Archaic cuneiform numbers

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1 Summary

2 Background

[TODO(egg): Restructure this. The internal references are all garbled.]

The Unicode Standard includes some cuneiform numbers: \[\frac{1}{4} \] 1–9(diš) and \[\frac{1}{4} \] 1–9(aš), \[\frac{1}{4} \] 1–5(u), \[\frac{1}{4} \] 1–9(neš₂), \[\frac{1}{4} \] 1–5(neš'u), etc., used in the Sumero-Akkadian Cuneiform script (ISO 15924: Xsux, Script property value long name: Cuneiform).

In the investigation that led to their encoding in Unicode Version 5.0, it was thought appropriate to unify these with the earlier curviform numerals -100 1-9 (100 10

In addition, these numerals will be needed for the representation of protocuneiform texts from the earlier archaic period. The non-numeric signs of protocuneiform (ISO 15924: Pcun) will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

The use of the curviform numeric signs is however understood, as we will discuss in Section 3; further, the conventions used for archaic numerals are also used when discussing ED numerals, see Section 7. As a result, the same numerals can be used when encoding archaic and ED texts, and in order to avoid issues ambiguities in representation when converting from transliteration, these should be unified. The overall picture of unifications and disunifications would be as follows:

	Uruk III & earlier	ED – Ur III	OB & later
Non-numeric signs	Future Pcun	Existing Xsux	
Numbers	This proposal	This proposal	Existing Xsux
		+ Existing Xsux	

3 Metrologies

```
时 約冊時 足口
时 上 4 日间 民 下 円垣
时 丁 角河 民 译 4 日勺垣
I want to write tablets: the tablet of 1 gur of barley to 600 gur; the tablet of 1 shekel of silver to 10 minas [...]
```

Edubba'a D

In order to explain why TODO:*n* more numerals are needed, it is useful to first recall why we have so many kinds of cuneiform numerals already.

As is well known¹ a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millenium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Robo8, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters \[\]_\frac{\frac{1}{1}}{1}, the multiples of ten (10–50) are \(-\frac{\frac{1}{1}}{1}, \) but the other digits 11–59 are sequences \(\[\] -\frac{\frac{1}{1}}{1}, \) in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a non-positional system, hereafter the cuneiform discrete counting system $S_{Ur\ III/OB}$, which had different signs for the units \[\] -\frac{\frac{1}{1}}{1}, \text{ tens} \(-\frac{1}{1}, \) sixties \[\] -\frac{\frac{1}{1}}{1} \] (with larger wedges than the units), six hundreds \(\] -\frac{1}{1}, three thousand six hundreds \(\) -\frac{1}{1}, and thirty-six thousands \(\Sigma - \frac{1}{1} \).

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram², where the num-

¹See, e.g., [Uni16, Section 22.3.3 "Non-Decimal Radix Systems", sub "Cuneiform Numerals"].

²These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

ber over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Frio7, p. 378; Rob19]:

$$\Leftrightarrow \stackrel{10}{\longleftrightarrow} \diamondsuit \stackrel{6}{\longleftrightarrow} \stackrel{1}{\longleftrightarrow} \stackrel{10}{\longleftrightarrow} \stackrel{3}{\longleftrightarrow} \stackrel{6}{\longleftrightarrow} \stackrel{6}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{1}{\longleftrightarrow} \stackrel{1}{\longleftrightarrow$$

Note that for the range of areas given above³, this system does not use any symbols separate from the numerals for the individual units ($ub\hat{u}m$, $ik\hat{u}m$, eblum, and $b\bar{u}rum$). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign \blacksquare functioning as punctuation, but the numerals are tied to the metrology; thus a surface of 5 $b\bar{u}r\bar{u}$ 1 eblum 4 $ik\hat{u}$ (100 $ik\hat{u}$, 36 ha) would be written⁴ $\ll \neg \Leftarrow \blacksquare$. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in "88 acres 3 roods 33 perches". Note also that the same signs are shared between multiple systems, with different relations; the ŠAR₂ sign \diamondsuit is equal to sixty times the U sign \lt in the area system, but to three hundred and sixty times \lt in the discrete counting system.

Another such system of note is the one for capacities⁵ [Frio7, p. 376; Rob19],

where the numerals for ban_2 are +, \neq , $\not\equiv$, and $\not\equiv$, and those for bariga are $\not\parallel$, $\not\parallel$, and $\not\parallel$ (contrast ordinary $\not\parallel$ and $\mid\parallel$ otherwise used with $\not\parallel$ -numerals). As described in [Hue11, p.585 with notes (b) and (f)], the sign GUR $\not\equiv$ \mid , while it is used

³For areas smaller than a quarter $ik\hat{u}m$, an overt unit is used, with 1 $m\bar{u}$ §arum (36 m²) written [\$\frac{1}{2}\], equal to one hundredth of an $ik\hat{u}m$, then sexigesimally subdivided in 60 $\frac{1}{2}$ (shekels). For areas greater than 3600 $b\bar{u}r\bar{u}$, the ♦- and ♦-numerals are reused with a suffix $\frac{1}{2}$ (gal, Sumerian: big), as follows [Robo8, p.295 with notes b and c; Frio7, p. 378; Rob19]:

⁴As in the surface of the field of **|| ← → →** (Apisal) reported on P102305 r. 1.

 $^{^5}$ Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{Ur\ III/OB}$ based on a height of one cubit, see[Pow87, p. 488; Robo8, p. 294; Rob19].

only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign ှ≯ if present, and after the word for "grain" if present, as in

$$W W \Leftrightarrow \sharp W \Rightarrow \sharp ^6$$
 3554 gur 3 ban₂ 6 sila₃ of grain.

Observe that while large numbers of gur follow⁷ system $S_{Ur\;III/OB}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as \checkmark would be 10 gur 1 bariga, and \checkmark \rightrightarrows would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system. To quote [Robo8, p. 78]: "The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity."

This intertwining of units and numerals explains the large number of alreadyencoded numeral series:

- I-I used in $S_{Ur\,III/OB}$ and the SPVS as well as with overt units;
- \leftarrow wsed in $G_{Ur \, III/OB}$, of which \leftarrow are also used in $S_{Ur \, III/OB}$ and the SPVS as well as with overt units;
- I-W used in $S_{Ur,III/OB}$ and the SPVS;
- \leftarrow used in $C_{Ur III/OB}$ as well as in the weight system;
- 十, 丰, ឝ, ឝ, 戡 used in C_{Ur III/OB};
- I, II, II used in $C_{Ur \, III/OB}$ —note the overlap with I-III;
- \prec and \bowtie used in $G_{Ur III/OB}$.

4 Arguments for curviform-cuneiform unification

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble in texts from Early Dynastic IIIa Šuruppag as in the character code charts, if later in the third millenium⁸, in Old Babylonian cursive, in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists⁹ and dictionaries¹⁰, and of composite texts¹¹. By being compatible with similarly diachronic transliteration practice (that is, by avoiding distinctions finer than those made in transliteration), the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as

⁶From P309594.

⁷A larger unit, the guru₇ (*karûm*, grain heap), is sometimes used instead, with **一旦冷無**<=**◇** 土! (1 *karûm* = 3600 kurrū). See [Frio7, p. 415; Rob19].

⁸Merging with U+1224E **☆**₩ NI₂.

⁹Notably the online edition of [Ryk10] in [Jim+23, Signs], as well as [VT+14].

¹⁰Notably the online edition of [Sch10] in [Jim+23, Dictionary], as well as [TJV17].

¹¹For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of W ⇒ ★ □ → , as well as Old Babylonian copies in both archaizing and cursive styles. Some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

[Rom24; JJ24]¹². The diachronic approach is also useful for pedagogical applications¹³.

In this context, the argument was made in [Ando4] as part of ongoing work on the cuneiform encoding¹⁴ that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters; some co-occurrence was known and acknowledged, but considered to be styling rather than plain text.

Indeed, some metrological systems from the Early Dynastic period match the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{\text{Ur III/OB}}$ [Frio7, p. 374; DE87, pp. 127, 165]:

$$\bullet \xleftarrow{10} \bullet \xleftarrow{6} \stackrel{10}{\longleftarrow} \xrightarrow{10} \stackrel{10}{\longleftarrow} \xleftarrow{6} \cdot \xleftarrow{10} \stackrel{10}{\longleftarrow} \qquad (S)$$

Likewise the area system used in the Early Dynastic IIIb period mirrors system $G_{\text{IIr III/OB}}$ [Frio7, p. 378; **Gombert2016**]:

$$\bullet \xleftarrow{10} \bullet \xleftarrow{6} \overset{10}{\Leftarrow} \xleftarrow{10} \bullet \xleftarrow{3} \bullet \xleftarrow{6} \triangleright, \qquad (G_{\text{ED IIIb}})$$

TODO(egg): words

Note that in [Rom24] [TODO(egg): Cite the GitHub repository], as in many other such analyses, numbers are removed as an early step in processing; these therefore would not benefit from diachrony in the encoding of numeric expressions.

¹²Attendees may recall the summary given on the third day of UTC #180, as recorded in [Con24]. Other readers may refer to [Svä+24, pp. 242, 148].

¹³For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Capo2].

5 Problems with unification: Early metrology

6 Problems with unification: Non-numeric usage

打害人名诺马 人名英马 阿克萨尼 订发 呱卜写度试 疑令问 使发生打罪 法客门证证 不过 不过过 罪某 三年 无过过 光过 加度分 那麼了 那么

The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'. Do you know its reading?

Fyamensteyt A

- 6.1 The case of ŠAR₂
- 7 Compatibility with transliteration
- 8 The necessity of ED-Uruk numeral identification
- 9 Characters not included in this proposal
- 9.1 Missing numerals

 $(N_{17}, 12N_{14}, \text{ etc.})$

9.2 Stacking patterns

(... are a mess, vary within Uruk, and are not transliterated/documented by Englund, so let's not go there for now.)

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