Twelve cuneiform *tenû* numerals

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Contents

1 Summary

This document proposes filling the Cuneiform Numbers and Punctuation block with twelve cuneiform numerals used in the third millennium.

Three of those are additional numerals in the AŠ (or DIŠ) *tenû* series, 7𒐗–9𒐗, where 1𒐗 = 𒐗 through 6𒐗 = 𒐼 are already encoded. Their glyphic range and usage, as well as possible reasons for their absence in the current version of the Standard, are discussed in §**??**.

The other proposed characters constitute a new series of numerals, formed by 𒐗 numerals crossing an 𒐕 wedge. They are discussed in §**??**.

2 Proposed changes to the Standard

2.1 Core specification text

No change is needed in the core specification.

2.2 Code charts

The code charts for the affected block, including the character names list with proposed informative aliases, cross references, and informative notes, are shown on the following pages. A plain text file containing the NamesList.txt lines is attached to this document.

2.3 Properties

Add to the respective UCD files the lines given in this section. These are available as plain text files attached to this document. Changes to derived files are not listed.

2.3.1 Name, General_Category, Numeric_Value, etc.

2.3.2 Line_Break

2.3.3 Script

2.3.4 Script_Extensions

2.3.5 Block

3 DIŠ *tenû* numerals

This section discusses the following proposed characters:

— U+1246F 𒑯 CUNEIFORM NUMERIC SIGN SEVEN ASH TENU
— U+12475 𒑵 CUNEIFORM NUMERIC SIGN EIGHT ASH TENU
— U+12476 𒑶 CUNEIFORM NUMERIC SIGN NINE ASH TENU

3.1 Name

The existing numerals in the 𒐗 series are named 𒐗 CUNEIFORM SIGN ASH ZIDA TENU for the first one and CUNEIFORM NUMERIC SIGN *n* ASH TENU for 𒐘–𒐼.

Some[1] technical terms used in cuneiform character names are derived originate from the structural descriptions of cuneiform signs by Akkadian-speaking scribes

[1] TODO also note gunû but contrast CROSSING rather than gi-li-mu-u, SQUARED rather than li-mu-bu i-gi-gu-ub-bu-u2

in late second and first millennium lexical texts. [TODO(egg): Cite Yushu Gong on tenu itself] In particular, the word *tenû* is used to describe slanted signs or parts of signs: thus 𒑎 is described as 𒁇 *tenû* in [**P365233**][2], 𒑏 as 𒑐 *tenû* in [**P391514**; **P467315**], 𒑑 as 𒐏 *tenû* in [**P391514**], 𒁮 as 𒁯 (containing) 𒀸 *tenû* in [**P365267**][3]. In most cases, the direction of the slant not explicitly specified. The terms *kaba tenû* and *zida tenû*, from Sumerian 𒆕 gab$_2$ "left" and 𒍣 zid "right" respectively, are used in [**P345960**], which contrasts 𒀺 described as *kaba tenû* and 𒀹 described as *zida tenû*.

In modern transliteration, 𒀹 numerals are described as 𒀸 *tenû* (ATF: `asz@t`) or 𒁹 *tenû* (ATF: `disz@t`), the latter being more common[4]. Informative aliases using *diš tenû* have been recommended for the existing characters in [**L2/24-239**]. The proposed names use ASH TENU for consistency with the already-encoded characters, and the proposed annotations include informative aliases with *diš tenû*.

3.2 Ur III usage

As described in [Sch35, p. 135] (see Figure 1), slanted signs are used in Ur III economic texts primarily in subtractive notation with 𒇲[5] lal[6], as well as for ordinals[7] and for ages of animals in years[8].

Accounts of animals giving their ages in years rarely go beyond three-year old animals. Subtractive notation, which appears in the ED IIIa period [Rob08, p. 77], is used to compactly express numbers close to a larger round number, *e.g.*, 𒌋𒇲 $10 - 1$ instead of 𒐝 for 9, 𒌍𒇲 $30 - 2$ instead of 𒌍𒐜 for 28, or 𒁹𒇲 $60 - 1$ instead of 𒐐𒐝 for 59; *cf.* IX instead of VIIII in Roman numerals. It is therefore usually limited to small subtrahends[9]. Larger subtrahends do occur for quantities close to a much larger unit; however in Ur III, they are often written 𒁹 numerals, as in [**P109346**] 𒐉𒂆𒇲𒐛𒊺𒆬𒄀 "4 shekels minus 7 grains of gold", a weight which would otherwise be written 𒐈𒑚𒂆𒐏𒐈𒊺 "$3 + \frac{2}{3}$ shekels and 53 grains", as 180𒊺 $= 1$𒂆.

Ordinals with 𒀹 numerals are also typically limited to small numbers or subtractive notation: many of the attestations of n𒀹𒄰 "nth" are in year names[10], such as 𒈬 𒋼𒄯𒆠 𒀀𒁕 𒀹𒄰𒀸 𒁀𒄭𒇻 mu kar$_2$-ḫar^{ki} a-ra$_2$ 2𒀹-kam-aš ba-ḫul "year Karhar was destroyed for the third time" (31st year of Šulgi's reign), 𒈬 𒋛𒈬𒊒𒌝𒆠 𒀀𒁕 𒀹𒄰𒀸 𒁀𒄭𒇻 mu si-mu-ru-um^{ki} a-ra$_2$ 3𒀹-kam-aš ba-ḫul "year Simurrum was destroyed for the third time" (32nd year of Šulgi's reign), or 𒈬 𒋛𒈬𒊒𒌝𒆠 𒌋 𒇷𒇷𒁍𒌝 𒀀𒁕 𒌋𒇲𒄰𒀸 𒁀𒄭𒇻 mu si-mu-ru-um^{ki} u$_3$

[2] Note that while the third millennium 𒑎 and 𒁇 are related by a 45° rotation, in the Neo-Assyrian style used by this list, these signs look like 𒑎 and 𒑐, so that only one wedge is slanted, as noted in [**Gong00**].

[3] TODO something on spelling out names 𒂵𒈾𒋼𒉡𒌑 *ga-na te-nu-u$_2$* and 𒊺𒋼𒉡𒌑 *še te-nu-u*; *ku te-nu-u*, etc.

[4] For an example of a transliteration using aš tenû, see [**Greco2021**]; note that only the HTML version uses aš tenû, the PDF uses diš.

[5] As noted in [L2/24-210R, p. 25 n. 40], the sign 𒇲 (lal, "minus") is often ligated with the following numerals, with the subtrahend placed under a sometimes considerably enlarged 𒇲, similar to the layout of the radical in modern mathematical notation, see, *e.g.*, [P020092, rev. 3 1, 2]. The font used in this document ligates or kerns 𒀹 subtrahends, but does not enlarge the 𒇲.

[6] Also transliterated la$_2$, as in [CDLI]. In the transliterated Ur III corpus on [CDLI], out of 3304 occurrences of `(disz@t)`, 1971 are in 𒇲n𒀹 `la2` n`(disz@t)`.

[7] 1583 out of 3304 occurrences are n𒀹𒄰 n`(disz@t)-kam`, including 647 after 𒇲

[8] 203 occurrences of `gu4`, `ab2`, `ansze`, or `dur3` n`(disz@t)`

[9] Of the 1971 Ur III occurrences of `lal` n`(disz@t)`, 1930 are with $n \leq 2$, of which 1823 with $n = 1$.

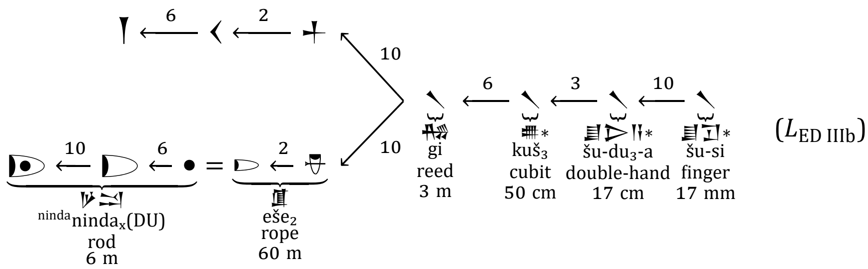
[10] 430 occurrences of n`(disz@t)-kam` are on lines starting with `mu`, of which 308 are in 𒌋𒇲.

lu-lu-bu-um^ki a-ra2 1𒌋 lal 1𒀹-kam-aš ba-ḫul "year Simurrum and Lullubum were destroyed for the ninth time" (44th year of Šulgi's reign). Larger ordinals are frequent, in particular for the day of the month, but these are written with 𒁹 numerals, thus 𒌓𒐌𒄰 for "the 7th day" or 𒌓𒎙𒐍𒄰 for "the 28th day".

The rarity of the higher 𒀹 numerals in the Ur III corpus likely explains the absence of 7𒀹–9𒀹 from the répertoire of Unicode Version 5.0, which was aiming to encode a répertoire appropriate for the Ur III period and later.

3.3 Early Dynastic usage

The situation is different in the Early Dynastic corpus. As described in [L2/24-210R], 𒀹 numerals are used in many Early Dynastic metrological systems, and in particular in the Early Dynastic IIIb length system



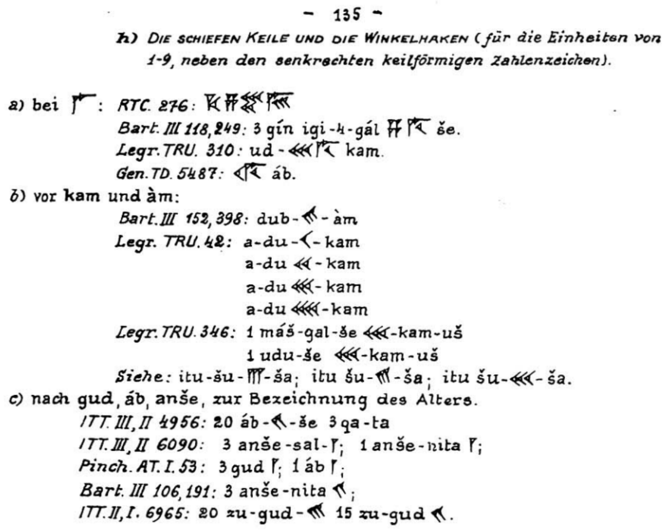
While these systems have a unit 1 𒃻𒁺 = 2 𒄀, lengths above 1 𒄀 are only expressed in 𒂠, or equivalently in tens of 𒃻𒁺, and in half-𒂠 equal to 10 𒄀. We can therefore expect 7–9 to occur, expressed using 𒀹 numerals. Indeed, 37 texts in the transliterated ED IIIb corpus on [CDLI] contain undamaged attestations of either 7𒀹𒄀 or 8𒀹𒄀[11]. TODO(egg): Figures. However, 9𒀹𒄀 is not attested, since instead subtractive notation is used, as in [3 ninda] [4 eše2] LAL gi in [P020129, obv. 3 3], [1 ninda] [5 eše2] 𒃻𒁺 LAL gi [4 kuš3] in [**P221272**], or 𒌋 LAL gi in [**P020304**].

The use of 𒀹 numerals for ordinals, especially for days, is more prevalent in the Early Dynastic period than in the Ur III period, and the use of subtractive notation is less frequent in these numbers. We therefore find many attestations of 7𒀹–9𒀹 in 𒌓*n*𒀹𒄰. TODO(egg) Figures.

In Ebla, the 𒀹 numerals are primarily used in subtractive notation, see [Gor24, p. 88 n. 298, p. 120 n. 465, p. 167 n. 739, p. 180 n. 801]. However, contrary to Ur III, 𒀹 numerals remain used for large subtrahends, thus [Gor24, p. 101 n. 355] cites occurrences of [40] LA 4𒀹 for 36 and [100] LA 6𒀹 *mi-at*[12] for 94. In particular, [Gor24, pp. 129 sq.] cites occurrences of LA 9𒀹 in Ebla: [10] LA 9𒀹 ma-na ku3-babbar "9 minas and 51 shekels of silver" in [**P241283**], and [2] LA 9𒀹 ma-na ku3-babbar "1 mina and 51 shekels of silver" in [**P241325**].

[11] Of those, 34 have 7𒀹𒄀 and 9 have 8𒀹𒄀.

[12] Recall that 𒌋𒄿𒀀𒀜 *mi-at* is Eblaite for "hundred", see [Arc15, p. 33; L2/24-210R, p. 27].



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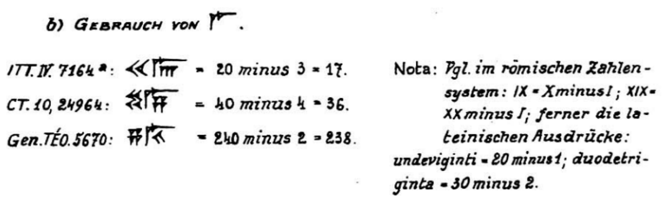
h) Die schiefen Keile und die Winkelhaken (für die Einheiten von 1-9, neben den senkrechten keilförmigen Zahlenzeichen).

a) bei : RTC. 276:
Bart. III 118, 249: 3 gín igi-4-gál še.
Legr. TRU. 310: ud - kam.
Gen. TD. 5487: áb.

b) vor kam und àm:
Bart. III 152, 398: dub - - àm
Legr. TRU. 42: a-du - - kam
a-du - kam
a-du - kam
a-du - kam
Legr. TRU. 346: 1 máš-gal-še -kam-uš
1 udu-še -kam-uš
Siehe: itu-šu- -ša; itu šu- -ša; itu šu- - ša.

c) nach gud, áb, anše, zur Bezeichnung des Alters.
ITT. III, II 4956: 20 áb - -še 3 qa-ta
ITT. III, II 6090: 3 anše-sal- ; 1 anše-nita ;
Pinch. AT. I. 53: 3 gud ; 1 áb ;
Bart. III 106, 191: 3 anše-nita ;
ITT. II, I. 6965: 20 zu-gud - 15 zu-gud .

Figure 1: [Sch35, p. 135]



b) Gebrauch von .

ITT. IV. 7164 a: = 20 minus 3 = 17.
CT. 10, 24964: = 40 minus 4 = 36.
Gen. TÉO. 5670: = 240 minus 2 = 238.

Nota: Vgl. im römischen Zahlensystem: IX = X minus I; XIX = XX minus I; ferner die lateinischen Ausdrücke: undeviginti = 20 minus 1; duodetriginta = 30 minus 2.

Figure 2: [Sch35, p. 132]

3.4 Stacking patterns

4 AŠ×(DIŠ *tenû*) numerals

https://cdli.mpiwg-berlin.mpg.de/artifacts/452986/reader/209489 https://cdli.mpiwg-berlin.mpg.de/artifacts/467743/reader/213564

4.1 Stacking patterns

The subsequent mu-iti system, which saw limited use at the end of the presargonic and the beginning of the Old Akkadian periods, seems, on its surface, to be a rational development from the system it replaced; the *basis* of a 30-day month carries on (cf. for instance the texts B. Foster, Umma in the Sargonic Period [Hamden 1982] pl. 18, Nr 37, discussed by J Friberg, Scientific American 250/2 [Feb. 1984] 114 and Foster, ASJ 4 [1982] 43 obv iii9-11) and, for a period at least, a graphically comparable method of representing year dates, with now vertical strokes impressed on either side of the long horizontal, was used (the date of the text BIN 8, 117,  , which both Powell, HUCA 49, 9 and B. Foster, Or.NS 48 (1979) 156 and USP p. 7 read 7 (mu) 1 (iti) 7 (ud), should be registered with some scepticism). Only here is the refinement of day added, so that documents

Figure 3: TODO note that this should cite BIN 8, 116, not 117.

A connection of IM with the later usage of im(.ma) (presargonic Lagash and later; Akkadian šaddaqdi/a(m) with lexical equivalent MU.IM.MA [MSL 5, 65:195]), meaning "previous (year)" is not apparent. For the latter usage cf. particularly DP 280 (= 281), a presargonic temple document which "loads onto the backs" (gu_2.ne.ne.a e.ne.gar) of the fisheries foremen Ne.sag and Lugal.$ša_3$.la_2.tuku the quota arrears of IM.IM.ma.kam  , IM.ma.kam  and mu.a.kam , that is of the year before last = year one (of the king Urukagina [second regnal year]), of last year = year 2 and of this year = year 3 Also DP 243 goats of various colors / maš IM.ma.kam / ditto / maš mu.a.kam and DP 94. maš IM.ma as delivery arrears noted after grown nannies (ud_5) and before maš $ša_3$.HI (//mu.a.kam, "of the current year"), further maš IM.ma = maš.gal.gal in the summation rev i2 (see footnote 17 to the notations of the type). A parallel usage is found in the Old Akkadian text ITT 2/1, 3078 obv 1-4. 3 1/2 ma.na sikI / [i]m.ma.kam / 1 gu_2 la_2.4 ma.na sikI / mu.a.kam. It would seem difficult to reconcile this clear usage IM = "previous year" with the often translated IM = "account tablet" (im = clay)

Figure 4: TODO

Acknowledgements

The *CuneiformComposite* font by Steve Tinney is used when referring to the reference glyphs for already-encoded cuneiform. *Noto Sans Cuneiform*, by Monotype Imaging, is used to for most of the cuneiform text in this document, with modifications (cuneiform glyph for ◇ $ŠAR_2$, corrected glyps for  UN and KALAM per [Uni16], alternate glyph for). Arabic text is set in *Scheherazade New* by SIL International; Traditional Chinese text is set in *Noto Serif TC* by Ken Lunde et al.; monospace text is set in *Consolas* by Luc(as) de Groot; the remainder of the text is set in *Cambria* and *Cambria Math* by Monotype Imaging and Tiro Typeworks.

References

Artefacts

[P020092] VAT 04428. Berlin, Germany: Vorderasiatisches Museum.
CDLI: P020092.
ORACC: epsd2/P020092.

[P020129] VAT 04713. Berlin, Germany: Vorderasiatisches Museum.
CDLI: P020129.
ORACC: epsd2/P020129.

ISO and Unicode documents

[L2/24-210R] R. Leroy, A. Pandey and S. Tinney. *Archaic cuneiform numerals*. 23rd Oct. 2024.
UTC: L2/24-210R.

[Uni16] The Unicode Consortium. *The Unicode Standard*. Version 16.0.0. The Unicode Consortium, 10th Sept. 2024.
ISBN: 978-1-936213-34-4.
https://www.unicode.org/versions/Unicode16.0.0/core-spec/.

Online corpora and related projects

[CDLI] É. Pagé-Perron, J. L. Dahl, B. Lafont, J. Renn, R. K. Englund and P. Damerow, eds. *Cuneiform Digital Library Initiative*. 2000–.
https://cdli.mpiwg-berlin.mpg.de.

Other documents

[Arc15] A. Archi. *Ebla and Its Archives. Texts, History, and Society*. Studies in ancient Near Eastern records 7. Walter de Gruyter, 2015.
ISBN: 978-1-61451-716-0.
DOI: 10.1515/9781614517887.

[Gor24] F. Gori. "Numeracy in Early Syro-Mesopotamia. A study of accounting practices from Fāra to Ebla". PhD thesis. Università degli studi di Verona, 2024.
https://iris.univr.it/bitstream/11562/1114808/1/Diss_Fiammetta_Gori.pdf.

[Rob08] E. Robson. *Mathematics in Ancient Iraq. A Social History*. Princeton University Press, 2008.
ISBN: 978-0-691-09182-2.

[Sch35] N. Schneider. *Die Keilschriftzeichen der Wirtschaftsurkunden von Ur III*. Editrice Pontificio Istituto Biblico, 1935.