

Archaic cuneiform numbers

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1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script¹ and the proto-cuneiform script². The proposed characters are listed in section 2. Most of them were listed in [L2/23-190]; however, the present document provides a more detailed rationale for their encoding and additional information about their identity.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

¹ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

²ISO 15924: Pcun, not yet encoded.

³Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: ▷, ▽ (small stylus), ▷, ▽ (large stylus). Some numerals are composed of multiple such impressions, e.g., ▷▷. The terms “curved”, “curviform”, and “round” can be found in the literature. We avoid the term “round” here as it has other meanings in the context of numbers. We use “curviform” in this document as, being the least common term, it is least likely to lead to confusion, and “CURVED” in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [inlineATF].

⁴Impressed into clay using a stylus with a trihedral end: — (stylus held horizontally), † (vertically), ↘ (diagonally) ‹ (with the head of the stylus), ‡ (stylus pressed deeper, forming a larger wedge), ‡ (combining † and ‹), etc.

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
Non-numeric signs	Future Pkun	Existing Xsux	

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties





2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [Powell1972] “in the same fashion as we use black and red ink”; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

3.1 The cuneiform encoding model

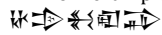
As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts,  later in the third millenium⁵,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists⁶ and dictionaries⁷, and of composite texts⁸. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which

⁵Merging with U+1224E CUNEIFORM SIGN NI2.

⁶Notably [OSL] and the online edition of [MZL] in [eBL, Signs].

⁷Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

⁸For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

has proven useful as a processing step in analyses such as [Rom24; JJ24]⁹. The diachronic approach is also useful for pedagogic applications¹⁰.

3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding¹¹ that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

1(ḫeš ₂)	1(u)	1/2(diš)	5(diš tenû)	gi	us ₂	sa ₂
	7.5 (ropes)		5	reed	side	equal
3(u)	6(diš tenû)	gi	saṇ	sa ₂		
3 (ropes)	6	reed	front	equal		
ašag-bi	1(bur ₃)	1(eše ₃)	1(iku)	1/2(iku)		
ašag=bi						
field=DEM ¹⁴						
				tug _x (LAK483)	si-ga-kam	
				tugsiga	=ak	=am -Ø
				ploughed=GEN=COP-3.SG.S		

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as¹⁵

465 metres, equal lengths
 198 metres, equal widths
 this field is 9, 18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further

⁹Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svā+24, pp. 242, 148].

¹⁰For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

¹¹At that time scoped to the repertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as ≠ , were informed by Early Dynastic distinctions.

¹²As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large — in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

¹³Note that ED IIIb < numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign in [P020054] looks more like Ur III .

¹⁴Alternatively: area=POSS.3.SG.NH, “its area”.

¹⁵We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

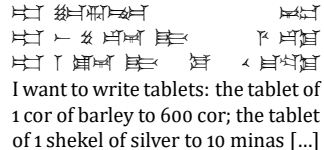
claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from \triangleright to ∇ numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere \triangleright/∇ switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 A primer on classic Ur III and Old Babylonian metrologies









I want to write tablets: the tablet of
1 cor of barley to 600 cor; the tablet
of 1 shekel of silver to 10 minas [...]

Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the contrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known¹⁶ a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters ∇ – 𒌦 , the multiples of ten (10–50) are 𒌦 – 𒌦 , but the other digits 11–59 are sequences 𒌦 – 𒌦 ; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$,



¹⁶See, *e.g.*, [Uni16, §22.3.3, sub “Cuneiform Numerals”].

which had different signs for the units \dashv -, tens \lessdot -, sixties \dashv - (with larger wedges than the units), multiples of six hundred \dashv -, multiples of three thousand six hundreds \diamond -, and multiples of thirty-six thousand \diamond -.

3.3.1 The discrete counting system






The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram¹⁷, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \blacktriangledown \xleftarrow{10} \blacktriangledown \xleftarrow{6} \blacktriangledown \xleftarrow{10} \blacktriangledown \quad (S_{\text{Ur III/OB}})$$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written  in the discrete counting system, and  in the sexagesimal place value system.

3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:

Note that for the range of areas given above¹⁸, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *bûrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign  functioning as punctuation¹⁹, but the numerals are tied to the metrology; thus a surface of 5 *bûr* 1 *ebel* 4 *ikû* (100 *ikû*, 36 ha) would be written²⁰    . Contrast this with systems where the same numerals are used for different units, and overt units are used, as in “88 acres 3 roods 33 perches” or 五頃八畝五分九厘. Note also that the same signs are shared between multiple

¹⁷These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

¹⁸For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mûšarum* (36 m²) written 𒌦𒍪, equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60 𒌦𒍪 (shekels). For areas greater than 3600 *būrû*, the 𒌦 and 𒍪 numerals are reused with a suffix 𒂗 (gal, Sumerian: big), as follows [Robo8, p. 295 n. b and c; Fri07, p. 378; Rob19]:

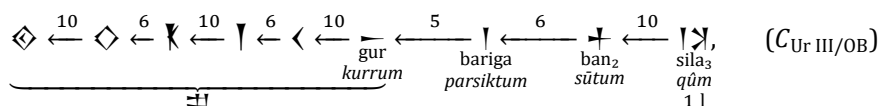
¹⁹This sign is sometimes interpreted as a measurement unit, and transliterated *iku*, see, *e.g.*, [Proust2020], or transliterations in [Feu04] discussed in section 3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system $G_{Ur III/Ob}$.

²⁰As in the surface of the field of (the city of Apisal) reported on [P102305, r. 1]






systems, with different relations; the sign \diamond is equal to sixty times \angle in the area system, but to three hundred and sixty times \angle in the discrete counting system.

3.3.3 The capacity system

Another such system of note is the one for capacities²¹ [Fri07, p. 376; Rob19],



where the numerals for ban₂ are 𐤁, 𐤂, 𐤃, 𐤄, and 𐤅, and those for bariga are 𐤆, 𐤇, 𐤈, and 𐤉 (contrast ordinary 𐤆 and 𐤇 otherwise used with 𐤆 numerals). As described in [Hue11, p. 585 n. (b) and (f)], the sign 𐤊 GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𐤋 if present, and after the word for “grain” if present, as in

 354 gur
  3 ban₂
  6
  sila₃
  of grain.

Observe that while large numbers of gur follow²³ system $S_{\text{Ur III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as $\langle \text{I} \text{---} \text{II} \rangle$ would be 10 gur 1 bariga, and $\langle \text{---} \text{I} \text{---} \text{II} \rangle$ would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

- \mathbb{I} - \mathbb{W} used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- \mathbb{L} - \mathbb{X} used in $G_{\text{Ur III/OB}}$, of which \mathbb{L} - \mathbb{X} are also used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- \mathbb{I} - \mathbb{W} used in $S_{\text{Ur III/OB}}$, and sometimes with overt units;
- \mathbb{K} - \mathbb{X} used in $S_{\text{Ur III/OB}}$;
- \mathbb{D} - \mathbb{D} used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \mathbb{E} - \mathbb{E} used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \mathbb{T} - \mathbb{T} used in $C_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
- \mathbb{T} , \mathbb{T} , \mathbb{T} , \mathbb{T} , \mathbb{T} used in $C_{\text{Ur III/OB}}$;
- \mathbb{I} , \mathbb{I} , \mathbb{W} , \mathbb{W} used in $C_{\text{Ur III/OB}}$ —note the overlap with \mathbb{I} - \mathbb{W} ;
- \mathbb{Y} and \mathbb{Z} used in $G_{\text{Ur III/OB}}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Robo8, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

²¹Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{Ur III/OB}$ based on a height of one cubit, see[[Pow87](#), p. 488; [Rob08](#), p. 294; [Rob19](#)].

²²From [P309594].

²³A larger unit, the guru₇ (*karûm*, grain heap), is sometimes used instead, with 𐎧𐎠𐏀𐎥𐎡𐏁 = 𐎧𐎠𐏀𐎥𐎡𐏁 (1 *karûm* = 3600 *kurrû*). See [Fri07, p. 415; Rob19].

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with 𐎶- and 𐎵 numerals with their system $S_{\text{Ur III/OB}}$ values²⁴. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Fri07, p. 118; Rob19]:

$$\begin{array}{ccccccc} \text{𐎶} \text{ 𐎶} & \xleftarrow{30} & \text{𐎶} & \xleftarrow{60} & \text{𐎶} & \xleftarrow{12} & \text{𐎶} \text{ 𐎶} \xleftarrow{30} \text{ 𐎶} \text{ 𐎶} \\ \text{danna} & & \text{UŠ}^{25} & & \text{nindan} & & \text{kuš}_3 & & \text{šu-si} \\ \text{bêrum} & & \text{cable} & & \text{nindānum} & & \text{ammatum} & & \text{ubānum} \\ \text{league} & & 360 \text{ m} & & \text{rod} & & \text{cubit} & & \text{finger} \\ 10,8 \text{ km} & & & & 6 \text{ m} & & 50 \text{ cm} & & 17 \text{ mm} \end{array} \quad (L_{\text{Ur III/OB}})$$

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:

$$\begin{array}{ccccccc} \text{𐎶} \text{ 𐎶} & \xleftarrow{30} & \text{𐎶} & \xleftarrow{6} & \text{𐎶} & \xleftarrow{10} & \text{𐎶} & \xleftarrow{2} & \text{𐎶} & \xleftarrow{6} & \text{𐎶} & \xleftarrow{30} & \text{𐎶} \text{ 𐎶} \\ & & & & \text{eše}_2 & & & & \text{gi} & & & & \\ & & & & \text{ašlum} & & & & \text{qānum} & & & & \\ & & & & \text{rope} & & & & \text{reed} & & & & \\ & & & & 60 \text{ m} & & & & 3 \text{ m} & & & & \end{array} \quad (\bar{L}_{\text{Ur III/OB}})$$

In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

Fractions of the *ikîm*, 𐎶 = $\frac{1}{2}$ and 𐎵 = $\frac{1}{4}$, have already been encountered. In other contexts, the fraction $\frac{1}{2}$ is written 𐎶, and the fractions $\frac{1}{3}$ and $\frac{2}{3}$ are written 𐎶 and 𐎶. The latter two signs are derived from curviform signs 𐎶 and 𐎶, which are already separately encoded; these are in turn derived from the sign 𐎶 (S_{U_2}), whose Early dynastic form resembles 𐎶, and 𐎶 numerals; see [Powell1971]. The 𐎶 is sometimes omitted, as in [P240545; P221530; P221531; P271238; P274845].

3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{\text{Ur III/OB}}$ [Fri07, p. 374; DE87, pp. 127, 165]:

$$\text{●} \xleftarrow{10} \text{●} \xleftarrow{6} \text{●} \xleftarrow{10} \text{●} \xleftarrow{6} \text{●} \xleftarrow{10} \text{●} \quad (S)$$

Likewise the area system used in the Early Dynastic IIIb period for areas of one *ikûm* and greater mirrors system $G_{\text{Ur III/OB}}$ [LAK, p. 72; NDE93, p. 63; Fri07, p. 378; Lec16]:

$$\text{●} \xleftarrow{10} \text{●} \xleftarrow{6} \text{●} \xleftarrow{10} \text{●} \xleftarrow{3} \text{●} \xleftarrow{6} \text{●} \quad (G_{\text{ED IIIb}})$$

with consistent use of the numerals: ● corresponds to 𐎶, ● to 𐎶, and ● to 𐎶. An exception to this correspondence, noted in [L2/04-099, p. 4] (see section 3.2), is

²⁴Adjacent units are no more than a factor of 60 apart, so higher numerals such as 𐎶 or 𐎶 are not used.

²⁵As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.

that the vertical \uparrow from $S_{Ur III/OB}$ corresponds to a horizontal \triangleright in system S . This is however far from the only case of such a reallocation of function. The earlier form of System G is [DE87, pp. 141, 165; Frio7, p. 378]:

$$\bullet \xleftarrow{6} \odot \xleftarrow{10} \bullet \xleftarrow{3} \triangleright \xleftarrow{6} \triangleright, \quad (G)$$

Observe that, as noted in [DE87, p. 142], \odot changes meaning from $10\bullet$ in system G to $600\bullet$ in system $G_{ED IIIb}$. System G is used in the Uruk period, but also in the ED I–II period (it is the “area 2” system in [Cha03], whereas $G_{ED IIIb}$ is the “area 1” system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb area system²⁶,

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \star \xleftarrow{10} \bullet \xleftarrow{3} \triangleright \xleftarrow{6} \triangleright \xleftarrow{2} \triangleright \xleftarrow{2} \triangleright \xleftarrow{2} \triangleright \xleftarrow{2} \triangleright, \quad (G_{ED IIIb})$$

with the numerals of a contemporaneous capacity system:

$$\underbrace{\triangleright \xleftarrow{10} \triangleright \xleftarrow{6} \bullet \xleftarrow{10} \triangleright \xleftarrow{4} \triangleright \xleftarrow{6} \triangleright}_{\text{gur san ḡal}_2} \quad (C_{\text{gur san ḡal}_2})$$

both described in [Lec16]. While the size of the gur san ḡal_2 in bariga is different from that of the Old Babylonian gur , the basic structure of the capacity system is recognizable, with \triangleright corresponding to \uparrow for bariga, \triangleright – \triangleright corresponding to \uparrow – \uparrow for ban₂, and the gur counted with \triangleright rather than \triangleright numerals. However, the half-*ikûm* is counted with the same \triangleright as the bariga, whereas it uses a different sign, \searrow , in the Old Babylonian system. As we will see, this cannot be handled as a split, by giving \searrow the glyph \triangleright in an Early Dynastic IIIb font, as the \searrow numeral series is also in use in that period.

3.4.1 Field lengths in Nirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign \triangleright is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign gur is not written either. Length shorter than one rope are expressed in half-rope using the 1/2 sign \uparrow (again with no gur), and then in reeds, with the sign gi . Effectively, this yields the following factor diagram:



$$\begin{array}{c} \uparrow \xleftarrow{6} \triangleleft \xleftarrow{2} \uparrow \xleftarrow{10} \searrow \text{gi} \\ \begin{array}{l} 1 \text{ eše}_2 = 10 \text{ nindan} \\ 1 \text{ rope} = 10 \text{ rods} \\ 60 \text{ m} \end{array} \quad \begin{array}{l} \text{reed} \\ 3 \text{ m} \end{array} \end{array} \quad (L_{ED IIIb})$$

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and most others from the same period, such as the ones discussed in [Lec20], areas are expressed in system $G_{ED IIIb}$, with

²⁶A variant is $\odot \xleftarrow{10} \bullet \xleftarrow{6} \star \xleftarrow{10} \bullet \xleftarrow{3} \triangleright \xleftarrow{6} \triangleright \xleftarrow{2} \triangleright \xleftarrow{2} \triangleright \xleftarrow{2} \triangleright \xleftarrow{2} \triangleright$, see [Powell1972].

²⁷The cuneiform counterpart is \downarrow .

³⁰Note that the reeds are counted using *tenû* numerals, \searrow , \searrow , \searrow , etc.

curviform numerals³¹; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope³², the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and thus the interpretation of its numerals between systems *G_{ED IIIb}* and *L_{ED IIIb}*. The sign , which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without . There are other such co-occurrences contrasting between metrological systems; for instance, [Kre98, p. 303 n. 686] mentions the use of cuneiform numerals for days and months³³.

3.4.2 Dyke lengths in Nirsu

[Pow87, p. 466] notes that reeds “are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus.” Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in CDLI finds many occurrences³⁴, such as:

- [P221305, obv. 1, 4]³⁵ 𣎵
— [P020129, rev. 2, 1] 𠂔
— [P221291, rev. 5, 1]³⁶ 𡗕
— [P221266, rev. 2, 1] 𢇛

These expressions use an explicit sign 𐤛𐤓 (counted in multiples of ten) or 𐤕. This notation—but not its use of curviform numerals—is remarked on in [Lec20, p. 290 n. 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. These³⁷ can be summarized by the following factor diagram:

$$\underbrace{\text{Diagram 1} \xleftarrow{10} \text{Diagram 2} \xleftarrow{6} \bullet}_{\text{Diagram 3}} = \underbrace{\text{Diagram 4} \xleftarrow{2} \text{Diagram 5}}_{\text{Diagram 6}} \xleftarrow{10} \text{Diagram 7} \xleftarrow{6} \text{Diagram 8} \xleftarrow{3} \text{Diagram 9} \quad (L'_{\text{ED IIIb}})$$

3.4.3 Cheese and wheat in Nirsu

A similar mixture of cuneiform and curviform numerals may be observed with the capacity system; indeed, the previously described 𐎶𐎵𐎲𐎠 system uses 𐎶

³¹A CDLI search for "(bur3)" (< numerals used for areas) currently returns 15 ED IIIB results, whereas one for "(bur3@c)" (• numerals used for areas) returns 206. Further, when dated, the tablets with cuneiform bur₃ are from the reigns of 𒂗𒀭𒂗𒀭𒂗𒀭 (variously transliterated iri-inim-gi-na, uru-ka-gi-na, etc.) and 𒂗𒀭𒂗𒀭𒂗𒀭𒂗𒀭 (lugal-zag-ge-si), the last two kings of ED IIIB Lagaš.

³²This is the case of the sides of the field in [P020054, obv. ii 2–3].

³³That note also mentions a contrast between the use of curviform numerals to count people and curviform numerals to count bread allotted to them in [P010876]; such contrasts are more akin to styling, and might not, on their own, justify the disunification.

³⁴A search for curviform numerals followed by some number of reeds counted in (*tenû*) cuneiform numerals currently finds 125 occurrences across 47 tablets.

³⁵CDLI only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the $\text{𒀭} \text{𒀭} \text{𒀭}$ is not visible. Lecompte notes that the copy is faithful; indeed another $\text{𒀭} \text{𒀭} \text{𒀭}$ can be seen both on the copy and the photo on obv. 2, 2.

³⁶From copy.

³⁷TODO Cite also DP 568, the one with and even though it has no reeds.

³⁸TODO(egg): Note that one unit may be omitted if the other is present

numerals for \searrow [Fri78, p. 43; Lec16]:

$$\underbrace{\text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \bullet \xleftarrow{10} \text{𐎴}}_{\text{𐎶𐎵𐎴}} \xleftarrow{4} \text{𐎶} \xleftarrow{6} \text{𐎶} \xleftarrow{6} \searrow, \quad (C_{\text{𐎶𐎵𐎴}})$$

as in [P020016; P020065; P020090; P020092; P020137] and others, where ban_2 counted with 𐎶 numerals are followed by sil_3 counted with \searrow numerals. Curviform numerals are also used to count sil_3 , but not³⁹ in combination with larger units. This contrast can be seen in [P220927], which counts butter (𐎶 , i_3) in $\text{𐎶𐎵𐎴} \text{𐎶}$, $\text{𐎶𐎵𐎴} \text{𐎶𐎵𐎴} \text{𐎶}$, and $\text{𐎶𐎵𐎴} \bullet \text{𐎶𐎵𐎴} \text{𐎶}$, pots of six, eight, and twelve sil_3 , themselves counted using the discrete counting system⁴⁰, but counts cheese (𐎶𐎵𐎴 , ga'ar) using the $\text{𐎶𐎵𐎴} \text{𐎶𐎵𐎴}$ capacity system, with \searrow numerals for the sil_3 .

It is not only pots that come in multiple sizes in ED IIIb Nirsu, but also 𐎶 . Another capacity system is the $\text{𐎶} \searrow \text{𐎶}$, the gur of two ul:

$$\bullet \xleftarrow{10} \text{𐎵} \xleftarrow{2} \text{𐎶} \xleftarrow{6} \text{𐎶} \xleftarrow{6} \searrow. \quad (C_{\text{𐎶} \searrow \text{𐎶}})$$

Here the $\text{𐎵} \searrow \text{𐎶}$ contrast occurs not only within the numerals of the system, but with its units; this is perhaps best illustrated by the expressions $\text{𐎶} \text{𐎵} \searrow \text{𐎶}$ $\text{𐎶} \searrow \text{𐎶}$ in [P221746] and $\text{𐎶} \text{𐎵} \bullet \text{𐎶} \searrow \text{𐎶}$ $\text{𐎶} \searrow \text{𐎶}$ in [P221814].

3.4.4 Grain in Ebla

The mixing of curviform and cuneiform numerals within a metrological system is not specific to Nirsu.

The system of grain⁴¹ capacities in Ebla uses the following units⁴²:

$$\text{𐎶𐎵} \text{𐎶} \xleftarrow{2} \text{𐎶𐎵} \text{𐎶} \xleftarrow{\frac{5}{2}} \text{𐎶𐎵} \xleftarrow{4} \text{𐎶𐎵} \text{𐎶} \xleftarrow{6} \text{𐎶𐎵} \text{𐎶}.$$

gu₂-bar ba-ri₂-zu ḡin₄ nin₂-sagšū an-zam_x

The $\text{𐎶𐎵} \text{𐎶}$ and $\text{𐎶𐎵} \text{𐎶}$ are generally counted using curviform numerals, and the smaller units using cuneiform 𐎶 numerals. Indeed, a search on [EbDA] for co-occurrences of either $\text{𐎶𐎵} \text{𐎶}$ or $\text{𐎶𐎵} \text{𐎶}$ with either of $\text{𐎶𐎵} \text{𐎶}$ or $\text{𐎶𐎵} \text{𐎶}$ finds the

³⁹As of this writing, the single occurrence of (ban2@c) followed by curviform numerals and sil_3 in CDLI, 4(ban2@c) 3(asz@c) sil_3 in [P221815], is incorrect: it should be 4(ban2@c) 3(disz@t) sil_3 .

⁴⁰Including the fraction $\frac{1}{3}$, and with subtractive notation, e.g., $\text{𐎶𐎵} \text{𐎶} \text{𐎶}$ “two pots (of one sil_3) minus one third (pot)”, or in the total, $\bullet \text{𐎶𐎵} \text{𐎶} \text{𐎶}$ ten minus one pots of six sil_3 . Such subtractive notation is common in most of the metrological systems discussed here; it appears in the ED IIIa period [Rob08, p. 77].

⁴¹Liquid capacities use a different system [Arc15, p. 229 n. 12]:

$$\text{𐎶𐎵} \text{𐎶} \xleftarrow{30} \searrow \xleftarrow{6} \text{𐎶𐎵} \text{𐎶}.$$

la-ḡa sil₃ an-zam_x

At a glance it seems that \searrow are counted with cuneiform numerals and higher units with curviform ones, thus

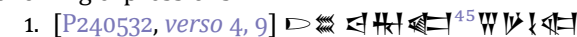
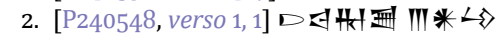

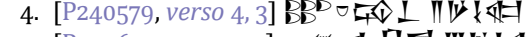
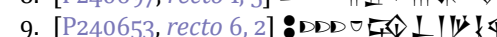
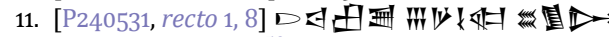

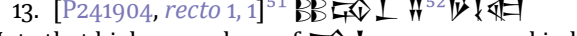

$$\text{𐎶𐎵} \text{𐎶} \xleftarrow{\frac{5}{3}} \text{𐎶𐎵} \text{𐎶} \xleftarrow{6} \bullet \xleftarrow{10} \text{𐎴} \xleftarrow{3} \text{𐎶} \xleftarrow{\frac{10}{3}} \text{𐎶} \xleftarrow{6} \text{𐎶𐎵} \text{𐎶},$$










𐎶𐎵𐎴 𐎶

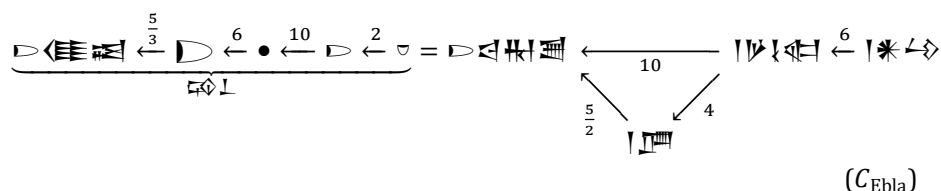
but we have not investigated this thoroughly.

⁴²TODO mention the other one citing Chambon and the footnote in Archi

following expressions⁴³:







1. [P240532, verso 4, 9] 
2. [P240548, verso 1, 1] 
3. [P240655, recto 7, 9] 
4. [P240579, verso 4, 3] 
5. [P240675, verso 2, 2] 
6. [P240609, verso 3, 1] 
7. [P240533, recto 3, 3] 
8. [P240697, recto 1, 5] 
9. [P240653, recto 6, 2] 
10. [P240654, recto 2, 6] 
11. [P240531, recto 1, 8] 
12. [P241708, recto 1, 1]⁵⁰ 
13. [P241904, recto 1, 1]⁵¹ 



Note that higher numbers of  are expressed in hundreds (*mi-at* ) and then thousands (*li-im* ) , as is typical in Ebla [Arc15, p. 33], e.g., in [P240532, verso 2, 3],      (100 + 60 + 30 + 5 = 195  of grain). These expressions correspond to the following factor diagram:



3.4.5 Use in modern publications


Because of their prevalence in the Uruk and Early Dynastic periods, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 3–16.


Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 16. The partial⁵³ transliteration “4  *a*₃-*da-um* 4  *aktum* 4  *ib*₂^{tu} × 3  *sa*₆ *gunu*₃” is used to illustrate a discussion of the interpretation of the contrast between  and  numerals. More conventional transliterations⁵⁴ might omit the numeral shapes


⁴³We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform  and larger with cuneiform  and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked⁴⁴ on a photograph (from EbDA unless noted otherwise).


⁴⁴As we will see in Section 3.7.2, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

⁴⁵ba-ri₂-zu₂, a variant spelling.

⁴⁶Short for .


⁴⁷Note the omitted .










⁴⁸Instead of the expected .

⁴⁹ not legible on the EbDA photo.

⁵⁰From CDLI photo.

⁵¹From photo in [Arc89, p. 6].

⁵²Laid out as ; on stacking patterns see Section 6.2.

⁵³The untransliterated text would be     ; note the atomically encoded *ib*₂ × 3  =  ×  = .

⁵⁴TODO cite the EbDA one.

$$\begin{cases} 4 \overline{\text{U}} 5 \text{o} = 24 \overline{\text{U}} 3 \text{o} & (\text{C } 234) \\ 5 \overline{\text{U}} 1 \text{o} 1 \overline{\text{U}} = 5 \overline{\text{U}} 7 \text{o} & (\text{C } 314) \\ 1 \overline{\text{U}} 1 \overline{\text{D}} 1 \overline{\text{D}} = 6 \text{o} 2 \overline{\text{U}} 1 \overline{\text{D}} & (\text{C } 27) . \end{cases}$$

These metrological equations for the "unknowns" $\overline{\text{U}}$, o , $\overline{\text{U}}$, etc., can be treated exactly as ordinary equations for unknowns x, y, z, \dots . In particular, the equations can be simplified by subtraction of equal amounts from both sides of the identities. In this way the three equations above can be reduced to:

$$\begin{aligned} 2 \text{o} &= 20 \overline{\text{U}} & (4 \overline{\text{U}} 3 \text{o} \text{ subtracted from both sides}) \\ 1 \overline{\text{U}} &= 6 \text{o} & (5 \overline{\text{U}} 1 \text{o} \text{ -- " -- }) \\ 1 \overline{\text{D}} &= 6 \text{o} 1 \overline{\text{U}} 9 \overline{\text{D}} & (1 \overline{\text{U}} 1 \overline{\text{D}} \text{ -- " -- }) \end{aligned}$$

We can now read off from the first equation that $1 \text{o} = 10 \overline{\text{U}}$, and from the second that $1 \overline{\text{U}} = 6 \text{o}$. Then the third equation can be simplified (by "substitution" of these values into the equation), to the following reduced form:

$$1 \overline{\text{D}} = 2 \overline{\text{U}} 9 \overline{\text{D}} .$$

The most likely solution to this last equation is, of course ,

$$1 \overline{\text{D}} = 2 \overline{\text{U}} , \quad 1 \overline{\text{D}} = 10 \overline{\text{D}} .$$

Figure 1: TODO [Fri78, p. 15]

Thus, for instance, the original set of fractions $\overline{\text{U}}$, o , and $\overline{\text{D}}$ ($1/2$, $1/4$ and $1/8$ of an iku) in the Sumerian GANA system, was after a time augmented through the addition of the new sub-unit SAR: $\overline{\text{D}}$, equal to $1/100$ of an iku ($\overline{\text{D}}$). Similarly, the Sumerian weight unit "ma-na" which originally may have had only the sub-units $\overline{\text{U}}$ ša-na (= $1/3$ mana) and $\overline{\text{U}}$ ša-na-bi (= $2/3$ mana), and perhaps also gin: $\overline{\text{U}}$ (= $1/60$ mana), seems to have acquired, at some time or other, also the smaller sub-units $\overline{\text{U}}$ (= $1/3$ gin), and $\overline{\text{U}}$ še (= $1/3 \times 1/60$ gin).

Figure 2: TODO [Fri78, p. 49]

entirely, *e.g.*, $4 \text{'a}_3\text{-da-um } 4 \text{ aktum } 4 \text{ ib}_2^{\text{tu}_9} \times 3 \text{ sa}_6 \text{ gunu}_3$, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, *e.g.*, $4(\text{aš}^\text{c}) \text{'a}_3\text{-da-um } 4(\text{diš}^\text{c}) \text{ aktum } 4(\text{aš}^\text{c}) \text{ ib}_2^{\text{tu}_9} \times 3(\text{diš}^\text{c}) \text{ sa}_6 \text{ gunu}_3$, but the result would be less readable. See Section 3.7.2 for a discussion of transliteration conventions for numerals.

⁵⁵TODO(egg): On the order cite TSS 188, Friberg2007 p. 148 and any of the usual suspects on the haphazard order of signs in early texts; contrast P274845, P241764.

there is in any case an important qualitative difference between IX for Latin novem and $\overline{\text{U}}$ for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 3: TODO [Englund1988]

of decreasing fractions $1/n$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus $\Xi = 1/2 N_{39}$, $\tilde{\Xi} = 1/3 N_{39}$, and so on. The first sign of the latter units, N_{34} ,

Figure 4: TODO [Eng98, p. 113]

For instance, the first line contains the notations $1N_{34} 1N_{390}; 2N_{20}$, which can be translated "60 of the (grain rations containing) Ξ (of grain); (grain involved): 2 \bullet (of ground barley)". This calculation contradicts the assumed numerical relationship $10N_1 = 1N_{14}$, since as was well known the measure represented by the sign N_{39} was $1/5$ of that represented by N_1 , so that $60 \times 1/5 = 12$ and not 20, as $2N_{14}$ would imply. Instead of relying on complicated

Figure 5: TODO [Eng98, p. 116]

Die halbkreisförmigen Griffelindrücke gehen manchmal in mehr oder weniger eckige Formen über (∇)⁶⁸⁵. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen (∇), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren⁶⁸⁶. Selten treten mit ∇ gebildete Zahlen auf⁶⁸⁷ (sie entsprechen den biga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 6: TODO [Kreg98, p. 303]

The calculations:

Obv. i	1	$60 \times 1/5 \nabla$	(Ξ)	=	$12 \times \nabla$	=	$2 \times \bullet$		
	2	$120 \times 1/10 \nabla$	(Ξ)	=	$12 \times \nabla$	=	$2 \times \bullet$		
	3	$120 \times 1/15 \nabla$	($\tilde{\Xi}$)	=	$8 \times \nabla$	=	$1 \times \bullet$	$2 \times \nabla$	
	4	$300 \times 1/20 \nabla$	($\tilde{\Xi}$)	=	$15 \times \nabla$	=	$2 \times \bullet$	$3 \times \nabla$	
	5	$600 \times 1/25 \nabla$	($\tilde{\Xi}$)	=	$24 \times \nabla$	=	$4 \times \bullet$		
Rev. i	1	1200			$1 \times \bullet$	$1 \times \bullet$	$5 \times \nabla$		
Obv. i	6	$6000 \times 1/30 \nabla$	(GAR+6N ₃₇)	=	$200 \times \nabla$	=	$1 \times \nabla$	$3 \times \bullet$	$2 \times \nabla$
ii	1	$120 \times \approx 1/4 \nabla$	(DUG _a +U _{2a})	=	$30 \times \nabla$	=	$5 \times \bullet$	$1 \times \nabla$	$1 \times \nabla$
	2	$180 \times 1/5 \nabla$	(DUG+AS _a)	=	$36 \times \nabla$	=	$6 \times \bullet$		
	3	$300 \times 1/15 \nabla$	(KAŠ _a)	=	$20 \times \nabla$	=	$3 \times \bullet$	$2 \times \nabla$	
Rev. i	3	600			$1 \times \bullet$	$4 \times \bullet$	$3 \times \nabla$		$1 \times \nabla$
					$1 \times \bullet$	$1 \times \bullet$	$5 \times \nabla$		
					$1 \times \nabla$	$3 \times \bullet$	$2 \times \nabla$		
					$1 \times \bullet$	$4 \times \bullet$	$3 \times \nabla$	$1 \times \nabla$	
Grand total of groats used:					$1 \times \nabla$	$2 \times \bullet$	$9 \times \bullet$	$4 \times \nabla$	$1 \times \nabla$
Grand total of malt used:					$1N_{47}$	$4N_{20}$	$3N_5$	$1N_{42a}$	(rev. i 3) $\times 3/5 \approx$
						$8 \times \bullet$	$4 \times \nabla$	$1 \times \nabla$	

Figure 6. Transliteration and calculations of MSVO 4, 66.

Figure 7: TODO [Englund2001]

strong similarities between "area" 1 and "area" 3 systems, the sign with two concentric discs (\bullet , notated N_{50} ²⁷) remains problematic. It never appears in any numerical combination with the sign with a single disc (\bullet),

Figure 8: TODO [Chao3, p. 6]

$1/15$, etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation $\text{𒄠} \text{𒄡}$ could be understood as $1 \frac{1}{5}$ gur.²⁷ However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs 𒄠 and 𒄡 cannot be considered klsamatograms.

Metrological tablets from the end of the 4th millennium (Nissen, Damerow and Englund 1993, 55-59, to *MSVO* 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

𒄠 1(iku) represents a surface of 3600m²
 𒄡 1(eše₃) represents a surface of 21,600m²
 etc.

The signs iku and eše₃ constitute by themselves measures of surface areas. These measures are usually followed by the sign GAN₂, which means either surface or field and

Figure 9: Discussion of Old Babylonian and fourth millenium area measures in [Proust2020]. The cuneiform text is Unicode-encoded.

formed by only two signs 𒄠 and 𒄡 , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60², 60³, 1/60, 1/60², etc. are written in the same way, with the vertical wedge 𒄠 . The third feature concerns the exact function of

Figure 10: TODO [Cha12, p. 58]

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with: $\text{𒄠} \text{𒄡} \text{𒄠} \text{𒄡}$ but never with the repetition $\text{𒄠} \text{𒄠} \text{𒄠} \text{𒄠}$.

Figure 11: TODO [Cha12, p. 59]⁵⁵

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

$\text{𒄠} \text{𒄠} \text{𒄠}$ še bar 𒄠 ba-ri-zu
 '3 gubar (capacity units) and 1 parisu'.

Figure 12: TODO [Cha12, p. 61]

This is particularly true of the signs 𒄠 , 𒄡 , 𒄢 and 𒄣 , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written 𒄠 in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign 𒄠 is most often associated with the parisu measure, while the signs 𒄠 , 𒄡 , 𒄢 and 𒄣 refer to 1, 2, 3,

Figure 13: TODO [Cha12, p. 64]

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., $\text{𒄠} \text{𒄠} \text{𒄠} \text{𒄠}$ transliterated as 2(šar₂) 1(geš'u) 3(u), means $2 \times 3600 + 1 \times 600 + 3 \times 10$). The system is based on an alternation of factors ten and

Figure 14: TODO

might think of one fabric and a half,¹¹ but the presence of notations with “2𐎶 2𐎵”, “3𐎶 3𐎵”, and “6𐎶 6𐎵” (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag’s weight measurement system.¹² The notation “1𐎶 gada” in o. ii 1 and r. vi 1, along with the total of “39



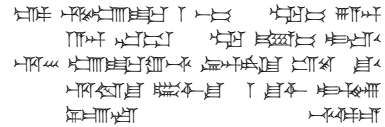
Fig. 1. Combinations of numerals attested in Š. 742.

Figure 15: Discussion of the contrast between 𐎶 and 𐎵 numerals in [Gor23, p. 162].

as, for example, in TM.75.G.3125 = ARET III 107 o. iv 1, “4𐎶 𐎶a₃-da-um^{tu9} 2 𐎶 4𐎵
aktum 4𐎶ib₂^{tu9} x3 𐎶 sa₆ gunu₃” (Fig. 2).

Figure 16: Transliteration in [Gor23, p. 163] of [P242293, recto 4, 1] incorporating untransliterated numerals.

3.5 Non-numeric usage



The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for ‘sixty’⁵⁶. Do you know its reading⁵⁷?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign 𐎶 has, *inter alia*, the values aš, rum, and dili. While the horizontal numerals are most frequently written 𐎶 in the Early Dynastic period⁵⁸, such non-numeric usage is almost⁵⁹ always written 𐎶, for instance:

- in personal names in administrative texts, such as the following, which all contain 𐎶 numerals:
 - 𐎶 in [P010424; P010458; P010459] from ED IIIa أبو صلايخ,
 - 𐎶 in [P010960] from ED IIIa Šuruppag,
 - 𐎶 in [P251641] from ED IIIb Adab,
 - 𐎶 in [P252866] from ED IIIb Adab,
 - 𐎶 in [P298637] from ED IIIb Umma;
- in the Sumerian word 𐎶 u₂-rum, “property” in ED IIIb Nirsu administrative

⁵⁶The reader will recall that 𐎶 is written 𐎶, with a larger wedge than 𐎶; however, these signs have merged by the time Examenstext A is composed.

⁵⁷Besides 𐎶, a look at [OSL] shows that the values diš, ge₃, makkaš, sanṭak₄, and tal₄ are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

⁵⁸A CDLI search for “(asz@c)” finds 3296 ED texts, while a search for “(asz)” finds 81 ED texts, of which 46 also contain “(asz@c)”.

⁵⁹Exceptions are discussed in section 3.7.1.

- texts which contain \triangleright numerals, such as [P020006; P020008; P020018; P020024; P020030];
- in lexical texts:
 - in the divine name $\text{𒌦 𒀭𒋼 𒀭𒋼} \text{— 𒌦}$ in the lexical texts [P010570; P010572], where the entries are prefixed with \triangleright .
 - in the word — dili , “small fish” in [P010578], witness to Early Dynastic Fish,
 - in the same word with a determinative, $\text{— 𒌦 dili}^{\text{ku}_6}$, in [P010586], witness to Early Dynastic Food, which starts with \triangleright numerals.

This is a clear contrast between — and \triangleright in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name — 𒀭𒋼 occurs on its own line in the aforementioned administrative texts; a line $\triangleright \text{— 𒀭𒋼}$ would instead be read as “one slave”.

3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as — that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [KWU, pp. 123 sqq.] lists all of — — — — — together with $\triangleright \text{— — — — —}$, while — , — , and — , and only those, appear at the beginning of the sign list, since they have non-numeric values⁶⁰. [PTACE, p. 58] has the numeric signs \triangleright , — , — , whereas non-numeric — is at the beginning of the sign list, where its values *aš* and *rum* are listed. For signs with both non-numeric and numeric usage, [LAK] writes *s. die Zahlz.* throughout the main list; LAK 1 — thus reappears at LAK 829 together with \triangleright , — , and — . One should note [MZL], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have — — — — — used in system $G_{\text{Ur III/OB}}$.

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes⁶¹ to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from \triangleright to — numerals prevents diachronic encoding even if \triangleright were unified with — . For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'–6']; however, they are written with — numerals, whereas they are written with \triangleright numerals in the Early Dynastic witnesses; since — and — are distinct⁶²

⁶⁰Non-numeric values of — were discussed in section 3.5; — has the values *man*₃ and *min*₅, and is used for the word *didli*, “several, various”; — has the value *eš*₆.

⁶¹TODO comment on the ED witnesses to the instructions of Šuruppak

⁶²Besides the contrasts in numeric usage mentioned in section 3.3.3, these characters are clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of — which are not shared with — have already been mentioned, but perhaps most striking is the fact that, in the

characters, the $\triangleright \rightarrow$ unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods⁶³, there is little opportunity for a diachronic representation of numeric quantities.



In the case of analyses such as [Romach2023], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

















3.7 Compatibility considerations



A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts⁶⁴ being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

3.7.1 The case of ŠAR₂

The character U+122B9  CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period⁶⁵, a contrast between non-numeric šar₂ written  and numeric 1(šar₂) written  can be observed, similar to the contrast between \rightarrow and \triangleright previously discussed in section 3.5. However, in lexical lists from Šuruppak and Ebla⁶⁶, as well as in the *Stèle des vautours*, non-numeric šar₂ is curviform:

-     and     in [P010566];
-   and   in [P010576];
-   in [P240986]⁶⁷;
-   in [P222399, obv. 17, 9, 18, 11, 22, 12]⁶⁸.














It *would* be disruptive to the diachronic representation of text if non-numeric šar₂ were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph , identical to the glyph of the proposed U+12579  CUNEIFORM NUMERIC SIGN ONE N45.

Neo-Assyrian period, \rightarrow is used for the preposition *ina*, “in”, and \uparrow for the preposition *ana*, “to”.

⁶³TODO cite a few things here.




⁶⁴Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

⁶⁵For example, in personal names:

-     in [P020019] from ED IIIb Nirsu;
-     in [P020182], also from ED IIIb Nirsu;
-   in [P222186] from ED IIIb Umma;
-    in [P235312] from Old Akkadian Umma.

⁶⁶TODO Mention other ways in which these are archaizing

⁶⁷From copy in [ELLes, No. 397].

⁶⁸Note however    on [P222399, obv. 6, 17]. Curviform non-numeric šar₂ is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here. TODO add a photo.

Since the archaizing style of texts wherein non-numeric šar₂ is curviform solidly predates the transition from ● to ◇ in the relevant metrological systems, there is no need to represent a ◇-● contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D ◇ CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

Note that in rare cases, such as [P222243] from ED IIIa Adab, non-numeric — (here with the value rum) is written ▷. It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 ▷ cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. In practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the CDLI transliteration (UR2-1(aš@c)) and the ePSD2 one (uru^{rum}) of this word disagree on that aspect. Since — has a cuneiform reference glyph, this does not pose any compatibility concerns.

3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL₂ can be described as 𒀭×𒀭 NE×KASKAL or 𒀭×𒀭 NE×PAP⁶⁹, they are typically all transliterated bil₂, and therefore are all represented by the character U+1224B 𒀭 CUNEIFORM SIGN NE SHESHIG, its name notwithstanding, as described in [UTR56, §2.5].

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, 𒄠𒀭𒄠 from [P305639] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in EbDA transliterations: the aforementioned 𒄠𒀭𒄠 from [P240533, recto 3, 3] is transliterated “20-1-1/2 gu₂-bar 7 nig₂-sagšu 2-1/2 an-zam_x⁷⁰ za”, reading both 𒄠 and 𒄠 as 1/2, but not distinguishing them.

In particular, these transliterations do not differentiate between — and 𒄠 numerals, nor between ▷ and ▽ numerals. For instance, the aforementioned 𒄠𒀭𒄠 from [P242293, recto 4, 1] is transliterated “4 'a₃-da-um^{tug₂}-II 4 aktum^{tug₂} 4 ib₂-III gun₃ sa₆^{tug₂}” in EbDA, with no distinction between the 𒄠 and 𒄠. Since — and 𒄠 numerals are separately encoded, the numeric expressions in such transliterations cannot be transformed into Unicode cuneiform without additional context, regardless of curviform–cuneiform unification.

In metrological systems such as systems G_{Ur III/OB} and C_{Ur III/OB} where some units are indicated by the type of numeral rather than an overt unit sign, it is common practice to add the unit in parentheses in transliteration; for instance, 𒄠𒀭𒄠 𒀭𒄠 𒀭𒄠






⁶⁹As on [P249253].




⁷⁰As of this writing, EbDA actually has an-zam_x, with U+1D6A GREEK SUBSCRIPT SMALL LETTER CHI.

4 Rationale for ED–Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millenium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millenium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.






This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs KAŠa, KAŠb, and KAŠc, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Englund2001]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered script.

As part of the development of these conventions, a classification of fourth millenium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter *N* with a numeric subscript (sometimes with an additional alphabetic subscript): N_1 is , N_{14} is , N_{34} is , etc. Transliterations of numeric expression then use those to identify the type of number used, thus $5N_1$ is , and $5N_{14}$ is .

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the CDLI transliteration of  or  in [P283802] currently uses (barig@c) for the vertical numerals, since  numerals are typically capacity measures; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millenium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I–II period, even though the Sumerian text uses classical assyriological transliteration conventions.

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millenium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in section 3.7.2; they are indeed used interchangeably, as in [P01104] which uses the notation u@f in [ePSD2], but N14@f in CDLI. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millenium would therefore induce confusion as to which numerals should be used in third millenium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millenium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance,  N_{30c} is listed as a variant of  N_{30a} in [DE87, p. 166], where the numeric value of either in relation to  N_{29a} is still unknown, but their values are found in [Englund2004] to be  = $\frac{1}{10}$ .

whereas $\boxtimes = \frac{1}{6} \cup$.

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban₂@c) and 5(ban₂@c).]

6 Characters not included in this proposal

6.1 Missing numerals

TODO N13 not attested in CDLI TODO (N_{17} not usefully numeric, $12N_{14}$ not encodable, etc.). Cite [DE87, p. 147] 7 and 8 (diš *tenû*) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.

6.2 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some *stacking patterns*; for instance 9𐎶 is encoded both as U+12446 𐎶 and as U+1240E 𐎶𐎵. This is in part due to contrastive usage of stacking patterns. For instance, besides 𐎶 and 𐎶 which are characteristic of bariga measures, four bariga is written 𐎶 even where 4𐎶 is written 𐎶𐎵, as in [P255010; P292843]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 < << <<< 𐎶 𐎶, and results in metrological systems, typically 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶 < << <<< 𐎶 𐎶, occasionally co-occurring as in [P142827; P142357]. This separate encoding is also for compatibility with distinctions made in reference works and in some non-numeric transliterations; for instance, 𐎶 is [MZL, No. 860] and has the value limmu, and 𐎶 is [MZL, No. 852] and has the value limmu_s. Numeric⁷³ transliterations occasionally distinguish the stacking patterns 𐎶 𐎶 𐎶 𐎶 𐎶 < 𐎶, as in the CDLI transliterations of the aforementioned tablets, although this is rare; often 4(diš) is 𐎶 in Ur III, but 𐎶 in the Neo-Assyrian period.

However, the stacking patterns from earlier periods are not separately encoded; for instance, in ED IIIb Nirsu, « 2(u) often has one < atop another. These older stacking patterns do not appear to be contrastive, are not marked in transliteration, and are not listed separately in sign lists nor assigned any different values. There is therefore no evidence of a need to encode them; instead, they should be considered style variants, and an ED IIIb Nirsu font should have an appropriate glyph for U+12399 « CUNEIFORM SIGN U U.

Likewise, many stacking patterns are attested for the curviform numerals proposed in this document, and it is not proposed to separately encode them; these distinctions would be incompatible with the state of the art in numeric transliterations, and are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier tablets, as they are structured in rectangular cases rather than lines, so that numerals may be

⁷³The Sumerian word limmu means “four”, so limmu and limmu₅ are still numbers. The distinction here is between usage in transliterations of phrases such as 𒌦 𒈪𒍪𒀭 𒊩𒌆𒍪𒅗𒄿 luḡal an-ub-da limmu₅-ba-ke₄ (king of the four quarters) or of names, and of numeric expressions such as 𒑭𒅗 4(diš) sila₃.



Figure 17: The layout of case [P011099]; the numeral 9 is rotated to fit the rounded corner of the tablet.



Figure 18: The layout of case [P020066]; the numeral 9 is spread across two lines. The text is read in the order 9 two dots two horizontal strokes hook-like, “twenty-two oxen, one year old”.

laid out across the case in whichever way fits the available space; this is illustrated in Figure 17. Note also that the numerals need to be considerably enlarged in order to reproduce the layout of the tablets, so that 9 often spans two lines of cuneiform signs, as shown in Figure 18. This is impractical when these numerals are set in text that contrasts them with the larger 𐎶, and inconsistent with actual practice when typesetting these numerals, as illustrated in Figure 3: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dynastic period, but that are also attested in the Uruk period; the Uruk period also frequently features numerals that use a more vertical layout, as illustrated in Figure 19. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millenium, usage in third millenium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents 𐎶, 𐎶, 9, and 9 must remain consistent across the numeral series, lest a 𐎶 numeral be confused with an 𐎶 numeral. Since the single indents are frequently used in running text, as illustrated in section 3.4-5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 19 uses OpenType stylistic alternates, and Figure 17 rotates the character 9, in both cases preserving the plain text backing.

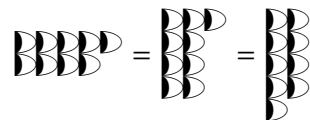


Figure 19: Three stacking patterns for U+12573 CUNEIFORM NUMERIC SIGN NINE N34. The one on the left is the reference glyph, used in Uruk III [P003499; P004430], and widely afterwards, *e.g.*, ED IIIa Šuruppag [P010678], ED IIIb Nirsu [P020057], Old Akkadian Umma [P212464]. The ones in the middle and right are used in two Uruk IV tablets [P001243; P004500]. All three Uruk examples are transliterated 9(N34) in CDLI.

6.3 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Englund2001].

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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