Archaic cuneiform numbers

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2024-09-09

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1 Summary

This document proposes encoding, at U+12550–U+12585, 310 numerals used in the fourth millennium (Uruk IV and Uruk III periods) and Early Dynastic period in conjunction with the Sumero-Akkadian cuneiform script[1] and the proto-cuneiform script[2]. The proposed characters are listed in §2. Most of them were listed in [L2/23-190]. The present document provides a more detailed rationale for their encoding and additional information about their identity and usage, both as part of the rationale and in §5. Some characters have been removed, in some cases because they are non-encodable variants, in others because their encodability should be considered as part of the proto-cuneiform proposal; these are discussed in §6. The glyphs have also been reworked, and additional characters used in the Early Dynastic period have been added.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model requires that the language of a text be understood to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in §4. The proposed "curved", or "curviform", numerals[3] should however *not* be unified with the already-encoded cuneiform numerals[4]. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in §3, including compatibility considerations in §3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in §2.3 reflect the overlap. Many of these numerals are also used in proto-Elamite[5] texts, where they are treated as identical characters in scholarship on proto-Elamite, so that they should

[1] [ISO15924]: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

[2] [ISO15924]: Pcun, not yet encoded.

[3] Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: 𒁹, 𒁺 (small stylus), 𒁻 (large stylus). Some numerals are composed of multiple such impressions, *e.g.*, 𒁼. The terms "curved", "curviform", "curvilinear", and "round" can be found in the literature. We avoid the term "round" here as it has other meanings in the context of numbers. We use "curviform" in this document as, being the least common term, it is least likely to lead to confusion, and "CURVED" in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [**inlineATF**].

[4] Impressed into clay using a stylus with a trihedral end: 𒁲 (stylus held horizontally), 𒁴 (vertically), 𒁶 (diagonally) 𒁾 (with the head of the stylus), 𒂊 (stylus pressed deeper, forming a larger wedge), 𒂗 (combining 𒂊 and 𒁾), etc.

[5] [ISO15924]: Pelm, not yet encoded.

be unified with the ones proposed in [L2/23-196]. However, in the interest of time, we do not provide a detailed rationale for this unification in this document, and we are not proposing that the numerals be given the corresponding Script_Extensions property value for now. Neither do we propose encoding any numerals that are solely attested in proto-Elamite texts, or well-attested in proto-Elamite texts but insufficiently attested in Uruk—those are discussed in §6.

<table>
<tr><td></td><td>Uruk III & earlier</td><td>ED – Ur III</td><td>OB & later</td></tr>
<tr><td rowspan="2">Numerals</td><td colspan="2">This proposal</td><td></td></tr>
<tr><td></td><td rowspan="2" colspan="2">Existing Xsux</td></tr>
<tr><td>Non-numeric signs</td><td>Future Pcun</td></tr>
</table>

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

2 Proposed changes to the Standard

2.1 Core specification text

Amend [Uni16, §11.1.2, *sub* "Cuneiform Numerals"], as follows:

> ***Cuneiform Numerals.*** In general, numerals that also have a phonetic, logographic, or determinative value are encoded in the main Cuneiform block; as a result, some series of numerals, such as 𒐗–𒐝 1(diš)–9(diš) or 𒐕–𒐛 1(u)–9(u), are split across the two blocks. Numerals have been encoded separately from signs that are visually identical but ~~semantically different~~ etymologically unrelated (for example, U+1244F 𒑏 CUNEIFORM NUMERIC SIGN ONE BAN2, U+12450 𒑐 CUNEIFORM NUMERIC SIGN TWO BAN2, and so on, versus U+12226 𒈦 CUNEIFORM SIGN MASH, U+1227A 𒉺 CUNEIFORM SIGN PA, and so on).
>
> The relation between series of numerals depends on the metrological system; for instance, when counting talents, written 𒁹 (a unit of weight, approximately 30 kg), 𒐼𒁹 is used for "one talent", and 𒐖𒁹 for "ten talents". However, when measuring areas, the area 𒆠 (one *būrum*) is eighteen times 𒐼𒆠 (one *ikûm*, approximately 3600 m^2). The Numeric_Value property assignment of a cuneiform numeral therefore reflects only its relation to the first numeral in its series, rather than the absolute numeric value that it might represent. For instance, the number "fifty" is written 𒐚, but U+12410 𒑐 CUNEIFORM NUMERIC SIGN FIVE U has Numeric_Value=5, as it is 5 × 𒐕.
>
> In the third millennium, and especially in the Early Dynastic period, some numerals are written using a cylindrical tool, rather than the cuneiform stylus, forming curved rather than cuneiform numerals (𒐽 rather than 𒐼). The cuneiform numerals are descended from these curved numerals. However, in the Early Dynastic period, the curved numerals contrast with the cuneiform ones, and are used together with them in several metrological systems; they are therefore separately encoded. Most curved numerals are encoded in the Archaic Cuneiform

Numerals block, with the exception of two fractions in the Cuneiform Numbers and Punctuation block: U+1245D 𒑝 CUNEIFORM NUMERIC SIGN ONE THIRD VARIANT FORM A and U+1245E 𒑞 CUNEIFORM NUMERIC SIGN TWO THIRDS VARIANT FORM A, the curved counterparts of U+1245A 𒑚 CUNEIFORM NUMERIC SIGN ONE THIRD DISH and U+1245B 𒑛 CUNEIFORM NUMERIC SIGN TWO THIRDS DISH.

Add after [Uni16, §11.1.3]:

11.1.4 Archaic Cuneiform Numerals: U+12550–U+1268F

This block contains numerals used in the fourth millennium and third millennium. The numerals that are used in the fourth millennium and Early Dynastic I–II period (2900–2700 BCE) are named according to the conventions of the Berlin *Archaische Texte aus Uruk* (ATU) project, with names such as U+12550 𒕐 CUNEIFORM NUMERIC SIGN ONE N01 or U+125B6 𒖶 CUNEIFORM NUMERIC SIGN ONE N39A. For the signs that are also used in the third millennium, informative aliases provide correspondences to more common third millennium conventions, such as "1 aš curved" for U+12550 𒕐 CUNEIFORM NUMERIC SIGN ONE N01. The numerals that are only used starting in the Early Dynastic III period, where the ATU notation is not used, are named in the same fashion as the numerals of the Cuneiform Numbers and Punctation block.

The curved numerals are produced using cylindrical tools of two different sizes, producing small curved indents (𒕐, 𒖶, and •), and large ones (𒐼, 𒑱, and ●). These can be combined, as in U+12574 𒕼 CUNEIFORM NUMERIC SIGN ONE N48 or U+12582 𒖂 CUNEIFORM NUMERIC SIGN ONE N50. Consistent sizing is important to identifying these characters, as there is no visual distinction other than size between, for instance, U+12566 𒕦 CUNEIFORM NUMERIC SIGN FIVE N14, U+1257D 𒖽 CUNEIFORM NUMERIC SIGN FIVE N45, or U+125A3 𒖣 CUNEIFORM NUMERIC SIGN ONE N54. The reference glyphs of some of the larger signs have been resized to fit in the code charts cells, but fonts for these characters should retain consistent size across the numeral series.

> *Editor's note: I have not yet done that resizing. The dashed-box convention for wide dashes, see [Uni16, §24.1.2,* sub *"Dashed Box Convention"] should probably not be extended to these, since numbers enclosed in a real dashed box are a thing in proto-Elamite texts.*

The Numeric_Value assignments follow the same principles as in the Cuneiform Numbers and Punctuation block. Numerals used in the third millennium have the Cuneiform script property value; numerals used only in the fourth millennium have the Proto-Cuneiform script property value. Numerals used in both the fourth and third millennium have both scripts in their Script_Extensions values.

The sign ŠAR$_2$. When used logographically, the sign ŠAR$_2$ has the same (cuneiform) appearance as U+1212D 𒄭 CUNEIFORM SIGN HI in all but the most archaizing Early Dynastic texts. The character U+122B9 CUNEIFORM SIGN SHAR2 should be used for logographic šar$_2$, whether cuneiform or curved. Most period-specific fonts will have the same cuneiform glyph

for U+122B9 and U+1212D. In the Early Dynastic period, numeric 1 $šar_2$ is typically written with a curved glyph, contrasting with logographic $šar_2$. U+12579 𒕹 CUNEIFORM NUMERIC SIGN ONE N45 should be used for curved 1 $šar_2$. In later periods, long after $ŠAR_2$ and $Ḫ$I have merged, even numeric 1 $šar_2$ has a cuneiform glyph. U+122B9 CUNEIFORM SIGN SHAR2 should be used for cuneiform 1 $šar_2$.

The reference glyph for U+122B9 𒊹 CUNEIFORM SIGN SHAR2 is curved, reflecting the rarer and more archaic practice, instead of cuneiform as it would be in the Ur III period, so as to distinguish it from U+1212D 𒉌 CUNEIFORM SIGN HI.

2.2 Code charts

The code charts for the proposed block, including the character names list with proposed informative aliases, cross references, and informative notes, are shown on the following pages.

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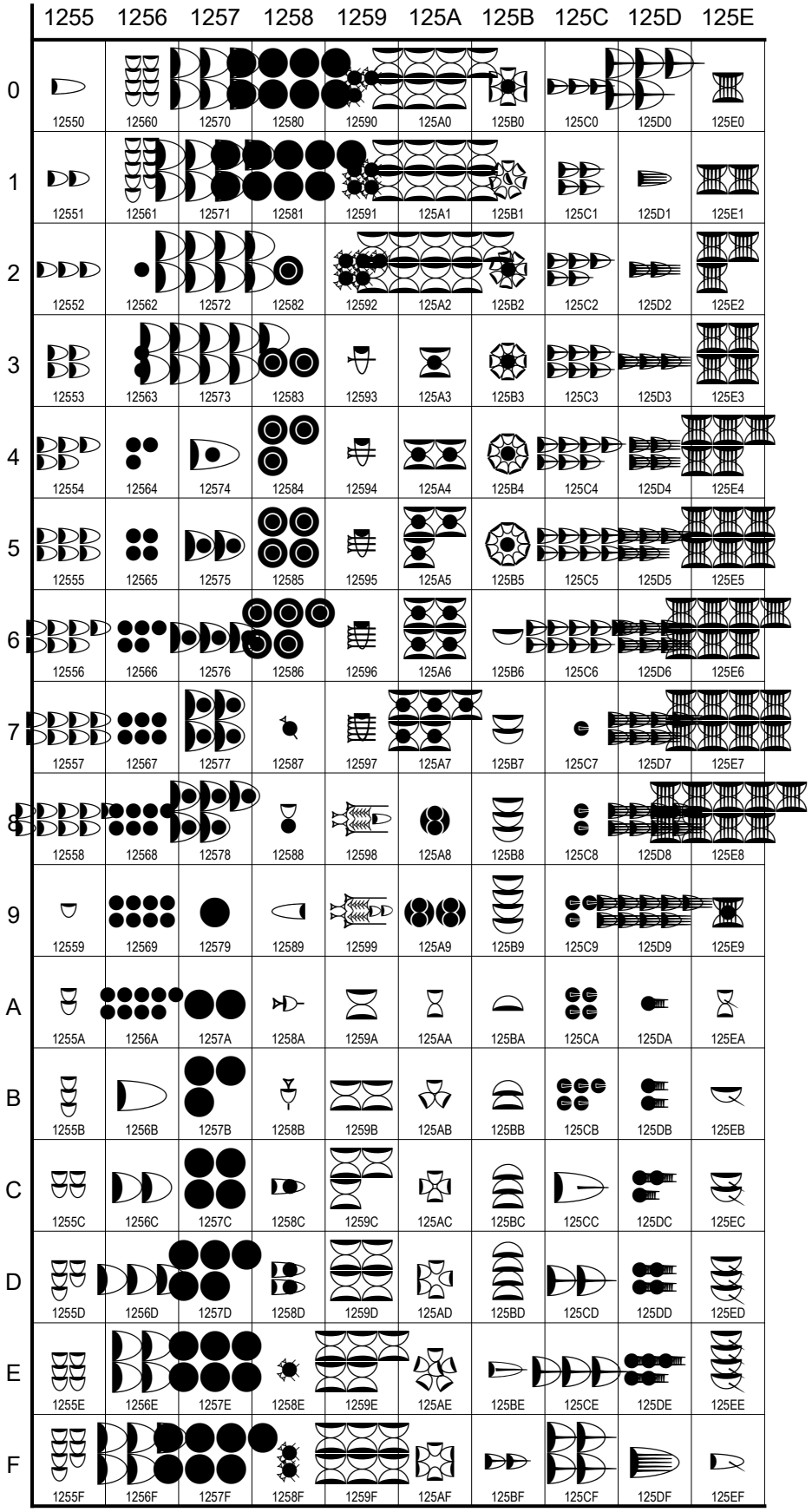
1255 1256 1257 1258 1259 125A 125B 125C 125D 125E
0 12550 12560 12570 12580 12590 125A0 125B0 125C0 125D0 125E0
1 12551 12561 12571 12581 12591 125A1 125B1 125C1 125D1 125E1
2 12552 12562 12572 12582 12592 125A2 125B2 125C2 125D2 125E2
3 12553 12563 12573 12583 12593 125A3 125B3 125C3 125D3 125E3
4 12554 12564 12574 12584 12594 125A4 125B4 125C4 125D4 125E4
5 12555 12565 12575 12585 12595 125A5 125B5 125C5 125D5 125E5
6 12556 12566 12576 12586 12596 125A6 125B6 125C6 125D6 125E6
7 12557 12567 12577 12587 12597 125A7 125B7 125C7 125D7 125E7
8 12558 12568 12578 12588 12598 125A8 125B8 125C8 125D8 125E8
9 12559 12569 12579 12589 12599 125A9 125B9 125C9 125D9 125E9
A 1255A 1256A 1257A 1258A 1259A 125AA 125BA 125CA 125DA 125EA
B 1255B 1256B 1257B 1258B 1259B 125AB 125BB 125CB 125DB 125EB
C 1255C 1256C 1257C 1258C 1259C 125AC 125BC 125CC 125DC 125EC
D 1255D 1256D 1257D 1258D 1259D 125AD 125BD 125CD 125DD 125ED
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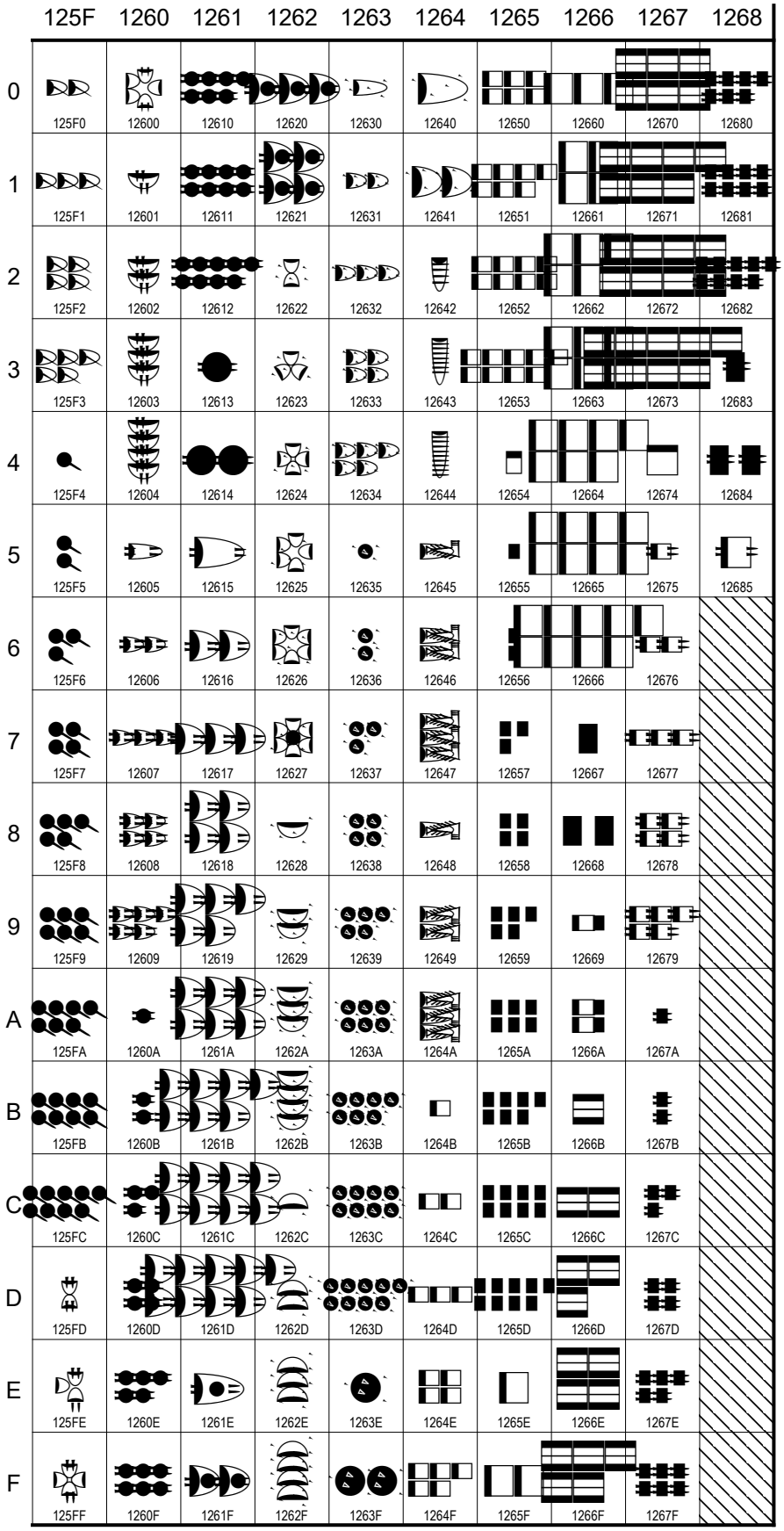
F 1255F 1256F 1257F 1258F 1259F 125AF 125BF 125CF 125DF 125EF


125F 1260 1261 1262 1263 1264 1265 1266 1267 1268
0 125F0 12600 12610 12620 12630 12640 12650 12660 12670 12680
1 125F1 12601 12611 12621 12631 12641 12651 12661 12671 12681
2 125F2 12602 12612 12622 12632 12642 12652 12662 12672 12682
3 125F3 12603 12613 12623 12633 12643 12653 12663 12673 12683
4 125F4 12604 12614 12624 12634 12644 12654 12664 12674 12684
5 125F5 12605 12615 12625 12635 12645 12655 12665 12675 12685
6 125F6 12606 12616 12626 12636 12646 12656 12666 12676
7 125F7 12607 12617 12627 12637 12647 12657 12667 12677
8 125F8 12608 12618 12628 12638 12648 12658 12668 12678
9 125F9 12609 12619 12629 12639 12649 12659 12669 12679
A 125FA 1260A 1261A 1262A 1263A 1264A 1265A 1266A 1267A
B 125FB 1260B 1261B 1262B 1263B 1264B 1265B 1266B 1267B
C 125FC 1260C 1261C 1262C 1263C 1264C 1265C 1266C 1267C
D 125FD 1260D 1261D 1262D 1263D 1264D 1265D 1266D 1267D
E 125FE 1260E 1261E 1262E 1263E 1264E 1265E 1266E 1267E
F 125FF 1260F 1261F 1262F 1263F 1264F 1265F 1266F 1267F

Many of the reference glyphs for the higher numbers (THREE and above) have been rescaled to fit the code chart cells. They should be sized consistently with the corresponding ONE numerals.

Common Numerals

Used in the sexagesimal discrete counting system and other metrological systems

12550 CUNEIFORM NUMERIC SIGN ONE N01
= 1 aš curved
→ 12038 𒁹 cuneiform sign ash
• often used instead of diš in Early Dynastic counterparts of cuneiform metrological systems
→ 12079 𒁴 cuneiform sign dish

12551 CUNEIFORM NUMERIC SIGN TWO N01
→ 12400 𒐙 cuneiform numeric sign two ash

12552 CUNEIFORM NUMERIC SIGN THREE N01

12553 CUNEIFORM NUMERIC SIGN FOUR N01

12554 CUNEIFORM NUMERIC SIGN FIVE N01

12555 CUNEIFORM NUMERIC SIGN SIX N01

12556 CUNEIFORM NUMERIC SIGN SEVEN N01

12557 CUNEIFORM NUMERIC SIGN EIGHT N01

12558 CUNEIFORM NUMERIC SIGN NINE N01

12559 CUNEIFORM NUMERIC SIGN ONE N08
= 1 diš curved
→ 12079 𒁴 cuneiform sign dish
= 1/2 iku curved
• used for one half in multiple metrological systems
→ 12039 𒁺 cuneiform sign ash zida tenu
→ 12226 𒈦 cuneiform sign mash
= 1 bariga curved
• used in Early Dynastic capacity systems

1255A CUNEIFORM NUMERIC SIGN TWO N08
→ 1222B 𒉋 cuneiform sign min
= 2 bariga curved
→ 12456 𒑖 cuneiform numeric sign nigidamin

1255B CUNEIFORM NUMERIC SIGN THREE N08
→ 12408 𒐋 cuneiform numeric sign three dish
• used in Early Dynastic capacity systems
= 3 bariga curved
→ 12457 𒑗 cuneiform numeric sign nigidaesh

1255C CUNEIFORM NUMERIC SIGN FOUR N08

1255D CUNEIFORM NUMERIC SIGN FIVE N08

1255E CUNEIFORM NUMERIC SIGN SIX N08

1255F CUNEIFORM NUMERIC SIGN SEVEN N08

12560 CUNEIFORM NUMERIC SIGN EIGHT N08

12561 CUNEIFORM NUMERIC SIGN NINE N08

12562 CUNEIFORM NUMERIC SIGN ONE N14
= 1 u curved
= 1 bur_3 curved
→ 1230B 𒌋 cuneiform sign u

12563 CUNEIFORM NUMERIC SIGN TWO N14
→ 12399 𒐗 cuneiform sign u u

12564 CUNEIFORM NUMERIC SIGN THREE N14
→ 1230D 𒌍 cuneiform sign u u u

12565 CUNEIFORM NUMERIC SIGN FOUR N14
→ 1240F 𒑏 cuneiform numeric sign four u

12566 CUNEIFORM NUMERIC SIGN FIVE N14

12567 CUNEIFORM NUMERIC SIGN SIX N14

12568 CUNEIFORM NUMERIC SIGN SEVEN N14

12569 CUNEIFORM NUMERIC SIGN EIGHT N14

1256A CUNEIFORM NUMERIC SIGN NINE N14

1256B CUNEIFORM NUMERIC SIGN ONE N34
= 1 $ŋeš_2$ curved
→ 12415 𒐕 cuneiform numeric sign one gesh2

1256C CUNEIFORM NUMERIC SIGN TWO N34

1256D CUNEIFORM NUMERIC SIGN THREE N34

1256E CUNEIFORM NUMERIC SIGN FOUR N34

1256F CUNEIFORM NUMERIC SIGN FIVE N34

12570 CUNEIFORM NUMERIC SIGN SIX N34

12571 CUNEIFORM NUMERIC SIGN SEVEN N34

12572 CUNEIFORM NUMERIC SIGN EIGHT N34

12573 CUNEIFORM NUMERIC SIGN NINE N34

12574 CUNEIFORM NUMERIC SIGN ONE N48
= 1 ŋeš'u curved
→ 1241E 𒐞 cuneiform numeric sign one geshu

12575 CUNEIFORM NUMERIC SIGN TWO N48

12576 CUNEIFORM NUMERIC SIGN THREE N48

12577 CUNEIFORM NUMERIC SIGN FOUR N48

12578 CUNEIFORM NUMERIC SIGN FIVE N48

12579 CUNEIFORM NUMERIC SIGN ONE N45
= 1 $šar_2$ curved
• 122B9 𒊹 should be used for cuneiform 1 $šar_2$
• 122B9 𒊹 should be used for logographic $šar_2$, even when curved
→ 122B9 𒊹 cuneiform sign shar2

1257A CUNEIFORM NUMERIC SIGN TWO N45

1257B CUNEIFORM NUMERIC SIGN THREE N45

1257C CUNEIFORM NUMERIC SIGN FOUR N45

1257D CUNEIFORM NUMERIC SIGN FIVE N45

1257E CUNEIFORM NUMERIC SIGN SIX N45

1257F CUNEIFORM NUMERIC SIGN SEVEN N45

12580 CUNEIFORM NUMERIC SIGN EIGHT N45

1258 CUNEIFORM NUMERIC SIGN NINE N45

12582 CUNEIFORM NUMERIC SIGN ONE N50
= 1 šar'u curved
→ 1242C 𒐬 cuneiform numeric sign one sharu
• used instead of 1258E 𒁾 in fourth millennium land area systems
→ 12434 𒐘 cuneiform numeric sign one buru

12583 CUNEIFORM NUMERIC SIGN TWO N50

12584 CUNEIFORM NUMERIC SIGN THREE N50

12585 CUNEIFORM NUMERIC SIGN FOUR N50

12586 CUNEIFORM NUMERIC SIGN FIVE N50

Numerals used for land areas

Together with N08, N01, N14, N45, and N50

12587 CUNEIFORM NUMERIC SIGN ONE EIGHTH IKU CURVED
→ 1245F 𒑟 cuneiform numeric sign one eighth ash

12588 CUNEIFORM NUMERIC SIGN ONE EIGHTH IKU CURVED VARIANT FORM

12589 CUNEIFORM NUMERIC SIGN ONE N01 REVERSED
= 1/4 iku curved
→ 12460 𒑠 cuneiform numeric sign one quarter ash

1258A CUNEIFORM NUMERIC SIGN ONE QUARTER IKU CURVED VARIANT FORM

1258B CUNEIFORM NUMERIC SIGN ONE HALF IKU CURVED VARIANT FORM
→ 12039 𒁺 cuneiform sign ash zida tenu

1258C CUNEIFORM NUMERIC SIGN ONE N22
= 1 $eše_3$ curved
→ 12458 𒑘 cuneiform numeric sign one eshe3

1258D CUNEIFORM NUMERIC SIGN TWO N22

1258E CUNEIFORM NUMERIC SIGN ONE BURU CURVED
→ 12434 𒐘 cuneiform numeric sign one buru

1258F CUNEIFORM NUMERIC SIGN TWO BURU CURVED

12590 CUNEIFORM NUMERIC SIGN THREE BURU CURVED

12591 CUNEIFORM NUMERIC SIGN FOUR BURU CURVED

12592 CUNEIFORM NUMERIC SIGN FIVE BURU CURVED

Early Dynastic capacity measures

12593 CUNEIFORM NUMERIC SIGN ONE BAN2 CURVED
- 1244F cuneiform numeric sign one ban2
= 1/2 aš curved
- used for one half in multiple metrological systems
→ 12226 cuneiform sign mash

12594 CUNEIFORM NUMERIC SIGN TWO BAN2 CURVED
12595 CUNEIFORM NUMERIC SIGN THREE BAN2 CURVED
12596 CUNEIFORM NUMERIC SIGN FOUR BAN2 CURVED
12597 CUNEIFORM NUMERIC SIGN FIVE BAN2 CURVED
12598 CUNEIFORM NUMERIC SIGN NINDA2 TIMES SHE PLUS ONE ASH CURVED
= 1/3 aš curved variant form
→ 1245D cuneiform numeric sign one third dish variant form a
→ 1245A cuneiform numeric sign one third dish
12599 CUNEIFORM NUMERIC SIGN NINDA2 TIMES SHE PLUS TWO ASH CURVED
= 2/3 aš curved variant form
→ 1245E cuneiform numeric sign two thirds dish variant form a
→ 1245B cuneiform numeric sign two thirds dish

Numerals used in the bisexagesimal system

Together with N08, N01, N14, and N34

1259A CUNEIFORM NUMERIC SIGN ONE N51
= 1 ŋeš$_2$ curved doubled, 1 ŋešmin curved
1259B CUNEIFORM NUMERIC SIGN TWO N51
1259C CUNEIFORM NUMERIC SIGN THREE N51
1259D CUNEIFORM NUMERIC SIGN FOUR N51
1259E CUNEIFORM NUMERIC SIGN FIVE N51
1259F CUNEIFORM NUMERIC SIGN SIX N51
125A0 CUNEIFORM NUMERIC SIGN SEVEN N51
125A1 CUNEIFORM NUMERIC SIGN EIGHT N51
125A2 CUNEIFORM NUMERIC SIGN NINE N51
125A3 CUNEIFORM NUMERIC SIGN ONE N54
= 1 ŋeš'u curved doubled, 1 ŋešmin'u curved
125A4 CUNEIFORM NUMERIC SIGN TWO N54
125A5 CUNEIFORM NUMERIC SIGN THREE N54
125A6 CUNEIFORM NUMERIC SIGN FOUR N54
125A7 CUNEIFORM NUMERIC SIGN FIVE N54
125A8 CUNEIFORM NUMERIC SIGN ONE N56
125A9 CUNEIFORM NUMERIC SIGN TWO N56

Fourth millennium capacity measures

125AA CUNEIFORM NUMERIC SIGN ONE N24
125AB CUNEIFORM NUMERIC SIGN ONE N26
125AC CUNEIFORM NUMERIC SIGN ONE N28
125AD CUNEIFORM NUMERIC SIGN ONE N29A
125AE CUNEIFORM NUMERIC SIGN ONE N29B
125AF CUNEIFORM NUMERIC SIGN ONE N30A
125B0 CUNEIFORM NUMERIC SIGN ONE N30C
125B1 CUNEIFORM NUMERIC SIGN ONE N30D
125B2 CUNEIFORM NUMERIC SIGN ONE N30E
125B3 CUNEIFORM NUMERIC SIGN ONE N31
125B4 CUNEIFORM NUMERIC SIGN ONE N32
125B5 CUNEIFORM NUMERIC SIGN ONE N33
125B6 CUNEIFORM NUMERIC SIGN ONE N39A
125B7 CUNEIFORM NUMERIC SIGN TWO N39A
125B8 CUNEIFORM NUMERIC SIGN THREE N39A
125B9 CUNEIFORM NUMERIC SIGN FOUR N39A
125BA CUNEIFORM NUMERIC SIGN ONE N39B
125BB CUNEIFORM NUMERIC SIGN TWO N39B
125BC CUNEIFORM NUMERIC SIGN THREE N39B
125BD CUNEIFORM NUMERIC SIGN FOUR N39B

Numerals of sexagesimal system S′

Used to count dead animals and jars of certain types of liquids

125BE CUNEIFORM NUMERIC SIGN ONE N02
125BF CUNEIFORM NUMERIC SIGN TWO N02
125C0 CUNEIFORM NUMERIC SIGN THREE N02
125C1 CUNEIFORM NUMERIC SIGN FOUR N02
125C2 CUNEIFORM NUMERIC SIGN FIVE N02
125C3 CUNEIFORM NUMERIC SIGN SIX N02
125C4 CUNEIFORM NUMERIC SIGN SEVEN N02
125C5 CUNEIFORM NUMERIC SIGN EIGHT N02
125C6 CUNEIFORM NUMERIC SIGN NINE N02
125C7 CUNEIFORM NUMERIC SIGN ONE N15
125C8 CUNEIFORM NUMERIC SIGN TWO N15
125C9 CUNEIFORM NUMERIC SIGN THREE N15
125CA CUNEIFORM NUMERIC SIGN FOUR N15
125CB CUNEIFORM NUMERIC SIGN FIVE N15
125CC CUNEIFORM NUMERIC SIGN ONE N35
125CD CUNEIFORM NUMERIC SIGN TWO N35
125CE CUNEIFORM NUMERIC SIGN THREE N35
125CF CUNEIFORM NUMERIC SIGN FOUR N35
125D0 CUNEIFORM NUMERIC SIGN FIVE N35

Numerals of bisexagesimal system B*

Used in the fourth millennium to count rations of an unclear nature

125D1 CUNEIFORM NUMERIC SIGN ONE N06
125D2 CUNEIFORM NUMERIC SIGN TWO N06
125D3 CUNEIFORM NUMERIC SIGN THREE N06
125D4 CUNEIFORM NUMERIC SIGN FOUR N06
125D5 CUNEIFORM NUMERIC SIGN FIVE N06
125D6 CUNEIFORM NUMERIC SIGN SIX N06
125D7 CUNEIFORM NUMERIC SIGN SEVEN N06
125D8 CUNEIFORM NUMERIC SIGN EIGHT N06
125D9 CUNEIFORM NUMERIC SIGN NINE N06
125DA CUNEIFORM NUMERIC SIGN ONE N21
125DB CUNEIFORM NUMERIC SIGN TWO N21
125DC CUNEIFORM NUMERIC SIGN THREE N21
125DD CUNEIFORM NUMERIC SIGN FOUR N21
125DE CUNEIFORM NUMERIC SIGN FIVE N21
125DF CUNEIFORM NUMERIC SIGN ONE N38
125E0 CUNEIFORM NUMERIC SIGN ONE N52
125E1 CUNEIFORM NUMERIC SIGN TWO N52
125E2 CUNEIFORM NUMERIC SIGN THREE N52
125E3 CUNEIFORM NUMERIC SIGN FOUR N52
125E4 CUNEIFORM NUMERIC SIGN FIVE N52
125E5 CUNEIFORM NUMERIC SIGN SIX N52
125E6 CUNEIFORM NUMERIC SIGN SEVEN N52
125E7 CUNEIFORM NUMERIC SIGN EIGHT N52
125E8 CUNEIFORM NUMERIC SIGN NINE N52
125E9 CUNEIFORM NUMERIC SIGN ONE N60

Numerals of capacity system Š′

Used in the fourth millennium to measure malted barley

125EA CUNEIFORM NUMERIC SIGN ONE N24A
125EB CUNEIFORM NUMERIC SIGN ONE N40
125EC CUNEIFORM NUMERIC SIGN TWO N40
125ED CUNEIFORM NUMERIC SIGN THREE N40
125EE CUNEIFORM NUMERIC SIGN FOUR N40
125EF CUNEIFORM NUMERIC SIGN ONE N03
125F0 CUNEIFORM NUMERIC SIGN TWO N03
125F1 CUNEIFORM NUMERIC SIGN THREE N03
125F2 CUNEIFORM NUMERIC SIGN FOUR N03
125F3 CUNEIFORM NUMERIC SIGN FIVE N03
125F4 CUNEIFORM NUMERIC SIGN ONE N18
125F5 CUNEIFORM NUMERIC SIGN TWO N18
125F6 CUNEIFORM NUMERIC SIGN THREE N18
125F7 CUNEIFORM NUMERIC SIGN FOUR N18
125F8 CUNEIFORM NUMERIC SIGN FIVE N18
125F9 CUNEIFORM NUMERIC SIGN SIX N18

125FA CUNEIFORM NUMERIC SIGN SEVEN N18
125FB CUNEIFORM NUMERIC SIGN EIGHT N18
125FC CUNEIFORM NUMERIC SIGN NINE N18

Numerals of capacity system Š″

Used in the fourth millennium to measure various kinds of emmer

125FD CUNEIFORM NUMERIC SIGN ONE N24B
125FE CUNEIFORM NUMERIC SIGN ONE N26B
125FF CUNEIFORM NUMERIC SIGN ONE N28B
12600 CUNEIFORM NUMERIC SIGN ONE N29AB
12601 CUNEIFORM NUMERIC SIGN ONE N41
12602 CUNEIFORM NUMERIC SIGN TWO N41
12603 CUNEIFORM NUMERIC SIGN THREE N41
12604 CUNEIFORM NUMERIC SIGN FOUR N41
12605 CUNEIFORM NUMERIC SIGN ONE N04
12606 CUNEIFORM NUMERIC SIGN TWO N04
12607 CUNEIFORM NUMERIC SIGN THREE N04
12608 CUNEIFORM NUMERIC SIGN FOUR N04
12609 CUNEIFORM NUMERIC SIGN FIVE N04
1260A CUNEIFORM NUMERIC SIGN ONE N19
1260B CUNEIFORM NUMERIC SIGN TWO N19
1260C CUNEIFORM NUMERIC SIGN THREE N19
1260D CUNEIFORM NUMERIC SIGN FOUR N19
1260E CUNEIFORM NUMERIC SIGN FIVE N19
1260F CUNEIFORM NUMERIC SIGN SIX N19
12610 CUNEIFORM NUMERIC SIGN SEVEN N19
12611 CUNEIFORM NUMERIC SIGN EIGHT N19
12612 CUNEIFORM NUMERIC SIGN NINE N19
12613 CUNEIFORM NUMERIC SIGN ONE N46
12614 CUNEIFORM NUMERIC SIGN TWO N46
12615 CUNEIFORM NUMERIC SIGN ONE N36
12616 CUNEIFORM NUMERIC SIGN TWO N36
12617 CUNEIFORM NUMERIC SIGN THREE N36
12618 CUNEIFORM NUMERIC SIGN FOUR N36
12619 CUNEIFORM NUMERIC SIGN FIVE N36
1261A CUNEIFORM NUMERIC SIGN SIX N36
1261B CUNEIFORM NUMERIC SIGN SEVEN N36
1261C CUNEIFORM NUMERIC SIGN EIGHT N36
1261D CUNEIFORM NUMERIC SIGN NINE N36
1261E CUNEIFORM NUMERIC SIGN ONE N49
1261F CUNEIFORM NUMERIC SIGN TWO N49
12620 CUNEIFORM NUMERIC SIGN THREE N49
12621 CUNEIFORM NUMERIC SIGN FOUR N49

Numerals of capacity system Š*

Used in the fourth millennium to measure barley groats

12622 CUNEIFORM NUMERIC SIGN ONE N25
12623 CUNEIFORM NUMERIC SIGN ONE N27
12624 CUNEIFORM NUMERIC SIGN ONE N28C
12625 CUNEIFORM NUMERIC SIGN ONE N29AC
12626 CUNEIFORM NUMERIC SIGN ONE N30AC
12627 CUNEIFORM NUMERIC SIGN ONE N30CC
12628 CUNEIFORM NUMERIC SIGN ONE N42A
12629 CUNEIFORM NUMERIC SIGN TWO N42A
1262A CUNEIFORM NUMERIC SIGN THREE N42A
1262B CUNEIFORM NUMERIC SIGN FOUR N42A
1262C CUNEIFORM NUMERIC SIGN ONE N42B
1262D CUNEIFORM NUMERIC SIGN TWO N42B
1262E CUNEIFORM NUMERIC SIGN THREE N42B
1262F CUNEIFORM NUMERIC SIGN FOUR N42B
12630 CUNEIFORM NUMERIC SIGN ONE N05
12631 CUNEIFORM NUMERIC SIGN TWO N05
12632 CUNEIFORM NUMERIC SIGN THREE N05
12633 CUNEIFORM NUMERIC SIGN FOUR N05
12634 CUNEIFORM NUMERIC SIGN FIVE N05
12635 CUNEIFORM NUMERIC SIGN ONE N20
12636 CUNEIFORM NUMERIC SIGN TWO N20
12637 CUNEIFORM NUMERIC SIGN THREE N20
12638 CUNEIFORM NUMERIC SIGN FOUR N20
12639 CUNEIFORM NUMERIC SIGN FIVE N20
1263A CUNEIFORM NUMERIC SIGN SIX N20
1263B CUNEIFORM NUMERIC SIGN SEVEN N20
1263C CUNEIFORM NUMERIC SIGN EIGHT N20
1263D CUNEIFORM NUMERIC SIGN NINE N20
1263E CUNEIFORM NUMERIC SIGN ONE N47
1263F CUNEIFORM NUMERIC SIGN TWO N47
12640 CUNEIFORM NUMERIC SIGN ONE N37
12641 CUNEIFORM NUMERIC SIGN TWO N37

Numerals of system EN

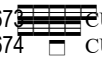
Only attested in the Uruk IV period

12642 CUNEIFORM NUMERIC SIGN ONE N09
12643 CUNEIFORM NUMERIC SIGN ONE N11
12644 CUNEIFORM NUMERIC SIGN ONE N12
12645 CUNEIFORM NUMERIC SIGN ONE N07A
12646 CUNEIFORM NUMERIC SIGN TWO N07A
12647 CUNEIFORM NUMERIC SIGN THREE N07A
12648 CUNEIFORM NUMERIC SIGN ONE N07B
12649 CUNEIFORM NUMERIC SIGN TWO N07B
1264A CUNEIFORM NUMERIC SIGN THREE N07B

Flat numerals

Rectangular numerals impressed with a flat tool, used in Ur in the Early Dynastic I–II period

1264B CUNEIFORM NUMERIC SIGN ONE N01 FLAT
= 1 aš flat
→ 12038 𒀸 cuneiform sign ash
1264C CUNEIFORM NUMERIC SIGN TWO N01 FLAT
1264D CUNEIFORM NUMERIC SIGN THREE N01 FLAT
1264E CUNEIFORM NUMERIC SIGN FOUR N01 FLAT
1264F CUNEIFORM NUMERIC SIGN FIVE N01 FLAT
12650 CUNEIFORM NUMERIC SIGN SIX N01 FLAT
12651 CUNEIFORM NUMERIC SIGN SEVEN N01 FLAT
12652 CUNEIFORM NUMERIC SIGN EIGHT N01 FLAT
12653 CUNEIFORM NUMERIC SIGN NINE N01 FLAT
12654 CUNEIFORM NUMERIC SIGN ONE N08 FLAT
12655 CUNEIFORM NUMERIC SIGN ONE N14 FLAT
= 1 u flat
→ 1230B 𒌋 cuneiform sign u
12656 CUNEIFORM NUMERIC SIGN TWO N14 FLAT
12657 CUNEIFORM NUMERIC SIGN THREE N14 FLAT
12658 CUNEIFORM NUMERIC SIGN FOUR N14 FLAT
12659 CUNEIFORM NUMERIC SIGN FIVE N14 FLAT
1265A CUNEIFORM NUMERIC SIGN SIX N14 FLAT
1265B CUNEIFORM NUMERIC SIGN SEVEN N14 FLAT
1265C CUNEIFORM NUMERIC SIGN EIGHT N14 FLAT
1265D CUNEIFORM NUMERIC SIGN NINE N14 FLAT
1265E CUNEIFORM NUMERIC SIGN ONE N34 FLAT
1265F CUNEIFORM NUMERIC SIGN TWO N34 FLAT
12660 CUNEIFORM NUMERIC SIGN THREE N34 FLAT
12661 CUNEIFORM NUMERIC SIGN FOUR N34 FLAT
12662 CUNEIFORM NUMERIC SIGN FIVE N34 FLAT
12663 CUNEIFORM NUMERIC SIGN SIX N34 FLAT
12664 CUNEIFORM NUMERIC SIGN SEVEN N34 FLAT
12665 CUNEIFORM NUMERIC SIGN EIGHT N34 FLAT
12666 CUNEIFORM NUMERIC SIGN NINE N34 FLAT
12667 CUNEIFORM NUMERIC SIGN ONE N45 FLAT
12668 CUNEIFORM NUMERIC SIGN TWO N45 FLAT
12669 CUNEIFORM NUMERIC SIGN ONE N22 FLAT
1266A CUNEIFORM NUMERIC SIGN TWO N22 FLAT
1266B CUNEIFORM NUMERIC SIGN ONE N51 FLAT
1266C CUNEIFORM NUMERIC SIGN TWO N51 FLAT
1266D CUNEIFORM NUMERIC SIGN THREE N51 FLAT
1266E CUNEIFORM NUMERIC SIGN FOUR N51 FLAT
1266F CUNEIFORM NUMERIC SIGN FIVE N51 FLAT
12670 CUNEIFORM NUMERIC SIGN SIX N51 FLAT
12671 CUNEIFORM NUMERIC SIGN SEVEN N51 FLAT
12672 CUNEIFORM NUMERIC SIGN EIGHT N51 FLAT

12673  CUNEIFORM NUMERIC SIGN NINE N51 FLAT
12674  CUNEIFORM NUMERIC SIGN ONE N34 FLAT TENU
= 1 n39a flat
12675 CUNEIFORM NUMERIC SIGN ONE N04 FLAT
12676 CUNEIFORM NUMERIC SIGN TWO N04 FLAT
12677 CUNEIFORM NUMERIC SIGN THREE N04 FLAT
12678 CUNEIFORM NUMERIC SIGN FOUR N04 FLAT
12679 CUNEIFORM NUMERIC SIGN FIVE N04 FLAT
1267A CUNEIFORM NUMERIC SIGN ONE N19 FLAT
1267B CUNEIFORM NUMERIC SIGN TWO N19 FLAT
1267C CUNEIFORM NUMERIC SIGN THREE N19 FLAT
1267D CUNEIFORM NUMERIC SIGN FOUR N19 FLAT
1267E CUNEIFORM NUMERIC SIGN FIVE N19 FLAT
1267F CUNEIFORM NUMERIC SIGN SIX N19 FLAT
12680 CUNEIFORM NUMERIC SIGN SEVEN N19 FLAT
12681 CUNEIFORM NUMERIC SIGN EIGHT N19 FLAT
12682 CUNEIFORM NUMERIC SIGN NINE N19 FLAT
12683 CUNEIFORM NUMERIC SIGN ONE N46 FLAT
12684 CUNEIFORM NUMERIC SIGN TWO N46 FLAT
12685 CUNEIFORM NUMERIC SIGN ONE N36 FLAT

2.3 Properties

3 Rationale for curviform–cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [Pow72, p. 215] “in the same fashion as we use black and red ink”; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts, later in the third millennium[6],  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists[7] and dictionaries[8], and of composite texts[9]. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24][10]. The diachronic approach is also useful for pedagogic applications[11].

3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding[12] that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early

[6] Merging with U+1224E CUNEIFORM SIGN NI2.

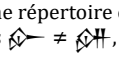
[7] Notably [OSL] and the online edition of [Bor10] in [eBL, Signs].

[8] Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

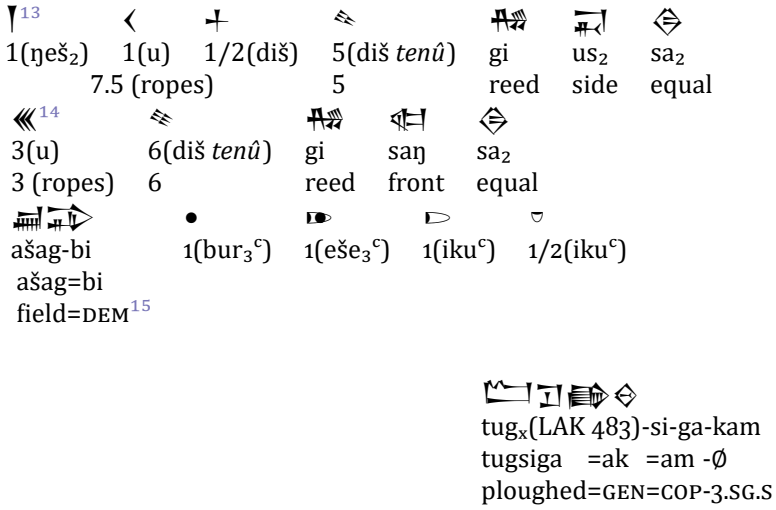
[9] For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of  , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

[10] Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svä+24, pp. 242, 148].

[11] For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

[12] At that time scoped to the répertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as  , were informed by Early Dynastic distinctions.

Dynastic IIIb administrative tablet from Ŋirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:



The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as[16]

> 465 metres, equal lengths
> 198 metres, equal widths
> this field is 9, 18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that "the number signs do not normally carry in their individual signs the meaning of what they are used to measure", and that curviform and cuneiform numerals "are not normally mixed together in a single numerical expression", noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from 𒌋 to 𒐐 numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in §3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that "The distinction between curved numerals and their cuneiform

[13] As noted in [Pow87, p. 466], this sign has a very short "tail" in this period, so that it is wider than it is tall, and can at first seem like a large 𒁹 in copies. The photos in [CDLI] clearly show that this is in fact a vertical wedge.

[14] Note that ED IIIb 𒐈 numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign 𒐋 in [P020054] looks more like Ur III 𒐉.

[15] Alternatively: area=POSS.3.SG.NH, "its area".

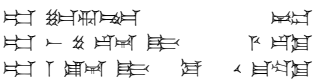
[16] We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases[17]."

The time has come to revisit this issue. As we will see in §3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In §3.4 we will see that in some periods:

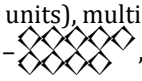
— the functions and use of the numerals vary beyond the mere 𒐕/𒐖 switch;
— the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
— some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 A primer on classic Ur III and Old Babylonian metrologies


I want to write tablets: the tablet of 1 cor of barley to 600 cor; the tablet of 1 shekel of silver to 10 minas [...]

Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known[18] a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters 𒐖–𒐚, the multiples of ten (10–50) are 𒌋–𒐏, but the other digits 11–59 are sequences 𒌋𒐖–𒐏𒐚; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units 𒐖–𒐚, tens 𒌋–𒐏, sixties 𒐗–𒐜 (with larger wedges than the units), multiples of six hundred 𒐝–𒐞, multiples of three thousand six hundreds 𒐠–  , and multiples of thirty-six thousand 𒐡–  .

3.3.1 The discrete counting system

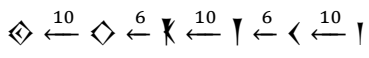
The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram[19], where the num-

[17] The cuneiform encoding process was planned in *stages* in [L2/03-162]. One might expect the second stage of encoding, which led to the creation of the Early Dynastic Cuneiform block, to incorporate the numerals needed for the representation and discussion of Early Dynastic texts; however, the proposal [L2/12-208] stated that "numerals have been omitted due to the complexity of numeral signs from this period. An expert in the metrology of this period must be consulted before these can be properly included."

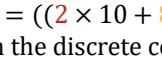
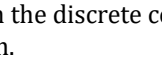
[18] See, *e.g.*, [Uni16, §22.3.3, *sub* "Cuneiform Numerals"].

[19] These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*, see Figure 4.

ber over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

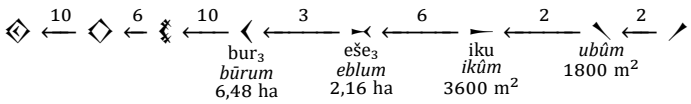


($S_{\text{Ur III/OB}}$)

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written  in the discrete counting system, and  in the sexagesimal place value system.

3.3.2 The area system

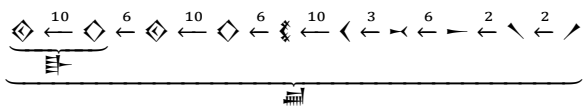
The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:



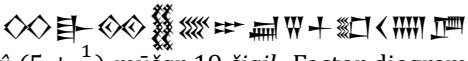
($G_{\text{Ur III/OB}}$)

Note that for the range of areas given above, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *būrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign  functioning as punctuation[20], but the numerals are tied to the metrology; thus a surface of 5 *būr* 1 *ebel* 4 *ikû* (100 *ikû*, 36 ha) would be written[21]  . Contrast this with systems where the same numerals are used for different units, and overt units are used, as in "88 acres 3 roods 33 perches" or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the sign  is equal to sixty times  in the area system, but to three hundred and sixty times in the discrete counting system.

For areas smaller than a quarter *ikûm*, an overt unit is used, with one  (sar, *mūšarum*), approximately 36 m^2, written , equal to one hundredth of an *ikîm*, then sexigesimally subdivided in 60 (giŋ$_4$, *šiqlum*, "shekel"). For areas greater than 3600 *būr*, the and numerals are reused with a suffix  (gal, "big"), as follows [Rob08, p. 295 nn. b, c; Fri07, p. 378; Rob19]:



($G_{\text{Ur III/OB}}$)

e.g.,  [22] for $(2 \times 3600 + 20 \times 60 + 49)$ *būr* 5 *ikû* $(5 + \frac{1}{2})$ *mūšar* 19 *šiqil*. Factor diagrams in this document will use bottom curly brackets in this fashion to separate numerals from units and other suffixes.

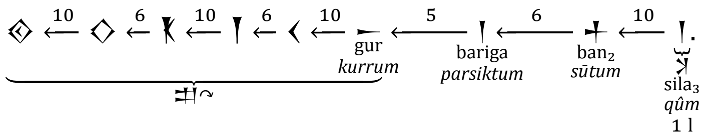
[20] This sign is sometimes interpreted as a measurement unit, and transliterated iku, see, *e.g.*, [Pro20, pp. 385 sqq.], or transliterations in [Feu04] discussed in §3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system $G_{\text{Ur III/OB}}$.

[21] As in the surface of the field of  (the city of Apisal) reported on [P102305, r. 1]

[22] From [P213162], which has an additional , two thirds (of a shekel), see §3.3.5.

3.3.3 The capacity system

Another such system of note is the one for capacities[23] [Fri07, p. 376; Rob19],



($C_{\mathrm{Ur\ III/OB}}$)

In the above diagram, the numerals for ban$_2$ are 𒐞, 𒐟, 𒐠, 𒐡, and 𒐣, and those for bariga are 𒁹, 𒑏, 𒑐, and 𒑑 (contrast ordinary 𒈫 and 𒐈 otherwise used with 𒁹 numerals). Further, we have used the symbol ↷ to express that, as described in [Hue11, p. 585 nn. (b), (f)], the sign 𒄥 GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𒋡 if present, and after the word for "grain" if present, as in

𒐐𒐊𒌋𒐼	𒐠	𒐋	𒋡	𒊺	𒄥[24]
3554 gur	3 ban$_2$	6	sila$_3$	of grain.	

Observe that while large numbers of gur follow[25] system $S_{\mathrm{Ur\ III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as 𒌋𒁹𒄥 would be 10 gur 1 bariga, and 𒌋𒀸𒄥 would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

— 𒁹–𒐎 used in $S_{\mathrm{Ur\ III/OB}}$ and the SPVS as well as with overt units;
— 𒌋–𒐐 used in $G_{\mathrm{Ur\ III/OB}}$, of which 𒌋–𒐏 are also used in $S_{\mathrm{Ur\ III/OB}}$ and the SPVS as well as with overt units;
— 𒐕–𒐝 used in $S_{\mathrm{Ur\ III/OB}}$, and sometimes with overt units;
— 𒐏–𒐝 used in $S_{\mathrm{Ur\ III/OB}}$;
— 𒊹–𒐬 used in $S_{\mathrm{Ur\ III/OB}}$ and $G_{\mathrm{Ur\ III/OB}}$;
— 𒐬–𒐲 used in $S_{\mathrm{Ur\ III/OB}}$ and $G_{\mathrm{Ur\ III/OB}}$;
— 𒀸–𒐼 used in $C_{\mathrm{Ur\ III/OB}}$ as well as with overt units of the weight system;
— 𒐞, 𒐟, 𒐠, 𒐡, 𒐣 used in $C_{\mathrm{Ur\ III/OB}}$;
— 𒁹, 𒑏, 𒑐, 𒑑 used in $C_{\mathrm{Ur\ III/OB}}$—note the overlap with 𒁹–𒐎;
— 𒑚 and 𒑛 used in $G_{\mathrm{Ur\ III/OB}}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Rob08, p. 78]: "The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity."

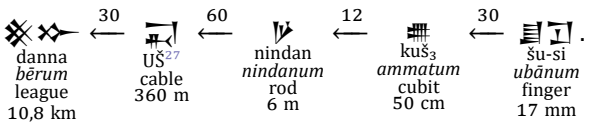
[23] Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{\mathrm{Ur\ III/OB}}$ based on a height of one cubit, see[Pow87, p. 488; Rob08, p. 294; Rob19].

[24] From [P309594, obv. 1 1].

[25] A larger unit, the guru$_7$ (*karûm*, grain heap), is sometimes used instead, with 𒀸𒄥𒊹𒌋𒐼𒄥 (1 *karûm* = 3600 kurrū). See [Fri07, p. 415; Rob19].

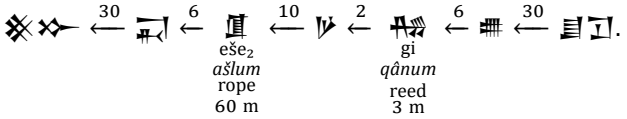
3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with 𒁹 and 𒌋 numerals with their system $S_{\text{Ur III/OB}}$ values. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in §3.4. Metrological tables use the following units[26] [Fri07, p. 118; Rob19]:



$(L_{\text{Ur III/OB}})$

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:



$(\bar{L}_{\text{Ur III/OB}})$

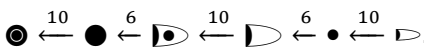
In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

Fractions of the *ikîm*, 𒍪 = $\frac{1}{2}$𒐼 and 𒍫 = $\frac{1}{4}$𒐼, have already been encountered. In other contexts, the fraction $\frac{1}{2}$ is written 𒊹, as in 𒊹𒄩. The fractions $\frac{1}{3}$ and $\frac{2}{3}$ are written 𒐻 and 𒐼. The latter two signs are derived from curviform signs 𒌷 and 𒌸, which are already separately encoded; these are in turn derived from the sign 𒋗 (ŠU_2), whose Early dynastic form resembles 𒐻, and 𒌷 numerals; see [Pow71, pp. 113, 134]. The 𒋗 is sometimes omitted, as in [P240545, *verso* 6 9; P221530; P221531; P271238; P274845].

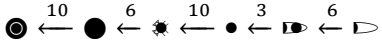
3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the fourth millennium) clearly mirrors system $S_{\text{Ur III/OB}}$ [Fri07, p. 374; DE87, pp. 127, 165]:



(S)

Likewise the area system used in the Early Dynastic IIIb period for areas of one iku and greater [Dei22, p. 72; NDE93, p. 63; Fri07, p. 378; Lec16],



$(G_{\text{ED IIIb}})$

[26] In this factor diagram and the next, we do not include the numerals. The units are no more than a factor of 60 apart, so higher numerals such as 𒐖 or 𒐗 are not used.

[27] As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.

mirrors system $G_{\text{Ur III/OB}}$, with consistent use of the numerals: • corresponds to 𒌋, ● to ◇, and ◎ to ◈. An exception to this correspondence, noted in [L2/04-099, p. 4] (see §3.2), is that the vertical 𒁹 from $S_{\text{Ur III/OB}}$ corresponds to a horizontal ▷ in system S. This is however far from the only case of such a reallocation of function. The earlier form of the area system is [DE87, pp. 141, 165; Fri07, p. 378]:

● $\xleftarrow{6}$ ◎ $\xleftarrow{10}$ • $\xleftarrow{3}$ ▶ $\xleftarrow{6}$ ▷, (G)

Observe that, as noted in [DE87, p. 142], ◎ changes meaning from 10• in system G to 600• in system $G_{\text{ED IIIb}}$. System G is used in the fourth millennium, but also in the ED I–II period (it is the "area 2" system in [Cha03], whereas $G_{\text{ED IIIb}}$ is the "area 1" system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb area system[28],

◎ $\xleftarrow{10}$ ● $\xleftarrow{6}$ ✹ $\xleftarrow{10}$ • $\xleftarrow{3}$ ▶ $\xleftarrow{6}$ ▷ $\xleftarrow{2}$ ▿ $\xleftarrow{2}$ ◁ $\xleftarrow{2}$ ◖[29], ($G_{\text{ED IIIb}}$)

with the numerals of a contemporaneous capacity system:

▶ $\xleftarrow{10}$ D $\xleftarrow{6}$ • $\xleftarrow{10}$ ▷ $\xleftarrow{4}$ ▿ $\xleftarrow{6}$ 𒑏, ($C_{𒄥𒊕𒅅}$)
(first four numerals braced: 𒄥𒊕𒅅~)

both described in [Lec16]. While the size of the 𒄥𒊕𒅅 (gur saŋ ŋal₂) in bariga is different from that of the Old Babylonian 𒄥, the basic structure of the capacity system is recognizable, with ▿ corresponding to 𒁹 for bariga, 𒑏–𒑐 corresponding to 𒐕–𒑔 for ban₂, and the 𒄥 counted with ▷ rather than 𒀸 numerals. However, the half-iku is counted with the same ▿ as the bariga, whereas it uses a different sign, 𒑊, in the Old Babylonian system. As we will see, this is cannot be handled as a split, by giving 𒑊 the glyph ▿ in an Early Dynastic IIIb font, as the 𒑊 numeral series is also in use in that period.

3.4.1 Field lengths in Ŋirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign 𒃻 is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign 𒂠 is not written either. Lengths shorter than one rope are expressed in half-ropes using the $\frac{1}{2}$ sign 𒈦 (again with no 𒂠), and then in reeds, *with* the sign 𒄀, as follows:

𒁹 $\xleftarrow{6}$ 𒌋 $\xleftarrow{2}$ 𒈦 $\xleftarrow{10}$ 𒑊[32]. ($L_{\text{ED IIIb}}$)

(under the first arrows: 1 eše₂=10 nindan / 1 rope=10 rods / 60 m; under 𒑊: 𒄀 / gi / reed / 3 m)

This is the system that was used to express the sides of the field in [P020054] discussed in §3.2. In that tablet and most others from the same period, such as the

[28] A variant is ◎ $\xleftarrow{10}$ ● $\xleftarrow{6}$ ✹ $\xleftarrow{10}$ • $\xleftarrow{3}$ ▶ $\xleftarrow{6}$ ▷ $\xleftarrow{2}$ ▿ $\xleftarrow{2}$ ◁ $\xleftarrow{2}$ ◖, see [Pow72, p. 218].

[29] The (fairly rare) cuneiform counterpart is 𒑋.

[32] The reeds are counted using *tenû* numerals, 𒑊, 𒑋, 𒑌, etc.

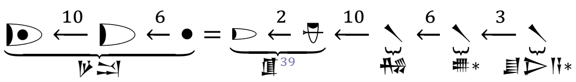
ones discussed in [Lec20], areas are expressed in system $G_{\text{ED IIIb}}$, with curviform numerals[33]; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope[34], the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and thus the interpretation of its numerals between systems $G_{\text{ED IIIb}}$ and $L_{\text{ED IIIb}}$. The sign 𒃷, which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without 𒃷. There are other such co-occurrences contrasting between metrological systems; for instance, [Kre98, p. 303 n. 686] mentions the use of cuneiform numerals for days and months[35].

3.4.2 Dyke lengths in Ŋirsu

[Pow87, p. 466] notes that reeds “are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus.” Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in [CDLI] finds many occurrences[36], such as:

— [P221305, obv. 1 4][37] [cuneiform]
— [P020129, rev. 2 1] [cuneiform]
— [P221291, rev. 5 1][38] [cuneiform]
— [P221266, rev. 2 1] [cuneiform]

These expressions use an explicit sign [cuneiform] (counted in multiples of ten) or [cuneiform]. This notation—but not its use of curviform numerals—is remarked on in [Lec20, p. 290 n. 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. The notation can be summarized by the following factor diagram, where prefix units have been marked by an asterisk:



($L'_{\text{ED IIIb}}$)

3.4.3 Butter, cheese and wheat in Ŋirsu

A similar mixture of cuneiform and curviform numerals may be observed with the capacity system; indeed, the previously described [cuneiform] system uses [cuneiform]

[33] A [CDLI] search for `"(bur3)"` (𒁓 numerals used for areas) currently returns 15 ED IIIb results, whereas one for `"(bur3@c)"` (● numerals used for areas) returns 206. Further, when dated, the tablets with cuneiform bur_3 are from the reigns of 𒌷𒅗𒄀𒈾 (variously transliterated iri-inim-gi-na, uru-ka-gi-na, etc.) and 𒈗𒍠𒄀𒋛 (lugal-zag-ge-si), the last two kings of ED IIIb Lagaš.

[34] This is the case of the sides of the field in [P020054, obv. ii 2–3].

[35] That note also mentions a contrast between the use of curviform numerals to count people and curviform numerals to count bread alotted to them in [P010876]; such contrasts are more akin to styling, and might not, on their own, justify the disunification.

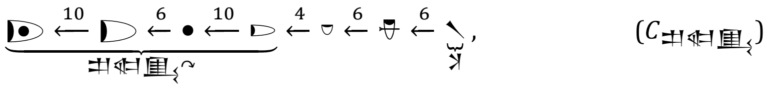
[36] A search for curviform numerals followed by some number of reeds counted in (*tenû*) cuneiform numerals currently finds 125 occurrences across 47 tablets.

[37] [CDLI] only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the [cuneiform] is not visible. Lecompte notes that the copy is faithful; indeed another [cuneiform] can be seen both on the copy and the photo on obv. 2 2.

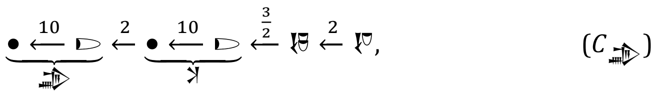
[38] From copy.

[39] With either unit omitted, as in the examples above, or both, as in [P020129, obv. 3 3] [cuneiform].

numerals for 𒋡 [Fri78, p. 43; Lec16]:



as in [P020016, rev. 1 4; P020065; P020090, obv. 1 3, rev. 2 1; P020092, rev. 3, 1; P020137, obv. 1 2] and others, where ban$_2$ counted with ▽ numerals are followed by sila$_3$ counted with ⟍ numerals. Curviform numerals are also used to count sila$_3$, but not[40] as part of the 𒄥 systems. This contrast can be seen in [P220927], which measures butter (𒉌, i$_3$) with a different capacity system, using the 𒂁 (dug, “pot”) of 20 𒋡, with ⊏ and ● numerals[41] for both the 𒂁 and the 𒋡, thus [Pow87, pp. 504 sq.]



but counts cheese (𒂵𒅈, ga’ar) using the 𒄥𒊕𒃲 capacity system, with ⟍ numerals for the 𒋡.

Another capacity system in ED IIIb Ŋirsu is the 𒄥𒐀𒌌, the gur of two ul [Lec16]:

Here the ⊏–⟍–▽ contrast occurs not only within the numerals of the system, but with its units; this is perhaps best illustrated by the expressions [illegible] 𒄥𒐀𒌌 in [P221746, rev. 2 2] and [illegible] 𒄥𒐀𒌌 in [P221814, rev. 1 5].

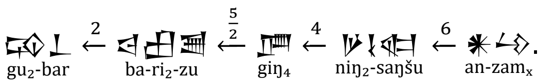
3.4.4 Grain in Ebla

The mixing of curviform and cuneiform numerals within a metrological system is not specific to Ŋirsu.

[40] As of this writing, the single occurrence of (ban2@c) followed by curviform numerals and sila3 in ED IIIb Ŋirsu transliterations on [CDLI], 4(ban2@c) 3(asz@c) sila3 in [P221815, obv. 4 7], is incorrect: it should be 4(ban2@c) 3(disz@t) sila3.

[41] This tablet also uses subtractive notation: ⊏⊏ 𒂁 𒇲 “two pots minus two thirds (sila$_3$)”, ● 𒇲 ⊏ 𒂁 𒐋 𒋡 “ten minus one pots, six sila$_3$”. Such subtractive notation is common in most of the metrological systems discussed here; it appears in the ED IIIa period [Rob08, p. 77]. It presents no complexity for character encoding, but it is noteworthy that the sign 𒇲 (lal, “minus”) is often ligated with the following numerals, with the subtrahend placed under a sometimes considerably enlarged 𒇲, similar to the layout of the radical in modern mathematical notation, see, *e.g.*, [P020092, rev. 3, 1, 2]. A good font could handle the very common −1 case, perhaps even −2 and −3; setting arbitrary numeric expressions under the 𒇲, or more generally replicating the layout of Early Dynastic tablets, is outside the realm of plain text; see also §6.2.

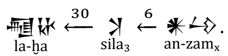
The system of grain[42] capacities in Ebla uses the following units[43]:



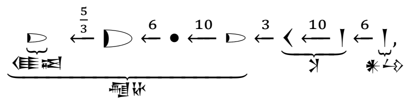
The 𒄘𒁇 and 𒁀𒌷𒍪 are generally counted using curviform numerals, and the smaller units using cuneiform 𒁹 numerals[44] Indeed, a search on [EbDA] for co-occurrences of either 𒀭𒍝 or 𒃻𒊕𒋗 with either of 𒄘𒁇 or 𒁀𒌷𒍪 finds the following expressions[45]:

1. [P240532, *verso* 4 9] ⊳≋ 𒁀𒊑𒅗[47] 𒐙 𒃻𒊕𒋗
2. [P240548, *verso* 1 1] ⊳𒁀𒊑𒍪 𒐗𒀭𒍝
3. [P240655, *recto* 7 9] ⊳⊳≋𒁇[48] 𒐙 𒃻𒊕𒋗
4. [P240579, *verso* 4 3] ⊳⊳⊳⊳⊳▽𒄘𒁇 𒐖𒃻𒊕𒋗
5. [P240675, *verso* 2 2] ⊳≋ 𒁀𒌷𒍪 𒐙𒃻𒊕𒋗
6. [P240609, *verso* 3 1] ⊳𒁀𒌷𒍪 𒐗𒀭𒍝
7. [P240533, *recto* 3 3] ●●⊳▽𒄘𒁇 𒐙𒃻𒊕𒋗 𒐖𒈦𒀭𒍝𒐘
8. [P240697, *recto* 1 5] ⊳▽[49]𒐖𒁾 𒐗𒀭𒍝
9. [P240653, *recto* 6 2] ●●⊳⊳⊳▽𒄘𒁇𒁹𒃻𒊕𒋗 𒐖𒈦𒀭𒍝
10. [P240654, *recto* 2 6] ⊳𒁀𒌷𒍪 𒐚𒋡[50] 𒐗𒀭𒍝[51]
11. [P240531, *recto* 1 8] ⊳𒁀𒌷𒍪 𒐚𒃻𒊕𒋗 ≋𒁾𒀳
12. [P241708, *recto* 1 1][52] ⊳⊳▽𒁾 𒄘𒁇 𒐙𒃻𒊕𒋗
13. [P241904, *recto* 1 1][53] ⊳⊳⊳⊳𒄘𒁇 𒐘[54]𒃻𒊕𒋗

[42]Liquid capacities use a different system [Arc15, p. 229 n. 12]:

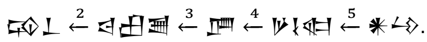


A glance it seems that 𒋡 are counted with cuneiform numerals and higher units with curviform ones, thus



but we have not investigated this thoroughly.

[43]Another system uses different values for the 𒁾 and 𒃻𒊕𒋗, see [Cha12, p. 62; Arc15, p. 229 n. 12]:



[44]As mentioned in [Cha12, p. 63], the 𒁾 is also counted using the 𒑖–𒑛 numeral series. Some instances of that usage are found transliterated $n/6$ in [EbDA]; in some cases the 𒁾 sign is omitted, and the 𒑖 numeral is then written before the 𒁇 unit, as in ⊳⊳𒑖≋𒁇 from [P240545, *verso* 1 3].

[45]We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform 𒁀𒌷𒍪 and larger with cuneiform 𒁾 and smaller. In all cases the transcriptions given here are based on the [EbDA] transliterations, but the shape and orientation of the numerals was checked[46]on a photograph (from [EbDA] unless noted otherwise).

[46]As we will see in §3.7.2, [CDLI] transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

[47]ba-ri₂-zu₂, a variant spelling.

[48]Short for 𒄘𒁇.

[49]Note the omitted 𒄘𒁇.

[50]Instead of the expected 𒃻𒊕𒋗.

[51]𒐗𒀭𒍝 not legible on the EbDA photo.

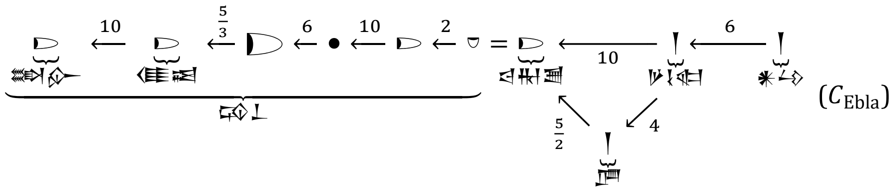
[52]From [CDLI] photo.

[53]From photo in [Arc89, p. 6].

[54]Laid out as 𒁹𒁹𒁹𒁹; on stacking patterns see §6.2.

14. [P240964, *recto* 3 2][55] [56] [57]

Note that higher numbers of are expressed in hundreds (*mi-at*) and then thousands (*li-im*), as is typical in Ebla [Arc15, p. 33], *e.g.*, in the last example above or in [P240532, *verso* 2 3], (100 + 60 + 30 + 5 = 195 of grain). These expressions correspond to the following factor diagram:



3.4.5 Use in modern publications

Because of their prevalence in the fourth millennium and Early Dynastic period, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 1–21.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 21. The partial[58] transliteration "4 *ʾa₃-da-um* 4 *aktum* 4 ib$_2^{\text{tu}_9}$×3 sa$_6$ gunu$_3$" is used to illustrate a discussion of the interpretation of the contrast between and numerals. More conventional transliterations might omit the numeral shapes entirely, *e.g.*, 4 *ʾa₃-da-um* 4 *aktum* 4 ib$_2^{\text{tu}_9}$×3 sa$_6$ gunu$_3$, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, *e.g.*, 4(aš^c) *ʾa₃-da-um* 4(diš^c) *aktum* 4(aš^c) ib$_2^{\text{tu}_9}$×3(diš) sa$_6$ gunu$_3$, but the result would be less readable. See §3.7.2 for a discussion of transliteration conventions for numerals.

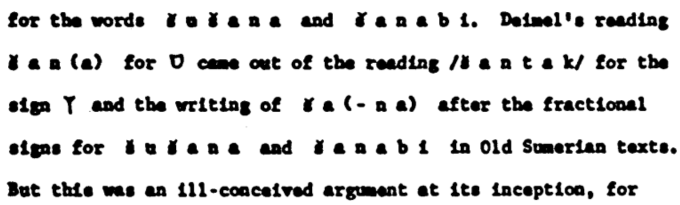

for the words š u š a n a and š a n a b i. Deimel's reading š a n (a) for came out of the reading /š a n t a k/ for the sign and the writing of š a (- n a) after the fractional signs for š u š a n a and š a n a b i in Old Sumerian texts. But this was an ill-conceived argument at its inception, for

Figure 1: Discussion of the readings of proposed and already-encoded in [Pow71, p. 107].

[55]From photo in [Arc89, p. 6]; see also the [CDLI] photo and the copy in [Fri86, p. 17]. This tablet features unusual usage of vertical numerals—"somewhat unorganized", as described by [Fri86, p. 16]—, such as or , but its and are consistently counted with cuneiform numerals, and the higher units with cuneiform numerals.

[56]Short for .

[57]ŠU$_2$+NIŊ$_2$-saŋ, an unusual variant spelling.

[58]The untransliterated text would be ; note the atomically encoded ib$_2$ × 3 = × = .

sions also. In example 6, the writing  may imply a reading /š a n a b i/,[1] whereas  in example 11 should be read */š u š a n a m i n/. Moreover, the question must be raised as to whether such writings as  k ù - b a b b a r + š a - n a[2] do not perhaps imply a linguistic resolution of */š u š a n a m i n/ rather than /š a n a b i/. I see no way of answering this question at present, but it is one which one

Figure 2: Discussion of the readings of proposed  and  as well as already-encoded and in [Pow71, p. 138].

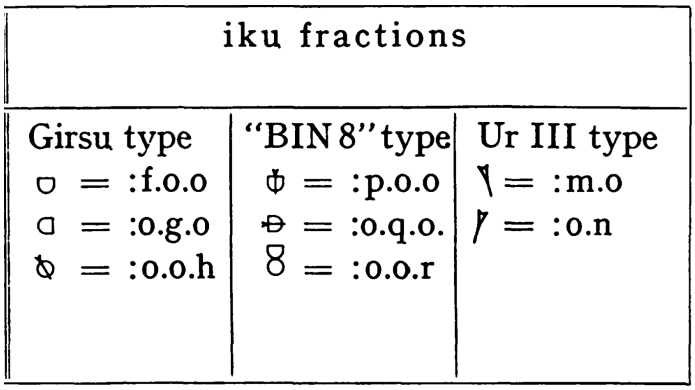
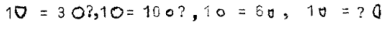


Figure 3: A transliteration system for the fractions of the iku in [Pow72, p. 216].

1 "big cup" = 3 "big disks". Hence we can infer from the two ŠE-texts BIN 8,4 and BIN 8,5 together, that the "ŠE-system" makes use of number signs whose values are related to each other through the equations



A more convenient way of saying the same thing is to write out the "steps" between the various ŠE-units in what we shall call a "step-diagram" for the "ŠE-system":

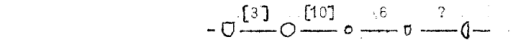


Figure 4: The first factor diagram, in [Fri78, p. 10].

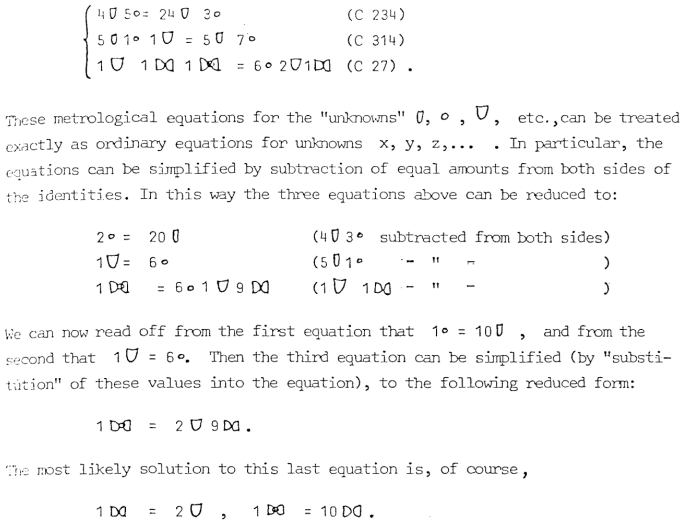


Figure 5: The derivation of the factors of the bisexagesimal system in [Fri78, p. 15][60].

[60]The bisexagesimal system is used alike in proto-Elamite and proto-cuneiform texts, see [Fri78, p. 38]; the derivation in [Fri78, p. 15] is based on proto-Elamite artefacts. There is a typo in the equation for C 27: the right-hand side should have 10 rather than 1, otherwise nothing could be deduced about . Note that in Friberg's early works [Fri78; Fri79; Fri86; Fri87], copies of fourth millennium and sometimes third millennium tablets are shown as vertical text (which they were for the scribes), and their numerals are written within horizontal text in the same orientation that they have if the tablet is taken as vertical text; in [UAX50] parlance, as if they had Vertical_Orientation=Upright. In addition, they are listed in these equations in the horizontal order in which they appear as vertical text (thus the rightmost numeral is the most significant, read first). Cuneiform is correctly Vertical_Orientation=Rotated, consistently both with modern practice and with the rotation between earlier vertical and later horizontal monumental inscriptions. Friberg's early conventions are not followed in later scholarship, and are abandoned in his own more recent works, such as [Fri07]; a more typical way to express the first equations might be

$$5\bullet + 4\text{▷} = 3\bullet + 24\text{▷} \qquad \text{(C 234)}$$

$$1\text{◗} + 1\bullet + 5\text{▷} = 7\bullet + 5\text{▷} \qquad \text{(C 314)}$$

$$1\text{⧖} + 1\text{⧗} + 1\text{◗} = 10\text{⧗} + 2\text{◗} + 6\bullet. \qquad \text{(C 27)}$$

A diplomatic edition of [Fri78] could rotate the numerals using a higher-level protocol:

$$\begin{cases} 4\text{▽}\,5\bullet = 24\text{▽}\,3\bullet & \text{(C 234)} \\ 5\text{▽}\,1\bullet\,1\text{▽} = 5\text{▽}\,7\bullet & \text{(C 314)} \\ 1\text{▽}\,1\text{▯}\,1\text{▯} = 6\bullet\,2\text{▽}\,1\text{▯} & \text{(C 27).} \end{cases}$$

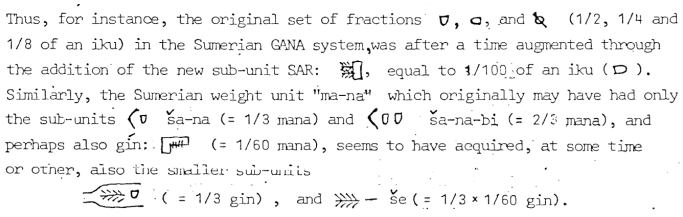

Thus, for instance, the original set of fractions 𒄀, 𒄁, and 𒄂 (1/2, 1/4 and 1/8 of an iku) in the Sumerian GANA system, was after a time augmented through the addition of the new sub-unit SAR: 𒄃, equal to 1/100 of an iku (𒌍). Similarly, the Sumerian weight unit "ma-na" which originally may have had only the sub-units 𒄀𒊭 ša-na (= 1/3 mana) and 𒄀𒄀𒊭 ša-na-bi (= 2/3 mana), and perhaps also gín: 𒄄 (= 1/60 mana), seems to have acquired, at some time or other, also the smaller sub-units 𒄅 (= 1/3 gín), and 𒄆 še (= 1/3 × 1/60 gín).

Figure 6: Discussion of proposed fractions 𒄀, 𒄁, 𒄂, and 𒄅, as well as already-encoded 𒐼 and 𒐽 in [Fri78, p. 49].

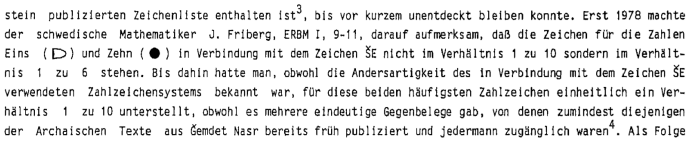

stein publizierten Zeichenliste enthalten ist[3], bis vor kurzem unentdeckt bleiben konnte. Erst 1978 machte der schwedische Mathematiker J. Friberg, ERBM I, 9-11, darauf aufmerksam, daß die Zeichen für die Zahlen Eins (𒌋) und Zehn (●) in Verbindung mit dem Zeichen ŠE nicht im Verhältnis 1 zu 10 sondern im Verhältnis 1 zu 6 stehen. Bis dahin hatte man, obwohl die Andersartigkeit des in Verbindung mit dem Zeichen ŠE verwendeten Zahlzeichensystems bekannt war, für diese beiden häufigsten Zahlzeichen einheitlich ein Verhältnis 1 zu 10 unterstellt, obwohl es mehrere eindeutige Gegenbelege gab, von denen zumindest diejenigen der Archaischen Texte aus Gemdet Nasr bereits früh publiziert und jedermann zugänglich waren[4]. Als Folge

Figure 7: Discussion in [DE87, p. 117] of the discovery in [Fri78, pp. 9–11] (see Figure 4) of the different relations between 𒌋 and ● in systems *G* and **??**.

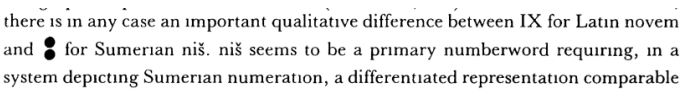

there is in any case an important qualitative difference between IX for Latin novem and ⁘ for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 8: The sign ⁘ used in a parallel with IX in [Eng88, pp. 131–133 n. 9], discussing an argument from [Pow72, p. 172] on the question of the language of the Uruk III texts.

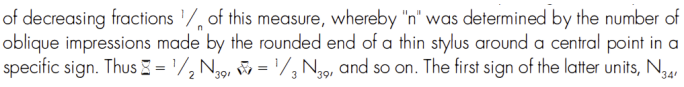

of decreasing fractions $^1/_n$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus 𒄇 = $^1/_2$ N_{39}, 𒄈 = $^1/_3$ N_{39}, and so on. The first sign of the latter units, N_{34},

Figure 9: Description of the fractions 𒄇 and 𒄈 in [Eng98, p. 113][61].

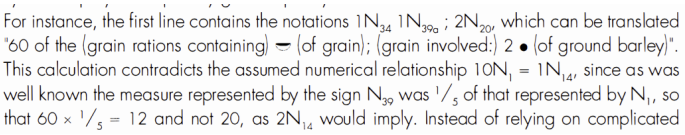

For instance, the first line contains the notations $1N_{34}$ $1N_{39a}$; $2N_{20}$, which can be translated "60 of the (grain rations containing) 𒌍 (of grain); (grain involved:) 2 ● (of ground barley)". This calculation contradicts the assumed numerical relationship $10N_1 = 1N_{14}$, since as was well known the measure represented by the sign N_{39} was $^1/_5$ of that represented by N_1, so that 60 × $^1/_5$ = 12 and not 20, as $2N_{14}$ would imply. Instead of relying on complicated

Figure 10: The sign 𒌍 used as a capacity measure within otherwise translated text in [Eng98, p. 116].

[61]The text erroneously has N_{34} instead of N_{24}.

Die halbkreisförmigen Griffeleindrücke gehen manchmal in mehr oder weniger eckige Formen über (▽)[685]. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen (𒐕), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren[686]. Selten treten mit ▽ gebildete Zahlen auf[687] (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 11: Discussion of co-occurrences and contrasts between 𒐕, 𒐖, and ▽ in [Kre98, p. 303].

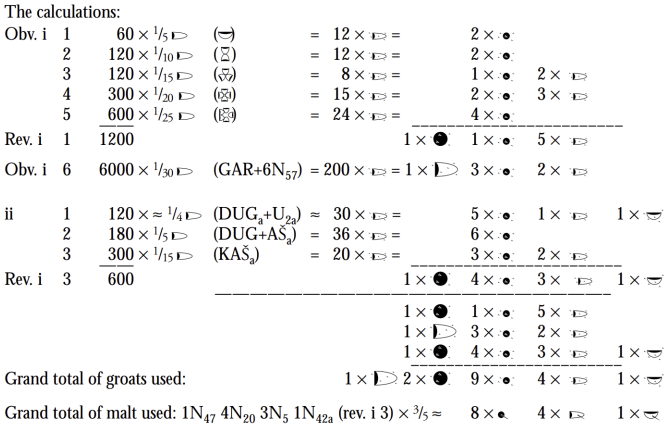


Figure 6. Transliteration and calculations of *MSVO* 4, 66.

Figure 12: Calculations from [P005468] transcribed in [Eng01, p. 132] using modern mathematical notation combined with some of the proposed characters.

strong similarities between "area" 1 and "area" 3 systems, the sign with two concentric discs (◎, notated N_{50}[27]) remains problematic. It never appears in any numerical combination with the sign with a single disc (●,

Figure 13: Discussion of ◎ and ●[62] in [Cha03, p. 6].

[62]The statement that these do not co-occur refers to the texts from ED I–II Ur; these signs co-occur both earlier and later in areas, with different relations as previously discussed.

1/15, etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation ⊢ 𒁹 𒄥 could be understood as 1 1/5 gur.[27] However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs ⊢ and 𒄥 cannot be considered klasmatograms.

Metrological tablets from the end of the 4th millennium (Nissen, Damerow and Englund 1993, 55-59, to *MSVO* 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

▷ 1 (iku) represents a surface of 3600m²
▷● 1 (eše3) represents a surface of 21,600m²
etc.

The signs iku and eše3 constitute by themselves measures of surface areas. These measures are usually followed by the sign GAN2, which means either surface or field and

Figure 14: Discussion of Old Babylonian[64] capacity and fourth millennium area measures in [Pro09, p. 9].

formed by only two signs 𒁹 and 𒌋, repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60², 60³, 1/60, 1/60², etc. are written in the same way, with the vertical wedge 𒁹. The third feature concerns the exact function of

Figure 15: Description of the SPVS in [Cha12, p. 58], using the already-encoded signs 𒁹 and 𒌋.

one step. The scribes of the Early Dynastic Period (*c.* 2600 BC), for instance, represented the number 648, 000 with: ⋿⊢●●● but never with the repetition ⋿⊢⋿⊢⋿⊢●●●.

Figure 16: Discussion of large numbers illustrated by ⋿⊢●●●[65] in [Cha12, p. 59]

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

▷▷▷ še bar ▽ *ba-ri-zu*

'3 *gubar* (capacity units) and 1 *parīsu*'.

Figure 17: Partial transliteration of [P240597, *recto* 5 3] ▷▷▷ 𒊺 𒁇 ▽ 𒁀𒊑𒍪 in [Cha12, p. 61].

[64] The cuneiform text is Unicode-encoded.

[65] Compare ◇⋿⊢ in system $G_{\text{Ur III/OB}}$. Sign order can be variable in early texts, see [Fox16, p. 8]. See [P010773], also discussed in [Fri07, p. 148], for an example of ⋿⊢●, and [P274845; P241764] for examples of n●⋿⊢.

This is particularly true of the signs , , and , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign is most often associated with the *parīsu* measure, while the signs , , and refer to 1, 2, 3,

Figure 18: Discussion in [Cha12, p. 64] of the relation between – and in Mesopotamia and in Ebla.

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., transliterated as 2(šar$_2$) 1(geš'u) 3(u), means 2 × 3600 + 1 × 600 + 3 × 10). The system is based on an alternation of factors ten and

Figure 19: Explanation of the structure of the number in [Pro20, p. 350].

might think of one fabric and a half,[11] but the presence of notations with "2 2", "3 3", and "6 6" (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag's weight measurement system.[12] The notation "1 gada" in o. ii 1 and r. vi 1, along with the total of "39

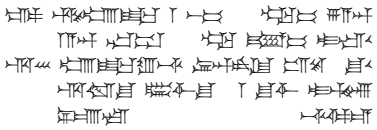
Fig. 1. Combinations of numerals attested in Š. 742.

Figure 20: Discussion of the contrast between and numerals in [Gor23, p. 162].

as, for example, in TM.75.G.3125 = *ARET* III 107 o. iv 1, "4 $'a_3$-*da-um*^{tu9}-2 4 aktum 4 ib$_2^{tu9}$ ×3 sa$_6$ gunu$_3$" (Fig. 2).

Figure 21: Transliteration in [Gor23, p. 163] of [P242293, *recto* 4 1] incorporating untransliterated numerals.

3.5 Non-numeric usage



The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'[66]. Do you know its reading[67]?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign 𒁹 has, *inter alia*, the values aš, rum, and dili. While the horizontal numerals are most frequently written 𒌋 in the Early Dynastic period[68], such non-numeric usage is almost[69] always written 𒁹, for instance:

- in personal names in administrative texts, such as the following, which all contain 𒌋 numerals:
  - 𒁹𒀀𒈾 in [P010424, rev. 1 5; P010458, obv. 1 5; P010459, obv. 2 5′] from ED IIIa أبو صلابيخ,
  - 𒁹𒄀 in [P010960, obv. 2 5] from ED IIIa Šuruppag,
  - 𒁹𒆤 in [P251641, obv. 4 3] from ED IIIb Adab,
  - 𒁹𒆤 in [P252866, rev. 2 3] from ED IIIb Adab,
  - 𒁹𒄀 in [P298637, rev. 2 4] from ED IIIb Umma;
- in the Sumerian word 𒌓𒁹 u_2-rum, "property" in ED IIIb Ŋirsu administrative texts which contain 𒌋 numerals, such as [P020006, obv. 2 3; P020008, rev. 1 2; P020018, rev. 1 2; P020024, obv. 1 4; P020030, obv. 3 1];
- in lexical texts:
  - in the divine name 𒀭𒂗𒍪𒁹𒆠 in the lexical texts [P010570, rev. 2 4; P010572, obv. 3 6], where the entries are prefixed with 𒌋.
  - in the word 𒁹 dili, "small fish" in [P010578, obv. 2 5], witness to Early Dynastic Fish,
  - in the same word with a determinative, 𒁹𒆜 $\text{dili}^{\text{ku}_6}$, in [P010586, obv. 4 4, 6], witness to Early Dynastic Food, which starts with 𒌋 numerals.

This is a clear contrast between 𒁹 and 𒌋 in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name 𒁹𒀀𒈾 occurs on its own line in the aforementioned administrative texts; a line 𒌋𒀀𒈾 would instead be read as "one slave".

3.6 The limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in §3.1. While these benefits are real and

[66] The reader will recall that ŋeš_2 is written 𒎙, with a larger wedge than 𒁕; however, these signs have merged by the time Examenstext A is composed.

[67] Besides ŋeš_2, a look at [OSL] shows that the values diš, ge_3, makkaš, saŋtak_4, and tal_4 are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

[68] A [CDLI] search for `"(asz@c)"` finds 3296 ED texts, while a search for `"(asz)"` finds 81 ED texts, of which 46 also contain `"(asz@c)"`.

[69] Exceptions are discussed in §3.7.1.

now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as 𒀸 that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [Sch35, pp. 123 sqq.] lists all of 𒀸–𒐉 together with ⊃–⊃⊃⊃⊃⊃, while 𒀸, 𒐀, and 𒐁, and only those, appear at the beginning of the sign list, since they have non-numeric values[70]. [Cat13, p. 58] has the numeric signs ⊃, 𒀹, 𒁹, whereas non-numeric 𒀸 is at the beginning of the sign list, where its values *aš* and *rum* are listed. For signs with both non-numeric and numeric usage, [Dei22] writes *s. die Zahlz.* throughout the main list; LAK 1 𒀸 thus reappears at LAK 829 together with ⊃, 𒀹, and 𒁹. One should note [Bor10], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have 𒌍–𒐏 used in system $G_{\text{Ur III/OB}}$.

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes[71] to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from ⊃ to 𒁹 numerals prevents diachronic encoding even if ⊃ were unified with 𒀸. For instance, the lexical list Early Dynastic Food, already mentioned in §3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1′–6′]; however, they are written with 𒁹 numerals, whereas they are written with ⊃ numerals in the Early Dynastic witnesses; since 𒁹 and 𒀸 are distinct[72] characters, the ⊃-𒀸 unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods[73], there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Rom23, *sub* "Adding Corpora"], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

[70]Non-numeric values of 𒀸 were discussed in §3.5; 𒐀 has the values man₃ and min₅, and is used for the word didli, "several, various"; 𒐁 has the value eš₆.

[71]Compare, *e.g.*, in the *Instructions of Šuruppag*, 𒋢𒆳𒊒𒆠 𒌉𒉌⊃ 𒈾𒈾𒊑 / 𒆠 𒆠𒁀𒁲𒁲 in the ED IIIa witness [P222243, obv. 2 7], also discussed in §3.7.1, and 𒋢𒆳𒊒𒆠𒂊 𒈾𒈾𒊑 𒆠 𒆠𒁀𒉆𒁲𒁲 in the OB composite [Q000782, 6] (translated "Šuruppag gave instructions to his son" in [ETCSL, t.5.6.1, 1–13]). It does not matter for the construction of a composite text whether this is encoded 𒌉𒉌⊃ or 𒌉𒉌𒀸, since that word is absent from other witnesses, and since the surrounding words differ.

[72]Besides the contrasts in numeric usage mentioned in §3.3.3, these (already-encoded) characters were clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of 𒀸 which are not shared with 𒁹 have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period, 𒀸 is used for the preposition *ina*, "in", and 𒁹 for the preposition *ana*, "to".

[73]See, *e.g.*, [Pow87, p. 493; Rob08, p. 55] on the unification of metrologies in the Old Akkadian period, resulting in the systems described in §3.3.

3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts[74] being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

3.7.1 The case of ŠAR$_2$

The character U+122B9 ● CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period[75], a contrast between non-numeric šar$_2$ written ◇ and numeric 1(šar$_2^c$) written ● can be observed, similar to the contrast between [cuneiform] and [cuneiform] previously discussed in §3.5. However, in lexical lists from Šuruppag and Ebla[76], as well as in the *Stèle des vautours*, non-numeric šar$_2$ is curviform:

— 𒀭 [cuneiform] ● and 𒀭 [cuneiform] ● [cuneiform] in [P010566, obv. 10 10, 11];
— ● [cuneiform] and 𒀭 ● [cuneiform] in [P010576, rev. 3 16, 17];
— ● [cuneiform] in [P240986, *recto* 3 3][77];
— ● [cuneiform] in [P222399, obv. 17 9, 18 11, 22 12][78].

It *would* be disruptive to the diachronic representation of text if non-numeric šar$_2$ were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph ◇, identical to the glyph of the proposed U+12579 ● CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric šar$_2$ is curviform solidly predates the transition from ● to ◇ in the relevant metrological systems, there is no need to represent a ◇-● contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D ◇ CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

---

[74] Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

[75] For example, in personal names:

— [cuneiform] ◇ [cuneiform] in [P020019, rev. 1 2] from ED IIIb Ŋirsu;
— [cuneiform] ◇ [cuneiform] in [P020182, obv. 2 9], also from ED IIIb Ŋirsu;
— [cuneiform] 𒀭 ◇ in [P222186, obv. 3 3] from ED IIIb Umma;
— [cuneiform] 𒀭 [cuneiform] ◇ in [P235312, obv. 16] from Old Akkadian Umma.

[76] These are archaizing in other ways, *e.g.*, they have a [cuneiform]–[cuneiform] (NAM$_2$–TUG$_2$) split.

[77] From copy in [Man81, ELLes 397].

[78] Note however 𒀭 [cuneiform] ◇ [cuneiform] on [P222399, obv. 6 17], see Figure 22. Curviform non-numeric šar$_2$ is clearly archaizing in ED IIIb Ŋirsu; one might suppose that the scribe slipped into their modern ways here.

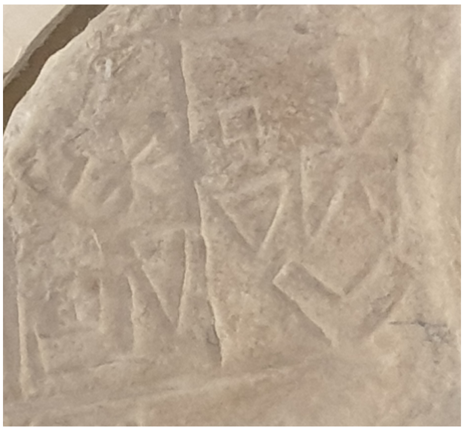


Figure 22: [P222399, obv. 6 16–17] / .

Note that in rare cases, such as [P222243, obv. 2 7] from ED IIIa Adab, non-numeric 𒀸 (here with the value rum) is written 𒕐. It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 𒕐 cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. in practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the [CDLI] transliteration (UR2-1(aš@c)) and the [ePSD2] one ($uru_8{}^{rum}$) of this word disagree on that aspect. Since 𒀸 has a cuneiform reference glyph, this does not pose any compatibility concerns.

3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL_2 can be described as 𒉈×𒆜 NE×KASKAL or 𒉈×𒉽 NE×PAP[79], they are typically all transliterated bil_2, and therefore are all represented by the character U+1224B 𒉋 CUNEIFORM SIGN NE SHESHIG, its name notwithstanting, as described in [UTR56, §2.5].

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, 𒁹𒌍𒐊𒄥 from [P305639, rev. 21] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in [EbDA] transliterations: the aforementioned from [P240533, *recto* 3 3] is transliterated "20-1-1/2 *gu_2-bar* 7 nig_2-sagšu 2-1/2 an-zam_x[80] *za*", reading both and as 1/2, but not distinguishing them.

[79] As on [P249253].

[80] As of this writing, [EbDA] actually has an-zam_χ, with U+1D6A GREEK SUBSCRIPT SMALL LETTER CHI.

In particular, these transliterations do not differentiate between 𒀸 and 𒁹 numerals, nor between curviform and cuneiform numerals. For instance, the aforementioned [P242293, *recto* 4 1] is transliterated "4 ʾa_3-*da-um*^{tug_2}-II 4 aktum^{tug_2} 4 ib_2-III gun_3 sa_6 ^{tug_2}" in [EbDA], with no distinction between the curviform and cuneiform numerals. Since 𒀸 and 𒁹 numerals are separately encoded, the numeric expressions in such transliterations cannot be transformed into Unicode cuneiform without additional context, regardless of curviform–cuneiform unification.

In metrological systems such as systems $G_{\mathrm{Ur\ III/OB}}$ and $C_{\mathrm{Ur\ III/OB}}$ where some units are indicated by the type of numeral rather than an overt unit sign, it is common practice to add the unit in parentheses in transliteration; for instance, the sequence from [P386847, obv. 1] is transliterated "1($eše_3$) 5½ iku[81] 7 sar" in [Feu04, vol. 2, p. 176], and the sequence from [P307255, obv. 12] is transliterated "1(n[82]) 2(b) 7 ½ $sila_3$" in [Feu04, vol. 2, p. 151].

This practice has been generalized to systematically indicate numeral shape; this is in particular the case in [CDLI], where the transliterations of some the above examples are "1(gesz2) 3(u) 5(asz) gur" for the first example, "1(esze3) 5(iku) 1/2(iku) GAN2 7(disz) sar" for the second, and "3(barig) 2(ban2) 7(disz) 1/2(disz) sila3" for the third. [CDLI] and [ePSD2] both distinguish curviform from cuneiform numerals in transliteration: the length from [P020129, rev. 2 1] is transliterated "6(gesz2@c) 3(u@c) {ninda}nindax(DU) 1/2(asz@c) 4(disz@t) gi" in [CDLI], and "6($geš_2^c$) 3(u^c) $^{ninda}ninda_x$(DU) 1/2($aš^c$) 4($diš^t$) gi" in [ePSD2]. Another example is [Mol14, p. 39], which uses 1*a* for 𒀸, 1*d* for 𒁹, 1*ac* for the curviform form, 1*dc* or ½*dc* for the curviform form depending on reading, etc. The literature on the Uruk and Early Dynastic I–II periods uses a different set of transliteration conventions that also disambiguate numeral shapes, as will be discussed in §4.

While there exist transliterations that distinguish 𒀸 from 𒁹 but not the curviform numeral from 𒀸, such as the ones used in [DCCMT], the trend, especially in more recent works in third millennium studies, seems to be to represent numeral shape; for example, [MV24] gave an example of the input syntax used by the new "Urban Economy Begins" project as "10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)" for the example sequence, with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are one form rather than the other numerals. The "tradition of cavalierly dispensing with numerical notations in notations of administrative documents", as [Eng04, p. 30] describes it, seems to be fading.

3.8 Conclusions

Co-occurences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications. The contrast frequently applies to individual numerals, rather than to the span of entire numeric expressions.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple

[81] The sign interpreted as a unit, as discussed in §3.3.

[82] short for nigida, an older reading of bariga; see [**Landsberger1950**; Pow75, p. 181; Fox22, p. 9].

cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted: 𒌍 is a length of three ropes, but ●●● is an area of three bur_3; ⊳𒁹 could be read as one 𒃻𒂗 and one 𒉺𒅅𒀴, where ⊳▿ would be one and a half 𒃻𒂗; 𒁹𒀴 is a personal name, but ⊳𒀴 would be "one slave".

In addition, there would be a risk of confusion about character identity should fontmakers attempt to treat the curviform and cuneiform numerals as unified. A designer concerned about the numeric-syllabic ⊳–𒁹 contrast, and wishing to support diachronic encoding between systems $S_{\text{Ur III/OB}}$ and S, might give the 𒁹 numeral series (which is typically only used numerically in the Early Dynastic period) the glyphs of the ⊳ numeral series, since the clear 𒁹–⊳ identification involves the same rotation; this would however make it impossible to represent capacity measures that use ▿. Similarly, in an effort to support diachronic encoding for 1/2(iku), one might be tempted to give 𒀹 the glyph of ▿, thereby rendering the font unusable for quantities measured using the 𒀹 numeral series; an ED I–II Ur font designer could decide to give 𒅅 the same glyph as ◇ (that of the proposed ◉), according to the older area system, making it impossible to represent the newer system.

At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millennium corpora. The oddity of ● requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.

4 Rationale for ED–Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millennium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millennium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs KAŠ_a, KAŠ_b, and KAŠ_c, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Eng01, pp. 34 sq.]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered scripts.

As part of the development of these conventions, a classification of fourth millennium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter N with a numeric subscript (sometimes with an additional alphabetic subscript): N_1 is ⊳, N_{14} is •, N_{34} is ⊳, etc. Transliterations of numeric expression then use those to identify the type of number used, thus $5N_1$ is ⊳⊳⊳⊳⊳, and $5N_{14}$ is •••••.

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where

numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the [CDLI] transliteration of or in [P283802, rev. 1 6, 2 2] currently uses (barig@c) for the vertical numerals, since numerals are typically capacity measures; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millennium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I–II period, even though the Sumerian text uses classical assyriological transliteration conventions; see [Cha03, p. 6 n. 27].

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millennium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in §3.7.2; they are indeed used interchangeably, as in [P011104] which uses the notation u@f in [ePSD2], but N14@f in [CDLI]. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millennium would therefore induce confusion as to which numerals should be used in third millennium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millennium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance, N_{30c} is listed as a variant of N_{30a} in [DE87, p. 166], where the numeric value of either in relation to N_{39a} is still unknown, but their values are found in [Eng04, p. 33] to be $= \frac{1}{10}$, whereas $= \frac{1}{6}$.

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban$_2$@c) and 5(ban$_2$@c).]

6 Characters not included in this proposal

6.1 Missing numerals

TODO "Ten of the sixty numerical signs contained in the list in figure 27, moreover, do not belong to any of the identified systems. Three of them were apparently scribbled by an awkward pupil. As to four of those remaining, we are not sure whether they constitute derivations of other, as yet unknown numerical signs or whether they are in fact numerical signs at all. For at least two of the ten signs, txi and we can affirm that each formed part of two additional systems, about which we know nothing due to the fact that no informative texts have been unearthed with notations in these systems." [NDE93, p. 27] TODO N10 described as coming from P001319 which does not have it anymore. TODO N13 not attested in CDLI TODO (N_{17} not usefully numeric, $12N_{14}$ not encodable, etc.). Cite [DE87, p. 147] N30Cb̆ not attested 7 and 8(diš *tenû*) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.

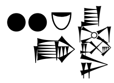


Figure 23: The layout of case [P011099, rev. 2 3]; the numeral ⁚ is rotated to fit the rounded corner of the tablet.

6.2 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some *stacking patterns*; for instance 9𒁹 is encoded both as U+12446 𒑆 and as U+1240E 𒐎. This is in part due to contrastive usage of stacking patterns. For instance, besides 𒁹 and 𒈫 which are characteristic of bariga measures, four bariga is written 𒐼 even where 4𒁹 is written 𒐉, as in [P255010, obv. 2 3, rev. 1 17; P292843, obv. 4, rev. 5]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often 𒁹 𒈫 𒐈 𒐉 𒐊 𒐋 𒐌 𒐍 𒐎 𒌋 𒎙 𒌍 𒐏 𒐐, and results in metrological systems, typically 𒁹 𒈫 𒐈 𒐼 𒐊 𒐋 𒑂 𒑄 𒑆 𒌋 𒎙 𒌍 𒑩 𒑪, occasionally co-occurring as in [P142827; P142357]. This separate encoding is also for compatibility with distinctions made in reference works and in some non-numeric transliterations; for instance, 𒐉 is [Bor10, MZL 860] and has the value limmu, whereas 𒐼 is [Bor10, MZL 852] and has the value limmu$_5$. Numeric[83] transliterations occasionally distinguish the stacking patterns 𒐉 𒐌 𒐍 𒐎 𒐏 𒐐, as in the [CDLI] transliterations of the aforementioned tablets, although this is rare; often 4(diš) is 𒐼 in Ur III, but 𒐉 in the Neo-Assyrian period.

However, the stacking patterns from earlier periods are not separately encoded; for instance, in ED IIIb Ŋirsu, 𒎙 2(u) often has one 𒌋 atop another. These older stacking patterns do not appear to be contrastive, are not marked in transliteration, and are not listed separately in sign lists nor assigned any different values. There is therefore no evidence of a need to encode them; instead, they should be considered style variants, and an ED IIIb Ŋirsu font should have an appropriate glyph for U+12399 𒎙 CUNEIFORM SIGN U U.

Likewise, many stacking patterns are attested for the curviform numerals proposed in this document, and it is not proposed to separately encode them. These distinctions would be incompatible with the state of the art in numeric transliterations, including those by Englund, who insisted on “a system of transliteration that reflects in a strict fashion the physical realities of the cuneiform inscriptions” [Eng04, p. 30], and they are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier tablets, as they are structured in rectangular cases rather than lines, so that numerals may be laid out across the case in whichever way fits the available space; this is illustrated in Figure 23. Note also that the numerals need to be considerably enlarged in order to reproduce the layout of the tablets, so that ⁚ often spans two lines of cuneiform signs, as shown in Figure 24. This is impractical when these numerals are set in text that contrasts them with the larger ◗, and inconsistent with actual practice when typesetting these numerals, as illustrated in Figure 8: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dyn-

[83]The Sumerian word limmu means “four”, so limmu and limmu$_5$ are still numbers. The distinction here is between usage in transliterations of phrases such as 𒈗 𒀭𒌒𒁕 𒐼𒁀𒆤 lugal an-ub-da limmu$_5$-ba-ke$_4$ (king of the four quarters) or of names, and of numeric expressions such as 𒐼𒋡 4(diš) sila$_3$.

Figure 24: The layout of case [P020066, obv. 1 1]; the numeral is spread across two lines. The text is read in the order , "twenty-two oxen, one year old".

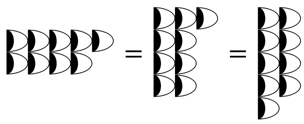


Figure 25: Three stacking patterns for U+12573 CUNEIFORM NUMERIC SIGN NINE N34. The one on the left is the reference glyph, used in Uruk III [P003499, obv. 1 1b; P004430, rev. 1 2], and widely afterwards, *e.g.*, ED IIIa Šuruppag [P010678, obv. 2], ED IIIb Ŋirsu [P020057, obv. 1 3], Old Akkadian Umma [P212464, obv. 11]. The ones in the middle and right are used in two Uruk IV tablets [P001243, rev. P004500, rev. 2]. All three Uruk examples are transliterated 9(N34) in [CDLI].

astic period, but that are also attested in the fourth millennium in the Uruk III period; the fourth millennium, especially the Uruk IV period, also frequently features numerals that use a more vertical layout, as illustrated in Figure 25. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millennium, usage in third millennium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents , , •, and ● must remain consistent across the numeral series, lest a numeral be confused with an numeral. Since the single indents are frequently used in running text, as illustrated in §3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 25 uses OpenType stylistic alternates, and Figure 23 rotates the character , in both cases preserving the plain text backing.

6.3 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Eng01].

Acknowledgements

TODO(egg): Something about the Vanséveren fonts; acknowledge the reviewers and whoever referred me to relevant literature; something about the proposal font.

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Artefacts

[P001243] VAT 14991. Berlin, Germany: Vorderasiatisches Museum.
CDLI: P001243.

[P003499] Excavation number W 20274,001.
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[P004500] IM 134965. Baghdad, Iraq: المتحف العراقي.
CDLI: P004500.

[P005468] IM 023426. Baghdad, Iraq: المتحف العراقي.
CDLI: P005468.

[P010424] IM 067642. Baghdad, Iraq: المتحف العراقي.
CDLI: P010424.
ORACC: epsd2/P010424.

[P010458] IM 081445. Baghdad, Iraq: المتحف العراقي.
CDLI: P010458.
ORACC: epsd2/P010458.

[P010459] IM 081449. Baghdad, Iraq: المتحف العراقي.
CDLI: P010459.
ORACC: epsd2/P010459.

[P010566] VAT 12760 +. Berlin, Germany: Vorderasiatisches Museum.
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ORACC: epsd2/P010566.
ORACC: dcclt/P010566.

[P010570] VAT 12626. Berlin, Germany: Vorderasiatisches Museum.
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ORACC: epsd2/P010570.
ORACC: dcclt/P010570.

[P010572] VAT 12644. Berlin, Germany: Vorderasiatisches Museum.
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ORACC: epsd2/P010572.
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ORACC: dcclt/P010576.

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ORACC: dccmt/P010678.

[P010773] Ist Š 0188. Istanbul, Turkey: İstanbul Arkeoloji Müzeleri.
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ORACC: dccmt/P010773.

[P010876] Ist Š 0648. Istanbul, Turkey: İstanbul Arkeoloji Müzeleri.
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ORACC: epsd2/P020066.

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[P142357] YBC 01793. New Haven, Connecticut, United States: Yale Babylonian Collection.
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[P142827] Ashm 1924-0667. Oxford, United Kingdom: Ashmolean Museum.
CDLI: P142827.
ORACC: epsd2/P142827.

[P212464] WML unn 003. Liverpool, United Kingdom: World Museum.
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[P213162] PULiège 0028. Université de Liège.
CDLI: P213162.
ORACC: epsd2/P213162.
ORACC: dccmt/P213162.

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ORACC: epsd2/P221531.

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ORACC: epsd2/P221746.

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ORACC: epsd2/P221814.

[P221815] Erm 14046. Saint Petersburg, Russia: Государственный Эрмитаж.
CDLI: P221815.
ORACC: epsd2/P221815.

[P222186] FLP 0003. Philadelphia, Pennsylvania, United States: Free Library of Philadelphia.
CDLI: P222186.
ORACC: epsd2/P222186.

[P222243] OIM A00645 + OIM A00649a–i. Chicago, Illinois, United States: Institute for the Study of Ancient Cultures, West Asia & North Africa (formerly Oriental Institute) Museum.
CDLI: P222243.
ORACC: epsd2/P222243.

[P222399] *Stèle des vautours*. BM 023580 (= AO 16109) + AO 00050 + AO 02436 + AO 02437 + AO 02438. Paris, France: Musée du Louvre.
CDLI: P222399.
ORACC: etcsri/Q001056.

[P232278] *Gudea E*. AO 00006. Paris, France: Musée du Louvre.
CDLI: P232278.
ORACC: etcsri/Q001544.

[P232280] *Gudea G*. AO 00007. Paris, France: Musée du Louvre.
CDLI: P232280.
ORACC: etcsri/Q001546.

[P235312] USC 6710. Los Angeles, California, United States: University of Southern California.
CDLI: P235312.
ORACC: epsd2/P235312.

[P240531] Excavation number TM.75.G.00265.
CDLI: P240531.
EbDA: 1415.

[P240532] Excavation number TM.75.G.00266.
CDLI: P240532.
EbDA: 1324.

[P240533] Excavation number TM.75.G.00267.
CDLI: P240533.
EbDA: 1379.

[P240545] Excavation number TM.75.G.00299.
CDLI: P240545.

[P240548] Excavation number TM.75.G.00302.
CDLI: P240548.
EbDA: 1350.

[P240579] Excavation number TM.75.G.00341.
CDLI: P240579.
EbDA: 1364.

[P240597] Excavation number TM.75.G.00407.
CDLI: P240597.

[P240609] Excavation number TM.75.G.00440.
CDLI: P240609.
EbDA: 1378.

[P240653] Excavation number TM.75.G.00535.
CDLI: P240653.
EbDA: 1382.

[P240654] Excavation number TM.75.G.00536.
CDLI: P240654.
EbDA: 1383.

[P240655] Excavation number TM.75.G.00537.
CDLI: P240655.
EbDA: 1358.

[P240675] Excavation number TM.75.G.00557.
CDLI: P240675.
EbDA: 1371.

[P240697] Excavation number TM.75.G.00579.
CDLI: P240697.
EbDA: 1381.

[P240964] Excavation number TM.75.G.01392.
CDLI: P240964.
ORACC: dccmt/P240964.
EbDA: 3184.

[P240986] Excavation number TM.75.G.01415.
CDLI: P240986.
ORACC: dcclt/P240986.

[P241708] Excavation number TM.75.G.02143.
CDLI: P241708.
EbDA: 3173.

[P241764] Excavation number TM.75.G.02200.
CDLI: P241764.

[P241904] Excavation number TM.75.G.02346.
CDLI: P241904.
EbDA: 3183.
ORACC: dccmt/P241904.

[P242293] Excavation number TM.75.G.03125.
CDLI: P242293.
EbDA: 217.

[P249253] *Code de Hammurabi*. Sb 00008. Paris, France: Musée du Louvre.
CDLI: P249253.

[P251641] MS 2464. Oslo, Norway: Schøyen Collection.
CDLI: P251641.
ORACC: epsd2/P251641.

[P252866] MS 3830. Oslo, Norway: Schøyen Collection.
CDLI: P252866.
ORACC: epsd2/P252866.

[P255010] YBC 04698. New Haven, Connecticut, United States: Yale Babylonian Collection.
CDLI: P255010.

[P271238] Anonymous.
CDLI: P271238.
ORACC: epsd2/P271238.

[P274845] CUNES 50-08-001. Ithaca, New York, United States: Department of Near Eastern Studies, Cornell University.
CDLI: P274845.

[P283802] Ist Š 0742. Arkeoloji Müzeleri.
CDLI: P283802.
ORACC: epsd2/P283802.

[P292843] NBC 05385. New Haven, Connecticut, United States: Nies Babylonian Collection, Yale Babylonian Collection.
CDLI: P292843.

[P298637] NBC 06978. New Haven, Connecticut, United States: Nies Babylonian Collection, Yale Babylonian Collection.
CDLI: P298637.
ORACC: epsd2/P298637.

[P305639] YBC 04398. New Haven, Connecticut, United States: Yale Babylonian Collection.
CDLI: P305639.

[P307255] YBC 06219. New Haven, Connecticut, United States: Yale Babylonian Collection.
CDLI: P307255.

[P309594] YBC 08761. New Haven, Connecticut, United States: Yale Babylonian Collection.
CDLI: P309594.
ORACC: epsd2/P309594.

[P386847] AO 06377. Paris, France: Musée du Louvre.
CDLI: P386847.

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CDLI: Q000782.
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