Archaic cuneiform numbers

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1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script¹ and the proto-cuneiform script². The proposed characters are listed in section 2. Most of them were listed in [L2/23-190]; however, the present document provides a more detailed rationale for their encoding and additional information about their identity.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed "curved", or "curviform", numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap. Many of these numerals are also used in proto-Elamite⁵ texts, where they are treated as identical characters in scholarship on proto-Elamite, so that

¹ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0. ²ISO 15924: Pcun, not yet encoded.

³Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding ● (small stylus) or ● (large stylus), or at a shallower angle: ▷, ▽ (small stylus), ▷ (large stylus). Some numerals are composed of multiple such impressions, *e.g.*, ▶ . The terms "curved", "curviform", and "round" can be found in the literature. We avoid the term "round" here as it has other meanings in the context of numbers. We use "curviform" in this document as, being the least common term, it is least likely to lead to confusion, and "CURVED" in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [inlineATF].

⁴Impressed into clay using a stylus with a trihedral end: — (stylus held horizontally), ↑ (vertically), ↑ (diagonally) ← (with the head of the stylus), ↑ (stylus pressed deeper, forming a larger wedge), ↑ (combining ↑ and ←), etc.

⁵ISO 15924: Pelm, not yet encoded.

they should be unified with the ones proposed in [L2/23-196]. However, in the interest of time, we do not provide a detailed rationale for this unification in this document, and we are not proposing that the numerals be given the corresponding Script_Extensions property value for now. Neither do we propose encoding any numerals that are solely attested in proto-Elamite texts, or well-attested in proto-Elamite texts but insufficiently attested in Uruk.

[TODO(egg): Mention the other sections here too.]

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
ivumerais		Existing Xsux	
Non-numeric signs	Future Pcun		

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

2 Proposed changes to the Standard

- 2.1 Summary of proposed characters
- 2.2 Properties
- 2.3 Character names list
- 2.4 Core specification text

3 Rationale for curviform-cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [Powell1972] "in the same fashion as we use black and red ink"; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble in texts from Early Dynastic IIIa Suruppag as in the character code charts, if later in the third millenium, in Old Babylonian cursive, in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

⁶Merging with U+1224E CUNEIFORM SIGN NI2.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists and dictionaries, and of composite texts. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]¹⁰. The diachronic approach is also useful for pedagogic applications.

3.2 Arguments for curviform-cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding 12 that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

```
<del>A</del>#
                                                                             ➾
                                      5(diš tenû)
1(\eta e \tilde{s}_2)
             1(u)
                       1/2(diš)
                                                         gi
                                                                    us_2
                                                                             sa_2
          7.5 (ropes)
                                                         reed
                                                                             equal
{{14}
                                                        ➾
                                              ∢⊟
3(u)
                6(diš tenû)
                                             saŋ
                                                        sa_2
3 (ropes)
                                   reed
                                             front
                                                       equal
量量
ašag-bi
                     1(bur<sub>3</sub>c)
                                   1(e\check{s}e_3^c)
                                                 1(iku<sup>c</sup>)
                                                             1/2(iku<sup>c</sup>)
 ašag=bi
 field=DEM15
```

tug_x(LAK483)-si-ga-kam tugsiga =ak =am -Ø ploughed=GEN=COP-3.SG.S

⁷Notably [OSL] and the online edition of [MZL] in [eBL, Signs].

⁸Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

⁹For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of ₩□ ★ ⟨ □ □ □ , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

 $^{^{10}}$ Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svä+24, pp. 242, 148].

¹¹For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

¹³As noted in [Pow87, p. 466], this sign has a very short "tail" in this period, so that it is wider than it is tall, and can at first seem like a large — in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

¹⁴Note that ED IIIb 〈 numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign ⋘ in [P020054] looks more like Ur III ❖.

¹⁵Alternatively: area=POSS.3.SG.NH, "its area".

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as 16

465 metres, equal lengths 198 metres, equal widths this field is 9, 18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that "the number signs do not normally carry in their individual signs the meaning of what they are used to measure", and that curviform and cuneiform numerals "are not normally mixed together in a single numerical expression", noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from to I numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that "The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases 17 ."

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere $\triangleright/!$ switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

 $^{^{16}}$ We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [Po20054], instead of a field of five by twenty-five metres.

¹⁷The cuneiform encoding process was planned in *stages* in [L2/03-162]. One might expect the second stage of encoding, which led to the creation of the Early Dynastic Cuneiform block, to incorporate the numerals needed for the representation and discussion of Early Dynastic texts; however, the proposal [**L2/12-208**] stated that "numerals have been omitted due to the complexity of numeral signs from this period. An expert in the metrology of this period must be consulted before these can be properly included."

3.3 A primer on classic Ur III and Old Babylonian metrologies

Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram¹⁹, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and

¹⁸See, e.g., [Uni16, §22.3.3, sub "Cuneiform Numerals"].

¹⁹These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

the approximate metric equivalent [Frio7, p. 378; Rob19]

$$\Leftrightarrow \stackrel{10}{\longleftrightarrow} \diamondsuit \stackrel{6}{\longleftrightarrow} \stackrel{10}{\diamondsuit} \stackrel{10}{\longleftrightarrow} \stackrel{3}{\longleftrightarrow} \stackrel{6}{\longleftrightarrow} \stackrel{10}{\longleftrightarrow} \stackrel{3}{\longleftrightarrow} \stackrel{6}{\longleftrightarrow} \stackrel{6}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{10}{\longleftrightarrow} \stackrel$$

Note that for the range of areas given above²⁰, this system does not use any symbols separate from the numerals for the individual units (ubûm, ikûm, eblum, and būrum). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign if functioning as punctuation²¹, but the numerals are tied to the metrology; thus a surface of 5 būr 1 ebel 4 ikû (100 ikû, 36 ha) would be written²² ≪ ← ₩ 🕍 . Contrast this with systems where the same numerals are used for different units, and overt units are used, as in "88 acres 3 roods 33 perches" or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations: the sign \diamondsuit is equal to sixty times \checkmark in the area system, but to three hundred and sixty times (in the discrete counting system.

3.3.3 The capacity system

Another such system of note is the one for capacities ²³ [Frio7, p. 376; Rob19],

where the numerals for ban₂ are +, \ddagger , \ddagger , \ddagger , and \ddagger , and those for bariga are \cdot , \cdot , \cdot , and \(\) (contrast ordinary \(\) and \(\) otherwise used with \(\) numerals). As described in [Hue11, p. 585 n. (b) and (f)], the sign ## GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign if present, and after the word for "grain" if present, as in

Observe that while large numbers of gur follow²⁵ system $S_{\text{Ur III}/OB}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as

$$\underbrace{\diamondsuit \not\models \stackrel{10}{\longleftrightarrow} \diamondsuit \not\models \stackrel{6}{\longleftrightarrow} \diamondsuit \stackrel{10}{\longleftrightarrow} \diamondsuit \stackrel{6}{\longleftrightarrow} \underbrace{^{10}}_{\longleftrightarrow} (\stackrel{3}{\longleftrightarrow} \overset{6}{\longleftrightarrow} -\stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow} \stackrel{10}{\longleftrightarrow} \stackrel{1}{\longleftrightarrow} \stackrel{$$

²⁰For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mūšarum* (36 m²) written [] equal to one hundredth of an ikûm, then sexigesimally subdivided in 60 [m] (shekels). For areas greater than 3600 *būrū*, the ♦ and ♦ numerals are reused with a suffix 🖹 (gal, Sumerian: big), as follows [Robo8, p. 295 n. b and c; Frio7, p. 378; Rob19]:

²¹This sign is sometimes interpreted as a measurement unit, and transliterated iku, see, e.g., [Proust2020], or transliterations in [Feu04] discussed in section 3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system $G_{\rm Ur~III/OB}$.

²As in the surface of the field of **|| ← | ⟩|** (the city of Apisal) reported on [P102305, r. 1]

²³Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{\text{Ur III/OB}}$ based on a height of one cubit, see [Pow87, p. 488; Robo8, p. 294; Rob19].
²⁴From [**P309594**].

²⁵A larger unit, the guru₁ (*karûm*, grain heap), is sometimes used instead, with **— ■冷**無<=◇ 井 (1 karûm = 3600 kurrū). See [Frio7, p. 415; Rob19].

⟨ | ♯ | would be 10 gur 1 bariga, and ⟨ − ♯ | would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of alreadyencoded numeral series:

- I-IIIII used in $S_{III}IIII/OB$ and the SPVS as well as with overt units;
- \leftarrow used in $G_{\text{Ur III}/OB}$, of which \leftarrow are also used in $S_{\text{Ur III}/OB}$ and the SPVS as well as with overt units;
- I-W used in $S_{Ur III/OB}$, and sometimes with overt units;
- \P - \P used in $S_{\text{Ur III/OB}}$;
- \diamond \diamond used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
 \diamond \diamond used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \leftarrow used in $\mathcal{C}_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
- 十, ‡, 肆, 戡 used in C_{Ur III/OB};
- I, I, I used in $C_{Ur III/OB}$ —note the overlap with I-III;
- \Join and \Join used in $G_{\text{Ur III}/\text{OB}}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Robo8, p. 78]: "The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity."

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with $\$ and $\$ numerals with their system $S_{\text{Ur} | \text{III}/\text{OB}}$ values²⁶. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Frio7, p. 118; Rob19]:

Two more units appear occasionally [Pow87, p. 459; Frio7, p. 118; Rob19]:

In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

 $^{^{26}}$ Adjacent units are no more than a factor of 60 apart, so higher numerals such as \P or \diamondsuit are not

²⁷As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.

3.3.5 Fractions

Fractions of the $ik\hat{n}m$, $N = \frac{1}{2}$ — and $N = \frac{1}{4}$ —, have already been encountered. In other contexts, the fraction $\frac{1}{2}$ is written $\frac{1}{4}$, and the fractions $\frac{1}{3}$ and $\frac{2}{3}$ are written $\frac{1}{4}$ and $\frac{1}{4}$. The latter two signs are derived from curviform signs \mathbb{F} and \mathbb{F} , which are already separately encoded; these are in turn derived from the sign \mathbb{F} ($\mathbb{S}U_2$), whose Early dynastic form resembles \mathbb{F} , and \mathbb{F} numerals; see [Powell1971]. The \mathbb{F} is sometimes omitted, as in [P240545; P221530; P221531; P271238; P274845].

3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{\text{Ur III}/OB}$ [Frio7, p. 374; DE87, pp. 127, 165]:

$$\bullet \stackrel{10}{\longleftarrow} \bullet \stackrel{6}{\longleftarrow} \stackrel{10}{\longleftarrow} \stackrel{10}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{10}{\longleftarrow} \stackrel{10}{\longleftarrow} . \tag{S}$$

Likewise the area system used in the Early Dynastic IIIb period for areas of one $ik\hat{u}m$ and greater mirrors system $G_{\text{Ur III}/\text{OB}}$ [LAK, p. 72; NDE93, p. 63; Frio7, p. 378; Lec16]:

$$\bullet \stackrel{10}{\longleftarrow} \bullet \stackrel{6}{\longleftarrow} * \stackrel{10}{\longleftarrow} \bullet \stackrel{3}{\longleftarrow} \stackrel{6}{\longleftarrow} \triangleright, \qquad (G_{\rm ED\,IIIb})$$

with consistent use of the numerals: \bullet corresponds to \langle , \bullet to \diamond , and \bullet to \diamond . An exception to this correspondence, noted in [L2/04-099, p. 4] (see section 3.2), is that the vertical I from $S_{\mathsf{Ur}\;\mathsf{III}/\mathsf{OB}}$ corresponds to a horizontal \triangleright in system S. This is however far from the only case of such a reallocation of function. The earlier form of System G is [DE87, pp. 141, 165; Frio7, p. 378]:

Observe that, as noted in [DE87, p. 142], \odot changes meaning from $10 \bullet$ in system G to $600 \bullet$ in system $G_{ED \, IIIb}$. System G is used in the Uruk period, but also in the ED I–II period (it is the "area 2" system in [Chao3], whereas $G_{ED \, IIIb}$ is the "area 1" system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb area system²⁸,

with the numerals of a contemporaneous capacity system:

$$\underbrace{\stackrel{10}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{10}{\longleftarrow} \stackrel{4}{\longleftarrow} \stackrel{6}{\longleftarrow} \stackrel{4}{\longleftarrow}, \qquad (C_{\exists \forall \exists \exists \exists \{\}})$$

²⁸A variant is \bigcirc $\stackrel{10}{\leftarrow}$ $\stackrel{6}{\leftarrow}$ $\stackrel{10}{\leftarrow}$ $\stackrel{3}{\leftarrow}$ $\stackrel{6}{\leftarrow}$ $\stackrel{2}{\leftarrow}$ $\stackrel{7}{\leftarrow}$ $\stackrel{2}{\leftarrow}$ $\stackrel{2}{\leftarrow}$ $\stackrel{2}{\leftarrow}$ $\stackrel{2}{\leftarrow}$ $\stackrel{7}{\leftarrow}$, see [Powell1972].

²⁹The cuneiform counterpart is **√**.

system is recognizable, with \neg corresponding to \P for bariga, \P - E corresponding to \P - E for ban₂, and the II counted with \square rather than II numerals. However, the half- $\operatorname{Ik}\hat{u}m$ is counted with the same \triangledown as the bariga, whereas it uses a different sign, $\char`$, in the Old Babylonian system. As we will see, this is cannot be handled as a split, by giving $\char`$ the glyph $\char`$ in an Early Dynastic IIIb font, as the $\char`$ numeral series is also in use in that period.

3.4.1 Field lengths in Nirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign \slash is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign \slash is not written either. Lengths shorter than one rope are expressed in half-ropes using the $\frac{1}{2}$ sign \slash (again with no \slash), and then in reeds, with the sign \slash , as follows:

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and most others from the same period, such as the ones discussed in [Lec20], areas are expressed in system $G_{\rm ED~IIIb}$, with curviform numerals³³; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope³⁴, the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and thus the interpretation of its numerals between systems $G_{\rm ED~IIIb}$ and $L_{\rm ED~IIIb}$. The sign \Box , which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without \Box . There are other such co-occurrences contrasting between metrological systems; for instance, [Kre98, p. 303 n. 686] mentions the use of cuneiform numerals for days and months³⁵.

3.4.2 Dyke lengths in Nirsu

[Pow87, p. 466] notes that reeds "are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus." Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in CDLI finds many occurrences³⁶, such as:

 $^{^{32} \}text{The reeds}$ are counted using $ten\hat{u}$ numerals, \nwarrow , \nwarrow , etc.

³⁴This is the case of the sides of the field in [P020054, obv. ii 2–3].

³⁵That note also mentions a contrast between the use of curviform numerals to count people and curviform numerals to count bread alotted to them in [P010876]; such contrasts are more akin to styling, and might not, on their own, justify the disunification.

 $^{^{36}}$ A search for curviform numerals followed by some number of reeds counted in ($ten\hat{u}$) cuneiform numerals currently finds 125 occurrences across 47 tablets.

These expressions use an explicit sign $V \cong I$ (counted in multiples of ten) or II. This notation—but not its use of curviform numerals—is remarked on in [Lec20, p. 290 n. 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. These can be summarized by the following factor diagram:

$$\underbrace{\bullet} \overset{10}{\longleftarrow} \underbrace{\bullet} \overset{6}{\longleftarrow} \bullet = \underbrace{\triangleright} \overset{2}{\longleftarrow} \overset{10}{\longleftarrow} \underbrace{\uparrow} \overset{10}{\longleftarrow} \underbrace{\uparrow} \overset{6}{\longleftarrow} \overset{4}{\longleftarrow} \underbrace{\uparrow} \overset{3}{\longleftarrow} \underbrace{\downarrow} \overset{1}{\longleftarrow} \underbrace{\longleftarrow} \overset{1}{\longleftarrow} \overset{1}{\longleftarrow} \underbrace{\longleftarrow} \overset{1}{\longleftarrow} \overset{1}{\longleftarrow} \underbrace{\longleftarrow} \overset{1}{\longleftarrow} \underbrace{\longleftarrow} \overset{1}{\longleftarrow} \underbrace{\longleftarrow} \overset{1}{\longleftarrow} \underbrace{\longleftarrow} \overset{1}{\longleftarrow} \overset{$$

Butter, cheese and wheat in Nirsu

A similar mixture of cuneiform and curviform numerals may be observed with the capacity system; indeed, the previously described $\exists t \notin \underline{\mathbb{H}}_{s}$ system uses \cdot numerals for \$\footnote{\chi}\$ [Fri78, p. 43; Lec16]:

as in [P020016; P020065; P020090; P020092; P020137] and others, where ban2 counted with ₹ numerals are followed by sila₃ counted with ↑ numerals. Curviform numerals are also used to count sila₃, but not⁴0 as part of the ♯ systems. This contrast can be seen in [P220927], which measures butter (\implies , i_3) with a different capacity system, using the ♠ (dug, "pot") of 20 ⋪, with ▷ and • numerals 41 for both the and the \$\,\text{, thus [Pow87, pp. 504 sq.]}

$$\underbrace{\stackrel{10}{\longleftrightarrow}}_{\longrightarrow} \stackrel{2}{\longleftrightarrow} \underbrace{\stackrel{10}{\longleftrightarrow}}_{\nearrow} \stackrel{1}{\smile} \underbrace{\stackrel{\frac{3}{2}}{\smile}}_{\nearrow} \stackrel{\mathbb{H}}{\longleftrightarrow} \stackrel{2}{\longleftrightarrow}_{\nearrow}, \qquad (C_{\longrightarrow})$$

but counts cheese (\bowtie , ga'ar) using the \rightrightarrows $\mathclap{\blacksquare}$ capacity system, with \searrow numerals for the 3.

 $^{^{37}}$ CDLI only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the $\blacksquare \triangleright \parallel \cdot \parallel$ is not visible. Lecompte notes that the copy is faithful; indeed another ▮▷!! < can be seen both on the copy and the photo on obv. 2, 2.

³⁸From copy.

³⁹With either unit omitted, as in the examples above, or both, as in [P020129, obv. 2, 1] ▶ • ⋄ ♣₩.

⁴⁰As of this writing, the single occurrence of (ban2@c) followed by curviform numerals and sila3 in ED IIIb Nirsu transliterations on CDLI, 4(ban2@c) 3(asz@c) sila3 in [P221815], is incorrect: it should be 4(ban2@c) 3(disz@t) sila3.

⁴¹This tablet also uses subtractive notation: ▶▶ 🛣 "two pots minus one third (sila₃)", • To provide the minus one pots, six sila₃". Such subtractive notation is common in most of the metrological systems discussed here; it appears in the ED IIIa period [Robo8, p. 77]. We shall not discuss it further, as presents no complexity for character encoding. It is noteworthy that the sign [(lal₂, "minus") is often ligated with the following numerals, with the subtrahend placed under a sometimes considerably enlarged Γ , similar to the layout of the radical in modern mathematical notation, see, e.g., [P020092]. A good font could handle the very common -1 case, perhaps even -2 and -3; setting arbitrary numeric expressions under the Γ , or more generally replicating the layout of Early Dynastic tablets, is outside the realm of plain text; see also section 6.2.

Another capacity system in ED IIIb Nirsu is the \(\pm\), the gur of two ul:

$$\underbrace{\stackrel{10}{\leftarrow} \stackrel{}{\smile}}_{\text{+1.5}} \stackrel{2}{\leftarrow} \stackrel{6}{\lor} \stackrel{6}{\leftarrow} \stackrel{4}{\lor} \stackrel{6}{\leftarrow} \stackrel{\checkmark}{\searrow}. \qquad (C_{\pm 1.5})$$

Here the $\triangleright - \setminus \neg \neg$ contrast occurs not only within the numerals of the system, but with its units; this is perhaps best illustrated by the expressions 🗮 🗲 •□母ミソ 出《囗 in [P221746] and 縱 医 •母冬ソ 出《囗 in [P221814].

3.4.4 Grain in Ebla

The mixing of curviform and cuneiform numerals within a metrological system is not specific to Nirsu.

The system of grain⁴² capacities in Ebla uses the following units⁴³:

The \Box and \Box \Box are generally counted using curviform numerals, and the smaller units using cuneiform I numerals44 Indeed, a search on [EbDA] for cooccurrences of either ★☆ or 🏿 🖈 with either of 🖈 🗘 or 🗗 🗏 finds the following expressions⁴⁵:

- 1. [P240532, verso 4, 9] ▷ ₩ 석 ₩ 4 □ 47 ₩ № { 4□
- 2. [P240548, verso 1, 1] ▷◁\\ ≡ \\ \★
- 3. [P240655, recto 7, 9] DD ₩ L 48 ₩ 12 14
- 4. [P240579, verso 4, 3] BP□□ □ L Ⅱ V {□ 5. [P240675, verso 2, 2] □ 無 日田 W V {□ 1
- 6. [P240609, verso 3, 1] ▷◁∄៕ \\\
- 7. [P240533, recto 3, 3] ♣▷▽ြ♠ L WV M I I + ★ ↔ H

$$\underset{\text{la-ha}}{\text{fill}} \overset{30}{\longleftarrow} \overset{3}{\underset{\text{sila}_3}{\bigvee}} \overset{6}{\longleftarrow} \overset{\text{#}}{\underset{\text{an-zam}_x}{\longleftarrow}}.$$

A glance it seems that \nearrow are counted with cuneiform numerals and higher units with curviform ones,

$$\underbrace{ \begin{array}{c} \underbrace{\frac{5}{3}}{} \\ \underbrace{\text{Fig.}} \end{array} \underbrace{\begin{array}{c} 6 \\ \bullet \end{array} \underbrace{\begin{array}{c} 10 \\ \bullet \end{array} \underbrace{\begin{array}{c} 10 \\ \bullet \end{array} \underbrace{\begin{array}{c} 6 \\ \bullet \end{array} \underbrace{\begin{array}{c} 1 \\ \bullet \end{array} \underbrace{\begin{array}{c} 6 \\ \bullet \end{array} \underbrace{\begin{array}{c} 1 \\$$

but we have not investigated this thoroughly.

¹³Another system uses different values for the 🎹 and 🏿 {♥☐, see [Cha12, p. 62; Arc15, p. 229 n. 12]:

⁴⁴As mentioned in [Cha12, p. 63], the 🎹 is also counted using the ₹-農 numeral series. Some instances of that usage are found transliterated n/6 in EbDA; in some cases the I sign is omitted, and the ₱ numeral is then written before the ⊥ unit, as in ▷▷ ₱ ፟ from [P240545].

⁴⁵We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform 🗗 🖽 and larger with cuneiform 🎹 and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked 46 on a photograph (from EbDA unless noted otherwise).

⁴⁶As we will see in Section 3.7.2, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

⁴²Liquid capacities use a different system [Arc15, p. 229 n. 12]:

⁴⁷ba-ri₂-zu₂, a variant spelling.

⁴⁸Short for □ L.

- 13. [P241904, recto 1, 1] 53 BB FQ L #54 W {4

3.4.5 Use in modern publications

Because of their prevalence in the Uruk and Early Dynastic periods, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 1–19.

⁵⁰Instead of the expected **₩**{�〓.

⁵¹ **III ★** ← ⇒ not legible on the EbDA photo.

⁵²From CDLI photo.

⁵³From photo in [Arc89, p. 6].

⁵⁴Laid out as [1]; on stacking patterns see Section 6.2.

⁵⁶Short for **⋘∤ ℘**—.

 $^{^{57}}$ ŠU₂+NIG₂-sag, an unusual variant spelling.

⁵⁸The untransliterated text would be 影解評過日本時間圖影整理過過; note the atomically encoded ib₂ × 3 | 三季 × ||| 三數.

for the words & u & a n a and & a n a b i. Deimel's reading & a n (a) for U came out of the reading / a n t a k/ for the sign Y and the writing of & a (-n a) after the fractional signs for & u & a n a and & a n a b i in Old Sumerian texts. But this was an ill-conceived argument at its inception, for

Figure 1: Discussion of the readings of proposed ∇ and already-encoded I in [Powell1971].

sions also. In example 6, the writing the may imply a reading /š a n a b i/, whereas the in example 11 should be read */š u š a n a m i n/. Moreover, the question must be raised as to whether such writings as < \cup \cup \cap k \under u - b a b b a r \under \under a - a a^2 do not perhaps imply a linguistic resolution of */š u \under a n a m i n/ rather than /\under a n a b i/. I see no way of answering this question at present, but it is one which one

Figure 2: Discussion of the readings of proposed \bowtie and \bowtie as well as already-encoded \triangledown and \triangledown in [Powell1971].

```
(C 234)
5010 1U = 50 70 (C 314)
1 ♥ 1 № 1 № = 6° 2 ♥ 1 № (C 27) .
```

These metrological equations for the "unknowns" \emptyset , \circ , $\overline{\mathbb{V}}$, etc., can be treated exactly as ordinary equations for unknowns $\ x,\ y,\ z,\dots$. In particular, the equations can be simplified by subtraction of equal amounts from both sides of the identities. In this way the three equations above can be reduced to:

```
(4\ensuremath{\mathsf{V}} 3. subtracted from both sides)
    2° = 20 🛭
1 \( \tau = 6 \cdot \) \( (5 \tau 1 \cdot \) \( \tau = \tau \) \( (1 \tau 1 \tau - \tau - \tau \) \( (1 \tau 1 \tau 1 \tau - \tau - \tau \) \( (1 \tau 1 \tau 1 \tau - \tau - \tau \) \( (1 \tau 1 \ta
```

We can now read off from the first equation that 10 = 100 , and from the second that 1 U = 6. Then the third equation can be simplified (by "substithtion" of these values into the equation), to the following reduced form:

```
100 = 2 U 9 Da.
```

The most likely solution to this last equation is, of course,

Figure 3: The derivation of the factors of the bisexagesimal system in [Fri78, p. 15]⁶⁰.

$$5 \bullet + 4 \triangleright = 3 \bullet + 24 \triangleright \tag{C 234}$$

$$1 \triangleright + 1 \bullet + 5 \triangleright = 7 \bullet + 5 \triangleright \tag{C 314}$$

$$1 \times + 1 \times + 1 \bigcirc = 1 \times + 2 \bigcirc + 6 \bullet. \tag{C 27}$$

A diplomatic edition of [Fri78] could rotate the numerals using a higher-level protocol:

$$(C 234)$$

$$1 \boxed{1} 1 \boxed{1} = 6 \cdot 2 \boxed{1} 1 \boxed{1} \tag{C 27}.$$

 $^{^{60}}$ The bisexagesimal system is used alike in proto-Elamite and proto-cuneiform texts, see [Fri78, p. 38]; the derivation in [Fri78, p. 15] is based on proto-Elamite artefacts. Note that in Friberg's early works [Fri78; Friberg1979; Friberg1986; Fri87], copies of fourth millenium and sometimes third millenium tablets are shown as vertical text (which they were for the scribes), and their numerals are written within horizontal text in the same orientation that they have if the tablet is taken as vertical text; in [UAX50] parlance, as if they had Vertical_Orientation=Upright. In addition, they are listed in these equations in the horizontal order in which they appear as vertical text (thus the rightmost numeral is the most significant, read first). Cuneiform is correctly Vertical_Orientation=Rotated, consistently both with modern practice and with the rotation between earlier vertical and later horizontal monumental inscriptions. Friberg's early conventions are not followed in later scholarship, and are abandoned in his own more recent works, such as [Frio7]; a more typical way to express the first equations might be

Thus, for instance, the original set of fractions ∇ , σ , and δ (1/2, 1/4 and 1/8 of an iku) in the Sumerian GANA system was after a time augmented through the addition of the new sub-unit SAR: Ξ , equal to 1/100 of an iku (D). Similarly, the Sumerian weight unit "ma-na" which originally may have had only the sub-units ∇ sa-na (= 1/3 mana) and ∇ ∇ sa-na-bi (= 2/3 mana), and perhaps also gin: ∇ (= 1/60 mana), seems to have acquired, at some time or other, also the smaller sub-units ∇ ∇ sa (= 1/3 gin), and ∇ seems to 1/4 × 1/60 gin).

Figure 4: Discussion of proposed fractions \triangledown , \triangleleft , \blacktriangleleft , and ">=, as well as already-encoded ">= and ">= in [Fri78, p. 49]

stein publizierten Zeicheniiste enthalten ist³, bis vor kurzem unentdeckt bleiben konnte. Erst 1978 machte der schwedische Mathematiker J. Friberg, ER8M I, 9-11, darauf aufmerksam, daß die Zeichen für die Zahlen Eins (D) und Zehn (ullet) in Verbindung mit dem Zeichen ŠE nicht im Verhältnis 1 zu 10 sondern im Verhältnis 1 zu 6 stehen. Bis dahin hatte man, obwohl die Andersartigkeit des in Verbindung mit dem Zeichen ŠE verwendeten Zahlzeichensystems bekannt war, für diese beiden häufigsten Zahlzeichen einheitlich ein Verhältnis 1 zu 10 unterstellt, obwohl es mehrere eindeutige Gegenbelege gab, von denen zumindest diejenigen der Archaischen Texte aus Gemdet Nasr bereits früh publiziert und jedermann zugänglich waren⁴. Als Folge

Figure 5: Discussion in [DE87, p. 117] of the discovery in [Fri78, pp. 9–11] of the different relations between \triangleright and \bullet in systems G and \ref{G} ?

3.5 Non-numeric usage

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign - has, *inter alia*, the values as, rum, and dili. While the

there is in any case an important qualitative difference between IX for Latin novem and $\ \ \ \$ for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 6: The sign \$\ \text{used in a parallel with IX in [Englund1988], discussing an argument from [Powell1972] on the question of the language of the Uruk III texts.

 $^{^{61}}$ The reader will recall that $\eta e \ddot{s}_2$ is written $\ref{1}$, with a larger wedge than $\ref{1}$; however, these signs have merged by the time Examenstext A is composed.

 $^{^{62}}$ Besides η e 82 , a look at [OSL] shows that the values dis, ge_3 , makkas, $sa\eta$ tak $_4$, and tal $_4$ are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word ana in the Neo-Assyrian period.

of decreasing fractions $^{1}/_{n}$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus $\Xi = ^{1}/_{n} N_{no}$, $\varpi = ^{1}/_{n} N_{no}$, and so on. The first sign of the latter units, N_{no} ,

Figure 7: Description of the fractions Ξ and $\overline{\diamondsuit}$ in [Eng98, p. 113]⁶⁵.

For instance, the first line contains the notations $1N_{34}$ $1N_{390}$; $2N_{20'}$ which can be translated "60 of the (grain rations containing) — (of grain); (grain involved:) $2 \bullet$ (of ground barley)". This calculation contradicts the assumed numerical relationship $10N_1 = 1N_{14'}$, since as was well known the measure represented by the sign N_{39} was $1/_5$ of that represented by N_1 , so that $60 \times 1/_5 = 12$ and not 20, as $2N_{14'}$ would imply. Instead of relying on complicated

Figure 8: The sign \bigcirc used as a capacity measure within otherwise translated text in [Eng98, p. 116].

Die halbkreisförmigen Griffeleindrücke gehen manchmal in mehr oder weniger eckige Formen über (\P)⁶⁸⁵. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen (\P), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren ⁶⁸⁶. Selten treten mit \neg gebildete Zahlen auf ⁶⁸⁷ (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

⁶⁵The text erroneously has N_{34} instead of N_{24} .

⁶⁶The statement that these do not co-occur refers to the texts from ED I–II Ur; these signs co-occur both earlier and later in areas, with different relations as previously discussed.

⁶⁸The cuneiform text is Unicode-encoded.

⁶⁹Compare \diamondsuit \biguplus discussed in section 3.3.2. The order is variable in the Early Dynastic period; see [P010773], also discussed in [Frio7, p. 148], for an example of \biguplus \blacksquare and [P274845; P241764] for examples of \biguplus \blacksquare \blacksquare . Sign order can be variable in early texts, see [Foxvog2016].

```
The calculations:
                                                                                                          60 × <sup>1</sup>/<sub>5</sub> ⊳
Obv. i 1
                                                                                                                                                                                                                                                                                                    = 12 × 100 =
                                                                                                      120 \times \frac{1}{10}
                                                                                                                                                                                                       (Z)
                                                                                                                                                                                                                                                                                                    = 12 × 1≥ =
                                                                                                120 \times \frac{1}{15} \bigcirc 300 \times \frac{1}{20} \bigcirc
                                                      3
                                                                                                                                                                                                                                                                                                                                 8 × 100 =
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 2 × 1
                                                                                                                                                                                                     (i⊠i)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                2 \times 10^{-1}
                                                                                                                                                                                                                                                                                                      = 15 × 155 =
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 3 × 10
                                                                                                  600 × 1/25 ₪ (図)
                                                                                                                                                                                                                                                                                                        = 24 × 100 =
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   4 \times 100
Rev. i 1
                                                                                       1200
                                                                                                                                                                                                                                                                                                                                                                                                1 × •
                                                                                                                                                                                                                                                                                                                                                                                                                                                              1 × :• :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 5 × 10
 Obv. i 6
                                                                                       6000 \times \frac{1}{30} \implies (GAR + 6N_{57}) = 200 \times 100 = 1 \times 100 \times 100 = 3 \times 100 \times 100 = 1 \times 100 \times 10
                                                                                                120 \times \approx 1/4 \bigcirc (DUG_a + U_{2a}) \approx 30 \times \bigcirc = 180 \times 1/5 \bigcirc (DUG + A\tilde{S}_a) = 36 \times \bigcirc = 180 \times 1/5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   5 × :•
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 1 × 10
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   6 \times 10^{\circ}
                                                                                                                                                                                                                                                                                                    = 20 × 1000 =
                                                                                                                                                                                                                                                                                                                                                                                                                                                                 3 \times 10^{\circ}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 2 × 🕞
                                                      3
                                                                                                  300 \times 1/_{15}
                                                                                                                                                                                                     (KAŠ<sub>a</sub>)
 Rev. i
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   4 × ...
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   3× ⋅ 1≥
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   5 × 10
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   1 × :•
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 2 × 100
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 3 × 🖘
                                                                                                                                                                                                                                                                                                                                         1 \times \bigcirc 2 \times \bigcirc
 Grand total of groats used:
                                                                                                                                                                                                                                                                                                                                                                                                                                                            9×:•
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 4 × 10
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               1 × 🖘
 Grand total of malt used: 1N_{47} 4N_{20} 3N_5 1N_{42a} (rev. i 3) \times 3/5 \approx
```

Figure 6. Transliteration and calculations of MSVO 4, 66.

Figure 10: Calculations from [P005468] transcribed in [Englund2001] using modern mathematical notation combined with some of the proposed characters.

> strong similarities between "area" 1 and "area" 3 systems, the sign with two concentric discs (\bigcirc , notated N_{50}^{27}) remains problematic. It never appears in any numerical combination with the sign with a single disc (,

Figure 11: Discussion of **②** and **③**⁶⁶ in [Chao₃, p. 6].

 $^{1}/_{15}$, etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation — I ## could be understood as 1 1/5 gur. ²⁷ However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs + and + cannot be considered klasmatograms

Metrological tablets from the end of the 4th millennium (Nissen, Damerow and Englund 1993, 55-59, to MSVO 1, nos. 2-3) contain a discrete set of numerical

1(iku) represents a surface of 3600m 1(eše₃) represents a surface of 21,600m²

The signs iku and eše $_3$ constitute by themselves measures of surface areas. These measures are usually followed by the sign GAN2, which means either surface or field and

Figure 12: Discussion of Old Babylonian 68 capacity and fourth millenium area measures in [Proust2009].

formed by only two signs \ and < , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60^2 , 60^3 , 1/60, $1/60^2$, etc. are written in the same way, with the vertical wedge . The third feature concerns the exact function of

Figure 13: Description of the SPVS in [Cha12, p. 58], using the already-encoded signs I and **⟨**.

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648, 000 with:

Figure 14: Discussion of large numbers illustrated by [69] in [Cha12, p. 59]

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

DDD še bar ∇ ba-rí-zu

'3 gubar (capacity units) and 1 parīsu'.

Figure 15: Partial transliteration of [**P240597**] DDD **★★ 上** □ ► ★ **★** ★ in [Cha12, p. 61].

This is particularly true of the signs \bowtie , \bowtie , \bowtie and \bowtie , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written \bigtriangledown in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign \bigtriangledown is most often associated with the *parīsu* measure, while the signs \bowtie , \bowtie , \bowtie , and \bowtie refer to 1, 2, 3,

Figure 16: Discussion in [Cha12, p. 64] of the relation between $\nabla - \not\equiv$ and ∇ in Mesopotamia and in Ebla.

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., transliterated as $2(\tilde{s}ar_2)$ $1(ge\tilde{s}'u)$ 3(u), means $2\times 3600+1\times 600+3\times 10$). The system is based on an alternation of factors ten and

Figure 17: Explanation of the structure of the number ●● ● • in [**Proust2020**].

might think of one fabric and a half, ¹¹ but the presence of notations with " $2^{D} 2^{U}$ ", " $3^{D} 3^{U}$ ", and " $6^{D} 6^{U}$ " (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag's weight measurement system. ¹² The notation " 1^{D} gada" in o. ii 1 and r. vi 1, along with the total of " 3^{D} 9.



Fig. 1. Combinations of numerals attested in Š. 742.

Figure 18: Discussion of the contrast between ▷ and ¬ numerals in [Gor23, p. 162].

as, for example, in TM.75.G.3125 = *ARET* III 107 o. iv 1, " $4 \ ^{\circ}$ ' a_3 ·da-um'" $-2 \ ^{\wedge}$ $4 \ ^{\circ}$ aktum $4 \ ^{\circ}$ Dib₂'" $-3 \ ^{\circ}$ sa₆ gunu₃" (Fig. 2).

Figure 19: Transliteration in [Gor23, p. 163] of [P242293, *recto* 4, 1] incorporating untransliterated numerals.

horizontal numerals are most frequently written ightharpoonup in the Early Dynastic period⁶³, such non-numeric usage is almost⁶⁴ always written ightharpoonup, for instance:

- in personal names in administrative texts, such as the following, which all contain
 □ numerals:
 - ← إلى صلابيخ in [**P010424**; **P010458**; **P010459**] from ED IIIa أبو صلابيخ
 - → in [P010960] from ED IIIa Šuruppag,
 - 種世ー in [P251641] from ED IIIb Adab,
 - < IIIb Adab, (P252866) from ED IIIb Adab,
 - 卦>> **這一** ⇒ in [**P298637**] from ED IIIb Umma;
- in the Sumerian word
 ^m − u₂-rum, "property" in ED IIIb Nirsu administrative texts which contain
 ^p numerals, such as [P020006; P020008; P020018; P020024; P020030];
- in lexical texts:
 - in the divine name *** □ □ □** in the lexical texts [**P010570**; **P010572**], where the entries are prefixed with **□**.
 - in the word ➤ dili, "small fish" in [P010578], witness to Early Dynastic Fish,
 - in the same word with a determinative, ¼ dili^{ku}, in [**P010586**], witness to Early Dynastic Food, which starts with ▷ numerals.

This is a clear contrast between - and \triangleright in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name - \blacktriangleleft occurs on its own line in the aforementioned administrative texts; a line \triangleright \blacktriangleleft would instead be read as "one slave".

3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

⁶³A CDLI search for "(asz@c)" finds 3296 ED texts, while a search for "(asz)" finds 81 ED texts, of which 46 also contain "(asz@c)".

⁶⁴Exceptions are discussed in section 3.7.1.

⁷⁰Non-numeric values of — were discussed in section 3.5; → has the values man₃ and min₅, and is used for the word didli, "several, various"; → has the value es₆.

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from to numerals prevents diachronic encoding even if were unified with -. For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'-6']; however, they are written with numerals, whereas they are written with numerals in the Early Dynastic witnesses; since and - are distinct characters, the --- unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods⁷³, there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Romach2023], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts⁷⁴ being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

3.7.1 The case of ŠAR₂

The character U+122B9

■ CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period

75, a con-

⁷¹Compare, e.g., in the Instructions of Šuruppag, 三文文 与ロト を文句 / 今 後代出出 in the ED IIIa witness [P222243], also discussed in section 3.7.1, and 三文文 自 を文句 や 後代書書 はいます。 (Cross and Endowed in the CD Composite [Q000782] (translated "Šuruppag gave instructions to his son" in [ETCSRI]). It does not matter for the construction of a composite text whether this is encoded マロー or トゥー, since that word is absent from other witnesses, and since the surrounding words differ.

⁷²Besides the contrasts in numeric usage mentioned in section 3.3.3, these (already-encoded) characters were clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of — which are not shared with I have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period, — is used for the preposition *ina*, "in", and I for the preposition *ana*, "to".

 $^{^{73}}$ See, e.g., [Robo8, p. 55] on the unification of metrologies under Sargon, resulting in the systems described in section 3.3.

 $^{^{74} \}rm Most$ prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

⁷⁵For example, in personal names:

[—] 具食心酊 in [P020019] from ED IIIb Nirsu;

[—] 戶下令司型 in [P020182], also from ED IIIb Nirsu;

^{— ♦★♦} in [P222186] from ED IIIb Umma;



Figure 20: [P222399, obv. 6, 16-17] 本文 [D] / 米文 〈顧].

trast between non-numeric $\$ar_2$ written \diamondsuit and numeric $1(\$ar_2^c)$ written \blacksquare can be observed, similar to the contrast between \vdash and \trianglerighteq previously discussed in section 3.5. However, in lexical lists from \$uruppag and &uruppag and &uruppag as well as in the $St\`ele$ des uruppag, non-numeric uruppag is curviform:

- * ਸ਼ 🌢 and * ਸ਼ 🕁 🖦 in [**P010566**];
- $\bullet \Rightarrow$ and $* \bullet \Rightarrow$ in [P010576];
- $\bullet + in [P240986]^{77};$
- \bullet \blacksquare \bullet in [P222399, obv. 17, 9, 18, 11, 22, 12]⁷⁸.

It would be disruptive to the diachronic representation of text if non-numeric Sar_2 were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph \diamondsuit , identical to the glyph of the proposed U+12579 \blacksquare CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric Sar_2 is curviform solidly predates the transition from \blacksquare to \diamondsuit in the relevant metrological systems, there is no need to represent a \diamondsuit - \blacksquare contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

Note that in rare cases, such as [P222243] from ED IIIa Adab, non-numeric \leftarrow (here with the value rum) is written \triangleright . It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 \triangleright cuneiform numeric sign one NO1, or as stylistic distinctions, encoded as

^{— 💾 ₩ 🍴 🌣} in [**P235312**] from Old Akkadian Umma.

 $^{^{76}}$ These are archaizing in other ways, *e.g.*, they have a 闰-闰 (NAM₂-TUG₂) split.

⁷⁷From copy in [ELLes, No. 397].

 $^{^{78}}$ Note however ***** \Rightarrow \diamondsuit 包 on [P222399, obv. 6, 17], see Figure 20. Curviform non-numeric šar₂ is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here

U+12038 CUNEIFORM SIGN ASH with a curviform glyph. in practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the CDLI transliteration (UR2-1(aš@c)) and the ePSD2 one (uru₈^{rum}) of this word disagree on that aspect. Since \vdash has a cuneiform reference glyph, this does not pose any compatibility concerns.

3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL₂ can be described as $\bowtie V \times \text{NE} \times \text{KASKAL}$ or $\bowtie V \times \text{NE} \times \text{PAP}^{79}$, they are typically all transliterated bil₂, and therefore are all represented by the character U+1224B $\bowtie V \times \text{CUNEIFORM SIGN NE SHESHIG}$, its name notwithstanting, as described in [UTR56, §2.5].

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, \P from [P305639] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in EbDA transliterations: the aforementioned \P from [P240533, recto 3, 3] is transliterated "20-1-1/2 gu_2 -bar 7 π nig $_2$ -sagšu 2-1/2 an-zam $_x$ π 0 π 0, reading both π 0 and π 1 as 1/2, but not distinguishing them.

⁷⁹As on [P249253]

⁸⁰As of this writing, EbDA actually has an-zam_X, with U+1D6A GREEK SUBSCRIPT SMALL LETTER CHI.

⁸¹ interpreted as a unit, as discussed in section 3.3.

⁸²short for nigida, an older reading of bariga; see [Landsberger1950; Powell1975; Foxvog2022].

 $\frac{1}{2}dc$ for ∇ depending on reading, etc. The literature on the Uruk and Early Dynastic I–II periods uses a different set of transliteration conventions that also disambiguate numeral shapes, as will be discussed in section 4.

While there exist transliterations that distinguish — from I but not \square from , such as the ones used in [DCCMT], the trend, especially in more recent works in third millenium studies, seems to be to represent numeral shape; for example, [Maiocchi2024] gave an example of the input syntax used by the new "Urban Economy Begins" project as "10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)" for \square with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are \square rather than \square numerals. The "tradition of cavalierly dispensing with numerical notations in notations of administrative documents", as [Englund2004] describes it, seems to be fading.

3.8 Conclusions

Co-occurences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications. The contrast frequently applies to individual numerals, rather than to the span of entire numeric expressions.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted: (is a length of three ropes, but is an area of three bur; could be read as one and one would be one and a half is a personal name, but would be "one slave".

In addition, there would be a risk of confusion about character identity should fontmakers attempt to treat the curviform and cuneiform numerals as unified. A designer concerned about the numeric-syllabic $\triangleright--$, and wishing to support diachronic encoding between systems $S_{\text{Ur III}/OB}$ and S, might give the ! numeral series (which is typically only used numerically in the Early Dynastic period) the glyphs of the \triangleright numeral series, but this would break capacity measures that use \triangledown ; in an effort to support diachronic use for 1/2(iku), one might be tempted to give \checkmark the glyph of \triangledown , thereby rendering the font unusable for quantities measured using the \checkmark numeral series; an ED I–II Ur font could decide to give \checkmark the same glyph as \diamondsuit (that of the proposed \bigcirc), according to the older area system, making it impossible to represent the newer one.

4 Rationale for ED-Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millenium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millenium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs KAŠ_a, KAŠ_b, and KAŠ_c, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [**Englund2001**]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered scripts.

As part of the development of these conventions, a classification of fourth millenium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter N with a numeric subscript (sometimes with an additional alphabetic subscript): N_1 is \triangleright , N_{14} is \bullet , N_{34} is \triangleright , etc. Transliterations of numeric expression then use those to identify the type of number used, thus $5N_1$ is \triangleright , and $5N_{14}$ is \bullet .

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the CDLI transliteration of portion of p

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millenium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in section 3.7.2; they are indeed used interchangeably, as in [P011104] which uses the notation u@f in [ePSD2], but N14@f in CDLI. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millenium would therefore induce confusion as to which numerals should be used in third millenium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millenium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance, \mathbb{R} N_{30c} is listed as a variant of \mathbb{R} N_{30a} in [DE87, p. 166], where the numeric value of either in relation to ∇N_{39a} is still unknown, but their values are found in [**Englund2004**] to be \mathbb{R} = $\frac{1}{10}\nabla$,

whereas $\boxtimes = \frac{1}{6} = -\infty$.

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban₂@c) and 5(ban₂@c).]

6 Characters not included in this proposal

6.1 Missing numerals

TODO N13 not attested in CDLI TODO (N_{17} not usefully numeric, $12N_{14}$ not encodable, etc.). Cite [DE87, p. 147] 7 and 8(diš $ten\hat{u}$) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.

6.2 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some $stacking\ patterns$; for instance 91 is encoded both as U+12446 $\mbox{\ H}\$ and as U+1240E $\mbox{\ HIII}\$. This is in part due to contrastive usage of stacking patterns. For instance, besides $\mbox{\ I}\$ and $\mbox{\ II}\$ which are characteristic of bariga measures, four bariga is written $\mbox{\ H}\$ even where 41 is written $\mbox{\ H}\$, as in [P255010; P292843]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often $\mbox{\ II}\$ $\mbox{\ III}\$ $\mbox{\ H}\$ $\mbox{$

Likewise, many stacking patterns are attested for the curviform numerals proposed in this document, and it is not proposed to separately encode them. These distinctions would be incompatible with the state of the art in numeric transliterations, including those by Englund, who insisted on "a system of transliteration that reflects in a strict fashion the physical realities of the cuneiform inscriptions" [Englund2004], and they are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier

⁸³The Sumerian word limmu means "four", so limmu and limmu₅ are still numbers. The distinction here is between usage in transliterations of phrases such as \clubsuit ★ � 🎞 \P Usage \P



Figure 21: The layout of case [**P011099**]; the numeral \$\frac{1}{2}\$ is rotated to fit the rounded corner of the tablet.



Figure 22: The layout of case [**P020066**]; the numeral \$\circ\$ is spread across two lines. The text is read in the order \$\bigcup \bigcup \langle \lan

tablets, as they are structured in rectangular cases rather than lines, so that numerals may be laid out across the case in whichever way fits the available space; this is illustrated in Figure 21. Note also that the numerals need to be considerably enlarged in order to reproduce the layout of the tablets, so that \$\frac{1}{2}\$ often spans two lines of cuneiform signs, as shown in Figure 22. This is impractical when these numerals are set in text that contrasts them with the larger \(\bigcup_{\text{,}} \), and inconsistent with actual practice when typesetting these numerals, as illustrated in Figure 6: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dynastic period, but that are also attested in the fourth millenium in the Uruk III period; the fourth millenium, especially the Uruk IV period, also frequently features numerals that use a more vertical layout, as illustrated in Figure 23. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millenium, usage in third millenium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of noncuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents \triangleright , \triangleright , \bullet , and \bullet must remain consistent across the numeral series, lest a \triangleright numeral be confused with an \triangleright numeral. Since the single indents are frequently used in running text, as illustrated in section 3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 23 uses OpenType stylistic alternates, and Figure 21 rotates the

Figure 23: Three stacking patterns for U+12573 CUNEIFORM NUMERIC SIGN NINE N34. The one on the left is the reference glyph, used in Uruk III [P003499; P004430], and widely afterwards, e.g., ED IIIa Šuruppag [P010678], ED IIIb Nirsu [P020057], Old Akkadian Umma [P212464]. The ones in the middle and right are used in two Uruk IV tablets [P001243; P004500]. All three Uruk examples are transliterated 9(N34) in CDLI.

character \$\,\ in both cases preserving the plain text backing.

6.3 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Englund2001].

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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[P221305] AO 13864. Musée du Louvre. CDLI: P221305.

ORACC: epsd2/corpus/P221305.

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ORACC: etcsri/Q001544.

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CDLI: P232280.

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