

# Archaic cuneiform numerals

Robin Leroy, Anshuman Pandey, and Steve Tinney

2024-09-13

## Contents

<b>1</b>	<b>Summary</b>	<b>2</b>
<b>2</b>	<b>Proposed changes to the Standard</b>	<b>3</b>
2.1	Core specification text . . . . .	3
2.2	Code charts . . . . .	5
2.3	Properties . . . . .	12
<b>3</b>	<b>Rationale for curviform–cuneiform disunification</b>	<b>12</b>
3.1	The cuneiform encoding model . . . . .	12
3.2	Arguments for curviform–cuneiform unification . . . . .	12
3.3	A primer on classic Ur III and Old Babylonian metrologies . . . . .	14
3.3.1	The discrete counting system . . . . .	14
3.3.2	The area system . . . . .	15
3.3.3	The capacity system . . . . .	16
3.3.4	The length system . . . . .	17
3.3.5	Fractions . . . . .	17
3.4	Curviform numerals in early metrologies . . . . .	17
3.4.1	Field lengths in Nirsu . . . . .	18
3.4.2	Dyke lengths in Nirsu . . . . .	19
3.4.3	Butter, cheese and wheat in Nirsu . . . . .	19
3.4.4	Grain in Ebla . . . . .	20
3.4.5	Use in modern publications . . . . .	22
3.5	Non-numeric usage . . . . .	29
3.6	The limited benefits of diachronic encoding for numerals . . . . .	29
3.7	Compatibility considerations . . . . .	31
3.7.1	The case of ŠAR <sub>2</sub> . . . . .	31
3.7.2	Transliteration . . . . .	32
3.8	Conclusions . . . . .	33
<b>4</b>	<b>Rationale for ED–Uruk numeral unification</b>	<b>34</b>
<b>5</b>	<b>Considerations on individual numeral series</b>	<b>35</b>

<b>6 Characters not included in this proposal</b>	<b>37</b>
6.1 Fourth millennium numerals . . . . .	37
6.2 Third millennium numerals . . . . .	38
6.3 Stacking patterns . . . . .	39
6.4 Other glyph variants not reflected in transliteration . . . . .	41
<b>Acknowledgements</b>	<b>41</b>
<b>References</b>	<b>41</b>
Artefacts . . . . .	41
ISO and Unicode documents . . . . .	47
Online corpora and related projects . . . . .	48
Other documents . . . . .	49

## 1 Summary

This document proposes encoding, at U+12550–U+12585, 310 numerals used in the fourth millennium (Uruk IV and Uruk III periods) and Early Dynastic period in conjunction with the Sumero-Akkadian cuneiform script<sup>1</sup> and the proto-cuneiform script<sup>2</sup>. The proposed characters are listed in §2. Most of them were listed in [L2/23-190]. The present document provides a more detailed rationale for their encoding and additional information about their identity and usage, both as part of the rationale and in §5. Some characters have been removed, in some cases because they are non-encodable variants, in others because their encodability should be considered as part of the proto-cuneiform proposal; these are discussed in §6. The glyphs have also been reworked, and additional characters used in the Early Dynastic period have been added.

The non-numeric signs of proto-cuneiform will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model requires an understanding of the text to correctly encode it.

However, the *numerals* used in proto-cuneiform should be unified with ones used in the Early Dynastic period, for the reasons set forth in §4. The proposed “curved”, or “curviform”, numerals<sup>3</sup> should however *not* be unified with the already-encoded cuneiform numerals<sup>4</sup>. Since the encoding proposals for the cuneiform script twenty years ago provisionally considered the curviform numerals to be

---

<sup>1</sup>[ISO15924]: Xsux, Script property value long name: Cuneiform; encoded since Unicode Version 5.0.  
<sup>2</sup>[ISO15924]: Pcus, not yet encoded.

<sup>3</sup>Impressed into clay using cylindrical stylus, held either perpendicular to the tablet, yielding • (small stylus) or ● (large stylus), or at a shallower angle: ▷, ◁ (small stylus), ▷ (large stylus). Some numerals are composed of multiple such impressions, e.g., ▷. The terms “curved”, “curviform”, “curvilinear”, and “round” can be found in the literature. We avoid the term “round” here as it has other meanings in the context of numbers. We use “curviform” in this document as, being the least common term, it is least likely to lead to confusion, and “CURVED” in the character names for consistency with documentation about the modifier @c used in machine readable ATF transliterations [[inlineATF](#)].

<sup>4</sup>Impressed into clay using a stylus with a trihedral end: — (stylus held horizontally), | (vertically), ↘ (diagonally) ↙ (with the head of the stylus), ┌ (stylus pressed deeper, forming a larger wedge), ┌ (combining ┌ and ↙), etc.

glyph variants of the cuneiform numerals, a detailed rationale is provided in §3, including compatibility considerations in §3.7.

The overall picture of unifications and disunifications over time is illustrated in table 1. The `Script_Extensions` property assignments in §2.3 reflect the overlap. Many of these numerals are also used in proto-Elamite<sup>5</sup> texts, where they are treated as identical characters in scholarship on proto-Elamite, so that they should be unified with the ones proposed in [L2/23-196]. However, in the interest of time, we do not provide a detailed rationale for this unification in this document, and we are not proposing that the numerals be given the corresponding `Script_Extensions` property value for now. Neither do we propose encoding any numerals that are solely attested in proto-Elamite texts, or well-attested in proto-Elamite texts but insufficiently attested in Uruk—those are discussed in §6.

	Uruk III & earlier	ED – Ur III	OB & later
Numerals	This proposal		
		Existing Xsux	
Non-numeric signs	Future Pcus		

Table 1: Usage of existing, proposed, and future characters across functions and time periods.

## **2 Proposed changes to the Standard**

## 2.1 Core specification text

Amend [Uni16, §11.1.2, sub “Cuneiform Numerals”], as follows:

**Cuneiform Numerals.** In general, numerals that also have a phonetic, logographic, or determinative value are encoded in the main Cuneiform block; as a result, some series of numerals, such as ፩-፭-፪ 1(diš)-9(diš) or ፻-፭-፪ 1(u)-9(u), are split across the two blocks. Numerals have been encoded separately from signs that are visually identical but semantically different etymologically unrelated (for example, U+1244F ፩ CUNEIFORM NUMERIC SIGN ONE BAN2, U+12450 ፪ CUNEIFORM NUMERIC SIGN TWO BAN2, and so on, versus U+12226 ፫ CUNEIFORM SIGN MASH, U+1227A ፬ CUNEIFORM SIGN PA, and so on).

The relation between series of numerals depends on the metrological system; for instance, when counting talents, written  (a unit of weight, approximately 30 kg),  is used for “one talent”, and  for “ten talents”. However, when measuring areas, the area  (one *burum*) is eighteen times  (one *ikum*, approximately 3600 m<sup>2</sup>). The Numeric\_Value property assignment of a cuneiform numeral therefore reflects only its relation to the first numeral in its series, rather than the absolute numeric value that it might represent. For instance, the number “fifty” is written , but U+12410  CUNEIFORM NUMERIC SIGN FIVE U has Numeric\_Value=5, as it is  $5 \times \text{$ .

<sup>5</sup>[ISO15924]: Pelm, not yet encoded.

In the third millennium, and especially in the Early Dynastic period, some numerals are written using a cylindrical tool, rather than the cuneiform stylus, forming curved rather than cuneiform numerals (☞ rather than ↗). The cuneiform numerals are descended from these curved numerals. However, in the Early Dynastic period, the curved numerals contrast with the cuneiform ones, and are used together with them in several metrological systems; they are therefore separately encoded. Most curved numerals are encoded in the Archaic Cuneiform Numerals block, with the exception of two fractions in the Cuneiform Numbers and Punctuation block: U+1245D ☞ CUNEIFORM NUMERIC SIGN ONE THIRD VARIANT FORM A and U+1245E ☚ CUNEIFORM NUMERIC SIGN TWO THIRDS VARIANT FORM A, the curved counterparts of U+1245A ☜ CUNEIFORM NUMERIC SIGN ONE THIRD DISH and U+1245B ☝ CUNEIFORM NUMERIC SIGN TWO THIRDS DISH.

Add after [Uni16, §11.1.3]:

#### 11.1.4 Archaic Cuneiform Numerals: U+12550–U+1268F

This block contains numerals used in the fourth millennium and third millennium. The numerals that are used in the fourth millennium and Early Dynastic I-II period (2900–2700 BCE) are named according to the conventions of the Berlin *Archaische Texte aus Uruk* (ATU) project, with names such as U+12550 ☞ CUNEIFORM NUMERIC SIGN ONE N01 or U+125B6 ☚ CUNEIFORM NUMERIC SIGN ONE N39A. For the signs that are also used in the third millennium, informative aliases provide correspondences to more common third millennium conventions, such as “1 as curved” for U+12550 ☞ CUNEIFORM NUMERIC SIGN ONE N01. The numerals that are only used starting in the Early Dynastic III period, where the ATU notation is not used, are named in the same fashion as the numerals of the Cuneiform Numbers and Punctuation block.

The curved numerals are produced using cylindrical tools of two different sizes, producing small curved indents (☞, ☚, and ☛), and large ones (☒, ☚, and ☝). These can be combined, as in U+12574 ☒ CUNEIFORM NUMERIC SIGN ONE N48, U+12582 ☚ CUNEIFORM NUMERIC SIGN ONE N50, or U+125A3 ☝ CUNEIFORM NUMERIC SIGN ONE N54. Consistent sizing is important to identifying these characters, as there is no visual distinction other than size between, for instance, U+12566 ☚ CUNEIFORM NUMERIC SIGN FIVE N14 and U+1257D ☚ CUNEIFORM NUMERIC SIGN FIVE N45. The reference glyphs of some of the larger signs have been resized to fit in the code charts cells, but fonts for these characters should retain consistent size across the numeral series.

*Editor's note: I have not yet done that resizing. The dashed-box convention for wide dashes, see [Uni16, §24.1.2, sub “Dashed Box Convention”] should probably not be extended to these, since numbers enclosed in a real dashed box are a thing in proto-Elamite texts.*

The Numeric\_Value assignments follow the same principles as in the Cuneiform Numbers and Punctuation block. Numerals used in the third millennium have the Cuneiform script property value; numerals

used only in the fourth millennium have the Proto-Cuneiform script property value. Numerals used in both the fourth and third millennium have both scripts in their Script\_Extensions values.

**The sign ŠAR₂.** When used logographically, the sign ŠAR₂ has the same (cuneiform) appearance as U+1212D ◇ CUNEIFORM SIGN HI in all but the most archaizing Early Dynastic texts. The character U+122B9 CUNEIFORM SIGN SHAR2 should be used for logographic šar₂, whether cuneiform or curved. Most period-specific fonts will have the same cuneiform glyph for U+122B9 and U+1212D. In the Early Dynastic period, numeric 1 šar₂ is typically written with a curved glyph, contrasting with logographic šar₂. U+12579 ● CUNEIFORM NUMERIC SIGN ONE N45 should be used for curved 1 šar₂. In later periods, long after ŠAR₂ and HI have merged, even numeric 1 šar₂ has a cuneiform glyph. U+122B9 CUNEIFORM SIGN SHAR2 should be used for cuneiform 1 šar₂.

The reference glyph for U+122B9 ● CUNEIFORM SIGN SHAR2 is curved, reflecting the rarer and more archaic practice, instead of cuneiform as it would be in the Ur III period, so as to distinguish it from U+1212D ◇ CUNEIFORM SIGN HI.

## 2.2 Code charts

The code charts for the proposed block, including the character names list with proposed informative aliases, cross references, and informative notes, are shown on the following pages.

This space for rent.

	1255	1256	1257	1258	1259	125A	125B	125C	125D	125E
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										
A										
B										
C										
D										
E										
F										

	125F	1260	1261	1262	1263	1264	1265	1266	1267	1268
0	125F0	12600	12610	12620	12630	12640	12650	12660	12670	12680
1	125F1	12601	12611	12621	12631	12641	12651	12661	12671	12681
2	125F2	12602	12612	12622	12632	12642	12652	12662	12672	12682
3	125F3	12603	12613	12623	12633	12643	12653	12663	12673	12683
4	125F4	12604	12614	12624	12634	12644	12654	12664	12674	12684
5	125F5	12605	12615	12625	12635	12645	12655	12665	12675	12685
6	125F6	12606	12616	12626	12636	12646	12656	12666	12676	
7	125F7	12607	12617	12627	12637	12647	12657	12667	12677	
8	125F8	12608	12618	12628	12638	12648	12658	12668	12678	
9	125F9	12609	12619	12629	12639	12649	12659	12669	12679	
A	125FA	1260A	1261A	1262A	1263A	1264A	1265A	1266A	1267A	
B	125FB	1260B	1261B	1262B	1263B	1264B	1265B	1266B	1267B	
C	125FC	1260C	1261C	1262C	1263C	1264C	1265C	1266C	1267C	
D	125FD	1260D	1261D	1262D	1263D	1264D	1265D	1266D	1267D	
E	125FE	1260E	1261E	1262E	1263E	1264E	1265E	1266E	1267E	
F	125FF	1260F	1261F	1262F	1263F	1264F	1265F	1266F	1267F	

Many of the reference glyphs for the higher numbers (THREE and above) have been rescaled to fit the code chart cells. They should be sized consistently with the corresponding ONE numerals.

## Common Numerals

Used in the sexagesimal discrete counting system and other metrological systems

- 12550 CUNEIFORM NUMERIC SIGN ONE N01  
= 1 aš curved  
→ 12038 ← cuneiform sign ash  
• often used instead of diš in Early Dynastic counterparts of cuneiform metrological systems  
→ 12079 T cuneiform sign dish
- 12551 CUNEIFORM NUMERIC SIGN TWO N01  
→ 12400 ↔ cuneiform numeric sign two ash
- 12552 CUNEIFORM NUMERIC SIGN THREE N01
- 12553 CUNEIFORM NUMERIC SIGN FOUR N01
- 12554 CUNEIFORM NUMERIC SIGN FIVE N01
- 12555 CUNEIFORM NUMERIC SIGN SIX N01
- 12556 CUNEIFORM NUMERIC SIGN SEVEN N01
- 12557 CUNEIFORM NUMERIC SIGN EIGHT N01
- 12558 CUNEIFORM NUMERIC SIGN NINE N01
- 12559 CUNEIFORM NUMERIC SIGN ONE N08  
= 1 diš curved  
→ 12079 T cuneiform sign dish  
= 1/2 iku curved  
• used for one half in multiple metrological systems  
→ 12039 ↘ cuneiform sign ash zida tenu  
→ 12226 † cuneiform sign mash  
= 1 bariga curved  
• used in Early Dynastic capacity systems
- 1255A CUNEIFORM NUMERIC SIGN TWO N08  
→ 1222B ¶ cuneiform sign min  
= 2 bariga curved  
→ 12456 ‡ cuneiform numeric sign nigidamin
- 1255B CUNEIFORM NUMERIC SIGN THREE N08  
→ 12408 III cuneiform numeric sign three dish  
• used in Early Dynastic capacity systems  
= 3 bariga curved  
→ 12457 ¶ cuneiform numeric sign nigidaesh
- 1255C CUNEIFORM NUMERIC SIGN FOUR N08
- 1255D CUNEIFORM NUMERIC SIGN FIVE N08
- 1255E CUNEIFORM NUMERIC SIGN SIX N08
- 1255F CUNEIFORM NUMERIC SIGN SEVEN N08
- 12560 CUNEIFORM NUMERIC SIGN EIGHT N08
- 12561 CUNEIFORM NUMERIC SIGN NINE N08
- 12562 CUNEIFORM NUMERIC SIGN ONE N14  
= 1 u curved  
= 1 bur<sub>3</sub> curved  
→ 1230B ↙ cuneiform sign u
- 12563 CUNEIFORM NUMERIC SIGN TWO N14  
→ 12399 ≪ cuneiform sign u u
- 12564 CUNEIFORM NUMERIC SIGN THREE N14  
→ 1230D ≪ cuneiform sign u u u
- 12565 CUNEIFORM NUMERIC SIGN FOUR N14  
→ 1240F ≪ cuneiform numeric sign four u
- 12566 CUNEIFORM NUMERIC SIGN FIVE N14
- 12567 CUNEIFORM NUMERIC SIGN SIX N14
- 12568 CUNEIFORM NUMERIC SIGN SEVEN N14
- 12569 CUNEIFORM NUMERIC SIGN EIGHT N14
- 1256A CUNEIFORM NUMERIC SIGN NINE N14
- 1256B CUNEIFORM NUMERIC SIGN ONE N34  
= 1 neš<sub>2</sub> curved  
→ 12415 Y cuneiform numeric sign one gesh2

- 1256C CUNEIFORM NUMERIC SIGN TWO N34
- 1256D CUNEIFORM NUMERIC SIGN THREE N34
- 1256E CUNEIFORM NUMERIC SIGN FOUR N34
- 1256F CUNEIFORM NUMERIC SIGN FIVE N34
- 12570 CUNEIFORM NUMERIC SIGN SIX N34
- 12571 CUNEIFORM NUMERIC SIGN SEVEN N34
- 12572 CUNEIFORM NUMERIC SIGN EIGHT N34
- 12573 CUNEIFORM NUMERIC SIGN NINE N34
- 12574 CUNEIFORM NUMERIC SIGN ONE N48  
= 1 neš<sub>2</sub> curved  
→ 1241E ¶ cuneiform numeric sign one geshu
- 12575 CUNEIFORM NUMERIC SIGN TWO N48
- 12576 CUNEIFORM NUMERIC SIGN THREE N48
- 12577 CUNEIFORM NUMERIC SIGN FOUR N48
- 12578 CUNEIFORM NUMERIC SIGN FIVE N48
- 12579 CUNEIFORM NUMERIC SIGN ONE N45  
= 1 šar<sub>2</sub> curved  
• 122B9 ● should be used for cuneiform 1 šar<sub>2</sub>  
• 122B9 ● should be used for logographic šar<sub>2</sub>, even when curved  
→ 122B9 ● cuneiform sign shar2
- 1257A CUNEIFORM NUMERIC SIGN TWO N45
- 1257B CUNEIFORM NUMERIC SIGN THREE N45
- 1257C CUNEIFORM NUMERIC SIGN FOUR N45
- 1257D CUNEIFORM NUMERIC SIGN FIVE N45
- 1257E CUNEIFORM NUMERIC SIGN SIX N45
- 1257F CUNEIFORM NUMERIC SIGN SEVEN N45
- 12580 CUNEIFORM NUMERIC SIGN EIGHT N45
- 12581 CUNEIFORM NUMERIC SIGN NINE N45
- 12582 CUNEIFORM NUMERIC SIGN ONE N50  
= 1 šar<sub>2</sub> curved  
→ 1242C ☰ cuneiform numeric sign one sharu  
• used instead of 1258E ● in fourth millennium land area systems  
→ 12434 § cuneiform numeric sign one buru
- 12583 CUNEIFORM NUMERIC SIGN TWO N50
- 12584 CUNEIFORM NUMERIC SIGN THREE N50
- 12585 CUNEIFORM NUMERIC SIGN FOUR N50
- 12586 CUNEIFORM NUMERIC SIGN FIVE N50

## Numerals used for land areas

Together with N08, N01, N14, N45, and N50

- 12587 CUNEIFORM NUMERIC SIGN ONE EIGHTH IKU CURVED  
→ 1245F ↗ cuneiform numeric sign one eighth ash
- 12588 CUNEIFORM NUMERIC SIGN ONE EIGHTH IKU CURVED VARIANT FORM
- 12589 CUNEIFORM NUMERIC SIGN ONE N01 REVERSED  
= 1/4 iku curved  
→ 12460 ↗ cuneiform numeric sign one quarter ash
- 1258A CUNEIFORM NUMERIC SIGN ONE QUARTER IKU CURVED VARIANT FORM
- 1258B CUNEIFORM NUMERIC SIGN ONE HALF IKU CURVED VARIANT FORM  
→ 12039 ↘ cuneiform sign ash zida tenu
- 1258C CUNEIFORM NUMERIC SIGN ONE N22  
= 1 eše<sub>3</sub> curved  
→ 12458 ↙ cuneiform numeric sign one eshe3
- 1258D CUNEIFORM NUMERIC SIGN TWO N22
- 1258E CUNEIFORM NUMERIC SIGN ONE BURU CURVED  
→ 12434 § cuneiform numeric sign one buru
- 1258F CUNEIFORM NUMERIC SIGN TWO BURU CURVED
- 12590 CUNEIFORM NUMERIC SIGN THREE BURU CURVED
- 12591 CUNEIFORM NUMERIC SIGN FOUR BURU CURVED
- 12592 CUNEIFORM NUMERIC SIGN FIVE BURU CURVED

## Early Dynastic capacity measures

- 12593 ፩ CUNEIFORM NUMERIC SIGN ONE BAN2 CURVED  
 • 1244F ፪ cuneiform numeric sign one ban2  
 = 1/2 aš curved  
 • used for one half in multiple metrological systems  
 → 12226 ፫ cuneiform sign mash
- 12594 ፪ CUNEIFORM NUMERIC SIGN TWO BAN2 CURVED  
 12595 ፫ CUNEIFORM NUMERIC SIGN THREE BAN2 CURVED  
 12596 ፬ CUNEIFORM NUMERIC SIGN FOUR BAN2 CURVED  
 12597 ፭ CUNEIFORM NUMERIC SIGN FIVE BAN2 CURVED  
 12598 ፮ CUNEIFORM NUMERIC SIGN NINDA2 TIMES SHE PLUS ONE ASH CURVED  
 = 1/3 aš curved variant form  
 → 1245D ፯ cuneiform numeric sign one third dish variant form a  
 → 1245A ፻ cuneiform numeric sign one third dish  
 12599 ፯ CUNEIFORM NUMERIC SIGN NINDA2 TIMES SHE PLUS TWO ASH CURVED  
 = 2/3 aš curved variant form  
 → 1245E ፰ cuneiform numeric sign two thirds dish variant form a  
 → 1245B ፼ cuneiform numeric sign two thirds dish

## Numerals used in the bisexagesimal system

Together with N08, N01, N14, and N34

- 1259A ፩ CUNEIFORM NUMERIC SIGN ONE N51  
 = 1 ḫeš<sub>2</sub> curved doubled, 1 ḫešmin curved
- 1259B ፪ CUNEIFORM NUMERIC SIGN TWO N51  
 1259C ፫ CUNEIFORM NUMERIC SIGN THREE N51  
 1259D ፬ CUNEIFORM NUMERIC SIGN FOUR N51  
 1259E ፭ CUNEIFORM NUMERIC SIGN FIVE N51  
 1259F ፮ CUNEIFORM NUMERIC SIGN SIX N51  
 125A0 ፯ CUNEIFORM NUMERIC SIGN SEVEN N51  
 125A1 ፰ CUNEIFORM NUMERIC SIGN EIGHT N51  
 125A2 ፱ CUNEIFORM NUMERIC SIGN NINE N51  
 125A3 ፲ CUNEIFORM NUMERIC SIGN ONE N54  
 = 1 ḫeš'ū curved doubled, 1 ḫešmin'ū curved
- 125A4 ፳ CUNEIFORM NUMERIC SIGN TWO N54  
 125A5 ፴ CUNEIFORM NUMERIC SIGN THREE N54  
 125A6 ፵ CUNEIFORM NUMERIC SIGN FOUR N54  
 125A7 ፶ CUNEIFORM NUMERIC SIGN FIVE N54  
 125A8 ፷ CUNEIFORM NUMERIC SIGN ONE N56  
 125A9 ፸ CUNEIFORM NUMERIC SIGN TWO N56

## Fourth millennium capacity measures

- 125AA ፩ CUNEIFORM NUMERIC SIGN ONE N24  
 125AB ፪ CUNEIFORM NUMERIC SIGN ONE N26  
 125AC ፫ CUNEIFORM NUMERIC SIGN ONE N28  
 125AD ፬ CUNEIFORM NUMERIC SIGN ONE N29A  
 125AE ፭ CUNEIFORM NUMERIC SIGN ONE N29B  
 125AF ፮ CUNEIFORM NUMERIC SIGN ONE N30A  
 125B0 ፯ CUNEIFORM NUMERIC SIGN ONE N30C  
 125B1 ፰ CUNEIFORM NUMERIC SIGN ONE N30D  
 125B2 ፱ CUNEIFORM NUMERIC SIGN ONE N30E  
 125B3 ፲ CUNEIFORM NUMERIC SIGN ONE N31  
 125B4 ፳ CUNEIFORM NUMERIC SIGN ONE N32  
 125B5 ፴ CUNEIFORM NUMERIC SIGN ONE N33  
 125B6 ፵ CUNEIFORM NUMERIC SIGN ONE N39A  
 125B7 ፶ CUNEIFORM NUMERIC SIGN TWO N39A  
 125B8 ፷ CUNEIFORM NUMERIC SIGN THREE N39A  
 125B9 ፸ CUNEIFORM NUMERIC SIGN FOUR N39A  
 125BA ፹ CUNEIFORM NUMERIC SIGN ONE N39B  
 125BB ፺ CUNEIFORM NUMERIC SIGN TWO N39B  
 125BC ፻ CUNEIFORM NUMERIC SIGN THREE N39B  
 125BD ፼ CUNEIFORM NUMERIC SIGN FOUR N39B

## Numerals of sexagesimal system S'

- Used to count dead animals and jars of certain types of liquids*
- 125BE ፩ CUNEIFORM NUMERIC SIGN ONE N02  
 125BF ፪ CUNEIFORM NUMERIC SIGN TWO N02  
 125C0 ፫ CUNEIFORM NUMERIC SIGN THREE N02  
 125C1 ፬ CUNEIFORM NUMERIC SIGN FOUR N02  
 125C2 ፭ CUNEIFORM NUMERIC SIGN FIVE N02  
 125C3 ፮ CUNEIFORM NUMERIC SIGN SIX N02  
 125C4 ፯ CUNEIFORM NUMERIC SIGN SEVEN N02  
 125C5 ፰ CUNEIFORM NUMERIC SIGN EIGHT N02  
 125C6 ፱ CUNEIFORM NUMERIC SIGN NINE N02  
 125C7 ፲ CUNEIFORM NUMERIC SIGN ONE N15  
 125C8 ፳ CUNEIFORM NUMERIC SIGN TWO N15  
 125C9 ፴ CUNEIFORM NUMERIC SIGN THREE N15  
 125CA ፵ CUNEIFORM NUMERIC SIGN FOUR N15  
 125CB ፶ CUNEIFORM NUMERIC SIGN FIVE N15  
 125CC ፷ CUNEIFORM NUMERIC SIGN ONE N35  
 125CD ፸ CUNEIFORM NUMERIC SIGN TWO N35  
 125CE ፹ CUNEIFORM NUMERIC SIGN THREE N35  
 125CF ፺ CUNEIFORM NUMERIC SIGN FOUR N35  
 125D0 ፻ CUNEIFORM NUMERIC SIGN FIVE N35

## Numerals of bisexagesimal system B\*

*Used in the fourth millennium to count ratios of an unclear nature*

- 125D1 ፩ CUNEIFORM NUMERIC SIGN ONE N06  
 125D2 ፪ CUNEIFORM NUMERIC SIGN TWO N06  
 125D3 ፫ CUNEIFORM NUMERIC SIGN THREE N06  
 125D4 ፬ CUNEIFORM NUMERIC SIGN FOUR N06  
 125D5 ፭ CUNEIFORM NUMERIC SIGN FIVE N06  
 125D6 ፮ CUNEIFORM NUMERIC SIGN SIX N06  
 125D7 ፯ CUNEIFORM NUMERIC SIGN SEVEN N06  
 125D8 ፰ CUNEIFORM NUMERIC SIGN EIGHT N06  
 125D9 ፱ CUNEIFORM NUMERIC SIGN NINE N06  
 125DA ፲ CUNEIFORM NUMERIC SIGN ONE N21  
 125DB ፳ CUNEIFORM NUMERIC SIGN TWO N21  
 125DC ፴ CUNEIFORM NUMERIC SIGN THREE N21  
 125DD ፵ CUNEIFORM NUMERIC SIGN FOUR N21  
 125DE ፶ CUNEIFORM NUMERIC SIGN FIVE N21  
 125DF ፷ CUNEIFORM NUMERIC SIGN ONE N38  
 125E0 ፸ CUNEIFORM NUMERIC SIGN ONE N52  
 125E1 ፹ CUNEIFORM NUMERIC SIGN TWO N52  
 125E2 ፺ CUNEIFORM NUMERIC SIGN THREE N52  
 125E3 ፻ CUNEIFORM NUMERIC SIGN FOUR N52  
 125E4 ፼ CUNEIFORM NUMERIC SIGN FIVE N52  
 125E5 ፻ CUNEIFORM NUMERIC SIGN SIX N52  
 125E6 ፽ CUNEIFORM NUMERIC SIGN SEVEN N52  
 125E7 ፾ CUNEIFORM NUMERIC SIGN EIGHT N52  
 125E8 ፷ CUNEIFORM NUMERIC SIGN NINE N52  
 125E9 ፸ CUNEIFORM NUMERIC SIGN ONE N60

## Numerals of capacity system S'

*Used in the fourth millennium to measure malted barley*

- 125EA ፩ CUNEIFORM NUMERIC SIGN ONE N24A  
 125EB ፪ CUNEIFORM NUMERIC SIGN ONE N40  
 125EC ፫ CUNEIFORM NUMERIC SIGN TWO N40  
 125ED ፬ CUNEIFORM NUMERIC SIGN THREE N40  
 125EE ፭ CUNEIFORM NUMERIC SIGN FOUR N40  
 125EF ፮ CUNEIFORM NUMERIC SIGN ONE N03  
 125F0 ፯ CUNEIFORM NUMERIC SIGN TWO N03  
 125F1 ፰ CUNEIFORM NUMERIC SIGN THREE N03  
 125F2 ፱ CUNEIFORM NUMERIC SIGN FOUR N03  
 125F3 ፲ CUNEIFORM NUMERIC SIGN FIVE N03  
 125F4 ፳ CUNEIFORM NUMERIC SIGN ONE N18  
 125F5 ፴ CUNEIFORM NUMERIC SIGN TWO N18  
 125F6 ፵ CUNEIFORM NUMERIC SIGN THREE N18  
 125F7 ፶ CUNEIFORM NUMERIC SIGN FOUR N18  
 125F8 ፷ CUNEIFORM NUMERIC SIGN FIVE N18  
 125F9 ፸ CUNEIFORM NUMERIC SIGN SIX N18

125FA CUNEIFORM NUMERIC SIGN SEVEN N18  
 125FB CUNEIFORM NUMERIC SIGN EIGHT N18  
 125FC CUNEIFORM NUMERIC SIGN NINE N18

## Numerals of capacity system Š"

*Used in the fourth millennium to measure various kinds of emmer*

125FD CUNEIFORM NUMERIC SIGN ONE N24B  
 125FE CUNEIFORM NUMERIC SIGN ONE N26B  
 125FF CUNEIFORM NUMERIC SIGN ONE N28B  
 12600 CUNEIFORM NUMERIC SIGN ONE N29AB  
 12601 CUNEIFORM NUMERIC SIGN ONE N41  
 12602 CUNEIFORM NUMERIC SIGN TWO N41  
 12603 CUNEIFORM NUMERIC SIGN THREE N41  
 12604 CUNEIFORM NUMERIC SIGN FOUR N41  
 12605 CUNEIFORM NUMERIC SIGN ONE N04  
 12606 CUNEIFORM NUMERIC SIGN TWO N04  
 12607 CUNEIFORM NUMERIC SIGN THREE N04  
 12608 CUNEIFORM NUMERIC SIGN FOUR N04  
 12609 CUNEIFORM NUMERIC SIGN FIVE N04  
 1260A CUNEIFORM NUMERIC SIGN ONE N19  
 1260B CUNEIFORM NUMERIC SIGN TWO N19  
 1260C CUNEIFORM NUMERIC SIGN THREE N19  
 1260D CUNEIFORM NUMERIC SIGN FOUR N19  
 1260E CUNEIFORM NUMERIC SIGN FIVE N19  
 1260F CUNEIFORM NUMERIC SIGN SIX N19  
 12610 CUNEIFORM NUMERIC SIGN SEVEN N19  
 12611 CUNEIFORM NUMERIC SIGN EIGHT N19  
 12612 CUNEIFORM NUMERIC SIGN NINE N19  
 12613 CUNEIFORM NUMERIC SIGN ONE N46  
 12614 CUNEIFORM NUMERIC SIGN TWO N46  
 12615 CUNEIFORM NUMERIC SIGN ONE N36  
 12616 CUNEIFORM NUMERIC SIGN TWO N36  
 12617 CUNEIFORM NUMERIC SIGN THREE N36  
 12618 CUNEIFORM NUMERIC SIGN FOUR N36  
 12619 CUNEIFORM NUMERIC SIGN FIVE N36  
 1261A CUNEIFORM NUMERIC SIGN SIX N36  
 1261B CUNEIFORM NUMERIC SIGN SEVEN N36  
 1261C CUNEIFORM NUMERIC SIGN EIGHT N36  
 1261D CUNEIFORM NUMERIC SIGN NINE N36  
 1261E CUNEIFORM NUMERIC SIGN ONE N49  
 1261F CUNEIFORM NUMERIC SIGN TWO N49  
 12620 CUNEIFORM NUMERIC SIGN THREE N49  
 12621 CUNEIFORM NUMERIC SIGN FOUR N49

## Numerals of capacity system Š\*

*Used in the fourth millennium to measure barley groats*

12622 CUNEIFORM NUMERIC SIGN ONE N25  
 12623 CUNEIFORM NUMERIC SIGN ONE N27  
 12624 CUNEIFORM NUMERIC SIGN ONE N28C  
 12625 CUNEIFORM NUMERIC SIGN ONE N29AC  
 12626 CUNEIFORM NUMERIC SIGN ONE N30AC  
 12627 CUNEIFORM NUMERIC SIGN ONE N30CC  
 12628 CUNEIFORM NUMERIC SIGN ONE N42A  
 12629 CUNEIFORM NUMERIC SIGN TWO N42A  
 1262A CUNEIFORM NUMERIC SIGN THREE N42A  
 1262B CUNEIFORM NUMERIC SIGN FOUR N42A  
 1262C CUNEIFORM NUMERIC SIGN ONE N42B  
 1262D CUNEIFORM NUMERIC SIGN TWO N42B  
 1262E CUNEIFORM NUMERIC SIGN THREE N42B  
 1262F CUNEIFORM NUMERIC SIGN FOUR N42B  
 12630 CUNEIFORM NUMERIC SIGN ONE N05  
 12631 CUNEIFORM NUMERIC SIGN TWO N05  
 12632 CUNEIFORM NUMERIC SIGN THREE N05  
 12633 CUNEIFORM NUMERIC SIGN FOUR N05  
 12634 CUNEIFORM NUMERIC SIGN FIVE N05  
 12635 CUNEIFORM NUMERIC SIGN ONE N20  
 12636 CUNEIFORM NUMERIC SIGN TWO N20  
 12637 CUNEIFORM NUMERIC SIGN THREE N20  
 12638 CUNEIFORM NUMERIC SIGN FOUR N20

12639 CUNEIFORM NUMERIC SIGN FIVE N20  
 1263A CUNEIFORM NUMERIC SIGN SIX N20  
 1263B CUNEIFORM NUMERIC SIGN SEVEN N20  
 1263C CUNEIFORM NUMERIC SIGN EIGHT N20  
 1263D CUNEIFORM NUMERIC SIGN NINE N20  
 1263E CUNEIFORM NUMERIC SIGN ONE N47  
 1263F CUNEIFORM NUMERIC SIGN TWO N47  
 12640 CUNEIFORM NUMERIC SIGN ONE N37  
 12641 CUNEIFORM NUMERIC SIGN TWO N37

## Numerals of system EN

*Only attested in the Uruk IV period*

12642 CUNEIFORM NUMERIC SIGN ONE N09  
 12643 CUNEIFORM NUMERIC SIGN ONE N11  
 12644 CUNEIFORM NUMERIC SIGN ONE N12  
 12645 CUNEIFORM NUMERIC SIGN ONE N07A  
 12646 CUNEIFORM NUMERIC SIGN TWO N07A  
 12647 CUNEIFORM NUMERIC SIGN THREE N07A  
 12648 CUNEIFORM NUMERIC SIGN ONE N07B  
 12649 CUNEIFORM NUMERIC SIGN TWO N07B  
 1264A CUNEIFORM NUMERIC SIGN THREE N07B

## Flat numerals

*Rectangular numerals impressed with a flat tool, used in Ur in the Early Dynastic I-II period*

1264B CUNEIFORM NUMERIC SIGN ONE N01 FLAT  
 = 1 aš flat  
 → 12038 ← cuneiform sign ash  
 1264C CUNEIFORM NUMERIC SIGN TWO N01 FLAT  
 1264D CUNEIFORM NUMERIC SIGN THREE N01 FLAT  
 1264E CUNEIFORM NUMERIC SIGN FOUR N01 FLAT  
 1264F CUNEIFORM NUMERIC SIGN FIVE N01 FLAT  
 12650 CUNEIFORM NUMERIC SIGN SIX N01 FLAT  
 12651 CUNEIFORM NUMERIC SIGN SEVEN N01 FLAT  
 12652 CUNEIFORM NUMERIC SIGN EIGHT N01 FLAT  
 12653 CUNEIFORM NUMERIC SIGN NINE N01 FLAT  
 12654 CUNEIFORM NUMERIC SIGN ONE N08 FLAT  
 12655 CUNEIFORM NUMERIC SIGN ONE N14 FLAT  
 = 1 u flat  
 → 1230B ↖ cuneiform sign u  
 12656 CUNEIFORM NUMERIC SIGN TWO N14 FLAT  
 12657 CUNEIFORM NUMERIC SIGN THREE N14 FLAT  
 12658 CUNEIFORM NUMERIC SIGN FOUR N14 FLAT  
 12659 CUNEIFORM NUMERIC SIGN FIVE N14 FLAT  
 1265A CUNEIFORM NUMERIC SIGN SIX N14 FLAT  
 1265B CUNEIFORM NUMERIC SIGN SEVEN N14 FLAT  
 1265C CUNEIFORM NUMERIC SIGN EIGHT N14 FLAT  
 1265D CUNEIFORM NUMERIC SIGN NINE N14 FLAT  
 1265E CUNEIFORM NUMERIC SIGN ONE N34 FLAT  
 1265F CUNEIFORM NUMERIC SIGN TWO N34 FLAT  
 12660 CUNEIFORM NUMERIC SIGN THREE N34 FLAT  
 12661 CUNEIFORM NUMERIC SIGN FOUR N34 FLAT  
 12662 CUNEIFORM NUMERIC SIGN FIVE N34 FLAT  
 12663 CUNEIFORM NUMERIC SIGN SIX N34 FLAT  
 12664 CUNEIFORM NUMERIC SIGN SEVEN N34 FLAT  
 12665 CUNEIFORM NUMERIC SIGN EIGHT N34 FLAT  
 12666 CUNEIFORM NUMERIC SIGN NINE N34 FLAT  
 12667 CUNEIFORM NUMERIC SIGN ONE N45 FLAT  
 12668 CUNEIFORM NUMERIC SIGN TWO N45 FLAT  
 12669 CUNEIFORM NUMERIC SIGN ONE N22 FLAT  
 1266A CUNEIFORM NUMERIC SIGN TWO N22 FLAT  
 1266B CUNEIFORM NUMERIC SIGN ONE N51 FLAT  
 1266C CUNEIFORM NUMERIC SIGN TWO N51 FLAT  
 1266D CUNEIFORM NUMERIC SIGN THREE N51 FLAT  
 1266E CUNEIFORM NUMERIC SIGN FOUR N51 FLAT  
 1266F CUNEIFORM NUMERIC SIGN FIVE N51 FLAT  
 12670 CUNEIFORM NUMERIC SIGN SIX N51 FLAT  
 12671 CUNEIFORM NUMERIC SIGN SEVEN N51 FLAT  
 12672 CUNEIFORM NUMERIC SIGN EIGHT N51 FLAT

12673	 CUNEIFORM NUMERIC SIGN NINE N51 FLAT
12674	 CUNEIFORM NUMERIC SIGN ONE N34 FLAT TENU = 1 n39a flat
12675	 CUNEIFORM NUMERIC SIGN ONE N04 FLAT
12676	 CUNEIFORM NUMERIC SIGN TWO N04 FLAT
12677	 CUNEIFORM NUMERIC SIGN THREE N04 FLAT
12678	 CUNEIFORM NUMERIC SIGN FOUR N04 FLAT
12679	 CUNEIFORM NUMERIC SIGN FIVE N04 FLAT
1267A	 CUNEIFORM NUMERIC SIGN ONE N19 FLAT
1267B	 CUNEIFORM NUMERIC SIGN TWO N19 FLAT
1267C	 CUNEIFORM NUMERIC SIGN THREE N19 FLAT
1267D	 CUNEIFORM NUMERIC SIGN FOUR N19 FLAT
1267E	 CUNEIFORM NUMERIC SIGN FIVE N19 FLAT
1267F	 CUNEIFORM NUMERIC SIGN SIX N19 FLAT
12680	 CUNEIFORM NUMERIC SIGN SEVEN N19 FLAT
12681	 CUNEIFORM NUMERIC SIGN EIGHT N19 FLAT
12682	 CUNEIFORM NUMERIC SIGN NINE N19 FLAT
12683	 CUNEIFORM NUMERIC SIGN ONE N46 FLAT
12684	 CUNEIFORM NUMERIC SIGN TWO N46 FLAT
12685	 CUNEIFORM NUMERIC SIGN ONE N36 FLAT

## 2.3 Properties

# 3 Rationale for curviform–cuneiform disunification

The numbering systems that use cuneiform numerals are descended from the ones that use curviform numerals, and many of the cuneiform signs have clear curviform counterparts across this transition. Co-occurrences are sometimes described by analogy to distinctions that are not the realm of plain text, as in [Pow72, p. 215] “in the same fashion as we use black and red ink”; however, we must bear in mind that such analogies are not made in the context of character encoding discussions. In 2004, the curviform numerals were deemed unencodable for the time being; however, closer inspection reveals that the distinction functions less like markup than was argued at the time, and that the unification is problematic.

## 3.1 The cuneiform encoding model

As outlined in, *e.g.*, [UTR56], the cuneiform encoding model is diachronic; each character may have wildly different glyphs depending on time period and region. For instance, the sign IM may resemble 𒂗 in texts from Early Dynastic IIIa Šuruppag as in the character code charts, 𒂗 later in the third millennium<sup>6</sup>, 𒀭 in Old Babylonian cursive, 𒀭 in Neo-Assyrian, but is always encoded as U+1214E CUNEIFORM SIGN IM.

This encoding model allows for the interoperable representation of editions of diachronic reference works such as sign lists<sup>7</sup> and dictionaries<sup>8</sup>, and of composite texts<sup>9</sup>. By being compatible with similarly diachronic transliteration practice, *i.e.*, by avoiding distinctions finer than those made in transliteration, the encoding model also allows for automated conversion of transliterated corpora to cuneiform, which has proven useful as a processing step in analyses such as [Rom24; JJ24]<sup>10</sup>. The diachronic approach is also useful for pedagogic applications<sup>11</sup>.

## 3.2 Arguments for curviform–cuneiform unification

In this context, the argument was made in [L2/04-099], as part of discussion of the cuneiform encoding<sup>12</sup> that the curviform numerals, which occasionally appear in the Ur III period and are used heavily in the Early Dynastic period, were a stylistic distinction unifiable with the cuneiform digits, and that an archaizing Ur III font or an Early Dynastic font could have curviform glyphs for the appropriate characters.

Some co-occurrence of curviform and cuneiform digits was known and acknowledged. [L2/04-099, p. 3] cites [NDE93, p. 62], which is a copy of [P020054], an Early

<sup>6</sup>Merging with U+1224E CUNEIFORM SIGN NI2.

<sup>7</sup>Notably [OSL] and the online edition of [Bor10] in [eBL, Signs].

<sup>8</sup>Notably [ePSD2] and the online edition of [Sch10] in [eBL, Dictionary].

<sup>9</sup>For example, there are Neo-Assyrian and Neo-Babylonian copies parts of the laws of 𒄑, 𒄑, 𒄑, 𒄑, as well as Old Babylonian copies in both archaizing and cursive styles. Because of damage on the stele [P249253], some sections are known only from those copies. See [Oel22, pp. 110 sqq.].

<sup>10</sup>Attendees may recall the summary given on the third day of UTC #180, as recorded in [L2/24-159]. Other readers may refer to [Svä+24, pp. 242, 148].

<sup>11</sup>For instance, Old Babylonian grammar may be taught in the Neo-Assyrian script, as in [Cap02].

<sup>12</sup>At that time scoped to the répertoire of the Ur III period and later, see [L2/03-162, p. 1], although many disunifications, such as 𒂗 ≠ 𒂗, were informed by Early Dynastic distinctions.

Dynastic IIIB administrative tablet from Nirsu. The excerpt cited, lines 1–3 of column 1 of the obverse, is as follows:

1(neš₂)	1(u)	1/2(diš)	5(diš <i>tenū</i> )	gi	us₂	sa₂	
7.5 (ropes)			5	reed	side	equal	
3(u)	6(diš <i>tenū</i> )	gi	sanj	sa₂			
3 (ropes)	6	reed	front	equal			
ašag=bi	1(bur₃)	1(eše₃)	1(iku)	1/2(iku)			
ašag=bi							
field=DEM <sup>15</sup>							

tugx(LAK 483)-si-ga-kam  
tugsiga =ak =am -∅  
ploughed=GEN=COP-3.SG.S

The argument made in [L2/04-099, p. 4] is that this is comparable to a stylistic distinction such as<sup>16</sup>

465 metres, equal lengths  
198 metres, equal widths  
this field is 9, 18 hectares of ploughed land

where the numerals have the same structure ([L2/04-099] contrasts this to the different structures of ASCII digits and roman numerals). That document further claims that “the number signs do not normally carry in their individual signs the meaning of what they are used to measure”, and that curviform and cuneiform numerals “are not normally mixed together in a single numerical expression”, noting the exceptions of [P232278; P232280]. In addition, [L2/04-099, p. 4] points out that the cuneiform numeric signs are descended from the curviform ones (this is undisputed), and claims there is only a small re-allocation of the function of signs (from to numerals). It therefore comes to the conclusion that the use of curviform numerals should be seen as a formatting distinction, rather than one that should be represented in plain text, and insists that the encoding should capture the lineal historical descent of those signs, presumably to take advantage of the benefits of diachronic encoding described in §3.1.

Although they had been part of the preliminary proposal [L2/03-393R], the curviform numerals were therefore removed from [L2/04-036] and [L2/04-189], which both state that “The distinction between curved numerals and their cuneiform

<sup>13</sup>As noted in [Pow87, p. 466], this sign has a very short “tail” in this period, so that it is wider than it is tall, and can at first seem like a large in copies. The photos in [CDLI] clearly show that this is in fact a vertical wedge.

<sup>14</sup>Note that ED IIIB numerals have a somewhat different appearance from those of the Ur III period used in this transcription; the sign in [P020054] looks more like Ur III .

<sup>15</sup>Alternatively: area=POSS.3.SG.NH, “its area”.

<sup>16</sup>We have taken the liberty of adjusting the analogy to use measures approximately equal to those in [P020054], instead of a field of five by twenty-five metres.

descendants is treated as glyptic for the purposes of the present proposal; this issue will need to be revisited in subsequent encoding phases<sup>17</sup>.”

The time has come to revisit this issue. As we will see in §3.3, numerals can only be interpreted in the context of what they measure, *i.e.*, as part of a metrological system. In §3.4 we will see that in some periods:

- the functions and use of the numerals vary beyond the mere ▷/◁ switch;
- the contrast between curviform and cuneiform numerals is commonly used to distinguish metrological systems;
- some metrological systems commonly mix curviform and cuneiform in single numerical expressions.

### 3.3 A primer on classic Ur III and Old Babylonian metrologies



I want to write tablets: the tablet of  
1 cor of barley to 600 cor; the tablet  
of 1 shekel of silver to 10 minas [...]

Edubba'a D

Before diving into the usage of the curviform numerals in the Early Dynastic period to explain the contrast with cuneiform numerals, it is useful to understand the usage of the already-encoded characters in the Ur III and Old Babylonian periods.

As is well known<sup>18</sup> a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millennium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems [Robo8, p. 76; Rob22]. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters ▷-VVV, the multiples of ten (10–50) are <-», but the other digits 11–59 are sequences <!-»VVV; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a *non-positional* system, hereafter the *cuneiform discrete counting system*  $S_{Ur\ III/OB}$ , which had different signs for the units ▷-VVV, tens <-», sixties ▷-VVV (with larger wedges than the units), multiples of six hundred <!-VV, multiples of three thousand six hundreds <!-VVV, and multiples of thirty-six thousand <!-VVV.

#### 3.3.1 The discrete counting system

The relations between the values of the signs in the cuneiform discrete counting system may be summarized by the following factor diagram<sup>19</sup>, where the num-

<sup>17</sup>The cuneiform encoding process was planned in *stages* in [L2/03-162]. One might expect the second stage of encoding, which led to the creation of the Early Dynastic Cuneiform block, to incorporate the numerals needed for the representation and discussion of Early Dynastic texts; however, the proposal [L2/12-208] stated that “numerals have been omitted due to the complexity of numeral signs from this period. An expert in the metrology of this period must be consulted before these can be properly included.”

<sup>18</sup>See, *e.g.*, [Uni16, §22.3.3, sub “Cuneiform Numerals”].

<sup>19</sup>These diagrams, which have become standard in discussions of Mesopotamian metrology, originate with [Fri78, p. 10], where they are called *step-diagrams*, see Figure 4.

ber over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \times \xleftarrow{10} \mid \xleftarrow{6} \leftarrow \times \xleftarrow{10} \mid \quad (S_{Ur\ III/OB})$$

For example, the number  $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$  would be written  in the discrete counting system, and  in the sexagesimal place value system.

### 3.3.2 The area system

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent [Fri07, p. 378; Rob19]:

$$\diamond \xleftarrow{10} \diamond \xleftarrow{6} \times \xleftarrow{10} \leftarrow \xleftarrow{3} \xleftarrow{6} \xleftarrow{2} \xleftarrow{2} \nearrow \quad (G_{Ur\ III/OB})$$

   
  
*bur<sub>3</sub>*      *eše<sub>3</sub>*      *iku*      *ubûm*  
*bûrûm*      *eblum*      *ikûm*       $1800\ m^2$

Note that for the range of areas given above, this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and *bûrûm*). As mentioned in [Rob19], the whole numeric expression for the area would be followed by the sign  functioning as punctuation<sup>20</sup>, but the numerals are tied to the metrology; thus a surface of 5 *bûr* 1 *ebl* 4 *ikû* (100 *ikû*, 36 ha) would be written<sup>21</sup>     . Contrast this with systems where the same numerals are used for different units, and overt units are used, as in “88 acres 3 roods 33 perches” or 五頃八畝五分九厘. Note also that the same signs are shared between multiple systems, with different relations; the sign  $\diamond$  is equal to sixty times  $\leftarrow$  in the area system, but to three hundred and sixty times  $\leftarrow$  in the discrete counting system.

For areas smaller than a quarter *ikûm*, an overt unit is used, with one  (*sar*, *mûšarum*), approximately  $36\ m^2$ , written , equal to one hundredth of an *ikûm*, then sexagesimally subdivided in 60  (*gîn<sub>4</sub>, šiqlum*, “shekel”). For areas greater than 3600 *bûr*, the  $\diamond$  and  numerals are reused with a suffix  (*gal*, “big”), as follows [Rob08, p. 295 nn. b, c; Fri07, p. 378; Rob19]:

$$\underbrace{\diamond \xleftarrow{10} \diamond \xleftarrow{6} \diamond \xleftarrow{10} \diamond \xleftarrow{6} \times \xleftarrow{10} \leftarrow \xleftarrow{3} \xleftarrow{6} \xleftarrow{2} \xleftarrow{2} \nearrow}_{\overbrace{\phantom{\diamond \xleftarrow{10} \diamond \xleftarrow{6} \diamond \xleftarrow{10} \diamond \xleftarrow{6}} \xleftarrow{2,5} \xleftarrow{10} \mid \xleftarrow{2} \xleftarrow{3} \xleftarrow{10} \mid}} \xleftarrow{2,5} \xleftarrow{10} \mid \xleftarrow{2} \xleftarrow{3} \xleftarrow{10} \mid \quad (G_{Ur\ III/OB})$$

e.g.,       for  $(2 \times 3600 + 20 \times 60 + 49) bûr$   
 $5 ikû (5 + \frac{1}{2}) mûšar$  19 *šiqil*. Factor diagrams in this document will use bottom curly brackets in this fashion to separate numerals from units and other suffixes.

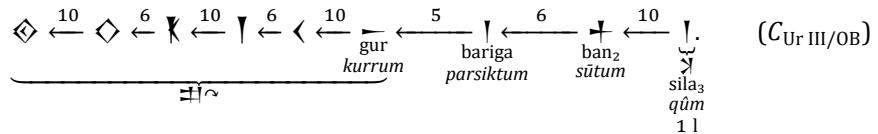
<sup>20</sup>This sign is sometimes interpreted as a measurement unit, and transliterated *iku*, see, e.g., [Pro20, pp. 385 sqq.], or transcriptions in [Feu04] discussed in §3.7.2. Even with this interpretation, the sequence of numerals used, and the interpretation of numerals shared with other metrological systems, is specific to system  $G_{Ur\ III/OB}$ .

<sup>21</sup>As in the surface of the field of  (the city of Apisal) reported on [P102305, r. 1]

<sup>22</sup>From [P213162], which has an additional , two thirds (of a shekel), see §3.3.5.

### 3.3.3 The capacity system

Another such system of note is the one for capacities<sup>23</sup> [Fri07, p. 376; Rob19],



In the above diagram, the numerals for  $\text{ban}_2$  are  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{1}{5}$ , and those for  $\text{bariga}$  are  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $\frac{1}{4}$  (contrast ordinary  $\frac{1}{1}$  and  $\frac{1}{5}$  otherwise used with  $\frac{1}{1}$  numerals). Further, we have used the symbol  $\sim$  to express that, as described in [Hue11, p. 585 nn. (b), (f)], the sign  $\frac{1}{1}$  GUR, while it is used only with volumes in excess of one gur, is written after the whole expression, after the overt unit sign  $\frac{1}{1}$  if present, and after the word for “grain” if present, as in

<sup>24</sup>

Observe that while large numbers of gur follow<sup>25</sup> system  $S_{\text{Ur III}/\text{OB}}$ , the use of horizontal (AŠ) numerals for the gur disambiguates from the vertical bariga, as  $\frac{1}{1}\frac{1}{1}$  would be 10 gur 1 bariga, and  $\frac{1}{1}\frac{1}{1}$  would be 11 gur; again even with some overt units, most of the numerals that participate in a metrological system have an interpretation dependent on that system.

This intertwining of units and numerals explains the large number of already-encoded numeral series:

- $\frac{1}{1}-\frac{1}{1}\frac{1}{1}\frac{1}{1}$  used in  $S_{\text{Ur III}/\text{OB}}$  and the SPVS as well as with overt units;
- $\frac{1}{1}-\frac{1}{1}\frac{1}{1}\frac{1}{1}$  used in  $G_{\text{Ur III}/\text{OB}}$ , of which  $\frac{1}{1}-\frac{1}{1}\frac{1}{1}$  are also used in  $S_{\text{Ur III}/\text{OB}}$  and the SPVS as well as with overt units;
- $\frac{1}{1}-\frac{1}{1}\frac{1}{1}\frac{1}{1}$  used in  $S_{\text{Ur III}/\text{OB}}$ , and sometimes with overt units;
- $\frac{1}{1}-\frac{1}{1}\frac{1}{1}$  used in  $S_{\text{Ur III}/\text{OB}}$ ;
- $\frac{1}{1}-\frac{1}{1}\frac{1}{1}\frac{1}{1}\frac{1}{1}\frac{1}{1}\frac{1}{1}$  used in  $S_{\text{Ur III}/\text{OB}}$  and  $G_{\text{Ur III}/\text{OB}}$ ;
- $\frac{1}{1}-\frac{1}{1}\frac{1}{1}\frac{1}{1}$  used in  $S_{\text{Ur III}/\text{OB}}$  and  $G_{\text{Ur III}/\text{OB}}$ ;
- $\frac{1}{1}-\frac{1}{1}\frac{1}{1}\frac{1}{1}$  used in  $C_{\text{Ur III}/\text{OB}}$  as well as with overt units of the weight system;
- $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$  used in  $C_{\text{Ur III}/\text{OB}}$ ;
- $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$  used in  $C_{\text{Ur III}/\text{OB}}$ —note the overlap with  $\frac{1}{1}-\frac{1}{1}\frac{1}{1}\frac{1}{1}$ ;
- $\frac{1}{1}$  and  $\frac{1}{1}$  used in  $G_{\text{Ur III}/\text{OB}}$ .

Only in the SPVS did numerals exist truly independently of metrology; to quote [Robo8, p. 78]: “The SPVS temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to [...] metrological system. [...] Once the calculation was done, the result was expressed in the most appropriate metrological units and thus re-entered the natural world as a concrete quantity.”

<sup>23</sup>Used for volumes of grain, but also oil, dairy products, beer, etc., as well as to express the capacity of boats; volumes of earthworks instead use system  $G_{\text{Ur III}/\text{OB}}$  based on a height of one cubit, see [Pow87, p. 488; Robo8, p. 294; Rob19].

<sup>24</sup>From [P309594, obv. 11].

<sup>25</sup>A larger unit, the guru<sub>7</sub> (*karûm*, grain heap), is sometimes used instead, with  $\leftarrow \frac{1}{1}\frac{1}{1}\frac{1}{1}\frac{1}{1} \leftarrow = \frac{1}{1}\frac{1}{1}$  (1 *karûm* = 3600 *kurrû*). See [Fri07, p. 415; Rob19].

### 3.3.4 The length system

In the Ur III and Old Babylonian periods, lengths are expressed using overt units counted with  $\text{I}$  and  $\text{K}$  numerals with their system  $S_{\text{Ur III/OB}}$  values. Since it does not have any unusual numerals, this system would not in itself be of much relevance to character encoding, but we present it here as background for its Early Dynastic counterpart presented in §3.4. Metrological tables use the following units<sup>26</sup> [Fri07, p. 118; Rob19]:

	$\xleftarrow{30}$		$\xleftarrow{60}$		$\xleftarrow{12}$		$\xleftarrow{30}$		.	$(L_{\text{Ur III/OB}})$
danna				nindan						
bērum		UŠ <sup>27</sup>		nindanum						
league		360 m		rod						
10,8 km				56 m						
ammatum				kuš <sub>3</sub>						
				cubit						
				50 cm						
				ubānum						
				su-si						
				finger						
				17 mm						

Two more units appear occasionally [Pow87, p. 459; Fri07, p. 118; Rob19]:

	$\xleftarrow{30}$		$\xleftarrow{6}$		$\xleftarrow{10}$		$\xleftarrow{2}$		$\xleftarrow{6}$		$\xleftarrow{30}$		.	$(\bar{L}_{\text{Ur III/OB}})$
eše <sub>2</sub>				ašlum				gi						
rope				60 m				reed						
								3 m						

In addition, there are Akkadian names for the half-rope and half-reed, see [Pow87, pp. 463 sq.].

### 3.3.5 Fractions

Fractions of the *ikim*,  $\text{↖} = \frac{1}{2}\text{←}$  and  $\text{↗} = \frac{1}{4}\text{←}$ , have already been encountered.

In other contexts, the fraction  $\frac{1}{2}$  is written  $\text{✚}$ , as in  $\text{✚} \text{❖}$ . The fractions  $\frac{1}{3}$  and  $\frac{2}{3}$  are written  $\text{✚}$  and  $\text{✚}$ . The latter two signs are derived from curviform signs  $\text{❖}$  and  $\text{❖}$ , which are already separately encoded; these are in turn derived from the sign  $\text{✚}$  ( $\text{SU}_2$ ), whose Early dynastic form resembles  $\text{✚}$ , and  $\text{◦}$  numerals; see [Pow71, pp. 113, 134]. The  $\text{✚}$  is sometimes omitted, as in [P240545, *verso* 6 9; P221530; P221531; P271238; P274845].

## 3.4 Curviform numerals in early metrologies

At first sight, the metrological systems from the Early Dynastic period resemble the ones previously mentioned. In particular, the discrete counting system used in the Early Dynastic period (and earlier in the fourth millennium) clearly mirrors system  $S_{\text{Ur III/OB}}$  [Fri07, p. 374; DE87, pp. 127, 165]:

$$\textcircled{O} \xleftarrow{10} \bullet \xleftarrow{6} \text{◐} \xleftarrow{10} \text{◑} \xleftarrow{6} \bullet \xleftarrow{10} \text{◑}. \quad (S)$$

Likewise the area system used in the Early Dynastic IIIb period for areas of one iku and greater [Dei22, p. 72; NDE93, p. 63; Fri07, p. 378; Lec16],

$$\textcircled{O} \xleftarrow{10} \bullet \xleftarrow{6} \text{◐} \xleftarrow{10} \bullet \xleftarrow{3} \text{◑} \xleftarrow{6} \text{◑} \quad (G_{\text{ED IIIb}})$$

<sup>26</sup>In this factor diagram and the next, we do not include the numerals. The units are no more than a factor of 60 apart, so higher numerals such as  $\text{❖}$  or  $\text{❖}$  are not used.

<sup>27</sup>As indicated by the capitalization, the reading of this sign is unknown; see [Pow87, pp. 465 sqq.] for a discussion of various hypotheses.

mirrors system  $G_{Ur\ III/OB}$ , with consistent use of the numerals: • corresponds to ↘, ● to ◇, and ○ to ◇. An exception to this correspondence, noted in [L2/04-099, p. 4] (see §3.2), is that the vertical | from  $S_{Ur\ III/OB}$  corresponds to a horizontal □ in system  $S$ . This is however far from the only case of such a reallocation of function. The earlier form of the area system is [DE87, pp. 141, 165; Frio7, p. 378]:

$$\bullet \xleftarrow{6} \circlearrowleft \bullet \xleftarrow{10} \bullet \xleftarrow{3} \square \xleftarrow{6} \square, \quad (G)$$

Observe that, as noted in [DE87, p. 142], ○ changes meaning from 10• in system  $G$  to 600● in system  $G_{ED\ IIIb}$ . System  $G$  is used in the fourth millennium, but also in the ED I-II period (it is the “area 2” system in [Chao3], whereas  $G_{ED\ IIIb}$  is the “area 1” system).

Another example of nontrivial correspondence between cuneiform and curviform numerals may be found by comparing the fractions the Early Dynastic IIIb area system<sup>28</sup>,

$$\circlearrowleft \xleftarrow{10} \bullet \xleftarrow{6} \star \xleftarrow{10} \bullet \xleftarrow{3} \square \xleftarrow{6} \square \xleftarrow{2} \square \xleftarrow{2} \square \xleftarrow{2} \star^{29}, \quad (G_{ED\ IIIb})$$

with the numerals of a contemporaneous capacity system:

$$\overbrace{\square \xleftarrow{10} \square \xleftarrow{6} \bullet \xleftarrow{10} \square \xleftarrow{4} \square \xleftarrow{6} \square}^{\text{半升量}} \quad (C_{半升量})$$

both described in [Lec16]. While the size of the  $\square \square \square$  (gur saŋ ḥal₂) in bariga is different from that of the Old Babylonian  $\square \square$ , the basic structure of the capacity system is recognizable, with  $\square$  corresponding to | for bariga,  $\square - \square$  corresponding to  $\perp - \square$  for ban₂, and the  $\square \square$  counted with  $\square$  rather than  $-$  numerals. However, the half-iku is counted with the same  $\square$  as the bariga, whereas it uses a different sign,  $\nwarrow$ , in the Old Babylonian system. As we will see, this is cannot be handled as a split, by giving  $\nwarrow$  the glyph  $\square$  in an Early Dynastic IIIb font, as the  $\nwarrow$  numeral series is also in use in that period.

### 3.4.1 Field lengths in Nirsu

The length system of the Early Dynastic IIIb state of Lagaš is of particular interest. As described in [Pow87, p. 466; Lec20, pp. 289 sq.], lengths are expressed in rods, but the unit sign  $\text{v}$  is generally omitted; in addition, only tens of rods are used; these are equal to one rope, but the sign  $\text{m}$  is not written either. Lengths shorter than one rope are expressed in half-ropes using the  $\frac{1}{2}$  sign  $\perp$  (again with no  $\text{m}$ ), and then in reeds, with the sign  $\text{r}$ , as follows:

$$| \xleftarrow{6} \nwarrow \xleftarrow{2} \perp \xleftarrow{10} \nwarrow^{32}. \quad (L_{ED\ IIIb})$$

$1\ eše_2 = 10\ nindan$   
 $1\ \text{rope} = 10\ \text{rods}$   
 $60\ \text{m}$   
 $\text{reed}$   
 $3\ \text{m}$

This is the system that was used to express the sides of the field in [PO20054] discussed in §3.2. In that tablet and most others from the same period, such as the

<sup>28</sup>A variant is  $\circlearrowleft \xleftarrow{10} \bullet \xleftarrow{6} \star \xleftarrow{10} \bullet \xleftarrow{3} \square \xleftarrow{6} \square \xleftarrow{2} \square \xleftarrow{2} \square \xleftarrow{2} \star$ , see [Pow72, p. 218].

<sup>29</sup>The (fairly rare) cuneiform counterpart is  $\perp$ .

<sup>32</sup>The reeds are counted using *tenū* numerals,  $\nwarrow$ ,  $\sim$ ,  $\diamond$ , etc.

ones discussed in [Lec20], areas are expressed in system  $G_{ED\ IIIb}$ , with curviform numerals<sup>33</sup>; in the absence of overt units, such as when dealing with length that are integer multiples of a half-rope<sup>34</sup>, the use of curviform or cuneiform numerals therefore disambiguates a numeric expression between an area and a length, and thus the interpretation of its numerals between systems  $G_{ED\ IIIb}$  and  $L_{ED\ IIIb}$ . The sign  $\text{𒉗}$ , which would also disambiguate the interpretation as an area, is sometimes used after areas in ED IIIb Lagaš, but not systematically; in particular the area of the first field in [P020054] does not use this suffix. See [Lec20] for many examples with and without  $\text{𒉗}$ . There are other such co-occurrences contrasting between metrological systems; for instance, [Kre98, p. 303 n. 686] mentions the use of cuneiform numerals for days and months<sup>35</sup>.

### 3.4.2 Dyke lengths in Nirsu

[Pow87, p. 466] notes that reeds “are regularly written with the normal, cuneiform end of the stylus. Higher units are usually written with the reversed (round) end of the stylus.” Powell does not elaborate on the specifics of this mixed use of numerals, but a cursory search in [CDLI] finds many occurrences<sup>36</sup>, such as:

- [P221305, obv. 1 4]<sup>37</sup>
- [P020129, rev. 2 1]
- [P221291, rev. 5 1]<sup>38</sup>
- [P221266, rev. 2 1]

These expressions use an explicit sign  $\text{𒉕 }$  (counted in multiples of ten) or  $\text{𒉗}$ . This notation—but not its use of curviform numerals—is remarked on in [Lec20, p. 290 n. 27], which cites several of the instances listed above. It seems to be typical of texts about dykes. The notation can be summarized by the following factor diagram, where prefix units have been marked by an asterisk:

$$\underbrace{\text{○} \leftarrow \text{○} \leftarrow \bullet}_{\text{𒉕 }} = \underbrace{\text{○} \leftarrow \text{○} \leftarrow \text{○}}_{\text{𒉗}} \leftarrow \underbrace{\text{○} \leftarrow \text{○} \leftarrow \text{○}}_{\text{𒉗}} \leftarrow \underbrace{\text{○} \leftarrow \text{○} \leftarrow \text{○}}_{\text{𒉗}} \quad (L'_{ED\ IIIb})$$

### 3.4.3 Butter, cheese and wheat in Nirsu

A similar mixture of cuneiform and curviform numerals may be observed with the capacity system; indeed, the previously described  $\text{𒂵 }$  system uses  $\text{ }$

<sup>33</sup>A [CDLI] search for "(bur3)" (< numerals used for areas) currently returns 15 ED IIIb results, whereas one for "(bur3@c)" (• numerals used for areas) returns 206. Further, when dated, the tablets with cuneiform bur<sub>3</sub> are from the reigns of  $\text{𒈗 }$  (variously transliterated iri-inim-gi-na, uruk-gi-na, etc.) and  $\text{𒈗 }$  (lugal-zag-ge-si), the last two kings of ED IIIb Lagaš.

<sup>34</sup>This is the case of the sides of the field in [P020054, obv. ii 2–3].

<sup>35</sup>That note also mentions a contrast between the use of curviform numerals to count people and curviform numerals to count bread allotted to them in [P010876]; such contrasts are more akin to styling, and might not, on their own, justify the disunification.

<sup>36</sup>A search for curviform numerals followed by some number of reeds counted in (*tenū*) cuneiform numerals currently finds 125 occurrences across 47 tablets.

<sup>37</sup>[CDLI] only has a copy, but a photo may be found in [Lec12, p. 82]. On that photo the  $\text{ }$  is not visible. Lecompte notes that the copy is faithful; indeed another  $\text{ }$  can be seen both on the copy and the photo on obv. 2 2.

<sup>38</sup>From copy.

<sup>39</sup>With either unit omitted, as in the examples above, or both, as in [P020129, obv. 3 3]

numerals for  $\aleph$  [Fri78, p. 43; Lec16]:

$$\overbrace{\bullet \leftarrow \square}^{\frac{10}{\text{慢}}} \leftarrow \overbrace{\bullet \leftarrow \square}^{\frac{10}{\text{快}}} \xrightarrow{\frac{3}{2}} \text{慢} \leftarrow \text{快}, \quad (\mathcal{C}_{\text{慢快}})$$

but counts cheese (迦珥, ga'ar) using the迦珥量 capacity system, with迦珥 numerals for the量.

Another capacity system in ED IIIb Nirsu is the **☷☱☲☱**, the gur of two ul [Lec16]:

$$\bullet \leftarrow \boxed{\square} \leftarrow \square \leftarrow \oplus \leftarrow \oplus \leftarrow \boxed{\square}.$$

Here the  $\square - \nwarrow - \triangleright$  contrast occurs not only within the numerals of the system, but with its units; this is perhaps best illustrated by the expressions  $\ll \leftarrow \diamond \triangleleft \boxtimes$  •  $\triangleright \boxtimes \ll \triangleleft \rightarrow$  in [P221746, rev. 2 2] and  $\ll \triangleleft \bullet \triangleright \boxtimes \ll \triangleleft \rightarrow$  in [P221814, rev. 1 5].

### 3.4.4 Grain in Ebla

The mixing of curviform and cuneiform numerals within a metrological system is not specific to Nirsu.

<sup>40</sup>As of this writing, the single occurrence of (banz@*c*) followed by curviform numerals and *sila3* in ED IIIb Nirsu transliterations on [CDLI], 4(banz@*c*) 3(asz@*c*) *sila3* in [P221815, obv. 4 7], is incorrect: it should be 4(banz@*c*) 3(disz@*t*) *sila3*.

<sup>41</sup>This tablet also uses subtractive notation: “two pots minus two thirds (*sila*<sub>3</sub>)”, • “ten minus one pots, six *sila*<sub>3</sub>”. Such subtractive notation is common in most of the metrological systems discussed here; it appears in the ED IIIa period [Robo08, p. 77]. It presents no complexity for character encoding, but it is noteworthy that the sign (lal, “minus”) is often ligated with the following numerals, with the subtrahend placed under a sometimes considerably enlarged , similar to the layout of the radical in modern mathematical notation, see, e.g., [Po20092, rev. 3, 1, 2]. A good font could handle the very common  $-1$  case, perhaps even  $-2$  and  $-3$ ; setting arbitrary numeric expressions under the , or more generally replicating the layout of Early Dynastic tablets, is outside the realm of plain text; see also §6.3.

The system of grain<sup>42</sup> capacities in Ebla uses the following units<sup>43</sup>:

$$\text{gu}_2\text{-bar} \xleftarrow{2} \text{ba-ri}_2\text{-zu} \xleftarrow{\frac{5}{2}} \text{giŋ}_4 \xleftarrow{4} \text{niŋ}_2\text{-saŋšu} \xleftarrow{6} \text{an-zam}_x.$$

The  $\text{gu}_2\text{-bar}$  and  $\text{ba-ri}_2\text{-zu}$  are generally counted using curviform numerals, and the smaller units using cuneiform  $\text{I}$  numerals<sup>44</sup>. Indeed, a search on [EbDA] for co-occurrences of either  $\text{* }$  or  $\text{V } \text{I }$  with either of  $\text{gu}_2\text{-bar}$  or  $\text{ba-ri}_2\text{-zu}$  finds the following expressions<sup>45</sup>:

1. [P240532, verso 4 9]  $\triangleright \text{gu}_2\text{-bar} \text{ V } \text{I }$ <sup>47</sup>
2. [P240548, verso 1 1]  $\triangleright \text{ba-ri}_2\text{-zu} \text{ V } \text{I }$ <sup>48</sup>
3. [P240655, recto 7 9]  $\triangleright \text{gu}_2\text{-bar} \text{ V } \text{I }$ <sup>49</sup>
4. [P240579, verso 4 3]  $\text{BBP} \triangleright \text{gu}_2\text{-bar} \text{ V } \text{I }$
5. [P240675, verso 2 2]  $\triangleright \text{ba-ri}_2\text{-zu} \text{ V } \text{I }$
6. [P240609, verso 3 1]  $\triangleright \text{ba-ri}_2\text{-zu} \text{ V } \text{I }$
7. [P240533, recto 3 3]  $\text{:} \triangleright \text{gu}_2\text{-bar} \text{ V } \text{I } \text{+ } \text{* }$
8. [P240697, recto 1 5]  $\triangleright \text{ba-ri}_2\text{-zu} \text{ V } \text{I }$
9. [P240653, recto 6 2]  $\text{:} \triangleright \text{gu}_2\text{-bar} \text{ V } \text{I } \text{+ } \text{* }$
10. [P240654, recto 2 6]  $\triangleright \text{ba-ri}_2\text{-zu} \text{ V } \text{I } \text{+ } \text{* }$ <sup>50</sup>
11. [P240531, recto 1 8]  $\triangleright \text{ba-ri}_2\text{-zu} \text{ V } \text{I } \text{+ } \text{I }$
12. [P241708, recto 1 1]<sup>52</sup>  $\triangleright \text{ba-ri}_2\text{-zu} \text{ V } \text{I }$
13. [P241904, recto 1 1]<sup>53</sup>  $\text{BB} \triangleright \text{gu}_2\text{-bar} \text{ V } \text{I }$ <sup>54</sup>

<sup>42</sup>Liquid capacities use a different system [Arc15, p. 229 n. 12]:

$$\text{la-ḥa} \xleftarrow{30} \text{sila}_3 \xleftarrow{6} \text{an-zam}_x.$$

A glance it seems that  $\text{I}$  are counted with cuneiform numerals and higher units with curviform ones, thus

$$\underbrace{\text{gu}_2\text{-bar} \xleftarrow{\frac{5}{3}} \text{D} \xleftarrow{6} \bullet \xleftarrow{10} \text{D} \xleftarrow{3} \underbrace{\text{I} \xleftarrow{10} \text{I}}_{\text{I}} \xleftarrow{6} \text{I},}_{\text{la-ḥa}}$$

but we have not investigated this thoroughly.

<sup>43</sup>Another system uses different values for the  $\text{I}$  and  $\text{V } \text{I }$ , see [Cha12, p. 62; Arc15, p. 229 n. 12]:

$$\text{gu}_2\text{-bar} \xleftarrow{2} \text{ba-ri}_2\text{-zu} \xleftarrow{3} \text{D} \xleftarrow{4} \text{V } \text{I } \xleftarrow{5} \text{* } \text{I}.$$

<sup>44</sup>As mentioned in [Cha12, p. 63], the  $\text{D}$  is also counted using the  $\text{V}-\text{I}$  numeral series. Some instances of that usage are found transliterated  $n/6$  in [EbDA]; in some cases the  $\text{D}$  sign is omitted, and the  $\text{V}$  numeral is then written before the  $\text{I}$  unit, as in  $\text{D} \text{ V } \text{I }$  from [P240545, verso 1 3].

<sup>45</sup>We cite here only one attestation per tablet; most tablets contain several expressions mixing curviform  $\text{gu}_2\text{-bar}$  and larger with cuneiform  $\text{D}$  and smaller. In all cases the transcriptions given here are based on the [EbDA] transliterations, but the shape and orientation of the numerals was checked<sup>46</sup> on a photograph (from [EbDA] unless noted otherwise).

<sup>46</sup>As we will see in §3.7.2, [CDLI] transliterations indicate numeral shape; however, as of this writing, they do so incorrectly on the Ebla corpus, claiming that all numerals are curviform, so we were not able to rely on them in this specific case.

<sup>47</sup>ba-ri<sub>2</sub>-zu<sub>2</sub>, a variant spelling.

<sup>48</sup>Short for  $\text{gu}_2\text{-bar}$ .

<sup>49</sup>Note the omitted  $\text{gu}_2\text{-bar}$ .

<sup>50</sup>Instead of the expected  $\text{V } \text{I }$ .

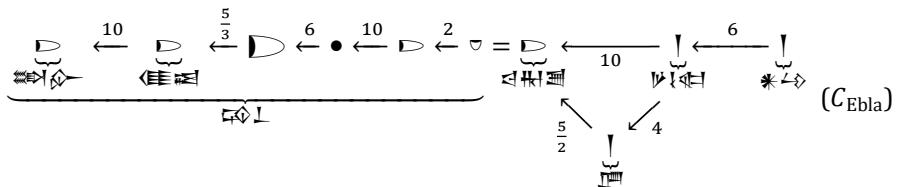
<sup>51</sup> $\text{V } \text{I }$  not legible on the EbDA photo.

<sup>52</sup>From [CDLI] photo.

<sup>53</sup>From photo in [Arc89, p. 6].

<sup>54</sup>Laid out as  $\text{V } \text{I }$ ; on stacking patterns see §6.3.

14. [P240964, recto 3 2]<sup>55</sup> BBB <sup>56</sup> D o L || <sup>57</sup> || \*



### 3.4.5 Use in modern publications

Because of their prevalence in the fourth millennium and Early Dynastic period, the proposed numerals are widely used in modern publications discussing metrology in those periods, as illustrated in Figures 1–21.

Since they contrast with the cuneiform numerals, they likewise appear contrastively in such publications. A remarkable example of that is found in Figure 21. The partial<sup>58</sup> transliteration “4▷ ’a<sub>3</sub>-da-um 4▽ aktum 4▷ ib<sub>2</sub><sup>tū<sub>9</sub></sup>×3! sa<sub>6</sub> gunu<sub>3</sub>” is used to illustrate a discussion of the interpretation of the contrast between ▷ and ▽ numerals. More conventional transliterations might omit the numeral shapes entirely, e.g., 4 ’a<sub>3</sub>-da-um 4 aktum 4 ib<sub>2</sub><sup>tū<sub>9</sub></sup>×3 sa<sub>6</sub> gunu<sub>3</sub>, which would obviously be inadequate in this context. There are transliteration conventions that are more explicit about numeral shape, e.g., 4(aš<sup>c</sup>) ’a<sub>3</sub>-da-um 4(diš<sup>c</sup>) aktum 4(aš<sup>c</sup>) ib<sub>2</sub><sup>tū<sub>9</sub></sup>×3(diš) sa<sub>6</sub> gunu<sub>3</sub>, but the result would be less readable. See §3.7.2 for a discussion of transliteration conventions for numerals.

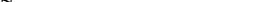
for the words 𒊩 𒊭 ဏ ဏ and 𒊩 ဏ ဏ ብ ி. Deimel's reading ସ ဏ (a) for U came out of the reading /ସ ဏ ဏ ା କ/ for the sign T and the writing of ସ ଆ (- ဏ ဏ) after the fractional signs for ସ ସ ဏ ဏ and ସ ဏ ဏ ብ ி in Old Sumerian texts. But this was an ill-conceived argument at its inception, for

Figure 1: Discussion of the readings of proposed  $\sqsupset$  and already-encoded  $\sqsubset$  in [Pow71, p. 107].

<sup>55</sup>From photo in [Arc89, p. 6]; see also the [CDLI] photo and the copy in [Fri86, p. 17]. This tablet features unusual usage of vertical numerals—“somewhat unorganized”, as described by [Fri86, p. 16]—such as 𒌩 𒌩 𒉺 or 𒌩 𒌩 𒉺, but its 𒌩 𒉺 and \* 𒉺 are consistently counted with cuneiform numerals, and the higher units with cuneiform numerals.

<sup>56</sup>Short for  .

<sup>57</sup> ŠU<sub>2</sub>+NIN<sub>2</sub>-sanj, an unusual variant spelling.

<sup>58</sup>The untransliterated text would be ; note the atomically encoded ib<sub>2</sub> × 3! = .

sions also. In example 6, the writing ~~ša~~ may imply a reading /š a n a b i/,<sup>1</sup> whereas ~~ša~~ in example 11 should be read \*/š u š a n a m i n/. Moreover, the question must be raised as to whether such writings as <ša/> kù - b a b b a r + š a - a a<sup>2</sup> do not perhaps imply a linguistic resolution of \*/š u š a n a m i n/ rather than /š a n a b i/. I see no way of answering this question at present, but it is one which one

Figure 2: Discussion of the readings of proposed ~~ša~~ and ~~ša~~ as well as already-encoded  $\overline{v}$  and  $\overline{v}$  in [Pow71, p. 138].

iku fractions		
Girsu type	"BIN 8" type	Ur III type
$\square = :f.o.o$	$\Phi = :p.o.o$	$\gamma = :m.o$
$\square = :o.g.o$	$\Theta = :o.q.o.$	$\gamma = :o.n$
$\Phi = :o.o.h$	$\Theta = :o.o.r$	

Figure 3: A transliteration system for the fractions of the iku in [Pow72, p. 216].

<sup>1</sup> "big cup" = 3 "big disks". Hence we can infer from the two ŠE-texts BIN 8,4 and BIN 8,5 together, that the "ŠE-system" makes use of number signs whose values are related to each other through the equations

$$1\square = 3 \circ, 1\circ = 10 \circ, 1\circ = 6 \circ, 1\circ = ? \circ$$

A more convenient way of saying the same thing is to write out the "steps" between the various ŠE-units in what we shall call a "step-diagram" for the "ŠE-system":

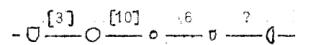


Figure 4: The first factor diagram, in [Fri78, p. 10].

$$\begin{cases} 4\bar{U} 5\bullet = 24\bar{U} 3\bullet & (\text{C 234}) \\ 5\bullet 1\bullet 1\bar{U} = 5\bar{U} 7\bullet & (\text{C 314}) \\ 1\bar{U} 1\bar{D}\bullet 1\bar{D}\bullet = 6\bullet 2\bar{U} 1\bar{D}\bullet & (\text{C 27}) \end{cases}$$

These metrological equations for the "unknowns"  $\bar{U}$ ,  $\bullet$ ,  $\bar{D}$ , etc., can be treated exactly as ordinary equations for unknowns  $x, y, z, \dots$ . In particular, the equations can be simplified by subtraction of equal amounts from both sides of the identities. In this way the three equations above can be reduced to:

$$\begin{array}{ll} 2\bullet = 20\bar{U} & (4\bar{U} 3\bullet \text{ subtracted from both sides}) \\ 1\bar{U} = 6\bullet & (5\bullet 1\bullet - " - ) \\ 1\bar{D}\bullet = 6\bullet 1\bar{U} 9\bar{D}\bullet & (1\bar{U} 1\bar{D}\bullet - " - ) \end{array}$$

We can now read off from the first equation that  $1\bullet = 10\bar{U}$ , and from the second that  $1\bar{U} = 6\bullet$ . Then the third equation can be simplified (by "substitution" of these values into the equation), to the following reduced form:

$$1\bar{D}\bullet = 2\bar{U} 9\bar{D}\bullet.$$

The most likely solution to this last equation is, of course,

$$1\bar{D}\bullet = 2\bar{U}, \quad 1\bar{D}\bullet = 10\bar{D}\bullet.$$

Figure 5: The derivation of the factors of the bisexagesimal system in [Fri78, p. 15]<sup>60</sup>.

<sup>60</sup>The bisexagesimal system is used alike in proto-Elamite and proto-cuneiform texts, see [Fri78, p. 38]; the derivation in [Fri78, p. 15] is based on proto-Elamite artefacts. There is a typo in the equation for C 27: the right-hand side should have  $10\bar{D}\bullet$  rather than  $1\bar{D}\bullet$ , otherwise nothing could be deduced about  $\bar{D}\bullet$ . Note that in Friberg's early works [Fri78; Fri79; Fri86; Fri87], copies of fourth millennium and sometimes third millennium tablets are shown as vertical text (which they were for the scribes), and their numerals are written within horizontal text in the same orientation that they have if the tablet is taken as vertical text; in [UAX50] parlance, as if they had Vertical\_Orientation=Upright. In addition, they are listed in these equations in the horizontal order in which they appear as vertical text (thus the rightmost numeral is the most significant, read first). Cuneiform is correctly Vertical\_Orientation=Rotated, consistently both with modern practice and with the rotation between earlier vertical and later horizontal monumental inscriptions. Friberg's early conventions are not followed in later scholarship, and are abandoned in his own more recent works, such as [Fri07]; a more typical way to express the first equations might be

$$\begin{array}{ll} 5\bullet + 4\bar{D} = 3\bullet + 24\bar{D} & (\text{C 234}) \\ 1\bar{D} + 1\bullet + 5\bar{D} = 7\bullet + 5\bar{D} & (\text{C 314}) \\ 1\bar{D} + 1\bullet + 1\bar{D} = 10\bar{D} + 2\bullet & (\text{C 27}) \end{array}$$

A diplomatic edition of [Fri78] could rotate the numerals using a higher-level protocol:

$$\begin{cases} 4\bar{U} 5\bullet = 24\bar{U} 3\bullet & (\text{C 234}) \\ 5\bar{U} 1\bullet 1\bar{U} = 5\bar{U} 7\bullet & (\text{C 314}) \\ 1\bar{U} 1\bar{D}\bullet 1\bar{D}\bullet = 6\bullet 2\bar{U} 1\bar{D}\bullet & (\text{C 27}) \end{cases}$$

Thus, for instance, the original set of fractions  $\text{v}$ ,  $\text{a}$ , and  $\text{b}$  ( $1/2$ ,  $1/4$  and  $1/8$  of an iku) in the Sumerian GANA system, was after a time augmented through the addition of the new sub-unit SAR:  $\text{s}$ , equal to  $1/100$  of an iku ( $\square$ ). Similarly, the Sumerian weight unit "ma-na" which originally may have had only the sub-units  $\text{v}$   $\text{sá-na}$  (=  $1/3$  mana) and  $\text{v}\text{v}$   $\text{sá-na-bi}$  (=  $2/3$  mana), and perhaps also  $\text{gín}$ :  $\text{v}\text{v}\text{v}$  (=  $1/60$  mana), seems to have acquired, at some time or other, also the smaller sub-units  $\text{v}\text{v}\text{v}\text{v}$  ( $= 1/3$  gin), and  $\text{v}\text{v}\text{v}\text{v}\text{v}$  ( $= 1/3 \times 1/60$  gin).

Figure 6: Discussion of proposed fractions  $\text{v}$ ,  $\text{a}$ ,  $\text{b}$ , and  $\text{s}$ , as well as already-encoded  $\text{v}$  and  $\text{s}$  in [Fri78, p. 49].

stein publizierten Zeichenliste enthalten ist<sup>3</sup>, bis vor kurzem unentdeckt bleiben konnte. Erst 1978 machte der schwedische Mathematiker J. Friberg, ERBM I, 9–11, darauf aufmerksam, daß die Zeichen für die Zahlen Eins ( $\square$ ) und Zehn ( $\bullet$ ) in Verbindung mit dem Zeichen  $\text{SE}$  nicht im Verhältnis 1 zu 10 sondern im Verhältnis 1 zu 6 stehen. Bis dahin hatte man, obwohl die Andersartigkeit des in Verbindung mit dem Zeichen  $\text{SE}$  verwendeten Zahlzeichenystems bekannt war, für diese beiden häufigsten Zahlzeichen einheitlich ein Verhältnis 1 zu 10 unterstellt, obwohl es mehrere eindeutige Gegenbelege gab, von denen zumindest diejenigen der Archaischen Texte aus Gēmet Nasr bereits früh publiziert und jedermann zugänglich waren<sup>4</sup>. Als Folge

Figure 7: Discussion in [DE87, p. 117] of the discovery in [Fri78, pp. 9–11] (see Figure 4) of the different relations between  $\square$  and  $\bullet$  in systems  $G$  and  $S$ .

there is in any case an important qualitative difference between IX for Latin novem and  $\bullet$  for Sumerian niš. niš seems to be a primary numberword requiring, in a system depicting Sumerian numeration, a differentiated representation comparable

Figure 8: The sign  $\bullet$  used in a parallel with IX in [Eng88, pp. 131–133 n. 9], discussing an argument from [Pow72, p. 172] on the question of the language of the Uruk III texts.

of decreasing fractions  $1/n$  of this measure, whereby "n" was determined by the number of oblique impressions made by the rounded end of a thin stylus around a central point in a specific sign. Thus  $\text{v} = 1/2 N_{39}$ ,  $\text{v}\text{v} = 1/3 N_{39}$ , and so on. The first sign of the latter units,  $N_{34}$ ,

Figure 9: Description of the fractions  $\text{v}$  and  $\text{v}\text{v}$  in [Eng98, p. 113]<sup>61</sup>.

For instance, the first line contains the notations  $1N_{34} 1N_{39} ; 2N_{20}$ , which can be translated "60 of the (grain rations containing)  $\text{v}$  (of grain); (grain involved) 2  $\bullet$  (of ground barley)". This calculation contradicts the assumed numerical relationship  $10N_1 = 1N_{14}$ , since as was well known the measure represented by the sign  $N_{39}$  was  $1/5$  of that represented by  $N_1$ , so that  $60 \times 1/5 = 12$  and not 20, as  $2N_{14}$  would imply. Instead of relying on complicated

Figure 10: The sign  $\text{v}$  used as a capacity measure within otherwise translated text in [Eng98, p. 116].

<sup>61</sup>The text erroneously has  $N_{34}$  instead of  $N_{24}$ .

Die halbkreisförmigen Griffleindrücke gehen manchmal in mehr oder weniger eckige Formen über (¶)<sup>685</sup>. Es gibt aber auch Einer in Form von regelrechten – meist mehr oder weniger schräggestellten – Keilen (＼), die öfters neben halbrunden Einern vorkommen und mit diesen kontrastieren<sup>686</sup>. Selten treten mit σ gebildete Zahlen auf<sup>687</sup> (sie entsprechen den bariga-Zahlen im Hohlmaßsystem, s.u. 7.4).

Figure 11: Discussion of co-occurrences and contrasts between ।, ↖, and σ in [Kre98, p. 303].

The calculations:						
Obv. i	1	$60 \times \frac{1}{5} \rightarrow$	(⊖)	=	$12 \times \rightarrow =$	$2 \times \bullet$
	2	$120 \times \frac{1}{10} \rightarrow$	(⊖)	=	$12 \times \rightarrow =$	$2 \times \bullet$
	3	$120 \times \frac{1}{15} \rightarrow$	(⊖)	=	$8 \times \rightarrow =$	$1 \times \bullet$ $2 \times \rightarrow$
	4	$300 \times \frac{1}{20} \rightarrow$	(⊖)	=	$15 \times \rightarrow =$	$2 \times \bullet$ $3 \times \rightarrow$
	5	$600 \times \frac{1}{25} \rightarrow$	(⊖)	=	$24 \times \rightarrow =$	$4 \times \bullet$
Rev. i	1	1200			$1 \times \bullet$	$1 \times \bullet$ $5 \times \rightarrow$
Obv. i	6	$6000 \times \frac{1}{30} \rightarrow$	(GAR+6N <sub>57</sub> )	=	$200 \times \rightarrow =$	$1 \times \rightarrow$ $3 \times \bullet$ $2 \times \rightarrow$
ii	1	$120 \approx \frac{1}{4} \rightarrow$	(DUG <sub>a</sub> +U <sub>2a</sub> )	≈	$30 \times \rightarrow =$	$5 \times \bullet$ $1 \times \rightarrow$ $1 \times \rightarrow$
	2	$180 \times \frac{1}{5} \rightarrow$	(DUG+AS <sub>a</sub> )	=	$36 \times \rightarrow =$	$6 \times \bullet$
	3	$300 \times \frac{1}{15} \rightarrow$	(KAŚ <sub>a</sub> )	=	$20 \times \rightarrow =$	$3 \times \bullet$ $2 \times \rightarrow$
Rev. i	3	600			$1 \times \bullet$	$4 \times \bullet$ $3 \times \rightarrow$ $1 \times \rightarrow$
					$1 \times \bullet$	$1 \times \bullet$ $5 \times \rightarrow$
					$1 \times \rightarrow$	$3 \times \bullet$ $2 \times \rightarrow$
					$1 \times \bullet$	$4 \times \bullet$ $3 \times \rightarrow$ $1 \times \rightarrow$
Grand total of groats used:						
					$1 \times \rightarrow$	$2 \times \bullet$ $9 \times \bullet$ $4 \times \rightarrow$ $1 \times \rightarrow$
Grand total of malt used: 1N <sub>47</sub> 4N <sub>20</sub> 3N <sub>5</sub> 1N <sub>42a</sub> (rev. i 3) × 3/5 ≈						
					$8 \times \bullet$	$4 \times \rightarrow$ $1 \times \rightarrow$

Figure 6. Transliteration and calculations of MSVO 4, 66.

Figure 12: Calculations from [Poo5468] transcribed in [Eng01, p. 132] using modern mathematical notation combined with some of the proposed characters.

strong similarities between “area” 1 and “area” 3 systems, the sign with two concentric discs (◎, notated N<sub>50</sub><sup>27</sup>) remains problematic. It never appears in any numerical combination with the sign with a single disc (●,

Figure 13: Discussion of ◎ and ●<sup>62</sup> in [Chao03, p. 6].

<sup>62</sup>The statement that these do not co-occur refers to the texts from ED I-II Ur; these signs co-occur both earlier and later in areas, with different relations as previously discussed.

<sup>1/15</sup>, etc., of gur, we would expect the metrogram gur to appear in sub-column ii. In a certain way, it does for larger measures: the notation  $\overline{\text{I}} \text{ } \text{I} \text{ } \text{I}$  could be understood as 1 1/5 gur.<sup>27</sup> However, the metrogram gur does not appear for lower measures. It would not be consistent to attribute different functions to the same grapheme, according to the relative importance (be it great or small) of the quantity, so the signs  $\text{I}$  and  $\text{I}$  cannot be considered klasmatograms.

Metrological tablets from the end of the 4<sup>th</sup> millennium (Nissen, Damerow and Englund 1993, 55-59, to *MSVO* 1, nos. 2-3) contain a discrete set of numerical signs with specific surface area reference:

- 1(iku) represents a surface of  $3600\text{m}^2$
  - 1(eše<sub>3</sub>) represents a surface of  $21,600\text{m}^2$   
etc.

The signs iku and eše<sub>3</sub> constitute by themselves measures of surface areas. These measures are usually followed by the sign GAN<sub>2</sub>, which means either surface or field and

Figure 14: Discussion of Old Babylonian<sup>64</sup> capacity and fourth millennium area measures in [Pro09, p. 9].

formed by only two signs  $\text{↑}$  and  $\text{↓}$ , repeated as many times as necessary; this type of notation is highly standardized. Second, the order of magnitude of the numbers noted in this system is not indicated: 1, 60,  $60^2$ ,  $60^3$ ,  $1/60$ ,  $1/60^2$ , etc. are written in the same way, with the vertical wedge  $\text{↑}$ . The third feature concerns the exact function of

Figure 15: Description of the SPVS in [Cha12, p. 58], using the already-encoded signs  $\sqcup$  and  $\sqsubset$ .

one step. The scribes of the Early Dynastic Period (c. 2600 BC), for instance, represented the number 648,000 with:    but never with the repetition   .

Figure 16: Discussion of large numbers illustrated by <sup>65</sup> in [Cha12, p. 59]

repetition of the same sign refers to both the capacity unit signified—often but not necessarily written immediately afterwards—and its value. The units of measurement are written in descending order from left to right—just as we would write 3 km, 120 m, 50 cm. For example:

**DDD** še bar **◻** *ba-ri-zu*  
'3 *gubar* (capacity units) and 1 *parīsu*'.

<sup>64</sup>The cuneiform text is Unicode-encoded.

<sup>65</sup>Compare  $\diamond\Box$ — in system [G<sub>UR III/OB</sub>](#). Sign order can be variable in early texts, see [Fox16, p. 8]. See [P010773], also discussed in [Fri07, p. 148], for an example of  $\Box\bullet$ , and [P274845; P241764] for examples of  $n\bullet\Box$ .

This is particularly true of the signs , ,  and , whose form explicitly denotes the fractions  $1/6$ ,  $2/6$ ,  $3/6$ , and  $4/6$  of the barig capacity measure written  in Mesopotamia—also transcribed by Assyriologists as 1 bán, 2 bán, 3 bán, and 4 bán with reference to the bán measure worth  $1/6$  of the barig. At Ebla, the sign  is most often associated with the *parisu* measure, while the signs , ,  and  refer to 1, 2, 3,

Figure 18: Discussion in [Cha12, p. 64] of the relation between  $\text{---}$  and  $\square$  in Mesopotamia and in Ebla.

shape. The principle of notation is additive: each sign is noted as many times as necessary (e.g.,  transliterated as 2(šar) 1(geš'u) 3(u), means  $2 \times 3600 + 1 \times 600 + 3 \times 10$ ). The system is based on an alternation of factors ten and

Figure 19: Explanation of the structure of the number **●○▷•** in [Pro20, p. 350].

might think of one fabric and a half,<sup>11</sup> but the presence of notations with “ $2\triangleright 2\blacktriangledown$ ”, “ $3\triangleright 3\blacktriangledown$ ”, and “ $6\triangleright 6\blacktriangledown$ ” (Fig. 1) elements excludes that one deals with fractions, as these notations are not consistent with those of Šuruppag’s weight measurement system.<sup>12</sup> The notation “ $1\triangleright$  gada” in o. ii 1 and r. vi 1, along with the total of “39



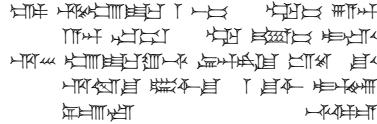
Fig. 1. Combinations of numerals attested in Š. 742.

Figure 20: Discussion of the contrast between  $\triangleright$  and  $\triangleright$  numerals in [Gor23, p. 162].

as, for example, in TM.75.G.3125 = ARET III 107 o. iv 1, “**4**  $\text{D}^{\text{a}_3\text{-da-um}^{\text{tu}^9\text{-2}}}$  **4**  $\text{D}^{\text{aktum} \text{4} \text{D}^{\text{tu}^9 \times 3} \text{T} \text{sa}_6 \text{ gunu}_3}$ ” (Fig. 2).

Figure 21: Transliteration in [Gor23, p. 163] of [P242293, recto 4 1] incorporating untransliterated numerals.

### 3.5 Non-numeric usage



The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'<sup>66</sup>. Do you know its reading<sup>67</sup>?

Examenstext A

Many of the cuneiform numerals are used with a logographic or phonetic value. For example, the sign — has, *inter alia*, the values aš, rum, and dili. While the horizontal numerals are most frequently written ▷ in the Early Dynastic period<sup>68</sup>, such non-numeric usage is almost<sup>69</sup> always written —, for instance:

- in personal names in administrative texts, such as the following, which all contain ▷ numerals:
  - — in [P010424, rev. 1 5; P010458, obv. 1 5; P010459, obv. 2 5'] from ED IIIa أبو صلبيخ,
  - — in [P010960, obv. 2 5] from ED IIIa Šuruppag,
  - — — in [P251641, obv. 4 3] from ED IIIb Adab,
  - — — in [P252866, rev. 2 3] from ED IIIb Adab,
  - — — in [P298637, rev. 2 4] from ED IIIb Umma;
- in the Sumerian word — u₂-rum, “property” in ED IIIb Nirsu administrative texts which contain ▷ numerals, such as [P020006, obv. 2 3; P020008, rev. 1 2; P020018, rev. 1 2; P020024, obv. 1 4; P020030, obv. 3 1];
- in lexical texts:
  - in the divine name — — — — in the lexical texts [P010570, rev. 2 4; P010572, obv. 3 6], where the entries are prefixed with ▷.
  - in the word — dili, “small fish” in [P010578, obv. 2 5], witness to Early Dynastic Fish,
  - in the same word with a determinative, — — dili<sup>ku</sup><sup>66</sup>, in [P010586, obv. 4 4, 6], witness to Early Dynastic Food, which starts with ▷ numerals.

This is a clear contrast between — and ▷ in this period, and genuine ambiguity can arise if it is lost; for instance, the personal name — — occurs on its own line in the aforementioned administrative texts; a line ▷ — would instead be read as “one slave”.

### 3.6 The limited benefits of diachronic encoding for numerals

The argument in favour of diachronic encoding is that it facilitates interoperability in a variety of use cases, as we have outlined in §3.1. While these benefits are real and

<sup>66</sup>The reader will recall that neš<sub>2</sub> is written !, with a larger wedge than !; however, these signs have merged by the time Examenstext A is composed.

<sup>67</sup>Besides neš<sub>2</sub>, a look at [OSL] shows that the values diš, ge<sub>3</sub>, makkaš, sajtak<sub>4</sub>, and tal<sub>4</sub> are attested both in [ePSD2] and in lexical lists. The sign is also used for the Akkadian word *ana* in the Neo-Assyrian period.

<sup>68</sup>A [CDLI] search for "(asz@c)" finds 3296 ED texts, while a search for "(asz)" finds 81 ED texts, of which 46 also contain "(asz@c)".

<sup>69</sup>Exceptions are discussed in §3.7.1.

now visible for cuneiform signs, similar considerations are not generally applicable to curviform numerals.

Diachronic reference works such as sign lists and dictionaries tend to not include numbers, or when they do, they treat them separately, and include signs such as  $\text{—}$  that have both numeric and non-numeric values in both the main list and the section on numbers. For instance, [Sch35, pp. 123 sqq.] lists all of  $\text{—}\text{—}\text{—}\text{—}\text{—}$  together with  $\text{—}\text{—}\text{—}\text{—}\text{—}$ , while  $\text{—}$ ,  $\text{--}$ , and  $\text{---}$ , and only those, appear at the beginning of the sign list, since they have non-numeric values<sup>70</sup>. [Cat13, p. 58] has the numeric signs  $\text{—}$ ,  $\text{—}$ ,  $\text{—}$ , whereas non-numeric  $\text{—}$  is at the beginning of the sign list, where its values *aš* and *rum* are listed. For signs with both non-numeric and numeric usage, [Dei22] writes *s. die Zahlz.* throughout the main list; LAK 1  $\text{—}$  thus reappears at LAK 829 together with  $\text{—}$ ,  $\text{—}$ , and  $\text{—}$ . One should note [Bor10], which has numbers throughout the sign list; but that sign list does not show glyphs predating the Old Babylonian period, nor does it comprehensively cover the numerals used in the Ur III and Old Babylonian periods, as, for instance, it does not have  $\text{—}\text{—}\text{—}\text{—}\text{—}$  used in system *G<sub>Ur III/OB</sub>*.

Composite texts rarely have witnesses both from the Early Dynastic period and later; the kinds of texts that do, chiefly lexical and literary texts, do not contain numbers to the extent that administrative texts do. Further, there tend to be changes<sup>71</sup> to the text between Early Dynastic and later witnesses that prevent a diachronic encoding of such composites. For numerals, the switch from  $\text{—}$  to  $\text{—}$  numerals prevents diachronic encoding even if  $\text{—}$  were unified with  $\text{—}$ . For instance, the lexical list Early Dynastic Food, already mentioned in §3.5, contains some numbers, and has a witness from the Old Akkadian period covering these numbers: [P215653, a 1'-6']; however, they are written with  $\text{—}$  numerals, whereas they are written with  $\text{—}$  numerals in the Early Dynastic witnesses; since  $\text{—}$  and  $\text{—}$  are distinct<sup>72</sup> characters, the  $\text{—}\text{—}$  unification does not help.

More generally, since numbers are so deeply tied to metrology, and since metrological systems change between the Early Dynastic and later periods<sup>73</sup>, there is little opportunity for a diachronic representation of numeric quantities.

In the case of analyses such as [Rom23, *sub “Adding Corpora”*], it is interesting to note that numeric expressions are removed prior to the conversion of the corpus to Unicode cuneiform for further analysis.

---

<sup>70</sup>Non-numeric values of  $\text{—}$  were discussed in §3.5;  $\text{--}$  has the values *man<sub>3</sub>* and *min<sub>5</sub>*, and is used for the word *didli*, “several, various”;  $\text{---}$  has the value *eš*.

<sup>71</sup>Compare, e.g., in the *Instructions of Šuruppak*, / in the ED IIIa witness [P222243, obv. 2 7], also discussed in §3.7.1, and in the OB composite [Q000782, 6] (translated “Šuruppak gave instructions to his son” in [ETCSL, t.5.6.1, 1-13]). It does not matter for the construction of a composite text whether this is encoded  $\text{—}\text{—}$  or  $\text{—}\text{—}$ , since that word is absent from other witnesses, and since the surrounding words differ.

<sup>72</sup>Besides the contrasts in numeric usage mentioned in §3.3.3, these (already-encoded) characters were clearly not unifiable because of the many contrasts in non-numeric usage between them; several values of  $\text{—}$  which are not shared with  $\text{—}$  have already been mentioned, but perhaps most striking is the fact that, in the Neo-Assyrian period,  $\text{—}$  is used for the preposition *ina*, “in”, and  $\text{—}$  for the preposition *ana*, “to”.

<sup>73</sup>See, e.g., [Pow87, p. 493; Rob08, p. 55] on the unification of metrologies in the Old Akkadian period, resulting in the systems described in §3.3.

## 3.7 Compatibility considerations

A disunification twenty years after the fact, affecting all numerals, would ordinarily be a serious compatibility issue. Fortunately, with the exception of one character discussed below, we are not aware of any font using curviform glyphs for the already-encoded numerals. In fact we are not aware of any font designed for a style earlier than Old Babylonian, except for fonts mimicking the representative glyphs from the code charts, which are primarily Ur III, but sometimes earlier or later, as described in [UTR56, §2.4]. The lack of dedicated Ur III fonts may be explainable by the chart-like fonts<sup>74</sup> being good enough for most purposes; the lack of Early Dynastic fonts, by the aforementioned issues with numeral unification making the representation of any text with numerals intractable.

### 3.7.1 The case of ŠAR₂

The character U+122B9 ● CUNEIFORM SIGN SHAR2 has a circular reference glyph.

In most texts from the Early Dynastic IIIb and Old Akkadian period<sup>75</sup>, a contrast between non-numeric šar₂ written ◇ and numeric 1(šar₂) written ● can be observed, similar to the contrast between — and ▷ previously discussed in §3.5. However, in lexical lists from Šuruppak and Ebla<sup>76</sup>, as well as in the *Stèle des vautours*, non-numeric šar₂ is curviform:

- \* 𒂗𒀭● and \* 𒂗●▷ in [P010566, obv. 10 10, 11];
- ●▷ and \* ●▷ in [P010576, rev. 3 16, 17];
- ●▷ in [P240986, recto 3 3]<sup>77</sup>;
- ●▷ in [P222399, obv. 17 9, 18 11, 22 12]<sup>78</sup>.

It would be disruptive to the diachronic representation of text if non-numeric šar₂ were to have two different representations. The character U+122B9 CUNEIFORM SIGN SHAR2 should therefore be used in those cases, with its curviform glyph ◇, identical to the glyph of the proposed U+12579 ● CUNEIFORM NUMERIC SIGN ONE N45. Since the archaizing style of texts wherein non-numeric šar₂ is curviform solidly predates the transition from ● to ◇ in the relevant metrological systems, there is no need to represent a ◇-● contrast, so these characters can have the same glyph in specialist archaizing Early Dynastic fonts.

Since cuneiform U+122B9 CUNEIFORM SIGN SHAR2 effectively merges with U+1212D ◇ CUNEIFORM SIGN HI, the reference glyph should remain as it is, *i.e.*, curviform, so that the contrast between reference glyphs within the Cuneiform block remains clear; see [UTR56, §2.4]. Since system fonts follow the reference glyphs, and since extant specialist fonts target styles where U+122B9 is unambiguously cuneiform, there are no compatibility issues.

<sup>74</sup>Most prominently Noto Sans Cuneiform, a system font on both Windows—as part of Segoe UI Historic—and macOS.

<sup>75</sup>For example, in personal names:

- 𒈗𒀭◇𒈗 in [P020019, rev. 1 2] from ED IIIb Nirsu;
- 𒈗𒀭◇𒈗 in [P020182, obv. 2 9], also from ED IIIb Nirsu;
- ▷\*◇ in [P222186, obv. 3 3] from ED IIIb Umma;
- 𒈗\*◇ in [P235312, obv. 16] from Old Akkadian Umma.

<sup>76</sup>These are archaizing in other ways, *e.g.*, they have a 𒉩-𒉪 (NAM₂-TUG₂) split.

<sup>77</sup>From copy in [Man81, ELLES 397].

<sup>78</sup>Note however \*▷◇ in [P222399, obv. 6 17], see Figure 22. Curviform non-numeric šar₂ is clearly archaizing in ED IIIb Nirsu; one might suppose that the scribe slipped into their modern ways here.

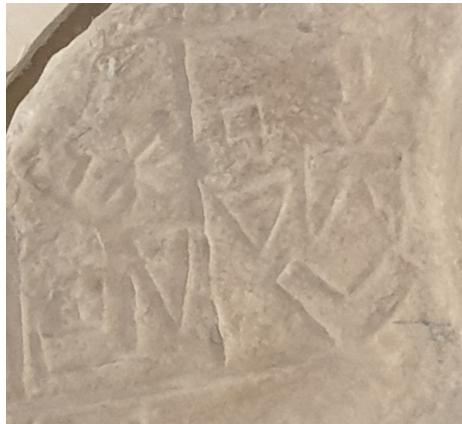


Figure 22: [P222399, obv. 6 16-17]  Σ  / \*  ◇ .

Note that in rare cases, such as [P222243, obv. 2 7] from ED IIIa Adab, non-numeric — (here with the value rum) is written ▷. It is out of scope for this proposal to decide whether such occurrences should be treated as anomalous spellings, encoded as U+12550 ▷ cuneiform numeric sign one N01, or as stylistic distinctions, encoded as U+12038 CUNEIFORM SIGN ASH with a curviform glyph. In practice this would often be determined by the transliteration from which the cuneiform text is generated; it is noteworthy that as of this writing, the [CDLI] transliteration (UR2-1(aš@c)) and the [ePSD2] one ( $uru_8^{rum}$ ) of this word disagree on that aspect. Since — has a cuneiform reference glyph, this does not pose any compatibility concerns.

### 3.7.2 Transliteration

An important feature of the encoding is that, in order to support input and bulk conversion of transliterated corpora to Unicode cuneiform, it should not represent distinctions that are finer than those recorded in typical transliterations; thus, while some older forms of BIL<sub>2</sub> can be described as 𒂔×× NE×KASKAL or 𒂔×× NE×PAP<sup>79</sup>, they are typically all transliterated bil<sub>2</sub>, and therefore are all represented by the character U+1224B 𒂔 CUNEIFORM SIGN NE SHESHIG, its name notwithstanding, as described in [UTR56, §2.5].

The situation is more complicated for numbers. Many transliterations do not represent the type of numeral used, instead interpreting the whole numeric expression and transcribing it with delimiters or units as needed to disambiguate. For instance, ፩፻፻፻ from [P305639, rev. 21] may be transliterated as 95 gur, as in [Feu04, vol. 2, p. 62]. The numerals may also be transliterated separately, but solely by their values in terms of the overt unit, as in [EbDA] transliterations: the aforementioned ፩፻፻፻፻ from [P240533, recto 3 3] is transliterated “20-1-1/2 *gu<sub>2</sub>-bar* 7 nig<sub>2</sub>-sagšu 2-1/2 an-zam<sub>x</sub><sup>80</sup> za”, reading both ፻ and ፻ as 1/2, but not distinguishing them.

<sup>79</sup>As on [P249253].

<sup>80</sup>As of this writing, [EbDA] actually has an-zam<sub>χ</sub>, with U+1D6A GREEK SUBSCRIPT SMALL LETTER CHI.

In particular, these transliterations do not differentiate between — and ! numerals, nor between ▷ and ◁ numerals. For instance, the aforementioned [P242293, recto 4 1] BB 𒉣 𒉤 𒉦 𒉪 𒉨 𒉩 𒉪 𒉨 𒉩 is transliterated “4 ‘a<sub>3</sub>-da-um<sup>tug<sub>2</sub></sup>-II 4 aktum<sup>tug<sub>2</sub></sup> 4 ib<sub>2</sub>-III gun<sub>3</sub> sa<sub>6</sub><sup>tug<sub>2</sub></sup>” in [EbDA], with no distinction between the BB and 𒉩. Since — and ! numerals are separately encoded, the numeric expressions in such transliterations cannot be transformed into Unicode cuneiform without additional context, regardless of curviform–cuneiform unification.

While there exist transliterations that distinguish  $\text{—}$  from  $\text{I}$  but not  $\text{BB}$  from  $\text{—}$ , such as the ones used in [DCCM<sup>T</sup>], the trend, especially in more recent works in third millennium studies, seems to be to represent numeral shape; for example, [MV24] gave an example of the input syntax used by the new “Urban Economy Begins” project as “10 + 5c(GUR) + 2(BARIGA) + 1(BAN2)” for  $\bullet\text{BBP}\text{I}\text{+}$ , with a c indicating that the GUR numerals are curviform, and the parenthetical GUR indicating that these are  $\square$  rather than  $\diamond$  numerals. The “tradition of cavalierly dispensing with numerical notations in notations of administrative documents”, as [Eng04, p. 30] describes it, seems to be fading.

### 3.8 Conclusions

Co-occurrences of curviform and cuneiform numerals are not anecdotal in the Early Dynastic period, nor are they the result of scribal idiosyncrasy. Instead, they represent systematic contrasts between metrological systems, between individual units within metrological system, and between numeric usage and phonetic or logographic usage. This contrastive usage is reflected in modern publications. The contrast frequently applies to individual numerals, rather than to the span of entire numeric expressions.

While it would be technically possible to handle this contrast as a stylistic distinction, this approach has no real benefit, and is highly inconvenient, as it would require any treatment of Early Dynastic administrative texts to use multiple

<sup>81</sup> [ ] interpreted as a unit, as discussed in §3.3.

<sup>82</sup>short for nigida, an older reading of bariga; see [Landsberger1950; Pow75, p. 181; Fox22, p. 9].

cuneiform fonts, often within single numeric expressions. Further, if that contrast is lost in plain-text interchange, the text can be misinterpreted: 𒃲 is a length of three ropes, but 𒃲• is an area of three bur<sub>3</sub>; 𒃲! could be read as one 𒃲 𒃲! and one 𒃲 𒃲!, where 𒃲! would be one and a half 𒃲 𒃲!; — 𒃲! is a personal name, but 𒃲 𒃲! would be “one slave”.

In addition, there would be a risk of confusion about character identity should fontmakers attempt to treat the curviform and cuneiform numerals as unified. A designer concerned about the numeric-syllabic 𒃲— 𒃲 contrast, and wishing to support diachronic encoding between systems *S<sub>Ur III/OB</sub>* and *S*, might give the 𒃲 numeral series (which is typically only used numerically in the Early Dynastic period) the glyphs of the 𒃲 numeral series, since the clear 𒃲— 𒃲 identification involves the same rotation; this would however make it impossible to represent capacity measures that use 𒃲. Similarly, in an effort to support diachronic encoding for 1/2(iku), one might be tempted to give 𒃲 the glyph of 𒃲, thereby rendering the font unusable for quantities measured using the 𒃲 numeral series; an ED I-II Ur font designer could decide to give 𒃲 the same glyph as 𒃲 (that of the proposed ●), according to the older area system, making it impossible to represent the newer system.

At the same time, contrary to most disunifications, the separate encoding of curviform numerals poses no serious compatibility issues for existing fonts or encoded corpora, nor does it, in general, introduce new issues with transliterated third millennium corpora. The oddity of ● requires some explanation, but does not pose any architectural issues, and is not fundamentally different from the other mergers and splits encountered in the cuneiform script.

## 4 Rationale for ED-Uruk numeral unification

A complete rationale for disunification between the non-numeric signs used in the fourth millennium and the already-encoded cuneiform signs will be given in the forthcoming proto-cuneiform encoding proposal. The core issue with extending the cuneiform script further back in time is that, since 1987, fourth millennium studies have used a different model of character identity and associated transliteration conventions, with names being given to structurally different glyphs, and no attempt being made at assigning phonetic values to them.

This is not a mere classification of glyph variants, as contrastive meanings of these systematic variants can often be reconstructed, with, *e.g.*, signs KAŠ<sub>a</sub>, KAŠ<sub>b</sub>, and KAŠ<sub>c</sub>, depicting filled jars with a spout (a), a handle (c), or neither (b), being understood as referring to containers of different substances, see [Eng01, pp. 34 sq.]. However, not all identified systematic variants are understood, and the general approach to character identity is closer to that used for undeciphered or partially deciphered scripts.

As part of the development of these conventions, a classification of fourth millennium numeric signs was developed; see [DE87]. This classification assigns to each unit numerals an identifier formed by the letter *N* with a numeric subscript (sometimes with an additional alphabetic subscript): *N*<sub>1</sub> is 𒃲, *N*<sub>14</sub> is •, *N*<sub>34</sub> is 𒃲, etc. Transliterations of numeric expression then use those to identify the type of number used, thus 5*N*<sub>1</sub> is ܰܰܰܰܰܰ, and 5*N*<sub>14</sub> is ܰܰ•ܰܰ•ܰܰ•ܰܰ•ܰܰ.

In contrast with the use of parenthetical unit names, this approach does not require interpreting the quantity being counted. This is valuable in contexts where

numerals are being used atypically, as conventional transliterations can otherwise force a dubious interpretation. For instance, the [CDLI] transliteration of or in [P283802, rev. 1 6, 2 2] currently uses (barig@c) for the vertical numerals, since  $\square$  numerals are typically capacity measures; but [Gor23] interprets these instead as counting linen textiles. As a result, the fourth millennium conventions for numeral transliteration are used in Early Dynastic texts, especially those from the ED I-II period, even though the Sumerian text uses classical assyriological transliteration conventions; see [Chao3, p. 6 n. 27].

While the non-numeric signs are treated as undeciphered, the metrological systems used in the fourth millennium are well understood, as can be seen in [DE87, p. 165]. As a result, contrary to the non-numeric proto-cuneiform conventions, these numeric transliteration conventions are compatible with the classical ones described in §3.7.2; they are indeed used interchangeably, as in [Po11104] which uses the notation u@f in [ePSD2], but N14@f in [CDLI]. Indeed, the numerals are used similarly in Early Dynastic metrological systems, and are visually identical.

A disunification of numerals between the third and fourth millennium would therefore induce confusion as to which numerals should be used in third millennium studies, and would needlessly duplicate the encoding of at least seventy characters; by splitting the attestations, these separate encoding proposals would run into additional difficulties to supply evidence for encoding.

Note that the structural variants designated by letters in fourth millennium notation have systematically been encoded, as they have occasionally been found to carry distinct numeric meaning. For instance,  $N_{30c}$  is listed as a variant of  $N_{30a}$  in [DE87, p. 166], where the numeric value of either in relation to  $\square N_{39a}$  is still unknown, but their values are found in [Eng04, p. 33] to be  $\square = \frac{1}{10} \square$ , whereas  $\square = \frac{1}{6} \square$ .

## 5 Considerations on individual numeral series

Usages of the characters U+12550–U+12597, under subheadings “Common Numerals”, “Numerals used for land areas”, and “Early Dynastic capacity measures”, have already been discussed in §3.4. The variant forms of fractions of the iku are not unifiable with the ordinary ones:  $\ddot{\square}$  is never used as a capacity measure, nor as one half in any other metrological system, contrary to  $\square$ .

The  $\frac{1}{2}$  and 1 ban<sub>2</sub> usages of  $\ddot{\square}$ , contrary to the disunification of U+12226  $\ddot{+}$  CUNEIFORM SIGN MASH and U+1244F  $\ddot{+}$  CUNEIFORM NUMERIC SIGN ONE BAN2. This disunification is motivated by the unrelated origins of maš (logographic, meaning “goat”), always resembling  $\ddot{+}$ , and 1 ban<sub>2</sub>, descended from  $\ddot{\square}$ . One could argue that based on their etymologies, U+1244F would make more sense as the sign used for  $\frac{1}{2}$ , but U+12226 is used as the transliteration MAŠ is frequent, see, e.g., [Hue11, p. 165].

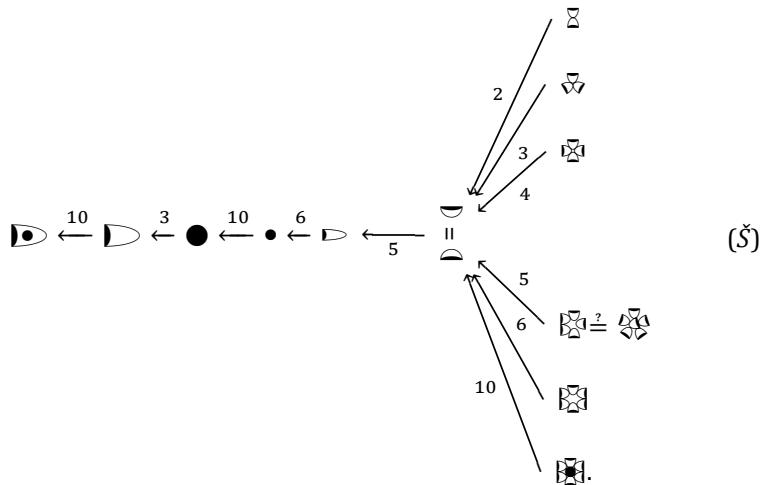
The signs U+12598 and U+12599 are used in the ED IIIb Nirsu weight system for fractions  $\frac{1}{3}$  and  $\frac{2}{3}$  of a shekel, with the already-encoded U+1245D  $\ddot{\times}$  and U+1245E  $\ddot{\times}$  used for fractions of a mina, see [Lec16]. Note that as usual, the description  $\square \times (\square + n\square)$  must be understood as allowing for free variation between  $\square$ , , and  $\times$ , the last one being the description in [Lec16]. Compare discussed in [UTR56, §2.5], šešsig =  $\times$   $\times$  =  $\times$  (the last one in, e.g., Ebla lexical texts).

The characters U+12550–U+12597 are used in the bisexagesimal counting system, whose factor diagram is as follows [Fri78, p. 15; DE87, p. 165; NDE93, p. 28], with □ being the unit:

$$\bullet \leftarrow \text{D} \leftarrow \text{C} \leftarrow \text{B} \leftarrow \text{A} \leftarrow \text{S}. \quad (B)$$

It is well attested in the fourth millennium, but is also attested in Early Dynastic IIIa Šuruppag. The reference glyph for 𒊩 is based on the design in [DE87; NDE93], rather than the one in [Eng04; Eng23], as the latter requires the use of grey, whereas the earlier one is black and white.

The characters U+12597–U+125B0, U+125B6–U+125BD are used in the grain capacity system [DE87, pp. 136–139, 165; NDE93, p. 28; Eng01, p. 4; Eng04, pp. 33, 39]:



The signs U+125B1–U+125B5 are listed together with them in [Eng01, p. 29] under “dry cereal products and rations: numerical signs in ideographic use”.

The characters U+125BE–U+125D0 are used in a variant of system *S* used to count dead animals, see [DE87, p. 139; NDE93, p. 28; Eng04, p. 40 n. 23]:

$$\text{D}\leftarrow \epsilon\text{D} \quad (S')$$

The highest attested number in this system is  , 306 dead sheep, in [P006365].

The characters U+125D1–U+125E9 are used in a variant of the bisexualesimal system, see [DE87, p. 165; NDE93, p. 28].

The characters U+125EA–U+125FC are used in a variant of system Š probably used to measure malted barley, see [DE87, p. 139; NDE93, p. 29; Engo1, p. 17 n. 30]. The characters U+125FD–U+12621 are used in a variant of system Š probably used to measure emmer, see [DE87, p. 140, p. 155 n. 67; NDE93, p. 29]. The characters U+12622–U+12641 are used in a variant of system Š probably used to measure barley groats, see [DE87, p. 141; NDE93, p. 29; Engo1, p. 3 n. 7, p. 17 n. 30].

The characters U+12642–U+1264A are used in a system whose function is unknown, attested only in the Uruk IV period, see [DE87, pp. 143 sq.; NDE93, pp. 27 sq.].

The “flat” characters U+1264B–U+12685 are used in various metrological systems in ED I-II Ur, see the factor diagrams in [Chao3, pp. 4 sq.].

## 6 Characters not included in this proposal

Some numerals previously proposed in [L2/23-190], as well as some other numerals known to exist in the third millennium, are not included in this proposal. Some should be proposed at a later date; others are likely not encodable.

### 6.1 Fourth millennium numerals

The following eight numeral series from [DE87, p. 166] are not included; they are all listed as under *Nichteinordnbare Zahlzeichen* in [DE87, p. 147]. On these (and two others, possibly  $N_{57}$  and  $N_{58}$ ), [NDE93, p. 27] write “Ten of the sixty numerical signs contained in the list in figure 27, moreover, do not belong to any of the identified systems. Three of them were apparently scribbled by an awkward pupil. As to four of those remaining, we are not sure whether they constitute derivations of other, as yet unknown numerical signs or whether they are in fact numerical signs at all. For at least two of the ten signs, [ $N_{23}$ ] and [ $N_{43}$ ], we can affirm that each formed part of two additional systems, about which we know nothing due to the fact that no informative texts have been unearthed with notations in these systems.”

- $N_{13}$   [DE87, p. 147] mentions [P002551] as the only attestation. [CDLI] now transliterates this 2(N04), not in [Eng23]. Presumably representable as a rotated , as here.
- $N_{16}$  and  $N_{17}$ . Described as “*vermutlich mit ideographischer Funktion*” in [DE87, p. 147].  $N_{17}$  is only attested in [P000524], a witness to [Q000028].  $N_{16}$  is attested in similar context in the same [P000524], but also in other artefacts, including [P283918] and [P283919] where notes by Englund in the [CDLI] transliteration suggest it is numeric, equal to  $\frac{1}{20}$ .  $N_{16}$  should probably be encoded in the Archaic Cuneiform Numerals block.
- $N_{23}$ . In the current transliterated [CDLI] corpus, attested in one Uruk V artefact, and four Uruk IV artefacts. Similar in shape to proto-Elamite  $N_{23}$ , which is well-attested (53 artefacts) and well-understood (part of a decimal system, where it means 100). Best encoded as part of a proto-Elamite proposal, where we would clearly have 1–9, and given appropriate Script\_Extensions.
- $N_{43}$ ; according to [DE87, p. 147], probably part of a variant of system .
- $N_{44}$ ,  $N_{53}$ , and  $N_{55}$ : Only attested in [P003855], which contains no other text. Presumably these are the “awkward pupil” signs.

In addition, the following are not included:

- $N_{10}$ . Only attested in [P001319] according to [DE87, p. 143], but that text now has  $N_{11}$  in its [CDLI] transliteration. Not in [Eng23].
- $N_{57}$  and  $N_{58}$ . Dependent on the main proto-cuneiform proposal, whose rationale will justify the disunification from — and !. These should be encoded in a different block to avoid confusion with — and !, since the Archaic Numerals block contains numerals unifiable between Pcun and Xsux.
- $N_{59}$ . Possibly a variant of  according to [DE87, p. 147].

- *N<sub>30b</sub>*. Not attested in [CDLI] transliterations, not included in Englund's more recent works such as [Eng01, p. 29], nor in [Eng23].

The well-understood U<sub>4</sub> numerals, documented in [Eng88, pp. 136 sqq.] and listed in [Eng23; L2/23-190], have not been included as they are likewise dependent on the disunification of proto-cuneiform, and should be encoded in a different block to avoid confusion with 劄. Some additional numeral series from [Eng23; L2/23-190] are not included due to lack of documentation on their usage. In general, only numerals that are part of a well-understood metrological system have been included. In particular, numerals such as 12• (attested in [P200010]) have not been encoded, since the metrological systems involving • numerals should allow for a numeral beyond ܰܰܰܰ (9•).

In addition, numerals that are not attested have not been included, unless they are part of a series where higher numerals in the metrological system are attested; thus the unattested  and , which are not in [Eng23], are included, because  is attested in [P006365], in a context where it is clearly used as part of system  $S'$ . However, 1(N30C~b), which is in [Eng23], and is the obvious counterpart of  in system  $\check{S}''$ , therefore presumably equal to  $\frac{1}{10}$  , is not included, as it is not attested in [CDLI] transliterations at this time.

## 6.2 Third millennium numerals

The metrological systems from Early Dynastic IIIb Nirsu discussed in §3.4 require the numerals 7 $\text{𒂗}$ , 8 $\text{𒂘}$ , and 9 $\text{𒂙}$ , whereas only  $\text{𒂗}-\text{𒂘}$  are encoded.  $\text{𒂗}$  numerals are also used in dates. The higher numerals are less frequent, as subtractive notation is often used instead, e.g.,  $\text{𒂗}\text{𒂘}$  rather than 9 $\text{𒂗}$ , or  $\text{𒂗}\text{𒂘}\text{𒂗}\text{𒂘}\text{𒂗}$  rather than  $\text{𒂗}\text{𒂘}\text{𒂗}\text{𒂘}\text{𒂘}$ , which is presumably why they are not yet encoded. However, they are clearly attested and understood enough to be encodable. They should be encoded in the Cuneiform Numbers and Punctuation block.

Early Dynastic IIIb Nirsu regnal years use  $\text{—} \times \nwarrow$  numerals (1–9). These are extremely well attested: a [CDLI] search for "( |ASZxDISZ@t | )" finds 1482 artefacts, all ED IIIb, of which 1447 are from Nirsu. These could be encoded in the Cuneiform Numbers and Punctuation block; together with 7–9 $\nwarrow$ , this would fill the block. There are three attestations of  $\text{—} \times \nwarrow$  numerals for regnal years of 𒂵, 𒂷, 𒂸, 𒂹, one of which is damaged, and none of which have photographs. Absent further evidence, these seem unifiable with  $\text{—} \times \nwarrow$ .

Some Old Akkadian artefacts have  x  (LAK 824) or  x , which has recently been found to mean 6000. The [CDLI] transliterations do not appear to distinguish the curviform and cuneiform versions of these signs. These signs appear to be associated with 6 or 6<img alt="Cuneiform sign consisting of a vertical wedge with a horizontal stroke through it" data-bbox="795 695 825 725/>. Further collation<sup>83</sup> is needed to understand exactly what needs to be encoded.

[Chao3, p. 5; Chal2, p. 61] mention a sign resembling a mirrored and rotated  $\ddot{\tau}$ , thus  $\ddot{\tau}\ddot{\tau}$ . It is transliterated  $N_1'$  in [Chao3, p. 5], which documents a metrological system that uses it. However, we have not been able to locate this sign in the ED I-II Ur corpus nor in the Ebla corpus, as it is unclear how it is transliterated in [CDLI] or [EbDA]. Once found, it will likely be proposed for encoding in the Archaic Cuneiform Numerals block.

---

<sup>83</sup>In the assyriological sense, not the Unicode sense.



Figure 23: The layout of case [P011099, rev. 2 3]; the numeral ♃ is rotated to fit the rounded corner of the tablet.

## 6.3 Stacking patterns

The already-encoded numerals in the Cuneiform Numbers and Punctuation block distinguish some *stacking patterns*; for instance 9 $\dagger$  is encoded both as U+12446 𒃏 and as U+1240E 𒃐. This is in part due to contrastive usage of stacking patterns. For instance, besides 𒃏 and 𒃐 which are characteristic of bariga measures, four bariga is written 𒃒 even where 4 $\dagger$  is written 𒃔, as in [P255010, obv. 2 3, rev. 1 17; P292843, obv. 4, rev. 5]. Another contrast is that between the stacking patterns used in scratch calculations in the SPVS, often 𒃏 𒃒 𒃔 𒃓 𒃕 𒃖 𒃗 𒃘 𒃙 < 𒃒 𒃔 𒃓 𒃖 𒃗 𒃘 𒃙, and results in metrological systems, typically 𒃏 𒃒 𒃔 𒃓 𒃕 𒃖 𒃗 𒃘 𒃙 𒃙 < 𒃒 𒃔 𒃓 𒃖 𒃗 𒃘 𒃙, occasionally co-occurring as in [P142827; P142357]. This separate encoding is also for compatibility with distinctions made in reference works and in some non-numeric transliterations; for instance, 𒃔 is [Bor10, MZL 860] and has the value limmu, whereas 𒃒 is [Bor10, MZL 852] and has the value limmu<sub>5</sub>. Numeric<sup>84</sup> transliterations occasionally distinguish the stacking patterns 𒃔 𒃓 𒃖 𒃗 𒃘 𒃙 as in the [CDLI] transliterations of the aforementioned tablets, although this is rare; often 4(diš) is 𒃒 in Ur III, but 𒃔 in the Neo-Assyrian period.

However, the stacking patterns from earlier periods are not separately encoded; for instance, in ED IIIb Nirsu, « 2(u) often has one 𠂇 atop another. These older stacking patterns do not appear to be contrastive, are not marked in transliteration, and are not listed separately in sign lists nor assigned any different values. There is therefore no evidence of a need to encode them; instead, they should be considered style variants, and an ED IIIb Nirsu font should have an appropriate glyph for U+12399 « CUNEIFORM SIGN U U.

Likewise, many stacking patterns are attested for the curviform numerals proposed in this document, and it is not proposed to separately encode them. These distinctions would be incompatible with the state of the art in numeric transliterations, including those by Englund, who insisted on “a system of transliteration that reflects in a strict fashion the physical realities of the cuneiform inscriptions” [Eng04, p. 30], and they are not needed to represent reference works. Idiosyncratic stacking patterns are in fact particularly common in Early Dynastic and earlier tablets, as they are structured in rectangular cases rather than lines, so that numerals may be laid out across the case in whichever way fits the available space; this is illustrated in Figure 23. Note also that the numerals need to be considerably enlarged in order to reproduce the layout of the tablets, so that ❶ often spans two lines of cuneiform signs, as shown in Figure 24. This is impractical when these numerals are set in text that contrasts them with the larger ❷, and inconsistent with actual practice when typesetting these numerals, as illustrated in Figure 8: reproducing the layout of tablets is not within the scope of plain text.

The reference glyphs use stacking patterns that are common in the Early Dyn-

<sup>84</sup>The Sumerian word limmu means “four”, so limmu and limmu<sub>5</sub> are still numbers. The distinction here is between usage in transliterations of phrases such as 𒂗 𒀭 𒈾 𒄑 𒄑 𒄑 lugal-an-ub-da limmu<sub>5</sub>-ba-ke<sub>4</sub> (king of the four quarters) or of names, and of numeric expressions such as 𒄑 4 (diš) sila<sub>3</sub>.



Figure 24: The layout of case [P020066, obv. 11]; the numeral ● is spread across two lines. The text is read in the order ● 二十 二 牛 一 年 老, “twenty-two oxen, one year old”.

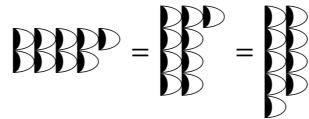


Figure 25: Three stacking patterns for U+12573 CUNEIFORM NUMERIC SIGN NINE N34. The one on the left is the reference glyph, used in Uruk III [P003499, obv. 1 1b; P004430, rev. 1 2], and widely afterwards, e.g., ED IIIa Šuruppag [P010678, obv. 2], ED IIIb Nirsu [P020057, obv. 1 3], Old Akkadian Umma [P212464, obv. 11]. The ones in the middle and right are used in two Uruk IV tablets [P001243, rev. P004500, rev. 2]. All three Uruk examples are transliterated 9(N34) in [CDLI].

astic period, but that are also attested in the fourth millennium in the Uruk III period; the fourth millennium, especially the Uruk IV period, also frequently features numerals that use a more vertical layout, as illustrated in Figure 25. The later, more horizontal styles were chosen for two reasons: for the numerals used in the third and fourth millennium, usage in third millennium scholarship will be more frequent; and the horizontal layout poses fewer layout difficulties when set in lines of non-cuneiform text, as most modern scholarship is. Indeed, the absolute size of the indents ▷, ▷, ●, and ● must remain consistent across the numeral series, lest a ▷ numeral be confused with an ▷ numeral. Since the single indents are frequently used in running text, as illustrated in §3.4.5, they need to be large enough that the vertical stacking patterns are impractical.

Variant stacking patterns, if needed, may be handled at a higher level as stylistic distinctions; Figure 25 uses OpenType stylistic alternates, and Figure 23 rotates the character ●, in both cases preserving the plain text backing.

$\text{IN}_{26}$	Uruk IV: <i>ATU</i> 5, pl. 32, W 8273 obv. i 1; pl. 34, W 9071.f obv. ii 1 (qualifying U <sub>0</sub> ); pl. 42, W 9169.c obv. i 5, ii 4 (?; qualifying DU <sub>0</sub> ), rev. i 1 (together with N <sub>34b</sub> , N <sub>24</sub> and N <sub>25</sub> ); pl. 108, W 9656.fm obv. i 1. Uruk III: <i>ATU</i> 2, pl. 57, W 16719 obv. i 4; <i>ATU</i> 5, pl. 1, W 5233.a obv. i 3; pl. 2, W 5233.b obv. i 3; pl. 2, W 5233.c obv. i 3; <i>BagM</i> 22, 143, W 24187 obv. i 3; W 16465 obv. i 4; W 17440 obv. i 4; <i>Archaic Bookkeeping</i> 42, fig. 38, obv. i 2a; <i>ATU</i> 1, 632 iv 4 (?); <i>MSVO</i> 4, 66 obv. i 3a.	$\text{IN}_{28a}$	Uruk IV: <i>ATU</i> 2, pl. 56, W 15920.a2 obv. i 1 (qualifying GAR); <i>ATU</i> 5, pl. 42, W 9169.c obv. ii 2 (?); Uruk III: <i>ATU</i> 3, pl. 88, W 16918.g obv. i 1 (possibly lexical); <i>Archaic Bookkeeping</i> 42, fig. 38, obv. i 4a; <i>MSVO</i> 4, 66 obv. i 5a.
------------------	--	-------------------	---

Figure 26: Variants of  $\text{\textcircled{N}}$  and  $\text{\textcircled{N}}$  from [Eng01, p. 31].

## 6.4 Other glyph variants not reflected in transliteration

In addition to stacking patterns proper, [L2/23-190, pp. 128 sq.] proposes separately encoding variant glyphs that are not distinguished in transliteration, but listed but listed together in [Eng23], thus proposing two characters for ܦܶ and two characters for ܵܶ. These are merely illustrative of a wide continuum of attested glyphs; there are additional variants, as shown in Figure 26, and a cursory search on [CDLI] will find many attestations with further variation in the same vein. They should not be encoded. Only the systematic structural variants, which have been distinguished in transliteration based on a suspicion of distinct semantics, should be encoded.

## Acknowledgements

TODO(egg): Something about the Vanséveren fonts; acknowledge the reviewers and whoever referred me to relevant literature; something about the proposal font.

## References

### Artefacts

- [P001243] VAT 14991. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P001243](#).
- [P003499] Excavation number W 20274,001.  
CDLI: [P003499](#).
- [P004430] IM 074345. Baghdad, Iraq: المتحف العراقي.  
CDLI: [P004430](#).
- [P004500] IM 134965. Baghdad, Iraq: المتحف العراقي.  
CDLI: [P004500](#).
- [P005468] IM 023426. Baghdad, Iraq: المتحف العراقي.  
CDLI: [P005468](#).
- [P010424] IM 067642. Baghdad, Iraq: المتحف العراقي.  
CDLI: [P010424](#).  
ORACC: [epsd2/P010424](#).
- [P010458] IM 081445. Baghdad, Iraq: المتحف العراقي.  
CDLI: [P010458](#).  
ORACC: [epsd2/P010458](#).
- [P010459] IM 081449. Baghdad, Iraq: المتحف العراقي.  
CDLI: [P010459](#).  
ORACC: [epsd2/P010459](#).
- [P010566] VAT 12760 +. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P010566](#).  
ORACC: [epsd2/P010566](#).  
ORACC: [dcclt/P010566](#).

- [P010570] VAT 12626. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P010570](#).  
ORACC: [epsd2/P010570](#).  
ORACC: [dcclt/P010570](#).
- [P010572] VAT 12644. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P010572](#).  
ORACC: [epsd2/P010572](#).  
ORACC: [dcclt/P010572](#).
- [P010576] VAT 12751 +. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P010576](#).  
ORACC: [dcclt/P010576](#).
- [P010578] VAT 12693. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P010578](#).  
ORACC: [dcclt/P010578](#).
- [P010586] VAT 12770. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P010586](#).  
ORACC: [dcclt/P010586](#).
- [P010678] VAT 12593. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P010678](#).  
ORACC: [dccmt/P010678](#).
- [P010773] Ist Š 0188. İstanbul, Turkey: İstanbul Arkeoloji Müzeleri.  
CDLI: [P010773](#).  
ORACC: [dccmt/P010773](#).
- [P010876] Ist Š 0648. İstanbul, Turkey: İstanbul Arkeoloji Müzeleri.  
CDLI: [P010876](#).  
ORACC: [epsd2/P010876](#).
- [P010960] VAT 12745. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P010960](#).  
ORACC: [epsd2/P010960](#).
- [P011099] VAT 12438. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P011099](#).  
ORACC: [epsd2/P011099](#).
- [P011104] VAT 12624. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P011104](#).  
ORACC: [epsd2/P011104](#).
- [P020006] VAT 04439. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020006](#).  
ORACC: [epsd2/P020006](#).
- [P020008] VAT 04430. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020008](#).  
ORACC: [epsd2/P020008](#).
- [P020016] VAT 04865. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020016](#).  
ORACC: [epsd2/P020016](#).

- [P020018] VAT 04800. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020018](#).  
ORACC: [epsd2/P020018](#).
- [P020019] VAT 04793. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020019](#).  
ORACC: [epsd2/P020019](#).
- [P020024] VAT 04795. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020024](#).  
ORACC: [epsd2/P020024](#).
- [P020030] VAT 04633. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020030](#).  
ORACC: [epsd2/P020030](#).
- [P020054] VAT 04731. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020054](#).  
ORACC: [epsd2/P020054](#).
- [P020057] VAT 04747. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020057](#).  
ORACC: [epsd2/P020057](#).
- [P020065] VAT 04639. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020065](#).  
ORACC: [epsd2/P020065](#).
- [P020066] VAT 04810. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020066](#).  
ORACC: [epsd2/P020066](#).
- [P020090] VAT 04609. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020090](#).  
ORACC: [epsd2/P020090](#).
- [P020092] VAT 04428. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020092](#).  
ORACC: [epsd2/P020092](#).
- [P020129] VAT 04713. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020129](#).  
ORACC: [epsd2/P020129](#).
- [P020137] VAT 04899. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020137](#).  
ORACC: [epsd2/P020137](#).
- [P020182] VAT 04405. Berlin, Germany: Vorderasiatisches Museum.  
CDLI: [P020182](#).  
ORACC: [epsd2/P020182](#).
- [P102305] X.3.139. Atlanta, Georgia, United States: Michael C. Carlos Museum,  
Emory University.  
CDLI: [P102305](#).  
ORACC: [epsd2/P102305](#).

- [P142357] YBC 01793. New Haven, Connecticut, United States: Yale Babylonian Collection.  
CDLI: [P142357](#).  
ORACC: [epsd2/P142357](#).
- [P142827] Ashm 1924-0667. Oxford, United Kingdom: Ashmolean Museum.  
CDLI: [P142827](#).  
ORACC: [epsd2/P142827](#).
- [P212464] WML unn 003. Liverpool, United Kingdom: World Museum.  
CDLI: [P212464](#).
- [P213162] PULiège 0028. Université de Liège.  
CDLI: [P213162](#).  
ORACC: [epsd2/P213162](#).  
ORACC: [dccmt/P213162](#).
- [P215653] AS 15375 21. Paris, France: Musée du Louvre.  
CDLI: [P215653](#).  
ORACC: [dcclt/P215653](#).  
Louvre Collections: [ark:/53355/cl010436723](#).
- [P220927] AO 13485. Paris, France: Musée du Louvre.  
CDLI: [P220927](#).  
ORACC: [epsd2/P220927](#).
- [P221266] AO 13825. Paris, France: Musée du Louvre.  
CDLI: [P221266](#).  
ORACC: [epsd2/P221266](#).  
Louvre Collections: [ark:/53355/cl010138527](#).
- [P221291] AO 13850. Paris, France: Musée du Louvre.  
CDLI: [P221291](#).  
ORACC: [epsd2/P221291](#).
- [P221305] AO 13864. Paris, France: Musée du Louvre.  
CDLI: [P221305](#).  
ORACC: [epsd2/P221305](#).
- [P221530] YBC 08446. New Haven, Connecticut, United States: Yale Babylonian Collection.  
CDLI: [P221530](#).  
ORACC: [epsd2/P221530](#).
- [P221531] YBC 08444. New Haven, Connecticut, United States: Yale Babylonian Collection.  
CDLI: [P221531](#).  
ORACC: [epsd2/P221531](#).
- [P221746] Erm 14039. Saint Petersburg, Russia: Государственный Эрмитаж.  
CDLI: [P221746](#).  
ORACC: [epsd2/P221746](#).
- [P221814] Erm 14045. Saint Petersburg, Russia: Государственный Эрмитаж.  
CDLI: [P221814](#).  
ORACC: [epsd2/P221814](#).

- [P221815] Erm 14046. Saint Petersburg, Russia: Государственный Эрмитаж.  
CDLI: [P221815](#).  
ORACC: [epsd2/P221815](#).
- [P222186] FLP 0003. Philadelphia, Pennsylvania, United States: Free Library of Philadelphia.  
CDLI: [P222186](#).  
ORACC: [epsd2/P222186](#).
- [P222243] OIM Aoo645 + OIM Aoo649a-i. Chicago, Illinois, United States: Institute for the Study of Ancient Cultures, West Asia & North Africa (formerly Oriental Institute) Museum.  
CDLI: [P222243](#).  
ORACC: [epsd2/P222243](#).
- [P222399] *Stèle des vautours.* BM 023580 (= AO 16109) + AO 00050 + AO 02436 + AO 02437 + AO 02438. Paris, France: Musée du Louvre.  
CDLI: [P222399](#).  
ORACC: [etcscri/Q001056](#).
- [P232278] *Gudea E.* AO 00006. Paris, France: Musée du Louvre.  
CDLI: [P232278](#).  
ORACC: [etcscri/Q001544](#).
- [P232280] *Gudea G.* AO 00007. Paris, France: Musée du Louvre.  
CDLI: [P232280](#).  
ORACC: [etcscri/Q001546](#).
- [P235312] USC 6710. Los Angeles, California, United States: University of Southern California.  
CDLI: [P235312](#).  
ORACC: [epsd2/P235312](#).
- [P240531] Excavation number TM.75.G.00265.  
CDLI: [P240531](#).  
EbDA: [1415](#).
- [P240532] Excavation number TM.75.G.00266.  
CDLI: [P240532](#).  
EbDA: [1324](#).
- [P240533] Excavation number TM.75.G.00267.  
CDLI: [P240533](#).  
EbDA: [1379](#).
- [P240545] Excavation number TM.75.G.00299.  
CDLI: [P240545](#).
- [P240548] Excavation number TM.75.G.00302.  
CDLI: [P240548](#).  
EbDA: [1350](#).
- [P240579] Excavation number TM.75.G.00341.  
CDLI: [P240579](#).  
EbDA: [1364](#).
- [P240597] Excavation number TM.75.G.00407.  
CDLI: [P240597](#).

- [P240609] Excavation number TM.75.G.00440.  
CDLI: [P240609](#).  
EbDA: [1378](#).
- [P240653] Excavation number TM.75.G.00535.  
CDLI: [P240653](#).  
EbDA: [1382](#).
- [P240654] Excavation number TM.75.G.00536.  
CDLI: [P240654](#).  
EbDA: [1383](#).
- [P240655] Excavation number TM.75.G.00537.  
CDLI: [P240655](#).  
EbDA: [1358](#).
- [P240675] Excavation number TM.75.G.00557.  
CDLI: [P240675](#).  
EbDA: [1371](#).
- [P240697] Excavation number TM.75.G.00579.  
CDLI: [P240697](#).  
EbDA: [1381](#).
- [P240964] Excavation number TM.75.G.01392.  
CDLI: [P240964](#).  
ORACC: [dccmt/P240964](#).  
EbDA: [3184](#).
- [P240986] Excavation number TM.75.G.01415.  
CDLI: [P240986](#).  
ORACC: [dcclt/P240986](#).
- [P241708] Excavation number TM.75.G.02143.  
CDLI: [P241708](#).  
EbDA: [3173](#).
- [P241764] Excavation number TM.75.G.02200.  
CDLI: [P241764](#).
- [P241904] Excavation number TM.75.G.02346.  
CDLI: [P241904](#).  
EbDA: [3183](#).  
ORACC: [dccmt/P241904](#).
- [P242293] Excavation number TM.75.G.03125.  
CDLI: [P242293](#).  
EbDA: [217](#).
- [P249253] *Code de Hammurabi.* Sb 00008. Paris, France: Musée du Louvre.  
CDLI: [P249253](#).
- [P251641] MS 2464. Oslo, Norway: Schøyen Collection.  
CDLI: [P251641](#).  
ORACC: [epsd2/P251641](#).
- [P252866] MS 3830. Oslo, Norway: Schøyen Collection.  
CDLI: [P252866](#).  
ORACC: [epsd2/P252866](#).

- [P255010] YBC 04698. New Haven, Connecticut, United States: Yale Babylonian Collection.  
CDLI: [P255010](#).
- [P271238] Anonymous.  
CDLI: [P271238](#).  
ORACC: [epsd2/P271238](#).
- [P274845] CUNES 50-08-001. Ithaca, New York, United States: Department of Near Eastern Studies, Cornell University.  
CDLI: [P274845](#).
- [P283802] Ist Š 0742. Arkeoloji Müzeleri.  
CDLI: [P283802](#).  
ORACC: [epsd2/P283802](#).
- [P292843] NBC 05385. New Haven, Connecticut, United States: Nies Babylonian Collection, Yale Babylonian Collection.  
CDLI: [P292843](#).
- [P298637] NBC 06978. New Haven, Connecticut, United States: Nies Babylonian Collection, Yale Babylonian Collection.  
CDLI: [P298637](#).  
ORACC: [epsd2/P298637](#).
- [P305639] YBC 04398. New Haven, Connecticut, United States: Yale Babylonian Collection.  
CDLI: [P305639](#).
- [P307255] YBC 06219. New Haven, Connecticut, United States: Yale Babylonian Collection.  
CDLI: [P307255](#).
- [P309594] YBC 08761. New Haven, Connecticut, United States: Yale Babylonian Collection.  
CDLI: [P309594](#).  
ORACC: [epsd2/P309594](#).
- [P386847] AO 06377. Paris, France: Musée du Louvre.  
CDLI: [P386847](#).
- [Q000782] *The instructions of Šuruppag*. Composite text.  
CDLI: [Q000782](#).  
ORACC: [epsd2/Q000782](#).  
ETCSL transliteration: [c.5.6.1](#); translation: [t.5.6.1](#).

## ISO and Unicode documents

- [ISO15924] ISO 15924/RA. “ISO 15924 Code Lists”. In: *Codes for the representation of names of scripts – Codes pour la représentation des noms d’écritures*. ISO 15924. <https://www.unicode.org/iso15924/codelists.html>.
- [L2/03-162] M. Everson and K. Feuerherm. *Basic principles for the encoding of Sumero-Akkadian Cuneiform*. 25th May 2003.  
UTC: [L2/03-162](#).  
ISO/IEC JTC 1/SC 2/WG 2: [N2585](#).

- [L2/03-393R] M. Everson, K. Feuerherm and S. Tinney. *Preliminary proposal to encode the Cuneiform script in the SMP of the UCS*. 3rd Nov. 2003.  
 UTC: [L2/03-393R](#).  
 ISO/IEC JTC 1/SC 2/WG 2: [N2664R](#).
- [L2/04-036] M. Everson, K. Feuerherm and S. Tinney. *Revised proposal to encode the Cuneiform script in the SMP of the UCS*. 29th Jan. 2004.  
 UTC: [L2/04-036](#).  
 ISO/IEC JTC 1/SC 2/WG 2: [N2698](#).
- [L2/04-099] L. Anderson. *Unification of Cuneiform Numbers*. 2004.  
 UTC: [L2/04-099](#).
- [L2/04-189] M. Everson, K. Feuerherm and S. Tinney. *Final proposal to encode the Cuneiform script in the SMP of the UCS*. 8th June 2004.  
 UTC: [L2/04-189](#).  
 ISO/IEC JTC 1/SC 2/WG 2: [N2786](#).
- [L2/12-208] M. Everson, C. Jay Crisostomo and S. Tinney. *Proposal for Early Dynastic Cuneiform*. 13th June 2012.  
 UTC: [L2/12-208](#).  
 ISO/IEC JTC 1/SC 2/WG 2: [N4278](#).
- [L2/23-190] A. Pandey. *Revised proposal to encode Proto-Cuneiform in Unicode*. 11th July 2023.  
 UTC: [L2/23-190](#).
- [L2/23-196] A. Pandey. *Proposal to encode Proto-Elamite in Unicode*. 18th Aug. 2023.  
 UTC: [L2/23-196](#).
- [L2/24-159] P. Constable, ed. *Minutes of UTC Meeting 180* (Redmond, 23rd–25th July 2024). 29th July 2024.  
 UTC: [L2/24-159](#).
- [UAX50] K. Lunde and K. Ishii, eds. *Unicode Vertical Text Layout*. Unicode Standard Annex #50. An integral part of *The Unicode Standard*. The Unicode Consortium.  
<https://www.unicode.org/reports/tr50/>.
- [Uni16] The Unicode Consortium. *The Unicode Standard*. Version 16.0.0. The Unicode Consortium, 10th Sept. 2024.  
 ISBN: 978-1-936213-34-4.  
<https://www.unicode.org/versions/Unicode16.0.0/core-spec/>.
- [UTR56] R. Leroy, ed. *Unicode Cuneiform Sign Lists*. Unicode Technical Report #56. The Unicode Consortium.  
<https://www.unicode.org/reports/tr56/>.

## Online corpora and related projects

- [CDLI] É. Pagé-Perron, J. L. Dahl, B. Lafont, J. Renn, R. K. Englund and P. Damerow, eds. *Cuneiform Digital Library Initiative*. 2000–.  
<https://cdli.mpiwg-berlin.mpg.de>.
- [DCCMT] E. Robson, ed. *The Digital Corpus of Cuneiform Mathematical Texts*. 2007–.  
 ORACC: [dccmt](#).

- [EbDA] L. Milano, M. Maiocchi, F. Di Filippo, R. Orsini, E. Scarpa, M. Surdi et al., eds. *Ebla Digital Archives*. 2007–. <http://ebda.cnr.it/>.
- [eBL] E. Jiménez, Z. Földi, A. Hätinens, A. Heinrich, T. Mitto, G. Rozzi, I. Khait, J. Laasonen, F. Simonjetz et al., eds. *electronic Babylonian Library*. 2023–. <https://www.ebl.lmu.de/>.
- [ePSD2] S. Tinney, P. Jones and N. Veldhuis, eds. *The electronic Pennsylvania Sumerian Dictionary*. 2nd ed. 2017–. <http://oracc.org/epsd2>.
- [ETCSL] J. A. Black, G. Cunningham, J. Ebeling, E. Flückiger-Hawker, E. Robson, J. Taylor and G. Zólyomi, eds. *The Electronic Text Corpus of Sumerian Literature*. Oxford, 1998–2006. <http://etcsl.orinst.ox.ac.uk/>.
- [OSL] N. Veldhuis, S. Tinney et al., eds. *Oracc Sign List*. 2014–. <http://oracc.org/osl/>.

## Other documents

- [Arc15] A. Archi. *Ebla and Its Archives. Texts, History, and Society*. Studies in ancient Near Eastern records 7. Walter de Gruyter, 2015.  
ISBN: 978-1-61451-716-0.  
DOI: [10.1515/9781614517887](https://doi.org/10.1515/9781614517887).
- [Arc89] A. Archi. “Tables de comptes eblaïtes”. In: *Revue d'assyriologie et d'archéologie orientale* 83.1 (1989). Ed. by P. Amiet and P. Garelli, pp. 1–6. ISSN: 0373-6032.
- [Bor10] R. Borger. *Mesopotamisches Zeichenlexikon*. Alter Orient und Altes Testament 305. Ugarit-Verlag, 2010.
- [Cap02] R. Caplice. *Introduction to Akkadian*. 4th ed. Editrice Pontificio Istituto Biblico, 2002.  
ISBN: 88-7653-566-7.
- [Cat13] A. Catagnoli. *La paleografia dei testi dell'amministrazione e della cancelleria di Ebla*. Quaderni di Semitistica 9. Università di Firenze, 2013.  
ISBN: 8890134054.
- [Chao3] G. Chambon. “Archaic Metrological Systems from Ur”. In: *Cuneiform Digital Library Journal* 2003.5 (23rd Dec. 2003). ISSN: 1540-8779. [http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003\\_005.html](http://cdli.ucla.edu/pubs/cdlj/2003/cdlj2003_005.html).
- [Cha12] G. Chambon. “Numeracy and Metrology”. In: *The Oxford Handbook of Cuneiform Culture*. Ed. by K. Radner and E. Robson. Oxford University Press, 18th Sept. 2012, pp. 51–67.  
ISBN: 9780199557301.  
DOI: [10.1093/oxfordhb/9780199557301.013.0003](https://doi.org/10.1093/oxfordhb/9780199557301.013.0003).

- [DE87] P. Damerow and R. K. Englund. "Die Zahlzeichensysteme der archaischen Texte aus Uruk". In: M. W. Green and H. J. Nissen. *Zeichenliste der archaischen Texte aus Uruk*. Archaische Texte aus Uruk 2. Gebr. Mann Verlag, 1987. Chap. 3, pp. 117–165. Repr.  
<https://cdli.mpiwg-berlin.mpg.de/files-up/publications/englund1987a.pdf>.
- [Dei22] A. Deimel. *Liste der archaischen Keilschriftzeichen von Fara*. Wissenschaftliche Veröffentlichungen der Deutschen Orient-Gesellschaft 40. J. C. Hinrichs'sche Buchhandlung, 1922.
- [Eng01] R. K. Englund. "Grain Accounting Practices in Archaic Mesopotamia". In: *Changing Views on Ancient Near Eastern Mathematics*. Ed. by J. Høyrup and P. Damerow. Berliner Beiträge zum Vorderen Orient 19. Dietrich Reimer Verlag, 2001, pp. 1–35.
- [Eng04] R. K. Englund. "Proto-Cuneiform Account-Books and Journals". In: *Creating Economic Order. Record-keeping, Standardization and the Development of Accounting in the Ancient Near East*. Ed. by M. Hudson and C. Wunsch. International Scholars Conference of Ancient Near Eastern Economies 4. CDL Press, 2004. Chap. 1, pp. 23–46.
- [Eng23] R. K. Englund. *Proto-cuneiform sign list*. 2023.  
[https://cdli-gh.github.io/proto-cuneiform\\_signs/](https://cdli-gh.github.io/proto-cuneiform_signs/).
- [Eng88] R. K. Englund. "Administrative Timekeeping in Ancient Mesopotamia". In: *Journal of the Economic and Social History of the Orient* 31.2 (1988).
- [Eng98] R. K. Englund. "Texts from the Late Uruk Period". In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 13–233.  
ISBN: 3-7278-1166-8.
- [Feu04] K. G. Feuerherm. "Abum-waqr and His Circle. A Prosopographical Study". PhD thesis. University of Toronto, 2004.
- [Fox16] D. A. Foxvog. "Introduction to Sumerian Grammar". In: *Cuneiform Digital Library Preprints* 2016.2 (4th Jan. 2016).  
<https://cdli.mpiwg-berlin.mpg.de/articles/cdlp/2.0>.
- [Fox22] D. A. Foxvog. "Elementary Sumerian Glossary (revised 2022)". In: *Cuneiform Digital Library Preprints* 2022.3.1 (11th Apr. 2022).  
<https://cdli.mpiwg-berlin.mpg.de/articles/cdlp/3.1>.
- [Fri07] J. Friberg. *A Remarkable Collection of Babylonian Mathematical Texts*. Sources and Studies in the History of Mathematics and Physical Sciences. Springer, 2007. Manuscripts in the Schøyen Collection Cuneiform Texts 1. Manuscripts in the Schøyen Collection 6.  
ISBN: 978-0-387-34543-7.
- [Fri78] J. Friberg. *A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and Proto-Elamite Semi-Pictographic Inscriptions*. The Third Millennium Roots of Babylonian Mathematics 1. Department of Mathematics, Chalmers University of Technology, 1978.

- [Fri79] J. Friberg. *The Early Roots of Babylonian Mathematics*. 2. Department of Mathematics, Chalmers University of Technology, 1979.
- [Fri86] J. Friberg. “Three Remarkable Texts from Ancient Ebla”. In: *Vicino Oriente* 6 (1986), pp. 3–25. ISSN: 0393-0300. The Early Roots of Babylonian Mathematics 3.
- [Fri87] J. Friberg. “Mathematik”. In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanuksabaš-Medizin. 1987–1990, pp. 531–585.
- [Gor23] F. Gori. “On Lapis Lazuli and Linen in Šuruppak Texts. An Analysis Through the Lens of Ebla Studies”. In: *Studia Eblaitica* 9 (2023), pp. 160–166. ISSN: 2364-7124.
- [Hue11] J. Huehnergard. *A Grammar of Akkadian*. 3rd ed. Brill, 2011. ISBN: 978-1-57506-941-8.
- [JJ24] T. Jauhiainen and H. Jauhiainen. “Advancing Cuneiform Text Dating Through Automatic Analysis”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 11th July 2024 14:00.
- [Kre98] M. Krebernik. “Die Texte aus Fāra und Tell Abū Ṣalābiḥ”. In: *Mesopotamien. Späturuk-Zeit und Frühdynastische Zeit*. Orbis Biblicus et Orientalis 160/1. 1998, pp. 235–427. ISBN: 3-7278-1166-8.
- [Lec12] C. Lecompte. “Des chiffres et des digues: à propos de deux textes présargoniques de Girsu et d'une notation numérique inhabituelle”. In: *Altorientalische Forschungen* 39.1 (Dec. 2012), pp. 81–86. DOI: [10.1524/aofo.2012.0006](https://doi.org/10.1524/aofo.2012.0006).
- [Lec16] C. Lecompte. “ED IIIb metrology: texts from Lagaš”. In: *CDLI:wiki. A Library of Knowledge of the Cuneiform Digital Library Initiative*. 12th Apr. 2016. [https://cdli.ox.ac.uk/wiki/doku.php?id=ed\\_iii\\_metrological\\_systems](https://cdli.ox.ac.uk/wiki/doku.php?id=ed_iii_metrological_systems).
- [Lec20] C. Lecompte. “The Measurement of Fields During the Pre-sargonic Period”. In: *Mathematics, Administrative and Economic Activities in Ancient Worlds*. Ed. by C. Michel and K. Chemla. Why the Sciences of the Ancient World Matter 5. Springer, 2020. Chap. 8, pp. 283–344.
- [Man81] P. Mander. “Lista dei segni dei testi lessicali di Ebla”. In: *Testi lessicali monolingui della biblioteca L. 2769*. Ed. by G. Pettinato. Materiali epigrafici di Ebla 3. Napoli: Istituto universitario orientale, 1981, pp. 285–382.
- [Mol14] M. Molina. *Sargonic Cuneiform Tablets in the Real Academia de la Historia. The Carl L. Lippmann Collection*. Real Academia de la Historia, 2014. ISBN: 978-84-15069-71-3.
- [MV24] M. Maiocchi and S. Volpi. “Reassessing Economic History in the Early Dynastic Period. Sources, Methods, and Perspectives within the frame of the “Urban Economy Begins” Project”. 69th Rencontre Assyriologique Internationale (8th–12th July 2024). 12th July 2024 16:00.

- [NDE93] H. J. Nissen, P. Damerow and R. K. Englund. *Archaic Bookkeeping. Early Writing and Techniques of Economic Administration in the Ancient Near East*. Trans. by P. Larsen. The University of Chicago Press, 1993.  
ISBN: 0-226-58659-6.
- [Oel22] J. Oelsner. *Der Kodex Hammu-rāpi*. dubsar 4. Zaphon, 2022.  
ISBN: 978-3-96327-008-6.
- [Pow71] M. Powell. "Sumerian Numeration and Metrology". PhD thesis. University of Minnesota, 1971.
- [Pow72] M. Powell. "Sumerian Area Measures and the Alleged Decimal Substratum". In: *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* 62.2 (1972), pp. 165–221. ISSN: 0084-5299.
- [Pow75] M. Powell. In: *Journal of Cuneiform Studies* 27.3 (July 1975), pp. 180–188. Rev. of H. Limet. *Étude de documents de la période d'Agadé appartenant à l'Université de Liège*. Bibliothèque de la Faculté de Philosophie et Lettres de l'Université de Liège 206. Paris: Les Belles Lettres, 1973.
- [Pow87] M. Powell. "Maße und Gewichte". In: *Reallexikon der Assyriologie und vorderasiatischen Archäologie*. Ed. by D. O. Edzard. Vol. 7 Libanukšabaš-Medizin. 1987–1990, pp. 457–530.
- [Pro09] C. Proust. "Numerical and Metrological Graphemes: From Cuneiform to Transliteration". In: *Cuneiform Digital Library Journal* 2009.1 (22nd June 2009). ISSN: 1540-8779.  
[http://cdli.ucla.edu/pubs/cdlj/2009/cdlj2009\\_001.html](http://cdli.ucla.edu/pubs/cdlj/2009/cdlj2009_001.html).
- [Pro20] C. Proust. "Early-Dynastic Tables from Southern Mesopotamia, or the Multiple Facets of the Quantification of Surfaces". In: *Mathematics, Administrative and Economic Activities in Ancient Worlds*. Ed. by C. Michel and K. Chemla. Why the Sciences of the Ancient World Matter 5. Springer, 2020. Chap. 9, pp. 345–395.
- [Robo8] E. Robson. *Mathematics in Ancient Iraq. A Social History*. Princeton University Press, 2008.  
ISBN: 978-0-691-09182-2.
- [Rob19] E. Robson. "Oracc metrology guidelines". In: *Oracc: The Open Richly Annotated Cuneiform Corpus*. 18th Dec. 2019.  
ORACC: <doc/help/editinginatf/metrology/metrologicaltables>.
- [Rob22] E. Robson. "Overview of Metrological Systems". In: *The Digital Corpus of Cuneiform Mathematical Texts*. 2022.  
ORACC: <dccmt/Metrology>.
- [Rom23] A. Romach. *Stylometric Analysis for Akkadian Cuneiform Texts*. 2023–  
<https://github.com/ARomach/Cuneiform-Stylometry>.
- [Rom24] A. Romach. "The Neo Assyrian Land Sale Documents from Dur-Katlimmu. A Stylometric Analysis of Their Scribal Features". 69th Rencontre Assyriologique Internationale (Helsinki, 8th–12th July 2024). 10th July 2024 12:00.

- [Sch10] W. Schramm. *Akkadische Logogramme*. Göttinger Beiträge zum Alten Orient 5. Universitätsverlag Göttingen, 2010.  
ISBN: 978-3-941875-65-4.  
DOI: [10.17875/gup2010-511](https://doi.org/10.17875/gup2010-511).
- [Sch35] N. Schneider. *Die Keilschriftzeichen der Wirtschaftsurkunden von Ur III*. Editrice Pontificio Istituto Biblico, 1935.
- [Svä+24] S. Svärd, M. Lorenzon, J. Töyräänuori, J. Valk, T. Alstola, E. Bennett, R. Uotila and T. Auranne, eds. *RAI 69 Abstracts*. July 2024.  
[https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäivätty\\_1.pdf](https://www.helsinki.fi/assets/drupal/2024-07/RaiAbstractBookAjoitettuJaPäivätty_1.pdf).