

Archaic cuneiform numbers

Robin Leroy, Anshuman Pandey, and Steve Tinney

2024-09-04

Contents

1	Summary	2
2	Proposed changes to the Standard	3
2.1	Summary of proposed characters	3
2.2	Properties	3
2.3	Character names list	3
2.4	Core specification text	3
3	Rationale for curviform–cuneiform disunification	3
3.1	The cuneiform encoding model	3
3.2	Arguments for curviform–cuneiform unification	4
3.3	A primer on classic Ur III and Old Babylonian metrologies	5
3.3.1	The discrete counting system	6
3.3.2	The area system	6
3.3.3	The capacity system	7
3.3.4	The length system	8
3.3.5	Fractions	8
3.4	Curviform numerals in early metrologies	8
3.4.1	Field lengths in Nirsu	9
3.4.2	Dyke lengths in Nirsu	10
3.4.3	Cheese and wheat in Nirsu	11
3.4.4	Grain in Ebla	11
3.4.5	Use in modern publications	12
3.5	Non-numeric usage	15
3.6	Limited benefits of diachronic encoding for numerals	17
3.7	Compatibility considerations	18
3.7.1	The case of ŠAR ₂	18
3.7.2	Transliteration	19
3.8	Conclusions	20
4	Rationale for ED–Uruk numeral unification	21
5	Considerations on individual numeral series	22

6 Characters not included in this proposal	22
6.1 Missing numerals	22
6.2 Stacking patterns	22
6.3 Other glyph variants not reflected in transliteration	24
Acknowledgements	24
References	24
Artefacts	24
Unicode documents	26
Major reference works and online projects	26
Other documents	27

1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script¹ and the proto-cuneiform script². The proposed characters are listed in section 2. Most of them were listed in [L2/23-190]; however, the present document provides a more detailed rationale for their encoding and additional information about their identity.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

¹ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

²ISO 15924: Pcun, not yet encoded.

³Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: ▷, ▽ (small stylus), ▷, ▽ (large stylus). Some numerals are composed of multiple such impressions, e.g., ▷▷. The terms “curved”, “curviform”, and “round” can be found in the literature. We avoid the term “round” here as it has other meanings in the context of numbers. We use “curviform” in this document as, being the least common term, it is least likely to lead to confusion, and “CURVED” in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [inlineATF].

⁴Impressed into clay using a stylus with a trihedral end: — (stylus held horizontally), † (vertically), ↘ (diagonally) ‹ (with the head of the stylus), ‡ (stylus pressed deeper, forming a larger wedge), ‡ (combining † and ‹), etc.

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
Non-numeric signs	Future Pcun	Existing Xsux	

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties





2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [Powell1972] “in the same fashion as we use black and red ink”; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

3.1 The cuneiform encoding model

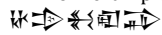
As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts,  later in the third millenium⁵,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists⁶ and dictionaries⁷, and of composite texts⁸. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which

⁵Merging with U+1224E CUNEIFORM SIGN NI2.

⁶Notably [OSL] and the online edition of [MZL] in [eBL, Signs].

⁷Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

⁸For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

has proven useful as a processing step in analyses such as [Rom24; JJ24]⁹. The diachronic approach is also useful for pedagogic applications¹⁰.

3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding¹¹ that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

1(ḫeš ₂)	1(u)	1/2(diš)	5(diš tenû)	gi	us ₂	sa ₂
	7.5 (ropes)		5	reed	side	equal
3(u)	6(diš tenû)	gi	saṇ	sa ₂		
3 (ropes)	6	reed	front	equal		
ašag-bi	1(bur ₃)	1(eše ₃)	1(iku)	1/2(iku)		
ašag=bi						
field=DEM ¹⁴						
				tug _x (LAK483)	si-ga-kam	
				tugsiga	=ak	=am -Ø
				ploughed=GEN=COP-3.SG.S		

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as¹⁵

465 metres, equal lengths
 198 metres, equal widths
 this field is 9, 18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further

⁹Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svā+24, pp. 242, 148].

¹⁰For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

¹¹At that time scoped to the repertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as ≠ , were informed by Early Dynastic distinctions.

¹²As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large — in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

¹³Note that ED IIIb < numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign in [P020054] looks more like Ur III .

¹⁴Alternatively: area=POSS.3.SG.NH, “its area”.

¹⁵We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

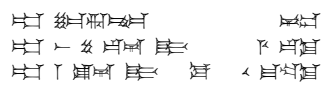
claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from \triangleright to ∇ numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases¹⁶.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere \triangleright / ∇ switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 A primer on classic Ur III and Old Babylonian metrologies



 I want to write tablets: the tablet of
 1 cor of barley to 600 cor; the tablet
 of 1 shekel of silver to 10 minas [...]
 —————
 Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the contrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known¹⁷ a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters ∇ – 𒀭 , the multiples of ten (10–50) are 𒀭 – 𒀭 , but the other digits 11–59 are sequences 𒀭 – 𒀭 ; in effect the base-sixty digits are

¹⁶The cuneiform encoding process was planned in *stages* in [L2/03-162]. One might expect the second stage of encoding, which led to the creation of the Early Dynastic Cuneiform block, to incorporate the numerals needed for the representation and discussion of Early Dynastic texts; however, the proposal [L2/12-208] stated that “numerals have omitted due to the complexity of numeral signs from this period. An expert in the metrology of this period must be consulted before these can be properly included.”



¹⁷See, *e.g.*, [Uni16, §22.3.3, sub “Cuneiform Numerals”].

themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $\mathcal{S}_{\text{UR III/OB}}$, which had different signs for the units $\text{I} - \text{𐎶}$, tens $\text{I} - \text{𐎵}$, sixties $\text{I} - \text{𐎶𐎵}$ (with larger wedges than the units), multiples of six hundred $\text{I} - \text{𐎶𐎵𐎶}$, multiples of three thousand six hundreds $\text{I} - \text{𐎶𐎵𐎶𐎵}$, and multiples of thirty-six thousand $\text{I} - \text{𐎶𐎵𐎶𐎵𐎶}$.

3.3.1 The discrete counting system


The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram¹⁸, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \blacktriangledown \xleftarrow{10} \blacktriangledown \xleftarrow{6} \blacktriangledown \xleftarrow{10} \blacktriangledown \quad (S_{\text{Ur III/OB}})$$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written  in the discrete counting system, and  in the sexagesimal place value system.

3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:

Note that for the range of areas given above¹⁹, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *bûrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign  functioning as punctuation²⁰, but the numerals are tied to the metrology; thus a surface of 5 *bûr* 1 *ebel* 4 *ikû* (100 *ikû*, 36 ha) would be

¹⁸These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

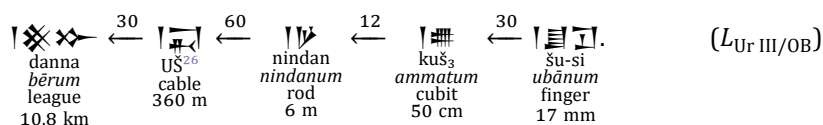
¹⁹For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mûšarum* (36 m²) written 𒍪𒍪, equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60 𒍪 (shekels). For areas greater than 3600 *būrû*, the 𐎶 and 𐎶 numerals are reused with a suffix 𐎶 (gal, Sumerian: big), as follows [Robo8, p. 295 n. b and c; Fri07, p. 378; Rob19]:

²⁰This sign is sometimes interpreted as a measurement unit, and transliterated *iku*, see, *e.g.*, [Proust2020], or transliterations in [Feu04] discussed in section 3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system $G_{\text{Ur III/OB}}$.

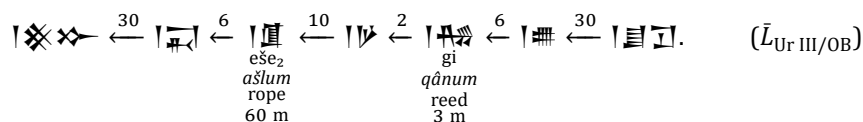
regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with 𐎶- and 𐎵-numerals with their system $S_{\text{Ur III/OB}}$ values²⁵. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Fri07, p. 118; Rob19]:



Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:



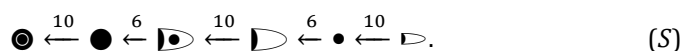
In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

Fractions of the *ikīm*, $\searrow = \frac{1}{2}$ and $\swarrow = \frac{1}{4}$, have already been encountered. In other contexts, the fraction $\frac{1}{2}$ is written ∇ , and the fractions $\frac{1}{3}$ and $\frac{2}{3}$ are written ∇ and ∇ . The latter two signs are derived from curviform signs ∇ and ∇ , which are already separately encoded; these are in turn derived from the sign ∇ (ŠU₂), whose Early dynastic form resembles ∇ , and ∇ numerals; see [Powell1971]. The ∇ is sometimes omitted, as in [P240545; P221530; P221531; P271238; P274845].

3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system $S_{\text{Ur III/OB}}$ [Fri07, p. 374; DE87, pp. 127, 165]:



Likewise the area system used in the Early Dynastic IIIb period for areas of one *ikûm* and greater mirrors system $G_{UR\ III/OB}$ [LAK, p. 72; NDE93, p. 63; Fri07, p. 378;

²⁵Adjacent units are no more than a factor of 60 apart, so higher numerals such as ♠ or ♦ are not used.

²⁶As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.

Lec16]:

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \star \xleftarrow{10} \bullet \xleftarrow{3} \blacktriangleright \xleftarrow{6} \blacktriangleright, \quad (G_{ED\ IIIb})$$

with consistent use of the numerals: \bullet corresponds to \blacktriangleleft , \bullet to \blacktriangle , and \odot to \blacktriangleleft . An exception to this correspondence, noted in [L2/04-099, p. 4] (see section 3.2), is that the vertical \uparrow from $S_{Ur\ III/OB}$ corresponds to a horizontal \blacktriangleright in system S . This is however far from the only case of such a reallocation of function. The earlier form of System G is [DE87, pp. 141, 165; Frio7, p. 378]:

$$\bullet \xleftarrow{6} \odot \xleftarrow{10} \bullet \xleftarrow{3} \blacktriangleright \xleftarrow{6} \blacktriangleright, \quad (G)$$

Observe that, as noted in [DE87, p. 142], \odot changes meaning from $10\bullet$ in system G to $600\bullet$ in system $G_{ED\ IIIb}$. System G is used in the Uruk period, but also in the ED I–II period (it is the “area 2” system in [Cha03], whereas $G_{ED\ IIIb}$ is the “area 1” system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb area system²⁷,

$$\odot \xleftarrow{10} \bullet \xleftarrow{6} \star \xleftarrow{10} \bullet \xleftarrow{3} \blacktriangleright \xleftarrow{6} \blacktriangleright \xleftarrow{2} \blacktriangleright \xleftarrow{2} \blacktriangleright \xleftarrow{2} \blacktriangleright \xleftarrow{2} \blacktriangleright, \quad (G_{ED\ IIIb})$$

with the numerals of a contemporaneous capacity system:

$$\underbrace{\blacktriangleright \xleftarrow{10} \blacktriangleright \xleftarrow{6} \bullet \xleftarrow{10} \blacktriangleright \xleftarrow{4} \blacktriangleright \xleftarrow{6} \blacktriangleright}_{\text{gur san } \eta al_2} \quad (C_{\text{gur san } \eta al_2})$$

both described in [Lec16]. While the size of the $\text{gur san } \eta al_2$ (gur san ηal_2) in bariga is different from that of the Old Babylonian gur , the basic structure of the capacity system is recognizable, with \blacktriangleright corresponding to \uparrow for bariga, \blacktriangleright – \blacktriangleright corresponding to gur – gur for ban₂, and the gur counted with \blacktriangleright rather than gur – gur numerals. However, the half-*ikûm* is counted with the same \blacktriangleright as the bariga, whereas it uses a different sign, \blacktriangleright , in the Old Babylonian system. As we will see, this cannot be handled as a split, by giving \blacktriangleright the glyph \blacktriangleright in an Early Dynastic IIIb font, as the \blacktriangleright numeral series is also in use in that period.

3.4.1 Field lengths in Nirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign \blacktriangleright is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign gur is not written either. Length shorter than one rope are expressed in half-ropes using the $1/2$ sign gur (again with no gur), and then in reeds, with the sign gur . Effectively, this yields the following factor diagram:

$$\begin{array}{c} \uparrow \xleftarrow{6} \blacktriangleleft \xleftarrow{2} \text{gur} \xleftarrow{10} \blacktriangleleft \text{gur} \\ \text{1 eše}_2 = 10 \text{ nindan} \\ \text{1 rope} = 10 \text{ rods} \\ \text{60 m} \end{array} \quad \begin{array}{c} \blacktriangleleft \text{gur} \text{ reed} \\ \text{3 m} \end{array} \quad (L_{ED\ IIIb})$$




²⁷A variant is $\odot \xleftarrow{10} \bullet \xleftarrow{6} \star \xleftarrow{10} \bullet \xleftarrow{3} \blacktriangleright \xleftarrow{6} \blacktriangleright \xleftarrow{2} \blacktriangleright \xleftarrow{2} \blacktriangleright \xleftarrow{2} \blacktriangleright$, see [Powell1972].

²⁸The cuneiform counterpart is \blacktriangleright .

1. [P240532, *verso 4, 9*] 
2. [P240548, *verso 1, 1*] 
3. [P240655, *recto 7, 9*] 
4. [P240579, *verso 4, 3*] 
5. [P240675, *verso 2, 2*] 
6. [P240609, *verso 3, 1*] 
7. [P240533, *recto 3, 3*] 
8. [P240697, *recto 1, 5*] 
9. [P240653, *recto 6, 2*] 
10. [P240654, *recto 2, 6*] 
11. [P240531, *recto 1, 8*] 
12. [P241708, *recto 1, 1*]⁵¹ 
13. [P241904, *recto 1, 1*]⁵² 

(C_{Ebla})

Because of their prevalence in the Uruk and Early Dynastic periods, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 3–16.

⁴⁴We cite here only one attestation per tablet; most tablets contain several expressions mixing cuneiform   and larger with cuneiform  and smaller. In all cases the transcriptions given here are based on the EbDA transliterations, but the shape and orientation of the numerals was checked⁴⁵ on a photograph (from EbDA unless noted otherwise).

⁴⁵As we will see in Section 3.7.2, CDLI transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

⁴⁶ba-ri₂-zu₂, a variant spelling.

⁴⁷Short for $\square \vdash$.

⁴⁸Note the omitted $\square \Diamond \perp$.

⁴⁹Instead of the expected 𐎶𐎠𐏀𐎧.

⁵⁰ not legible on the EbDA photo.

⁵¹From CDLI photo.

⁵²From photo in [Arc89, p. 6].

⁵³Laid out as $\begin{bmatrix} | & | & | \\ | & | & | \end{bmatrix}$; on stacking patterns see Section 6.2.

These metrological equations for the "unknowns" \bar{U} , σ , \bar{V} , etc., can be treated exactly as ordinary equations for unknowns x , y , z ,... . In particular, the equations can be simplified by subtraction of equal amounts from both sides of the identities. In this way the three equations above can be reduced to:

We can now read off from the first equation that $1^\circ = 10^\circ$, and from the second that $1^\circ = 6^\circ$. Then the third equation can be simplified (by "substitution" of these values into the equation), to the following reduced form:

The most likely solution to this last equation is, of course,

Figure 1: TODO [Fri78, p. 15]

 (= 1/3 gin), and - še (= 1/3 × 1/60 gin).

Figure 2: TODO [Fri78, p. 49]

⁵⁴The untransliterated text would be 𐎠𐎢𐏀𐎧𐎶𐎡𐏁𐎫𐎵𐎥𐏃𐎤𐎩𐎦𐎰𐎺𐎠𐎮𐏁𐎣𐎲𐎪𐎭𐎴𐎬𐎷𐎨𐎻𐎽𐎿𐎱𐎼𐎾𐎿; note the atomically encoded $\text{ib}_7 \times 3! = \overline{\text{M}} \times \text{III} = \overline{\text{MM}}$.

⁵⁵TODO cite the EbDA one.

there is in any case an important qualitative difference between IX for Latin novem and 𒌦 for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 3: TODO [Englund1988]

of decreasing fractions $1/n$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus $\Xi = 1/2 N_{39}$, $\tilde{\Xi} = 1/3 N_{39}$, and so on. The first sign of the latter units, N_{34} ,

Figure 4: TODO [Eng98, p. 113]

For instance, the first line contains the notations $1N_{34} 1N_{390}; 2N_{20}$, which can be translated "60 of the (grain rations containing) Ξ (of grain); (grain involved): 2 \bullet (of ground barley)". This calculation contradicts the assumed numerical relationship $10N_1 = 1N_{14}$, since as was well known the measure represented by the sign N_{39} was $1/5$ of that represented by N_1 , so that $60 \times 1/5 = 12$ and not 20, as $2N_{14}$ would imply. Instead of relying on complicated

Figure 5: TODO [Eng98, p. 116]

Die halbkreisförmigen Griffelindrücke gehen manchmal in mehr oder weniger eckige Formen über (∇)⁶⁸⁵. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen (∇), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren⁶⁸⁶. Selten treten mit ∇ gebildete Zahlen auf⁶⁸⁷ (sie entsprechen den biga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 6: TODO [Kreg98, p. 303]

The calculations:

Obv. i	1	$60 \times 1/5 \nabla$	(Ξ)	=	$12 \times \nabla$	=	$2 \times \bullet$		
	2	$120 \times 1/10 \nabla$	(Ξ)	=	$12 \times \nabla$	=	$2 \times \bullet$		
	3	$120 \times 1/15 \nabla$	($\tilde{\Xi}$)	=	$8 \times \nabla$	=	$1 \times \bullet$	$2 \times \nabla$	
	4	$300 \times 1/20 \nabla$	($\tilde{\Xi}$)	=	$15 \times \nabla$	=	$2 \times \bullet$	$3 \times \nabla$	
	5	$600 \times 1/25 \nabla$	($\tilde{\Xi}$)	=	$24 \times \nabla$	=	$4 \times \bullet$		
Rev. i	1	1200					$1 \times \bullet$	$1 \times \bullet$	$5 \times \nabla$
Obv. i	6	$6000 \times 1/30 \nabla$	(GAR+6N ₃₇)	=	$200 \times \nabla$	=	$1 \times \nabla$	$3 \times \bullet$	$2 \times \nabla$
ii	1	$120 \times \approx 1/4 \nabla$	(DUG _a +U _{2a})	=	$30 \times \nabla$	=	$5 \times \bullet$	$1 \times \nabla$	$1 \times \nabla$
	2	$180 \times 1/5 \nabla$	(DUG+AS _a)	=	$36 \times \nabla$	=	$6 \times \bullet$		
	3	$300 \times 1/15 \nabla$	(KAŠ _a)	=	$20 \times \nabla$	=	$3 \times \bullet$	$2 \times \nabla$	
Rev. i	3	600					$1 \times \bullet$	$4 \times \bullet$	$3 \times \nabla$
							$1 \times \bullet$	$1 \times \bullet$	$5 \times \nabla$
							$1 \times \nabla$	$3 \times \bullet$	$2 \times \nabla$
							$1 \times \bullet$	$4 \times \bullet$	$3 \times \nabla$
							$1 \times \nabla$		$1 \times \nabla$
Grand total of groats used:							$1 \times \nabla$	$2 \times \bullet$	$9 \times \bullet$
Grand total of malt used:							$8 \times \bullet$	$4 \times \nabla$	$1 \times \nabla$

Figure 6. Transliteration and calculations of MSVO 4, 66.

Figure 7: TODO [Englund2001]

strong similarities between "area" 1 and "area" 3 systems, the sign with two concentric discs (\bullet , notated N_{50} ²⁷) remains problematic. It never appears in any numerical combination with the sign with a single disc (\bullet),

Figure 8: TODO [Chao3, p. 6]

$1/15$, etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation $\text{gur} \text{ } \text{gur}$ could be understood as $1 \frac{1}{5}$ gur.²⁷ However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs gur and gur cannot be considered klasmatograms.

Metrological tablets from the end of the 4th millennium (Nissen, Damerow and Englund 1993, 55-59, to *MSVO* 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

iku represents a surface of 3600m²
 eše_3 represents a surface of 21,600m²
 etc.

The signs iku and eše_3 constitute by themselves measures of surface areas. These measures are usually followed by the sign GAN_2 , which means either surface or field and

Figure 9: Discussion of Old Babylonian and fourth millenium area measures in [Proust2020]. The cuneiform text is Unicode-encoded.

formed by only two signs gur and gur , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60², 60³, 1/60, 1/60², etc. are written in the same way, with the vertical wedge gur . The third feature concerns the exact function of

Figure 10: TODO [Cha12, p. 58]

3.5 Non-numeric usage

The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'⁵⁷. Do you know its reading⁵⁸?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign gur has, *inter alia*, the values aš , rum , and dili . While the horizontal numerals are most frequently written gur in the Early Dynastic period⁵⁹, such non-numeric usage is almost⁶⁰ always written gur , for instance:

⁵⁶TODO(egg): On the order cite TSS 188, Friberg2007 p. 148 and any of the usual suspects on the haphazard order of signs in early texts; contrast P274845, P241764.

⁵⁷The reader will recall that gur is written gur , with a larger wedge than gur ; however, these signs have merged by the time Examenstext A is composed.

⁵⁸Besides gur , a look at [OSL] shows that the values diš , ge_3 , makkaš , $\text{san} \text{tak}_4$, and tal_4 are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

⁵⁹A CDLI search for "(asz@c)" finds 3296 ED texts, while a search for "(asz)" finds 81 ED texts, of which 46 also contain "(asz@c)".

⁶⁰Exceptions are discussed in section 3.7.1.

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with: $\text{gur} \text{ } \text{gur} \text{ } \text{gur}$ but never with the repetition $\text{gur} \text{ } \text{gur} \text{ } \text{gur}$.

Figure 11: TODO [Cha12, p. 59]⁵⁶

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

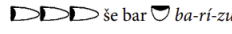
 *še bar ba-rí-zu*
 ‘3 *gubar* (capacity units) and 1 *parisu*’.

Figure 12: TODO [Cha12, p. 61]











This is particularly true of the signs , , , and , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written  in Mesopotamia—also transcribed by Assyriologists as 1 *bán*, 2 *bán*, 3 *bán*, and 4 *bán* with reference to the *bán* measure worth 1/6 of the barig. At Ebla, the sign  is most often associated with the *parisu* measure, while the signs , ,  and  refer to 1, 2, 3,

Figure 13: TODO [Cha12, p. 64]


shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g.,  transliterated as 2(*šar*₂) 1(*geš*^u) 3(*u*), means 2 × 3600 + 1 × 600 + 3 × 10). The system is based on an alternation of factors ten and

Figure 14: TODO





might think of one fabric and a half,¹¹ but the presence of notations with “2 ”, “3 ”, and “6 ” (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag’s weight measurement system.¹² The notation “1  *gada*” in o. ii 1 and r. vi 1, along with the total of “39



Fig. 1. Combinations of numerals attested in Š. 742.

Figure 15: Discussion of the contrast between  and  numerals in [Gor23, p. 162].



as, for example, in TM.75.G.3125 = ARET III 107 o. iv 1, “4  *a₃-da-um^{u9}* 2  4 
 aktum 4  *ib₂^{u9} × 3*  *sa₆ gunu₃*” (Fig. 2).

Figure 16: Transliteration in [Gor23, p. 163] of [P242293, recto 4, 1] incorporating untransliterated numerals.

- in personal names in administrative texts, such as the following, which all contain \triangleright numerals:
 - 𒀭𒀭 in [P010424; P010458; P010459] from ED IIIa أبو صلابيخ ,
 - 𒀭𒀭 in [P010960] from ED IIIa Šuruppag,
 - 𒀭𒀭𒀭𒀭 in [P251641] from ED IIIb Adab,
 - 𒀭𒀭𒀭𒀭𒀭𒀭 in [P252866] from ED IIIb Adab,
 - 𒀭𒀭𒀭𒀭𒀭𒀭 in [P298637] from ED IIIb Umma;
- in the Sumerian word 𒀭𒀭 $\text{u}_2\text{-rum}$, “property” in ED IIIb Nirsu administrative texts which contain \triangleright numerals, such as [P020006; P020008; P020018; P020024; P020030];
- in lexical texts:
 - in the divine name 𒀭𒀭𒀭𒀭𒀭𒀭 in the lexical texts [P010570; P010572], where the entries are prefixed with \triangleright .
 - in the word 𒀭𒀭 dili, “small fish” in [P010578], witness to Early Dynastic Fish,
 - in the same word with a determinative, 𒀭𒀭𒀭 dili^{ku₆}, in [P010586], witness to Early Dynastic Food, which starts with \triangleright numerals.

This is a clear contrast between 𒀭𒀭 and \triangleright in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name 𒀭𒀭𒀭 occurs on its own line in the aforementioned administrative texts; a line $\triangleright \text{𒀭𒀭𒀭}$ would instead be read as “one slave”.

3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as 𒀭𒀭 that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [KWU, pp. 123 sqq.] lists all of 𒀭𒀭 together with $\triangleright\text{𒀭𒀭𒀭𒀭}$, while 𒀭𒀭 , 𒀭𒀭 , and 𒀭𒀭 , and only those, appear at the beginning of the sign list, since they have non-numeric values⁶¹. [PTACE, p. 58] has the numeric signs \triangleright , 𒀭𒀭 , 𒀭𒀭 , whereas non-numeric 𒀭𒀭 is at the beginning of the sign list, where its values *aš* and *rum* are listed. For signs with both non-numeric and numeric usage, [LAK] writes *s. die Zahlz.* throughout the main list; LAK 1 𒀭𒀭 thus reappears at LAK 829 together with \triangleright , 𒀭𒀭 , and 𒀭𒀭 . One should note [MZL], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have 𒀭𒀭 – 𒀭𒀭 used in system $G_{\text{Ur III/OB}}$.

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes⁶² to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from \triangleright to 𒀭𒀭 numerals

⁶¹Non-numeric values of 𒀭𒀭 were discussed in section 3.5; 𒀭𒀭 has the values *man₃* and *min₅*, and is used for the word *didli*, “several, various”; 𒀭𒀭 has the value *eš₆*.

⁶²TODO comment on the ED witnesses to the instructions of Šuruppag

prevents diachronic encoding even if \triangleright were unified with \blacktriangleright . For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'–6']; however, they are written with \uparrow numerals, whereas they are written with \triangleright numerals in the Early Dynastic witnesses; since \uparrow and \blacktriangleright are distinct⁶³ characters, the \triangleright - \blacktriangleright unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods⁶⁴, there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Romach2023], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts⁶⁵ being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

3.7.1 The case of ŠAR₂

The character U+122B9 \bullet CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period⁶⁶, a contrast between non-numeric šar₂ written \diamond and numeric 1(šar₂) written \bullet can be observed, similar to the contrast between \blacktriangleright and \triangleright previously discussed in section 3.5. However, in lexical lists from Šuruppak and Ebla⁶⁷, as well as in the *Stèle des vautours*, non-numeric šar₂ is curviform:

- $\ast \text{𒌦} \text{𒌦} \text{𒌦} \bullet$ and $\ast \text{𒌦} \bullet \text{𒌦} \text{𒌦}$ in [P010566];
- $\bullet \text{𒌦} \text{𒌦}$ and $\ast \bullet \text{𒌦} \text{𒌦}$ in [P010576];
- $\bullet \text{𒌦}$ in [P240986]⁶⁸;

⁶³Besides the contrasts in numeric usage mentioned in section 3.3.3, these characters are clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of \blacktriangleright which are not shared with \uparrow have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period, \blacktriangleright is used for the preposition *ina*, “in”, and \uparrow for the preposition *ana*, “to”.

⁶⁴TODO cite a few things here.

⁶⁵Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

⁶⁶For example, in personal names:

- $\text{𒌦} \text{𒌦} \text{𒌦} \text{𒌦}$ in [P020019] from ED IIIb Nirsu;
- $\text{𒌦} \text{𒌦} \text{𒌦} \text{𒌦}$ in [P020182], also from ED IIIb Nirsu;
- $\text{𒌦} \text{𒌦} \text{𒌦}$ in [P222186] from ED IIIb Umma;
- $\text{𒌦} \text{𒌦} \text{𒌦}$ in [P235312] from Old Akkadian Umma.

⁶⁷TODO Mention other ways in which these are archaizing

⁶⁸From copy in [ELLes, No. 397].

— ● 𐎶𐎵 in [P222399, obv. 17, 9, 18, 11, 22, 12]⁶⁹.

It *would* be disruptive to the diachronic representation of text if non-numeric šar₂ were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph 𐎶, identical to the glyph of the proposed U+12579 ● CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric šar₂ is curviform solidly predates the transition from ● to 𐎶 in the relevant metrological systems, there is no need to represent a 𐎶-● contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D 𐎶 CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

Note that in rare cases, such as [P222243] from ED IIIa Adab, non-numeric — (here with the value rum) is written 𐎶. It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 𐎶 cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. In practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the CDLI transliteration (UR2-1(aš@c)) and the ePSD2 one (uru₈^{rum}) of this word disagree on that aspect. Since — has a cuneiform reference glyph, this does not pose any compatibility concerns.

3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL₂ can be described as 𐎶𐎶𐎶𐎶 NE×KASKAL or 𐎶𐎶𐎶𐎶 NE×PAP⁷⁰, they are typically all transliterated bil₂, and therefore are all represented by the character U+1224B 𐎶 CUNEIFORM SIGN NE SHESHIG, its name notwithstanding, as described in [UTR56, §2.5].

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, 𐎶𐎶𐎶𐎶 from [P305639] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in EbDA transliterations: the aforementioned 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 from [P240533, recto 3, 3] is transliterated “20-1-1/2 gu₂-bar 7 nig₂-sagšu 2-1/2 an-zam_x⁷¹ za”, reading both 𐎶 and 𐎶 as 1/2, but not distinguishing them.

In particular, these transliterations do not differentiate between — and | numerals, nor between 𐎶 and 𐎶 numerals. For instance, the aforementioned 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶 𐎶𐎶𐎶𐎶

⁶⁹Note however 𐎶𐎶 𐎶𐎶 on [P222399, obv. 6, 17]. Curviform non-numeric šar₂ is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here. TODO add a photo.

⁷⁰As on [P249253].

⁷¹As of this writing, EbDA actually has an-zam_x, with U+1D6A GREEK SUBSCRIPT SMALL LETTER CHI.

𐎶𐎵𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 𐎶𐎵 from [P242293, recto 4, 1] is transliterated “4 a_3 -*dam*^{tug₂}-II 4 aktum^{tug₂} 4 ib₂-III gun₃ sa₆^{tug₂}” in EbDA, with no distinction between the 𐎶𐎵 and 𐎶𐎵. Since 𐎶 and 𐎵 numerals are separately encoded, the numeric expressions in such transliterations cannot be transformed into Unicode cuneiform without additional context, regardless of curviform–cuneiform unification.

In metrological systems such as systems $G_{Ur III/Ob}$ and $C_{Ur III/Ob}$ where some units are indicated by the type of numeral rather than an overt unit sign, it is common practice to add the unit in parentheses in transliteration; for instance, 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵 from [P386847] is transliterated “1(eše₃) 5½ iku⁷² 7 sar” in [Feu04, vol. 2, p. 176], and 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵 from [P307255] is transliterated “1(n⁷³) 2(b) 7 ½ sila₃” in [Feu04, vol. 2, p. 151].

This practice has been generalized to systematically indicate numeral shape; this is in particular the case in CDLI, where the transliterations of some of the above examples are “1(gez2) 3(u) 5(asz) gur” for 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵, “1(esze3) 5(iku) 1/2(iku) GAN2 7(disz) sar” for 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵, and “3(barig) 2(ban2) 7(disz) 1/2(disz) sila3” for 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵. CDLI and ePSD2 both distinguish curviform from cuneiform numerals in transliteration: the length 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵 from [P020129, rev. 2, 1] is transliterated “6(gez2@c) 3(u@c) {ninda}nindax(DU) 1/2(asz@c) 4(disz@t) gi” in CDLI, and “6(geš₂^c) 3(u^c) ninda_{ninda_x}(DU) 1/2(aš^c) 4(diš^t) gi” in ePSD2. Another example is [Molina2014], which uses 1a for 𐎶, 1d for 𐎵, 1ac for 𐎶𐎵, 1dc or ½dc for 𐎶𐎵 depending on reading, etc. The literature on the Uruk and Early Dynastic I–II periods uses a different set of transliteration conventions that also disambiguate numeral shapes, as will be discussed in section 4.

While there exist transliterations that distinguish 𐎶 from 𐎵 but not 𐎶𐎵 from 𐎶𐎵, such as the ones used in [DCCMT], the trend, especially in more recent works in third millennium studies, seems to be to represent numeral shape; for example, [Maiocchi2024] gave an example of the input syntax used by the new “Urban Economy Begins” project as “10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)” for 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵, with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are 𐎶𐎵 rather than 𐎶𐎵 numerals.


3.8 Conclusions


Co-occurrences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idiosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications. The contrast frequently applies to individual numerals, rather than to the span of entire numeric expressions.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted: 𐎶𐎵 is a length of three ropes, but 𐎶𐎵 is an area of three bur₃; 𐎶𐎵𐎶𐎵 could be read as one 𐎶𐎵 𐎶𐎵 and one 𐎶𐎵𐎶𐎵, where 𐎶𐎵𐎶𐎵 would be one and a half 𐎶𐎵 𐎶𐎵; 𐎶𐎵𐎶𐎵 is a personal name,

⁷²TODO say something about this reading

⁷³TODO comment on nigida.






but  would be “one slave”.




At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millenium corpora. The oddity of  requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.

4 Rationale for ED–Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millenium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millenium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs KAŠa, KAŠb, and KAŠc, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Englund2001]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered script.



As part of the development of these conventions, a classification of fourth millenium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter *N* with a numeric subscript (sometimes with an additional alphabetic subscript): N_1 is , N_{14} is , N_{34} is , etc. Transliterations of numeric expression then use those to identify the type of number used, thus $5N_1$ is , and $5N_{14}$ is .

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the CDLI transliteration of  or  in [P283802] currently uses (barig@c) for the vertical numerals, since  numerals are typically capacity measures; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millenium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I–II period, even though the Sumerian text uses classical assyriological transliteration conventions.

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millenium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in section 3.7.2; they are indeed used interchangeably, as in [P011104] which uses the notation u@f in [ePSD2], but N_{14} @f in CDLI. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millenium would therefore induce confusion as to which numerals should be used in third millenium

studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millenium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance,  N_{30c} is listed as a variant of  N_{30a} in [DE87, p. 166], where the numeric value of either in relation to $\ominus N_{29a}$ is still unknown, but their values are found in [Englund2004] to be $\frac{1}{10} \ominus$, whereas $\frac{1}{6} \ominus$.

5 Considerations on individual numeral series













































[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban₂@c) and 5(ban₂@c).]

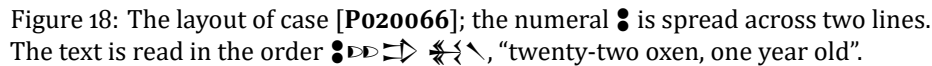
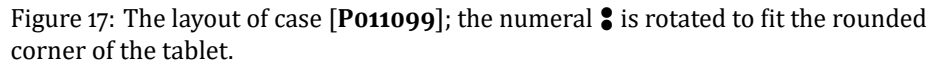
6 Characters not included in this proposal

6.1 Missing numerals

TODO N13 not attested in CDLI TODO (N_{17} not usefully numeric, $12N_{14}$ not encodable, etc.). Cite [DE87, p. 147] 7 and 8(diš *tenû*) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.

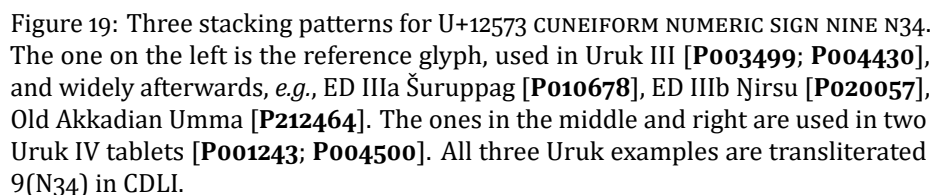
6.2 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some *stacking patterns*; for instance 9| is encoded both as U+12446  and as U+1240E . This is in part due to contrastive usage of stacking patterns. For instance, besides  and  which are characteristic of bariga measures, four bariga is written  even where 4| is written , as in [P255010; P292843]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often                                      



Likewise, many stacking patterns are attested for the cuneiform numerals proposed in this document, and it is not proposed to separately encode them; these distinctions would be incompatible with the state of the art in numeric transliterations, and are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier tablets, as they are structured in rectangular cases rather than lines, so that numerals may be laid out across the case in whichever way fits the available space; this is illustrated in Figure 17. Note also that the numerals need to be considerably enlarged in order to reproduce the layout of the tablets, so that 𐎶 often spans two lines of cuneiform signs, as shown in Figure 18. This is impractical when these numerals are set in text that contrasts them with the larger 𐎵, and inconsistent with actual practice when typesetting these numerals, as illustrated in Figure 3: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dynastic period, but that are also attested in the Uruk period; the Uruk period also frequently features numerals that use a more vertical layout, as illustrated in Figure 19. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millenium, usage in third millenium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed,



the absolute size of the indents \triangleright , \triangleright , \bullet , and \bullet must remain consistent across the numeral series, lest a \triangleright numeral be confused with an \triangleright numeral. Since the single indents are frequently used in running text, as illustrated in section 3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 19 uses OpenType stylistic alternates, and Figure 17 rotates the character \bullet , in both cases preserving the plain text backing.

6.3 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Englund2001].

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

References

Artefacts

- [P020054] VAT 4731. [För16, 40 p.14]. Vorderasiatisches Museum.
CDLI: [P020054](#).
- [P020129] VAT 04713. Vorderasiatisches Museum.
CDLI: [P020129](#).
ORACC: [epsd2/corpus/P020129](#).
- [P102305] X.3.139. Michael C. Carlos Museum, Emory University.
CDLI: [P102305](#).
- [P215653] AS 15375 21. Musée du Louvre.
CDLI: [P215653](#).
ORACC: [dcclt/corpus/P215653](#).
Louvre Collections: [ark:/53355/cl010436723](#).
- [P221266] AO 13825. Musée du Louvre.
CDLI: [P221266](#).
ORACC: [epsd2/corpus/P221266](#).
Louvre Collections: [ark:/53355/cl010138527](#).
- [P221291] AO 13850. Musée du Louvre.
CDLI: [P221291](#).
ORACC: [epsd2/corpus/P221291](#).
- [P221305] AO 13864. Musée du Louvre.
CDLI: [P221305](#).
ORACC: [epsd2/corpus/P221305](#).
- [P222399] *Stèle des vautours*. AO 50; AO 2346; AO 2347; AO 2348; AO 16109. Musée du Louvre.
CDLI: [P222399](#).

- [P232278] *Gudea E.* AO 6. Musée du Louvre.
CDLI: [P232278](#).
ORACC: [etc/sri/Q001544](#).
- [P232280] *Gudea G.* AO 7. Musée du Louvre.
CDLI: [P232280](#).
ORACC: [etc/sri/Q001546](#).
- [P240531] TM.75.G.00265. Idlib, Syria: National Museum of Syria.
CDLI: [P240531](#).
EbDA: [1415](#).
- [P240532] TM.75.G.00266. Idlib, Syria: National Museum of Syria.
CDLI: [P240532](#).
EbDA: [1324](#).
- [P240533] TM.75.G.00267. Idlib, Syria: National Museum of Syria.
CDLI: [P240533](#).
EbDA: [1379](#).
- [P240548] TM.75.G.00302. Idlib, Syria: National Museum of Syria.
CDLI: [P240548](#).
EbDA: [1350](#).
- [P240579] TM.75.G.00341. Idlib, Syria: National Museum of Syria.
CDLI: [P240579](#).
EbDA: [1364](#).
- [P240609] TM.75.G.00440. Idlib, Syria: National Museum of Syria.
CDLI: [P240609](#).
EbDA: [1378](#).
- [P240653] TM.75.G.00535. Idlib, Syria: National Museum of Syria.
CDLI: [P240653](#).
EbDA: [1382](#).
- [P240654] TM.75.G.00536. Idlib, Syria: National Museum of Syria.
CDLI: [P240654](#).
EbDA: [1383](#).
- [P240655] TM.75.G.00537. Idlib, Syria: National Museum of Syria.
CDLI: [P240655](#).
EbDA: [1358](#).
- [P240675] TM.75.G.00557. Idlib, Syria: National Museum of Syria.
CDLI: [P240675](#).
EbDA: [1371](#).
- [P240697] TM.75.G.00579. Idlib, Syria: National Museum of Syria.
CDLI: [P240697](#).
EbDA: [1381](#).
- [P241708] TM.75.G.02143. Idlib, Syria: National Museum of Syria.
CDLI: [P241708](#).
EbDA: [3173](#).
- [P241904] TM.75.G.02346. [[Arc89](#), p. 6]. Idlib, Syria: National Museum of Syria.
CDLI: [P241904](#).
EbDA: [3183](#).

- [P242293] TM.75.G.03125. Idlib, Syria: National Museum of Syria.
CDLI: [P242293](#).
EbDA: [217](#).
- [P249253] *Code de Hammurabi*. Sb 8. Musée du Louvre.
CDLI: [P249253](#).

Unicode documents

- [L2/03-162] M. Everson and K. Feuerherm. *Basic principles for the encoding of Sumero-Akkadian Cuneiform*. 25th May 2003.
UTC: [L2/03-162](#).
- [L2/03-393R] M. Everson, K. Feuerherm and S. Tinney. *Preliminary proposal to encode the Cuneiform script in the SMP of the UCS*. 3rd Nov. 2003.
UTC: [L2/03-393R](#).
- [L2/04-036] M. Everson, K. Feuerherm and S. Tinney. *Revised proposal to encode the Cuneiform script in the SMP of the UCS*. 29th Jan. 2004.
UTC: [L2/04-036](#).
- [L2/04-099] L. Anderson. *Unification of Cuneiform Numbers*. 2004.
UTC: [L2/04-099](#).
- [L2/04-189] M. Everson, K. Feuerherm and S. Tinney. *Final proposal to encode the Cuneiform script in the SMP of the UCS*. 8th June 2004.
UTC: [L2/04-189](#).
- [L2/24-159] P. Constable, ed. *Minutes of UTC Meeting 180* (23rd–25th July 2024). 29th July 2024.
UTC: [L2/24-159](#).
- [Uni16] The Unicode Consortium. *The Unicode Standard*. Version 16.0.0. The Unicode Consortium, 10th Sept. 2024.
ISBN: 978-1-936213-34-4.
eprint: <https://www.unicode.org/versions/Unicode16.0.0/core-spec/>.
- [UTR56] R. Leroy, ed. *Unicode Cuneiform Sign Lists*. Unicode Technical Report #56.
eprint: <https://www.unicode.org/reports/tr56/>.

Major reference works and online projects

- [EbDA] L. Milano, M. Maiocchi, F. Di Filippo, R. Orsini, E. Scarpa, M. Surdi et al., eds. *Ebla Digital Archives*. 2007–.
eprint: <http://ebda.cnr.it/>.
- [eBL] E. Jiménez, Z. Földi, A. Härtinen, A. Heinrich, T. Mitto, G. Rozzi, I. Khait, J. Laasonen, F. Simonjetz et al., eds. *electronic Babylonian Library*. 2023–.
eprint: <https://www.ebl.lmu.de/>.
- [ELLes] P. Mander. “Lista dei segni dei testi lessicali di Ebla”. In: *Testi lessicali monolingui della biblioteca L. 2769*. Ed. by G. Pettinato. Materiali epigrafici di Ebla 3. Napoli: Istituto universitario orientale, 1981, pp. 285–382.

- [ePSD2] S. Tinney, P. Jones and N. Veldhuis, eds. *The electronic Pennsylvania Sumerian Dictionary*. 2nd ed. 2017–. eprint: <http://oracc.org/epsd2>.
- [KWU] N. Schneider. *Die Keilschriftzeichen der Wirtschaftsurkunden von Ur III*. Editrice Pontificio Istituto Biblico, 1935.
- [LAK] A. Deimel. *Liste der archaischen Keilschriftzeichen von Fara*. Wissenschaftliche Veröffentlichungen der Deutschen Orient-Gesellschaft 40. J. C. Hinrichs'sche Buchhandlung, 1922.
- [MZL] R. Borger. *Mesopotamisches Zeichenlexikon*. Alter Orient und Altes Testament 305. Ugarit-Verlag, 2010.
- [OSL] N. Veldhuis, S. Tinney et al., eds. *Oracc Sign List*. 2014–. eprint: <http://oracc.org/osl/>.
- [PTACE] A. Catagnoli. *La paleografia dei testi dell'amministrazione e della cancelleria di Ebla*. Quaderni di Semitistica 9. Università di Firenze, 2013. ISBN: 8890134054.

Other documents

- [Arc15] A. Archi. *Ebla and Its Archives. Texts, History, and Society*. Studies in ancient Near Eastern records 7. Walter de Gruyter, 2015. ISBN: 978-1-61451-716-0. DOI: [10.1515/9781614517887](https://doi.org/10.1515/9781614517887).
- [Arc89] A. Archi. "Tables de comptes eblaïtes". In: *Revue d'assyriologie et d'archéologie orientale* 83.1 (1989). Ed. by P. Amiet and P. Garelli, pp. 1–6. ISSN: 0373-6032.
- [Cap02] R. Caplice. *Introduction to Akkadian*. 4th ed. Editrice Pontificio Istituto Biblico, 2002. ISBN: 88-7653-566-7.
- [Cha03] G. Chambon. "Archaic Metrological Systems from Ur". In: *Cuneiform Digital Library Journal* 2003.5 (23rd Dec. 2003). ISSN: 1540-8779. eprint: http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003_005.html.
- [Cha12] G. Chambon. "Numeracy and Metrology". In: *The Oxford Handbook of Cuneiform Culture*. Ed. by K. Radner and E. Robson. Oxford University Press, 18th Sept. 2012, pp. 51–67. ISBN: 9780199557301. DOI: [10.1093/oxfordhb/9780199557301.013.0003](https://doi.org/10.1093/oxfordhb/9780199557301.013.0003).
- [DE87] P. Damerow and R. K. Englund. "Die Zahlzeichensysteme der archaischen Texte aus Uruk". In: M. W. Green and H. J. Nissen. *Zeichenliste der archaischen Texte aus Uruk*. Archaische Texte aus Uruk 2. An offprint of this chapter is available at <https://cdli.mpiwg-berlin.mpg.de/files-up/publications/englund1987a.pdf>. Gebr. Mann Verlag, 1987. Chap. 3, pp. 117–165.
- [Eng98] R. K. Englund. "Texts from the Late Uruk Period". In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 13–233. ISBN: 3-7278-1166-8.

- [Feu04] K. G. Feuerherm. “Abum-waqar and His Circle. A Prosopographical Study”. PhD thesis. University of Toronto, 2004.
- [För16] W. Förtsch. *Altbabylonische Wirtschaftstexte aus der Zeit Lugalanda’s und Urukagina’s*. Vorderasiatische Schriftdenkmäler der Königlichen Museen zu Berlin 14. J. C. Hinrichs, 1916.
- [Fri07] J. Friberg. *A Remarkable Collection of Babylonian Mathematical Texts. Manuscripts in the Schøyen Collection: Cuneiform Texts I*. Sources and Studies in the History of Mathematics and Physical Sciences. Springer, 2007.
ISBN: 978-0-387-34543-7.
- [Fri78] J. Friberg. *A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and Proto-Elamite Semi-Pictographic Inscriptions*. The Third Millenium Roots of Babylonian Mathematics 1. Department of Mathematics, Chalmers University of Technology, 1978.
- [Gor23] F. Gori. “On Lapis Lazuli and Linen in Šuruppag Texts. An Analysis Through the Lens of Ebla Studies”. In: *Studia Eblaitica* 9 (2023), pp. 160–166. ISSN: 2364-7124.
- [Hue11] J. Huehnergard. *A Grammar of Akkadian*. 3rd ed. Brill, 2011.
ISBN: 978-1-57506-941-8.
- [JJ24] T. Jauhiainen and H. Jauhiainen. “Advancing Cuneiform Text Dating Through Automatic Analysis”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 11th July 2024 14:00.
- [Kre98] M. Krebernik. “Die Texte aus Fāra und Tell Abū Šalābīḥ”. In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 235–427.
ISBN: 3-7278-1166-8.
- [Lec12] C. Lecompte. “Des chiffres et des digues: à propos de deux textes présargoniques de Ġirsu et d’une notation numérique inhabituelle”. In: *Altorientalische Forschungen* 39.1 (Dec. 2012), pp. 81–86.
DOI: [10.1524/aof.2012.0006](https://doi.org/10.1524/aof.2012.0006).
- [Lec16] C. Lecompte. “ED IIIb metrology: texts from Lagaš”. In: *CDLI:wiki. A Library of Knowledge of the Cuneiform Digital Library Initiative*. 12th Apr. 2016.
eprint: https://cdli.ox.ac.uk/wiki/doku.php?id=ed_iii_metrological_systems.
- [Lec20] C. Lecompte. “The Measurement of Fields During the Pre-sargonic Period”. In: *Mathematics, Administrative and Economic Activities in Ancient Worlds*. Ed. by C. Michel and K. Chemla. Why the Sciences of the Ancient World Matter 5. Springer, 2020.
- [NDE93] H. J. Nissen, P. Damerow and R. K. Englund. *Archaic Bookkeeping. Early Writing and Techniques of Economic Administration in the Ancient Near East*. Trans. by P. Larsen. The University of Chicago Press, 1993.
ISBN: 0-226-58659-6.
- [Oel22] J. Oelsner. *Der Kodex Ḥammu-rāpi*. dubsar 4. Zaphon, 2022.

- [Pow87] M. Powell. “Maße und Gewichte”. In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš–Medizin. 1987–1990, pp. 457–530.
- [Rob08] E. Robson. *Mathematics in Ancient Iraq. A Social History*. Princeton University Press, 2008.
ISBN: 978-0-691-09182-2.
- [Rob19] E. Robson. “Oracc metrology guidelines”. In: *Oracc: The Open Richly Annotated Cuneiform Corpus*. 18th Dec. 2019.
ORACC: [doc/help/editinginf/metrology/metrologicaltables](https://oracc.berkeley.edu/doc/help/editinginf/metrology/metrologicaltables).
- [Rob22] E. Robson. “Overview of Metrological Systems”. In: *The Digital Corpus of Cuneiform Mathematical Texts*. 2022.
ORACC: [dccmt/Metrology](https://oracc.berkeley.edu/dccmt/Metrology).
- [Rom24] A. Romach. “The Neo Assyrian Land Sale Documents from Dur-Katlimmu: A Stylometric Analysis of Their Scribal Features”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 10th July 2024 12:00.
- [Sch10] W. Schramm. *Akkadische Logogramme*. Göttinger Beiträge zum Alten Orient 5. Universitätsverlag Göttingen, 2010.
ISBN: 978-3-941875-65-4.
DOI: [10.17875/gup2010-511](https://doi.org/10.17875/gup2010-511).
- [Svä+24] S. Svärd, M. Lorenzon, J. Töyräänvuori, J. Valk, T. Alstola, E. Bennett, R. Uotila and T. Auranne, eds. *RAI 69 Abstracts*. July 2024.
eprint: https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäiväty_1.pdf.