Archaic cuneiform numbers

Robin Leroy, Anshuman Pandey, and Steve Tinney

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Contents

1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script[1] and the proto-cuneiform script[2]. The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed "curved", or "curviform", numerals[3] should however *not* be unified with the already-encoded cuneiform numerals[4]. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

<table>
<tr><td></td><td>Uruk III & earlier</td><td>ED – Ur III</td><td>OB & later</td></tr>
<tr><td rowspan="2">Numerals</td><td colspan="2">This proposal</td><td></td></tr>
<tr><td></td><td colspan="2" rowspan="2">Existing Xsux</td></tr>
<tr><td>Non-numeric signs</td><td>Future Pcun</td></tr>
</table>

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

[1] ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.
[2] ISO 15924: Pcun, not yet encoded.
[3] – 1–9(aš^c = N_1), – 1–5(u^c = N_{14}), – 1–9(ŋeš$_2$^{c} = N_{34}), – 1–5(ŋešʾu^c = N_{48}), etc.
[4] – 1–9(aš), – 1–5(u), – 1–9(ŋeš$_2$), – 1–5(ŋešʾu), etc.

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties

2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

TODO(egg): blurb.

3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble 𒅎 in texts from Early Dynastic IIIa Šuruppag as in the character code charts, 𒅎 later in the third millenium[5], 𒅎 in Old Babylonian cursive, 𒅎 in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists[6] and dictionaries[7], and of composite texts[8]. By being compatible with similarly diachronic transliteration practice (that is, by avoiding distinctions finer than those made in transliteration), the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24][9]. The diachronic approach is also useful for pedagogical applications[10].

3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [And04], as part of discussion of the cuneiform encoding[11] that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [And04, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Ŋirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

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[5] Merging with U+1224E 𒉎 NI₂.

[6] Notably [VT+14] and the online edition of [Bor10] in [Jim+23, Signs].

[7] Notably [TJV17] and the online edition of [Sch10] in [Jim+23, Dictionary].

[8] For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of 𒄩𒄠𒈬𒊏𒁉, as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

[9] Attendees may recall the summary given on the third day of UTC #180, as recorded in [Con24]. Other readers may refer to [Svä+24, pp. 242, 148].

[10] For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

[11] At that time scoped to the répertoire of the Ur III period and later, see [EF03, p. 1], although many disunifications, such as 𒅎 ≠ 𒉎, were informed by Early Dynastic distinctions.

1(ŊEŠ$_2$)[12]	1(U)	1/2(DIŠ)	5(DIŠ *tenû*)	gi	us$_2$	sa$_2$
	7.5 (ropes)		5	reed	side	equal

3(U)[13]	6(DIŠ *tenû*)	gi	saŋ	sa$_2$
3(ropes)	6	reed	front	equal

ašag-bi	1(BUR$_3^c$)	1(EŠE$_3^c$)	1(IKU^c)	1/2(IKU^c)
this field				

tug$_x$(LAK483)-si-ga-kam[14]
deep ploughing

The argument made in [And04, p. 4] is that this is comparable to a stylistic distinction such as[15]

> 465 metres, equal lengths
> 198 metres, equal widths
> this field: 9,18 hectares, deeply ploughed

where the numerals have the same structure ([And04] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that "the number signs do not normally carry in their individual signs the meaning of what they are used to measure", and that curviform and cuneiform numerals "are not normally mixed together in a single numerical expression", noting the exceptions of [P232278; P232280]. In addition, [And04, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from -- to -numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [EFT03], the curviform numerals were therefore removed from [EFT04b] and [EFT04a], which both state that "The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases."

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

— the functions and use of the numerals vary beyond the mere / switch;

[12] As noted in [Pow87, p. 466], this sign has a very short "tail" in this period, so that it is wider than it is tall, and can at first seem like a large in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

[13] Note that ED IIIb numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign in [P020054] looks more like Ur III .

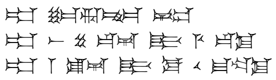
[14] Transliteration after [Lec20, p. 8].

[15] We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

— the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;

— some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 Metrology



I want to write tablets: the tablet of 1 gur *of barley to 600* gur*; the tablet of 1 shekel of silver to 10 minas [...]*

Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known[16] a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millenium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters 𒁹–𒐎, the multiples of ten (10–50) are 𒌋–𒐐, but the other digits 11–59 are sequences 𒌋𒁹–𒐐𒐎; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units 𒁹–𒐎, tens 𒌋–𒐐, sixties 𒐕–𒐝 (with larger wedges than the units), six hundreds 𒐞–𒐢, three thousand six hundreds 𒊹–𒐫, and thirty-six thousands 𒐬–𒐱.

3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram[17], where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\text{𒐬} \xleftarrow{10} \text{𒊹} \xleftarrow{6} \text{𒐞} \xleftarrow{10} \text{𒐕} \xleftarrow{6} \text{𒌋} \xleftarrow{10} \text{𒁹} \qquad (S_{\text{Ur III/OB}})$$

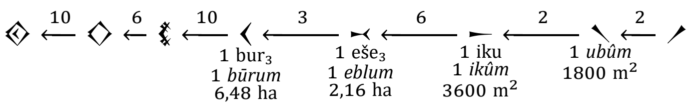
For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written 𒐟𒐜𒐏𒐎 in the discrete counting system, and 𒎙𒐍𒐏𒐎 in the sexagesimal place value system.

[16] See, *e.g.*, [Uni16, Section 22.3.3 "Non-Decimal Radix Systems", *sub* "Cuneiform Numerals"].

[17] These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:

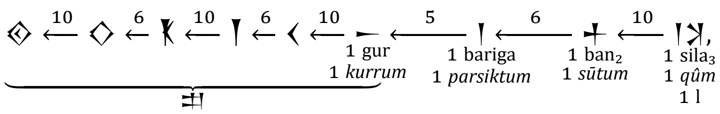


($G_{\text{Ur III/OB}}$)

Note that for the range of areas given above[18], this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *būrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign 𒂷 functioning as punctuation[19], but the numerals are tied to the metrology; thus a surface of 5 *būrū* 1 *eblum* 4 *ikû* (100 *ikû*, 36 ha) would be written[20] 𒐼𒑘𒐉𒂷. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in “88 acres 3 roods 33 perches” or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the ŠAR$_2$ sign 𒊹 is equal to sixty times the U sign 𒌋 in the area system, but to three hundred and sixty times 𒌋 in the discrete counting system.

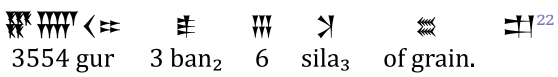
3.3.3 The capacity system

Another such system of note is the one for capacities[21] [Fri07, p. 376; Rob19],

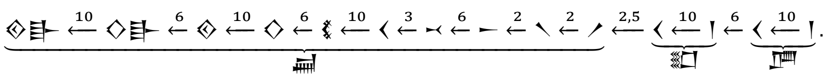


($C_{\text{Ur III/OB}}$)

where the numerals for ban$_2$ are 𒑏, 𒑐, 𒐊, 𒑓, and 𒑕, and those for bariga are 𒁹, 𒑖, 𒑗, and 𒑘 (contrast ordinary 𒐖 and 𒐗 otherwise used with 𒁹-numerals). As described in [Hue11, p. 585 with notes (b) and (f)], the sign GUR 𒄥, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𒋡 if present, and after the word for “grain” if present, as in



[18]For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mūšarum* (36 m^2) written 𒁹𒊬, equal to one hundredth of an *ikûm*, then sexigesimally subdivided in 60 𒂆 (shekels). For areas greater than 3600 *būrū*, the 𒊹- and 𒐬-numerals are reused with a suffix 𒃲 (gal, Sumerian: big), as follows [Rob08, p. 295 with notes b and c; Fri07, p. 378; Rob19]:



[19]TODO(egg): acknowledge Proust 2020 but note that this is irrelevant to encoding concerns

[20]As in the surface of the field of 𒀀𒉺𒄥𒋡𒆠 (the city of Apisal) reported on [P102305, r. 1]

[21]Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{\text{Ur III/OB}}$ based on a height of one cubit, see[Pow87, p. 488; Rob08, p. 294; Rob19].

Observe that while large numbers of gur follow[23] system $S_{\text{Ur III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as 𒌋𒁹𒄥 would be 10 gur 1 bariga, and 𒌋𒀸𒄥 would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

— 𒁹–𒐎 used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
— 𒌋–𒐐 used in $G_{\text{Ur III/OB}}$, of which 𒌋–𒐏 are also used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
— 𒐕–𒐝 used in $S_{\text{Ur III/OB}}$, and sometimes with overt units;
— 𒐞–𒐢 used in $S_{\text{Ur III/OB}}$;
— 𒊹–𒐪 used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
— 𒐫–𒐯 used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
— 𒀸–𒐃 used in $C_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
— 𒑘, 𒑙, 𒑚, 𒑛, 𒑜 used in $C_{\text{Ur III/OB}}$;
— 𒁹, 𒑝, 𒑞, 𒑟 used in $C_{\text{Ur III/OB}}$—note the overlap with 𒁹–𒐎;
— 𒑡 and 𒑠 used in $G_{\text{Ur III/OB}}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Rob08, p. 78]: "The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to […] metrological system. […] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity."

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with 𒁹- and 𒌋-numerals with their system $S_{\text{Ur III/OB}}$ values[24]. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Fri07, p. 118; Rob19]:

$$\underset{\substack{\text{danna}\\ \textit{bērum}\\ \text{league}\\ \text{10,8 km}}}{\text{𒁹𒆜}} \xleftarrow{60} \underset{\substack{\text{UŠ}^{25}\\ \\ \text{cable}\\ \text{360 m}}}{\text{𒁹𒍑}} \xleftarrow{10} \underset{\substack{\text{nindan}\\ \textit{nindanum}\\ \text{rod}\\ \text{6 m}}}{\text{𒁹𒃻}} \xleftarrow{12} \underset{\substack{\text{kuš}_3\\ \textit{ammatum}\\ \text{cubit}\\ \text{50 cm}}}{\text{𒁹𒌑}} \xleftarrow{30} \underset{\substack{\text{šu-si}\\ \textit{ubānum}\\ \text{finger}\\ \text{17 mm}}}{\text{𒁹𒋗𒋛}}. \qquad (L_{\text{Ur III/OB}})$$

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:

$$\text{𒁹𒆜} \xleftarrow{30} \text{𒁹𒍑} \xleftarrow{6} \underset{\substack{\text{eše}_2\\ \textit{ašlum}\\ \text{rope}\\ \text{60 m}}}{\text{𒁹𒂠}} \xleftarrow{10} \text{𒁹𒃻} \xleftarrow{2} \underset{\substack{\text{gi}\\ \textit{qânum}\\ \text{reed}\\ \text{3 m}}}{\text{𒁹𒄀}} \xleftarrow{6} \text{𒁹𒌑} \xleftarrow{30} \text{𒁹𒋗𒋛}. \qquad (\bar{L}_{\text{Ur III/OB}})$$

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[22] From P309594.

[23] A larger unit, the guru$_7$ (*karûm*, grain heap), is sometimes used instead, with 𒀸𒄥𒐃=𒊹𒄥 (1 *karûm* = 3600 kurrū). See [Fri07, p. 415; Rob19].

[24] Adjacent units are no more than a factor of 60 apart, so higher numerals such as 𒐞 or 𒊹 are not used.

[25] TODO

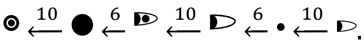
In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

TODO

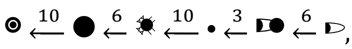
3.4 Early metrology

At first sight, the metrological systems from the Early Dynastic period match the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{\text{Ur III/OB}}$ [Fri07, p. 374; DE87, pp. 127, 165]:



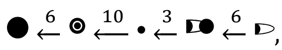
$$(S)$$

Likewise the area system used in the Early Dynastic IIIb period mirrors system $G_{\text{Ur III/OB}}$ [Dei22, p. 72; NDE93, p. 63; Fri07, p. 378; Gom16]:



$$(G_{\text{ED IIIb}})$$

As noted in [And04, p. 4] (see section 3.2), the vertical 𒁹 from $S_{\text{Ur III/OB}}$ becomes a horizontal 𒀸 in system S. It is however far from the only case of such a reallocation of function. The earlier form of System G was [DE87, pp. 141, 165; Fri07, p. 378]:

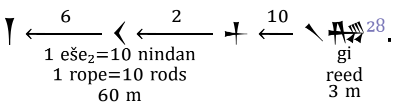


$$(G)$$

Observe that, as noted in [DE87, p. 142], ⊚ changes meaning from 10• in system G to 10• in system $G_{\text{ED IIIb}}$. System G is used in the Uruk period, but also in the ED I–II period (it is the “area 2” system in [Cha03], whereas $G_{\text{ED IIIb}}$ is the “area 1” system).

3.4.1 Field lengths in Ŋirsu

The length system Early Dynastic IIIb of the state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign 𒃻 is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign 𒂠 is not written either. Length shorter than one rope are expressed in half-ropes using the 1/2 sign 𒈦 (again with no 𒂠), and then in reeds, *with* the sign 𒄀. Effectively, this yields the following factor diagram:



$$(L_{\text{ED IIIb}})$$

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and others from the same period, such as the ones discussed in [Lec20] areas are expressed in system $G_{\text{ED IIIb}}$, with curviform numerals[29]; in the absence of overt units, such as when dealing with length that

[28]Note that the reeds are counted using *tenû* numerals, 𒀹, ..., etc.
[29]TODO(egg): Note the handful of late Urukagina tablets that start to have cuneiform areas.

are integer multiples of a half-rope[30], the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and therefore the interpretation of its numerals between systems $G_{\text{ED IIIb}}$ and $L_{\text{ED IIIb}}$. The sign GAN$_2$, which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without.

3.4.2 Dyke lengths in Ŋirsu

[Pow87, p. 466] notes that reeds “are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus.” [TODO(egg): also mention Krebernik 1998 p. 303 with note 686.] Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in CDLI finds many occurrences[31], such as:

- **[P221305]**
- **[P221305]**
- **[P020129]**
- **[P221291]**
- **[P221266]**

These expressions use an explicit sign (counted in multiples of ten) or. This notation—but not its use of curviform numerals—is remarked on in [Lec20, p. 290 with note 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. These match the following factor diagrams:

$$\underbrace{\;\xleftarrow{2}\;}\;\xleftarrow{10}\;\;\xleftarrow{6}\;\;\xleftarrow{3}\;. \qquad (L'_{\text{ED IIIb}})$$

$$\underbrace{\;\xleftarrow{10}\;\;\xleftarrow{6}\;\bullet}\;\xleftarrow{2}\;{}^{32}\;\xleftarrow{10}\;. \qquad (L''_{\text{ED IIIb}})$$

3.4.3 Grain in Ebla

Lengths of Early Dynastic IIIb dykes from Ŋirsu are far from the only numeric expressions that mix curviform and cuneiform numerals.

[30] This is the case of the sides of the field in [P020054, obv. ii 2–3].

[31] A search for curviform numerals followed by some number of reeds counted in (*tenû*) cuneiform numerals currently finds 125 occurrences across 47 tablets.

[32] TODO(egg): Note that one unit may be omitted if the other is present

The system of grain[33] capacities in Ebla uses the following units[34]:

$$\text{gu}_2\text{-bar} \xleftarrow{2} \text{ba-ri}_2\text{-zu} \xleftarrow{\frac{5}{2}} \text{ŋin}_4 \xleftarrow{4} \text{niŋ}_2\text{-sagšu} \xleftarrow{6} \text{an-zam}_x.$$

The gu$_2$-bar and ba-ri$_2$-zu are generally counted using curviform numerals, and the smaller units using cuneiform numerals. Indeed, a search on [Mil+07] for co-occurrences of an-zam$_x$ with either of gu$_2$-bar or ba-ri$_2$-zu finds the following expressions[35]:

1. [P240532, *verso* 4, 9] [37]
2. [P240548, *verso* 1, 1]
3. [P240655, *recto* 7, 9] [38]
4. [P240579, *verso* 4, 3]
5. [P240675, *verso* 2, 2]
6. [P240609, *verso* 3, 1]
7. [P240533, *recto* 3, 3]
8. [P240697, *recto* 1, 5] [39]
9. [P240653, *recto* 6, 2]
10. [P240654, *recto* 2, 6] [40] [41]
11. [P240531, *recto* 1, 8]
12. [P241708, *recto* 1, 1][42]
13. [P241904, *recto* 1, 1][43] [44]

Note that higher numbers of gu$_2$-bar are expressed in hundreds (*mi-at*) and then thousands (*li-im*), as is typical in Ebla [Arc15, p. 33], *e.g.*, in [P240532, *verso* 2, 3], (100 + 60 + 30 + 5 = 195 gu$_2$-bar of grain).

[33]Liquid capacities use a different system [Arc15, p. 229 with note 12]:

$$\text{la-ḫa} \xleftarrow{30} \text{sila}_3 \xleftarrow{6} \text{an-zam}_x.$$

At a glance it seems that sila$_3$ are counted with cuneiform numerals and higher units with curviform ones, thus

$$\underbrace{\cdot \xleftarrow{\frac{5}{3}} \cdot \xleftarrow{6} \cdot \xleftarrow{10} \cdot}_{\text{la-ḫa}} \xleftarrow{3} \underbrace{\cdot \xleftarrow{10} \cdot}_{\text{sila}_3} \xleftarrow{6} \text{an-zam}_x,$$

but we have not investigated this thoroughly.

[34]TODO mention the other one citing Chambon and the footnote in Archi

[35]We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform ba-ri$_2$-zu and larger with cuneiform ŋin$_4$ and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked[36] on a photograph (from EbDA unless noted otherwise).

[36]As we will see in Section 3.6.1, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

[37]ba-ri$_2$-zu$_2$, a variant spelling.

[38]Short for gu$_2$-bar.

[39]Note the omitted gu$_2$-bar.

[40]Instead of the expected niŋ$_2$-sagšu.

[41] not legible on the EbDA photo.

[42]From CDLI photo.

[43]From photo in [Arc89, p. 6].

[44]Laid out as ; on stacking patterns see Section 6.2.

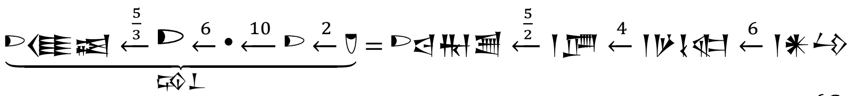
of decreasing fractions $^1/_n$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus ⧖ = $^1/_2$ N_{39}, ⧖ = $^1/_3$ N_{39}, and so on. The first sign of the latter units, N_{34},

Figure 1: TODO [Eng98, p. 113]

For instance, the first line contains the notations 1N_{34} 1N_{39a} ; 2N_{20}, which can be translated "60 of the (grain rations containing) ⌓ (of grain); (grain involved:) 2 • (of ground barley)". This calculation contradicts the assumed numerical relationship 10N_1 = 1N_{14}, since as was well known the measure represented by the sign N_{39} was $^1/_5$ of that represented by N_1, so that 60 × $^1/_5$ = 12 and not 20, as 2N_{14} would imply. Instead of relying on complicated

Figure 2: TODO [Eng98, p. 116]

These expressions match the following factor diagram:



(C_{Ebla})

3.4.4 Use in modern publications

Because of their prevalence in the Uruk and Early Dynastic periods, the proposed numerals are used modern publications discussing metrology in those periods, as illustrated in Figures 1–10.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 10. The partial[45] transliteration "4▷ *ʾa₃-da-um* 4▽ *aktum* 4▷ ib$_2^{\mathrm{tu_9}}$×3𒁹 sa$_6$ gunu$_3$" is used to illustrate a discussion of the interpretation of the contrast between ▷ and ▽ numerals. More conventional transliterations[46] might omit the numeral shapes entirely, *e.g.*, 4 *ʾa₃-da-um* 4 *aktum* 4 ib$_2^{\mathrm{tu_9}}$×3 sa$_6$ gunu$_3$, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, *e.g.*, 4(aš^c) *ʾa₃-da-um* 4(diš^c) *aktum* 4(aš^c) ib$_2^{\mathrm{tu_9}}$×3(diš) sa$_6$ gunu$_3$, but the result would be less readable. See Section 3.6.1 for a discussion of transliteration conventions for numerals.

[45]The untransliterated text would be [cuneiform]; note the atomically encoded ib$_2$ × 3𒁹 = [cuneiform] × 𒐈 = [cuneiform].

[46]TODO cite the EbDA one.

[47]TODO(egg): On the order cite TSS 188, Friberg2007 p. 148 and any of the usual suspects on the haphazard order of signs in early texts.

Die halbkreisförmigen Griffeleindrücke gehen manchmal in mehr oder weniger eckige Formen über (▽)[685]. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen (∖), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren[686]. Selten treten mit ▿ gebildete Zahlen auf[687] (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 3: TODO [Kre98, p. 303]

formed by only two signs and , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60^2, 60^3, 1/60, $1/60^2$, etc. are written in the same way, with the vertical wedge . The third feature concerns the exact function of

Figure 4: TODO

one step. The scribes of the Early Dynastic Period (*c.* 2600 BC), for instance, represented the number 648, 000 with: but never with the repetition .

Figure 5: TODO[47]

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

še bar *ba-rí-zu*

'3 *gubar* (capacity units) and 1 *parīsu*'.

Figure 6: TODO

This is particularly true of the signs , , and , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign is most often associated with the *parīsu* measure, while the signs , , and refer to 1, 2, 3,

Figure 7: TODO

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., transliterated as 2(šar$_2$) 1(geš'u) 3(u), means $2 \times 3600 + 1 \times 600 + 3 \times 10$). The system is based on an alternation of factors ten and

Figure 8: TODO

might think of one fabric and a half,[11] but the presence of notations with "2 2", "3 3", and "6 6" (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag's weight measurement system.[12] The notation "1 gada" in o. ii 1 and r. vi 1, along with the total of "39

Fig. 1. Combinations of numerals attested in Š. 742.

Figure 9: Discussion of the contrast between and numerals in [Gor23, p. 162].

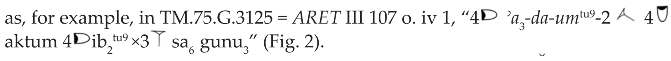
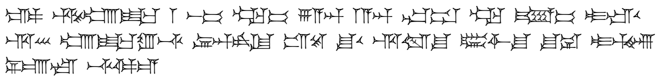


Figure 10: Transliteration in [Gor23, p. 163] of [**P242293**] incorporating untransliterated numerals.

3.5 Non-numeric usage



The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'. Do you know its reading?

Examenstext A

3.6 Limited benefits of diachronic encoding for numerals

[TODO Composite texts dating back to the period where curved numerals are in use tend to be limited to lexical texts, which do not usually have numbers. When they do, diachronic encoding is prevented by diš-aš distincitons anyway. Administrative texts, which are where numbers are most prominent, are not composite.]

[TODO Diachronic reference works tend to not include numbers, or when they do, to treat them specially (for intance, they are shown at the end of sign lists such as TODO).]

[TODO The overarching goal of having consistent representation for equivalent numeric expressions from different periods is quickly foiled by changes in metrology.]

Note that in [Rom24] [TODO(egg): Cite the GitHub repository], as in many other such analyses, numbers are removed as an early step in processing; these therefore would not benefit from diachrony in the encoding of numeric expressions.

3.6.1 Compatibility with transliteration

TODO words [Rob08, p. 295] TODO cite [**Molina2014**]

3.7 Compatibility considerations

3.7.1 The case of ŠAR$_2$

[TODO explain why this isn't a problem, effectively anyone who needs to cuneify 1(šar$_2$^{c}) will also need to cuneify some of the numerals proposed here and will therefore not be using Unicode cuneiform.] [TODO U+122B9 CUNEIFORM SIGN SHAR2 represents both 1(šar$_2$) and non-numeric šar$_2$; it looks like ◇ (so, like ◇) in all but lexical texts from Ebla and Šuruppag (and the archaizing vulture stele, where note that the scribe slipped into his modern ways once), where it looks like (TODO: the proposed character). The proposed character is to be used for 1(šar$_2$^{c}). 1(šar$_2$) does not exist back when non-numeric šar$_2$ is curviform, so it works out.]

4 Rationale for ED–Uruk numeral unification

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban$_2$@c) and 5(ban$_2$@c).]

6 Characters not included in this proposal

6.1 Missing numerals

(N_{17}, 12N_{14}, etc.) 7(diš *tenû*)

6.2 Stacking patterns

(… are a mess, vary within Uruk, and are not transliterated/documented by Englund, so let's not go there for now.)

6.3 Matters for higher-level protocols

Rotated bits: https://cdli.mpiwg-berlin.mpg.de/artifacts/101087

7 Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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