

# Archaic cuneiform numbers

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2024-08-28

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## 1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script<sup>1</sup> and the proto-cuneiform script<sup>2</sup>. The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals<sup>3</sup> should however *not* be unified with the already-encoded cuneiform numerals<sup>4</sup>. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script\_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

<sup>1</sup>ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

<sup>2</sup>ISO 15924: Pcun, not yet encoded.

<sup>3</sup>Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: ◻, ◻ (small stylus), ◻ (large stylus). Some numerals are composed of multiple such impressions, e.g., ◻◻.

<sup>4</sup>Impressed into clay using a stylus with a trihedral end: — (stylus held horizontally), † (vertically), † (diagonally) † (diagonally with the stylus rotated along its axis), † (stylus pressed deeper, forming a larger wedge), † (combining † and †), etc.

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
Non-numeric signs	Future Pkun	Existing Xsux	

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

## 2 Proposed changes to the Standard

### 2.1 Summary of proposed characters

### 2.2 Properties





### 2.3 Character names list

### 2.4 Core specification text

## 3 Rationale for curviform–cuneiform disunification

TODO(egg): blurb.

### 3.1 The cuneiform encoding model

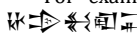
As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Suruppak as in the character code charts,  later in the third millennium<sup>5</sup>,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists<sup>6</sup> and dictionaries<sup>7</sup>, and of composite texts<sup>8</sup>. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]<sup>9</sup>. The diachronic approach is also useful for pedagogic applications<sup>10</sup>.

<sup>5</sup>Merging with U+1224E cuneiform sign ni2.

<sup>6</sup>Notably [OSL] and the online edition of [MZL] in [eBL, Signs].

<sup>7</sup>Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

<sup>8</sup>For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

<sup>9</sup>Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svā+24, pp. 242, 148].

<sup>10</sup>For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

### 3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding<sup>11</sup> that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

† <sup>12</sup>	<	+	≡	𐎶	𐎶	𐎶
1(ḫeš <sub>2</sub> )	1(u)	1/2(diš)	5(diš <i>tenû</i> )	gi	us <sub>2</sub>	sa <sub>2</sub>
	7.5 (ropes)		5	reed	side	equal
« <sup>13</sup>	≡	𐎶	𐎶	𐎶		
3(u)	6(diš <i>tenû</i> )	gi	saṇ	sa <sub>2</sub>		
3(ropes)	6	reed	front	equal		
𐎶𐎶𐎶	•	◁	▷	◡		
ašag-bi	1(bur <sub>3</sub> <sup>c</sup> )	1(eše <sub>3</sub> <sup>c</sup> )	1(iku <sup>c</sup> )	1/2(iku <sup>c</sup> )		
this field						

𐎶𐎶𐎶𐎶𐎶  
tug<sub>x</sub>(LAK483)-si-ga-kam<sup>14</sup>  
deep ploughing

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as<sup>15</sup>

465 metres, equal lengths  
198 metres, equal widths  
this field: 𐎶, 𐎶 hectares, deeply ploughed

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from ▷ to † numerals). It therefore comes to the conclusion that the use of

<sup>11</sup>At that time scoped to the répertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as 𐎶 ≠ 𐎶𐎶, were informed by Early Dynastic distinctions.

<sup>12</sup>As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large — in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

<sup>13</sup>Note that ED IIIb < numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign « in [P020054] looks more like Ur III 𐎶.

<sup>14</sup>Transliteration after [Lec20, p. 325].

<sup>15</sup>We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.


curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere  $\triangleright$ / $\lvert$  switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

### 3.3 A primer on classic Ur III and Old Babylonian metrologies

  
 I want to write tablets: the tablet of  
 1 cor of barley to 600 cor; the tablet  
 of 1 shekel of silver to 10 minas [...]  
 Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the contrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known<sup>16</sup> a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters  $\lvert$ – $\text{||||}$ , the multiples of ten (10–50) are  $\text{<}$ – $\text{<||||}$ , but the other digits 11–59 are sequences  $\text{<}$ – $\text{<||||}$ ; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system*  $S_{\text{Ur III/OB}}$ , which had different signs for the units  $\lvert$ – $\text{||||}$ , tens  $\text{<}$ – $\text{<||||}$ , sixties  $\text{||||}$  (with larger wedges than the units), multiples of six hundred  $\text{||||}$ , multiples of three thousand six hundred  $\text{||||}$ , and multiples of thirty-six thousand  $\text{||||}$ .

<sup>16</sup>See, e.g., [Uni16, §22.3.3, sub “Cuneiform Numerals”].

### 3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram<sup>17</sup>, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \quad (S_{\text{Ur III/OB}})$$

For example, the number  $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$  would be written 𐎶𐎵𐎶𐎵𐎶𐎵 in the discrete counting system, and 𐎶𐎵𐎶𐎵𐎶𐎵 in the sexagesimal place value system.

### 3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:

$$\begin{array}{ccccccc} \diamond & \xleftarrow{10} & \diamond & \xleftarrow{6} & \text{𐎶} & \xleftarrow{10} & \text{𐎵} \xleftarrow{3} \text{𐎶} \xleftarrow{6} \text{𐎵} \xleftarrow{2} \text{𐎶} \xleftarrow{2} \text{𐎵} \\ & & & & \text{bur}_3 & & \text{eše}_3 & & \text{iku} & & \text{ubûm} \\ & & & & \text{bûrum} & & \text{eblum} & & \text{ikûm} & & 1800 \text{ m}^2 \\ & & & & 6,48 \text{ ha} & & 2,16 \text{ ha} & & 3600 \text{ m}^2 & & \end{array} \quad (G_{\text{Ur III/OB}})$$

Note that for the range of areas given above<sup>18</sup>, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *bûrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign 𐎶𐎵 functioning as punctuation<sup>19</sup>, but the numerals are tied to the metrology; thus a surface of 5 *bûr* 1 *ebel* 4 *ikû* (100 *ikû*, 36 ha) would be written<sup>20</sup> 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in “88 acres 3 roods 33 perches” or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the sign  $\diamond$  is equal to sixty times  $\text{𐎶}$  in the area system, but to three hundred and sixty times  $\text{𐎶}$  in the discrete counting system.

<sup>17</sup>These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

<sup>18</sup>For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mûšarum* (36 m<sup>2</sup>) written 𐎶𐎵𐎶, equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60 𐎶𐎵 (shekels). For areas greater than 3600 *bûrû*, the  $\diamond$  and  $\text{𐎶}$  numerals are reused with a suffix 𐎶𐎵 (gal, Sumerian: big), as follows [Rob08, p. 295 n. b and c; Fri07, p. 378; Rob19]:

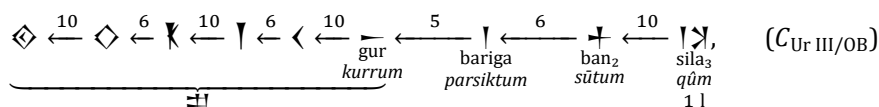
$$\begin{array}{cccccccccccccccccccc} \diamond & \text{𐎶𐎵} & \xleftarrow{10} & \diamond & \text{𐎶𐎵} & \xleftarrow{6} & \diamond & \xleftarrow{10} & \diamond & \xleftarrow{6} & \text{𐎶} & \xleftarrow{10} & \text{𐎵} & \xleftarrow{3} & \text{𐎶} & \xleftarrow{6} & \text{𐎵} & \xleftarrow{2} & \text{𐎶} & \xleftarrow{2} & \text{𐎵} & \xleftarrow{2,5} & \text{𐎶𐎵} & \xleftarrow{10} & \text{𐎶} & \xleftarrow{10} & \text{𐎵} \\ & \end{array}$$

<sup>19</sup>TODO(egg): acknowledge Proust 2020 but note that this is irrelevant to encoding concerns






<sup>20</sup>As in the surface of the field of 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵 (the city of Apisal) reported on [P102305, r. 1]

### 3.3.3 The capacity system

Another such system of note is the one for capacities<sup>21</sup> [Fri07, p. 376; Rob19],



where the numerals for ban<sub>2</sub> are 𐤁, 𐤂, 𐤃, 𐤄, and 𐤅, and those for bariga are 𐤆, 𐤇, 𐤈, and 𐤉 (contrast ordinary 𐤈 and 𐤉 otherwise used with 𐤆 numerals). As described in [Hue11, p. 585 n. (b) and (f)], the sign 𐤆𐤇 GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𐤇 if present, and after the word for “grain” if present, as in

 354 gur   
  3 ban<sub>2</sub>   
  6   
  sila<sub>3</sub>   
  of grain.

Observe that while large numbers of gur follow<sup>23</sup> system  $S_{\text{Ur III/OB}}$ , the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as  $\langle \text{I} \text{—} \text{II} \rangle$  would be 10 gur 1 bariga, and  $\langle \text{—} \text{II} \rangle$  would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

- $\mathbb{I}$ - $\mathbb{W}$  used in  $S_{\text{Ur III/OB}}$  and the SPVS as well as with overt units;
- $\mathbb{L}$ - $\mathbb{X}$  used in  $G_{\text{Ur III/OB}}$ , of which  $\mathbb{L}$ - $\mathbb{X}$  are also used in  $S_{\text{Ur III/OB}}$  and the SPVS as well as with overt units;
- $\mathbb{I}$ - $\mathbb{W}$  used in  $S_{\text{Ur III/OB}}$ , and sometimes with overt units;
- $\mathbb{K}$ - $\mathbb{X}$  used in  $S_{\text{Ur III/OB}}$ ;
- $\mathbb{O}$ - $\mathbb{X}$  used in  $S_{\text{Ur III/OB}}$  and  $G_{\text{Ur III/OB}}$ ;
- $\mathbb{O}$ - $\mathbb{X}$  used in  $S_{\text{Ur III/OB}}$  and  $G_{\text{Ur III/OB}}$ ;
- $\mathbb{L}$ - $\mathbb{X}$  used in  $C_{\text{Ur III/OB}}$  as well as with overt units of the weight system;
- $\mathbb{L}$ ,  $\mathbb{L}$ ,  $\mathbb{L}$ ,  $\mathbb{L}$ ,  $\mathbb{L}$  used in  $C_{\text{Ur III/OB}}$ ;
- $\mathbb{L}$ ,  $\mathbb{L}$ ,  $\mathbb{L}$ ,  $\mathbb{L}$  used in  $C_{\text{Ur III/OB}}$ —note the overlap with  $\mathbb{I}$ - $\mathbb{W}$ ;
- $\mathbb{L}$  and  $\mathbb{L}$  used in  $G_{\text{Ur III/OB}}$ .

Only in the SPVS did numerals exist truly independently of metrology; to quote [Robo8, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

<sup>21</sup>Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system  $G_{Ur III/OB}$  based on a height of one cubit, see[[Pow87](#), p. 488; [Rob08](#), p. 294; [Rob19](#)].

<sup>22</sup>From [P309594].

<sup>23</sup>A larger unit, the guru<sub>7</sub> (*karûm*, grain heap), is sometimes used instead, with 𐎠𐎡𐏁𐎧𐎶𐎥𐎺𐎠 = 𐎠𐎡𐏁𐎧𐎶𐎥𐎺𐎠 (1 *karûm* = 3600 *kurrû*). See [Fri07, p. 415; Rob19].

8







of decreasing fractions  $\frac{1}{n}$  of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus  $\text{N}_{30} = \frac{1}{2} \text{N}_{30}$ ,  $\text{N}_{30} = \frac{1}{3} \text{N}_{30}$ , and so on. The first sign of the latter units,  $\text{N}_{34}$ ,

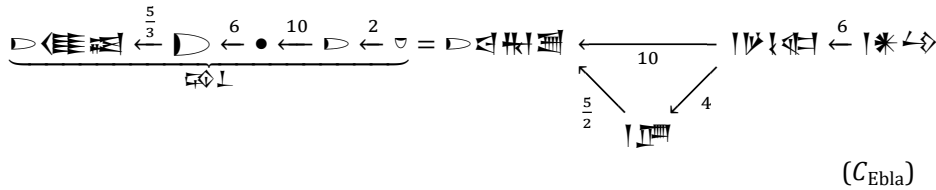
Figure 1: TODO [Eng98, p. 113]

For instance, the first line contains the notations  $1\text{N}_{34} 1\text{N}_{30} : 2\text{N}_{20}$ , which can be translated '60 of the (grain rations containing) (of grain); (grain involved:) 2 (of ground barley)'. This calculation contradicts the assumed numerical relationship  $10\text{N}_1 = 1\text{N}_{14}$ , since as was well known the measure represented by the sign  $\text{N}_{30}$  was  $\frac{1}{5}$  of that represented by  $\text{N}_1$ , so that  $60 \times \frac{1}{5} = 12$  and not 20, as  $2\text{N}_{14}$  would imply. Instead of relying on complicated

Figure 2: TODO [Eng98, p. 116]

11. [P240531, recto 1, 8]
12. [P241708, recto 1, 1]<sup>45</sup>
13. [P241904, recto 1, 1]<sup>46</sup>

Note that higher numbers of are expressed in hundreds (*mi-at* ) and then thousands (*li-im* ) as is typical in Ebla [Arc15, p. 33], e.g., in [P240532, verso 2, 3], (100 + 60 + 30 + 5 = 195 of grain). These expressions correspond to the following factor diagram:



### 3.4.5 Use in modern publications

Because of their prevalence in the Uruk and Early Dynastic periods, the proposed numerals are used modern publications discussing metrology in those periods, as illustrated in Figures 1–11.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 11. The partial<sup>48</sup> transliteration “4 'a<sub>3</sub>-da-um 4 aktum 4 ib<sub>2</sub><sup>tu</sup> × 3 sa<sub>6</sub> gunu<sub>3</sub>” is used to illustrate a discussion of the interpretation of the contrast between and numerals. More conventional transliterations<sup>49</sup> might omit the numeral shapes entirely, e.g., 4 'a<sub>3</sub>-da-um 4 aktum 4 ib<sub>2</sub><sup>tu</sup> × 3 sa<sub>6</sub> gunu<sub>3</sub>, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, e.g., 4(aš<sup>c</sup>) 'a<sub>3</sub>-da-um 4(diš<sup>c</sup>) aktum 4(aš<sup>c</sup>) ib<sub>2</sub><sup>tu</sup> × 3(diš<sup>c</sup>) sa<sub>6</sub> gunu<sub>3</sub>, but the result would be less readable. See Section 3.7.2 for a discussion of transliteration conventions for numerals.

<sup>45</sup>From CDLI photo.

<sup>46</sup>From photo in [Arc89, p. 6].

<sup>47</sup>Laid out as ; on stacking patterns see Section 6.3.

<sup>48</sup>The untransliterated text would be ; note the atomically

Die halbkreisförmigen Griffelindrücke gehen manchmal in mehr oder weniger eckige Formen über ( $\nabla$ )<sup>685</sup>. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen ( $\backslash$ ), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren<sup>686</sup>. Selten treten mit  $\nabla$  gebildete Zahlen auf<sup>687</sup> (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 3: TODO [Kre98, p. 303]

TODO figure

Figure 4: TODO [Cha03, p. 6]

formed by only two signs  $\nabla$  and  $\backslash$ , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60<sup>2</sup>, 60<sup>3</sup>, 1/60, 1/60<sup>2</sup>, etc. are written in the same way, with the vertical wedge  $\nabla$ . The third feature concerns the exact function of

Figure 5: TODO [Cha12, p. 58]

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with:  $\nabla\nabla\nabla\nabla$  but never with the repetition  $\nabla\nabla\nabla\nabla$ .

Figure 6: TODO [Cha12, p. 59]<sup>50</sup>

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

$\nabla\nabla\nabla$  še bar  $\nabla$  ba-ri-zu

‘3 gubar (capacity units) and 1 parisu’.

Figure 7: TODO [Cha12, p. 61]

This is particularly true of the signs  $\nabla$ ,  $\nabla$ ,  $\nabla$  and  $\nabla$ , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written  $\nabla$  in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign  $\nabla$  is most often associated with the *parisu* measure, while the signs  $\nabla$ ,  $\nabla$ ,  $\nabla$  and  $\nabla$  refer to 1, 2, 3,

Figure 8: TODO [Cha12, p. 64]

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g.,  $\nabla\nabla\nabla\nabla$  transliterated as 2(šar<sub>2</sub>) 1(geš'u) 3(u), means  $2 \times 3600 + 1 \times 600 + 3 \times 10$ ). The system is based on an alternation of factors ten and

Figure 9: TODO

might think of one fabric and a half,<sup>11</sup> but the presence of notations with “2▷2▽”, “3▷3▽”, and “6▷6▽” (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag’s weight measurement system.<sup>12</sup> The notation “1▷gada” in o. ii 1 and r. vi 1, along with the total of “39



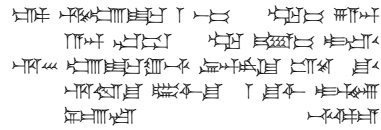
Fig. 1. Combinations of numerals attested in Š. 742.

Figure 10: Discussion of the contrast between ▷ and ▽ numerals in [Gor23, p. 162].

as, for example, in TM.75.G.3125 = ARET III 107 o. iv 1, “4▷ ‘a<sub>3</sub>-da-um<sup>u9</sup>-2 4▽  
aktum 4▷ib<sub>2</sub><sup>u9</sup>×3▽ sa<sub>6</sub> gunu<sub>3</sub>” (Fig. 2).

Figure 11: Transliteration in [Gor23, p. 163] of [P242293, recto 4, 1] incorporating untransliterated numerals.

### 3.5 Non-numeric usage



The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for ‘sixty’<sup>51</sup>. Do you know its reading<sup>52</sup>?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign — has, *inter alia*, the values aš, rum, and dili. While the horizontal numerals are most frequently written ▷ in the Early Dynastic period<sup>53</sup>, such non-numeric usage is almost<sup>54</sup> always written —, for instance:

- in personal names in administrative texts, such as the following, which all contain ▷ numerals:
  - — in [P010424; P010458; P010459] from ED IIIa أبو صلايخ,
  - — in [P010960] from ED IIIa Šuruppag,
  - — in [P251641] from ED IIIb Adab,

encoded  $ib_2 \times 3! = \text{𐎶} \times \text{𐎶} = \text{𐎶𐎶}$ .

<sup>49</sup>TODO cite the EbDA one.











<sup>50</sup>TODO(egg): On the order cite TSS 188, Friberg2007 p. 148 and any of the usual suspects on the haphazard order of signs in early texts; contrast P274845, P241764.

<sup>51</sup>The reader will recall that 𐎶 is written 𐎶, with a larger wedge than 𐎶; however, these signs have merged by the time Examenstext A is composed.

<sup>52</sup>Besides 𐎶, a look at [OSL] shows that the values diš, ge<sub>3</sub>, makkaš, saṇtak<sub>4</sub>, and tal<sub>4</sub> are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

<sup>53</sup>A CDLI search for “(asz@c)” finds 3296 ED texts, while a search for “(asz)” finds 81 ED texts, of which 46 also contain “(asz@c)”.





<sup>54</sup>Exceptions are discussed in section 3.7.1.

-    in [P252866] from ED IIIb Adab,
-    in [P298637] from ED IIIb Umma;
- in the Sumerian word   $\text{u}_2\text{-rum}$ , “property” in ED IIIb Nirsu administrative texts which contain  $\triangleright$  numerals, such as [P020006; P020008; P020018; P020024; P020030];
- in lexical texts:
  - in the divine name     $\text{—} \text{L}$  in the lexical texts [P010570; P010572], where the entries are prefixed with  $\triangleright$ .
  - in the word  $\text{—} \text{dili}$ , “small fish” in [P010578], witness to Early Dynastic Fish,
  - in the same word with a determinative,  $\text{—} \text{dili}^{\text{ku}_6}$ , in [P010586], witness to Early Dynastic Food, which starts with  $\triangleright$  numerals.

This is a clear contrast between  $\text{—}$  and  $\triangleright$  in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name  $\text{—} \text{L}$  occurs on its own line in the aforementioned administrative texts; a line  $\triangleright \text{L}$  would instead be read as “one slave”.

### 3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as  $\text{—}$  that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [KWU, pp. 123 sqq.] lists all of  $\text{—}$   together with  $\triangleright$  , while  $\text{—}$ ,  $\text{--}$ , and  $\text{---}$ , and only those, appear at the beginning of the sign list, since they have non-numeric values<sup>55</sup>. [PTACE, p. 58] has the numeric signs  $\triangleright$ ,  $\text{—}$ ,  $\text{L}$ , whereas non-numeric  $\text{—}$  is at the beginning of the sign list, where its values *as* and *rum* are listed. For signs with both non-numeric and numeric usage, [LAK] writes *s. die Zahlz.* throughout the main list; LAK 1  $\text{—}$  thus reappears at LAK 829 together with  $\triangleright$ ,  $\text{—}$ , and  $\text{L}$ . One should note [MZL], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have   $\text{—}$   used in system  $G_{\text{Ur III/OB}}$ .

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes<sup>56</sup> to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from  $\triangleright$  to  $\text{L}$  numerals prevents diachronic encoding even if  $\triangleright$  were unified with  $\text{—}$ . For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'–6']; however, they are written with  $\text{L}$  numerals, whereas they are writ-

<sup>55</sup>Non-numeric values of  $\text{—}$  were discussed in section 3.5;  $\text{--}$  has the values  $\text{man}_3$  and  $\text{min}_5$ , and is used for the word *didli*, “several, various”;  $\text{---}$  has the value  $\text{eš}_6$ .

<sup>56</sup>TODO comment on the ED witnesses to the instructions of Šuruppak

ten with  $\triangleright$  numerals in the Early Dynastic witnesses; since  $\uparrow$  and  $\leftarrow$  are distinct<sup>57</sup> characters, the  $\triangleright \leftarrow$  unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods<sup>58</sup>, there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Romach2023], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

### 3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts<sup>59</sup> being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

#### 3.7.1 The case of ŠAR<sub>2</sub>

The character U+122B9  $\bullet$  CUNEIFORM SIGN ŠAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period<sup>60</sup>, a contrast between non-numeric šar<sub>2</sub> written  $\diamond$  and numeric 1(šar<sub>2</sub>) written  $\bullet$  can be observed, similar to the contrast between  $\leftarrow$  and  $\triangleright$  previously discussed in section 3.5. However, in lexical lists from Šuruppak and Ebla<sup>61</sup>, as well as in the *Stèle des vautours*, non-numeric šar<sub>2</sub> is curviform:

- $\ast \text{𒌦} \text{𒌦} \text{𒌦} \bullet$  and  $\ast \text{𒌦} \bullet \text{𒌦} \text{𒌦}$  in [P010566];
- $\bullet \text{𒌦} \text{𒌦}$  and  $\ast \bullet \text{𒌦} \text{𒌦}$  in [P010576];
- $\bullet \text{𒌦}$  in [P240986]<sup>62</sup>;
- $\bullet \text{𒌦} \text{𒌦}$  in [P222399, obv. 17, 9, 18, 11, 22, 12]<sup>63</sup>.

<sup>57</sup>Besides the contrasts in numeric usage mentioned in section 3.3.3, these characters are clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of  $\leftarrow$  which are not shared with  $\uparrow$  have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period,  $\leftarrow$  is used for the preposition *ina*, “in”, and  $\uparrow$  for the preposition *ana*, “to”.

<sup>58</sup>TODO cite a few things here.

<sup>59</sup>Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

<sup>60</sup>For example, in personal names:







- $\text{𒌦} \text{𒌦} \text{𒌦} \text{𒌦}$  in [P020019] from ED IIIb Nirsu;
- $\text{𒌦} \text{𒌦} \text{𒌦} \text{𒌦}$  in [P020182], also from ED IIIb Nirsu;
- $\text{𒌦} \text{𒌦}$  in [P222186] from ED IIIb Umma;
- $\text{𒌦} \text{𒌦} \text{𒌦}$  in [P235312] from Old Akkadian Umma.


<sup>61</sup>TODO Mention other ways in which these are archaizing





<sup>62</sup>From copy in [ELLES, No. 397].

<sup>63</sup>Note however  $\ast \text{𒌦} \text{𒌦} \text{𒌦}$  on [P222399, obv. 6, 17]. Curviform non-numeric šar<sub>2</sub> is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here. TODO add a photo.













It *would* be disruptive to the diachronic representation of text if non-numeric šar<sub>2</sub> were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph , identical to the glyph of the proposed U+12579  CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric šar<sub>2</sub> is curviform solidly predates the transition from  to  in the relevant metrological systems, there is no need to represent a - contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.











Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D  CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

Note that in rare cases, such as [P222243] from ED IIIa Adab, non-numeric  (here with the value rum) is written . It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550  cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. In practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the CDLI transliteration (UR2-1(aš@c)) and the ePSD2 one (uru<sub>8</sub><sup>rum</sup>) of this word disagree on that aspect. Since  has a cuneiform reference glyph, this does not pose any compatibility concerns.

### 3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL<sub>2</sub> can be described as  NE×KASKAL or  NE×PAP<sup>64</sup>, they are typically all transliterated bil<sub>2</sub>, and therefore are all represented by the character U+1224B  CUNEIFORM SIGN NE SHESHIG, its name notwithstanding, as described in [UTR56, §2.5].

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance,  from [P305639] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in EbDA transliterations: the aforementioned     7 nīg<sub>2</sub>-sagšu 2-1/2 an-zam<sub>x</sub><sup>65</sup> za”, reading both  and  as 1/2, but not distinguishing them.

In particular, these transliterations do not differentiate between  and  numerals, nor between  and  numerals. For instance, the aforementioned  -da-um<sup>tug<sub>2</sub></sup>-II 4 aktum<sup>tug<sub>2</sub></sup> 4 ib<sub>2</sub>-III gun<sub>3</sub> sa<sub>6</sub><sup>tug<sub>2</sub></sup>” in EbDA, with no distinction between the  and . Since  and  numerals are separately encoded, the numeric ex-

<sup>64</sup>As on [P249253].

<sup>65</sup>As of this writing, EbDA actually has an-zam<sub>x</sub>, with U+1D6A GREEK SUBSCRIPT SMALL LETTER CHI.



pressions in such transliterations cannot be transformed into Unicode cuneiform without additional context, regardless of curviform–cuneiform unification.

In metrological systems such as systems  $G_{Ur III/OB}$  and  $C_{Ur III/OB}$  where some units are indicated by the type of numeral rather than an overt unit sign, it is common practice to add the unit in parentheses in transliteration; for instance,  $\text{𒀭} \text{𒌷} \text{𒌷} \text{𒌷}$  from [P386847] is transliterated “1(eše<sub>3</sub>) 5½ iku<sup>66</sup> 7 sar” in [Feu04, vol. 2, p. 176], and  $\text{𒌷} \text{𒌷} \text{𒌷} \text{𒌷}$  from [P307255] is transliterated “1(n<sup>67</sup>) 2(b) 7 ½ sila<sub>3</sub>” in [Feu04, vol. 2, p. 151].

This practice has been generalized to systematically indicate numeral shape; this is in particular the case in CDLI, where the transliterations of some of the above examples are “1(gešz2) 3(u) 5(asz) gur” for  $\text{𒀭} \text{𒌷} \text{𒌷} \text{𒌷}$ , “1(esze3) 5(iku) 1/2(iku) GAN2 7(disz) sar” for  $\text{𒀭} \text{𒌷} \text{𒌷} \text{𒌷}$ , and “3(barig) 2(ban2) 7(disz) 1/2(disz) sila3” for  $\text{𒌷} \text{𒌷} \text{𒌷} \text{𒌷}$ . CDLI and ePSD2 both distinguish curviform from cuneiform numerals in transliteration: the length  $\text{𒀭} \text{𒌷} \text{𒌷} \text{𒌷}$  from [P020129, rev. 2, 1] is transliterated “6(gešz2@c) 3(u@c) {ninda}nindax(DU) 1/2(asz@c) 4(disz@t) gi” in CDLI, and “6(gešz<sub>2</sub><sup>c</sup>) 3(u<sup>c</sup>) ninda ninda<sub>x</sub>(DU) 1/2(aš<sup>c</sup>) 4(diš<sup>t</sup>) gi” in ePSD2. Another example is [Molina2014], which uses *1a* for  $\text{𒀭}$ , *1d* for  $\text{𒌷}$ , *1ac* for  $\text{𒌷}$ , *1dc* or  $\frac{1}{2}dc$  for  $\text{𒌷}$  depending on reading, etc. The literature on the Uruk and Early Dynastic I–II periods uses a different set of transliteration conventions that also disambiguate numeral shapes, as will be discussed in section 4.

While there exist transliterations that distinguish  $\text{𒀭}$  from  $\text{𒌷}$  but not  $\text{𒀭}$  from  $\text{𒌷}$ , such as the ones used in [DCCMT], the trend, especially in more recent works in third millennium studies, seems to be to represent numeral shape; for example, [Maiocchi2024] gave an example of the input syntax used by the new “Urban Economy Begins” project as “10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)” for  $\text{𒀭} \text{𒌷} \text{𒌷} \text{𒌷}$ , with a *c* indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are  $\text{𒀭}$  rather than  $\text{𒌷}$  numerals.

### 3.8 Conclusions

Co-occurrences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idiosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted:  $\text{𒀭}$  is a length of three ropes, but  $\text{𒀭}$  is an area of three bur<sub>3</sub>;  $\text{𒀭} \text{𒌷}$  could be read as one  $\text{𒀭} \text{𒌷}$  and one  $\text{𒌷} \text{𒌷}$ , where  $\text{𒀭} \text{𒌷}$  would be one and a half  $\text{𒀭} \text{𒌷}$ ;  $\text{𒀭} \text{𒌷}$  is a personal name, but  $\text{𒀭} \text{𒌷}$  would be “one slave”.

At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millennium corpora. The oddity of  $\text{𒀭}$  requires some explanation, but does not

<sup>66</sup>TODO say something about this reading

<sup>67</sup>TODO comment on nigida.

pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.

## 4 Rationale for ED-Uruk numeral unification

TODO mention the bariga silliness in the CDLI transliteration of Gori's paper.

## 5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban<sub>2</sub>@c) and 5(ban<sub>2</sub>@c).]

## 6 Characters not included in this proposal

### 6.1 The fractions $\frac{1}{3}$ and $\frac{2}{3}$

TODO ¶, ¶. Note the occasional omission of ¶.

## 6.2 Missing numerals

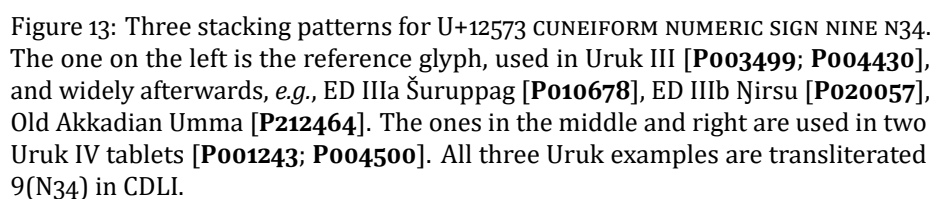
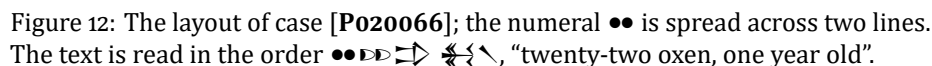
TODO N13 not attested in CDLI TODO ( $N_{17}$  not usefully numeric,  $12N_{14}$  not encodable, etc.). 7 and 8 (diš *tenû*) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.

### 6.3 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some *stacking patterns*; for instance 9I is encoded both as U+12446 𐎶𐎵 and as U+1240E 𐎶𐎵𐎫. This is in part due to contrastive usage of stacking patterns. For instance, besides I and II which are characteristic of bariga measures, four bariga is written IIII even where 4I is written IIII, as in [P255010; P292843]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often I II III IV V VI VII VIII IX X XI XII XIII XIV XV XVI XVII XVIII XIX XX XXI XXII XXIII XXIV XXV XXVI XXVII XXVIII XXIX XXX XXXI XXXII XXXIII XXXIV XXXV XXXVI XXXVII XXXVIII XXXIX XL XLI XLII XLIII XLIV XLV XLVI XLVII XLVIII XLIX L LI LII LIII LIV LV LVI LVII LVIII LX LXI LXII LXIII LXIV LXV LXVI LXVII LXVIII LXIX LXX LXXI LXXII LXXIII LXXIV LXXV LXXVI LXXVII LXXVIII LXXIX LXXX LXXXI LXXXII LXXXIII LXXXIV LXXXV LXXXVI LXXXVII LXXXVIII LXXXIX XC XCI XCII XCIII XCIV XCV XCVI XCVII XCVIII XCIX C CI CII CIII CIV CV CVI CVII CVIII CX CXI CXII CXIII CXIV CXV CXVI CXVII CXVIII CXIX CC CCII CCIII CCIV CCV CCVI CCVII CCVIII CCIX CCC CCCII CCCIII CCCIV CCCV CCCVI CCCVII CCCVIII CCCIX CCCC CCCCC CCCCCCC CCCCCC < << <<< > >> >>>, and results in metrological systems, typically I II III IV V VI VII VIII IX X XI XII XIII XIV XV XVI XVII XVIII XIX XX XXI XXII XXIII XXIV XXV XXVI XXVII XXVIII XXIX XXX XXXI XXXII XXXIII XXXIV XXXV XXXVI XXXVII XXXVIII XXXIX XL XLI XLII XLIII XLIV XLV XLVI XLVII XLVIII XLIX L LI LII LIII LIV LV LVI LVII LVIII LX LXI LXII LXIII LXIV LXV LXVI LXVII LXVIII LXIX LXX LXXI LXXII LXXIII LXXIV LXXV LXXVI LXXVII LXXVIII LXXIX LXXX LXXXI LXXXII LXXXIII LXXXIV LXXXV LXXXVI LXXXVII LXXXVIII LXXXIX XC XCI XCII XCIII XCIV XCV XCVI XCVII XCVIII XCIX C CI CII CIII CIV CV CVI CVII CVIII CX CXI CXII CXIII CXIV CXV CXVI CXVII CXVIII CXIX CC CCII CCIII CCIV CCV CCVI CCVII CCVIII CCIX CCC CCCII CCCIII CCCIV CCCV CCCVI CCCVII CCCVIII CCCIX CCCC CCCCC CCCCCCC CCCCCC < << <<< > >> >>>, occasionally co-occurring as in [P142827; P142357]. This separate encoding is also for compatibility with distinctions made in reference works and in some non-numeric transliterations; for instance, III is [MZL, No. 860] and has the value limmu, and II is [MZL, No. 852] and has the value limmu<sub>s</sub>. Numeric<sup>68</sup> transliterations occasionally distinguish the stacking patterns III IIII < > >> >>>, as in the CDLI transliterations of the aforementioned tablets, although this is rare; often 4(diš) is III in Ur III, but IIII in the Neo-Assyrian period.

However, the stacking patterns from earlier periods are not separately encoded; for instance, in ED IIIb Nirsu, « 2(u) often has one < atop another. These older stacking patterns do not appear to be contrastive, are not marked in transliteration,

<sup>68</sup>The Sumerian word limmu means “four”, so limmu and limmu<sub>5</sub> are still numbers. The distinction here is between usage in transliterations of phrases such as 𒌦 𒈪 𒂗 𒀭 𒄩 𒅆 𒊕 𒋼 lugal an-ub-da limmu<sub>5</sub>-ba-ke<sub>4</sub> (king of the four quarters) or of names, and of numeric expressions such as 𒁫 𒉺 4(diš) sila<sub>3</sub>.



and are not listed separately in sign lists nor assigned any different values. There is therefore no evidence of a need to encode them; instead, they should be considered style variants, and an ED IIIb Nirsu font should have an appropriate glyph for U+12399 « CUNEIFORM SIGN U U.

Likewise, many stacking patterns are attested for the curviform numerals proposed in this document, and it is not proposed to separately encode them; these distinctions would be incompatible with the state of the art in numeric transliterations, and are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier tablets, as they are structured in rectangular cases rather than lines, so that numerals may be laid out across the case in whichever way fits the available space; this is illustrated in Figure 12. Note that in that figure, the numerals need to be considerably enlarged in order to reproduce the layout of the tablet:  $\triangleright$  has the same height as  $\triangleright\triangleright$ . This is impractical when these numerals are set in text that contrasts them with the larger  $\bigtriangleright$ , and inconsistent with actual practice when typesetting these numerals, as illustrated in section 3.4.5: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dynastic period, but that are also attested in the Uruk period; the Uruk period also frequently features numerals that use a more vertical layout, as illustrated in Figure 13. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millenium, usage in third millenium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents  $\triangleright$ ,  $\triangleright$ ,  $\bullet$ , and  $\bullet$  must remain consistent across the numeral series, lest a  $\triangleright$  numeral be confused with an  $\triangleright$  numeral. Since the single indents are frequently used in running text, as illustrated in section 3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic

distinctions; Figure 13 uses OpenType stylistic alternates, and Figure 12 rotates the character ••, in both cases preserving the plain text backing.

## Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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- [Svä+24] S. Svärd, M. Lorenzon, J. Töyräänvuori, J. Valk, T. Alstola, E. Bennett, R. Uotila and T. Auranne, eds. *RAI 69 Abstracts*. July 2024.  
eprint: [https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäiväty\\_1.pdf](https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäiväty_1.pdf).