

Archaic cuneiform numbers

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	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
Non-numeric signs	Future Pcun	Existing Xsux	

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script¹ and the proto-cuneiform script². The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties

2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

TODO(egg): blurb.





¹ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

²ISO 15924: Pcun, not yet encoded.

³ 𐎶 1-9(aš^c = N_1), 𐎷 1-5(u^c = N_{14}), 𐎸 1-9(ḫeš₂^c = N_{34}), 𐎹 1-5(ḫeš^cu^c = N_{48}), etc.

⁴ 𐎶 1-9(aš), 𐎷 1-5(u), 𐎸 1-9(ḫeš₂), 𐎹 1-5(ḫeš^cu), etc.

3.1 The cuneiform encoding model



















As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts,  later in the third millennium⁵,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.


This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists⁶ and dictionaries⁷, and of composite texts⁸. By being compatible with similarly diachronic transliteration practice (that is, by avoiding distinctions finer than those made in transliteration), the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]⁹. The diachronic approach is also useful for pedagogical applications¹⁰.

3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [Ando4], as part of discussion of the cuneiform encoding¹¹ that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.
































Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [Ando4, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:


 ¹²						
1(NEŠ ₂)	1(U)	1/2(DIŠ)	5(DIŠ <i>tenû</i>)	gi	us ₂	sa ₂
	7.5 (ropes)		5	reed	side	equal
 ¹³						
3(U)	6(DIŠ <i>tenû</i>)	gi	saṇ	sa ₂		
3(ropes)	6	reed	front	equal		
						
ašag-bi	1(BUR ₃ ^c)	1(EŠE ₃ ^c)	1(IKU ^c)	1/2(IKU ^c)		
this field						

⁵Merging with U+1224E  NI₂.

⁶Notably [VT+14] and the online edition of [Bor10] in [Jim+23, Signs].



⁷Notably [TJV17] and the online edition of [Sch10] in [Jim+23, Dictionary].

⁸For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of                               


 tug_x(LAK483)-si-ga-kam¹⁴
 deep ploughing



The argument made in [Ando4, p. 4] is that this is comparable to a stylistic distinction such as¹⁵



465 metres, equal lengths
 198 metres, equal widths
 this field: 9, 18 hectares, deeply ploughed

where the numerals have the same structure ([Ando4] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [Ando4, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from  to -numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [EFT03], the curviform numerals were therefore removed from [EFT04b] and [EFT04a], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere / switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

used in this transcription; the sign  in [P020054] looks more like Ur III .

¹⁴Transliteration after [Lec20, p. 8].

¹⁵We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

3.3 Metrology

正 終 正 正 正
 正 一 名 正 正 下 正
 正 一 正 正 正 正

I want to write tablets: the tablet of 1 gur of barley to 600 gur; the tablet of 1 shekel of silver to 10 minas [...]

Edubba'a D



Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known¹⁰ a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millenium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Robo8, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters $\text{I}-\text{IX}$, the multiples of ten (10–50) are $\text{X}-\text{XL}$, but the other digits 11–59 are sequences $\text{X}-\text{X} \text{I}-\text{X} \text{L}$; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units $\text{I}-\text{IX}$, tens $\text{X}-\text{XL}$, sixties $\text{I}-\text{LXX}$ (with larger wedges than the units), six hundreds $\text{X}-\text{LXX}$, three thousand six hundreds $\text{X}-\text{LXX}$, and thirty-six thousands $\text{X}-\text{LXX}$.

3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram¹⁷, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \blacktriangledown \xleftarrow{10} \blacktriangledown \xleftarrow{6} \blacktriangleleft \xleftarrow{10} \blacktriangledown \quad (S_{\text{Ur III/OB}})$$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written  in the discrete counting system, and  in the sexagesimal place value system.


















3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name

¹⁶See, e.g., [Uni16, Section 22.3.3 “Non-Decimal Radix Systems”, sub “Cuneiform Numerals”].

¹⁷These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:






Note that for the range of areas given above¹⁸, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *bûrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign  functioning as punctuation¹⁹, but the numerals are tied to the metrology; thus a surface of 5 *bûrû* 1 *eblum* 4 *ikû* (100 *ikû*, 36 ha) would be written²⁰             . Contrast this with systems where the same numerals are used for different units, and overt units are used, as in “88 acres 3 roods 33 perches”. Note also that the same signs are shared between multiple systems, with different relations; the ŠAR₂ sign  is equal to sixty times the U sign  in the area system, but to three hundred and sixty times  in the discrete counting system.

3.3.3 The capacity system

Another such system of note is the one for capacities²¹ [Fri07, p. 376; Rob19],

(C_{Ur} III/OB)

where the numerals for ban₂ are 𐎶, 𐎷, 𐎸, 𐎹, and 𐎺, and those for bariga are 𐎶, 𐎷, 𐎸, and 𐎹 (contrast ordinary 𐎶 and 𐎷 otherwise used with 𐎶-numerals). As described in [Hue11, p.585 with notes (b) and (f)], the sign GUR 𐎶𐎶, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𐎶 if present, and after the word for “grain” if present, as in

 354 gur
  3 ban₂
  6
  sila₃
  of grain.

Observe that while large numbers of gur follow²³ system $\mathcal{S}_{\text{Ur III/OB}}$, the use of horizontal ($\check{\text{A}}^{\text{S}}$) numerals for the gur disambiguates from the vertical bariga, as $\langle \text{I} \frac{1}{2} \rangle$

¹⁸For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mûšarum* (36 m²) written 𐎶𐎵, equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60 𐎶𐎵 (shekels). For areas greater than 3600 *būrû*, the 𐎶- and 𐎵-numerals are reused with a suffix 𐎶𐎵 (gal, Sumerian: big), as follows [Robo8, p.295 with notes b and c; Fri07, p. 378; Rob19]:

¹⁹TODO(egg): acknowledge Proust 2020 but note that this is irrelevant to encoding concerns

²⁰As in the surface of the field of (the city of Apisal) reported on [P102305, r. 1]

²¹Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{UR III/08}$ based on a height of one cubit, see [Pow87, p. 488; Rob08, p. 294; Rob19].

²²From P309594.

²³A larger unit, the *guru*₇ (*karûm*, grain heap), is sometimes used instead, with 𐎠𐎡𐎢𐎣𐎤𐎥𐎦𐎧𐎨𐎩𐎪𐎫𐎬𐎭𐎮𐎯𐎰𐎱𐎲𐎳𐎴𐎵𐎶𐎷𐎸𐎹𐎺𐎻𐎼𐎽𐎾𐎿𐏀𐏁𐏂𐏃𐏄𐏅𐏆𐏇𐏈𐏉𐏊𐏋𐏌𐏍𐏎𐏏𐏐𐏑𐏒𐏓𐏔𐏕𐏖𐏗𐏘𐏙𐏚𐏛𐏜𐏝𐏞𐏟𐏠𐏡𐏢𐏣𐏤𐏥𐏦𐏧𐏨𐏩𐏪𐏫𐏬𐏭𐏮𐏯𐏰𐏱𐏲𐏳𐏴𐏵𐏶𐏷𐏸𐏹𐏺𐏻𐏼𐏽𐏾𐏿𐐀𐐁𐐂𐐃𐐄𐐅𐐆𐐇𐐈𐐉𐐊𐐋𐐌𐐍𐐎𐐏𐐐𐐑𐐒𐐓𐐔𐐕𐐖𐐗𐐘𐐙𐐚𐐛𐐜𐐝𐐞𐐟𐐠𐐡𐐢𐐣𐐤𐐥𐐦𐐧𐐨𐐩𐐪𐐫𐐬𐐭𐐮𐐯𐐰𐐱𐐲𐐳𐐴𐐵𐐶𐐷𐐸𐐹𐐺𐐻𐐼𐐽𐐾𐐿𐑀𐑁𐑂𐑃𐑄𐑅𐑆𐑇𐑈𐑉𐑊𐑋𐑌𐑍𐑎𐑏𐑐𐑑𐑒𐑓𐑔𐑕𐑖𐑗𐑘𐑙𐑚𐑛𐑜𐑝𐑞𐑟𐑠𐑡𐑢𐑣𐑤𐑥𐑦𐑧𐑨𐑩𐑪𐑫𐑬𐑭𐑮𐑯𐑰𐑱𐑲𐑳𐑴𐑵𐑶𐑷𐑸𐑹𐑺𐑻𐑼𐑽𐑾𐑿𐒀𐒁𐒂𐒃𐒄𐒅𐒆𐒇𐒈𐒉𐒊𐒋𐒌𐒍𐒎𐒏𐒐𐒑𐒒𐒓𐒔𐒕𐒖𐒗𐒘𐒙𐒚𐒛𐒜𐒝𐒞𐒟𐒠𐒡𐒢𐒣𐒤𐒥𐒦𐒧𐒨𐒩𐒪𐒫𐒬𐒭𐒮𐒯𐒰𐒱𐒲𐒳𐒴𐒵𐒶𐒷𐒸𐒹𐒺𐒻𐒼𐒽𐒾𐒿𐓀𐓁𐓂𐓃𐓄𐓅𐓆𐓇𐓈𐓉𐓊𐓋𐓌𐓍𐓎𐓏𐓐𐓑𐓒𐓓𐓔𐓕𐓖𐓗𐓘𐓙𐓚𐓛𐓜𐓝𐓞𐓟𐓠𐓡𐓢𐓣𐓤𐓥𐓦𐓧𐓨𐓩𐓪𐓫𐓬𐓭𐓮𐓯𐓰𐓱𐓲𐓳𐓴𐓵𐓶𐓷𐓸𐓹𐓺𐓻𐓼𐓽𐓾𐓿𐔀𐔁𐔂𐔃𐔄𐔅𐔆𐔇𐔈𐔉𐔊𐔋𐔌𐔍𐔎𐔏𐔐𐔑𐔒𐔓𐔔𐔕𐔖𐔗𐔘𐔙𐔚𐔛𐔜𐔝𐔞𐔟𐔠𐔡𐔢𐔣𐔤𐔥𐔦𐔧𐔨𐔩𐔪𐔫𐔬𐔭𐔮𐔯𐔰𐔱𐔲𐔳𐔴𐔵𐔶𐔷𐔸𐔹𐔺𐔻𐔼𐔽𐔾𐔿𐕀𐕁𐕂𐕃𐕄𐕅𐕆𐕇𐕈𐕉𐕊𐕋𐕌𐕍𐕎𐕏𐕐𐕑𐕒𐕓𐕔𐕕𐕖𐕗𐕘𐕙𐕚𐕛𐕜𐕝𐕞𐕟𐕠𐕡𐕢𐕣𐕤𐕥𐕦𐕧𐕨𐕩𐕪𐕫𐕬𐕭𐕮𐕯𐕰𐕱𐕲𐕳𐕴𐕵𐕶𐕷𐕸𐕹𐕺𐕻𐕼𐕽𐕾𐕿𐖀𐖁𐖂𐖃𐖄𐖅𐖆𐖇𐖈𐖉𐖊𐖋𐖌𐖍𐖎𐖏𐖐𐖑𐖒𐖓𐖔𐖕𐖖𐖗𐖘𐖙𐖚𐖛𐖜𐖝𐖞𐖟𐖠𐖡𐖢𐖣𐖤𐖥𐖦𐖧𐖨𐖩𐖪𐖫𐖬𐖭𐖮𐖯𐖰𐖱𐖲𐖳𐖴𐖵𐖶𐖷𐖸𐖹𐖺𐖻𐖼𐖽𐖾𐖿𐗀𐗁𐗂𐗃𐗄𐗅𐗆𐗇𐗈𐗉𐗊𐗋𐗌𐗍𐗎𐗏𐗐𐗑𐗒𐗓𐗔𐗕𐗖𐗗𐗘𐗙𐗚𐗛𐗜𐗝𐗞𐗟𐗠𐗡𐗢𐗣𐗤𐗥𐗦𐗧𐗨𐗩𐗪𐗫𐗬𐗭𐗮𐗯𐗰𐗱𐗲𐗳𐗴𐗵𐗶𐗷𐗸𐗹𐗺𐗻𐗼𐗽𐗾𐗿𐘀𐘁𐘂𐘃𐘄𐘅𐘆𐘇𐘈𐘉𐘊𐘋𐘌𐘍𐘎𐘏𐘐𐘑𐘒𐘓𐘔𐘕𐘖𐘗𐘘𐘙𐘚𐘛𐘜𐘝𐘞𐘟𐘠𐘡𐘢𐘣𐘤𐘥𐘦𐘧𐘨𐘩𐘪𐘫𐘬𐘭𐘮𐘯𐘰𐘱𐘲𐘳𐘴𐘵𐘶𐘷𐘸𐘹𐘺𐘻𐘼𐘽𐘾𐘿𐙀𐙁𐙂𐙃𐙄𐙅𐙆𐙇𐙈𐙉𐙊𐙋𐙌𐙍𐙎𐙏𐙐𐙑𐙒𐙓𐙔𐙕𐙖𐙗𐙘𐙙𐙚𐙛𐙜𐙝𐙞𐙟𐙠𐙡𐙢𐙣𐙤𐙥𐙦𐙧𐙨𐙩𐙪𐙫𐙬𐙭𐙮𐙯𐙰𐙱𐙲𐙳𐙴𐙵𐙶𐙷𐙸𐙹𐙺𐙻𐙼𐙽𐙾𐙿𐚀𐚁𐚂𐚃𐚄𐚅𐚆𐚇𐚈𐚉𐚊𐚋𐚌𐚍𐚎𐚏𐚐𐚑𐚒𐚓𐚔𐚕𐚖𐚗𐚘𐚙𐚚𐚛𐚜𐚝𐚞𐚟𐚠𐚡𐚢𐚣𐚤𐚥𐚦𐚧𐚨𐚩𐚪𐚫𐚬𐚭𐚮𐚯𐚰𐚱𐚲𐚳𐚴𐚵𐚶𐚷𐚸𐚹𐚺𐚻𐚼𐚽𐚾𐚿𐛀𐛁𐛂𐛃𐛄𐛅𐛆𐛇𐛈𐛉𐛊𐛋𐛌𐛍𐛎𐛏𐛐𐛑𐛒𐛓𐛔𐛕𐛖𐛗𐛘𐛙𐛚𐛛𐛜𐛝𐛞𐛟𐛠𐛡𐛢𐛣𐛤𐛥𐛦𐛧𐛨𐛩𐛪𐛫𐛬𐛭𐛮𐛯𐛰𐛱𐛲𐛳𐛴𐛵𐛶𐛷𐛸𐛹𐛺𐛻𐛼𐛽𐛾𐛿𐜀𐜁𐜂𐜃𐜄𐜅𐜆𐜇𐜈𐜉𐜊𐜋𐜌𐜍𐜎𐜏𐜐𐜑𐜒𐜓𐜔𐜕𐜖𐜗𐜘𐜙𐜚𐜛𐜜𐜝𐜞𐜟𐜠𐜡𐜢𐜣𐜤𐜥𐜦𐜧𐜨𐜩𐜪𐜫𐜬𐜭𐜮𐜯𐜰𐜱𐜲𐜳𐜴𐜵𐜶𐜷𐜸𐜹𐜺𐜻𐜼𐜽𐜾𐜿𐝀𐝁𐝂𐝃𐝄𐝅𐝆𐝇𐝈𐝉𐝊𐝋𐝌𐝍𐝎𐝏𐝐𐝑𐝒𐝓𐝔𐝕𐝖𐝗𐝘𐝙𐝚𐝛𐝜𐝝𐝞𐝟𐝠𐝡𐝢𐝣𐝤𐝥𐝦𐝧𐝨𐝩𐝪𐝫𐝬𐝭𐝮𐝯𐝰𐝱𐝲𐝳𐝴𐝵𐝶𐝷𐝸𐝹𐝺𐝻𐝼𐝽𐝾𐝿𐞀𐞁𐞂𐞃𐞄𐞅𐞆𐞇𐞈𐞉𐞊𐞋𐞌𐞍𐞎𐞏𐞐𐞑𐞒𐞓

would be 10 gur 1 bariga, and $\leftarrow \text{𒍪}$ would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system. To quote [Robo8, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

This intertwining of units and numerals explains the large number of already-encoded numeral series:

- $\text{I}-\text{𒍪}$ used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- $\leftarrow \text{𒍪}$ used in $G_{\text{Ur III/OB}}$, of which $\leftarrow \text{𒍪}$ are also used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- $\text{I}-\text{𒍪}$ used in $S_{\text{Ur III/OB}}$, and sometimes with overt units;
- $\text{K}-\text{𒍪}$ used in $S_{\text{Ur III/OB}}$;
- $\diamond-\text{𒍪}$ used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- $\diamond-\text{𒍪}$ used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- $\text{I}-\text{𒍪}$ used in $C_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
- $\text{I}, \text{I}, \text{I}, \text{I}, \text{I}$ used in $C_{\text{Ur III/OB}}$;
- $\text{I}, \text{I}, \text{I}, \text{I}$ used in $C_{\text{Ur III/OB}}$ —note the overlap with $\text{I}-\text{𒍪}$;
- I and I used in $G_{\text{Ur III/OB}}$.

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with I - and \leftarrow -numerals with their system $S_{\text{Ur III/OB}}$ values²⁴. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Fri07, p. 118; Rob19]:

$\text{I}-\text{𒍪}$	$\leftarrow 60$	$\text{I}-\text{𒍪}$	$\leftarrow 10$	$\text{I}-\text{𒍪}$	$\leftarrow 12$	$\text{I}-\text{𒍪}$	$\leftarrow 30$	$\text{I}-\text{𒍪}$	
danna		US ²⁵		nindan		kuš ₃		šu-si	
bērum		cable		nindanum		ammatum		ubānum	
league		360 m		rod		cubit		finger	
10,8 km				6 m		50 cm		17 mm	

($L_{\text{Ur III/OB}}$)

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:

$\text{I}-\text{𒍪}$	$\leftarrow 30$	$\text{I}-\text{𒍪}$	$\leftarrow 6$	$\text{I}-\text{𒍪}$	$\leftarrow 10$	$\text{I}-\text{𒍪}$	$\leftarrow 2$	$\text{I}-\text{𒍪}$	$\leftarrow 6$	$\text{I}-\text{𒍪}$	$\leftarrow 30$	$\text{I}-\text{𒍪}$	
		eše ₂		ašlum		gi		qānum					
		rope				reed							
		60 m				3 m							

($\bar{L}_{\text{Ur III/OB}}$)

In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

²⁴Adjacent units are no more than a factor of 60 apart, so higher numerals such as K or \diamond are not used.

²⁵TODO

3.3.5 Fractions

TODO

3.4 Early metrology

At first sight, the metrological systems from the Early Dynastic period match the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{Ur III/OB}$ [Fri07, p. 374; DE87, pp. 127, 165]:

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \blacktriangleright \xleftarrow{10} \blacktriangleright \xleftarrow{6} \bullet \xleftarrow{10} \blacktriangleright. \quad (S)$$

Likewise the area system used in the Early Dynastic IIIb period mirrors system $G_{Ur III/OB}$ [Dei22, p. 72; NDE93, p. 63; Fri07, p. 378; Gom16]:

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \odot \xleftarrow{10} \bullet \xleftarrow{3} \blacktriangleright \xleftarrow{6} \blacktriangleright, \quad (G_{ED IIIb})$$

As noted in [And04, p. 4] (see section 3.2), the vertical \uparrow from $S_{Ur III/OB}$ becomes a horizontal \blacktriangleright in system S . It is however far from the only case of such a reallocation of function. The earlier form of System G was [DE87, pp. 141, 165; Fri07, p. 378]:

$$\bullet \xleftarrow{6} \odot \xleftarrow{10} \bullet \xleftarrow{3} \blacktriangleright \xleftarrow{6} \blacktriangleright, \quad (G)$$

Observe that, as noted in [DE87, p. 142], \odot changes meaning from $10\bullet$ in system G to $10\bullet$ in system $G_{ED IIIb}$. System G is used in the Uruk period, but also in the ED I–II period (it is the “area 2” system in [Cha03], whereas $G_{ED IIIb}$ is the “area 1” system).


The length system Early Dynastic IIIb of the state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign \blacktriangleright is generally omitted; in addition, only ten of rods are used; these are equal to one rope, but the sign \blacktriangleright is not written either. Length shorter than one rope are expressed in half-rope using the $1/2$ sign \blacktriangleright (again with no \blacktriangleright), and then in reeds, *with* the sign \blacktriangleright . Effectively, this yields the following factor diagram:



$$\begin{array}{c} \uparrow \xleftarrow{6} \blacktriangleright \xleftarrow{2} \blacktriangleright \xleftarrow{10} \blacktriangleright \blacktriangleright \\ \begin{array}{l} 1 \text{ eše}_2 = 10 \text{ nindan} \\ 1 \text{ rope} = 10 \text{ rods} \\ 60 \text{ m} \end{array} \quad \begin{array}{l} \text{gi} \\ \text{reed} \\ 3 \text{ m} \end{array} \end{array} \quad (L_{ED IIIb})$$

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and others from the same period, such as the ones, areas are expressed in system $G_{ED IIIb}$, with curviform numerals²⁶; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope²⁷, the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and therefore the interpretation of its numerals between systems $G_{ED IIIb}$ and $L_{ED IIIb}$. The sign GAN_2 \blacktriangleright , which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field

²⁶TODO(egg): Note the handful of late Urukagina tablets that start to have cuneiform areas.

²⁷This is the case of the sides of the field in [P020054, obv. ii 2–3].

in [P020054] does not use this suffix. See [Lec20] for many examples with and without .

[Pow87, p. 466] notes that reeds “are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus.” Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in CDLI finds many occurrences: [TODO(egg): cite VS 14, 100, 130, 74, VS25, 83... Just search the downloaded ATF pile for @c.*disz@t\) gi] These expressions use an explicit sign  (counted in multiples of ten) or . This notation—but not its use of curviform numerals—is remarked on in [Lec20, p.290 with note 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. These match the following factor diagrams:

$$\begin{array}{c} \text{D} \xleftarrow{2} \text{U} \xleftarrow{10} \text{A} \\ \hline \text{U} \end{array} \quad (L'_{\text{ED IIIb}})$$

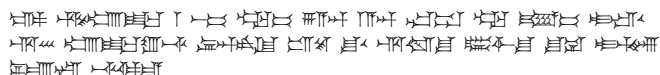
$$\begin{array}{c} \text{D} \xleftarrow{10} \text{D} \xleftarrow{6} \cdot \xleftarrow{2} \text{U} \xleftarrow{28} \text{U} \xleftarrow{10} \text{A} \\ \hline \text{U} \end{array} \quad (L''_{\text{ED IIIb}})$$

Lengths of Early Dynastic IIIb dykes from Nirsu are far from the only numeric expressions that mix curviform and cuneiform numerals: [TODO(egg):Eblaite capacities [P240964] TODO(egg): There are $\eta\epsilon\check{s}_2$ gubar]

$$\begin{array}{c} \text{D} \xleftarrow{10} \text{D} \xleftarrow{10} \cdot \xleftarrow{10} \text{U} \xleftarrow{10} \text{U} \xleftarrow{6} \text{U} \\ \hline \text{U} \end{array} \quad (C_{\text{Ebla}})$$

3.4.1 Use in modern publications

3.5 Non-numeric usage



The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for ‘sixty’. Do you know its reading?

Examenstext A

3.6 Limited benefits of diachronic encoding for numerals

[Composite texts dating back to the period where curved numerals are in use tend to be limited to lexical texts, which do not usually have numbers. When they do, diachronic encoding is prevented by diš-aš distinctions anyway. Administrative texts, which are where numbers are most prominent, are not composite.]

[Diachronic reference works tend to not include numbers, or when they do, to treat them specially (for instance, they are shown at the end of sign lists such as TODO).]

[The overarching goal of having consistent representation for equivalent numeric expressions from different periods is quickly foiled by changes in metrology.]

Note that in [Rom24] [TODO(egg): Cite the GitHub repository], as in many other such analyses, numbers are removed as an early step in processing; these therefore would not benefit from diachrony in the encoding of numeric expressions.

²⁸TODO(egg): Note that one unit may be omitted if the other is present

3.6.1 Compatibility with transliteration

3.7 Compatibility considerations

3.7.1 The case of ŠAR₂

4 Rationale for ED–Uruk numeral unification

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N₉ and N₁₀ from 4(ban₂@c) and 5(ban₂@c).]

6 Characters not included in this proposal

6.1 Missing numerals

(N₁₇, 12N₁₄, etc.) 7(diš tenû)

6.2 Stacking patterns

(... are a mess, vary within Uruk, and are not transliterated/documented by Englund, so let's not go there for now.)

6.3 Matters for higher-level protocols

Rotated bits: <https://cdli.mpiwg-berlin.mpg.de/artifacts/101087>

7 Acknowledgements

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