface charge density on the belt.

3. Isolated Sphere An isolated conducting sphere has a 10 cm radius. One wire carries a current of 1.000 002 0 A into it. Another wire carries a current of 1.000 000 0 A out of it. How long would it take for the sphere to increase in potential by 1000 V?

SEC. 26-5 ■ RESISTANCE AND OHM'S LAW

4. Electrical Cable An electrical cable consists of 125 strands of fine wire, each having 2.65 $\mu\Omega$ resistance. The same potential difference is applied between the ends of all the strands and results in a total current of 0.750 A. (a) What is the current in each strand? (b) What is the applied potential difference? (c) What is the resistance of the cable?

5. Electrocution A human being can be electrocuted if a current as small as 50 mA passes near the heart. An electrician working with sweaty hands makes good contact with the two conductors he is holding, one in each hand. If his resistance is 2000 Ω, what might the fatal voltage be?

SEC. 26-6 ■ RESISTANCE AND RESISTIVITY

6. Trolley Car A steel trolley-car rail has a cross-sectional area of 56.0 cm^2. What is the resistance of 10.0 km of rail? The resistivity of the steel is $3.00 \times 10^{-7}\ \Omega \cdot \text{m}$.

7. Conducting Wire A conducting wire has a 1.0 mm diameter, a 2.0 m length, and a 50 mΩ resistance. What is the resistivity of the material?

8. A Wire A wire 4.00 m long and 6.00 mm in diameter has a resistance of 15.0 mΩ. A potential difference of 23.0 V is applied between the ends. (a) What is the current in the wire? (b) Calculate the resistivity of the wire material. Identify the material. (Use Table 26-2.)

9. A Coil A coil is formed by winding 250 turns of insulated 16-gauge copper wire (diameter = 1.3 mm) in a single layer on a cylindrical form of radius 12 cm. What is the resistance of the coil? Neglect the thickness of the insulation (Use Table 26-2.)

10. What Temperature (a) At what temperature would the resistance of a copper conductor be double its resistance at 20.0°C? (Use 20.0°C as the reference point in Eq. 26-9; compare your answer with Fig. 26-23.) (b) Does this same "doubling temperature" hold for all copper conductors regardless of shape or size?

11. Longer Wire A wire with a resistance of 6.0 Ω is drawn out through a die so that its new length is three times its original length. Find the resistance of the longer wire, assuming that the resistivity and density of the material are unchanged.

12. A Certain Wire A certain wire has a resistance R. What is the resistance of a second wire, made of the same material, that is half as long and has half the diameter?

13. Two Conductors Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1.0 mm. Conductor B is a hollow tube of outside diameter 2.0 mm and inside diameter 1.0 mm. What is the resistance ratio R_A/R_B, measured between their ends?

14. Flashlight Bulb A common flashlight bulb is rated at 0.30 A and 2.9 V (the values of the current and voltage under operating conditions). If the resistance of the bulb filament at room temperature (20°C) is 1.1 Ω, what is the temperature of the filament when the bulb is on? The filament is made of tungsten.

15. Metal Rod When a metal rod is heated, not only its resistance but also its length and its cross-sectional area change. The relation $R = \rho L/A$ suggests that all three factors should be taken into account in measuring ρ at various temperatures. (a) If the temperature changes by 1.0 C°, what percentage changes in R, L, and A occur for a copper conductor? (b) The coefficient of linear expansion for copper is 1.7×10^{-5}/K. What conclusion do you draw?

16. Gauge Number If the gauge number of a wire is increased by 6, the diameter is halved; if a gauge number is increased by 1, the diameter decreases by the factor $2^{1/6}$ (see the table in Problem 32). Knowing this, and knowing that 1000 ft of 10-gauge copper wire has a resistance of approximately 1.00 Ω, estimate the resistance of 25 ft of 22-gauge copper wire.

SEC. 26-7 ■ POWER IN ELECTRIC CIRCUITS

17. X-Ray Tube A certain x-ray tube operates at a current of 7.0 mA and a potential difference of 80 kV. What is its power in watts?

18. A Student A student kept his 9.0 V, 7.0 W radio turned on at full volume from 9:00 P.M. until 2:00 A.M. How much charge went through it?

19. Space Heater A 120 V potential difference is applied to a space heater whose resistance is 14 Ω when hot. (a) At what rate is electric energy transferred to heat? (b) At 5.0¢/kW·h, what does it cost to operate the device for 5.0 h?

20. Thermal Energy Thermal energy is produced in a resistor at a rate of 100 W when the current is 3.00 A What is the resistance?

21. Energy Is Dissipated An unknown resistor is connected between the terminals of a 3.00 V battery. Energy is dissipated in the resistor at the rate of 0.540 W. The same resistor is then connected between the terminals of a 1.50 V battery. At what rate is energy now dissipated?

22. Space Heater Two A 120 V potential difference is applied to a space heater that dissipates 500 W during operation. (a) What is its resistance during operation? (b) At what rate do electrons flow through any cross section of the heater element?

23. Radiant Heater A 1250 W radiant heater is constructed to operate at 115 V. (a) What will be the current in the heater? (b) What is the resistance of the heating coil? (c) How much thermal energy is produced in 1.0 h by the heater?

24. Heating Element A heating element is made by maintaining a potential difference of 75.0 V across the length of a Nichrome wire that has a 2.60×10^{-6} m^2 cross section. Nichrome has a resistivity of 5.00×10^{-7} $\Omega \cdot$m. (a) If the element dissipates 5000 W, what is its length? (b) If a potential difference of 100 V is used to obtain the same dissipation rate, what should the length be?

25. Nichrome Heater A Nichrome heater dissipates 500 W when the applied potential difference is 110 V and the wire temperature is 800°C. What would be the dissipation rate if the wire temperature were held at 200°C by immersing the wire in a bath of cooling oil? The applied potential difference remains the same, and α for Nichrome at 800°C is 4.0×10^{-4}/K.

26. 100 W Lightbulb A 100 W lightbulb is plugged into a standard 120 V outlet. (a) How much does it cost per month to leave the light turned on continuously? Assume electric energy costs 12¢/kW$\cdot$h. (b) What is the resistance of the bulb? (c) What is the current in the bulb? (d) Is the resistance different when the bulb is turned off?

27. Linear Accelerator A linear accelerator produces a pulsed beam of electrons. The pulse current is 0.50 A, and each pulse has a duration of 0.10 μs. (a) How many electrons are accelerated per pulse? (b) What is the average current for an accelerator operating at 500 pulses/s? (c) If the electrons are accelerated to an energy of 50 MeV, what are the average and peak powers of the accelerator?

28. Cylindrical Resistor A cylindrical resistor of radius 5.0 mm and length 2.0 cm is made of material that has a resistivity of 3.5×10^{-5} $\Omega \cdot$m. What is the potential difference when the energy dissipation rate in the resistor is 1.0 W?

29. Copper Wire A copper wire of cross-sectional area 2.0×10^{-6} m^2 and length 4.0 m has a current of 2.0 A uniformly distributed across that area. How much electric energy is transferred to thermal energy in 30 min?

SEC. 26-8 ■ CURRENT DENSITY IN A CONDUCTOR

30. Small But Measurable A small but measurable current of 1.2×10^{-10} A exists in a copper wire whose diameter is 2.5 mm. Assuming the current is uniform, calculate (a) the current density and (b) the average electron speed.

31. A Beam A beam contains 2.0×10^8 doubly charged positive ions per cubic centimeter, all of which are moving north with a speed of 1.0×10^5 m/s. (a) What are the magnitude and direction of the current density $\vec{J}$? (b) Can you calculate the total current i in this ion beam? If not what additional information is needed?

32. The U.S. Electric Code The (United States) National Electric Code, which sets maximum safe currents for insulated copper wires of various diameters, is given (in part) in the table. Plot the safe current density as a function of diameter. Which wire gauge has the maximum safe current density? ("Gauge" is a way of identifying wire diameters, and 1 mil $= 10^{-3}$ in.)

Gauge	4	6	8	10	12	14	16	18
Diameter, mils	204	162	129	102	81	64	51	40
Safe current, A	70	50	35	25	20	15	6	3

33. A Fuse A fuse in an electric circuit is a wire that is designed to melt, and thereby open the circuit, if the current exceeds a predetermined value. Suppose that the material to be used in a fuse melts when the current density rises to 440 A/cm^2. What diameter of cylindrical wire should be used to make a fuse that will limit the current to 0.50 A?

34. Near Earth Near the Earth, the density of protons in the solar wind (a stream of particles from the Sun) is 8.70 cm^{-3}, and their speed is 470 km/s. (a) Find the current density of these protons. (b) If the Earth's magnetic field did not deflect them, the protons would strike the planet. What total current would the Earth then receive?

35. Steady Beam A steady beam of alpha particles ($q = +2e$) traveling with constant kinetic energy 20 MeV carries a current of 0.25 μA. (a) If the beam is directed perpendicular to a plane surface, how many alpha particles strike the surface in 3.0 s? (b) At any instant, how many alpha particles are there in a given 20 cm length of the beam? (c) Through what potential difference is it necessary to accelerate each alpha particle from rest to bring it to an energy of 20 MeV?

36. Current Density (a) The current density across a cylindrical conductor of radius R varies in magnitude according to the equation

$$J = J_0\left(1 - \frac{r}{R}\right),$$

where r is the distance from the central axis. Thus, the current density has a maximum magnitude of $J_0 = |\vec{J}_0|$ at that axis ($r = 0$) and decreases linearly to zero at the surface ($r = R$). Calculate the current in terms of J_0 and the conductor's cross-sectional area $A = \pi R^2$. (b) Suppose that, instead, the current density is a maximum J_0 at the cylinder's surface and decreases linearly to zero at the axis: $J = J_0\, r/R$. Calculate the magnitude of the current. Why is the result different from that in (a)?

37. How Long How long does it take electrons to get from a car battery to the starting motor? Assume the current is 300 A and the electrons travel through a copper wire with cross-sectional area 0.21 cm^2 and length 0.85 m. (*Hint:* Assume one conduction electron per atom and take the number density of copper atoms to be 8.5×10^{28} atoms/m^2.)

38. Nichrome A wire of Nichrome (a nickel-chromium-iron alloy commonly used in heating elements) is 1.0 m long and 1.0 mm^2 in cross-sectional area. It carries a current of 4.0 A when a 2.0 V potential difference is applied between its ends. Calculate the conductivity σ of Nichrome.

39. When Applied When 115 V is applied across a wire that is 10 m long and has a 0.30 mm radius, the current density is 1.4×10^4 A/m^2. Find the resistivity of the wire.

40. Truncated Right-Circular Cone A resistor has the shape of a truncated right-circular cone (Fig. 26-29). The end radii are a and b, and the altitude is L. If the taper is small, we may assume that the current density is uniform across any cross section. (a) Calculate the resistance of this object. (b) Show that your answer reduces to $\rho(L/A)$ for the special case of zero taper (that is, for $a = b$).

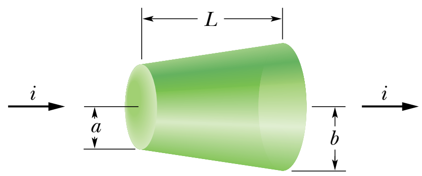


FIGURE 26-29 ■ Problem 40.

SEC. 26-10 ■ A MICROSCOPIC VIEW OF CURRENT AND RESISTANCE

41. Gas Discharge Tube A current is established in a gas discharge tube when a sufficiently high potential difference is applied across the two electrodes in the tube. The gas ionizes; electrons move toward the positive terminal and singly charged positive ions toward the negative terminal. (a) What is the magnitude of the current in a hydrogen discharge tube in which 3.1×10^{18} electrons and 1.1×10^{18} protons move past a cross-sectional area of the tube each second? (b) What is the direction of the current density $\vec{J}$?

42. A Block A block in the shape of a rectangular solid has a cross-sectional area of 3.50 cm^2 across its width, a front-to-rear length of 15.8 cm, and a resistance of 935 Ω. The material of which the block is made has 5.33×10^{22} conduction electrons/m^3. A potential difference of 35.8 V is maintained between its front and rear faces. (a) What is the current in the block? (b) If the current density is uniform, what is its value? (c) What is the average or drift speed of the conduction electrons? (d) What is the magnitude of the electric field in the block?

43. Earth's Lower Atmosphere Earth's lower atmosphere contains negative and positive ions that are produced by radioactive elements in the soil and cosmic rays from space. In a certain region, the atmospheric electric field strength is 120 V/m, directed vertically down. This field causes singly charged positive ions, at a density of 620/cm^3, to drift downward and singly charged negative ions, at a density of 550/cm^3, to drift upward (Fig. 26-30). The measured conductivity of the air in that region is $2.70 \times 10^{-14}(1/\Omega \cdot \text{m})$. Calculate (a) the average ion speed, assumed to be the same for positive and negative ions, and (b) the current density.

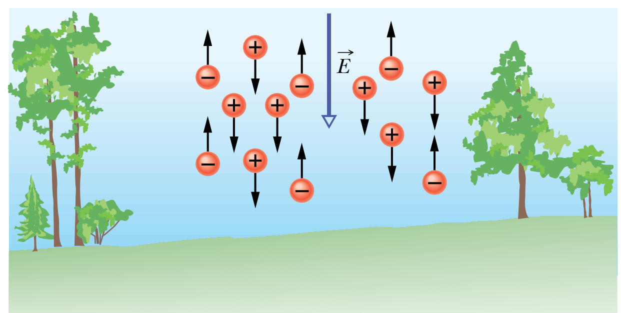


FIGURE 26-30 ■ Problem 43.

Additional Problems

44. Saving on Your Electric Bill Fluorescent bulbs deliver the same amount of light using much less power. If one kW-hr costs 12¢, estimate the amount of money you would save each month by replacing all the 75 W incandescent bulbs in your house by 10 W fluorescent ones than incandescent ones. *Be sure to clearly state your assumptions.*

45. Building a Water Heater The nickel-chromium alloy Nichrome has a resistivity of about 10^{-6} Ω-m. Suppose you want to build a small heater out of a coil of Nichrome wire and a 6 V battery in order to heat 30 ml of water from a temperature of 20 C to 40 C in 1 min. Assume the battery has negligible internal resistance.
(a) How much heat energy (in joules) do you need to do this?
(b) How much power (in watts) do you need to do it in the time indicated?
(c) What resistance should your Nichrome coil have in order to produce this much power in heat?
(d) Can you create a coil having these properties? (*Hint:* Can you find a plausible length and cross-sectional area for your wire that will give you the resistance you need?)
(e) If the internal resistance of the battery were 1/3 Ω, how would it affect your calculation? (Only explain what you would have to do; don't recalculate the size of your coil.)

46. A Confusing Thing One of the most confusing things about wiring circuits and figuring out what you've done is that many arrangements are electrically equivalent. Unless you have unusual powers of visualization it is often hard to recognize this. For example, three of the circuits shown in Fig. 26-31 are electrically equivalent and one is not. Answer questions (a) through (d) that follow.
(a) Which circuit is not like the others? Explain why it's different.
(b) Draw circuit diagrams for each of the arrangements and label each diagram as A, B, C, or D. (c) Examine your diagrams. Is it possible for neat circuit diagrams that look superficially different to represent the same set of electrical connections?

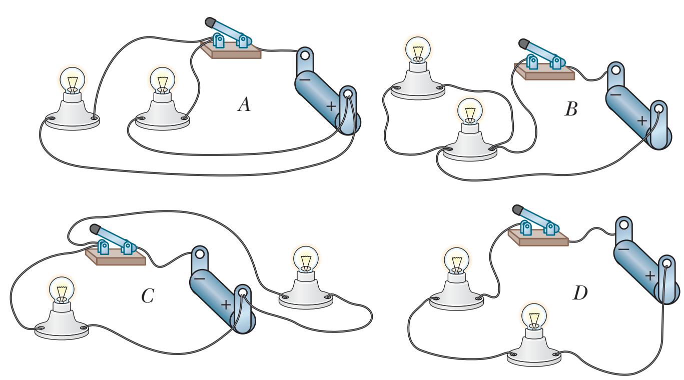


FIGURE 26-31 ■ Problem 46.

47. Draw the Circuit Diagram Draw a neat circuit diagram for each of the two circuits shown in Fig. 26-32 using the standard symbols for bulbs, batteries, and switches.

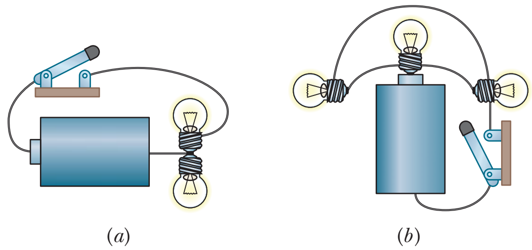


FIGURE 26-32 ■ Problem 47.

48. Charge Through Conductor The charge passing through a conductor increases over time as $q(t) = (1.6\ \mathrm{C/s^2})t^2 + (2.2\ \mathrm{C/s})t$, where t is in seconds. (a) What equation describes the current in the circuit as a function of time? (b) What is the current in the conductor at $t = 0.0$ s and at $t = 2.0$ s?

49. Increases Over Time The charge passing through a conductor increases over time as $q(t) = (1.5\ \mathrm{C/s^3})t^3 - (4.5\ \mathrm{C/s^2})t^2 + (2\ \mathrm{C/s})t$, where t is in seconds. (a) What equation describes the current in the circuit as a function of time? (b) What is the current in the conductor at $t = 0.0$ s and at $t = 1.0$ s?

50. 1994 Honda Accord Consider a 1994 Honda Accord with a battery that is rated at 52 ampere-hours. This battery is supposed to be able to deliver 1 ampere of current to electrical devices in a car for at least 52 hours or 2 amperes for 26 hours, and so on. Suppose you leave the car lights turned on when you park the car and the car lights draw 20 amperes of current. How long will it be before your battery is dead?

51. The Resistance of a Pocket Calculator A typical AAA battery delivers a nearly constant voltage of 1.5 V and stores about 3 kJ of energy. From the time it takes you to use up the batteries in your calculator, estimate the resistance of your calculator. (If you don't have a calculator of this type, make a plausible estimate of how long it might take to use up the batteries. Give some reason for your estimate.)