Archaic cuneiform numbers

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Contents

1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script[1] and the proto-cuneiform script[2].

[1] ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.
[2] ISO 15924: Pcun, not yet encoded.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed curviform numerals[3] should however *not* be unified with the already-encoded cuneiform numerals[4]. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is as follows:

	Uruk III & earlier	ED – Ur III	OB & later
Non-numeric signs	Future Pcun	Existing Xsux	
Numbers	This proposal	This proposal + Existing Xsux	Existing Xsux

2 Proposed changes to the Standard

2.1 New characters

2.2 Properties

2.3 Names list

2.4 Core specification

3 Rationale for curviform–cuneiform disunification

3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble 𒅎 in texts from Early Dynastic IIIa Šuruppag as in the character code charts, 𒅎 later in the third millenium[5], 𒅎 in Old Babylonian cursive, 𒅎 in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists[6] and dictionaries[7], and of composite texts[8]. By being compatible with similarly diachronic transliteration prac-

3 – 1–9(aš^c = N_1), – 1–5(u^c = N_{14}), – 1–9(ŋeš$_2$^{c} = N_{34}), – 1–5(ŋešʾu^c = N_{48}), etc.

4 – 1–9(aš), – 1–5(u), – 1–9(ŋeš$_2$), – 1–5(ŋešʾu), etc.

5 Merging with U+1224E 𒉎 NI$_2$.

6 Notably [VT+14] and the online edition of [Bor10] in [Jim+23, Signs].

7 Notably [TJV17] and the online edition of [Sch10] in [Jim+23, Dictionary].

8 For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of 𒄩𒄠𒈬𒊏𒁉, as well as Old Babylonian copies in both archaizing and cursive styles. Some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

tice (that is, by avoiding distinctions finer than those made in transliteration), the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24][9]. The diachronic approach is also useful for pedagogical applications[10].

3.2 Arguments for curviform–cuneiform unification

[TODO(egg): Present the argument from L2/04-099 detail, including the artifact cited.]

In this context, the argument was made in [And04] as part of ongoing work on the cuneiform encoding[11] that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters; some co-occurrence was known and acknowledged, but considered to be styling rather than plain text. Although they had been part of the preliminary proposal [EFT03], they were therefore removed from [EFT04b] and [EFT04a], which both state that "The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases."

Indeed, some metrological systems from the Early Dynastic period match the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{\text{Ur III/OB}}$ [Fri07, p. 374; DE87, pp. 127, 165]:

$$⊚ \xleftarrow{10} ● \xleftarrow{6} ◗ \xleftarrow{10} ⊳ \xleftarrow{6} • \xleftarrow{10} ⊳. \tag{S}$$

Likewise the area system used in the Early Dynastic IIIb period mirrors system $G_{\text{Ur III/OB}}$ [**Deimel1922**; NDE93, p. 63; Fri07, p. 378; Gom16]:

$$⊚ \xleftarrow{10} ● \xleftarrow{6} ✹ \xleftarrow{10} • \xleftarrow{3} ⊳● \xleftarrow{6} ⊳, \tag{$G_{\text{ED IIIb}}$}$$

The reader will have noticed that in system S, the vertical from $S_{\text{Ur III/OB}}$ becomes a horizontal ⊳. This is noted in [And04, p. 4]. It is however far from the only case of such a reallocation of function. The earlier form of System G was [DE87, pp. 141, 165; Fri07, p. 378]:

$$● \xleftarrow{6} ⊚ \xleftarrow{10} • \xleftarrow{3} ⊳● \xleftarrow{6} ⊳, \tag{G}$$

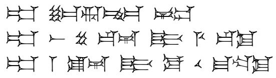
Observe that, as noted in [DE87, p. 142], ⊚ changes meaning from 10● in system G to 10✹ in system $G_{\text{ED IIIb}}$. System G is used in the Uruk period, but also in the ED I–II period (it is the "area 2" system in [**Chambon2003**], whereas $G_{\text{ED IIIb}}$ is the "area 1" system).

[9] Attendees may recall the summary given on the third day of UTC #180, as recorded in [Con24]. Other readers may refer to [Svä+24, pp. 242, 148].

[10] For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

[11] At that time scoped to the répertoire of the Ur III period and later, see [EF03, p. 1], although many disunifications, such as ≠, were informed by Early Dynastic distinctions.

3.3 Metrology



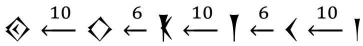
I want to write tablets: the tablet of 1 gur *of barley to 600* gur*; the tablet of 1 shekel of silver to 10 minas [...]*

Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with curviform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known[12] a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millenium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters 𒁹–𒐎, the multiples of ten (10–50) are 𒌋–𒐐, but the other digits 11–59 are sequences 𒌋𒁹–𒐐𒐎; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units 𒁹–𒐎, tens 𒌋–𒐐, sixties 𒐕–𒐝 (with larger wedges than the units), six hundreds 𒐞–𒐣, three thousand six hundreds 𒊹–𒐤, and thirty-six thousands 𒐬–𒐭.

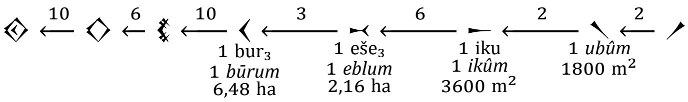
The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram[13], where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).



$(S_{\text{Ur III/OB}})$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written 𒐞𒐚𒐏𒐎 in the discrete counting system, and 𒎙𒐍𒐏𒐎 in the sexagesimal place value system.

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:



$(G_{\text{Ur III/OB}})$

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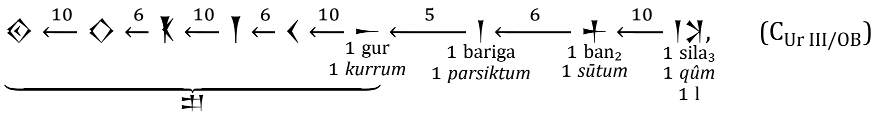
[12] See, *e.g.*, [Uni16, Section 22.3.3 "Non-Decimal Radix Systems", *sub* "Cuneiform Numerals"].

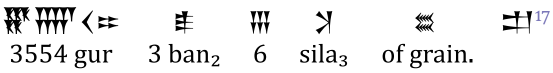
[13] These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

Note that for the range of areas given above[14], this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *būrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign 𒃷 functioning as punctuation, but the numerals are tied to the metrology; thus a surface of 5 *būrū* 1 *eblum* 4 *ikû* (100 *ikû*, 36 ha) would be written[15] 𒃷. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in "88 acres 3 roods 33 perches". Note also that the same signs are shared between multiple systems, with different relations; the ŠAR$_2$ sign 𒊹 is equal to sixty times the U sign 𒌋 in the area system, but to three hundred and sixty times 𒌋 in the discrete counting system.

Another such system of note is the one for capacities[16] [Fri07, p. 376; Rob19],



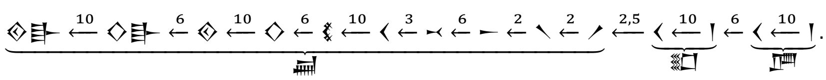
where the numerals for ban$_2$ are 𒑏, 𒑐, 𒑑, 𒑒, and 𒑔, and those for bariga are (contrast ordinary 𒈫 and 𒐈 otherwise used with 𒁹-numerals). As described in [Hue11, p.585 with notes (b) and (f)], the sign GUR 𒄥, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𒋡 if present, and after the word for "grain" if present, as in



Observe that while large numbers of gur follow[18] system $S_{\text{Ur III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as would be 10 gur 1 bariga, and would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system. To quote [Rob08, p. 78]: "The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity."

This intertwining of units and numerals explains the large number of already-encoded numeral series:

[14] For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mūšarum* (36 m^2) written, equal to one hundredth of an *ikûm*, then sexigesimally subdivided in 60 (shekels). For areas greater than 3600 *būrū*, the 𒊹- and -numerals are reused with a suffix (gal, Sumerian: big), as follows [Rob08, p.295 with notes b and c; Fri07, p. 378; Rob19]:



[15] As in the surface of the field of (the city of Apisal) reported on P102305 r. 1.

[16] Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{\text{Ur III/OB}}$ based on a height of one cubit, see[Pow87, p. 488; Rob08, p. 294; Rob19].

[17] From P309594.

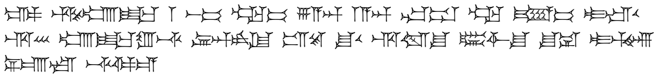
[18] A larger unit, the guru$_7$ (*karûm*, grain heap), is sometimes used instead, with = 𒊹 𒄥 (1 *karûm* = 3600 kurrū). See [Fri07, p. 415; Rob19].

— 𒁹–𒐈 used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
— 𒌋–𒐐 used in $G_{\text{Ur III/OB}}$, of which 𒌋–𒌍 are also used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
— 𒐕–𒐝 used in $S_{\text{Ur III/OB}}$ and the SPVS;
— 𒀸–𒐂 used in $C_{\text{Ur III/OB}}$ as well as in the weight system;
— 𒑏, 𒑐, 𒑑, 𒑒, 𒑔 used in $C_{\text{Ur III/OB}}$;
— 𒁹, 𒐖, 𒐗, 𒐼 used in $C_{\text{Ur III/OB}}$—note the overlap with 𒁹–𒐈;
— 𒑖 and 𒑗 used in $G_{\text{Ur III/OB}}$.

3.3.1 Fractions

3.4 Problems with unification: Early metrology

3.5 Problems with unification: Non-numeric usage



The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'. Do you know its reading?

Examenstext A

3.6 Limited benefits of diachronic encoding for numerals

[Composite texts dating back to the period where curved numerals are in use tend to be limited to lexical texts, which do not usually have numbers. When they do, diachronic encoding is prevented by diš-aš distincitons anyway. Administrative texts, which are where numbers are most prominent, are not composite.]

[Diachronic reference works tend to not include numbers, or when they do, to treat them specially (for intance, they are shown at the end of sign lists such as TODO).]

[The overarching goal of having consistent representation for equivalent numeric expressions from different periods is quickly foiled by changes in metrology.]

Note that in [Rom24] [TODO(egg): Cite the GitHub repository], as in many other such analyses, numbers are removed as an early step in processing; these therefore would not benefit from diachrony in the encoding of numeric expressions.

3.6.1 Compatibility with transliteration

3.7 Compatibility considerations

3.7.1 The case of ŠAR$_2$

4 Rationale for ED–Uruk numeral unification

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban$_2$@c) and 5(ban$_2$@c).]

6 Characters not included in this proposal

6.1 Missing numerals

(N_{17}, $12N_{14}$, etc.) 7(diš *tenû*)

6.2 Stacking patterns

(… are a mess, vary within Uruk, and are not transliterated/documented by Englund, so let's not go there for now.)

7 Acknowledgements

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