

# Archaic cuneiform numbers

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2024-09-06

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## 1 Summary

This document proposes encoding some numerals used in the fourth millenium (Uruk IV and Uruk III) and Early Dynastic period in conjunction with the Sumero-Akkadian cuneiform script<sup>1</sup> and the proto-cuneiform script<sup>2</sup>. The proposed characters are listed in section 2. Most of them were listed in [L2/23-190]; however, the present document provides a more detailed rationale for their encoding and additional information about their identity.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals<sup>3</sup> should however *not* be unified with the already-encoded cuneiform numerals<sup>4</sup>. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script\_Extensions property assignments in section 2.2 reflect the overlap. Many of these numerals are also used in proto-Elamite<sup>5</sup> texts, where they are treated as identical characters in scholarship on proto-Elamite, so that

<sup>1</sup>ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

<sup>2</sup>ISO 15924: Pcun, not yet encoded.

<sup>3</sup>Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: ◻, ◻ (small stylus), ◻ (large stylus). Some numerals are composed of multiple such impressions, e.g., ◻◻. The terms “curved”, “curviform”, and “round” can be found in the literature. We avoid the term “round” here as it has other meanings in the context of numbers. We use “curviform” in this document as, being the least common term, it is least likely to lead to confusion, and “CURVED” in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [inlineATF].

<sup>4</sup>Impressed into clay using a stylus with a trihedral end: ┐ (stylus held horizontally), ↑ (vertically), ↘ (diagonally) < (with the head of the stylus), ↑ (stylus pressed deeper, forming a larger wedge), ⚡ (combining ↑ and <), etc.

<sup>5</sup>ISO 15924: Pelm, not yet encoded.

they should be unified with the ones proposed in [L2/23-196]. However, in the interest of time, we do not provide a detailed rationale for this unification in this document, and we are not proposing that the numerals be given the corresponding Script\_Extensions property value for now. Neither do we propose encoding any numerals that are solely attested in proto-Elamite texts, or well-attested in proto-Elamite texts but insufficiently attested in Uruk.

[TODO(egg): Mention the other sections here too.]

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
	Existing Xsux		
Non-numeric signs	Future Pcun		

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

## 2 Proposed changes to the Standard

### 2.1 Summary of proposed characters

### 2.2 Properties





### 2.3 Character names list

### 2.4 Core specification text

## 3 Rationale for curviform–cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [Powell1972] “in the same fashion as we use black and red ink”; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

### 3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts,  later in the third millenium<sup>6</sup>,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.


















<sup>6</sup>Merging with U+1224E CUNEIFORM SIGN NI2.





This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists<sup>7</sup> and dictionaries<sup>8</sup>, and of composite texts<sup>9</sup>. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]<sup>10</sup>. The diachronic approach is also useful for pedagogic applications<sup>11</sup>.

### 3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding<sup>12</sup> that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.


























Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

						
1(ḡeš <sub>2</sub> )	1(u)	1/2(diš)	5(diš <i>tenû</i> )	gi	us <sub>2</sub>	sa <sub>2</sub>
	7.5 (ropes)		5	reed	side	equal
						
3(u)	6(diš <i>tenû</i> )	gi	saṇ	sa <sub>2</sub>		
3 (ropes)	6	reed	front	equal		
						
ašag-bi	1(bur <sub>3</sub> )	1(eše <sub>3</sub> )	1(iku)	1/2(iku)		
ašag=bi						
field=DEM <sup>15</sup>						

     
tug<sub>x</sub>(LAK483)-si-ga-kam  
tugsiga =ak =am -Ø  
ploughed=GEN=COP-3.SG.S

<sup>7</sup>Notably [OSL] and the online edition of [MZL] in [eBL, Signs].

<sup>8</sup>Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

<sup>9</sup>For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of                           

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as<sup>16</sup>

465 metres, equal lengths  
198 metres, equal widths  
this field is 9, 18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from  $\triangleright$  to  $\uparrow$  numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases<sup>17</sup>.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

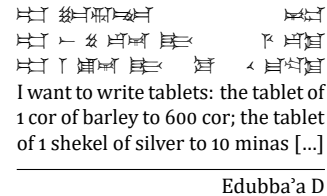
- the functions and use of the numerals vary beyond the mere  $\triangleright/\uparrow$  switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

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<sup>16</sup>We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

<sup>17</sup>The cuneiform encoding process was planned in *stages* in [L2/03-162]. One might expect the second stage of encoding, which led to the creation of the Early Dynastic Cuneiform block, to incorporate the numerals needed for the representation and discussion of Early Dynastic texts; however, the proposal [L2/12-208] stated that “numerals have been omitted due to the complexity of numeral signs from this period. An expert in the metrology of this period must be consulted before these can be properly included.”

### 3.3 A primer on classic Ur III and Old Babylonian metrologies





Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known<sup>16</sup> a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millenium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Robo8, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters 𐎶–𐎶𐎵, the multiples of ten (10–50) are 𐎶–𐎶𐎵, but the other digits 11–59 are sequences 𐎶–𐎶𐎵𐎶𐎵; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system*  $\mathcal{S}_{\text{Ur III/OB}}$ , which had different signs for the units 𐎶–𐎶𐎵, tens 𐎶–𐎶𐎵, sixties 𐎶–𐎶𐎵𐎶𐎵 (with larger wedges than the units), multiples of six hundred 𐎶–𐎶𐎵𐎶𐎵, multiples of three thousand six hundreds 𐎶–𐎶𐎵𐎶𐎵𐎶𐎵, and multiples of thirty-six thousand 𐎶–𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵.

### 3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram<sup>19</sup>, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \blacktriangledown \xleftarrow{10} \blacktriangledown \xleftarrow{6} \blacktriangledown \xleftarrow{10} \blacktriangledown \quad (S_{\text{Ur III/OB}})$$

For example, the number  $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$  would be written  in the discrete counting system, and  in the sexagesimal place value system.

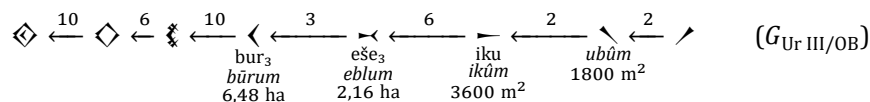
### 3.3.2 The area system
































The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and






<sup>18</sup>See, *e.g.*, [Uni16, §22.3.3, sub “Cuneiform Numerals”].

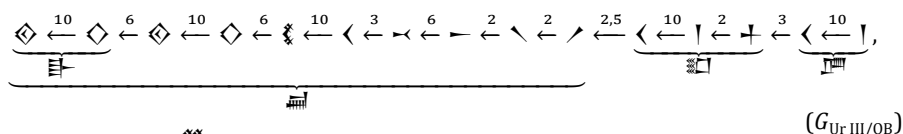
<sup>19</sup>These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*, see Figure 3.


the approximate metric equivalent [Fri07, p. 378; Rob19]:



Note that for the range of areas given above, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *bûrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign  functioning as punctuation<sup>20</sup>, but the numerals are tied to the metrology; thus a surface of 5 *bûr* 1 *ebel* 4 *ikû* (100 *ikû*, 36 ha) would be written<sup>21</sup>                                

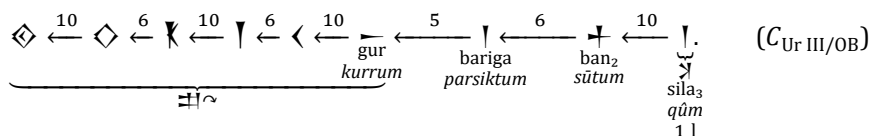
For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 sar = 1 *mûšarum*  $\approx 36 \text{ m}^2$  written , equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60  ( $\eta\eta_{4,}$ , *šiklû*, shekels). For areas greater than 3600 *bûr*, the  and  numerals are reused with a suffix  (*gal*, “big”), as follows [Rob08, p. 295 n. b and c; Fri07, p. 378; Rob19]:



e.g.,  for  $(2 \times 3600 + 20 \times 60 + 49) \text{ bur } 5 \text{ ikū } (5 + \frac{1}{2}) \text{ mūšar } 19 \text{ šiqil}$ . Factor diagrams in this document will use bottom curly brackets in this fashion to separate numerals from units and other suffixes.

### 3.3.3 The capacity system

Another such system of note is the one for capacities<sup>23</sup> [Fri07, p. 376; Rob19],



In the above diagram, the numerals for  $\text{ban}_2$  are 𐎶, 𐎶𐎵, 𐎶𐎵𐎶, 𐎶𐎵𐎶𐎵, and 𐎶𐎵𐎶𐎵𐎶, and those for  $\text{bariga}$  are 𐎶, 𐎶𐎵, 𐎶𐎵𐎶, and 𐎶𐎵𐎶𐎵 (contrast ordinary 𐎶𐎵 and 𐎶𐎵𐎶 otherwise used with 𐎶 numerals). Further, we have used the symbol  $\sim$  to express that, as described in [Hue11, p. 585 n. (b) and (f)], the sign 𐎶𐎵 GUR, while it is used only with volumes in


<sup>20</sup>This sign is sometimes interpreted as a measurement unit, and transliterated *iku*, see, *e.g.*, [Proust2020], or transliterations in [Feu04] discussed in section 3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system  $G_{\text{Ur III/OB}}$ .

<sup>21</sup>As in the surface of the field of   (the city of Apisal) reported on [P102305, r. 1]

<sup>22</sup>From [P213162], which has an additional , two thirds (of a shekel), see section 3.3.5

<sup>23</sup>Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system  $G_{Ur III/OB}$  based on a height of one cubit, see[[Pow87](#), p. 488; [Rob08](#), p. 294; [Rob19](#)].

excess of one gur, is written after the whole expression, after the overt unit sign  $\text{𒍪}$  if present, and after the word for “grain” if present, as in

  
 3554 gur    3 ban<sub>2</sub>    6    sila<sub>3</sub>    of grain.

Observe that while large numbers of gur follow<sup>25</sup> system  $S_{Ur\ III/OB}$ , the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as  $\text{𒍪} \text{𒍪}$  would be 10 gur 1 bariga, and  $\text{𒍪} \text{𒍪}$  would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

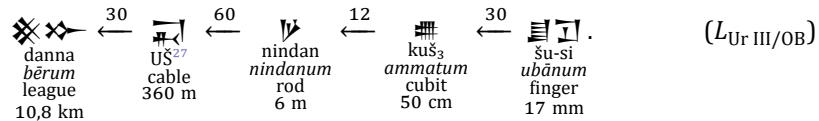
This intertwining of units and numerals explains the large number of already-encoded numeral series:

- $\text{𒍪} \text{𒍪}$  used in  $S_{Ur\ III/OB}$  and the SPVS as well as with overt units;
- $\text{𒍪} \text{𒍪}$  used in  $G_{Ur\ III/OB}$ , of which  $\text{𒍪} \text{𒍪}$  are also used in  $S_{Ur\ III/OB}$  and the SPVS as well as with overt units;
- $\text{𒍪} \text{𒍪}$  used in  $S_{Ur\ III/OB}$ , and sometimes with overt units;
- $\text{𒍪} \text{𒍪}$  used in  $S_{Ur\ III/OB}$ ;
- $\text{𒍪} \text{𒍪}$  used in  $S_{Ur\ III/OB}$  and  $G_{Ur\ III/OB}$ ;
- $\text{𒍪} \text{𒍪}$  used in  $S_{Ur\ III/OB}$  and  $G_{Ur\ III/OB}$ ;
- $\text{𒍪} \text{𒍪}$  used in  $C_{Ur\ III/OB}$  as well as with overt units of the weight system;
- $\text{𒍪}, \text{𒍪}, \text{𒍪}, \text{𒍪}, \text{𒍪}$  used in  $C_{Ur\ III/OB}$ ;
- $\text{𒍪}, \text{𒍪}, \text{𒍪}, \text{𒍪}$  used in  $C_{Ur\ III/OB}$ —note the overlap with  $\text{𒍪} \text{𒍪}$ ;
- $\text{𒍪}$  and  $\text{𒍪}$  used in  $G_{Ur\ III/OB}$ .

Only in the SPVS did numerals exist truly independently of metrology; to quote [Robo8, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

### 3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with  $\text{𒍪}$  and  $\text{𒍪}$  numerals with their system  $S_{Ur\ III/OB}$  values. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units<sup>26</sup> [Fri07, p. 118; Rob19]:



<sup>24</sup>From [P309594].

<sup>25</sup>A larger unit, the guru<sub>7</sub> (karûm, grain heap), is sometimes used instead, with  $\text{𒍪} \text{𒍪} \text{𒍪} \text{𒍪} \text{𒍪} \text{𒍪}$  (1 karûm = 3600 kurrû). See [Fri07, p. 415; Rob19].

<sup>26</sup>In this factor diagram and the next, we do not include the numerals. The units are no more than a factor of 60 apart, so higher numerals such as  $\text{𒍪}$  or  $\text{𒍪}$  are not used.



Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:

$$\begin{array}{ccccccc} \text{⌘} \text{⌘} \leftarrow^{30} \text{⌘} \leftarrow^6 \text{⌘} \leftarrow^{10} \text{⌘} \leftarrow^2 \text{⌘} \leftarrow^6 \text{⌘} \leftarrow^{30} \text{⌘} \text{⌘} & (\bar{L}_{\text{Ur III/OB}}) \\ \text{eše}_2 & & \text{gi} & & & & \\ \text{ašlum} & & \text{qānum} & & & & \\ \text{rope} & & \text{reed} & & & & \\ 60 \text{ m} & & 3 \text{ m} & & & & \end{array}$$

In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

### 3.3.5 Fractions

Fractions of the *ikîm*,  $\text{↖} = \frac{1}{2}$  and  $\text{↗} = \frac{1}{4}$ , have already been encountered. In other contexts, the fraction  $\frac{1}{2}$  is written  $\text{⌘}$ , as in  $\text{⌘} \text{⌘}$ . The fractions  $\frac{1}{3}$  and  $\frac{2}{3}$  are written  $\text{⌘}$  and  $\text{⌘}$ . The latter two signs are derived from curviform signs  $\text{⌘}$  and  $\text{⌘}$ , which are already separately encoded; these are in turn derived from the sign  $\text{⌘}$  (ŠU<sub>2</sub>), whose Early dynastic form resembles  $\text{⌘}$ , and  $\text{⌘}$  numerals; see [Powell1971]. The  $\text{⌘}$  is sometimes omitted, as in [P240545; P221530; P221531; P271238; P274845].

## 3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the fourth millenium) clearly mirrors system  $\bar{S}_{\text{Ur III/OB}}$  [Fri07, p. 374; DE87, pp. 127, 165]:

$$\text{⊙} \leftarrow^{10} \text{●} \leftarrow^6 \text{⌘} \leftarrow^{10} \text{⌘} \leftarrow^6 \text{●} \leftarrow^{10} \text{⌘}. \quad (\bar{S})$$

Likewise the area system used in the Early Dynastic IIIb period for areas of one iku and greater mirrors system  $G_{\text{Ur III/OB}}$  [LAK, p. 72; NDE93, p. 63; Fri07, p. 378; Lec16]:

$$\text{⊙} \leftarrow^{10} \text{●} \leftarrow^6 \text{⌘} \leftarrow^{10} \text{●} \leftarrow^3 \text{⌘} \leftarrow^6 \text{⌘}, \quad (G_{\text{ED IIIb}})$$

with consistent use of the numerals:  $\text{●}$  corresponds to  $\text{⌘}$ ,  $\text{●}$  to  $\text{⌘}$ , and  $\text{⊙}$  to  $\text{⌘}$ . An exception to this correspondence, noted in [L2/04-099, p. 4] (see section 3.2), is that the vertical  $\text{⌘}$  from  $\bar{S}_{\text{Ur III/OB}}$  corresponds to a horizontal  $\text{⌘}$  in system  $\bar{S}$ . This is however far from the only case of such a reallocation of function. The earlier form of System G is [DE87, pp. 141, 165; Fri07, p. 378]:

$$\text{●} \leftarrow^6 \text{⊙} \leftarrow^{10} \text{●} \leftarrow^3 \text{⌘} \leftarrow^6 \text{⌘}, \quad (G)$$

Observe that, as noted in [DE87, p. 142],  $\text{⊙}$  changes meaning from  $10\text{●}$  in system  $G$  to  $600\text{●}$  in system  $G_{\text{ED IIIb}}$ . System  $G$  is used in the fourth millenium, but also in the ED I–II period (it is the “area 2” system in [Cha03], whereas  $G_{\text{ED IIIb}}$  is the “area 1” system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb area system<sup>28</sup>,

$$\text{⊙} \leftarrow^{10} \text{●} \leftarrow^6 \text{⌘} \leftarrow^{10} \text{●} \leftarrow^3 \text{⌘} \leftarrow^6 \text{⌘} \leftarrow^2 \text{⌘} \leftarrow^2 \text{⌘} \leftarrow^2 \text{⌘} \leftarrow^2 \text{⌘} \leftarrow^2 \text{⌘}, \quad (G_{\text{ED IIIb}})$$

<sup>27</sup>As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.

<sup>28</sup>A variant is  $\text{⊙} \leftarrow^{10} \text{●} \leftarrow^6 \text{⌘} \leftarrow^{10} \text{●} \leftarrow^3 \text{⌘} \leftarrow^6 \text{⌘} \leftarrow^2 \text{⌘} \leftarrow^2 \text{⌘} \leftarrow^2 \text{⌘} \leftarrow^2 \text{⌘}$ , see [Powell1972].

with the numerals of a contemporaneous capacity system:

$$\underbrace{\text{𒀭} \leftarrow^{10} \text{𒀭} \leftarrow^6 \bullet \leftarrow^{10} \text{𒀭} \leftarrow^4 \text{𒀭} \leftarrow^6 \text{𒀭}}_{\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}} \quad (C \text{ 𒀭} \text{𒀭} \text{𒀭})$$

both described in [Lec16]. While the size of the 𒀭𒀭𒀭𒀭 (gur saṇ ṇal<sub>2</sub>) in bariga is different from that of the Old Babylonian 𒀭𒀭, the basic structure of the capacity system is recognizable, with 𒀭 corresponding to 𒀭 for bariga, 𒀭-𒀭 corresponding to 𒀭-𒀭 for ban<sub>2</sub>, and the 𒀭 counted with 𒀭 rather than 𒀭 numerals. However, the half-iku is counted with the same 𒀭 as the bariga, whereas it uses a different sign, 𒀭, in the Old Babylonian system. As we will see, this is cannot be handled as a split, by giving 𒀭 the glyph 𒀭 in an Early Dynastic IIIb font, as the 𒀭 numeral series is also in use in that period.

### 3.4.1 Field lengths in Nirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign 𒀭 is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign 𒀭 is not written either. Lengths shorter than one rope are expressed in half-rope using the  $\frac{1}{2}$  sign 𒀭 (again with no 𒀭), and then in reeds, *with* the sign 𒀭, as follows:

$$\text{𒀭} \leftarrow^6 \text{𒀭} \leftarrow^2 \text{𒀭} \leftarrow^{10} \text{𒀭} \leftarrow^{32} \text{𒀭} \quad (L_{ED IIIb})$$

$\frac{1}{2}$  eše<sub>2</sub>=10 nindan  
 1 rope=10 rods  
 60 m

𒀭<sub>gi</sub>  
 reed  
 3 m

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and most others from the same period, such as the ones discussed in [Lec20], areas are expressed in system  $G_{ED IIIb}$ , with curviform numerals<sup>33</sup>; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope<sup>34</sup>, the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and thus the interpretation of its numerals between systems  $G_{ED IIIb}$  and  $L_{ED IIIb}$ . The sign 𒀭, which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without 𒀭. There are other such co-occurrences contrasting between metrological systems; for instance, [Kre98, p. 303 n. 686] mentions the use of cuneiform numerals for days and months<sup>35</sup>.

<sup>29</sup>The cuneiform counterpart is 𒀭.



<sup>32</sup>The reeds are counted using *tenû* numerals, 𒀭, 𒀭, 𒀭, etc.

<sup>33</sup>A CDLI search for "(bur3)" (< numerals used for areas) currently returns 15 ED IIIb results, whereas one for "(bur3@c)" (• numerals used for areas) returns 206. Further, when dated, the tablets with cuneiform bur<sub>3</sub> are from the reigns of 𒀭𒀭𒀭𒀭𒀭 (variously transliterated iri-inim-gi-na, uru-ka-gi-na, etc.) and 𒀭𒀭𒀭𒀭𒀭 (lugal-zag-ge-si), the last two kings of ED IIIb Lagaš.





<sup>34</sup>This is the case of the sides of the field in [P020054, obv. ii 2–3].


<sup>35</sup>That note also mentions a contrast between the use of curviform numerals to count people and curviform numerals to count bread allotted to them in [P010876]; such contrasts are more akin to styling, and might not, on their own, justify the disunification.
















both the  and the , thus [Pow87, pp. 504 sq.]

$$\begin{array}{c} \bullet \xleftarrow{10} \text{D} \xleftarrow{2} \bullet \xleftarrow{10} \text{D} \xleftarrow{\frac{3}{2}} \text{V} \xleftarrow{2} \text{V}, \\ \text{C} \end{array} \quad (C \text{ } \text{D})$$

but counts cheese (, ga'ar) using the  capacity system, with  numerals for the .

Another capacity system in ED IIIb Nirsu is the , the gur of two ul:

$$\begin{array}{c} \bullet \xleftarrow{10} \text{D} \xleftarrow{2} \text{V} \xleftarrow{6} \text{V} \xleftarrow{6} \text{V}. \\ \text{H} \end{array} \quad (C \text{ } \text{H})$$








Here the -- contrast occurs not only within the numerals of the system, but with its units; this is perhaps best illustrated by the expressions      in [P221746] and      in [P221814].

### 3.4.4 Grain in Ebla

The mixing of curviform and cuneiform numerals within a metrological system is not specific to Nirsu.


The system of grain<sup>42</sup> capacities in Ebla uses the following units<sup>43</sup>:

$$\begin{array}{c} \text{gu}_2\text{-bar} \xleftarrow{2} \text{ba-ri}_2\text{-zu} \xleftarrow{\frac{5}{2}} \text{jin}_4 \xleftarrow{4} \text{nin}_2\text{-san}_3\text{su} \xleftarrow{6} \text{an-zam}_x. \end{array}$$

The  and  are generally counted using curviform numerals, and the smaller units using cuneiform  numerals<sup>44</sup>. Indeed, a search on [EbDA] for co-occurrences of either  or  with either of  or  finds the following expressions<sup>45</sup>:

<sup>42</sup>Liquid capacities use a different system [Arc15, p. 229 n. 12]:

$$\begin{array}{c} \text{la-ha} \xleftarrow{30} \text{sil}_3 \xleftarrow{6} \text{an-zam}_x. \end{array}$$








A glance it seems that  are counted with cuneiform numerals and higher units with curviform ones, thus



$$\begin{array}{c} \text{D} \xleftarrow{\frac{5}{3}} \text{D} \xleftarrow{6} \bullet \xleftarrow{10} \text{D} \xleftarrow{3} \text{D} \xleftarrow{\frac{10}{3}} \text{D} \xleftarrow{6} \text{D}, \\ \text{H} \end{array}$$

but we have not investigated this thoroughly.

<sup>43</sup>Another system uses different values for the  and , see [Cha12, p. 62; Arc15, p. 229 n. 12]:

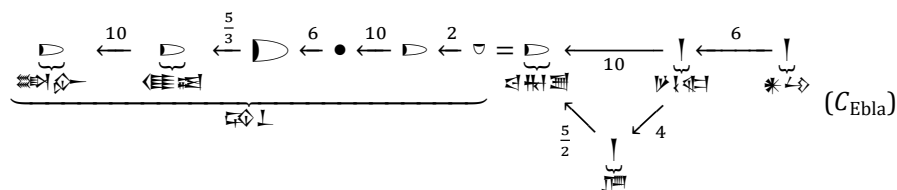
$$\begin{array}{c} \text{gu}_2\text{-bar} \xleftarrow{2} \text{ba-ri}_2\text{-zu} \xleftarrow{3} \text{jin}_4 \xleftarrow{4} \text{nin}_2\text{-san}_3\text{su} \xleftarrow{5} \text{an-zam}_x. \end{array}$$

<sup>44</sup>As mentioned in [Cha12, p. 63], the  is also counted using the - numeral series. Some instances of that usage are found transliterated *n/6* in EbDA; in some cases the  sign is omitted, and the  numeral is then written before the  unit, as in  from [P240545].

<sup>45</sup>We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform  and larger with cuneiform  and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked<sup>46</sup> on a photograph (from EbDA unless noted otherwise).

<sup>46</sup>As we will see in Section 3.7.2, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

- Note that higher numbers of  $\text{𐎶} \perp$  are expressed in hundreds (*mi-at*  $\text{𒈾}$ ) and then thousands (*li-im*  $\text{𒂗}$ ), as is typical in Ebla [Arc15, p. 33], e.g., in the last example above or in [P240532, verso 2, 3],  $\text{𐎵} \text{𒈾} \text{𐎠} : \text{𐎴} \text{𐎣} \text{𐎢} \text{𐎡} \text{𐎶} \perp$  (100 + 60 + 30 + 5 = 195  $\text{𐎶} \perp$  of grain). These expressions correspond to the following factor diagram:




Because of their prevalence in the fourth millennium and Early Dynastic period, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 1–20.

<sup>47</sup>ba-ri<sub>2</sub>-zu<sub>2</sub>, a variant spelling.

<sup>48</sup>Short for  $\square \vdash$ .

<sup>49</sup>Note the omitted  $\square \downarrow$ .











<sup>50</sup>Instead of the expected  $\nabla \{ \nabla \}$ .

<sup>51</sup>  not legible on the EbDA photo.

<sup>52</sup>From CDLI photo.

<sup>53</sup>From photo in [Arc89, p. 6].

<sup>54</sup>Laid out as  $\begin{smallmatrix} | & | & | & | \end{smallmatrix}$ ; on stacking patterns see Section 6.2.

<sup>55</sup>From photo in [Arc89, p. 6]; see also the CDLI photo and the copy in [Friberg1986]. This tablet features unusual usage of vertical numerals—"somewhat unorganized", as described by [Friberg1986]—, such as    or   , but its   and   are consistently counted with cuneiform numerals, and the higher units with cuneiform numerals.



<sup>56</sup>Short for .

<sup>57</sup>ŠU<sub>2</sub>+NIN<sub>2</sub>-san, an unusual variant spelling.

<sup>58</sup>The untranslated text would be 𐎠𐎡𐏁𐎧𐎢𐎫𐎲𐎣𐎵𐎶𐎷𐎸𐎹𐎺𐎻𐎼𐎽𐎾𐎿𐌱𐌴𐌰𐍂𐌓𐌀𐌀𐌆𐌇𐌈𐌉𐌊𐌋𐌌𐌍𐌎𐌏𐌐𐌑𐌒𐌓𐌔𐌕𐌖𐌗𐌘𐌙𐌚𐌛𐌜𐌝𐌞𐌟𐌠𐌡𐌢𐌣𐌤𐌥𐌦𐌧𐌨𐌩𐌪𐌫𐌬𐌭𐌮𐌯𐌰𐌱𐌲𐌳𐌴𐌵𐌶𐌷𐌸𐌹; note the atomically encoded  $\text{ib}_7 \times 3! = \frac{\infty}{\infty} \times III = \frac{\infty}{III}$ .

entirely, *e.g.*, 4 'a<sub>3</sub>-da-um 4 aktum 4 ib<sub>2</sub><sup>tu<sub>9</sub></sup>×3 sa<sub>6</sub> gunu<sub>3</sub>, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, *e.g.*, 4(aš<sup>c</sup>) 'a<sub>3</sub>-da-um 4(diš<sup>c</sup>) aktum 4(aš<sup>c</sup>) ib<sub>2</sub><sup>tu<sub>9</sub></sup>×3(diš<sup>c</sup>) sa<sub>6</sub> gunu<sub>3</sub>, but the result would be less readable. See Section 3.7.2 for a discussion of transliteration conventions for numerals.

for the words š u š a n a and š a n a b i. Deimel's reading š a n (a) for U came out of the reading /š a n t a k/ for the sign Y and the writing of š a (- n a) after the fractional signs for š u š a n a and š a n a b i in Old Sumerian texts. But this was an ill-conceived argument at its inception, for

Figure 1: Discussion of the readings of proposed  and already-encoded  in [Powell1971].







sions also. In example 6, the writing  may imply a reading /š a n a b i/,<sup>1</sup> whereas  in example 11 should be read \*/š u š a n a m i n/. Moreover, the question must be raised as to whether such writings as <U><B> k ù - b a b b a r + š a - n a<sup>2</sup> do not perhaps imply a linguistic resolution of \*/š u š a n a m i n/ rather than /š a n a b i/. I see no way of answering this question at present, but it is one which one

Figure 2: Discussion of the readings of proposed  and  as well as already-encoded  and  in [Powell1971].

<sup>1</sup> "big cup" = 3 "big disks". Hence we can infer from the two ŠE-texts BIN 8,<sup>4</sup> and BIN 8,5 together, that the "ŠE-system" makes use of number signs whose values are related to each other through the equations

$$1\text{U} = 3\text{O}, 1\text{O} = 10\text{o}, 1\text{o} = 6\text{v}, 1\text{v} = ?\text{Q}$$

A more convenient way of saying the same thing is to write out the "steps" between the various ŠE-units in what we shall call a "step-diagram" for the "ŠE-system":

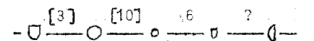


Figure 3: The first factor diagram, in [Fri78, p. 10].

$$\begin{cases} 4\overline{\cup} 5\bullet = 24\overline{\cup} 3\bullet & (\text{C } 234) \\ 5\overline{\cup} 1\bullet 1\overline{\cup} = 5\overline{\cup} 7\bullet & (\text{C } 314) \\ 1\overline{\cup} 1\overline{\cup} 1\overline{\cup} 1\overline{\cup} = 6\bullet 2\overline{\cup} 1\overline{\cup} & (\text{C } 27) . \end{cases}$$

These metrological equations for the "unknowns"  $\overline{\cup}$ ,  $\bullet$ ,  $\overline{\cup}$ , etc., can be treated exactly as ordinary equations for unknowns  $x, y, z, \dots$ . In particular, the equations can be simplified by subtraction of equal amounts from both sides of the identities. In this way the three equations above can be reduced to:

$$\begin{aligned} 2\bullet &= 20\overline{\cup} & (4\overline{\cup} 3\bullet \text{ subtracted from both sides}) \\ 1\overline{\cup} &= 6\bullet & (5\overline{\cup} 1\bullet - " = ) \\ 1\overline{\cup} &= 6\bullet 1\overline{\cup} 9\overline{\cup} & (1\overline{\cup} 1\overline{\cup} - " = ) \end{aligned}$$

We can now read off from the first equation that  $1\bullet = 10\overline{\cup}$ , and from the second that  $1\overline{\cup} = 6\bullet$ . Then the third equation can be simplified (by "substitution" of these values into the equation), to the following reduced form:

$$1\overline{\cup} = 2\overline{\cup} 9\overline{\cup} .$$

The most likely solution to this last equation is, of course,

$$1\overline{\cup} = 2\overline{\cup} , \quad 1\overline{\cup} = 10\overline{\cup} .$$

Figure 4: The derivation of the factors of the bisexagesimal system in [Fri78, p. 15]<sup>60</sup>.

<sup>60</sup>The bisexagesimal system is used alike in proto-Elamite and proto-cuneiform texts, see [Fri78, p. 38]; the derivation in [Fri78, p. 15] is based on proto-Elamite artefacts. Note that in Friberg's early works [Fri78; Friberg1979; Friberg1986; Fri87], copies of fourth millenium and sometimes third millenium tablets are shown as vertical text (which they were for the scribes), and their numerals are written within horizontal text in the same orientation that they have if the tablet is taken as vertical text; in [UAX50] parlance, as if they had Vertical\_Orientation=Upright. In addition, they are listed in these equations in the horizontal order in which they appear as vertical text (thus the rightmost numeral is the most significant, read first). Cuneiform is correctly Vertical\_Orientation=Rotated, consistently both with modern practice and with the rotation between earlier vertical and later horizontal monumental inscriptions. Friberg's early conventions are not followed in later scholarship, and are abandoned in his own more recent works, such as [Fri07]; a more typical way to express the first equations might be

$$\begin{aligned} 5\bullet + 4\overline{\cup} &= 3\bullet + 24\overline{\cup} & (\text{C } 234) \\ 1\overline{\cup} + 1\bullet + 5\overline{\cup} &= 7\bullet + 5\overline{\cup} & (\text{C } 314) \\ 1\overline{\cup} + 1\overline{\cup} + 1\overline{\cup} &= 1\overline{\cup} + 2\overline{\cup} + 6\bullet & (\text{C } 27) \end{aligned}$$

A diplomatic edition of [Fri78] could rotate the numerals using a higher-level protocol:

$$\begin{cases} 4\overline{\cup} 5\bullet = 24\overline{\cup} 3\bullet & (\text{C } 234) \\ 5\overline{\cup} 1\bullet 1\overline{\cup} = 5\overline{\cup} 7\bullet & (\text{C } 314) \\ 1\overline{\cup} 1\overline{\cup} 1\overline{\cup} = 6\bullet 2\overline{\cup} 1\overline{\cup} & (\text{C } 27) . \end{cases}$$

Thus, for instance, the original set of fractions  $\nabla$ ,  $\triangleleft$ , and  $\blacktriangledown$  (1/2, 1/4 and 1/8 of an iku) in the Sumerian GANA system, was after a time augmented through the addition of the new sub-unit SAR:  $\text{𒌦}$ , equal to 1/100 of an iku ( $\nabla$ ). Similarly, the Sumerian weight unit "na-na" which originally may have had only the sub-units  $\text{𒌶}$  ša-na (= 1/3 mana) and  $\text{𒌶𒌶}$  ša-na-bi (= 2/3 mana), and perhaps also gin:  $\text{𒌶𒌶𒌶}$  (= 1/60 mana), seems to have acquired, at some time or other, also the smaller sub-units  $\text{𒌶𒌶𒌶𒌶}$  (= 1/3 gin), and  $\text{𒌶𒌶𒌶𒌶𒌶}$  = še (= 1/3 × 1/60 gin).

Figure 5: Discussion of proposed fractions  $\nabla$ ,  $\triangleleft$ ,  $\blacktriangledown$ , and  $\text{𒌦}$ , as well as already-encoded  $\text{𒌶}$  and  $\text{𒌶𒌶}$  in [Fri78, p. 49].

stein publizierten Zeichenliste enthalten ist<sup>3</sup>, bis vor kurzem unentdeckt bleiben konnte. Erst 1978 machte der schwedische Mathematiker J. Friberg, ERBM I, 9-11, darauf aufmerksam, daß die Zeichen für die Zahlen Eins ( $\nabla$ ) und Zehn ( $\bullet$ ) in Verbindung mit dem Zeichen Še nicht im Verhältnis 1 zu 10 sondern im Verhältnis 1 zu 6 stehen. Bis dahin hatte man, obwohl die Andersartigkeit des in Verbindung mit dem Zeichen Še verwendeten Zahlzeichensystems bekannt war, für diese beiden häufigsten Zahlzeichen einheitlich ein Verhältnis 1 zu 10 unterstellt, obwohl es mehrere eindeutige Gegenbelege gab, von denen zumindest diejenigen der Archaischen Texte aus Gmdet Nasr bereits früh publiziert und jedermann zugänglich waren<sup>4</sup>. Als Folge

Figure 6: Discussion in [DE87, p. 117] of the discovery in [Fri78, pp. 9–11] (see Figure 3) of the different relations between  $\nabla$  and  $\bullet$  in systems G and ??.

there is in any case an important qualitative difference between IX for Latin novem and  $\bullet$  for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 7: The sign  $\bullet$  used in a parallel with IX in [Englund1988], discussing an argument from [Powell1972] on the question of the language of the Uruk III texts.

of decreasing fractions  $1/n$  of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus  $\text{𒌶} = 1/2 N_{30}$ ,  $\text{𒌶𒌶} = 1/3 N_{30}$ , and so on. The first sign of the latter units,  $N_{34}$ ,

Figure 8: Description of the fractions  $\text{𒌶}$  and  $\text{𒌶𒌶}$  in [Eng98, p. 113]<sup>61</sup>.

For instance, the first line contains the notations  $1N_{34} 1N_{300} ; 2N_{20}$ , which can be translated "60 of the (grain rations containing)  $\text{𒌶}$  (of grain); (grain involved): 2  $\bullet$  (of ground barley)". This calculation contradicts the assumed numerical relationship  $10N_1 = 1N_{14}$ , since as was well known the measure represented by the sign  $N_{30}$  was  $1/5$  of that represented by  $N_1$ , so that  $60 \times 1/5 = 12$  and not 20, as  $2N_{14}$  would imply. Instead of relying on complicated

Figure 9: The sign  $\text{𒌶}$  used as a capacity measure within otherwise translated text in [Eng98, p. 116].

<sup>61</sup>The text erroneously has  $N_{34}$  instead of  $N_{24}$ .



Die halbkreisförmigen Griffelindrücke gehen manchmal in mehr oder weniger eckige Formen über ( $\nabla$ )<sup>65</sup>. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen ( $\searrow$ ), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren<sup>66</sup>. Selten treten mit  $\nabla$  gebildete Zahlen auf<sup>67</sup> (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 10: Discussion of co-occurrences and contrasts between  $\nabla$ ,  $\searrow$ , and  $\nabla$  in [Kre98, p. 303].

The calculations:

Obv. i	1	$60 \times \frac{1}{5} \nabla$	( $\nabla$ )	=	$12 \times \nabla$	=	$2 \times \bullet$
	2	$120 \times \frac{1}{10} \nabla$	( $\nabla$ )	=	$12 \times \nabla$	=	$2 \times \bullet$
	3	$120 \times \frac{1}{15} \nabla$	( $\nabla$ )	=	$8 \times \nabla$	=	$1 \times \bullet$ $2 \times \nabla$
	4	$300 \times \frac{1}{20} \nabla$	( $\nabla$ )	=	$15 \times \nabla$	=	$2 \times \bullet$ $3 \times \nabla$
	5	$600 \times \frac{1}{25} \nabla$	( $\nabla$ )	=	$24 \times \nabla$	=	$4 \times \bullet$
Rev. i	1	1200			$1 \times \bullet$	$1 \times \bullet$	$5 \times \nabla$
Obv. i	6	$6000 \times \frac{1}{30} \nabla$	(GAR+6N <sub>57</sub> )	=	$200 \times \nabla = 1 \times \nabla$	$3 \times \bullet$	$2 \times \nabla$
ii	1	$120 \times \approx \frac{1}{4} \nabla$	(DUG <sub>5</sub> +U <sub>29</sub> )	=	$30 \times \nabla$	=	$5 \times \bullet$ $1 \times \nabla$ $1 \times \nabla$
	2	$180 \times \frac{1}{5} \nabla$	(DUG+AS <sub>9</sub> )	=	$36 \times \nabla$	=	$6 \times \bullet$
	3	$300 \times \frac{1}{15} \nabla$	(KAS <sub>9</sub> )	=	$20 \times \nabla$	=	$3 \times \bullet$ $2 \times \nabla$
Rev. i	3	600			$1 \times \bullet$	$4 \times \bullet$	$3 \times \nabla$ $1 \times \nabla$
					$1 \times \bullet$	$1 \times \bullet$	$5 \times \nabla$
					$1 \times \nabla$	$3 \times \bullet$	$2 \times \nabla$
					$1 \times \bullet$	$4 \times \bullet$	$3 \times \nabla$ $1 \times \nabla$
Grand total of groats used:					$1 \times \nabla$	$2 \times \bullet$	$9 \times \bullet$ $4 \times \nabla$ $1 \times \nabla$
Grand total of malt used:					$1N_{47}$ $4N_{20}$ $3N_5$ $1N_{42a}$ (rev. i 3) $\times \frac{3}{5} \approx$	$8 \times \bullet$	$4 \times \nabla$ $1 \times \nabla$

Figure 6. Transliteration and calculations of *MSVO* 4, 66.

Figure 11: Calculations from [P005468] transcribed in [Englund2001] using modern mathematical notation combined with some of the proposed characters.

strong similarities between “area” 1 and “area” 3 systems, the sign with two concentric discs ( $\odot$ , notated N<sub>50</sub><sup>27</sup>) remains problematic. It never appears in any numerical combination with the sign with a single disc ( $\bullet$ ,

Figure 12: Discussion of  $\odot$  and  $\bullet$ <sup>62</sup> in [Cha03, p. 6].

<sup>62</sup>The statement that these do not co-occur refers to the texts from ED I–II Ur; these signs co-occur both earlier and later in areas, with different relations as previously discussed.

$1/15$ , etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation  $\text{𒄩} \text{𒄩}$  could be understood as  $1 \frac{1}{5}$  gur.<sup>64</sup> However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs  $\text{𒄩}$  and  $\text{𒄩}$  cannot be considered klsmatograms.

Metrological tablets from the end of the 4<sup>th</sup> millennium (Nissen, Damerow and Englund 1993, 55-59, to *MSVO* 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

$\text{𒄩}$  1(iku) represents a surface of 3600m<sup>2</sup>  
 $\text{𒄩}$  1(eše<sub>3</sub>) represents a surface of 21,600m<sup>2</sup>  
 etc.

The signs iku and eše<sub>3</sub> constitute by themselves measures of surface areas. These measures are usually followed by the sign GAN<sub>2</sub>, which means either surface or field and

Figure 13: Discussion of Old Babylonian<sup>64</sup> capacity and fourth millenium area measures in [Proust2009].

formed by only two signs  $\text{𒄩}$  and  $\text{𒄩}$ , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60<sup>2</sup>, 60<sup>3</sup>, 1/60, 1/60<sup>2</sup>, etc. are written in the same way, with the vertical wedge  $\text{𒄩}$ . The third feature concerns the exact function of

Figure 14: Description of the SPVS in [Cha12, p. 58], using the already-encoded signs  $\text{𒄩}$  and  $\text{𒄩}$ .

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with:  $\text{𒄩} \text{𒄩} \text{𒄩}$  but never with the repetition  $\text{𒄩} \text{𒄩} \text{𒄩}$ .

Figure 15: Discussion of large numbers illustrated by  $\text{𒄩} \text{𒄩} \text{𒄩}$ <sup>65</sup> in [Cha12, p. 59]

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

$\text{𒄩} \text{𒄩} \text{𒄩}$  še bar  $\text{𒄩}$  ba-ri-zu  
 ‘3 gubar (capacity units) and 1 parisu’.

Figure 16: Partial transliteration of [P240597]  $\text{𒄩} \text{𒄩} \text{𒄩} \text{𒄩} \text{𒄩} \text{𒄩}$  in [Cha12, p. 61].

This is particularly true of the signs  $\text{𒄩}$ ,  $\text{𒄩}$ ,  $\text{𒄩}$  and  $\text{𒄩}$ , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written  $\text{𒄩}$  in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign  $\text{𒄩}$  is most often associated with the parisu measure, while the signs  $\text{𒄩}$ ,  $\text{𒄩}$ ,  $\text{𒄩}$  and  $\text{𒄩}$  refer to 1, 2, 3,

Figure 17: Discussion in [Cha12, p. 64] of the relation between  $\text{𒄩}$ – $\text{𒄩}$  and  $\text{𒄩}$  in Mesopotamia and in Ebla.

<sup>64</sup>The cuneiform text is Unicode-encoded.

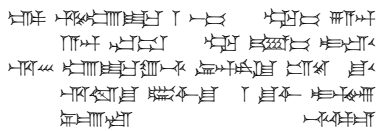
<sup>65</sup>Compare  $\text{𒄩} \text{𒄩}$  discussed in section 3.3.2. The order is variable in the Early Dynastic period; see [P010773], also discussed in [Fri07, p. 148], for an example of  $\text{𒄩} \text{𒄩}$ , and [P274845; P241764] for examples of  $\text{𒄩} \text{𒄩}$ . Sign order can be variable in early texts, see [Foxvog2016].

$\triangleright$  and  $\triangleright$

—

P242293, re

## Non-numeric usage



The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'<sup>66</sup>. Do you know its reading<sup>67</sup>?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign  $\text{—}$  has, *inter alia*, the values aš, rum, and dili. While the horizontal numerals are most frequently written  $\text{▷}$  in the Early Dynastic period<sup>68</sup>, such non-numeric usage is almost<sup>69</sup> always written  $\text{—}$ , for instance:

<sup>66</sup>The reader will recall that ꝛęš₂ is written ꝛ, with a larger wedge than ꝛ; however, these signs have merged by the time Examenstext A is composed.

<sup>67</sup>Besides *neš<sub>2</sub>*, a look at [OSL] shows that the values *diš*, *ge<sub>3</sub>*, *makkaš*, *sanjak<sub>4</sub>*, and *tal<sub>4</sub>* are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

<sup>68</sup>A CDLI search for "(asz@c)" finds 3296 ED texts, while a search for "(asz)" finds 81 ED texts, of which 46 also contain "(asz@c)".

<sup>69</sup>Exceptions are discussed in section 3.7.1.

- in personal names in administrative texts, such as the following, which all contain  $\triangleright$  numerals:
  - $\text{𒀭𒂗𒍪}$  in [P010424; P010458; P010459] from ED IIIa  $\text{أبو صلابيخ}$ ,
  - $\text{𒀭𒂗𒍪}$  in [P010960] from ED IIIa Šuruppag,
  - $\text{𒀭𒂗𒍪𒀭𒂗𒍪}$  in [P251641] from ED IIIb Adab,
  - $\text{𒀭𒂗𒍪𒀭𒂗𒍪𒀭𒂗𒍪}$  in [P252866] from ED IIIb Adab,
  - $\text{𒀭𒂗𒍪𒀭𒂗𒍪𒀭𒂗𒍪}$  in [P298637] from ED IIIb Umma;
- in the Sumerian word  $\text{𒌦}$   $\text{u}_2$ -rum, “property” in ED IIIb Nirsu administrative texts which contain  $\triangleright$  numerals, such as [P020006; P020008; P020018; P020024; P020030];
- in lexical texts:
  - in the divine name  $\text{𒀭𒂗𒍪𒀭𒂗𒍪𒀭𒂗𒍪}$  in the lexical texts [P010570; P010572], where the entries are prefixed with  $\triangleright$ .
  - in the word  $\text{𒀭𒂗𒍪}$  dili, “small fish” in [P010578], witness to Early Dynastic Fish,
  - in the same word with a determinative,  $\text{𒀭𒂗𒍪𒀭𒂗𒍪}$  dili<sup>ku</sup>, in [P010586], witness to Early Dynastic Food, which starts with  $\triangleright$  numerals.

This is a clear contrast between  $\text{𒀭𒂗𒍪}$  and  $\triangleright$  in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name  $\text{𒀭𒂗𒍪}$  occurs on its own line in the aforementioned administrative texts; a line  $\triangleright \text{𒀭𒂗𒍪}$  would instead be read as “one slave”.

### 3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as  $\text{𒀭𒂗𒍪}$  that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [KWU, pp. 123 sqq.] lists all of  $\text{𒀭𒂗𒍪}$  together with  $\triangleright$ - $\text{𒀭𒂗𒍪}$ , while  $\text{𒀭𒂗𒍪}$ ,  $\text{𒀭𒂗𒍪}$ , and  $\text{𒀭𒂗𒍪}$ , and only those, appear at the beginning of the sign list, since they have non-numeric values<sup>70</sup>. [PTACE, p. 58] has the numeric signs  $\triangleright$ ,  $\text{𒀭𒂗𒍪}$ ,  $\text{𒀭𒂗𒍪}$ , whereas non-numeric  $\text{𒀭𒂗𒍪}$  is at the beginning of the sign list, where its values *as* and *rum* are listed. For signs with both non-numeric and numeric usage, [LAK] writes *s. die Zahlz.* throughout the main list; LAK 1  $\text{𒀭𒂗𒍪}$  thus reappears at LAK 829 together with  $\triangleright$ ,  $\text{𒀭𒂗𒍪}$ , and  $\text{𒀭𒂗𒍪}$ . One should note [MZL], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have  $\text{𒀭𒂗𒍪}$ – $\text{𒀭𒂗𒍪}$  used in system  $G_{\text{Ur III/OB}}$ .

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes<sup>71</sup>

<sup>70</sup>Non-numeric values of  $\text{𒀭𒂗𒍪}$  were discussed in section 3.5;  $\text{𒀭𒂗𒍪}$  has the values *man*<sub>3</sub> and *min*<sub>5</sub>, and is used for the word *didli*, “several, various”;  $\text{𒀭𒂗𒍪}$  has the value *es*<sub>6</sub>.

<sup>71</sup>Compare, e.g., in the *Instructions of Šuruppag*,  $\text{𒀭𒂗𒍪𒀭𒂗𒍪𒀭𒂗𒍪}$  /  $\text{𒀭𒂗𒍪𒀭𒂗𒍪𒀭𒂗𒍪}$  in the ED IIIa witness [P222243], also discussed in section 3.7.1, and  $\text{𒀭𒂗𒍪𒀭𒂗𒍪𒀭𒂗𒍪}$  in the OB composite [Q000782] (translated “Šuruppag gave instructions to his son”

to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from  $\triangleright$  to  $\uparrow$  numerals prevents diachronic encoding even if  $\triangleright$  were unified with  $\leftarrow$ . For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'–6']; however, they are written with  $\uparrow$  numerals, whereas they are written with  $\triangleright$  numerals in the Early Dynastic witnesses; since  $\uparrow$  and  $\leftarrow$  are distinct<sup>72</sup> characters, the  $\triangleright\leftarrow$  unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods<sup>73</sup>, there is little opportunity for a diachronic representation of numeric quantities.


In the case of analyses such as [Romach2023], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

### 3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts<sup>74</sup> being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

#### 3.7.1 The case of ŠAR<sub>2</sub>

The character U+122B9  CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period<sup>75</sup>, a contrast between non-numeric šar<sub>2</sub> written  $\diamond$  and numeric 1(šar<sub>2</sub>) written  can be observed, similar to the contrast between  $\leftarrow$  and  $\triangleright$  previously discussed in

in [ETCSRI]). It does not matter for the construction of a composite text whether this is encoded  $\leftarrow\triangleright$  or  $\leftarrow\leftarrow$ , since that word is absent from other witnesses, and since the surrounding words differ.

<sup>72</sup>Besides the contrasts in numeric usage mentioned in section 3.3.3, these (already-encoded) characters were clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of  $\leftarrow$  which are not shared with  $\uparrow$  have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period,  $\leftarrow$  is used for the preposition *ina*, “in”, and  $\uparrow$  for the preposition *ana*, “to”.

<sup>73</sup>See, e.g., [Rob08, p. 55] on the unification of metrologies under Sargon, resulting in the systems described in section 3.3.

<sup>74</sup>Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

<sup>75</sup>For example, in personal names:

- $\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow$  in [P020019] from ED IIIb Nirsu;
- $\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow$  in [P020182], also from ED IIIb Nirsu;
- $\leftarrow\leftarrow\leftarrow\leftarrow$  in [P222186] from ED IIIb Umma;
- $\leftarrow\leftarrow\leftarrow\leftarrow$  in [P235312] from Old Akkadian Umma.

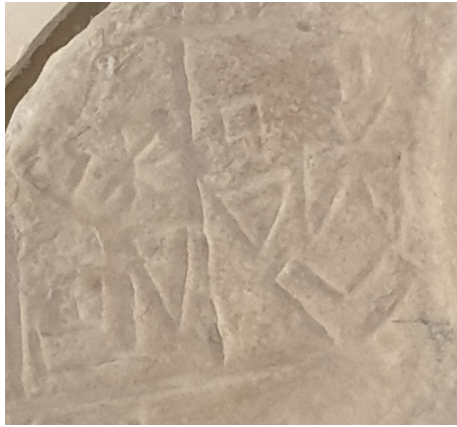


Figure 21: [P222399, obv. 6, 16–17] 𒌦𒌦𒌦𒌦 / 𒌦𒌦𒌦𒌦.

section 3.5. However, in lexical lists from Šuruppag and Ebla<sup>76</sup>, as well as in the *Stèle des vautours*, non-numeric šar<sub>2</sub> is curviform:

- 𒌦𒌦𒌦𒌦 and 𒌦𒌦𒌦𒌦 in [P010566];
- 𒌦𒌦 and 𒌦𒌦 in [P010576];
- 𒌦𒌦 in [P240986]<sup>77</sup>;
- 𒌦𒌦 in [P222399, obv. 17, 9, 18, 11, 22, 12]<sup>78</sup>.

It *would* be disruptive to the diachronic representation of text if non-numeric šar<sub>2</sub> were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph 𒌦, identical to the glyph of the proposed U+12579 𒌦 CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric šar<sub>2</sub> is curviform solidly predates the transition from 𒌦 to 𒌦 in the relevant metrological systems, there is no need to represent a 𒌦-𒌦 contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D 𒌦 CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

Note that in rare cases, such as [P222243] from ED IIIa Adab, non-numeric 𒌦 (here with the value rum) is written 𒌦. It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 𒌦 cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. In practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the CDLI transliteration (UR2-1(aš@c)) and the

<sup>76</sup>These are archaizing in other ways, *e.g.*, they have a 𒌦-𒌦 (NAM<sub>2</sub>-TUG<sub>2</sub>) split.

<sup>77</sup>From copy in [ELLeS, No. 397].

<sup>78</sup>Note however 𒌦𒌦 on [P222399, obv. 6, 17], see Figure 21. Curviform non-numeric šar<sub>2</sub> is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here.





While there exist transliterations that distinguish  $\blacktriangleleft$  from  $\blacktriangleright$  but not  $\mathbb{B}$  from  $\blacktriangleleft$ , such as the ones used in [DCCMT], the trend, especially in more recent works in third millenium studies, seems to be to represent numeral shape; for example, [Maiocchi2024] gave an example of the input syntax used by the new “Urban Economy Begins” project as “10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)” for  $\bullet\mathbb{B}\mathbb{B}\blacktriangleright\blacktriangleright$ , with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are  $\triangleright$  rather than  $\triangleright$  numerals. The “tradition of cavalierly dispensing with numerical notations in notations of administrative documents”, as [Englund2004] describes it, seems to be fading.

### 3.8 Conclusions

Co-occurrences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idiosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications. The contrast frequently applies to individual numerals, rather than to the span of entire numeric expressions.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted:  $\llcorner$  is a length of three ropes, but  $\bullet\bullet$  is an area of three bur<sub>3</sub>;  $\triangleright\blacktriangleright$  could be read as one  $\blacktriangleleft\blacktriangleright$  and one  $\blacktriangleleft\blacktriangleright$ , where  $\triangleright\triangleright$  would be one and a half  $\blacktriangleleft\blacktriangleright$ ;  $\blacktriangleleft\blacktriangleright$  is a personal name, but  $\triangleright\blacktriangleleft$  would be “one slave”.

In addition, there would be a risk of confusion about character identity should fontmakers attempt to treat the curviform and cuneiform numerals as unified. A designer concerned about the numeric-syllabic  $\triangleright\blacktriangleleft$  contrast, and wishing to support diachronic encoding between systems  $S_{Ur III/OB}$  and  $S$ , might give the  $\blacktriangleright$  numeral series (which is typically only used numerically in the Early Dynastic period) the glyphs of the  $\triangleright$  numeral series, since the clear  $\blacktriangleright\blacktriangleright$  identification involves the same rotation; this would however make it impossible to represent capacity measures that use  $\triangleright$ . Similarly, in an effort to support diachronic encoding for 1/2(iku), one might be tempted to give  $\blacktriangleleft$  the glyph of  $\triangleright$ , thereby rendering the font unusable for quantities measured using the  $\blacktriangleleft$  numeral series; an ED I–II Ur font designer could decide to give  $\blacktriangleleft$  the same glyph as  $\blacktriangleleft$  (that of the proposed  $\bullet$ ), according to the older area system, making it impossible to represent the newer system.

At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millenium corpora. The oddity of  $\bullet$  requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.



## 4 Rationale for ED–Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millenium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millenium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs  $KA\check{S}_a$ ,  $KA\check{S}_b$ , and  $KA\check{S}_c$ , depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Englund2001]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered scripts.

As part of the development of these conventions, a classification of fourth millenium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter  $N$  with a numeric subscript (sometimes with an additional alphabetic subscript):  $N_1$  is  $\triangleright$ ,  $N_{14}$  is  $\bullet$ ,  $N_{34}$  is  $\triangleright$ , etc. Transliterations of numeric expression then use those to identify the type of number used, thus  $5N_1$  is  $\mathbb{B}\mathbb{P}$ , and  $5N_{14}$  is  $\mathbb{B}\mathbb{P}$ .

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the CDLI transliteration of  $\mathbb{B}\mathbb{P}\mathbb{B}\mathbb{P}$  or  $\mathbb{B}\mathbb{B}\mathbb{P}$  in [P283802] currently uses (barig@c) for the vertical numerals, since  $\triangleright$  numerals are typically capacity measures; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millenium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I–II period, even though the Sumerian text uses classical assyriological transliteration conventions; see [Cha03, p. 6 n. 27].

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millenium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in section 3.7.2; they are indeed used interchangeably, as in [P011104] which uses the notation  $u@f$  in [ePSD2], but  $N_{14}@f$  in CDLI. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millenium would therefore induce confusion as to which numerals should be used in third millenium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millenium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance,  $\mathbb{B}\mathbb{P}$   $N_{30c}$  is listed as a variant of  $\mathbb{B}\mathbb{P}$   $N_{30a}$  in [DE87, p. 166], where the numeric value of either in relation to  $\triangleright$   $N_{39a}$  is still unknown, but their values are found in [Englund2004] to be  $\mathbb{B}\mathbb{P} = \frac{1}{10} \triangleright$ ,

whereas  $\boxtimes = \frac{1}{6} \cup$ .

## 5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban<sub>7</sub>@c) and 5(ban<sub>7</sub>@c).]

## 6 Characters not included in this proposal

## 6.1 Missing numerals

TODO N13 not attested in CDLI TODO ( $N_{17}$  not usefully numeric,  $12N_{14}$  not encodable, etc.). Cite [DE87, p. 147] 7 and 8 (diš *tenû*) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.

## 6.2 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some *stacking patterns*; for instance 9𐎶 is encoded both as U+12446 𐎶 and as U+1240E 𐎵𐎺. This is in part due to contrastive usage of stacking patterns. For instance, besides 𐎧 and 𐎨 which are characteristic of bariga measures, four bariga is written 𐎴 even where 4𐎶 is written 𐎲, as in [P255010; P292843]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often 𐎶 𐎷 𐎸 𐎹 𐏁 𐏂 𐏃 𐏄 𐏅 < 𐏆 𐏇 𐏈 𐏉 𐏊, and results in metrological systems, typically 𐎶 𐎷 𐎸 𐎹 𐏁 𐏂 𐏃 𐏄 𐏅 𐏆 𐏇 𐏈 𐏉 𐏊<, occasionally co-occurring as in [P142827; P142357]. This separate encoding is also for compatibility with distinctions made in reference works and in some non-numeric transliterations; for instance, 𐎶 is [MZL, No. 860] and has the value limmu, and 𐎷 is [MZL, No. 852] and has the value limmu<sub>s</sub>. Numeric<sup>83</sup> transliterations occasionally distinguish the stacking patterns 𐎶 𐎷 𐎸 𐎹 𐏁 𐏂 𐏃 𐏄 𐏅 < 𐏆 𐏇 𐏈 𐏉 𐏊, as in the CDLI transliterations of the aforementioned tablets, although this is rare; often 4(diš) is 𐎴 in Ur III, but 𐎲 in the Neo-Assyrian period.

However, the stacking patterns from earlier periods are not separately encoded; for instance, in ED IIIB Nirsu, « 2(u) often has one < atop another. These older stacking patterns do not appear to be contrastive, are not marked in transliteration, and are not listed separately in sign lists nor assigned any different values. There is therefore no evidence of a need to encode them; instead, they should be considered style variants, and an ED IIIB Nirsu font should have an appropriate glyph for U+12399 « CUNEIFORM SIGN U U.

Likewise, many stacking patterns are attested for the curviform numerals proposed in this document, and it is not proposed to separately encode them. These distinctions would be incompatible with the state of the art in numeric transliterations, including those by Englund, who insisted on “a system of transliteration that reflects in a strict fashion the physical realities of the cuneiform inscriptions” [Englund2004], and they are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier

<sup>83</sup>The Sumerian word limmu means “four”, so limmu and limmu<sub>5</sub> are still numbers. The distinction here is between usage in transliterations of phrases such as 𒌦𒍪 𒈾𒀭𒂗𒅗𒄩 lugal an-ub-da limmu<sub>5</sub>-ba-ke<sub>4</sub> (king of the four quarters) or of names, and of numeric expressions such as 𒁕𒉺 4(diš) sila<sub>3</sub>.



Figure 22: The layout of case [P011099]; the numeral 9 is rotated to fit the rounded corner of the tablet.



Figure 23: The layout of case [P020066]; the numeral 9 is spread across two lines. The text is read in the order 9 9 9 9 9, “twenty-two oxen, one year old”.

tablets, as they are structured in rectangular cases rather than lines, so that numerals may be laid out across the case in whichever way fits the available space; this is illustrated in Figure 22. Note also that the numerals need to be considerably enlarged in order to reproduce the layout of the tablets, so that 9 often spans two lines of cuneiform signs, as shown in Figure 23. This is impractical when these numerals are set in text that contrasts them with the larger 10, and inconsistent with actual practice when typesetting these numerals, as illustrated in Figure 7: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dynastic period, but that are also attested in the fourth millenium in the Uruk III period; the fourth millenium, especially the Uruk IV period, also frequently features numerals that use a more vertical layout, as illustrated in Figure 24. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millenium, usage in third millenium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents 10, 100, 1000, and 10000 must remain consistent across the numeral series, lest a 10 numeral be confused with an 100 numeral. Since the single indents are frequently used in running text, as illustrated in section 3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 24 uses OpenType stylistic alternates, and Figure 22 rotates the

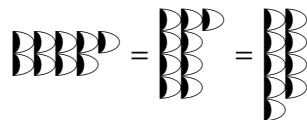


Figure 24: Three stacking patterns for U+12573 CUNEIFORM NUMERIC SIGN NINE N34. The one on the left is the reference glyph, used in Uruk III [P003499; P004430], and widely afterwards, *e.g.*, ED IIIa Šuruppag [P010678], ED IIIb Nirsu [P020057], Old Akkadian Umma [P212464]. The ones in the middle and right are used in two Uruk IV tablets [P001243; P004500]. All three Uruk examples are transliterated 9(N34) in CDLI.

character **•**, in both cases preserving the plain text backing.

### 6.3 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Englund2001].

## Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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