

Archaic cuneiform numbers

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1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script¹ and the proto-cuneiform script². The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
			Existing Xsux
Non-numeric signs	Future Pcun		

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

¹ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

²ISO 15924: Pcun, not yet encoded.

³ 𐎶 1-9(aš^c = N₁), 𐎶 1-5(u^c = N₁₄), 𐎶 1-9(ḫeš₂^c = N₃₄), 𐎶 1-5(ḫeš^cu^c = N₄₈), etc.

⁴ 𐎶 1-9(aš), 𐎶 1-5(u), 𐎶 1-9(ḫeš₂), 𐎶 1-5(ḫeš^cu), etc.

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties





2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

TODO(egg): blurb.

3.1 The cuneiform encoding model


As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts,  later in the third millennium⁵,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists⁶ and dictionaries⁷, and of composite texts⁸. By being compatible with similarly diachronic transliteration practice (that is, by avoiding distinctions finer than those made in transliteration), the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]⁹. The diachronic approach is also useful for pedagogic applications¹⁰.

3.2 Arguments for curviform–cuneiform unification






















In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding¹¹ that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

















Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:





⁵Merging with U+1224E  NI₂.

⁶Notably [VT+14] and the online edition of [Bor10] in [Jim+23, Signs].

⁷Notably [TJV17] and the online edition of [Sch10] in [Jim+23, Dictionary].

⁸For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of                     

						
1(NĚŠ ₂)	1(U)	1/2(DIŠ)	5(DIŠ <i>tenû</i>)	gi	us ₂	sa ₂
	7.5 (ropes)		5	reed	side	equal
						
3(U)	6(DIŠ <i>tenû</i>)	gi	saŋ	sa ₂		
3(ropes)	6	reed	front	equal		
	•					
ašag-bi	1(BUR ₃ ^c)	1(EŠE ₃ ^c)	1(IKU ^c)	1/2(IKU ^c)		
this field						





 tug_x(LAK483)-si-ga-kam
 deep ploughing

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as¹⁵

465 metres, equal lengths
198 metres, equal widths
this field: 9,18 hectares, deeply ploughed

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from \mathbb{P} to \mathbb{I} numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere \mathbb{D}/\mathbb{I} switch;

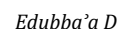
¹²As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large — in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

¹³Note that ED IIIb < numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign \llcorner in [P020054] looks more like Ur III \star .

¹⁴Transliteration after [Lec20, p. 8].

¹⁵ We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

- ### 3.3 A primer on classic Ur III and Old Babylonian metrologies



As is well known¹⁶ a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millenium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters I – III , the multiples of ten (10–50) are < – X , but the other digits 11–59 are sequences <I – XIII ; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $\mathcal{S}_{\text{Ur III/OB}}$, which had different signs for the units I – III , tens < – X , sixties I – XIII (with larger wedges than the units), six hundreds K – XK , three thousand six hundreds D – XXK , and thirty-six thousands D – XXK .

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \blacktriangledown \xleftarrow{10} \blacktriangledown \xleftarrow{6} \blacktriangleleft \xleftarrow{10} \blacktriangledown \quad (S_{\text{Ur III/OB}})$$

5

Observe that while large numbers of gur follow²³ system $S_{Ur\ III/OB}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as <|𐎶| would be 10 gur 1 bariga, and <𐎶-𐎶| would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

- |𐎶| used in $S_{Ur\ III/OB}$ and the SPVS as well as with overt units;
- <𐎶-𐎶| used in $G_{Ur\ III/OB}$, of which <𐎶-𐎶 are also used in $S_{Ur\ III/OB}$ and the SPVS as well as with overt units;
- |𐎶-𐎶| used in $S_{Ur\ III/OB}$, and sometimes with overt units;
- 𐎶-𐎶 used in $S_{Ur\ III/OB}$;
- 𐎶-𐎶 used in $S_{Ur\ III/OB}$ and $G_{Ur\ III/OB}$;
- 𐎶-𐎶 used in $S_{Ur\ III/OB}$ and $G_{Ur\ III/OB}$;
- 𐎶-𐎶 used in $C_{Ur\ III/OB}$ as well as with overt units of the weight system;
- 𐎶, 𐎶, 𐎶, 𐎶, 𐎶 used in $C_{Ur\ III/OB}$;
- |, |, |, | used in $C_{Ur\ III/OB}$ —note the overlap with |𐎶|;
- 𐎶 and 𐎶 used in $G_{Ur\ III/OB}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Rob08, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with | and < numerals with their system $S_{Ur\ III/OB}$ values²⁴. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Fri07, p. 118; Rob19]:

𐎶𐎶𐎶𐎶	← 60	𐎶𐎶	← 10	𐎶𐎶	← 12	𐎶𐎶	← 30	𐎶𐎶𐎶𐎶	($L_{Ur\ III/OB}$)
danna		US ²⁵		nindan		kuš ₃		šu-si	
bērum		cable		nindanum		ammatum		ubānum	
league		360 m		rod		cubit		finger	
10,8 km				6 m		50 cm		17 mm	

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:

𐎶𐎶𐎶𐎶	← 30	𐎶𐎶	← 6	𐎶𐎶	← 10	𐎶𐎶	← 2	𐎶𐎶	← 6	𐎶𐎶	← 30	𐎶𐎶𐎶𐎶	($\bar{L}_{Ur\ III/OB}$)
				eše ₂		gi		qānum					
				ašlum		reed		3 m					
				rope									
				60 m									

²²From [P309594].

²³A larger unit, the guru₇ (karûm, grain heap), is sometimes used instead, with 𐎶𐎶𐎶𐎶𐎶𐎶𐎶𐎶 (1 karûm = 3600 kurrû). See [Fri07, p. 415; Rob19].

²⁴Adjacent units are no more than a factor of 60 apart, so higher numerals such as 𐎶 or 𐎶 are not used.

²⁵TODO

In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

TODO

3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period match the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{Ur III/Ob}$ [Fri07, p. 374; DE87, pp. 127, 165]:

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \text{D} \xleftarrow{10} \text{D} \xleftarrow{6} \bullet \xleftarrow{10} \text{D}, \quad (S)$$

Likewise the area system used in the Early Dynastic IIIb period mirrors system $G_{Ur III/Ob}$ [Dei22, p. 72; NDE93, p. 63; Fri07, p. 378; Gom16]:

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \text{D} \xleftarrow{10} \bullet \xleftarrow{3} \text{D} \xleftarrow{6} \text{D}, \quad (G_{ED IIIb})$$

As noted in [L2/04-099, p. 4] (see section 3.2), the vertical \uparrow from $S_{Ur III/Ob}$ becomes a horizontal D in system S . It is however far from the only case of such a reallocation of function. The earlier form of System G was [DE87, pp. 141, 165; Fri07, p. 378]:

$$\bullet \xleftarrow{6} \odot \xleftarrow{10} \bullet \xleftarrow{3} \text{D} \xleftarrow{6} \text{D}, \quad (G)$$

Observe that, as noted in [DE87, p. 142], \odot changes meaning from $10\bullet$ in system G to $10\bullet$ in system $G_{ED IIIb}$. System G is used in the Uruk period, but also in the ED I–II period (it is the “area 2” system in [Cha03], whereas $G_{ED IIIb}$ is the “area 1” system).

3.4.1 Field lengths in Nirsu

The length system Early Dynastic IIIb of the state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign D is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign D is not written either. Length shorter than one rope are expressed in half-rope using the $1/2$ sign D (again with no D), and then in reeds, *with* the sign D . Effectively, this yields the following factor diagram:

$$\begin{array}{c} \uparrow \xleftarrow{6} \text{D} \xleftarrow{2} \text{D} \xleftarrow{10} \text{D} \xleftarrow{28} \text{D} \\ \begin{array}{l} 1 \text{ eše}_2 = 10 \text{ nindan} \\ 1 \text{ rope} = 10 \text{ rods} \\ 60 \text{ m} \end{array} \quad \begin{array}{l} \text{gl} \\ \text{reed} \\ 3 \text{ m} \end{array} \end{array} \quad (L_{ED IIIb})$$

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and others from the same period, such as the ones discussed in [Lec20], areas are expressed in system $G_{ED IIIb}$, with curviform numerals²⁹; in the absence of overt units, such as when dealing with length that

²⁸Note that the reeds are counted using *tenû* numerals, D , D , D , etc.

²⁹TODO(egg): Note the handful of late Urukagina tablets that start to have cuneiform areas.

3.4.2 Dyke lengths in Nirsu

— [P221305, obv. 1, 4]³² 𐎶𐎠𐏀𐎵𐎲𐎢𐏁𐎧𐎺𐎡𐏁𐎫𐎼𐎪𐎥𐎴𐎣𐎤𐎬𐎦
— [P020129, rev. 2, 1] 𐎶𐎠𐏀𐎵𐎲𐎢𐏁𐎧𐎺𐎡𐏁𐎫𐎼𐎪𐎥𐎴𐎣𐎤𐎬𐎦
— [P221291, rev. 5, 1]³³ 𐎶𐎠𐏀𐎵𐎲𐎢𐏁𐎧𐎺𐎡𐏁𐎫𐎼𐎪𐎥𐎴𐎣𐎤𐎬𐎦
— [P221266, rev. 2, 1] 𐎶𐎠𐏀𐎵𐎲𐎢𐏁𐎧𐎺𐎡𐏁𐎫𐎼𐎪𐎥𐎴𐎣𐎤𐎬𐎦

$$\underbrace{\text{E} \xleftarrow{10} \text{D} \xleftarrow{6} \bullet}_{\text{Ib}} = \underbrace{\text{D} \xleftarrow{2} \text{F}}_{\text{35}} \xleftarrow{10} \text{A} \xleftarrow{6} \text{B} \xleftarrow{3} \text{C} \xleftarrow{10} \text{D} \xleftarrow{6} \text{E} \quad (L'_{\text{ED IIIb}})$$

³⁰This is the case of the sides of the field in [P020054, obv. ii 2–3].

³²CDLI only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the $\text{𒀭} \text{𒀵} \text{𒀭}$ is not visible. Lecompte notes that the copy is faithful; indeed another $\text{𒀭} \text{𒀵} \text{𒀭}$ can be seen both on the copy and the photo on obv. 2, 2.

³³From copy.

³⁴TODO Cite also DP 568, the one with and even though it has no reeds.

³⁵TODO(egg): Note that one unit may be omitted if the other is present

The system of grain³⁶ capacities in Ebla uses the following units³⁷:

$$\begin{array}{ccccccc} \text{𐎗𐎗𐎕} & \xleftarrow{2} & \text{𐎗𐎗𐎕} & \xleftarrow{\frac{5}{2}} & \text{𐎗𐎗𐎕} & \xleftarrow{4} & \text{𐎗𐎗𐎕} & \xleftarrow{6} & \text{𐎗𐎗𐎕} \\ \text{gu}_2\text{-bar} & & \text{ba-ri}_2\text{-zu} & & \text{ḡin}_4 & & \text{niḡ}_2\text{-sagšū} & & \text{an-zam}_x \end{array}$$

The 𐎗𐎗𐎕 and 𐎗𐎗𐎕 are generally counted using curviform numerals, and the smaller units using cuneiform 𐎗 numerals. Indeed, a search on [Mil+07] for co-occurrences of 𐎗𐎗𐎕 with either of 𐎗𐎗𐎕 or 𐎗𐎗𐎕 finds the following expressions³⁸:

1. [P240532, verso 4, 9] 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕⁴⁰ 𐎗𐎗𐎕
2. [P240548, verso 1, 1] 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕
3. [P240655, recto 7, 9] 𐎗𐎗𐎕 𐎗𐎗𐎕⁴¹ 𐎗𐎗𐎕
4. [P240579, verso 4, 3] 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕
5. [P240675, verso 2, 2] 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕
6. [P240609, verso 3, 1] 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕
7. [P240533, recto 3, 3] 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕
8. [P240697, recto 1, 5] 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕
9. [P240653, recto 6, 2] 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕
10. [P240654, recto 2, 6] 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕⁴³ 𐎗𐎗𐎕⁴⁴
11. [P240531, recto 1, 8] 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕
12. [P241708, recto 1, 1]⁴⁵ 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕
13. [P241904, recto 1, 1]⁴⁶ 𐎗𐎗𐎕 𐎗𐎗𐎕⁴⁷ 𐎗𐎗𐎕

Note that higher numbers of 𐎗𐎗𐎕 are expressed in hundreds (*mi-at* 𐎗𐎗𐎕) and then thousands (*li-im* 𐎗𐎗𐎕), as is typical in Ebla [Arc15, p. 33], *e.g.*, in [P240532, verso 2, 3], 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 𐎗𐎗𐎕 (100 + 60 + 30 + 5 = 195 𐎗𐎗𐎕 of grain).

³⁶Liquid capacities use a different system [Arc15, p. 229 with note 12]:

$$\begin{array}{ccc} \text{𐎗𐎗𐎕} & \xleftarrow{30} & \text{𐎗} & \xleftarrow{6} & \text{𐎗𐎗𐎕} \\ \text{la-ḡa} & & \text{sil}_3 & & \text{an-zam}_x \end{array}$$

At a glance it seems that 𐎗 are counted with cuneiform numerals and higher units with curviform ones, thus

$$\begin{array}{ccccccc} \text{𐎗𐎗𐎕} & \xleftarrow{\frac{5}{3}} & \text{𐎗} & \xleftarrow{6} & \text{𐎗} & \xleftarrow{10} & \text{𐎗} & \xleftarrow{3} & \text{𐎗} & \xleftarrow{10} & \text{𐎗} & \xleftarrow{6} & \text{𐎗𐎗𐎕} \\ & & & & & & & & & & & & \text{𐎗} \end{array}$$

but we have not investigated this thoroughly.

³⁷TODO mention the other one citing Chambon and the footnote in Archi

³⁸We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform 𐎗𐎗𐎕 and larger with cuneiform 𐎗 and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked³⁹ on a photograph (from EbDA unless noted otherwise).

³⁹As we will see in Section 3.6.1, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

⁴⁰ba-ri₂-zu₂, a variant spelling.

⁴¹Short for 𐎗𐎗𐎕.

⁴²Note the omitted 𐎗𐎗𐎕.

⁴³Instead of the expected 𐎗𐎗𐎕.

⁴⁴𐎗𐎗𐎕 not legible on the EbDA photo.

⁴⁵From CDLI photo.

⁴⁶From photo in [Arc89, p. 6].

⁴⁷Laid out as 𐎗𐎗𐎕; on stacking patterns see Section 6.2.

formed by only two signs Υ and \triangleleft , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60², 60³, 1/60, 1/60², etc. are written in the same way, with the vertical wedge Υ . The third feature concerns the exact function of

Figure 4: TODO [Cha12, p. 58]

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with: $\Upsilon\Upsilon\Upsilon\bullet\bullet\bullet$ but never with the repetition $\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon$.

Figure 5: TODO [Cha12, p. 59]⁵⁰

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

$\Upsilon\Upsilon\Upsilon$ še bar Υ ba-rí-zu
‘3 gubar (capacity units) and 1 parisu’.

Figure 6: TODO [Cha12, p. 61]

This is particularly true of the signs Υ , Υ , Υ and Υ , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written Υ in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign Υ is most often associated with the *parisu* measure, while the signs Υ , Υ , Υ and Υ refer to 1, 2, 3,

Figure 7: TODO [Cha12, p. 64]

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., $\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon$ transliterated as 2(šar₂) 1(geš'u) 3(u), means 2 × 3600 + 1 × 600 + 3 × 10). The system is based on an alternation of factors ten and

Figure 8: TODO

might think of one fabric and a half,¹¹ but the presence of notations with “2 Υ 2 Υ ”, “3 Υ 3 Υ ”, and “6 Υ 6 Υ ” (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag’s weight measurement system.¹² The notation “1 Υ gada” in o. ii 1 and r. vi 1, along with the total of “39



Fig. 1. Combinations of numerals attested in Š. 742.

Figure 9: Discussion of the contrast between Υ and Υ numerals in [Gor23, p. 162].

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can arise if it is lost; for instance, the personal name 𒀭𒂗𒂊 occurs on its own line in the aforementioned administrative texts; a line 𒀭𒂗𒂊 would instead be read as “one slave”.

3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as 𒀭 that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [Sch35, pp. 123 sqq.] lists all of 𒀭 together with 𒀭𒀭𒀭 , while 𒀭 , 𒀭𒀭 , and 𒀭𒀭𒀭 , and only those, appear at the beginning of the sign list, since they have non-numeric values⁵⁵. [Cat13, p. 58] has the numeric signs 𒀭 , 𒀭𒀭 , 𒀭𒀭𒀭 , whereas non-numeric 𒀭 is at the beginning of the sign list, where its values *aš* and *rum* are listed. For signs with both non-numeric and numeric usage, [Dei22] writes *s. die Zahlz.* throughout the main list; LAK 1 𒀭 thus reappears at LAK 829 together with 𒀭 , 𒀭𒀭 , and 𒀭𒀭𒀭 . One should note [Bor10], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have 𒀭𒀭𒀭 – 𒀭𒀭𒀭𒀭 used in system $G_{\text{Ur III/OB}}$.

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes⁵⁶ to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from 𒀭 to 𒀭𒀭 numerals prevents diachronic encoding even if 𒀭 were unified with 𒀭 . For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'–6']; however, they are written with 𒀭 numerals, whereas they are written with 𒀭𒀭 numerals in the Early Dynastic witnesses; since 𒀭 and 𒀭 are distinct⁵⁷ characters, the 𒀭 – 𒀭 unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods⁵⁸, there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Romach2023], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

⁵⁵Non-numeric values of 𒀭 were discussed in section 3.5; 𒀭𒀭 has the values *man*₃ and *min*₅, and is used for the word *didli*, “several, various”; 𒀭𒀭𒀭 has the value *eš*₆.

⁵⁶TODO comment on the ED witnesses to the instructions of Šuruppak

⁵⁷Besides the contrasts in numeric usage mentioned in section 3.3.3, these characters are clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of 𒀭 which are not shared with 𒀭 have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period, 𒀭 is used for the preposition *ina*, “in”, and 𒀭 for the preposition *ana*, “to”.

⁵⁸TODO cite a few things here.



3.6.1 Compatibility with transliteration

TODO words [Robo8, p. 295] TODO cite [Molina2014]

3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with one exception discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts⁵⁹ being good enough; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

3.7.1 The case of ŠAR₂

[TODO explain why this isn't a problem, effectively anyone who needs to cuneify 1(šar₂) will also need to cuneify some of the numerals proposed here and will therefore not be using Unicode cuneiform.] [TODO U+122B9 CUNEIFORM SIGN SHAR2 represents both 1(šar₂) and non-numeric šar₂; it looks like  (so, like ) in all but lexical texts from Ebla and Šuruppak (and the archaizing vulture stele, where note that the scribe slipped into his modern ways once), where it looks like (TODO: the proposed character). The proposed character is to be used for 1(šar₂). 1(šar₂) does not exist back when non-numeric šar₂ is curviform, so it works out.] [TODO Mention P222243]

3.8 Conclusions

Mention something about potential for confusion in character identity.

4 Rationale for ED–Uruk numeral unification

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban₂@c) and 5(ban₂@c).]

6 Characters not included in this proposal

6.1 Missing numerals

(N₁₇, 12N₁₄, etc.) 7(diš *tenû*)

⁵⁹Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

6.2 Stacking patterns

(... are a mess, vary within Uruk, and are not transliterated/documentated by Englund, so let's not go there for now.)

6.3 Matters for higher-level protocols

Rotated bits: <https://cdli.mpiwg-berlin.mpg.de/artifacts/101087>

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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