

# Archaic cuneiform numbers

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2024-09-05

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## 1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script<sup>1</sup> and the proto-cuneiform script<sup>2</sup>. The proposed characters are listed in section 2. Most of them were listed in [L2/23-190]; however, the present document provides a more detailed rationale for their encoding and additional information about their identity.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals<sup>3</sup> should however *not* be unified with the already-encoded cuneiform numerals<sup>4</sup>. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script\_Extensions property assignments in section 2.2 reflect the overlap. Many of these numerals are also used in proto-Elamite<sup>5</sup> texts, where they are treated as identical characters in scholarship on proto-Elamite, so that

<sup>1</sup>ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

<sup>2</sup>ISO 15924: Pcun, not yet encoded.

<sup>3</sup>Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: ◻, ◻ (small stylus), ◻ (large stylus). Some numerals are composed of multiple such impressions, e.g., ◻◻. The terms “curved”, “curviform”, and “round” can be found in the literature. We avoid the term “round” here as it has other meanings in the context of numbers. We use “curviform” in this document as, being the least common term, it is least likely to lead to confusion, and “CURVED” in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [inlineATF].

<sup>4</sup>Impressed into clay using a stylus with a trihedral end: — (stylus held horizontally), † (vertically), † (diagonally) † (with the head of the stylus), † (stylus pressed deeper, forming a larger wedge), † (combining † and †), etc.

<sup>5</sup>ISO 15924: Pelm, not yet encoded.

they should be unified with the ones proposed in [L2/23-196]. However, in the interest of time, we do not provide a detailed rationale for this unification in this document, and we are not proposing that the numerals be given the corresponding Script\_Extensions property value for now. Neither do we propose encoding any numerals that are solely attested in proto-Elamite texts, or well-attested in proto-Elamite texts but insufficiently attested in Uruk.

[TODO(egg): Mention the other sections here too.]

|                   | Uruk III & earlier | ED – Ur III   | OB & later |
|-------------------|--------------------|---------------|------------|
| Numerals          | This proposal      |               |            |
| Non-numeric signs | Future Pcun        | Existing Xsux |            |

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

## 2 Proposed changes to the Standard

### 2.1 Summary of proposed characters

### 2.2 Properties





### 2.3 Character names list

### 2.4 Core specification text

## 3 Rationale for curviform–cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [Powell1972] “in the same fashion as we use black and red ink”; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

### 3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts,  later in the third millennium<sup>6</sup>,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.


















<sup>6</sup>Merging with U+1224E CUNEIFORM SIGN NI2.





This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists<sup>7</sup> and dictionaries<sup>8</sup>, and of composite texts<sup>9</sup>. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]<sup>10</sup>. The diachronic approach is also useful for pedagogic applications<sup>11</sup>.

### 3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding<sup>12</sup> that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.


























Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

|   |   |   |   |   |  |  |
|---|---|---|---|---|--|--|
|  <sup>13</sup>  |   |   |   |    |  |  |
| 1(ḫeš <sub>2</sub> )  | 1(u)  | 1/2(diš)  | 5(diš <i>tenû</i> )   | gi  | us <sub>2</sub>  | sa <sub>2</sub>  |
|   | 7.5 (ropes)   |   | 5   | reed  | side   | equal  |
|  <sup>14</sup> |  |  |  |   |  |  |
| 3(u)  | 6(diš <i>tenû</i> )   | gi  | saṇ   | sa <sub>2</sub>   |  |  |
| 3 (ropes)   | 6   | reed  | front   | equal   |  |  |
|                |  |  |  |  |  |  |
| ašag-bi   | 1(bur <sub>3</sub> )  | 1(eše <sub>3</sub> )  | 1(iku)  | 1/2(iku)  |  |  |
| ašag=bi   |   |   |   |   |  |  |
| field=DEM <sup>15</sup>   |   |   |   |   |  |  |

     
tug<sub>x</sub>(LAK483)-si-ga-kam  
tugsiga =ak =am -Ø  
ploughed=GEN=COP-3.SG.S

<sup>7</sup>Notably [OSL] and the online edition of [MZL] in [eBL, Signs].

<sup>8</sup>Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

<sup>9</sup>For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of                           

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as<sup>16</sup>

465 metres, equal lengths  
198 metres, equal widths  
this field is 9, 18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from  $\triangleright$  to  $\uparrow$  numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases<sup>17</sup>.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere  $\triangleright/\uparrow$  switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

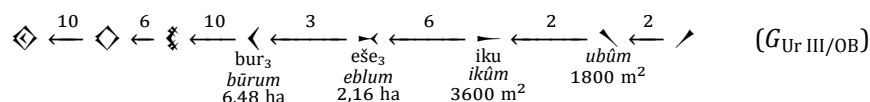
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<sup>16</sup>We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

<sup>17</sup>The cuneiform encoding process was planned in *stages* in [L2/03-162]. One might expect the second stage of encoding, which led to the creation of the Early Dynastic Cuneiform block, to incorporate the numerals needed for the representation and discussion of Early Dynastic texts; however, the proposal [L2/12-208] stated that “numerals have been omitted due to the complexity of numeral signs from this period. An expert in the metrology of this period must be consulted before these can be properly included.”



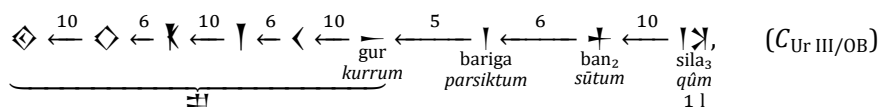
the approximate metric equivalent [Fri07, p. 378; Rob19]:




Note that for the range of areas given above<sup>20</sup>, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *bûrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign  functioning as punctuation<sup>21</sup>, but the numerals are tied to the metrology; thus a surface of 5 *bûr* 1 *ebel* 4 *ikû* (100 *ikû*, 36 ha) would be written<sup>22</sup>    . Contrast this with systems where the same numerals are used for different units, and overt units are used, as in “88 acres 3 roods 33 perches” or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the sign  is equal to sixty times  in the area system, but to three hundred and sixty times  in the discrete counting system.

### 3.3.3 The capacity system

Another such system of note is the one for capacities<sup>23</sup> [Fri07, p. 376; Rob19],

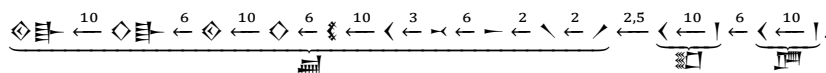


where the numerals for ban<sub>2</sub> are 𐤁, 𐤂, 𐤃, 𐤄, and 𐤅, and those for bariga are 𐤆, 𐤇, 𐤈, and 𐤉 (contrast ordinary 𐤆 and 𐤇 otherwise used with 𐤆 numerals). As described in [Hue11, p. 585 n. (b) and (f)], the sign 𐤆𐤇 GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𐤆 if present, and after the word for “grain” if present, as in

 3554 gur   
  3 ban<sub>2</sub>   
  6   
  sila<sub>3</sub>   
  of grain.

Observe that while large numbers of gur follow<sup>25</sup> system  $\mathcal{S}_{\text{Ur III/OB}}$ , the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as

<sup>20</sup>For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mūšarum* (36 m<sup>2</sup>) written 𒄩𒄪, equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60 𒄩𒄪 (shekels). For areas greater than 3600 *būrû*, the 𒄩 and 𒄪 numerals are reused with a suffix 𒄩𒄪 (gal, Sumerian: big), as follows [Robo8, p. 295 n. b and c; Fri07, p. 378; Rob19]:



<sup>21</sup>This sign is sometimes interpreted as a measurement unit, and transliterated *iku*, see, *e.g.*, [Proust2020], or transliterations in [Feu04] discussed in section 3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system  $G_{Ur III/OB}$ .

<sup>22</sup>As in the surface of the field of   (the city of Apisal) reported on [P102305, r. 1]

<sup>23</sup>Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system  $G_{Ur III/OB}$  based on a height of one cubit, see[[Pow87](#), p. 488; [Rob08](#), p. 294; [Rob19](#)].

<sup>24</sup>From [P309594].

<sup>25</sup>A larger unit, the *guru*<sub>7</sub> (*karûm*, grain heap), is sometimes used instead, with 𐎠𐎡𐎢𐎣𐎤𐎥𐎦𐎧𐎨𐎩𐎪𐎫𐎬𐎭𐎮𐎯𐎰𐎱𐎲𐎳𐎴𐎵𐎶𐎷𐎸𐎹𐎺𐎻𐎼𐎽𐎾𐎿𐏀𐏁𐏂𐏃𐏄𐏅𐏆𐏇𐏈𐏉𐏊𐏋𐏌𐏍𐏎𐏏𐏐𐏑𐏒𐏓𐏔𐏕𐏖𐏗𐏘𐏙𐏚𐏛𐏜𐏝𐏞𐏟𐏠𐏡𐏢𐏣𐏤𐏥𐏦𐏧𐏨𐏩𐏪𐏫𐏬𐏭𐏮𐏯𐏰𐏱𐏲𐏳𐏴𐏵𐏶𐏷𐏸𐏹𐏺𐏻𐏼𐏽𐏾𐏿𐐀𐐁𐐂𐐃𐐄𐐅𐐆𐐇𐐈𐐉𐐊𐐋𐐌𐐍𐐎𐐏𐐐𐐑𐐒𐐓𐐔𐐕𐐖𐐗𐐘𐐙𐐚𐐛𐐜𐐝𐐞𐐟𐐠𐐡𐐢𐐣𐐤𐐥𐐦𐐧𐐨𐐩𐐪𐐫𐐬𐐭𐐮𐐯𐐰𐐱𐐲𐐳𐐴𐐵𐐶𐐷𐐸𐐹𐐺𐐻𐐼𐐽𐐾𐐿𐑀𐑁𐑂𐑃𐑄𐑅𐑆𐑇𐑈𐑉𐑊𐑋𐑌𐑍𐑎𐑏𐑐𐑑𐑒𐑓𐑔𐑕𐑖𐑗𐑘𐑙𐑚𐑛𐑜𐑝𐑞𐑟𐑠𐑡𐑢𐑣𐑤𐑥𐑦𐑧𐑨𐑩𐑪𐑫𐑬𐑭𐑮𐑯𐑰𐑱𐑲𐑳𐑴𐑵𐑶𐑷𐑸𐑹𐑺𐑻𐑼𐑽𐑾𐑿𐒀𐒁𐒂𐒃𐒄𐒅𐒆𐒇𐒈𐒉𐒊𐒋𐒌𐒍𐒎𐒏𐒐𐒑𐒒𐒓𐒔𐒕𐒖𐒗𐒘𐒙𐒚𐒛𐒜𐒝𐒞𐒟𐒠𐒡𐒢𐒣𐒤𐒥𐒦𐒧𐒨𐒩𐒪𐒫𐒬𐒭𐒮𐒯𐒰𐒱𐒲𐒳𐒴𐒵𐒶𐒷𐒸𐒹𐒺𐒻𐒼𐒽𐒾𐒿𐓀𐓁𐓂𐓃𐓄𐓅𐓆𐓇𐓈𐓉𐓊𐓋𐓌𐓍𐓎𐓏𐓐𐓑𐓒𐓓𐓔𐓕𐓖𐓗𐓘𐓙𐓚𐓛𐓜𐓝𐓞𐓟𐓠𐓡𐓢𐓣𐓤𐓥𐓦𐓧𐓨𐓩𐓪𐓫𐓬𐓭𐓮𐓯𐓰𐓱𐓲𐓳𐓴𐓵𐓶𐓷𐓸𐓹𐓺𐓻𐓼𐓽𐓾𐓿𐔀𐔁𐔂𐔃𐔄𐔅𐔆𐔇𐔈𐔉𐔊𐔋𐔌𐔍𐔎𐔏𐔐𐔑𐔒𐔓𐔔𐔕𐔖𐔗𐔘𐔙𐔚𐔛𐔜𐔝𐔞𐔟𐔠𐔡𐔢𐔣𐔤𐔥𐔦𐔧𐔨𐔩𐔪𐔫𐔬𐔭𐔮𐔯𐔰𐔱𐔲𐔳𐔴𐔵𐔶𐔷𐔸𐔹𐔺𐔻𐔼𐔽𐔾𐔿𐕀𐕁𐕂𐕃𐕄𐕅𐕆𐕇𐕈𐕉𐕊𐕋𐕌𐕍𐕎𐕏𐕐𐕑𐕒𐕓𐕔𐕕𐕖𐕗𐕘𐕙𐕚𐕛𐕜𐕝𐕞𐕟𐕠𐕡𐕢𐕣𐕤𐕥𐕦𐕧𐕨𐕩𐕪𐕫𐕬𐕭𐕮𐕯𐕰𐕱𐕲𐕳𐕴𐕵𐕶𐕷𐕸𐕹𐕺𐕻𐕼𐕽𐕾𐕿𐖀𐖁𐖂𐖃𐖄𐖅𐖆𐖇𐖈𐖉𐖊𐖋𐖌𐖍𐖎𐖏𐖐𐖑𐖒𐖓𐖔𐖕𐖖𐖗𐖘𐖙𐖚𐖛𐖜𐖝𐖞𐖟𐖠𐖡𐖢𐖣𐖤𐖥𐖦𐖧𐖨𐖩𐖪𐖫𐖬𐖭𐖮𐖯𐖰𐖱𐖲𐖳𐖴𐖵𐖶𐖷𐖸𐖹𐖺𐖻𐖼𐖽𐖾𐖿𐗀𐗁𐗂𐗃𐗄𐗅𐗆𐗇𐗈𐗉𐗊𐗋𐗌𐗍𐗎𐗏𐗐𐗑𐗒𐗓𐗔𐗕𐗖𐗗𐗘𐗙𐗚𐗛𐗜𐗝𐗞𐗟𐗠𐗡𐗢𐗣𐗤𐗥𐗦𐗧𐗨𐗩𐗪𐗫𐗬𐗭𐗮𐗯𐗰𐗱𐗲𐗳𐗴𐗵𐗶𐗷𐗸𐗹𐗺𐗻𐗼𐗽𐗾𐗿𐘀𐘁𐘂𐘃𐘄𐘅𐘆𐘇𐘈𐘉𐘊𐘋𐘌𐘍𐘎𐘏𐘐𐘑𐘒𐘓𐘔𐘕𐘖𐘗𐘘𐘙𐘚𐘛𐘜𐘝𐘞𐘟𐘠𐘡𐘢𐘣𐘤𐘥𐘦𐘧𐘨𐘩𐘪𐘫𐘬𐘭𐘮𐘯𐘰𐘱𐘲𐘳𐘴𐘵𐘶𐘷𐘸𐘹𐘺𐘻𐘼𐘽𐘾𐘿𐙀𐙁𐙂𐙃𐙄𐙅𐙆𐙇𐙈𐙉𐙊𐙋𐙌𐙍𐙎𐙏𐙐𐙑𐙒𐙓𐙔𐙕𐙖𐙗𐙘𐙙𐙚𐙛𐙜𐙝𐙞𐙟𐙠𐙡𐙢𐙣𐙤𐙥𐙦𐙧𐙨𐙩𐙪𐙫𐙬𐙭𐙮𐙯𐙰𐙱𐙲𐙳𐙴𐙵𐙶𐙷𐙸𐙹𐙺𐙻𐙼𐙽𐙾𐙿𐚀𐚁𐚂𐚃𐚄𐚅𐚆𐚇𐚈𐚉𐚊𐚋𐚌𐚍𐚎𐚏𐚐𐚑𐚒𐚓𐚔𐚕𐚖𐚗𐚘𐚙𐚚𐚛𐚜𐚝𐚞𐚟𐚠𐚡𐚢𐚣𐚤𐚥𐚦𐚧𐚨𐚩𐚪𐚫𐚬𐚭𐚮𐚯𐚰𐚱𐚲𐚳𐚴𐚵𐚶𐚷𐚸𐚹𐚺𐚻𐚼𐚽𐚾𐚿𐛀𐛁𐛂𐛃𐛄𐛅𐛆𐛇𐛈𐛉𐛊𐛋𐛌𐛍𐛎𐛏𐛐𐛑𐛒𐛓𐛔𐛕𐛖𐛗𐛘𐛙𐛚𐛛𐛜𐛝𐛞𐛟𐛠𐛡𐛢𐛣𐛤𐛥𐛦𐛧𐛨𐛩𐛪𐛫𐛬𐛭𐛮𐛯𐛰𐛱𐛲𐛳𐛴𐛵𐛶𐛷𐛸𐛹𐛺𐛻𐛼𐛽𐛾𐛿𐜀𐜁𐜂𐜃𐜄𐜅𐜆𐜇𐜈𐜉𐜊𐜋𐜌𐜍𐜎𐜏𐜐𐜑𐜒𐜓𐜔𐜕𐜖𐜗𐜘𐜙𐜚𐜛𐜜𐜝𐜞𐜟𐜠𐜡𐜢𐜣𐜤𐜥𐜦𐜧𐜨𐜩𐜪𐜫𐜬𐜭𐜮𐜯𐜰𐜱𐜲𐜳𐜴𐜵𐜶𐜷𐜸𐜹𐜺𐜻𐜼𐜽𐜾𐜿𐝀𐝁𐝂𐝃𐝄𐝅𐝆𐝇𐝈𐝉𐝊𐝋𐝌𐝍𐝎𐝏𐝐𐝑𐝒𐝓𐝔𐝕𐝖𐝗𐝘𐝙𐝚𐝛𐝜𐝝𐝞𐝟𐝠𐝡𐝢𐝣𐝤𐝥𐝦𐝧𐝨𐝩𐝪𐝫𐝬𐝭𐝮𐝯𐝰𐝱𐝲𐝳𐝴𐝵𐝶𐝷𐝸𐝹𐝺𐝻𐝼𐝽𐝾𐝿𐞀𐞁𐞂𐞃𐞄𐞅𐞆𐞇𐞈𐞉𐞊𐞋𐞌𐞍𐞎𐞏𐞐𐞑𐞒𐞓

◁|𐎶 would be 10 gur 1 bariga, and ◁—𐎶 would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

- |—𐎶 used in  $S_{Ur III/OB}$  and the SPVS as well as with overt units;
- ◁—𐎶 used in  $G_{Ur III/OB}$ , of which ◁—𐎶 are also used in  $S_{Ur III/OB}$  and the SPVS as well as with overt units;
- |—𐎶 used in  $S_{Ur III/OB}$ , and sometimes with overt units;
- 𐎶—𐎶 used in  $S_{Ur III/OB}$ ;
- ◁—𐎶 used in  $S_{Ur III/OB}$  and  $G_{Ur III/OB}$ ;
- ◁—𐎶 used in  $S_{Ur III/OB}$  and  $G_{Ur III/OB}$ ;
- —𐎶 used in  $C_{Ur III/OB}$  as well as with overt units of the weight system;
- 𐎶, 𐎶, 𐎶, 𐎶, 𐎶 used in  $C_{Ur III/OB}$ ;
- |, |, |, | used in  $C_{Ur III/OB}$ —note the overlap with |—𐎶;
- 𐎶 and 𐎶 used in  $G_{Ur III/OB}$ .

Only in the SPVS did numerals exist truly independently of metrology; to quote [Robo8, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

### 3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with | and ◁ numerals with their system  $S_{Ur III/OB}$  values<sup>26</sup>. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in section 3.4. Metrological tables use the following units [Fri07, p. 118; Rob19]:

|         |      |                  |      |          |      |                  |      |        |  |                     |
|---------|------|------------------|------|----------|------|------------------|------|--------|--|---------------------|
| 𐎶𐎶𐎶     | ← 30 | 𐎶𐎶               | ← 60 | 𐎶𐎶       | ← 12 | 𐎶𐎶               | ← 30 | 𐎶𐎶𐎶    |  | ( $L_{Ur III/OB}$ ) |
| danna   |      | UŠ <sup>27</sup> |      | nindan   |      | kuš <sub>3</sub> |      | šu-si  |  |                     |
| bērum   |      | cable            |      | nindanum |      | ammatum          |      | ubānum |  |                     |
| league  |      | 360 m            |      | rod      |      | cubit            |      | finger |  |                     |
| 10,8 km |      |                  |      | 6 m      |      | 50 cm            |      | 17 mm  |  |                     |

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:

|     |      |    |     |                  |      |      |     |       |     |    |      |     |  |                           |
|-----|------|----|-----|------------------|------|------|-----|-------|-----|----|------|-----|--|---------------------------|
| 𐎶𐎶𐎶 | ← 30 | 𐎶𐎶 | ← 6 | 𐎶𐎶               | ← 10 | 𐎶𐎶   | ← 2 | 𐎶𐎶    | ← 6 | 𐎶𐎶 | ← 30 | 𐎶𐎶𐎶 |  | ( $\bar{L}_{Ur III/OB}$ ) |
|     |      |    |     | eše <sub>2</sub> |      | gi   |     | qānum |     |    |      |     |  |                           |
|     |      |    |     | ašlum            |      | reed |     | reed  |     |    |      |     |  |                           |
|     |      |    |     | rope             |      | 3 m  |     |       |     |    |      |     |  |                           |
|     |      |    |     | 60 m             |      |      |     |       |     |    |      |     |  |                           |

In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

<sup>26</sup>Adjacent units are no more than a factor of 60 apart, so higher numerals such as 𐎶 or ◁ are not used.

<sup>27</sup>As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.



### 3.3.5 Fractions

Fractions of the *ikûm*,  $\swarrow = \frac{1}{2}$  and  $\nearrow = \frac{1}{4}$ , have already been encountered. In other contexts, the fraction  $\frac{1}{2}$  is written  $\nwarrow$ , and the fractions  $\frac{1}{3}$  and  $\frac{2}{3}$  are written  $\llcorner$  and  $\lrcorner$ . The latter two signs are derived from curviform signs  $\text{𒌷}$  and  $\text{𒌸}$ , which are already separately encoded; these are in turn derived from the sign  $\text{𒌵}$  ( $\text{ŠU}_2$ ), whose Early dynastic form resembles  $\text{𒌵}$ , and  $\varnothing$  numerals; see [Powell1971]. The  $\text{𒌵}$  is sometimes omitted, as in [P240545; P221530; P221531; P271238; P274845].

### 3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the Uruk period) clearly mirrors system  $S_{\text{Ur III/OB}}$  [Fri07, p. 374; DE87, pp. 127, 165]:

$$\text{𒌵} \xleftarrow{10} \bullet \xleftarrow{6} \text{𒌷} \xleftarrow{10} \text{𒌸} \xleftarrow{6} \bullet \xleftarrow{10} \text{𒌵}. \quad (S)$$

Likewise the area system used in the Early Dynastic IIIb period for areas of one *ikûm* and greater mirrors system  $G_{\text{Ur III/OB}}$  [LAK, p. 72; NDE93, p. 63; Fri07, p. 378; Lec16]:

$$\text{𒌵} \xleftarrow{10} \bullet \xleftarrow{6} \text{𒌷} \xleftarrow{10} \bullet \xleftarrow{3} \text{𒌷} \xleftarrow{6} \text{𒌵}, \quad (G_{\text{ED IIIb}})$$

with consistent use of the numerals:  $\bullet$  corresponds to  $\swarrow$ ,  $\bullet$  to  $\nwarrow$ , and  $\text{𒌵}$  to  $\text{𒌵}$ . An exception to this correspondence, noted in [L2/04-099, p. 4] (see section 3.2), is that the vertical  $\text{𒌵}$  from  $S_{\text{Ur III/OB}}$  corresponds to a horizontal  $\text{𒌵}$  in system  $S$ . This is however far from the only case of such a reallocation of function. The earlier form of System G is [DE87, pp. 141, 165; Fri07, p. 378]:

$$\bullet \xleftarrow{6} \text{𒌵} \xleftarrow{10} \bullet \xleftarrow{3} \text{𒌷} \xleftarrow{6} \text{𒌵}, \quad (G)$$

Observe that, as noted in [DE87, p. 142],  $\text{𒌵}$  changes meaning from  $10\bullet$  in system  $G$  to  $600\bullet$  in system  $G_{\text{ED IIIb}}$ . System  $G$  is used in the Uruk period, but also in the ED I–II period (it is the “area 2” system in [Cha03], whereas  $G_{\text{ED IIIb}}$  is the “area 1” system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb area system<sup>28</sup>,

$$\text{𒌵} \xleftarrow{10} \bullet \xleftarrow{6} \text{𒌷} \xleftarrow{10} \bullet \xleftarrow{3} \text{𒌷} \xleftarrow{6} \text{𒌵} \xleftarrow{2} \varnothing \xleftarrow{2} \varnothing \xleftarrow{2} \text{𒌷}^{29}, \quad (G_{\text{ED IIIb}})$$

with the numerals of a contemporaneous capacity system:

$$\text{𒌵} \xleftarrow{10} \text{𒌷} \xleftarrow{6} \bullet \xleftarrow{10} \text{𒌵} \xleftarrow{4} \varnothing \xleftarrow{6} \text{𒌷}, \quad (C_{\text{𒌵𒌷𒌵}})$$

both described in [Lec16]. While the size of the  $\text{𒌵𒌷𒌵}$  (gur san ḡal<sub>2</sub>) in bariga is different from that of the Old Babylonian  $\text{𒌵}$ , the basic structure of the capacity

<sup>28</sup>A variant is  $\text{𒌵} \xleftarrow{10} \bullet \xleftarrow{6} \text{𒌷} \xleftarrow{10} \bullet \xleftarrow{3} \text{𒌷} \xleftarrow{6} \text{𒌵} \xleftarrow{2} \varnothing \xleftarrow{2} \varnothing \xleftarrow{2} \text{𒌷}$ , see [Powell1972].

<sup>29</sup>The cuneiform counterpart is  $\text{𒌵}$ .

system is recognizable, with  $\varnothing$  corresponding to  $\text{I}$  for bariga,  $\text{𒀭}$ – $\text{𒀮}$  corresponding to  $\text{𒀭}$ – $\text{𒀮}$  for ban<sub>2</sub>, and the  $\text{𒀭}$  counted with  $\text{𒀭}$  rather than  $\text{𒀭}$  numerals. However, the half-*ikûm* is counted with the same  $\varnothing$  as the bariga, whereas it uses a different sign,  $\text{𒀭}$ , in the Old Babylonian system. As we will see, this is cannot be handled as a split, by giving  $\text{𒀭}$  the glyph  $\varnothing$  in an Early Dynastic IIIb font, as the  $\text{𒀭}$  numeral series is also in use in that period.

### 3.4.1 Field lengths in Nirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign  $\text{𒀭}$  is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign  $\text{𒀭}$  is not written either. Lengths shorter than one rope are expressed in half-rope using the  $\frac{1}{2}$  sign  $\text{𒀭}$  (again with no  $\text{𒀭}$ ), and then in reeds, *with* the sign  $\text{𒀭}$ , as follows:

$$\text{𒀭} \xleftarrow{6} \text{𒀭} \xleftarrow{2} \text{𒀭} \xleftarrow{10} \text{𒀭} \text{𒀭}^{32}. \quad (L_{\text{ED IIIb}})$$

$\begin{array}{l} 1 \text{ eše}_2 = 10 \text{ nindan} \\ 1 \text{ rope} = 10 \text{ rods} \\ 60 \text{ m} \end{array} \quad \begin{array}{l} \text{gi} \\ \text{reed} \\ 3 \text{ m} \end{array}$

This is the system that was used to express the sides of the field in [P020054] discussed in section 3.2. In that tablet and most others from the same period, such as the ones discussed in [Lec20], areas are expressed in system  $G_{\text{ED IIIb}}$ , with curviform numerals<sup>33</sup>; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope<sup>34</sup>, the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and thus the interpretation of its numerals between systems  $G_{\text{ED IIIb}}$  and  $L_{\text{ED IIIb}}$ . The sign  $\text{𒀭}$ , which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without  $\text{𒀭}$ . There are other such co-occurrences contrasting between metrological systems; for instance, [Kre98, p. 303 n. 686] mentions the use of cuneiform numerals for days and months<sup>35</sup>.

### 3.4.2 Dyke lengths in Nirsu

[Pow87, p. 466] notes that reeds “are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus.” Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in CDLI finds many occurrences<sup>36</sup>, such as:

<sup>32</sup>The reeds are counted using *tenû* numerals,  $\text{𒀭}$ ,  $\text{𒀭}$ ,  $\text{𒀭}$ , etc.

<sup>33</sup>A CDLI search for “(bur3)” (< numerals used for areas) currently returns 15 ED IIIb results, whereas one for “(bur3@c)” (• numerals used for areas) returns 206. Further, when dated, the tablets with cuneiform bur<sub>3</sub> are from the reigns of  $\text{𒀭}$ – $\text{𒀭}$  (variously transliterated iri-inim-gi-na, uru-ka-gi-na, etc.) and  $\text{𒀭}$ – $\text{𒀭}$  (lugal-zag-ge-si), the last two kings of ED IIIb Lagaš.

<sup>34</sup>This is the case of the sides of the field in [P020054, obv. ii 2–3].

<sup>35</sup>That note also mentions a contrast between the use of curviform numerals to count people and curviform numerals to count bread allotted to them in [P010876]; such contrasts are more akin to styling, and might not, on their own, justify the disunification.

<sup>36</sup>A search for curviform numerals followed by some number of reeds counted in (*tenû*) cuneiform numerals currently finds 125 occurrences across 47 tablets.


- [P221305, obv. 1, 4]<sup>37</sup> 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵  
 — [P020129, rev. 2, 1] 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵  
 — [P221291, rev. 5, 1]<sup>38</sup> 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵  
 — [P221266, rev. 2, 1] 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵 𣎵

These expressions use an explicit sign 𐤁𐤁 (counted in multiples of ten) or 𐤁. This notation—but not its use of curviform numerals—is remarked on in [Lec20, p. 290 n. 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. These can be summarized by the following factor diagram:

$$\underbrace{\begin{array}{c} 10 \\ \leftarrow \\ \text{[Diagram 1]} \end{array}}_{\text{[Diagram 2]}} \xleftarrow{6} \bullet = \underbrace{\begin{array}{c} 2 \\ \leftarrow \\ \text{[Diagram 3]} \end{array}}_{\text{[Diagram 4]}} \xleftarrow{10} \text{[Diagram 5]} \xleftarrow{6} \text{[Diagram 6]} \xleftarrow{3} \text{[Diagram 7]} \quad (L'_{\text{ED IIIb}})$$

### 3.4.3 Butter, cheese and wheat in Nirsu

A similar mixture of cuneiform and curviform numerals may be observed with the capacity system; indeed, the previously described 𐎶𐎵𐎴𐎳 system uses 𐎶 numerals for 𐎶 [Fri78, p. 43; Lec16]:


(C 出右圖)

as in [P020016; P020065; P020090; P020092; P020137] and others, where  $\text{ban}_2$  counted with  $\nabla$  numerals are followed by  $\text{sil}_3$  counted with  $\searrow$  numerals. Curviform numerals are also used to count  $\text{sil}_3$ , but not<sup>40</sup> as part of the  $\nabla$  systems. This contrast can be seen in [P220927], which measures butter ( $\nabla$ ,  $i_3$ ) with a different capacity system, using the  $\nabla$  (dug, “pot”) of 20  $\searrow$ , with  $\triangleright$  and  $\bullet$  numerals<sup>41</sup> for both the  $\nabla$  and the  $\searrow$ , thus [Pow87, pp. 504 sq.]

$$\bullet \xleftarrow{10} \text{D} \xleftarrow{2} \bullet \xleftarrow{10} \text{D} \xleftarrow{\frac{3}{2}} \text{D} \xleftarrow{2} \text{D}, \quad (C)$$

but counts cheese (𐌺𐌰𐌹𐌸𐌰, ga'ar) using the 𐌹𐌺𐌰𐌽𐌰 capacity system, with 𐌺 numerals for the 𐌹.

Another capacity system in ED IIIb Nirsu is the  $\text{𒍪} \text{𒍪}$ , the gur of two ul:











$$\bullet \xleftarrow{10} \underbrace{\bigcirc \xleftarrow{2} \bigcirc \xleftarrow{6} \bigcirc \xleftarrow{6} \bigcirc \xleftarrow{6} \bigcirc \xleftarrow{6} \bigcirc}_{\mathbb{H}_s(\mathbb{H})} \quad (C_{\mathbb{H}_s(\mathbb{H})})$$

<sup>37</sup>CDLI only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the  $\text{𒀭𒀭𒀭𒀭𒀭}$  is not visible. Lecompte notes that the copy is faithful; indeed another  $\text{𒀭𒀭𒀭𒀭𒀭}$  can be seen both on the copy and the photo on obv. 2, 2.

<sup>38</sup>From copy.

<sup>39</sup>With either unit omitted, as in the examples above, or both, as in [P020129, obv. 2, 1]   .

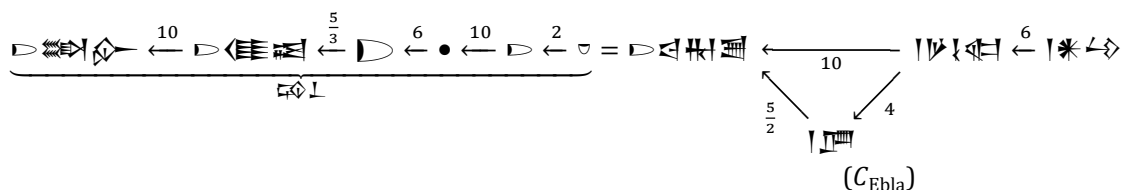
<sup>40</sup>As of this writing, the single occurrence of {ban2@c} followed by curviform numerals and sila3 in ED IIIb Nirsu transliterations on CDLI, 4{ban2@c} 3{asz@c} sila3 in [P221815], is incorrect: it should be 4(ban2@c) 3(disz@t) sila3.

<sup>41</sup>This tablet also uses subtractive notation:     “two pots minus one third (silā<sub>3</sub>)”,     “ten minus one pots, six silā<sub>2</sub>”. Such subtractive notation is common in most of the metrological systems discussed here; it appears in the ED IIIa period [Robo8, p. 77]. We shall not discuss it further, as presents no complexity for character encoding. It is noteworthy that the sign  (la<sub>2</sub>, “minus”) is often ligated with the following numerals, with the subtrahend placed under a sometimes considerably enlarged , similar to the layout of the radical in modern mathematical notation, see, e.g., [P020092].



11. [P240531, recto 1, 8]
12. [P241708, recto 1, 1]<sup>52</sup>
13. [P241904, recto 1, 1]<sup>53</sup>
14. [P240964, recto 3, 2]<sup>54</sup>

Note that higher numbers of are expressed in hundreds (*mi-at* ) and then thousands (*li-im* ), as is typical in Ebla [Arc15, p. 33], e.g., in the last example above or in [P240532, verso 2, 3], (100 + 60 + 30 + 5 = 195 of grain). These expressions correspond to the following factor diagram:



### 3.4.5 Use in modern publications

Because of their prevalence in the Uruk and Early Dynastic periods, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 3–16.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 16. The partial<sup>58</sup> transliteration “4 'a<sub>3</sub>-da-um 4 aktum 4 ib<sub>2</sub><sup>tu<sub>9</sub></sup>×3 sa<sub>6</sub> gunu<sub>3</sub>” is used to illustrate a discussion of the interpretation of the contrast between and numerals. More conventional transliterations might omit the numeral shapes entirely, e.g., 4 'a<sub>3</sub>-da-um 4 aktum 4 ib<sub>2</sub><sup>tu<sub>9</sub></sup>×3 sa<sub>6</sub> gunu<sub>3</sub>, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, e.g., 4(aš<sup>c</sup>) 'a<sub>3</sub>-da-um 4(diš<sup>c</sup>) aktum 4(aš<sup>c</sup>) ib<sub>2</sub><sup>tu<sub>9</sub></sup>×3(diš<sup>c</sup>) sa<sub>6</sub> gunu<sub>3</sub>, but the result would be less readable. See Section 3.7.2 for a discussion of transliteration conventions for numerals.

<sup>52</sup>From CDLI photo.

<sup>53</sup>From photo in [Arc89, p. 6].

<sup>54</sup>Laid out as ; on stacking patterns see Section 6.2.

<sup>55</sup>From photo in [Arc89, p. 6]; see also the CDLI photo and the copy in [Friberg1986]. This tablet features unusual usage of vertical numerals—“somewhat unorganized”, as described by [Friberg1986]—, such as or , but its and are consistently counted with cuneiform numerals, and the higher units with cuneiform numerals.

<sup>56</sup>Short for .

<sup>57</sup>ŠU<sub>2</sub>+NIG<sub>2</sub>-sag, an unusual variant spelling.

<sup>58</sup>The untransliterated text would be ; note the atomically encoded ib<sub>2</sub> × 3 = × = .

$$\begin{cases} 4\overline{\text{U}} 5\bullet = 24\overline{\text{U}} 3\bullet & (\text{C } 234) \\ 5\overline{\text{U}} 1\bullet 1\overline{\text{U}} = 5\overline{\text{U}} 7\bullet & (\text{C } 314) \\ 1\overline{\text{U}} 1\overline{\text{D}} 1\overline{\text{D}} = 6\bullet 2\overline{\text{U}} 1\overline{\text{D}} & (\text{C } 27) \end{cases}$$

These metrological equations for the "unknowns"  $\overline{\text{U}}$ ,  $\bullet$ ,  $\overline{\text{U}}$ , etc., can be treated exactly as ordinary equations for unknowns  $x, y, z, \dots$ . In particular, the equations can be simplified by subtraction of equal amounts from both sides of the identities. In this way the three equations above can be reduced to:

$$\begin{aligned} 2\bullet &= 20\overline{\text{U}} & (4\overline{\text{U}} 3\bullet \text{ subtracted from both sides}) \\ 1\overline{\text{U}} &= 6\bullet & (5\overline{\text{U}} 1\bullet - " = " = ) \\ 1\overline{\text{D}} &= 6\bullet 1\overline{\text{U}} 9\overline{\text{D}} & (1\overline{\text{U}} 1\overline{\text{D}} - " = " = ) \end{aligned}$$

We can now read off from the first equation that  $1\bullet = 10\overline{\text{U}}$ , and from the second that  $1\overline{\text{U}} = 6\bullet$ . Then the third equation can be simplified (by "substitution" of these values into the equation), to the following reduced form:

$$1\overline{\text{D}} = 2\overline{\text{U}} 9\overline{\text{D}}.$$

The most likely solution to this last equation is, of course,

$$1\overline{\text{D}} = 2\overline{\text{U}} \quad , \quad 1\overline{\text{D}} = 10\overline{\text{D}}.$$

Figure 1: The derivation of the factors of the bisexagesimal system in [Fri78, p. 15]<sup>60</sup>.

<sup>60</sup>The bisexagesimal system is used alike in proto-Elamite and proto-cuneiform texts, see [Fri78, p. 38]; the derivation in [Fri78, p. 15] is based on proto-Elamite artefacts. Note that in Friberg's early works [Fri78; Friberg1979; Friberg1986; Fri87], copies of fourth millenium and sometimes third millenium tablets are shown as vertical text (which they were for the scribes), and their numerals are written within horizontal text in the same orientation that they have if the tablet is taken as vertical text; in [UAX50] parlance, as if they had Vertical\_Orientation=Upright. In addition, they are listed in these equations in the horizontal order in which they appear as vertical text (thus the rightmost numeral is the most significant, read first). Cuneiform is correctly Vertical\_Orientation=Rotated, consistently both with modern practice and with the rotation between earlier vertical and later horizontal monumental inscriptions. Friberg's early conventions are not followed in later scholarship, and are abandoned in his own more recent works, such as [Fri07]; a more typical way to express the first equations might be

$$\begin{aligned} 5\bullet + 4\overline{\text{D}} &= 3\bullet + 24\overline{\text{D}} & (\text{C } 234) \\ 1\overline{\text{D}} + 1\bullet + 5\overline{\text{D}} &= 7\bullet + 5\overline{\text{D}} & (\text{C } 314) \\ 1\overline{\text{D}} + 1\overline{\text{D}} + 1\overline{\text{D}} &= 1\overline{\text{D}} + 2\overline{\text{D}} + 6\bullet & (\text{C } 27) \end{aligned}$$

A diplomatic edition of [Fri78] could rotate the numerals using a higher-level protocol:

$$\begin{cases} 4\overline{\text{U}} 5\bullet = 24\overline{\text{U}} 3\bullet & (\text{C } 234) \\ 5\overline{\text{U}} 1\bullet 1\overline{\text{U}} = 5\overline{\text{U}} 7\bullet & (\text{C } 314) \\ 1\overline{\text{U}} 1\overline{\text{D}} 1\overline{\text{D}} = 6\bullet 2\overline{\text{U}} 1\overline{\text{D}} & (\text{C } 27). \end{cases}$$

Thus, for instance, the original set of fractions  $\frac{1}{2}$ ,  $\frac{1}{4}$ , and  $\frac{1}{8}$  (1/2, 1/4 and 1/8 of an iku) in the Sumerian GANA system, was after a time augmented through the addition of the new sub-unit SAR:  $\frac{1}{100}$ , equal to 1/100 of an iku ( $\frac{1}{100}$ ). Similarly, the Sumerian weight unit "ma-na" which originally may have had only the sub-units  $\frac{1}{3}$  sa-na (= 1/3 mana) and  $\frac{2}{3}$  sa-na-bi (= 2/3 mana), and perhaps also gin:  $\frac{1}{60}$  (= 1/60 mana), seems to have acquired, at some time or other, also the smaller sub-units  $\frac{1}{3}$  gin, and  $\frac{1}{3} \times \frac{1}{60}$  gin.

Figure 2: TODO [Fri78, p. 49]

there is in any case an important qualitative difference between IX for Latin novem and  $\bullet$  for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 3: TODO [Englund1988]

### 3.5 Non-numeric usage

The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'<sup>61</sup>. Do you know its reading<sup>62</sup>?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign  $\neg$  has, *inter alia*, the values aš, rum, and dili. While the horizontal numerals are most frequently written  $\triangleright$  in the Early Dynastic period<sup>64</sup>, such non-numeric usage is almost<sup>65</sup> always written  $\neg$ , for instance:

<sup>63</sup>TODO(egg): On the order cite TSS 188, Friberg2007 p. 148 and any of the usual suspects on the haphazard order of signs in early texts; contrast P274845, P241764.

<sup>61</sup>The reader will recall that  $\eta\epsilon\delta_2$  is written  $\neg$ , with a larger wedge than  $\neg$ ; however, these signs have merged by the time Examenstext A is composed.

<sup>62</sup>Besides  $\eta\epsilon\delta_2$ , a look at [OSL] shows that the values diš, ge<sub>3</sub>, makkaš, saṇtak<sub>4</sub>, and tal<sub>4</sub> are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

<sup>64</sup>A CDLI search for "(asz@c)" finds 3296 ED texts, while a search for "(asz)" finds 81 ED texts, of which 46 also contain "(asz@c)".

<sup>65</sup>Exceptions are discussed in section 3.7.1.

of decreasing fractions  $\frac{1}{n}$  of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus  $\Xi = \frac{1}{2} N_{39}$ ,  $\sqrt{\neg} = \frac{1}{3} N_{39}$ , and so on. The first sign of the latter units,  $N_{34}$ ,

Figure 4: TODO [Eng98, p. 113]

For instance, the first line contains the notations  $1N_{34} 1N_{30} ; 2N_{20}$ , which can be translated "60 of the (grain rations containing)  $\ominus$  (of grain); (grain involved:)  $2 \bullet$  (of ground barley)". This calculation contradicts the assumed numerical relationship  $10N_1 = 1N_{14}$ , since as was well known the measure represented by the sign  $N_{30}$  was  $1/5$  of that represented by  $N_1$ , so that  $60 \times 1/5 = 12$  and not 20, as  $2N_{14}$  would imply. Instead of relying on complicated

Figure 5: TODO [Eng98, p. 116]

Die halbkreisförmigen Griffleindrücke gehen manchmal in mehr oder weniger eckige Formen über ( $\nabla$ )<sup>685</sup>. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen ( $\backslash$ ), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren<sup>686</sup>. Selten treten mit  $\nabla$  gebildete Zahlen auf<sup>687</sup> (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 6: TODO [Kre98, p. 303]

The calculations:

|                             |   |  |  |   |                                 |                                 |                           |                           |                           |
|-----------------------------|---|--|--|---|---------------------------------|---------------------------------|---------------------------|---------------------------|---------------------------|
| Obv. i                      | 1 | $60 \times 1/5 \triangleright (\ominus)$     | =  | $12 \times \triangleright =$                          | $2 \times \bullet$              |                                 |                           |                           |                           |
|                             | 2 | $120 \times 1/10 \triangleright (\boxminus)$ | =  | $12 \times \triangleright =$                          | $2 \times \bullet$              |                                 |                           |                           |                           |
|                             | 3 | $120 \times 1/15 \triangleright (\boxdot)$   | =  | $8 \times \triangleright =$                           | $1 \times \bullet$              | $2 \times \triangleright$       |                           |                           |                           |
|                             | 4 | $300 \times 1/20 \triangleright (\boxplus)$  | =  | $15 \times \triangleright =$                          | $2 \times \bullet$              | $3 \times \triangleright$       |                           |                           |                           |
|                             | 5 | $600 \times 1/25 \triangleright (\boxtimes)$ | =  | $24 \times \triangleright =$                          | $4 \times \bullet$              |                                 |                           |                           |                           |
| Rev. i                      | 1 | 1200   |  |   | $1 \times \bullet$              | $1 \times \bullet$              | $5 \times \triangleright$ |                           |                           |
| Obv. i                      | 6 | $6000 \times 1/30 \triangleright$            | (GAR+6N <sub>37</sub> ) =                      | $200 \times \triangleright = 1 \times \triangleright$ | $3 \times \bullet$              | $2 \times \triangleright$       |                           |                           |                           |
| ii                          | 1 | $120 \times \approx 1/4 \triangleright$      | (DUG <sub>a</sub> +U <sub>2a</sub> ) $\approx$ | $30 \times \triangleright =$                          | $5 \times \bullet$              | $1 \times \triangleright$       | $1 \times \triangleright$ |                           |                           |
|                             | 2 | $180 \times 1/5 \triangleright$              | (DUG+AS <sub>a</sub> ) =                       | $36 \times \triangleright =$                          | $6 \times \bullet$              |                                 |                           |                           |                           |
|                             | 3 | $300 \times 1/15 \triangleright$             | (KAS <sub>4</sub> ) =                          | $20 \times \triangleright =$                          | $3 \times \bullet$              | $2 \times \triangleright$       |                           |                           |                           |
| Rev. i                      | 3 | 600  |  |   | $1 \times \bullet$              | $4 \times \bullet$              | $3 \times \triangleright$ | $1 \times \triangleright$ |                           |
|                             |   |  |  |   | $1 \times \bullet$              | $1 \times \bullet$              | $5 \times \triangleright$ |                           |                           |
|                             |   |  |  |   | $1 \times \triangleright$       | $3 \times \bullet$              | $2 \times \triangleright$ |                           |                           |
|                             |   |  |  |   | $1 \times \bullet$              | $4 \times \bullet$              | $3 \times \triangleright$ | $1 \times \triangleright$ |                           |
| Grand total of groats used: |   |  |  |   | $1 \times \triangleright$       | $2 \times \bullet$              | $9 \times \bullet$        | $4 \times \triangleright$ | $1 \times \triangleright$ |
| Grand total of malt used:   |   |  |  |   | $1N_{47} 4N_{20} 3N_5 1N_{42a}$ | (rev. i 3) $\times 3/5 \approx$ | $8 \times \bullet$        | $4 \times \triangleright$ | $1 \times \triangleright$ |

Figure 6. Transliteration and calculations of MSVO 4, 66.

Figure 7: TODO [Englund2001]

strong similarities between “area” 1 and “area” 3 systems, the sign with two concentric discs ( $\bullet$ ), notated  $N_{50}$ <sup>27</sup> remains problematic. It never appears in any numerical combination with the sign with a single disc ( $\bullet$ ),

Figure 8: TODO [Chao3, p. 6]



$1/15$ , etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation  $\text{gur} \text{ } \text{gur}$  could be understood as  $1 \frac{1}{5}$  gur.<sup>27</sup> However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs  $\text{gur}$  and  $\text{gur}$  cannot be considered klatmatograms.

Metrological tablets from the end of the 4<sup>th</sup> millennium (Nissen, Damerow and Englund 1993, 55-59, to MSVO 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

$\text{iku}$  represents a surface of 3600m<sup>2</sup>  
 $\text{eše}_3$  represents a surface of 21,600m<sup>2</sup>  
 etc.

The signs  $\text{iku}$  and  $\text{eše}_3$  constitute by themselves measures of surface areas. These measures are usually followed by the sign  $\text{GAN}_2$ , which means either surface or field and

Figure 9: Discussion of Old Babylonian and fourth millenium area measures in [Proust2020]. The cuneiform text is Unicode-encoded.

formed by only two signs  $\text{gur}$  and  $\text{gur}$ , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60<sup>2</sup>, 60<sup>3</sup>, 1/60, 1/60<sup>2</sup>, etc. are written in the same way, with the vertical wedge  $\text{gur}$ . The third feature concerns the exact function of

Figure 10: TODO [Cha12, p. 58]

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with:  $\text{gur}$  but never with the repetition  $\text{gur}$ .

Figure 11: TODO [Cha12, p. 59]<sup>63</sup>

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

$\text{gur}$  še bar  $\text{gur}$  ba-ri-zu

'3 gubar (capacity units) and 1 parisu'.

Figure 12: TODO [Cha12, p. 61]

This is particularly true of the signs  $\text{gur}$ ,  $\text{gur}$ ,  $\text{gur}$  and  $\text{gur}$ , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written  $\text{gur}$  in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign  $\text{gur}$  is most often associated with the *parisu* measure, while the signs  $\text{gur}$ ,  $\text{gur}$ ,  $\text{gur}$  and  $\text{gur}$  refer to 1, 2, 3,

Figure 13: TODO [Cha12, p. 64]

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g.,  $\text{gur}$  transliterated as 2(šar<sub>2</sub>) 1(geš'u) 3(u), means  $2 \times 3600 + 1 \times 600 + 3 \times 10$ ). The system is based on an alternation of factors ten and

Figure 14: TODO

might think of one fabric and a half,<sup>11</sup> but the presence of notations with “2▷ 2◡”, “3▷ 3◡”, and “6▷ 6◡” (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag’s weight measurement system.<sup>12</sup> The notation “1▷ gada” in o. ii 1 and r. vi 1, along with the total of “39



Fig. 1. Combinations of numerals attested in Š. 742.

Figure 15: Discussion of the contrast between ▷ and ◡ numerals in [Gor23, p. 162].

as, for example, in TM.75.G.3125 = *ARET* III 107 o. iv 1, “4▷ ‘a<sub>3</sub>-da-um<sup>tu9</sup>-2 4◡  
aktum 4▷ib<sub>2</sub><sup>tu9</sup>×3 3 sa<sub>6</sub> gunu<sub>3</sub>” (Fig. 2).

Figure 16: Transliteration in [Gor23, p. 163] of [P242293, *recto* 4, 1] incorporating untransliterated numerals.

- in personal names in administrative texts, such as the following, which all contain ▷ numerals:
  - — in [P010424; P010458; P010459] from ED IIIa أبو صلابيخ,
  - — in [P010960] from ED IIIa Šuruppag,
  - — in [P251641] from ED IIIb Adab,
  - — in [P252866] from ED IIIb Adab,
  - — in [P298637] from ED IIIb Umma;
- in the Sumerian word — u<sub>2</sub>-rum, “property” in ED IIIb Nirsu administrative texts which contain ▷ numerals, such as [P020006; P020008; P020018; P020024; P020030];
- in lexical texts:
  - in the divine name \* in the lexical texts [P010570; P010572], where the entries are prefixed with ▷.
  - in the word — dili, “small fish” in [P010578], witness to Early Dynastic Fish,
  - in the same word with a determinative, — dili<sup>ku<sub>6</sub></sup>, in [P010586], witness to Early Dynastic Food, which starts with ▷ numerals.

This is a clear contrast between — and ▷ in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name — occurs on its own line in the aforementioned administrative texts; a line ▷ — would instead be read as “one slave”.

### 3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as — that have both numeric and non-numeric values in both the main list and the

section on numbers. For instance, [KWU, pp. 123 sqq.] lists all of  $\text{𐎶}$ – $\text{𐎶𐎵}$  together with  $\text{𐎶}$ – $\text{𐎶𐎵}$ , while  $\text{𐎶}$ ,  $\text{𐎶𐎵}$ , and  $\text{𐎶𐎵𐎶}$ , and only those, appear at the beginning of the sign list, since they have non-numeric values<sup>66</sup>. [PTACE, p. 58] has the numeric signs  $\text{𐎶}$ ,  $\text{𐎶𐎵}$ ,  $\text{𐎶𐎵𐎶}$ , whereas non-numeric  $\text{𐎶}$  is at the beginning of the sign list, where its values *aš* and *rum* are listed. For signs with both non-numeric and numeric usage, [LAK] writes *s. die Zahlz.* throughout the main list; LAK 1  $\text{𐎶}$  thus reappears at LAK 829 together with  $\text{𐎶}$ ,  $\text{𐎶𐎵}$ , and  $\text{𐎶𐎵𐎶}$ . One should note [MZL], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have  $\text{𐎶𐎶}$ – $\text{𐎶𐎶𐎶}$  used in system  $G_{\text{Ur III/OB}}$ .

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes<sup>67</sup> to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from  $\text{𐎶}$  to  $\text{𐎶𐎵}$  numerals prevents diachronic encoding even if  $\text{𐎶}$  were unified with  $\text{𐎶}$ . For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'–6']; however, they are written with  $\text{𐎶}$  numerals, whereas they are written with  $\text{𐎶}$  numerals in the Early Dynastic witnesses; since  $\text{𐎶}$  and  $\text{𐎶}$  are distinct<sup>68</sup> characters, the  $\text{𐎶}$ – $\text{𐎶}$  unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods<sup>69</sup>, there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Romach2023], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

### 3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts<sup>70</sup> being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation

<sup>66</sup>Non-numeric values of  $\text{𐎶}$  were discussed in section 3.5;  $\text{𐎶𐎵}$  has the values *man*<sub>3</sub> and *min*<sub>5</sub>, and is used for the word *didli*, “several, various”;  $\text{𐎶𐎵𐎶}$  has the value *eš*<sub>6</sub>.

<sup>67</sup>TODO comment on the ED witnesses to the instructions of Šuruppak

<sup>68</sup>Besides the contrasts in numeric usage mentioned in section 3.3.3, these (already-encoded) characters were clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of  $\text{𐎶}$  which are not shared with  $\text{𐎶}$  have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period,  $\text{𐎶}$  is used for the preposition *ina*, “in”, and  $\text{𐎶}$  for the preposition *ana*, “to”.





<sup>69</sup>TODO cite a few things here.


















<sup>70</sup>Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.







of any text with numerals intractable.


### 3.7.1 The case of ŠAR<sub>2</sub>





The character U+122B9  CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period<sup>71</sup>, a contrast between non-numeric šar<sub>2</sub> written  and numeric 1(šar<sub>2</sub>) written  can be observed, similar to the contrast between  and  previously discussed in section 3.5. However, in lexical lists from Šuruppag and Ebla<sup>72</sup>, as well as in the *Stèle des vautours*, non-numeric šar<sub>2</sub> is curviform:














-     and     in [P010566];
-   and    in [P010576];
-   in [P240986]<sup>73</sup>;
-   in [P22399, obv. 17, 9, 18, 11, 22, 12]<sup>74</sup>.

It *would* be disruptive to the diachronic representation of text if non-numeric šar<sub>2</sub> were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph , identical to the glyph of the proposed U+12579  CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric šar<sub>2</sub> is curviform solidly predates the transition from  to  in the relevant metrological systems, there is no need to represent a - contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D  CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.




Note that in rare cases, such as [P22243] from ED IIIa Adab, non-numeric  (here with the value rum) is written . It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550  cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. in practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the CDLI transliteration (UR2-1(aš@c)) and the ePSD2 one (uru<sub>8</sub><sup>rum</sup>) of this word disagree on that aspect. Since  has a cuneiform reference glyph, this does not pose any compatibility concerns.

<sup>71</sup>For example, in personal names:







-     in [P020019] from ED IIIb Nirsu;
-     in [P020182], also from ED IIIb Nirsu;
-   in [P222186] from ED IIIb Umma;
-    in [P235312] from Old Akkadian Umma.

<sup>72</sup>TODO Mention other ways in which these are archaizing

<sup>73</sup>From copy in [ELLes, No. 397].










<sup>74</sup>Note however    on [P22399, obv. 6, 17]. Curviform non-numeric šar<sub>2</sub> is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here. TODO add a photo.




[Maiocchi2024] gave an example of the input syntax used by the new “Urban Economy Begins” project as “10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)” for    , with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are  rather than  numerals.

### 3.8 Conclusions

Co-occurrences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idiosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications. The contrast frequently applies to individual numerals, rather than to the span of entire numeric expressions.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted:  is a length of three ropes, but  is an area of three bur<sub>3</sub>;  could be read as one  and one , where  would be one and a half ;  is a personal name, but  would be “one slave”.

TODO mention the risk of confusion about character identity.

At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millenium corpora. The oddity of  requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.

## 4 Rationale for ED-Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millenium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millenium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs KAŠ<sub>a</sub>, KAŠ<sub>b</sub>, and KAŠ<sub>c</sub>, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Englund2001]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered scripts.

As part of the development of these conventions, a classification of fourth millenium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter *N* with a numeric subscript

(sometimes with an additional alphabetic subscript):  $N_1$  is  $\text{𒐠}$ ,  $N_{14}$  is  $\bullet$ ,  $N_{34}$  is  $\text{𒐠}$ , etc. Transliterations of numeric expression then use those to identify the type of number used, thus  $5N_1$  is  $\text{𒐠𒐠}$ , and  $5N_{14}$  is  $\bullet\bullet\bullet$ .

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the CDLI transliteration of  $\text{𒐠𒐠𒐠}$  or  $\text{𒐠𒐠𒐠}$  in [P283802] currently uses (barig@c) for the vertical numerals, since  $\text{𒐠}$  numerals are typically capacity measures; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millenium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I–II period, even though the Sumerian text uses classical assyriological transliteration conventions.

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millenium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in section 3.7.2; they are indeed used interchangeably, as in [P011104] which uses the notation u@f in [ePSD2], but N14@f in CDLI. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millenium would therefore induce confusion as to which numerals should be used in third millenium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millenium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance,  $\text{𒐠}$   $N_{30c}$  is listed as a variant of  $\text{𒐠}$   $N_{30a}$  in [DE87, p. 166], where the numeric value of either in relation to  $\text{𒐠}$   $N_{29a}$  is still unknown, but their values are found in [Englund2004] to be  $\text{𒐠} = \frac{1}{10} \text{𒐠}$ , whereas  $\text{𒐠} = \frac{1}{6} \text{𒐠}$ .

## 5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban<sub>2</sub>@c) and 5(ban<sub>2</sub>@c).]

## 6 Characters not included in this proposal

### 6.1 Missing numerals

TODO N13 not attested in CDLI TODO ( $N_{17}$  not usefully numeric,  $12N_{14}$  not encodable, etc.). Cite [DE87, p. 147] 7 and 8(diš *tenû*) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.









Figure 18: The layout of case [P020066]; the numeral 𐎶 is spread across two lines. The text is read in the order 𐎶𐎵𐎶𐎶𐎶𐎶, “twenty-two oxen, one year old”.

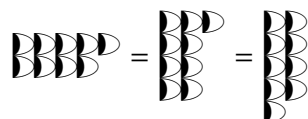


Figure 19: Three stacking patterns for U+12573 CUNEIFORM NUMERIC SIGN NINE N34. The one on the left is the reference glyph, used in Uruk III [P003499; P004430], and widely afterwards, *e.g.*, ED IIIa Šuruppag [P010678], ED IIIb Nirsu [P020057], Old Akkadian Umma [P212464]. The ones in the middle and right are used in two Uruk IV tablets [P001243; P004500]. All three Uruk examples are transliterated 9(N34) in CDLI.

numerals used in the third and fourth millenium, usage in third millenium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents 𐎶, 𐎵, 𐎶, and 𐎶 must remain consistent across the numeral series, lest a 𐎵 numeral be confused with an 𐎶 numeral. Since the single indents are frequently used in running text, as illustrated in section 3.4-5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 19 uses OpenType stylistic alternates, and Figure 17 rotates the character 𐎶, in both cases preserving the plain text backing.

### 6.3 Other glyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Englund2001].

## Acknowledgements

TODO(egg): Something about the Vanséveren fonts

## References

### Artefacts

[P020054] VAT 4731. [För16, 40 p.14]. Vorderasiatisches Museum.  
CDLI: P020054.

- [P020129] VAT 04713. Vorderasiatisches Museum.  
CDLI: [P020129](#).  
ORACC: [epsd2/corpus/P020129](#).
- [P102305] X.3.139. Michael C. Carlos Museum, Emory University.  
CDLI: [P102305](#).
- [P215653] AS 15375 21. Musée du Louvre.  
CDLI: [P215653](#).  
ORACC: [dcclt/corpus/P215653](#).  
Louvre Collections: [ark:/53355/cl010436723](#).
- [P221266] AO 13825. Musée du Louvre.  
CDLI: [P221266](#).  
ORACC: [epsd2/corpus/P221266](#).  
Louvre Collections: [ark:/53355/cl010138527](#).
- [P221291] AO 13850. Musée du Louvre.  
CDLI: [P221291](#).  
ORACC: [epsd2/corpus/P221291](#).
- [P221305] AO 13864. Musée du Louvre.  
CDLI: [P221305](#).  
ORACC: [epsd2/corpus/P221305](#).
- [P222399] *Stèle des vautours*. AO 50; AO 2346; AO 2347; AO 2348; AO 16109.  
Musée du Louvre.  
CDLI: [P222399](#).
- [P232278] *Gudea E*. AO 6. Musée du Louvre.  
CDLI: [P232278](#).  
ORACC: [etcstri/Q001544](#).
- [P232280] *Gudea G*. AO 7. Musée du Louvre.  
CDLI: [P232280](#).  
ORACC: [etcstri/Q001546](#).
- [P240531] TM.75.G.00265. Idlib, Syria: National Museum of Syria.  
CDLI: [P240531](#).  
EbDA: [1415](#).
- [P240532] TM.75.G.00266. Idlib, Syria: National Museum of Syria.  
CDLI: [P240532](#).  
EbDA: [1324](#).
- [P240533] TM.75.G.00267. Idlib, Syria: National Museum of Syria.  
CDLI: [P240533](#).  
EbDA: [1379](#).
- [P240548] TM.75.G.00302. Idlib, Syria: National Museum of Syria.  
CDLI: [P240548](#).  
EbDA: [1350](#).
- [P240579] TM.75.G.00341. Idlib, Syria: National Museum of Syria.  
CDLI: [P240579](#).  
EbDA: [1364](#).
- [P240609] TM.75.G.00440. Idlib, Syria: National Museum of Syria.  
CDLI: [P240609](#).  
EbDA: [1378](#).

- [P240653] TM.75.G.00535. Idlib, Syria: National Museum of Syria.  
CDLI: [P240653](#).  
EbDA: [1382](#).
- [P240654] TM.75.G.00536. Idlib, Syria: National Museum of Syria.  
CDLI: [P240654](#).  
EbDA: [1383](#).
- [P240655] TM.75.G.00537. Idlib, Syria: National Museum of Syria.  
CDLI: [P240655](#).  
EbDA: [1358](#).
- [P240675] TM.75.G.00557. Idlib, Syria: National Museum of Syria.  
CDLI: [P240675](#).  
EbDA: [1371](#).
- [P240697] TM.75.G.00579. Idlib, Syria: National Museum of Syria.  
CDLI: [P240697](#).  
EbDA: [1381](#).
- [P240964] TM.75.G.1392. Idlib, Syria: National Museum of Syria.  
CDLI: [P240964](#).
- [P241708] TM.75.G.02143. Idlib, Syria: National Museum of Syria.  
CDLI: [P241708](#).  
EbDA: [3173](#).
- [P241904] TM.75.G.02346. [[Arc89](#), p. 6]. Idlib, Syria: National Museum of Syria.  
CDLI: [P241904](#).  
EbDA: [3183](#).
- [P242293] TM.75.G.03125. Idlib, Syria: National Museum of Syria.  
CDLI: [P242293](#).  
EbDA: [217](#).
- [P249253] *Code de Hammurabi*. Sb 8. Musée du Louvre.  
CDLI: [P249253](#).

## Unicode documents

- [L2/03-162] M. Everson and K. Feuerherm. *Basic principles for the encoding of Sumero-Akkadian Cuneiform*. 25th May 2003.  
UTC: [L2/03-162](#).
- [L2/03-393R] M. Everson, K. Feuerherm and S. Tinney. *Preliminary proposal to encode the Cuneiform script in the SMP of the UCS*. 3rd Nov. 2003.  
UTC: [L2/03-393R](#).
- [L2/04-036] M. Everson, K. Feuerherm and S. Tinney. *Revised proposal to encode the Cuneiform script in the SMP of the UCS*. 29th Jan. 2004.  
UTC: [L2/04-036](#).
- [L2/04-099] L. Anderson. *Unification of Cuneiform Numbers*. 2004.  
UTC: [L2/04-099](#).
- [L2/04-189] M. Everson, K. Feuerherm and S. Tinney. *Final proposal to encode the Cuneiform script in the SMP of the UCS*. 8th June 2004.  
UTC: [L2/04-189](#).

- [L2/24-159] P. Constable, ed. *Minutes of UTC Meeting 180* (23rd–25th July 2024). 29th July 2024.  
UTC: [L2/24-159](#).
- [Uni16] The Unicode Consortium. *The Unicode Standard*. Version 16.0.0. The Unicode Consortium, 10th Sept. 2024.  
ISBN: 978-1-936213-34-4.  
eprint: <https://www.unicode.org/versions/Unicode16.0.0/core-spec/>.
- [UTR56] R. Leroy, ed. *Unicode Cuneiform Sign Lists*. Unicode Technical Report #56.  
eprint: <https://www.unicode.org/reports/tr56/>.

### Major reference works and online projects

- [EbDA] L. Milano, M. Maiocchi, F. Di Filippo, R. Orsini, E. Scarpa, M. Surdi et al., eds. *Ebla Digital Archives*. 2007–.  
eprint: <http://ebda.cnr.it/>.
- [eBL] E. Jiménez, Z. Földi, A. Härtinen, A. Heinrich, T. Mitto, G. Rozzi, I. Khait, J. Laasonen, F. Simonjetz et al., eds. *electronic Babylonian Library*. 2023–.  
eprint: <https://www.ebl.lmu.de/>.
- [ELLes] P. Mander. “Lista dei segni dei testi lessicali di Ebla”. In: *Testi lessicali monolingui della biblioteca L. 2769*. Ed. by G. Pettinato. Materiali epigrafici di Ebla 3. Napoli: Istituto universitario orientale, 1981, pp. 285–382.
- [ePSD2] S. Tinney, P. Jones and N. Veldhuis, eds. *The electronic Pennsylvania Sumerian Dictionary*. 2nd ed. 2017–.  
eprint: <http://oracc.org/epsd2>.
- [KWU] N. Schneider. *Die Keilschriftzeichen der Wirtschaftsurkunden von Ur III*. Editrice Pontificio Istituto Biblico, 1935.
- [LAK] A. Deimel. *Liste der archaischen Keilschriftzeichen von Fara*. Wissenschaftliche Veröffentlichungen der Deutschen Orient-Gesellschaft 40. J. C. Hinrichs’sche Buchhandlung, 1922.
- [MZL] R. Borger. *Mesopotamisches Zeichenlexikon*. Alter Orient und Altes Testament 305. Ugarit-Verlag, 2010.
- [OSL] N. Veldhuis, S. Tinney et al., eds. *Oracc Sign List*. 2014–.  
eprint: <http://oracc.org/osl/>.
- [PTACE] A. Catagnoti. *La paleografia dei testi dell’amministrazione e della cancelleria di Ebla*. Quaderni di Semitistica 9. Università di Firenze, 2013.  
ISBN: 8890134054.

## Other documents

- [Arc15] A. Archi. *Ebla and Its Archives. Texts, History, and Society*. Studies in ancient Near Eastern records 7. Walter de Gruyter, 2015.  
ISBN: 978-1-61451-716-0.  
DOI: [10.1515/9781614517887](https://doi.org/10.1515/9781614517887).
- [Arc89] A. Archi. "Tables de comptes eblaïtes". In: *Revue d'assyriologie et d'archéologie orientale* 83.1 (1989). Ed. by P. Amiet and P. Garelli, pp. 1–6. ISSN: 0373-6032.
- [Cap02] R. Caplice. *Introduction to Akkadian*. 4th ed. Editrice Pontificio Istituto Biblico, 2002.  
ISBN: 88-7653-566-7.
- [Cha03] G. Chambon. "Archaic Metrological Systems from Ur". In: *Cuneiform Digital Library Journal* 2003.5 (23rd Dec. 2003). ISSN: 1540-8779.  
eprint: [http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003\\_005.html](http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003_005.html).
- [Cha12] G. Chambon. "Numeracy and Metrology". In: *The Oxford Handbook of Cuneiform Culture*. Ed. by K. Radner and E. Robson. Oxford University Press, 18th Sept. 2012, pp. 51–67.  
ISBN: 9780199557301.  
DOI: [10.1093/oxfordhb/9780199557301.013.0003](https://doi.org/10.1093/oxfordhb/9780199557301.013.0003).
- [DE87] P. Damerow and R. K. Englund. "Die Zahlzeichensysteme der archaischen Texte aus Uruk". In: M. W. Green and H. J. Nissen. *Zeichenliste der archaischen Texte aus Uruk*. Archaische Texte aus Uruk 2. An offprint of this chapter is available at <https://cdli.mpiwg-berlin.mpg.de/files-up/publications/englund1987a.pdf>. Gebr. Mann Verlag, 1987. Chap. 3, pp. 117–165.
- [Eng98] R. K. Englund. "Texts from the Late Uruk Period". In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 13–233.  
ISBN: 3-7278-1166-8.
- [Feu04] K. G. Feuerherm. "Abum-waqar and His Circle. A Prosopographical Study". PhD thesis. University of Toronto, 2004.
- [För16] W. Förtsch. *Altbabylonische Wirtschaftstexte aus der Zeit Lugalanda's und Urukagina's*. Vorderasiatische Schriftdenkmäler der Königlichen Museen zu Berlin 14. J. C. Hinrichs, 1916.
- [Fri07] J. Friberg. *A Remarkable Collection of Babylonian Mathematical Texts. Manuscripts in the Schøyen Collection: Cuneiform Texts I*. Sources and Studies in the History of Mathematics and Physical Sciences. Springer, 2007.  
ISBN: 978-0-387-34543-7.
- [Fri78] J. Friberg. *A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and Proto-Elamite Semi-Pictographic Inscriptions*. The Third Millenium Roots of Babylonian Mathematics 1. Department of Mathematics, Chalmers University of Technology, 1978.

- [Fri87] J. Friberg. “Mathematik”. In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš-Medizin. 1987–1990, pp. 531–585.
- [Gor23] F. Gori. “On Lapis Lazuli and Linen in Šuruppag Texts. An Analysis Through the Lens of Ebla Studies”. In: *Studia Eblaitica* 9 (2023), pp. 160–166. ISSN: 2364-7124.
- [Hue11] J. Huehnergard. *A Grammar of Akkadian*. 3rd ed. Brill, 2011. ISBN: 978-1-57506-941-8.
- [JJ24] T. Jauhiainen and H. Jauhiainen. “Advancing Cuneiform Text Dating Through Automatic Analysis”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 11th July 2024 14:00.
- [Kre98] M. Krebern timer. “Die Texte aus Fāra und Tell Abū Šalābīḥ”. In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 235–427. ISBN: 3-7278-1166-8.
- [Lec12] C. Lecompte. “Des chiffres et des digues: à propos de deux textes présargoniques de Ġirsu et d’une notation numérique inhabituelle”. In: *Altorientalische Forschungen* 39.1 (Dec. 2012), pp. 81–86. DOI: [10.1524/aof.2012.0006](https://doi.org/10.1524/aof.2012.0006).
- [Lec16] C. Lecompte. “ED IIIb metrology: texts from Lagaš”. In: *CDLI:wiki. A Library of Knowledge of the Cuneiform Digital Library Initiative*. 12th Apr. 2016. eprint: [https://cdli.ox.ac.uk/wiki/doku.php?id=ed\\_iii\\_metrological\\_systems](https://cdli.ox.ac.uk/wiki/doku.php?id=ed_iii_metrological_systems).
- [Lec20] C. Lecompte. “The Measurement of Fields During the Pre-sargonic Period”. In: *Mathematics, Administrative and Economic Activities in Ancient Worlds*. Ed. by C. Michel and K. Chemla. Why the Sciences of the Ancient World Matter 5. Springer, 2020.
- [NDE93] H. J. Nissen, P. Damerow and R. K. Englund. *Archaic Bookkeeping. Early Writing and Techniques of Economic Administration in the Ancient Near East*. Trans. by P. Larsen. The University of Chicago Press, 1993. ISBN: 0-226-58659-6.
- [Oel22] J. Oelsner. *Der Kodex Ḥammu-rāpi*. dubsar 4. Zaphon, 2022.
- [Pow87] M. Powell. “Maße und Gewichte”. In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš-Medizin. 1987–1990, pp. 457–530.
- [Rob08] E. Robson. *Mathematics in Ancient Iraq. A Social History*. Princeton University Press, 2008. ISBN: 978-0-691-09182-2.
- [Rob19] E. Robson. “Oracc metrology guidelines”. In: *Oracc: The Open Richly Annotated Cuneiform Corpus*. 18th Dec. 2019. ORACC: [doc/help/editinginatf/metrology/metrologicaltables](https://oracc.berkeley.edu/doc/help/editinginatf/metrology/metrologicaltables).
- [Rob22] E. Robson. “Overview of Metrological Systems”. In: *The Digital Corpus of Cuneiform Mathematical Texts*. 2022. ORACC: [dccmt/Metrology](https://oracc.berkeley.edu/dccmt/Metrology).

- [Rom24] A. Romach. “The Neo Assyrian Land Sale Documents from Dur-Katlimmu: A Stylometric Analysis of Their Scribal Features”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 10th July 2024 12:00.
- [Sch10] W. Schramm. *Akkadische Logogramme*. Göttinger Beiträge zum Alten Orient 5. Universitätsverlag Göttingen, 2010.  
ISBN: 978-3-941875-65-4.  
DOI: [10.17875/gup2010-511](https://doi.org/10.17875/gup2010-511).
- [Svä+24] S. Svärd, M. Lorenzon, J. Töyräänvuori, J. Valk, T. Alstola, E. Bennett, R. Uotila and T. Auranne, eds. *RAI 69 Abstracts*. July 2024.  
eprint: [https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäiväty\\_1.pdf](https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäiväty_1.pdf).