

Archaic cuneiform numbers

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Contents

1	Summary	2
2	Proposed changes to the Standard	3
2.1	Summary of proposed characters	3
2.2	Properties	3
2.3	Character names list	3
2.4	Core specification text	3
3	Rationale for curviform–cuneiform disunification	3
3.1	The cuneiform encoding model	3
3.2	Arguments for curviform–cuneiform unification	3
3.3	Metrology	5
3.3.1	The discrete counting system	5
3.3.2	The area system	6
3.3.3	The capacity system	6
3.3.4	The length system	7
3.3.5	Fractions	8
3.4	Early metrology	8
3.4.1	Field lengths in Nirsu	8
3.4.2	Dyke lengths in Nirsu	9
3.4.3	Grain in Ebla	9
3.4.4	Use in modern publications	10
3.5	Non-numeric usage	11
3.6	Limited benefits of diachronic encoding for numerals	11
3.6.1	Compatibility with transliteration	12
3.7	Compatibility considerations	12
3.7.1	The case of ŠAR ₂	12
4	Rationale for ED–Uruk numeral unification	12
5	Considerations on individual numeral series	12
6	Characters not included in this proposal	12
6.1	Missing numerals	12
6.2	Stacking patterns	12
6.3	Matters for higher-level protocols	12

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
Non-numeric signs	Future Pcun	Existing Xsux	

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

7 Acknowledgements

12

1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script¹ and the proto-cuneiform script². The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

¹ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

²ISO 15924: Pcun, not yet encoded.

³𐎶 1-9(aš^c = N_1), 𐎷 1-5(u^c = N_{14}), 𐎸 1-9(ḫeš₂^c = N_{34}), 𐎹 1-5(ḫeš^cu^c = N_{48}), etc.

⁴𐎶 1-9(aš), 𐎷 1-5(u), 𐎸 1-9(ḫeš₂), 𐎹 1-5(ḫeš^cu), etc.

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties





2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

TODO(egg): blurb.

3.1 The cuneiform encoding model


As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts,  later in the third millennium⁵,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists⁶ and dictionaries⁷, and of composite texts⁸. By being compatible with similarly diachronic transliteration practice (that is, by avoiding distinctions finer than those made in transliteration), the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]⁹. The diachronic approach is also useful for pedagogical applications¹⁰.

3.2 Arguments for curviform–cuneiform unification


In this context, the argument was made in [Ando4], as part of discussion of the cuneiform encoding¹¹ that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [Ando4, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

⁵Merging with U+1224E  NI₂.



⁶Notably [VT+14] and the online edition of [Bor10] in [Jim+23, Signs].

















⁷Notably [TJV17] and the online edition of [Sch10] in [Jim+23, Dictionary].





⁸For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

⁹Attendees may recall the summary given on the third day of UTC #180, as recorded in [Con24]. Other readers may refer to [Svā+24, pp. 242, 148].

¹⁰For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

¹¹At that time scoped to the repertoire of the Ur III period and later, see [EF03, p. 1], although many disunifications, such as  ≠ , were informed by Early Dynastic distinctions.



 ¹²						
1(NĖŠ ₂)	1(U)	1/2(DIŠ)	5(DIŠ <i>tenû</i>)	gi	us ₂	sa ₂
	7.5 (ropes)		5	reed	side	equal
 ¹³						
3(U)	6(DIŠ <i>tenû</i>)	gi	saṇ	sa ₂		
3(ropes)	6	reed	front	equal		
	•					
ašag-bi	1(BUR ₃)	1(EŠE ₃)	1(IKU ^c)	1/2(IKU ^c)		
this field						

tug_x(LAK483)-si-ga-kam¹⁴
deep ploughing



The argument made in [Ando4, p. 4] is that this is comparable to a stylistic distinction such as¹⁵


465 metres, equal lengths
198 metres, equal widths
this field: 9, 18 hectares, deeply ploughed




where the numerals have the same structure ([Ando4] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [Ando4, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from  to -numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [EFT03], the curviform numerals were therefore removed from [EFT04b] and [EFT04a], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere / switch;

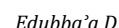
¹²As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large  in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

¹³Note that ED IIIb  numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign  in [P020054] looks more like Ur III .

¹⁴Transliteration after [Lec20, p. 8].

¹⁵We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

- ### 3.3 Metrology



As is well known¹⁶ a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Robo8, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters $\text{I} - \text{III}$, the multiples of ten (10–50) are $\text{<} - \text{X}$, but the other digits 11–59 are sequences $\text{<} \text{I} - \text{X}$; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units $\text{I} - \text{III}$, tens $\text{<} - \text{X}$, sixties $\text{I} - \text{X}$ (with larger wedges than the units), six hundreds $\text{K} - \text{XX}$, three thousand six hundred $\diamond - \diamond\diamond\diamond\diamond\diamond\diamond$, and thirty-six thousands $\diamond - \diamond\diamond\diamond\diamond\diamond\diamond$.






$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \nmid \xleftarrow{10} \nmid \xleftarrow{6} \prec \xleftarrow{10} \nmid \quad (S_{\text{Ur III/OB}})$$

¹⁷These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.








This intertwining of units and numerals explains the large number of already-encoded numeral series:

- \mathbb{I} - \mathbb{W} used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- \mathbb{L} - \mathbb{X} used in $G_{\text{Ur III/OB}}$, of which \mathbb{L} - \mathbb{X} are also used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- \mathbb{Y} - \mathbb{Z} used in $S_{\text{Ur III/OB}}$, and sometimes with overt units;
- \mathbb{A} - \mathbb{B} used in $S_{\text{Ur III/OB}}$;
- \mathbb{C} - \mathbb{D} used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \mathbb{E} - \mathbb{F} used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \mathbb{G} - \mathbb{H} used in $C_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
- \mathbb{I} , \mathbb{J} , \mathbb{K} , \mathbb{L} , \mathbb{M} used in $C_{\text{Ur III/OB}}$;
- \mathbb{N} , \mathbb{O} , \mathbb{P} , \mathbb{Q} used in $C_{\text{Ur III/OB}}$ —note the overlap with \mathbb{I} - \mathbb{W} ;
- \mathbb{R} and \mathbb{S} used in $G_{\text{Ur III/OB}}$.

3.3.4 The length system

 ← 60  ← 10  ← 12  ← 30 . (*L*_{Ur III/OB})

danna	US ²⁵	nindan	kuš ₃	šu-si
bērum		nindanum	ammatum	ubānum
league		rod	cubit	finger
10.8 km	cable	6 m	50 cm	17 mm

30 ← 6 ← 10 ← 2 ← 6 ← 30 ← 1

eše₂ qānum
 rope reed
 60 m 3 m

(L_{Ur III/OB})

²³A larger unit, the *guru*₇ (*karûm*, grain heap), is sometimes used instead, with 𐎠𐎡𐎢𐎣𐎤𐎥𐎦𐎧𐎨𐎩𐎪𐎫𐎬𐎭𐎮𐎯𐎰𐎱𐎲𐎳𐎴𐎵𐎶𐎷𐎸𐎹𐎺𐎻𐎼𐎽𐎾𐎿𐏀𐏁𐏂𐏃𐏄𐏅𐏆𐏇𐏈𐏉𐏊𐏋𐏌𐏍𐏎𐏏𐏐𐏑𐏒𐏓𐏔𐏕𐏖𐏗𐏘𐏙𐏚𐏛𐏜𐏝𐏞𐏟𐏠𐏡𐏢𐏣𐏤𐏥𐏦𐏧𐏨𐏩𐏪𐏫𐏬𐏭𐏮𐏯𐏰𐏱𐏲𐏳𐏴𐏵𐏶𐏷𐏸𐏹𐏺𐏻𐏼𐏽𐏾𐏿𐐀𐐁𐐂𐐃𐐄𐐅𐐆𐐇𐐈𐐉𐐊𐐋𐐌𐐍𐐎𐐏𐐐𐐑𐐒𐐓𐐔𐐕𐐖𐐗𐐘𐐙𐐚𐐛𐐜𐐝𐐞𐐟𐐠𐐡𐐢𐐣𐐤𐐥𐐦𐐧𐐨𐐩𐐪𐐫𐐬𐐭𐐮𐐯𐐰𐐱𐐲𐐳𐐴𐐵𐐶𐐷𐐸𐐹𐐺𐐻𐐼𐐽𐐾𐐿𐑀𐑁𐑂𐑃𐑄𐑅𐑆𐑇𐑈𐑉𐑊𐑋𐑌𐑍𐑎𐑏𐑐𐑑𐑒𐑓𐑔𐑕𐑖𐑗𐑘𐑙𐑚𐑛𐑜𐑝𐑞𐑟𐑠𐑡𐑢𐑣𐑤𐑥𐑦𐑧𐑨𐑩𐑪𐑫𐑬𐑭𐑮𐑯𐑰𐑱𐑲𐑳𐑴𐑵𐑶𐑷𐑸𐑹𐑺𐑻𐑼𐑽𐑾𐑿𐒀𐒁𐒂𐒃𐒄𐒅𐒆𐒇𐒈𐒉𐒊𐒋𐒌𐒍𐒎𐒏𐒐𐒑𐒒𐒓𐒔𐒕𐒖𐒗𐒘𐒙𐒚𐒛𐒜𐒝𐒞𐒟𐒠𐒡𐒢𐒣𐒤𐒥𐒦𐒧𐒨𐒩𐒪𐒫𐒬𐒭𐒮𐒯𐒰𐒱𐒲𐒳𐒴𐒵𐒶𐒷𐒸𐒹𐒺𐒻𐒼𐒽𐒾𐒿𐓀𐓁𐓂𐓃𐓄𐓅𐓆𐓇𐓈𐓉𐓊𐓋𐓌𐓍𐓎𐓏𐓐𐓑𐓒𐓓𐓔𐓕𐓖𐓗𐓘𐓙𐓚𐓛𐓜𐓝𐓞𐓟𐓠𐓡𐓢𐓣𐓤𐓥𐓦𐓧𐓨𐓩𐓪𐓫𐓬𐓭𐓮𐓯𐓰𐓱𐓲𐓳𐓴𐓵𐓶𐓷𐓸𐓹𐓺𐓻𐓼𐓽𐓾𐓿𐔀𐔁𐔂𐔃𐔄𐔅𐔆𐔇𐔈𐔉𐔊𐔋𐔌𐔍𐔎𐔏𐔐𐔑𐔒𐔓𐔔𐔕𐔖𐔗𐔘𐔙𐔚𐔛𐔜𐔝𐔞𐔟𐔠𐔡𐔢𐔣𐔤𐔥𐔦𐔧𐔨𐔩𐔪𐔫𐔬𐔭𐔮𐔯𐔰𐔱𐔲𐔳𐔴𐔵𐔶𐔷𐔸𐔹𐔺𐔻𐔼𐔽𐔾𐔿𐕀𐕁𐕂𐕃𐕄𐕅𐕆𐕇𐕈𐕉𐕊𐕋𐕌𐕍𐕎𐕏𐕐𐕑𐕒𐕓𐕔𐕕𐕖𐕗𐕘𐕙𐕚𐕛𐕜𐕝𐕞𐕟𐕠𐕡𐕢𐕣𐕤𐕥𐕦𐕧𐕨𐕩𐕪𐕫𐕬𐕭𐕮𐕯𐕰𐕱𐕲𐕳𐕴𐕵𐕶𐕷𐕸𐕹𐕺𐕻𐕼𐕽𐕾𐕿𐖀𐖁𐖂𐖃𐖄𐖅𐖆𐖇𐖈𐖉𐖊𐖋𐖌𐖍𐖎𐖏𐖐𐖑𐖒𐖓𐖔𐖕𐖖𐖗𐖘𐖙𐖚𐖛𐖜𐖝𐖞𐖟𐖠𐖡𐖢𐖣𐖤𐖥𐖦𐖧𐖨𐖩𐖪𐖫𐖬𐖭𐖮𐖯𐖰𐖱𐖲𐖳𐖴𐖵𐖶𐖷𐖸𐖹𐖺𐖻𐖼𐖽𐖾𐖿𐗀𐗁𐗂𐗃𐗄𐗅𐗆𐗇𐗈𐗉𐗊𐗋𐗌𐗍𐗎𐗏𐗐𐗑𐗒𐗓𐗔𐗕𐗖𐗗𐗘𐗙𐗚𐗛𐗜𐗝𐗞𐗟𐗠𐗡𐗢𐗣𐗤𐗥𐗦𐗧𐗨𐗩𐗪𐗫𐗬𐗭𐗮𐗯𐗰𐗱𐗲𐗳𐗴𐗵𐗶𐗷𐗸𐗹𐗺𐗻𐗼𐗽𐗾𐗿𐘀𐘁𐘂𐘃𐘄𐘅𐘆𐘇𐘈𐘉𐘊𐘋𐘌𐘍𐘎𐘏𐘐𐘑𐘒𐘓𐘔𐘕𐘖𐘗𐘘𐘙𐘚𐘛𐘜𐘝𐘞𐘟𐘠𐘡𐘢𐘣𐘤𐘥𐘦𐘧𐘨𐘩𐘪𐘫𐘬𐘭𐘮𐘯𐘰𐘱𐘲𐘳𐘴𐘵𐘶𐘷𐘸𐘹𐘺𐘻𐘼𐘽𐘾𐘿𐙀𐙁𐙂𐙃𐙄𐙅𐙆𐙇𐙈𐙉𐙊𐙋𐙌𐙍𐙎𐙏𐙐𐙑𐙒𐙓𐙔𐙕𐙖𐙗𐙘𐙙𐙚𐙛𐙜𐙝𐙞𐙟𐙠𐙡𐙢𐙣𐙤𐙥𐙦𐙧𐙨𐙩𐙪𐙫𐙬𐙭𐙮𐙯𐙰𐙱𐙲𐙳𐙴𐙵𐙶𐙷𐙸𐙹𐙺𐙻𐙼𐙽𐙾𐙿𐚀𐚁𐚂𐚃𐚄𐚅𐚆𐚇𐚈𐚉𐚊𐚋𐚌𐚍𐚎𐚏𐚐𐚑𐚒𐚓𐚔𐚕𐚖𐚗𐚘𐚙𐚚𐚛𐚜𐚝𐚞𐚟𐚠𐚡𐚢𐚣𐚤𐚥𐚦𐚧𐚨𐚩𐚪𐚫𐚬𐚭𐚮𐚯𐚰𐚱𐚲𐚳𐚴𐚵𐚶𐚷𐚸𐚹𐚺𐚻𐚼𐚽𐚾𐚿𐛀𐛁𐛂𐛃𐛄𐛅𐛆𐛇𐛈𐛉𐛊𐛋𐛌𐛍𐛎𐛏𐛐𐛑𐛒𐛓𐛔𐛕𐛖𐛗𐛘𐛙𐛚𐛛𐛜𐛝𐛞𐛟𐛠𐛡𐛢𐛣𐛤𐛥𐛦𐛧𐛨𐛩𐛪𐛫𐛬𐛭𐛮𐛯𐛰𐛱𐛲𐛳𐛴𐛵𐛶𐛷𐛸𐛹𐛺𐛻𐛼𐛽𐛾𐛿𐜀𐜁𐜂𐜃𐜄𐜅𐜆𐜇𐜈𐜉𐜊𐜋𐜌𐜍𐜎𐜏𐜐𐜑𐜒𐜓𐜔𐜕𐜖𐜗𐜘𐜙𐜚𐜛𐜜𐜝𐜞𐜟𐜠𐜡𐜢𐜣𐜤𐜥𐜦𐜧𐜨𐜩𐜪𐜫𐜬𐜭𐜮𐜯𐜰𐜱𐜲𐜳𐜴𐜵𐜶𐜷𐜸𐜹𐜺𐜻𐜼𐜽𐜾𐜿𐝀𐝁𐝂𐝃𐝄𐝅𐝆𐝇𐝈𐝉𐝊𐝋𐝌𐝍𐝎𐝏𐝐𐝑𐝒𐝓𐝔𐝕𐝖𐝗𐝘𐝙𐝚𐝛𐝜𐝝𐝞𐝟𐝠𐝡𐝢𐝣𐝤𐝥𐝦𐝧𐝨𐝩𐝪𐝫𐝬𐝭𐝮𐝯𐝰𐝱𐝲𐝳𐝴𐝵𐝶𐝷𐝸𐝹𐝺𐝻𐝼𐝽𐝾𐝿𐞀𐞁𐞂𐞃𐞄𐞅𐞆𐞇𐞈𐞉𐞊𐞋𐞌𐞍𐞎𐞏𐞐𐞑𐞒𐞓

²⁵TODO

In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

TODO

3.4 Early metrology

At first sight, the metrological systems from the Early Dynastic period match the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{Ur III/OB}$ [Fri07, p. 374; DE87, pp. 127, 165]:

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \blacktriangleright \xleftarrow{10} \blacktriangleright \xleftarrow{6} \bullet \xleftarrow{10} \blacktriangleright. \quad (S)$$

Likewise the area system used in the Early Dynastic IIIb period mirrors system $G_{Ur III/OB}$ [Dei22, p. 72; NDE93, p. 63; Fri07, p. 378; Gom16]:

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \bullet \xleftarrow{10} \bullet \xleftarrow{3} \blacktriangleright \xleftarrow{6} \blacktriangleright, \quad (G_{ED IIIb})$$

As noted in [And04, p. 4] (see section 3.2), the vertical \uparrow from $S_{Ur III/OB}$ becomes a horizontal \blacktriangleright in system S . It is however far from the only case of such a reallocation of function. The earlier form of System G was [DE87, pp. 141, 165; Fri07, p. 378]:

$$\bullet \xleftarrow{6} \odot \xleftarrow{10} \bullet \xleftarrow{3} \blacktriangleright \xleftarrow{6} \blacktriangleright, \quad (G)$$

Observe that, as noted in [DE87, p. 142], \odot changes meaning from $10\bullet$ in system G to $10\bullet$ in system $G_{ED IIIb}$. System G is used in the Uruk period, but also in the ED I–II period (it is the “area 2” system in [Cha03], whereas $G_{ED IIIb}$ is the “area 1” system).

3.4.1 Field lengths in Nirsu



The length system Early Dynastic IIIb of the state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign \blacktriangleright is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign \blacktriangleright is not written either. Length shorter than one rope are expressed in half-ropes using the $1/2$ sign \blacktriangleright (again with no \blacktriangleright), and then in reeds, *with* the sign \blacktriangleright . Effectively, this yields the following factor diagram:

$$\begin{array}{c} \uparrow \xleftarrow{6} \blacktriangleright \xleftarrow{2} \blacktriangleright \xleftarrow{10} \blacktriangleright \blacktriangleright \\ \text{1 eše}_2 = 10 \text{ nindan} \\ \text{1 rope} = 10 \text{ rods} \\ \text{60 m} \end{array} \quad \begin{array}{c} \text{gi} \\ \text{reed} \\ \text{3 m} \end{array}. \quad (L_{ED IIIb})$$

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and others from the same period, such as the ones discussed in [Lec20] areas are expressed in system $G_{ED IIIb}$, with curviform numerals²⁶; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope²⁷, the use of curviform or cuneiform numerals

























²⁶TODO(egg): Note the handful of late Urukagina tablets that start to have cuneiform areas.

²⁷This is the case of the sides of the field in [P020054, obv. ii 2–3].

therefore disambiguates a numeric expression between an area and a length, and therefore the interpretation of its numerals between systems $G_{ED IIIb}$ and $L_{ED IIIb}$. The sign GAN_2 , which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without .

3.4.2 Dyke lengths in Nirsu

[Pow87, p. 466] notes that reeds “are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus.” Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in CDLI finds many occurrences²⁸, such as:

- [P221305]                         <

The 𒂗𒂗𒂗 and 𒂗𒂗𒂗 are generally counted using curviform numerals, and the smaller units using cuneiform 𒂗 numerals. Indeed, a search on [Mil+07] for co-occurrences of 𒂗𒂗𒂗 with either of 𒂗𒂗𒂗 or 𒂗𒂗𒂗 finds the following expressions³²:

1. [P240532, verso 4, 9] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$ ³⁴
2. [P240548, verso 1, 1] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$
3. [P240655, recto 7, 9] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$ ³⁵
4. [P240579, verso 4, 3] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$
5. [P240675, verso 2, 2] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$
6. [P240609, verso 3, 1] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$
7. [P240533, recto 3, 3] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$
8. [P240697, recto 1, 5] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$ ³⁶
9. [P240653, recto 6, 2] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$
10. [P240654, recto 2, 6] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$ ³⁷
11. [P240531, recto 1, 8] $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$
12. [P241708, recto 1, 1]³⁹ $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$
13. [P241904, recto 1, 1]⁴⁰ $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$ ⁴¹

Note that higher numbers of 𒂗𒂗𒂗 are expressed in hundreds (*mi-at* 𒂗𒂗𒂗) and then thousands (*li-im* 𒂗𒂗𒂗), as is typical in Ebla [Arc15, p. 33], e.g., in [P240532, verso 2, 3], $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$ (100 + 60 + 30 + 5 = 195 𒂗𒂗𒂗 of grain). These expressions match the following factor diagram:

$$\underbrace{\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}}_{\text{𒂗𒂗𒂗}} \xleftarrow{\frac{5}{3}} \text{𒂗} \xleftarrow{6} \text{𒂗} \xleftarrow{10} \text{𒂗} \xleftarrow{2} \text{𒂗} = \text{𒂗𒂗𒂗} \xleftarrow{\frac{5}{2}} \text{𒂗} \xleftarrow{4} \text{𒂗𒂗𒂗} \xleftarrow{6} \text{𒂗𒂗𒂗}$$

($\mathcal{C}_{\text{Ebla}}$)

3.4.4 Use in modern publications

Because of their prevalence in the Early Dynastic period, the proposed numerals are used modern publications discussing Metrology from that period, as illustrated in figures TODO.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 2. The partial⁴² transliteration “4^𒂗 *a₃-da-um* 4^𒂗 *aktum* 4^𒂗 *ib₂^{tu} × 3* *sa₆ gunu₃*” is used to illustrate a discussion of the interpretation of the contrast between 𒂗 and

³²We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform 𒂗𒂗𒂗 and larger with cuneiform 𒂗𒂗𒂗 and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked³³ on a photograph (from EbDA unless noted otherwise).

³³As we will see in Section 3.6.1, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

³⁴ba-ri₂-zu₂, a variant spelling.

³⁵Short for 𒂗𒂗𒂗 .

³⁶Note the omitted 𒂗𒂗𒂗 .

³⁷Instead of the expected 𒂗𒂗𒂗 .

³⁸ 𒂗𒂗𒂗 not legible on the EbDA photo.

³⁹From CDLI photo.

⁴⁰From photo in [Arc89, p. 6].

⁴¹Laid out as 𒂗𒂗𒂗 ; on stacking patterns see Section 6.2.

⁴²The untransliterated text would be $\text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗} \text{𒂗𒂗𒂗}$; note the atomically encoded $\text{ib}_2 \times 3$ = $\text{𒂗𒂗} \times \text{𒂗}$ = 𒂗𒂗𒂗 .

3.6.1 Compatibility with transliteration

3.7 Compatibility considerations

3.7.1 The case of ŠAR₂

4 Rationale for ED–Uruk numeral unification

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N₉ and N₁₀ from 4(ban₂@c) and 5(ban₂@c).]

6 Characters not included in this proposal

6.1 Missing numerals

(N₁₇, 12N₁₄, etc.) 7(diš *tenû*)

6.2 Stacking patterns

(... are a mess, vary within Uruk, and are not transliterated/documented by Englund, so let's not go there for now.)

6.3 Matters for higher-level protocols

Rotated bits: <https://cdli.mpiwg-berlin.mpg.de/artifacts/101087>

7 Acknowledgements

TODO(egg): Something about the Vanséveren fonts

References

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