

Archaic cuneiform numbers

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1 Summary

This document proposes encoding some numerals used in the fourth millennium (Uruk IV and Uruk III) and Early Dynastic period in conjunction with the Sumero-Akkadian cuneiform script¹ and the proto-cuneiform script². The proposed characters are listed in §2. Most of them were listed in [L2/23-190]. The present document provides a more detailed rationale for their encoding and additional information about their identity. Some characters have been removed, in some cases because they are non-encodable variants, in others because their encodability should be considered as part of the proto-cuneiform proposal; these are discussed in §???. The glyphs have also been reworked, and additional characters used in the Early Dynastic period have been added.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in §4. The proposed “curved”, or “curviform”, numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in §3, including compatibility considerations in §3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in §2.2 reflect the overlap. Many of these numerals are also used in proto-Elamite⁵ texts, where they are treated as identical characters in scholarship on proto-Elamite, so that they should be unified with the ones proposed in [L2/23-196]. However, in the interest of time,

¹ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

²ISO 15924: Pcun, not yet encoded.

³Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: ◩, ◪ (small stylus), ◩, ◪ (large stylus). Some numerals are composed of multiple such impressions, e.g., ◩◩. The terms “curved”, “curviform”, “curvilinear”, and “round” can be found in the literature. We avoid the term “round” here as it has other meanings in the context of numbers. We use “curviform” in this document as, being the least common term, it is least likely to lead to confusion, and “CURVED” in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [inlineATF].

⁴Impressed into clay using a stylus with a trihedral end: ◡ (stylus held horizontally), † (vertically), ◡ (diagonally) ◡ (with the head of the stylus), † (stylus pressed deeper, forming a larger wedge), ◡ (combining † and ◡), etc.

⁵ISO 15924: Pelm, not yet encoded.

we do not provide a detailed rationale for this unification in this document, and we are not proposing that the numerals be given the corresponding Script_Extensions property value for now. Neither do we propose encoding any numerals that are solely attested in proto-Elamite texts, or well-attested in proto-Elamite texts but insufficiently attested in Uruk.

[TODO(egg): Mention the other sections here too.]

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
Non-numeric signs	Future Pcun	Existing Xsux	

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

2 Proposed changes to the Standard

2.1 Code charts

2.2 Properties

2.3 Core specification text

Amend [Uni16, §11.1.2, sub “Cuneiform Numerals”], as follows:

Cuneiform Numerals. In general, numerals that also have a phonetic, logographic, or determinative value are encoded in the main Cuneiform block; as a result, some series of numerals, such as 𐎶–𐎶𐎶 1(diš)–9(diš) or 𐎶–𐎶𐎶𐎶 1(u)–9(u), are split across the two blocks. Numerals have been encoded separately from signs that are visually identical but ~~semantically different~~ etymologically unrelated (for example, U+1244F 𐎶 CUNEIFORM NUMERIC SIGN ONE BAN2, U+12450 𐎶 CUNEIFORM NUMERIC SIGN TWO BAN2, and so on, versus U+12226 𐎶 CUNEIFORM SIGN MASH, U+1227A 𐎶 CUNEIFORM SIGN PA, and so on).

The relation between series of numerals depends on the metrological system; for instance, when counting talents, written 𐎶𐎶 (a unit of weight, approximately 30 kg), 𐎶𐎶 is used for “one talent”, and 𐎶𐎶𐎶 for “ten talents”. However, when measuring areas, the area 𐎶𐎶 (one *būrum*) is eighteen times 𐎶𐎶 (one *ikûm*, approximately 3600 m²). The Numeric_Value property assignment of a cuneiform numeral therefore reflects only its relation to the first numeral in its series, rather than the absolute numeric value that it might represent. For instance, the number “fifty” is written 𐎶𐎶, but U+12410 𐎶 CUNEIFORM NUMERIC SIGN FIVE U has Numeric_Value=5, as it is 5 × 𐎶.

In the third millennium, and especially in the Early Dynastic period, some numerals are written using a cylindrical tool, rather than the cuneiform stylus, forming curved rather than cuneiform numerals (𐎶 rather than 𐎶). The cuneiform numerals are descended from these curved numerals. However, in the Early Dynastic period, the curved

numerals contrast with the cuneiform ones, and are used together with them in several metrological systems; they are therefore separately encoded. Most curved numerals are encoded in the Archaic Cuneiform Numerals block, with the exception of two fractions in the Cuneiform Numbers and Punctuation block: U+1245D 𐎧 CUNEIFORM NUMERIC SIGN ONE THIRD VARIANT FORM A and U+1245E 𐎨 CUNEIFORM NUMERIC SIGN TWO THIRDS VARIANT FORM A, the curved counterparts of U+1245A 𐎡 CUNEIFORM NUMERIC SIGN ONE THIRD DISH and U+1245B 𐎢 CUNEIFORM NUMERIC SIGN TWO THIRDS DISH.

Add after [Uni16, §11.1.3]:

11.1.4 Archaic Cuneiform Numerals: U+12550–U+TODO

TODO something about consistent size

The sign ŠAR₂.

Amend [Uni16, §24.1.2, sub “Dashed Box Convention”], third paragraph, as follows:

In a few cases of very wide ~~punctuation~~ characters that do not naturally fit into a code chart cell, the representative glyph may be shown with an artificially narrow shape, displayed inside the dashed box, with or without additional annotation, to indicate this adjustment of shape.

3 Rationale for curviform–cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [Pow72, p. 215] “in the same fashion as we use black and red ink”; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble 𐎶 in texts from Early Dynastic IIIa Šuruppak as in the character code charts, 𐎶 later in the third millennium⁶, 𐎶 in Old Babylonian cursive, 𐎶 in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists⁷ and dictionaries⁸, and of composite

⁶Merging with U+1224E CUNEIFORM SIGN NI2.

⁷Notably [OSL] and the online edition of [Borio] in [eBL, Signs].

⁸Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

texts⁹. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]¹⁰. The diachronic approach is also useful for pedagogic applications¹¹.

3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding¹² that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

1 ¹³	<	+	≡	𐎶	𐎵	𐎶
1(ḫeš ₂)	1(u)	1/2(diš)	5(diš <i>tenû</i>)	gi	us ₂	sa ₂
	7.5 (ropes)		5	reed	side	equal
𐎶 ¹⁴	≡	𐎶	𐎶	𐎶		
3(u)	6(diš <i>tenû</i>)	gi	saḫ	sa ₂		
3 (ropes)	6	reed	front	equal		
𐎶 𐎶 𐎶	•	𐎶	𐎶	𐎶		
ašag-bi	1(bur ₃ ^c)	1(eše ₃ ^c)	1(iku ^c)	1/2(iku ^c)		
ašag=bi						
field=DEM ¹⁵						

𐎶 𐎶 𐎶 𐎶
tug_x(LAK 483)-si-ga-kam
tugsiga =ak =am -Ø
ploughed=GEN=COP-3.SG.S

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as¹⁶

⁹For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of 𐎶 𐎶 𐎶 𐎶 𐎶 𐎶, as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

¹⁰Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svā+24, pp. 242, 148].

¹¹For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

¹²At that time scoped to the repertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as 𐎶 ≠ 𐎶, were informed by Early Dynastic distinctions.

¹³As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large 𐎶 in copies. The photos in [CDLI] clearly show that this is in fact a vertical wedge.

¹⁴Note that ED IIIb < numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign 𐎶 in [P020054] looks more like Ur III 𐎶.

¹⁵Alternatively: area=POSS.3.SG.NH, “its area”.

¹⁶We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

465 metres, equal lengths
 198 metres, equal widths
 this field is 9, 18 hectares of ploughed land


where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from \triangleright to \uparrow numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in §3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases¹⁷.”

The time has come to revisit this issue. As we will see in §3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In §3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere \triangleright / \uparrow switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 A primer on classic Ur III and Old Babylonian metrologies


 I want to write tablets: the tablet of
 1 cor of barley to 600 cor; the tablet
 of 1 shekel of silver to 10 minas [...]

 Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the contrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known¹⁸ a sexagesimal place value system (SPVS) was used in Meso-

¹⁷The cuneiform encoding process was planned in *stages* in [L2/03-162]. One might expect the second stage of encoding, which led to the creation of the Early Dynastic Cuneiform block, to incorporate the numerals needed for the representation and discussion of Early Dynastic texts; however, the proposal [L2/12-208] stated that “numerals have been omitted due to the complexity of numeral signs from this period. An expert in the metrology of this period must be consulted before these can be properly included.”

¹⁸See, *e.g.*, [Unit6, §22.3.3, sub “Cuneiform Numerals”].

potamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters $\text{I}-\text{IIII}$, the multiples of ten (10–50) are $\text{X}-\text{XL}$, but the other digits 11–59 are sequences $\text{X}-\text{XL}-\text{IIII}$; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units $\text{I}-\text{IIII}$, tens $\text{X}-\text{XL}$, sixties $\text{I}-\text{IIII}$ (with larger wedges than the units), multiples of six hundred $\text{K}-\text{KK}$, multiples of three thousand six hundreds $\text{D}-\text{DDDD}$, and multiples of thirty-six thousand $\text{D}-\text{DDDD}$.

3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram¹⁹, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\text{D} \xleftarrow{10} \text{D} \xleftarrow{6} \text{K} \xleftarrow{10} \text{I} \xleftarrow{6} \text{X} \xleftarrow{10} \text{I} \quad (S_{\text{Ur III/OB}})$$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written $\text{K} \text{X} \text{IIII} \text{X} \text{IIII}$ in the discrete counting system, and $\text{XX} \text{III} \text{XL} \text{III}$ in the sexagesimal place value system.

3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:

$$\text{D} \xleftarrow{10} \text{D} \xleftarrow{6} \text{K} \xleftarrow{10} \text{X} \xleftarrow{3} \text{X} \xleftarrow{6} \text{I} \xleftarrow{2} \text{I} \xleftarrow{2} \text{I} \quad (G_{\text{Ur III/OB}})$$







bur_3 eše_3 iku ubûm
 bûrum eblum ikûm ubûm
 6,48 ha 2,16 ha 3600 m² 1800 m²

Note that for the range of areas given above, this system does not use any symbols separate from the numerals for the individual units (ubûm , ikûm , eblum , and bûrum). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign 𒀭 functioning as punctuation²⁰, but the numerals are tied to the metrology; thus a surface of 5 bûr 1 ebel 4 ikû (100 ikû , 36 ha) would be written²¹ $\text{XL} \text{X} \text{IIII} \text{X} \text{IIII}$. Contrast this with systems where the same numerals are used

¹⁹These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*, see Figure 3.

²⁰This sign is sometimes interpreted as a measurement unit, and transliterated iku , see, e.g., [Pro20, pp. 385 sqq.], or transliterations in [Feu04] discussed in §3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system $G_{\text{Ur III/OB}}$.

²¹As in the surface of the field of $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$ (the city of Apisal) reported on [P102305, r. 1]

For areas smaller than a quarter *ikûm*, an overt unit is used, with one  (*sar*, *mûšarum*), approximately 36 m², written , equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60  (*gin₄*, *šiqlum*, “shekel”). For areas greater than 3600 *bûr*, the  and  numerals are reused with a suffix  (*gal*, “big”), as follows [Robo8, p. 295 nn. b, c; Fri07, p. 378; Rob19]:

e.g., for $(2 \times 3600 + 20 \times 60 + 49) b\bar{u}r$
 $5 ik\dot{u}$ ($5 + \frac{1}{2}$) $m\bar{u}s\bar{a}r$ 19 $\dot{s}iqil$. Factor diagrams in this document will use bottom curly brackets in this fashion to separate numerals from units and other suffixes.


Another such system of note is the one for capacities²³ [Fri07, p. 376; Rob19],

In the above diagram, the numerals for ban₂ are 𐤁, 𐤂, 𐤃, 𐤄, and 𐤅, and those for bariga are 𐤆, 𐤇, 𐤈, and 𐤉 (contrast ordinary 𐤈 and 𐤉 otherwise used with 𐤆 numerals). Further, we have used the symbol \sim to express that, as described in [Hue11, p. 585 nn. (b), (f)], the sign 𐤁𐤆 GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𐤁 if present, and after the word for “grain” if present, as in

3554 gur 3 ban₂ 6 sila₃ of grain.








Observe that while large numbers of gur follow²⁵ system $S_{Ur III/OB}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as $\langle 1 \overline{\text{AŠ}} \rangle$ would be 10 gur 1 bariga, and $\langle \text{—} \overline{\text{AŠ}} \rangle$ would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

- **I-III** used in $S_{\text{IF, III/OR}}$ and the SPVS as well as with overt units;

²²From [P213162], which has an additional , two thirds (of a shekel), see §3.3.5.

²³Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system *G*_{Ur III/0B} based on a height of one cubit, see [Pow87, p. 488; Robo8, p. 294; Rob19].

²⁴From [P309594, obv. 11].

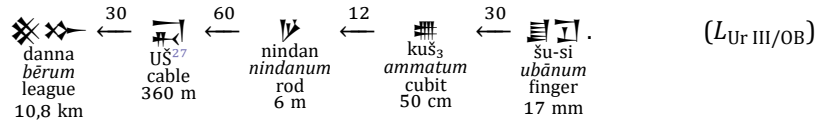
²⁵A larger unit, the *guru*₇ (*karûm*, grain heap), is sometimes used instead, with        (1 *karûm* = 3600 kurrû). See [Fri07, p. 415; Rob19].

- \leftarrow - \lll used in $G_{\text{Ur III/OB}}$, of which \leftarrow - \ll are also used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- \uparrow - \lll used in $S_{\text{Ur III/OB}}$, and sometimes with overt units;
- K - \lll used in $S_{\text{Ur III/OB}}$;
- \diamond - \lll used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \diamond - \lll used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \leftarrow - \lll used in $C_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
- + , + , + , + , + used in $C_{\text{Ur III/OB}}$;
- I , I , I , I used in $C_{\text{Ur III/OB}}$ —note the overlap with I - \lll ;
- + and + used in $G_{\text{Ur III/OB}}$.

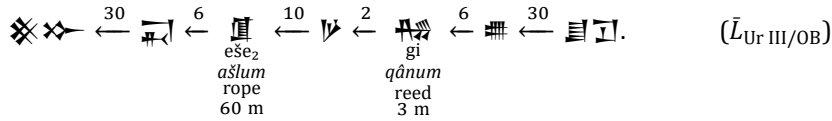
Only in the SPVS did numerals exist truly independently of metrology; to quote [Rob08, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with \uparrow and \leftarrow numerals with their system $S_{\text{Ur III/OB}}$ values. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in §3.4. Metrological tables use the following units²⁶ [Fri07, p. 118; Rob19]:



Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:



In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

Fractions of the *ikîm*, $\text{↖} = \frac{1}{2}$ and $\text{↗} = \frac{1}{4}$, have already been encountered. In other contexts, the fraction $\frac{1}{2}$ is written + , as in + \lll . The fractions $\frac{1}{3}$ and $\frac{2}{3}$ are written + and + . The latter two signs are derived from curviform signs +

²⁶In this factor diagram and the next, we do not include the numerals. The units are no more than a factor of 60 apart, so higher numerals such as K or \diamond are not used.

²⁷As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.

and 𐎶 , which are already separately encoded; these are in turn derived from the sign 𐎶 ($\text{\$U}_2$), whose Early dynastic form resembles 𐎶 , and $\text{\textcircled{v}}$ numerals; see [Pow71, pp. 113, 134]. The 𐎶 is sometimes omitted, as in [P240545, verso 6 9; P221530; P221531; P271238; P274845].

3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the fourth millennium) clearly mirrors system $S_{\text{Ur III/OB}}$ [Fri07, p. 374; DE87, pp. 127, 165]:

$$\text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎴} \xleftarrow{10} \text{𐎳} \xleftarrow{6} \text{𐎲} \xleftarrow{10} \text{𐎱}. \quad (S)$$

Likewise the area system used in the Early Dynastic IIIb period for areas of one iku and greater [Dei22, p. 72; NDE93, p. 63; Fri07, p. 378; Lec16],

$$\text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎴} \xleftarrow{10} \text{𐎳} \xleftarrow{3} \text{𐎲} \xleftarrow{6} \text{𐎱} \quad (G_{\text{ED IIIb}})$$

mirrors system $G_{\text{Ur III/OB}}$, with consistent use of the numerals: \bullet corresponds to 𐎶 , 𐎵 to 𐎶 , and 𐎶 to 𐎶 . An exception to this correspondence, noted in [L2/04-099, p. 4] (see §3.2), is that the vertical 𐎶 from $S_{\text{Ur III/OB}}$ corresponds to a horizontal 𐎶 in system S . This is however far from the only case of such a reallocation of function. The earlier form of the area system is [DE87, pp. 141, 165; Fri07, p. 378]:

$$\text{𐎶} \xleftarrow{6} \text{𐎵} \xleftarrow{10} \text{𐎴} \xleftarrow{3} \text{𐎲} \xleftarrow{6} \text{𐎱}, \quad (G)$$

Observe that, as noted in [DE87, p. 142], 𐎶 changes meaning from $10\bullet$ in system G to $600\bullet$ in system $G_{\text{ED IIIb}}$. System G is used in the fourth millennium, but also in the ED I–II period (it is the “area 2” system in [Cha03], whereas $G_{\text{ED IIIb}}$ is the “area 1” system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb area system²⁸,

$$\text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎴} \xleftarrow{10} \text{𐎳} \xleftarrow{3} \text{𐎲} \xleftarrow{6} \text{𐎱} \xleftarrow{2} \text{𐎰} \xleftarrow{2} \text{𐎯} \xleftarrow{2} \text{𐎮} \xleftarrow{2} \text{𐎭}, \quad (G_{\text{ED IIIb}})$$

with the numerals of a contemporaneous capacity system:






$$\text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎴} \xleftarrow{10} \text{𐎳} \xleftarrow{4} \text{𐎲} \xleftarrow{6} \text{𐎱} \xleftarrow{2} \text{𐎰} \xleftarrow{2} \text{𐎯} \xleftarrow{2} \text{𐎮} \xleftarrow{2} \text{𐎭}, \quad (C_{\text{ED IIIb}})$$

both described in [Lec16]. While the size of the $\text{𐎶} \text{𐎵} \text{𐎴} \text{𐎳} \text{𐎲} \text{𐎱} \text{𐎰} \text{𐎯} \text{𐎮} \text{𐎭}$ (gur san ḡal₂) in bariga is different from that of the Old Babylonian $\text{𐎶} \text{𐎵} \text{𐎴} \text{𐎳} \text{𐎲} \text{𐎱}$, the basic structure of the capacity system is recognizable, with 𐎶 corresponding to 𐎶 for bariga, $\text{𐎵} \text{𐎴} \text{𐎳}$ corresponding to $\text{𐎶} \text{𐎵} \text{𐎴}$ for ban₂, and the 𐎶 counted with 𐎶 rather than 𐎶 numerals. However, the half-iku is counted with the same 𐎶 as the bariga, whereas it uses a different sign, 𐎶 , in the Old Babylonian system. As we will see, this is cannot be handled as a split, by giving 𐎶 the glyph 𐎶 in an Early Dynastic IIIb font, as the 𐎶 numeral series is also in use in that period.



²⁸A variant is $\text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎴} \xleftarrow{10} \text{𐎳} \xleftarrow{3} \text{𐎲} \xleftarrow{6} \text{𐎱} \xleftarrow{2} \text{𐎰} \xleftarrow{2} \text{𐎯} \xleftarrow{2} \text{𐎮} \xleftarrow{2} \text{𐎭}$, see [Pow72, p. 218].

²⁹The (fairly rare) cuneiform counterpart is 𐎶 .

3.4.1 Field lengths in Nirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign  is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign  is not written either. Lengths shorter than one rope are expressed in half-rope using the $\frac{1}{2}$ sign  (again with no ) , and then in reeds, *with* the sign , as follows:

$$\begin{array}{c} \text{I} \leftarrow \frac{6}{1 \text{ eše}_2 = 10 \text{ nindan}} \leftarrow \frac{2}{1 \text{ rope} = 10 \text{ rods}} \leftarrow \frac{10}{60 \text{ m}} \leftarrow \frac{32}{\text{gi reed } 3 \text{ m}} \end{array} \quad (L_{\text{ED IIIb}})$$

This is the system that was used to express the sides of the field in [P020054] discussed in §3.2. In that tablet and most others from the same period, such as the ones discussed in [Lec20], areas are expressed in system $G_{\text{ED IIIb}}$, with curviform numerals³³; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope³⁴, the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and thus the interpretation of its numerals between systems $G_{\text{ED IIIb}}$ and $L_{\text{ED IIIb}}$. The sign , which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without . There are other such co-occurrences contrasting between metrological systems; for instance, [Kre98, p. 303 n. 686] mentions the use of cuneiform numerals for days and months³⁵.

3.4.2 Dyke lengths in Nirsu

[Pow87, p. 466] notes that reeds “are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus.” Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in [CDLI] finds many occurrences³⁶, such as:

- [P221305, obv. 1 4]³⁷                                            

$$\underbrace{\left(\begin{array}{c} \text{eye} \\ \text{arrow} \end{array} \right)}_{10} \leftarrow \underbrace{\left(\begin{array}{c} \text{arrow} \\ \text{dot} \end{array} \right)}_{6} = \underbrace{\left(\begin{array}{c} \text{arrow} \\ \text{dot} \end{array} \right)}_{2} \leftarrow \underbrace{\left(\begin{array}{c} \text{arrow} \\ \text{dot} \end{array} \right)}_{10} \leftarrow \underbrace{\left(\begin{array}{c} \text{arrow} \\ \text{dot} \end{array} \right)}_{6} \leftarrow \underbrace{\left(\begin{array}{c} \text{arrow} \\ \text{dot} \end{array} \right)}_{3} \quad (L'_{\text{ED IIIb}})$$

3.4.3 Butter, cheese and wheat in Nirsu

(C中白黑)

as in [P020016, rev. 1 4; P020065; P020090, obv. 1 3, rev. 2 1; P020092, rev. 3, 1; P020137, obv. 1 2] and others, where ban_2 counted with ∇ numerals are followed by sil_3 counted with \searrow numerals. Curviform numerals are also used to count sil_3 , but not⁴⁰ as part of the ∇ systems. This contrast can be seen in [P220927], which measures butter (∇ , i_3) with a different capacity system, using the ∇ (dug, “pot”) of 20 ∇ , with \triangleright and \bullet numerals⁴¹ for both the ∇ and the ∇ , thus [Pow87, pp. 504 sq.]



but counts cheese ([tʃi:z] , ga'ar) using the [tʃi:z] capacity system, with [tʃi:z] numerals for the [tʃi:z] .

[illegible]

Here the $\triangleright-\triangleleft-\sqcup$ contrast occurs not only within the numerals of the system, but with its units; this is perhaps best illustrated by the expressions $\lll \lll \lll \lll \lll \lll \lll \lll \lll \lll$ $\bullet \lll \lll \lll \lll \lll \lll \lll \lll \lll$ in [P221746, rev. 2 2] and $\lll \lll \bullet \lll \lll \lll \lll \lll \lll \lll$ in [P221814, rev. 15].

³⁹With either unit omitted, as in the examples above, or both, as in [Po20129, obv. 3 3] .

⁴⁰As of this writing, the single occurrence of (ban2@c) followed by curviform numerals and sila3 in ED IIIb Nirsu transliterations on [CDLI], 4(ban2@c) 3(asz@c) sila3 in [P221815, obv. 4 7], is incorrect: it should be 4(ban2@c) 3(disz@t) sila3.

⁴¹This tablet also uses subtractive notation:  “two pots minus two thirds (sila₃)”,  “ten minus one pots, six sila₃”. Such subtractive notation is common in most of the metrological systems discussed here; it appears in the ED IIIa period [Rob08, p. 77]. It presents no complexity for character encoding, but it is noteworthy that the sign Γ (‘al,’ ‘minus’) is often ligated with the following numerals, with the subtrahend placed under a sometimes considerably enlarged Γ , similar to the layout of the radical in modern mathematical notation, see, e.g., [P020092, rev. 3, 1, 2]. A good font could handle the very common -1 case, perhaps even -2 and -3 ; setting arbitrary numeric expressions under the Γ , or more generally replicating the layout of Early Dynastic tablets, is outside the realm of plain text; see also §6.2.

3.4.4 Grain in Ebla

The mixing of curviform and cuneiform numerals within a metrological system is not specific to Nirsu.

The system of grain⁴² capacities in Ebla uses the following units⁴³:

$$\begin{array}{ccccccc} \text{𐎶𐎵} \text{ } \text{𐎶} & \xleftarrow{2} & \text{𐎶𐎵} & \xleftarrow{\frac{5}{2}} & \text{𐎶} & \xleftarrow{4} & \text{𐎶𐎵} \xleftarrow{6} \text{ } \text{𐎶} \\ \text{gu}_2\text{-bar} & & \text{ba-ri}_2\text{-zu} & & \text{gīn}_4 & & \text{nīn}_2\text{-sañšu} \quad \text{an-zam}_x \end{array}$$

The 𐎶𐎵 and 𐎶 are generally counted using curviform numerals, and the smaller units using cuneiform 𐎶 numerals⁴⁴. Indeed, a search on [EbDA] for co-occurrences of either 𐎶𐎵 or 𐎶 with either of 𐎶𐎵 or 𐎶 finds the following expressions⁴⁵:

1. [P240532, verso 4 9] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
2. [P240548, verso 1 1] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
3. [P240655, recto 7 9] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
4. [P240579, verso 4 3] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
5. [P240675, verso 2 2] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
6. [P240609, verso 3 1] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
7. [P240533, recto 3 3] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
8. [P240697, recto 1 5] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
9. [P240653, recto 6 2] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
10. [P240654, recto 2 6] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
11. [P240531, recto 1 8] 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵
12. [P241708, recto 1 1]⁵² 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵

⁴²Liquid capacities use a different system [Arc15, p. 229 n. 12]:

$$\begin{array}{ccc} \text{𐎶𐎵} & \xleftarrow{30} & \text{𐎶} \xleftarrow{6} \text{ } \text{𐎶} \\ \text{la-ha} & & \text{sila}_3 \quad \text{an-zam}_x \end{array}$$

A glance it seems that 𐎶 are counted with cuneiform numerals and higher units with curviform ones, thus

$$\begin{array}{ccccccc} \text{𐎶} & \xleftarrow{\frac{5}{3}} & \text{𐎶} & \xleftarrow{6} & \text{𐎶} & \xleftarrow{10} & \text{𐎶} \xleftarrow{3} \text{ } \text{𐎶} \xleftarrow{\frac{10}{3}} \text{ } \text{𐎶} \xleftarrow{6} \text{ } \text{𐎶} \\ \text{𐎶} & & \text{𐎶} & & \text{𐎶} & & \text{𐎶} \end{array}$$

but we have not investigated this thoroughly.

⁴³Another system uses different values for the 𐎶 and 𐎶𐎵, see [Cha12, p. 62; Arc15, p. 229 n. 12]:

$$\text{𐎶} \text{ } \text{𐎶} \xleftarrow{2} \text{ } \text{𐎶} \xleftarrow{3} \text{ } \text{𐎶} \xleftarrow{4} \text{ } \text{𐎶} \xleftarrow{5} \text{ } \text{𐎶}.$$

⁴⁴As mentioned in [Cha12, p. 63], the 𐎶 is also counted using the 𐎶–𐎶 numeral series. Some instances of that usage are found transliterated *n*/6 in [EbDA]; in some cases the 𐎶 sign is omitted, and the 𐎶 numeral is then written before the 𐎶 unit, as in 𐎶𐎶𐎶 from [P240545, verso 1 3].

⁴⁵We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform 𐎶𐎵 and larger with cuneiform 𐎶 and smaller. In all cases the transcriptions given here are based on the [EbDA] transliterations, but the shape and orientation of the numerals was checked⁴⁶ on a photograph (from [EbDA] unless noted otherwise).

⁴⁶As we will see in §3.7.2, [CDLI] transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

⁴⁷ba-ri₂-zu₂, a variant spelling.

⁴⁸Short for 𐎶𐎵.




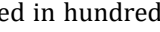

⁴⁹Note the omitted 𐎶𐎵.

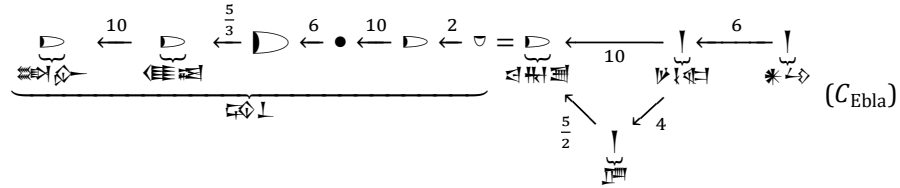
⁵⁰Instead of the expected 𐎶𐎵.

⁵¹𐎶𐎶𐎶 not legible on the EbDA photo.

⁵²From [CDLI] photo.







13. [P241904, recto 11]⁵³  ⁵⁴ 
 14. [P240964, recto 3 2]⁵⁵  ⁵⁶  ⁵⁷ 

Note that higher numbers of  are expressed in hundreds (*mi-at* ) and then thousands (*li-im* ) as is typical in Ebla [Arc15, p. 33], e.g., in the last example above or in [P240532, verso 2 3],  (100 + 60 + 30 + 5 = 195  of grain). These expressions correspond to the following factor diagram:



3.4.5 Use in modern publications

Because of their prevalence in the fourth millennium and Early Dynastic period, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 1–20.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 20. The partial⁵⁸ transliteration “4  *a₃-da-um* 4  *aktum* 4  *ib₂^{tu} × 3*  *sa₆ gunu₃*” is used to illustrate a discussion of the interpretation of the contrast between  and  numerals. More conventional transliterations might omit the numeral shapes entirely, e.g., 4 *a₃-da-um* 4 *aktum* 4 *ib₂^{tu} × 3* *sa₆ gunu₃*, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, e.g., 4(aš^c) *a₃-da-um* 4(diš^c) *aktum* 4(aš^c) *ib₂^{tu} × 3* (diš) *sa₆ gunu₃*, but the result would be less readable. See §3.7.2 for a discussion of transliteration conventions for numerals.











for the words *š u š a n a* and *š a n a b i*. Deimel's reading
š a n (a) for  came out of the reading /š a n t a k/ for the
 sign  and the writing of *š a (- n a)* after the fractional
 signs for *š u š a n a* and *š a n a b i* in Old Sumerian texts.
 But this was an ill-conceived argument at its inception, for

Figure 1: Discussion of the readings of proposed  and already-encoded  in [Pow71, p. 107].





⁵³From photo in [Arc89, p. 6].

⁵⁴Laid out as ; on stacking patterns see §6.2.

⁵⁵From photo in [Arc89, p. 6]; see also the [CDLI] photo and the copy in [Fri86, p. 17]. This tablet features unusual usage of vertical numerals—“somewhat unorganized”, as described by [Fri86, p. 16]—, such as  or , but its  and  are consistently counted with cuneiform numerals, and the higher units with cuneiform numerals.

⁵⁶Short for .

⁵⁷ŠU₂+NIN₂-sa₂, an unusual variant spelling.

⁵⁸The untransliterated text would be ; note the atomically encoded *ib₂ × 3* =  ×  = .

Thus, for instance, the original set of fractions ∇ , \triangleleft , and \blacktriangledown (1/2, 1/4 and 1/8 of an iku) in the Sumerian GANA system, was after a time augmented through the addition of the new sub-unit SAR: 𒌦 , equal to 1/100 of an iku (∇). Similarly, the Sumerian weight unit "ma-na" which originally may have had only the sub-units 𒌦 ša-na (= 1/3 mana) and 𒌦𒌦 ša-na-bi (= 2/3 mana), and perhaps also gin: 𒌦𒌦𒌦 (= 1/60 mana), seems to have acquired, at some time or other, also the smaller sub-units 𒌦𒌦𒌦𒌦 (= 1/3 gin), and 𒌦𒌦𒌦𒌦𒌦 še (= 1/3 × 1/60 gin).

Figure 5: Discussion of proposed fractions ∇ , \triangleleft , \blacktriangledown , and 𒌦 , as well as already-encoded 𒌦 and 𒌦 in [Fri78, p. 49].

stein publizierten Zeichenliste enthalten ist³, bis vor kurzem unentdeckt bleiben konnte. Erst 1978 machte der schwedische Mathematiker J. Friberg, EBM 1, 9-11, darauf aufmerksam, daß die Zeichen für die Zahlen Eins (∇) und Zehn (\bullet) in Verbindung mit dem Zeichen ŠE nicht im Verhältnis 1 zu 10 sondern im Verhältnis 1 zu 6 stehen. Bis dahin hatte man, obwohl die Andersartigkeit des in Verbindung mit dem Zeichen ŠE verwendeten Zahlzeichensystems bekannt war, für diese beiden häufigsten Zahlzeichen einheitlich ein Verhältnis 1 zu 10 unterstellt, obwohl es mehrere eindeutige Gegenbelege gab, von denen zumindest diejenigen der Archaischen Texte aus Gendet Nasr bereits früh publiziert und jedermann zugänglich waren⁴. Als Folge

Figure 6: Discussion in [DE87, p. 117] of the discovery in [Fri78, pp. 9–11] (see Figure 3) of the different relations between ∇ and \bullet in systems G and ??.

there is in any case an important qualitative difference between IX for Latin novem and \bullet for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 7: The sign \bullet used in a parallel with IX in [Eng88, pp. 131–133 n. 9], discussing an argument from [Pow72, p. 172] on the question of the language of the Uruk III texts.

the derivation in [Fri78, p. 15] is based on proto-Elamite artefacts. Note that in Friberg's early works [Fri78; Fri79; Fri86; Fri87], copies of fourth millennium and sometimes third millennium tablets are shown as vertical text (which they were for the scribes), and their numerals are written within horizontal text in the same orientation that they have if the tablet is taken as vertical text; in [UAX50] parlance, as if they had Vertical_Orientation=Upright. In addition, they are listed in these equations in the horizontal order in which they appear as vertical text (thus the rightmost numeral is the most significant, read first). Cuneiform is correctly Vertical_Orientation=Rotated, consistently both with modern practice and with the rotation between earlier vertical and later horizontal monumental inscriptions. Friberg's early conventions are not followed in later scholarship, and are abandoned in his own more recent works, such as [Fri07]; a more typical way to express the first equations might be

$$\begin{aligned} 5\bullet + 4\nabla &= 3\bullet + 24\nabla & (\text{C } 234) \\ 1\nabla + 1\bullet + 5\nabla &= 7\bullet + 5\nabla & (\text{C } 314) \\ 1\text{𒌦} + 1\text{𒌦} + 1\nabla &= 1\text{𒌦} + 2\nabla + 6\bullet & (\text{C } 27) \end{aligned}$$

A diplomatic edition of [Fri78] could rotate the numerals using a higher-level protocol:

$$\begin{aligned} 4\text{𒌦} 5\bullet &= 24\text{𒌦} 3\bullet & (\text{C } 234) \\ 5\text{𒌦} 1\bullet 1\text{𒌦} &= 5\text{𒌦} 7\bullet & (\text{C } 314) \\ 1\text{𒌦} 1\text{𒌦} 1\text{𒌦} &= 6\bullet 2\text{𒌦} 1\text{𒌦} & (\text{C } 27). \end{aligned}$$

of decreasing fractions $1/n$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus $\Xi = 1/2 N_{39}$, $\var� = 1/3 N_{39}$, and so on. The first sign of the latter units, N_{34} ,

Figure 8: Description of the fractions Ξ and $\var�$ in [Eng98, p. 113]⁶¹.

For instance, the first line contains the notations $1N_{34} 1N_{39a}$; $2N_{20}$, which can be translated "60 of the (grain rations containing) Ξ (of grain); (grain involved:) $2 \bullet$ (of ground barley)". This calculation contradicts the assumed numerical relationship $10N_1 = 1N_{14}$, since as was well known the measure represented by the sign N_{39} was $1/5$ of that represented by N_1 , so that $60 \times 1/5 = 12$ and not 20, as $2N_{14}$ would imply. Instead of relying on complicated

Figure 9: The sign Ξ used as a capacity measure within otherwise translated text in [Eng98, p. 116].

Die halbkreisförmigen Griffleindrücke gehen manchmal in mehr oder weniger eckige Formen über (∇)⁶⁸⁵. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen (∇), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren⁶⁸⁶. Selten treten mit $\var�$ gebildete Zahlen auf⁶⁸⁷ (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 10: Discussion of co-occurrences and contrasts between ∇ , ∇ , and $\var�$ in [Kre98, p. 303].

The calculations:									
Obv. i	1	$60 \times 1/5 \nabla$	(Ξ)	=	$12 \times \nabla$	=	$2 \times \bullet$		
	2	$120 \times 1/10 \nabla$	(Ξ)	=	$12 \times \nabla$	=	$2 \times \bullet$		
	3	$120 \times 1/15 \nabla$	($\var�$)	=	$8 \times \nabla$	=	$1 \times \bullet$	$2 \times \nabla$	
	4	$300 \times 1/20 \nabla$	(Ξ)	=	$15 \times \nabla$	=	$2 \times \bullet$	$3 \times \nabla$	
	5	$600 \times 1/25 \nabla$	(Ξ)	=	$24 \times \nabla$	=	$4 \times \bullet$		
Rev. i	1	1200			$1 \times \bullet$	$1 \times \bullet$	$5 \times \nabla$		
Obv. i	6	$6000 \times 1/30 \nabla$	(GAR+6N ₅₇)	=	$200 \times \nabla$	=	$1 \times \nabla$	$3 \times \bullet$	$2 \times \nabla$
ii	1	$120 \times \approx 1/4 \nabla$	(DUG ₃ +U ₂₃)	\approx	$30 \times \nabla$	=	$5 \times \bullet$	$1 \times \nabla$	$1 \times \Xi$
	2	$180 \times 1/5 \nabla$	(DUG+AS ₉)	=	$36 \times \nabla$	=	$6 \times \bullet$		
	3	$300 \times 1/15 \nabla$	(KAS ₉)	=	$20 \times \nabla$	=	$3 \times \bullet$	$2 \times \nabla$	
Rev. i	3	600			$1 \times \bullet$	$4 \times \bullet$	$3 \times \nabla$		$1 \times \Xi$
					$1 \times \nabla$	$1 \times \bullet$	$5 \times \nabla$		
					$1 \times \nabla$	$3 \times \bullet$	$2 \times \nabla$		
					$1 \times \bullet$	$4 \times \bullet$	$3 \times \nabla$		$1 \times \Xi$
Grand total of groats used:					$1 \times \nabla$	$2 \times \bullet$	$9 \times \bullet$	$4 \times \nabla$	$1 \times \Xi$
Grand total of malt used: $1N_{47} 4N_{20} 3N_5 1N_{42a}$ (rev. i 3) $\times 3/5$					=	$8 \times \bullet$	$4 \times \nabla$	$1 \times \Xi$	

Figure 6. Transliteration and calculations of MSVO 4, 66.

Figure 11: Calculations from [P005468] transcribed in [Eng01, p. 132] using modern mathematical notation combined with some of the proposed characters.

⁶¹The text erroneously has N_{34} instead of N_{24} .





strong similarities between “area” 1 and “area” 3 systems, the sign with two concentric discs (, notated N_{50} ²⁷) remains problematic. It never appears in any numerical combination with the sign with a single disc (,

Figure 12: Discussion of  and ⁶² in [Cha03, p. 6].

$1/15$, etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation $\text{gur} \text{---} \text{gur}$ could be understood as $1 \frac{1}{5}$ gur.²⁷ However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs gur^+ and gur^- cannot be considered klatmatograms.

Metrological tablets from the end of the 4th millennium (Nissen, Damerow and Englund 1993, 55-59, to *MSVO* 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

 1(iku) represents a surface of 3600m²
 1(eše₃) represents a surface of 21,600m²
 etc.

The signs iku and eše₃ constitute by themselves measures of surface areas. These measures are usually followed by the sign GAN₂, which means either surface or field and

Figure 13: Discussion of Old Babylonian⁶⁴ capacity and fourth millennium area measures in [Pro09, p. 9].

formed by only two signs gur^+ and gur^- , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60², 60³, $1/60$, $1/60^2$, etc. are written in the same way, with the vertical wedge gur^+ . The third feature concerns the exact function of

Figure 14: Description of the SPVS in [Cha12, p. 58], using the already-encoded signs gur^+ and gur^- .













one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648, 000 with:    but never with the repetition   .

Figure 15: Discussion of large numbers illustrated by   ⁶⁵ in [Cha12, p. 59]

⁶²The statement that these do not co-occur refers to the texts from ED I-II Ur; these signs co-occur both earlier and later in areas, with different relations as previously discussed.

⁶⁴The cuneiform text is Unicode-encoded.

⁶⁵Compare   in system *G_{Ur III}/08*. Sign order can be variable in early texts, see [Fox16, p. 8]. See [P010773], also discussed in [Fri07, p. 148], for an example of  , and [P274845; P241764] for examples of n .

𐎶𐎶𐎶 še bar 𐎶 ba-rí-zu
'3 gubar (capacity units) and 1 parīšu'.

DDD ㉔ ㉕ ㉖ ㉗ ㉘ ㉙ ㉚ ㉛ ㉜ ㉝ ㉞ ㉟ ㊀ ㊁ ㊂ ㊃ ㊄ ㊅ ㊆ ㊇ ㊈ ㊉ ㊊ ㊋ ㊌ ㊍ ㊎ ㊏ ㊐ ㊑ ㊒ ㊓ ㊔ ㊕ ㊖ ㊗ ㊘ ㊙ ㊚ ㊛ ㊜ ㊝ ㊞ ㊟ ㊠ ㊡ ㊢ ㊣ ㊤ ㊦ ㊧ ㊨ ㊩ ㊪ ㊫ ㊬ ㊭ ㊮ ㊯ ㊰ ㊱ ㊲ ㊳ ㊴ ㊵ ㊶ ㊷ ㊸ ㊹ ㊺ ㊻ ㊼ ㊽ ㊾ ㊿

●●▷● in [Pro20, p. 350].

might think of one fabric and a half,¹¹ but the presence of notations with “22 20” “33 30”, and “66 60” (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Śuruppāg’s weight measurement system.¹² The notation “10 gada” in o. ii 1 and r. vi 1, along with the total of “39



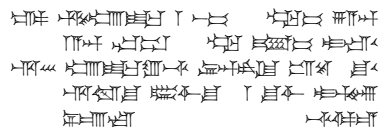
Fig. 1. Combinations of numerals attested in Š. 742.

 \supset and \supsetneq numerals in [Gor23, p. 162].

as, for example, in TM.75.G.3125 = *ARET* III 107 o. iv 1, “4^D 'a₃-da-um^{tu9}-2 ^A 4^U
aktum 4^Dib^{tu9}×3^T sa₆ gunu₃” (Fig. 2).

ecto 4 1] incorporating

3.5 Non-numeric usage



The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for ‘sixty’⁶⁶. Do you know its reading⁶⁷?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign has, *inter alia*, the values *aš*, *rum*, and *dili*. While the horizontal numerals are most frequently written in the Early Dynastic period⁶⁸, such non-numeric usage is almost⁶⁹ always written , for instance:

- in personal names in administrative texts, such as the following, which all contain numerals:
 - in [P010424, rev. 1 5; P010458, obv. 1 5; P010459, obv. 2 5'] from ED IIIa *أبو صلابيخ*,
 - in [P010960, obv. 2 5] from ED IIIa Šuruppag,
 - in [P251641, obv. 4 3] from ED IIIb Adab,
 - in [P252866, rev. 2 3] from ED IIIb Adab,
 - in [P298637, rev. 2 4] from ED IIIb Umma;
- in the Sumerian word *u₂-rum*, “property” in ED IIIb Nirsu administrative texts which contain numerals, such as [P020006, obv. 2 3; P020008, rev. 1 2; P020018, rev. 1 2; P020024, obv. 1 4; P020030, obv. 3 1];
- in lexical texts:
 - in the divine name in the lexical texts [P010570, rev. 2 4; P010572, obv. 3 6], where the entries are prefixed with .
 - in the word *dili*, “small fish” in [P010578, obv. 2 5], witness to Early Dynastic Fish,
 - in the same word with a determinative, *dili*^{ku₆}, in [P010586, obv. 4 4, 6], witness to Early Dynastic Food, which starts with numerals.

This is a clear contrast between and in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name occurs on its own line in the aforementioned administrative texts; a line would instead be read as “one slave”.

3.6 The limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in §3.1. While these benefits are real and

⁶⁶The reader will recall that *neš₂* is written , with a larger wedge than ; however, these signs have merged by the time Examenstext A is composed.

⁶⁷Besides *neš₂*, a look at [OSL] shows that the values *diš*, *ge₃*, *makkaš*, *san₂ak₄*, and *tal₄* are attested both in [ePSD₂] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

⁶⁸A [CDLI] search for “(asz@c)” finds 3296 ED texts, while a search for “(asz)” finds 81 ED texts, of which 46 also contain “(asz@c)”.





⁶⁹Exceptions are discussed in §3.7.1.


















3.7 Compatibility considerations







A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts⁷⁴ being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.


3.7.1 The case of ŠAR₂

The character U+122B9  CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period⁷⁵, a contrast between non-numeric šar₂ written  and numeric 1(šar₂) written  can be observed, similar to the contrast between  and  previously discussed in §3.5. However, in lexical lists from Šuruppag and Ebla⁷⁶, as well as in the *Stèle des vautours*, non-numeric šar₂ is curviform:












-     and     in [P010566, obv. 10 10, 11];
-   and    in [P010576, rev. 3 16, 17];
-   in [P240986, recto 3 3]⁷⁷;
-   in [P222399, obv. 17 9, 18 11, 22 12]⁷⁸.


It *would* be disruptive to the diachronic representation of text if non-numeric šar₂ were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph , identical to the glyph of the proposed U+12579  CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric šar₂ is curviform solidly predates the transition from  to  in the relevant metrological systems, there is no need to represent a - contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D  CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.




⁷⁴Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

⁷⁵For example, in personal names:

-    in [P020019, rev. 1 2] from ED IIIb Nirsu;
-    in [P020182, obv. 2 9], also from ED IIIb Nirsu;
-   in [P222186, obv. 3 3] from ED IIIb Umma;
-    in [P235312, obv. 16] from Old Akkadian Umma.

⁷⁶These are archaizing in other ways, *e.g.*, they have a  (NAM₂-TUG₂) split.

⁷⁷From copy in [Man81, Elles 397].

⁷⁸Note however    on [P222399, obv. 6 17], see Figure 21. Curviform non-numeric šar₂ is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here.

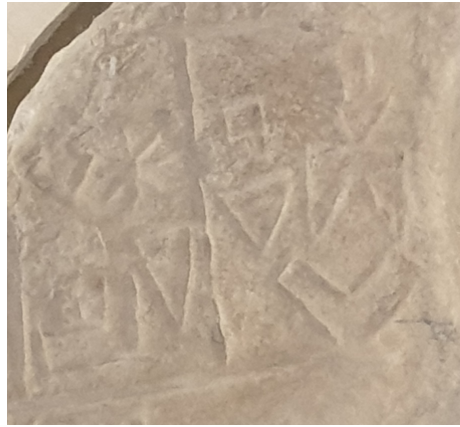


Figure 21: [P222399, obv. 6 16–17] 𒀭𒀭𒀭𒀭𒀭𒀭 / 𒀭𒀭𒀭𒀭𒀭𒀭.

Note that in rare cases, such as [P222243, obv. 2 7] from ED IIIa Adab, non-numeric 𒀭 (here with the value rum) is written 𒀭. It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 𒀭 cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. In practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the [CDLI] transliteration (UR2-1(aš@c)) and the [ePSD2] one (uru₈^{rum}) of this word disagree on that aspect. Since 𒀭 has a cuneiform reference glyph, this does not pose any compatibility concerns.




3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL₂ can be described as 𒀭𒀭𒀭𒀭 NE×KASKAL or 𒀭𒀭𒀭𒀭 NE×PAP⁷⁹, they are typically all transliterated bil₂, and therefore are all represented by the character U+1224B 𒀭 CUNEIFORM SIGN NE SHESHIG, its name notwithstanding, as described in [UTR56, §2.5].


The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, 𒀭𒀭𒀭𒀭 from [P305639, rev. 21] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in [EbDA] transliterations: the aforementioned 𒀭𒀭𒀭𒀭 𒀭𒀭𒀭𒀭 𒀭𒀭𒀭𒀭 𒀭𒀭𒀭𒀭 𒀭𒀭𒀭𒀭 from [P240533, recto 3 3] is transliterated “20-1-1/2 gu₂-bar 7 nig₂-sagšu 2-1/2 an-zam_x⁸⁰ za”, reading both 𒀭 and 𒀭 as 1/2, but not distinguishing them.


⁷⁹As on [P249253].

⁸⁰As of this writing, [EbDA] actually has an-zam_x, with U+1D6A GREEK SUBSCRIPT SMALL LETTER CHI.

In particular, these transliterations do not differentiate between \neg and \intercal numerals, nor between \triangleright and \triangleright numerals. For instance, the aforementioned [P242293, *recto* 4 1]  is transliterated “4 ‘a₃-da-um^{tug₂}-II 4 aktum^{tug₂} 4 ib₂-III gun₃ sa₆^{tug₂}” in [EbDA], with no distinction between the  and . Since \neg and \intercal numerals are separately encoded, the numeric expressions in such transliterations cannot be transformed into Unicode cuneiform without additional context, regardless of curviform–cuneiform unification.

In metrological systems such as systems $G_{Ur III/Ob}$ and $C_{Ur III/Ob}$ where some units are indicated by the type of numeral rather than an overt unit sign, it is common practice to add the unit in parentheses in transliteration; for instance, $\neg \neg \neg \neg \neg \neg \neg \neg$ from [P386847, obv. 1] is transliterated “1(eše₃) 5½ iku⁸¹ 7 sar” in [Feu04, vol. 2, p. 176], and $\neg \neg \neg \neg \neg \neg \neg \neg$ from [P307255, obv. 12] is transliterated “1(n⁸²) 2(b) 7 ½ sila₃” in [Feu04, vol. 2, p. 151].


This practice has been generalized to systematically indicate numeral shape; this is in particular the case in [CDLI], where the transliterations of some of the above examples are “1(gesz₂) 3(u) 5(asz) gur” for $\neg \neg \neg \neg \neg \neg \neg \neg$, “1(esze₃) 5(iku) 1/2(iku) GAN₂ 7(disz) sar” for $\neg \neg \neg \neg \neg \neg \neg \neg$, and “3(barig) 2(ban₂) 7(disz) 1/2(disz) sila₃” for $\neg \neg \neg \neg \neg \neg \neg \neg$. [CDLI] and [ePSD2] both distinguish curviform from cuneiform numerals in transliteration: the length  from [P020129, rev. 2 1] is transliterated “6(gesz₂@c) 3(u@c) {ninda}ninda_x(DU) 1/2(asz@c) 4(disz@t) gi” in [CDLI], and “6(geš₂^c) 3(u^c) ninda_xninda_x(DU) 1/2(aš^c) 4(diš^b) gi” in [ePSD2]. Another example is [Mol14, p. 39], which uses 1a for \neg , 1d for \intercal , 1ac for \triangleright , 1dc or ½dc for \triangleright depending on reading, etc. The literature on the Uruk and Early Dynastic I–II periods uses a different set of transliteration conventions that also disambiguate numeral shapes, as will be discussed in §4.

While there exist transliterations that distinguish \neg from \intercal but not  from \neg , such as the ones used in [DCCMT], the trend, especially in more recent works in third millennium studies, seems to be to represent numeral shape; for example, [MV24] gave an example of the input syntax used by the new “Urban Economy Begins” project as “10 + 5c(GUR) + 2(BARIGA) + 1(BAN₂)” for $\bullet \bullet \bullet \neg \neg \neg$, with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are \triangleright rather than \triangleright numerals. The “tradition of cavalierly dispensing with numerical notations in notations of administrative documents”, as [Eng04, p. 30] describes it, seems to be fading.

3.8 Conclusions

Co-occurrences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idiosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications. The contrast frequently applies to individual numerals, rather than to the span of entire numeric expressions.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple

⁸¹  interpreted as a unit, as discussed in §3.3.

⁸² short for nigida, an older reading of bariga; see [Landsberger1950; Pow75, p. 181; Fox22, p. 9].

cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted: \llcorner is a length of three ropes, but $\bullet\bullet$ is an area of three bur₃; $\triangleright\lrcorner$ could be read as one $\llcorner\lrcorner$ and one $\lrcorner\lrcorner$, where $\triangleright\lrcorner$ would be one and a half $\llcorner\lrcorner$; $\lrcorner\lrcorner$ is a personal name, but $\triangleright\lrcorner$ would be “one slave”.

In addition, there would be a risk of confusion about character identity should fontmakers attempt to treat the curviform and cuneiform numerals as unified. A designer concerned about the numeric-syllabic $\triangleright\lrcorner$ contrast, and wishing to support diachronic encoding between systems $S_{\text{Ur III/OB}}$ and S , might give the \lrcorner numeral series (which is typically only used numerically in the Early Dynastic period) the glyphs of the \triangleright numeral series, since the clear $\lrcorner\lrcorner$ identification involves the same rotation; this would however make it impossible to represent capacity measures that use \lrcorner . Similarly, in an effort to support diachronic encoding for 1/2(iku), one might be tempted to give \lrcorner the glyph of \lrcorner , thereby rendering the font unusable for quantities measured using the \lrcorner numeral series; an ED I–II Ur font designer could decide to give \lrcorner the same glyph as \lrcorner (that of the proposed \bullet), according to the older area system, making it impossible to represent the newer system.

At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millennium corpora. The oddity of \bullet requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.







4 Rationale for ED–Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millennium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millennium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs $KA\check{S}_a$, $KA\check{S}_b$, and $KA\check{S}_c$, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Eng01, pp. 34 sq.]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered scripts.








As part of the development of these conventions, a classification of fourth millennium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter N with a numeric subscript (sometimes with an additional alphabetic subscript): N_1 is \triangleright , N_{14} is \bullet , N_{34} is \lrcorner , etc. Transliterations of numeric expression then use those to identify the type of number used, thus $5N_1$ is $\lrcorner\lrcorner\lrcorner$, and $5N_{14}$ is $\bullet\bullet\bullet$.

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where

numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the [CDLI] transliteration of    or   in [P283802, rev. 1 6, 2 2] currently uses (barig@c) for the vertical numerals, since  numerals are typically capacity measures; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millennium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I–II period, even though the Sumerian text uses classical assyriological transliteration conventions; see [Cha03, p. 6 n. 27].

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millennium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in §3.7.2; they are indeed used interchangeably, as in [P01104] which uses the notation u@f in [ePSD2], but N14@f in [CDLI]. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millennium would therefore induce confusion as to which numerals should be used in third millennium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millennium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance,  N_{30c} is listed as a variant of  N_{30a} in [DE87, p. 166], where the numeric value of either in relation to  N_{39a} is still unknown, but their values are found in [Eng04, p. 33] to be  = $\frac{1}{10}$ , whereas  = $\frac{1}{6}$ .

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban₂@c) and 5(ban₂@c).]

6 Characters not included in this proposal

6.1 Missing numerals

TODO “Ten of the sixty numerical signs contained in the list in figure 27, moreover, do not belong to any of the identified systems. Three of them were apparently scribbled by an awkward pupil. As to four of those remaining, we are not sure whether they constitute derivations of other, as yet unknown numerical signs or whether they are in fact numerical signs at all. For at least two of the ten signs, txi and we can affirm that each formed part of two additional systems, about which we know nothing due to the fact that no informative texts have been unearthed with notations in these systems.” [NDE93, p. 27] TODO N10 described as coming from P001319 which does not have it anymore. TODO N13 not attested in CDLI TODO (N₁₇ not usefully numeric, 12N₁₄ not encodable, etc.). Cite [DE87, p. 147] N30C5 not attested 7 and 8(diš *tenû*) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.



Figure 23: The layout of case [P020066, obv. 1 1]; the numeral 22 is spread across two lines. The text is read in the order 22 1 year, “twenty-two oxen, one year old”.

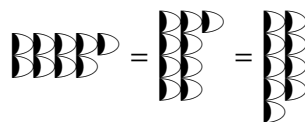


Figure 24: Three stacking patterns for U+12573 CUNEIFORM NUMERIC SIGN NINE N34. The one on the left is the reference glyph, used in Uruk III [P003499, obv. 1 1b; P004430, rev. 1 2], and widely afterwards, *e.g.*, ED IIIa Šuruppag [P010678, obv. 2], ED IIIb Nirsu [P020057, obv. 1 3], Old Akkadian Umma [P212464, obv. 11]. The ones in the middle and right are used in two Uruk IV tablets [P001243, rev. P004500, rev. 2]. All three Uruk examples are transliterated 9(N34) in [CDLI].

astic period, but that are also attested in the fourth millennium in the Uruk III period; the fourth millennium, especially the Uruk IV period, also frequently features numerals that use a more vertical layout, as illustrated in Figure 24. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millennium, usage in third millennium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents 9, 9, •, and • must remain consistent across the numeral series, lest a 9 numeral be confused with an 9 numeral. Since the single indents are frequently used in running text, as illustrated in §3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 24 uses OpenType stylistic alternates, and Figure 22 rotates the character 22, in both cases preserving the plain text backing.

6.3 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Engo1].

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

References

Artefacts

- [P001243] VAT 14991. Vorderasiatisches Museum.
CDLI: [P001243](#).
- [P003499] Excavation number W 20274,001.
CDLI: [P003499](#).
- [P004430] IM 074345. المتحف العراقي.
CDLI: [P004430](#).
- [P004500] IM 134965. المتحف العراقي.
CDLI: [P004500](#).
- [P005468] IM 023426. المتحف العراقي.
CDLI: [P005468](#).
- [P010424] IM 067642. المتحف العراقي.
CDLI: [P010424](#).
ORACC: [epsd2/P010424](#).
- [P010458] IM 081445. المتحف العراقي.
CDLI: [P010458](#).
ORACC: [epsd2/P010458](#).
- [P010459] IM 081449. المتحف العراقي.
CDLI: [P010459](#).
ORACC: [epsd2/P010459](#).
- [P010566] VAT 12760 +. Vorderasiatisches Museum.
CDLI: [P010566](#).
ORACC: [epsd2/P010566](#).
ORACC: [dcclt/P010566](#).
- [P010570] VAT 12626. Vorderasiatisches Museum.
CDLI: [P010570](#).
ORACC: [epsd2/P010570](#).
ORACC: [dcclt/P010570](#).
- [P010572] VAT 12644. Vorderasiatisches Museum.
CDLI: [P010572](#).
ORACC: [epsd2/P010572](#).
ORACC: [dcclt/P010572](#).
- [P010576] VAT 12751 +. Vorderasiatisches Museum.
CDLI: [P010576](#).
ORACC: [dcclt/P010576](#).
- [P010578] VAT 12693. Vorderasiatisches Museum.
CDLI: [P010578](#).
ORACC: [dcclt/P010578](#).
- [P010586] VAT 12770. Vorderasiatisches Museum.
CDLI: [P010586](#).
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[P010678]	VAT 12593. Vorderasiatisches Museum. CDLI: P010678 . ORACC: dccmt/P010678 .
[P010773]	Ist Š 0188. İstanbul Arkeoloji Müzeleri. CDLI: P010773 . ORACC: dccmt/P010773 .
[P010876]	Ist Š 0648. İstanbul Arkeoloji Müzeleri. CDLI: P010876 . ORACC: epsd2/P010876 .
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[P020006]	VAT 04439. Vorderasiatisches Museum. CDLI: P020006 . ORACC: epsd2/P020006 .
[P020008]	VAT 04430. Vorderasiatisches Museum. CDLI: P020008 . ORACC: epsd2/P020008 .
[P020016]	VAT 04865. Vorderasiatisches Museum. CDLI: P020016 . ORACC: epsd2/P020016 .
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- [P020065] VAT 04639. Vorderasiatisches Museum.
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ORACC: [epsd2/P020065](#).
- [P020066] VAT 04810. Vorderasiatisches Museum.
CDLI: [P020066](#).
ORACC: [epsd2/P020066](#).
- [P020090] VAT 04609. Vorderasiatisches Museum.
CDLI: [P020090](#).
ORACC: [epsd2/P020090](#).
- [P020092] VAT 04428. Vorderasiatisches Museum.
CDLI: [P020092](#).
ORACC: [epsd2/P020092](#).
- [P020129] VAT 04713. Vorderasiatisches Museum.
CDLI: [P020129](#).
ORACC: [epsd2/P020129](#).
- [P020137] VAT 04899. Vorderasiatisches Museum.
CDLI: [P020137](#).
ORACC: [epsd2/P020137](#).
- [P020182] VAT 04405. Vorderasiatisches Museum.
CDLI: [P020182](#).
ORACC: [epsd2/P020182](#).
- [P102305] X.3.139. Michael C. Carlos Museum, Emory University.
CDLI: [P102305](#).
ORACC: [epsd2/P102305](#).
- [P142357] YBC 01793. Yale Babylonian Collection.
CDLI: [P142357](#).
ORACC: [epsd2/P142357](#).
- [P142827] Ashm 1924-0667. Ashmolean Museum.
CDLI: [P142827](#).
ORACC: [epsd2/P142827](#).
- [P212464] WML unn 003. World Museum Liverpool.
CDLI: [P212464](#).
- [P213162] PULiège 0028. Université de Liège.
CDLI: [P213162](#).
ORACC: [epsd2/P213162](#).
ORACC: [dccmt/P213162](#).
- [P215653] AS 15375 21. Musée du Louvre.
CDLI: [P215653](#).
ORACC: [dcclt/P215653](#).
Louvre Collections: [ark:/53355/cl010436723](#).
- [P220927] AO 13485. Musée du Louvre.
CDLI: [P220927](#).
ORACC: [epsd2/P220927](#).

- [P221266] AO 13825. Musée du Louvre.
CDLI: [P221266](#).
ORACC: [epsd2/P221266](#).
Louvre Collections: [ark:/53355/cl010138527](#).
- [P221291] AO 13850. Musée du Louvre.
CDLI: [P221291](#).
ORACC: [epsd2/P221291](#).
- [P221305] AO 13864. Musée du Louvre.
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- [P221530] YBC 08446. Yale Babylonian Collection.
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- [P221531] YBC 08444. Yale Babylonian Collection.
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- [P221746] Erm 14039. Государственный Эрмитаж.
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- [P221814] Erm 14045. Государственный Эрмитаж.
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- [P221815] Erm 14046. Государственный Эрмитаж.
CDLI: [P221815](#).
ORACC: [epsd2/P221815](#).
- [P222186] FLP 0003. Free Library of Philadelphia.
CDLI: [P222186](#).
ORACC: [epsd2/P222186](#).
- [P222243] OIM A00645 + OIM A00649a–i. Institute for the Study of Ancient Cultures, West Asia & North Africa (formerly Oriental Institute) Museum.
CDLI: [P222243](#).
ORACC: [epsd2/P222243](#).
- [P222399] *Stèle des vautours*. BM 023580 (= AO 16109) + AO 00050 + AO 02436 + AO 02437 + AO 02438. Musée du Louvre.
CDLI: [P222399](#).
ORACC: [etcsri/Q001056](#).
- [P232278] *Gudea E*. AO 00006. Musée du Louvre.
CDLI: [P232278](#).
ORACC: [etcsri/Q001544](#).
- [P232280] *Gudea G*. AO 00007. Musée du Louvre.
CDLI: [P232280](#).
ORACC: [etcsri/Q001546](#).
- [P235312] USC 6710. University of Southern California.
CDLI: [P235312](#).
ORACC: [epsd2/P235312](#).

- [P240531] Excavation number TM.75.G.00265.
CDLI: [P240531](#).
EbDA: [1415](#).
- [P240532] Excavation number TM.75.G.00266.
CDLI: [P240532](#).
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- [P240533] Excavation number TM.75.G.00267.
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EbDA: [1379](#).
- [P240545] Excavation number TM.75.G.00299.
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- [P240548] Excavation number TM.75.G.00302.
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- [P240579] Excavation number TM.75.G.00341.
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- [P240597] Excavation number TM.75.G.00407.
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- [P240609] Excavation number TM.75.G.00440.
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- [P240653] Excavation number TM.75.G.00535.
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- [P240654] Excavation number TM.75.G.00536.
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- [P240655] Excavation number TM.75.G.00537.
CDLI: [P240655](#).
EbDA: [1358](#).
- [P240675] Excavation number TM.75.G.00557.
CDLI: [P240675](#).
EbDA: [1371](#).
- [P240697] Excavation number TM.75.G.00579.
CDLI: [P240697](#).
EbDA: [1381](#).
- [P240964] Excavation number TM.75.G.01392.
CDLI: [P240964](#).
ORACC: [dccmt/P240964](#).
EbDA: [3184](#).
- [P240986] Excavation number TM.75.G.01415.
CDLI: [P240986](#).
ORACC: [dcclt/P240986](#).

- [P241708] Excavation number TM.75.G.02143.
CDLI: [P241708](#).
EbDA: [3173](#).
- [P241764] Excavation number TM.75.G.02200.
CDLI: [P241764](#).
- [P241904] Excavation number TM.75.G.02346.
CDLI: [P241904](#).
EbDA: [3183](#).
ORACC: [dccmt/P241904](#).
- [P242293] Excavation number TM.75.G.03125.
CDLI: [P242293](#).
EbDA: [217](#).
- [P249253] *Code de Hammurabi*. Sb 00008. Musée du Louvre.
CDLI: [P249253](#).
- [P251641] MS 2464. Schøyen Collection.
CDLI: [P251641](#).
ORACC: [epsd2/P251641](#).
- [P252866] MS 3830. Schøyen Collection.
CDLI: [P252866](#).
ORACC: [epsd2/P252866](#).
- [P255010] YBC 04698. Yale Babylonian Collection.
CDLI: [P255010](#).
- [P271238] Anonymous.
CDLI: [P271238](#).
ORACC: [epsd2/P271238](#).
- [P274845] CUNES 50-08-001. Department of Near Eastern Studies, Cornell University.
CDLI: [P274845](#).
- [P283802] Ist Š 0742. Arkeoloji Müzeleri.
CDLI: [P283802](#).
ORACC: [epsd2/P283802](#).
- [P292843] NBC 05385. Nies Babylonian Collection, Yale Babylonian Collection.
CDLI: [P292843](#).
- [P298637] NBC 06978. Nies Babylonian Collection, Yale Babylonian Collection.
CDLI: [P298637](#).
ORACC: [epsd2/P298637](#).
- [P305639] YBC 04398. Yale Babylonian Collection.
CDLI: [P305639](#).
- [P307255] YBC 06219. Yale Babylonian Collection.
CDLI: [P307255](#).
- [P309594] YBC 08761. Yale Babylonian Collection.
CDLI: [P309594](#).
ORACC: [epsd2/P309594](#).
- [P386847] AO 06377. Musée du Louvre.
CDLI: [P386847](#).

- [Q000782] *The instructions of Šuruppag*. Composite text.
CDLI: [Q000782](#).
ORACC: [epsd2/Q000782](#).
ETCSL transliteration: [c.5.6.1](#); translation: [t.5.6.1](#).

Unicode documents

- [L2/03-162] M. Everson and K. Feuerherm. *Basic principles for the encoding of Sumero-Akkadian Cuneiform*. 25th May 2003.
UTC: [L2/03-162](#).
ISO/IEC JTC 1/SC 2/WG 2: [N2585](#).
- [L2/03-393R] M. Everson, K. Feuerherm and S. Tinney. *Preliminary proposal to encode the Cuneiform script in the SMP of the UCS*. 3rd Nov. 2003.
UTC: [L2/03-393R](#).
ISO/IEC JTC 1/SC 2/WG 2: [N2664R](#).
- [L2/04-036] M. Everson, K. Feuerherm and S. Tinney. *Revised proposal to encode the Cuneiform script in the SMP of the UCS*. 29th Jan. 2004.
UTC: [L2/04-036](#).
ISO/IEC JTC 1/SC 2/WG 2: [N2698](#).
- [L2/04-099] L. Anderson. *Unification of Cuneiform Numbers*. 2004.
UTC: [L2/04-099](#).
- [L2/04-189] M. Everson, K. Feuerherm and S. Tinney. *Final proposal to encode the Cuneiform script in the SMP of the UCS*. 8th June 2004.
UTC: [L2/04-189](#).
ISO/IEC JTC 1/SC 2/WG 2: [N2786](#).
- [L2/12-208] M. Everson, C. Jay Crisostomo and S. Tinney. *Proposal for Early Dynastic Cuneiform*. 13th June 2012.
UTC: [L2/12-208](#).
ISO/IEC JTC 1/SC 2/WG 2: [N4278](#).
- [L2/23-190] A. Pandey. *Revised proposal to encode Proto-Cuneiform in Unicode*. 11th July 2023.
UTC: [L2/23-190](#).
- [L2/23-196] A. Pandey. *Proposal to encode Proto-Elamite in Unicode*. 18th Aug. 2023.
UTC: [L2/23-196](#).
- [L2/24-159] P. Constable, ed. *Minutes of UTC Meeting 180* (Redmond, 23rd–25th July 2024). 29th July 2024.
UTC: [L2/24-159](#).
- [UAX50] K. Lunde and K. Ishii, eds. *Unicode Vertical Text Layout*. Unicode Standard Annex #50. An integral part of *The Unicode Standard*. The Unicode Consortium.
<https://www.unicode.org/reports/tr50/>.
- [Uni16] The Unicode Consortium. *The Unicode Standard*. Version 16.0.0. The Unicode Consortium, 10th Sept. 2024.
ISBN: 978-1-936213-34-4.
<https://www.unicode.org/versions/Unicode16.0.0/core-spec/>.

- [UTR56] R. Leroy, ed. *Unicode Cuneiform Sign Lists*. Unicode Technical Report #56. The Unicode Consortium.
<https://www.unicode.org/reports/tr56/>.

Online corpora and related projects

- [CDLI] É. Pagé-Perron, J. L. Dahl, B. Lafont, J. Renn, R. K. Englund and P. Damerow, eds. *Cuneiform Digital Library Initiative*. 2000–.
<https://cdli.mpiwg-berlin.mpg.de>.
- [DCCMT] E. Robson, ed. *The Digital Corpus of Cuneiform Mathematical Texts*. 2007–.
ORACC: [dccmt](https://oracc.org/dccmt).
- [EbDA] L. Milano, M. Maiocchi, F. Di Filippo, R. Orsini, E. Scarpa, M. Surdi et al., eds. *Ebla Digital Archives*. 2007–.
<http://ebda.cnr.it/>.
- [eBL] E. Jiménez, Z. Földi, A. Härtinen, A. Heinrich, T. Mitto, G. Rozzi, I. Khait, J. Laasonen, F. Simonjetz et al., eds. *electronic Babylonian Library*. 2023–.
<https://www.ebl.lmu.de/>.
- [ePSD2] S. Tinney, P. Jones and N. Veldhuis, eds. *The electronic Pennsylvania Sumerian Dictionary*. 2nd ed. 2017–.
<http://oracc.org/epsd2>.
- [ETCSL] J. A. Black, G. Cunningham, J. Ebeling, E. Flückiger-Hawker, E. Robson, J. Taylor and G. Zólyomi, eds. *The Electronic Text Corpus of Sumerian Literature*. Oxford, 1998–2006.
<http://etcs1.orinst.ox.ac.uk/>.
- [OSL] N. Veldhuis, S. Tinney et al., eds. *Oracc Sign List*. 2014–.
<http://oracc.org/osl/>.

Other documents

- [Arc15] A. Archi. *Ebla and Its Archives. Texts, History, and Society*. Studies in ancient Near Eastern records 7. Walter de Gruyter, 2015.
ISBN: 978-1-61451-716-0.
DOI: [10.1515/9781614517887](https://doi.org/10.1515/9781614517887).
- [Arc89] A. Archi. “Tables de comptes eblaïtes”. In: *Revue d’assyriologie et d’archéologie orientale* 83.1 (1989). Ed. by P. Amiet and P. Garelli, pp. 1–6. ISSN: 0373-6032.
- [Bor10] R. Borger. *Mesopotamisches Zeichenlexikon*. Alter Orient und Altes Testament 305. Ugarit-Verlag, 2010.
- [Cap02] R. Caplice. *Introduction to Akkadian*. 4th ed. Editrice Pontificio Istituto Biblico, 2002.
ISBN: 88-7653-566-7.
- [Cat13] A. Catagnoli. *La paleografia dei testi dell’amministrazione e della cancelleria di Ebla*. Quaderni di Semitistica 9. Università di Firenze, 2013.
ISBN: 8890134054.

- [Cha03] G. Chambon. "Archaic Metrological Systems from Ur". In: *Cuneiform Digital Library Journal* 2003.5 (23rd Dec. 2003). ISSN: 1540-8779. http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003_005.html.
- [Cha12] G. Chambon. "Numeracy and Metrology". In: *The Oxford Handbook of Cuneiform Culture*. Ed. by K. Radner and E. Robson. Oxford University Press, 18th Sept. 2012, pp. 51–67. ISBN: 9780199557301. DOI: [10.1093/oxfordhb/9780199557301.013.0003](https://doi.org/10.1093/oxfordhb/9780199557301.013.0003).
- [DE87] P. Damerow and R. K. Englund. "Die Zahlzeichensysteme der archaischen Texte aus Uruk". In: M. W. Green and H. J. Nissen. *Zeichenliste der archaischen Texte aus Uruk*. Archaische Texte aus Uruk 2. Gebr. Mann Verlag, 1987. Chap. 3, pp. 117–165. Repr. <https://cdli.mpiwg-berlin.mpg.de/files-up/publications/englund1987a.pdf>.
- [Dei22] A. Deimel. *Liste der archaischen Keilschriftzeichen von Fara*. Wissenschaftliche Veröffentlichungen der Deutschen Orient-Gesellschaft 40. J. C. Hinrichs'sche Buchhandlung, 1922.
- [Eng01] R. K. Englund. "Grain Accounting Practices in Archaic Mesopotamia". In: *Changing Views on Ancient Near Eastern Mathematics*. Ed. by J. Høyrup and P. Damerow. Berliner Beiträge zum Vorderen Orient 19. Dietrich Reimer Verlag, 2001, pp. 1–35.
- [Eng04] R. K. Englund. "Proto-Cuneiform Account-Books and Journals". In: *Creating Economic Order. Record-keeping, Standardization and the Development of Accounting in the Ancient Near East*. Ed. by M. Hudson and C. Wunsch. International Scholars Conference of Ancient Near Eastern Economies 4. CDL Press, 2004. Chap. 1, pp. 23–46.
- [Eng88] R. K. Englund. "Administrative Timekeeping in Ancient Mesopotamia". In: *Journal of the Economic and Social History of the Orient* 31.2 (1988).
- [Eng98] R. K. Englund. "Texts from the Late Uruk Period". In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 13–233. ISBN: 3-7278-1166-8.
- [Feu04] K. G. Feuerherm. "Abum-waqar and His Circle. A Prosopographical Study". PhD thesis. University of Toronto, 2004.
- [Fox16] D. A. Foxvog. "Introduction to Sumerian Grammar". In: *Cuneiform Digital Library Preprints* 2016.2 (4th Jan. 2016). <https://cdli.mpiwg-berlin.mpg.de/articles/cdlp/2.0>.
- [Fox22] D. A. Foxvog. "Elementary Sumerian Glossary (revised 2022)". In: *Cuneiform Digital Library Preprints* 2022.3.1 (11th Apr. 2022). <https://cdli.mpiwg-berlin.mpg.de/articles/cdlp/3.1>.
- [Fri07] J. Friberg. *A Remarkable Collection of Babylonian Mathematical Texts*. Sources and Studies in the History of Mathematics and Physical Sciences. Springer, 2007. Manuscripts in the Schøyen Collection Cuneiform Texts 1. Manuscripts in the Schøyen Collection 6. ISBN: 978-0-387-34543-7.

- [Fri78] J. Friberg. *A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and Proto-Elamite Semi-Pictographic Inscriptions*. The Third Millenium Roots of Babylonian Mathematics 1. Department of Mathematics, Chalmers University of Technology, 1978.
- [Fri79] J. Friberg. *The Early Roots of Babylonian Mathematics*. 2. Department of Mathematics, Chalmers University of Technology, 1979.
- [Fri86] J. Friberg. “Three Remarkable Texts from Ancient Ebla”. In: *Vicino Oriente* 6 (1986), pp. 3–25. ISSN: 0393-0300. The Early Roots of Babylonian Mathematics 3.
- [Fri87] J. Friberg. “Mathematik”. In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš–Medizin. 1987–1990, pp. 531–585.
- [Gor23] F. Gori. “On Lapis Lazuli and Linen in Šuruppag Texts. An Analysis Through the Lens of Ebla Studies”. In: *Studia Eblaitica* 9 (2023), pp. 160–166. ISSN: 2364-7124.
- [Hue11] J. Huehnergard. *A Grammar of Akkadian*. 3rd ed. Brill, 2011. ISBN: 978-1-57506-941-8.
- [JJ24] T. Jauhiainen and H. Jauhiainen. “Advancing Cuneiform Text Dating Through Automatic Analysis”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 11th July 2024 14:00.
- [Kre98] M. Krebern timer. “Die Texte aus Fāra und Tell Abū Šalābīḥ”. In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 235–427. ISBN: 3-7278-1166-8.
- [Lec12] C. Lecompte. “Des chiffres et des digues: à propos de deux textes présargoniques de Ġirsu et d’une notation numérique inhabituelle”. In: *Altorientalische Forschungen* 39.1 (Dec. 2012), pp. 81–86. DOI: [10.1524/aof.2012.0006](https://doi.org/10.1524/aof.2012.0006).
- [Lec16] C. Lecompte. “ED IIIb metrology: texts from Lagaš”. In: *CDLI:wiki. A Library of Knowledge of the Cuneiform Digital Library Initiative*. 12th Apr. 2016. https://cdli.ox.ac.uk/wiki/doku.php?id=ed_iii_metrological_systems.
- [Lec20] C. Lecompte. “The Measurement of Fields During the Pre-sargonic Period”. In: *Mathematics, Administrative and Economic Activities in Ancient Worlds*. Ed. by C. Michel and K. Chemla. Why the Sciences of the Ancient World Matter 5. Springer, 2020. Chap. 8, pp. 283–344.
- [Man81] P. Mander. “Lista dei segni dei testi lessicali di Ebla”. In: *Testi lessicali monolingui della biblioteca L. 2769*. Ed. by G. Pettinato. Materiali epigrafici di Ebla 3. Napoli: Istituto universitario orientale, 1981, pp. 285–382.
- [Mol14] M. Molina. *Sargonic Cuneiform Tablets in the Real Academia de la Historia. The Carl L. Lippmann Collection*. Real Academia de la Historia, 2014. ISBN: 978-84-15069-71-3.

- [MV24] M. Maiocchi and S. Volpi. “Reassessing Economic History in the Early Dynastic Period. Sources, Methods, and Perspectives within the frame of the “Urban Economy Begins” Project”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 12th July 2024 16:00.
- [NDE93] H. J. Nissen, P. Damerow and R. K. Englund. *Archaic Bookkeeping. Early Writing and Techniques of Economic Administration in the Ancient Near East*. Trans. by P. Larsen. The University of Chicago Press, 1993.
ISBN: 0-226-58659-6.
- [Oel22] J. Oelsner. *Der Kodex Hammu-rāpi*. dubsar 4. Zaphon, 2022.
ISBN: 978-3-96327-008-6.
- [Pow71] M. Powell. “Sumerian Numeration and Metrology”. PhD thesis. University of Minnesota, 1971.
- [Pow72] M. Powell. “Sumerian Area Measures and the Alleged Decimal Substratum”. In: *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* 62.2 (1972), pp. 165–221. ISSN: 0084-5299.
- [Pow75] M. Powell. In: *Journal of Cuneiform Studies* 27.3 (July 1975), pp. 180–188. Rev. of H. Limet. *Étude de documents de la période d'Agadé appartenant à l'Université de Liège*. Bibliothèque de la Faculté de Philosophie et Lettres de l'Université de Liège 206. Paris: Les Belles Lettres, 1973.
- [Pow87] M. Powell. “Maße und Gewichte”. In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš–Medizin. 1987–1990, pp. 457–530.
- [Pro09] C. Proust. “Numerical and Metrological Graphemes: From Cuneiform to Transliteration”. In: *Cuneiform Digital Library Journal* 2009.1 (22nd June 2009). ISSN: 1540-8779.
http://cdli.ucla.edu/pubs/cdlj/2009/cdlj2009_001.html.
- [Pro20] C. Proust. “Early-Dynastic Tables from Southern Mesopotamia, or the Multiple Facets of the Quantification of Surfaces”. In: *Mathematics, Administrative and Economic Activities in Ancient Worlds*. Ed. by C. Michel and K. Chemla. Why the Sciences of the Ancient World Matter 5. Springer, 2020. Chap. 9, pp. 345–395.
- [Rob08] E. Robson. *Mathematics in Ancient Iraq. A Social History*. Princeton University Press, 2008.
ISBN: 978-0-691-09182-2.
- [Rob19] E. Robson. “Oracc metrology guidelines”. In: *Oracc: The Open Richly Annotated Cuneiform Corpus*. 18th Dec. 2019.
ORACC: [doc/help/editinginf/metrology/metrologicaltables](http://oracc.berkeley.edu/doc/help/editinginf/metrology/metrologicaltables).
- [Rob22] E. Robson. “Overview of Metrological Systems”. In: *The Digital Corpus of Cuneiform Mathematical Texts*. 2022.
ORACC: [dccmt/Metrology](http://oracc.berkeley.edu/dccmt/Metrology).
- [Rom23] A. Romach. *Stylometric Analysis for Akkadian Cuneiform Texts*. 2023–.
<https://github.com/ARomach/Cuneiform-Stylometry>.

- [Rom24] A. Romach. “The Neo Assyrian Land Sale Documents from Dur-Katlimmu. A Stylometric Analysis of Their Scribal Features”. 69th Rencontre Assyriologique Internationale (Helsinki, 8th–12th July 2024). 10th July 2024 12:00.
- [Sch10] W. Schramm. *Akkadische Logogramme*. Göttinger Beiträge zum Alten Orient 5. Universitätsverlag Göttingen, 2010.
ISBN: 978-3-941875-65-4.
DOI: [10.17875/gup2010-511](https://doi.org/10.17875/gup2010-511).
- [Sch35] N. Schneider. *Die Keilschriftzeichen der Wirtschaftsurkunden von Ur III*. Editrice Pontificio Istituto Biblico, 1935.
- [Svä+24] S. Svärd, M. Lorenzon, J. Töyräänvuori, J. Valk, T. Alstola, E. Bennett, R. Uotila and T. Auranne, eds. *RAI 69 Abstracts*. July 2024.
https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäiväty_1.pdf.