Archaic cuneiform numbers

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2024-08-16

Contents

1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script[1] and the proto-cuneiform script[2]. The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed "curved", or "curviform", numerals[3] should however *not* be unified with the already-encoded cuneiform numerals[4]. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

<table>
<tr><td></td><td>Uruk III & earlier</td><td>ED – Ur III</td><td>OB & later</td></tr>
<tr><td rowspan="2">Numerals</td><td colspan="2">This proposal</td><td></td></tr>
<tr><td></td><td rowspan="2" colspan="2">Existing Xsux</td></tr>
<tr><td>Non-numeric signs</td><td>Future Pcun</td></tr>
</table>

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

[1] ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.
[2] ISO 15924: Pcun, not yet encoded.
[3] 𒐩–𒐼 1–9(aš^c = N_1), 𒐕–𒐳 1–5(u^c = N_{14}), 𒐩–𒐹 1–9(ŋeš$_2{}^c$ = N_{34}), 𒐩–𒐺 1–5(ŋeš'u^c = N_{48}), etc.
[4] 𒐗–𒐽 1–9(aš), 𒐖–𒐾 1–5(u), 𒐜–𒐿 1–9(ŋeš$_2$), 𒐝–𒑀 1–5(ŋeš'u), etc.

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties

2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

TODO(egg): blurb.

3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts,  later in the third millenium[5],  in Old Babylonian cursive, in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists[6] and dictionaries[7], and of composite texts[8]. By being compatible with similarly diachronic transliteration practice (that is, by avoiding distinctions finer than those made in transliteration), the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24][9]. The diachronic approach is also useful for pedagogical applications[10].

3.2 Arguments for curviform–cuneiform unification

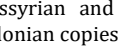
In this context, the argument was made in [And04], as part of discussion of the cuneiform encoding[11] that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [And04, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Ŋirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

[5] Merging with U+1224E NI_2.

[6] Notably [VT+14] and the online edition of [Bor10] in [Jim+23, Signs].

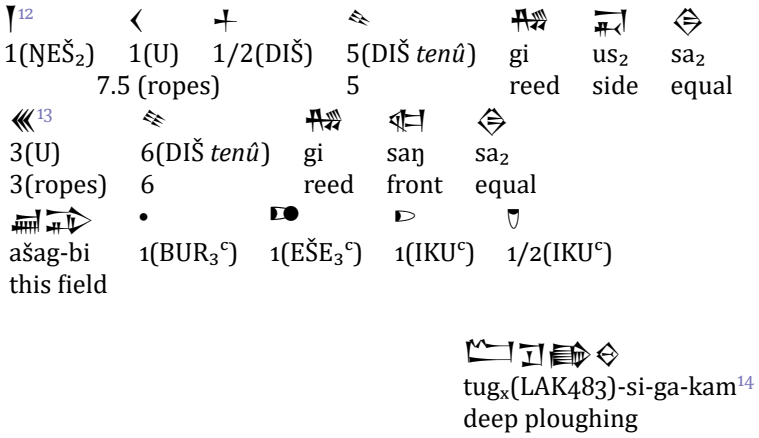
[7] Notably [TJV17] and the online edition of [Sch10] in [Jim+23, Dictionary].

[8] For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of  , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

[9] Attendees may recall the summary given on the third day of UTC #180, as recorded in [Con24]. Other readers may refer to [Svä+24, pp. 242, 148].

[10] For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

[11] At that time scoped to the répertoire of the Ur III period and later, see [EF03, p. 1], although many disunifications, such as ≠ , were informed by Early Dynastic distinctions.



The argument made in [And04, p. 4] is that this is comparable to a stylistic distinction such as[15]

> 465 metres, equal lengths
> 198 metres, equal widths
> this field: 9, 18 hectares, deeply ploughed

where the numerals have the same structure ([And04] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that "the number signs do not normally carry in their individual signs the meaning of what they are used to measure", and that curviform and cuneiform numerals "are not normally mixed together in a single numerical expression", noting the exceptions of [P232278; P232280]. In addition, [And04, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from 𒐙- to 𒐐-numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [EFT03], the curviform numerals were therefore removed from [EFT04b] and [EFT04a], which both state that "The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases."

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

— the functions and use of the numerals vary beyond the mere 𒐙/𒐐 switch;

[12] As noted in [Pow87, p. 466], this sign has a very short "tail" in this period, so that it is wider than it is tall, and can at first seem like a large 𒁹 in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

[13] Note that ED IIIb 𒌋 numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign 𒌋 in [P020054] looks more like Ur III 𒐼.

[14] Transliteration after [Lec20, p. 8].

[15] We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

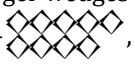
— the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
— some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 Metrology


I want to write tablets: the tablet of 1 gur *of barley to 600* gur*; the tablet of 1 shekel of silver to 10 minas […]*

Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known[16] a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millenium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Robo8, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters 𒁹–𒐙, the multiples of ten (10–50) are 𒌋–𒐚, but the other digits 11–59 are sequences 𒌋𒁹–𒐚𒐙; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units 𒁹–𒐙, tens 𒌋–𒐚, sixties 𒁹–𒑱 (with larger wedges than the units), six hundreds 𒊹–𒐼, three thousand six hundreds 𒇽–  , and thirty-six thousands 𒳡–  .

3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram[17], where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\text{𒳡} \xleftarrow{10} \text{𒇽} \xleftarrow{6} \text{𒊹} \xleftarrow{10} \text{𒁹} \xleftarrow{6} \text{𒌋} \xleftarrow{10} \text{𒁹} \qquad (S_{\text{Ur III/OB}})$$

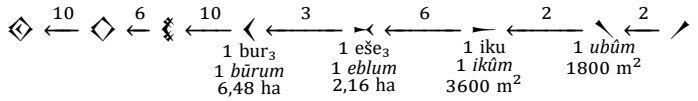
For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written 𒊹𒐚𒐘𒐙 in the discrete counting system, and 𒄭𒐙𒐘𒐙 in the sexagesimal place value system.

[16] See, *e.g.*, [Uni16, Section 22.3.3 "Non-Decimal Radix Systems", *sub* "Cuneiform Numerals"].

[17] These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:

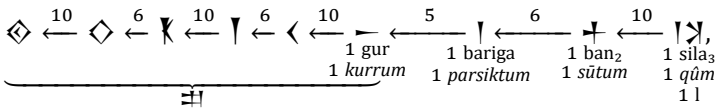


$(G_{\text{Ur III/OB}})$

Note that for the range of areas given above[18], this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *būrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign 𒁹 functioning as punctuation[19], but the numerals are tied to the metrology; thus a surface of 5 *būrū* 1 *eblum* 4 *ikû* (100 *ikû*, 36 ha) would be written[20] 𒐼𒌋𒁹. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in "88 acres 3 roods 33 perches" or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the ŠAR_2 sign 𒊹 is equal to sixty times the U sign 𒌋 in the area system, but to three hundred and sixty times 𒌋 in the discrete counting system.

3.3.3 The capacity system

Another such system of note is the one for capacities[21] [Fri07, p. 376; Rob19],

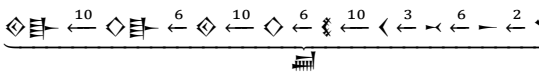


$(C_{\text{Ur III/OB}})$

where the numerals for ban_2 are 𒊺, 𒊺𒊺, 𒊺𒊺𒊺, 𒊺𒊺𒊺𒊺, and 𒊺𒊺𒊺𒊺𒊺, and those for bariga are 𒁹, 𒁹𒁹, 𒁹𒁹𒁹, and 𒁹𒁹𒁹𒁹 (contrast ordinary 𒁹𒁹 and 𒁹𒁹𒁹 otherwise used with 𒁹-numerals). As described in [Hue11, p. 585 with notes (b) and (f)], the sign GUR 𒄑, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𒅆 if present, and after the word for "grain" if present, as in

[22]

[18]For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mūšarum* (36 m^2) written 𒁹𒌍, equal to one hundredth of an *ikûm*, then sexigesimally subdivided in 60 𒂗 (shekels). For areas greater than 3600 *būrū*, the 𒊹- and 𒁲-numerals are reused with a suffix 𒃲 (gal, Sumerian: big), as follows [Rob08, p. 295 with notes b and c; Fri07, p. 378; Rob19]:



[19]TODO(egg): acknowledge Proust 2020 but note that this is irrelevant to encoding concerns

[20]As in the surface of the field of 𒌨𒄩𒅆𒆳 (the city of Apisal) reported on [P102305, r. 1]

[21]Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{\text{Ur III/OB}}$ based on a height of one cubit, see[Pow87, p. 488; Rob08, p. 294; Rob19].

Observe that while large numbers of gur follow[23] system $S_{\text{Ur III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as 𒐹𒐼𒐽 would be 10 gur 1 bariga, and 𒐹𒐻𒐽 would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

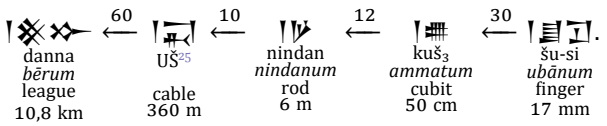
This intertwining of units and numerals explains the large number of already-encoded numeral series:

— 𒐹–𒐳 used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
— 𒐹–𒐴 used in $G_{\text{Ur III/OB}}$, of which 𒐹–𒐵 are also used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
— 𒐹–𒐶 used in $S_{\text{Ur III/OB}}$, and sometimes with overt units;
— 𒐹–𒐷 used in $S_{\text{Ur III/OB}}$;
— 𒐹–𒐸 used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
— 𒐹–𒐺 used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
— 𒐻–𒐿 used in $C_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
— 𒑀, 𒑁, 𒑂, 𒑃, 𒑄 used in $C_{\text{Ur III/OB}}$;
— 𒑅, 𒑆, 𒑇, 𒑈 used in $C_{\text{Ur III/OB}}$—note the overlap with 𒐹–𒐳;
— 𒑉 and 𒑊 used in $G_{\text{Ur III/OB}}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Rob08, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

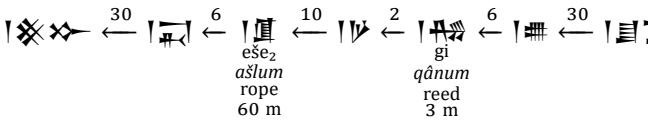
3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with 𒐹- and 𒐻-numerals with their system $S_{\text{Ur III/OB}}$ values[24]. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Fri07, p. 118; Rob19]:



($L_{\text{Ur III/OB}}$)

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:



($\tilde{L}_{\text{Ur III/OB}}$)

[22] From P309594.

[23] A larger unit, the guru7 (*karûm*, grain heap), is sometimes used instead, with 𒐼𒐽𒐾𒐿𒐻=𒐸𒐹 (1 *karûm* = 3600 kurrū). See [Fri07, p. 415; Rob19].

[24] Adjacent units are no more than a factor of 60 apart, so higher numerals such as 𒐷 or 𒐸 are not used.

[25] TODO

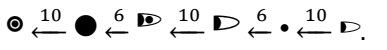
In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

TODO

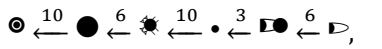
3.4 Early metrology

At first sight, the metrological systems from the Early Dynastic period match the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{\text{Ur III/OB}}$ [Fri07, p. 374; DE87, pp. 127, 165]:



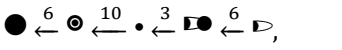
(S)

Likewise the area system used in the Early Dynastic IIIb period mirrors system $G_{\text{Ur III/OB}}$ [Dei22, p. 72; NDE93, p. 63; Fri07, p. 378; Gom16]:



$(G_{\text{ED IIIb}})$

As noted in [And04, p. 4] (see section 3.2), the vertical 𒁹 from $S_{\text{Ur III/OB}}$ becomes a horizontal 𒁺 in system S. It is however far from the only case of such a reallocation of function. The earlier form of System G was [DE87, pp. 141, 165; Fri07, p. 378]:

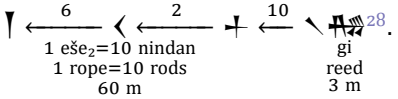


(G)

Observe that, as noted in [DE87, p. 142], 𒌍 changes meaning from 10• in system G to 10• in system $G_{\text{ED IIIb}}$. System G is used in the Uruk period, but also in the ED I–II period (it is the "area 2" system in [Cha03], whereas $G_{\text{ED IIIb}}$ is the "area 1" system).

3.4.1 Field lengths in Ŋirsu

The length system Early Dynastic IIIb of the state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign 𒁹 is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign 𒈨 is not written either. Length shorter than one rope are expressed in half-ropes using the 1/2 sign 𒊹 (again with no 𒈨), and then in reeds, *with* the sign 𒂗. Effectively, this yields the following factor diagram:



[28].

$(L_{\text{ED IIIb}})$

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and others from the same period, such as the ones discussed in [Lec20] areas are expressed in system $G_{\text{ED IIIb}}$, with curviform numerals[29]; in the absence of overt units, such as when dealing with length that

[28] Note that the reeds are counted using *tenû* numerals, 𒐗, 𒐘, 𒐙, etc.

[29] TODO(egg): Note the handful of late Urukagina tablets that start to have cuneiform areas.

are integer multiples of a half-rope[30], the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and therefore the interpretation of its numerals between systems $G_{\text{ED IIIb}}$ and $L_{\text{ED IIIb}}$. The sign GAN_2 𒄭, which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without 𒄭.

3.4.2 Dyke lengths in Ŋirsu

[Pow87, p. 466] notes that reeds "are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus." [TODO(egg): also mention Krebernik 1998 p. 303 with note 686.] Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in CDLI finds many occurrences[31], such as:

— [P221305, obv. 1, 4][32] 𒐏𒁕𒌋𒁹𒈫𒌋𒁹𒄑𒁾
— [P020129, rev. 2, 1] 𒐊𒐊𒐊𒁻𒄑𒁹𒌋𒁹
— [P221291, rev. 5, 1][33] 𒐏𒐏𒐏𒁻𒁕𒄑𒁹𒌋𒁹
— [P221266, rev. 2, 1] 𒐏𒁻𒁕𒁹𒌋

These expressions use an explicit sign 𒄑 (counted in multiples of ten) or 𒁕. This notation—but not its use of curviform numerals—is remarked on in [Lec20, p. 290 with note 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. These match the following factor diagram:

$$\underbrace{\text{𒐏}\xleftarrow{10}\text{𒐏}\xleftarrow{6}\text{•}}_{\text{𒄑}} = \underbrace{\text{𒐏}\xleftarrow{2}\text{𒁻}}_{\text{𒁕}[34]}\xleftarrow{10}\text{𒌋}\xleftarrow{6}\text{𒌋}\xleftarrow{3}\text{𒄑}\text{𒁾}\text{𒌋} \qquad (L'_{\text{ED IIIb}})$$

3.4.3 Grain in Ebla

Lengths of Early Dynastic IIIb dykes from Ŋirsu are far from the only numeric expressions that mix curviform and cuneiform numerals.

[30] This is the case of the sides of the field in [P020054, obv. ii 2–3].

[31] A search for curviform numerals followed by some number of reeds counted in (*tenû*) cuneiform numerals currently finds 125 occurrences across 47 tablets.

[32] CDLI only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the 𒄑𒁾𒌋 is not visible. Lecompte notes that the copy is faithful; indeed another 𒄑𒁾𒌋 can be seen both on the copy and the photo on obv. 2, 2.

[33] From copy.

[34] TODO(egg): Note that one unit may be omitted if the other is present

The system of grain[35] capacities in Ebla uses the following units[36]:

$$\underset{\text{gu}_2\text{-bar}}{𒄘𒁇} \xleftarrow{2} \underset{\text{ba-ri}_2\text{-zu}}{𒁀𒌷𒍪} \xleftarrow{\frac{5}{2}} \underset{\text{ŋin}_4}{𒂷} \xleftarrow{4} \underset{\text{niŋ}_2\text{-sagšu}}{𒃻𒋗𒊕} \xleftarrow{6} \underset{\text{an-zam}_x}{𒀭𒍠}.$$

The 𒄘𒁇 and 𒁀𒌷𒍪 are generally counted using curviform numerals, and the smaller units using cuneiform 𒁹 numerals. Indeed, a search on [Mil+07] for co-occurrences of 𒀭𒍠 with either of 𒄘𒁇 or 𒁀𒌷𒍪 finds the following expressions[37]:

1. [P240532, *verso* 4, 9] ◗𒐏 𒁀𒌷𒍪[39] 𒐈 𒃻𒋗𒊕
2. [P240548, *verso* 1, 1] ◗𒁀𒌷𒍪 𒐈𒀭𒍠
3. [P240655, *recto* 7, 9] ◗◗𒐏𒁇 [40] 𒐈 𒃻𒋗𒊕
4. [P240579, *verso* 4, 3] ◗◗◗◗◗◗▽𒄘𒁇 𒈫𒃻𒋗𒊕
5. [P240675, *verso* 2, 2] ◗𒐏 𒁀𒌷𒍪 𒐈 𒃻𒋗𒊕
6. [P240609, *verso* 3, 1] ◗𒁀𒌷𒍪 𒐈𒀭𒍠
7. [P240533, *recto* 3, 3] ••◗▽𒄘𒁇 𒐊𒃻𒋗𒊕 𒈫𒈦𒀭𒍠𒐉
8. [P240697, *recto* 1, 5] ◗▽[41]𒈫𒂷 𒐈𒀭𒍠
9. [P240653, *recto* 6, 2] ••◗◗◗▽𒄘𒁇𒁹𒃻𒋗𒊕 𒈫𒈦𒀭𒍠
10. [P240654, *recto* 2, 6] ◗𒁀𒌷𒍪 𒐊𒋡[42] 𒐈𒀭𒍠[43]
11. [P240531, *recto* 1, 8] ◗𒁀𒌷𒍪 𒐊𒃻𒋗𒊕 𒐏𒆷𒋡
12. [P241708, *recto* 1, 1][44] ◗◗▽𒆕 𒄘𒁇 𒐊𒃻𒋗𒊕
13. [P241904, *recto* 1, 1][45] ◗◗◗◗𒄘𒁇 𒐉[46]𒃻𒋗𒊕

Note that higher numbers of 𒄘𒁇 are expressed in hundreds (*mi-at* 𒈪𒀜) and then thousands (*li-im* 𒇷𒅎), as is typical in Ebla [Arc15, p. 33], *e.g.*, in [P240532, *verso* 2, 3], ◗𒈪𒀜◗•◗◗◗◗◗𒐏 𒄘𒁇 (100 + 60 + 30 + 5 = 195 𒄘𒁇 of grain).

[35]Liquid capacities use a different system [Arc15, p. 229 with note 12]:

$$\underset{\text{la-ḫa}}{𒆷𒄩} \xleftarrow{30} \underset{\text{sila}_3}{𒋡} \xleftarrow{6} \underset{\text{an-zam}_x}{𒀭𒍠}.$$

At a glance it seems that 𒋡 are counted with cuneiform numerals and higher units with curviform ones, thus

$$\underbrace{◗𒈪𒀜 \xleftarrow{\frac{5}{3}} ◗ \xleftarrow{6} • \xleftarrow{10} ◗}_{𒆷𒄩} \xleftarrow{3} \underbrace{𒌋 \xleftarrow{10} 𒁹}_{𒋡} \xleftarrow{6} 𒁹𒀭𒍠,$$

but we have not investigated this thoroughly.

[36]TODO mention the other one citing Chambon and the footnote in Archi

[37]We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform 𒁀𒌷𒍪 and larger with cuneiform 𒂷 and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked[38]on a photograph (from EbDA unless noted otherwise).

[38]As we will see in Section 3.6.1, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

[39]ba-ri$_2$-zu$_2$, a variant spelling.

[40]Short for 𒄘𒁇.

[41]Note the omitted 𒄘𒁇.

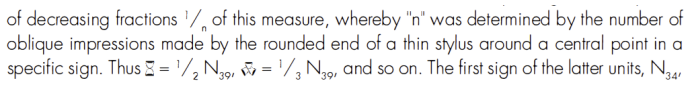
[42]Instead of the expected 𒃻𒋗𒊕.

[43]𒐈𒀭𒍠 not legible on the EbDA photo.

[44]From CDLI photo.

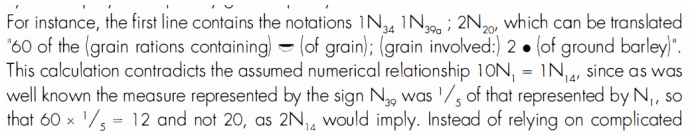
[45]From photo in [Arc89, p. 6].

[46]Laid out as 𒁹𒁹𒁹𒁹; on stacking patterns see Section 6.2.



of decreasing fractions $^{1}/_{n}$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus $\overline{\Xi} = {}^{1}/_{2}\ N_{39}$, $\overline{\overline{\Xi}} = {}^{1}/_{3}\ N_{39}$, and so on. The first sign of the latter units, N_{34},

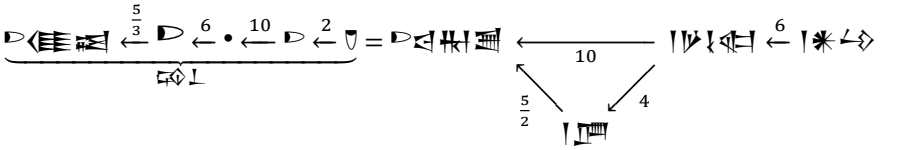
Figure 1: TODO [Eng98, p. 113]



For instance, the first line contains the notations $1N_{34}\ 1N_{39a}$; $2N_{20}$, which can be translated "60 of the (grain rations containing) ⏤ (of grain); (grain involved:) 2 • (of ground barley)". This calculation contradicts the assumed numerical relationship $10N_{1} = 1N_{14}$, since as was well known the measure represented by the sign N_{39} was $^{1}/_{5}$ of that represented by N_{1}, so that $60 \times {}^{1}/_{5} = 12$ and not 20, as $2N_{14}$ would imply. Instead of relying on complicated

Figure 2: TODO [Eng98, p. 116]

These expressions match the following factor diagram:



(C_{Ebla})

3.4.4 Use in modern publications

Because of their prevalence in the Uruk and Early Dynastic periods, the proposed numerals are used modern publications discussing metrology in those periods, as illustrated in Figures 1–10.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 10. The partial[47] transliteration "4 *ʾa₃-da-um* 4 *aktum* 4 $ib_2^{tu_9}$×3 sa_6 $gunu_3$" is used to illustrate a discussion of the interpretation of the contrast between and numerals. More conventional transliterations[48] might omit the numeral shapes entirely, *e.g.*, 4 *ʾa₃-da-um* 4 *aktum* 4 $ib_2^{tu_9}$×3 sa_6 $gunu_3$, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, *e.g.*, 4(aš^{c}) *ʾa₃-da-um* 4(diš^{c}) *aktum* 4(aš^{c}) $ib_2^{tu_9}$×3(diš) sa_6 $gunu_3$, but the result would be less readable. See Section 3.6.1 for a discussion of transliteration conventions for numerals.

[47] The untransliterated text would be ; note the atomically

Die halbkreisförmigen Griffeleindrücke gehen manchmal in mehr oder weniger eckige Formen über ( )[685]. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen ( ), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren[686]. Selten treten mit ∪ gebildete Zahlen auf[687] (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 3: TODO [Kre98, p. 303]

formed by only two signs 𒁹 and 𒌋, repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60^2, 60^3, 1/60, $1/60^2$, etc. are written in the same way, with the vertical wedge 𒁹. The third feature concerns the exact function of

Figure 4: TODO [Cha12, p. 58]

one step. The scribes of the Early Dynastic Period (*c.* 2600 BC), for instance, represented the number 648,000 with: 𒐼𒐐𒐐𒐐 but never with the repetition 𒐼𒐼𒐼 𒐼𒐐𒐐𒐐.

Figure 5: TODO [Cha12, p. 59][49]

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

𒁕𒁕𒁕 še bar 𒁍 *ba-rí-zu*

'3 *gubar* (capacity units) and 1 *parīsu*'.

Figure 6: TODO [Cha12, p. 61]

This is particularly true of the signs 𒁍, 𒁍, 𒁍 and 𒁍, whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written 𒁍 in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign 𒁍 is most often associated with the *parīsu* measure, while the signs 𒁍, 𒁍, 𒁍 and 𒁍 refer to 1, 2, 3,

Figure 7: TODO [Cha12, p. 64]

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., 𒐈𒐈𒐕𒐊𒐊𒐊𒐊 transliterated as 2(šar$_2$) 1(geš'u) 3(u), means $2 \times 3600 + 1 \times 600 + 3 \times 10$). The system is based on an alternation of factors ten and

Figure 8: TODO

might think of one fabric and a half,[11] but the presence of notations with "2𒁕 2𒁹", "3𒁕 3𒁹", and "6𒁕 6𒁹" (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag's weight measurement system.[12] The notation "1𒁕 gada" in o. ii 1 and r. vi 1, along with the total of "39

Fig. 1. Combinations of numerals attested in Š. 742.

Figure 9: Discussion of the contrast between 𒁕 and 𒁹 numerals in [Gor23, p. 162].

as, for example, in TM.75.G.3125 = *ARET* III 107 o. iv 1, “4 a_3-*da-um*^{tu9}-2 4 aktum 4 ib_2^{tu9} ×3 sa_6 $gunu_3$” (Fig. 2).

Figure 10: Transliteration in [Gor23, p. 163] of [P242293, *recto* 4, 1] incorporating untransliterated numerals.

3.5 Non-numeric usage

The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for ‘sixty’. Do you know its reading?

Examenstext A

3.6 Limited benefits of diachronic encoding for numerals

[TODO Composite texts dating back to the period where curved numerals are in use tend to be limited to lexical texts, which do not usually have numbers. When they do, diachronic encoding is prevented by diš-aš distincitons anyway. Administrative texts, which are where numbers are most prominent, are not composite.]

[TODO Diachronic reference works tend to not include numbers, or when they do, to treat them specially (for intance, they are shown at the end of sign lists such as TODO).]

[TODO The overarching goal of having consistent representation for equivalent numeric expressions from different periods is quickly foiled by changes in metrology.]

Note that in [Rom24] [TODO(egg): Cite the GitHub repository], as in many other such analyses, numbers are removed as an early step in processing; these therefore would not benefit from diachrony in the encoding of numeric expressions.

3.6.1 Compatibility with transliteration

TODO words [Rob08, p. 295] TODO cite [**Molina2014**]

3.7 Compatibility considerations

3.7.1 The case of ŠAR$_2$

[TODO explain why this isn’t a problem, effectively anyone who needs to cuneify 1(šar$_2$^{c}) will also need to cuneify some of the numerals proposed here and will therefore not be using Unicode cuneiform.] [TODO U+122B9 CUNEIFORM SIGN SHAR2 represents both 1(šar$_2$) and non-numeric šar$_2$; it looks like 𒊹 (so, like 𒊹) in all but lexical texts from Ebla and Šuruppag (and the archaizing vulture stele, where note that the scribe slipped into his modern ways once), where it looks like (TODO: the proposed character). The proposed character is to be used for 1(šar$_2$^{c}). 1(šar$_2$) does not exist back when non-numeric šar$_2$ is curviform, so it works out.]

encoded $ib_2 \times 3$ = 𒁀 × 𒐛 = 𒁇.

[48] TODO cite the EbDA one.

[49] TODO(egg): On the order cite TSS 188, Friberg2007 p. 148 and any of the usual suspects on the haphazard order of signs in early texts; contrast P274845, P241764.

4 Rationale for ED–Uruk numeral unification

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban$_2$@c) and 5(ban$_2$@c).]

6 Characters not included in this proposal

6.1 Missing numerals

(N_{17}, $12N_{14}$, etc.) 7(diš *tenû*)

6.2 Stacking patterns

(… are a mess, vary within Uruk, and are not transliterated/documented by Englund, so let's not go there for now.)

6.3 Matters for higher-level protocols

Rotated bits: https://cdli.mpiwg-berlin.mpg.de/artifacts/101087

7 Acknowledgements

TODO(egg): Something about the Vanséveren fonts

References

[And04] L. Anderson. *Unification of Cuneiform Numbers*. 2004.
UTC: L2/04-099.

[Arc15] A. Archi. *Ebla and Its Archives. Texts, History, and Society*. Studies in ancient Near Eastern records 7. Walter de Gruyter, 2015.
ISBN: 978-1-61451-716-0.
DOI: 10.1515/9781614517887.

[Arc89] A. Archi. "Tables de comptes eblaïtes". In: *Revue d'assyriologie et d'archéologie orientale* 83.1 (1989). Ed. by P. Amiet and P. Garelli, pp. 1–6. ISSN: 0373-6032.

[Bor10] R. Borger. *Mesopotamisches Zeichenlexikon*. Alter Orient und Altes Testament 305. Ugarit-Verlag, 2010.

[Cap02] R. Caplice. *Introduction to Akkadian*. 4th ed. Editrice Pontificio Istituto Biblico, 2002.
ISBN: 88-7653-566-7.

[Cha03] G. Chambon. "Archaic Metrological Systems from Ur". In: *Cuneiform Digital Library Journal* 2003.5 (Dec. 23, 2003). ISSN: 1540-8779.
eprint: http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003_005.html.

[Cha12] G. Chambon. “Numeracy and Metrology”. In: *The Oxford Handbook of Cuneiform Culture*. Ed. by K. Radner and E. Robson. Oxford University Press, Sept. 18, 2012, pp. 51–67.
ISBN: 9780199557301.
DOI: 10.1093/oxfordhb/9780199557301.013.0003.

[Con24] P. Constable, ed. *Minutes of UTC Meeting 180* (July 23–25, 2024). July 29, 2024.
UTC: L2/24-159.

[DE87] P. Damerow and R. K. Englund. “Die Zahlzeichensysteme der archaischen Texte aus Uruk”. In: M. W. Green and H. J. Nissen. *Zeichenliste der archaischen Texte aus Uruk*. Archaische Texte aus Uruk 2. An offprint of this chapter is available at https://cdli.mpiwg-berlin.mpg.de/files-up/publications/englund1987a.pdf. Gebr. Mann Verlag, 1987. Chap. 3, pp. 117–165.

[Dei22] A. Deimel. *Liste der archaischen Keilschriftzeichen von Fara*. Wissenschaftliche Veröffentlichungen der Deutschen Orient-Gesellschaft 40. J. C. Hinrichs’sche Buchhandlung, 1922.

[EF03] M. Everson and K. Feuerherm. *Basic principles for the encoding of Sumero-Akkadian Cuneiform*. May 25, 2003.
UTC: L2/03-162.

[EFT03] M. Everson, K. Feuerherm, and S. Tinney. *Preliminary proposal to encode the Cuneiform script in the SMP of the UCS*. Nov. 3, 2003.
UTC: L2/03-393R.

[EFT04a] M. Everson, K. Feuerherm, and S. Tinney. *Final proposal to encode the Cuneiform script in the SMP of the UCS*. June 8, 2004.
UTC: L2/04-189.

[EFT04b] M. Everson, K. Feuerherm, and S. Tinney. *Revised proposal to encode the Cuneiform script in the SMP of the UCS*. Jan. 29, 2004.
UTC: L2/04-036.

[Eng98] R. K. Englund. “Texts from the Late Uruk Period”. In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Vol. 160. Orbis Biblicus et Orientalis 1. 1998, pp. 13–233.
ISBN: 3-7278-1166-8.

[För16] W. Förtsch. *Altbabylonische Wirtschaftstexte aus der Zeit Lugalanda’s und Urukagina’s*. Vorderasiatische Schriftdenkmäler der Königlichen Museen zu Berlin 14. J. C. Hinrichs, 1916.

[Fri07] J. Friberg. *A Remarkable Collection of Babylonian Mathematical Texts. Manuscripts in the Schøyen Collection: Cuneiform Texts I*. Sources and Studies in the History of Mathematics and Physical Sciences. Springer, 2007.
ISBN: 978-0-387-34543-7.

[Fri78] J. Friberg. *A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and Proto-Elamite Semi-Pictographic Inscriptions*. The Third Millenium Roots of Babylonian Mathematics 1. Department of Mathematics, Chalmers University of Technology, 1978.

[Gom16] B. Gombert. “ED IIIb metrology: texts from Lagaš”. In: *CDLI:wiki. A Library of Knowledge of the Cuneiform Digital Library Initiative*. Apr. 12, 2016.
eprint: https://cdli.ox.ac.uk/wiki/doku.php?id=ed_iii_metrological_systems.

[Gor23] F. Gori. “On Lapis Lazuli and Linen in Šuruppag Texts. An Analysis Through the Lens of Ebla Studies”. In: *Studia Eblaitica* 9 (2023), pp. 160–166. ISSN: 2364-7124.

[Hue11] J. Huehnergard. *A Grammar of Akkadian*. 3rd ed. Brill, 2011.
ISBN: 978-1-57506-941-8.

[Jim+23] E. Jiménez, Z. Földi, A. Hätinen, A. Heinrich, T. Mitto, G. Rozzi, I. Khait, J. Laasonen, F. Simonjetz, et al., eds. *electronic Babylonian Library*. 2023–.
eprint: https://www.ebl.lmu.de/.

[JJ24] T. Jauhiainen and H. Jauhiainen. “Advancing Cuneiform Text Dating Through Automatic Analysis”. 69th Rencontre Assyriologique Internationale (July 8–12, 2024). July 11, 2024 14:00.

[Kre98] M. Krebernik. “Die Texte aus Fāra und Tell Abū Ṣalābīḫ”. In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 235–427.
ISBN: 3-7278-1166-8.

[Lec12] C. Lecompte. “Des chiffres et des digues: à propos de deux textes présargoniques de Ĝirsu et d’une notation numérique inhabituelle”. In: *Altorientalische Forschungen* 39.1 (Dec. 2012), pp. 81–86.
DOI: 10.1524/aofo.2012.0006.

[Lec20] C. Lecompte. “The Measurement of Fields During the Pre-sargonic Period”. In: *Mathematics, Administrative and Economic Activities in Ancient Worlds*. Ed. by C. Michel and K. Chemla. Why the Sciences of the Ancient World Matter 5. Springer, 2020.

[Mil+07] L. Milano, M. Maiocchi, F. Di Filippo, R. Orsini, E. Scarpa, M. Surdi, et al., eds. *Ebla Digital Archives*. 2007–.
eprint: http://ebda.cnr.it/.

[NDE93] H. J. Nissen, P. Damerow, and R. K. Englund. *Archaic Bookkeeping. Early Writing and Techniques of Economic Administration in the Ancient Near East*. Trans. by P. Larsen. The University of Chicago Press, 1993.
ISBN: 0-226-58659-6.

[Oel22] J. Oelsner. *Der Kodex Ḫammu-rāpi*. dubsar 4. Zaphon, 2022.

[P020054] VAT 4731. [För16, 40 p.14]. Vorderasiatisches Museum.
CDLI: P020054.

[P020129] VAT 04713. Vorderasiatisches Museum.
CDLI: P020129.
ORACC: epsd2/corpus/P020129.

[P102305] X.3.139. Michael C. Carlos Museum, Emory University.
CDLI: P102305.

[P221266] AO 13825. Musée du Louvre.
CDLI: P221266.
ORACC: epsd2/corpus/P221266.

Louvre Collections: ark:/53355/cl010138527.

[P221291] AO 13850. Musée du Louvre.
CDLI: P221291.
ORACC: epsd2/corpus/P221291.

[P221305] AO 13864. Musée du Louvre.
CDLI: P221305.
ORACC: epsd2/corpus/P221305.

[P232278] *Gudea E*. AO 6. Musée du Louvre.
CDLI: P232278.
ORACC: etcsri/Q001544.

[P232280] *Gudea G*. AO 7. Musée du Louvre.
CDLI: P232280.
ORACC: etcsri/Q001546.

[P240531] TM.75.G.00265. Idlib, Syria: National Museum of Syria.
CDLI: P240531.
EbDA: 1415.

[P240532] TM.75.G.00266. Idlib, Syria: National Museum of Syria.
CDLI: P240532.
EbDA: 1324.

[P240533] TM.75.G.00267. Idlib, Syria: National Museum of Syria.
CDLI: P240533.
EbDA: 1379.

[P240548] TM.75.G.00302. Idlib, Syria: National Museum of Syria.
CDLI: P240548.
EbDA: 1350.

[P240579] TM.75.G.00341. Idlib, Syria: National Museum of Syria.
CDLI: P240579.
EbDA: 1364.

[P240609] TM.75.G.00440. Idlib, Syria: National Museum of Syria.
CDLI: P240609.
EbDA: 1378.

[P240653] TM.75.G.00535. Idlib, Syria: National Museum of Syria.
CDLI: P240653.
EbDA: 1382.

[P240654] TM.75.G.00536. Idlib, Syria: National Museum of Syria.
CDLI: P240654.
EbDA: 1383.

[P240655] TM.75.G.00537. Idlib, Syria: National Museum of Syria.
CDLI: P240655.
EbDA: 1358.

[P240675] TM.75.G.00557. Idlib, Syria: National Museum of Syria.
CDLI: P240675.
EbDA: 1371.

[P240697] TM.75.G.00579. Idlib, Syria: National Museum of Syria.
CDLI: P240697.
EbDA: 1381.

[P241708] TM.75.G.02143. Idlib, Syria: National Museum of Syria.
CDLI: P241708.
EbDA: 3173.

[P241904] TM.75.G.02346. [Arc89, p. 6]. Idlib, Syria: National Museum of Syria.
CDLI: P241904.
EbDA: 3183.

[P242293] TM.75.G.03125. Idlib, Syria: National Museum of Syria.
CDLI: P242293.
EbDA: 217.

[P249253] *Code de Hammurabi*. Sb 8. Musée du Louvre.
CDLI: P249253.

[Pow87] M. Powell. "Maße und Gewichte". In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš–Medizin. 1987–1990, pp. 457–530.

[Rob08] E. Robson. *Mathematics in Ancient Iraq. A Social History*. Princeton University Press, 2008.
ISBN: 978-0-691-09182-2.

[Rob19] E. Robson. "Oracc metrology guidelines". In: *Oracc: The Open Richly Annotated Cuneiform Corpus*. Dec. 18, 2019.
eprint: http://oracc.org/doc/help/editinginatf/metrology/metrologicaltables/.

[Rob22] E. Robson. "Overview of Metrological Systems". In: *The Digital Corpus of Cuneiform Mathematical Texts*. 2022.
eprint: http://oracc.org/dccmt/Metrology/.

[Rom24] A. Romach. "The Neo Assyrian Land Sale Documents from Dur-Katlimmu: A Stylometric Analysis of Their Scribal Features". 69th Rencontre Assyriologique Internationale (July 8–12, 2024). July 10, 2024 12:00.

[Sch10] W. Schramm. *Akkadische Logogramme*. Göttinger Beiträge zum Alten Orient 5. Universitätsverlag Göttingen, 2010.
ISBN: 978-3-941875-65-4.
DOI: 10.17875/gup2010-511.

[Svä+24] S. Svärd, M. Lorenzon, J. Töyräänvuori, J. Valk, T. Alstola, E. Bennett, R. Uotila, and T. Auranne, eds. *RAI 69 Abstracts*. July 2024.
eprint: https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäivätty_1.pdf.

[TJV17] S. Tinney, P. Jones, and N. Veldhuis, eds. *The electronic Pennsylvania Sumerian Dictionary*. 2nd ed. 2017–.
eprint: http://oracc.org/epsd2.

[Uni16] The Unicode Consortium. *The Unicode Standard*. Version 16.0.0. The Unicode Consortium, Sept. 10, 2024.
ISBN: 978-1-936213-34-4.
eprint: https://www.unicode.org/versions/Unicode16.0.0/core-spec/.

[UTR56] R. Leroy, ed. *Unicode Cuneiform Sign Lists*. Unicode Technical Report #56.
eprint: https://www.unicode.org/reports/tr56/.

[VT+14] N. Veldhuis, S. Tinney, et al., eds. *Oracc Sign List*. 2014–.
eprint: http://oracc.org/osl/.