

Archaic cuneiform numbers

Robin Leroy, Anshuman Pandey, and Steve Tinney

2024-08-18

Contents

1	Summary	2
2	Proposed changes to the Standard	3
2.1	Summary of proposed characters	3
2.2	Properties	3
2.3	Character names list	3
2.4	Core specification text	3
3	Rationale for curviform–cuneiform disunification	3
3.1	The cuneiform encoding model	3
3.2	Arguments for curviform–cuneiform unification	3
3.3	A primer on classic Ur III and Old Babylonian metrologies	5
3.3.1	The discrete counting system	5
3.3.2	The area system	6
3.3.3	The capacity system	6
3.3.4	The length system	7
3.3.5	Fractions	8
3.4	Curviform numerals in early metrologies	8
3.4.1	Field lengths in Nirsu	8
3.4.2	Dyke lengths in Nirsu	9
3.4.3	Grain in Ebla	9
3.4.4	Use in modern publications	11
3.5	Non-numeric usage	13
3.6	Limited benefits of diachronic encoding for numerals	14
3.6.1	Compatibility with transliteration	14
3.7	Compatibility considerations	14
3.7.1	The case of ŠAR ₂	15
3.8	Conclusions	15
4	Rationale for ED–Uruk numeral unification	15
5	Considerations on individual numeral series	15
6	Characters not included in this proposal	15
6.1	Missing numerals	15
6.2	Stacking patterns	15
6.3	Matters for higher-level protocols	15

Acknowledgements	15
References	16
Artefacts	16
Unicode documents	17
Other documents	18

1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script¹ and the proto-cuneiform script². The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
			Existing Xsux
Non-numeric signs	Future Pcun		

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

¹ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

²ISO 15924: Pcun, not yet encoded.

³𐎶 1-9(aš^c = N₁), 𐎶 1-5(u^c = N₁₄), 𐎶 1-9(ḫeš₂^c = N₃₄), 𐎶 1-5(ḫeš^cu^c = N₄₈), etc.

⁴𐎶 1-9(aš), 𐎶 1-5(u), 𐎶 1-9(ḫeš₂), 𐎶 1-5(ḫeš^cu), etc.

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties





2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

TODO(egg): blurb.

3.1 The cuneiform encoding model


As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Suruppag as in the character code charts,  later in the third millennium⁵,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists⁶ and dictionaries⁷, and of composite texts⁸. By being compatible with similarly diachronic transliteration practice (that is, by avoiding distinctions finer than those made in transliteration), the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]⁹. The diachronic approach is also useful for pedagogical applications¹⁰.

3.2 Arguments for curviform–cuneiform unification

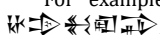
In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding¹¹ that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an

⁵Merging with U+1224E  NI₂.



⁶Notably [VT+14] and the online edition of [Bor10] in [Jim+23, Signs].

⁷Notably [TJV17] and the online edition of [Sch10] in [Jim+23, Dictionary].

















⁸For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].





⁹Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svā+24, pp. 242, 148].

¹⁰For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

¹¹At that time scoped to the repertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as  ≠ , were informed by Early Dynastic distinctions.



Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

 ¹²						
1(NEŠ ₂)	1(U)	1/2(DIŠ)	5(DIŠ <i>tenû</i>)	gi	us ₂	sa ₂
	7.5 (ropes)		5	reed	side	equal
 ¹³						
3(U)	6(DIŠ <i>tenû</i>)	gi	saṇ	sa ₂		
3(ropes)	6	reed	front	equal		
	•					
ašag-bi	1(BUR ₃)	1(EŠE ₃)	1(IKU)	1/2(IKU)		
this field						

   
tug_x(LAK483)-si-ga-kam¹⁴
deep ploughing


The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as¹⁵




465 metres, equal lengths
198 metres, equal widths
this field: 9, 18 hectares, deeply ploughed

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from - to -numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

¹²As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large  in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

¹³Note that ED IIIb  numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign  in [P020054] looks more like Ur III .

¹⁴Transliteration after [Lec20, p. 8].

¹⁵We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

- the functions and use of the numerals vary beyond the mere ∇ / \intercal switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 A primer on classic Ur III and Old Babylonian metrologies

𒂗𒍪 𒂗𒍪𒂗𒍪𒂗𒍪 𒂗𒍪
 𒂗𒍪 𒂗𒍪𒂗𒍪𒂗𒍪𒂗𒍪𒂗𒍪𒂗𒍪
 𒂗𒍪 𒂗𒍪𒂗𒍪𒂗𒍪𒂗𒍪𒂗𒍪𒂗𒍪𒂗𒍪

I want to write tablets: the tablet of 1 gur of barley to 600 gur; the tablet of 1 shekel of silver to 10 minas [...]

Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the contrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known¹⁶ a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Robo8, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters \intercal – 𒌦 , the multiples of ten (10–50) are \leftarrow – 𒌶 , but the other digits 11–59 are sequences \leftarrow \intercal – 𒌶 𒌦 ; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units \intercal – 𒌦 , tens \leftarrow – 𒌶 , sixties 𒌦 – 𒌶 (with larger wedges than the units), six hundreds 𒌦 – 𒌶 , three thousand six hundreds 𒌦 – 𒌶 , and thirty-six thousands 𒌦 – 𒌶 .

3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram¹⁷, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\text{𒌦} \xleftarrow{10} \text{𒌦} \xleftarrow{6} \text{𒌦} \xleftarrow{10} \text{𒌦} \xleftarrow{6} \text{𒌦} \xleftarrow{10} \text{𒌦} \quad (S_{\text{Ur III/OB}})$$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written $\text{𒌦} \text{𒌦} \text{𒌦} \text{𒌦} \text{𒌦}$ in the discrete counting system, and $\text{𒌦} \text{𒌦} \text{𒌦} \text{𒌦}$ in the sexagesimal place value system.

¹⁶See, e.g., [Uni16, §22.3.3, sub “Cuneiform Numerals”].

¹⁷These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

Observe that while large numbers of gur follow²³ system $S_{Ur\ III/OB}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as $\langle \text{I} \text{AŠ} \rangle$ would be 10 gur 1 bariga, and $\langle \text{—} \text{AŠ} \rangle$ would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

- $\text{I} \text{—} \text{AŠ}$ used in $S_{Ur\ III/OB}$ and the SPVS as well as with overt units;
- $\langle \text{—} \text{AŠ} \rangle$ used in $G_{Ur\ III/OB}$, of which $\langle \text{—} \text{AŠ} \rangle$ are also used in $S_{Ur\ III/OB}$ and the SPVS as well as with overt units;
- $\text{I} \text{—} \text{AŠ}$ used in $S_{Ur\ III/OB}$, and sometimes with overt units;
- $\text{I} \text{—} \text{AŠ}$ used in $S_{Ur\ III/OB}$;
- $\text{I} \text{—} \text{AŠ}$ used in $S_{Ur\ III/OB}$ and $G_{Ur\ III/OB}$;
- $\text{I} \text{—} \text{AŠ}$ used in $S_{Ur\ III/OB}$ and $G_{Ur\ III/OB}$;
- $\text{I} \text{—} \text{AŠ}$ used in $C_{Ur\ III/OB}$ as well as with overt units of the weight system;
- $\text{I} \text{—} \text{AŠ}$, $\text{I} \text{—} \text{AŠ}$, $\text{I} \text{—} \text{AŠ}$, $\text{I} \text{—} \text{AŠ}$, $\text{I} \text{—} \text{AŠ}$ used in $C_{Ur\ III/OB}$;
- I , I , I , I used in $C_{Ur\ III/OB}$ —note the overlap with $\text{I} \text{—} \text{AŠ}$;
- I and I used in $G_{Ur\ III/OB}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Rob08, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with I - and $\langle \text{—} \rangle$ -numerals with their system $S_{Ur\ III/OB}$ values²⁴. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Fri07, p. 118; Rob19]:

$\text{I} \text{—} \text{AŠ}$	$\xleftarrow{60}$	$\text{I} \text{—} \text{AŠ}$	$\xleftarrow{10}$	$\text{I} \text{—} \text{AŠ}$	$\xleftarrow{12}$	$\text{I} \text{—} \text{AŠ}$	$\xleftarrow{30}$	$\text{I} \text{—} \text{AŠ}$	
danna bērum league 10,8 km		US ²⁵ cable 360 m		nindan nindanum rod 6 m		kuš ₃ ammatum cubit 50 cm		šu-si ubānum finger 17 mm	($L_{Ur\ III/OB}$)

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:

$\text{I} \text{—} \text{AŠ}$	$\xleftarrow{30}$	$\text{I} \text{—} \text{AŠ}$	$\xleftarrow{6}$	$\text{I} \text{—} \text{AŠ}$	$\xleftarrow{10}$	$\text{I} \text{—} \text{AŠ}$	$\xleftarrow{2}$	$\text{I} \text{—} \text{AŠ}$	$\xleftarrow{6}$	$\text{I} \text{—} \text{AŠ}$	$\xleftarrow{30}$	$\text{I} \text{—} \text{AŠ}$	
			eše ₂ ašlum rope 60 m				gi qānum reed 3 m						($\bar{L}_{Ur\ III/OB}$)

²²From [P309594].

²³A larger unit, the guru₇ (karûm, grain heap), is sometimes used instead, with $\text{—} \text{AŠ} \text{—} \text{AŠ} \text{—} \text{AŠ} \text{—} \text{AŠ} \text{—} \text{AŠ} \text{—} \text{AŠ}$ (1 karûm = 3600 kurrû). See [Fri07, p. 415; Rob19].

²⁴Adjacent units are no more than a factor of 60 apart, so higher numerals such as I or I are not used.

²⁵TODO

3.4.2 Dyke lengths in Nirsu

— [P221305, obv. 1, 4]³²

— [P020129, rev. 2, 1]

— [P221291, rev. 5, 1]³³

— [P221266, rev. 2, 1]

$$\underbrace{\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array}}_{\substack{\text{Diagram 3} \\ 10}} \cdot \underbrace{\begin{array}{c} \text{Diagram 4} \\ \text{Diagram 5} \end{array}}_{\substack{\text{Diagram 6} \\ 35}} = \text{Diagram 7} \quad (L'_{\text{ED IIIb}})$$

³⁰This is the case of the sides of the field in [P020054, obv. ii 2–3].

³²CDLI only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the $\Xi \triangleright \Pi \approx$ is not visible. Lecompte notes that the copy is faithful; indeed another $\Xi \triangleright \Pi \approx$ can be seen both on the copy and the photo on obv. 2, 2.

³³From copy.

³⁴TODO Cite also DP 568, the one with and even though it has no reeds.

³⁵TODO(egg): Note that one unit may be omitted if the other is present

The system of grain³⁶ capacities in Ebla uses the following units³⁷:

$$\begin{array}{ccccccc} \text{𐎗𐎗𐎗} & \xleftarrow{2} & \text{𐎗𐎗𐎗} & \xleftarrow{\frac{5}{2}} & \text{𐎗𐎗} & \xleftarrow{4} & \text{𐎗𐎗𐎗} & \xleftarrow{6} & \text{𐎗𐎗𐎗} \\ \text{gu}_2\text{-bar} & & \text{ba-ri}_2\text{-zu} & & \eta\text{in}_4 & & \text{ni}\eta_2\text{-sagšû} & & \text{an-zam}_x \end{array}$$

The 𐎗𐎗𐎗 and 𐎗𐎗𐎗 are generally counted using curviform numerals, and the smaller units using cuneiform 𐎗 numerals. Indeed, a search on [Mil+07] for co-occurrences of 𐎗𐎗𐎗 with either of 𐎗𐎗𐎗 or 𐎗𐎗𐎗 finds the following expressions³⁸:

1. [P240532, verso 4, 9] 𐎗𐎗𐎗 𐎗𐎗𐎗 𐎗𐎗𐎗⁴⁰ 𐎗𐎗𐎗
2. [P240548, verso 1, 1] 𐎗𐎗𐎗 𐎗𐎗𐎗 𐎗𐎗𐎗
3. [P240655, recto 7, 9] 𐎗𐎗𐎗⁴¹ 𐎗𐎗𐎗
4. [P240579, verso 4, 3] 𐎗𐎗𐎗 𐎗𐎗𐎗
5. [P240675, verso 2, 2] 𐎗𐎗𐎗 𐎗𐎗𐎗
6. [P240609, verso 3, 1] 𐎗𐎗𐎗 𐎗𐎗𐎗
7. [P240533, recto 3, 3] 𐎗𐎗𐎗 𐎗𐎗𐎗 𐎗𐎗𐎗 𐎗𐎗𐎗
8. [P240697, recto 1, 5] 𐎗𐎗𐎗⁴² 𐎗𐎗𐎗 𐎗𐎗𐎗
9. [P240653, recto 6, 2] 𐎗𐎗𐎗 𐎗𐎗𐎗 𐎗𐎗𐎗 𐎗𐎗𐎗
10. [P240654, recto 2, 6] 𐎗𐎗𐎗 𐎗𐎗𐎗⁴³ 𐎗𐎗𐎗⁴⁴
11. [P240531, recto 1, 8] 𐎗𐎗𐎗 𐎗𐎗𐎗 𐎗𐎗𐎗
12. [P241708, recto 1, 1]⁴⁵ 𐎗𐎗𐎗 𐎗𐎗𐎗
13. [P241904, recto 1, 1]⁴⁶ 𐎗𐎗𐎗⁴⁷ 𐎗𐎗𐎗

Note that higher numbers of 𐎗𐎗𐎗 are expressed in hundreds (*mi-at* 𐎗𐎗𐎗) and then thousands (*li-im* 𐎗𐎗𐎗), as is typical in Ebla [Arc15, p. 33], *e.g.*, in [P240532, verso 2, 3], 𐎗𐎗𐎗 𐎗𐎗𐎗 𐎗𐎗𐎗 𐎗𐎗𐎗 (100 + 60 + 30 + 5 = 195 𐎗𐎗𐎗 of grain).

³⁶Liquid capacities use a different system [Arc15, p. 229 with note 12]:

$$\begin{array}{ccc} \text{𐎗𐎗𐎗} & \xleftarrow{30} & \text{𐎗} & \xleftarrow{6} & \text{𐎗𐎗𐎗} \\ \text{la-ḥa} & & \text{sil}_3 & & \text{an-zam}_x \end{array}$$

At a glance it seems that 𐎗 are counted with cuneiform numerals and higher units with curviform ones, thus

$$\begin{array}{ccccccc} \text{𐎗𐎗𐎗} & \xleftarrow{\frac{5}{3}} & \text{𐎗} & \xleftarrow{6} & \text{𐎗} & \xleftarrow{10} & \text{𐎗} & \xleftarrow{3} & \text{𐎗} & \xleftarrow{10} & \text{𐎗} & \xleftarrow{6} & \text{𐎗𐎗𐎗} \\ & & & & & & & & & & & & \text{𐎗} \end{array}$$

but we have not investigated this thoroughly.

³⁷TODO mention the other one citing Chambon and the footnote in Archi

³⁸We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform 𐎗𐎗𐎗 and larger with cuneiform 𐎗 and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked³⁹ on a photograph (from EbDA unless noted otherwise).

³⁹As we will see in Section 3.6.1, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

⁴⁰ba-ri₂-zu₂, a variant spelling.

⁴¹Short for 𐎗𐎗𐎗.

⁴²Note the omitted 𐎗𐎗𐎗.

⁴³Instead of the expected 𐎗𐎗𐎗.

⁴⁴𐎗𐎗𐎗 not legible on the EbDA photo.

⁴⁵From CDLI photo.

⁴⁶From photo in [Arc89, p. 6].

⁴⁷Laid out as 𐎗𐎗𐎗; on stacking patterns see Section 6.2.

formed by only two signs Υ and \triangleleft , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60², 60³, 1/60, 1/60², etc. are written in the same way, with the vertical wedge Υ . The third feature concerns the exact function of

Figure 4: TODO [Cha12, p. 58]

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with: $\Upsilon\Upsilon\Upsilon\bullet\bullet\bullet$ but never with the repetition $\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon$.

Figure 5: TODO [Cha12, p. 59]⁵⁰

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

$\Upsilon\Upsilon\Upsilon$ še bar Υ ba-rí-zu
‘3 gubar (capacity units) and 1 parisu’.

Figure 6: TODO [Cha12, p. 61]

This is particularly true of the signs Υ , Υ , Υ and Υ , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written Υ in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign Υ is most often associated with the *parisu* measure, while the signs Υ , Υ , Υ and Υ refer to 1, 2, 3,

Figure 7: TODO [Cha12, p. 64]

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., $\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon$ transliterated as 2(šar₂) 1(geš’u) 3(u), means 2 × 3600 + 1 × 600 + 3 × 10). The system is based on an alternation of factors ten and



Figure 8: TODO

might think of one fabric and a half,¹¹ but the presence of notations with “2 Υ 2 Υ ”, “3 Υ 3 Υ ”, and “6 Υ 6 Υ ” (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag’s weight measurement system.¹² The notation “1 Υ gada” in o. ii 1 and r. vi 1, along with the total of “39
















Fig. 1. Combinations of numerals attested in Š. 742.

Figure 9: Discussion of the contrast between Υ and Υ numerals in [Gor23, p. 162].

can arise if it is lost; for instance, the personal name  occurs on its own line in the aforementioned administrative texts; a line  would instead be read as “one slave”.

3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as  that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [Schneider1935] lists all of  together with , while , , and , and only those, appear at the beginning of the sign list, since they have non-numeric values⁵⁵; [Catagnoti2010] has the numeric signs , , , whereas non-numeric  is at the beginning of the sign list given with its values *aš* and *rum*; [Dei22] starts with . One should note [Bor10], which has numbers throughout the sign list; but that sign list does not use glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have - used in *G_{Ur III}/OB*.

[TODO Composite texts dating back to the period where curved numerals are in use tend to be limited to lexical texts, which do not usually have numbers. When they do, diachronic encoding is prevented by diš-aš distinctions anyway. Administrative texts, which are where numbers are most prominent, are not composite.]

[TODO The overarching goal of having consistent representation for equivalent numeric expressions from different periods is quickly foiled by changes in metrology.]




Note that in [Rom24] [TODO(egg): Cite the GitHub repository], as in many other such analyses, numbers are removed as an early step in processing; these therefore would not benefit from diachrony in the encoding of numeric expressions.

3.6.1 Compatibility with transliteration

TODO words [Rob08, p. 295] TODO cite [Molina2014]

3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with one exception discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in

⁵⁵Non-numeric values of  were discussed in section 3.5;  has the values *man*₃ and *min*₅, and is used for the word *didli*, “several, various”;  has the value *eš*₆.

[UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts⁵⁶ being good enough; the lack of Early Dynastic fonts, by the aforementioned issues with numeral identification making the representation of any text with numerals intractable.

3.7.1 The case of ŠAR₂

[TODO explain why this isn't a problem, effectively anyone who needs to cuneify 1(šar₂^c) will also need to cuneify some of the numerals proposed here and will therefore not be using Unicode cuneiform.] [TODO U+122B9 CUNEIFORM SIGN SHAR2 represents both 1(šar₂) and non-numeric šar₂; it looks like ◇ (so, like ◇) in all but lexical texts from Ebla and Šuruppak (and the archaizing vulture stele, where note that the scribe slipped into his modern ways once), where it looks like (TODO: the proposed character). The proposed character is to be used for 1(šar₂^c). 1(šar₂) does not exist back when non-numeric šar₂ is curviform, so it works out.] [TODO Mention P222243]

3.8 Conclusions

Mention something about potential for confusion in character identity.

4 Rationale for ED–Uruk numeral unification

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban₂@c) and 5(ban₂@c).]

6 Characters not included in this proposal

6.1 Missing numerals

(N₁₇, 12N₁₄, etc.) 7(diš *tenû*)

6.2 Stacking patterns

(... are a mess, vary within Uruk, and are not transliterated/documented by Englund, so let's not go there for now.)

6.3 Matters for higher-level protocols

Rotated bits: <https://cdli.mpiwg-berlin.mpg.de/artifacts/101087>

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

⁵⁶Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

References

Artefacts

- [P020054] VAT 4731. [[För16](#), 40 p.14]. Vorderasiatisches Museum.
CDLI: [P020054](#).
- [P020129] VAT 04713. Vorderasiatisches Museum.
CDLI: [P020129](#).
ORACC: [epsd2/corpus/P020129](#).
- [P102305] X.3.139. Michael C. Carlos Museum, Emory University.
CDLI: [P102305](#).
- [P221266] AO 13825. Musée du Louvre.
CDLI: [P221266](#).
ORACC: [epsd2/corpus/P221266](#).
Louvre Collections: [ark:/53355/cl010138527](#).
- [P221291] AO 13850. Musée du Louvre.
CDLI: [P221291](#).
ORACC: [epsd2/corpus/P221291](#).
- [P221305] AO 13864. Musée du Louvre.
CDLI: [P221305](#).
ORACC: [epsd2/corpus/P221305](#).
- [P232278] *Gudea E*. AO 6. Musée du Louvre.
CDLI: [P232278](#).
ORACC: [etcstri/Q001544](#).
- [P232280] *Gudea G*. AO 7. Musée du Louvre.
CDLI: [P232280](#).
ORACC: [etcstri/Q001546](#).
- [P240531] TM.75.G.00265. Idlib, Syria: National Museum of Syria.
CDLI: [P240531](#).
EbDA: [1415](#).
- [P240532] TM.75.G.00266. Idlib, Syria: National Museum of Syria.
CDLI: [P240532](#).
EbDA: [1324](#).
- [P240533] TM.75.G.00267. Idlib, Syria: National Museum of Syria.
CDLI: [P240533](#).
EbDA: [1379](#).
- [P240548] TM.75.G.00302. Idlib, Syria: National Museum of Syria.
CDLI: [P240548](#).
EbDA: [1350](#).
- [P240579] TM.75.G.00341. Idlib, Syria: National Museum of Syria.
CDLI: [P240579](#).
EbDA: [1364](#).
- [P240609] TM.75.G.00440. Idlib, Syria: National Museum of Syria.
CDLI: [P240609](#).
EbDA: [1378](#).

- [P240653] TM.75.G.00535. Idlib, Syria: National Museum of Syria.
CDLI: [P240653](#).
EbDA: [1382](#).
- [P240654] TM.75.G.00536. Idlib, Syria: National Museum of Syria.
CDLI: [P240654](#).
EbDA: [1383](#).
- [P240655] TM.75.G.00537. Idlib, Syria: National Museum of Syria.
CDLI: [P240655](#).
EbDA: [1358](#).
- [P240675] TM.75.G.00557. Idlib, Syria: National Museum of Syria.
CDLI: [P240675](#).
EbDA: [1371](#).
- [P240697] TM.75.G.00579. Idlib, Syria: National Museum of Syria.
CDLI: [P240697](#).
EbDA: [1381](#).
- [P241708] TM.75.G.02143. Idlib, Syria: National Museum of Syria.
CDLI: [P241708](#).
EbDA: [3173](#).
- [P241904] TM.75.G.02346. [[Arc89](#), p. 6]. Idlib, Syria: National Museum of Syria.
CDLI: [P241904](#).
EbDA: [3183](#).
- [P242293] TM.75.G.03125. Idlib, Syria: National Museum of Syria.
CDLI: [P242293](#).
EbDA: [217](#).
- [P249253] *Code de Hammurabi*. Sb 8. Musée du Louvre.
CDLI: [P249253](#).

Unicode documents

- [L2/03-162] M. Everson and K. Feuerherm. *Basic principles for the encoding of Sumero-Akkadian Cuneiform*. 25th May 2003.
UTC: [L2/03-162](#).
- [L2/03-393R] M. Everson, K. Feuerherm and S. Tinney. *Preliminary proposal to encode the Cuneiform script in the SMP of the UCS*. 3rd Nov. 2003.
UTC: [L2/03-393R](#).
- [L2/04-036] M. Everson, K. Feuerherm and S. Tinney. *Revised proposal to encode the Cuneiform script in the SMP of the UCS*. 29th Jan. 2004.
UTC: [L2/04-036](#).
- [L2/04-099] L. Anderson. *Unification of Cuneiform Numbers*. 2004.
UTC: [L2/04-099](#).
- [L2/04-189] M. Everson, K. Feuerherm and S. Tinney. *Final proposal to encode the Cuneiform script in the SMP of the UCS*. 8th June 2004.
UTC: [L2/04-189](#).
- [L2/24-159] P. Constable, ed. *Minutes of UTC Meeting 180* (23rd–25th July 2024).
29th July 2024.
UTC: [L2/24-159](#).

- [Uni16] The Unicode Consortium. *The Unicode Standard*. Version 16.0.0. The Unicode Consortium, 10th Sept. 2024.
ISBN: 978-1-936213-34-4.
eprint: <https://www.unicode.org/versions/Unicode16.0.0/core-spec/>.
- [UTR56] R. Leroy, ed. *Unicode Cuneiform Sign Lists*. Unicode Technical Report #56.
eprint: <https://www.unicode.org/reports/tr56/>.

Other documents

- [Arc15] A. Archi. *Ebla and Its Archives. Texts, History, and Society*. Studies in ancient Near Eastern records 7. Walter de Gruyter, 2015.
ISBN: 978-1-61451-716-0.
DOI: [10.1515/9781614517887](https://doi.org/10.1515/9781614517887).
- [Arc89] A. Archi. "Tables de comptes eblaïtes". In: *Revue d'assyriologie et d'archéologie orientale* 83.1 (1989). Ed. by P. Amiet and P. Garelli, pp. 1–6. ISSN: 0373-6032.
- [Bor10] R. Borger. *Mesopotamisches Zeichenlexikon*. Alter Orient und Altes Testament 305. Ugarit-Verlag, 2010.
- [Cap02] R. Caplice. *Introduction to Akkadian*. 4th ed. Editrice Pontificio Istituto Biblico, 2002.
ISBN: 88-7653-566-7.
- [Cha03] G. Chambon. "Archaic Metrological Systems from Ur". In: *Cuneiform Digital Library Journal* 2003.5 (23rd Dec. 2003). ISSN: 1540-8779.
eprint: http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003_005.html.
- [Cha12] G. Chambon. "Numeracy and Metrology". In: *The Oxford Handbook of Cuneiform Culture*. Ed. by K. Radner and E. Robson. Oxford University Press, 18th Sept. 2012, pp. 51–67.
ISBN: 9780199557301.
DOI: [10.1093/oxfordhb/9780199557301.013.0003](https://doi.org/10.1093/oxfordhb/9780199557301.013.0003).
- [DE87] P. Damerow and R. K. Englund. "Die Zahlzeichensysteme der archaischen Texte aus Uruk". In: M. W. Green and H. J. Nissen. *Zeichenliste der archaischen Texte aus Uruk*. Archaische Texte aus Uruk 2. An offprint of this chapter is available at <https://cdli.mpiwg-berlin.mpg.de/files-up/publications/englund1987a.pdf>. Gebr. Mann Verlag, 1987. Chap. 3, pp. 117–165.
- [Dei22] A. Deimel. *Liste der archaischen Keilschriftzeichen von Fara*. Wissenschaftliche Veröffentlichungen der Deutschen Orient-Gesellschaft 40. J. C. Hinrichs'sche Buchhandlung, 1922.
- [Eng98] R. K. Englund. "Texts from the Late Uruk Period". In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Vol. 160. Orbis Biblicus et Orientalis 1. 1998, pp. 13–233.
ISBN: 3-7278-1166-8.
- [För16] W. Förtsch. *Altbabylonische Wirtschaftstexte aus der Zeit Lugalanda's und Urukagina's*. Vorderasiatische Schriftdenkmäler der Königlichen Museen zu Berlin 14. J. C. Hinrichs, 1916.

- [Fri07] J. Friberg. *A Remarkable Collection of Babylonian Mathematical Texts. Manuscripts in the Schøyen Collection: Cuneiform Texts I*. Sources and Studies in the History of Mathematics and Physical Sciences. Springer, 2007.
ISBN: 978-0-387-34543-7.
- [Fri78] J. Friberg. *A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and Proto-Elamite Semi-Pictographic Inscriptions*. The Third Millenium Roots of Babylonian Mathematics 1. Department of Mathematics, Chalmers University of Technology, 1978.
- [Gom16] B. Gombert. “ED IIIb metrology: texts from Lagaš”. In: *CDLI:wiki. A Library of Knowledge of the Cuneiform Digital Library Initiative*. 12th Apr. 2016.
eprint: https://cdli.ox.ac.uk/wiki/doku.php?id=ed_iii_metrological_systems.
- [Gor23] F. Gori. “On Lapis Lazuli and Linen in Šuruppak Texts. An Analysis Through the Lens of Ebla Studies”. In: *Studia Eblaitica* 9 (2023), pp. 160–166. ISSN: 2364-7124.
- [Hue11] J. Huehnergard. *A Grammar of Akkadian*. 3rd ed. Brill, 2011.
ISBN: 978-1-57506-941-8.
- [Jim+23] E. Jiménez, Z. Földi, A. Härtinen, A. Heinrich, T. Mitto, G. Rozzi, I. Khait, J. Laasonen, F. Simonjetz et al., eds. *electronic Babylonian Library*. 2023–.
eprint: <https://www.ebl.lmu.de/>.
- [JJ24] T. Jauhiainen and H. Jauhiainen. “Advancing Cuneiform Text Dating Through Automatic Analysis”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 11th July 2024 14:00.
- [Kre98] M. Krebern timer. “Die Texte aus Fāra und Tell Abū Šalābīḥ”. In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 235–427.
ISBN: 3-7278-1166-8.
- [Lec12] C. Lecompte. “Des chiffres et des digues: à propos de deux textes présargoniques de Ġirsu et d’une notation numérique inhabituelle”. In: *Altorientalische Forschungen* 39.1 (Dec. 2012), pp. 81–86.
DOI: [10.1524/aofo.2012.0006](https://doi.org/10.1524/aofo.2012.0006).
- [Lec20] C. Lecompte. “The Measurement of Fields During the Pre-sargonic Period”. In: *Mathematics, Administrative and Economic Activities in Ancient Worlds*. Ed. by C. Michel and K. Chemla. Why the Sciences of the Ancient World Matter 5. Springer, 2020.
- [Mil+07] L. Milano, M. Maiocchi, F. Di Filippo, R. Orsini, E. Scarpa, M. Surdi et al., eds. *Ebla Digital Archives*. 2007–.
eprint: <http://ebda.cnr.it/>.
- [NDE93] H. J. Nissen, P. Damerow and R. K. Englund. *Archaic Bookkeeping. Early Writing and Techniques of Economic Administration in the Ancient Near East*. Trans. by P. Larsen. The University of Chicago Press, 1993.
ISBN: 0-226-58659-6.

- [Oel22] J. Oelsner. *Der Kodex Ḫammu-rāpi*. dubsar 4. Zaphon, 2022.
- [Pow87] M. Powell. “Maße und Gewichte”. In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš-Medizin. 1987–1990, pp. 457–530.
- [Rob08] E. Robson. *Mathematics in Ancient Iraq. A Social History*. Princeton University Press, 2008.
ISBN: 978-0-691-09182-2.
- [Rob19] E. Robson. “Oracc metrology guidelines”. In: *Oracc: The Open Richly Annotated Cuneiform Corpus*. 18th Dec. 2019.
eprint: <http://oracc.org/doc/help/editinginf/metrology/metrologicaltables/>.
- [Rob22] E. Robson. “Overview of Metrological Systems”. In: *The Digital Corpus of Cuneiform Mathematical Texts*. 2022.
eprint: <http://oracc.org/dccmt/Metrology/>.
- [Rom24] A. Romach. “The Neo Assyrian Land Sale Documents from Dur-Katlimmu: A Stylometric Analysis of Their Scribal Features”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 10th July 2024 12:00.
- [Sch10] W. Schramm. *Akkadische Logogramme*. Göttinger Beiträge zum Alten Orient 5. Universitätsverlag Göttingen, 2010.
ISBN: 978-3-941875-65-4.
DOI: [10.17875/gup2010-511](https://doi.org/10.17875/gup2010-511).
- [Svä+24] S. Svärd, M. Lorenzon, J. Töyräänvuori, J. Valk, T. Alstola, E. Bennett, R. Uotila and T. Auranne, eds. *RAI 69 Abstracts*. July 2024.
eprint: https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäiväty_1.pdf.
- [TJV17] S. Tinney, P. Jones and N. Veldhuis, eds. *The electronic Pennsylvania Sumerian Dictionary*. 2nd ed. 2017–.
eprint: <http://oracc.org/epsd2>.
- [VT+14] N. Veldhuis, S. Tinney et al., eds. *Oracc Sign List*. 2014–.
eprint: <http://oracc.org/osl/>.