

Archaic cuneiform numbers

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1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script¹ and the proto-cuneiform script². The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

¹ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

²ISO 15924: Pcun, not yet encoded.

³Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: ▷, ▽ (small stylus), ▢, ▣ (large stylus). Some numerals are composed of multiple such impressions, e.g., ▢▷. The terms “curved”, “curviform”, and “round” can be found in the literature. We avoid the term “round” here as it has other meanings in the context of numbers. We use “curviform” in this document as, being the least common term, it is least likely to lead to confusion, and “CURVED” in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [inlineATF].

⁴Impressed into clay using a stylus with a trihedral end: — (stylus held horizontally), † (vertically), ↘ (diagonally) < (with the head of the stylus), ‡ (stylus pressed deeper, forming a larger wedge), ¶ (combining † and <), etc.

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
Non-numeric signs	Future Pkun	Existing Xsux	

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties





2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

TODO(egg): blurb.

3.1 The cuneiform encoding model

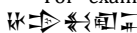
As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts,  later in the third millennium⁵,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists⁶ and dictionaries⁷, and of composite texts⁸. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]⁹. The diachronic approach is also useful for pedagogic applications¹⁰.

⁵Merging with U+1224E cuneiform sign ni2.

⁶Notably [OSL] and the online edition of [MZL] in [eBL, Signs].

⁷Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

⁸For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

⁹Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svā+24, pp. 242, 148].

¹⁰For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding¹¹ that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

† ¹²	<	+	≡	𐎶	𐎶	𐎶
1(ḡeš ₂)	1(u)	1/2(diš)	5(diš <i>tenû</i>)	gi	us ₂	sa ₂
	7.5 (ropes)		5	reed	side	equal
≡ ¹³	≡	𐎶	𐎶	𐎶		
3(u)	6(diš <i>tenû</i>)	gi	saḡ	sa ₂		
3(ropes)	6	reed	front	equal		
𐎶 𐎶 𐎶	•	•	▷	◁		
ašag-bi	1(bur ₃ ^c)	1(eše ₃ ^c)	1(iku ^c)	1/2(iku ^c)		
field-this						

𐎶 𐎶 𐎶 𐎶 𐎶
tug_x(LAK483)-si-ga-kam
tugsiga =ak =am
ploughed =GEN =COP

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as¹⁴

465 metres, equal lengths
198 metres, equal widths
this field is 9, 18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from ▷ to † numerals). It therefore comes to the conclusion that the use of

¹¹At that time scoped to the répertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as 𐎶 ≠ 𐎶, were informed by Early Dynastic distinctions.

¹²As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large — in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

¹³Note that ED IIIb < numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign ≡ in [P020054] looks more like Ur III +.

¹⁴We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

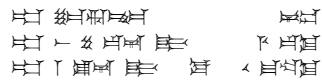
curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere \triangleright / \lvert switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 A primer on classic Ur III and Old Babylonian metrologies


 I want to write tablets: the tablet of
 1 cor of barley to 600 cor; the tablet
 of 1 shekel of silver to 10 minas [...]
 Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the contrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known¹⁵ a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters \lvert – |||| , the multiples of ten (10–50) are < – <|||| , but the other digits 11–59 are sequences < – <|||| ; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units \lvert – |||| , tens < – <|||| , sixties |||| (with larger wedges than the units), multiples of six hundred |||| , multiples of three thousand six hundred |||| , and multiples of thirty-six thousand |||| .

¹⁵See, *e.g.*, [Uni16, §22.3.3, sub “Cuneiform Numerals”].

3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram¹⁶, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \quad (S_{\text{Ur III/OB}})$$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written 𐎶𐎵𐎶𐎵𐎶𐎵 in the discrete counting system, and 𐎶𐎵𐎶𐎵𐎶𐎵 in the sexagesimal place value system.

3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{3} \text{𐎶} \xleftarrow{6} \text{𐎵} \xleftarrow{2} \text{𐎶} \xleftarrow{2} \text{𐎵} \quad (G_{\text{Ur III/OB}})$$

bur_3	eše_3	iku	ubûm
bûrum	eblum	ikûm	
6,48 ha	2,16 ha	3600 m ²	1800 m ²

Note that for the range of areas given above¹⁷, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *bûrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign 𐎶𐎵 functioning as punctuation¹⁸, but the numerals are tied to the metrology; thus a surface of 5 *bûr* 1 *ebel* 4 *ikû* (100 *ikû*, 36 ha) would be written¹⁹ 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in “88 acres 3 roods 33 perches” or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the sign \diamond is equal to sixty times 𐎶 in the area system, but to three hundred and sixty times 𐎶 in the discrete counting system.

¹⁶These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

¹⁷For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mûšarum* (36 m²) written 𐎶𐎵𐎶𐎵, equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60 𐎶𐎵 (shekels). For areas greater than 3600 *bûrû*, the \diamond and 𐎶 numerals are reused with a suffix 𐎶𐎵 (gal, Sumerian: big), as follows [Rob08, p. 295 n. b and c; Fri07, p. 378; Rob19]:

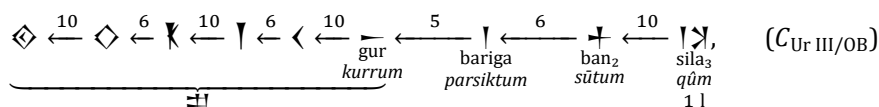
$$\diamond \text{𐎶𐎵} \xleftarrow{10} \diamond \text{𐎶𐎵} \xleftarrow{6} \diamond \text{𐎶𐎵} \xleftarrow{10} \diamond \text{𐎶𐎵} \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{3} \text{𐎶} \xleftarrow{6} \text{𐎵} \xleftarrow{2} \text{𐎶} \xleftarrow{2} \text{𐎵} \xleftarrow{2,5} \text{𐎶𐎵} \xleftarrow{10} \text{𐎶𐎵} \xleftarrow{6} \text{𐎶𐎵} \xleftarrow{10} \text{𐎶𐎵}.$$

¹⁸TODO(egg): acknowledge Proust 2020 but note that this is irrelevant to encoding concerns






¹⁹As in the surface of the field of 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵 (the city of Apisal) reported on [P102305, r. 1]

3.3.3 The capacity system

Another such system of note is the one for capacities²⁰ [Fri07, p. 376; Rob19],



where the numerals for ban₂ are 𐤁, 𐤂, 𐤃, 𐤄, and 𐤅, and those for bariga are 𐤆, 𐤇, 𐤈, and 𐤉 (contrast ordinary 𐤈 and 𐤉 otherwise used with 𐤆 numerals). As described in [Hue11, p. 585 n. (b) and (f)], the sign 𐤆𐤇 GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𐤇 if present, and after the word for “grain” if present, as in

 354 gur
  3 ban₂
  6
  sila₃
  of grain.

Observe that while large numbers of gur follow²² system $S_{\text{Ur III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as $\langle \text{I} \text{---} \text{II} \rangle$ would be 10 gur 1 bariga, and $\langle \text{---} \text{II} \rangle$ would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

- \mathbb{I} – \mathbb{W} used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- \mathbb{L} – \mathbb{X} used in $G_{\text{Ur III/OB}}$, of which \mathbb{L} – \mathbb{X} are also used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- \mathbb{Y} – \mathbb{W} used in $S_{\text{Ur III/OB}}$, and sometimes with overt units;
- \mathbb{K} – \mathbb{W} used in $S_{\text{Ur III/OB}}$;
- \mathbb{D} – \mathbb{X} used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \mathbb{E} – \mathbb{R} used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \mathbb{T} – \mathbb{M} used in $C_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
- \mathbb{P} , \mathbb{Q} , \mathbb{R} , \mathbb{S} , \mathbb{T} used in $C_{\text{Ur III/OB}}$;
- \mathbb{U} , \mathbb{V} , \mathbb{W} , \mathbb{X} used in $C_{\text{Ur III/OB}}$ —note the overlap with \mathbb{I} – \mathbb{W} ;
- \mathbb{Z} and \mathbb{Z} used in $G_{\text{Ur III/OB}}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Robo8, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

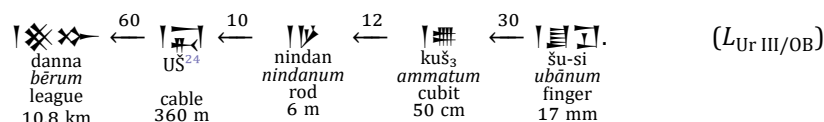
²⁰Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{Ur III/OB}$ based on a height of one cubit, see[[Pow87](#), p. 488; [Rob08](#), p. 294; [Rob19](#)].

²¹From [P309594].

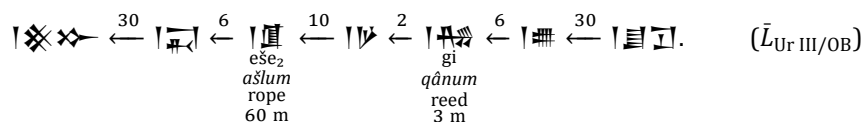
²²A larger unit, the *guru*₇ (*karûm*, grain heap), is sometimes used instead, with 𐎠𐎡𐎢𐎣𐎤𐎥𐎦𐎧 = 𐎠𐎡𐎢𐎣𐎤𐎥𐎦𐎧𐎨 (1 *karûm* = 3600 *kurrû*). See [Fri07, p. 415; Rob19].

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with 𒌦- and 𒌶 numerals with their system $S_{\text{Ur III/OB}}$ values²³. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Fri07, p. 118; Rob19]:



Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:



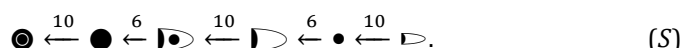
In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

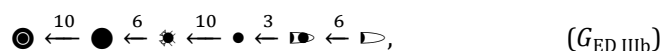
TODO

3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period match the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{UR\ III/OB}$ [Fri07, p. 374; DE87, pp. 127, 165]:



Likewise the area system used in the Early Dynastic IIIb period mirrors system $G_{Ur III/0B}$ [LAK, p. 72; NDE93, p. 63; Fri07, p. 378; Lec16]:



As noted in [L2/04-099, p. 4] (see section 3.2), the vertical \Uparrow from $S_{\text{Ur III/OB}}$ becomes a horizontal \triangleright in system S . It is however far from the only case of such a reallocation of function. The earlier form of System G is [DE87, pp. 141, 165; Fri07, p. 378]:



Observe that, as noted in [DE87, p. 142], \odot changes meaning from $10\bullet$ in system G to $600\bullet$ in system $G_{\text{ED IIIb}}$. System G is used in the Uruk period, but also in the ED I–II period (it is the “area 2” system in [Chao3], whereas $G_{\text{ED IIIb}}$ is the “area 1” system).

²³Adjacent units are no more than a factor of 60 apart, so higher numerals such as 𐎧 or 𐎡 are not used.

24TODO

3.4.3 Grain in Nirsu

TODO note the $\text{𒀭} \text{𒀭}$. Also cite Friberg1978 p. 43 and reproduce its factor diagram.

3.4.4 Grain in Ebla

Lengths of Early Dynastic IIIb dykes from Nirsu are far from the only numeric expressions that mix curviform and cuneiform numerals.

The system of grain³⁵ capacities in Ebla uses the following units³⁶:

$$\text{𒀭} \text{𒀭} \text{𒀭} \xleftarrow{2} \text{𒀭} \text{𒀭} \text{𒀭} \xleftarrow{\frac{5}{2}} \text{𒀭} \xleftarrow{4} \text{𒀭} \text{𒀭} \xleftarrow{6} \text{𒀭} \text{𒀭}.$$

gu₂-bar ba-ri₂-zu nin₄ nin₂-sagš_u an-zam_x

The $\text{𒀭} \text{𒀭} \text{𒀭}$ and $\text{𒀭} \text{𒀭} \text{𒀭}$ are generally counted using curviform numerals, and the smaller units using cuneiform 𒀭 numerals. Indeed, a search on [EbDA] for co-occurrences of either $\text{𒀭} \text{𒀭} \text{𒀭}$ or $\text{𒀭} \text{𒀭} \text{𒀭}$ with either of $\text{𒀭} \text{𒀭} \text{𒀭}$ or $\text{𒀭} \text{𒀭} \text{𒀭}$ finds the following expressions³⁷:

1. [P240532, verso 4, 9] $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$ ³⁹
2. [P240548, verso 1, 1] $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$
3. [P240655, recto 7, 9] $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$ ⁴⁰
4. [P240579, verso 4, 3] $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$
5. [P240675, verso 2, 2] $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$
6. [P240609, verso 3, 1] $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$
7. [P240533, recto 3, 3] $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$
8. [P240697, recto 1, 5] $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$ ⁴¹
9. [P240653, recto 6, 2] $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$
10. [P240654, recto 2, 6] $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$ ⁴² $\text{𒀭} \text{𒀭} \text{𒀭}$ ⁴³

³⁴TODO(egg): Note that one unit may be omitted if the other is present

³⁵Liquid capacities use a different system [Arc15, p. 229 n. 12]:

$$\text{𒀭} \text{𒀭} \text{𒀭} \xleftarrow{30} \text{𒀭} \xleftarrow{6} \text{𒀭} \text{𒀭} \text{𒀭}.$$

la-ha sila₃ an-zam_x

At a glance it seems that 𒀭 are counted with cuneiform numerals and higher units with curviform ones, thus

$$\text{𒀭} \text{𒀭} \text{𒀭} \xleftarrow{\frac{5}{3}} \text{𒀭} \xleftarrow{6} \text{𒀭} \xleftarrow{10} \text{𒀭} \xleftarrow{3} \text{𒀭} \xleftarrow{\frac{10}{3}} \text{𒀭} \xleftarrow{6} \text{𒀭} \text{𒀭} \text{𒀭},$$

𒀭 𒀭 𒀭

but we have not investigated this thoroughly.

³⁶TODO mention the other one citing Chambon and the footnote in Archi

³⁷We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform $\text{𒀭} \text{𒀭} \text{𒀭}$ and larger with cuneiform 𒀭 and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked³⁸ on a photograph (from EbDA unless noted otherwise).

³⁸As we will see in Section 3.7.2, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

³⁹ba-ri₂-zu₂, a variant spelling.

⁴⁰Short for $\text{𒀭} \text{𒀭} \text{𒀭}$.

⁴¹Note the omitted $\text{𒀭} \text{𒀭} \text{𒀭}$.

⁴²Instead of the expected $\text{𒀭} \text{𒀭} \text{𒀭}$.

⁴³ $\text{𒀭} \text{𒀭} \text{𒀭}$ not legible on the EbDA photo.

$$\begin{cases} 4\overline{\text{U}}\ 5\text{o} = 24\overline{\text{U}}\ 3\text{o} & (\text{C } 234) \\ 5\overline{\text{U}}\ 1\text{o} + 1\overline{\text{U}} = 5\overline{\text{U}}\ 7\text{o} & (\text{C } 314) \\ 1\overline{\text{U}} + 1\overline{\text{D}}\ 1\overline{\text{D}}\ 1\overline{\text{D}} = 6\text{o} + 2\overline{\text{U}}\ 1\overline{\text{D}} & (\text{C } 27) \end{cases}$$

These metrological equations for the "unknowns" $\overline{\text{U}}$, o , $\overline{\text{U}}$, etc., can be treated exactly as ordinary equations for unknowns x, y, z, \dots . In particular, the equations can be simplified by subtraction of equal amounts from both sides of the identities. In this way the three equations above can be reduced to:

$$\begin{aligned} 2\text{o} &= 20\overline{\text{U}} & (4\overline{\text{U}}\ 3\text{o} \text{ subtracted from both sides}) \\ 1\overline{\text{U}} &= 6\text{o} & (5\overline{\text{U}}\ 1\text{o} - " = ") \\ 1\overline{\text{D}} &= 6\text{o} + 1\overline{\text{U}}\ 9\overline{\text{D}} & (1\overline{\text{U}} + 1\overline{\text{D}} - " = ") \end{aligned}$$

We can now read off from the first equation that $1\text{o} = 10\overline{\text{U}}$, and from the second that $1\overline{\text{U}} = 6\text{o}$. Then the third equation can be simplified (by "substitution" of these values into the equation), to the following reduced form:

$$1\overline{\text{D}} = 2\overline{\text{U}}\ 9\overline{\text{D}}.$$

The most likely solution to this last equation is, of course,

$$1\overline{\text{D}} = 2\overline{\text{U}}, \quad 1\overline{\text{D}} = 10\overline{\text{D}}.$$

Figure 1: TODO [Fri78, p. 15]

Thus, for instance, the original set of fractions $\overline{\text{U}}$, o , and $\overline{\text{D}}$ ($1/2$, $1/4$ and $1/8$ of an iku) in the Sumerian GANA system, was after a time augmented through the addition of the new sub-unit SAR: $\overline{\text{D}}$, equal to $1/100$ of an iku ($\overline{\text{D}}$). Similarly, the Sumerian weight unit "ma-na" which originally may have had only the sub-units $\overline{\text{U}}$ sa-na (= $1/3$ mana) and $\overline{\text{U}}\overline{\text{U}}$ sa-na-bi (= $2/3$ mana), and perhaps also gin: $\overline{\text{U}}\overline{\text{U}}\overline{\text{U}}$ (= $1/60$ mana), seems to have acquired, at some time or other, also the smaller sub-units $\overline{\text{U}}\overline{\text{U}}\overline{\text{U}}\overline{\text{U}}$ (= $1/3$ gin), and $\overline{\text{U}}\overline{\text{U}}\overline{\text{U}}\overline{\text{U}}\overline{\text{U}}$ se (= $1/3 \times 1/60$ gin).

Figure 2: TODO [Fri78, p. 49]

there is in any case an important qualitative difference between IX for Latin novem and $\overline{\text{U}}$ for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 3: TODO [Englund1988]

of decreasing fractions $1/n$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus $\overline{\text{U}} = 1/2 N_{30}$, $\overline{\text{U}}\overline{\text{U}} = 1/3 N_{30}$, and so on. The first sign of the latter units, N_{34} ,

Figure 4: TODO [Eng98, p. 113]

For instance, the first line contains the notations $1N_{34} 1N_{300}$; $2N_{20}$, which can be translated "60 of the (grain rations containing) $\overline{\text{U}}$ (of grain); (grain involved): $2 \bullet$ (of ground barley)". This calculation contradicts the assumed numerical relationship $10N_1 = 1N_{14}$, since as was well known the measure represented by the sign N_{30} was $1/5$ of that represented by N_1 , so that $60 \times 1/5 = 12$ and not 20, as $2N_{14}$ would imply. Instead of relying on complicated

Figure 5: TODO [Eng98, p. 116]

Die halbkreisförmigen Griffleindrücke gehen manchmal in mehr oder weniger eckige Formen über (∇)⁶⁸⁵. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen (\blacktriangledown), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren⁶⁸⁶. Selten treten mit ∇ gebildete Zahlen auf⁶⁸⁷ (sie entsprechen den *bariga*-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 6: TODO [Kre98, p. 303]

The calculations:									
Obsv. i	1	$60 \times 1/5 \sqsupset (\ominus)$	$= 12 \times \sqsupset =$	$2 \times \bullet$					
	2	$120 \times 1/10 \sqsupset (\boxminus)$	$= 12 \times \sqsupset =$	$2 \times \bullet$					
	3	$120 \times 1/15 \sqsupset (\boxplus)$	$= 8 \times \sqsupset =$	$1 \times \bullet$	$2 \times \sqsupset$				
	4	$300 \times 1/20 \sqsupset (\boxtimes)$	$= 15 \times \sqsupset =$	$2 \times \bullet$	$3 \times \sqsupset$				
	5	$600 \times 1/25 \sqsupset (\boxdot)$	$= 24 \times \sqsupset =$	$4 \times \bullet$					
Rev. i	1	1200		$1 \times \bullet$	$1 \times \bullet$	$5 \times \sqsupset$			
Obsv. i	6	$6000 \times 1/30 \sqsupset (\text{GAR}+6\text{N}_{57})$	$= 200 \times \sqsupset = 1 \times \sqsupset$	$3 \times \bullet$	$2 \times \sqsupset$				
ii	1	$120 \times \approx 1/4 \sqsupset (\text{DUG}_a + \text{U}_{2a})$	$\approx 30 \times \sqsupset =$	$5 \times \bullet$	$1 \times \sqsupset$	$1 \times \sqsupset$			
	2	$180 \times 1/5 \sqsupset (\text{DUG} + \text{AS}_a)$	$= 36 \times \sqsupset =$	$6 \times \bullet$					
	3	$300 \times 1/15 \sqsupset (\text{KAS}_a)$	$= 20 \times \sqsupset =$	$3 \times \bullet$	$2 \times \sqsupset$				
Rev. i	3	600		$1 \times \bullet$	$4 \times \bullet$	$3 \times \sqsupset$			
				$1 \times \bullet$	$1 \times \bullet$	$5 \times \sqsupset$			
				$1 \times \sqsupset$	$3 \times \bullet$	$2 \times \sqsupset$			
				$1 \times \bullet$	$4 \times \bullet$	$3 \times \sqsupset$			
Grand total of groats used:			$1 \times \sqsupset$	$2 \times \bullet$	$9 \times \bullet$	$4 \times \sqsupset$			$1 \times \sqsupset$
Grand total of malt used:	1N_{47}	4N_{20}	3N_5	1N_{42a}	(rev. i 3)	$\times 3/5 =$	$8 \times \bullet$	$4 \times \sqsupset$	$1 \times \sqsupset$

Figure 6. Transliteration and calculations of *MSVO* 4, 66.

Figure 7: TODO [Englund2001]



strong similarities between “area” 1 and “area” 3 systems, the sign with two concentric discs (, noted N₅₀²⁷) remains problematic. It never appears in any numerical combination with the sign with a single disc (,

Figure 8: TODO [Cha03, p. 6]

formed by only two signs \Uparrow and \lhd , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: $1, 60, 60^2, 60^3, 1/60, 1/60^2$, etc. are written in the same way, with the vertical wedge \Uparrow . The third feature concerns the exact function of

Figure 9: TODO [Cha12, p. 58]





one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with:   but never with the repetition  .

Figure 10: TODO [Cha12, p. 59]⁴⁹

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

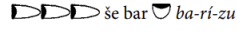

 še bar  *ba-rí-zu*
‘3 *gubar* (capacity units) and 1 *parisu*’.

Figure 11: TODO [Cha12, p. 61]











This is particularly true of the signs , ,  and , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written  in Mesopotamia—also transcribed by Assyriologists as 1 *bán*, 2 *bán*, 3 *bán*, and 4 *bán* with reference to the *bán* measure worth 1/6 of the barig. At Ebla, the sign  is most often associated with the *parisu* measure, while the signs , ,  and  refer to 1, 2, 3,

Figure 12: TODO [Cha12, p. 64]




shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g.,    transliterated as 2(*šar*₂) 1(*geš*^u) 3(*u*), means $2 \times 3600 + 1 \times 600 + 3 \times 10$). The system is based on an alternation of factors ten and

Figure 13: TODO









might think of one fabric and a half,¹¹ but the presence of notations with “2  2 ”, “3  3 ”, and “6  6 ” (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag’s weight measurement system.¹² The notation “1  *gada*” in o. ii 1 and r. vi 1, along with the total of “39



Fig. 1. Combinations of numerals attested in Š. 742.

Figure 14: Discussion of the contrast between  and  numerals in [Gor23, p. 162].





as, for example, in TM.75.G.3125 = *ARET* III 107 o. iv 1, “4  *a*₃-*da-um*^{u9}-2  4 
aktum 4  *ib*₂^{u9} × 3  *sa*₆ *gunu*₃” (Fig. 2).

Figure 15: Transliteration in [Gor23, p. 163] of [P242293, *recto* 4, 1] incorporating untransliterated numerals.

real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as 𒀭 that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [KWU, pp. 123 sqq.] lists all of 𒀭 – 𒀭𒀭𒀭𒀭 together with 𒀭 – 𒀭𒀭𒀭𒀭 , while 𒀭 , 𒀭𒀭 , and 𒀭𒀭𒀭 , and only those, appear at the beginning of the sign list, since they have non-numeric values⁵⁴. [PTACE, p. 58] has the numeric signs 𒀭 , 𒀭𒀭 , 𒀭𒀭𒀭 , whereas non-numeric 𒀭 is at the beginning of the sign list, where its values *aš* and *rum* are listed. For signs with both non-numeric and numeric usage, [LAK] writes *s. die Zahlz.* throughout the main list; LAK 1 𒀭 thus reappears at LAK 829 together with 𒀭 , 𒀭𒀭 , and 𒀭𒀭𒀭 . One should note [MZL], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have 𒀭𒀭𒀭 – 𒀭𒀭𒀭𒀭 used in system $G_{\text{Ur III/OB}}$.

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes⁵⁵ to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from 𒀭 to 𒀭𒀭 numerals prevents diachronic encoding even if 𒀭 were unified with 𒀭 . For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'–6']; however, they are written with 𒀭 numerals, whereas they are written with 𒀭 numerals in the Early Dynastic witnesses; since 𒀭 and 𒀭 are distinct⁵⁶ characters, the 𒀭 – 𒀭 unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods⁵⁷, there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Romach2023], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described

⁵⁴Non-numeric values of 𒀭 were discussed in section 3.5; 𒀭 has the values *man*₃ and *min*₅, and is used for the word *didli*, “several, various”; 𒀭𒀭 has the value *eš*₆.

⁵⁵TODO comment on the ED witnesses to the instructions of Šuruppak





⁵⁶Besides the contrasts in numeric usage mentioned in section 3.3.3, these characters are clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of 𒀭 which are not shared with 𒀭 have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period, 𒀭 is used for the preposition *ina*, “in”, and 𒀭 for the preposition *ana*, “to”.


















⁵⁷TODO cite a few things here.







in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts⁵⁸ being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.


3.7.1 The case of ŠAR₂




The character U+122B9  CUNEIFORM SIGN ŠAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period⁵⁹, a contrast between non-numeric šar₂ written  and numeric 1(šar₂) written  can be observed, similar to the contrast between  and  previously discussed in section 3.5. However, in lexical lists from Šuruppag and Ebla⁶⁰, as well as in the *Stèle des vautours*, non-numeric šar₂ is curviform:

-     and     in [P010566];
-   and    in [P010576];
-   in [P240986]⁶¹;
-   in [P222399, obv. 17, 9, 18, 11, 22, 12]⁶².











It would be disruptive to the diachronic representation of text if non-numeric šar₂ were to have two different representations. The character U+122B9 CUNEIFORM SIGN ŠAR2 should therefore be used in those cases, with its curviform glyph , identical to the glyph of the proposed U+12579 . Since the archaizing style of texts wherein non-numeric šar₂ is curviform solidly predates the transition from  to  in the relevant metrological systems, there is no need to represent a - contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN ŠAR2 effectively merges with U+1212D , the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

Note that in rare cases, such as [P222243] from ED IIIa Adab, non-numeric  (here with the value rum) is written . It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550  cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. In practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the CDLI transliteration (UR2-1(aš@c)) and the




⁵⁸Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

⁵⁹For example, in personal names:

-    in [P020019] from ED IIIb Nirsu;
-    in [P020182], also from ED IIIb Nirsu;
-   in [P222186] from ED IIIb Umma;
-   in [P235312] from Old Akkadian Umma.

⁶⁰TODO Mention other ways in which these are archaizing

⁶¹From copy in [ELLEs, No. 397].

⁶²Note however    on [P222399, obv. 6, 17]. Curviform non-numeric šar₂ is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here. TODO add a photo.

ePSD2 one (uru₈^{rum}) of this word disagree on that aspect. Since — has a cuneiform reference glyph, this does not pose any compatibility concerns.

3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL₂ can be described as 𐎠𐎫𐎠𐎫 NE×KASKAL or 𐎠𐎫𐎠𐎫 NE×PAP⁶³, they are typically all transliterated bil₂, and therefore are all represented by the character U+1224B 𐎠𐎫 CUNEIFORM SIGN NE SHESHIG, its name notwithstanding, as described in [UTR56, §2.5].

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, 𐎠𐎫𐎠𐎫 from [P305639] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in EbDA transliterations: the aforementioned 𐎠𐎫𐎠𐎫 from [P240533, recto 3, 3] is transliterated “20-1-1/2 gu₂-bar 7 nig₂-sagšu 2-1/2 an-zam_x⁶⁴ za”, reading both 𐎠 and 𐎠 as 1/2, but not distinguishing them.

In particular, these transliterations do not differentiate between — and 𐎠 numerals, nor between 𐎠 and 𐎠 numerals. For instance, the aforementioned 𐎠𐎫𐎠𐎫 from [P242293, recto 4, 1] is transliterated “4 ‘a₃-da-um^{tug₂}-II 4 aktum^{tug₂} 4 ib₂-III gun₃ sa₆^{tug₂}” in EbDA, with no distinction between the 𐎠 and 𐎠. Since — and 𐎠 numerals are separately encoded, the numeric expressions in such transliterations cannot be transformed into Unicode cuneiform without additional context, regardless of curviform–cuneiform unification.

In metrological systems such as systems $G_{Ur III/Ob}$ and $C_{Ur III/Ob}$ where some units are indicated by the type of numeral rather than an overt unit sign, it is common practice to add the unit in parentheses in transliteration; for instance, 𐎠𐎫𐎠𐎫 from [P386847] is transliterated “1(eše₃) 5½ iku⁶⁵ 7 sar” in [Feu04, vol. 2, p. 176], and 𐎠𐎫𐎠𐎫 from [P307255] is transliterated “1(n⁶⁶) 2(b) 7 ½ sila₃” in [Feu04, vol. 2, p. 151].

This practice has been generalized to systematically indicate numeral shape; this is in particular the case in CDLI, where the transliterations of some the above examples are “1(gesz2) 3(u) 5(asz) gur” for 𐎠𐎫𐎠𐎫, “1(esze3) 5(iku) 1/2(iku) GAN2 7(disz) sar” for 𐎠𐎫𐎠𐎫, and “3(barig) 2(ban2) 7(disz) 1/2(disz) sila3” for 𐎠𐎫𐎠𐎫. CDLI and ePSD2 both distinguish curviform from cuneiform numerals in transliteration: the length 𐎠𐎫𐎠𐎫 from [P020129, rev. 2, 1] is transliterated “6(gesz2@c) 3(u@c) {ninda}nindax(DU) 1/2(asz@c) 4(disz@t) gi” in CDLI, and “6(geš₂°) 3(u°) ninda_x(DU) 1/2(aš°) 4(diš°) gi” in ePSD2. Another example is [Molina2014], which uses 1a for —, 1d for 𐎠, 1ac for 𐎠, 1dc or ½dc for 𐎠 depending on reading, etc. The literature on the Uruk and Early Dynastic I–II periods uses a different set of transliteration conventions that also disambiguate numeral shapes, as will be discussed in section 4.

⁶³As on [P249253].

⁶⁴As of this writing, EbDA actually has an-zam_x, with U+1D6A GREEK SUBSCRIPT SMALL LETTER CHI.

⁶⁵TODO say something about this reading

⁶⁶TODO comment on nigida.

While there exist transliterations that distinguish \neg from \updownarrow but not \mathbb{B} from \neg , such as the ones used in [DCCMT], the trend, especially in more recent works in third millenium studies, seems to be to represent numeral shape; for example, [Maiocchi2024] gave an example of the input syntax used by the new “Urban Economy Begins” project as “10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)” for $\bullet\mathbb{B}\mathbb{B}\updownarrow\updownarrow$, with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are \triangleright rather than \triangleright numerals.

3.8 Conclusions

Co-occurrences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idiosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted: \lll is a length of three ropes, but $\bullet\bullet$ is an area of three bur₃; $\triangleright\updownarrow$ could be read as one $\mathbb{B}\mathbb{B}\updownarrow$ and one $\updownarrow\mathbb{B}\mathbb{B}$, where $\triangleright\triangleright$ would be one and a half $\mathbb{B}\mathbb{B}\updownarrow$; $\neg\mathbb{B}\mathbb{B}$ is a personal name, but $\triangleright\mathbb{B}\mathbb{B}$ would be “one slave”.

At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millenium corpora. The oddity of \bullet requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.

4 Rationale for ED–Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millenium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millenium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs KAŠa, KAŠb, and KAŠc, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Englund2001]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered script.

As part of the development of these conventions, a classification of fourth millenium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter *N* with a numeric subscript

(sometimes with an additional alphabetic subscript): N_1 is \triangleright , N_{14} is \bullet , N_{34} is \triangleright , etc. Transliterations of numeric expression then use those to identify the type of number used, thus $5N_1$ is $\mathbb{B}\triangleright$, and $5N_{14}$ is $\mathbb{B}\bullet$.

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where the exact meaning of the choice of numeral type is not fully understood, as the transliterations can otherwise force a dubious interpretation. For instance, the CDLI transliteration of $\mathbb{B}\triangleright\mathbb{B}\mathbb{B}\mathbb{B}$ or $\mathbb{B}\mathbb{B}\mathbb{B}\mathbb{B}$ in [P283802] currently uses (barig@c) for the vertical numerals, suggesting a capacity measure; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millenium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I-II period, even though the Sumerian text uses classical assyriological transliteration conventions.

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millenium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in section 3.7.2; they are indeed used interchangeably, as in [P011104] which uses the notation u@f in [ePSD2], but N14@f in CDLI. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millenium would therefore induce confusion as to which numerals should be used in third millenium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millenium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance, $\mathbb{B}\mathbb{B}$ N_{30c} is listed as a variant of $\mathbb{B}\mathbb{B}$ N_{30a} in [DE87, p. 166], where the numeric value of either in relation to \mathbb{B} N_{29a} is still unknown, but their values are found in [Englund2004] to be $\mathbb{B}\mathbb{B} = \frac{1}{10}\mathbb{B}$, whereas $\mathbb{B}\mathbb{B} = \frac{1}{6}\mathbb{B}$.

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban₂@c) and 5(ban₂@c).]

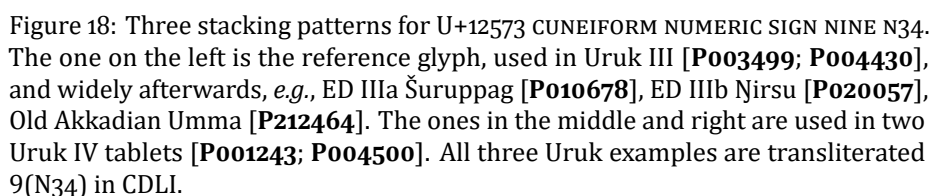
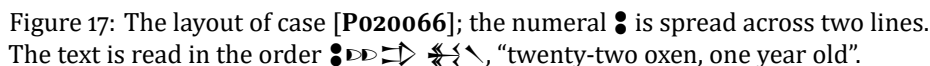
6 Characters not included in this proposal

6.1 The fractions $\frac{1}{3}$ and $\frac{2}{3}$

TODO \mathbb{B} , \mathbb{B} . Note the occasional omission of \mathbb{B} , see citations in email to steve plus <https://cdli.mpiwg-berlin.mpg.de/artifacts/274845/reader/51537>.

6.2 Missing numerals

TODO N13 not attested in CDLI TODO (N_{17} not usefully numeric, $12N_{14}$ not encodable, etc.). Cite [DE87, p. 147] 7 and 8(diš *tenû*) encodable, but not today; want to go



The reference glyphs use stacking patterns that are common in the Early Dynastic period, but that are also attested in the Uruk period; the Uruk period also frequently features numerals that use a more vertical layout, as illustrated in Figure 18. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millennium, usage in third millennium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents \triangleright , \triangleright , \bullet , and \bullet must remain consistent across the numeral series, lest a \triangleright numeral be confused with an \triangleright numeral. Since the single indents are frequently used in running text, as illustrated in section 3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 18 uses OpenType stylistic alternates, and Figure 16 rotates the character **•**, in both cases preserving the plain text backing.

6.4 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Englund2001].

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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