

Archaic cuneiform numbers

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1 Summary

2 Background

[TODO(egg): Restructure this. The internal references are all garbled.]

The Unicode Standard includes some cuneiform numbers: 𐎶–𐎶𐎵 1–9(diš) and 𐎶–𐎶𐎵𐎶 1–9(aš), 𐎶–𐎶𐎵 1–5(u), 𐎶–𐎶𐎵𐎶 1–9(ḫeš₂), 𐎶–𐎶𐎵 1–5(ḫeš^ou), etc., used in the Sumero-Akkadian Cuneiform script (ISO 15924: Xsux, Script property value long name: Cuneiform).

In the investigation that led to their encoding in Unicode Version 5.0, it was thought appropriate to unify these with the earlier curviform numerals 𐎶–𐎶𐎵 1–9(aš^c = N_1), 𐎶–𐎶𐎵 1–5(u^c = N_{14}), 𐎶–𐎶𐎵𐎶 1–9(ḫeš₂^c = N_{34}), 𐎶–𐎶𐎵 1–5(ḫeš^ou^c = N_{48}), etc. It has now become apparent that a distinction needs to be made for the adequate representation of Early Dynastic (ED) texts and scholarship pertaining to them.

ber over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎴} \xleftarrow{10} \text{𐎳} \quad (S_{Ur III/OB})$$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written 𐎶𐎵𐎶𐎴𐎶𐎳 in the discrete counting system, and 𐎶𐎵𐎶𐎴𐎶𐎳 in the sexagesimal place value system.

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{3} \text{𐎴} \xleftarrow{6} \text{𐎳} \xleftarrow{2} \text{𐎲} \xleftarrow{2} \text{𐎱} \quad (G_{Ur III/OB})$$

1 bur ₃	1 eše ₃	1 iku	1 ubûm
1 būrum	1 eblum	1 ikûm	1800 m ²
6,48 ha	2,16 ha	3600 m ²	

Note that for the range of areas given above³, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *būrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign 𐎶𐎵 functioning as punctuation, but the numerals are tied to the metrology; thus a surface of 5 *būrû* 1 *eblum* 4 *ikû* (100 *ikû*, 36 ha) would be written⁴ 𐎶𐎵𐎶𐎴𐎶𐎳𐎶𐎴𐎶𐎳. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in “88 acres 3 roods 33 perches”. Note also that the same signs are shared between multiple systems, with different relations; the ŠAR₂ sign ◇ is equal to sixty times the U sign < in the area system, but to three hundred and sixty times < in the discrete counting system.

Another such system of note is the one for capacities⁵ [Fri07, p. 376; Rob19],

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎴} \xleftarrow{10} \text{𐎳} \xleftarrow{5} \text{𐎲} \xleftarrow{6} \text{𐎱} \xleftarrow{10} \text{𐎰} \quad (C_{Ur III/OB})$$

1 gur	1 bariga	1 ban ₂	1 sila ₃
1 kurrum	1 parsiktum	1 sūtum	1 qûm
			1 l

where the numerals for ban₂ are 𐎶, 𐎵, 𐎴, 𐎳, and 𐎲, and those for bariga are 𐎶, 𐎵, 𐎴, and 𐎳 (contrast ordinary 𐎶 and 𐎵 otherwise used with !-numerals). As described in [Hue11, p.585 with notes (b) and (f)], the sign GUR 𐎶𐎵, while it is used

³For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mūšarum* (36 m²) written 𐎶𐎵, equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60 𐎶𐎵 (shekels). For areas greater than 3600 *būrû*, the ◇- and 𐎶- numerals are reused with a suffix 𐎶𐎵 (gal, Sumerian: big), as follows [Rob08, p.295 with notes b and c; Fri07, p. 378; Rob19]:

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎴} \xleftarrow{10} \text{𐎳} \xleftarrow{3} \text{𐎲} \xleftarrow{6} \text{𐎱} \xleftarrow{2} \text{𐎰} \xleftarrow{2} \text{𐎯} \xleftarrow{2,5} \text{𐎮} \xleftarrow{10} \text{𐎭} \xleftarrow{6} \text{𐎬} \xleftarrow{10} \text{𐎫}.$$

⁴As in the surface of the field of 𐎶𐎵𐎶𐎴𐎶𐎳 (Apisal) reported on P102305 r. 1.

⁵Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system G_{Ur III/OB} based on a height of one cubit, see [Pow87, p. 488; Rob08, p. 294; Rob19].

[Rom24; JJ24]¹². The diachronic approach is also useful for pedagogical applications¹³.

In this context, the argument was made in [Ando4] as part of ongoing work on the cuneiform encoding¹⁴ that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters; some co-occurrence was known and acknowledged, but considered to be styling rather than plain text.

Indeed, some metrological systems from the Early Dynastic period match the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{\text{Ur III/OB}}$ [Fri07, p. 374; DE87, pp. 127, 165]:

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \text{D} \xleftarrow{10} \text{D} \xleftarrow{6} \bullet \xleftarrow{10} \text{D}, \quad (\text{S})$$

Likewise the area system used in the Early Dynastic IIIb period mirrors system $G_{\text{Ur III/OB}}$ [Fri07, p. 378; Gombert2016]:

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \text{D} \xleftarrow{10} \bullet \xleftarrow{3} \text{D} \xleftarrow{6} \text{D}, \quad (G_{\text{ED IIIb}})$$

TODO(egg): words

Note that in [Rom24] [TODO(egg): Cite the GitHub repository], as in many other such analyses, numbers are removed as an early step in processing; these therefore would not benefit from diachrony in the encoding of numeric expressions.

¹²Attendees may recall the summary given on the third day of UTC #180, as recorded in [Con24]. Other readers may refer to [Svā+24, pp. 242, 148].

¹³For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Capo2].

¹⁴At that time scoped to the repertoire of the Ur III period and later, see [EF03, p. 1], although many disunifications, such as $\text{𒀭} \neq \text{𒀭}$, were informed by Early Dynastic distinctions.

6 Problems with unification: Non-numeric usage

The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'. Do you know its reading?

6.1 The case of ŠAR₂

7 Compatibility with transliteration

8 The necessity of ED-Uruk numeral identification

9 Characters not included in this proposal

9.1 Missing numerals

 $(N_{17}, 12N_{14}, \text{etc.})$

9.2 Stacking patterns

(... are a mess, vary within Uruk, and are not transliterated/documented by Englund, so let's not go there for now.)

References

- [And04] L. Anderson. *Unification of Cuneiform Numbers*. 2004.
UTC: [L2/04-099](#).
- [Cap02] R. Caplice. *Introduction to Akkadian*. 4th ed. Editrice Pontificio Istituto Biblico, 2002.
ISBN: 88-7653-566-7.
- [Con24] P. Constable, ed. *Minutes of UTC Meeting 180* (July 23–25, 2024). July 29, 2024.
UTC: [L2/24-159](#).
- [DE87] P. Damerow and R. K. Englund. “Die Zahlzeichensysteme der archaischen Texte aus Uruk”. In: M. W. Green and H. J. Nissen. *Zeichenliste der archaischen Texte aus Uruk*. Archaische Texte aus Uruk 2. An offprint of this chapter is available at <https://cdli.mpiwg-berlin.mpg.de/files-up/publications/englund1987a.pdf>. Gebr. Mann Verlag, 1987. Chap. 3, pp. 117–165.
- [EF03] M. Everson and K. Feuerherm. *Basic principles for the encoding of Sumero-Akkadian Cuneiform*. May 25, 2003.
UTC: [L2/03-162](#).

- [Fri07] J. Friberg. *A Remarkable Collection of Babylonian Mathematical Texts. Manuscripts in the Schøyen Collection: Cuneiform Texts I*. Sources and Studies in the History of Mathematics and Physical Sciences. Springer, 2007.
ISBN: 978-0-387-34543-7.
- [Fri78] J. Friberg. *A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and Proto-Elamite Semi-Pictographic Inscriptions*. The Third Millenium Roots of Babylonian Mathematics 1. Department of Mathematics, Chalmers University of Technology, 1978.
- [Hue11] J. Huehnergard. *A Grammar of Akkadian*. 3rd ed. Brill, 2011.
ISBN: 978-1-57506-941-8.
- [Jim+23] E. Jiménez, Z. Földi, A. Hättinen, A. Heinrich, T. Mitto, G. Rozzi, I. Khait, J. Laasonen, F. Simonjetz, et al., eds. *electronic Babylonian Library*. 2023-.
eprint: <https://www.ebl.lmu.de/>.
- [JJ24] T. Jauhiainen and H. Jauhiainen. “Advancing Cuneiform Text Dating Through Automatic Analysis”. 69th Rencontre Assyriologique Internationale (July 8–12, 2024). July 11, 2024.
- [Oel22] J. Oelsner. *Der Kodex Hammu-rāpi*. dubsar 4. Zaphon, 2022.
- [Pow87] M. Powell. “Maße und Gewichte”. In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš–Medizin. 1987–1990, pp. 457–530.
- [Rob08] E. Robson. *Mathematics in Ancient Iraq. A Social History*. Princeton University Press, 2008.
ISBN: 978-0-691-09182-2.
- [Rob19] E. Robson. “Oracc metrology guidelines”. In: *Oracc: The Open Richly Annotated Cuneiform Corpus*. Dec. 18, 2019.
eprint: <http://oracc.org/doc/help/editinginf/metrology/metrologicaltables/>.
- [Rob22] E. Robson. “Overview of Metrological Systems”. In: *The Digital Corpus of Cuneiform Mathematical Texts*. 2022.
eprint: <http://oracc.org/dccmt/Metrology/>.
- [Rom24] A. Romach. “The Neo Assyrian Land Sale Documents from Dur-Katlimmu: A Stylometric Analysis of Their Scribal Features”. 69th Rencontre Assyriologique Internationale (July 8–12, 2024). July 10, 2024.
- [Ryk10] B. Rykle. *Mesopotamisches Zeichenlexikon*. Alter Orient und Altes Testament 305. Ugarit-Verlag, 2010.
- [Sch10] W. Schramm. *Akkadische Logogramme*. Göttinger Beiträge zum Alten Orient 5. Universitätsverlag Göttingen, 2010.
ISBN: 9783941875654.
DOI: [10.17875/gup2010-511](https://doi.org/10.17875/gup2010-511).
- [Svä+24] S. Svärd, M. Lorenzon, J. Töyräänvuori, J. Valk, T. Alstola, E. Bennett, R. Uotila, and T. Auranen, eds. *RAI 69 Abstracts*. July 2024.
eprint: https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäiväty_1.pdf.

- [TJV17] S. Tinney, P. Jones, and N. Veldhuis, eds. *The electronic Pennsylvania Sumerian Dictionary*. 2nd ed. 2017–.
eprint: <http://oracc.org/epsd2>.
- [Uni16] The Unicode Consortium. *The Unicode Standard*. Version 16.0.0. The Unicode Consortium, Sept. 10, 2024.
ISBN: 978-1-936213-34-4.
eprint: <https://www.unicode.org/versions/Unicode16.0.0/core-spec/>.
- [UTR56] R. Leroy, ed. *Unicode Cuneiform Sign Lists*. Unicode Technical Report #56.
eprint: <https://www.unicode.org/reports/tr56/>.
- [VT+14] N. Veldhuis, S. Tinney, et al., eds. *Oracc Sign List*. 2014–.
eprint: <http://oracc.org/osl/>.