

Archaic cuneiform numbers

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1 Summary

This document proposes encoding some numerals used in the Uruk and Early Dynastic periods in conjunction with the Sumero-Akkadian cuneiform script¹ and the proto-cuneiform script². The proposed characters are listed in section 2.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in section 4. The proposed “curved”, or “curviform”, numerals³ should however *not* be unified with the already-encoded cuneiform numerals⁴. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be glyph variants of the cuneiform numerals, a detailed rationale is provided in section 3, including compatibility considerations in section 3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The Script_Extensions property assignments in section 2.2 reflect the overlap.

[TODO(egg): Mention the other sections here too.]

¹ISO 15924: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.

²ISO 15924: Pcun, not yet encoded.

³Impressed into clay using cylindrical styli, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: ▷, ▽ (small stylus), ▢, ▣ (large stylus). Some numerals are composed of multiple such impressions, e.g., ▢▷. The terms “curved”, “curviform”, and “round” can be found in the literature. We avoid the term “round” here as it has other meanings in the context of numbers. We use “curviform” in this document as, being the least common term, it is least likely to lead to confusion, and “CURVED” in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [inlineATF].

⁴Impressed into clay using a stylus with a trihedral end: — (stylus held horizontally), † (vertically), ‡ (diagonally) ‹ (with the head of the stylus), † (stylus pressed deeper, forming a larger wedge), ‡ (combining † and ‹), etc.

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
Non-numeric signs	Future Pkun	Existing Xsux	

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

2 Proposed changes to the Standard

2.1 Summary of proposed characters

2.2 Properties





2.3 Character names list

2.4 Core specification text

3 Rationale for curviform–cuneiform disunification

TODO(egg): blurb.

3.1 The cuneiform encoding model

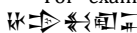
As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble  in texts from Early Dynastic IIIa Šuruppag as in the character code charts,  later in the third millennium⁵,  in Old Babylonian cursive,  in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists⁶ and dictionaries⁷, and of composite texts⁸. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]⁹. The diachronic approach is also useful for pedagogic applications¹⁰.

⁵Merging with U+1224E cuneiform sign ni2.

⁶Notably [OSL] and the online edition of [MZL] in [eBL, Signs].

⁷Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

⁸For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of , as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

⁹Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svā+24, pp. 242, 148].

¹⁰For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding¹¹ that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early Dynastic IIIb administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

† ¹²	<	+	≡	𐎶	𐎶	𐎶
1(ḫeš ₂)	1(u)	1/2(diš)	5(diš <i>tenû</i>)	gi	us ₂	sa ₂
	7.5 (ropes)		5	reed	side	equal
« ¹³	≡	𐎶	𐎶	𐎶		
3(u)	6(diš <i>tenû</i>)	gi	saṇ	sa ₂		
3(ropes)	6	reed	front	equal		
𐎶𐎶𐎶	•	◁	▷	◡		
ašag-bi	1(bur ₃ ^c)	1(eše ₃ ^c)	1(iku ^c)	1/2(iku ^c)		
this field						

𐎶𐎶𐎶𐎶𐎶
tug_x(LAK483)-si-ga-kam¹⁴
deep ploughing

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as¹⁵

465 metres, equal lengths
198 metres, equal widths
this field: 𐎶, 𐎶 hectares, deeply ploughed

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from ▷ to † numerals). It therefore comes to the conclusion that the use of

¹¹At that time scoped to the répertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as 𐎶 ≠ 𐎶𐎶, were informed by Early Dynastic distinctions.

¹²As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large — in copies. The photos in CDLI clearly show that this is in fact a vertical wedge.

¹³Note that ED IIIb < numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign « in [P020054] looks more like Ur III 𐎶.

¹⁴Transliteration after [Lec20, p. 325].

¹⁵We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

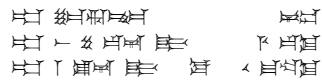
curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in section 3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform descendants is treated as glyphic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases.”

The time has come to revisit this issue. As we will see in section 3.3, numerals can only be interpreted in the context of what they measure *i.e.*, as part of a metrological system. In section 3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere \triangleright / \lvert switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

3.3 A primer on classic Ur III and Old Babylonian metrologies


 I want to write tablets: the tablet of
 1 cor of barley to 600 cor; the tablet
 of 1 shekel of silver to 10 minas [...]
 Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the contrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known¹⁶ a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Rob08, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters \lvert – |||| , the multiples of ten (10–50) are < – <|||| , but the other digits 11–59 are sequences < – <|||| ; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system* $S_{\text{Ur III/OB}}$, which had different signs for the units \lvert – |||| , tens < – <|||| , sixties |||| (with larger wedges than the units), multiples of six hundred |||| , multiples of three thousand six hundreds |||| , and multiples of thirty-six thousand |||| .

¹⁶See, e.g., [Uni16, §22.3.3, sub “Cuneiform Numerals”].

3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram¹⁷, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{6} \text{𐎴} \xleftarrow{10} \text{𐎳} \quad (S_{\text{Ur III/OB}})$$

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written 𐎶𐎵𐎴𐎳𐎲 in the discrete counting system, and 𐎶𐎵𐎴𐎳𐎲 in the sexagesimal place value system.

3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{3} \text{𐎴} \xleftarrow{6} \text{𐎳} \xleftarrow{2} \text{𐎲} \xleftarrow{2} \text{𐎱} \quad (G_{\text{Ur III/OB}})$$

<i>bur₃</i>	<i>eše₃</i>	<i>iku</i>	<i>ubûm</i>
<i>bûrum</i>	<i>eblum</i>	<i>ikûm</i>	
6,48 ha	2,16 ha	3600 m ²	1800 m ²

Note that for the range of areas given above¹⁸, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *bûrum*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign 𐎶 functioning as punctuation¹⁹, but the numerals are tied to the metrology; thus a surface of 5 *bûr* 1 *ebel* 4 *ikû* (100 *ikû*, 36 ha) would be written²⁰ 𐎶𐎵𐎴𐎳𐎲. Contrast this with systems where the same numerals are used for different units, and overt units are used, as in “88 acres 3 roods 33 perches” or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the sign \diamond is equal to sixty times 𐎴 in the area system, but to three hundred and sixty times 𐎴 in the discrete counting system.

¹⁷These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*.

¹⁸For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mûšarum* (36 m²) written 𐎶𐎵, equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60 𐎶 (shekels). For areas greater than 3600 *bûrû*, the \diamond and \diamond numerals are reused with a suffix 𐎶 (gal, Sumerian: big), as follows [Rob08, p. 295 n. b and c; Fri07, p. 378; Rob19]:

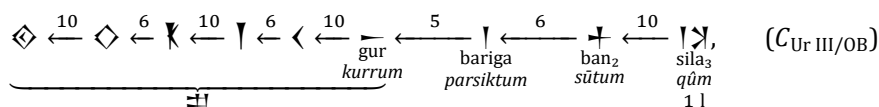
$$\diamond \text{𐎶} \xleftarrow{10} \diamond \text{𐎶} \xleftarrow{6} \diamond \text{𐎶} \xleftarrow{10} \diamond \text{𐎶} \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎵} \xleftarrow{3} \text{𐎴} \xleftarrow{6} \text{𐎳} \xleftarrow{2} \text{𐎲} \xleftarrow{2} \text{𐎱} \xleftarrow{2,5} \text{𐎶} \xleftarrow{10} \text{𐎶} \xleftarrow{6} \text{𐎶} \xleftarrow{10} \text{𐎶}.$$

¹⁹TODO(egg): acknowledge Proust 2020 but note that this is irrelevant to encoding concerns






²⁰As in the surface of the field of 𐎶𐎵𐎴𐎳𐎲 (the city of Apisal) reported on [P102305, r. 1]

3.3.3 The capacity system

Another such system of note is the one for capacities²¹ [Fri07, p. 376; Rob19],



where the numerals for ban₂ are 𐤁, 𐤂, 𐤃, 𐤄, and 𐤅, and those for bariga are 𐤆, 𐤇, 𐤈, and 𐤉 (contrast ordinary 𐤈 and 𐤉 otherwise used with 𐤆 numerals). As described in [Hue11, p. 585 n. (b) and (f)], the sign 𐤆𐤇 GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign 𐤇 if present, and after the word for “grain” if present, as in

 354 gur
  3 ban₂
  6
  sila₃
  of grain.

Observe that while large numbers of gur follow²³ system $\mathcal{S}_{\text{Ur III/OB}}$, the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as $\langle \text{I} \text{—} \text{II} \rangle$ would be 10 gur 1 bariga, and $\langle \text{—} \text{II} \rangle$ would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

- \mathbb{I} – \mathbb{W} used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- \mathbb{L} – \mathbb{X} used in $G_{\text{Ur III/OB}}$, of which \mathbb{L} – \mathbb{X} are also used in $S_{\text{Ur III/OB}}$ and the SPVS as well as with overt units;
- \mathbb{Y} – \mathbb{W} used in $S_{\text{Ur III/OB}}$, and sometimes with overt units;
- \mathbb{K} – \mathbb{W} used in $S_{\text{Ur III/OB}}$;
- \mathbb{D} – \mathbb{X} used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \mathbb{E} – \mathbb{R} used in $S_{\text{Ur III/OB}}$ and $G_{\text{Ur III/OB}}$;
- \mathbb{T} – \mathbb{M} used in $C_{\text{Ur III/OB}}$ as well as with overt units of the weight system;
- \mathbb{P} , \mathbb{Q} , \mathbb{R} , \mathbb{S} , \mathbb{T} used in $C_{\text{Ur III/OB}}$;
- \mathbb{U} , \mathbb{V} , \mathbb{W} , \mathbb{X} used in $C_{\text{Ur III/OB}}$ —note the overlap with \mathbb{Y} – \mathbb{W} ;
- \mathbb{Z} and \mathbb{Z} used in $G_{\text{Ur III/OB}}$.

Only in the SPVS did numerals exist truly independently of metrology; to quote [Robo8, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

²¹Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system $G_{Ur III/OB}$ based on a height of one cubit, see[[Pow87](#), p. 488; [Rob08](#), p. 294; [Rob19](#)].

²²From [P309594].

²³A larger unit, the *guru*₇ (*karûm*, grain heap), is sometimes used instead, with 𐎠𐎡𐎢𐎣𐎤𐎥𐎦𐎧𐎨𐎩𐎪𐎫𐎬𐎭𐎮𐎯𐎰𐎱𐎲𐎳𐎴𐎵𐎶𐎷𐎸𐎹𐎺𐎻𐎼𐎽𐎾𐎿𐏀𐏁𐏂𐏃𐏄𐏅𐏆𐏇𐏈𐏉𐏊𐏋𐏌𐏍𐏎𐏏𐏐𐏑𐏒𐏓𐏔𐏕𐏖𐏗𐏘𐏙𐏚𐏛𐏜𐏝𐏞𐏟𐏠𐏡𐏢𐏣𐏤𐏥𐏦𐏧𐏨𐏩𐏪𐏫𐏬𐏭𐏮𐏯𐏰𐏱𐏲𐏳𐏴𐏵𐏶𐏷𐏸𐏹𐏺𐏻𐏼𐏽𐏾𐏿𐐀𐐁𐐂𐐃𐐄𐐅𐐆𐐇𐐈𐐉𐐊𐐋𐐌𐐍𐐎𐐏𐐐𐐑𐐒𐐓𐐔𐐕𐐖𐐗𐐘𐐙𐐚𐐛𐐜𐐝𐐞𐐟𐐠𐐡𐐢𐐣𐐤𐐥𐐦𐐧𐐨𐐩𐐪𐐫𐐬𐐭𐐮𐐯𐐰𐐱𐐲𐐳𐐴𐐵𐐶𐐷𐐸𐐹𐐺𐐻𐐼𐐽𐐾𐐿𐑀𐑁𐑂𐑃𐑄𐑅𐑆𐑇𐑈𐑉𐑊𐑋𐑌𐑍𐑎𐑏𐑐𐑑𐑒𐑓𐑔𐑕𐑖𐑗𐑘𐑙𐑚𐑛𐑜𐑝𐑞𐑟𐑠𐑡𐑢𐑣𐑤𐑥𐑦𐑧𐑨𐑩𐑪𐑫𐑬𐑭𐑮𐑯𐑰𐑱𐑲𐑳𐑴𐑵𐑶𐑷𐑸𐑹𐑺𐑻𐑼𐑽𐑾𐑿𐒀𐒁𐒂𐒃𐒄𐒅𐒆𐒇𐒈𐒉𐒊𐒋𐒌𐒍𐒎𐒏𐒐𐒑𐒒𐒓𐒔𐒕𐒖𐒗𐒘𐒙𐒚𐒛𐒜𐒝𐒞𐒟𐒠𐒡𐒢𐒣𐒤𐒥𐒦𐒧𐒨𐒩𐒪𐒫𐒬𐒭𐒮𐒯𐒰𐒱𐒲𐒳𐒴𐒵𐒶𐒷𐒸𐒹𐒺𐒻𐒼𐒽𐒾𐒿𐓀𐓁𐓂𐓃𐓄𐓅𐓆𐓇𐓈𐓉𐓊𐓋𐓌𐓍𐓎𐓏𐓐𐓑𐓒𐓓𐓔𐓕𐓖𐓗𐓘𐓙𐓚𐓛𐓜𐓝𐓞𐓟𐓠𐓡𐓢𐓣𐓤𐓥𐓦𐓧𐓨𐓩𐓪𐓫𐓬𐓭𐓮𐓯𐓰𐓱𐓲𐓳𐓴𐓵𐓶𐓷𐓸𐓹𐓺𐓻𐓼𐓽𐓾𐓿𐔀𐔁𐔂𐔃𐔄𐔅𐔆𐔇𐔈𐔉𐔊𐔋𐔌𐔍𐔎𐔏𐔐𐔑𐔒𐔓𐔔𐔕𐔖𐔗𐔘𐔙𐔚𐔛𐔜𐔝𐔞𐔟𐔠𐔡𐔢𐔣𐔤𐔥𐔦𐔧𐔨𐔩𐔪𐔫𐔬𐔭𐔮𐔯𐔰𐔱𐔲𐔳𐔴𐔵𐔶𐔷𐔸𐔹𐔺𐔻𐔼𐔽𐔾𐔿𐕀𐕁𐕂𐕃𐕄𐕅𐕆𐕇𐕈𐕉𐕊𐕋𐕌𐕍𐕎𐕏𐕐𐕑𐕒𐕓𐕔𐕕𐕖𐕗𐕘𐕙𐕚𐕛𐕜𐕝𐕞𐕟𐕠𐕡𐕢𐕣𐕤𐕥𐕦𐕧𐕨𐕩𐕪𐕫𐕬𐕭𐕮𐕯𐕰𐕱𐕲𐕳𐕴𐕵𐕶𐕷𐕸𐕹𐕺𐕻𐕼𐕽𐕾𐕿𐖀𐖁𐖂𐖃𐖄𐖅𐖆𐖇𐖈𐖉𐖊𐖋𐖌𐖍𐖎𐖏𐖐𐖑𐖒𐖓𐖔𐖕𐖖𐖗𐖘𐖙𐖚𐖛𐖜𐖝𐖞𐖟𐖠𐖡𐖢𐖣𐖤𐖥𐖦𐖧𐖨𐖩𐖪𐖫𐖬𐖭𐖮𐖯𐖰𐖱𐖲𐖳𐖴𐖵𐖶𐖷𐖸𐖹𐖺𐖻𐖼𐖽𐖾𐖿𐗀𐗁𐗂𐗃𐗄𐗅𐗆𐗇𐗈𐗉𐗊𐗋𐗌𐗍𐗎𐗏𐗐𐗑𐗒𐗓𐗔𐗕𐗖𐗗𐗘𐗙𐗚𐗛𐗜𐗝𐗞𐗟𐗠𐗡𐗢𐗣𐗤𐗥𐗦𐗧𐗨𐗩𐗪𐗫𐗬𐗭𐗮𐗯𐗰𐗱𐗲𐗳𐗴𐗵𐗶𐗷𐗸𐗹𐗺𐗻𐗼𐗽𐗾𐗿𐘀𐘁𐘂𐘃𐘄𐘅𐘆𐘇𐘈𐘉𐘊𐘋𐘌𐘍𐘎𐘏𐘐𐘑𐘒𐘓𐘔𐘕𐘖𐘗𐘘𐘙𐘚𐘛𐘜𐘝𐘞𐘟𐘠𐘡𐘢𐘣𐘤𐘥𐘦𐘧𐘨𐘩𐘪𐘫𐘬𐘭𐘮𐘯𐘰𐘱𐘲𐘳𐘴𐘵𐘶𐘷𐘸𐘹𐘺𐘻𐘼𐘽𐘾𐘿𐙀𐙁𐙂𐙃𐙄𐙅𐙆𐙇𐙈𐙉𐙊𐙋𐙌𐙍𐙎𐙏𐙐𐙑𐙒𐙓𐙔𐙕𐙖𐙗𐙘𐙙𐙚𐙛𐙜𐙝𐙞𐙟𐙠𐙡𐙢𐙣𐙤𐙥𐙦𐙧𐙨𐙩𐙪𐙫𐙬𐙭𐙮𐙯𐙰𐙱𐙲𐙳𐙴𐙵𐙶𐙷𐙸𐙹𐙺𐙻𐙼𐙽𐙾𐙿𐚀𐚁𐚂𐚃𐚄𐚅𐚆𐚇𐚈𐚉𐚊𐚋𐚌𐚍𐚎𐚏𐚐𐚑𐚒𐚓𐚔𐚕𐚖𐚗𐚘𐚙𐚚𐚛𐚜𐚝𐚞𐚟𐚠𐚡𐚢𐚣𐚤𐚥𐚦𐚧𐚨𐚩𐚪𐚫𐚬𐚭𐚮𐚯𐚰𐚱𐚲𐚳𐚴𐚵𐚶𐚷𐚸𐚹𐚺𐚻𐚼𐚽𐚾𐚿𐛀𐛁𐛂𐛃𐛄𐛅𐛆𐛇𐛈𐛉𐛊𐛋𐛌𐛍𐛎𐛏𐛐𐛑𐛒𐛓𐛔𐛕𐛖𐛗𐛘𐛙𐛚𐛛𐛜𐛝𐛞𐛟𐛠𐛡𐛢𐛣𐛤𐛥𐛦𐛧𐛨𐛩𐛪𐛫𐛬𐛭𐛮𐛯𐛰𐛱𐛲𐛳𐛴𐛵𐛶𐛷𐛸𐛹𐛺𐛻𐛼𐛽𐛾𐛿𐜀𐜁𐜂𐜃𐜄𐜅𐜆𐜇𐜈𐜉𐜊𐜋𐜌𐜍𐜎𐜏𐜐𐜑𐜒𐜓𐜔𐜕𐜖𐜗𐜘𐜙𐜚𐜛𐜜𐜝𐜞𐜟𐜠𐜡𐜢𐜣𐜤𐜥𐜦𐜧𐜨𐜩𐜪𐜫𐜬𐜭𐜮𐜯𐜰𐜱𐜲𐜳𐜴𐜵𐜶𐜷𐜸𐜹𐜺𐜻𐜼𐜽𐜾𐜿𐝀𐝁𐝂𐝃𐝄𐝅𐝆𐝇𐝈𐝉𐝊𐝋𐝌𐝍𐝎𐝏𐝐𐝑𐝒𐝓𐝔𐝕𐝖𐝗𐝘𐝙𐝚𐝛𐝜𐝝𐝞𐝟𐝠𐝡𐝢𐝣𐝤𐝥𐝦𐝧𐝨𐝩𐝪𐝫𐝬𐝭𐝮𐝯𐝰𐝱𐝲𐝳𐝴𐝵𐝶𐝷𐝸𐝹𐝺𐝻𐝼𐝽𐝾𐝿𐞀𐞁𐞂𐞃𐞄𐞅𐞆𐞇𐞈𐞉𐞊𐞋𐞌𐞍𐞎𐞏𐞐𐞑𐞒𐞓

8

of decreasing fractions $\frac{1}{n}$ of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus $\text{N}_{30} = \frac{1}{2} \text{N}_{30}$, $\text{N}_{30} = \frac{1}{3} \text{N}_{30}$, and so on. The first sign of the latter units, N_{34} ,

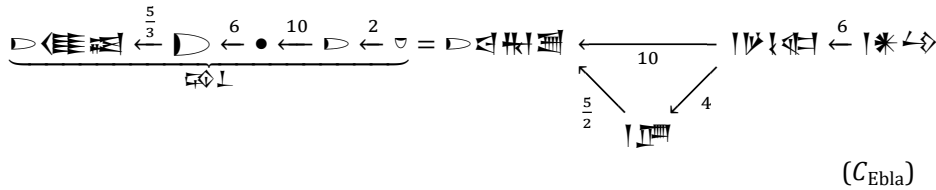
Figure 1: TODO [Eng98, p. 113]

For instance, the first line contains the notations $1\text{N}_{34} 1\text{N}_{30} : 2\text{N}_{20}$, which can be translated '60 of the (grain rations containing) (of grain); (grain involved:) 2 (of ground barley)'. This calculation contradicts the assumed numerical relationship $10\text{N}_1 = 1\text{N}_{14}$, since as was well known the measure represented by the sign N_{30} was $\frac{1}{5}$ of that represented by N_1 , so that $60 \times \frac{1}{5} = 12$ and not 20, as 2N_{14} would imply. Instead of relying on complicated

Figure 2: TODO [Eng98, p. 116]

11. [P240531, recto 1, 8]
12. [P241708, recto 1, 1]⁴⁵
13. [P241904, recto 1, 1]⁴⁶

Note that higher numbers of are expressed in hundreds (*mi-at*) and then thousands (*li-im*), as is typical in Ebla [Arc15, p. 33], e.g., in [P240532, verso 2, 3], (100 + 60 + 30 + 5 = 195 of grain). These expressions correspond to the following factor diagram:



3.4.5 Use in modern publications

Because of their prevalence in the Uruk and Early Dynastic periods, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 1–11.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 11. The partial⁴⁸ transliteration “4 'a₃-da-um 4 aktum 4 ib₂^{tu}×3 sa₆ gunu₃” is used to illustrate a discussion of the interpretation of the contrast between and numerals. More conventional transliterations⁴⁹ might omit the numeral shapes entirely, e.g., 4 'a₃-da-um 4 aktum 4 ib₂^{tu}×3 sa₆ gunu₃, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, e.g., 4(aš^c) 'a₃-da-um 4(diš^c) aktum 4(aš^c) ib₂^{tu}×3(diš^c) sa₆ gunu₃, but the result would be less readable. See Section 3.7.2 for a discussion of transliteration conventions for numerals.

⁴⁵From CDLI photo.

⁴⁶From photo in [Arc89, p. 6].

⁴⁷Laid out as ; on stacking patterns see Section 6.3.

⁴⁸The untransliterated text would be ; note the atomically

Die halbkreisförmigen Griffelindrücke gehen manchmal in mehr oder weniger eckige Formen über (∇)⁶⁸⁵. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen (\backslash), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren⁶⁸⁶. Selten treten mit ∇ gebildete Zahlen auf⁶⁸⁷ (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 3: TODO [Kre98, p. 303]

strong similarities between “area” 1 and “area” 3 systems, the sign with two concentric discs (\odot , notated N_{50} ²⁷) remains problematic. It never appears in any numerical combination with the sign with a single disc (\bullet),

Figure 4: TODO [Chao3, p. 6]

formed by only two signs Υ and \angle , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60, 60², 60³, 1/60, 1/60², etc. are written in the same way, with the vertical wedge Υ . The third feature concerns the exact function of

Figure 5: TODO [Cha12, p. 58]

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with: $\Upsilon\Upsilon\Upsilon\bullet\bullet\bullet$ but never with the repetition $\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon\Upsilon$.

Figure 6: TODO [Cha12, p. 59]⁵⁰

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

$\Upsilon\Upsilon\Upsilon$ še bar \cup ba-rí-zu
‘3 gubar (capacity units) and 1 parisu’.

Figure 7: TODO [Cha12, p. 61]

This is particularly true of the signs Υ , Υ , Υ and Υ , whose form explicitly denotes the fractions 1/6, 2/6, 3/6, and 4/6 of the barig capacity measure written \cup in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth 1/6 of the barig. At Ebla, the sign \cup is most often associated with the *parisu* measure, while the signs Υ , Υ , Υ and Υ refer to 1, 2, 3,

Figure 8: TODO [Cha12, p. 64]

F

"



Fi

1

3



T

H

17

er

49

h.

m

b

as
al

I

horizontal numerals are most frequently written \triangleright in the Early Dynastic period⁵³, such non-numeric usage is almost⁵⁴ always written \neg , for instance:

- in personal names in administrative texts, such as the following, which all contain \triangleright numerals:
 - 𒀭 in [P010424; P010458; P010459] from ED IIIa أبو صلابيخ,
 - 𒀭 in [P010960] from ED IIIa Šuruppag,
 - 𒀭 in [P251641] from ED IIIb Adab,
 - 𒀭 in [P252866] from ED IIIb Adab,
 - 𒀭 in [P298637] from ED IIIb Umma;
- in the Sumerian word 𒀭 — u₂-rum, “property” in ED IIIb Nirsu administrative texts which contain \triangleright numerals, such as [P020006; P020008; P020018; P020024; P020030];
- in lexical texts:
 - in the divine name 𒀭 — 𒀭 in the lexical texts [P010570; P010572], where the entries are prefixed with \triangleright .
 - in the word 𒀭 dili, “small fish” in [P010578], witness to Early Dynastic Fish,
 - in the same word with a determinative, 𒀭 dili^{ku₆}, in [P010586], witness to Early Dynastic Food, which starts with \triangleright numerals.

This is a clear contrast between 𐎧 and 𐎡 in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name 𐎧𐎡𐎴 occurs on its own line in the aforementioned administrative texts; a line 𐎡𐎡𐎴 would instead be read as “one slave”,

3.6 Limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in section 3.1. While these benefits are real and now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as — that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [KWU, pp. 123 sqq.] lists all of — – — together with — – — , while — , — , and — , and only those, appear at the beginning of the sign list, since they have non-numeric values⁵⁵. [PTACE, p. 58] has the numeric signs — , — , — , whereas non-numeric — is at the beginning of the sign list, where its values *aš* and *rum* are listed. For signs with both non-numeric and numeric usage, [LAK] writes *s. die Zahlz.* throughout the main list; LAK 1 — thus reappears at LAK 829 together with — , — , and — . One should note [MZL], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have — – — used in system $G_{\text{Ur III/OB}}$.

⁵³A CDLI search for "(asz@c)" finds 3296 ED texts, while a search for "(asz)" finds 81 ED texts, of which 46 also contain "(asz@c)".

⁵⁴Exceptions are discussed in section 3.7.1.

⁵⁵Non-numeric values of \rightarrow were discussed in section 3.5; \rightarrow has the values man_3 and min_5 , and is used for the word *didli*, ‘several, various’; $\rightarrow\rightarrow$ has the value $\text{e}\$6$.

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes⁵⁶ to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from \triangleright to \uparrow numerals prevents diachronic encoding even if \triangleright were unified with \leftarrow . For instance, the lexical list Early Dynastic Food, already mentioned in section 3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'–6']; however, they are written with \uparrow numerals, whereas they are written with \triangleright numerals in the Early Dynastic witnesses; since \uparrow and \leftarrow are distinct⁵⁷ characters, the \triangleright - \leftarrow unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods⁵⁸, there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Romach2023], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts⁵⁹ being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

3.7.1 The case of ŠAR₂

The character U+122B9 \bullet CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period⁶⁰, a contrast between non-numeric šar₂ written \diamond and numeric 1(šar₂) written \bullet can be observed, similar to the contrast between \leftarrow and \triangleright previously discussed in

⁵⁶TODO comment on the ED witnesses to the instructions of Šuruppag

⁵⁷Besides the contrasts in numeric usage mentioned in section 3.3.3, these characters are clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of \leftarrow which are not shared with \uparrow have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period, \leftarrow is used for the preposition *ina*, “in”, and \uparrow for the preposition *ana*, “to”.

⁵⁸TODO cite a few things here.

⁵⁹Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

⁶⁰For example, in personal names:

- $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$ in [P020019] from ED IIIb Nirsu;
- $\text{𒀭} \text{𒀭} \text{𒀭} \text{𒀭}$ in [P020182], also from ED IIIb Nirsu;
- $\text{𒀭} \text{𒀭} \text{𒀭}$ in [P222186] from ED IIIb Umma;
- $\text{𒀭} \text{𒀭} \text{𒀭}$ in [P235312] from Old Akkadian Umma.

section 3.5. However, in lexical lists from Šuruppag and Ebla⁶¹, as well as in the *Stèle des vautours*, non-numeric šar₂ is curviform:

- * 𒌦 𒌦 𒌦 𒌦 and * 𒌦 𒌦 𒌦 𒌦 in [P010566];
- 𒌦 𒌦 and * 𒌦 𒌦 in [P010576];
- 𒌦 𒌦 in [P240986]⁶²;
- 𒌦 𒌦 in [P222399, obv. 17, 9, 18, 11, 22, 12]⁶³.

It *would* be disruptive to the diachronic representation of text if non-numeric šar₂ were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph 𒌦, identical to the glyph of the proposed U+12579 𒌦 CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric šar₂ is curviform solidly predates the transition from 𒌦 to 𒌦 in the relevant metrological systems, there is no need to represent a 𒌦-𒌦 contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D 𒌦 CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

Note that in rare cases, such as [P222243] from ED IIIa Adab, non-numeric 𒌦 (here with the value rum) is written 𒌦. It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 𒌦 cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. In practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the CDLI transliteration (UR2-1(aš@c)) and the ePSD2 one (uru₈^{rum}) of this word disagree on that aspect. Since 𒌦 has a cuneiform reference glyph, this does not pose any compatibility concerns.

3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL₂ can be described as 𒌦 𒌦 𒌦 𒌦 NE×KASKAL or 𒌦 𒌦 𒌦 𒌦 NE×PAP⁶⁴, they are typically all transliterated bil₂, and therefore are all represented by the character U+1224B 𒌦 CUNEIFORM SIGN NE SHESHIG, its name notwithstanding, as described in [UTR56, §2.5].










The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, 𒌦 𒌦 𒌦 𒌦 from [P305639] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by


⁶¹TODO Mention other ways in which these are archaizing

⁶²From copy in [ELLes, No. 397].

⁶³Note however * 𒌦 𒌦 𒌦 𒌦 on [P222399, obv. 6, 17]. Curviform non-numeric šar₂ is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here. TODO add a photo.

⁶⁴As on [P249253].






would require any treatment of Early Dynastic administrative texts to use multiple cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted:  is a length of three ropes, but  is an area of three bur₃;  could be read as one  and one , where  would be one and a half ;  is a personal name, but  would be “one slave”.



At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millenium corpora. The oddity of  requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.

4 Rationale for ED-Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millenium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millenium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs KAŠa, KAŠb, and KAŠc, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Englund2001]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered script.

As part of the development of these conventions, a classification of fourth millenium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter *N* with a numeric subscript (sometimes with an additional alphabetic subscript): N_1 is , N_{14} is , N_{34} is , etc. Transliterations of numeric expression then use those to identify the type of number used, thus $5N_1$ is , and $5N_{14}$ is .

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where the exact meaning of the choice of numeral type is not fully understood, as the transliterations can otherwise force a dubious interpretation. For instance, the CDLI transliteration of  or  in [P283802] currently uses (barig@c) for the vertical numerals, suggesting a capacity measure; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millenium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I–II period, even though the Sumerian text uses classical assyriological transliteration conventions.

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millenium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones

described in section 3.7.2; they are indeed used interchangeably, as in TODO which uses the notation TODO in [ePSD2], but TODO in CDLI. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millenium would therefore induce confusion as to which numerals should be used in third millenium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millenium notation have systematically been encoded, as they have occasionally be found to carry distinct numeric meaning. For instance, $\boxtimes N_{30c}$ is listed as a variant of $\boxtimes N_{30a}$ in [DE87, p. 166], where the numeric value of either in relation to $\ominus N_{29a}$ is still unknown, but their values are found in [Englund2004] to be $\boxtimes = \frac{1}{10} \ominus$, whereas $\boxtimes = \frac{1}{6} \ominus$.

5 Considerations on individual numeral series

[TODO Document to the extent possible the metrological systems in which each sign is used. Note the disunification of N9 and N10 from 4(ban₂@c) and 5(ban₂@c).]

6 Characters not included in this proposal

6.1 The fractions $\frac{1}{3}$ and $\frac{2}{3}$

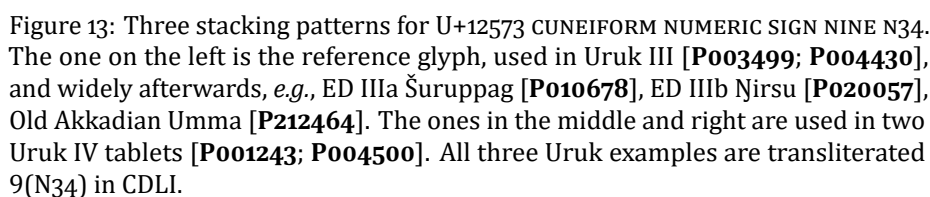
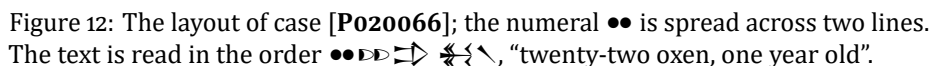
TODO ¶, ¶. Note the occasional omission of ¶, see citations in email to steve plus <https://cdli.mpiwg-berlin.mpg.de/artifacts/274845/reader/51537>.

6.2 Missing numerals

TODO N13 not attested in CDLI TODO (N_{17} not usefully numeric, $12N_{14}$ not encodable, etc.). 7 and 8 (diš *tenû*) encodable, but not today; want to go into the Cuneiform Numbers and Punctuation block for sanity.

6.3 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some *stacking patterns*; for instance 9𐎶 is encoded both as U+12446 𐎶 and as U+1240E 𐎶𐎵. This is in part due to contrastive usage of stacking patterns. For instance, besides 𐎶 and 𐎶𐎵 which are characteristic of bariga measures, four bariga is written 𐎶 even where 4𐎶 is written 𐎶𐎵, as in [P255010; P292843]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often 𐎶 𐎶𐎵 𐎶𐎵𐎶 𐎶𐎵𐎶𐎵 𐎶𐎵𐎶𐎵𐎶 𐎶𐎵𐎶𐎵𐎶𐎵 < << 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵, and results in metrological systems, typically 𐎶 𐎶𐎵 𐎶𐎵𐎶 𐎶𐎵𐎶𐎵 𐎶𐎵𐎶𐎵𐎶 𐎶𐎵𐎶𐎵𐎶𐎵 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵 < << 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵, occasionally co-occurring as in [P142827; P142357]. This separate encoding is also for compatibility with distinctions made in reference works and in some non-numeric transliterations; for instance, 𐎶 is [MZL, No. 860] and has the value limmu, and 𐎶𐎵 is [MZL, No. 852] and has the value limmu.



Numeric⁶⁸ transliterations occasionally distinguish the stacking patterns Ψ $\overline{\Psi}$ $\overline{\overline{\Psi}}$ $\overline{\overline{\overline{\Psi}}}$ \diamond \diamond , as in the CDLI transliterations of the aforementioned tablets, although this is rare; often 4(diš) is $\overline{\overline{\Psi}}$ in Ur III, but Ψ in the Neo-Assyrian period.

However, the stacking patterns from earlier periods are not separately encoded; for instance, in ED IIIb Nirsu, « 2(u) often has one < atop another. These older stacking patterns do not appear to be contrastive, are not marked in transliteration, and are not listed separately in sign lists nor assigned any different values. There is therefore no evidence of a need to encode them; instead, they should be considered style variants, and an ED IIIb Nirsu font should have an appropriate glyph for U+12399 « CUNEIFORM SIGN U U.

Likewise, many stacking patterns are attested for the curviform numerals proposed in this document, and it is not proposed to separately encode them; these distinctions would be incompatible with the state of the art in numeric transliterations, and are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier tablets, as they are structured in rectangular cases rather than lines, so that numerals may be laid out across the case in whichever way fits the available space; this is illustrated in Figure 12. Note that in that figure, the numerals need to be considerably enlarged in order to reproduce the layout of the tablet: \triangleright has the same height as $\triangleright\triangleright$. This is impractical when these numerals are set in text that contrasts them with the larger \bigtriangleright , and inconsistent with actual practice when typesetting these numerals, as illustrated in section 3.4.5: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dynastic period, but that are also attested in the Uruk period; the Uruk period also

⁶⁸The Sumerian word limmu means “four”, so limmu and limmu₅ are still numbers. The distinction here is between usage in transliterations of phrases such as 𒌦 𒈪 𒊩𒌆𒍪 lugal an-ub-da limmu₅-ba-ke₄ (king of the four quarters) or of names, and of numeric expressions such as 𒄠 𒅗 4(diš) sila₃.

frequently features numerals that use a more vertical layout, as illustrated in Figure 13. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millenium, usage in third millenium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents 𐎶, 𐎷, •, and ● must remain consistent across the numeral series, lest a 𐎷 numeral be confused with an 𐎶 numeral. Since the single indents are frequently used in running text, as illustrated in section 3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 13 uses OpenType stylistic alternates, and Figure 12 rotates the character ●, in both cases preserving the plain text backing.

6.4 Other lyph variants not reflected in transliteration

TODO Comment on the nameless variant glyphs from L2/23-190 and note that they are illustrating an even wider glyphic range as shown in [Englund2001].

Acknowledgements

TODO(egg): Something about the Vanséveren fonts

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ORACC: [epsd2/corpus/P020129](#).
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CDLI: [P102305](#).
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ORACC: [epsd2/corpus/P221266](#).
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CDLI: [P240531](#).
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CDLI: [P240654](#).
EbDA: [1383](#).
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