Archaic cuneiform numbers

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The Unicode Standard includes some cuneiform numbers: \[\frac{1}{1} \] 1-9(diš) and \[\frac{1}{1} \] 1-9(aš), \[\frac{1}{1} \] 1-5(u), \[\frac{1}{1} \] 1-9(neš₂), \[\frac{1}{1} \] 1-5(neš'u), etc., used in the Sumero-Akkadian Cuneiform script (ISO 15924: Xsux, Script property value long name: Cuneiform).

In the investigation that led to their encoding in Unicode Version 5.0, it was thought appropriate to unify these with the earlier curviform numerals -10 1-9(10 1-9(10 1-9(10 1-9(10 1-9(10 1-5) 1-5(10 1-5(10 1-5) 1-5(10 1-5(10 1-5) 1-5(10 1-5(10 1-5) 1-5(10 1-5(10 1-5) 1-5(10 1-5(10 1-5(10 1-5) 1-5(10 1-5(10 1-5(10 1-5) 1-5(10 1-5(1

In addition, these numerals will be needed for the representation of protocuneiform texts from the earlier archaic period. The non-numeric signs of protocuneiform (ISO 15924: Pcun) will be the subject of a separate proposal; we need only note here that the divergence between the approaches to character identity in modern scholarship requires that proto-cuneiform be disunified from cuneiform: proto-cuneiform is effectively treated as an undeciphered script. In contrast, the cuneiform encoding model is semantic, requiring an understanding of the text to correctly encode it.

The use of the curviform numeric signs is however understood, as we will discuss in Section 1; further, the conventions used for archaic numerals are also used when discussing ED numerals, see Section 5. As a result, the same numerals can be used when encoding archaic and ED texts, and in order to avoid issues ambiguities in representation when converting from transliteration, these should be unified. The overall picture of unifications and disunifications would be as follows:

	Uruk III & earlier	ED – Ur III	OB & later
Non-numeric signs	Future Pcun	Existing Xsux	
Numbers	This proposal	This proposal	Existing Xsux
		+ Existing Xsux	

1 Metrologies

开门 筆重 帶 宜 五点时 开口 笨蛋更加 有打

I want to write tablets: the tablet of 1 gur of barley to 600 gur; the tablet of 1 shekel of silver to 10 minas [...]

Edubba'a D

In order to explain why TODO:*n* more numerals are needed, it is useful to first recall why we have so many kinds of cuneiform numerals already.

As is well known¹ a sexagesimal place value system (SPVS) was used in Mesopotamia from the late third millenium onwards. One should bear in mind, however, that other systems were used; the SPVS was primarily used in calculations, with results being expressed in non-positional systems. The digits 1–59 of the SPVS have inner structure which is reflected in the encoding: the digits 1–9 are the individual characters !—\;; the multiples of ten (10–50) are <-\;, but the other digits 11–59 are sequences <!-\;; in effect the base-sixty digits are themselves written in base ten, with a different set of symbols for the tens place. This reflects the origin of the sexagesimal place value system; it derives from a non-positional system, hereafter the cuneiform discrete counting system S_{Ur III/OB}, which had different signs for the units !—\;; tens <-\;; sixties !—\; (with larger wedges than the units), six hundreds \.—\;; three thousand six hundreds \.—\; and thirty-six thousands \.—\; here

The relations between the values of the signs in the cuneiform discrete counting system may be summarized as follows, where the number over arrow indicates the multiple of the preceding sign (right of the arrow) corresponding to the following sign (left).

For example, the number $1729 = ((2 \times 10 + 8) \times 6 + 4) \times 10 + 9 = 28 \times 60 + 49$ would be written $\mbox{$\mathbb{K}$} \mbox{$\mathbb{H}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{H}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{H}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{H}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{H}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{H}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{H}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{H}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{K}$} \mbox{$\mathbb{H}$} \mbox{$\mathbb{K}$} \$

The discrete counting system was not the only non-positional system in use in the Ur III and Old Babylonian periods; different systems were in use depending on what was being counted or measured. For instance, field areas were measured using the following system, where for the named units we have provided the name of the unit in transliterated Sumerian, normalized Old Babylonian Akkadian, and the approximate metric equivalent:

Note that for the range of areas given above², this system does not use any symbols separate from the numerals for the individual units (*ubûm*, *ikûm*, *eblum*, and

¹See, e.g., The Unicode Standard, Version 16.0, Section 22.3.3 Non-Decimal Radix Systems, sub "Cuneiform Numerals".

²For areas smaller than a quarter *ikûm*, an overt unit is used, with 1 *mūšarum* (36 m²) written ¹ ≋□, equal to one hundredth of an *ikûm*, then sexigesimally subdivided in 60 □ (shekels). For areas greater

 $b\bar{u}rum$). The whole numeric expression for the area would be followed by the sign functioning as punctuation, but the numerals are tied to the metrology; thus a surface of 5 $b\bar{u}r\bar{u}$ 1 eblum 4 $ik\hat{u}$ (100 $ik\hat{u}$, 36 ha) would be written³ ≪ \leftarrow \rightleftharpoons \equiv . Contrast this with systems where the same numerals are used for different units, and overt units are used, as in "88 acres 3 roods 33 perches". Note also that the same signs are shared between multiple systems, with different relations; the ŠAR₂ sign \diamondsuit is equal to sixty times the U sign \lt in the area system, but to three hundred and sixty times \lt in the discrete counting system.

Another such system of note is the one for volumes,

$$\bigotimes \stackrel{10}{\longleftarrow} \bigotimes \stackrel{6}{\longleftarrow} \bigvee \stackrel{10}{\longleftarrow} \bigvee \stackrel{6}{\longleftarrow} \bigvee \stackrel{10}{\longleftarrow} \bigvee \stackrel{1}{\longleftarrow} \stackrel{1}{\longleftarrow} \stackrel{1}{\longleftarrow} \bigvee \stackrel{1}{\longleftarrow} \stackrel{1}{\longleftarrow} \stackrel{10}{\longleftarrow} \bigvee \stackrel{1}{\longrightarrow} \stackrel{1}{\longrightarrow}$$

This intertwining of units and numerals explains the large number of alreadyencoded numeral series:

- !-₩ used in S_{Ur III/OB} and the SPVS as well as with overt units;
- <-₩ used in G_{Ur III/OB}, of which <-₩ are also used in S_{Ur III/OB} and the SPVS as well as with overt units;
- ¶-₩₩ used in S_{Ur III/OB} and the SPVS;
- ← used in C as well as in the weight system;
- **+**, **‡**, **‡**, **⊉**, **数** used in TODO;
- I, I, II, II used in C—note the overlap with I–III;
- \prec and \rightleftarrows used in $G_{Ur III/OB}$.

than 3600 *būrū*, the ♦- and ♦-numerals are reused with a suffix 🖹 (gal, Sumerian: big), as follows:

³As in the surface of the field of **|** | **|** ★ **|** ★ **|** ★ **|** (Apisal) reported on P102305 r. 1.

⁴From P309594.

⁵A larger unit, the guru₇ (*karûm*, grain heap), is sometimes used instead, with ← **□/>□** □ □ (1 *karûm* = 3600 kurrū).

- 2 Arguments for curviform-cuneiform unification
- 3 Problems with unification: Early metrology
- 4 Problems with unification: Non-numeric usage

The beginning of the scribal art is a single wedge. That one has six pronunciations; it also stands for 'sixty'. Do you know its reading?

Examenstext A

- 4.1 The case of ŠAR₂
- 5 Compatibility with transliteration
- 6 The necessity of ED-Uruk numeral identification
- 7 Characters not included in this proposal
- 7.1 Missing numerals

ED 《十二年本本

7.2 Stacking patterns