Archaic cuneiform numbers

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1 Summary

This document proposes encoding, at U+12550–U+12585, 310 numerals used in the fourth millennium (Uruk IV and Uruk III periods) and Early Dynastic period in conjunction with the Sumero-Akkadian cuneiform script and the proto-cuneiform script. The proposed characters are listed in §2. Most of them were listed in [L2/23-190]. The present document provides a more detailed rationale for their encoding and additional information about their identity and usage, both as part of the rationale and in §5. Some characters have been removed, in some cases because they are non-encodable variants, in others because their encodability should be considered as part of the proto-cuneiform proposal; these are discussed in §6. The glyphs have also been reworked, and additional characters used in the Early Dynastic period have been added.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model requires that the language of a text be understood to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in §4. The proposed "curved", or "curviform", numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in §3, including compatibility considerations in §3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in §2.3 reflect the overlap. Many of these numerals are also used in proto-Elamite⁵ texts, where they are treated as identical characters in scholarship on proto-Elamite, so that they should

¹[ISO15924]: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0. ²[ISO15924]: Pcun, not yet encoded.

³Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding ● (small stylus) or ● (large stylus), or at a shallower angle: ▷, ▽ (small stylus), ▷ (large stylus). Some numerals are composed of multiple such impressions, e.g., ▶ . The terms "curved", "curviform", "curvilinear", and "round" can be found in the literature. We avoid the term "round" here as it has other meanings in the context of numbers. We use "curviform" in this document as, being the least common term, it is least likely to lead to confusion, and "CURVED" in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [inlineATF].

⁴Impressed into clay using a stylus with a trihedral end: — (stylus held horizontally), ↑ (vertically), ↑ (diagonally) ≺ (with the head of the stylus), ↑ (stylus pressed deeper, forming a larger wedge), ↑ (combining ↑ and ≺), etc.

⁵[ISO₁₅₉₂₄]: Pelm, not yet encoded.

be unified with the ones proposed in [L2/23-196]. However, in the interest of time, we do not provide a detailed rationale for this unification in this document, and we are not proposing that the numerals be given the corresponding Script_Extensions property value for now. Neither do we propose encoding any numerals that are solely attested in proto-Elamite texts, or well-attested in proto-Elamite texts but insufficiently attested in Uruk—those are discussed in §2.

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
Numerais		Fyictin	a Yeny
Non-numeric signs	Future Pcun	Existing Xsux	

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

2 Proposed changes to the Standard

2.1 Core specification text

Amend [Uni16, §11.1.2, *sub* "Cuneiform Numerals"], as follows:

Cuneiform Numerals. In general, numerals that also have a phonetic, logographic, or determinative value are encoded in the main Cuneiform block; as a result, some series of numerals, such as Y-₩ΨΥ 1(diš)-9(diš) or ✓-₩ 1(u)-9(u), are split across the two blocks. Numerals have been encoded separately from signs that are visually identical but-semantically different etymologically unrelated (for example, U+1244F + CUNEIFORM NUMERIC SIGN ONE BAN2, U+12450 + CUNEIFORM NUMERIC SIGN TWO BAN2, and so on, versus U+12226 + CUNEIFORM SIGN MASH, U+1227A + CUNEIFORM SIGN PA, and so on).

The relation between series of numerals depends on the metrological system; for instance, when counting talents, written (a unit of weight, approximately 30 kg), (a unit of weight, approximately 30 kg), is used for "one talent", and (a v) for "ten talents". However, when measuring areas, the area (a v) (one (a v)) is eighteen times (a v) (one (a v)). The Numeric_Value property assignment of a cuneiform numeral therefore reflects only its relation to the first numeral in its series, rather than the absolute numeric value that it might represent. For instance, the number "fifty" is written (a v), but U+12410 (a v) CUNEIFORM NUMERIC SIGN FIVE U has Numeric_Value=5, as it is (a v) is (a v).

In the third millennium, and especially in the Early Dynastic period, some numerals are written using a cylindrical tool, rather than the cuneiform stylus, forming curved rather than cuneiform numerals (▷ rather than ▶). The cuneiform numerals are descended from these curved numerals. However, in the Early Dynastic period, the curved numerals contrast with the cuneiform ones, and are used together with them in several metrological systems; they are therefore separately encoded. Most curved numerals are encoded in the Archaic Cuneiform

Numerals block, with the exception of two fractions in the Cuneiform Numbers and Punctuation block: U+1245D (TO CUNEIFORM NUMERIC SIGN ONE THIRD VARIANT FORM A and U+1245E (TO CUNEIFORM NUMERIC SIGN TWO THIRDS VARIANT FORM A, the curved counterparts of U+1245A (TO CUNEIFORM NUMERIC SIGN ONE THIRD DISH and U+1245B (TO CUNEIFORM NUMERIC SIGN TWO THIRDS DISH.)

Add after [Uni16, §11.1.3]:

11.1.4 Archaic Cuneiform Numerals: U+12550-U+1268F

This block contains numerals used in the fourth millennium and third millennium. The numerals that are used in the fourth millennium and Early Dynastic I–II period (2900–2700 BCE) are named according to the conventions of the Berlin *Archaische Texte aus Uruk* (ATU) project, with names such as U+12550 \triangleright CUNEIFORM NUMERIC SIGN ONE N01 or U+125B6 $\stackrel{\frown}{}$ CUNEIFORM NUMERIC SIGN ONE N39A. For the signs that are also used in the third millennium, informative aliases provide correspondences to more common third millennium conventions, such as "1 aš curved" for U+12550 $\stackrel{\frown}{}$ CUNEIFORM NUMERIC SIGN ONE N01. The numerals that are only used starting in the Early Dynastic III period, where the ATU notation is not used, are named in the same fashion as the numerals of the Cuneiform Numbers and Punctation block.

The curved numerals are produced using cylindrical tools of two different sizes, producing small curved indents (▷, ▽, and •), and large ones (▷, ▽, and •). These can be combined, as in U+12574 ▷ CUNEIFORM NUMERIC SIGN ONE N48 or U+12582 ⓒ CUNEIFORM NUMERIC SIGN ONE N50. Consistent sizing is important to identifying these characters, as there is no visual distinction other than size between, for instance, U+12566 ** CUNEIFORM NUMERIC SIGN FIVE N14, U+1257D CUNEIFORM NUMERIC SIGN FIVE N45, or U+125A3 ☒ CUNEIFORM NUMERIC SIGN ONE N54. The reference glyphs of some of the larger signs have been resized to fit in the code charts cells, but fonts for these characters should retain consistent size across the numeral series.

Editor's note: I have not yet done that resizing. The dashed-box convention for wide dashes, see [Uni16, §24.1.2, sub "Dashed Box Convention"] should probably not be extended to these, since numbers enclosed in a real dashed box are a thing in proto-Elamite texts.

The Numeric_Value assignments follow the same principles as in the Cuneiform Numbers and Punctuation block. Numerals used in the third millennium have the Cuneiform script property value; numerals used only in the fourth millennium have the Proto-Cuneiform script property value. Numerals used in both the fourth and third millennium have both scripts in their Script_Extensions values.

The sign ŠAR₂. When used logographically, the sign ŠAR₂ has the same (cuneiform) appearance as U+1212D \diamondsuit CUNEIFORM SIGN HI in all but the most archaizing Early Dynastic texts. The character U+122B9 CUNEIFORM SIGN SHAR2 should be used for logographic šar₂, whether cuneiform or curved. Most period-specific fonts will have the same cuneiform glyph

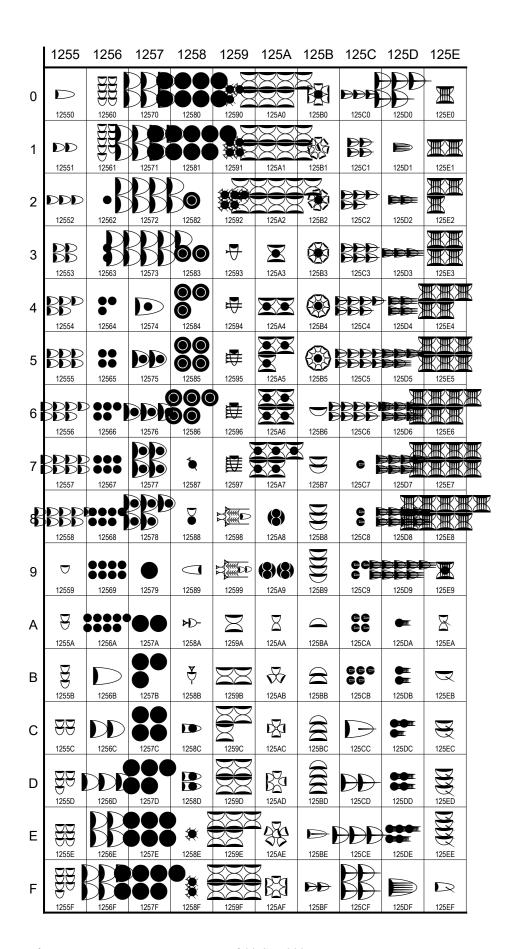
for U+122B9 and U+1212D. In the Early Dynastic period, numeric 1 $\$ar_2$ is typically written with a curved glyph, contrasting with logographic $\$ar_2$. U+12579 • CUNEIFORM NUMERIC SIGN ONE N45 should be used for curved 1 $\$ar_2$. In later periods, long after $\$AR_2$ and \$H have merged, even numeric 1 $\$ar_2$ has a cuneiform glyph. U+122B9 CUNEIFORM SIGN SHAR2 should be used for cuneiform 1 $\$ar_2$.

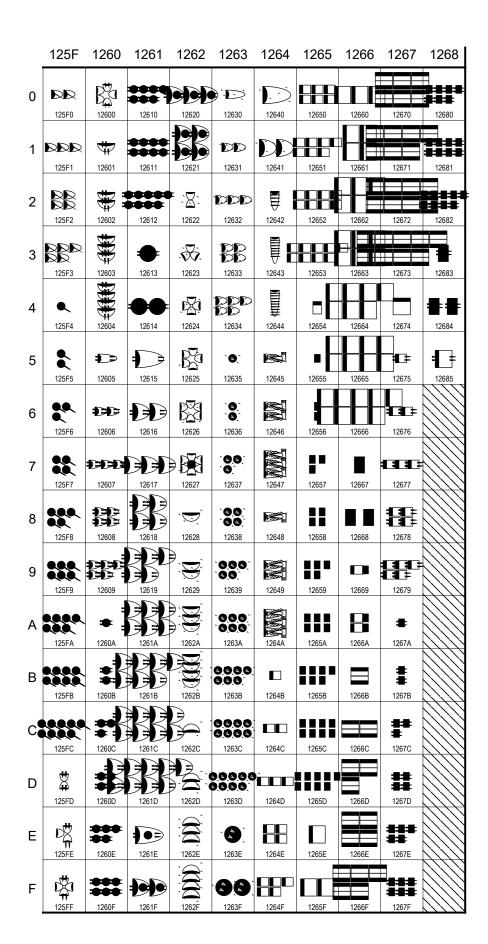
The reference glyph for U+122B9 CUNEIFORM SIGN SHAR2 is curved, reflecting the rarer and more archaic practice, instead of cuneiform as it would be in the Ur III period, so as to distinguish it from U+1212D ♦ CUNEIFORM SIGN HI.

2.2 Code charts

The code charts for the proposed block, including the character names list with proposed informative aliases, cross references, and informative notes, are shown on the following pages.

This space for rent.





Many of the reference glyphs for the higher numbers (THREE and above) have been rescaled to fit the code chart cells. They should be sized consistently with the corresponding ONE numerals.

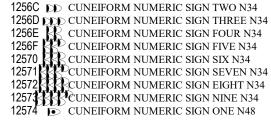
Common Numerals

Used in the sexagesimal discrete counting system and other metrological systems

12550 ▷ CUNEIFORM NUMERIC SIGN ONE N01

- = 1 aš curved
- → 12038 cuneiform sign ash
- often used instead of diš in Early Dynastic counterparts of cuneiform metrological systems
- → 12079 T cuneiform sign dish
- 12551 DD CUNEIFORM NUMERIC SIGN TWO N01
 - → 12400 » cuneiform numeric sign two ash
- 12552 DDD CUNEIFORM NUMERIC SIGN THREE N01
- 12553 BE CUNEIFORM NUMERIC SIGN FOUR N01
- 12554 ₽₽ CUNEIFORM NUMERIC SIGN FIVE N01
- 12555 BBB CUNEIFORM NUMERIC SIGN SIX N01
- 12556 BBBD CUNEIFORM NUMERIC SIGN SEVEN NO1
- 12557 BBB CUNEIFORM NUMERIC SIGN EIGHT N01 12558 BEEEP CUNEIFORM NUMERIC SIGN NINE N01
- 12559 alpha CUNEIFORM NUMERIC SIGN ONE N08
 - = 1 diš curved
 - → 12079 T cuneiform sign dish
 - = 1/2 iku curved
 - used for one half in multiple metrological systems
 - → 12039 < cuneiform sign ash zida tenu
 - → 12226 ¥ cuneiform sign mash
 - = 1 bariga curved
 - used in Early Dynastic capacity systems

- → 1222B Tr cuneiform sign min
- = 2 bariga curved
- → 12456 ¥ cuneiform numeric sign nigidamin
- 1255B ₹ CUNEIFORM NUMERIC SIGN THREE N08
 - → 12408 TT cuneiform numeric sign three dish
 - used in Early Dynastic capacity systems
 - = 3 bariga curved
 - → 12457 भ cuneiform numeric sign nigidaesh
- 1255C CUNEIFORM NUMERIC SIGN FOUR N08
- 1255D CUNEIFORM NUMERIC SIGN FIVE N08 1255E CUNEIFORM NUMERIC SIGN SIX N08
- 1255F CUNEIFORM NUMERIC SIGN SEVEN N08
- 12560 CUNEIFORM NUMERIC SIGN EIGHT N08
- 12561 CUNEIFORM NUMERIC SIGN NINE N08
- 12562 CUNEIFORM NUMERIC SIGN ONE N14
 - = 1 u curved
 - = 1 bur₃ curved
 - → 1230B < cuneiform sign u
- 12563 CUNEIFORM NUMERIC SIGN TWO N14
 - → 12399 « cuneiform sign u u
- 12564 CUNEIFORM NUMERIC SIGN THREE N14
 - → 1230D « cuneiform sign u u u
- 12565 :: CUNEIFORM NUMERIC SIGN FOUR N14
 - → 1240F \ cuneiform numeric sign four u
- 12566 ******* CUNEIFORM NUMERIC SIGN FIVE N14
- 12567 ******* CUNEIFORM NUMERIC SIGN SIX N14
- 12568 ::: CUNEIFORM NUMERIC SIGN SEVEN N14 12569 :::: CUNEIFORM NUMERIC SIGN EIGHT N14
- 1256A **** CUNEIFORM NUMERIC SIGN NINE N14
- 1256B CUNEIFORM NUMERIC SIGN ONE N34
 - = 1 neš₂ curved
 - → 12415 Y cuneiform numeric sign one gesh2



- = 1 ŋeš'u curved
- → 1241E 🏌 cuneiform numeric sign one geshu
- 12575 DE CUNEIFORM NUMERIC SIGN TWO N48 12576 CUNEIFORM NUMERIC SIGN THREE N48 12577 CUNEIFORM NUMERIC SIGN FOUR N48 12578 CUNEIFORM NUMERIC SIGN FIVE N48 12579 CUNEIFORM NUMERIC SIGN ONE N45
 - = 1 šar₂ curved
 - 122B9 should be used for cuneiform 1 šar₂
 - 122B9 should be used for logographic šar₂, even when curved
 - → 122B9 cuneiform sign shar2
- 1257A CUNEIFORM NUMERIC SIGN TWO N45 1257B **CUNEIFORM NUMERIC SIGN THREE N45** CUNEIFORM NUMERIC SIGN FOUR N45 1257C 1257D CUNEIFORM NUMERIC SIGN SIX N45
 1257E CUNEIFORM NUMERIC SIGN SEVEN N

 OUTPIEFORM NUMERIC SIGN SEVEN N CUNEIFORM NUMERIC SIGN FIVE N45 1257F CUNEIFORM NUMERIC SIGN SEVEN N45 12580 CUNEIFORM NUMERIC SIGN EIGHT N45 1258 CUNEIFORM NUMERIC SIGN NINE N45 12582 CUNEIFORM NUMERIC SIGN ONE N50
 - = 1 šar'u curved
 - → 1242C Ø cuneiform numeric sign one sharu
 - used instead of 1258E * in fourth millennium land

area systems

→ 12434 & cuneiform numeric sign one buru

12583 CUNEIFORM NUMERIC SIGN TWO N50 12584 CUNEIFORM NUMERIC SIGN THREE N50 12585 CUNEIFORM NUMERIC SIGN FOUR N50 12586 CUNEIFORM NUMERIC SIGN FIVE N50

Numerals used for land areas

Together with N08, N01, N14, N45, and N50

- CUNEIFORM NUMERIC SIGN ONE EIGHTH IKU 12587 **CURVED**
 - → 1245F Հ cuneiform numeric sign one eighth ash
- 12588 CUNEIFORM NUMERIC SIGN ONE EIGHTH IKU CURVED VARIANT FORM
- 12589 CUNEIFORM NUMERIC SIGN ONE N01 REVERSED = 1/4 iku curved

 - → 12460 / cuneiform numeric sign one quarter ash
- 1258A CUNEIFORM NUMERIC SIGN ONE QUARTER IKU CURVED VARIANT FORM
- 1258B CUNEIFORM NUMERIC SIGN ONE HALF IKU CURVED VARIANT FORM
 - → 12039 < cuneiform sign ash zida tenu
- 1258C ■ CUNEIFORM NUMERIC SIGN ONE N22
 - = 1 eše₃ curved
 - → 12458 × cuneiform numeric sign one eshe3
- CUNEIFORM NUMERIC SIGN TWO N22 1258D
- 1258E CUNEIFORM NUMERIC SIGN ONE BURU CURVED
- → 12434 🛊 cuneiform numeric sign one buru 1258F CUNEIFORM NUMERIC SIGN TWO BURU CURVED
- 12590 CUNEIFORM NUMERIC SIGN THREE BURU CURVED
- 12591 CUNEIFORM NUMERIC SIGN FOUR BURU CURVED
- **CUNEIFORM NUMERIC SIGN FIVE BURU CURVED** 12592

Early Dynastic capacity measures

- 12593 ₱ CUNEIFORM NUMERIC SIGN ONE BAN2 CURVED
 - 1244F 并 cuneiform numeric sign one ban2
 - = 1/2 aš curved
 - used for one half in multiple metrological systems
 - → 12226 ¥ cuneiform sign mash

- 12598 E CUNEIFORM NUMERIC SIGN NINDA2 TIMES SHE PLUS
 - ONE ASH CURVED
 - = 1/3 aš curved variant form
 - → 1245D ₹ cuneiform numeric sign one third dish variant form a
 - \rightarrow 1245A \coprod cuneiform numeric sign one third dish
- 12599 🖆 CUNEIFORM NUMERIC SIGN NINDA2 TIMES SHE PLUS TWO ASH CURVED
 - = 2/3 aš curved variant form
 - → 1245E ₹ cuneiform numeric sign two thirds dish
 - → 1245B 🗗 cuneiform numeric sign two thirds dish

Numerals used in the bisexagesimal system

Together with N08, N01, N14, and N34

□ CUNEIFORM NUMERIC SIGN ONE N51

= 1 ŋeš₂ curved doubled, 1 ŋešmin curved

1259B CUNEIFORM NUMERIC SIGN TWO N51

1259C CUNEIFORM NUMERIC SIGN THREE N51

1259D == **CUNEIFORM NUMERIC SIGN FOUR N51**

CUNEIFORM NUMERIC SIGN FIVE N51 1259E 1259F

🗮 CUNEIFORM NUMERIC SIGN SIX N51

125A0 CUNEIFORM NUMERIC SIGN SEVEN N51 125A1 CUNEIFORM NUMERIC SIGN EIGHT N51 125A2 CUNEIFORM NUMERIC SIGN NINE N51

125A3

▼ CUNEIFORM NUMERIC SIGN ONE N54

= 1 neš'u curved doubled, 1 nešmin'u curved

125A4 CUNEIFORM NUMERIC SIGN TWO N54
125A5 CUNEIFORM NUMERIC SIGN THREE N54
125A6 CUNEIFORM NUMERIC SIGN FOUR N54

125A7 CUNEIFORM NUMERIC SIGN FOUR N54

125A8 CUNEIFORM NUMERIC SIGN ONE N56 125A9 🚱 CUNEIFORM NUMERIC SIGN TWO N56

Fourth millennium capacity measures

125AA ⊠ **CUNEIFORM NUMERIC SIGN ONE N24**

125AB ↔ CUNEIFORM NUMERIC SIGN ONE N26 125AC ☑ CUNEIFORM NUMERIC SIGN ONE N28

125AD 🖾 CUNEIFORM NUMERIC SIGN ONE N29A

125AE R CUNEIFORM NUMERIC SIGN ONE N29B

125AF 🛱 CUNEIFORM NUMERIC SIGN ONE N30A

125B0 📳 CUNEIFORM NUMERIC SIGN ONE N30C

125B1 CUNEIFORM NUMERIC SIGN ONE N30D 125B2 CUNEIFORM NUMERIC SIGN ONE N30E

◉ CUNEIFORM NUMERIC SIGN ONE N31 125B3

125B4 **⊗** CUNEIFORM NUMERIC SIGN ONE N32

125B5 **CUNEIFORM NUMERIC SIGN ONE N33** 125B6 CUNEIFORM NUMERIC SIGN ONE N39A

125B7 CUNEIFORM NUMERIC SIGN TWO N39A

125B8 CUNEIFORM NUMERIC SIGN THREE N39A

125B9 CUNEIFORM NUMERIC SIGN FOUR N39A

125BA CUNEIFORM NUMERIC SIGN ONE N39B 125BB CUNEIFORM NUMERIC SIGN TWO N39B

125BC CUNEIFORM NUMERIC SIGN THREE N39B

125BD CUNEIFORM NUMERIC SIGN FOUR N39B

Numerals of sexagesimal system S'

Used to count dead animals and jars of certain types of liquids

125BF ₱ CUNEIFORM NUMERIC SIGN TWO N02

125C0 DDD CUNEIFORM NUMERIC SIGN THREE NO2

125C1 CUNEIFORM NUMERIC SIGN FOUR N02

125C2 CUNEIFORM NUMERIC SIGN FIVE N02

125C3 CUNEIFORM NUMERIC SIGN SIX N02

125C4 CUNEIFORM NUMERIC SIGN SEVEN N02

125C5 ESSE CUNEIFORM NUMERIC SIGN EIGHT N02

125C6 CUNEIFORM NUMERIC SIGN NINE NO2

125C7 CUNEIFORM NUMERIC SIGN ONE N15

125C8 **CUNEIFORM NUMERIC SIGN TWO N15** 125C9 : CUNEIFORM NUMERIC SIGN THREE N15

125CA **\$\$** CUNEIFORM NUMERIC SIGN FOUR N15

125CB **SSO** CUNEIFORM NUMERIC SIGN FIVE N15

125CC

☐ CUNEIFORM NUMERIC SIGN ONE N35

125CD → CUNEIFORM NUMERIC SIGN TWO N35

125CE CUNEIFORM NUMERIC SIGN THREE N35 125CF CUNEIFORM NUMERIC SIGN FOUR N35 125D0 CUNEIFORM NUMERIC SIGN FIVE N35

Numerals of bisexagesimal system B*

Used in the fourth millennium to count rations of an unclear nature

125D1 ► CUNEIFORM NUMERIC SIGN ONE N06

125D2 ➡ CUNEIFORM NUMERIC SIGN TWO N06

125D3 CUNEIFORM NUMERIC SIGN THREE N06

125D4 E CUNEIFORM NUMERIC SIGN FOUR N06

125D5 CUNEIFORM NUMERIC SIGN FIVE N06

125D6 CUNEIFORM NUMERIC SIGN SIX N06

125D7 CUNEIFORM NUMERIC SIGN SEVEN N06

125D8 CUNEIFORM NUMERIC SIGN EIGHT N06

125D9 CUNEIFORM NUMERIC SIGN NINE N06

125DA ← CUNEIFORM NUMERIC SIGN ONE N21

125DB **№** CUNEIFORM NUMERIC SIGN TWO N21

125DC 🛫 CUNEIFORM NUMERIC SIGN THREE N21

125DD 😄 CUNEIFORM NUMERIC SIGN FOUR N21 125DE 😅 CUNEIFORM NUMERIC SIGN FIVE N21

125DF **▶** CUNEIFORM NUMERIC SIGN ONE N38

125E0

■ CUNEIFORM NUMERIC SIGN ONE N52

125E1
125E1
125E2
125E2
125E3
125E3
125E4
125E4
125E5
125E5
125E5
125E6
125E6
125E6
125E7
125E7
125E7
125E7
125E7
125E8
125E8
125E8
125E9

Numerals of capacity system S'

Used in the fourth millennium to measure malted barley

125EA ⊠ CUNEIFORM NUMERIC SIGN ONE N24A 125EB CUNEIFORM NUMERIC SIGN ONE N40

125EC **CUNEIFORM NUMERIC SIGN TWO N40** CUNEIFORM NUMERIC SIGN THREE N40

125ED § 125EE CUNEIFORM NUMERIC SIGN FOUR N40

125EF CUNEIFORM NUMERIC SIGN ONE N03

125F0 ► CUNEIFORM NUMERIC SIGN TWO N03

125F1 ► CUNEIFORM NUMERIC SIGN THREE N03

125F3 CUNEIFORM NUMERIC SIGN FIVE N03

125F4 • CUNEIFORM NUMERIC SIGN ONE N18

125F5 CUNEIFORM NUMERIC SIGN TWO N18 125F6 🐏 CUNEIFORM NUMERIC SIGN THREE N18

125F7 CUNEIFORM NUMERIC SIGN FOUR N18 ** 125F8 CUNEIFORM NUMERIC SIGN FIVE N18

125F9 🗱 CUNEIFORM NUMERIC SIGN SIX N18

```
125FA **** CUNEIFORM NUMERIC SIGN SEVEN N18
125FB **** CUNEIFORM NUMERIC SIGN EIGHT N18
125FC **** CUNEIFORM NUMERIC SIGN NINE N18
```

Numerals of capacity system S"

Used in the fourth millennium to measure various kinds of emmer

```
125FD # CUNEIFORM NUMERIC SIGN ONE N24B
125FE
            CUNEIFORM NUMERIC SIGN ONE N26B
125FF
            CUNEIFORM NUMERIC SIGN ONE N28B
        ₿ CUNEIFORM NUMERIC SIGN ONE N29AB
12600
12601
            CUNEIFORM NUMERIC SIGN ONE N41
12602
            CUNEIFORM NUMERIC SIGN TWO N41
12603
            CUNEIFORM NUMERIC SIGN THREE N41
12604
           CUNEIFORM NUMERIC SIGN FOUR N41
12605
        CUNEIFORM NUMERIC SIGN ONE N04
12606
       CUNEIFORM NUMERIC SIGN TWO N04
12607 CUNEIFORM NUMERIC SIGN THREE N04
12608 據 CUNEIFORM NUMERIC SIGN FOUR N04
12609 R CUNEIFORM NUMERIC SIGN FIVE N04
1260A
            CUNEIFORM NUMERIC SIGN ONE N19
1260B CUNEIFORM NUMERIC SIGN TWO N19
1260C # CUNEIFORM NUMERIC SIGN THREE N19
1260D ## CUNEIFORM NUMERIC SIGN FOUR N19
1260E # CUNEIFORM NUMERIC SIGN FIVE N19
1260F EXECUTE CUNEIFORM NUMERIC SIGN SIX N19
12610 *** CUNEIFORM NUMERIC SIGN SEVEN N19
12611 EXECUTE CUNEIFORM NUMERIC SIGN EIGHT N19
12612 CUNEIFORM NUMERIC SIGN NINE N19
12613 • CUNEIFORM NUMERIC SIGN ONE N46
12614 •• CUNEIFORM NUMERIC SIGN TWO N46
12615 ₽ CUNEIFORM NUMERIC SIGN ONE N36
12616 ▶ CUNEIFORM NUMERIC SIGN TWO N36
12616 DE CUNEIFORM NUMERIC SIGN TWO N36
12617 LEE CUNEIFORM NUMERIC SIGN THREE N36
12618 CUNEIFORM NUMERIC SIGN FOUR N36
12619 CUNEIFORM NUMERIC SIGN FIVE N36
12610 CUNEIFORM NUMERIC SIGN SEVEN N36
12610 CUNEIFORM NUMERIC SIGN SEVEN N36
12610 CUNEIFORM NUMERIC SIGN NINE N36
12611 CUNEIFORM NUMERIC SIGN ONE N36
12611 CUNEIFORM NUMERIC SIGN ONE N36
12612 CUNEIFORM NUMERIC SIGN ONE N36
1261F CUNEIFORM NUMERIC SIGN TWO N49
12620 CUNEIFORM NUMERIC SIGN THREE N49
12621 CUNEIFORM NUMERIC SIGN FOUR N49
```

Numerals of capacity system S*

Used in the fourth millennium to measure barley groats

```
12622
     E CUNEIFORM NUMERIC SIGN ONE N25
12623
         CUNEIFORM NUMERIC SIGN ONE N27
      Š
12624
      12625
      № CUNEIFORM NUMERIC SIGN ONE N29AC
12626

⊠ CUNEIFORM NUMERIC SIGN ONE N30AC

12627
         CUNEIFORM NUMERIC SIGN ONE N30CC
12628
         CUNEIFORM NUMERIC SIGN ONE N42A
12629
         CUNEIFORM NUMERIC SIGN TWO N42A
1262A
         CUNEIFORM NUMERIC SIGN THREE N42A
1262B
         CUNEIFORM NUMERIC SIGN FOUR N42A
1262C
         CUNEIFORM NUMERIC SIGN ONE N42B
1262D
         CUNEIFORM NUMERIC SIGN TWO N42B
1262E
      CUNEIFORM NUMERIC SIGN THREE IN CUNEIFORM NUMERIC SIGN FOUR N4:
CUNEIFORM NUMERIC SIGN ONE N05
         CUNEIFORM NUMERIC SIGN THREE N42B
1262F
         CUNEIFORM NUMERIC SIGN FOUR N42B
12630
12631
      CUNEIFORM NUMERIC SIGN TWO N05
12632
     CUNEIFORM NUMERIC SIGN THREE N05
12633  CUNEIFORM NUMERIC SIGN FOUR N05
12634 ₽₽ CUNEIFORM NUMERIC SIGN FIVE N05
12635
      • CUNEIFORM NUMERIC SIGN ONE N20
12636
         CUNEIFORM NUMERIC SIGN TWO N20
         CUNEIFORM NUMERIC SIGN THREE N20
12637
12638
      CUNEIFORM NUMERIC SIGN FOUR N20
```

```
12639 CUNEIFORM NUMERIC SIGN FIVE N20
1263A SSS CUNEIFORM NUMERIC SIGN SIX N20
1263B CUNEIFORM NUMERIC SIGN SEVEN N20
1263C CUNEIFORM NUMERIC SIGN EIGHT N20
1263D CUNEIFORM NUMERIC SIGN NINE N20
1263E • CUNEIFORM NUMERIC SIGN ONE N47
1263F • CUNEIFORM NUMERIC SIGN TWO N47
12640 CUNEIFORM NUMERIC SIGN ONE N37
12641 DO CUNEIFORM NUMERIC SIGN TWO N37
```

Numerals of system EN

Only attested in the Uruk IV period

```
12642
        CUNEIFORM NUMERIC SIGN ONE N09
12643
        CUNEIFORM NUMERIC SIGN ONE N11
12644
      ı
         CUNEIFORM NUMERIC SIGN ONE N12
12645
        CUNEIFORM NUMERIC SIGN ONE N07A
     12646
        CUNEIFORM NUMERIC SIGN TWO N07A
12647
        CUNEIFORM NUMERIC SIGN THREE N07A
12648
     CUNEIFORM NUMERIC SIGN ONE N07B
12649
        CUNEIFORM NUMERIC SIGN TWO N07B
1264A
     CUNEIFORM NUMERIC SIGN THREE N07B
```

Flat numerals

Rectangular numerals impressed with a flat tool, used in Ur in the Early Dynastic I-II period

```
1264B □ CUNEIFORM NUMERIC SIGN ONE N01 FLAT
         = 1 aš flat
         → 12038 ← cuneiform sign ash
1264C 
CUNEIFORM NUMERIC SIGN TWO N01 FLAT
1264D CUNEIFORM NUMERIC SIGN THREE N01 FLAT
1264E 
☐ CUNEIFORM NUMERIC SIGN FOUR N01 FLAT
1264F HP CUNEIFORM NUMERIC SIGN FIVE N01 FLAT
```

12650 **HH** CUNEIFORM NUMERIC SIGN SIX N01 FLAT 12651 CUNEIFORM NUMERIC SIGN SEVEN N01 FLAT 12652 HHH CUNEIFORM NUMERIC SIGN EIGHT N01 FLAT 12653 HTT CUNEIFORM NUMERIC SIGN NINE N01 FLAT □ CUNEIFORM NUMERIC SIGN ONE N08 FLAT 12654 12655 CUNEIFORM NUMERIC SIGN ONE N14 FLAT

= 1 u flat

→ 1230B < cuneiform sign u

```
12656
               CUNEIFORM NUMERIC SIGN TWO N14 FLAT
         .
 12657
               CUNEIFORM NUMERIC SIGN THREE N14 FLAT
         # CUNEIFORM NUMERIC SIGN FOUR N14 FLAT
 12658
 12659 LIP CUNEIFORM NUMERIC SIGN FIVE N14 FLAT
 1265A *** CUNEIFORM NUMERIC SIGN SIX N14 FLAT
1265B **** CUNEIFORM NUMERIC SIGN SEVEN N14 FLAT
1265C **** CUNEIFORM NUMERIC SIGN EIGHT N14 FLAT
1265D ***** CUNEIFORM NUMERIC SIGN NINE N14 FLAT
 1265E ☐ CUNEIFORM NUMERIC SIGN ONE N34 FLAT
12661 CUNEIFORM NUMERIC SIGN THREE N34 FLAT
12662 CUNEIFORM NUMERIC SIGN FOUR N34 FLAT
12663 CUNEIFORM NUMERIC SIGN FIVE N34 FLAT
12664 CUNEIFORM NUMERIC SIGN SIX N34 FLAT
12665 CUNEIFORM NUMERIC SIGN SEVEN N34 FLAT
12666 CUNEIFORM NUMERIC SIGN EIGHT N34 FLAT
12667 CUNEIFORM NUMERIC SIGN NINE N34 FLAT
12667 CUNEIFORM NUMERIC SIGN ONE N45 FLAT
12667 CUNEIFORM NUMERIC SIGN ONE N45 FLAT
 12668 ■■ CUNEIFORM NUMERIC SIGN TWO N45 FLAT
 12669
          ■ CUNEIFORM NUMERIC SIGN ONE N22 FLAT
 1266A
          ■ CUNEIFORM NUMERIC SIGN TWO N22 FLAT
 1266B
               CUNEIFORM NUMERIC SIGN ONE N51 FLAT
 1266C
               CUNEIFORM NUMERIC SIGN TWO N51 FLAT
1266D CUNEIFORM NUMERIC SIGN THREE N51 FLAT
1266E CUNEIFORM NUMERIC SIGN FOUR N51 FLAT
               CUNEIFORM NUMERIC SIGN THREE N51 FLAT
1266E CUNEIFORM NUMERIC SIGN FOUR N51 FLAT
1266F CUNEIFORM NUMERIC SIGN FIVE N51 FLAT
12670 CUNEIFORM NUMERIC SIGN SIX N51 FLAT
12671 CUNEIFORM NUMERIC SIGN SEVEN N51 FLAT
12672 CUNEIFORM NUMERIC SIGN EIGHT N51 FLAT
```

12673 UNEIFORM NUMERIC SIGN NINE N51 FLAT 12674 CUNEIFORM NUMERIC SIGN ONE N34 FLAT TENU = 1 n39a flat 12675 ■ CUNEIFORM NUMERIC SIGN ONE N04 FLAT 12676 **CUNEIFORM NUMERIC SIGN TWO N04 FLAT** 12677 CUNEIFORM NUMERIC SIGN THREE N04 FLAT 12678 **T** CUNEIFORM NUMERIC SIGN FOUR N04 FLAT 12679 **EF** CUNEIFORM NUMERIC SIGN FIVE N04 FLAT 1267A * CUNEIFORM NUMERIC SIGN ONE N19 FLAT 1267B **‡** CUNEIFORM NUMERIC SIGN TWO N19 FLAT 1267C # CUNEIFORM NUMERIC SIGN THREE N19 FLAT 1267D ## CUNEIFORM NUMERIC SIGN FOUR N19 FLAT 1267E # CUNEIFORM NUMERIC SIGN FIVE N19 FLAT 1267F ## CUNEIFORM NUMERIC SIGN SIX N19 FLAT 12680 ******** CUNEIFORM NUMERIC SIGN SEVEN N19 FLAT 12681 ******** CUNEIFORM NUMERIC SIGN EIGHT N19 FLAT 12682 ***** CUNEIFORM NUMERIC SIGN NINE N19 FLAT 12683 **•** CUNEIFORM NUMERIC SIGN ONE N46 FLAT 12684 **##** CUNEIFORM NUMERIC SIGN TWO N46 FLAT 12685 E CUNEIFORM NUMERIC SIGN ONE N36 FLAT

2.3 Properties

3 Rationale for curviform-cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [Pow72, p. 215] "in the same fashion as we use black and red ink"; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble — in texts from Early Dynastic IIIa Šuruppag as in the character code charts, That later in the third millennium⁶, — in Old Babylonian cursive, — in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists and dictionaries, and of composite texts. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]¹⁰. The diachronic approach is also useful for pedagogic applications.

3.2 Arguments for curviform-cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding 12 that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early

⁶Merging with U+1224E CUNEIFORM SIGN NI2.

⁷Notably [OSL] and the online edition of [Bor10] in [eBL, Signs].

⁸Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

⁹For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of

⟨⟨⟨□| → ⟨⟨□| → ⟩⟩, as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage of the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

 $^{^{10}}$ Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svä+24, pp. 242, 148].

¹¹For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

```
13
                                      S.
                                                        RP.
                                                                          ♦
          1(u) 1/2(diš)
                                     5(diš tenû)
1(neš_2)
                                                       gi
                                                                 us_2
                                                                          sa<sub>2</sub>
         7.5 (ropes)
                                     5
                                                       reed
                                                                 side
                                                                          equal
W<sup>14</sup>
                                            ➾
3(u)
                6(diš tenû)
                                            saŋ
                                                      sa_2
3 (ropes)
                                  reed
                                            front
                                                      equal
ašag-bi
                    1(bur<sub>3</sub><sup>c</sup>)
                                  1(e\check{s}e_3^c)
                                               1(iku<sup>c</sup>) 1/2(iku<sup>c</sup>)
 ašag=bi
 field=DEM15
```

tug_x(LAK 483)-si-ga-kam tugsiga =ak =am -Ø ploughed=gen=cop-3.sg.s

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as 16

465 metres, equal lengths 198 metres, equal widths this field is 9, 18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that "the number signs do not normally carry in their individual signs the meaning of what they are used to measure", and that curviform and cuneiform numerals "are not normally mixed together in a single numerical expression", noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from \triangleright to I numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in §3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that "The distinction between curved numerals and their cuneiform

 $^{^{13}}$ As noted in [Pow87, p. 466], this sign has a very short "tail" in this period, so that it is wider than it is tall, and can at first seem like a large — in copies. The photos in [CDLI] clearly show that this is in fact a vertical wedge.

¹⁴Note that ED IIIb ≺ numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign **≪** in [Po20054] looks more like Ur III ❖.

¹⁵Alternatively: area=POSS.3.SG.NH, "its area".

¹⁶We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases¹⁷."

The time has come to revisit this issue. As we will see in $\S3.3$, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In $\S3.4$ we will see that in some periods:

- the functions and use of the numerals vary beyond the mere \triangleright / | switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 A primer on classic Ur III and Old Babylonian metrologies

Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known 18 a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Robo8, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters [-\frac{1}{2}], the multiples of ten (10–50) are \(-\frac{1}{2}, \) but the other digits 11–59 are sequences \(\frac{1}{2} - \frac{1}{2} \) in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a non-positional system, hereafter the cuneiform discrete counting system $S_{Ur\ III/OB}$, which had different signs for the units [-\frac{1}{2}], tens \(-\frac{1}{2}, \) sixties [-\frac{1}{2}] (with larger wedges than the units), multiples of six hundred [-\frac{1}{2}], multiples of three thousand six hundreds \(-\frac{1}{2}, \) and multiples of thirty-six thousand \(\frac{1}{2} - \frac{1}{2} \).

3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram¹⁹, where the num-

 $^{^{17}}$ The cuneiform encoding process was planned in *stages* in [L2/03-162]. One might expect the second stage of encoding, which led to the creation of the Early Dynastic Cuneiform block, to incorporate the numerals needed for the representation and discussion of Early Dynastic texts; however, the proposal [L2/12-208] stated that "numerals have been omitted due to the complexity of numeral signs from this period. An expert in the metrology of this period must be consulted before these can be properly included"

¹⁸See, *e.g.*, [Uni16, §22.3.3, *sub* "Cuneiform Numerals"].

¹⁹These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*, see Figure 4.

ber over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Frio7, p. 378; Rob19]:

Note that for the range of areas given above, this system does not use any symbols separate from the numerals for the individual units ($ub\hat{u}m$, $ik\hat{u}m$, eblum, and $b\bar{u}rum$). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign \blacksquare functioning as punctuation²⁰, but the numerals are tied to the metrology; thus a surface of 5 $b\bar{u}r$ 1 ebel 4 $ik\hat{u}$ (100 $ik\hat{u}$, 36 ha) would be written²¹ 《 \prec \equiv . Contrast this with systems where the same numerals are used for different units, and overt units are used, as in "88 acres 3 roods 33 perches" or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the sign \diamondsuit is equal to sixty times \lt in the area system, but to three hundred and sixty times \lt in the discrete counting system.

For areas smaller than a quarter $ik\hat{u}m$, an overt unit is used, with one \mathbb{Z} (sar, $m\bar{u}$ sarum), approximately 36 m², written \mathbb{Z} , equal to one hundredth of an $ik\hat{u}m$, then sexigesimally subdivided in 60 \mathbb{Z} (gin₄, siqlum, "shekel"). For areas greater than 3600 $b\bar{u}r$, the \diamondsuit and \diamondsuit numerals are reused with a suffix \mathbb{Z} (gal, "big"), as follows [Robo8, p. 295 nn. b, c; Frio7, p. 378; Rob19]:

 $^{^{20}}$ This sign is sometimes interpreted as a measurement unit, and transliterated iku, see, e.g., [Pro20, pp. 385 sqq.], or transliterations in [Feu04] discussed in §3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system $G_{\rm Ur\,III/OB}$.

²¹As in the surface of the field of [14] (the city of Apisal) reported on [P102305, r. 1]

²²From [P213162], which has an additional ♣ ₺, two thirds (of a shekel), see §3.3.5.

3.3.3 The capacity system

Another such system of note is the one for capacities²³ [Frio7, p. 376; Rob19],

In the above diagram, the numerals for ban₂ are +, \ddagger , \ddagger , \ddagger , and \ddagger , and those for bariga are |, |, ||, and || (contrast ordinary || and || otherwise used with | numerals). Further, we have used the symbol \circ to express that, as described in [Hue11, p. 585 nn. (b), (f)], the sign \(\pm\) GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign > if present, and after the word for "grain" if present, as in

$$3554 \, \text{gur}$$
 $3 \, \text{ban}_2$ $6 \, \text{sila}_3$ of grain.

Observe that while large numbers of gur follow²⁵ system $S_{\text{Ur III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as < would be 10 gur 1 bariga, and <-♯ would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of alreadyencoded numeral series:

- I-I-I used in $S_{Ur III/OB}$ and the SPVS as well as with overt units;
- \leftarrow wsed in $G_{\text{Ur III}/\text{OB}}$, of which \leftarrow are also used in $S_{\text{Ur III}/\text{OB}}$ and the SPVS as well as with overt units;
- I-I used in $S_{Ur \, III/OB}$, and sometimes with overt units;
- $(-)^{*}$ used in $S_{\text{Hr III/OR}}$;
- \diamondsuit \diamondsuit used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;

 \diamondsuit \diamondsuit used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- - used in $C_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
- 十, 丰, 肆, 赴, 戡 used in C_{Ur III/OB};
- \P , \P , \P used in $\mathcal{C}_{\text{Ur III/OB}}$ —note the overlap with \P - \P ;
- \Join and \Join used in $G_{\text{Ur III/OB}}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Robo8, p. 78]: "The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity."

²³Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{\text{Ur III/OB}}$ based on a height of one cubit, see [Pow87, p. 488; Robo8, p. 294; Rob19].

²⁴From [P309594, obv. 11].

²⁵A larger unit, the guru₇ ($kar\hat{u}m$, grain heap), is sometimes used instead, with - \square \square \square \square \square (1 karûm = 3600 kurrū). See [Frio7, p. 415; Rob19].

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with I and C numerals with their system $S_{\rm Ur\,III/OB}$ values. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in §3.4. Metrological tables use the following units ²⁶ [Frio7, p. 118; Rob19]:

Two more units appear occasionally [Pow87, p. 459; Frio7, p. 118; Rob19]:

In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

Fractions of the $ik\hat{n}m$, $N = \frac{1}{2}$ —and $N = \frac{1}{4}$ —, have already been encountered. In other contexts, the fraction $\frac{1}{2}$ is written $\frac{1}{4}$, as in $\frac{1}{4}$ \mathbb{Z} . The fractions $\frac{1}{3}$ and $\frac{2}{3}$ are written \mathbb{Z} and \mathbb{Z} . The latter two signs are derived from curviform signs \mathbb{Z} and \mathbb{Z} , which are already separately encoded; these are in turn derived from the sign \mathbb{Z} , whose Early dynastic form resembles \mathbb{Z} , and \mathbb{Z} numerals; see [Pow71, pp. 113, 134]. The \mathbb{Z} is sometimes omitted, as in [P240545, *verso* 6 9; P221530; P221531; P271238; P274845].

3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the fourth millennium) clearly mirrors system $S_{\text{Ur III/OB}}$ [Frio7, p. 374; DE87, pp. 127, 165]:

$$\bullet \stackrel{10}{\leftarrow} \bullet \stackrel{6}{\leftarrow} \bullet \stackrel{10}{\leftarrow} \triangleright \stackrel{6}{\leftarrow} \bullet \stackrel{10}{\leftarrow} \triangleright. \tag{S}$$

Likewise the area system used in the Early Dynastic IIIb period for areas of one iku and greater [Dei22, p. 72; NDE93, p. 63; Frio7, p. 378; Lec16],

²⁶In this factor diagram and the next, we do not include the numerals. The units are no more than a factor of 60 apart, so higher numerals such as ₹ or ♦ are not used.

²⁷As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.

mirrors system $G_{\text{Ur III}/OB}$, with consistent use of the numerals: • corresponds to \langle , • to \Diamond , and • to \Diamond . An exception to this correspondence, noted in [L2/04-099, p. 4] (see §3.2), is that the vertical I from $S_{\text{Ur III}/OB}$ corresponds to a horizontal \triangleright in system S. This is however far from the only case of such a reallocation of function. The earlier form of the area system is [DE87, pp. 141, 165; Frio7, p. 378]:

$$\bullet \stackrel{6}{\leftarrow} \bullet \stackrel{10}{\leftarrow} \stackrel{3}{\leftarrow} \stackrel{6}{\leftarrow} \triangleright,$$
(G)

Observe that, as noted in [DE87, p. 142], \odot changes meaning from $10 \bullet$ in system G to $600 \bullet$ in system $G_{\text{ED IIIb}}$. System G is used in the fourth millennium, but also in the ED I–II period (it is the "area 2" system in [Chao3], whereas $G_{\text{ED IIIb}}$ is the "area 1" system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb area system 28 ,

$$\bullet \stackrel{10}{\longleftarrow} \bullet \stackrel{6}{\longleftarrow} \stackrel{10}{\longleftarrow} \stackrel{3}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{2}{\longleftarrow} \stackrel{2}{\longleftarrow} \stackrel{2}{\longleftarrow} \stackrel{2}{\longleftarrow} \stackrel{29}{\longleftarrow}, \qquad (G_{\rm ED\,IIIb})$$

with the numerals of a contemporaneous capacity system:

$$\underbrace{\stackrel{10}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{10}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{10}{\longleftarrow} \stackrel{4}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{7}{\lnot}, \qquad (C_{\pm 1} \rightleftharpoons 1)}_{= \pm 1}$$

3.4.1 Field lengths in Nirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign V is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign I is not written either. Lengths shorter than one rope are expressed in half-ropes using the $\frac{1}{2}$ sign I (again with no I), and then in reeds, with the sign I as follows:

This is the system that was used to express the sides of the field in [P020054] discussed in §3.2. In that tablet and most others from the same period, such as the

²⁸A variant is \bigcirc $\stackrel{10}{\longleftarrow}$ \bigcirc $\stackrel{6}{\longleftarrow}$ $\stackrel{10}{\longleftarrow}$ $\stackrel{3}{\longleftarrow}$ $\stackrel{6}{\longleftarrow}$ \bigcirc $\stackrel{2}{\smile}$ $\stackrel{2}{\smile}$ $\stackrel{2}{\smile}$ $\stackrel{2}{\smile}$ $\stackrel{2}{\smile}$ $\stackrel{2}{\smile}$ $\stackrel{2}{\smile}$, see [Pow72, p. 218].

²⁹The (fairly rare) cuneiform counterpart is \checkmark .

 $^{^{32}}$ The reeds are counted using $ten\hat{u}$ numerals, \nwarrow , \nwarrow , etc.

ones discussed in [Lec20], areas are expressed in system $G_{\rm ED\,IIIb}$, with curviform numerals³³; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope³⁴, the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and thus the interpretation of its numerals between systems $G_{\rm ED\,IIIb}$ and $L_{\rm ED\,IIIb}$. The sign which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without . There are other such co-occurrences contrasting between metrological systems; for instance, [Kre98, p. 303 n. 686] mentions the use of cuneiform numerals for days and months³⁵.

3.4.2 Dyke lengths in Nirsu

[Pow87, p. 466] notes that reeds "are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus." Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in [CDLI] finds many occurrences³⁶, such as:

$$\underbrace{\begin{array}{c} \stackrel{10}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{\bullet}{\longleftarrow} = \underbrace{\begin{array}{c} \stackrel{2}{\longleftarrow} \stackrel{\uparrow}{\longleftarrow} \stackrel{10}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{3}{\longleftarrow} \stackrel{3}{\longleftarrow} \stackrel{1}{\longleftarrow} \stackrel{1}{\longrightarrow} \stackrel$$

3.4.3 Butter, cheese and wheat in Nirsu

A similar mixture of cuneiform and curviform numerals may be observed with the capacity system; indeed, the previously described #1 #1 #1 \$\sqrt{1}\$ system uses \

³³A [CDLI] search for "(bur3)" (⟨ numerals used for areas) currently returns 15 ED IIIb results, whereas one for "(bur3@c)" (● numerals used for areas) returns 206. Further, when dated, the tablets with cuneiform bur3 are from the reigns of 田 智 報 報 (variously transliterated iri-inim-gi-na, uru-ka-gi-na, etc.) and 計論 剛 報 [(lugal-zag-ge-si), the last two kings of ED IIIb Lagaš.

³⁴This is the case of the sides of the field in [P020054, obv. ii 2–3].

³⁵That note also mentions a contrast between the use of curviform numerals to count people and curviform numerals to count bread alotted to them in [P010876]; such contrasts are more akin to styling, and might not, on their own, justify the disunification.

 $^{^{36}}$ A search for curviform numerals followed by some number of reeds counted in ($ten\hat{u}$) cuneiform numerals currently finds 125 occurrences across 47 tablets.

 $^{^{37}}$ [CDLI] only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the $\blacksquare \triangleright \parallel \cdot$ is not visible. Lecompte notes that the copy is faithful; indeed another $\blacksquare \triangleright \parallel \cdot$ can be seen both on the copy and the photo on obv. 2 2.

³⁸From copy.

³⁹With either unit omitted, as in the examples above, or both, as in [P020129, obv. 3 3] DDD ♣ ₹.

numerals for ≯ [Fri78, p. 43; Lec16]:

$$\underbrace{\stackrel{10}{\longrightarrow} \stackrel{6}{\longleftarrow} \stackrel{10}{\longleftarrow} \stackrel{0}{\longleftarrow} \stackrel{4}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{5}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{5}{\longleftarrow} \stackrel{6}{\longrightarrow} \stackrel{7}{\longrightarrow} \stackrel{7}{\longrightarrow} \stackrel{7}{\longrightarrow} \stackrel{10}{\longrightarrow} \stackrel{$$

as in [P020016, rev. 1 4; P020065; P020090, obv. 1 3, rev. 2 1; P020092, rev. 3, 1; P020137, obv. 1 2] and others, where ban₂ counted with \triangledown numerals are followed by sila₃ counted with $\char`$ numerals. Curviform numerals are also used to count sila₃, but not⁴⁰ as part of the \rightrightarrows systems. This contrast can be seen in [P220927], which measures butter ($\char`$, i₃) with a different capacity system, using the $\char`$ (dug, "pot") of 20 \char , with \bowtie and \bullet numerals⁴¹ for both the \char and the \char , thus [Pow87, pp. 504 sq.]

$$\underbrace{\bullet \overset{10}{\longleftrightarrow} \bowtie}_{\stackrel{1}{\Longrightarrow}} \stackrel{2}{\longleftrightarrow} \underbrace{\bullet \overset{10}{\longleftrightarrow} \bowtie}_{\stackrel{1}{\Longrightarrow}} \stackrel{\frac{3}{2}}{\longleftrightarrow} \stackrel{17}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{17}{\longleftrightarrow}, \qquad (C_{\stackrel{1}{\Longrightarrow}})$$

but counts cheese (\bowtie , ga'ar) using the \bowtie \bowtie capacity system, with \searrow numerals for the \nearrow .

Another capacity system in ED IIIb Nirsu is the $\sharp \$ $\Leftrightarrow \Leftrightarrow \$, the gur of two ul [Lec16]:

$$\underbrace{\stackrel{10}{\longleftarrow} \stackrel{2}{\triangleright}}_{\text{$\stackrel{?}{\rightleftharpoons}}} \stackrel{6}{\triangledown} \stackrel{6}{\rightleftharpoons} \stackrel{5}{\longleftarrow} \stackrel{6}{\searrow}. \qquad (C_{\text{$\stackrel{?}{\rightleftharpoons}}} \stackrel{6}{\longleftarrow})$$

Here the ローヘーマ contrast occurs not only within the numerals of the system, but with its units; this is perhaps best illustrated by the expressions 無 (主意 医・マ母ミソ ゴミ仁) in [P221746, rev. 2 2] and 無 医・母ミソ ゴミ仁) in [P221814, rev. 1 5].

3.4.4 Grain in Ebla

The mixing of curviform and cuneiform numerals within a metrological system is not specific to Nirsu.

⁴⁰As of this writing, the single occurrence of (ban2@c) followed by curviform numerals and sila3 in ED IIIb Nirsu transliterations on [CDLI], 4(ban2@c) 3(asz@c) sila3 in [P221815, obv. 47], is incorrect: it should be 4(ban2@c) 3(disz@t) sila3.

The system of grain⁴² capacities in Ebla uses the following units⁴³:

The \implies \bot and \bowtie \boxminus are generally counted using curviform numerals, and the smaller units using cuneiform | numerals44 Indeed, a search on [EbDA] for cooccurrences of either ★ ↔ or ▶ { ☐ with either of ☐ ↓ or ☐ ☐ ☐ finds the following expressions⁴⁵:

- 1. [P240532, verso 4 9] ▷ 🗮 ടി 🚻 😂 ⁴ Կ ખ ໄ 🔄
- 2. [P240548, verso 1 1] ▷◁\\ ≡ \\ \★
- 3. [P240655, recto 7 9] DD ₩ L 48 ₩ \ \ \
- 4. [P240579, verso 4 3] BBD □ □ □ L IV L □
- 5. [P240675, verso 2 2] ▷ 縱 日田 ₩以付
- 6. [P240609, verso 3 1] ▷◁∄邇 Ⅲ※↔
- 8. [P240697, recto 1 5] ▷ ▽ ⁴⁹ ||] □ || || * ← ◆

A glance it seems that 🕽 are counted with cuneiform numerals and higher units with curviform ones,

but we have not investigated this thoroughly.

¹³Another system uses different values for the 🎹 and 🎶 【ଐ, see [Cha12, p. 62; Arc15, p. 229 n. 12]:

$$\Box$$
 \bot $\overset{2}{\leftarrow}$ 日出 $\overset{3}{\leftarrow}$ \Box $\overset{4}{\leftarrow}$ \lor \downarrow \longleftrightarrow $\overset{5}{\leftarrow}$ $*$ \longleftrightarrow $\overset{5}{\leftarrow}$

⁴⁴As mentioned in [Cha12, p. 63], the 🎹 is also counted using the ₹-農 numeral series. Some instances of that usage are found transliterated n/6 in [EbDA]; in some cases the \mathbf{P} sign is omitted, and the ₱ numeral is then written before the ⊥ unit, as in ▶ ₱ ₡ ⊥ from [P240545, verso 13].

⁴⁵We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform \preceq \coprod and larger with cuneiform \coprod and smaller. In all cases the transcriptions given here are based on the [EbDA] transliterations, but the shape and orientation of the numerals was checked 46 on a photograph (from [EbDA] unless noted otherwise).

⁶As we will see in §3.7.2, [CDLI] transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

- ⁴⁷ba-ri₂-zu₂, a variant spelling.
- 48 Short for $\Box \diamondsuit \bot$.
- ⁵⁰Instead of the expected **Ⅳ** { ◀ □.
- ⁵¹ **Ⅲ *** → not legible on the EbDA photo.
- 52 From [CDLI] photo.
- ⁵³From photo in [Arc89, p. 6].
- ⁵⁴Laid out as []]; on stacking patterns see §6.2.

⁴²Liquid capacities use a different system [Arc15, p. 229 n. 12]:

Note that higher numbers of $\Box \Delta L$ are expressed in hundreds (*mi-at* ($\blacksquare \Xi L$) and then thousands (*li-im* $\Xi L \Delta L$), as is typical in Ebla [Arc15, p. 33], e.g., in the last example above or in [P240532, verso 2 3], $\Box L \Xi L \Delta L \Delta L$ (100 + 60 + 30 + 5 = 195 $\Box \Delta L$ of grain). These expressions correspond to the following factor diagram:

3.4.5 Use in modern publications

Because of their prevalence in the fourth millennium and Early Dynastic period, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 1–21.

for the words & u & a n a and & a n a b i. Deimel's reading & a n (a) for U came out of the reading /& a n t a k/ for the sign Y and the writing of & a (- n a) after the fractional signs for & u & a n a and & a n a b i in Old Sumerian texts. But this was an ill-conceived argument at its inception, for

Figure 1: Discussion of the readings of proposed ¬ and already-encoded | in [Pow71, p. 107].

^{**}SFrom photo in [Arc89, p. 6]; see also the [CDLI] photo and the copy in [Fri86, p. 17]. This tablet features unusual usage of vertical numerals—"somewhat unorganized", as described by [Fri86, p. 16]—, such as $\exists \Box \Diamond \bot$ or $\exists \Box \Diamond \bot$ or $\exists \Box \Diamond \bot$ are consistently counted with cuneiform numerals, and the higher units with cuneiform numerals.

⁶⁶Short for **⋘∤**�

⁵⁷ŠU₂+NIN₂-saŋ, an unusual variant spelling.

⁵⁸The untransliterated text would be 图 阿可可含粉狀圖 图 文集 回 文集 回 文章 词 to the atomically encoded ib2 × 31 = 文本 × III = 文本 · III = 工 · III =

sions also. In example 6, the writing the may imply a reading /8 a n a b i/, whereas the in example 11 should be read */8 u 8 a n a m i n/. Moreover, the question must be raised as to whether such writings as <00/6 k ù - b a b b a r * 8 a - m a d onot perhaps imply a linguistic resolution of */8 u 8 a n a m i n/ rather than /8 a n a b i/. I see no way of answering this question at present, but it is one which one

Figure 2: Discussion of the readings of proposed " and " as well as already-encoded " and " in [Pow71, p. 138].

iku fractions						
Girsu type 0 = :f.o.o 0 = :o.g.o 0 = :o.o.h	"BIN 8" type Φ = :p.o.o ⊕ = :o.q.o. 8 = :o.o.r	₹= :m.o				

Figure 3: A transliteration system for the fractions of the iku in [Pow72, p. 216].

1 "big cup" = 3 "big disks". Hence we can infer from the two ŠE-texts BIN 8,4 and BIN 8,5 together, that the "ŠE-system" makes use of number signs whose values are related to each other through the equations

A more convenient way of saying the same thing is to write out the "steps" between the various ŠE-units in what we shall call a "step-diagram" for the "ŠE-system":

Figure 4: The first factor diagram, in [Fri78, p. 10].

```
(C 234)
501010 = 5070 (C 314)
1 U 1 DQ 1 DQ = 6 · 2 U1DQ (C 27).
```

These metrological equations for the "unknowns" 0, o , $\overline{\text{U}}\,,\,\,$ etc.,can be treated exactly as ordinary equations for unknowns $\ x,\ y,\ z,\dots$. In particular, the equations can be simplified by subtraction of equal amounts from both sides of the identities. In this way the three equations above can be reduced to:

```
2° = 20 Ū
          (403° subtracted from both sides)
```

We can now read off from the first equation that $1 \circ = 10 \, \text{U}$, and from the second that 1 U = 6 o. Then the third equation can be simplified (by "substithition" of these values into the equation), to the following reduced form:

The most likely solution to this last equation is, of course,

Figure 5: The derivation of the factors of the bisexagesimal system in [Fri78, p. 15]⁶⁰.

$$5 \bullet + 4 \triangleright = 3 \bullet + 24 \triangleright \tag{C 234}$$

$$1 \bigcirc +1 \bullet +5 \bigcirc =7 \bullet +5 \bigcirc \tag{C 314}$$

$$1 \mathbf{\Xi} + 1 \mathbf{\Xi} + 1 \mathbf{\triangleright} = 10 \mathbf{\Xi} + 2 \mathbf{\triangleright} + 6 \bullet. \tag{C 27}$$

A diplomatic edition of [Fri78] could rotate the numerals using a higher-level protocol:

$$\begin{cases}
4 \[\] 5 \bullet = 24 \[\] 3 \bullet \\
5 \[\] 1 \bullet 1 \[\] = 5 \[\] 7 \bullet \\
1 \[\] 1 \[\] 1 \[\] = 6 \bullet 2 \[\] 1 \[\]
\end{cases}$$
(C 234)

$$\begin{cases} 5\sqrt{1} \cdot 1 \sqrt{1} = 5\sqrt{7} \cdot 6 & (C314) \end{cases}$$

$$|1 | 1 | 1 | 1 | = 6 \cdot 2 | 1 | 1 | 1 | (C 27).$$

⁶⁰The bisexagesimal system is used alike in proto-Elamite and proto-cuneiform texts, see [Fri78, p. 38]; the derivation in [Fri78, p. 15] is based on proto-Elamite artefacts. There is a typo in the equation for C 27: the right-hand side should have 10[X] rather than 1[X], otherwise nothing could be deduced about [X]. Note that in Friberg's early works [Fri78; Fri86; Fri86; Fri87], copies of fourth millennium and sometimes third millennium tablets are shown as vertical text (which they were for the scribes), and their numerals are written within horizontal text in the same orientation that they have if the tablet is taken as vertical text; in [UAX50] parlance, as if they had Vertical_Orientation=Upright. In addition, they are listed in these equations in the horizontal order in which they appear as vertical text (thus the rightmost numeral is the most significant, read first). Cuneiform is correctly Vertical_Orientation=Rotated, consistently both with modern practice and with the rotation between earlier vertical and later horizontal monumental inscriptions. Friberg's early conventions are not followed in later scholarship, and are abandoned in his own more recent works, such as [Frio7]; a more typical way to express the first equations might be

Figure 6: Discussion of proposed fractions ∇ , \triangleleft , \triangleleft , and $\not\equiv$, as well as already-encoded $\not\vdash$ and $\not\vdash$ in [Fri78, p. 49].

stein publizierten Zeicheniiste enthalten ist³, bis vor kurzem unentdeckt bleiben konnte. Erst 1978 machte der schwedische Mathematiker J. Friberg, ERBM I, 9-11, darauf aufmerksam, daß die Zeichen für die Zahlen Eins (□) und Zehn (●) in Verbindung mit dem Zeichen ŠE nicht im Verhältnis 1 zu 10 sondern im Verhältnis 1 zu 6 stehen. Bis dahin hatte man, obwohl die Andersartigkeit des in Verbindung mit dem Zeichen ŠE verwendeten Zahlzeichensystems bekannt war, für diese beiden häufigsten Zahlzeichen einheitlich ein Verhältnis 1 zu 10 unterstellt, obwohl es mehrere eindeutige Gegenbelege geb, vom denen zumindest diejenigen der Archaischen Texte aus Gemdet Nasr bereits früh publiziert und jedermann zugänglich waren⁴. Als Folge

Figure 7: Discussion in [DE87, p. 117] of the discovery in [Fri78, pp. 9–11] (see Figure 4) of the different relations between \triangleright and \bullet in systems G and \ref{G} ?.

Figure 8: The sign \$\ \text{used in a parallel with IX in [Eng88, pp. 131–133 n. 9], discussing an argument from [Pow72, p. 172] on the question of the language of the Uruk III texts.

of decreasing fractions $^1/_n$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus $\Xi=^1/_2$ N_{39} , $\varpi=^1/_3$ N_{39} , and so on. The first sign of the latter units, N_{34} ,

Figure 9: Description of the fractions Σ and $\overline{\otimes}$ in [Eng98, p. 113]⁶¹.

For instance, the first line contains the notations $1N_{34}$ $1N_{36_0}$; $2N_{20}$, which can be translated "60 of the (grain rations containing) = (of grain); (grain involved:) $2 \bullet$ (of ground barley)". This calculation contradicts the assumed numerical relationship $10N_1 = 1N_{14}$, since as was well known the measure represented by the sign N_{39} was 1/5 of that represented by N_1 , so that $60 \times 1/5 = 12$ and not 20, as $2N_{14}$ would imply. Instead of relying on complicated

Figure 10: The sign \bigcirc used as a capacity measure within otherwise translated text in [Eng98, p. 116].

⁶¹The text erroneously has N_{34} instead of N_{24} .

Die halbkreisförmigen Griffeleindrücke gehen manchmal in mehr oder weniger eckige Formen über $\{\P\}^{085}$. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen $\{\P\}$, die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren 086 . Selten treten mit \odot gebildete Zahlen auf 087 (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 11: Discussion of co-occurrences and contrasts between $\$, $\$, and $\$ in [Kre98, p. 303].

```
The calculations:
Obv. i
                       \begin{array}{ccc} 60 \times {}^{1}\!/_{\!5} \rhd & (\leftrightarrows) \\ 120 \times {}^{1}\!/_{\!10} \rhd & (\Xi) \\ 120 \times {}^{1}\!/_{\!15} \rhd & (\leftrightarrows) \end{array}
                        60 \times {}^{1}/_{5} \triangleright
                                                                    = 12 × 100 = = 8 × 100 = =
                        300 × ½0 ₪ (🖾)
                                                                     = 15 × 100 =
                       600 \times 1/_{25} ▷
Rev. i
                     1200
                                                                                                        1 × •
Obv. i 6
                     6000 \times \frac{1}{30} (GAR+6N<sub>57</sub>) = 200 \times 100 = 1 \times 100 3 × 10
                        120 \times \approx 1/4   (DUG<sub>a</sub>+U<sub>2a</sub>) \approx 30 \times 100   =
                       180 × ¹/₅ ⊳
                                              (DUG+AŠ_a) = 36 \times E = 
                                                                                                         6 \times 10^{\circ}
            3
                                                                   = 20 × 100 =
                                                                                                                         2 × 10
                       300 \times 1/15   (KAŠ<sub>a</sub>)
                                                                                                         3 \times 300
                                                                                          1 × ¹●.
Rev. i 3
                       600
                                                                                                        4 \times 100
                                                                                                                        3 × ⋾⇨
                                                                                                                                         1 × 🖘
                                                                                                                        3× 100
                                                                                          1 × •
                                                                                                         4 \times 0
Grand total of groats used:
                                                                             1 \times 1 \longrightarrow 2 \times 1 
                                                                                                        9×...
                                                                                                                        4 × ₁⊳
Grand total of malt used: 1N_{47} 4N_{20} 3N_5 1N_{42a} (rev. i 3) \times ^3/_5 \approx 8 \times •.
```

Figure 6. Transliteration and calculations of MSVO 4, 66.

Figure 12: Calculations from [P005468] transcribed in [Eng01, p. 132] using modern mathematical notation combined with some of the proposed characters.

strong similarities between "area" 1 and "area" 3 systems, the sign with two concentric discs (\bigcirc , notated N₅₀²⁷) remains problematic. It never appears in any numerical combination with the sign with a single disc (\bigcirc ,

Figure 13: Discussion of \bullet and \bullet ⁶² in [Chao3, p. 6].

⁶²The statement that these do not co-occur refers to the texts from ED I–II Ur; these signs co-occur both earlier and later in areas, with different relations as previously discussed.

 $^{1}/_{15}$, etc., of gur, we would expect the metrogram gur tent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs + and + cannot be of surface areas. These measures are usually followed by considered klasmatograms.

Metrological tablets from the end of the 4th millennium (Nissen, Damerow and Englund 1993, 55-59, to MSVO 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

> 1(iku) represents a surface of 3600m² 1(eše₃) represents a surface of 21,600m² etc.

the sign GAN2, which means either surface or field and

Figure 14: Discussion of Old Babylonian⁶⁴ capacity and fourth millennium area measures in [Proo9, p. 9].

formed by only two signs \(\) and \(\), repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: $1, 60, 60^2, 60^3, 1/60, 1/60^2$, etc. are written in the same way, with the vertical wedge \(\tilde{\cappa} \). The third feature concerns the exact function of

Figure 15: Description of the SPVS in [Cha12, p. 58], using the already-encoded signs I and **⟨**.

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648, 000 with:

Figure 16: Discussion of large numbers illustrated by \$_{\cup}^{\infty}\$ in [Cha12, p. 59]

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

DDD še bar ∇ ba-rí-zu

'3 gubar (capacity units) and 1 parīsu'.

[Cha12, p. 61].

⁶⁴The cuneiform text is Unicode-encoded.

⁶⁵Compare \diamondsuit \biguplus in system $G_{\text{Ur III}/OB}$. Sign order can be variable in early texts, see [Fox16, p. 8]. See [P010773], also discussed in [Fri07, p. 148], for an example of

→ and [P274845; P241764] for examples of $n \bullet \not\models$.

This is particularly true of the signs \bowtie , \bowtie , \bowtie and \bowtie , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written \bigcirc in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign \bigcirc is most often associated with the *parisu* measure, while the signs \bowtie , \bowtie , and \bowtie refer to 1, 2, 3,

Figure 18: Discussion in [Cha12, p. 64] of the relation between \P - \blacksquare and \triangledown in Mesopotamia and in Ebla.

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., transliterated as $2(\$ar_2)$ 1(ge\$'u) 3(u), means $2 \times 3600 + 1 \times 600 + 3 \times 10$). The system is based on an alternation of factors ten and

Figure 19: Explanation of the structure of the number ●● ● in [Pro20, p. 350].

might think of one fabric and a half, 11 but the presence of notations with "2 D 2 $\overline{\mathbb{U}}$ ", "3 D 3 $\overline{\mathbb{U}}$ ", and "6 D 6 $\overline{\mathbb{U}}$ " (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag's weight measurement system. 12 The notation "1 D gada" in o. ii 1 and r. vi 1, along with the total of "39



Fig. 1. Combinations of numerals attested in Š. 742.

Figure 20: Discussion of the contrast between ▷ and ¬ numerals in [Gor23, p. 162].

as, for example, in TM.75.G.3125 = *ARET* III 107 o. iv 1, " 4^{\square} " a_3 -da-um^{tu9}-2 $\stackrel{\wedge}{\sim}$ 4^{\square} aktum 4^{\square} ib₂^{tu9} ×3 $\stackrel{\wedge}{\rightarrow}$ sa₆ gunu₃" (Fig. 2).

Figure 21: Transliteration in [Gor23, p. 163] of [P242293, *recto* 4 1] incorporating untransliterated numerals.

3.5 Non-numeric usage

The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty' ⁶⁶. Do you know its reading ⁶⁷?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign — has, *inter alia*, the values aš, rum, and dili. While the horizontal numerals are most frequently written □ in the Early Dynastic period ⁶⁸, such non-numeric usage is almost ⁶⁹ always written —, for instance:

- in personal names in administrative texts, such as the following, which all contain
 □ numerals:
 - ﴿ in [P010424, rev. 1 5; P010458, obv. 1 5; P010459, obv. 2 5′] from ED IIIa أبو صلابيخ,
 - → in [P010960, obv. 2 5] from ED IIIa Šuruppag,
 - 通じー in [P251641, obv. 43] from ED IIIb Adab,
 - < i in [P252866, rev. 2 3] from ED IIIb Adab,
 - 卦>> **道一** in [P298637, rev. 2 4] from ED IIIb Umma;
- in the Sumerian word

 □ u₂-rum, "property" in ED IIIb Nirsu administrative texts which contain

 numerals, such as [P020006, obv. 2 3; P020008, rev. 1 2; P020018, rev. 1 2; P020018, rev. 1 2; P020024, obv. 1 4; P020030, obv. 3 1];
- in lexical texts:
 - in the divine name *** □ □** in the lexical texts [P010570, rev. 2 4; P010572, obv. 3 6], where the entries are prefixed with □.
 - in the word ➤ dili, "small fish" in [P010578, obv. 2 5], witness to Early Dynastic Fish,
 - in the same word with a determinative, ¬ ¼ dili^{ku6}, in [P010586, obv. 4 4, 6], witness to Early Dynastic Food, which starts with □ numerals.

This is a clear contrast between — and \triangleright in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name — \blacktriangleleft occurs on its own line in the aforementioned administrative texts; a line \triangleright \blacktriangleleft would instead be read as "one slave".

3.6 The limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in $\S 3.1$. While these benefits are real and

 $^{^{66}}$ The reader will recall that η es $_2$ is written \P , with a larger wedge than \P ; however, these signs have merged by the time Examenstext A is composed.

 $^{^{67}}$ Besides η es₂, a look at [OSL] shows that the values diš, ge₃, makkaš, sa η tak₄, and tal₄ are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

⁶⁸A [CDLI] search for "(asz@c)" finds 3296 ED texts, while a search for "(asz)" finds 81 ED texts, of which 46 also contain "(asz@c)".

⁶⁹Exceptions are discussed in §3.7.1.

now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as — that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [Sch35, pp. 123 sqq.] lists all of ——***** together with \bigcirc ——**** together with \bigcirc ——**** together with \bigcirc —, while \longrightarrow , and \longrightarrow , and only those, appear at the beginning of the sign list, since they have non-numeric values 70 . [Cat13, p. 58] has the numeric signs \bigcirc , \, \, \, \, whereas non-numeric — is at the beginning of the sign list, where its values a and c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usage, [Dei22] writes c and c are listed. For signs with both non-numeric and numeric usag

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes⁷¹ to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from \triangleright to I numerals prevents diachronic encoding even if \triangleright were unified with -. For instance, the lexical list Early Dynastic Food, already mentioned in §3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'-6']; however, they are written with I numerals, whereas they are written with \triangleright numerals in the Early Dynastic witnesses; since I and - are distinct⁷² characters, the \triangleright -- unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods⁷³, there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Rom23, *sub* "Adding Corpora"], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

 $^{^{70}}$ Non-numeric values of — were discussed in §3.5; → has the values man₃ and min₅, and is used for the word didli, "several, various"; → has the value eš₆.

⁷²Besides the contrasts in numeric usage mentioned in §3.3.3, these (already-encoded) characters were clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of — which are not shared with ↑ have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period, — is used for the preposition *ina*, "in", and ↑ for the preposition *ana*, "to".

⁷³See, *e.g.*, [Pow87, p. 493; Robo8, p. 55] on the unification of metrologies in the Old Akkadian period, resulting in the systems described in §3.3.

3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chartlike fonts⁷⁴ being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

3.7.1 The case of ŠAR₂

The character U+122B9

■ CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period⁷⁵, a contrast between non-numeric $\$ar_2$ written \diamondsuit and numeric $1(\$ar_2^c)$ written \blacksquare can be observed, similar to the contrast between \vdash and \trianglerighteq previously discussed in \$3.5. However, in lexical lists from \S uruppag and $Ebla^{76}$, as well as in the *Stèle des vautours*, non-numeric $\$ar_2$ is curviform:

- *,其\$● and *,其●+ =* in [P010566, obv. 10 10, 11];
- ● → and * → in [P010576, rev. 3 16, 17];
- $\bullet + 1$ in [P240986, recto 3 3]⁷⁷;
- \blacksquare **♠** in [P222399, obv. 17 9, 18 11, 22 12]⁷⁸.

It would be disruptive to the diachronic representation of text if non-numeric $\operatorname{\check{s}ar_2}$ were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph \diamondsuit , identical to the glyph of the proposed U+12579 \blacksquare CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric $\operatorname{\check{s}ar_2}$ is curviform solidly predates the transition from \blacksquare to \diamondsuit in the relevant metrological systems, there is no need to represent a \diamondsuit - \blacksquare contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

 $^{^{74} \}rm Most$ prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

⁷⁵For example, in personal names:

[—] 具食◇幫 in [P020019, rev. 12] from ED IIIb Nirsu;

[—] 澤卜◇和料 in [P020182, obv. 2 9], also from ED IIIb Nirsu;

^{— ▶ ★ ♦} in [P222186, obv. 3 3] from ED IIIb Umma;

 $^{^{76}}$ These are archaizing in other ways, e.g., they have a 且-囯 (NAM2-TUG2) split.

⁷⁷From copy in [Man81, ELLes 397].

 $^{^{78}}$ Note however ***** \Rightarrow \diamondsuit 何 on [P222399, obv. 6 17], see Figure 22. Curviform non-numeric šar $_2$ is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here



Note that in rare cases, such as [P222243, obv. 2 7] from ED IIIa Adab, nonnumeric — (here with the value rum) is written \square . It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 \square cuneiform numeric sign one NO1, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. in practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the [CDLI] transliteration (UR2-1(aš@c)) and the [ePSD2] one (uru8 rum) of this word disagree on that aspect. Since — has a cuneiform reference glyph, this does not pose any compatibility concerns.

3.7.2 Transliteration

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, \P from [P305639, rev. 21] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in [EbDA] transliterations: the aforementioned \P values in terms of the overt unit, as in [P240533, recto 3 3] is transliterated "20-1-1/2 gu_2 -bar 7 gu_2 -sagšu 2-1/2 an-zam 80 gu_2 ", reading both gu_2 and gu_3 as 1/2, but not distinguishing them.

⁷⁹As on [P249253].

 $^{^{80}\}text{As}$ of this writing, [EbDA] actually has an-zam $_\chi$, with U+1D6A greek subscript small letter chi.

This practice has been generalized to systematically indicate numeral shape; this is in particular the case in [CDLI], where the transliterations of some the above examples are "1(gesz2) 3(u) 5(asz) gur" for $\mbox{\ \ \ }\mbox{\ \ \ }\mbox{\ \ }\mbox{$

While there exist transliterations that distinguish — from I but not \mathbb{R} from —, such as the ones used in [DCCMT], the trend, especially in more recent works in third millennium studies, seems to be to represent numeral shape; for example, [MV24] gave an example of the input syntax used by the new "Urban Economy Begins" project as "10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)" for • \mathbb{R} , with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are \mathbb{R} rather than \mathbb{R} numerals. The "tradition of cavalierly dispensing with numerical notations in notations of administrative documents", as [Eng04, p. 30] describes it, seems to be fading.

3.8 Conclusions

Co-occurences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications. The contrast frequently applies to individual numerals, rather than to the span of entire numeric expressions.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple

⁸¹ \blacksquare interpreted as a unit, as discussed in §3.3.

⁸²short for nigida, an older reading of bariga; see [**Landsberger1950**; Pow75, p. 181; Fox22, p. 9].

cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted: (is a length of three ropes, but is an area of three bur; could be read as one and one where would be one and a half is a personal name, but would be "one slave".

In addition, there would be a risk of confusion about character identity should fontmakers attempt to treat the curviform and cuneiform numerals as unified. A designer concerned about the numeric-syllabic $\triangleright - \blacktriangleright$ contrast, and wishing to support diachronic encoding between systems $S_{\text{Ur III}/OB}$ and S, might give the I numeral series (which is typically only used numerically in the Early Dynastic period) the glyphs of the \triangleright numeral series, since the clear $\blacksquare - \blacksquare$ identification involves the same rotation; this would however make it impossible to represent capacity measures that use \triangledown . Similarly, in an effort to support diachronic encoding for 1/2(iku), one might be tempted to give \char the glyph of \char , thereby rendering the font unusable for quantities measured using the \char numeral series; an ED I–II Ur font designer could decide to give \char the same glyph as \circledR (that of the proposed \blacksquare), according to the older area system, making it impossible to represent the newer system.

4 Rationale for ED-Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millennium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millennium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, e.g., signs KAŠ_a, KAŠ_b, and KAŠ_c, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Eng01, pp. 34 sq.]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered scripts.

As part of the development of these conventions, a classification of fourth millennium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter N with a numeric subscript (sometimes with an additional alphabetic subscript): N_1 is \triangleright , N_{14} is \bullet , N_{34} is \triangleright , etc. Transliterations of numeric expression then use those to identify the type of number used, thus $5N_1$ is \triangleright , and $5N_{14}$ is \bullet .

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where

numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the [CDLI] transliteration of ▷▷▷ 등 or IPP® 등 in [P283802, rev. 1 6, 2 2] currently uses (barig@c) for the vertical numerals, since or numerals are typically capacity measures; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millennium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I–II period, even though the Sumerian text uses classical assyriological transliteration conventions; see [Chao3, p. 6 n. 27].

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millennium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in §3.7.2; they are indeed used interchangeably, as in [P011104] which uses the notation u@f in [ePSD2], but N14@f in [CDLI]. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millennium would therefore induce confusion as to which numerals should be used in third millennium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millennium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance, [A] N_{30c} is listed as a variant of [A] N_{30a} in [DE87, p. 166], where the numeric value of either in relation to [A] [A]

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from $4(ban_2@c)$ and $5(ban_2@c)$.]

6 Characters not included in this proposal

6.1 Missing numerals

TODO "Ten of the sixty numerical signs contained in the list in figure 27, moreover, do not belong to any of the identified systems. Three of them were apparently scribbled by an awkward pupil. As to four of those remaining, we are not sure whether they constitute derivations of other, as yet unknown numerical signs or whether they are in fact numerical signs at all. For at least two of the ten signs, txi and we can affirm that each formed part of two additional systems, about which we know nothing due to the fact that no informative texts have been unearthed with notations in these systems." [NDE93, p. 27] TODO N10 described as coming from P001319 which does not have it anymore. TODO N13 not attested in CDLI TODO (N_{17} not usefully numeric, $12N_{14}$ not encodable, etc.). Cite [DE87, p. 147] N30Cb not attested 7 and 8(diš $ten\hat{u}$) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.



Figure 23: The layout of case [P011099, rev. 23]; the numeral \$\frac{1}{2}\$ is rotated to fit the rounded corner of the tablet.

6.2 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some $stacking\ patterns$; for instance 91 is encoded both as U+12446 \ref{W} and as U+1240E \ref{W} . This is in part due to contrastive usage of stacking patterns. For instance, besides \ref{W} and \ref{W} which are characteristic of bariga measures, four bariga is written \ref{W} even where 41 is written \ref{W} , as in [P255010, obv. 2 3, rev. 1 17; P292843, obv. 4, rev. 5]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often \ref{W} \ref{W}

Likewise, many stacking patterns are attested for the curviform numerals proposed in this document, and it is not proposed to separately encode them. These distinctions would be incompatible with the state of the art in numeric transliterations, including those by Englund, who insisted on "a system of transliteration that reflects in a strict fashion the physical realities of the cuneiform inscriptions" [Engo4, p. 30], and they are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier tablets, as they are structured in rectangular cases rather than lines, so that numerals may be laid out across the case in whichever way fits the available space; this is illustrated in Figure 23. Note also that the numerals need to be considerably enlarged in order to reproduce the layout of the tablets, so that \$\frac{1}{2}\$ often spans two lines of cuneiform signs, as shown in Figure 24. This is impractical when these numerals are set in text that contrasts them with the larger \(\bigcirc\), and inconsistent with actual practice when typesetting these numerals, as illustrated in Figure 8: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dyn-

Figure 25: Three stacking patterns for U+12573 CUNEIFORM NUMERIC SIGN NINE N34. The one on the left is the reference glyph, used in Uruk III [P003499, obv. 1 1b; P004430, rev. 1 2], and widely afterwards, *e.g.*, ED IIIa Šuruppag [P010678, obv. 2], ED IIIb Nirsu [P020057, obv. 1 3], Old Akkadian Umma [P212464, obv. 11]. The ones in the middle and right are used in two Uruk IV tablets [P001243, rev. P004500, rev. 2]. All three Uruk examples are transliterated 9(N34) in [CDLI].

astic period, but that are also attested in the fourth millennium in the Uruk III period; the fourth millennium, especially the Uruk IV period, also frequently features numerals that use a more vertical layout, as illustrated in Figure 25. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millennium, usage in third millennium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents \triangleright , \triangleright , \bullet , and \bullet must remain consistent across the numeral series, lest a \triangleright numeral be confused with an \triangleright numeral. Since the single indents are frequently used in running text, as illustrated in §3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 25 uses OpenType stylistic alternates, and Figure 23 rotates the character \$\frac{1}{2}\$, in both cases preserving the plain text backing.

6.3 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Eng01].

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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ORACC: etcsri/Q001546.

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ern California. CDLI: P235312.

ORACC: epsd2/P235312.

[P240531] Excavation number TM.75.G.00265.

CDLI: P240531. Ebda: 1415.

[P240532] Excavation number TM.75.G.00266.

CDLI: P240532. Ebda: 1324.

[P240533] Excavation number TM.75.G.00267.

CDLI: P240533. Ebda: 1379.

[P240545] Excavation number TM.75.G.00299.

CDLI: P240545.

[P240548] Excavation number TM.75.G.00302.

CDLI: P240548. Ebda: 1350.

[P240579] Excavation number TM.75.G.00341.

CDLI: P240579. Ebda: 1364.

[P240597] Excavation number TM.75.G.00407.

CDLI: P240597.

[P240609] Excavation number TM.75.G.00440.

CDLI: P240609. Ebda: 1378.

[P240653] Excavation number TM.75.G.00535.

CDLI: P240653. Ebda: 1382.

[P240654] Excavation number TM.75.G.00536.

CDLI: P240654. Ebda: 1383.

[P240655] Excavation number TM.75.G.00537.

CDLI: P240655. Ebda: 1358.

[P240675] Excavation number TM.75.G.00557.

CDLI: P240675. Ebda: 1371.

[P240697] Excavation number TM.75.G.00579.

CDLI: P240697. Ebda: 1381.

[P240964] Excavation number TM.75.G.01392.

CDLI: P240964.

ORACC: dccmt/P240964.

Ebda: 3184.

Excavation number TM.75.G.01415. [P240986] CDLI: P240986. ORACC: dcclt/P240986. [P241708] Excavation number TM.75.G.02143. CDLI: P241708. Ebda: 3173. [P241764] Excavation number TM.75.G.02200. CDLI: P241764. Excavation number TM.75.G.02346. [P241904] CDLI: P241904. Ebda: 3183. ORACC: dccmt/P241904. [P242293] Excavation number TM.75.G.03125. CDLI: P242293. Ebda: 217. Code de Hammurabi. Sb 00008. Paris, France: Musée du Louvre. [P249253] CDLI: P249253. [P251641] MS 2464. Oslo, Norway: Schøyen Collection. CDLI: P251641. ORACC: epsd2/P251641. MS 3830. Oslo, Norway: Schøyen Collection. [P252866] CDLI: P252866. ORACC: epsd2/P252866. YBC 04698. New Haven, Connecticut, United States: Yale Babylonian [P255010] Collection. CDLI: P255010. [P271238] Anonymous. CDLI: P271238. ORACC: epsd2/P271238. CUNES 50-08-001. Ithaca, New York, United States: Department of [P274845] Near Eastern Studies, Cornell University. CDLI: P274845. [P283802] Ist Š 0742. Arkeoloji Müzeleri. CDLI: P283802. ORACC: epsd2/P283802. NBC 05385. New Haven, Connecticut, United States: Nies Babylonian [P292843] Collection, Yale Babylonian Collection. CDLI: P292843. [P298637] NBC 06978. New Haven, Connecticut, United States: Nies Babylonian Collection, Yale Babylonian Collection. CDLI: P298637. ORACC: epsd2/P298637.

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Collection. CDLI: P305639.

YBC 04398. New Haven, Connecticut, United States: Yale Babylonian

[P307255] YBC 06219. New Haven, Connecticut, United States: Yale Babylonian

Collection.

CDLI: P307255.

[P309594] YBC 08761. New Haven, Connecticut, United States: Yale Babylonian

Collection. CDLI: P309594.

ORACC: epsd2/P309594.

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CDLI: P386847.

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