# Archaic cuneiform numbers

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# **Contents**

1	Sun	nmary		2		
2	Pro	posed	changes to the Standard	3		
	2.1	Sumn	nary of proposed characters	3		
	2.2	Prope	erties	3		
	2.3	Chara	cter names list	3		
	2.4		specification text			
3	Rationale for curviform-cuneiform disunification					
	3.1	The c	uneiform encoding model	3		
	3.2	Argur	nents for curviform-cuneiform unification	3		
	3.3	A prin	ner on classic Ur III and Old Babylonian metrologies	5		
		3.3.1	The discrete counting system	5		
		3.3.2	The area system	6		
		3.3.3	The capacity system	6		
		3.3.4	The length system	7		
		3.3.5	Fractions	8		
	3.4 Curviform numerals in early metrologies					
		3.4.1	Field lengths in Nirsu	8		
		3.4.2	Dyke lengths in Nirsu	9		
		3.4.3	Grain in Nirsu	9		
		3.4.4	Grain in Ebla	9		
		3.4.5	Use in modern publications	11		
	3.5	Non-r	numeric usage	13		
	3.6	Limited benefits of diachronic encoding for numerals				
	3.7	Comp	atibility considerations	15		
		3.7.1	The case of $SAR_2$	15		
		3.7.2	Transliteration	16		
	3.8	Concl	usions	17		
4	Rat	ionale	for ED-Uruk numeral unification	18		
5	Con	sidera	tions on individual numeral series	18		

6	Characters not included in this proposal				
	6.1	Missing numerals	18		
	6.2	Stacking patterns	18		
	6.3	Matters for higher-level protocols	18		
Ac	knov	vledgements	18		
Re	fere	nces	18		
		efacts			
	Unio	code documents	20		
	Oth	er documents	21		

# 1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script<sup>1</sup> and the proto-cuneiform script<sup>2</sup>. The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed "curved", or "curviform", numerals<sup>3</sup> should however *not* be unified with the already-encoded cuneiform numerals<sup>4</sup>. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script\_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
Numerais		Fyictin	a Yeny
Non-numeric signs	Future Pcun	- Existing Xsux	

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

 $<sup>^1 \</sup>rm ISO$ 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.  $^2 \rm ISO$ 15924: Pcun, not yet encoded.

 $<sup>^{3} \</sup>bowtie -1111 \ 1-9 (a\$^{c}=N_{1}), \ \cdot -\sharp \ 1-5 (u^{c}=N_{14}), \ \bowtie -\sharp \ 1-9 (\eta e\$_{2}{}^{c}=N_{34}), \ \bowtie -\sharp \ 1-5 (\eta e\$'u^{c}=N_{48}), \ \text{etc.}$ 

<sup>&</sup>lt;sup>4</sup> -- - → 1-9(aš), <- 1-5(u), - 1-5(u), 1-9(neš<sub>2</sub>), - 1-5(neš²u), etc.

# 2 Proposed changes to the Standard

- 2.1 Summary of proposed characters
- 2.2 Properties
- 2.3 Character names list
- 2.4 Core specification text

# 3 Rationale for curviform-cuneiform disunification

TODO(egg): blurb.

# 3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble in texts from Early Dynastic IIIa Šuruppag as in the character code charts, if later in the third millenium, if in Old Babylonian cursive, in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists<sup>6</sup> and dictionaries<sup>7</sup>, and of composite texts<sup>8</sup>. By being compatible with similarly diachronic transliteration practice (that is, by avoiding distinctions finer than those made in transliteration), the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]<sup>9</sup>. The diachronic approach is also useful for pedagogic applications<sup>10</sup>.

# 3.2 Arguments for curviform-cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding<sup>11</sup> that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

<sup>&</sup>lt;sup>5</sup>Merging with U+1224E **☆**₩ NI<sub>2</sub>.

<sup>&</sup>lt;sup>6</sup>Notably [VT+14] and the online edition of [Bor10] in [Jim+23, Signs].

<sup>&</sup>lt;sup>7</sup>Notably [T]V<sub>17</sub>] and the online edition of [Sch<sub>10</sub>] in [lim+2<sub>3</sub>, Dictionary].

<sup>\*</sup>For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of the law of the stellar, as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

<sup>&</sup>lt;sup>9</sup>Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svä+24, pp. 242, 148].

<sup>&</sup>lt;sup>10</sup>For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

<sup>&</sup>lt;sup>11</sup>At that time scoped to the répertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as & = \$ £ \$ £ \$ were informed by Early Dynastic distinctions.

```
1(NEŠ_2)
             1(U)
                                    5(DIŠ tenû)
                     1/2(DIŠ)
                                                              us<sub>2</sub>
                                                                       sa_2
         7.5 (ropes)
                                                     reed
                                                              side
                                                                      equal
((13
                                                  3(U)
              6(DIŠ tenû)
                                         saŋ
                                gi
                                                  sa_2
3(ropes)
                                        front
                                                  equal
直立
                                                     V
              1(BUR_3^c)
                           1(EŠE_3^c)
                                         1(IKU<sup>c</sup>)
                                                     1/2(IKU<sup>c</sup>)
ašag-bi
this field
                                                     tug<sub>x</sub>(LAK483)-si-ga-kam<sup>14</sup>
```

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as  $^{15}$ 

deep ploughing

```
465 metres, equal lengths
198 metres, equal widths
this field: 9, 18 hectares, deeply ploughed
```

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that "the number signs do not normally carry in their individual signs the meaning of what they are used to measure", and that curviform and cuneiform numerals "are not normally mixed together in a single numerical expression", noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from to I numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that "The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases."

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

— the functions and use of the numerals vary beyond the mere P/I switch;

<sup>&</sup>lt;sup>12</sup>As noted in [Pow87, p. 466], this sign has a very short "tail" in this period, so that it is wider than it is tall, and can at first seem like a large ← in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

<sup>&</sup>lt;sup>14</sup>Transliteration after [Lec20, p. 325].

<sup>&</sup>lt;sup>15</sup>We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

# 3.3 A primer on classic Ur III and Old Babylonian metrologies

Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

### 3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram<sup>17</sup>, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

<sup>&</sup>lt;sup>16</sup>See, e.g., [Uni16, §22.3.3, sub "Cuneiform Numerals"].

 $<sup>^{17}</sup>$ These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called step-diagrams.

### 3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Frio7, p. 378; Rob19]:

$$\Leftrightarrow \stackrel{10}{\longleftrightarrow} \diamondsuit \stackrel{6}{\longleftrightarrow} \stackrel{1}{\longleftrightarrow} \stackrel{10}{\longleftrightarrow} \stackrel{3}{\longleftrightarrow} \stackrel{6}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{1}{\longleftrightarrow} \stackrel{1}{\longleftrightarrow} \stackrel{10}{\longleftrightarrow} \stackrel{1}{\longleftrightarrow} \stackrel{10}{\longleftrightarrow} \stackrel{10}{\longleftrightarrow}$$

Note that for the range of areas given above  $^{18}$ , this system does not use any symbols separate from the numerals for the individual units ( $ub\hat{u}m$ ,  $ik\hat{u}m$ , eblum, and  $b\bar{u}rum$ ). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign  $\blacksquare$  functioning as punctuation  $^{19}$ , but the numerals are tied to the metrology; thus a surface of 5  $b\bar{u}r\bar{u}$  1 eblum 4  $ik\hat{u}$  (100  $ik\hat{u}$ , 36 ha) would be written  $^{20}$  《  $\prec$   $\vDash$   $\blacksquare$ . Contrast this with systems where the same numerals are used for different units, and overt units are used, as in "88 acres 3 roods 33 perches" or  $\Xi$  頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the ŠAR $_2$  sign  $\diamondsuit$  is equal to sixty times the U sign  $\lt$  in the area system, but to three hundred and sixty times  $\lt$  in the discrete counting system.

# 3.3.3 The capacity system

Another such system of note is the one for capacities<sup>21</sup> [Frio7, p. 376; Rob19],

where the numerals for  $ban_2$  are +,  $\ddagger$ ,  $\ddagger$ ,  $\ddagger$ , and  $\ddagger$ , and those for bariga are  $\cdot$ ,  $\cdot$ ,  $\cdot$ , and  $\cdot$  (contrast ordinary  $\cdot$ ) and  $\cdot$  (the sign GUR  $\cdot$ ). As described in [Hue11, p. 585 with notes (b) and (f)], the sign GUR  $\cdot$ , while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign  $\cdot$  if present, and after the word for "grain" if present, as in

$$\underbrace{\otimes \biguplus \stackrel{10}{\longleftrightarrow} \Diamond \biguplus \stackrel{6}{\longleftrightarrow} \Diamond \stackrel{10}{\longleftrightarrow} \Diamond \stackrel{6}{\longleftrightarrow} \stackrel{10}{\longleftrightarrow} \Diamond \stackrel{6}{\longleftrightarrow} \stackrel{10}{\longleftrightarrow} \Diamond \stackrel{3}{\longleftrightarrow} \stackrel{6}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2.5}{\longleftrightarrow} \underbrace{\stackrel{10}{\longleftrightarrow} \underbrace{\stackrel{1}{\longleftrightarrow}} \underbrace{\stackrel{1}{\longleftrightarrow} \underbrace{\stackrel{10}{\longleftrightarrow}} \underbrace{\stackrel{1}{\longleftrightarrow}} \underbrace{\stackrel{10}{\longleftrightarrow} \underbrace{\stackrel{1}{\longleftrightarrow}} \underbrace{\stackrel{10}{\longleftrightarrow}} \underbrace{\stackrel{1}{\longleftrightarrow} \underbrace{\stackrel{10}{\longleftrightarrow}} \underbrace{\stackrel{1}{\longleftrightarrow}} \underbrace{\stackrel{10}{\longleftrightarrow}} \underbrace{\stackrel{1}{\longleftrightarrow} \underbrace{\stackrel{10}{\longleftrightarrow}} \underbrace{\stackrel{1}{\longleftrightarrow}} \underbrace{\stackrel{10}{\longleftrightarrow}} \underbrace{\stackrel{10}{\longleftrightarrow$$

<sup>&</sup>lt;sup>18</sup>For areas smaller than a quarter  $ik\hat{u}m$ , an overt unit is used, with 1  $m\bar{u}$ sarum (36 m²) written !  $\stackrel{\blacksquare}{\cong}$ 1, equal to one hundredth of an  $ik\hat{u}m$ , then sexigesimally subdivided in 60  $\stackrel{\blacksquare}{=}$ 1 (shekels). For areas greater than 3600  $b\bar{u}r\bar{u}$ , the  $\diamondsuit$  and  $\diamondsuit$  numerals are reused with a suffix  $\stackrel{\blacksquare}{=}$ 4 (gal, Sumerian: big), as follows [Robo8, p. 295 with notes b and c; Frio7, p. 378; Rob19]:

<sup>&</sup>lt;sup>19</sup>TODO(egg): acknowledge Proust 2020 but note that this is irrelevant to encoding concerns

<sup>&</sup>lt;sup>20</sup>As in the surface of the field of **|** ★ **|** ★ (the city of Apisal) reported on [P102305, r. 1]

<sup>&</sup>lt;sup>21</sup>Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system  $G_{\text{Ur III}/OB}$  based on a height of one cubit, see[Pow87, p. 488; Robo8, p. 294; Rob19].

Observe that while large numbers of gur follow<sup>23</sup> system  $S_{\text{Ur III}/OB}$ , the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as ⟨!\pm would be 10 gur 1 bariga, and ⟨-\pm would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of alreadyencoded numeral series:

- I-# used in  $S_{Ur III/OB}$  and the SPVS as well as with overt units;
- $\leftarrow$  used in  $G_{\text{Ur III}/\text{OB}}$ , of which  $\leftarrow$  are also used in  $S_{\text{Ur III}/\text{OB}}$  and the SPVS as well as with overt units;
- I-W used in  $S_{\text{Ur III/OB}}$ , and sometimes with overt units;
- K-W used in  $S_{\text{Ur III/OB}}$ ;
- $\diamond$   $\diamond$  used in  $S_{\text{Ur III/OB}}$  and  $G_{\text{Ur III/OB}}$ ;
    $\diamond$   $\diamond$  used in  $S_{\text{Ur III/OB}}$  and  $G_{\text{Ur III/OB}}$ ;
- -- used in  $C_{\text{Ur III/OB}}$  as well as with overt units of the weight system;
- 十, 丰, 隼, 卧, 鼣 used in  $C_{\text{Ur III}/OB}$ ;
- $I, I, II, II used in C_{Ur III/OB}$ —note the overlap with I-III;
- $\prec$  and  $\rightleftarrows$  used in  $G_{Ur III/OB}$ .

Only in the SPVS did numerals exist truly independently of metrology; to quote [Robo8, p. 78]: "The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity."

#### 3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with  $\$  - and  $\$  numerals with their system  $S_{\text{Ur III}/OB}$  values  $^{24}$ . Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Frio7, p. 118; Rob19]:

Two more units appear occasionally [Pow87, p. 459; Frio7, p. 118; Rob19]:

| \*\* \*\* 
$$\leftarrow$$
 \*\*  $\leftarrow$  \*\*

<sup>&</sup>lt;sup>22</sup>From [**P309594**].

<sup>&</sup>lt;sup>23</sup>A larger unit, the guru<sub>7</sub> ( $kar\hat{u}m$ , grain heap), is sometimes used instead, with -  $\square \lozenge \square \triangleleft \square$ (1 karûm = 3600 kurrū). See [Frio7, p. 415; Rob19].

<sup>&</sup>lt;sup>24</sup>Adjacent units are no more than a factor of 60 apart, so higher numerals such as ₹ or ♦ are not

<sup>&</sup>lt;sup>25</sup>TODO

In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

## 3.3.5 Fractions

**TODO** 

# 3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period match the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system  $S_{\text{Ur III/OB}}$  [Frio7, p. 374; DE87, pp. 127, 165]:

$$\bullet \xleftarrow{10} \bullet \xleftarrow{6} \bullet \xrightarrow{10} \overset{10}{\smile} \overset{6}{\smile} \bullet \xrightarrow{10} \smile. \tag{S}$$

Likewise the area system used in the Early Dynastic IIIb period mirrors system  $G_{\text{Ur III}/OB}$  [Dei22, p. 72; NDE93, p. 63; Fri07, p. 378; Gom16]:

$$\bullet \xleftarrow{10} \bullet \xleftarrow{6} * \xleftarrow{10} \bullet \xleftarrow{3} \bullet \xleftarrow{6} \triangleright, \qquad (G_{\rm ED\,IIIb})$$

As noted in [L2/04-099, p. 4] (see section 3.2), the vertical  $\P$  from  $S_{\text{Ur III/OB}}$  becomes a horizontal  $\P$  in system S. It is however far from the only case of such a reallocation of function. The earlier form of System G was [DE87, pp. 141, 165; Frio7, p. 378]:

$$\bullet \stackrel{6}{\leftarrow} \bullet \stackrel{10}{\leftarrow} \bullet \stackrel{3}{\leftarrow} \bullet \stackrel{6}{\leftarrow} \triangleright, \tag{G}$$

Observe that, as noted in [DE87, p. 142],  $^{\odot}$  changes meaning from  $10^{\bullet}$  in system G to  $10^{\bullet}$  in system G is used in the Uruk period, but also in the ED I–II period (it is the "area 2" system in [Chao3], whereas  $G_{\rm ED~IIIb}$  is the "area 1" system).

### 3.4.1 Field lengths in Nirsu

The length system Early Dynastic IIIb of the state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign  $\slash$  is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign  $\slash$  is not written either. Length shorter than one rope are expressed in half-ropes using the 1/2 sign  $\slash$  (again with no  $\slash$ ), and then in reeds, with the sign  $\slash$ 3. Effectively, this yields the following factor diagram:

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and others from the same period, such as the ones discussed in [Lec20], areas are expressed in system  $G_{\rm ED~IIIb}$ , with curviform numerals<sup>29</sup>; in the absence of overt units, such as when dealing with length that

 $<sup>^{28}</sup>$  Note that the reeds are counted using  $ten\hat{u}$  numerals,  $\nwarrow$  ,  $\diamondsuit$  , etc.

<sup>&</sup>lt;sup>29</sup>TODO(egg): Note the handful of late Urukagina tablets that start to have cuneiform areas.

are integer multiples of a half-rope<sup>30</sup>, the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and therefore the interpretation of its numerals between systems  $G_{\rm ED\,IIIb}$  and  $L_{\rm ED\,IIIb}$ . The sign  $_{\rm HI}$ , which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without  $_{\rm HI}$ .

# 3.4.2 Dyke lengths in Nirsu

[Pow87, p. 466] notes that reeds "are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus." [TODO(egg): also mention Krebernik 1998 p. 303 with note 686.] Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in CDLI finds many occurrences<sup>31</sup>, such as:

These expressions use an explicit sign  $\slash$  (counted in multiples of ten) or  $\slash$ . This notation—but not its use of curviform numerals—is remarked on in [Lec20, p. 290 with note 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. These 34 can be summarized by the following factor diagram:

$$\underbrace{\stackrel{10}{\longrightarrow}}_{V \cong I} \underbrace{\stackrel{6}{\longrightarrow}}_{I \cong I} = \underbrace{\stackrel{2}{\longrightarrow}}_{I \cong I} \underbrace{\stackrel{10}{\longrightarrow}}_{I \cong I} \underbrace{\stackrel{6}{\longrightarrow}}_{I \cong I} \underbrace{\stackrel{3}{\longrightarrow}}_{I \cong I} \underbrace{\stackrel{1}{\longrightarrow}}_{I \cong I} \underbrace{\stackrel{10}{\longrightarrow}}_{I \cong I} \underbrace{\stackrel{6}{\longrightarrow}}_{I \cong I} \underbrace{\stackrel{1}{\longrightarrow}}_{I \cong I} \underbrace{\stackrel{10}{\longrightarrow}}_{I \cong I} \underbrace{\stackrel{10}{$$

# 3.4.3 Grain in Nirsu

TODO note the **♯** \* **☼**.

# 3.4.4 Grain in Ebla

Lengths of Early Dynastic IIIb dykes from Nirsu are far from the only numeric expressions that mix curviform and cuneiform numerals.

<sup>&</sup>lt;sup>30</sup>This is the case of the sides of the field in [P020054, obv. ii 2–3].

 $<sup>^{31}</sup>$ A search for curviform numerals followed by some number of reeds counted in ( $ten\hat{u}$ ) cuneiform numerals currently finds 125 occurrences across 47 tablets.

 $<sup>^{32}</sup>$ CDLI only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the  $\blacksquare \triangleright \parallel \cdot$  is not visible. Lecompte notes that the copy is faithful; indeed another  $\blacksquare \triangleright \parallel \cdot$  can be seen both on the copy and the photo on obv. 2, 2.

<sup>&</sup>lt;sup>33</sup>From copy.

 $<sup>^{34}</sup>$ TODO Cite also DP 568, the one with  $^{\square}$  and  $^{\square}$  $^{\square}$  even though it has no reeds.

<sup>&</sup>lt;sup>35</sup>TODO(egg): Note that one unit may be omitted if the other is present

The system of grain<sup>36</sup> capacities in Ebla uses the following units<sup>37</sup>:

The  $\Box \triangle \bot$  and  $\lhd \Box \Box \Box$  are generally counted using curviform numerals, and the smaller units using cuneiform I numerals. Indeed, a search on [Mil+07] for cooccurrences of  $* \leftrightarrow$  with either of  $\Box \land \bot$  or  $\lhd H =$  finds the following expres-

- 2. [P240548, verso 1, 1] ▷되冊 Ⅲ米↔
- 3. [P240655, recto 7, 9] DD ₩ L 41 ₩ W I 41

- 8. [P240697, recto 1, 5] \(\nabla^{42}\) \(\nabla^{42}\) \(\nabla^{42}\)

Note that higher numbers of □ L are expressed in hundreds (mi-at ( and ) and 

**恒以** 
$$\stackrel{30}{\leftarrow}$$
  $\stackrel{1}{\underset{\text{la-ha}}{\leftarrow}}$   $\stackrel{6}{\underset{\text{sila}_3}{\leftarrow}}$  #  $\stackrel{4}{\underset{\text{an-zam}_x}{\leftarrow}}$ .

At a glance it seems that 🕽 are counted with cuneiform numerals and higher units with curviform ones,

$$\underbrace{\overset{\frac{5}{3}}{\mathbb{P}} \overset{\frac{5}{6}}{\bullet} \overset{10}{\bullet} \overset{\mathbb{P}}{\bullet}}_{\text{al}} \overset{3}{\leftarrow} \underbrace{\overset{10}{\leftarrow} \overset{1}{\bullet}}_{\text{al}} \overset{6}{\leftarrow} \overset{1*}{\leftrightarrow}}_{\text{al}},$$

but we have not investigated this thoroughly.

<sup>37</sup>TODO mention the other one citing Chambon and the footnote in Archi

<sup>&</sup>lt;sup>36</sup>Liquid capacities use a different system [Arc15, p. 229 with note 12]:

<sup>&</sup>lt;sup>38</sup>We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform ≰ 💾 🗃 and larger with cuneiform 🏴 and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked<sup>39</sup>on a photograph (from EbDA unless noted otherwise).

As we will see in Section 3.7.2, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

<sup>&</sup>lt;sup>40</sup>ba-ri<sub>2</sub>-zu<sub>2</sub>, a variant spelling.

<sup>&</sup>lt;sup>41</sup>Short for ⋤� L.

<sup>43</sup>Instead of the expected ₩ [44].

<sup>&</sup>lt;sup>44</sup> **Ⅲ \*** ← not legible on the EbDA photo.

<sup>&</sup>lt;sup>45</sup>From CDLI photo.

<sup>&</sup>lt;sup>46</sup>From photo in [Arc89, p. 6].

<sup>&</sup>lt;sup>47</sup>Laid out as [1]]; on stacking patterns see Section 6.2.

of decreasing fractions  $^1/_n$  of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus  $\Xi=^1/_2$   $N_{39}$ ,  $\varpi=^1/_3$   $N_{39}$ , and so on. The first sign of the latter units,  $N_{34}$ .

Figure 1: TODO [Eng98, p. 113]

For instance, the first line contains the notations  $1N_{34}$   $1N_{39_0}$ ;  $2N_{20}$ , which can be translated "60 of the (grain rations containing) = (of grain); (grain involved:)  $2 \bullet$  (of ground barley)". This calculation contradicts the assumed numerical relationship  $10N_1 = 1N_{14}$ , since as was well known the measure represented by the sign  $N_{39}$  was 1/5 of that represented by  $N_1$ , so that  $60 \times 1/5 = 12$  and not 20, as  $2N_{14}$  would imply. Instead of relying on complicated

Figure 2: TODO [Eng98, p. 116]

These expressions correspond to the following factor diagram:

$$\begin{array}{c|c}
\hline
 & \begin{array}{c}
 & \begin{array}{c}
 & \begin{array}{c}
 & \begin{array}{c}
 & \\
 & \end{array}
\end{array}
\end{array}
\begin{array}{c}
 & \begin{array}{c}
 & \begin{array}{c}
 & \\
 & \end{array}
\end{array}
\end{array}
\begin{array}{c}
 & \begin{array}{c}
 & \\
 & \end{array}
\end{array}
\begin{array}{c}
 & \\
 & \end{array}
\begin{array}{c}
 & \\
 & \end{array}$$

$$\begin{array}{c}
 & \\
 & \end{array}
\begin{array}{c}
 & \\
 & \end{array}
\end{array}
\begin{array}{c}
 & \\
 & \end{array}
\end{array}
\begin{array}{c}
 & \\
 & \end{array}
\end{array}
\begin{array}{c}
 & \\
 & \end{array}
\end{array}
\begin{array}{c}
 & \\
 & \end{array}
\end{array}
\begin{array}{c}
 & \\
 & \\
 & \end{array}
\begin{array}{c}
 & \\\\ & \end{array}
\begin{array}{c}$$

# 3.4.5 Use in modern publications

Because of their prevalence in the Uruk and Early Dynastic periods, the proposed numerals are used modern publications discussing metrology in those periods, as illustrated in Figures 1–11.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 11. The partial transliteration " $4^{\mbox{$\sim$}}$  ' $a_3$ -da-um  $4^{\mbox{$\sim$}}$  aktum  $4^{\mbox{$\sim$}}$  ib  $_2$  tu  $_9\times 3^{\mbox{$\sim$}}$  sa gunu  $_3$ " is used to illustrate a discussion of the interpretation of the contrast between  $^{\mbox{$\sim$}}$  and  $^{\mbox{$\sim$}}$  numerals. More conventional transliterations  $^{49}$  might omit the numeral shapes entirely, e.g.,  $^{\mbox{$\sim$}}$  about  $^{\mbox{$\sim$}}$  4 aktum  $^{\mbox{$\sim$}}$  ib  $^{\mbox{$\sim$}}$  tu  $^{\mbox{$\sim$}}$  and  $^{\mbox{$\sim$}}$  which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, e.g.,  $^{\mbox{$\sim$}}$  ( $^{\mbox{$\sim$}}$  about  $^{\mbox{$\sim$}}$  ( $^{\mbox{$\sim$}}$  about numeral shape, e.g.,  $^{\mbox{$\sim$}}$  and  $^{\mbox{$\sim$}}$  ( $^{\mbox{$\sim$}}$  about numeral shape, e.g.,  $^{\mbox{$\sim$}}$  and  $^{\mbox{$\sim$}}$  and  $^{\mbox{$\sim$}}$  ib  $^{\mbox{$\sim$}}$  ib  $^{\mbox{$\sim$}}$  and  $^{\mbox{$\sim$}}$  in  $^{\mbox{$\sim$}}$  and  $^{\mbox{$\sim$}}$  in  $^{\mbox{$\sim$}}$  in  $^{\mbox{$\sim$}}$  and  $^{\mbox{$\sim$}}$  in  $^{\mbox{$\sim$}}$  and  $^{\mbox{$\sim$}}$  in  $^{\mbox{$\sim$}}$ 

Die halbkreisförmigen Griffeleindrücke gehen manchmal in mehr oder weniger eckige Formen über ( $\P$ ) $^{o85}$ . Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen ( $\P$ ), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren $^{o86}$ . Selten treten mit  $\triangledown$  gebildete Zahlen auf $^{o87}$  (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 3: TODO [Kre98, p. 303]

<sup>&</sup>lt;sup>48</sup>The untransliterated text would be 器層耳:回目≤器 II 圖器 型盘 II ■ iii ii note the atomically

formed by only two signs  $\Gamma$  and  $\checkmark$ , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60², 60³, 1/60, 1/60², etc. are written in the same way, with the vertical wedge  $\Gamma$ . The third feature concerns the exact function of

Figure 4: TODO [Cha12, p. 58]

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with:

Figure 5: TODO [Cha12, p. 59]<sup>50</sup>

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

DDD še bar ∇ ba-rí-zu

'3 gubar (capacity units) and 1 parīsu'.

Figure 6: TODO [Cha12, p. 61]

This is particularly true of the signs  $\nearrow$ ,  $\rightleftharpoons$ ,  $\rightleftharpoons$  and  $\rightleftharpoons$ , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written  $\bigcirc$  in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign  $\bigcirc$  is most often associated with the *parīsu* measure, while the signs  $\rightleftharpoons$ ,  $\rightleftharpoons$ ,  $\rightleftharpoons$ , and  $\rightleftharpoons$  refer to 1, 2, 3,

Figure 7: TODO [Cha12, p. 64]

TODO figure

Figure 8: TODO [Chambon2016]

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., transliterated as  $2(\bar{s}ar_2)$   $1(ge\bar{s}'u)$  3(u), means  $2 \times 3600 + 1 \times 600 + 3 \times 10$ ). The system is based on an alternation of factors ten and

Figure 9: TODO

might think of one fabric and a half, \$^{11}\$ but the presence of notations with "2\$\subseteq 2\$\subseteq 7", "3\$\subseteq 3\$\subseteq 7", and "6\$\subseteq 6\$\subseteq 7" (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag's weight measurement system.12 The notation "1 pagada" in o. ii 1 and r. vi 1, along with the total of "39



Fig. 1. Combinations of numerals attested in Š. 742.

Figure 10: Discussion of the contrast between ▶ and ¬numerals in [Gor23, p. 162].

```
as, for example, in TM.75.G.3125 = ARET III 107 o. iv 1, "4^{\circ}2 ^{\circ}4 ^{\circ}4 ^{\circ}4 ^{\circ}5 4 ^{\circ
aktum 4 Dib, tu9 ×3 T sa, gunu," (Fig. 2).
```

Figure 11: Transliteration in [Gor23, p. 163] of [P242293, recto 4, 1] incorporating untransliterated numerals.

#### Non-numeric usage 3.5

其事 化克克 江 下河 直耳条字 世际 三字 在过 古 西斯 事 去 不其 阿米斯 阿克斯 医 玉鱼属 群争叫 阻阻 事奉者 计算点

The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'51. Do you know its reading<sup>52</sup>?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign ► has, inter alia, the values aš, rum, and dili. While the horizontal numerals are most frequently written written in the Early Dynastic period<sup>53</sup>, such non-numeric usage is almost  $^{54}$  always written -, for instance:

- in personal names in administrative texts, such as the following, which all contain ▶ numerals:
  - ﴿ in [**P010424**; **P010458**; **P010459**] from ED IIIa أبو صلابيخ in [**P010960**] from ED IIIa Šuruppag,

  - 🛨 📹 🖷 Ң in [P010964] from ED IIIa Šuruppag,

encoded ib<sub>2</sub> × 3 $! = \mathbb{R} \times \mathbb{M} = \mathbb{R}$ .

<sup>&</sup>lt;sup>9</sup>TODO cite the EbDA one.

 $<sup>^{50}</sup>$ TODO(egg): On the order cite TSS 188, Friberg2007 p. 148 and any of the usual suspects on the haphazard order of signs in early texts; contrast P274845, P241764.

 $<sup>^{51}</sup>$ The reader will recall that  $\eta$ eš $_2$  is written  $\P$ , with a larger wedge than  $\P$ ; however, these signs have merged by the Old Babylonian period when the first witnesses of Examenstext A are attested.

<sup>&</sup>lt;sup>52</sup>Besides  $9e_2$ , a look at [VT+14] shows that the values diš,  $9e_3$ , makkaš, sa $9e_4$ , and tal $4e_4$  are attested both in [TJV17] and in lexical lists. The sign is also used for the Akkadian word ana in the Neo-Assyrian

<sup>&</sup>lt;sup>53</sup>A CDLI search for "(asz@c)" finds 3296 ED texts, while a search for "(asz)" finds 81 ED texts, of which 46 also contain "(asz@c)".

<sup>&</sup>lt;sup>54</sup>Exceptions are discussed in section 3.7.1.

- 編世ー園 in [P251641] from ED IIIb Adab,
- < III → IIII → IIIII → IIII → IIII → IIII → IIII → IIII → IIIII → IIIII → IIII → IIII → IIIII → IIII → IIII → IIII → IIII → IIII → IIIII → IIII → I
- 卦>> **I** → in [**P298637**] from ED IIIb Umma;
- in the Sumerian word # ← u<sub>2</sub>-rum, "property" in ED IIIb Nirsu administrative texts which contain pumerals, such as [P020006; P020008; P020018; P020024; P020030];
- in lexical texts:
  - in the divine name  $\# \bowtie \bot$  in the lexical texts [**P010570**; **P010572**], where the entries are prefixed with  $\triangleright$ .
  - in the word ➤ dili, "small fish" in [P010578], witness to Early Dynastic Fish.
  - in the same word with a determinative, ¼ dili<sup>ku</sup>, in [**P010586**], witness to Early Dynastic Food, which starts with □ numerals.

This is a clear contrast between — and  $\square$  in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name —  $\blacktriangleleft$  occurs on its own line in the aforementioned administrative texts; a line  $\square$   $\blacktriangleleft$  would instead be read as "one slave".

# 3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as — that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [Sch35, pp. 123 sqq.] lists all of —— together with P—— together with phase with phase per at the beginning of the sign list, since they have non-numeric values sign list, since they have non-numeric values sign list, pp. 58] has the numeric signs P,  $\rightarrow$ , whereas non-numeric — is at the beginning of the sign list, where its values as and rum are listed. For signs with both non-numeric and numeric usage, [Dei22] writes s. die Zahlz. throughout the main list; LAK 1 — thus reappears at LAK 829 together with P,  $\rightarrow$ , and  $\rightarrow$ . One should note [Bor10], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have - we used in system  $G_{\text{UF},\text{III}/OB}$ .

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from to I numerals prevents diachronic encoding even if were unified with the For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'-6']; however, they are written with I numerals, whereas they are written with

 $<sup>^{55}</sup>$ Non-numeric values of — were discussed in section 3.5; — has the values man<sub>3</sub> and min<sub>5</sub>, and is used for the word didli, "several, various"; — has the value e $\S_6$ .

<sup>&</sup>lt;sup>56</sup>TODO comment on the ED witnesses to the instructions of Suruppag

ho numerals in the Early Dynastic witnesses; since ho and ho are distinct ho characters, the ho-ho unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods<sup>58</sup>, there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Romach2023], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

# 3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with one exception discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

# 3.7.1 The case of ŠAR<sub>2</sub>

The character U+122B9 ■ CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period<sup>60</sup>, a contrast between non-numeric  $\operatorname{Sar}_2$  written  $\diamondsuit$  and numeric  $\operatorname{1}(\operatorname{Sar}_2^c)$  written  $\blacksquare$  can be observed, similar to the contrast between  $\vdash$  and  $\vdash$  previously discussed in section 3.5. However, in lexical lists from  $\operatorname{Suruppag}$  and  $\operatorname{Ebla}^{61}$ , as well as in the *Stèle des vautours*, non-numeric  $\operatorname{Sar}_2$  is curviform:

- ☀୍ਸ਼ੑ♦● and ☀ਸ਼ੑ●+==== in [**P010566**];
- $\bullet \Rightarrow$  and  $* \bullet \Rightarrow$  in [P010576];
- **<del>↓</del>** [P240986];
- ● **紅**� in [P222399, obv. 17, 9, 18, 11, 22, 12]<sup>62</sup>.

- 具食◇酊 in [P020019] from ED IIIb Nirsu;
- 海上◇町料 in [P020182], also from ED IIIb Nirsu;
- ♦ ★ ♦ in [P222186] from ED IIIb Umma;

<sup>&</sup>lt;sup>57</sup>Besides the contrasts in numeric usage mentioned in section 3,3,3, these characters are clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of ← which are not shared with | have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period, ← is used for the preposition *ina*, "in", and | for the preposition *ana*, "to".

<sup>&</sup>lt;sup>58</sup>TODO cite a few things here.

 $<sup>^{59} \</sup>rm Most$  prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

<sup>&</sup>lt;sup>60</sup>For example, in personal names:

<sup>&</sup>lt;sup>61</sup>TODO Mention other ways in which these are archaizing

<sup>62</sup> Note however ♣ → ♦ ᡚ on [P222399, obv. 6, 17]. Curviform non-numeric šar₂ is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here. TODO add a photo.

It would be disruptive to the diachronic representation of text if non-numeric  $\operatorname{\check{s}ar_2}$  were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph  $\bullet$ , identical to the glyph of (TODO: the proposed character). Since the archaizing style of texts wherein non-numeric  $\operatorname{\check{s}ar_2}$  is curviform solidly predates the transition from  $\bullet$  to  $\diamondsuit$  in the relevant metrological systems, there is no need to represent a  $\diamondsuit$ - $\bullet$  contrast, so these characters can have the same glyph.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

Note that in rare cases, such as [P222243] from ED IIIa Adab, non-numeric — (here with the value rum) is written . It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings and encoded as (TODO: the proposed characters) or as stylistic distinctions and encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. in practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the CDLI transliteration (UR2-1(aš@c)) and the ePSD2 one (uru<sub>8</sub><sup>rum</sup>) of this word disagree on that aspect. Since — has a cuneiform reference glyph, this does not pose any compatibility concerns.

## 3.7.2 Transliteration

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance,  $\P$  from  $\P$  from  $\P$  from  $\P$  may be transliterated as 95 gur, as in  $\P$  in  $\P$  from  $\P$  and  $\P$  as in  $\P$  from  $\P$  and  $\P$  as 1/2, but not distinguishing them.

<sup>&</sup>lt;sup>63</sup>As on [P249253].

 $<sup>^{64}\!\</sup>text{As}$  of this writing, EbDA actually has an-zam  $_{\chi}\!$  , with U+1D6A greek subscript small letter chi.

In metrological systems such as systems  $G_{\text{Ur III/OB}}$  and  $C_{\text{Ur III/OB}}$  where some units are indicated by the type of numeral rather than an overt unit sign, it is common practice to add the unit in parentheses in transliteration; for instance,  $\prec \Leftrightarrow \land = \Downarrow \Downarrow \Downarrow \square$  from [P386847] is transliterated "1(eše<sub>3</sub>) 5½ iku<sup>65</sup> 7 sar" in [Feuerherm2004], and  $\Downarrow \rightleftharpoons \Downarrow \dashv \searrow$  from [P307255] is transliterated "1(n<sup>66</sup>) 2(b) 7½ sila<sub>3</sub>" in [Feuerherm2004].

While there exist transliterations that distinguish — from ! but not  $\mathbb{R}$  from —, such as the ones used in [**DCCMT**], the trend, especially in more recent works in third millenium studies, seems to be to represent numeral shape; for example, [**Maiocchi2024**] gave an example of the input syntax used by the new "Urban Economy Begins" project as "10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)" for \*\*P\*I+\*, with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are  $\mathbb{P}$  rather than  $\mathbb{T}$  numerals.

## 3.8 Conclusions

Co-occurences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period. Instead, they represent contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications.

While it would be technically possible to handle this contrast as a stylistic distinction, this is highly inconvenient, as it requires single numeric expressions to systematically use multiple fonts. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted: ( is a length of three ropes, but  $^{\bullet}$  is an area of three bur<sub>3</sub>; ( could be read as one ( and one ( would be one and a half ( would be "one slave".

At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millenium corpora. The oddity of  $\odot$  requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.

 $<sup>^{65}\</sup>text{TODO}$  say something about this reading

<sup>&</sup>lt;sup>66</sup>TODO comment on nigida.

# 4 Rationale for ED-Uruk numeral unification

TODO mention the bariga silliness in the CDLI transliteration of Gori's paper.

# 5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban<sub>2</sub>@c) and 5(ban<sub>2</sub>@c).]

# 6 Characters not included in this proposal

# 6.1 Missing numerals

N<sub>13</sub> not attested in CDLI ( $N_{17}$ , 12 $N_{14}$ , etc.) 7(diš  $ten\hat{u}$ )

# 6.2 Stacking patterns

(... are a mess, vary within Uruk, and are not transliterated/documented by Englund, so let's not go there for now.)

# 6.3 Matters for higher-level protocols

Rotated bits: https://cdli.mpiwg-berlin.mpg.de/artifacts/101087

# Acknowledgements

TODO(egg): Something about the Vanséveren fonts

# References

### **Artefacts**

[P020054] VAT 4731. [För16, 40 p.14]. Vorderasiatisches Museum.

CDLI: P020054.

[P020129] VAT 04713. Vorderasiatisches Museum.

CDLI: P020129.

ORACC: epsd2/corpus/P020129.

[P102305] X.3.139. Michael C. Carlos Museum, Emory University.

CDLI: P102305.

[P215653] AS 15375 21. Musée du Louvre.

CDLI: P215653.

ORACC: dcclt/corpus/P215653.

Louvre Collections: ark:/53355/cl010436723.

[P221266] AO 13825. Musée du Louvre.

CDLI: P221266.

ORACC: epsd2/corpus/P221266.

Louvre Collections: ark:/53355/clo10138527.

[P221291] AO 13850. Musée du Louvre.

CDLI: P221291.

ORACC: epsd2/corpus/P221291.

[P221305] AO 13864. Musée du Louvre.

CDLI: P221305.

ORACC: epsd2/corpus/P221305.

[P222399] Stèle des vautours. AO 50; AO 2346; AO 2347; AO 2348; AO 16109.

Musée du Louvre. CDLI: P222399.

[P232278] Gudea E. AO 6. Musée du Louvre.

CDLI: P232278.

ORACC: etcsri/Q001544.

[P232280] Gudea G. AO 7. Musée du Louvre.

CDLI: P232280.

ORACC: etcsri/Q001546.

[P240531] TM.75.G.00265. Idlib, Syria: National Museum of Syria.

CDLI: P240531. Ebda: 1415.

[P240532] TM.75.G.00266. Idlib, Syria: National Museum of Syria.

CDLI: P240532. Ebda: 1324.

[P240533] TM.75.G.00267. Idlib, Syria: National Museum of Syria.

CDLI: P240533. Ebda: 1379.

[P240548] TM.75.G.00302. Idlib, Syria: National Museum of Syria.

CDLI: P240548. Ebda: 1350.

[P240579] TM.75.G.00341. Idlib, Syria: National Museum of Syria.

CDLI: P240579. Ebda: 1364.

[P240609] TM.75.G.00440. Idlib, Syria: National Museum of Syria.

CDLI: P240609. Ebda: 1378.

[P240653] TM.75.G.00535. Idlib, Syria: National Museum of Syria.

CDLI: P240653. Ebda: 1382.

[P240654] TM.75.G.00536. Idlib, Syria: National Museum of Syria.

CDLI: P240654. Ebda: 1383.

[P240655] TM.75.G.00537. Idlib, Syria: National Museum of Syria.

CDLI: P240655. Ebda: 1358. [P240675] TM.75.G.00557. Idlib, Syria: National Museum of Syria.

CDLI: P240675. Ebda: 1371.

[P240697] TM.75.G.00579. Idlib, Syria: National Museum of Syria.

CDLI: P240697. Ebda: 1381.

[P241708] TM.75.G.02143. Idlib, Syria: National Museum of Syria.

CDLI: P241708. Ebda: 3173.

[P241904] TM.75.G.02346. [Arc89, p. 6]. Idlib, Syria: National Museum of Syria.

CDLI: P241904. Ebda: 3183.

[P242293] TM.75.G.03125. Idlib, Syria: National Museum of Syria.

CDLI: P242293. Ebda: 217.

[P249253] Code de Hammurabi. Sb 8. Musée du Louvre.

CDLI: P249253.

## **Unicode documents**

[L2/03-162] M. Everson and K. Feuerherm. *Basic principles for the encoding of Sumero-Akkadian Cuneiform*. 25th May 2003.

UTC: L2/03-162.

[L2/03-393R] M. Everson, K. Feuerherm and S. Tinney. *Preliminary proposal to* 

encode the Cuneiform script in the SMP of the UCS. 3rd Nov. 2003.

UTC: L2/03-393R.

[L2/04-036] M. Everson, K. Feuerherm and S. Tinney. Revised proposal to encode

the Cuneiform script in the SMP of the UCS. 29th Jan. 2004.

UTC: L2/04-036.

[L2/04-099] L. Anderson. *Unification of Cuneiform Numbers*. 2004.

UTC: L2/04-099.

[L2/04-189] M. Everson, K. Feuerherm and S. Tinney. Final proposal to encode

the Cuneiform script in the SMP of the UCS. 8th June 2004.

UTC: L2/04-189.

[L2/24-159] P. Constable, ed. *Minutes of UTC Meeting 180* (23rd–25th July 2024).

29th July 2024. UTC: L2/24-159.

[Uni16] The Unicode Consortium. *The Unicode Standard*. Version 16.0.0. The

Unicode Consortium, 10th Sept. 2024.

ISBN: 978-1-936213-34-4.

eprint: https://www.unicode.org/versions/Unicode16.0.0/core-

spec/.

[UTR56] R. Leroy, ed. *Unicode Cuneiform Sign Lists*. Unicode Technical Report

#56.

eprint: https://www.unicode.org/reports/tr56/.

# Other documents

[Arc15]

ancient Near Eastern records 7. Walter de Gruyter, 2015. ISBN: 978-1-61451-716-0. DOI: 10.1515/9781614517887. [Arc89] A. Archi. "Tables de comptes eblaïtes". In: Revue d'assyriologie et d'archéologie orientale 83.1 (1989). Ed. by P. Amiet and P. Garelli, pp. 1-6. ISSN: 0373-6032. [Bor10] R. Borger. Mesopotamisches Zeichenlexikon. Alter Orient und Altes Testament 305. Ugarit-Verlag, 2010. [Cap02] R. Caplice. Introduction to Akkadian. 4th ed. Editrice Pontificio Istituto Biblico, 2002. ISBN: 88-7653-566-7. [Cat13] A. Catagnoti. La paleografia dei testi dell'amministrazione e della cancelleria di Ebla. Quaderni di Semitistica 9. Università di Firenze, 2013. ISBN: 8890134054. [Chao3] G. Chambon. "Archaic Metrological Systems from Ur". In: Cuneiform Digital Library Journal 2003.5 (23rd Dec. 2003). ISSN: 1540-8779. eprint: http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003\_005.html. [Cha12] G. Chambon. "Numeracy and Metrology". In: *The Oxford Handbook of* Cuneiform Culture. Ed. by K. Radner and E. Robson. Oxford University Press, 18th Sept. 2012, pp. 51-67. ISBN: 9780199557301. DOI: 10.1093/oxfordhb/9780199557301.013.0003. [DE87] P. Damerow and R. K. Englund. "Die Zahlzeichensysteme der archaischen Texte aus Uruk". In: M. W. Green and H. J. Nissen. Zeichenliste der archaischen Texte aus Uruk. Archaische Texte aus Uruk 2. An offprint of this chapter is available at https://cdli.mpiwgberlin.mpg.de/files-up/publications/englund1987a.pdf. Gebr. Mann Verlag, 1987. Chap. 3, pp. 117-165. [Dei22] A. Deimel. Liste der archaischen Keilschriftzeichen von Fara. Wissenschaftliche Veröffentlichungen der Deutschen Orient-Gesellschaft 40. J. C. Hinrichs'sche Buchhandlung, 1922. [Eng98] R. K. Englund. "Texts from the Late Uruk Period". In: Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit. Orbis Biblicus et Orientalis 160/1. 1998, pp. 13-233. ISBN: 3-7278-1166-8. [För16] W. Förtsch. Altbabylonische Wirtschaftstexte aus der Zeit Lugalanda's und Urukagina's. Vorderasiatische Schriftdenkmäler der Königlichen Museen zu Berlin 14. J. C. Hinrichs, 1916. [Frio7] J. Friberg. A Remarkable Collection of Babylonian Mathematical Texts. Manuscripts in the Schøyen Collection: Cuneiform Texts I. Sources and Studies in the History of Mathematics and Physical Sciences. Springer, 2007.

A. Archi. Ebla and Its Archives. Texts, History, and Society. Studies in

ISBN: 978-0-387-34543-7.

- [Fri78] J. Friberg. A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and Proto-Elamite Semi-Pictographic Inscriptions. The Third Millenium Roots of Babylonian Mathematics 1. Department of Mathematics, Chalmers University of Technology, 1978.
- [Gom16] B. Gombert. "ED IIIb metrology: texts from Lagaš". In: *CDLI:wiki. A Library of Knowledge of the Cuneiform Digital Library Initiative*. 12th Apr. 2016.

  eprint: https://cdli.ox.ac.uk/wiki/doku.php?id=ed\_iii\_metrological\_systems.
- [Gor23] F. Gori. "On Lapis Lazuli and Linen in Šuruppag Texts. An Analysis Through the Lens of Ebla Studies". In: *Studia Eblaitica* 9 (2023), pp. 160–166. ISSN: 2364-7124.
- [Hue11] J. Huehnergard. *A Grammar of Akkadian*. 3rd ed. Brill, 2011. ISBN: 978-1-57506-941-8.
- [Jim+23] E. Jiménez, Z. Földi, A. Hätinen, A. Heinrich, T. Mitto, G. Rozzi, I. Khait, J. Laasonen, F. Simonjetz et al., eds. *electronic Babylonian Library*. 2023–. eprint: https://www.ebl.lmu.de/.
- [JJ24] T. Jauhiainen and H. Jauhiainen. "Advancing Cuneiform Text Dating Through Automatic Analysis". 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 11th July 2024 14:00.
- [Kre98] M. Krebernik. "Die Texte aus F\u00e4ra und Tell Ab\u00fc \u00e7al\u00e5b\u00e7\u00e5. In: Mesopotamien. Sp\u00e4turuk-Zeit und Fr\u00e4hdynastische Zeit. Orbis Biblicus et Orientalis 160/1. 1998, pp. 235-427.
  ISBN: 3-7278-1166-8.
- [Lec12] C. Lecompte. "Des chiffres et des digues: à propos de deux textes présargoniques de Ĝirsu et d'une notation numérique inhabituelle". In: *Altorientalische Forschungen* 39.1 (Dec. 2012), pp. 81–86. DOI: 10.1524/aofo.2012.0006.
- [Lec20] C. Lecompte. "The Measurement of Fields During the Pre-sargonic Period". In: *Mathematics, Administrative and Economic Activities in Ancient Worlds*. Ed. by C. Michel and K. Chemla. Why the Sciences of the Ancient World Matter 5. Springer, 2020.
- [Mil+07] L. Milano, M. Maiocchi, F. Di Filippo, R. Orsini, E. Scarpa, M. Surdi et al., eds. *Ebla Digital Archives*. 2007–. eprint: http://ebda.cnr.it/.
- [NDE93] H. J. Nissen, P. Damerow and R. K. Englund. Archaic Bookkeeping.
   Early Writing and Techniques of Economic Administration in the
   Ancient Near East. Trans. by P. Larsen. The University of Chicago
   Press, 1993.

   ISBN: 0-226-58659-6.
- [Oel22] J. Oelsner. Der Kodex Hammu-rāpi. dubsar 4. Zaphon, 2022.
- [Pow87] M. Powell. "Maße und Gewichte". In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš–Medizin. 1987–1990, pp. 457–530.

[Robo8] E. Robson. Mathematics in Ancient Iraq. A Social History. Princeton University Press, 2008. ISBN: 978-0-691-09182-2. [Rob19] E. Robson. "Oracc metrology guidelines". In: Oracc: The Open Richly Annotated Cuneiform Corpus. 18th Dec. 2019. ORACC: doc/help/editinginatf/metrology/metrologicaltables. [Rob22] E. Robson. "Overview of Metrological Systems". In: The Digital Corpus of Cuneiform Mathematical Texts. 2022. ORACC: dccmt/Metrology. [Rom24] A. Romach. "The Neo Assyrian Land Sale Documents from Dur-Katlimmu: A Stylometric Analysis of Their Scribal Features". 69th  $Rencontre\,Assyriologique\,Internationale\,(8th-12th\,July\,2024).\,10th\,July$ 2024 12:00. [Sch10] W. Schramm. Akkadische Logogramme. Göttinger Beiträge zum Alten Orient 5. Universitätsverlag Göttingen, 2010. ISBN: 978-3-941875-65-4. DOI: 10.17875/gup2010-511. N. Schneider. Die Keilschriftzeichen der Wirtschaftsurkunden von Ur [Sch35] III. Editrice Pontificio Istituto Biblico, 1935. S. Svärd, M. Lorenzon, J. Töyräänvuori, J. Valk, T. Alstola, E. Bennett, [Svä+24] R. Uotila and T. Auranne, eds. RAI 69 Abstracts. July 2024. eprint: https://www.helsinki.fi/assets/drupal/2024-07/RaiA bstractBookAjoitettuJaPäivätty\_1.pdf. [TJV17] S. Tinney, P. Jones and N. Veldhuis, eds. The electronic Pennsylvania Sumerian Dictionary. 2nd ed. 2017-. eprint: http://oracc.org/epsd2. [VT+14] N. Veldhuis, S. Tinney et al., eds. Oracc Sign List. 2014-.

eprint: http://oracc.org/osl/.