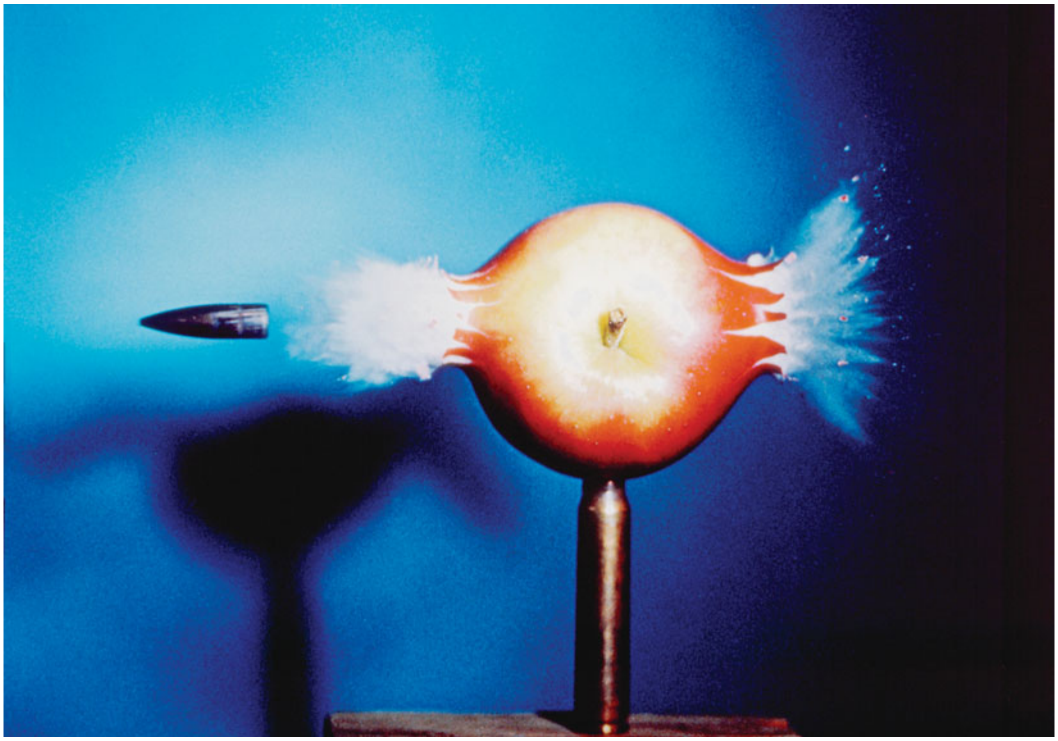
28 Capacitance



In 1964 Harold "Doc" Edgerton of MIT, who was renowned for his ability to take high-quality stop-action photos, captured this image of a bullet penetrating an apple. This stop-action photo was made by leaving the camera shutter open and tripping a high-speed electronic flash device at just the right time to illuminate the apple and bullet. Since the bullet was moving at 900 m/s, Edgerton used a flash with a duration of only 0.3 μs. This meant that the bullet only moved 0.3 mm during the flash. If we were doing ordinary photography we would probably illuminate the apple with a 100 W lightbulb using an exposure time of 1/20 s to provide about 5 J of energy for illumination. But providing the necessary 5 J of electrical energy for the illumination in a time period of 0.3 μs requires 15 MW of power.

How is it possible to provide the energy needed to stop a bullet's action when it is only illuminated for a tiny fraction of a second?

The answer is in this chapter.

28-1 The Uses of Capacitors

In Chapter 26 we discussed transferring excess charge to a pair of metal plates as shown in Fig. 26-1. The pair of metal plates is an example of the basic component of a **capacitor.** A capacitor can be constructed using any two conductors separated by an insulator. If we connect each conductor making up a capacitor to one of the terminals of a source of potential difference such as a battery, one conductor acquires a net positive charge while the other conductor acquires the same amount of net negative charge. The conductors can be any shape. Figure 28-1 shows some possible capacitor geometries. No matter what shape or size a capacitor's conductors are, we often casually refer to the conductors as "plates."

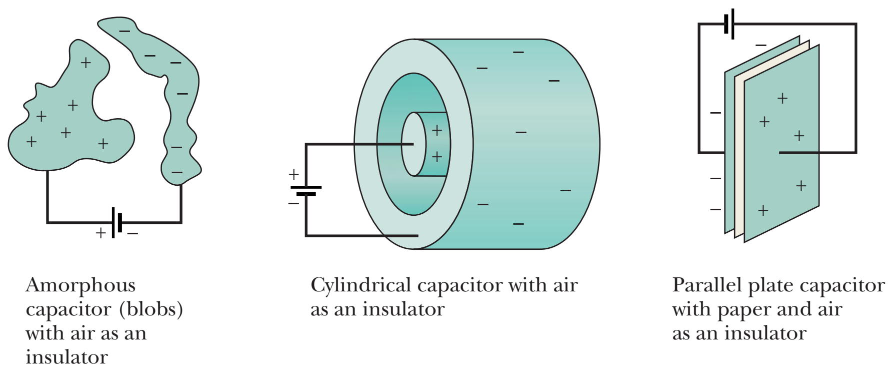


FIGURE 28-1 ■ Three capacitors of different sizes and shapes have been connected to a battery. They each consist of a pair of conductors separated by an insulator. In each case the battery removes electrons from one of the two conductors, leaving it with excess positive charge and forces the same number of electrons to the opposite conductor.

There are many reasons for constructing and studying capacitors: they are useful circuit elements and they can store energy.

FIGURE 28-2 ■ An assortment of capacitors commonly found in electrical circuits. The structures of these devices are hidden.

Capacitors in Electrical Circuits

Since a capacitor consists of conductors separated by an insulator, no current can flow *through* it. So at first glance, it doesn't seem to make sense to use a capacitor as a circuit element. Surprisingly, capacitors have very interesting and useful properties in circuits with changing currents through their other components. For example, variable capacitors are vital elements that enable us to tune radio and television receivers. They are found in most household electrical devices. Capacitors are used to control the frequency of the flashing lights used for warning signals at construction sites. The coaxial cables used to carry high-frequency microwave and radio signals are cylindrical capacitors. Microscopic capacitors are used in communications and computers to shape the timing and strength of time-varying signal transmissions. Figure 28-2 shows some of the many sizes and shapes of capacitors commonly found in electric circuits.

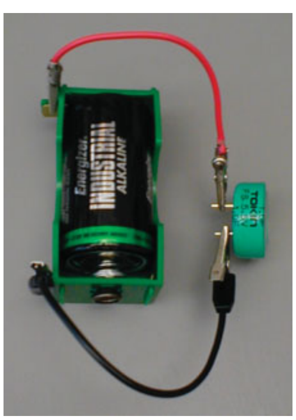


FIGURE 28-3 ■ When a battery is connected across the terminals of a capacitor, the capacitor stores electrical energy.

Capacitors as Energy Storage Devices

Just as you can store potential energy by pulling a bowstring, stretching a spring, compressing a gas, or lifting a book, you can also store electrical energy in the electric field found inside a "charged" capacitor as shown in Figs. 28-3 and 28-4. For example, energy storage in microscopic capacitors enables them to function as memory devices in modern digital computers and in the charge-coupled devices (CCDs) used in video cameras. Energy stored in capacitors can also be used to keep computer circuits running smoothly during brief power outages. A much larger capacitor lies at the heart of a battery-powered photoflash unit. This capacitor accumulates electrical energy relatively slowly during the time between flashes, building up an electric field as it does so.

The electric field across the capacitor plates stores energy that can be released rapidly to create an intensive flash of light. (It is important to note that because capacitors are storehouses for electrical energy, some electrical devices can give you a nasty shock if you open them and accidentally touch both terminals of a capacitor—even when the device is turned off.)

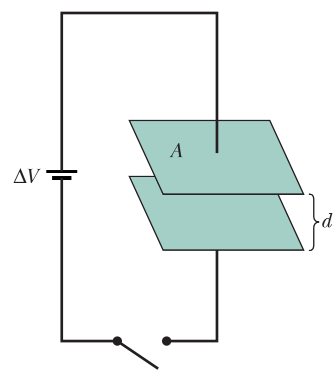
FIGURE 28-4 ■ When a "charged" capacitor is disconnected from its battery and wired in series with a bulb, the energy stored in it can light the bulb for a short period of time.

28-2 Capacitance

Figure 28-5 shows a capacitor made from a conventional arrangement of a pair of metal plates. A device consisting of two parallel conducting plates of area A separated by a distance d is called a *parallel-plate capacitor*. The circuit symbol we use to represent a capacitor (⊣⊢) is based on the structure of a parallel-plate capacitor but is used for capacitors of all shapes. For the purpose of defining capacitance in a simple manner, we will consider an ideal capacitor as two flat parallel conductors (or **plates**) with a perfect insulator between its plates. This perfect insulator allows absolutely no current to pass between them. For simplicity, at first we choose to consider the situation where there is no matter (such as air, glass, or plastic) between the capacitor plates. We just have a vacuum between the plates. We further assume we will charge our capacitor with an ideal battery. Recall that an ideal battery has no internal resistance, so its emf and the potential difference across its terminals are always the same. In Section 28-6 and those following we will relax some of these idealized restrictions.

Equal and Opposite Excess Charge on Plates

When capacitor plates of any shape are connected to a battery or some other voltage source, electrons flow from the negative terminal of the battery through the connecting wire and onto one plate of the capacitor. Meanwhile, the positive terminal of the battery attracts electrons from the other plate. These electrons are pulled through the wires of the circuit, away from the capacitor plate, and leave behind an excess of positive metal atoms with missing electrons. During this process, we cannot find an electric field outside of the capacitor so the overall capacitor seems to be electrically neutral. Hence, we must conclude that at any given time one plate has net or excess charge of $+q$ while the other has a net charge of $-q$. The chemical reactions taking place in the battery are complex, so the electrons pulled off one plate are not necessarily the same ones being pushed through the wires of the circuit onto the other plate. However, the battery does deposit one electron on the negative plate for every one it pulls off the positive plate. We will call this process *charge separation*. Sometimes the process is called *charging*.



d = spacing
A = area
ΔV = potential difference

FIGURE 28-5 ■ A parallel-plate capacitor with identical plates of area A and spacing d is connected to a battery with potential difference ΔV. The plates have equal and opposite excess charges of amount $|q|$ on their facing surfaces.

> To understand how a capacitor works, it is important to note that charge separation occurs as a result of charge flow in the wires of the circuit. Charges are not transferred from one plate to the other inside an ideal capacitor.

Why Do Capacitor Plates Stop Accumulating Charge?

Observations show that the battery eventually stops pulling electrons off the positively charged plate and depositing electrons on the negatively charged plate. This is because as electrons build up on the negative plate, they oppose the battery's action and start repelling the flow of additional electrons. Similarly, it becomes harder and harder for the battery to pull electrons off the positive plate as the atoms carrying positive net charge pull back on them. When enough charge has accumulated on the plates, the force exerted on an electron by the battery and the oppositely directed forces exerted on it by the other charges on a plate cancel each other. No more electrons can flow from one plate to the other. We can use a high-quality voltmeter to measure the potential difference across a capacitor just disconnected from a battery. This measurement shows that *charge separation stops when the potential difference across a capacitor is the same as the potential difference across the battery*.

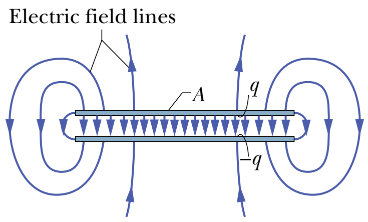


FIGURE 28-6 ■ As the field lines show, the electric field due to the charged plates is uniform in the central region between the plates. The field is not uniform at the edges of the plates, as indicated by the "fringing" of the field lines there.

Factors Affecting Charge Separation Capacity

By convention we refer to the *charge on a capacitor* as $|q|$, the absolute value of the net charge on each plate. Although we refer to a capacitor with charges q and $-q$ on its plates as "charged," a capacitor is electrically neutral so we are actually describing its charge separation created by a voltage source. What factors might affect the capacity for charge separation in a parallel-plate capacitor? We can use our knowledge of electrostatics to explore the effects of several factors. In particular, we will explore how we expect charge to depend on the potential difference across the battery terminals and on geometric factors such as the area of the plates and their spacing (Fig. 28-6):

1. **Potential Difference, ΔV:** For a given capacitor of any shape, we would expect the charge separation to be larger when the potential difference the battery places across the capacitor plates is larger. How much larger? Consider a group of n charges distributed on the plates of a capacitor. Since the plates are conductors, each one is an equipotential surface. According to Eq. 25-25 we can find the electric potential at a given point on a plate relative to infinity. We just need to know the locations of the group of n charges distributed on the capacitor plates. The potential is given by

$$V = k\sum_{i=1}^{n}\frac{q_i}{r_i} \qquad \text{(Eq. 25-25)}$$

where r_i represents the radial distance between the point where the potential is being calculated and the location of the ith charge. By examining this equation we can see that if the potential is to be doubled, there needs to be twice as much charge at each location on the capacitor plates. We expect the amount of the charge separation on a capacitor to be proportional to the potential difference across its plates. We predict

$$|q| \propto |\Delta V|.$$

As you will see in the next subsection, the constant of proportionality between the amount of excess charge on each plate and the potential difference across the plates for a given capacitor is known as its *capacitance*. We will deal more formally with the definition of capacitance and its units in the next section.

2. **Influence of Plate Area, A:** Consider a parallel-plate capacitor. For a given potential difference and plate spacing, d, how do we expect the charge separation capacity to depend on the area of the plates? When the plates have a large area, the electrons the battery is trying to push on the negative plate have more room to spread out. Likewise, the unneutralized atoms left behind when electrons are pulled off the positive plate can be distributed further apart. We expect that as the area of the plates increases, it will be easier to remove or deposit electrons on them.

A simple experiment can be done to show that the charge separation capacity is in fact directly proportional to area. In this experiment, two sheets of aluminum

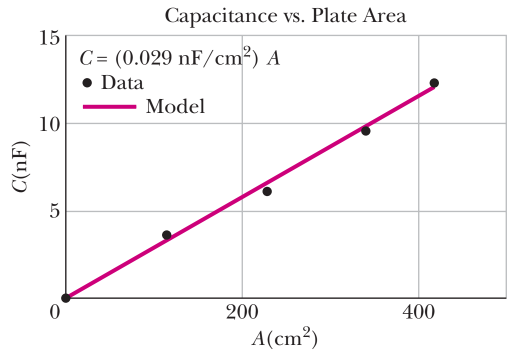


FIGURE 28-7 Two rectangular pieces of aluminum foil are wedged between the insulating pages of a book. A multimeter is used to measure the capacitance of the system. The result shows capacitance increasing in direct proportion to the area of the conducting aluminum plates.

foil are placed opposite each other and separated by the insulating pages of a book. A multimeter like that described in Section 26-4 can be used to measure capacitance. Measurements are taken for different areas of foil. The results are shown in Fig. 28-7. We derive this relationship theoretically in the next section.

3. **Influence of Plate Spacing, *d*:** Once again we consider a parallel-plate capacitor. For a given potential difference and plate area, A, how do we expect the charge separation capacity to depend on the spacing between the plates? When the plates have a small spacing, the excess positive charges on one plate are quite close to the excess negative charges on the other plate. Since opposite charges attract each other, these charges pull on each other across the insulating gap even though they cannot cross the gap. This attraction helps to counterbalance the repulsion between the like charges on each plate. As the spacing between plates becomes smaller, we expect the overall capacity for the charge separation caused by the battery action to become larger.

   A simple experiment can be done to show that the charge separation capacity does in fact increase as the spacing between plates decreases. In this experiment, two sheets of aluminum foil are placed opposite each other and separated by the insulating pages of a book. A multimeter is used to measure capacitance as different numbers of pages are inserted between the foil plates. The results are shown in Fig. 28-8. This graph shows that the capacitance of the foil plate system is inversely proportional to the spacing, d, between the plates. We derive this relationship theoretically in the next section.

Defining Capacitance

As we just discussed in the last subsection, the amount of the excess charge on each plate of a capacitor, $|q|$, and the size of the potential difference, $|\Delta V|$, across it should be proportional to each other, so

$$|q| = C|\Delta V|. \tag{28-1}$$

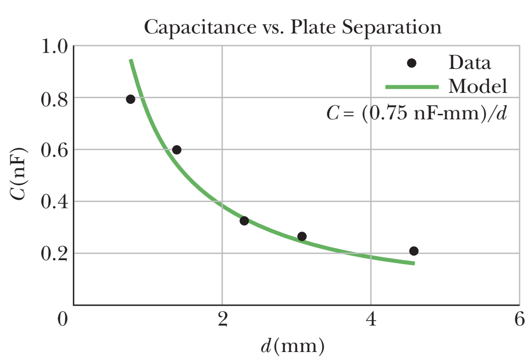


FIGURE 28-8 Two rectangular pieces of aluminum foil are wedged between the insulating pages of a book. The capacitance of the system is measured as a function of the spacing between the plates. The result shows that capacitance is inversely proportional to the spacing.

The proportionality constant C is defined as the **capacitance.** The capacitance is a measure of how much excess charge must be put on each of the plates to produce a certain potential difference between them: the greater the capacitance, the larger the charge separation created by a given potential difference.

For a parallel-plate capacitor, experimental results have shown us its capacitance depends directly on the plate areas and inversely on the spacing between plates. We will see in Sections 28-6 and 28-7 that capacitance will also depend on the nature of the insulating material inserted between the plates. Capacitors having different shapes will not have the same simple relationships between plate area and spacing. In the next section, we will use the definition of electric potential and Gauss' law to identify the theoretical geometric factors for several different types of capacitors including parallel-plate, cylindrical, and spherical capacitors.

Capacitance Units

The SI unit of capacitance following from this expression is the coulomb per volt. This unit occurs so often that it is given a special name, the *farad* (F):

$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb per volt} = 1 \text{ C/V}. \tag{28-2}$$

As you will see, the farad is a very large unit. Fractions of the farad, such as the microfarad ($1\ \mu\text{F} = 10^{-6}$ F) and the picofarad ($1 \text{ pF} = 10^{-12}$ F), are more convenient units in practice. A summary of units and their common notations is shown in Table 28-1.

TABLE 28-1
Units of Capacitance

microfarad: $10^{-6}\text{ F} = 1\ \mu\text{F}$
nanofarad: $10^{-9}\text{ F} = 1\text{ nF} = 1000\ \mu\mu\text{F}$
picofarad: $10^{-12}\text{ F} = 1\text{ pF} = 1\ \mu\mu\text{F}$

READING EXERCISE 28-1: Does the capacitance C of a capacitor increase, decrease, or remain the same (a) when the excess charge of amount $|q|$ on its plates is doubled and (b) when the potential difference ΔV_c across it is tripled?

28-3 Calculating the Capacitance

Our task here is to calculate the capacitance of a capacitor once we know its geometry. Because we will consider a number of different geometries, it seems wise to develop a general plan to simplify the work. In brief, our plan is as follows:

1. Assume a charge of amount $|q|$ on each of the "plates."
2. Calculate the electric field $\vec{E}$ between the plates in terms of this amount of charge, using Gauss' law.
3. Knowing $\vec{E}$, calculate the potential difference ΔV between the plates from

$$V_2 - V_1 = -\int_1^2 \vec{E} \cdot d\vec{s}. \tag{Eq. 25-16}$$

4. Calculate C from $|q| = C|\Delta V|$ (Eq. 28-1).

Before we start, we can simplify the calculation of both the electric field and the potential difference by making certain assumptions. We discuss each in turn.

Calculating the Electric Field

To relate the electric field $\vec{E}$ between the plates of a capacitor to the amount of excess charge $|q|$ on either plate, we shall use Gauss' law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q^{\text{net}} = q. \tag{28-3}$$

Here q is the net charge enclosed by a Gaussian surface, and $\oint \vec{E}\cdot d\vec{A}$ is the net electric flux through that surface. In all cases we shall consider, the Gaussian surface will be such whenever electric flux passes through it, $\vec{E}$ will have a uniform magnitude $E = |\vec{E}|$, and the vectors $\vec{E}$ and $d\vec{A}$ will be parallel. This equation will then reduce to

$$|q| = \varepsilon_0 EA \qquad \text{(special case of Eq. 28-3)}, \tag{28-4}$$

in which A is the area of the part of the Gaussian surface through which flux passes. For convenience, we shall always draw the Gaussian surface in such a way it completely encloses the charge on the positive plate; see Fig. 28-9 for an example.

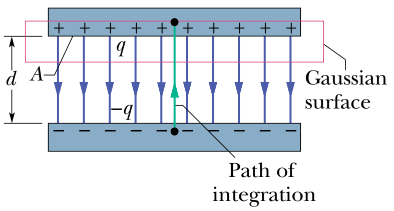


FIGURE 28-9 ■ A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration of Eq. 28-6 is taken along a path extending directly from the negative plate to the positive plate.

Calculating the Potential Difference

In the notation of Chapter 25 (Eq. 25-16), the potential difference between the plates of a capacitor is related to the field $\vec{E}$ by

$$V_2 - V_1 = -\int_1^2 \vec{E}\cdot d\vec{s}, \tag{28-5}$$

in which the integral is to be evaluated along any path starting on one plate and ending on the other. We shall always choose a path following an electric field line, from the negative plate to the positive plate. For this path, the vectors $\vec{E}$ and $d\vec{s}$ will have opposite directions, so the dot product $\vec{E}\cdot d\vec{s}$ will be equal to $-|\vec{E}||d\vec{s}|$. The right side of this equation will then be positive. Letting ΔV represent the difference, $V_2 - V_1$, we can then recast the relationship as

$$\Delta V = -\int_-^+ |\vec{E}||d\vec{s}| \qquad \text{(special case of Eq. 28-5)}, \tag{28-6}$$

in which the " $-$ " and " $+$ " remind us that our path of integration starts on the negative plate and ends on the positive plate.

We are now ready to apply $|q| = \varepsilon_0 EA$ (Eq. 28-4) and $\Delta V = -\int_-^+ |\vec{E}||d\vec{s}|$ (Eq. 28-6) to some particular cases.

A Parallel-Plate Capacitor

We assume, as Fig. 28-9 suggests, that the plates of our parallel-plate capacitor are so large and so close together we can neglect the fringing of the electric field at the edges of the plates, taking $\vec{E}$ to be constant throughout the region between the plates. This configuration was used in old-time radios. As we will see in Chapter 33, the frequency of an oscillating circuit depends on the capacitance. In old radios (those built before the time that tiny transistors became ubiquitous), the dial was connected to a set of nested metal plates. When the dial was turned, some of the plates rotated while others stayed fixed. By turning the dial, the overlap of the plates changed, changing the capacitance and thereby the frequency of the signal selected.

We draw a Gaussian surface enclosing just the excess charge q on the positive plate, as in Fig. 28-9. Recall from above that

$$|q| = \varepsilon_0 EA, \tag{28-7}$$

where A is the area of each of the plates.

Equation 28-6 yields

$$\Delta V = \int_-^+ |\vec{E}||d\vec{s}| = |\vec{E}|\int_0^d ds = Ed. \tag{28-8}$$

Here, $|\vec{E}| = E$ can be placed outside the integral because it is a constant; the second integral then is simply the plate separation d.

Combining these two expressions with the relation $|q| = C|\Delta V|$ (Eq. 28-1), we find

$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor).} \tag{28-9}$$

This theoretical relationship matches the results of the experiments we presented in the last section. The capacitance does indeed depend only on geometrical factors—namely, the plate area A and the plate separation d. Note that C increases as we increase the plate area A or decrease the separation d.

As an aside, we point out that this expression suggests one of our reasons for writing the electrostatic constant in Coulomb's law in the form $1/4\pi\varepsilon_0$. If we had not done so, the expression for the capacitance of a parallel-plate capacitor above—which is used more often in engineering practice than Coulomb's law—would have been less simple in form. We note further that it permits us to express the permittivity constant ε_0 in a unit more appropriate for use in problems involving capacitors; namely,

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{F/m} = 8.85\ \text{pF/m}. \tag{28-10}$$

We have previously expressed this constant as

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2. \tag{28-11}$$

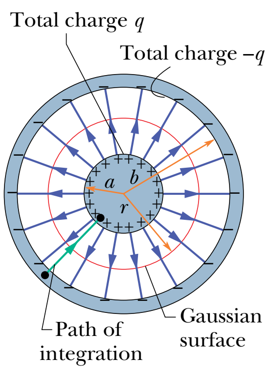


FIGURE 28-10 ■ A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive "plate") and the radial path of integration along which Eq. 28-6 is to be applied. If we visualize the central conductor as the cross section of a sphere rather than that of a long cylindrical wire then this figure also illustrates a spherical capacitor.

A Cylindrical Capacitor

Fig. 28-10 shows, in cross section, a cylindrical capacitor of length L formed by two coaxial cylinders of radii a and b. We assume $L \gg b$ so we can neglect the fringing of the electric field occurring at the ends of the cylinders. Each plate contains an amount of excess charge $|q|$. This configuration is important because coaxial cables are used in the communications industry for the long distance transmission of electrical signals (Fig 28-11).

The electric field inside the cylinder is highly symmetrical, so we can use Gauss's law to determine its values. As a Gaussian surface, we choose a cylinder

FIGURE 28-11 ■ Coaxial cables and connectors are used for long-distance transmission of television and radio signals. The cable consists of a central conducting wire surrounded by a layer of insulation and then a cylindrical conductor. All three elements are centered on the same axis. Coaxial cables are good examples of cylindrical capacitors.

of length L and radius r, closed by end caps and placed as is shown in Fig. 28-10. Then

$$|q| = \varepsilon_0 |\vec{E}| A = \varepsilon_0 |\vec{E}| (2\pi r L),$$

in which $2\pi rL$ is the area of the curved part of the Gaussian surface. There is no flux through the end caps. Solving for $|\vec{E}|$ yields

$$|\vec{E}| = \frac{|q|}{2\pi\varepsilon_0 L r}. \qquad (28\text{-}12)$$

Substitution of this result into our general expression for potential difference yields

$$\Delta V = \int_{-}^{+} \vec{E} \cdot d\vec{s} = -\frac{q}{2\pi\varepsilon_0 L} \int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right), \qquad (28\text{-}13)$$

where here $ds = -dr$ (we integrated radially inward). From the relation $C = |q/\Delta V|$, we then have

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor).} \qquad (28\text{-}14)$$

We see that the capacitance of a cylindrical capacitor, like that of a parallel-plate capacitor, depends only on geometrical factors, in this case L, b, and a.

A Spherical Capacitor

Fig. 28-10 can also serve as a central cross section of a capacitor consisting of two concentric spherical shells, of radii a and b. As a Gaussian surface we draw a sphere of radius r concentric with the two shells; then

$$|q| = \varepsilon_0 E A = \varepsilon_0 E (4\pi r^2),$$

in which $4\pi r^2$ is the area of the spherical Gaussian surface. We solve this equation for $|\vec{E}|$, obtaining

$$E = |\vec{E}| = k\frac{|q|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}, \qquad (28\text{-}15)$$

which we recognize as the expression for the electric field due to a uniform spherical charge distribution from Chapter 24.

If we substitute this expression into Eq. 28-6, we find

$$\Delta V = \int_{-}^{+} \vec{E} \cdot d\vec{s} = -\frac{|q|}{4\pi\varepsilon_0} \int_b^a \frac{dr}{r^2} = \frac{|q|}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{|q|}{4\pi\varepsilon_0}\frac{b-a}{ab}, \qquad (28\text{-}16)$$

where again we have substituted $-dr$ for ds. If we now substitute this into $|q| = C|\Delta V|$ (Eq. 28-1) and solve for C, we find

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \quad \text{(spherical capacitor).} \qquad (28\text{-}17)$$

An Isolated Sphere

We can assign a capacitance to a *single* isolated spherical conductor of radius R by assuming that the "missing plate" is a conducting sphere of infinite radius. After all, the field lines leaving the surface of a positively charged isolated conductor must end somewhere; the walls of the room in which the conductor is housed can serve effectively as our sphere of infinite radius.

To find the capacitance of the isolated conductor, we first rewrite the expression for a spherical capacitor above as

$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b}.$$

If we then let $b \to \infty$ and substitute R for a, we find

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere)}. \tag{28-18}$$

Note that this formula and the others we have derived for capacitance (Eqs. 28-9, 28-14, and 28-17) involve the constant ε_0 multiplied by a quantity having the dimensions of a length.

READING EXERCISE 28-2: Consider capacitors charged by and then removed from the same battery. Does the charge on the capacitor plates increase, decrease, or remain the same in each of the following situations? (a) The plate separation of a parallel-plate capacitor is increased. (b) The radius of the inner cylinder of a cylindrical capacitor is increased. (c) The radius of the outer spherical shell of a spherical capacitor is increased.

READING EXERCISE 28-3: Consider capacitors charged by identical batteries. If the capacitors stay connected to the batteries, does the amount of excess charge on the capacitor plates increase, decrease, or remain the same in each of the following situations? (a) The plate separation of a parallel-plate capacitor is increased. (b) The radius of the inner cylinder of a cylindrical capacitor is increased. (c) The radius of the outer spherical shell of a spherical capacitor is increased.

28-4 Capacitors in Parallel and in Series

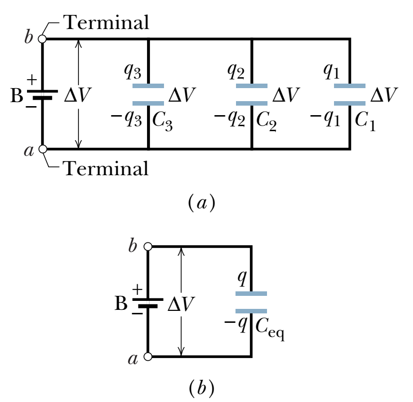


FIGURE 28-12 (*a*) Three capacitors connected in parallel to battery B. The battery maintains a positive potential difference $\Delta V = V_b - V_a$ across its terminals and thus across each fully charged capacitor. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the parallel combination.

When there is a combination of capacitors in a circuit, we can often replace that combination with an **equivalent capacitor**—that is, a single capacitor having the same behavior as the actual combination of capacitors. With such a replacement, we can simplify circuits This is similar to the approach we took with resistors in Chapter 27. In addition, circuits often have what is termed *stray capacitance* due to the presence of conductors and insulators in other types of circuit elements. Knowing how the effective capacitance of such elements might combine with each other and other capacitors in the vicinity is vital to the design of high-performance circuits. In this section we discuss the behavior of two basic types of capacitor combinations—parallel and series.

Capacitors in Parallel

Figure 28-12*a* shows an electric circuit in which three capacitors are connected *in parallel* with battery B. This description has little to do with where the capacitor plates appear in the diagram. Rather, "in parallel" means that one plate of each capacitor is wired directly to one plate of the other capacitors. The opposite plates of the capacitors are also wired to each other. When the parallel combination is connected to a battery, the battery's potential difference ΔV_B is applied across all three capacitors as shown in Fig. 28-12*a*.

We can anticipate how the parallel combination will behave by considering the special case in which all three capacitors are parallel-plate capacitors with the same spacing. What happens in this case is that the effective area of the plates of the combined network of capacitors is equal to the sum of the three areas. Using Eq. 28-9 we see

$$C_{eq} = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0(A_1 + A_2 + A_3)}{d} = \frac{\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d} + \frac{\varepsilon_0 A_3}{d} = C_1 + C_2 + C_3.$$

Even if the three capacitors are of different types with each having a different geometry, we expect the effective area of the combination will be increased. The proof of the pudding is in the experiment. It turns out that a multimeter set to measure capacitance can be used to verify

$$C_{eq} = C_1 + C_2 + C_3,$$

for parallel combinations of three capacitors of all sorts of different types. Since the potential difference across a parallel combination of capacitors connected to a voltage source is the same, we can use the expression $|q| = C|\Delta V|$ (Eq. 28-1) to show that if $C_{eq} = C_1 + C_2 + C_3$, then

$$\frac{|q_{eq}|}{|\Delta V|} = \frac{|q_1|}{|\Delta V|} + \frac{|q_2|}{|\Delta V|} + \frac{|q_3|}{|\Delta V|},$$

so that

$$|q_{eq}| = |q_1| + |q_2| + |q_3|.$$

In general,

> When a potential difference ΔV is applied across several capacitors connected in parallel, that potential difference ΔV is applied across each capacitor. The total amount of the excess charge $|q|$ found on each plate of the equivalent capacitor is equal to the sum of the excess charge amounts found on each of the capacitors.

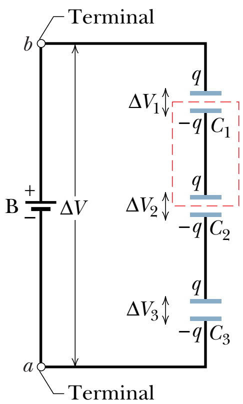
When we analyze a circuit of capacitors in parallel, we can simplify it with this mental replacement:

> Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same total charge $|q|$ and the same potential difference ΔV as the actual capacitors.

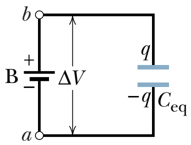
We can easily extend our method for finding the equivalent capacitance for three capacitors to any number of capacitors. For n capacitors wired in parallel,

$$C_{eq} = \sum_{j=1}^{n} C_j \quad \text{(n capacitors in parallel).} \tag{28-19}$$

To find the equivalent capacitance of a parallel combination, we simply add the individual capacitances.



(a)



(b)

FIGURE 28-13 ■ (a) Three capacitors connected in series to battery B. The battery maintains a positive potential difference ΔV between the top and bottom plates of the series combination. (b) The equivalent capacitor, with capacitance C_{eq}, replaces the series combination.

Capacitors in Series

Figure 28-13*a* shows three capacitors connected *in series* to battery B. This description has little to do with where the capacitors are located on the drawing. Rather, "in series" means the capacitors are wired serially, one after the other, so a battery can set up a potential difference ΔV across the two ends of the series as shown in Fig. 28-13*a*.

Let's consider what goes on with the charges on the capacitor plates by following a *chain reaction* of events, in which the charging of each capacitor causes the charging of the next capacitor. We start with capacitor 3 and work upward to capacitor 1. When the battery is first connected to the series of capacitors, it produces a net charge $-q$ on the bottom plate of capacitor 3. That charge then repels negative charge from the top plate of capacitor 3 (leaving it with a net or excess charge $+q$). The repelled negative charge moves to the bottom plate of capacitor 2 (giving it charge $-q$). That excess negative charge on the bottom plate of capacitor 2 then repels negative charge from the top plate of capacitor 2 (leaving it with charge $+q$) to the bottom plate of capacitor 1 (giving it a net charge $-q$). Finally the excess charge on the bottom plate of capacitor 1 helps move negative charge from the top plate of capacitor 1 to the battery, leaving that top plate with net charge $+q$. We see then that the potential differences existing across the capacitors in the series produce identical amounts of excess charge $|q|$ on their plates.

Since the amounts of excess charge on each pair of plates in a series connection are the same, we can use Eq. 28-1, $|q| = C|\Delta V|$, to summarize our reasoning in equation form:

$$|q_1| = |q_2| = |q_3| = |q|,$$

and so

$$|\Delta V_1| = \frac{|q|}{C_1}, \quad |\Delta V_2| = \frac{|q|}{C_2}, \quad \text{and} \quad |\Delta V_3| = \frac{|q|}{C_3}.$$

The total potential difference ΔV due to the battery is the sum of these three potential differences. Thus,

$$|\Delta V| = |\Delta V_1| + |\Delta V_2| + |\Delta V_3|,$$

so that

$$\frac{|q|}{C_{eq}} = \frac{|q|}{C_1} + \frac{|q|}{C_2} + \frac{|q|}{C_3}.$$

The equivalent capacitance is then

$$C_{eq} = \frac{|q|}{|\Delta V|} \quad \text{and also} \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

> When a potential difference of size $|\Delta V|$ is applied across several capacitors connected in series, each of the capacitors has the same amount of excess charge $|q|$ on its plates. The sum of the potential differences across the entire network of capacitors is equal to the size of the applied potential difference $|\Delta V|$.

Here is an important point about capacitors in series: When charge is shifted from one capacitor to another in a series of capacitors, it can move along only one route, such as from capacitor 3 to capacitor 2 in Fig. 28-13*a*. If there are additional routes, the capacitors are not in series. Hence, when we analyze a circuit of capacitors in series, we can simplify it with this mental replacement:

TABLE 28-2
Series and Parallel Resistors and Capacitors

Resistors		Capacitors	
Series	**Parallel**	**Series**	**Parallel**
$R_{eq} = \sum_{j=1}^{n} R_j$	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$	$C_{eq} = \sum_{j=1}^{n} C_j$
Eq. 27-4	Eq. 27-12	Eq. 28-20	Eq. 28-19
1. Same current through all resistors	1. Same potential difference across all resistors	1. Same excess charge on all capacitors	1. Same potential difference across all capacitors
2. Potential differences across each resistor add	2. Currents through each resistor add	2. Potential differences across each capacitor add	2. Excess charges on attached plates add

Capacitors connected in series can be replaced with an equivalent capacitor having the same amount of excess charge $|q|$ on each plate and the same size of potential difference $|\Delta V|$ as the size of the total potential differences across the individual capacitors.

We can easily extend our method of determining the equivalent capacitance of a set of capacitors wired in series from three capacitors to n capacitors by using the expression

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}). \tag{28-20}$$

Using this expression, you can show that the equivalent of a series of capacitances is always less than the least capacitance in the series. This can also be predicted qualitatively since the effective insulated separation between the top and bottom plate increases since $d = d_1 + d_2 + d_3$. According to Eq. 28-9, capacitance is inversely proportional to plate separation.

Table 28-2 summarizes the equivalence relations for resistors and capacitors in series and in parallel. It also presents the information about potential differences and charges on the combinations we determined by thinking about the physics of how the charges move and distribute themselves in these different geometrical configurations.

READING EXERCISE 28-4: A battery with a potential difference ΔV is used to store an amount of excess charge $|q|$ on each of two identical capacitors and is then disconnected. The two capacitors are then connected to each other. What is the potential difference across each capacitor and the amount of excess charge on each capacitor plate when the capacitors are wired (a) in parallel and (b) in series? ■

TOUCHSTONE EXAMPLE 28-1: Equivalent Capacitance

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 28-14*a*, across which potential difference ΔV is applied. Assume

$$C_1 = 12.0\ \mu\text{F}, \quad C_2 = 5.30\ \mu\text{F}, \quad \text{and} \quad C_3 = 4.50\ \mu\text{F}.$$

SOLUTION ■ The **Key Idea** here is that any capacitors connected in series can be replaced with their equivalent capacitor, and any capacitors connected in parallel can be replaced with their equivalent capacitor. Therefore, we should first check whether any of the capacitors in Fig. 28-14*a* are in parallel or series.

Capacitors 1 and 3 are connected one after the other, but are they in series? No. The potential ΔV that is applied to the capacitors forces excess charge on the bottom plate of capacitor 3. That charge causes charge to shift from the top plate of capacitor 3. However, note that the shifting charge can move to the bottom

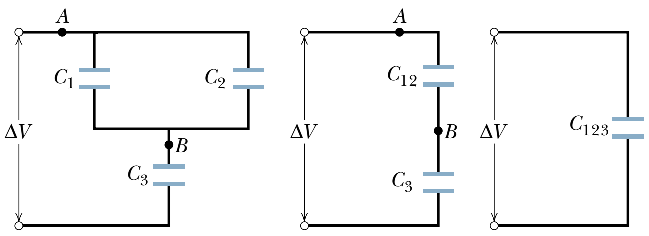


FIGURE 28-14 ■ (*a*) Three capacitors. (*b*) C_1 and C_2, a parallel combination, are replaced by C_{12}. (*c*) C_{12} and C_3, a series combination, are replaced by the equivalent capacitance C_{123}.

plates of both capacitor 1 and capacitor 2. Because there is more than one route for the shifting charge, capacitor 3 is *not* in series with capacitor 1 (or capacitor 2).

Are capacitor 1 and capacitor 2 in parallel? Yes. Their top plates are directly wired together and their bottom plates are directly wired together, and electric potential is applied between the top-plate pair and the bottom-plate pair. Thus, capacitor 1 and capacitor 2 are in parallel, and Eq. 28-19 tells us that their equivalent capacitance C_{12} is

$$C_{12} = C_1 + C_2 = 12.0\ \mu\text{F} + 5.30\ \mu\text{F} = 17.3\ \mu\text{F}.$$

In Fig. 28-14*b*, we have replaced capacitors 1 and 2 with their equivalent capacitor, call it capacitor 12 (say "one two"). (The connections at points *A* and *B* are exactly the same in Figs. 28-14*a* and *b*.)

Is capacitor 12 in series with capacitor 3? Again applying the test for series capacitances, we see that the charge that shifts from the top plate of capacitor 3 must entirely go to the bottom plate of capacitor 12. Thus, capacitor 12 and capacitor 3 are in series, and we can replace them with their equivalent C_{123}, as shown in Fig. 28-14*c*.

From Eq. 28-20, we have

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{17.3\ \mu\text{F}} + \frac{1}{4.50\ \mu\text{F}} = 0.280\ \mu\text{F}^{-1},$$

from which

$$C_{123} = \frac{1}{0.280\ \mu\text{F}^{-1}} = 3.57\ \mu\text{F}. \qquad \text{(Answer)}$$

(b) The potential difference that is applied to the input terminals in Fig. 28-14*a* is $V = 12.5$ V. What is the excess charge on each plate of C_1?

SOLUTION ■ One **Key Idea** here is that, to get the excess charge q_1 on each plate of capacitor 1, we now have to work backward to that capacitor, starting with the equivalent capacitor 123. Since the given potential difference $\Delta V = 12.5$ V is applied across the actual combination of three capacitors in Fig. 28-14*a*, it is also applied across capacitor 123 in Fig. 28-14*c*. Thus, Eq. 28-1 ($|q| = C|\Delta V|$) gives us

$$|q_{123}| = C_{123}|\Delta V| = (3.57\ \mu\text{F})(12.5\ \text{V}) = 44.6\ \mu\text{C}.$$

A second **Key Idea** is that the series capacitors 12 and 3 in Fig. 28-1*b* have the same charge as their equivalent capacitor 123. Thus, capacitor 12 has charge $q_{12} = q_{123} = 44.6\ \mu$C. From Eq. 28-1, the potential difference across capacitor 12 must be

$$|\Delta V_{12}| = \frac{|q_{12}|}{C_{12}} = \frac{44.6\ \mu\text{C}}{17.3\ \mu\text{F}} = 2.58\ \text{V}.$$

A third **Key Idea** is that the parallel capacitors 1 and 2 both have the same potential difference as their equivalent capacitor 12. Thus, capacitor 1 has the potential difference $\Delta V_1 = \Delta V_{12} = 2.58$ V. Thus, from Eq. 28-1, the excess charge on each plate of capacitor 1 must be

$$|q_1| = C_1|\Delta V_1| = (12.0\ \mu\text{F})(2.58\ \text{V})$$
$$= 31.0\ \mu\text{C}. \qquad \text{(Answer)}$$

28-5 Energy Stored in an Electric Field

Work must be done by an external agent to charge a capacitor. Starting with an uncharged capacitor, for example, imagine—using "magic tweezers"—that you remove electrons from one plate and deposit them one at a time to the other plate. The electric field building up in the space between the plates has a direction that tends to oppose further separation of charge. As excess charge accumulates on the capacitor plates, you have to do increasingly larger amounts of work to transfer additional electrons. In practice, this work is done not by "magic tweezers" but by a battery, at the expense of its store of chemical energy.

We visualize the work required to charge a capacitor as being stored in the form of **electric potential energy** *U* in the electric field between the plates. You can recover this energy at will, by discharging the capacitor in a circuit, just as you can recover the potential energy stored in a stretched bow by releasing the bowstring to transfer the energy to the kinetic energy of an arrow. Another example is carrying rocks up a hill against gravity. Energy is stored because of the hill's height and can be recovered by letting the rocks fall down again. In a capacitor, we can recover the stored energy by connecting wires to the ends.

Suppose that at a given instant, a charge $|q'|$ has been moved from one plate of a capacitor, through the wires in the circuit, to the other plate. The amount of the potential difference $|\Delta V'|$ between the plates at that instant will be $|q'|/C$. If an extra increment of charge $|dq'|$ is then removed from one plate and deposited on the other, the amount of the increment of work required will be (from Chapter 25)

$$|dW| = |\Delta V'||dq'| = \frac{|q'|}{C}|dq'|.$$

The work required to bring the total capacitor charge separation up to a final value $|q|$ is

$$W = \int dW = \frac{1}{C}\int_0^q q'|dq'| = \frac{|q|^2}{2C}.$$

This work is stored as potential energy U in the capacitor, and since $q^2 = |q|^2$

$$U = \frac{q^2}{2C} \qquad \text{(potential energy)}. \tag{28-21}$$

From $|q| = C|\Delta V|$, we can also write this as

$$U = \tfrac{1}{2}C(\Delta V)^2 = \tfrac{1}{2}q\Delta V \qquad \text{(potential energy)}. \tag{28-22}$$

These relations hold no matter what the geometry of the capacitor is.

To gain some physical insight into energy storage, consider two parallel-plate capacitors identical except that capacitor 1 has twice the plate separation of capacitor 2. Then capacitor 1 has twice the volume between its plates and also, from Eq. 28-9, half the capacitance of capacitor 2. Equation 28-4 tells us that if both capacitors have the same amount of charge $|q|$, the electric fields between their plates are identical. Equation 28-21 tells us capacitor 1 has twice the stored potential energy of capacitor 2. Of two otherwise identical capacitors with the same charge and same electric field, the one with twice the volume between its plates has twice the stored potential energy. Arguments like this tend to verify our earlier assumption:

> The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

A High-Speed Electronic Flash Unit

The ability of a capacitor to store potential energy is the basis of *high-speed electronic flash* devices, like those used in stop-action photography. In an electronic flash unit, a battery charges a capacitor relatively slowly to a high potential difference, storing a large amount of energy in the capacitor. The battery maintains only a modest potential difference; an electronic circuit repeatedly uses that potential difference to greatly increase the potential difference of the capacitor. The power, or rate of energy transfer, during this process is also modest.

When a high-speed flash unit fires, the capacitor releases its stored energy by sending a burst of electric current through a Xenon gas discharge tube that gives off a brief flash of white light. As an example, when a 200 μF capacitor in a high-speed flash unit is charged to 300 V, Eq. 28-22 gives the energy stored in the capacitor as

$$U = \tfrac{1}{2}C(\Delta V)^2 = \tfrac{1}{2}(200 \times 10^{-6}\ \text{F})(300\ \text{V})^2 = 9\ \text{J}.$$

As mentioned in the puzzler at the beginning of this chapter, this should be more than enough energy to provide the illumination needed to take a photograph with ordinary film. Suppose the flashtube in the high-speed flash unit Edgerton used to take the photo of the bullet passing through the apple has a very rapid discharge rate. If the Xenon tube takes only one-third of a microsecond to discharge, then the power associated with the discharge is

$$P = \frac{U}{t} = \frac{9\ \mathrm{J}}{0.33 \times 10^{-6}\ \mathrm{s}} = 27 \times 10^6\ \mathrm{W} = 27\ \mathrm{MW}.$$

Energy Density

In a parallel-plate capacitor, neglecting fringing, the electric field has the same value at all points between the plates. The **energy density** u—that is, the potential energy per unit volume between the plates—should also be uniform. We can find u by dividing the total potential energy by the volume Ad of the space between the plates. Using Eq. 28-22, we obtain

$$u = \frac{U}{Ad} = \frac{C(\Delta V)^2}{2Ad}.$$

With Eq. 28-9 ($C = \varepsilon_0 A/d$), this result becomes

$$u = \tfrac{1}{2}\varepsilon_0\left(\frac{\Delta V}{d}\right)^2.$$

However, from Eq. 25-39, $\Delta V/d$ equals the electric field magnitude $|\vec{E}| = E$, so

$$u = \tfrac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density).} \qquad (28\text{-}23)$$

Although we derived this result for the special case of a parallel-plate capacitor, it holds generally, whatever may be the source of the electric field. If an electric field $\vec{E}$ exists at any point in space, we can think of that point as a site of electric potential energy whose amount per unit volume is given by Eq. 28-23.

TOUCHSTONE EXAMPLE 28-2: Redistributing Charge

(a) Capacitor 1, with $C_1 = 3.55\ \mu\mathrm{F}$, is charged to a potential difference $\Delta V_0 = 6.30\ \mathrm{V}$, using a 6.30 V battery. The battery is then removed and the capacitor is connected as in Fig. 28-15 to an uncharged capacitor 2, with $C_2 = 8.95\ \mu\mathrm{F}$. When switch S is closed, charge flows between the capacitors until they have the same potential difference ΔV. Find ΔV.

SOLUTION ■ The situation here differs from Touchstone Example 28-1 because an applied electric potential is *not* maintained across a combination of capacitors by a battery or some other source. Here, just after switch S is closed, the only applied electric potential is that of capacitor 1 on capacitor 2, and that potential is decreasing. Thus, although the capacitors in Fig. 28-15 are connected end to end, in this situation they are not *in series;* and although they are drawn parallel, in this situation they are not *in parallel.*

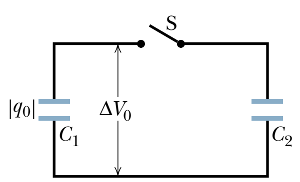


FIGURE 28-15 ■ A potential difference ΔV_0 is applied to capacitor 1 and the charging battery is removed. Switch S is then closed so that the charge on capacitor 1 is shared with capacitor 2.

To find the final electric potential (when the system comes to equilibrium and charge stops flowing), we use this **Key Idea**: After the switch is closed, the original excess charge $|q_0|$ on each plate of capacitor 1 is redistributed (shared) between capacitor 1 and capacitor 2. When equilibrium is reached, we can relate the original charge $|q_0|$ with the final charges $|q_1|$ and q_2 by writing

$$|q_0| = |q_1| + |q_2|.$$

Applying the relation $|q| = C|\Delta V|$ (Eq. 28-1) to each term of this equation yields

$$C_1|\Delta V_0| = C_1|\Delta V| + C_2|\Delta V|,$$

from which

$$|\Delta V| = |\Delta V_0|\frac{C_1}{C_1 + C_2} = \frac{(6.30\text{ V})(3.55\ \mu\text{F})}{3.55\ \mu\text{F} + 8.95\ \mu\text{F}}$$
$$= 1.79\text{ V}. \qquad \text{(Answer)}$$

When the capacitors reach this steady value of electric potential difference, the charge flow stops.

(b) How much energy is stored in the original capacitor when it is first charged up?

SOLUTION ■ The **Key Idea** here is that the potential energy stored in a capacitor, given by Eq. 28-22, is just

$$U = (\tfrac{1}{2})C(\Delta V)^2$$
$$= (\tfrac{1}{2})(3.55\ \mu\text{F})(6.30\text{ V})^2$$
$$= 70.4\ \mu\text{J}. \qquad \text{(Answer)}$$

(c) How much energy is stored in the two capacitors after they are connected together?

SOLUTION ■ The **Key Idea** here is that the potential energy stored in *each* capacitor, given by Eq. 28-22, so that

$$U^{\text{total}} = U_1 + U_2 = (\tfrac{1}{2})C_1(\Delta V_1)^2 + (\tfrac{1}{2})C_2(\Delta V_2)^2$$
$$= (\tfrac{1}{2})((3.55\ \mu\text{F}) + (8.95\ \mu\text{F}))(1.79\text{ V})^2$$
$$= 20.0\ \mu\text{J}. \qquad \text{(Answer)}$$

But how can this be? Before the second capacitor was placed across the first one, there was over 70 μJ of energy stored in the system. What happened to the 50 μJ of energy that seems to have vanished when the second capacitor was charged from the first one? You might argue that the "lost" energy must have been dissipated as heat in the resistance of the wires connecting the two capacitors. But suppose we used superconducting wires with zero resistance? Then where does the missing energy go? The answer, as you will learn in Chapters 33 and 34, is that the charge would oscillate back and forth between the two capacitors until the 50 μJ of "excess" energy was radiated away in the form of electromagnetic waves.

28-6 Capacitor with a Dielectric

If you fill the space between the plates of a capacitor with a *dielectric,* which is usually an insulating material such as mineral oil or plastic, what happens to the capacitance? Michael Faraday—to whom the whole concept of capacitance is largely due and for whom the SI unit of capacitance is named—first looked into this matter in 1837. Using simple equipment much like that shown in Fig. 28-16, he found that the capacitance *increased* by a numerical factor κ, which he called the dielectric constant of the insulating material. Table 28-3 shows some dielectric materials and their dielectric constants. The dielectric constant of a vacuum is unity by definition. Because air is mostly empty space, its measured dielectric constant is only slightly greater than unity.

Another effect of the introduction of a dielectric is to limit the potential difference that can be applied between the plates to a certain value ΔV^{max}, called the *breakdown potential*. If this value is substantially exceeded, the dielectric material will break down and form a conducting path between the plates. That is, when the capacitor is filled with a dielectric, the charge separation you can maintain with a given potential difference increases. Every dielectric material has a characteristic *dielectric strength,* which is the maximum value of the electric field that it can tolerate without breakdown. A few such values are listed in Table 28-3.

FIGURE 28-16 ■ The simple electrostatic apparatus used by Faraday. An assembled apparatus (second from left) forms a spherical capacitor consisting of a central brass ball and a concentric brass shell. Faraday placed dielectric materials in the space between the ball and the shell.

As we discussed in connection with Eq. 28-18, the capacitance of any capacitor can be written in the form

$$C = \varepsilon_0 L, \qquad (28\text{-}24)$$

in which L has the dimensions of a length. For example, $L = A/d$ for a parallel-plate capacitor. Faraday's discovery was, with a dielectric *completely* filling the space between the plates, Eq. 28-24 becomes

$$C = \kappa\varepsilon_0 L = \kappa C_{\text{air}}, \qquad (28\text{-}25)$$

where C_{air} is the value of the capacitance with only air between the plates.

TABLE 28-3
Some Properties of Dielectrics[a]

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Tantalum oxide	11.6	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)[b]	80.4	
Water (25°C)[b]	78.5	
Titania ceramic	130	
Strontium titanate	310	8
For a vacuum, κ = unity.		

[a]Measured at room temperature, except for the water.
[b]Note that water is not an insulating material. It is listed because it has dielectric properties.

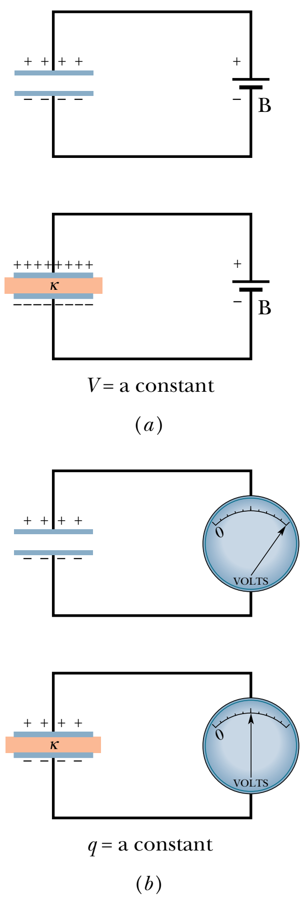


FIGURE 28-17 (a) If the potential difference between the plates of a capacitor is maintained, as by battery B, the effect of a dielectric is to increase the excess charge on each plate. (b) If the charge on the capacitor plates is maintained, as in this case, the effect of a dielectric is to reduce the potential difference between the plates. The scale shown is that of a *potentiometer,* a device used to measure potential difference (here, between the plates). A capacitor cannot discharge through a potentiometer.

Figure 28-17 provides some insight into Faraday's experiments. In Fig. 28-17a the battery ensures that the potential difference ΔV between the plates will remain constant. When a dielectric slab is inserted between the plates, the excess amount of charge $|q|$ on the plates increases by a factor of κ, where κ is always greater than 1; the additional charge is delivered to the capacitor plates by the battery. In Fig. 28-17b there is no battery and therefore the amount of excess charge $|q|$ must remain constant when the dielectric slab is inserted; then the potential difference ΔV between the plates decreases by a factor of κ. Both these observations are consistent (through the relation $|q| = C|\Delta V|$) with the increase in capacitance caused by the dielectric.

Comparison of Eqs. 28-24 and 28-25 suggests that the effect of a dielectric can be summed up in more general terms:

> In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

A point charge inside a dielectric produces an electric field that, by Coulomb's law, has the magnitude

$$|\vec{E}| = \frac{1}{4\pi\kappa\varepsilon_0}\frac{|q|}{r^2}. \qquad (28\text{-}26)$$

Also, the expression for the electric field just outside an isolated conductor immersed in a dielectric (see Eq. 24-20) becomes

$$|\vec{E}| = \frac{|\sigma|}{\kappa\varepsilon_0}. \qquad (28\text{-}27)$$

Both these equations show that *for a fixed distribution of charges, the effect of a dielectric is to weaken the magnitude of the electric field that would otherwise be present.* In addition, the amount of energy stored is reduced because work must be done by the field to pull in the dielectric.

TOUCHSTONE EXAMPLE 28-3: A Dielectric's Energetics

A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference $\Delta V = 12.5$ V between its plates. The charging battery is now disconnected and a porcelain slab ($\kappa = 6.50$) is slipped between the plates. What is the potential energy of the device, both before and after the slab is put into place?

SOLUTION ■ The **Key Idea** here is that we can relate the potential energy U of the capacitor to the capacitance C and either the potential ΔV (with Eq. 28-22) or the capacitor charge $|q|$ (with Eq. 28-21):

$$U_1 = \tfrac{1}{2}C\Delta V^2 = \frac{q^2}{2C}.$$

Because we are given the initial potential $\Delta V(=12.5\text{V})$, we use Eq. 28-22 to find the initial stored energy:

$$U_1 = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(13.5 \times 10^{-12}\ \text{F})(12.5\ \text{V})^2$$
$$= 1.055 \times 10^{-9}\ \text{J} = 1055\ \text{pJ} \approx 1100\ \text{pJ}. \quad \text{(Answer)}$$

To find the final potential energy U_2 (after the slab is introduced), we need another **Key Idea**: Because the battery has been disconnected, the amount of excess charge on each capacitor plate cannot change when the dielectric is inserted. However, the potential *does* change. Thus, we must now use Eq. 28-21 (based on q) to write the final potential energy U_2, but now that the slab is within the capacitor, the capacitance is κC. We then have

$$U_2 = \frac{q^2}{2\kappa C} = \frac{U_1}{\kappa} = \frac{1055\ \text{pJ}}{6.50} = 162\ \text{pJ} \approx 160\ \text{pJ}. \quad \text{(Answer)}$$

When the slab is introduced, the potential energy decreases by a factor of κ.

The "missing" energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_1 - U_2 = (1055 - 162)\ \text{pJ} = 893\ \text{pJ}.$$

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a (constant) mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.

28-7 Dielectrics: An Atomic View

What happens, in atomic and molecular terms, when we put a dielectric in an electric field? There are two possibilities, depending on the nature of the molecules:

1. *Polar dielectrics.* The molecules of some dielectrics, like water, have permanent electric dipole moments. In such materials (called *polar dielectrics*), the electric dipoles tend to line up with an external electric field as in Fig. 28-18. Because the molecules are continuously jostling each other as a result of their random thermal motion, this alignment is not complete, but it becomes more complete as the magnitude of the applied field is increased (or as the temperature, and thus the jostling, is decreased). The alignment of the electric dipoles produces an electric field directed opposite the applied field and smaller in magnitude.
2. *Nonpolar dielectrics.* Regardless of whether they have permanent electric dipole moments, molecules acquire dipole moments by induction when placed in an external electric field. In Section 25-9 (see Fig. 25-16), we saw that this occurs because the external field tends to "stretch" the molecules, slightly separating the centers of negative and positive charge.

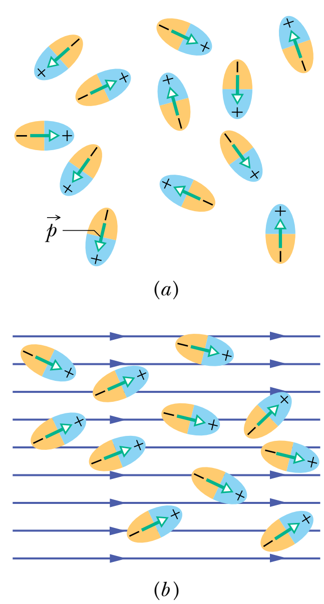


FIGURE 28-18 ■ (*a*) Molecules with a permanent electric dipole moment, showing their random orientation in the absence of an external electric field. (*b*) An electric field is applied, producing partial alignment of the dipoles. Thermal agitation prevents complete alignment.

Figure 28-19*a* shows a nonpolar dielectric slab with no external electric field applied. An electric field $\vec{E}_0$ is present due to the excess charges shown on the capacitor plates in Fig. 28-19*a*. The result is a slight separation of the centers of the positive and negative charge distributions within the slab, producing positive charge on one face of the slab (due to the positive ends of dipoles there) and negative charge on the opposite

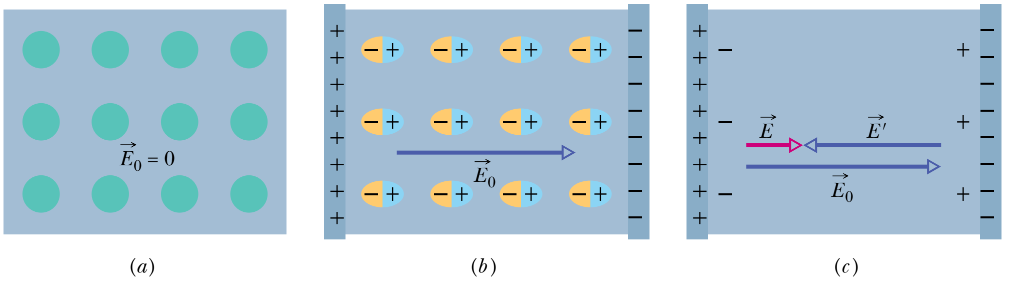


FIGURE 28-19 ■ (*a*) A nonpolar dielectric slab. The circles represent the electrically neutral atoms within the slab. (*b*) An electric field is applied via charged capacitor plates; the field slightly stretches the atoms, separating the centers of positive and negative charge. (c) The separation produces surface charges on the slab faces. These charges set up a field $\vec{E}'$, which opposes the applied field $\vec{E}_0$. The resultant field $\vec{E}$ inside the dielectric (the vector sum of $\vec{E}_0$ and $\vec{E}'$) has the same direction as $\vec{E}_0$ but smaller magnitude.

face (due to the negative ends of dipoles there). The slab as a whole remains electrically neutral and—within the slab—there is no excess charge in any volume element.

Figure 28-19*c* shows that the induced surface charges on the faces produce an electric field $\vec{E}'$, in the direction opposite the applied electric field $\vec{E}_0$. The resultant field $\vec{E}$ inside the dielectric (the vector sum of fields $\vec{E}_0$ and $\vec{E}'$) has the direction of $\vec{E}_0$ but is smaller in magnitude.

Both the field $\vec{E}'$ produced by the surface charges in Fig. 28-19*c* and the electric field produced by the permanent electric dipoles in Fig. 28-18 act in the same way—they oppose the applied field $\vec{E}$. (Inside the material, the $\vec{E}$ field fluctuates wildly, depending on whether you are close to one side of a molecule or another. The effects we are looking at are the average effects of the molecules.) Thus, the effect of both polar and nonpolar dielectrics is to weaken any applied field within them, as between the plates of a capacitor. As a result, a given charge separation can be maintained at a lower potential difference, ΔV, with a dielectric than with a vacuum. This means that a capacitor with a dielectric added has a higher capacitance.

We can now see why the dielectric porcelain slab in Touchstone Example 28-3 is pulled into the capacitor: As it enters the space between the plates, the excess surface charge appearing on each slab face has a sign that is opposite to that of the excess charge on the nearby capacitor plate. Thus, slab and plates attract each other.

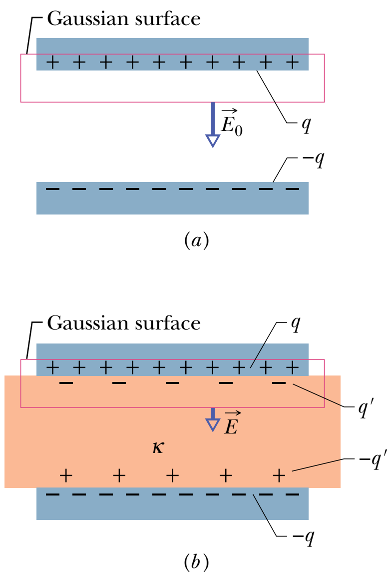


FIGURE 28-20 ■ A parallel-plate capacitor (*a*) without and (*b*) with a dielectric slab inserted. The excess charge q on the plates is assumed to be the same in both cases.

28-8 Dielectrics and Gauss' Law

In our discussion of Gauss' law in Chapter 24, we assumed that the charges existed in a vacuum. Here we shall see how to modify and generalize that law if dielectric materials, such as those listed in Table 28-3, are present. Figure 28-20 shows a parallel-plate capacitor of plate area A, both with and without a dielectric. We assume the amount of excess charge $|q|$ on the plates is the same in both situations. Note the field between the plates induces charge buildup on the faces of the dielectric by one of the methods discussed in Section 28-7.

For the situation of Fig. 28-20*a*, without a dielectric, we can find the electric field $\vec{E}_0$ between the plates as we did in Fig. 28-9: We enclose the excess charge q on the top plate with a Gaussian surface and then apply Gauss' law. Letting $E_0 = |\vec{E}_0|$ represent the magnitude of the field, we find

$$\left|\varepsilon_0 \oint \vec{E} \cdot d\vec{A}\right| = \varepsilon_0 E_0 A = |q^{\text{net}}| = |q|, \tag{28-28}$$

or

$$E_0 = \frac{|q|}{\varepsilon_0 A}. \tag{28-29}$$

In Fig. 28-20*b*, with the dielectric in place, we can find the electric field between the plates (and within the dielectric) by using the same Gaussian surface. However, now the surface encloses two types of charge: it still encloses a net charge q on the top plate but it now also encloses the induced charge q' on the top face of the dielectric. The excess charge on each conducting plate is said to be *free charge* because it can move through the circuit if we change the electric potential of the plate. The induced charge on the surfaces of the dielectric is bound charge. It's stuck to the molecules of an insulator. It can only be displaced from its original position by microscopic amounts and cannot move from the surface.

The amount of net charge enclosed by the Gaussian surface in Fig. 28-20*b* is $|q + q'|$, so Gauss' law now gives

$$\left|\varepsilon_0 \oint \vec{E} \cdot d\vec{A}\right| = \varepsilon_0 EA = |q + q'|, \tag{28-30}$$

or

$$E = \frac{|q + q'|}{\varepsilon_0 A}. \tag{28-31}$$

Since q' and q have different signs, this means that the effect of the dielectric is to weaken the original field E_0 by a factor of κ, so we may write

$$E = \frac{E_0}{\kappa} = \frac{|q|}{\kappa \varepsilon_0 A}. \tag{28-32}$$

Comparison of Eqs. 28-31 and 28-32 shows

$$|q^{\text{net}}| = |q + q'| = \frac{|q|}{\kappa}. \tag{28-33}$$

Equation 28-33 shows correctly that the amount of induced surface charge is less than that of the excess free charge and is zero if no dielectric is present (then, $\kappa = 1$ in Eq. 28-33).

By substituting for $|q + q'|$ from Eq. 28-33 in Eq. 28-30, we can write Gauss' law in the form

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \qquad \text{(Gauss' law with dielectric)}, \tag{28-34}$$

where q is the net free charge on the plate of interest. Here we drop the absolute value sign to account for the fact that the excess charge on a plate of interest, q, can be either positive or negative.

This important equation, although derived for a parallel-plate capacitor, is true generally and is the most general form in which Gauss' law can be written. Note the following:

1. The flux integral now involves $\kappa\vec{E}$, not just $\vec{E}$. (The vector $\varepsilon_0\kappa\vec{E}$ is sometimes called the electric displacement $\vec{D}$, so Eq. 28-34 can be written in the form $\oint \vec{D} \cdot d\vec{A} = q$).
2. The amount of excess charge $|q|$ enclosed by the Gaussian surface is now taken to be the free charge only. The induced surface charge is deliberately ignored on the right side of Eq. 28-34, having been taken fully into account by introducing the dielectric constant κ on the left side.
3. Equation 28-34 differs from Eq. 24-7, our original statement of Gauss' law, only in that ε_0 in the latter equation has been replaced by $\kappa\varepsilon_0$. We keep κ inside the integral of Eq. 28-34 to allow for cases in which κ is not constant over the entire Gaussian surface.

Gauss's law still holds when charged molecules are present, but it's hard to use, since we don't know where those molecular charges are. We only know their average effect, which is summarized by the measured constant κ. Here, we saw how to create a form of Gauss's law including the effect of the molecules automatically, and this allows us to work only with the charges we control directly—the "free" charges.

TOUCHSTONE EXAMPLE 28-4: Adding a Dielectric

Figure 28-21 shows a parallel-plate capacitor of plate area A and plate separation d. A potential difference ΔV_0 is applied between the plates. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume

$$A = 115\ \text{cm}^2,\quad d = 1.24\ \text{cm},\quad \Delta V_0 = 85.5\ \text{V},$$
$$b = 0.780\ \text{cm},\quad \text{and}\quad \kappa = 2.61.$$

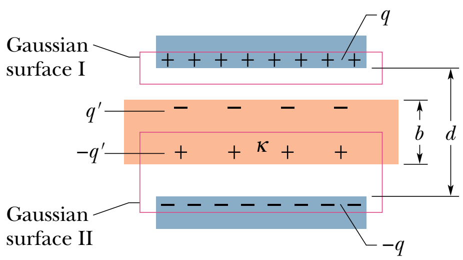


FIGURE 28-21 ■ A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.

(a) What is the capacitance C_0 before the dielectric slab is inserted?

SOLUTION ■ From Eq. 28-9 we have

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(115 \times 10^{-4}\ \text{m}^2)}{1.24 \times 10^{-2}\ \text{m}}$$
$$= 8.21 \times 10^{-12}\ \text{F} = 8.21\ \text{pF}. \qquad \text{(Answer)}$$

(b) What is the amount of free excess charge that appears on each plate?

SOLUTION ■ From Eq. 28-1,

$$|q| = C_0|\Delta V_0| = (8.21 \times 10^{-12}\ \text{F})(85.5\ \text{V})$$
$$= 7.02 \times 10^{-10}\ \text{C} = 702\ \text{pC}. \qquad \text{(Answer)}$$

Because the charging battery was disconnected before the slab was introduced, the free charge remains unchanged as the slab is put into place.

(c) What is the magnitude of the electric field E_0 in the gaps between the plates and the dielectric slab?

SOLUTION ■ A **Key Idea** here is to apply Gauss' law, in the form of Eq. 28-34, to Gaussian surface I in Fig. 28-21—that surface passes through the gap, and so it encloses *only* the free charge on the upper capacitor plate. Because the area vector $d\vec{A}$ and the field vector $\vec{E}_0$ are both directed downward, the dot product in Eq. 28-34 becomes

$$\vec{E}_0 \cdot d\vec{A} = |\vec{E}_0| dA \cos 0° = E_0 dA.$$

Equation 28-34 then becomes

$$\varepsilon_0 \kappa E_0 \oint dA = q.$$

The integration now simply gives the surface area A of the plate. Thus, we obtain

$$\varepsilon_0 \kappa |\vec{E}_0| \oint dA = q,$$

or

$$E_0 = \frac{q}{\varepsilon_0 \kappa A}.$$

One more **Key Idea** is needed before we evaluate E_0; that is, we must put $\kappa = 1$ here because Gaussian surface I does not pass through the dielectric. Since the charge q on the upper plate is positive, we have

$$E_0 = \frac{q}{\varepsilon_0 \kappa A} = \frac{7.02 \times 10^{-10}\ \text{C}}{(8.85 \times 10^{-12}\ \text{F/m})(1)(115 \times 10^{-4}\ \text{m}^2)}$$
$$= 6900\ \text{V/m} = 6.90\ \text{kV/m}. \qquad \text{(Answer)}$$

Note that the value of E_0 does not change when the slab is introduced because the amount of charge enclosed by Gaussian surface I in Fig. 28-21 does not change.

(d) What is the magnitude of the electric field E_1 in the dielectric slab?

SOLUTION ■ The **Key Idea** here is to apply Eq. 28-34 to Gaussian surface II in Fig. 28-21. That surface encloses free charge $-q$ and induced charge $-q'$, but we ignore the latter when we use Eq. 28-34. We find

$$\varepsilon_0 \oint \kappa \vec{E}_1 \cdot d\vec{A} = -\varepsilon_0 \kappa E_1 A = -q. \qquad (28\text{-}35)$$

(The first minus sign in this equation comes from the dot product $\vec{E}_1 \cdot d\vec{A}$, because now the field vector $\vec{E}_1$ is directed downward and the area vector dA' is directed upward.) Equation 28-35 gives us

$$E_1 = \frac{q}{\varepsilon_0 \kappa A} = \frac{E_0}{\kappa} = \frac{6.90\text{kV/m}}{2.61} = 2.64\ \text{kV/m}. \qquad \text{(Answer)}$$

(e) What is the potential difference ΔV between the plates after the slab has been introduced?

SOLUTION ■ The **Key Idea** here is to find ΔV by integrating along a straight-line path extending directly from the bottom plate to the top plate. Within the dielectric, the path length is b and the electric field is E_1. Within the two gaps above and below the dielectric, the total path length is $d - b$ and the electric field is E_0. Equation 28-6 then yields

$$\begin{aligned}\Delta V &= \int_-^+ |\vec{E}||d\vec{s}| = E_0(d-b) + E_1 b \\ &= (6900 \text{ V/m})(0.0124 \text{ m} - 0.00780 \text{ m}) \\ &\quad + (2640 \text{ V/m})(0.00780 \text{ m}) \\ &= 52.3 \text{ V}. \qquad \text{(Answer)}\end{aligned}$$

This is less than the original potential difference of 85.5 V.

(f) What is the capacitance with the slab in place?

SOLUTION ■ The **Key Idea** now is that the capacitance C is related to the free charge q and the potential difference ΔV via Eq. 28-1, just as when a dielectric is not in place. Taking q from (b) and ΔV from (e), we have

$$\begin{aligned}C &= \frac{|q|}{|\Delta V|} = \frac{7.02 \times 10^{-10} \text{ C}}{52.3 \ V} \\ &= 1.34 \times 10^{-11} \text{ F} = 13.4 \text{ pF}. \qquad \text{(Answer)}\end{aligned}$$

This is greater than the original capacitance of 8.21 pF.

28-9 *RC* Circuits

In preceding sections we dealt only with circuits in which the currents did not vary with time. Here we begin a discussion of time-varying currents.

Charging a Capacitor

The capacitor of capacitance C in Fig. 28-22 is initially uncharged. To charge it, we close switch S on point a. This completes an *RC series circuit* consisting of the capacitor, an ideal battery of emf ε, and a resistance R. Since an ideal battery has no internal resistance, its emf is the same as the potential difference across the battery, ΔV_B.

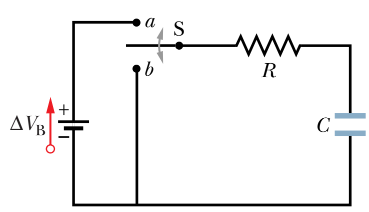


FIGURE 28-22 ■ When switch S is closed on a, the capacitor is *charged* through the resistor. When the switch is afterward closed on b, the capacitor *discharges* through the resistor.

From Section 28-2, we already know that as soon as the circuit is complete, charge begins to flow (current exists) between a capacitor plate and a battery terminal on each side of the capacitor. This current increases the amount of excess charge on the plates, q and the size of the potential difference $|\Delta V_C| = |q|/C$ across the capacitor. When that potential difference across the capacitor equals the potential difference across the battery (which here is equal to the emf of the battery, ΔV_B), the current is zero. From Eq. 28-1 ($|q| = C|\Delta V_C|$), the *equilibrium* (final) amount of excess *charge* on each plate of the fully charged capacitor is equal to $C|\Delta V_B|$.

Here we want to examine the charging process. In particular we want to know how the amount of excess charge $|q(t)|$ on each capacitor plate, the potential difference $\Delta V_C(t)$ across the capacitor, and the current $i(t)$ in the circuit vary with time during the charging process. We begin by applying the loop rule to the circuit, traversing it clockwise from the negative terminal of the battery. We find

$$\Delta V_B - iR - \frac{q}{C} = 0, \qquad (28\text{-}36)$$

where q represents the excess charge on the top plate of the capacitor, which is positive in this case.

The last term on the left side represents the potential difference across the capacitor. The term is negative because the capacitor's top plate, which is connected to the battery's positive terminal, is at a higher potential than the lower plate. Thus, there is a drop in potential as we move down through the capacitor.

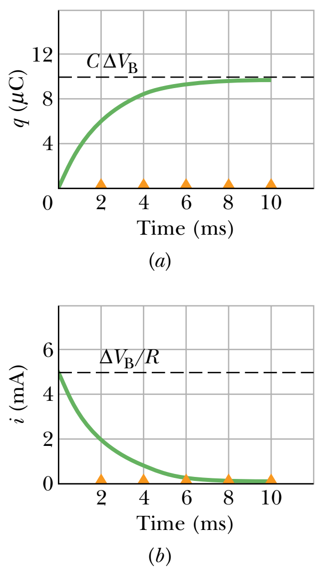


FIGURE 28-23 ■ (*a*) A plot of Eq. 28-39, which shows the buildup of excess charge on the capacitor plates of Fig. 28-22. (*b*) A plot of Eq. 28-40. The charging current in the circuit of Fig. 28-22 declines as the capacitor becomes more fully charged. The curves are plotted for $R = 2000\ \Omega$, $C = 1\,\mu$F, and $\Delta V_B = 10$ V. The small triangles represent successive intervals of one time constant τ.

We cannot immediately solve Eq. 28-36 because it contains two variables, i and q. However, those variables are not independent but are related by

$$i = \frac{dq}{dt}. \tag{28-37}$$

Substituting this for i and rearranging, we find

$$R\frac{dq}{dt} + \frac{q}{C} = \Delta V_B \qquad \text{(charging equation).} \tag{28-38}$$

This differential equation describes the time variation of the excess positive charge q on the top plate of the capacitor shown in Fig. 28-23. To solve it, we need to find the function $q(t)$ that satisfies this equation and also satisfies the condition the capacitor be initially uncharged: $q = 0$ C at $t = 0$ s.

The solution to Eq. 28-38 is

$$q = C\Delta V_B(1 - e^{-t/RC}) \qquad \text{(charging a capacitor).} \tag{28-39}$$

(Here e is the exponential base, 2.718 . . . , and not the elementary charge.) You can verify by substitution that Eq. 28-39 is indeed a solution to Eq. 28-38. We can see that this expression does indeed satisfy our required initial condition, because at $t = 0$ the term $e^{-t/RC}$ is unity, so the equation gives $q = 0$. Note also that as t goes to ∞ (that is, a long time later), the term $e^{-t/RC}$ goes to zero; so the equation gives the proper value for the full (equilibrium) excess charge on the positive plate of the capacitor—namely, $q = C\Delta V_B$. A plot of $q(t)$ for the charging process is given in Fig. 28-23*a*.

The derivative of $q(t)$ is the positive current $i(t)$ charging the capacitor:

$$i = \frac{dq}{dt} = \left(\frac{\Delta V_B}{R}\right)e^{-t/RC} \qquad \text{(charging a capacitor).} \tag{28-40}$$

A plot of $i(t)$ for the charging process is given in Fig. 28-23*b*. Note that the current has the initial value $\Delta V_B/R$ and it decreases to zero as the capacitor becomes fully charged.

> A capacitor being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

By combining $|q| = C|\Delta V_C|$ (Eq. 28-1) and $q = C\Delta V_B(1 - e^{-t/RC})$ (Eq. 28-39), we find the potential difference $\Delta V_C(t)$ across the capacitor during the charging process is

$$|\Delta V_C| = \frac{q}{C} = |\Delta V_B(1 - e^{-t/RC})| \qquad \text{(charging a capacitor).} \tag{28-41}$$

This tells us $\Delta V_C = 0$ at $t = 0$ and $\Delta V_C = \Delta V_B$ when the capacitor is fully charged as the time approaches infinity ($t \to \infty$).

The Time Constant

The product RC appearing in the equations above has the dimensions of time (both because the argument of an exponential must be dimensionless and because, in fact, $1.0\ \Omega \times 1.0\ \text{F} = 1.0$ s). RC is called the **capacitive time constant** of the circuit and is represented with the symbol τ.

$$\tau = RC \qquad \text{(time constant).} \tag{28-42}$$

From the expression for the excess charge as a function of time on one plate of a charging capacitor $q = C\Delta V_B(1 - e^{-t/RC})$ (Eq. 28-39), we can now see that at time $t = \tau\,(=RC)$, the excess charge on the top plate of the initially uncharged capacitor of Fig. 28-22 has increased from zero to

$$q = C\Delta V_B(1 - e^{-1}) = 0.63C\Delta V_B. \tag{28-43}$$

In words, after the first time constant, τ, the amount of excess charge has increased from zero to 63% of its final value, $C\Delta V_B$. In Fig. 28-22, the small triangles along the time axes mark successive intervals of one time constant during the charging of the capacitor. The charging times for RC circuits are often stated in terms of τ. The greater τ is, the greater is the charging time.

Discharging a Capacitor

Assume that now the capacitor of Fig. 28-22 is fully charged to a potential ΔV_0 equal to the potential difference, ΔV_B, of the battery. At a new time $t = 0$, switch S is thrown from a to b so the capacitor can *discharge* through resistance R. How do the excess charge $q(t)$ on the top plate of the capacitor and the current $i(t)$ through the discharge loop of capacitor and resistance now vary with time?

The differential equation describing $q(t)$ in this case is similar to the one we worked with for the case of charging Eq. 28-38, except now there is no battery in the discharge loop and so $\Delta V_B = 0$. Thus,

$$R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad \text{(discharging equation)}, \tag{28-44}$$

where the current term, dq/dt, and the voltage across the capacitor, q/C, can be positive or negative. The solution to this differential equation is

$$q = q_0 e^{-t/RC} \qquad \text{(discharging a capacitor)}, \tag{28-45}$$

where $|q_0|(=C|\Delta V_0|)$ is the initial amount of excess charge on the capacitor plates. You can verify by substitution that Eq. 28-45 is indeed a solution of Eq. 28-44.

Equation 28-45 tells us that the amount of excess charge on each capacitor plate decreases exponentially with time, at a rate set by the capacitive time constant $\tau = RC$. At time $t = \tau$, the capacitor's excess charge has been reduced to $|q_0|e^{-1}$, or about 37% of the initial value. That is, the amount of excess charge on the plates has decreased by 63%. Note that a greater τ means a greater discharge time.

Differentiating Eq. 28-45 gives us the current $i(t)$:

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC} \qquad \text{(discharging a capacitor)}. \tag{28-46}$$

This tells us the current also decreases exponentially with time, at a rate set by τ. The initial current i_0 is equal to q_0/RC. Note that you can find i_0 by simply applying the loop rule to the circuit at $t = 0$ the moment when the capacitor's initial potential ΔV_0 is connected across the resistance R. So the current must be

$$i_0 = \frac{\Delta V_0}{R} = \frac{(q_0/C)}{R} = \frac{q_0}{RC}.$$

The minus sign in the discharging capacitor expression (Eq. 28-46) can be ignored; it merely means the amount of excess charge on the plate is decreasing.

READING EXERCISE 28-5: The table gives four sets of values for the circuit elements in Fig. 28-22. Rank the sets according to (a) the initial current (as the switch is closed on *a*) and (b) the time required for the current to decrease to half its initial value, greatest first.

	1	2	3	4
ΔV_B (V)	12.0	12.0	10.0	10.0
R (Ω)	2.0	3.0	10.0	5.0
C (μF)	3.0	2.0	0.5	2.0

TOUCHSTONE EXAMPLE 28-5: Discharging a Capacitor

A capacitor of capacitance C is discharging through a resistor of resistance R.

(a) In terms of the time constant $\tau = RC$, when will the excess charge on each plate of the capacitor be half its initial value?

SOLUTION The **Key Idea** here is that the excess charge on each plate of the capacitor varies according to Eq. 28-45,

$$q = q_0 e^{-t/RC},$$

in which q_0 is the initial charge. We are asked to find the time t at which $q = \frac{1}{2}q_0$ or at which

$$\tfrac{1}{2}q_0 = q_0 e^{-t/RC}. \quad (28\text{-}47)$$

After canceling q_0, we realize that the time t we seek is "buried" inside an exponential function. To expose the symbol t in Eq. 28-47, we take the natural logarithms of both sides of the equation. (The natural logarithm is the inverse function of the exponential function.) We find

$$\ln\tfrac{1}{2} = \ln(e^{-t/RC}) = -\frac{t}{RC},$$

or

$$t = (-\ln\tfrac{1}{2})RC = 0.69RC = 0.69\tau. \quad \text{(Answer)}$$

(b) When will the energy stored in the capacitor be half its initial value?

SOLUTION There are two **Key Ideas** here. First, the energy U stored in a capacitor is related to the charge $|q|$ on the each plate according to Eq. 28-21 ($U = q^2/2C$). Second, that charge is decreasing according to Eq. 28-45. Combining these two ideas gives us

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2C}e^{-2t/RC} = U_0 e^{-2t/RC},$$

in which U_0 is the initial stored energy. We are asked to find the time at which $U = \frac{1}{2}U_0$, or at which

$$\tfrac{1}{2}U_0 = U_0 e^{-2t/RC}.$$

Canceling U_0 and taking the natural logarithms of both sides, we obtain

$$\ln\tfrac{1}{2} = -\frac{2t}{RC},$$

or

$$t = -RC\frac{\ln\frac{1}{2}}{2} = 0.35RC = 0.35\tau. \quad \text{(Answer)}$$

It takes longer (0.69τ versus 0.35τ) for the *charge* to fall to half its initial value than for the *stored energy* to fall to half its initial value. Does this result surprise you?

Problems

SEC. 28-2 ■ CAPACITANCE

1. Electrometer An electrometer is a device used to measure static charge—an unknown excess charge is placed on the plates of the meter's capacitor, and the potential difference is measured. What minimum charge can be measured by an electrometer with a capacitance of 50 pF and a voltage sensitivity of 0.15 V?

2. Two Metal Objects The two metal objects in Fig. 28-24 have net (or excess) charges of +70 pC and −70 pC, which result in a 20 V potential difference between them. (a) What is the capacitance of the system? (b) If the excess charges are changed to +200 pC and −200 pC, what does the capacitance become? (c) What does the potential difference become?

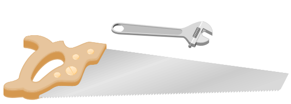


FIGURE 28-24 ■ Problem 2.

3. Initially Uncharged The capacitor in Fig. 28-25 has a capacitance of

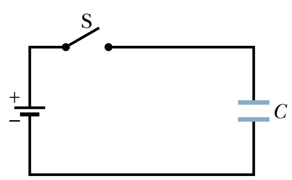


FIGURE 28-25 ■ Problem 3.

25 μF and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?

SEC. 28-3 ■ CALCULATING THE CAPACITANCE

4. Show That If we solve Eq. 28-9 for ε_0 we see that its SI unit is the farad per meter. Show that this unit is equivalent to that obtained earlier for ε_0—namely, the coulomb squared per newton-meter squared ($C^2/N\cdot m^2$).

5. Circular Plates A parallel-plate capacitor has circular plates of 8.2 cm radius and 1.3 mm separation. (a) Calculate the capacitance. (b) What excess charge will appear on each of the plates if a potential difference of 120 V is applied?

6. Two Flat Metal Plates You have two flat metal plates, each of area 1.00 m^2, with which to construct a parallel-plate capacitor. If the capacitance of the device is to be 1.00 F, what must be the separation between the plates? Could this capacitor actually be constructed?

7. Spherical Drop of Mercury A spherical drop of mercury of radius R has a capacitance given by $C = 4\pi\varepsilon_0 R$. If two such drops combine to form a single larger drop what is its capacitance?

8. Spherical Capacitor The plates of a spherical capacitor have radii 38.0 mm and 40.0 mm. (a) Calculate the capacitance. (b) What must be the plate area of a parallel-plate capacitor with the same plate separation and capacitance?

9. Two Spherical Shells Suppose that the two spherical shells of a spherical capacitor have approximately equal radii. Under these conditions the device approximates a parallel-plate capacitor with $b - a = d$. Show that Eq. 28-17 does indeed reduce to Eq. 28-9 in this case.

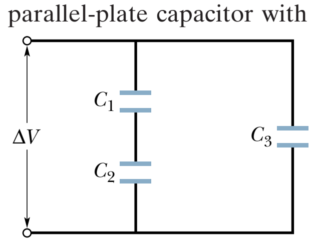


FIGURE 28-26 ■ Problems 10 and 30.

SEC. 28-4 ■ CAPACITORS IN PARALLEL AND IN SERIES

10. Equivalent In Fig. 28-26, find the equivalent capacitance of the combination. Assume that $C_1 = 10.0$ μF, $C_2 = 5.00$ μF, and $C_3 = 4.00$ μF.

11. How Many How many 1.00 μF capacitors must be connected in parallel to store an excess charge of 1.00 C with a potential of 110 V across the capacitors?

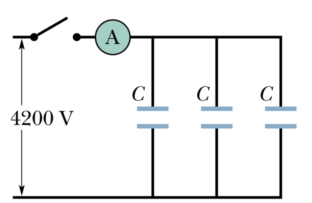


FIGURE 28-27 ■ Problem 12.

12. Each Uncharged Each of the uncharged capacitors in Fig. 28-27 has a capacitance of 25.0 μF. A potential difference of 4200 V is established when the switch is closed. How many coulombs of charge then pass through meter A?

13. Combo In Fig. 28-28 find the equivalent capacitance of the combination. Assume that $C_1 = 10.0$ μF, $C_2 = 5.00$ μF, and $C_3 = 4.00$ μF.

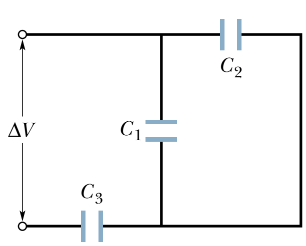


FIGURE 28-28 ■ Problems 13, 14, and 28.

14. Breaks Down In Fig. 28-28 suppose that capacitor 3 breaks down electrically, becoming equivalent to a conducting path. What *changes* in (a) the amount of excess charge and (b) the potential difference occur for capacitor 1? Assume that $\Delta V = 100$ V.

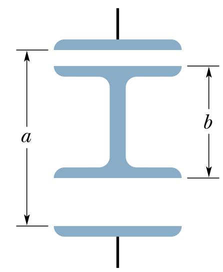


FIGURE 28-29 ■ Problem 15.

15. Two in Series Figure 28-29 shows two capacitors in series; the center section of length b is movable vertically. Show that the equivalent capacitance of this series combination is independent of the position of the center section and is given by $C = \varepsilon_0 A/(a - b)$, where A is the plate area.

16. Battery Potential In Fig. 28-30, the battery has a potential difference of 10 V and the five capacitors each have a capacitance of 10 μF. What is the excess charge on (a) capacitor 1 and (b) capacitor 2?

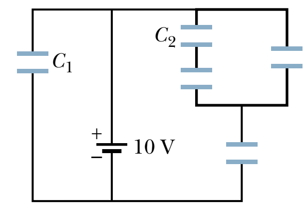


FIGURE 28-30 ■ Problem 16.

17. Parallel with Second 100 pF capacitor is charged to a potential difference of 50 V, and the charging battery is disconnected. The capacitor is then connected in parallel with a second (initially uncharged) capacitor. If the potential difference across the first capacitor drops to 35 V, what is the capacitance of this second capacitor?

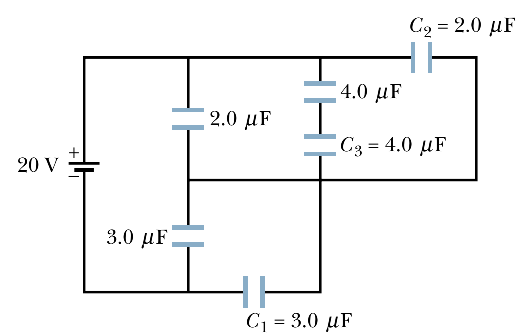


FIGURE 28-31 ■ Problem 18.

18. Charge Stored In Fig. 28-31, the battery has a potential difference of 20 V. Find (a) the equivalent capacitance of all the capacitors and (b) the excess charge stored by that equivalent capacitance. Find the potential across and charge on (c) capacitor 1, (d) capacitor 2, and (e) capacitor 3.

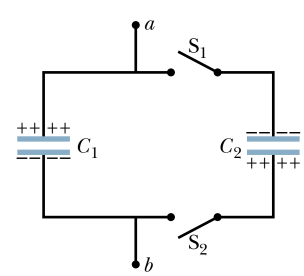


FIGURE 28-32 ■ Problem 19.

19. Opposite Polarity In Fig. 28-32, the capacitances are $C_1 = 1.0$ μF and $C_2 = 3.0$ μF and both capacitors are charged to a potential difference of $\Delta V = 100$ V but with opposite polarity as shown. Switches S_1, and S_2 are now closed. (a) What is now the potential difference between points a and b? What are now the amounts of excess charge on capacitors (b) 1 and (c) 2?

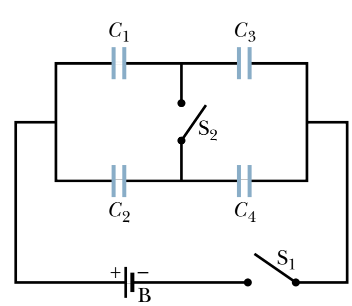


FIGURE 28-33 ■ Problem 20.

20. Battery Supplies In Fig. 28-33, battery B supplies 12 V. Find the excess charge on each capacitor (a) first when only switch S_1

switch S_2 is also closed. Take $C_1 = 1.0$ μF, $C_2 = 2.0$ μF, $C_3 = 3.0$ μF, and $C_4 = 4.0$ μF.

21. Switch Is Thrown When switch S is thrown to the left in Fig. 28-34, the plates of capacitor 1 acquire a potential difference ΔV_0. Capacitors 2 and 3 are initially uncharged. The switch is now thrown to the right. What are the final amounts of excess charge $|q_1|$, $|q_2|$, and $|q_3|$ on the capacitors?

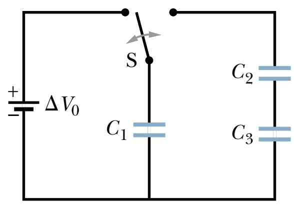


FIGURE 28-34 ■ Problem 21.

SEC. 28-5 ■ ENERGY STORED IN AN ELECTRIC FIELD

22. Air How much energy is stored in one cubic meter of air due to the "fair weather" electric field of magnitude 150 V/m?

23. Capacitance Required What capacitance is required to store an energy of 10 kW · h at a potential difference of 1000 V?

24. Air-Filled Capacitor A parallel-plate air-filled capacitor having area 40 cm^2 and plate spacing 1.0 mm is charged to a potential difference of 600 V. Find (a) the capacitance, (b) the amount of excess charge on each plate, (c) the stored energy, (d) the electric field between the plates, and (e) the energy density between the plates.

25. Two Capacitors Two capacitors, of 2.0 and 4.0 μF capacitance, are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

26. Connected Bank A parallel-connected bank of 5.00 μF capacitors is used to store electric energy. What does it cost to charge the 2000 capacitors of the bank to 50,000 V assuming 12.0¢/kW · h?

27. One Capacitor One capacitor is charged until its stored energy is 4.0 J. A second uncharged capacitor is then connected to it in parallel. (a) If the charge distributes equally, what is now the total energy stored in the electric fields? (b) Where did the excess energy go?

28. Find In Fig. 28-28 find (a) the excess charge, (b) the potential difference, and (c) the stored energy for each capacitor. Assume the numerical values of Problem 13, with $\Delta V = 100$ V.

29. Plates of Area A A parallel-plate capacitor has plates of area A and separation d and is charged to a potential difference ΔV. The charging battery is then disconnected, and the plates are pulled apart until their separation is $2d$. Derive expressions in terms of A, d, and ΔV for (a) the new potential difference; (b) the initial and final stored energies, U_i and U_f and (c) the work required to separate the plates.

30. Find the Charge In Fig. 28-26, find (a) the excess charge, (b) the potential difference, and (c) the stored energy for each capacitor. Assume the numerical values of Problem 10, with $\Delta V = 100$ V.

31. Cylindrical Capacitor A cylindrical capacitor has radii a and b as in Fig. 28-10. Show that half the stored electric potential energy lies within a cylinder whose radius is $r = \sqrt{ab}$.

32. Metal Sphere A charged isolated metal sphere of diameter 10 cm has a potential of 8000 V relative to $V = 0$ at infinity. Calculate the energy density in the electric field near the surface of the sphere.

33. Force of Magnitude (a) Show that the plates of a parallel-plate capacitor attract each other with a force of magnitude given by $F = q^2/2\varepsilon_0 A$. Do so by calculating the work needed to increase the plate separation from x to $x + dx$, with the excess charge $|q|$ remaining constant. (b) Next show that the magnitude of the force per unit area (the *electrostatic stress*) acting on either capacitor plate is given by $\frac{1}{2}\varepsilon_0 E^2$. (Actually, this is the force per unit area on *any* conductor of *any* shape with an electric field $\vec{E}$ at its surface.)

SEC. 28-6 ■ CAPACITOR WITH A DIELECTRIC

34. Wax An air-filled parallel-plate capacitor has a capacitance of 1.3 pF. The separation of the plates is doubled and wax is inserted between them. The new capacitance is 2.6 pF. Find the dielectric constant of the wax.

35. Convert It Given a 7.4 pF air-filled capacitor, you are asked to convert it to a capacitor that can store up to 7.4 μJ with a maximum potential difference of 652 V. What dielectric in Table 28-3 should you use to fill the gap in the air capacitor if you do not allow for a margin of error?

36. Separation A parallel-plate air-filled capacitor has a capacitance of 50 pF. (a) If each of its plates has an area of 0.35 m^2, what is the separation? (b) If the region between the plates is now filled with material having $\kappa = 5.6$, what is the capacitance?

37. Coaxial Cable A coaxial cable used in a transmission line has an inner radius of 0.10 mm and an outer radius of 0.60 mm. Calculate the capacitance per meter for the cable. Assume that the space between the conductors is filled with polystyrene.

38. Construct a Capacitor You are asked to construct a capacitor having a capacitance near 1 nF and a breakdown potential in excess of 10 000 V. You think of using the sides of a tall Pyrex drinking glass as a dielectric, lining the inside and outside curved surfaces with aluminum foil to act as the plates. The glass is 15 cm tall with an inner radius of 3.6 cm and an outer radius of 3.8 cm. What are the (a) capacitance and (b) breakdown potential of this capacitor?

39. Certain Substance A certain substance has a dielectric constant of 2.8 and a dielectric strength of 18 MV/m. If it is used as the dielectric material in a parallel-plate capacitor, what minimum area should the plates of the capacitor have to obtain a capacitance of 7.0×10^{-2} μF and to ensure that the capacitor will be able to withstand a potential difference of 4.0 kV?

40. Two Dielectrics A parallel-plate capacitor of plate area A is filled with two dieletrics as in Fig. 28-35*a*. Show that the capacitance is

$$C = \frac{\varepsilon_0 A}{d}\,\frac{\kappa_1 + \kappa_2}{2}.$$

Check this formula for limiting cases. (*Hint:* Can you justify this arrangement as being two capacitors in parallel?)

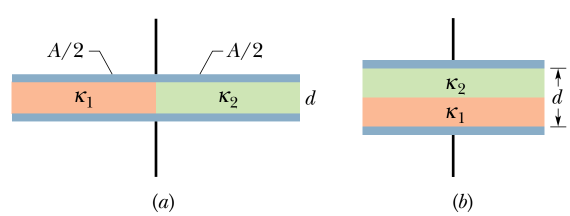


FIGURE 28-35 ■ Problems 40 and 41.

41. Limiting Cases A parallel-plate capacitor of plate area A is filled with two dielectrics as in Fig. 28-35*b*. Show that the capacitance is

$$C = \frac{2\varepsilon_0 A}{d}\frac{\kappa_1\kappa_2}{\kappa_1 + \kappa_2}.$$

Check this formula for limiting cases. (*Hint:* Can you justify this arrangement as being two capacitors in series?)

42. What is Capacitance What is the capacitance of the capacitor, of plate area A, shown in Fig. 28-36? (*Hint:* See Problems 40 and 41.)

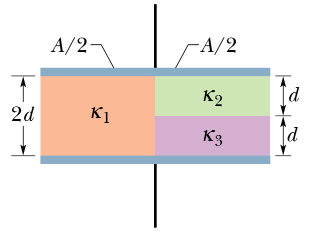


FIGURE 28-36 ■ Problem 42.

SEC. 28-8 ■ DIELECTRICS AND GAUSS' LAW

43. Mica A parallel-plate capacitor has a capacitance of 100 pF, a plate area of 100 cm^2, and a mica dielectric ($\kappa = 5.4$) completely filling the space between the plates. At 50 V potential difference, calculate (a) the electric field magnitude E in the mica, (b) the amount of excess free charge on each plate, and (c) the amount of induced surface charge on the mica.

44. Electric Field Two parallel plates of area 100 cm^2 are given excess charges of equal amounts 8.9×10^{-7} C but opposite signs. The electric field within the dielectric material filling the space between the plates is 1.4×10^6 V/m. (a) Calculate the dielectric constant of the material. (b) Determine the amount of bound charge induced on each dielectric surface.

45. Concentric Conducting Shells The space between two concentric conducting spherical shells of radii b and a (where $b > a$) is filled with a substance of dielectric constant κ. A potential difference ΔV exists between the inner and outer shells. Determine (a) the capacitance of the device, (b) the excess free charge q on the inner shell, and (c) the charge q' induced along the surface of the inner shell.

SEC. 28-9 ■ *RC* CIRCUITS

46. Initial Charge A capacitor with initial excess charge of amount $|q_0|$ is discharged through a resistor. In terms of the time constant τ, how long is required for the capacitor to lose (a) the first one-third of its charge and (b) two-thirds of its charge?

47. How Many Time Constants How many time constants must elapse for an initially uncharged capacitor in an RC series circuit to be charged to 99.0% of its equilibrium charge?

48. Leaky Capacitor The potential difference between the plates of a leaky (meaning that charges leak directly across the "insulated" space between the plates) 2.0 μF capacitor drops to one-fourth its initial value in 2.0 s. What is the equivalent resistance between the capacitor plates?

49. Time Constant A 15.0 kΩ resistor and a capacitor are connected in series and then a 12.0 V potential difference is suddenly applied across them. The potential difference across the capacitor rises to 5.00 V in 1.30 μs. (a) Calculate the time constant of the circuit. (b) Find the capacitance of the capacitor.

50. Flashing Lamp Figure 28-37 shows the circuit of a flashing lamp, like those attached to barrels at highway construction sites. The fluorescent lamp L (of negligible capacitance) is connected in parallel across the capacitor C of an RC circuit. There is a current through the lamp only when the potential difference across it reaches the breakdown voltage V_L; in this event, the capacitor discharges completely through the lamp and the lamp flashes briefly. Suppose that two flashes per second are needed. For a lamp with breakdown voltage $\Delta V_L = 72.0$ V, wired to a 95.0 V ideal battery and a 0.150 μF capacitor, what should be the resistance R?

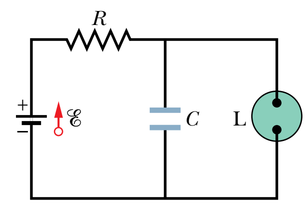


FIGURE 28-37 ■ Problem 50.

51. Initial Potential Difference A capacitor with an initial potential difference of 100 V is discharged through a resistor when a switch between them is closed at $t = 0$. At $t = 10.0$ s, the potential difference across the capacitor is 1.00 V. (a) What is the time constant of the circuit? (b) What is the potential difference across the capacitor at $t = 17.0$ s?

52. Electronic Arcade Game A controller on an electronics arcade games consists of a variable resistor connected across the plates of a 0.220 μF capacitor. The capacitor is charged to 5.00 V, then discharged through the resistor. The time for the potential difference across the plates to decrease to 0.800 V is measured by a clock inside the game. If the range of discharge times that can be handled effectively is from 10.0 μs to 6.00 ms, what should be the resistance range of the resistor?

53. Initial Stored Energy A 1.0 μF capacitor with an initial stored energy of 0.50 J is discharged through a 1.0 MΩ resistor. (a) What is the initial amount of excess charge on the capacitor plates? (b) What is the current through the resistor when the discharge starts? (c) Determine ΔV_C, the potential difference across the capacitor, and ΔV_R, the potential difference across the resistor, as functions of time. (d) Express the production rate of thermal energy in the resistor as a function of time.

Additional Problems

54. Capacitance (a) What is the physical definition and description of a capacitor? (b) What is the mathematical definition of capacitance? (c) Based on the physical description of a capacitor, why would you expect it to hold more excess charge on each of its conducting surfaces when the voltage difference between the two pieces of conductor increases?

55. Net Charge What is the net charge on a capacitor in a circuit? Is it ever possible for the amount of excess charge on one conductor to be different from the amount of excess charge on the other conductor? Explain.

56. Attraction and Repulsion Consider the attraction and repulsion of different types of charge. (a) Explain why you expect to find that the amount of excess charge a battery can pump onto a parallel-plate capacitor will double if the area of each plate doubles. (b) Explain why you expect to find that the amount of excess charge a battery can pump onto a parallel-plate capacitor will be cut in half if the distance between each plate doubles.

57. Three Parallel-Plate Capacitors Suppose you have three parallel-plate capacitors as follows:

Capacitor 1: Area A, spacing d
Capacitor 2: Area A, spacing $2d$
Capacitor 3: Area $2A$, spacing d

The three graph lines (labeled a, b, and c) in Fig. 28-38 represent data for the amounts of excess of charge on the plates of each capacitor as a function of the potential difference across it. Which capacitor {1, 2, or 3} belongs to which line {a, b, and c}? Explain your reasoning carefully.

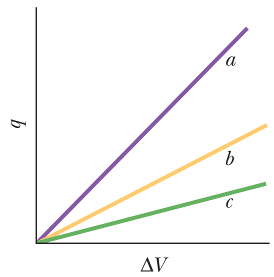


FIGURE 28-38 ■ Problem 57.

58. Capacitors in Series Give as clear an explanation as possible as to why it is physically reasonable to expect that two identical parallel-plate capacitors that are placed in series ought to have half the capacitance as one capacitor. *Hints:* What happens to the effective spacing between the first plate of capacitor 1 and the second plate of capacitor 2 when they are wired in series? What does the fact that like charges repel and opposites attract have to do with anything?

59. Capacitors in Parallel Give as clear an explanation as possible as to why it is physically reasonable to expect that two identical parallel-plate capacitors placed in parallel ought to have twice the capacitance as one capacitor. *Hints:* What happens to the effective area of capacitors wired in parallel? What does the fact that like charges repel and opposites attract have to do with anything?

60. *Charge Ratios on Capacitors* (Adapted from a TYC WS Project ranking task by D. Takahashi). Eight capacitor circuits are shown in Fig. 28-39. All of the capacitors are identical and all are fully charged. The batteries are also identical. In each circuit, one capacitor is labeled C_1 and another is labeled C_2. Assuming $|q_1|$ denotes the amount of excess charge on C_1, $|q_2|$ denotes the amount of excess charge on C_2, and the value of the ratio is denoted $|q_1/q_2|$, rank the circuit in which the value of the ratio $|q_1/q_2|$ is largest *first,* and rank the circuit in which the value of the ratio is the smallest *last*. If two or more circuits result in identical values for the ratio, give these circuits equal ranking. Express your ranking symbolically. (For example, suppose the ratio was highest for D and G and lowest for A and E with the in-between ratios being equal, then the symbolic ranking would be

$$\mathrm{D = G > B = C = H = F > A = E}$$

(*Beware:* This is only a sample, not a correct answer!)

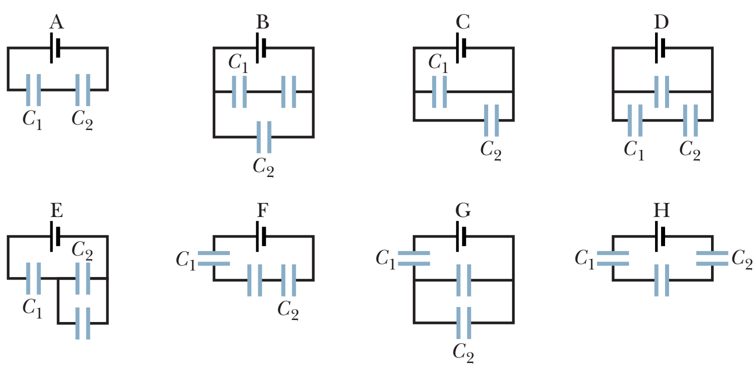


FIGURE 28-39 ■ Problem 60.

61. Physicists Claim Physicists claim that charge <u>never</u> flows *through* an ideal capacitor. Yet when an uncharged capacitor is first placed in series with a resistor and a battery, current flows through the battery and the resistor. Explain how this is possible.

62. Voltage Graphs Figure 28-40 shows plots of voltage across the capacitor as a function of time for three different capacitors that have each been separately discharged through the same resistor. Rank the plots according to the capacitances, the greatest first. Explain the reasons for your rankings.

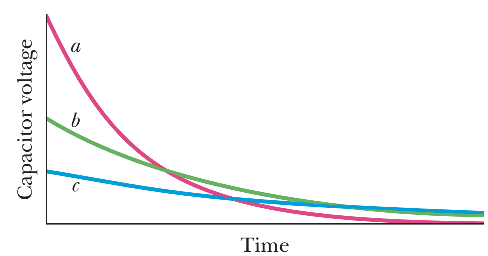


FIGURE 28-40 ■ Problem 62.

63. A Cell Membrane The inner and outer surfaces of a cell membrane carry excess negative and positive charge, respectively. Because of these charges, a potential difference of about 70 mV exists across the membrane. The thickness of the membrane is 8 nm.

(a) If the membrane were empty (filled with air), what would be the magnitude of the electric field inside the membrane?
(b) If the dielectric constant of the membrane were $\kappa = 3$ what would the field be inside the membrane?
(c) Cells can carry ions across a membrane *against the field* ("uphill") using a variety of active transport mechanisms. One mechanism does so by using up some of the cell's stored energy converting ATP to ADP. How much work does it take to carry one sodium ion (charge $= +e$) across the membrane against the field? Calculate your answer in eV, joules, and kcal/mole (the last for 1 mole of sodium ions).