Archaic cuneiform numbers

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The Unicode Standard includes some cuneiform numbers: 𒁹–𒐎 1–9(diš) and 𒀸–𒐇 1–9(aš), 𒌋–𒐐 1–5(u), 𒐕–𒐝 1–9(ŋeš$_2$), 𒐞–𒐢 1–5(ŋeš’u), etc., used in the Sumero-Akkadian Cuneiform script (ISO 15924: Xsux, Script property value long name: Cuneiform).

In the investigation that led to their encoding in Unicode Version 5.0, it was thought appropriate to unify these with the earlier curviform numerals – 1–9(aš^c = N_1), – 1–5(u^c = N_{14}), – 1–9(ŋeš$_2$^{c} = N_{34}), – 1–5(ŋeš’u^c = N_{48}), etc., see L2/04-099. While the curviform numerals sometimes co-occur with the cuneiform ones, this was analysed as a stylistic distinction which should not be encoded in plain text. It has now become apparent that a distinction needs to be made for the adequate representation of Early Dynastic (ED) texts and scholarship pertaining to them.

In addition, these numerals will be needed for the representation of proto-cuneiform texts from the earlier archaic period. The non-numeric signs of proto-cuneiform (ISO 15924: Pcun) will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

The use of the curviform numeric signs is however understood, as we will discuss in Section 1; further, the conventions used for archaic numerals are also used when discussing ED numerals, see Section 5. As a result, the same numerals can be used when encoding archaic and ED texts, and in order to avoid issues ambiguities in representation when converting from transliteration, these should be unified. The overall picture of unifications and disunifications would be as follows:

	Uruk III & earlier	ED – Ur III	OB & later
Non-numeric signs	Future Pcun	Existing Xsux	
Numbers	This proposal	This proposal + Existing Xsux	Existing Xsux

1 Metrologies

𒁾 𒊬𒊏𒁉 𒁾
𒁾 𒀸 𒃷 𒊺𒋫 𒐚 𒃷𒂠
𒁾 𒁹 𒂆𒆬 𒋫 𒌋 𒈠𒈾𒂠
I want to write tablets: the tablet of 1 gur *of barley to 600* gur*; the tablet of 1 shekel of silver to 10 minas [...]*

Edubba'a D

In order to explain why TODO:n more numerals are needed, it is useful to first recall why we have so many kinds of cuneiform numerals already.

As is well known[1] a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millenium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters 𒁹–𒐎, the multiples of ten (10–50) are 𒌋–𒐐, but the other digits 11–59 are sequences 𒌋𒁹–𒐐𒐎; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $\mathrm{S}_{\text{Ur III/OB}}$, which had different signs for the units 𒁹–𒐎, tens 𒌋–𒐐, sixties 𒐕–𒐞 (with larger wedges than the units), six hundreds 𒐟–𒐤, three thousand six hundreds 𒊹–𒐫, and thirty-six thousands 𒐬–𒐲.

The relations between the values of the signs in the cuneiform discrete counting system may be summarized as follows, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\text{𒐬} \xleftarrow{10} \text{𒊹} \xleftarrow{6} \text{𒐟} \xleftarrow{10} \text{𒐕} \xleftarrow{6} \text{𒌋} \xleftarrow{10} \text{𒁹} \qquad (\mathrm{S}_{\text{Ur III/OB}})$$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written 𒐠𒐝𒐏𒐎 in the discrete counting system, and 𒎙𒐍𒐏𒐎 in the sexagesimal place value system.

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent:

$$\text{𒐬} \xleftarrow{10} \text{𒊹} \xleftarrow{6} \text{𒐗} \xleftarrow{10} \underset{\substack{1\ \text{bur}_3 \\ 1\ \textit{būrum} \\ 6{,}48\ \text{ha}}}{\text{𒌋}} \xleftarrow{3} \underset{\substack{1\ \text{eše}_3 \\ 1\ \textit{eblum} \\ 2{,}16\ \text{ha}}}{\text{𒑘}} \xleftarrow{6} \underset{\substack{1\ \text{iku} \\ 1\ \textit{ikûm} \\ 3600\ \text{m}^2}}{\text{𒀸}} \xleftarrow{2} \underset{\substack{1\ \textit{ubûm} \\ 1800\ \text{m}^2}}{\text{𒑚}} \xleftarrow{2} \text{𒑛} \qquad (\mathrm{G}_{\text{Ur III/OB}})$$

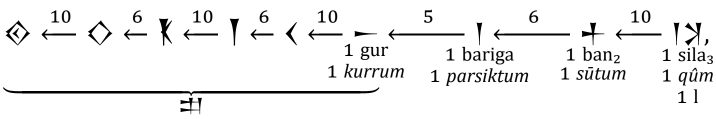
Note that for the range of areas given above[2], this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and

[1] See, *e.g.*, *The Unicode Standard*, Version 16.0, Section 22.3.3 *Non-Decimal Radix Systems*, *sub* "Cuneiform Numerals".

[2] For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mūšarum* ($36\ \text{m}^2$) written 𒁹𒊬, equal to one hundredth of an *ikûm*, then sexigesimally subdivided in 60 𒂆 (shekels). For areas greater

būrum). The whole numeric expression for the area would be followed by the sign 𒃷 functioning as punctuation, but the numerals are tied to the metrology; thus a surface of 5 *būrū* 1 *eblum* 4 *ikû* (100 *ikû*, 36 ha) would be written[3] 𒐼𒑘𒐂𒃷. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in “88 acres 3 roods 33 perches”. Note also that the same signs are shared between multiple systems, with different relations; the ŠAR$_2$ sign 𒊹 is equal to sixty times the U sign 𒌋 in the area system, but to three hundred and sixty times 𒌋 in the discrete counting system.

Another such systems of note is the one for volumes,



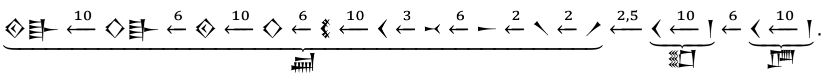
(C)

where the numerals for ban$_2$ are 𒑏, 𒑐, 𒑑, 𒑓, and 𒑕, and those for bariga are 𒐕, 𒑖, 𒑗, and 𒑘 (contrast ordinary 𒐖 and 𒐗 otherwise used with 𒁹-numerals). Note that while it is used only with volumes in excess of one gur, the sign GUR 𒄥 is written after the whole expression, after the overt unit sign 𒋡 if present, and after the word for “grain” if present, as in 𒐼𒐋𒌋𒐀𒑑𒐊𒋡𒊺𒄥 (3554 gur 3 ban$_2$ 6 sila$_3$ of grain[4]). Observe that while large number of gur follow[5] system S$_{\text{Ur III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates with the vertical bariga, as 𒌋𒁹𒄥 would be 10 gur 1 bariga, and 𒌋𒀸𒄥 would be 11 gur; again even with some overt units, most of the numerals are tied to the metrology.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

— 𒁹–𒑆 used in S$_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
— 𒌋–𒐏 used in G$_{\text{Ur III/OB}}$, of which 𒌋–𒐏 are also used in S$_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
— 𒁹–𒐎 used in S$_{\text{Ur III/OB}}$ and the SPVS;
— 𒀸–𒐃 used in C as well as in the weight system;
— 𒑏, 𒑐, 𒑑, 𒑓, 𒑕 used in TODO;
— 𒐕, 𒑖, 𒑗, 𒑘 used in C—note the overlap with 𒁹–𒑆;
— 𒑘 and 𒑙 used in G$_{\text{Ur III/OB}}$.

than 3600 *būrū*, the 𒊹- and 𒐞-numerals are reused with a suffix 𒃲 (gal, Sumerian: big), as follows:



[3]As in the surface of the field of 𒐖𒋛𒁀𒋡𒊹 (Apisal) reported on P102305 r. 1.
[4]From P309594.
[5]A larger unit, the guru$_7$ (*karûm*, grain heap), is sometimes used instead, with 𒀸𒄥𒌋=𒊹𒄥 (1 *karûm* = 3600 kurrū).

2 The case for unification

3 Early metrology

4 Non-numeric usage

> [illegible]
> *The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'. Do you know its reading?*
>
> *Examenstext A*

4.1 The ŠAR$_2$ problem

5 Compatibility with transliteration

6 Characters not included in this proposal

6.1 Missing numerals

ED [illegible]

6.2 Stacking patterns