Archaic cuneiform numbers

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1 Summary

This document proposes encoding, at U+12550–U+12585, 310 numerals used in the fourth millennium (Uruk IV and Uruk III periods) and Early Dynastic period in conjunction with the Sumero-Akkadian cuneiform script[1] and the proto-cuneiform script[2]. The proposed characters are listed in §2. Most of them were listed in [L2/23-190]. The present document provides a more detailed rationale for their encoding and additional information about their identity and usage, both as part of the rationale and in §5. Some characters have been removed, in some cases because they are non-encodable variants, in others because their encodability should be considered as part of the proto-cuneiform proposal; these are discussed in §6. The glyphs have also been reworked, and additional characters used in the Early Dynastic period have been added.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model requires that the language of a text be understood to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in §4. The proposed "curved", or "curviform", numerals[3] should however *not* be unified with the already-encoded cuneiform numerals[4]. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in §3, including compatibility considerations in §3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in §2.3 reflect the overlap. Many of these numerals are also used in proto-Elamite[5] texts, where they are treated as identical characters in scholarship on proto-Elamite, so that they should

[1][ISO15924]: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

[2][ISO15924]: Pcun, not yet encoded.

[3]Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: , (small stylus), (large stylus). Some numerals are composed of multiple such impressions, *e.g.*, . The terms "curved", "curviform", "curvilinear", and "round" can be found in the literature. We avoid the term "round" here as it has other meanings in the context of numbers. We use "curviform" in this document as, being the least common term, it is least likely to lead to confusion, and "CURVED" in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [**inlineATF**].

[4]Impressed into clay using a stylus with a trihedral end: (stylus held horizontally), (vertically), (diagonally) (with the head of the stylus), (stylus pressed deeper, forming a larger wedge), (combining and), etc.

[5][ISO15924]: Pelm, not yet encoded.

be unified with the ones proposed in [L2/23-196]. However, in the interest of time, we do not provide a detailed rationale for this unification in this document, and we are not proposing that the numerals be given the corresponding Script_Extensions property value for now. Neither do we propose encoding any numerals that are solely attested in proto-Elamite texts, or well-attested in proto-Elamite texts but insufficiently attested in Uruk—those are discussed in §2.

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal	This proposal	
Numerals		Existing Xsux	Existing Xsux
Non-numeric signs	Future Pcun	Existing Xsux	Existing Xsux

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

2 Proposed changes to the Standard

2.1 Core specification text

Amend [Uni16, §11.1.2, *sub* "Cuneiform Numerals"], as follows:

> ***Cuneiform Numerals.*** In general, numerals that also have a phonetic, logographic, or determinative value are encoded in the main Cuneiform block; as a result, some series of numerals, such as 𒁹–𒐎 1(diš)–9(diš) or 𒌋–𒐔 1(u)–9(u), are split across the two blocks. Numerals have been encoded separately from signs that are visually identical but ~~semantically different~~ etymologically unrelated (for example, U+1244F 𒑏 CUNEIFORM NUMERIC SIGN ONE BAN2, U+12450 𒑐 CUNEIFORM NUMERIC SIGN TWO BAN2, and so on, versus U+12226 𒈦 CUNEIFORM SIGN MASH, U+1227A 𒉺 CUNEIFORM SIGN PA, and so on).
>
> The relation between series of numerals depends on the metrological system; for instance, when counting talents, written 𒄘 (a unit of weight, approximately 30 kg), 𒀸𒄘 is used for "one talent", and 𒌋𒄘 for "ten talents". However, when measuring areas, the area 𒌋𒃷 (one *būrum*) is eighteen times 𒀸𒃷 (one *ikûm*, approximately 3600 m^2). The Numeric_Value property assignment of a cuneiform numeral therefore reflects only its relation to the first numeral in its series, rather than the absolute numeric value that it might represent. For instance, the number "fifty" is written 𒐐, but U+12410 𒐐 CUNEIFORM NUMERIC SIGN FIVE U has Numeric_Value=5, as it is 5 × 𒌋.
>
> In the third millennium, and especially in the Early Dynastic period, some numerals are written using a cylindrical tool, rather than the cuneiform stylus, forming curved rather than cuneiform numerals (rather than 𒀸). The cuneiform numerals are descended from these curved numerals. However, in the Early Dynastic period, the curved numerals contrast with the cuneiform ones, and are used together with them in several metrological systems; they are therefore separately encoded. Most curved numerals are encoded in the Archaic Cuneiform

Numerals block, with the exception of two fractions in the Cuneiform Numbers and Punctuation block: U+1245D 𒑝 CUNEIFORM NUMERIC SIGN ONE THIRD VARIANT FORM A and U+1245E 𒑞 CUNEIFORM NUMERIC SIGN TWO THIRDS VARIANT FORM A, the curved counterparts of U+1245A 𒑚 CUNEIFORM NUMERIC SIGN ONE THIRD DISH and U+1245B 𒑛 CUNEIFORM NUMERIC SIGN TWO THIRDS DISH.

Add after [Uni16, §11.1.3]:

11.1.4 Archaic Cuneiform Numerals: U+12550–U+1268F

TODO something about consistent size

The sign ŠAR$_2$.

Amend [Uni16, §24.1.2, *sub* "Dashed Box Convention"], third paragraph, as follows:

> In a few cases of very wide ~~punctuation~~ characters that do not naturally fit into a code chart cell, the representative glyph may be shown with an artificially narrow shape, displayed inside the dashed box, with or without additional annotation, to indicate this adjustment of shape.

2.2 Code charts

The code charts for the proposed block, including the character names list with proposed informative aliases, cross references, and informative notes, are shown on the following pages.

This space for rent.

	1255	1256	1257	1258	1259	125A	125B	125C	125D	125E
0	12550	12560	12570	12580	12590	125A0	125B0	125C0	125D0	125E0
1	12551	12561	12571	12581	12591	125A1	125B1	125C1	125D1	125E1
2	12552	12562	12572	12582	12592	125A2	125B2	125C2	125D2	125E2
3	12553	12563	12573	12583	12593	125A3	125B3	125C3	125D3	125E3
4	12554	12564	12574	12584	12594	125A4	125B4	125C4	125D4	125E4
5	12555	12565	12575	12585	12595	125A5	125B5	125C5	125D5	125E5
6	12556	12566	12576	12586	12596	125A6	125B6	125C6	125D6	125E6
7	12557	12567	12577	12587	12597	125A7	125B7	125C7	125D7	125E7
8	12558	12568	12578	12588	12598	125A8	125B8	125C8	125D8	125E8
9	12559	12569	12579	12589	12599	125A9	125B9	125C9	125D9	125E9
A	1255A	1256A	1257A	1258A	1259A	125AA	125BA	125CA	125DA	125EA
B	1255B	1256B	1257B	1258B	1259B	125AB	125BB	125CB	125DB	125EB
C	1255C	1256C	1257C	1258C	1259C	125AC	125BC	125CC	125DC	125EC
D	1255D	1256D	1257D	1258D	1259D	125AD	125BD	125CD	125DD	125ED
E	1255E	1256E	1257E	1258E	1259E	125AE	125BE	125CE	125DE	125EE
F	1255F	1256F	1257F	1258F	1259F	125AF	125BF	125CF	125DF	125EF

	125F	1260	1261	1262	1263	1264	1265	1266	1267	1268
0	125F0	12600	12610	12620	12630	12640	12650	12660	12670	12680
1	125F1	12601	12611	12621	12631	12641	12651	12661	12671	12681
2	125F2	12602	12612	12622	12632	12642	12652	12662	12672	12682
3	125F3	12603	12613	12623	12633	12643	12653	12663	12673	12683
4	125F4	12604	12614	12624	12634	12644	12654	12664	12674	12684
5	125F5	12605	12615	12625	12635	12645	12655	12665	12675	12685
6	125F6	12606	12616	12626	12636	12646	12656	12666	12676	
7	125F7	12607	12617	12627	12637	12647	12657	12667	12677	
8	125F8	12608	12618	12628	12638	12648	12658	12668	12678	
9	125F9	12609	12619	12629	12639	12649	12659	12669	12679	
A	125FA	1260A	1261A	1262A	1263A	1264A	1265A	1266A	1267A	
B	125FB	1260B	1261B	1262B	1263B	1264B	1265B	1266B	1267B	
C	125FC	1260C	1261C	1262C	1263C	1264C	1265C	1266C	1267C	
D	125FD	1260D	1261D	1262D	1263D	1264D	1265D	1266D	1267D	
E	125FE	1260E	1261E	1262E	1263E	1264E	1265E	1266E	1267E	
F	125FF	1260F	1261F	1262F	1263F	1264F	1265F	1266F	1267F	

Common Numerals

Used in the sexagesimal discrete counting system and other metrological systems

12550 CUNEIFORM NUMERIC SIGN ONE N01
= 1 aš curved
→ 12038 cuneiform sign ash
• often used instead of diš in Early Dynastic counterparts of cuneiform metrological systems
→ 12079 cuneiform sign dish
12551 CUNEIFORM NUMERIC SIGN TWO N01
→ 12400 cuneiform numeric sign two ash
12552 CUNEIFORM NUMERIC SIGN THREE N01
12553 CUNEIFORM NUMERIC SIGN FOUR N01
12554 CUNEIFORM NUMERIC SIGN FIVE N01
12555 CUNEIFORM NUMERIC SIGN SIX N01
12556 CUNEIFORM NUMERIC SIGN SEVEN N01
12557 CUNEIFORM NUMERIC SIGN EIGHT N01
12558 CUNEIFORM NUMERIC SIGN NINE N01
12559 CUNEIFORM NUMERIC SIGN ONE N08
= 1 diš curved
→ 12079 cuneiform sign dish
= 1/2 iku curved
• used for one half in multiple metrological systems
→ 12039 cuneiform sign ash zida tenu
→ 12226 cuneiform sign mash
= 1 bariga curved
• used in Early Dynastic capacity systems
1255A CUNEIFORM NUMERIC SIGN TWO N08
→ 1222B cuneiform sign min
= 2 bariga curved
→ 12456 cuneiform numeric sign nigidamin
1255B CUNEIFORM NUMERIC SIGN THREE N08
→ 12408 cuneiform numeric sign three dish
• used in Early Dynastic capacity systems
= 3 bariga curved
→ 12457 cuneiform numeric sign nigidaesh
1255C CUNEIFORM NUMERIC SIGN FOUR N08
1255D CUNEIFORM NUMERIC SIGN FIVE N08
1255E CUNEIFORM NUMERIC SIGN SIX N08
1255F CUNEIFORM NUMERIC SIGN SEVEN N08
12560 CUNEIFORM NUMERIC SIGN EIGHT N08
12561 CUNEIFORM NUMERIC SIGN NINE N08
12562 CUNEIFORM NUMERIC SIGN ONE N14
= 1 u curved
= 1 bur$_3$ curved
→ 1230B cuneiform sign u
12563 CUNEIFORM NUMERIC SIGN TWO N14
→ 12399 cuneiform sign u u
12564 CUNEIFORM NUMERIC SIGN THREE N14
→ 1230D cuneiform sign u u u
12565 CUNEIFORM NUMERIC SIGN FOUR N14
→ 1240F cuneiform numeric sign four u
12566 CUNEIFORM NUMERIC SIGN FIVE N14
12567 CUNEIFORM NUMERIC SIGN SIX N14
12568 CUNEIFORM NUMERIC SIGN SEVEN N14
12569 CUNEIFORM NUMERIC SIGN EIGHT N14
1256A CUNEIFORM NUMERIC SIGN NINE N14
1256B CUNEIFORM NUMERIC SIGN ONE N34
= 1 ŋeš$_2$ curved
→ 12415 cuneiform numeric sign one gesh2
1256C CUNEIFORM NUMERIC SIGN TWO N34
1256D CUNEIFORM NUMERIC SIGN THREE N34
1256E CUNEIFORM NUMERIC SIGN FOUR N34
1256F CUNEIFORM NUMERIC SIGN FIVE N34
12570 CUNEIFORM NUMERIC SIGN SIX N34
12571 CUNEIFORM NUMERIC SIGN SEVEN N34
12572 CUNEIFORM NUMERIC SIGN EIGHT N34
12573 CUNEIFORM NUMERIC SIGN NINE N34
12574 CUNEIFORM NUMERIC SIGN ONE N48
= 1 ŋešʾu curved
→ 1241E cuneiform numeric sign one geshu
12575 CUNEIFORM NUMERIC SIGN TWO N48
12576 CUNEIFORM NUMERIC SIGN THREE N48
12577 CUNEIFORM NUMERIC SIGN FOUR N48
12578 CUNEIFORM NUMERIC SIGN FIVE N48
12579 CUNEIFORM NUMERIC SIGN ONE N45
= 1 šar$_2$ curved
• 122B9 should be used for cuneiform 1 šar$_2$
• 122B9 should be used for logographic šar$_2$, even when curved
→ 122B9 cuneiform sign shar2
1257A CUNEIFORM NUMERIC SIGN TWO N45
1257B CUNEIFORM NUMERIC SIGN THREE N45
1257C CUNEIFORM NUMERIC SIGN FOUR N45
1257D CUNEIFORM NUMERIC SIGN FIVE N45
1257E CUNEIFORM NUMERIC SIGN SIX N45
1257F CUNEIFORM NUMERIC SIGN SEVEN N45
12580 CUNEIFORM NUMERIC SIGN EIGHT N45
12581 CUNEIFORM NUMERIC SIGN NINE N45
12582 CUNEIFORM NUMERIC SIGN ONE N50
= 1 šarʾu curved
→ 1242C cuneiform numeric sign one sharu
• used instead of 1258E in fourth millennium land area systems
→ 12434 cuneiform numeric sign one buru
12583 CUNEIFORM NUMERIC SIGN TWO N50
12584 CUNEIFORM NUMERIC SIGN THREE N50
12585 CUNEIFORM NUMERIC SIGN FOUR N50
12586 CUNEIFORM NUMERIC SIGN FIVE N50

Numerals used for land areas

Together with N08, N01, N14, N45, and N50

12587 CUNEIFORM NUMERIC SIGN ONE EIGHTH IKU CURVED
→ 1245F cuneiform numeric sign one eighth ash
12588 CUNEIFORM NUMERIC SIGN ONE EIGHTH IKU CURVED VARIANT FORM
12589 CUNEIFORM NUMERIC SIGN ONE N01 REVERSED
= 1/4 iku curved
→ 12460 cuneiform numeric sign one quarter ash
1258A CUNEIFORM NUMERIC SIGN ONE QUARTER IKU CURVED VARIANT FORM
1258B CUNEIFORM NUMERIC SIGN ONE HALF IKU CURVED VARIANT FORM
→ 12039 cuneiform sign ash zida tenu
1258C CUNEIFORM NUMERIC SIGN ONE N22
= 1 eše$_3$ curved
→ 12458 cuneiform numeric sign one eshe3
1258D CUNEIFORM NUMERIC SIGN TWO N22
1258E CUNEIFORM NUMERIC SIGN ONE BURU CURVED
→ 12434 cuneiform numeric sign one buru
1258F CUNEIFORM NUMERIC SIGN TWO BURU CURVED
12590 CUNEIFORM NUMERIC SIGN THREE BURU CURVED
12591 CUNEIFORM NUMERIC SIGN FOUR BURU CURVED
12592 CUNEIFORM NUMERIC SIGN FIVE BURU CURVED

Early Dynastic capacity measures

12593 CUNEIFORM NUMERIC SIGN ONE BAN2 CURVED
- • 1244F cuneiform numeric sign one ban2
- = 1/2 aš curved
- • used for one half in multiple metrological systems
- → 12226 cuneiform sign mash

12594 CUNEIFORM NUMERIC SIGN TWO BAN2 CURVED
12595 CUNEIFORM NUMERIC SIGN THREE BAN2 CURVED
12596 CUNEIFORM NUMERIC SIGN FOUR BAN2 CURVED
12597 CUNEIFORM NUMERIC SIGN FIVE BAN2 CURVED
12598 CUNEIFORM NUMERIC SIGN NINDA2 TIMES SHE PLUS ONE ASH CURVED
- = 1/3 aš curved variant form
- → 1245D cuneiform numeric sign one third dish variant form a
- → 1245A cuneiform numeric sign one third dish

12599 CUNEIFORM NUMERIC SIGN NINDA2 TIMES SHE PLUS TWO ASH CURVED
- = 2/3 aš curved variant form
- → 1245E cuneiform numeric sign two thirds dish variant form a
- → 1245B cuneiform numeric sign two thirds dish

Numerals used in the bisexagesimal system

Together with N08, N01, N14, and N34

1259A CUNEIFORM NUMERIC SIGN ONE N51
- = 1 ŋeš₂ curved doubled, 1 ŋešmin curved

1259B CUNEIFORM NUMERIC SIGN TWO N51
1259C CUNEIFORM NUMERIC SIGN THREE N51
1259D CUNEIFORM NUMERIC SIGN FOUR N51
1259E CUNEIFORM NUMERIC SIGN FIVE N51
1259F CUNEIFORM NUMERIC SIGN SIX N51
125A0 CUNEIFORM NUMERIC SIGN SEVEN N51
125A1 CUNEIFORM NUMERIC SIGN EIGHT N51
125A2 CUNEIFORM NUMERIC SIGN NINE N51
125A3 CUNEIFORM NUMERIC SIGN ONE N54
- = 1 ŋešʾu curved doubled, 1 ŋešminʾu curved

125A4 CUNEIFORM NUMERIC SIGN TWO N54
125A5 CUNEIFORM NUMERIC SIGN THREE N54
125A6 CUNEIFORM NUMERIC SIGN FOUR N54
125A7 CUNEIFORM NUMERIC SIGN FIVE N54
125A8 CUNEIFORM NUMERIC SIGN ONE N56
125A9 CUNEIFORM NUMERIC SIGN TWO N56

Fourth millennium capacity measures

125AA CUNEIFORM NUMERIC SIGN ONE N24
125AB CUNEIFORM NUMERIC SIGN ONE N26
125AC CUNEIFORM NUMERIC SIGN ONE N28
125AD CUNEIFORM NUMERIC SIGN ONE N29A
125AE CUNEIFORM NUMERIC SIGN ONE N29B
125AF CUNEIFORM NUMERIC SIGN ONE N30A
125B0 CUNEIFORM NUMERIC SIGN ONE N30C
125B1 CUNEIFORM NUMERIC SIGN ONE N30D
125B2 CUNEIFORM NUMERIC SIGN ONE N30E
125B3 CUNEIFORM NUMERIC SIGN ONE N31
125B4 CUNEIFORM NUMERIC SIGN ONE N32
125B5 CUNEIFORM NUMERIC SIGN ONE N33
125B6 CUNEIFORM NUMERIC SIGN ONE N39A
125B7 CUNEIFORM NUMERIC SIGN TWO N39A
125B8 CUNEIFORM NUMERIC SIGN THREE N39A
125B9 CUNEIFORM NUMERIC SIGN FOUR N39A
125BA CUNEIFORM NUMERIC SIGN ONE N39B
125BB CUNEIFORM NUMERIC SIGN TWO N39B
125BC CUNEIFORM NUMERIC SIGN THREE N39B
125BD CUNEIFORM NUMERIC SIGN FOUR N39B

Numerals of sexagesimal system S′

Used to count dead animals and jars of certain types of liquids

125BE CUNEIFORM NUMERIC SIGN ONE N02
125BF CUNEIFORM NUMERIC SIGN TWO N02
125C0 CUNEIFORM NUMERIC SIGN THREE N02
125C1 CUNEIFORM NUMERIC SIGN FOUR N02
125C2 CUNEIFORM NUMERIC SIGN FIVE N02
125C3 CUNEIFORM NUMERIC SIGN SIX N02
125C4 CUNEIFORM NUMERIC SIGN SEVEN N02
125C5 CUNEIFORM NUMERIC SIGN EIGHT N02
125C6 CUNEIFORM NUMERIC SIGN NINE N02
125C7 CUNEIFORM NUMERIC SIGN ONE N15
125C8 CUNEIFORM NUMERIC SIGN TWO N15
125C9 CUNEIFORM NUMERIC SIGN THREE N15
125CA CUNEIFORM NUMERIC SIGN FOUR N15
125CB CUNEIFORM NUMERIC SIGN FIVE N15
125CC CUNEIFORM NUMERIC SIGN ONE N35
125CD CUNEIFORM NUMERIC SIGN TWO N35
125CE CUNEIFORM NUMERIC SIGN THREE N35
125CF CUNEIFORM NUMERIC SIGN FOUR N35
125D0 CUNEIFORM NUMERIC SIGN FIVE N35

Numerals of bisexagesimal system B*

Used in the fourth millennium to count rations of an unclear nature

125D1 CUNEIFORM NUMERIC SIGN ONE N06
125D2 CUNEIFORM NUMERIC SIGN TWO N06
125D3 CUNEIFORM NUMERIC SIGN THREE N06
125D4 CUNEIFORM NUMERIC SIGN FOUR N06
125D5 CUNEIFORM NUMERIC SIGN FIVE N06
125D6 CUNEIFORM NUMERIC SIGN SIX N06
125D7 CUNEIFORM NUMERIC SIGN SEVEN N06
125D8 CUNEIFORM NUMERIC SIGN EIGHT N06
125D9 CUNEIFORM NUMERIC SIGN NINE N06
125DA CUNEIFORM NUMERIC SIGN ONE N21
125DB CUNEIFORM NUMERIC SIGN TWO N21
125DC CUNEIFORM NUMERIC SIGN THREE N21
125DD CUNEIFORM NUMERIC SIGN FOUR N21
125DE CUNEIFORM NUMERIC SIGN FIVE N21
125DF CUNEIFORM NUMERIC SIGN ONE N38
125E0 CUNEIFORM NUMERIC SIGN ONE N52
125E1 CUNEIFORM NUMERIC SIGN TWO N52
125E2 CUNEIFORM NUMERIC SIGN THREE N52
125E3 CUNEIFORM NUMERIC SIGN FOUR N52
125E4 CUNEIFORM NUMERIC SIGN FIVE N52
125E5 CUNEIFORM NUMERIC SIGN SIX N52
125E6 CUNEIFORM NUMERIC SIGN SEVEN N52
125E7 CUNEIFORM NUMERIC SIGN EIGHT N52
125E8 CUNEIFORM NUMERIC SIGN NINE N52
125E9 CUNEIFORM NUMERIC SIGN ONE N60

Numerals of capacity system Š′

Used in the fourth millennium to measure malted barley

125EA CUNEIFORM NUMERIC SIGN ONE N24A
125EB CUNEIFORM NUMERIC SIGN ONE N40
125EC CUNEIFORM NUMERIC SIGN TWO N40
125ED CUNEIFORM NUMERIC SIGN THREE N40
125EE CUNEIFORM NUMERIC SIGN FOUR N40
125EF CUNEIFORM NUMERIC SIGN ONE N03
125F0 CUNEIFORM NUMERIC SIGN TWO N03
125F1 CUNEIFORM NUMERIC SIGN THREE N03
125F2 CUNEIFORM NUMERIC SIGN FOUR N03
125F3 CUNEIFORM NUMERIC SIGN FIVE N03
125F4 CUNEIFORM NUMERIC SIGN ONE N18
125F5 CUNEIFORM NUMERIC SIGN TWO N18
125F6 CUNEIFORM NUMERIC SIGN THREE N18
125F7 CUNEIFORM NUMERIC SIGN FOUR N18
125F8 CUNEIFORM NUMERIC SIGN FIVE N18
125F9 CUNEIFORM NUMERIC SIGN SIX N18

125FA	CUNEIFORM NUMERIC SIGN SEVEN N18
125FB	CUNEIFORM NUMERIC SIGN EIGHT N18
125FC	CUNEIFORM NUMERIC SIGN NINE N18

Numerals of capacity system Š″

Used in the fourth millennium to measure various kinds of emmer

125FD	CUNEIFORM NUMERIC SIGN ONE N24B
125FE	CUNEIFORM NUMERIC SIGN ONE N26B
125FF	CUNEIFORM NUMERIC SIGN ONE N28B
12600	CUNEIFORM NUMERIC SIGN ONE N29AB
12601	CUNEIFORM NUMERIC SIGN ONE N41
12602	CUNEIFORM NUMERIC SIGN TWO N41
12603	CUNEIFORM NUMERIC SIGN THREE N41
12604	CUNEIFORM NUMERIC SIGN FOUR N41
12605	CUNEIFORM NUMERIC SIGN ONE N04
12606	CUNEIFORM NUMERIC SIGN TWO N04
12607	CUNEIFORM NUMERIC SIGN THREE N04
12608	CUNEIFORM NUMERIC SIGN FOUR N04
12609	CUNEIFORM NUMERIC SIGN FIVE N04
1260A	CUNEIFORM NUMERIC SIGN ONE N19
1260B	CUNEIFORM NUMERIC SIGN TWO N19
1260C	CUNEIFORM NUMERIC SIGN THREE N19
1260D	CUNEIFORM NUMERIC SIGN FOUR N19
1260E	CUNEIFORM NUMERIC SIGN FIVE N19
1260F	CUNEIFORM NUMERIC SIGN SIX N19
12610	CUNEIFORM NUMERIC SIGN SEVEN N19
12611	CUNEIFORM NUMERIC SIGN EIGHT N19
12612	CUNEIFORM NUMERIC SIGN NINE N19
12613	CUNEIFORM NUMERIC SIGN ONE N46
12614	CUNEIFORM NUMERIC SIGN TWO N46
12615	CUNEIFORM NUMERIC SIGN ONE N36
12616	CUNEIFORM NUMERIC SIGN TWO N36
12617	CUNEIFORM NUMERIC SIGN THREE N36
12618	CUNEIFORM NUMERIC SIGN FOUR N36
12619	CUNEIFORM NUMERIC SIGN FIVE N36
1261A	CUNEIFORM NUMERIC SIGN SIX N36
1261B	CUNEIFORM NUMERIC SIGN SEVEN N36
1261C	CUNEIFORM NUMERIC SIGN EIGHT N36
1261D	CUNEIFORM NUMERIC SIGN NINE N36
1261E	CUNEIFORM NUMERIC SIGN ONE N49
1261F	CUNEIFORM NUMERIC SIGN TWO N49
12620	CUNEIFORM NUMERIC SIGN THREE N49
12621	CUNEIFORM NUMERIC SIGN FOUR N49

Numerals of capacity system Š*

Used in the fourth millennium to measure barley groats

12622	CUNEIFORM NUMERIC SIGN ONE N25
12623	CUNEIFORM NUMERIC SIGN ONE N27
12624	CUNEIFORM NUMERIC SIGN ONE N28C
12625	CUNEIFORM NUMERIC SIGN ONE N29AC
12626	CUNEIFORM NUMERIC SIGN ONE N30AC
12627	CUNEIFORM NUMERIC SIGN ONE N30CC
12628	CUNEIFORM NUMERIC SIGN ONE N42A
12629	CUNEIFORM NUMERIC SIGN TWO N42A
1262A	CUNEIFORM NUMERIC SIGN THREE N42A
1262B	CUNEIFORM NUMERIC SIGN FOUR N42A
1262C	CUNEIFORM NUMERIC SIGN ONE N42B
1262D	CUNEIFORM NUMERIC SIGN TWO N42B
1262E	CUNEIFORM NUMERIC SIGN THREE N42B
1262F	CUNEIFORM NUMERIC SIGN FOUR N42B
12630	CUNEIFORM NUMERIC SIGN ONE N05
12631	CUNEIFORM NUMERIC SIGN TWO N05
12632	CUNEIFORM NUMERIC SIGN THREE N05
12633	CUNEIFORM NUMERIC SIGN FOUR N05
12634	CUNEIFORM NUMERIC SIGN FIVE N05
12635	CUNEIFORM NUMERIC SIGN ONE N20
12636	CUNEIFORM NUMERIC SIGN TWO N20
12637	CUNEIFORM NUMERIC SIGN THREE N20
12638	CUNEIFORM NUMERIC SIGN FOUR N20
12639	CUNEIFORM NUMERIC SIGN FIVE N20
1263A	CUNEIFORM NUMERIC SIGN SIX N20
1263B	CUNEIFORM NUMERIC SIGN SEVEN N20
1263C	CUNEIFORM NUMERIC SIGN EIGHT N20
1263D	CUNEIFORM NUMERIC SIGN NINE N20
1263E	CUNEIFORM NUMERIC SIGN ONE N47
1263F	CUNEIFORM NUMERIC SIGN TWO N47
12640	CUNEIFORM NUMERIC SIGN ONE N37
12641	CUNEIFORM NUMERIC SIGN TWO N37

Numerals of system EN

Only attested in the Uruk IV period

12642	CUNEIFORM NUMERIC SIGN ONE N09
12643	CUNEIFORM NUMERIC SIGN ONE N11
12644	CUNEIFORM NUMERIC SIGN ONE N12
12645	CUNEIFORM NUMERIC SIGN ONE N07A
12646	CUNEIFORM NUMERIC SIGN TWO N07A
12647	CUNEIFORM NUMERIC SIGN THREE N07A
12648	CUNEIFORM NUMERIC SIGN ONE N07B
12649	CUNEIFORM NUMERIC SIGN TWO N07B
1264A	CUNEIFORM NUMERIC SIGN THREE N07B

Flat numerals

Rectangular numerals impressed with a flat tool, used in Ur in the Early Dynastic I–II period

1264B	CUNEIFORM NUMERIC SIGN ONE N01 FLAT
	= 1 aš flat
	→ 12038 cuneiform sign ash
1264C	CUNEIFORM NUMERIC SIGN TWO N01 FLAT
1264D	CUNEIFORM NUMERIC SIGN THREE N01 FLAT
1264E	CUNEIFORM NUMERIC SIGN FOUR N01 FLAT
1264F	CUNEIFORM NUMERIC SIGN FIVE N01 FLAT
12650	CUNEIFORM NUMERIC SIGN SIX N01 FLAT
12651	CUNEIFORM NUMERIC SIGN SEVEN N01 FLAT
12652	CUNEIFORM NUMERIC SIGN EIGHT N01 FLAT
12653	CUNEIFORM NUMERIC SIGN NINE N01 FLAT
12654	CUNEIFORM NUMERIC SIGN ONE N08 FLAT
12655	CUNEIFORM NUMERIC SIGN ONE N14 FLAT
	= 1 u flat
	→ 1230B cuneiform sign u
12656	CUNEIFORM NUMERIC SIGN TWO N14 FLAT
12657	CUNEIFORM NUMERIC SIGN THREE N14 FLAT
12658	CUNEIFORM NUMERIC SIGN FOUR N14 FLAT
12659	CUNEIFORM NUMERIC SIGN FIVE N14 FLAT
1265A	CUNEIFORM NUMERIC SIGN SIX N14 FLAT
1265B	CUNEIFORM NUMERIC SIGN SEVEN N14 FLAT
1265C	CUNEIFORM NUMERIC SIGN EIGHT N14 FLAT
1265D	CUNEIFORM NUMERIC SIGN NINE N14 FLAT
1265E	CUNEIFORM NUMERIC SIGN ONE N34 FLAT
1265F	CUNEIFORM NUMERIC SIGN TWO N34 FLAT
12660	CUNEIFORM NUMERIC SIGN THREE N34 FLAT
12661	CUNEIFORM NUMERIC SIGN FOUR N34 FLAT
12662	CUNEIFORM NUMERIC SIGN FIVE N34 FLAT
12663	CUNEIFORM NUMERIC SIGN SIX N34 FLAT
12664	CUNEIFORM NUMERIC SIGN SEVEN N34 FLAT
12665	CUNEIFORM NUMERIC SIGN EIGHT N34 FLAT
12666	CUNEIFORM NUMERIC SIGN NINE N34 FLAT
12667	CUNEIFORM NUMERIC SIGN ONE N45 FLAT
12668	CUNEIFORM NUMERIC SIGN TWO N45 FLAT
12669	CUNEIFORM NUMERIC SIGN ONE N22 FLAT
1266A	CUNEIFORM NUMERIC SIGN TWO N22 FLAT
1266B	CUNEIFORM NUMERIC SIGN ONE N51 FLAT
1266C	CUNEIFORM NUMERIC SIGN TWO N51 FLAT
1266D	CUNEIFORM NUMERIC SIGN THREE N51 FLAT
1266E	CUNEIFORM NUMERIC SIGN FOUR N51 FLAT
1266F	CUNEIFORM NUMERIC SIGN FIVE N51 FLAT
12670	CUNEIFORM NUMERIC SIGN SIX N51 FLAT
12671	CUNEIFORM NUMERIC SIGN SEVEN N51 FLAT
12672	CUNEIFORM NUMERIC SIGN EIGHT N51 FLAT

12673	CUNEIFORM NUMERIC SIGN NINE N51 FLAT
12674	CUNEIFORM NUMERIC SIGN ONE N34 FLAT TENU = 1 n39a flat
12675	CUNEIFORM NUMERIC SIGN ONE N04 FLAT
12676	CUNEIFORM NUMERIC SIGN TWO N04 FLAT
12677	CUNEIFORM NUMERIC SIGN THREE N04 FLAT
12678	CUNEIFORM NUMERIC SIGN FOUR N04 FLAT
12679	CUNEIFORM NUMERIC SIGN FIVE N04 FLAT
1267A	CUNEIFORM NUMERIC SIGN ONE N19 FLAT
1267B	CUNEIFORM NUMERIC SIGN TWO N19 FLAT
1267C	CUNEIFORM NUMERIC SIGN THREE N19 FLAT
1267D	CUNEIFORM NUMERIC SIGN FOUR N19 FLAT
1267E	CUNEIFORM NUMERIC SIGN FIVE N19 FLAT
1267F	CUNEIFORM NUMERIC SIGN SIX N19 FLAT
12680	CUNEIFORM NUMERIC SIGN SEVEN N19 FLAT
12681	CUNEIFORM NUMERIC SIGN EIGHT N19 FLAT
12682	CUNEIFORM NUMERIC SIGN NINE N19 FLAT
12683	CUNEIFORM NUMERIC SIGN ONE N46 FLAT
12684	CUNEIFORM NUMERIC SIGN TWO N46 FLAT
12685	CUNEIFORM NUMERIC SIGN ONE N36 FLAT

2.3 Properties

3 Rationale for curviform–cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [Pow72, p. 215] “in the same fashion as we use black and red ink”; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble 𒅎 in texts from Early Dynastic IIIa Šuruppag as in the character code charts, 𒅎 later in the third millennium[6], 𒅎 in Old Babylonian cursive, 𒅎 in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists[7] and dictionaries[8], and of composite texts[9]. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24][10]. The diachronic approach is also useful for pedagogic applications[11].

3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding[12] that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early

[6] Merging with U+1224E CUNEIFORM SIGN NI2.

[7] Notably [OSL] and the online edition of [Bor10] in [eBL, Signs].

[8] Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

[9] For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of 𒄩𒄠𒈬𒊏𒁉, as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

[10] Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svä+24, pp. 242, 148].

[11] For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

[12] At that time scoped to the répertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as 𒅎 ≠ 𒉎, were informed by Early Dynastic distinctions.

Dynastic IIIb administrative tablet from Ŋirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

[13]						
1(ŋeš$_2$)	1(u)	1/2(diš)	5(diš *tenû*)	gi	us$_2$	sa$_2$
	7.5 (ropes)		5	reed	side	equal

[14]				
3(u)	6(diš *tenû*)	gi	saŋ	sa$_2$
3 (ropes)	6	reed	front	equal

ašag-bi	1(bur$_3$^{c})	1(eše$_3$^{c})	1(iku^c)	1/2(iku^c)
ašag=bi				
field=DEM[15]				

tug$_x$(LAK 483)-si-ga-kam
tugsiga =ak =am -Ø
ploughed=GEN=COP-3.SG.S

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as[16]

> 465 metres, equal lengths
> 198 metres, equal widths
> this field is 9,18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that "the number signs do not normally carry in their individual signs the meaning of what they are used to measure", and that curviform and cuneiform numerals "are not normally mixed together in a single numerical expression", noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from ⊳ to 𒁹 numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in §3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that "The distinction between curved numerals and their cuneiform

[13]As noted in [Pow87, p. 466], this sign has a very short "tail" in this period, so that it is wider than it is tall, and can at first seem like a large 𒀸 in copies. The photos in [CDLI] clearly show that this is in fact a vertical wedge.

[14]Note that ED IIIb 𒌋 numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign 𒌍 in [P020054] looks more like Ur III 𒌍.

[15]Alternatively: area=POSS.3.SG.NH, "its area".

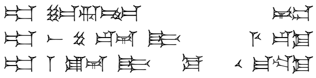
[16]We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases[17].”

The time has come to revisit this issue. As we will see in §3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In §3.4 we will see that in some periods:

— the functions and use of the numerals vary beyond the mere ⊃/𒁹 switch;
— the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
— some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 A primer on classic Ur III and Old Babylonian metrologies



I want to write tablets: the tablet of 1 cor of barley to 600 cor; the tablet of 1 shekel of silver to 10 minas [...]

Edubba’a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the constrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known[18] a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters 𒁹–𒑆, the multiples of ten (10–50) are 𒌋–𒐐, but the other digits 11–59 are sequences 𒌋𒁹–𒐐𒑆; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units 𒁹–𒑆, tens 𒌋–𒐐, sixties 𒐕–𒐝 (with larger wedges than the units), multiples of six hundred 𒐞–𒐣, multiples of three thousand six hundreds 𒊹–𒐬, and multiples of thirty-six thousand 𒐭–𒐯.

3.3.1 The discrete counting system

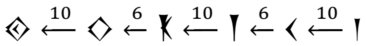
The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram[19], where the num-

[17]The cuneiform encoding process was planned in *stages* in [L2/03-162]. One might expect the second stage of encoding, which led to the creation of the Early Dynastic Cuneiform block, to incorporate the numerals needed for the representation and discussion of Early Dynastic texts; however, the proposal [L2/12-208] stated that “numerals have been omitted due to the complexity of numeral signs from this period. An expert in the metrology of this period must be consulted before these can be properly included.”

[18]See, *e.g.*, [Uni16, §22.3.3, *sub* “Cuneiform Numerals”].

[19]These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*, see Figure 4.

ber over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

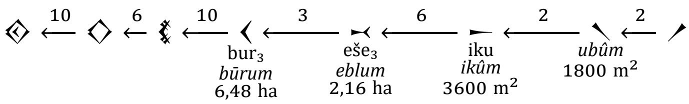


($S_{\text{Ur III/OB}}$)

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written 𒐖𒐜𒐏𒐝 in the discrete counting system, and 𒎙𒐜𒐏𒐝 in the sexagesimal place value system.

3.3.2 The area system

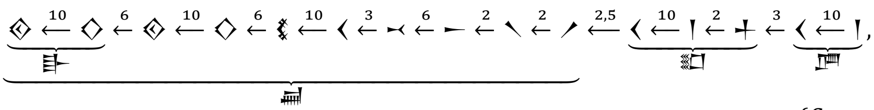
The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:



($G_{\text{Ur III/OB}}$)

Note that for the range of areas given above, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *būrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign 𒃷 functioning as punctuation[20], but the numerals are tied to the metrology; thus a surface of 5 *būr* 1 *ebel* 4 *ikû* (100 *ikû*, 36 ha) would be written[21] 𒐐𒑘𒐉𒃷. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in "88 acres 3 roods 33 perches" or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the sign ◇ is equal to sixty times 𒌋 in the area system, but to three hundred and sixty times 𒌋 in the discrete counting system.

For areas smaller than a quarter *ikûm*, an overt unit is used, with one 𒊬 (sar, *mūšarum*), approximately 36 m², written 𒁹𒊬, equal to one hundredth of an *ikûm*, then sexigesimally subdivided in 60 𒂆 (giŋ₄, *šiqlum*, "shekel"). For areas greater than 3600 *būr*, the ◇ and ◈ numerals are reused with a suffix 𒃲 (gal, "big"), as follows [Rob08, p. 295 nn. b, c; Fri07, p. 378; Rob19]:



($G_{\text{Ur III/OB}}$)

e.g., ◇◇𒃲 ◈◈ 𒐏𒐌𒃷 𒐋 𒊬 𒌋𒐝 𒂆[22] for $(2 \times 3600 + 20 \times 60 + 49)$ *būr* 5 *ikû* $(5 + \frac{1}{2})$ *mūšar* 19 *šiqil*. Factor diagrams in this document will use bottom curly brackets in this fashion to separate numerals from units and other suffixes.

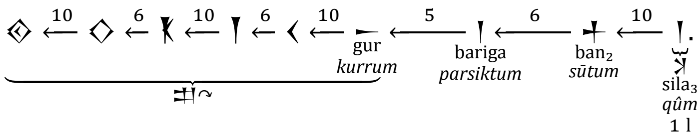
[20] This sign is sometimes interpreted as a measurement unit, and transliterated iku, see, *e.g.*, [Pro20, pp. 385 sqq.], or transliterations in [Feu04] discussed in §3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system $G_{\text{Ur III/OB}}$.

[21] As in the surface of the field of 𒁹𒀀𒉺𒉌𒆠 (the city of Apisal) reported on [P102305, r. 1]

[22] From [P213162], which has an additional 𒑚𒂆, two thirds (of a shekel), see §3.3.5.

3.3.3 The capacity system

Another such system of note is the one for capacities[23] [Fri07, p. 376; Rob19],



$(C_{\text{Ur III/OB}})$

In the above diagram, the numerals for ban$_2$ are 𒑏, 𒑐, 𒑑, 𒑒, and 𒑔, and those for bariga are 𒁹, 𒑖, 𒑗, and 𒑘 (contrast ordinary 𒈫 and 𒐈 otherwise used with 𒁹 numerals). Further, we have used the symbol ↷ to express that, as described in [Hue11, p. 585 nn. (b), (f)], the sign 𒄥 GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𒋡 if present, and after the word for "grain" if present, as in

𒐢𒐝𒌋𒐂	𒑑	𒐋	𒋡	𒊺	𒄥[24]
3554 gur	3 ban$_2$	6	sila$_3$	of grain.	

Observe that while large numbers of gur follow[25] system $S_{\text{Ur III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as 𒌋𒁹𒄥 would be 10 gur 1 bariga, and 𒌋𒀸𒄥 would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

— 𒁹–𒐎 used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
— 𒌋–𒐔 used in $G_{\text{Ur III/OB}}$, of which 𒌋–𒐐 are also used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
— 𒐕–𒐝 used in $S_{\text{Ur III/OB}}$, and sometimes with overt units;
— 𒐞–𒐢 used in $S_{\text{Ur III/OB}}$;
— 𒊹–𒐪 used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
— 𒐫–𒐯 used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
— 𒀸–𒐇 used in $C_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
— 𒑏, 𒑐, 𒑑, 𒑒, 𒑔 used in $C_{\text{Ur III/OB}}$;
— 𒁹, 𒑖, 𒑗, 𒑘 used in $C_{\text{Ur III/OB}}$—note the overlap with 𒁹–𒐉;
— 𒑚 and 𒑛 used in $G_{\text{Ur III/OB}}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Rob08, p. 78]: "The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity."

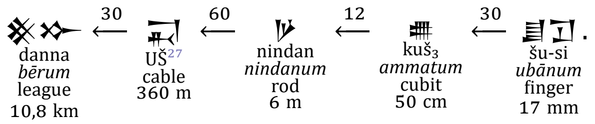
[23] Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{\text{Ur III/OB}}$ based on a height of one cubit, see[Pow87, p. 488; Rob08, p. 294; Rob19].

[24] From [P309594, obv. 1 1].

[25] A larger unit, the guru$_7$ (*karûm*, grain heap), is sometimes used instead, with 𒀸𒂃 = 𒊹𒄥 (1 *karûm* = 3600 kurrū). See [Fri07, p. 415; Rob19].

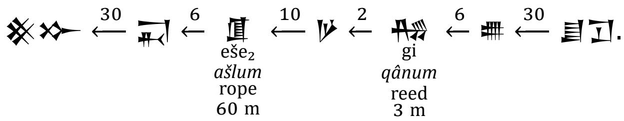
3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with 𒁹 and 𒌋 numerals with their system $S_{\text{Ur III/OB}}$ values. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in §3.4. Metrological tables use the following units[26] [Fri07, p. 118; Rob19]:



$(L_{\text{Ur III/OB}})$

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:



$(\bar{L}_{\text{Ur III/OB}})$

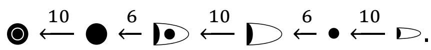
In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

3.3.5 Fractions

Fractions of the *ikîm*, 𒑠 = $\frac{1}{2}$𒃷 and 𒑡 = $\frac{1}{4}$𒃷, have already been encountered. In other contexts, the fraction $\frac{1}{2}$ is written 𒈦, as in 𒈦 𒊬. The fractions $\frac{1}{3}$ and $\frac{2}{3}$ are written 𒑚 and 𒑛. The latter two signs are derived from curviform signs 𒑚 and 𒑛, which are already separately encoded; these are in turn derived from the sign 𒋙 (ŠU$_2$), whose Early dynastic form resembles 𒑚, and ▿ numerals; see [Pow71, pp. 113, 134]. The 𒋙 is sometimes omitted, as in [P240545, *verso* 6 9; P221530; P221531; P271238; P274845].

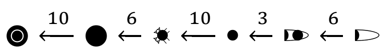
3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the fourth millennium) clearly mirrors system $S_{\text{Ur III/OB}}$ [Fri07, p. 374; DE87, pp. 127, 165]:



(S)

Likewise the area system used in the Early Dynastic IIIb period for areas of one iku and greater [Dei22, p. 72; NDE93, p. 63; Fri07, p. 378; Lec16],



$(G_{\text{ED IIIb}})$

[26] In this factor diagram and the next, we do not include the numerals. The units are no more than a factor of 60 apart, so higher numerals such as 𒐕 or ◇ are not used.

[27] As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.

mirrors system $G_{\text{Ur III/OB}}$, with consistent use of the numerals: • corresponds to 𒌋, ● to ◇, and ◉ to ◈. An exception to this correspondence, noted in [L2/04-099, p. 4] (see §3.2), is that the vertical 𒁹 from $S_{\text{Ur III/OB}}$ corresponds to a horizontal ⊃ in system S. This is however far from the only case of such a reallocation of function. The earlier form of the area system is [DE87, pp. 141, 165; Fri07, p. 378]:

$$● \xleftarrow{6} ◉ \xleftarrow{10} • \xleftarrow{3} ⊃• \xleftarrow{6} ⊃, \qquad (G)$$

Observe that, as noted in [DE87, p. 142], ◉ changes meaning from 10• in system G to 600• in system $G_{\text{ED IIIb}}$. System G is used in the fourth millennium, but also in the ED I–II period (it is the "area 2" system in [Cha03], whereas $G_{\text{ED IIIb}}$ is the "area 1" system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb area system[28],

$$◉ \xleftarrow{10} ● \xleftarrow{6} ✹ \xleftarrow{10} • \xleftarrow{3} ⊃• \xleftarrow{6} ⊃ \xleftarrow{2} ▽ \xleftarrow{2} ◁ \xleftarrow{2} •^{29}, \qquad (G_{\text{ED IIIb}})$$

with the numerals of a contemporaneous capacity system:

$$\underbrace{⊃• \xleftarrow{10} ⊃ \xleftarrow{6} • \xleftarrow{10} ⊃}_{𒄥𒊕𒅅} \xleftarrow{4} ▽ \xleftarrow{6} ▽, \qquad (C_{𒄥𒊕𒅅})$$

both described in [Lec16]. While the size of the 𒄥𒊕𒅅 (gur saŋ ŋal₂) in bariga is different from that of the Old Babylonian 𒄥, the basic structure of the capacity system is recognizable, with ▽ corresponding to 𒁹 for bariga, ▽–𒁓 corresponding to 𒈦–𒁓 for ban₂, and the 𒄥 counted with ⊃ rather than 𒀸 numerals. However, the half-iku is counted with the same ▽ as the bariga, whereas it uses a different sign, 𒀹, in the Old Babylonian system. As we will see, this is cannot be handled as a split, by giving 𒀹 the glyph ▽ in an Early Dynastic IIIb font, as the 𒀹 numeral series is also in use in that period.

3.4.1 Field lengths in Ŋirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign 𒃻 is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign 𒂠 is not written either. Lengths shorter than one rope are expressed in half-ropes using the $\frac{1}{2}$ sign 𒈦 (again with no 𒂠), and then in reeds, *with* the sign 𒄀, as follows:

$$\underset{\substack{1\ \text{eše}_2\text{=10 nindan}\\ 1\ \text{rope=10 rods}\\ 60\ \text{m}}}{𒁹 \xleftarrow{\quad 6 \quad} 𒌋} \xleftarrow{\quad 2 \quad} 𒈦 \xleftarrow{10} \underbrace{𒀹^{32}}_{\substack{𒄀\\ \text{gi}\\ \text{reed}\\ 3\ \text{m}}}. \qquad (L_{\text{ED IIIb}})$$

This is the system that was used to express the sides of the field in [P020054] discussed in §3.2. In that tablet and most others from the same period, such as the

[28] A variant is $◉ \xleftarrow{10} ● \xleftarrow{6} ✹ \xleftarrow{10} • \xleftarrow{3} ⊃• \xleftarrow{6} ⊃ \xleftarrow{2} ▽ \xleftarrow{2} ◁ \xleftarrow{2} •$, see [Pow72, p. 218].

[29] The (fairly rare) cuneiform counterpart is 𒀹.

[32] The reeds are counted using *tenû* numerals, 𒀹, 𒀹𒀹, 𒀹𒀹𒀹, etc.

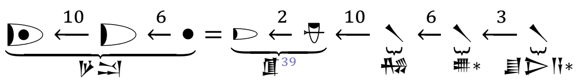
ones discussed in [Lec20], areas are expressed in system $G_{\text{ED IIIb}}$, with curviform numerals[33]; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope[34], the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and thus the interpretation of its numerals between systems $G_{\text{ED IIIb}}$ and $L_{\text{ED IIIb}}$. The sign , which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without . There are other such co-occurrences contrasting between metrological systems; for instance, [Kre98, p. 303 n. 686] mentions the use of cuneiform numerals for days and months[35].

3.4.2 Dyke lengths in Ŋirsu

[Pow87, p. 466] notes that reeds "are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus." Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in [CDLI] finds many occurrences[36], such as:

— [P221305, obv. 1 4][37]
— [P020129, rev. 2 1]
— [P221291, rev. 5 1][38]
— [P221266, rev. 2 1]

These expressions use an explicit sign (counted in multiples of ten) or . This notation—but not its use of curviform numerals—is remarked on in [Lec20, p. 290 n. 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. The notation can be summarized by the following factor diagram, where prefix units have been marked by an asterisk:


$(L'_{\text{ED IIIb}})$

3.4.3 Butter, cheese and wheat in Ŋirsu

A similar mixture of cuneiform and curviform numerals may be observed with the capacity system; indeed, the previously described system uses

[33] A [CDLI] search for "(bur3)" (numerals used for areas) currently returns 15 ED IIIb results, whereas one for "(bur3@c)" (• numerals used for areas) returns 206. Further, when dated, the tablets with cuneiform bur₃ are from the reigns of (variously transliterated iri-inim-gi-na, uru-ka-gi-na, etc.) and (lugal-zag-ge-si), the last two kings of ED IIIb Lagaš.

[34] This is the case of the sides of the field in [P020054, obv. ii 2–3].

[35] That note also mentions a contrast between the use of curviform numerals to count people and curviform numerals to count bread alotted to them in [P010876]; such contrasts are more akin to styling, and might not, on their own, justify the disunification.

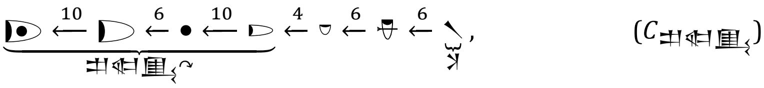
[36] A search for curviform numerals followed by some number of reeds counted in (*tenû*) cuneiform numerals currently finds 125 occurrences across 47 tablets.

[37] [CDLI] only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the is not visible. Lecompte notes that the copy is faithful; indeed another can be seen both on the copy and the photo on obv. 2 2.

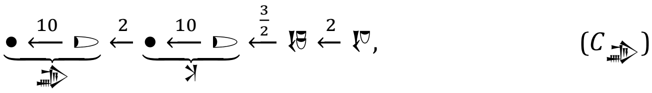
[38] From copy.

[39] With either unit omitted, as in the examples above, or both, as in [P020129, obv. 3 3] .

numerals for 𒋡 [Fri78, p. 43; Lec16]:

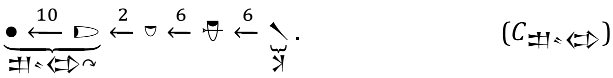


as in [P020016, rev. 1 4; P020065; P020090, obv. 1 3, rev. 2 1; P020092, rev. 3, 1; P020137, obv. 1 2] and others, where ban_2 counted with numerals are followed by $sila_3$ counted with numerals. Curviform numerals are also used to count $sila_3$, but not[40] as part of the 𒄥 systems. This contrast can be seen in [P220927], which measures butter (𒉌, i_3) with a different capacity system, using the 𒂁 (dug, "pot") of 20 𒋡, with and numerals[41] for both the 𒂁 and the 𒋡, thus [Pow87, pp. 504 sq.]



but counts cheese (, ga'ar) using the capacity system, with numerals for the 𒋡.

Another capacity system in ED IIIb Ŋirsu is the , the gur of two ul [Lec16]:



Here the –– contrast occurs not only within the numerals of the system, but with its units; this is perhaps best illustrated by the expressions in [P221746, rev. 2 2] and in [P221814, rev. 1 5].

3.4.4 Grain in Ebla

The mixing of curviform and cuneiform numerals within a metrological system is not specific to Ŋirsu.

[40] As of this writing, the single occurrence of (ban2@c) followed by curviform numerals and sila3 in ED IIIb Ŋirsu transliterations on [CDLI], 4(ban2@c) 3(asz@c) sila3 in [P221815, obv. 4 7], is incorrect: it should be 4(ban2@c) 3(disz@t) sila3.

[41] This tablet also uses subtractive notation: "two pots minus two thirds ($sila_3$)", "ten minus one pots, six $sila_3$". Such subtractive notation is common in most of the metrological systems discussed here; it appears in the ED IIIa period [Rob08, p. 77]. It presents no complexity for character encoding, but it is noteworthy that the sign 𒇲 (lal, "minus") is often ligated with the following numerals, with the subtrahend placed under a sometimes considerably enlarged 𒇲, similar to the layout of the radical in modern mathematical notation, see, *e.g.*, [P020092, rev. 3, 1, 2]. A good font could handle the very common −1 case, perhaps even −2 and −3; setting arbitrary numeric expressions under the 𒇲, or more generally replicating the layout of Early Dynastic tablets, is outside the realm of plain text; see also §6.2.

The system of grain[42] capacities in Ebla uses the following units[43]:

$$\underset{\text{gu}_2\text{-bar}}{𒄘𒁇} \xleftarrow{2} \underset{\text{ba-ri}_2\text{-zu}}{𒁀𒌷𒍪} \xleftarrow{\frac{5}{2}} \underset{\text{giŋ}_4}{𒂷} \xleftarrow{4} \underset{\text{niŋ}_2\text{-saŋšu}}{𒃻𒊕𒋗} \xleftarrow{6} \underset{\text{an-zam}_x}{𒀭𒍠}.$$

The 𒄘𒁇 and 𒁀𒌷𒍪 are generally counted using curviform numerals, and the smaller units using cuneiform 𒁹 numerals[44] Indeed, a search on [EbDA] for co-occurrences of either 𒀭𒍠 or 𒃻𒊕𒋗 with either of 𒄘𒁇 or 𒁀𒌷𒍪 finds the following expressions[45]:

1. [P240532, *verso* 4 9] 𒁀𒌷𒅗[47] 𒐋 𒃻𒊕𒋗
2. [P240548, *verso* 1 1] 𒁀𒌷𒍪 𒐈 𒀭𒍠
3. [P240655, *recto* 7 9] 𒁇[48] 𒐋 𒃻𒊕𒋗
4. [P240579, *verso* 4 3] 𒄘𒁇 𒈫 𒃻𒊕𒋗
5. [P240675, *verso* 2 2] 𒁀𒌷𒍪 𒐋 𒃻𒊕𒋗
6. [P240609, *verso* 3 1] 𒁀𒌷𒍪 𒐈 𒀭𒍠
7. [P240533, *recto* 3 3] 𒄘𒁇 𒐋 𒃻𒊕𒋗 𒈫 𒀭𒍠
8. [P240697, *recto* 1 5] [49] 𒈫 𒂷 𒐈 𒀭𒍠
9. [P240653, *recto* 6 2] 𒄘𒁇 𒁹 𒃻𒊕𒋗 𒈫 𒀭𒍠
10. [P240654, *recto* 2 6] 𒁀𒌷𒍪 𒐋 𒋡[50] 𒐈 𒀭𒍠[51]
11. [P240531, *recto* 1 8] 𒁀𒌷𒍪 𒐋 𒃻𒊕𒋗
12. [P241708, *recto* 1 1][52] 𒄘𒁇 𒐋 𒃻𒊕𒋗
13. [P241904, *recto* 1 1][53] 𒄘𒁇 𒐉[54] 𒃻𒊕𒋗

[42]Liquid capacities use a different system [Arc15, p. 229 n. 12]:

$$\underset{\text{la-ḫa}}{𒆷𒄩} \xleftarrow{30} \underset{\text{sila}_3}{𒋡} \xleftarrow{6} \underset{\text{an-zam}_x}{𒀭𒍠}.$$

A glance it seems that 𒋡 are counted with cuneiform numerals and higher units with curviform ones, thus

but we have not investigated this thoroughly.

[43]Another system uses different values for the 𒂷 and 𒃻𒊕𒋗, see [Cha12, p. 62; Arc15, p. 229 n. 12]:

$$𒄘𒁇 \xleftarrow{2} 𒁀𒌷𒍪 \xleftarrow{3} 𒂷 \xleftarrow{4} 𒃻𒊕𒋗 \xleftarrow{5} 𒀭𒍠.$$

[44]As mentioned in [Cha12, p. 63], the 𒂷 is also counted using the 𒑚–𒑛 numeral series. Some instances of that usage are found transliterated $n/6$ in [EbDA]; in some cases the 𒂷 sign is omitted, and the 𒑚 numeral is then written before the 𒁇 unit, as in 𒑚𒁇 from [P240545, *verso* 1 3].

[45]We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform 𒁀𒌷𒍪 and larger with cuneiform 𒂷 and smaller. In all cases the transcriptions given here are based on the [EbDA] transliterations, but the shape and orientation of the numerals was checked[46]on a photograph (from [EbDA] unless noted otherwise).

[46]As we will see in §3.7.2, [CDLI] transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

[47]ba-ri$_2$-zu$_2$, a variant spelling.

[48]Short for 𒄘𒁇.

[49]Note the omitted 𒄘𒁇.

[50]Instead of the expected 𒃻𒊕𒋗.

[51]𒐈 𒀭𒍠 not legible on the EbDA photo.

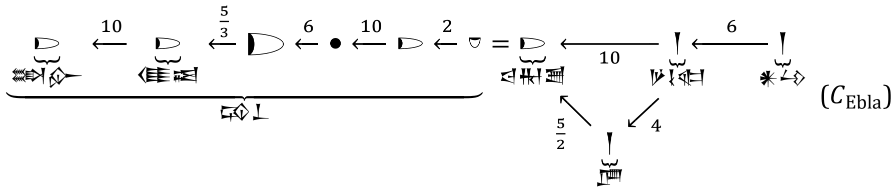
[52]From [CDLI] photo.

[53]From photo in [Arc89, p. 6].

[54]Laid out as 𒁹𒁹𒁹𒁹; on stacking patterns see §6.2.

14. [P240964, *recto* 3 2][55] 𒐈𒇷[56] 𒐞𒐕𒄘𒁇 𒐖𒋙𒊕[57] 𒐖𒀭𒅗

Note that higher numbers of 𒄘𒁇 are expressed in hundreds (*mi-at* 𒈪𒀜) and then thousands (*li-im* 𒇷𒅎), as is typical in Ebla [Arc15, p. 33], *e.g.*, in the last example above or in [P240532, *verso* 2 3], 𒐕𒈪𒀜𒐞𒐈𒌋 𒄘𒁇 (100 + 60 + 30 + 5 = 195 𒄘𒁇 of grain). These expressions correspond to the following factor diagram:



3.4.5 Use in modern publications

Because of their prevalence in the fourth millennium and Early Dynastic period, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 1–21.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 21. The partial[58] transliteration "4𒁹 *ʾa₃-da-um* 4𒐕 *aktum* 4𒁹 $\text{ib}_2^{\text{tu}_9}$×3𒁹 sa_6 gunu_3" is used to illustrate a discussion of the interpretation of the contrast between 𒁹 and 𒐕 numerals. More conventional transliterations might omit the numeral shapes entirely, *e.g.*, 4 *ʾa₃-da-um* 4 *aktum* 4 $\text{ib}_2^{\text{tu}_9}$×3 sa_6 gunu_3, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, *e.g.*, 4(aš^{c}) *ʾa₃-da-um* 4(diš^{c}) *aktum* 4(aš^{c}) $\text{ib}_2^{\text{tu}_9}$×3(diš) sa_6 gunu_3, but the result would be less readable. See §3.7.2 for a discussion of transliteration conventions for numerals.

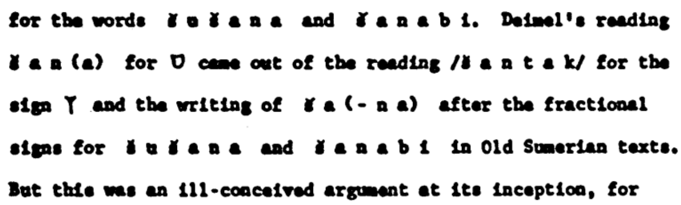

for the words š u š a n a and š a n a b i. Deimel's reading
š a n (a) for ᗜ came out of the reading /š a n t a k/ for the
sign Y and the writing of š a (- n a) after the fractional
signs for š u š a n a and š a n a b i in Old Sumerian texts.
But this was an ill-conceived argument at its inception, for

Figure 1: Discussion of the readings of proposed 𒐕 and already-encoded 𒁹 in [Pow71, p. 107].

[55] From photo in [Arc89, p. 6]; see also the [CDLI] photo and the copy in [Fri86, p. 17]. This tablet features unusual usage of vertical numerals—"somewhat unorganized", as described by [Fri86, p. 16]—, such as 𒐊𒄘𒁇 or 𒐋𒈪, but its 𒋙𒊕 and 𒀭𒅗 are consistently counted with cuneiform numerals, and the higher units with cuneiform numerals.

[56] Short for 𒇷𒅎.

[57] ŠU_2+NIŊ_2-saŋ, an unusual variant spelling.

[58] The untransliterated text would be 𒐉𒀀𒁕𒌝𒐉𒀝𒌈𒐉𒅁𒁉𒌆𒊷𒂅; note the atomically encoded ib_2 × 3𒁹 = 𒅁 × 𒐈 = 𒌈.

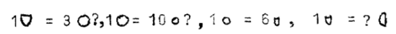
sions also. In example 6, the writing may imply a reading /š a n a b i/,[1] whereas in example 11 should be read */š u š a n a m i n/. Moreover, the question must be raised as to whether such writings as kù - b a b b a r + š a - n a[2] do not perhaps imply a linguistic resolution of */š u š a n a m i n/ rather than /š a n a b i/. I see no way of answering this question at present, but it is one which one

Figure 2: Discussion of the readings of proposed and as well as already-encoded and in [Pow71, p. 138].

iku fractions		
Girsu type	"BIN 8" type	Ur III type
= :f.o.o	= :p.o.o	= :m.o
= :o.g.o	= :o.q.o.	= :o.n
= :o.o.h	= :o.o.r	

Figure 3: A transliteration system for the fractions of the iku in [Pow72, p. 216].

1 "big cup" = 3 "big disks". Hence we can infer from the two ŠE-texts BIN 8,4 and BIN 8,5 together, that the "ŠE-system" makes use of number signs whose values are related to each other through the equations



A more convenient way of saying the same thing is to write out the "steps" between the various ŠE-units in what we shall call a "step-diagram" for the "ŠE-system":

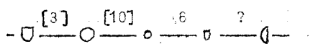
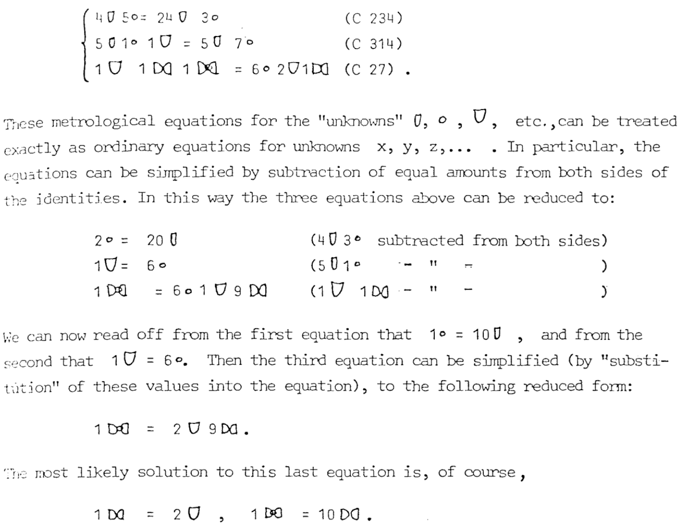


Figure 4: The first factor diagram, in [Fri78, p. 10].


$$\begin{cases} 4ʊ\ 5∘ = 24ʊ\ 3∘ & \text{(C 234)} \\ 5ʊ\ 1∘\ 1Ʊ = 5ʊ\ 7∘ & \text{(C 314)} \\ 1Ʊ\ \ 1⋈\ 1⧓ = 6∘\ 2Ʊ\ 1⋈ & \text{(C 27) .} \end{cases}$$

These metrological equations for the "unknowns" ʊ, ∘ , Ʊ, etc.,can be treated exactly as ordinary equations for unknowns x, y, z,... . In particular, the equations can be simplified by subtraction of equal amounts from both sides of the identities. In this way the three equations above can be reduced to:

2∘ = 20ʊ (4ʊ 3∘ subtracted from both sides)
1Ʊ = 6∘ (5ʊ 1∘ - " -)
1⧓ = 6∘ 1Ʊ 9⋈ (1Ʊ 1⋈ - " -)

We can now read off from the first equation that 1∘ = 10ʊ , and from the second that 1Ʊ = 6∘. Then the third equation can be simplified (by "substitution" of these values into the equation), to the following reduced form:

1⧓ = 2Ʊ 9⋈.

The most likely solution to this last equation is, of course,

1⋈ = 2Ʊ , 1⧓ = 10⋈.

Figure 5: The derivation of the factors of the bisexagesimal system in [Fri78, p. 15][60].

[60]The bisexagesimal system is used alike in proto-Elamite and proto-cuneiform texts, see [Fri78, p. 38]; the derivation in [Fri78, p. 15] is based on proto-Elamite artefacts. There is a typo in the equation for C 27: the right-hand side should have 10⋈ rather than 1⋈, otherwise nothing could be deduced about ⋈. Note that in Friberg's early works [Fri78; Fri79; Fri86; Fri87], copies of fourth millennium and sometimes third millennium tablets are shown as vertical text (which they were for the scribes), and their numerals are written within horizontal text in the same orientation that they have if the tablet is taken as vertical text; in [UAX50] parlance, as if they had Vertical_Orientation=Upright. In addition, they are listed in these equations in the horizontal order in which they appear as vertical text (thus the rightmost numeral is the most significant, read first). Cuneiform is correctly Vertical_Orientation=Rotated, consistently both with modern practice and with the rotation between earlier vertical and later horizontal monumental inscriptions. Friberg's early conventions are not followed in later scholarship, and are abandoned in his own more recent works, such as [Fri07]; a more typical way to express the first equations might be

$$\begin{aligned} 5● + 4⊳ &= 3● + 24⊳ && \text{(C 234)} \\ 1▶ + 1● + 5⊳ &= 7● + 5⊳ && \text{(C 314)} \\ 1⧗ + 1⧖ + 1▶ &= 10⧖ + 2▶ + 6●. && \text{(C 27)} \end{aligned}$$

A diplomatic edition of [Fri78] could rotate the numerals using a higher-level protocol:

$$\begin{cases} 4ʊ\,5● = 24ʊ\,3● & \text{(C 234)} \\ 5ʊ\,1●\,1Ʊ = 5ʊ\,7● & \text{(C 314)} \\ 1Ʊ\ 1⋈\ 1⧓ = 6●\,2Ʊ\ 1⋈ & \text{(C 27).} \end{cases}$$

Thus, for instance, the original set of fractions , , and (1/2, 1/4 and 1/8 of an iku) in the Sumerian GANA system was after a time augmented through the addition of the new sub-unit SAR: , equal to 1/100 of an iku ( ). Similarly, the Sumerian weight unit "ma-na" which originally may have had only the sub-units ša-na (= 1/3 mana) and ša-na-bi (= 2/3 mana), and perhaps also gín: (= 1/60 mana), seems to have acquired, at some time or other, also the smaller sub-units (= 1/3 gín), and še (= 1/3 × 1/60 gín).

Figure 6: Discussion of proposed fractions , , , and , as well as already-encoded and in [Fri78, p. 49].

stein publizierten Zeichenliste enthalten ist[3], bis vor kurzem unentdeckt bleiben konnte. Erst 1978 machte der schwedische Mathematiker J. Friberg, ERBM I, 9-11, darauf aufmerksam, daß die Zeichen für die Zahlen Eins ( ) und Zehn ( ) in Verbindung mit dem Zeichen ŠE nicht im Verhältnis 1 zu 10 sondern im Verhältnis 1 zu 6 stehen. Bis dahin hatte man, obwohl die Andersartigkeit des in Verbindung mit dem Zeichen ŠE verwendeten Zahlzeichensystems bekannt war, für diese beiden häufigsten Zahlzeichen einheitlich ein Verhältnis 1 zu 10 unterstellt, obwohl es mehrere eindeutige Gegenbelege gab, von denen zumindest diejenigen der Archaischen Texte aus Ğemdet Nasr bereits früh publiziert und jedermann zugänglich waren[4]. Als Folge

Figure 7: Discussion in [DE87, p. 117] of the discovery in [Fri78, pp. 9–11] (see Figure 4) of the different relations between and • in systems *G* and **??**.

there is in any case an important qualitative difference between IX for Latin novem and for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 8: The sign used in a parallel with IX in [Eng88, pp. 131–133 n. 9], discussing an argument from [Pow72, p. 172] on the question of the language of the Uruk III texts.

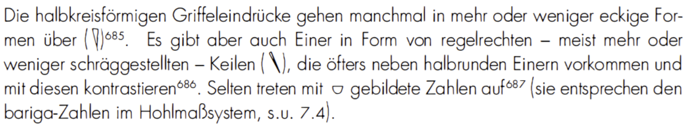
of decreasing fractions $^1/_n$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus = $^1/_2$ N_{39}, = $^1/_3$ N_{39}, and so on. The first sign of the latter units, N_{34},

Figure 9: Description of the fractions and in [Eng98, p. 113][61].

For instance, the first line contains the notations 1N_{34} 1N_{39a} ; 2N_{20}, which can be translated "60 of the (grain rations containing) (of grain); (grain involved:) 2 • (of ground barley)". This calculation contradicts the assumed numerical relationship 10N_1 = 1N_{14}, since as was well known the measure represented by the sign N_{39} was $^1/_5$ of that represented by N_1, so that 60 × $^1/_5$ = 12 and not 20, as 2N_{14} would imply. Instead of relying on complicated

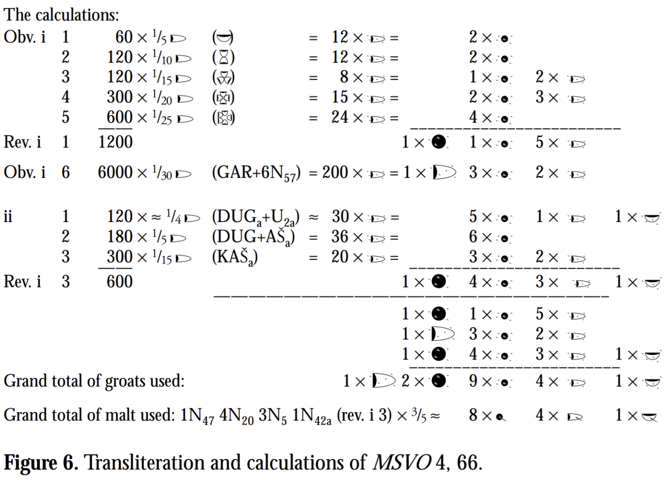
Figure 10: The sign used as a capacity measure within otherwise translated text in [Eng98, p. 116].

[61]The text erroneously has N_{34} instead of N_{24}.



Die halbkreisförmigen Griffeleindrücke gehen manchmal in mehr oder weniger eckige Formen über (…)[685]. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen (…), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren[686]. Selten treten mit ▽ gebildete Zahlen auf[687] (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 11: Discussion of co-occurrences and contrasts between ⁊, ⟍, and ▽ in [Kre98, p. 303].

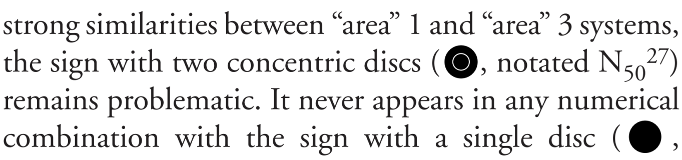


The calculations:

Obv. i	1	$60 \times 1/5$		$= 12 \times\ =$		$2 \times$		
	2	$120 \times 1/10$		$= 12 \times\ =$		$2 \times$		
	3	$120 \times 1/15$		$= 8 \times\ =$		$1 \times$	$2 \times$	
	4	$300 \times 1/20$		$= 15 \times\ =$		$2 \times$	$3 \times$	
	5	$600 \times 1/25$		$= 24 \times\ =$		$4 \times$		
Rev. i	1	1200			$1 \times$	$1 \times$	$5 \times$	
Obv. i	6	$6000 \times 1/30$	($GAR+6N_{57}$)	$= 200 \times\ =$	$1 \times$	$3 \times$	$2 \times$	
ii	1	$120 \times \approx 1/4$	(DUG_a+U_{2a})	$\approx 30 \times\ =$		$5 \times$	$1 \times$	$1 \times$
	2	$180 \times 1/5$	(DUG+AŠ$_a$)	$= 36 \times\ =$		$6 \times$		
	3	$300 \times 1/15$	(KAŠ$_a$)	$= 20 \times\ =$		$3 \times$	$2 \times$	
Rev. i	3	600			$1 \times$	$4 \times$	$3 \times$	$1 \times$
					$1 \times$	$1 \times$	$5 \times$	
					$1 \times$	$3 \times$	$2 \times$	
					$1 \times$	$4 \times$	$3 \times$	$1 \times$
Grand total of groats used:					$1 \times$ $2 \times$	$9 \times$	$4 \times$	$1 \times$
Grand total of malt used: $1N_{47}$ $4N_{20}$ $3N_5$ $1N_{42a}$ (rev. i 3) $\times 3/5 \approx$						$8 \times$	$4 \times$	$1 \times$

Figure 6. Transliteration and calculations of *MSVO* 4, 66.

Figure 12: Calculations from [P005468] transcribed in [Eng01, p. 132] using modern mathematical notation combined with some of the proposed characters.



strong similarities between "area" 1 and "area" 3 systems, the sign with two concentric discs (◉, notated N_{50}[27]) remains problematic. It never appears in any numerical combination with the sign with a single disc (●,

Figure 13: Discussion of ◉ and ●[62] in [Cha03, p. 6].

[62] The statement that these do not co-occur refers to the texts from ED I–II Ur; these signs co-occur both earlier and later in areas, with different relations as previously discussed.

$^1/_{15}$, etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation could be understood as 1 $^1/_5$ gur.[27] However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs and cannot be considered klasmatograms.

Metrological tablets from the end of the 4th millennium (Nissen, Damerow and Englund 1993, 55-59, to *MSVO* 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

1(iku) represents a surface of 3600m²
1(eše$_3$) represents a surface of 21,600m²
etc.

The signs iku and eše$_3$ constitute by themselves measures of surface areas. These measures are usually followed by the sign GAN$_2$, which means either surface or field and

Figure 14: Discussion of Old Babylonian[64] capacity and fourth millennium area measures in [Pro09, p. 9].

formed by only two signs 𒁹 and 𒌋, repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60^2, 60^3, 1/60, $1/60^2$, etc. are written in the same way, with the vertical wedge 𒁹. The third feature concerns the exact function of

Figure 15: Description of the SPVS in [Cha12, p. 58], using the already-encoded signs 𒁹 and 𒌋.

one step. The scribes of the Early Dynastic Period (*c.* 2600 BC), for instance, represented the number 648, 000 with: ●●● but never with the repetition ●●●.

Figure 16: Discussion of large numbers illustrated by ●●●[65] in [Cha12, p. 59]

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

še bar *ba-ri-zu*

'3 *gubar* (capacity units) and 1 *parīsu*'.

Figure 17: Partial transliteration of [P240597, *recto* 5 3] in [Cha12, p. 61].

[64]The cuneiform text is Unicode-encoded.

[65]Compare ◇ in system $G_{\text{Ur III/OB}}$. Sign order can be variable in early texts, see [Fox16, p. 8]. See [P010773], also discussed in [Fri07, p. 148], for an example of ●, and [P274845; P241764] for examples of n●.

This is particularly true of the signs , , and , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign is most often associated with the *parīsu* measure, while the signs , , and refer to 1, 2, 3,

Figure 18: Discussion in [Cha12, p. 64] of the relation between – and in Mesopotamia and in Ebla.

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g., transliterated as 2(šar$_2$) 1(geš'u) 3(u), means 2 × 3600 + 1 × 600 + 3 × 10). The system is based on an alternation of factors ten and

Figure 19: Explanation of the structure of the number in [Pro20, p. 350].

might think of one fabric and a half,[11] but the presence of notations with "2 2", "3 3", and "6 6" (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag's weight measurement system.[12] The notation "1 gada" in o. ii 1 and r. vi 1, along with the total of "39

Fig. 1. Combinations of numerals attested in Š. 742.

Figure 20: Discussion of the contrast between and numerals in [Gor23, p. 162].

as, for example, in TM.75.G.3125 = *ARET* III 107 o. iv 1, "4 ʾa$_3$-da-um^{tu9}-2 4 aktum 4 ib$_2^{tu9}$ ×3 sa$_6$ gunu$_3$" (Fig. 2).

Figure 21: Transliteration in [Gor23, p. 163] of [P242293, *recto* 4 1] incorporating untransliterated numerals.

3.5 Non-numeric usage

[illegible]

The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'[66]. Do you know its reading[67]?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign 𒀸 has, *inter alia*, the values aš, rum, and dili. While the horizontal numerals are most frequently written [illegible] in the Early Dynastic period[68], such non-numeric usage is almost[69] always written 𒀸, for instance:

- in personal names in administrative texts, such as the following, which all contain [illegible] numerals:
  - 𒀸𒀴 in [P010424, rev. 1 5; P010458, obv. 1 5; P010459, obv. 2 5′] from ED IIIa أبو صلابيخ,
  - 𒀸[illegible] in [P010960, obv. 2 5] from ED IIIa Šuruppag,
  - [illegible] in [P251641, obv. 4 3] from ED IIIb Adab,
  - [illegible] in [P252866, rev. 2 3] from ED IIIb Adab,
  - [illegible] in [P298637, rev. 2 4] from ED IIIb Umma;
- in the Sumerian word 𒌑𒀸 u_2-rum, "property" in ED IIIb Ŋirsu administrative texts which contain [illegible] numerals, such as [P020006, obv. 2 3; P020008, rev. 1 2; P020018, rev. 1 2; P020024, obv. 1 4; P020030, obv. 3 1];
- in lexical texts:
  - in the divine name [illegible] in the lexical texts [P010570, rev. 2 4; P010572, obv. 3 6], where the entries are prefixed with [illegible].
  - in the word 𒀸 dili, "small fish" in [P010578, obv. 2 5], witness to Early Dynastic Fish,
  - in the same word with a determinative, 𒀸𒄩 dili^{ku_6}, in [P010586, obv. 4 4, 6], witness to Early Dynastic Food, which starts with [illegible] numerals.

This is a clear contrast between 𒀸 and [illegible] in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name 𒀸𒀴 occurs on its own line in the aforementioned administrative texts; a line [illegible]𒀴 would instead be read as "one slave".

3.6 The limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in §3.1. While these benefits are real and

[66] The reader will recall that $ŋeš_2$ is written [illegible], with a larger wedge than [illegible]; however, these signs have merged by the time Examenstext A is composed.

[67] Besides $ŋeš_2$, a look at [OSL] shows that the values diš, ge_3, makkaš, $saŋtak_4$, and tal_4 are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

[68] A [CDLI] search for `"(asz@c)"` finds 3296 ED texts, while a search for `"(asz)"` finds 81 ED texts, of which 46 also contain `"(asz@c)"`.

[69] Exceptions are discussed in §3.7.1.

now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as 𒀸 that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [Sch35, pp. 123 sqq.] lists all of 𒀸–𒐃 together with 𒐕–𒐝, while 𒀸, 𒐀, and 𒐁, and only those, appear at the beginning of the sign list, since they have non-numeric values[70]. [Cat13, p. 58] has the numeric signs 𒐕, 𒀹, 𒁹, whereas non-numeric 𒀸 is at the beginning of the sign list, where its values *aš* and *rum* are listed. For signs with both non-numeric and numeric usage, [Dei22] writes *s. die Zahlz.* throughout the main list; LAK 1 𒀸 thus reappears at LAK 829 together with 𒐕, 𒀹, and 𒁹. One should note [Bor10], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have 𒐼–𒐽 used in system $G_{\text{Ur III/OB}}$.

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes[71] to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from 𒐕 to 𒁹 numerals prevents diachronic encoding even if 𒐕 were unified with 𒀸. For instance, the lexical list Early Dynastic Food, already mentioned in §3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1′–6′]; however, they are written with 𒁹 numerals, whereas they are written with 𒐕 numerals in the Early Dynastic witnesses; since 𒁹 and 𒀸 are distinct[72] characters, the 𒐕-𒀸 unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods[73], there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Rom23, *sub* "Adding Corpora"], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

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[70] Non-numeric values of 𒀸 were discussed in §3.5; 𒐀 has the values man₃ and min₅, and is used for the word didli, "several, various"; 𒐁 has the value eš₆.

[71] Compare, *e.g.*, in the *Instructions of Šuruppag*, 𒋢𒆳𒊒𒆠 𒌉𒉌𒐕 𒌉𒉌𒊏 / 𒈾 𒈾𒈬𒊑𒊑 in the ED IIIa witness [P222243, obv. 2 7], also discussed in §3.7.1, and 𒋢𒆳𒊒𒆠𒂊 𒌉𒉌𒊏 𒈾 𒈾𒈬𒌦𒉈𒉈𒊑𒊑 in the OB composite [Q000782, 6] (translated "Šuruppag gave instructions to his son" in [ETCSL, t.5.6.1, 1–13]). It does not matter for the construction of a composite text whether this is encoded 𒌉𒉌𒐕 or 𒌉𒉌𒀸, since that word is absent from other witnesses, and since the surrounding words differ.

[72] Besides the contrasts in numeric usage mentioned in §3.3.3, these (already-encoded) characters were clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of 𒀸 which are not shared with 𒁹 have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period, 𒀸 is used for the preposition *ina*, "in", and 𒁹 for the preposition *ana*, "to".

[73] See, *e.g.*, [Pow87, p. 493; Rob08, p. 55] on the unification of metrologies in the Old Akkadian period, resulting in the systems described in §3.3.

3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts[74] being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

3.7.1 The case of ŠAR$_2$

The character U+122B9 ● CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period[75], a contrast between non-numeric šar$_2$ written ◇ and numeric 1(šar$_2$^{c}) written ● can be observed, similar to the contrast between [cuneiform] and [cuneiform] previously discussed in §3.5. However, in lexical lists from Šuruppag and Ebla[76], as well as in the *Stèle des vautours*, non-numeric šar$_2$ is curviform:

— ✳[cuneiform]● and ✳[cuneiform]●[cuneiform] in [P010566, obv. 10 10, 11];
— ●[cuneiform] and ✳●[cuneiform] in [P010576, rev. 3 16, 17];
— ●[cuneiform] in [P240986, *recto* 3 3][77];
— ●[cuneiform] in [P222399, obv. 17 9, 18 11, 22 12][78].

It *would* be disruptive to the diachronic representation of text if non-numeric šar$_2$ were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph ◇, identical to the glyph of the proposed U+12579 ● CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric šar$_2$ is curviform solidly predates the transition from ● to ◇ in the relevant metrological systems, there is no need to represent a ◇-● contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D ◇ CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

[74] Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

[75] For example, in personal names:
— [cuneiform]◇[cuneiform] in [P020019, rev. 1 2] from ED IIIb Ŋirsu;
— [cuneiform]◇[cuneiform] in [P020182, obv. 2 9], also from ED IIIb Ŋirsu;
— [cuneiform]✳◇ in [P222186, obv. 3 3] from ED IIIb Umma;
— [cuneiform]✳[cuneiform]◇ in [P235312, obv. 16] from Old Akkadian Umma.

[76] These are archaizing in other ways, *e.g.*, they have a [cuneiform]–[cuneiform] (NAM$_2$–TUG$_2$) split.

[77] From copy in [Man81, ELLes 397].

[78] Note however ✳[cuneiform] ◇[cuneiform] on [P222399, obv. 6 17], see Figure 22. Curviform non-numeric šar$_2$ is clearly archaizing in ED IIIb Ŋirsu; one might suppose that the scribe slipped into their modern ways here.

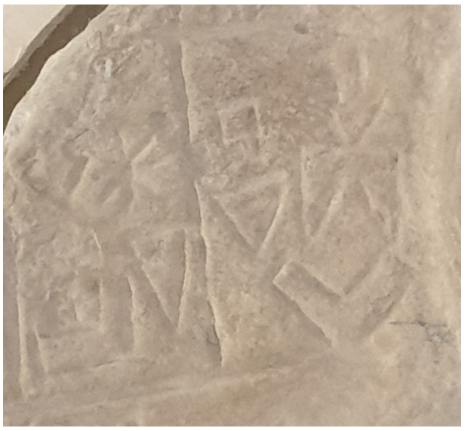

Figure 22: [P222399, obv. 6 16–17] 𒆬𒌑𒆬 / 𒀭𒁉 𒄭𒆠.

Note that in rare cases, such as [P222243, obv. 2 7] from ED IIIa Adab, non-numeric 𒀸 (here with the value rum) is written 𒕐. It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 𒕐 cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. in practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the [CDLI] transliteration (UR2-1(aš@c)) and the [ePSD2] one (uru_8^{rum}) of this word disagree on that aspect. Since 𒀸 has a cuneiform reference glyph, this does not pose any compatibility concerns.

3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL_2 can be described as 𒉈×𒆜 NE×KASKAL or 𒉈×𒉽 NE×PAP[79], they are typically all transliterated bil_2, and therefore are all represented by the character U+1224B 𒉋 CUNEIFORM SIGN NE SHESHIG, its name notwithstanding, as described in [UTR56, §2.5].

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, 𒁹𒌍𒐊𒄥 from [P305639, rev. 21] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in [EbDA] transliterations: the aforementioned 𒕚𒕐𒑲𒄘𒁇 𒐌𒃻𒊕𒋗 𒐖𒈦𒀭𒍠𒍝 from [P240533, *recto* 3 3] is transliterated “20-1-1/2 *gu*$_2$-*bar* 7 nig_2-sagšu 2-1/2 an-zam_x[80] *za*”, reading both 𒑲 and 𒈦 as 1/2, but not distinguishing them.

[79]As on [P249253].

[80]As of this writing, [EbDA] actually has an-zam_χ, with U+1D6A GREEK SUBSCRIPT SMALL LETTER CHI.

In particular, these transliterations do not differentiate between 𒀸 and 𒁹 numerals, nor between and numerals. For instance, the aforementioned [P242293, *recto* 4 1] is transliterated "4 ʾ*a*$_3$-*da*-*um*^{tug_2}-II 4 aktum^{tug_2} 4 ib$_2$-III gun$_3$ sa$_6$ ^{tug_2}" in [EbDA], with no distinction between the and . Since 𒀸 and 𒁹 numerals are separately encoded, the numeric expressions in such transliterations cannot be transformed into Unicode cuneiform without additional context, regardless of curviform–cuneiform unification.

In metrological systems such as systems $G_{\text{Ur III/OB}}$ and $C_{\text{Ur III/OB}}$ where some units are indicated by the type of numeral rather than an overt unit sign, it is common practice to add the unit in parentheses in transliteration; for instance, from [P386847, obv. 1] is transliterated "1(eše$_3$) 5½ iku[81] 7 sar" in [Feu04, vol. 2, p. 176], and from [P307255, obv. 12] is transliterated "1(n[82]) 2(b) 7 ½ sila$_3$" in [Feu04, vol. 2, p. 151].

This practice has been generalized to systematically indicate numeral shape; this is in particular the case in [CDLI], where the transliterations of some the above examples are "1(gesz2) 3(u) 5(asz) gur" for , "1(esze3) 5(iku) 1/2(iku) GAN2 7(disz) sar" for , and "3(barig) 2(ban2) 7(disz) 1/2(disz) sila3" for . [CDLI] and [ePSD2] both distinguish curviform from cuneiform numerals in transliteration: the length from [P020129, rev. 2 1] is transliterated "6(gesz2@c) 3(u@c) {ninda}nindax(DU) 1/2(asz@c) 4(disz@t) gi" in [CDLI], and "6(geš$_2^c$) 3(u^c) $^{\text{ninda}}$ninda$_x$(DU) 1/2(aš^c) 4(diš^t) gi" in [ePSD2]. Another example is [Mol14, p. 39], which uses 1*a* for 𒀸, 1*d* for 𒁹, 1*ac* for , 1*dc* or ½*dc* for depending on reading, etc. The literature on the Uruk and Early Dynastic I–II periods uses a different set of transliteration conventions that also disambiguate numeral shapes, as will be discussed in §4.

While there exist transliterations that distinguish 𒀸 from 𒁹 but not from 𒀸, such as the ones used in [DCCMT], the trend, especially in more recent works in third millennium studies, seems to be to represent numeral shape; for example, [MV24] gave an example of the input syntax used by the new "Urban Economy Begins" project as "10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)" for , with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are rather than numerals. The "tradition of cavalierly dispensing with numerical notations in notations of administrative documents", as [Eng04, p. 30] describes it, seems to be fading.

3.8 Conclusions

Co-occurences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications. The contrast frequently applies to individual numerals, rather than to the span of entire numeric expressions.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple

[81] interpreted as a unit, as discussed in §3.3.

[82] short for nigida, an older reading of bariga; see [**Landsberger1950**; Pow75, p. 181; Fox22, p. 9].

cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted: is a length of three ropes, but is an area of three bur_3; could be read as one and one , where would be one and a half ; is a personal name, but would be "one slave".

In addition, there would be a risk of confusion about character identity should fontmakers attempt to treat the curviform and cuneiform numerals as unified. A designer concerned about the numeric-syllabic – contrast, and wishing to support diachronic encoding between systems $S_{\mathrm{Ur\ III/OB}}$ and S, might give the numeral series (which is typically only used numerically in the Early Dynastic period) the glyphs of the numeral series, since the clear – identification involves the same rotation; this would however make it impossible to represent capacity measures that use . Similarly, in an effort to support diachronic encoding for 1/2(iku), one might be tempted to give the glyph of , thereby rendering the font unusable for quantities measured using the numeral series; an ED I–II Ur font designer could decide to give the same glyph as (that of the proposed), according to the older area system, making it impossible to represent the newer system.

At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millennium corpora. The oddity of ● requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.

4 Rationale for ED–Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millennium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millennium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs KAŠ_a, KAŠ_b, and KAŠ_c, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Eng01, pp. 34 sq.]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered scripts.

As part of the development of these conventions, a classification of fourth millennium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter N with a numeric subscript (sometimes with an additional alphabetic subscript): N_1 is , N_{14} is •, N_{34} is , etc. Transliterations of numeric expression then use those to identify the type of number used, thus $5N_1$ is , and $5N_{14}$ is .

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the [CDLI] transliteration of or in [P283802, rev. 1 6, 2 2] currently uses (barig@c) for the vertical numerals, since numerals are typically capacity measures; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millennium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I–II period, even though the Sumerian text uses classical assyriological transliteration conventions; see [Cha03, p. 6 n. 27].

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millennium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in §3.7.2; they are indeed used interchangeably, as in [P011104] which uses the notation u@f in [ePSD2], but N14@f in [CDLI]. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millennium would therefore induce confusion as to which numerals should be used in third millennium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millennium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance, N_{30c} is listed as a variant of N_{30a} in [DE87, p. 166], where the numeric value of either in relation to N_{39a} is still unknown, but their values are found in [Eng04, p. 33] to be $= \frac{1}{10}$, whereas $= \frac{1}{6}$.

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban$_2$@c) and 5(ban$_2$@c).]

6 Characters not included in this proposal

6.1 Missing numerals

TODO "Ten of the sixty numerical signs contained in the list in figure 27, moreover, do not belong to any of the identified systems. Three of them were apparently scribbled by an awkward pupil. As to four of those remaining, we are not sure whether they constitute derivations of other, as yet unknown numerical signs or whether they are in fact numerical signs at all. For at least two of the ten signs, txi and we can affirm that each formed part of two additional systems, about which we know nothing due to the fact that no informative texts have been unearthed with notations in these systems." [NDE93, p. 27] TODO N10 described as coming from P001319 which does not have it anymore. TODO N13 not attested in CDLI TODO (N_{17} not usefully numeric, $12N_{14}$ not encodable, etc.). Cite [DE87, p. 147] N30Cb̆ not attested 7 and 8(diš *tenû*) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.

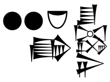

Figure 23: The layout of case [P011099, rev. 2 3]; the numeral ⁚ is rotated to fit the rounded corner of the tablet.

6.2 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some *stacking patterns*; for instance 9𒁹 is encoded both as U+12446 𒑆 and as U+1240E 𒐎. This is in part due to contrastive usage of stacking patterns. For instance, besides 𒑘 and 𒑙 which are characteristic of bariga measures, four bariga is written 𒐼 even where 4𒁹 is written 𒐉, as in [P255010, obv. 2 3, rev. 1 17; P292843, obv. 4, rev. 5]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often 𒁹 𒈫 𒐈 𒐉 𒐊 𒑂 𒑄 𒑆 𒐎 𒌋 𒎙 𒌍 𒐏 𒐐, and results in metrological systems, typically 𒁹 𒈫 𒐈 𒐼 𒐊 𒐋 𒐌 𒐍 𒐎 𒌋 𒎙 𒌍 𒑩 𒑪, occasionally co-occurring as in [P142827; P142357]. This separate encoding is also for compatibility with distinctions made in reference works and in some non-numeric transliterations; for instance, 𒐉 is [Bor10, MZL 860] and has the value limmu, whereas 𒐼 is [Bor10, MZL 852] and has the value limmu_5. Numeric[83] transliterations occasionally distinguish the stacking patterns 𒐉 𒑂 𒑄 𒑆 𒐏 𒐐, as in the [CDLI] transliterations of the aforementioned tablets, although this is rare; often 4(diš) is 𒐼 in Ur III, but 𒐉 in the Neo-Assyrian period.

However, the stacking patterns from earlier periods are not separately encoded; for instance, in ED IIIb Ŋirsu, 𒎙 2(u) often has one 𒌋 atop another. These older stacking patterns do not appear to be contrastive, are not marked in transliteration, and are not listed separately in sign lists nor assigned any different values. There is therefore no evidence of a need to encode them; instead, they should be considered style variants, and an ED IIIb Ŋirsu font should have an appropriate glyph for U+12399 𒎙 CUNEIFORM SIGN U U.

Likewise, many stacking patterns are attested for the curviform numerals proposed in this document, and it is not proposed to separately encode them. These distinctions would be incompatible with the state of the art in numeric transliterations, including those by Englund, who insisted on "a system of transliteration that reflects in a strict fashion the physical realities of the cuneiform inscriptions" [Eng04, p. 30], and they are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier tablets, as they are structured in rectangular cases rather than lines, so that numerals may be laid out across the case in whichever way fits the available space; this is illustrated in Figure 23. Note also that the numerals need to be considerably enlarged in order to reproduce the layout of the tablets, so that ⁚ often spans two lines of cuneiform signs, as shown in Figure 24. This is impractical when these numerals are set in text that contrasts them with the larger ⊃, and inconsistent with actual practice when typesetting these numerals, as illustrated in Figure 8: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dyn-

[83] The Sumerian word limmu means "four", so limmu and limmu_5 are still numbers. The distinction here is between usage in transliterations of phrases such as 𒈗 𒀭𒌒𒁕 𒐼𒁀𒆤 lugal an-ub-da limmu_5-ba-ke_4 (king of the four quarters) or of names, and of numeric expressions such as 𒐼𒋡 4(diš) sila_3.

Figure 24: The layout of case [P020066, obv. 1 1]; the numeral is spread across two lines. The text is read in the order , "twenty-two oxen, one year old".

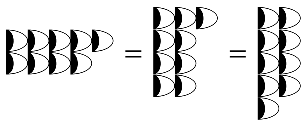

Figure 25: Three stacking patterns for U+12573 CUNEIFORM NUMERIC SIGN NINE N34. The one on the left is the reference glyph, used in Uruk III [P003499, obv. 1 1b; P004430, rev. 1 2], and widely afterwards, *e.g.*, ED IIIa Šuruppag [P010678, obv. 2], ED IIIb Ŋirsu [P020057, obv. 1 3], Old Akkadian Umma [P212464, obv. 11]. The ones in the middle and right are used in two Uruk IV tablets [P001243, rev. P004500, rev. 2]. All three Uruk examples are transliterated 9(N34) in [CDLI].

astic period, but that are also attested in the fourth millennium in the Uruk III period; the fourth millennium, especially the Uruk IV period, also frequently features numerals that use a more vertical layout, as illustrated in Figure 25. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millennium, usage in third millennium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents , , •, and ● must remain consistent across the numeral series, lest a numeral be confused with an numeral. Since the single indents are frequently used in running text, as illustrated in §3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 25 uses OpenType stylistic alternates, and Figure 23 rotates the character , in both cases preserving the plain text backing.

6.3 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Eng01].

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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[P005468] IM 023426. Baghdad, Iraq: المتحف العراقي.
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[P010424] IM 067642. Baghdad, Iraq: المتحف العراقي.
CDLI: P010424.
ORACC: epsd2/P010424.

[P010458] IM 081445. Baghdad, Iraq: المتحف العراقي.
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ORACC: epsd2/P010458.

[P010459] IM 081449. Baghdad, Iraq: المتحف العراقي.
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[P010566] VAT 12760 +. Berlin, Germany: Vorderasiatisches Museum.
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ORACC: epsd2/P010566.
ORACC: dcclt/P010566.

[P010570] VAT 12626. Berlin, Germany: Vorderasiatisches Museum.
CDLI: P010570.
ORACC: epsd2/P010570.
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[P010572] VAT 12644. Berlin, Germany: Vorderasiatisches Museum.
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[P235312] USC 6710. Los Angeles, California, United States: University of Southern California.
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CDLI: P240531.
EbDA: 1415.

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EbDA: 1324.

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EbDA: 1364.

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CDLI: P240697.
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ORACC: dccmt/P240964.
EbDA: 3184.

[P240986] Excavation number TM.75.G.01415.
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CDLI: P249253.

[P251641] MS 2464. Oslo, Norway: Schøyen Collection.
CDLI: P251641.
ORACC: epsd2/P251641.

[P252866] MS 3830. Oslo, Norway: Schøyen Collection.
CDLI: P252866.
ORACC: epsd2/P252866.

[P255010] YBC 04698. New Haven, Connecticut, United States: Yale Babylonian Collection.
CDLI: P255010.

[P271238] Anonymous.
CDLI: P271238.
ORACC: epsd2/P271238.

[P274845] CUNES 50-08-001. Ithaca, New York, United States: Department of Near Eastern Studies, Cornell University.
CDLI: P274845.

[P283802] Ist Š 0742. Arkeoloji Müzeleri.
CDLI: P283802.
ORACC: epsd2/P283802.

[P292843] NBC 05385. New Haven, Connecticut, United States: Nies Babylonian Collection, Yale Babylonian Collection.
CDLI: P292843.

[P298637] NBC 06978. New Haven, Connecticut, United States: Nies Babylonian Collection, Yale Babylonian Collection.
CDLI: P298637.
ORACC: epsd2/P298637.

[P305639] YBC 04398. New Haven, Connecticut, United States: Yale Babylonian Collection.
CDLI: P305639.

[P307255] YBC 06219. New Haven, Connecticut, United States: Yale Babylonian Collection.
CDLI: P307255.

[P309594] YBC 08761. New Haven, Connecticut, United States: Yale Babylonian Collection.
CDLI: P309594.
ORACC: epsd2/P309594.

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