

# Archaic cuneiform numbers

Robin Leroy, Anshuman Pandey, and Steve Tinney

2024-08-17

## Contents

<b>1</b>	<b>Summary</b>	<b>2</b>
<b>2</b>	<b>Proposed changes to the Standard</b>	<b>3</b>
2.1	Summary of proposed characters . . . . .	3
2.2	Properties . . . . .	3
2.3	Character names list . . . . .	3
2.4	Core specification text . . . . .	3
<b>3</b>	<b>Rationale for curviform–cuneiform disunification</b>	<b>3</b>
3.1	The cuneiform encoding model . . . . .	3
3.2	Arguments for curviform–cuneiform unification . . . . .	3
3.3	Metrology . . . . .	5
3.3.1	The discrete counting system . . . . .	5
3.3.2	The area system . . . . .	6
3.3.3	The capacity system . . . . .	6
3.3.4	The length system . . . . .	7
3.3.5	Fractions . . . . .	8
3.4	Early metrology . . . . .	8
3.4.1	Field lengths in Nirsu . . . . .	8
3.4.2	Dyke lengths in Nirsu . . . . .	9
3.4.3	Grain in Ebla . . . . .	9
3.4.4	Use in modern publications . . . . .	11
3.5	Non-numeric usage . . . . .	13
3.6	Limited benefits of diachronic encoding for numerals . . . . .	13
3.6.1	Compatibility with transliteration . . . . .	13
3.7	Compatibility considerations . . . . .	13
3.7.1	The case of ŠAR <sub>2</sub> . . . . .	13
3.8	Conclusions . . . . .	14
<b>4</b>	<b>Rationale for ED–Uruk numeral unification</b>	<b>14</b>
<b>5</b>	<b>Considerations on individual numeral series</b>	<b>14</b>
<b>6</b>	<b>Characters not included in this proposal</b>	<b>14</b>
6.1	Missing numerals . . . . .	14
6.2	Stacking patterns . . . . .	14
6.3	Matters for higher-level protocols . . . . .	14

Acknowledgements	14
References	14
Artefacts	14
Unicode documents	16
Other documents	16

## 1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script<sup>1</sup> and the proto-cuneiform script<sup>2</sup>. The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals<sup>3</sup> should however *not* be unified with the already-encoded cuneiform numerals<sup>4</sup>. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script\_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
	Future Pcun		Existing Xsux
Non-numeric signs			

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

<sup>1</sup>ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

<sup>2</sup>ISO 15924: Pcun, not yet encoded.

<sup>3</sup>𐎶 1-9(aš<sup>c</sup> = N<sub>1</sub>), 𐎶 1-5(u<sup>c</sup> = N<sub>14</sub>), 𐎶 1-9(𐎶š<sub>2</sub><sup>c</sup> = N<sub>34</sub>), 𐎶 1-5(𐎶š<sub>2</sub>’u<sup>c</sup> = N<sub>48</sub>), etc.

<sup>4</sup>𐎶 1-9(aš), 𐎶 1-5(u), 𐎶 1-9(𐎶š<sub>2</sub>), 𐎶 1-5(𐎶š<sub>2</sub>’u), etc.

## 2 Proposed changes to the Standard

## 2.1 Summary of proposed characters

## 2.2 Properties





### 2.3 Character names list

## 2.4 Core specification text

### 3 Rationale for curviform–cuneiform disunification

TODO(egg): blurb.

### 3.1 The cuneiform encoding model


As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Suruppak as in the character code charts,  later in the third millennium<sup>5</sup>,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists<sup>6</sup> and dictionaries<sup>7</sup>, and of composite texts<sup>8</sup>. By being compatible with similarly diachronic transliteration practice (that is, by avoiding distinctions finer than those made in transliteration), the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]<sup>9</sup>. The diachronic approach is also useful for pedagogical applications<sup>10</sup>.

### 3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding<sup>11</sup> that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

<sup>5</sup>Merging with U+1224E  NI<sub>2</sub>.

<sup>6</sup>Notably [VT+14] and the online edition of [Bor10] in [Jim+23, Signs].


















<sup>7</sup>Notably [TJV17] and the online edition of [Sch10] in [Jim+23, Dictionary].

<sup>8</sup>For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵, as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

<sup>9</sup>Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svä+24, pp. 242, 148].



<sup>10</sup>For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

<sup>11</sup>At that time scoped to the répertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as  $\text{𒀭} \neq \text{𒀭}$ , were informed by Early Dynastic distinctions.

 <sup>12</sup>						
1(NEŠ <sub>2</sub> )	1(U)	1/2(DIŠ)	5(DIŠ <i>tenû</i> )	gi	us <sub>2</sub>	sa <sub>2</sub>
	7.5 (ropes)		5	reed	side	equal
 <sup>13</sup>						
3(U)	6(DIŠ <i>tenû</i> )	gi	saṇ	sa <sub>2</sub>		
3(ropes)	6	reed	front	equal		
	•					
ašag-bi	1(BUR <sub>3</sub> )	1(EŠE <sub>3</sub> )	1(IKU <sup>c</sup> )	1/2(IKU <sup>c</sup> )		
this field						
						
				tug <sub>x</sub> (LAK483)-si-ga-kam <sup>14</sup>		
				deep ploughing		

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as<sup>15</sup>


465 metres, equal lengths  
198 metres, equal widths  
this field: 9, 18 hectares, deeply ploughed




where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from  to -numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere / switch;

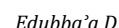
<sup>12</sup>As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large  in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

<sup>13</sup>Note that ED IIIb  numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign  in [P020054] looks more like Ur III .

<sup>14</sup>Transliteration after [Lec20, p. 8].

<sup>15</sup>We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

- ### 3.3 Metrology



As is well known<sup>16</sup> a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Robo8, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters  $\text{I}$ – $\text{III}$ , the multiples of ten (10–50) are  $\text{<}$ – $\text{X}$ , but the other digits 11–59 are sequences  $\text{<}$ – $\text{X}$   $\text{III}$ ; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system*  $\mathcal{S}_{\text{Ur III/OB}}$ , which had different signs for the units  $\text{I}$ – $\text{III}$ , tens  $\text{<}$ – $\text{X}$ , sixties  $\text{I}$ – $\text{III}$  (with larger wedges than the units), six hundreds  $\text{I}$ – $\text{III}$ , three thousand six hundreds  $\text{◇}$ – $\text{◇◇◇◇◇◇}$ , and thirty-six thousands  $\text{◇}$ – $\text{◇◇◇◇◇◇◇◇◇◇}$ .






## 5











This intertwining of units and numerals explains the large number of already-encoded numeral series:

- $\mathbb{I}$ - $\mathbb{W}$  used in  $S_{\text{Ur III/OB}}$  and the SPVS as well as with overt units;
- $\mathbb{L}$ - $\mathbb{X}$  used in  $G_{\text{Ur III/OB}}$ , of which  $\mathbb{L}$ - $\mathbb{X}$  are also used in  $S_{\text{Ur III/OB}}$  and the SPVS as well as with overt units;
- $\mathbb{Y}$ - $\mathbb{Z}$  used in  $S_{\text{Ur III/OB}}$ , and sometimes with overt units;
- $\mathbb{A}$ - $\mathbb{B}$  used in  $S_{\text{Ur III/OB}}$ ;
- $\mathbb{C}$ - $\mathbb{D}$  used in  $S_{\text{Ur III/OB}}$  and  $G_{\text{Ur III/OB}}$ ;
- $\mathbb{E}$ - $\mathbb{F}$  used in  $S_{\text{Ur III/OB}}$  and  $G_{\text{Ur III/OB}}$ ;
- $\mathbb{G}$ - $\mathbb{H}$  used in  $C_{\text{Ur III/OB}}$  as well as with overt units of the weight system;
- $\mathbb{I}$ ,  $\mathbb{J}$ ,  $\mathbb{K}$ ,  $\mathbb{L}$ ,  $\mathbb{M}$  used in  $C_{\text{Ur III/OB}}$ ;
- $\mathbb{N}$ ,  $\mathbb{O}$ ,  $\mathbb{P}$ ,  $\mathbb{Q}$  used in  $C_{\text{Ur III/OB}}$ —note the overlap with  $\mathbb{I}$ - $\mathbb{W}$ ;
- $\mathbb{R}$  and  $\mathbb{S}$  used in  $G_{\text{Ur III/OB}}$ .

### 3.3.4 The length system

 ← 60     ← 10     ← 12     ← 30    .    (*L*<sub>Ur III/OB</sub>)

danna	US <sup>25</sup>	nindan	kuš <sub>3</sub>	šu-si
bērum		nindanum	ammatum	ubānum
league		rod	cubit	finger
10.8 km	cable	6 m	50 cm	17 mm



 $\leftarrow$  30
 
 $\leftarrow$  6
 
 $\leftarrow$  10
 
 $\leftarrow$  2
 
 $\leftarrow$  6
 
 $\leftarrow$  30
 
 ( $\bar{L}_{\text{Ur III/OB}}$ )

eše<sub>2</sub>  
 ašlum  
 rope  
 60 m

gi  
 qānum  
 reed  
 3 m

<sup>23</sup>A larger unit, the *guru*<sub>7</sub> (*karûm*, grain heap), is sometimes used instead, with 𐎠𐎡𐎢𐎣𐎤𐎥𐎦𐎧𐎨𐎩𐎪𐎫𐎬𐎭𐎮𐎯𐎰𐎱𐎲𐎳𐎴𐎵𐎶𐎷𐎸𐎹𐎺𐎻𐎼𐎽𐎾𐎿𐏀𐏁𐏂𐏃𐏄𐏅𐏆𐏇𐏈𐏉𐏊𐏋𐏌𐏍𐏎𐏏𐏐𐏑𐏒𐏓𐏔𐏕𐏖𐏗𐏘𐏙𐏚𐏛𐏜𐏝𐏞𐏟𐏠𐏡𐏢𐏣𐏤𐏥𐏦𐏧𐏨𐏩𐏪𐏫𐏬𐏭𐏮𐏯𐏰𐏱𐏲𐏳𐏴𐏵𐏶𐏷𐏸𐏹𐏺𐏻𐏼𐏽𐏾𐏿𐐀𐐁𐐂𐐃𐐄𐐅𐐆𐐇𐐈𐐉𐐊𐐋𐐌𐐍𐐎𐐏𐐐𐐑𐐒𐐓𐐔𐐕𐐖𐐗𐐘𐐙𐐚𐐛𐐜𐐝𐐞𐐟𐐠𐐡𐐢𐐣𐐤𐐥𐐦𐐧𐐨𐐩𐐪𐐫𐐬𐐭𐐮𐐯𐐰𐐱𐐲𐐳𐐴𐐵𐐶𐐷𐐸𐐹𐐺𐐻𐐼𐐽𐐾𐐿𐑀𐑁𐑂𐑃𐑄𐑅𐑆𐑇𐑈𐑉𐑊𐑋𐑌𐑍𐑎𐑏𐑐𐑑𐑒𐑓𐑔𐑕𐑖𐑗𐑘𐑙𐑚𐑛𐑜𐑝𐑞𐑟𐑠𐑡𐑢𐑣𐑤𐑥𐑦𐑧𐑨𐑩𐑪𐑫𐑬𐑭𐑮𐑯𐑰𐑱𐑲𐑳𐑴𐑵𐑶𐑷𐑸𐑹𐑺𐑻𐑼𐑽𐑾𐑿𐒀𐒁𐒂𐒃𐒄𐒅𐒆𐒇𐒈𐒉𐒊𐒋𐒌𐒍𐒎𐒏𐒐𐒑𐒒𐒓𐒔𐒕𐒖𐒗𐒘𐒙𐒚𐒛𐒜𐒝𐒞𐒟𐒠𐒡𐒢𐒣𐒤𐒥𐒦𐒧𐒨𐒩𐒪𐒫𐒬𐒭𐒮𐒯𐒰𐒱𐒲𐒳𐒴𐒵𐒶𐒷𐒸𐒹𐒺𐒻𐒼𐒽𐒾𐒿𐓀𐓁𐓂𐓃𐓄𐓅𐓆𐓇𐓈𐓉𐓊𐓋𐓌𐓍𐓎𐓏𐓐𐓑𐓒𐓓𐓔𐓕𐓖𐓗𐓘𐓙𐓚𐓛𐓜𐓝𐓞𐓟𐓠𐓡𐓢𐓣𐓤𐓥𐓦𐓧𐓨𐓩𐓪𐓫𐓬𐓭𐓮𐓯𐓰𐓱𐓲𐓳𐓴𐓵𐓶𐓷𐓸𐓹𐓺𐓻𐓼𐓽𐓾𐓿𐔀𐔁𐔂𐔃𐔄𐔅𐔆𐔇𐔈𐔉𐔊𐔋𐔌𐔍𐔎𐔏𐔐𐔑𐔒𐔓𐔔𐔕𐔖𐔗𐔘𐔙𐔚𐔛𐔜𐔝𐔞𐔟𐔠𐔡𐔢𐔣𐔤𐔥𐔦𐔧𐔨𐔩𐔪𐔫𐔬𐔭𐔮𐔯𐔰𐔱𐔲𐔳𐔴𐔵𐔶𐔷𐔸𐔹𐔺𐔻𐔼𐔽𐔾𐔿𐕀𐕁𐕂𐕃𐕄𐕅𐕆𐕇𐕈𐕉𐕊𐕋𐕌𐕍𐕎𐕏𐕐𐕑𐕒𐕓𐕔𐕕𐕖𐕗𐕘𐕙𐕚𐕛𐕜𐕝𐕞𐕟𐕠𐕡𐕢𐕣𐕤𐕥𐕦𐕧𐕨𐕩𐕪𐕫𐕬𐕭𐕮𐕯𐕰𐕱𐕲𐕳𐕴𐕵𐕶𐕷𐕸𐕹𐕺𐕻𐕼𐕽𐕾𐕿𐖀𐖁𐖂𐖃𐖄𐖅𐖆𐖇𐖈𐖉𐖊𐖋𐖌𐖍𐖎𐖏𐖐𐖑𐖒𐖓𐖔𐖕𐖖𐖗𐖘𐖙𐖚𐖛𐖜𐖝𐖞𐖟𐖠𐖡𐖢𐖣𐖤𐖥𐖦𐖧𐖨𐖩𐖪𐖫𐖬𐖭𐖮𐖯𐖰𐖱𐖲𐖳𐖴𐖵𐖶𐖷𐖸𐖹𐖺𐖻𐖼𐖽𐖾𐖿𐗀𐗁𐗂𐗃𐗄𐗅𐗆𐗇𐗈𐗉𐗊𐗋𐗌𐗍𐗎𐗏𐗐𐗑𐗒𐗓𐗔𐗕𐗖𐗗𐗘𐗙𐗚𐗛𐗜𐗝𐗞𐗟𐗠𐗡𐗢𐗣𐗤𐗥𐗦𐗧𐗨𐗩𐗪𐗫𐗬𐗭𐗮𐗯𐗰𐗱𐗲𐗳𐗴𐗵𐗶𐗷𐗸𐗹𐗺𐗻𐗼𐗽𐗾𐗿𐘀𐘁𐘂𐘃𐘄𐘅𐘆𐘇𐘈𐘉𐘊𐘋𐘌𐘍𐘎𐘏𐘐𐘑𐘒𐘓𐘔𐘕𐘖𐘗𐘘𐘙𐘚𐘛𐘜𐘝𐘞𐘟𐘠𐘡𐘢𐘣𐘤𐘥𐘦𐘧𐘨𐘩𐘪𐘫𐘬𐘭𐘮𐘯𐘰𐘱𐘲𐘳𐘴𐘵𐘶𐘷𐘸𐘹𐘺𐘻𐘼𐘽𐘾𐘿𐙀𐙁𐙂𐙃𐙄𐙅𐙆𐙇𐙈𐙉𐙊𐙋𐙌𐙍𐙎𐙏𐙐𐙑𐙒𐙓𐙔𐙕𐙖𐙗𐙘𐙙𐙚𐙛𐙜𐙝𐙞𐙟𐙠𐙡𐙢𐙣𐙤𐙥𐙦𐙧𐙨𐙩𐙪𐙫𐙬𐙭𐙮𐙯𐙰𐙱𐙲𐙳𐙴𐙵𐙶𐙷𐙸𐙹𐙺𐙻𐙼𐙽𐙾𐙿𐚀𐚁𐚂𐚃𐚄𐚅𐚆𐚇𐚈𐚉𐚊𐚋𐚌𐚍𐚎𐚏𐚐𐚑𐚒𐚓𐚔𐚕𐚖𐚗𐚘𐚙𐚚𐚛𐚜𐚝𐚞𐚟𐚠𐚡𐚢𐚣𐚤𐚥𐚦𐚧𐚨𐚩𐚪𐚫𐚬𐚭𐚮𐚯𐚰𐚱𐚲𐚳𐚴𐚵𐚶𐚷𐚸𐚹𐚺𐚻𐚼𐚽𐚾𐚿𐛀𐛁𐛂𐛃𐛄𐛅𐛆𐛇𐛈𐛉𐛊𐛋𐛌𐛍𐛎𐛏𐛐𐛑𐛒𐛓𐛔𐛕𐛖𐛗𐛘𐛙𐛚𐛛𐛜𐛝𐛞𐛟𐛠𐛡𐛢𐛣𐛤𐛥𐛦𐛧𐛨𐛩𐛪𐛫𐛬𐛭𐛮𐛯𐛰𐛱𐛲𐛳𐛴𐛵𐛶𐛷𐛸𐛹𐛺𐛻𐛼𐛽𐛾𐛿𐜀𐜁𐜂𐜃𐜄𐜅𐜆𐜇𐜈𐜉𐜊𐜋𐜌𐜍𐜎𐜏𐜐𐜑𐜒𐜓𐜔𐜕𐜖𐜗𐜘𐜙𐜚𐜛𐜜𐜝𐜞𐜟𐜠𐜡𐜢𐜣𐜤𐜥𐜦𐜧𐜨𐜩𐜪𐜫𐜬𐜭𐜮𐜯𐜰𐜱𐜲𐜳𐜴𐜵𐜶𐜷𐜸𐜹𐜺𐜻𐜼𐜽𐜾𐜿𐝀𐝁𐝂𐝃𐝄𐝅𐝆𐝇𐝈𐝉𐝊𐝋𐝌𐝍𐝎𐝏𐝐𐝑𐝒𐝓𐝔𐝕𐝖𐝗𐝘𐝙𐝚𐝛𐝜𐝝𐝞𐝟𐝠𐝡𐝢𐝣𐝤𐝥𐝦𐝧𐝨𐝩𐝪𐝫𐝬𐝭𐝮𐝯𐝰𐝱𐝲𐝳𐝴𐝵𐝶𐝷𐝸𐝹𐝺𐝻𐝼𐝽𐝾𐝿𐞀𐞁𐞂𐞃𐞄𐞅𐞆𐞇𐞈𐞉𐞊𐞋𐞌𐞍𐞎𐞏𐞐𐞑𐞒𐞓

<sup>25</sup>TODO

### 3.3.5 Fractions

### 3.4 Early metrology

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \odot \xleftarrow{10} \triangleright \xleftarrow{6} \bullet \xleftarrow{10} \triangleright. \quad (S)$$

Diagram of the GED IIIb system showing a sequence of components with distances in AU: a central star, a planet at 10 AU, a gas giant at 6 AU, a star at 10 AU, a planet at 3 AU, a gas giant at 6 AU, and a final planet.

$$\bullet \xleftarrow{6} \odot \xleftarrow{10} \bullet \xleftarrow{3} \blacksquare \xleftarrow{6} \triangleright, \quad (G)$$

### 3.4.1 Field lengths in Nirsu

$\mathbf{I} \xleftarrow{6} \mathbf{I} \xleftarrow{2} \mathbf{I} \xleftarrow{10} \mathbf{I} \xleftarrow{28} \mathbf{I}$

1 eše<sub>2</sub>=10 nindan  
 1 rope=10 rods  
 60 m

gi  
 reed  
 3 m

(L<sub>ED IIIb</sub>)

<sup>28</sup>Note that the reeds are counted using *tenû* numerals, 𐎧, 𐎡, 𐎢, etc.

8



### 3.4.2 Dyke lengths in Nirsu

[illegible][illegible]

<sup>30</sup>This is the case of the sides of the field in [P020054, obv. ii 2–3].






<sup>32</sup>CDLI only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the  $\text{𒀭} \text{𒀵} \text{𒌦}$  is not visible. Lecompte notes that the copy is faithful; indeed another  $\text{𒀭} \text{𒀵} \text{𒌦}$  can be seen both on the copy and the photo on obv. 2, 2.

<sup>33</sup>From copy.

<sup>34</sup>TODO Cite also DP 568, the one with  and  even though it has no reeds.

9

The system of grain<sup>36</sup> capacities in Ebla uses the following units<sup>37</sup>:

  $\xleftarrow{2}$    $\xleftarrow{\frac{5}{2}}$    $\xleftarrow{4}$    $\xleftarrow{6}$  




gu<sub>2</sub>-bar      ba-ri<sub>2</sub>-zu      nin<sub>4</sub>      nin<sub>2</sub>-sagšu      an-zam<sub>x</sub>

The  $\text{𐤀𐤁}$  and  $\text{𐤁𐤁𐤁}$  are generally counted using curviform numerals, and the smaller units using cuneiform  $\text{𐤁}$  numerals. Indeed, a search on [Mil+07] for co-occurrences of  $\text{𐤁𐤁}$  with either of  $\text{𐤀𐤁}$  or  $\text{𐤁𐤁𐤁}$  finds the following expressions<sup>38</sup>:

1. [P240532, verso 4, 9] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
2. [P240548, verso 1, 1] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
3. [P240655, recto 7, 9] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
4. [P240579, verso 4, 3] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
5. [P240675, verso 2, 2] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
6. [P240609, verso 3, 1] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
7. [P240533, recto 3, 3] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
8. [P240697, recto 1, 5] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
9. [P240653, recto 6, 2] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
10. [P240654, recto 2, 6] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
11. [P240531, recto 1, 8] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
12. [P241708, recto 1, 1] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖
13. [P241904, recto 1, 1] 𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖𪛗𪛖

Note that higher numbers of  $\text{𒀭}$  are expressed in hundreds (*mi-at*  $\text{𒈹𒄀}$ ) and then thousands (*li-im*  $\text{𒊕𒁺𒀭}$ ), as is typical in Ebla [Arc15, p. 33], e.g., in [P240532, verso 2, 3],  $\text{𒄠𒋼𒃶𒉈𒅗𒄣𒂍𒀭𒇻𒀭𒆸𒄣𒀭}$  (100 + 60 + 30 + 5 = 195  $\text{𒀭}$  of grain).

<sup>36</sup>Liquid capacities use a different system [Arc15, p. 229 with note 12]:




 $\xleftarrow{30}$ 

 $\xleftarrow{6}$ 


At a glance it seems that  $\bowtie$  are counted with cuneiform numerals and higher units with curviform ones, thus

$\frac{5}{3}$  6 10 3 10 6 \*

but we have not investigated this thoroughly.

<sup>37</sup>TODO mention the other one citing Chambon and the footnote in Archi

<sup>38</sup>We cite here only one attestation per tablet; most tablets contain several expressions mixing cursive  and larger with cuneiform  and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked<sup>39</sup> on a photograph (from EbDA unless noted otherwise).

<sup>39</sup>As we will see in Section 3.6.1, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

<sup>40</sup>ba-ri<sub>2</sub>-zu<sub>2</sub>, a variant spelling.

<sup>41</sup>Short for  $\Box \vdash$ .

<sup>42</sup>Note the omitted  $\begin{smallmatrix} \square & \diamond \\ \hline \square & \diamond \end{smallmatrix} \perp$ .

<sup>43</sup>Instead of the expected  $\mathbb{P}\{X \in \mathbb{Z}\}$ .

<sup>44</sup>  not legible on the EbDA photo.

<sup>45</sup>From CDLI photo.

<sup>46</sup>From photo in [Arc89, p. 6].

<sup>47</sup>Laid out as  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ ; on stacking patterns see Section 6.2.



formed by only two signs  $\Upsilon$  and  $\triangleleft$ , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60<sup>2</sup>, 60<sup>3</sup>, 1/60, 1/60<sup>2</sup>, etc. are written in the same way, with the vertical wedge  $\Upsilon$ . The third feature concerns the exact function of

Figure 4: TODO [Cha12, p. 58]

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with:  $\Upsilon\Upsilon\Upsilon\bullet\bullet\bullet$  but never with the repetition  $\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon$ .

Figure 5: TODO [Cha12, p. 59]<sup>50</sup>

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

$\Upsilon\Upsilon\Upsilon$  še bar  $\Upsilon$  ba-rí-zu  
‘3 gubar (capacity units) and 1 parisu’.

Figure 6: TODO [Cha12, p. 61]

This is particularly true of the signs  $\Upsilon$ ,  $\Upsilon$ ,  $\Upsilon$  and  $\Upsilon$ , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written  $\Upsilon$  in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign  $\Upsilon$  is most often associated with the *parisu* measure, while the signs  $\Upsilon$ ,  $\Upsilon$ ,  $\Upsilon$  and  $\Upsilon$  refer to 1, 2, 3,

Figure 7: TODO [Cha12, p. 64]

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g.,  $\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon$  transliterated as 2(šar<sub>2</sub>) 1(geš’u) 3(u), means 2 × 3600 + 1 × 600 + 3 × 10). The system is based on an alternation of factors ten and

Figure 8: TODO

might think of one fabric and a half,<sup>11</sup> but the presence of notations with “2 $\Upsilon$  2 $\Upsilon$ ”, “3 $\Upsilon$  3 $\Upsilon$ ”, and “6 $\Upsilon$  6 $\Upsilon$ ” (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag’s weight measurement system.<sup>12</sup> The notation “1 $\Upsilon$  gada” in o. ii 1 and r. vi 1, along with the total of “39



Fig. 1. Combinations of numerals attested in Š. 742.

Figure 9: Discussion of the contrast between  $\Upsilon$  and  $\Upsilon$  numerals in [Gor23, p. 162].

Y

### 3.8 Conclusions

## 4 Rationale for ED-Uruk numeral unification

## 5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N<sub>9</sub> and N<sub>10</sub> from 4(ban<sub>2</sub>@c) and 5(ban<sub>2</sub>@c).]

## 6 Characters not included in this proposal

### 6.1 Missing numerals

(N<sub>17</sub>, 12N<sub>14</sub>, etc.) 7(diš tenû)

### 6.2 Stacking patterns

(... are a mess, vary within Uruk, and are not transliterated/documented by Englund, so let's not go there for now.)

### 6.3 Matters for higher-level protocols

Rotated bits: <https://cdli.mpiwg-berlin.mpg.de/artifacts/101087>

## Acknowledgements

TODO(egg): Something about the Vanséveren fonts

## References

### Artefacts

- [P020054] VAT 4731. [För16, 40 p.14]. Vorderasiatisches Museum.  
CDLI: [P020054](#).
- [P020129] VAT 04713. Vorderasiatisches Museum.  
CDLI: [P020129](#).  
ORACC: [epsd2/corpus/P020129](#).
- [P102305] X.3.139. Michael C. Carlos Museum, Emory University.  
CDLI: [P102305](#).
- [P221266] AO 13825. Musée du Louvre.  
CDLI: [P221266](#).  
ORACC: [epsd2/corpus/P221266](#).  
Louvre Collections: [ark:/53355/cl010138527](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-63868-p0101-3).
- [P221291] AO 13850. Musée du Louvre.  
CDLI: [P221291](#).  
ORACC: [epsd2/corpus/P221291](#).

- [P221305] AO 13864. Musée du Louvre.  
CDLI: [P221305](#).  
ORACC: [epsd2/corpus/P221305](#).
- [P232278] *Gudea E*. AO 6. Musée du Louvre.  
CDLI: [P232278](#).  
ORACC: [etcstri/Q001544](#).
- [P232280] *Gudea G*. AO 7. Musée du Louvre.  
CDLI: [P232280](#).  
ORACC: [etcstri/Q001546](#).
- [P240531] TM.75.G.00265. Idlib, Syria: National Museum of Syria.  
CDLI: [P240531](#).  
EbDA: [1415](#).
- [P240532] TM.75.G.00266. Idlib, Syria: National Museum of Syria.  
CDLI: [P240532](#).  
EbDA: [1324](#).
- [P240533] TM.75.G.00267. Idlib, Syria: National Museum of Syria.  
CDLI: [P240533](#).  
EbDA: [1379](#).
- [P240548] TM.75.G.00302. Idlib, Syria: National Museum of Syria.  
CDLI: [P240548](#).  
EbDA: [1350](#).
- [P240579] TM.75.G.00341. Idlib, Syria: National Museum of Syria.  
CDLI: [P240579](#).  
EbDA: [1364](#).
- [P240609] TM.75.G.00440. Idlib, Syria: National Museum of Syria.  
CDLI: [P240609](#).  
EbDA: [1378](#).
- [P240653] TM.75.G.00535. Idlib, Syria: National Museum of Syria.  
CDLI: [P240653](#).  
EbDA: [1382](#).
- [P240654] TM.75.G.00536. Idlib, Syria: National Museum of Syria.  
CDLI: [P240654](#).  
EbDA: [1383](#).
- [P240655] TM.75.G.00537. Idlib, Syria: National Museum of Syria.  
CDLI: [P240655](#).  
EbDA: [1358](#).
- [P240675] TM.75.G.00557. Idlib, Syria: National Museum of Syria.  
CDLI: [P240675](#).  
EbDA: [1371](#).
- [P240697] TM.75.G.00579. Idlib, Syria: National Museum of Syria.  
CDLI: [P240697](#).  
EbDA: [1381](#).
- [P241708] TM.75.G.02143. Idlib, Syria: National Museum of Syria.  
CDLI: [P241708](#).  
EbDA: [3173](#).

- [P241904] TM.75.G.02346. [Arc89, p. 6]. Idlib, Syria: National Museum of Syria.  
CDLI: [P241904](#).  
EbDA: [3183](#).
- [P242293] TM.75.G.03125. Idlib, Syria: National Museum of Syria.  
CDLI: [P242293](#).  
EbDA: [217](#).
- [P249253] *Code de Hammurabi*. Sb 8. Musée du Louvre.  
CDLI: [P249253](#).

## Unicode documents

- [L2/03-162] M. Everson and K. Feuerherm. *Basic principles for the encoding of Sumero-Akkadian Cuneiform*. 25th May 2003.  
UTC: [L2/03-162](#).
- [L2/03-393R] M. Everson, K. Feuerherm and S. Tinney. *Preliminary proposal to encode the Cuneiform script in the SMP of the UCS*. 3rd Nov. 2003.  
UTC: [L2/03-393R](#).
- [L2/04-036] M. Everson, K. Feuerherm and S. Tinney. *Revised proposal to encode the Cuneiform script in the SMP of the UCS*. 29th Jan. 2004.  
UTC: [L2/04-036](#).
- [L2/04-099] L. Anderson. *Unification of Cuneiform Numbers*. 2004.  
UTC: [L2/04-099](#).
- [L2/04-189] M. Everson, K. Feuerherm and S. Tinney. *Final proposal to encode the Cuneiform script in the SMP of the UCS*. 8th June 2004.  
UTC: [L2/04-189](#).
- [L2/24-159] P. Constable, ed. *Minutes of UTC Meeting 180* (23rd–25th July 2024). 29th July 2024.  
UTC: [L2/24-159](#).
- [Uni16] The Unicode Consortium. *The Unicode Standard*. Version 16.0.0. The Unicode Consortium, 10th Sept. 2024.  
ISBN: 978-1-936213-34-4.  
eprint: <https://www.unicode.org/versions/Unicode16.0.0/core-spec/>.
- [UTR56] R. Leroy, ed. *Unicode Cuneiform Sign Lists*. Unicode Technical Report #56.  
eprint: <https://www.unicode.org/reports/tr56/>.

## Other documents

- [Arc15] A. Archi. *Ebla and Its Archives. Texts, History, and Society*. Studies in ancient Near Eastern records 7. Walter de Gruyter, 2015.  
ISBN: 978-1-61451-716-0.  
DOI: [10.1515/9781614517887](https://doi.org/10.1515/9781614517887).
- [Arc89] A. Archi. “Tables de comptes eblaïtes”. In: *Revue d’assyriologie et d’archéologie orientale* 83.1 (1989). Ed. by P. Amiet and P. Garelli, pp. 1–6. ISSN: 0373-6032.



- [Bor10] R. Borger. *Mesopotamisches Zeichenlexikon*. Alter Orient und Altes Testament 305. Ugarit-Verlag, 2010.
- [Cap02] R. Caplice. *Introduction to Akkadian*. 4th ed. Editrice Pontificio Istituto Biblico, 2002.  
ISBN: 88-7653-566-7.
- [Cha03] G. Chambon. “Archaic Metrological Systems from Ur”. In: *Cuneiform Digital Library Journal* 2003.5 (23rd Dec. 2003). ISSN: 1540-8779.  
eprint: [http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003\\_005.html](http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003_005.html).
- [Cha12] G. Chambon. “Numeracy and Metrology”. In: *The Oxford Handbook of Cuneiform Culture*. Ed. by K. Radner and E. Robson. Oxford University Press, 18th Sept. 2012, pp. 51–67.  
ISBN: 9780199557301.  
DOI: [10.1093/oxfordhb/9780199557301.013.0003](https://doi.org/10.1093/oxfordhb/9780199557301.013.0003).
- [DE87] P. Damerow and R. K. Englund. “Die Zahlzeichensysteme der archaischen Texte aus Uruk”. In: M. W. Green and H. J. Nissen. *Zeichenliste der archaischen Texte aus Uruk*. Archaische Texte aus Uruk 2. An offprint of this chapter is available at <https://cdli.mpiwg-berlin.mpg.de/files-up/publications/englund1987a.pdf>. Gebr. Mann Verlag, 1987. Chap. 3, pp. 117–165.
- [Dei22] A. Deimel. *Liste der archaischen Keilschriftzeichen von Fara*. Wissenschaftliche Veröffentlichungen der Deutschen Orient-Gesellschaft 40. J. C. Hinrichs’sche Buchhandlung, 1922.
- [Eng98] R. K. Englund. “Texts from the Late Uruk Period”. In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Vol. 160. Orbis Biblicus et Orientalis 1. 1998, pp. 13–233.  
ISBN: 3-7278-1166-8.
- [För16] W. Förtsch. *Altbabylonische Wirtschaftstexte aus der Zeit Lugalanda’s und Urukagina’s*. Vorderasiatische Schriftdenkmäler der Königlichen Museen zu Berlin 14. J. C. Hinrichs, 1916.
- [Fri07] J. Friberg. *A Remarkable Collection of Babylonian Mathematical Texts. Manuscripts in the Schøyen Collection: Cuneiform Texts I*. Sources and Studies in the History of Mathematics and Physical Sciences. Springer, 2007.  
ISBN: 978-0-387-34543-7.
- [Fri78] J. Friberg. *A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and Proto-Elamite Semi-Pictographic Inscriptions*. The Third Millenium Roots of Babylonian Mathematics 1. Department of Mathematics, Chalmers University of Technology, 1978.
- [Gom16] B. Gombert. “ED IIIb metrology: texts from Lagaš”. In: *CDLI:wiki. A Library of Knowledge of the Cuneiform Digital Library Initiative*. 12th Apr. 2016.  
eprint: [https://cdli.ox.ac.uk/wiki/doku.php?id=ed\\_iii\\_metrological\\_systems](https://cdli.ox.ac.uk/wiki/doku.php?id=ed_iii_metrological_systems).
- [Gor23] F. Gori. “On Lapis Lazuli and Linen in Šuruppak Texts. An Analysis Through the Lens of Ebla Studies”. In: *Studia Eblaitica* 9 (2023), pp. 160–166. ISSN: 2364-7124.

- [Hue11] J. Huehnergard. *A Grammar of Akkadian*. 3rd ed. Brill, 2011. ISBN: 978-1-57506-941-8.
- [Jim+23] E. Jiménez, Z. Földi, A. Härtinen, A. Heinrich, T. Mitto, G. Rozzi, I. Khait, J. Laasonen, F. Simonjetz et al., eds. *electronic Babylonian Library*. 2023–. eprint: <https://www.ebl.lmu.de/>.
- [JJ24] T. Jauhiainen and H. Jauhiainen. “Advancing Cuneiform Text Dating Through Automatic Analysis”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 11th July 2024 14:00.
- [Kre98] M. Krebern timer. “Die Texte aus Fāra und Tell Abū Šalābīḥ”. In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 235–427. ISBN: 3-7278-1166-8.
- [Lec12] C. Lecompte. “Des chiffres et des digues: à propos de deux textes présargoniques de Ġirsu et d’une notation numérique inhabituelle”. In: *Altorientalische Forschungen* 39.1 (Dec. 2012), pp. 81–86. DOI: [10.1524/aof.2012.0006](https://doi.org/10.1524/aof.2012.0006).
- [Lec20] C. Lecompte. “The Measurement of Fields During the Pre-sargonic Period”. In: *Mathematics, Administrative and Economic Activities in Ancient Worlds*. Ed. by C. Michel and K. Chemla. Why the Sciences of the Ancient World Matter 5. Springer, 2020.
- [Mil+07] L. Milano, M. Maiocchi, F. Di Filippo, R. Orsini, E. Scarpa, M. Surdi et al., eds. *Ebla Digital Archives*. 2007–. eprint: <http://ebda.cnr.it/>.
- [NDE93] H. J. Nissen, P. Damerow and R. K. Englund. *Archaic Bookkeeping. Early Writing and Techniques of Economic Administration in the Ancient Near East*. Trans. by P. Larsen. The University of Chicago Press, 1993. ISBN: 0-226-58659-6.
- [Oel22] J. Oelsner. *Der Kodex Ḥammu-rāpi*. dubsar 4. Zaphon, 2022.
- [Pow87] M. Powell. “Maße und Gewichte”. In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš-Medizin. 1987–1990, pp. 457–530.
- [Rob08] E. Robson. *Mathematics in Ancient Iraq. A Social History*. Princeton University Press, 2008. ISBN: 978-0-691-09182-2.
- [Rob19] E. Robson. “Oracc metrology guidelines”. In: *Oracc: The Open Richly Annotated Cuneiform Corpus*. 18th Dec. 2019. eprint: <http://oracc.org/doc/help/editinginf/metrology/metrologicaltables/>.
- [Rob22] E. Robson. “Overview of Metrological Systems”. In: *The Digital Corpus of Cuneiform Mathematical Texts*. 2022. eprint: <http://oracc.org/dccmt/Metrology/>.

- [Rom24] A. Romach. “The Neo Assyrian Land Sale Documents from Dur-Katlimmu: A Stylometric Analysis of Their Scribal Features”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 10th July 2024 12:00.
- [Sch10] W. Schramm. *Akkadische Logogramme*. Göttinger Beiträge zum Alten Orient 5. Universitätsverlag Göttingen, 2010.  
ISBN: 978-3-941875-65-4.  
DOI: [10.17875/gup2010-511](https://doi.org/10.17875/gup2010-511).
- [Svä+24] S. Svärd, M. Lorenzon, J. Töyräänvuori, J. Valk, T. Alstola, E. Bennett, R. Uotila and T. Auranne, eds. *RAI 69 Abstracts*. July 2024.  
eprint: [https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäiväty\\_1.pdf](https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäiväty_1.pdf).
- [TJV17] S. Tinney, P. Jones and N. Veldhuis, eds. *The electronic Pennsylvania Sumerian Dictionary*. 2nd ed. 2017–.  
eprint: <http://oracc.org/epsd2>.
- [VT+14] N. Veldhuis, S. Tinney et al., eds. *Oracc Sign List*. 2014–.  
eprint: <http://oracc.org/osl/>.