Archaic cuneiform numbers

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Contents

1 Summary

This document proposes encoding some numerals used in the fourth millenium (Uruk IV and Uruk III) and Early Dynastic period in conjunction with the Sumero-Akkadian cuneiform script[1] and the proto-cuneiform script[2]. The proposed characters are listed in section 2. Most of them were listed in [**L2/23-190**]; however, the present document provides a more detailed rationale for their encoding and additional information about their identity.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed "curved", or "curviform", numerals[3] should however *not* be unified with the already-encoded cuneiform numerals[4]. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap. Many of these numerals are also used in proto-Elamite[5] texts, where they are treated as identical characters in scholarship on proto-Elamite, so that

[1]ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

[2]ISO 15924: Pcun, not yet encoded.

[3]Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: ▻, ▿ (small stylus), ▷ (large stylus). Some numerals are composed of multiple such impressions, *e.g.*, ▷•. The terms "curved", "curviform", and "round" can be found in the literature. We avoid the term "round" here as it has other meanings in the context of numbers. We use "curviform" in this document as, being the least common term, it is least likely to lead to confusion, and "CURVED" in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [**inlineATF**].

[4]Impressed into clay using a stylus with a trihedral end: 𒀸 (stylus held horizontally), 𒁹 (vertically), 𒀹 (diagonally) 𒌋 (with the head of the stylus), 𒁹 (stylus pressed deeper, forming a larger wedge), 𒁹𒌋 (combining 𒁹 and 𒌋), etc.

[5]ISO 15924: Pelm, not yet encoded.

they should be unified with the ones proposed in [**L2/23-196**]. However, in the interest of time, we do not provide a detailed rationale for this unification in this document, and we are not proposing that the numerals be given the corresponding Script_Extensions property value for now. Neither do we propose encoding any numerals that are solely attested in proto-Elamite texts, or well-attested in proto-Elamite texts but insufficiently attested in Uruk.

[TODO(egg): Mention the other sections here too.]

<table>
<tr><td></td><td>Uruk III & earlier</td><td>ED – Ur III</td><td>OB & later</td></tr>
<tr><td rowspan="2">Numerals</td><td colspan="2">This proposal</td><td></td></tr>
<tr><td></td><td colspan="2" rowspan="2">Existing Xsux</td></tr>
<tr><td>Non-numeric signs</td><td>Future Pcun</td></tr>
</table>

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties

2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [**Powell1972**] “in the same fashion as we use black and red ink”; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble 𒅎 in texts from Early Dynastic IIIa Šuruppag as in the character code charts, 𒅎 later in the third millenium[6], 𒅎 in Old Babylonian cursive, 𒅎 in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

[6]Merging with U+1224E CUNEIFORM SIGN NI2.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists[7] and dictionaries[8], and of composite texts[9]. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24][10]. The diachronic approach is also useful for pedagogic applications[11].

3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding[12] that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Ŋirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

[13]						
1(ŋeš$_2$)	1(u)	1/2(diš)	5(diš *tenû*)	gi	us$_2$	sa$_2$
7.5 (ropes)			5	reed	side	equal

[14]				
3(u)	6(diš *tenû*)	gi	saŋ	sa$_2$
3 (ropes)	6	reed	front	equal

ašag-bi	1(bur$_3$^{c})	1(eše$_3$^{c})	1(iku^c)	1/2(iku^c)
ašag=bi				
field=DEM[15]				

tug$_x$(LAK483)-si-ga-kam
tugsiga =ak =am -∅
ploughed=GEN=COP-3.SG.S

[7]Notably [OSL] and the online edition of [MZL] in [eBL, Signs].

[8]Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

[9]For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

[10]Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svä+24, pp. 242, 148].

[11]For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

[12]At that time scoped to the répertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as ≠ , were informed by Early Dynastic distinctions.

[13]As noted in [Pow87, p. 466], this sign has a very short "tail" in this period, so that it is wider than it is tall, and can at first seem like a large in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

[14]Note that ED IIIb numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign in [P020054] looks more like Ur III .

[15]Alternatively: area=POSS.3.SG.NH, "its area".

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as[16]

> 465 metres, equal lengths
> 198 metres, equal widths
> this field is 9, 18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that "the number signs do not normally carry in their individual signs the meaning of what they are used to measure", and that curviform and cuneiform numerals "are not normally mixed together in a single numerical expression", noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from ⊃ to 𒁹 numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that "The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases[17]."

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

— the functions and use of the numerals vary beyond the mere ⊃/𒁹 switch;
— the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
— some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

[16] We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

[17] The cuneiform encoding process was planned in *stages* in [L2/03-162]. One might expect the second stage of encoding, which led to the creation of the Early Dynastic Cuneiform block, to incorporate the numerals needed for the representation and discussion of Early Dynastic texts; however, the proposal [**L2/12-208**] stated that "numerals have been omitted due to the complexity of numeral signs from this period. An expert in the metrology of this period must be consulted before these can be properly included."

3.3 A primer on classic Ur III and Old Babylonian metrologies

> I want to write tablets: the tablet of 1 cor of barley to 600 cor; the tablet of 1 shekel of silver to 10 minas [...]
>
> Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known[18] a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millenium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters 𒁹–𒑆, the multiples of ten (10–50) are 𒌋–𒐐, but the other digits 11–59 are sequences 𒌋𒁹–𒐐𒑆; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units 𒁹–𒑆, tens 𒌋–𒐐, sixties 𒐕–𒐝 (with larger wedges than the units), multiples of six hundred 𒐞–𒐢, multiples of three thousand six hundreds 𒊹–𒐫, and multiples of thirty-six thousand 𒐬–𒐰.

3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram[19], where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\text{𒐬} \xleftarrow{10} \text{𒊹} \xleftarrow{6} \text{𒐞} \xleftarrow{10} \text{𒐕} \xleftarrow{6} \text{𒌋} \xleftarrow{10} \text{𒁹} \qquad (S_{\text{Ur III/OB}})$$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written 𒐟𒐜𒐏𒐎 in the discrete counting system, and 𒎙𒐍𒐏𒐎 in the sexagesimal place value system.

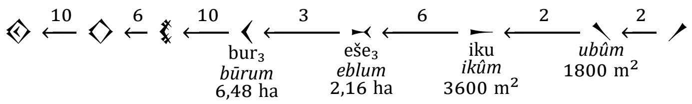
3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and

[18] See, *e.g.*, [Uni16, §22.3.3, *sub* "Cuneiform Numerals"].

[19] These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*, see Figure 3.

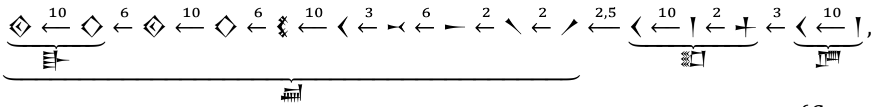
the approximate metric equivalent [Fri07, p. 378; Rob19]:



$(G_{\text{Ur III/OB}})$

Note that for the range of areas given above, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *būrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign 𒃷 functioning as punctuation[20], but the numerals are tied to the metrology; thus a surface of 5 *būr* 1 *ebel* 4 *ikû* (100 *ikû*, 36 ha) would be written[21] 𒐊𒑏𒐋𒃷. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in "88 acres 3 roods 33 perches" or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the sign ◇ is equal to sixty times 𒌋 in the area system, but to three hundred and sixty times 𒌋 in the discrete counting system.

For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 sar $=$ 1 *mūšarum* $\approx$ 36 m^2 written 𒁹𒊬, equal to one hundredth of an *ikûm*, then sexigesimally subdivided in 60 𒂆 (ŋin₄, *šiqlū*, shekels). For areas greater than 3600 *būr*, the ◇ and ◈ numerals are reused with a suffix 𒃲 (gal, "big"), as follows [Rob08, p. 295 n. b and c; Fri07, p. 378; Rob19]:

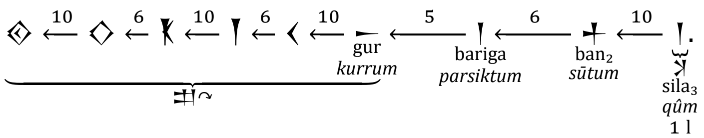


$(G_{\text{Ur III/OB}})$

e.g., ◇◇𒃲 ◈◈𒐏𒐏 𒑏𒃷 𒐊𒑏𒊬 𒌋𒐝𒂆[22] for $(2 \times 3600 + 20 \times 60 + 49)$ *būr* 5 *ikû* $(5 + \frac{1}{2})$ *mūšar* 19 *šiqil*. Factor diagrams in this document will use bottom curly brackets in this fashion to separate numerals from units and other suffixes.

3.3.3 The capacity system

Another such system of note is the one for capacities[23] [Fri07, p. 376; Rob19],



$(C_{\text{Ur III/OB}})$

In the above diagram, the numerals for ban₂ are 𒑏, 𒑐, 𒑑, 𒑒, and 𒑓, and those for bariga are 𒁹, 𒑖, 𒑗, and 𒑘 (contrast ordinary 𒈫 and 𒐈 otherwise used with 𒁹 numerals). Further, we have used the symbol ↷ to express that, as described in [Hue11, p. 585 n. (b) and (f)], the sign 𒄥 GUR, while it is used only with volumes in

[20] This sign is sometimes interpreted as a measurement unit, and transliterated iku, see, *e.g.*, [**Proust2020**], or transliterations in [Feu04] discussed in section 3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system $G_{\text{Ur III/OB}}$.

[21] As in the surface of the field of 𒁹𒀀𒉿𒊩𒆠 (the city of Apisal) reported on [P102305, r. 1]

[22] From [P213162], which has an additional 𒐞𒂆, two thirds (of a shekel), see section 3.3.5.

[23] Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{\text{Ur III/OB}}$ based on a height of one cubit, see[Pow87, p. 488; Rob08, p. 294; Rob19].

excess of one gur, is written after the whole expression, after the overt unit sign 𒋡 if present, and after the word for “grain” if present, as in

𒐋𒐊 𒌋𒄥	𒑏	𒐋	𒋡	𒊺	𒄥[24]
3554 gur	3 ban_2	6	$sila_3$	of grain.	

Observe that while large numbers of gur follow[25] system $S_{\mathrm{Ur\ III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as 𒌋𒁹𒄥 would be 10 gur 1 bariga, and 𒌋𒀸𒄥 would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

— 𒁹–𒑆 used in $S_{\mathrm{Ur\ III/OB}}$ and the SPVS as well as with overt units;
— 𒌋–𒐐 used in $G_{\mathrm{Ur\ III/OB}}$, of which 𒌋–𒐏 are also used in $S_{\mathrm{Ur\ III/OB}}$ and the SPVS as well as with overt units;
— 𒐕–𒐜 used in $S_{\mathrm{Ur\ III/OB}}$, and sometimes with overt units;
— 𒐞–𒐣 used in $S_{\mathrm{Ur\ III/OB}}$;
— 𒊹–𒐦 used in $S_{\mathrm{Ur\ III/OB}}$ and $G_{\mathrm{Ur\ III/OB}}$;
— 𒐬–𒐳 used in $S_{\mathrm{Ur\ III/OB}}$ and $G_{\mathrm{Ur\ III/OB}}$;
— 𒀸–𒐌 used in $C_{\mathrm{Ur\ III/OB}}$ as well as with overt units of the weight system;
— 𒑖, 𒑗, 𒑘, 𒑙, 𒑚 used in $C_{\mathrm{Ur\ III/OB}}$;
— 𒑛, 𒑜, 𒑝, 𒑞 used in $C_{\mathrm{Ur\ III/OB}}$—note the overlap with 𒁹–𒑆;
— 𒑟 and 𒑠 used in $G_{\mathrm{Ur\ III/OB}}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Rob08, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with 𒁹 and 𒌋 numerals with their system $S_{\mathrm{Ur\ III/OB}}$ values. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units[26] [Fri07, p. 118; Rob19]:

$$\underset{\substack{\text{danna}\\ \textit{bērum}\\ \text{league}\\ \text{10,8 km}}}{𒂆𒁕𒈾} \xleftarrow{30} \underset{\substack{\text{UŠ}^{27}\\ \text{cable}\\ \text{360 m}}}{𒍑} \xleftarrow{60} \underset{\substack{\text{nindan}\\ \textit{nindanum}\\ \text{rod}\\ \text{6 m}}}{𒃻} \xleftarrow{12} \underset{\substack{\text{kuš}_3\\ \textit{ammatum}\\ \text{cubit}\\ \text{50 cm}}}{𒌑} \xleftarrow{30} \underset{\substack{\text{šu-si}\\ \textit{ubānum}\\ \text{finger}\\ \text{17 mm}}}{𒋗𒋛}. \qquad (L_{\mathrm{Ur\ III/OB}})$$

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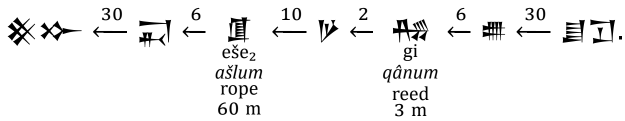
[24] From [P309594, obv. 1, 1].
[25] A larger unit, the $guru_7$ (*karûm*, grain heap), is sometimes used instead, with 𒀸𒄯𒊒𒄥 (1 *karûm* = 3600 kurrū). See [Fri07, p. 415; Rob19].
[26] In this factor diagram and the next, we do not include the numerals. The units are no more than a factor of 60 apart, so higher numerals such as 𒐞 or 𒊹 are not used.

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:



($\bar{L}_{\text{Ur III/OB}}$)

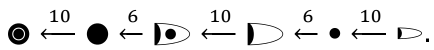
In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

Fractions of the *ikûm*, ⟍ $= \frac{1}{2}$ ⟝ and ⟋ $= \frac{1}{4}$ ⟝, have already been encountered. In other contexts, the fraction $\frac{1}{2}$ is written, as in. The fractions $\frac{1}{3}$ and $\frac{2}{3}$ are written and. The latter two signs are derived from curviform signs and, which are already separately encoded; these are in turn derived from the sign (ŠU$_2$), whose Early dynastic form resembles, and numerals; see [**Powell1971**]. The is sometimes omitted, as in [P240545, *verso* 6, 9; P221530; P221531; P271238; P274845].

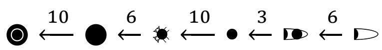
3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the fourth millenium) clearly mirrors system $S_{\text{Ur III/OB}}$ [Fri07, p. 374; DE87, pp. 127, 165]:



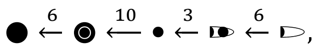
(S)

Likewise the area system used in the Early Dynastic IIIb period for areas of one iku and greater [LAK, p. 72; NDE93, p. 63; Fri07, p. 378; Lec16],



($G_{\text{ED IIIb}}$)

mirrors system $G_{\text{Ur III/OB}}$, with consistent use of the numerals: • corresponds to ⟨, ● to ◇, and ◉ to ◈. An exception to this correspondence, noted in [L2/04-099, p. 4] (see section 3.2), is that the vertical from $S_{\text{Ur III/OB}}$ corresponds to a horizontal in system S. This is however far from the only case of such a reallocation of function. The earlier form of the area system is [DE87, pp. 141, 165; Fri07, p. 378]:



(G)

Observe that, as noted in [DE87, p. 142], ◉ changes meaning from 10• in system G to 600• in system $G_{\text{ED IIIb}}$. System G is used in the fourth millenium, but also in the ED I–II period (it is the "area 2" system in [Cha03], whereas $G_{\text{ED IIIb}}$ is the "area 1" system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb

[27] As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.

area system[28],

$$◎ \xleftarrow{10} ● \xleftarrow{6} ✹ \xleftarrow{10} • \xleftarrow{3} ⊃\!• \xleftarrow{6} ⊃ \xleftarrow{2} ▽ \xleftarrow{2} ⊲ \xleftarrow{2} •^{29}, \qquad (G_{\text{ED IIIb}})$$

with the numerals of a contemporaneous capacity system:

$$\underbrace{⊃\!• \xleftarrow{10} ⊃ \xleftarrow{6} • \xleftarrow{10} ⊃}_{𒄥𒊕𒅅} \xleftarrow{4} ▽ \xleftarrow{6} 𒁇, \qquad (C_{𒄥𒊕𒅅})$$

both described in [Lec16]. While the size of the 𒄥𒊕𒅅 (gur saŋ ŋal$_2$) in bariga is different from that of the Old Babylonian 𒄥, the basic structure of the capacity system is recognizable, with ▽ corresponding to 𒁹 for bariga, 𒁇–𒁇 corresponding to 𒈦–𒈦 for ban$_2$, and the 𒄥 counted with ⊃ rather than 𒐕 numerals. However, the half-iku is counted with the same ▽ as the bariga, whereas it uses a different sign, 𒀹, in the Old Babylonian system. As we will see, this is cannot be handled as a split, by giving 𒀹 the glyph ▽ in an Early Dynastic IIIb font, as the 𒀹 numeral series is also in use in that period.

3.4.1 Field lengths in Ŋirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign 𒃻 is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign 𒂠 is not written either. Lengths shorter than one rope are expressed in half-ropes using the $\frac{1}{2}$ sign 𒈦 (again with no 𒂠), and then in reeds, *with* the sign 𒄀, as follows:

$$\underbrace{𒁹 \xleftarrow{6} 𒌋}_{\substack{1\ \text{eše}_2\text{=10 nindan} \\ 1\ \text{rope=10 rods} \\ 60\ \text{m}}} \xleftarrow{2} 𒈦 \xleftarrow{10} \underbrace{𒀹^{32}}_{\substack{𒄀 \\ \text{gi} \\ \text{reed} \\ 3\ \text{m}}}. \qquad (L_{\text{ED IIIb}})$$

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and most others from the same period, such as the ones discussed in [Lec20], areas are expressed in system $G_{\text{ED IIIb}}$, with curviform numerals[33]; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope[34], the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and thus the interpretation of its numerals between systems $G_{\text{ED IIIb}}$ and $L_{\text{ED IIIb}}$. The sign 𒃷, which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without 𒃷. There are other such co-occurrences contrasting

[28]A variant is $◎ \xleftarrow{10} ● \xleftarrow{6} ✹ \xleftarrow{10} • \xleftarrow{3} ⊃\!• \xleftarrow{6} ⊃ \xleftarrow{2} ▽ \xleftarrow{2} ⊳ \xleftarrow{2} •$, see [**Powell1972**].

[29]The cuneiform counterpart is 𒀹.

[32]The reeds are counted using *tenû* numerals, 𒀹, 𒀹, 𒀹, etc.

[33]A CDLI search for `"(bur3)"` (𒌋 numerals used for areas) currently returns 15 ED IIIb results, whereas one for `"(bur3@c)"` (● numerals used for areas) returns 206. Further, when dated, the tablets with cuneiform bur$_3$ are from the reigns of 𒌷𒅎𒄀𒈾 (variously transliterated iri-inim-gi-na, uru-ka-gi-na, etc.) and 𒈗𒍠𒄀𒋛 (lugal-zag-ge-si), the last two kings of ED IIIb Lagaš.

[34]This is the case of the sides of the field in [P020054, obv. ii 2–3].

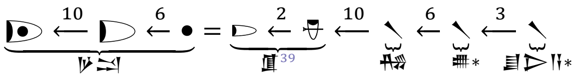
between metrological systems; for instance, [Kre98, p. 303 n. 686] mentions the use of cuneiform numerals for days and months[35].

3.4.2 Dyke lengths in Ŋirsu

[Pow87, p. 466] notes that reeds "are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus." Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in CDLI finds many occurrences[36], such as:

- [P221305, obv. 1, 4][37]
- [P020129, rev. 2, 1]
- [P221291, rev. 5, 1][38]
- [P221266, rev. 2, 1]

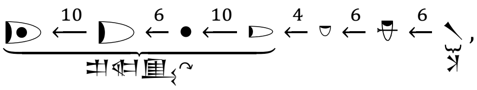
These expressions use an explicit sign (counted in multiples of ten) or . This notation—but not its use of curviform numerals—is remarked on in [Lec20, p. 290 n. 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. The notation can be summarized by the following factor diagram, where prefix units have been marked by an asterisk:



$(L'_{\text{ED IIIb}})$

3.4.3 Butter, cheese and wheat in Ŋirsu

A similar mixture of cuneiform and curviform numerals may be observed with the capacity system; indeed, the previously described system uses numerals for [Fri78, p. 43; Lec16]:



as in [P020016, rev. 1, 4; P020065; P020090, obv. 1, 3, rev. 2, 1; P020092, rev. 3, 1; P020137, obv. 1, 2] and others, where ban$_2$ counted with numerals are followed by sila$_3$ counted with numerals. Curviform numerals are also used to count sila$_3$, but not[40] as part of the systems. This contrast can be seen in [P220927], which measures butter (, i$_3$) with a different capacity system, using the

[35] That note also mentions a contrast between the use of curviform numerals to count people and curviform numerals to count bread alotted to them in [P010876]; such contrasts are more akin to styling, and might not, on their own, justify the disunification.

[36] A search for curviform numerals followed by some number of reeds counted in (*tenû*) cuneiform numerals currently finds 125 occurrences across 47 tablets.

[37] CDLI only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the is not visible. Lecompte notes that the copy is faithful; indeed another can be seen both on the copy and the photo on obv. 2, 2.

[38] From copy.

[39] With either unit omitted, as in the examples above, or both, as in [P020129, obv. 3, 3] .

[40] As of this writing, the single occurrence of (ban2@c) followed by curviform numerals and sila3 in ED IIIb Ŋirsu transliterations on CDLI, 4(ban2@c) 3(asz@c) sila3 in [P221815, obv. 4, 7], is incorrect: it should be 4(ban2@c) 3(disz@t) sila3.

(dug, “pot”) of 20 sila₃, with ⊳ and ● numerals[41] for both the dug and the sila₃, thus [Pow87, pp. 504 sq.]

$$\underbrace{\bullet \xleftarrow{10} \rhd}_{\text{dug}} \xleftarrow{2} \underbrace{\bullet \xleftarrow{10} \rhd}_{\text{sila}_3} \xleftarrow{\frac{3}{2}} \cdot \xleftarrow{2} \cdot, \qquad (C_{\text{dug}})$$

but counts cheese (ga’ar) using the capacity system, with ⟍ numerals for the sila₃.

Another capacity system in ED IIIb Ŋirsu is the gur of two ul:

$$\underbrace{\bullet \xleftarrow{10} \rhd}_{\sim} \xleftarrow{2} \triangledown \xleftarrow{6} \cdot \xleftarrow{6} \underbrace{\diagdown}_{\text{sila}_3}. \qquad (C)$$

Here the ⊳–⟍–▽ contrast occurs not only within the numerals of the system, but with its units; this is perhaps best illustrated by the expressions ●▽ in [P221746, rev. 2, 2] and ● in [P221814, rev. 1, 5].

3.4.4 Grain in Ebla

The mixing of curviform and cuneiform numerals within a metrological system is not specific to Ŋirsu.

The system of grain[42] capacities in Ebla uses the following units[43]:

$$\underset{\text{gu}_2\text{-bar}}{\cdot} \xleftarrow{2} \underset{\text{ba-ri}_2\text{-zu}}{\cdot} \xleftarrow{\frac{5}{2}} \underset{\text{ŋin}_4}{\cdot} \xleftarrow{4} \underset{\text{niŋ}_2\text{-saŋšu}}{\cdot} \xleftarrow{6} \underset{\text{an-zam}_x}{\cdot}.$$

The gu₂-bar and ba-ri₂-zu are generally counted using curviform numerals, and the smaller units using cuneiform numerals[44] Indeed, a search on [EbDA] for co-

[41] This tablet also uses subtractive notation: “two pots minus two thirds (sila₃)”, “ten minus one pots, six sila₃”. Such subtractive notation is common in most of the metrological systems discussed here; it appears in the ED IIIa period [Rob08, p. 77]. It presents no complexity for character encoding, but it is noteworthy that the sign ⌜ (lal, “minus”) is often ligated with the following numerals, with the subtrahend placed under a sometimes considerably enlarged ⌜, similar to the layout of the radical in modern mathematical notation, see, *e.g.*, [P020092, rev. 3, 1, 2]. A good font could handle the very common −1 case, perhaps even −2 and −3; setting arbitrary numeric expressions under the ⌜, or more generally replicating the layout of Early Dynastic tablets, is outside the realm of plain text; see also section 6.2.

[42] Liquid capacities use a different system [Arc15, p. 229 n. 12]:

$$\underset{\text{la-ḫa}}{\cdot} \xleftarrow{30} \underset{\text{sila}_3}{\cdot} \xleftarrow{6} \underset{\text{an-zam}_x}{\cdot}.$$

A glance it seems that sila₃ are counted with cuneiform numerals and higher units with curviform ones, thus

$$\underbrace{\rhd \xleftarrow{\frac{5}{3}} D \xleftarrow{6} \bullet \xleftarrow{10} \rhd}_{\text{la-ḫa}} \xleftarrow{3} \underbrace{\langle \xleftarrow{10} |}_{\text{sila}_3} \xleftarrow{6} \underbrace{|}_{\text{an-zam}_x},$$

but we have not investigated this thoroughly.

[43] Another system uses different values for the ŋin₄ and niŋ₂-saŋšu, see [Cha12, p. 62; Arc15, p. 229 n. 12]:

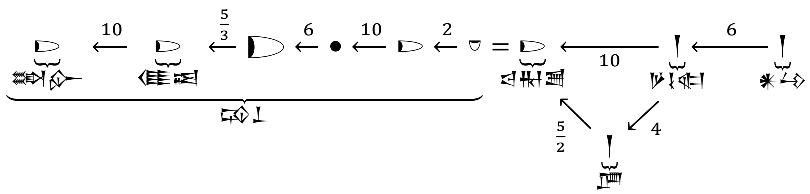
$$\cdot \xleftarrow{2} \cdot \xleftarrow{3} \cdot \xleftarrow{4} \cdot \xleftarrow{5} \cdot.$$

[44] As mentioned in [Cha12, p. 63], the ŋin₄ is also counted using the numeral series. Some instances of that usage are found transliterated $n/6$ in EbDA; in some cases the ŋin₄ sign is omitted, and the numeral is then written before the gu₂-bar unit, as in from [P240545, *verso* 1, 3].

occurrences of either 𒀭𒍝 or 𒋙𒃻𒊕 with either of 𒄘𒁇 or 𒁀𒊑𒂵 finds the following expressions[45]:

1. [P240532, *verso* 4, 9] ⊳𒌍 𒁀𒊑𒍪[47]𒐛𒋙𒃻𒊕
2. [P240548, *verso* 1, 1] ⊳𒁀𒊑𒂵 𒐗𒀭𒍝
3. [P240655, *recto* 7, 9] ⊳⊳𒌍𒁇[48]𒐛𒋙𒃻𒊕
4. [P240579, *verso* 4, 3] ◗◗⊳▿𒄘𒁇 𒐖𒋙𒃻𒊕
5. [P240675, *verso* 2, 2] ⊳𒌍 𒁀𒊑𒂵 𒐛𒋙𒃻𒊕
6. [P240609, *verso* 3, 1] ⊳𒁀𒊑𒂵 𒐗𒀭𒍝
7. [P240533, *recto* 3, 3] ●⊳▿𒄘𒁇 𒐛𒋙𒃻𒊕 𒐖+𒀭𒍝𒐖
8. [P240697, *recto* 1, 5] ⊳▿[49]𒐖𒃻 𒐗𒀭𒍝
9. [P240653, *recto* 6, 2] ●⊳⊳⊳▿𒄘𒁇𒐕𒋙𒃻𒊕 𒐖+𒀭𒍝
10. [P240654, *recto* 2, 6] ⊳𒁀𒊑𒂵 𒐛𒋙[50] 𒐗𒀭𒍝[51]
11. [P240531, *recto* 1, 8] ⊳𒁀𒊑𒂵 𒐛𒋙𒃻𒊕 𒌍𒂟𒁇
12. [P241708, *recto* 1, 1][52] ⊳⊳▿𒁀 𒄘𒁇 𒐛𒋙𒃻𒊕
13. [P241904, *recto* 1, 1][53] ◗◗𒄘𒁇 𒐘[54]𒋙𒃻𒊕
14. [P240964, *recto* 3, 2][55] ◗◗◗𒇷[56] ◗▿𒄘𒁇 𒐖𒋙𒃻𒊕[57] 𒐖𒀭𒍝

Note that higher numbers of 𒄘𒁇 are expressed in hundreds (*mi-at* 𒈪𒀜) and then thousands (*li-im* 𒇷𒅎), as is typical in Ebla [Arc15, p. 33], *e.g.*, in the last example above or in [P240532, *verso* 2, 3], ⊳𒈪𒀜◗●◗◗⊳𒌍 𒄘𒁇 ($100 + 60 + 30 + 5 = 195$ 𒄘𒁇 of grain). These expressions correspond to the following factor diagram:



(C_{Ebla})

3.4.5 Use in modern publications

Because of their prevalence in the fourth millenium and Early Dynastic period, the proposed numerals are widely used in modern publications discussing metrology

[45] We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform 𒁀𒊑𒂵 and larger with cuneiform 𒃻 and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked[46] on a photograph (from EbDA unless noted otherwise).

[46] As we will see in Section 3.7.2, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

[47] ba-ri$_2$-zu$_2$, a variant spelling.

[48] Short for 𒄘𒁇.

[49] Note the omitted 𒄘𒁇.

[50] Instead of the expected 𒋙𒃻𒊕.

[51] 𒐗𒀭𒍝 not legible on the EbDA photo.

[52] From CDLI photo.

[53] From photo in [Arc89, p. 6].

[54] Laid out as 𒐕𒐕𒐕𒐕; on stacking patterns see Section 6.2.

[55] From photo in [Arc89, p. 6]; see also the CDLI photo and the copy in [**Friberg1986**]. This tablet features unusual usage of vertical numerals—"somewhat unorganized", as described by [**Friberg1986**]—, such as 𒐗𒄘𒁇 or 𒐚𒈪, but its 𒋙𒃻𒊕 and 𒀭𒍝 are consistently counted with cuneiform numerals, and the higher units with cuneiform numerals.

[56] Short for 𒇷𒅎.

[57] ŠU$_2$+NIŊ$_2$-saŋ, an unusual variant spelling.

in those periods, as illustrated in Figures 1–20.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 20. The partial[58] transliteration "4⊳ *ʾa*$_3$*-da-um* 4▿ *aktum* 4⊳ ib$_2^{tu_9}$×3𒁹 sa$_6$ gunu$_3$" is used to illustrate a discussion of the interpretation of the contrast between ⊳ and ▿ numerals. More conventional transliterations might omit the numeral shapes entirely, *e.g.*, 4 *ʾa*$_3$*-da-um* 4 *aktum* 4 ib$_2^{tu_9}$×3 sa$_6$ gunu$_3$, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, *e.g.*, 4(aš^c) *ʾa*$_3$*-da-um* 4(diš^c) *aktum* 4(aš^c) ib$_2^{tu_9}$×3(diš) sa$_6$ gunu$_3$, but the result would be less readable. See Section 3.7.2 for a discussion of transliteration conventions for numerals.

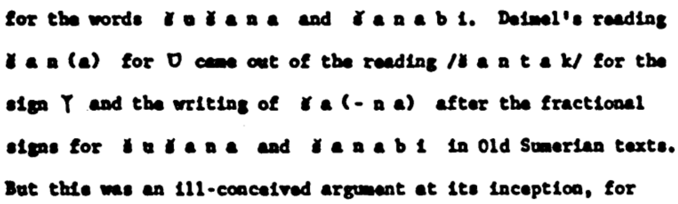

for the words š u š a n a and š a n a b i. Deimel's reading š a n (a) for ▿ came out of the reading /š a n t a k/ for the sign 𒁹 and the writing of š a (- n a) after the fractional signs for š u š a n a and š a n a b i in Old Sumerian texts. But this was an ill-conceived argument at its inception, for

Figure 1: Discussion of the readings of proposed ▿ and already-encoded 𒁹 in [**Powell1971**].

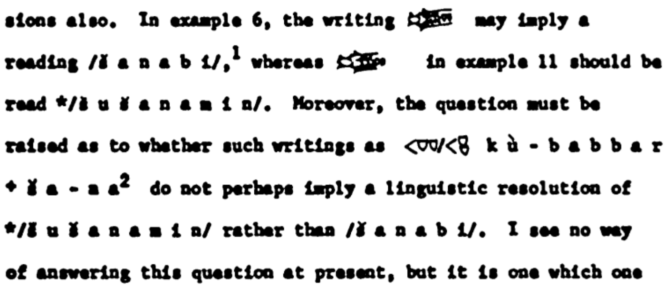
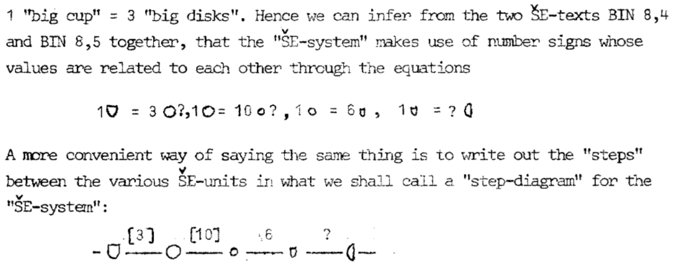

sions also. In example 6, the writing [cuneiform] may imply a reading /š a n a b i/,[1] whereas [cuneiform] in example 11 should be read */š u š a n a m i n/. Moreover, the question must be raised as to whether such writings as <▿▿/<[cuneiform] k ù - b a b b a r + š a - n a[2] do not perhaps imply a linguistic resolution of */š u š a n a m i n/ rather than /š a n a b i/. I see no way of answering this question at present, but it is one which one

Figure 2: Discussion of the readings of proposed [cuneiform] and [cuneiform] as well as already-encoded [cuneiform] and [cuneiform] in [**Powell1971**].

[58]The untransliterated text would be [cuneiform]; note the atomically encoded ib$_2$ × 3𒁹 = [cuneiform] × 𒐈 = [cuneiform].



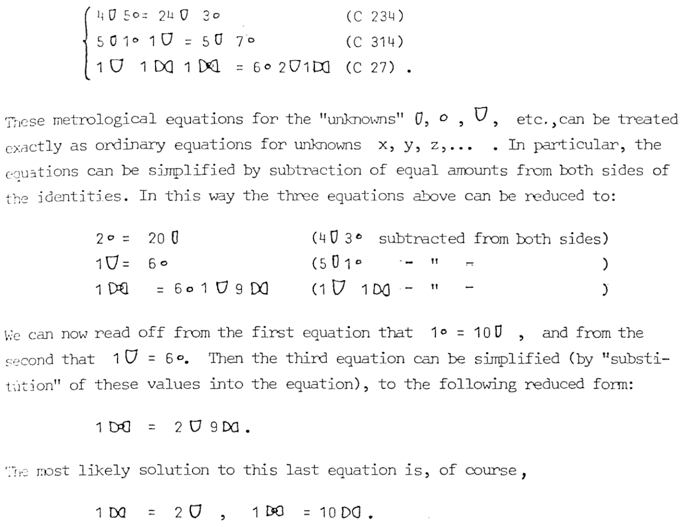
1 "big cup" = 3 "big disks". Hence we can infer from the two ŠE-texts BIN 8,4 and BIN 8,5 together, that the "ŠE-system" makes use of number signs whose values are related to each other through the equations

1Ʊ = 3 O?, 1O = 10 o?, 1 o = 6 ʊ, 1 ʊ = ? 0

A more convenient way of saying the same thing is to write out the "steps" between the various ŠE-units in what we shall call a "step-diagram" for the "ŠE-system":

– Ʊ —[3]— O —[10]— o —6— ʊ —?— 0 –

Figure 3: The first factor diagram, in [Fri78, p. 10].



4 ʊ 5 o = 24 ʊ 3 o (C 234)
5 ʊ 1 o 1 Ʊ = 5 ʊ 7 o (C 314)
1 Ʊ 1 ⋈ 1 ⋈ = 6 o 2 Ʊ 1 ⋈ (C 27) .

These metrological equations for the "unknowns" ʊ, o , Ʊ , etc., can be treated exactly as ordinary equations for unknowns x, y, z,... . In particular, the equations can be simplified by subtraction of equal amounts from both sides of the identities. In this way the three equations above can be reduced to:

2 o = 20 ʊ (4 ʊ 3 o subtracted from both sides)
1 Ʊ = 6 o (5 ʊ 1 o " " ")
1 ⋈ = 6 o 1 Ʊ 9 ⋈ (1 Ʊ 1 ⋈ " " ")

We can now read off from the first equation that 1 o = 10 ʊ , and from the second that 1 Ʊ = 6 o. Then the third equation can be simplified (by "substitution" of these values into the equation), to the following reduced form:

1 ⋈ = 2 Ʊ 9 ⋈ .

The most likely solution to this last equation is, of course,

1 ⋈ = 2 Ʊ , 1 ⋈ = 10 ⋈ .

Figure 4: The derivation of the factors of the bisexagesimal system in [Fri78, p. 15][60].

[60]The bisexagesimal system is used alike in proto-Elamite and proto-cuneiform texts, see [Fri78, p. 38]; the derivation in [Fri78, p. 15] is based on proto-Elamite artefacts. Note that in Friberg's early works [Fri78; **Friberg1979**; **Friberg1986**; Fri87], copies of fourth millenium and sometimes third millenium tablets are shown as vertical text (which they were for the scribes), and their numerals are written within horizontal text in the same orientation that they have if the tablet is taken as vertical text; in [**UAX50**] parlance, as if they had Vertical_Orientation=Upright. In addition, they are listed in these equations in the horizontal order in which they appear as vertical text (thus the rightmost numeral is the most significant, read first). Cuneiform is correctly Vertical_Orientation=Rotated, consistently both with modern practice and with the rotation between earlier vertical and later horizontal monumental inscriptions. Friberg's early conventions are not followed in later scholarship, and are abandoned in his own more recent works, such as [Fri07]; a more typical way to express the first equations might be

5● + 4⊳ = 3● + 24⊳ (C 234)

1▶ + 1● + 5⊳ = 7● + 5⊳ (C 314)

1⧗ + 1⧖ + 1▶ = 1⧖ + 2▶ + 6●. (C 27)

Thus, for instance, the original set of fractions ▽, ⊲, and ... (1/2, 1/4 and 1/8 of an iku) in the Sumerian GANA system, was after a time augmented through the addition of the new sub-unit SAR: ..., equal to 1/100 of an iku (⊃). Similarly, the Sumerian weight unit "ma-na" which originally may have had only the sub-units ... ša-na (= 1/3 mana) and ... ša-na-bi (= 2/3 mana), and perhaps also gin: ... (= 1/60 mana), seems to have acquired, at some time or other, also the smaller sub-units ... (= 1/3 gin), and ... še (= 1/3 × 1/60 gin).

Figure 5: Discussion of proposed fractions ▽, ⊲, ..., and ..., as well as already-encoded ... and ... in [Fri78, p. 49].

stein publizierten Zeichenliste enthalten ist[3], bis vor kurzem unentdeckt bleiben konnte. Erst 1978 machte der schwedische Mathematiker J. Friberg, ERBM I, 9-11, darauf aufmerksam, daß die Zeichen für die Zahlen Eins (⊃) und Zehn (●) in Verbindung mit dem Zeichen ŠE nicht im Verhältnis 1 zu 10 sondern im Verhältnis 1 zu 6 stehen. Bis dahin hatte man, obwohl die Andersartigkeit des in Verbindung mit dem Zeichen ŠE verwendeten Zahlzeichensystems bekannt war, für diese beiden häufigsten Zahlzeichen einheitlich ein Verhältnis 1 zu 10 unterstellt, obwohl es mehrere eindeutige Gegenbelege gab, von denen zumindest diejenigen der Archaischen Texte aus Ğemdet Nasr bereits früh publiziert und jedermann zugänglich waren[4]. Als Folge

Figure 6: Discussion in [DE87, p. 117] of the discovery in [Fri78, pp. 9–11] (see Figure 3) of the different relations between ⊃ and • in systems *G* and **??**.

there is in any case an important qualitative difference between IX for Latin novem and ... for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 7: The sign ... used in a parallel with IX in [**Englund1988**], discussing an argument from [**Powell1972**] on the question of the language of the Uruk III texts.

of decreasing fractions $^1/_n$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus ... = $^1/_2$ N_{39}, ... = $^1/_3$ N_{39}, and so on. The first sign of the latter units, N_{34},

Figure 8: Description of the fractions ... and ... in [Eng98, p. 113][61].

A diplomatic edition of [Fri78] could rotate the numerals using a higher-level protocol:

$$\begin{cases} 4▽\,5\bullet = 24▽\,3\bullet & \text{(C 234)} \\ 5▽\,1\bullet\,1▽ = 5▽\,7\bullet & \text{(C 314)} \\ 1▽\,1⋈\,1⋈ = 6\bullet\,2▽\,1⋈ & \text{(C 27).} \end{cases}$$

[61]The text erroneously has N_{34} instead of N_{24}.

For instance, the first line contains the notations $1N_{34}$ $1N_{39a}$; $2N_{20}$, which can be translated "60 of the (grain rations containing) ⌣ (of grain); (grain involved:) 2 • (of ground barley)". This calculation contradicts the assumed numerical relationship $10N_1 = 1N_{14}$, since as was well known the measure represented by the sign N_{39} was $^1/_5$ of that represented by N_1, so that $60 \times {}^1/_5 = 12$ and not 20, as $2N_{14}$ would imply. Instead of relying on complicated

Figure 9: The sign ⌣ used as a capacity measure within otherwise translated text in [Eng98, p. 116].

Die halbkreisförmigen Griffeleindrücke gehen manchmal in mehr oder weniger eckige Formen über ( )[685]. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen ( ), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren[686]. Selten treten mit ▽ gebildete Zahlen auf[687] (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 10: Discussion of co-occurrences and contrasts between , , and ▽ in [Kre98, p. 303].

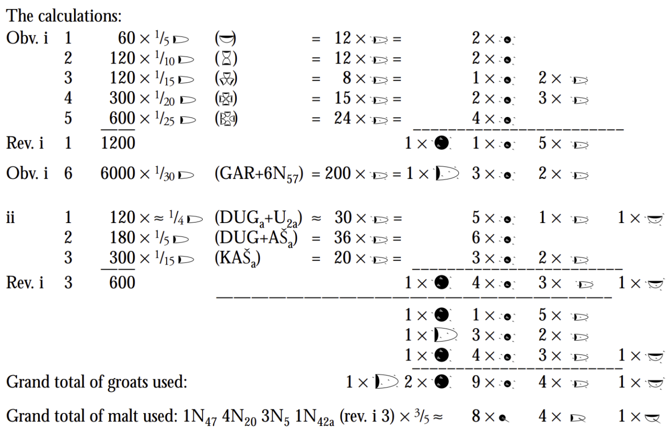


Figure 6. Transliteration and calculations of *MSVO* 4, 66.

Figure 11: Calculations from [P005468] transcribed in [**Englund2001**] using modern mathematical notation combined with some of the proposed characters.

strong similarities between "area" 1 and "area" 3 systems, the sign with two concentric discs (◉, notated N_{50}[27]) remains problematic. It never appears in any numerical combination with the sign with a single disc (●,

Figure 12: Discussion of ◉ and ●[62] in [Cha03, p. 6].

1/15, etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation 𒑏 𒁹 𒄥 could be understood as 1 1/5 gur.[27] However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs 𒑏 and 𒄥 cannot be considered klasmatograms.

Metrological tablets from the end of the 4th millennium (Nissen, Damerow and Englund 1993, 55-59, to *MSVO* 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

1(iku) represents a surface of 3600m²
1(eše$_3$) represents a surface of 21,600m²
etc.

The signs iku and eše$_3$ constitute by themselves measures of surface areas. These measures are usually followed by the sign GAN$_2$, which means either surface or field and

Figure 13: Discussion of Old Babylonian[64] capacity and fourth millenium area measures in [**Proust2009**].

formed by only two signs 𒁹 and 𒌋, repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60², 60³, 1/60, 1/60², etc. are written in the same way, with the vertical wedge 𒁹. The third feature concerns the exact function of

Figure 14: Description of the SPVS in [Cha12, p. 58], using the already-encoded signs 𒁹 and 𒌋.

one step. The scribes of the Early Dynastic Period (*c.* 2600 BC), for instance, represented the number 648,000 with: 𒊹𒐏●●● but never with the repetition 𒊹𒐏𒊹𒐏𒊹𒐏●●●.

Figure 15: Discussion of large numbers illustrated by 𒊹𒐏●●●[65] in [Cha12, p. 59]

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

𒑏𒑏𒑏 še bar 𒑏 *ba-rí-zu*

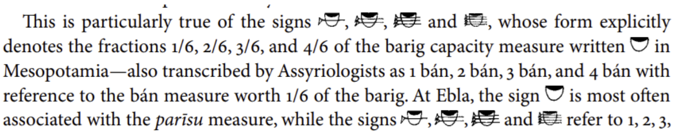
'3 *gubar* (capacity units) and 1 *parīsu*'.

Figure 16: Partial transliteration of [P240597, *recto* 5, 3] 𒑏𒑏𒑏 𒊺 𒁇 𒑏 𒁀𒊑𒍪 in [Cha12, p. 61].

[62]The statement that these do not co-occur refers to the texts from ED I–II Ur; these signs co-occur both earlier and later in areas, with different relations as previously discussed.

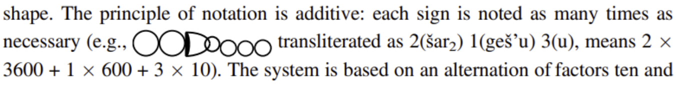
[64]The cuneiform text is Unicode-encoded.

[65]Compare ◇𒊹𒐏 discussed in section 3.3.2. The order is variable in the Early Dynastic period; see [P010773], also discussed in [Fri07, p. 148], for an example of 𒊹𒐏●, and [P274845; P241764] for examples of 𒊹𒐏 n●. Sign order can be variable in early texts, see [**Foxvog2016**].



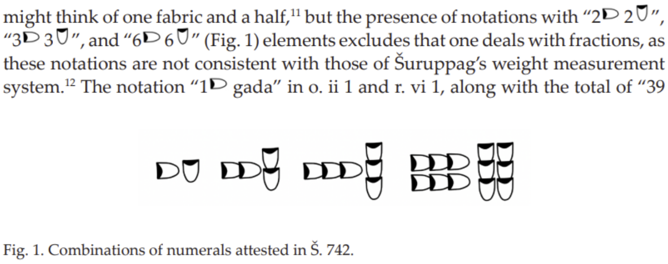
This is particularly true of the signs , , and , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign is most often associated with the *parīsu* measure, while the signs , , and refer to 1, 2, 3,

Figure 17: Discussion in [Cha12, p. 64] of the relation between – and in Mesopotamia and in Ebla.



shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., transliterated as 2(šar$_2$) 1(geš'u) 3(u), means 2 × 3600 + 1 × 600 + 3 × 10). The system is based on an alternation of factors ten and

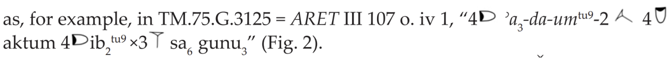
Figure 18: Explanation of the structure of the number in [**Proust2020**].



might think of one fabric and a half,[11] but the presence of notations with "2 2", "3 3", and "6 6" (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag's weight measurement system.[12] The notation "1 gada" in o. ii 1 and r. vi 1, along with the total of "39

Fig. 1. Combinations of numerals attested in Š. 742.

Figure 19: Discussion of the contrast between and numerals in [Gor23, p. 162].



as, for example, in TM.75.G.3125 = *ARET* III 107 o. iv 1, "4 ʾ*a*$_3$-*da-um*^{tu9}-2 4 aktum 4 ib$_2^{tu9}$ ×3 sa$_6$ gunu$_3$" (Fig. 2).

Figure 20: Transliteration in [Gor23, p. 163] of [P242293, *recto* 4, 1] incorporating untransliterated numerals.

3.5 Non-numeric usage

[illegible]

The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'[66]. Do you know its reading[67]?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign 𒀸 has, *inter alia*, the values aš, rum, and dili. While the horizontal numerals are most frequently written [illegible] in the Early Dynastic period[68], such non-numeric usage is almost[69] always written 𒀸, for instance:

- in personal names in administrative texts, such as the following, which all contain [illegible] numerals:
  - 𒀸[illegible] in [P010424, rev. 1, 5; P010458, obv. 1, 5; P010459, obv. 2, 5′] from ED IIIa أبو صلابيخ,
  - 𒀸[illegible] in [P010960, obv. 2, 5] from ED IIIa Šuruppag,
  - [illegible] in [P251641, obv. 4, 3] from ED IIIb Adab,
  - [illegible] in [P252866, rev. 2, 3] from ED IIIb Adab,
  - [illegible] in [P298637, rev. 2, 4] from ED IIIb Umma;
- in the Sumerian word [illegible]𒀸 u_2-rum, "property" in ED IIIb Ŋirsu administrative texts which contain [illegible] numerals, such as [P020006, obv. 2, 3; P020008, rev. 1, 2; P020018, rev. 1, 2; P020024, obv. 1, 4; P020030, obv. 3, 1];
- in lexical texts:
  - in the divine name [illegible] in the lexical texts [P010570, rev. 2, 4; P010572, obv. 3, 6], where the entries are prefixed with [illegible].
  - in the word 𒀸 dili, "small fish" in [P010578, obv. 2, 5], witness to Early Dynastic Fish,
  - in the same word with a determinative, 𒀸[illegible] $dili^{ku_6}$, in [P010586, obv. 4, 4 & 6], witness to Early Dynastic Food, which starts with [illegible] numerals.

This is a clear contrast between 𒀸 and [illegible] in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name 𒀸[illegible] occurs on its own line in the aforementioned administrative texts; a line [illegible] would instead be read as "one slave".

3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are

[66] The reader will recall that $ŋeš_2$ is written 𒁹, with a larger wedge than [illegible]; however, these signs have merged by the time Examenstext A is composed.

[67] Besides $ŋeš_2$, a look at [OSL] shows that the values diš, ge_3, makkaš, $saŋtak_4$, and tal_4 are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

[68] A CDLI search for `"(asz@c)"` finds 3296 ED texts, while a search for `"(asz)"` finds 81 ED texts, of which 46 also contain `"(asz@c)"`.

[69] Exceptions are discussed in section 3.7.1.

real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as 𒀸 that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [KWU, pp. 123 sqq.] lists all of 𒀸–𒐇 together with 𒑉–𒑑, while 𒀸, 𒐀, and 𒐁, and only those, appear at the beginning of the sign list, since they have non-numeric values[70]. [PTACE, p. 58] has the numeric signs 𒑉, 𒑊, 𒁹, whereas non-numeric 𒀸 is at the beginning of the sign list, where its values *aš* and *rum* are listed. For signs with both non-numeric and numeric usage, [LAK] writes *s. die Zahlz.* throughout the main list; LAK 1 𒀸 thus reappears at LAK 829 together with 𒑉, 𒑊, and 𒁹. One should note [MZL], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have 𒐾–𒑀 used in system $G_{\text{Ur III/OB}}$.

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes[71] to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from 𒑉 to 𒁹 numerals prevents diachronic encoding even if 𒑉 were unified with 𒀸. For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1′–6′]; however, they are written with 𒁹 numerals, whereas they are written with 𒑉 numerals in the Early Dynastic witnesses; since 𒁹 and 𒀸 are distinct[72] characters, the 𒑉-𒀸 unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods[73], there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [**Romach2023**], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character

[70]Non-numeric values of 𒀸 were discussed in section 3.5; 𒐀 has the values man$_3$ and min$_5$, and is used for the word didli, "several, various"; 𒐁 has the value eš$_6$.

[71]Compare, *e.g.*, in the *Instructions of Šuruppag*, 𒋢𒆳𒊒𒆠 𒌉𒑉 𒈾𒈾𒈬 / 𒉌 𒉌𒅗𒊑𒊑 in the ED IIIa witness [P222243, obv. 2, 7], also discussed in section 3.7.1, and 𒋢𒆳𒊒𒆠𒆤 𒈾𒈾𒈬 𒉌 𒉌𒅗𒁉𒀸𒊑𒊑 in the OB composite [**Q000782**] (translated "Šuruppag gave instructions to his son" in [**ETCSRI**]). It does not matter for the construction of a composite text whether this is encoded 𒌉𒑉 or 𒌉𒀸, since that word is absent from other witnesses, and since the surrounding words differ.

[72]Besides the contrasts in numeric usage mentioned in section 3.3.3, these (already-encoded) characters were clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of 𒀸 which are not shared with 𒁹 have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period, 𒀸 is used for the preposition *ina*, "in", and 𒁹 for the preposition *ana*, "to".

[73]See, *e.g.*, [Rob08, p. 55] on the unification of metrologies under Sargon, resulting in the systems described in section 3.3.

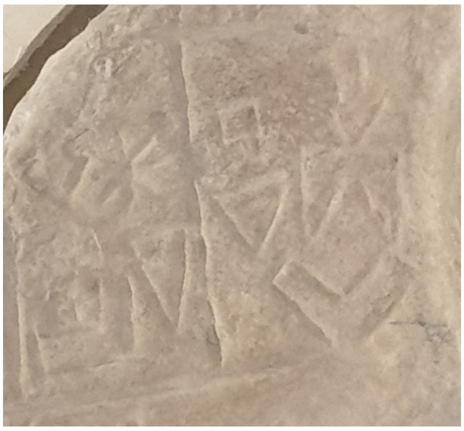


Figure 21: [P222399, obv. 6, 16–17] [cuneiform] / 𒀭[cuneiform] ◇[cuneiform].

discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts[74] being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

3.7.1 The case of ŠAR$_2$

The character U+122B9 ● CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period[75], a contrast between non-numeric šar$_2$ written ◇ and numeric 1(šar$_2$^{c}) written ● can be observed, similar to the contrast between [cuneiform] and [cuneiform] previously discussed in section 3.5. However, in lexical lists from Šuruppag and Ebla[76], as well as in the *Stèle des vautours*, non-numeric šar$_2$ is curviform:

- 𒀭[cuneiform]● and 𒀭[cuneiform]●[cuneiform] in [P010566, obv. 10, 10 & 11];
- ●[cuneiform] and 𒀭●[cuneiform] in [P010576, rev. 3, 16 & 17];
- ●[cuneiform] in [P240986, *recto* 3, 3][77];
- ●[cuneiform] in [P222399, obv. 17, 9, 18, 11, 22, 12][78].

[74]Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

[75]For example, in personal names:

- [cuneiform]◇[cuneiform] in [P020019, rev. 1, 2] from ED IIIb Ŋirsu;
- [cuneiform]◇[cuneiform] in [P020182, obv. 2, 9], also from ED IIIb Ŋirsu;
- [cuneiform]𒀭◇ in [P222186, obv. 3, 3] from ED IIIb Umma;
- [cuneiform]𒀭[cuneiform]◇ in [P235312, obv. 16] from Old Akkadian Umma.

[76]These are archaizing in other ways, *e.g.*, they have a [cuneiform]–[cuneiform] (NAM$_2$–TUG$_2$) split.

[77]From copy in [ELLes, No. 397].

[78]Note however 𒀭[cuneiform] ◇[cuneiform] on [P222399, obv. 6, 17], see Figure 21. Curviform non-numeric šar$_2$ is clearly archaizing in ED IIIb Ŋirsu; one might suppose that the scribe slipped into their modern ways here.

It *would* be disruptive to the diachronic representation of text if non-numeric $\check{s}ar_2$ were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph ◇, identical to the glyph of the proposed U+12579 ● CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric $\check{s}ar_2$ is curviform solidly predates the transition from ● to ◇ in the relevant metrological systems, there is no need to represent a ◇-● contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D ◇ CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

Note that in rare cases, such as [P222243, obv. 2, 7] from ED IIIa Adab, non-numeric 𒀸 (here with the value rum) is written 𒕐. It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 𒕐 cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. in practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the CDLI transliteration (UR2-1(aš@c)) and the ePSD2 one ($\mathrm{uru}_8{}^{\mathrm{rum}}$) of this word disagree on that aspect. Since 𒀸 has a cuneiform reference glyph, this does not pose any compatibility concerns.

3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL_2 can be described as NE×KASKAL or NE×PAP[79], they are typically all transliterated bil_2, and therefore are all represented by the character U+1224B 𒉋 CUNEIFORM SIGN NE SHESHIG, its name notwithstanting, as described in [UTR56, §2.5].

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, from [P305639, rev. 21] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in EbDA transliterations: the aforementioned from [P240533, *recto* 3, 3] is transliterated "20-1-1/2 gu_2-*bar* 7 nig_2-sagšu 2-1/2 an-zam_x[80] *za*", reading both ▽ and ┼ as 1/2, but not distinguishing them.

In particular, these transliterations do not differentiate between 𒀸 and 𒁹 numerals, nor between 𒕐 and ▽ numerals. For instance, the aforementioned from [P242293, *recto* 4, 1] is transliterated "4 *ʾa*$_3$-*da*-*um*$^{\mathrm{tug_2}}$-II 4 aktum$^{\mathrm{tug_2}}$ 4 ib_2-III gun_3 sa_6 $^{\mathrm{tug_2}}$" in EbDA, with no distinction between

[79] As on [P249253].

[80] As of this writing, EbDA actually has an-zam_χ, with U+1D6A GREEK SUBSCRIPT SMALL LETTER CHI.

the and . Since and numerals are separately encoded, the numeric expressions in such transliterations cannot be transformed into Unicode cuneiform without additional context, regardless of curviform–cuneiform unification.

In metrological systems such as systems $G_{\text{Ur III/OB}}$ and $C_{\text{Ur III/OB}}$ where some units are indicated by the type of numeral rather than an overt unit sign, it is common practice to add the unit in parentheses in transliteration; for instance, from [P386847, obv. 1] is transliterated “1(eše$_3$) 5½ iku[81] 7 sar” in [Feu04, vol. 2, p. 176], and from [P307255, obv. 12] is transliterated “1(n[82]) 2(b) 7 ½ sila$_3$” in [Feu04, vol. 2, p. 151].

This practice has been generalized to systematically indicate numeral shape; this is in particular the case in CDLI, where the transliterations of some the above examples are “1(gesz2) 3(u) 5(asz) gur” for , “1(esze3) 5(iku) 1/2(iku) GAN2 7(disz) sar” for , and “3(barig) 2(ban2) 7(disz) 1/2(disz) sila3” for . CDLI and ePSD2 both distinguish curviform from cuneiform numerals in transliteration: the length from [P020129, rev. 2, 1] is transliterated “6(gesz2@c) 3(u@c) {ninda}nindax(DU) 1/2(asz@c) 4(disz@t) gi” in CDLI, and “6(geš$_2^{c}$) 3(u^{c}) $^{\text{ninda}}$ninda$_x$(DU) 1/2(aš^{c}) 4(diš^{t}) gi” in ePSD2. Another example is [**Molina2014**], which uses 1a for , 1d for , 1ac for , 1dc or ½dc for depending on reading, etc. The literature on the Uruk and Early Dynastic I–II periods uses a different set of transliteration conventions that also disambiguate numeral shapes, as will be discussed in section 4.

While there exist transliterations that distinguish from but not from , such as the ones used in [**DCCMT**], the trend, especially in more recent works in third millenium studies, seems to be to represent numeral shape; for example, [**Maiocchi2024**] gave an example of the input syntax used by the new “Urban Economy Begins” project as “10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)” for , with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are rather than numerals. The “tradition of cavalierly dispensing with numerical notations in notations of administrative documents”, as [**Englund2004**] describes it, seems to be fading.

3.8 Conclusions

Co-occurences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications. The contrast frequently applies to individual numerals, rather than to the span of entire numeric expressions.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted: is a length of three ropes, but is an area of three bur$_3$; could be read as one and one , where would be one and a half ; is a personal name,

[81] interpreted as a unit, as discussed in section 3.3.

[82] short for nigida, an older reading of bariga; see [**Landsberger1950**; **Powell1975**; **Foxvog2022**].

but would be "one slave".

In addition, there would be a risk of confusion about character identity should fontmakers attempt to treat the curviform and cuneiform numerals as unified. A designer concerned about the numeric-syllabic – contrast, and wishing to support diachronic encoding between systems $S_{\text{Ur III/OB}}$ and S, might give the numeral series (which is typically only used numerically in the Early Dynastic period) the glyphs of the numeral series, since the clear – identification involves the same rotation; this would however make it impossible to represent capacity measures that use . Similarly, in an effort to support diachronic encoding for 1/2(iku), one might be tempted to give the glyph of , thereby rendering the font unusable for quantities measured using the numeral series; an ED I–II Ur font designer could decide to give the same glyph as (that of the proposed), according to the older area system, making it impossible to represent the newer system.

At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millenium corpora. The oddity of ● requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.

4 Rationale for ED–Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millenium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millenium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs KAŠ_{a}, KAŠ_{b}, and KAŠ_{c}, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [**Englund2001**]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered scripts.

As part of the development of these conventions, a classification of fourth millenium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter N with a numeric subscript (sometimes with an additional alphabetic subscript): N_1 is , N_{14} is •, N_{34} is , etc. Transliterations of numeric expression then use those to identify the type of number used, thus $5N_1$ is , and $5N_{14}$ is .

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the CDLI transliteration of or in [P283802, rev. 1, 6, 2, 2] currently uses (barig@c) for the vertical numerals, since numerals are typically capacity measures; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millenium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I–II period, even though the Sumerian text uses classical assyriological transliteration conventions; see [Cha03, p. 6 n. 27].

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millenium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in section 3.7.2; they are indeed used interchangeably, as in [P011104] which uses the notation u@f in [ePSD2], but N14@f in CDLI. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millenium would therefore induce confusion as to which numerals should be used in third millenium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millenium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance, N_{30c} is listed as a variant of N_{30a} in [DE87, p. 166], where the numeric value of either in relation to N_{39a} is still unknown, but their values are found in [**Englund2004**] to be $= \frac{1}{10}$, whereas $= \frac{1}{6}$.

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban$_2$@c) and 5(ban$_2$@c).]

6 Characters not included in this proposal

6.1 Missing numerals

TODO N13 not attested in CDLI TODO (N_{17} not usefully numeric, $12N_{14}$ not encodable, etc.). Cite [DE87, p. 147] 7 and 8(diš *tenû*) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.

6.2 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some *stacking patterns*; for instance 9𒁹 is encoded both as U+12446 𒑆 and as U+1240E 𒐎. This is in part due to contrastive usage of stacking patterns. For instance, besides and which are characteristic of bariga measures, four bariga is written even where 4𒁹 is written, as in [P255010, obv. 2, 3, rev. 1, 17; P292843, obv. 4, rev. 5]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often , and results in metrological systems, typically , occasionally co-occurring as in [P142827; P142357]. This separate encoding is also for compatibility with distinctions made in reference works and in some non-numeric

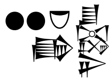


Figure 22: The layout of case [P011099, rev. 2, 3]; the numeral is rotated to fit the rounded corner of the tablet.

Figure 23: The layout of case [P020066, obv. 1, 1]; the numeral is spread across two lines. The text is read in the order , “twenty-two oxen, one year old”.

transliterations; for instance, 𒐉 is [MZL, No. 860] and has the value limmu, and 𒑀 is [MZL, No. 852] and has the value limmu_5. Numeric[83] transliterations occasionally distinguish the stacking patterns 𒐉 𒐊 𒑀 𒑁 𒑂 𒑃, as in the CDLI transliterations of the aforementioned tablets, although this is rare; often 4(diš) is 𒑀 in Ur III, but 𒐉 in the Neo-Assyrian period.

However, the stacking patterns from earlier periods are not separately encoded; for instance, in ED IIIb Ŋirsu, 𒎙 2(u) often has one 𒌋 atop another. These older stacking patterns do not appear to be contrastive, are not marked in transliteration, and are not listed separately in sign lists nor assigned any different values. There is therefore no evidence of a need to encode them; instead, they should be considered style variants, and an ED IIIb Ŋirsu font should have an appropriate glyph for U+12399 𒎙 CUNEIFORM SIGN U U.

Likewise, many stacking patterns are attested for the curviform numerals proposed in this document, and it is not proposed to separately encode them. These distinctions would be incompatible with the state of the art in numeric transliterations, including those by Englund, who insisted on “a system of transliteration that reflects in a strict fashion the physical realities of the cuneiform inscriptions” [**Englund2004**], and they are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier tablets, as they are structured in rectangular cases rather than lines, so that numerals may be laid out across the case in whichever way fits the available space; this is illustrated in Figure 22. Note also that the numerals need to be considerably enlarged in order to reproduce the layout of the tablets, so that often spans two lines of cuneiform signs, as shown in Figure 23. This is impractical when these numerals are set in text that contrasts them with the larger , and inconsistent with actual practice when typesetting these numerals, as illustrated in Figure 7: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dynastic period, but that are also attested in the fourth millenium in the Uruk III period; the fourth millenium, especially the Uruk IV period, also frequently features numerals that use a more vertical layout, as illustrated in Figure 24. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millenium, usage in third millenium scholarship will be more frequent;

[83] The Sumerian word limmu means “four”, so limmu and limmu_5 are still numbers. The distinction here is between usage in transliterations of phrases such as 𒈗 𒀭𒌒𒁕 𒑀𒁀𒆤 lugal an-ub-da limmu_5-ba-ke_4 (king of the four quarters) or of names, and of numeric expressions such as 𒑀𒋡 4(diš) sila_3.

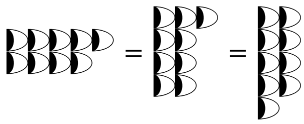


Figure 24: Three stacking patterns for U+12573 CUNEIFORM NUMERIC SIGN NINE N34. The one on the left is the reference glyph, used in Uruk III [P003499, obv. 1, 1b; P004430, rev. 1, 2], and widely afterwards, *e.g.*, ED IIIa Šuruppag [P010678, obv. 2], ED IIIb Ŋirsu [P020057, obv. 1, 3], Old Akkadian Umma [P212464, obv. 11]. The ones in the middle and right are used in two Uruk IV tablets [P001243, rev. P004500, rev. 2]. All three Uruk examples are transliterated 9(N34) in CDLI.

and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents , , , and must remain consistent across the numeral series, lest a numeral be confused with an numeral. Since the single indents are frequently used in running text, as illustrated in section 3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 24 uses OpenType stylistic alternates, and Figure 22 rotates the character , in both cases preserving the plain text backing.

6.3 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [**Englund2001**].

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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Artefacts

[P001243] TODO.
CDLI: P001243.

[P003499] TODO.
CDLI: P003499.

[P004430] TODO.
CDLI: P004430.

[P004500] TODO.
CDLI: P004500.

[P005468] TODO.
CDLI: P005468.

[P010424] TODO.
CDLI: P010424.

[P010458] TODO.
CDLI: P010458.

[P010459] TODO.
CDLI: P010459.

[P010566] TODO.
CDLI: P010566.

[P010570] TODO.
CDLI: P010570.

[P010572] TODO.
CDLI: P010572.

[P010576] TODO.
CDLI: P010576.

[P010578] TODO.
CDLI: P010578.

[P010586] TODO.
CDLI: P010586.

[P010678] TODO.
CDLI: P010678.

[P010773] TODO.
CDLI: P010773.

[P010876] TODO.
CDLI: P010876.

[P010960] TODO.
CDLI: P010960.

[P011099] TODO.
CDLI: P011099.

[P011104] TODO.
CDLI: P011104.

[P020006] TODO.
CDLI: P020006.

[P020008] TODO.
CDLI: P020008.

[P020016] TODO.
CDLI: P020016.

[P020018] TODO.
CDLI: P020018.

[P020019] TODO.
CDLI: P020019.

[P020024] TODO.
CDLI: P020024.

[P020030] TODO.
CDLI: P020030.

[P020054] VAT 4731. [För16, 40 p.14]. Vorderasiatisches Museum.
CDLI: P020054.

[P020057] TODO.
CDLI: P020057.

[P020065] TODO.
CDLI: P020065.

[P020066] TODO.
CDLI: P020066.

[P020090] TODO.
CDLI: P020090.

[P020092] TODO.
CDLI: P020092.

[P020129] VAT 04713. Vorderasiatisches Museum.
CDLI: P020129.
ORACC: epsd2/P020129.

[P020137] TODO.
CDLI: P020137.

[P020182] TODO.
CDLI: P020182.

[P102305] X.3.139. Michael C. Carlos Museum, Emory University.
CDLI: P102305.

[P142357] TODO.
CDLI: P142357.

[P142827] TODO.
CDLI: P142827.

[P212464] TODO.
CDLI: P212464.

[P213162] TODO.
CDLI: P213162.

[P215653] AS 15375 21. Musée du Louvre.
CDLI: P215653.
ORACC: dcclt/corpus/P215653.
Louvre Collections: ark:/53355/cl010436723.

[P220927] TODO.
CDLI: P220927.

[P221266] AO 13825. Musée du Louvre.
CDLI: P221266.
ORACC: epsd2/P221266.
Louvre Collections: ark:/53355/cl010138527.

[P221291] AO 13850. Musée du Louvre.
CDLI: P221291.
ORACC: epsd2/P221291.

[P221305] AO 13864. Musée du Louvre.
CDLI: P221305.
ORACC: epsd2/P221305.

[P221530] TODO.
CDLI: P221530.

[P221531] TODO.
CDLI: P221531.

[P221746] TODO.
CDLI: P221746.

[P221814] TODO.
CDLI: P221814.

[P221815] TODO.
CDLI: P221815.

[P222186] TODO.
CDLI: P222186.

[P222243] TODO.
CDLI: P222243.

[P222399] *Stèle des vautours*. AO 50; AO 2346; AO 2347; AO 2348; AO 16109. Musée du Louvre.
CDLI: P222399.

[P232278] *Gudea E*. AO 6. Musée du Louvre.
CDLI: P232278.
ORACC: etcsri/Q001544.

[P232280] *Gudea G*. AO 7. Musée du Louvre.
CDLI: P232280.
ORACC: etcsri/Q001546.

[P235312] TODO.
CDLI: P235312.

[P240531] TM.75.G.00265. Idlib, Syria: National Museum of Syria.
CDLI: P240531.
EbDA: 1415.

[P240532] TM.75.G.00266. Idlib, Syria: National Museum of Syria.
CDLI: P240532.
EbDA: 1324.

[P240533] TM.75.G.00267. Idlib, Syria: National Museum of Syria.
CDLI: P240533.
EbDA: 1379.

[P240545] TODO.
CDLI: P240545.

[P240548] TM.75.G.00302. Idlib, Syria: National Museum of Syria.
CDLI: P240548.
EbDA: 1350.

[P240579] TM.75.G.00341. Idlib, Syria: National Museum of Syria.
CDLI: P240579.
EbDA: 1364.

[P240597] TODO.
CDLI: P240597.

[P240609] TM.75.G.00440. Idlib, Syria: National Museum of Syria.
CDLI: P240609.
EbDA: 1378.

[P240653] TM.75.G.00535. Idlib, Syria: National Museum of Syria.
CDLI: P240653.
EbDA: 1382.

[P240654] TM.75.G.00536. Idlib, Syria: National Museum of Syria.
CDLI: P240654.
EbDA: 1383.

[P240655] TM.75.G.00537. Idlib, Syria: National Museum of Syria.
CDLI: P240655.
EbDA: 1358.

[P240675] TM.75.G.00557. Idlib, Syria: National Museum of Syria.
CDLI: P240675.
EbDA: 1371.

[P240697] TM.75.G.00579. Idlib, Syria: National Museum of Syria.
CDLI: P240697.
EbDA: 1381.

[P240964] TM.75.G.1392. Idlib, Syria: National Museum of Syria.
CDLI: P240964.

[P240986] TODO.
CDLI: P240986.

[P241708] TM.75.G.02143. Idlib, Syria: National Museum of Syria.
CDLI: P241708.
EbDA: 3173.

[P241764] TODO.
CDLI: P241764.

[P241904] TM.75.G.02346. [Arc89, p. 6]. Idlib, Syria: National Museum of Syria.
CDLI: P241904.
EbDA: 3183.

[P242293] TM.75.G.03125. Idlib, Syria: National Museum of Syria.
CDLI: P242293.
EbDA: 217.

[P249253] *Code de Hammurabi*. Sb 8. Musée du Louvre.
CDLI: P249253.

[P251641] TODO.
CDLI: P251641.

[P252866] TODO.
CDLI: P252866.

[P255010] TODO.
CDLI: P255010.

[P271238] TODO.
CDLI: P271238.

[P274845] TODO.
CDLI: P274845.

[P283802] TODO.
CDLI: P283802.

[P292843] TODO.
CDLI: P292843.

[P298637] TODO.
CDLI: P298637.

[P305639] TODO.
CDLI: P305639.

[P307255] TODO.
CDLI: P307255.

[P309594] TODO.
CDLI: P309594.

[P386847] TODO.
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