Homework 2

Name:

Session (ROB 599 or ROB 498):

Reading Assignment:

YouTube Playlist: https://www.youtube.com/playlist?list=PLEYKx4BGriISagN3Ihc-9L6Cw44vtzTD9

Submission: Export this file as a PDF and submit to Canvas.

The following Videos are also requried to be submitted for this homework:

- "JointSpace"
- "TaskSpace_method1"
- "TaskSpace_method2"
- "ContactSim"

Deadline: 10/8/2025

- If you have any questions about the assignment or software, please use the Discussion function on Canvas to seek help!
- If your question was not addressed on Canvas, then reach out to GSI: Yulun Zhuang
- The live script for homework is still evolving, please check Canvas periodically to make sure you are working on the latest version

Written Assignment

Use LaTex for homework

Use LaTex syntax to anwer the written part of the assignment

Latex expressions can be brought up by pressing ctrl + shift + L in Windows or cmd + shift + L in Mac

Table of Contents

Problem 1: Manipulability and Force Ellipsoids [2 pts]

1a) [1 pt] A manipulability ellipsoid chracterizes the end-effector's ability to generate velocity. It consists of the set of velocity vectors $\dot{x}$ that satisfies $\dot{x}^\top A^{-1}\dot{x} = 1$. Derive the expression for matrix A.

$$\textit{Given} \;\; \dot{x} = J\dot{q} \;\; \textit{and} \;\; \dot{q}^\top \dot{q} = 1$$

$$\dot{x}^\top (JJ^\top)^{-1}\dot{x} = (J\dot{q})^\top (JJ^\top)^{-1}(J\dot{q}) = \dot{q}^\top \dot{q} = 1.$$

Matching $\dot{x}^\top A^{-1}\dot{x} = 1$ gives

$$A = JJ^\top$$

1b) [1 pt] A force ellipsoid chracterizes the end-effector's ability to generate force. It consists of the set of forces f that satisfies $f^\top B^{-1} f = 1$. Derive the expression for matrix B.

Given

$$\tau = J^\top f.$$

$$\textit{Assume} \;\; \tau^\top \tau = 1.$$

Then

$$(J^\top f)^\top (J^\top f) = f^\top JJ^\top f = 1.$$

Comparing this with the definition $f^\top B^{-1} f = 1$, we obtain

$$B = (JJ^\top)^{-1}$$

Problem 2: Inverse Velocity Kinematics [2 pts]

Denote the joint space variable q and task space variable p, then a redundant robot is defined as: $q \in \mathbb{R}^n, p \in \mathbb{R}^m, m < n$.

2a) [1pt] The jacobian matrix J relates joint velocity $\dot{q}$ to task space velocity $V = J\dot{q}$. What is the expression of the Moore-Penrose Pseudoinverse matrix $J^\dagger$?

Given

$\mathrm{V} = \mathrm{J}\,\dot{\mathrm{q}}, where\ \ \mathrm{J}\ \in R^{m\times n}$ and $m < n$. Then

$\dot{q} = J^{\dagger}V,$

where

$J^{\dagger} = J^{\top}(JJ^{\top})^{-1}.$

2b) [1pt] What is special about the solution $\dot{q}$ given by this specific matrix $J^{\dagger}$?

$\dot{q} = J^{\dagger}V$

is the minimum-norm solution among all joint velocities satisfying

$J\dot{q} = V,$

it has the smallest Euclidean norm

$\|\dot{q}\|_2 = \min.$

Problem 3: Least-Square Problem [4 pts]

Least-square problem is a type of unconstrained optimization problem with an objective which is a sum of squares

$$\text{minimize} \quad f_0(x) = \|Ax - b\|_2^2 = \sum_{i=1}^{k} \left(a_i^T x - b_i\right)^2.$$

3a) [2 pts] Suppose we have $A \in \mathbb{R}^{m\times n}, m > n$ as in a typical regression problem. What is the analytical solution of the least-square problem?

$x^* = (A^{\top}A)^{-1}A^{\top}b.$

3b) [2 pts] We can solve numerical IK problems as nonlinear least-square problems in the following form:

$$\text{minimize} \quad f(q) = e(q)^{\top}e(q),$$

where $e(q) = FK(q) - P_d$ and P_d is the desired end-effector pose.

Derive the gradient of the objective $f(q)$ with respect to decision variable q

$$f(q) = e(q)^{\top}e(q), \quad \mathrm{d}f = 2e^{\top}\mathrm{d}e, \quad \mathrm{d}e = J\,\mathrm{d}q \Rightarrow \nabla_q f(q) = 2\,J(q)^{\top}e(q).$$

Problem 4: Task-Space Control [6 pts]

The manipulator EoM is: $M\ddot{q} + h = \tau$. Assume that the Jacobian matrix J is invertable.

4a) [2 pts] Derive the task-space inertia matrix Λ as a function of M and J from the perspective of conservation of kinetic energy.

$\Lambda = (J M^{-1} J^\top)^{-1}$.

4b) [2 pts] Given desired task-space position p_d, velocity $\dot{p}_d$ and acceleration $\ddot{p}_d$, what is the task-space dynamics that follows the mass-spring-damper sytem?

Note: Assume taks-space inertia Λ, stiffness K_p and damping K_d

$\ddot{p}_{\text{des}} = \ddot{p}_d - \Lambda^{-1}\big(K_d(\dot{p} - \dot{p}_d) + K_p(p - p_d)\big)$.

4c) [2 pts] How to realize TSC by solving a system of linear equations? Derive the linear system in the standard form of $Ax = b$

$$\begin{bmatrix} M & -J^T \\ J & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F \end{bmatrix} = \begin{bmatrix} -h \\ \ddot{p}_{\text{des}} - \dot{J}\dot{q} \end{bmatrix}$$
$$\tau = J^\top F$$

Coding Assignment

Codebase Structure & Setup

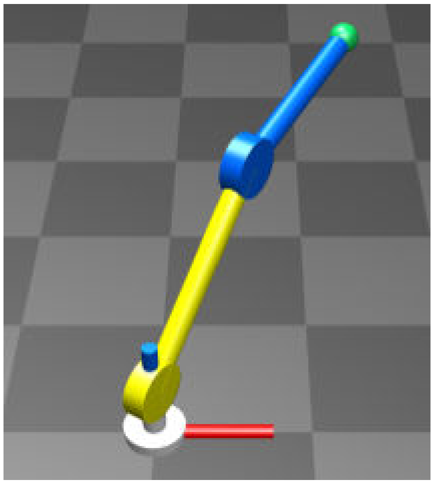
Execute the `setup.m` script **separately** to add paths of `spatical_v2` and check installation status of CasADi.

```
clear; close all force;
run ../codeBase/setup.m
```

```
Setup Success!
```

```
p.robot = model_inverted_leg();
p.linkLength = p.robot.linkLength';
p.robot_viz = model_inverted_leg_viz();
```

Assignment 1: Task Space Control [5 pts]



1) [2 pts] Derive the expression of using the function

```
regenerate_symbolic = true;

if regenerate_symbolic == true

% forward kinematics
syms q dq l [3,1] real;

T01 = [rot(q1, 'z'), [0, 0, 0]';
       zeros(1, 3), 1]

T12 = [rot(q2, 'y'), [0, 0, l1]';
       zeros(1, 3), 1]

T23 = [rot(q3, 'y'), [0, 0, l2]';
       zeros(1, 3), 1]

T34 = [eye(3), [0, 0, l3]';
       zeros(1, 3), 1]

T_ee = T01 * T12 * T23 * T34;
pos_ee_htm = simplify(T_ee(1:3, 4));
J_ee = simplify(jacobian(pos_ee_htm, q));
vel_ee = J_ee * dq;
```

Hint: the time derivative of Jacobian: $\dot{J} = \frac{\partial J}{\partial q}\dot{q}$ is a 3-dimensional tensor, the product $\dot{J}\dot{q}$ is a 2-dimensional matrix

```
% Compute dJdq_ee for the end-effector
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% YOUR CODE STARTS

dJdq_ee = sym(zeros(size(J_ee)));
```

```
for i = 1:length(q)
    dJdq_ee = dJdq_ee + diff(J_ee, q(i)) * dq(i);
end

dJdq_ee = simplify(dJdq_ee);

% YOUR CODE ENDS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% generate kinematics functions
matlabFunction(pos_ee_htm, 'File', 'fcn_pos_ee.m', 'Vars', {q, l});
matlabFunction(J_ee, 'File', 'fcn_J_ee.m', 'Vars', {q, l});
matlabFunction(vel_ee, 'File', 'fcn_vel_ee.m', 'Vars', {q, dq, l});
matlabFunction(dJdq_ee, 'File', 'fcn_dJdq_ee.m', 'Vars', {q, dq, l});

end
```

T01 =

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T12 =

$$\begin{pmatrix} \cos(q_2) & 0 & \sin(q_2) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(q_2) & 0 & \cos(q_2) & l_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T23 =

$$\begin{pmatrix} \cos(q_3) & 0 & \sin(q_3) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(q_3) & 0 & \cos(q_3) & l_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T34 =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2) [3 pts] The main tasks of this assignment is to implement 2 controllers in the **Function to be implemented** section below:

- JointSpace_controller
- TaskSpace_controller

Generate animation and plots to get a qualitative/quantitative comparison between two controllers

```
FLAG_controller = 'JointSpace';
[tout, Xout, XDesout] = run_controller(FLAG_controller, p);
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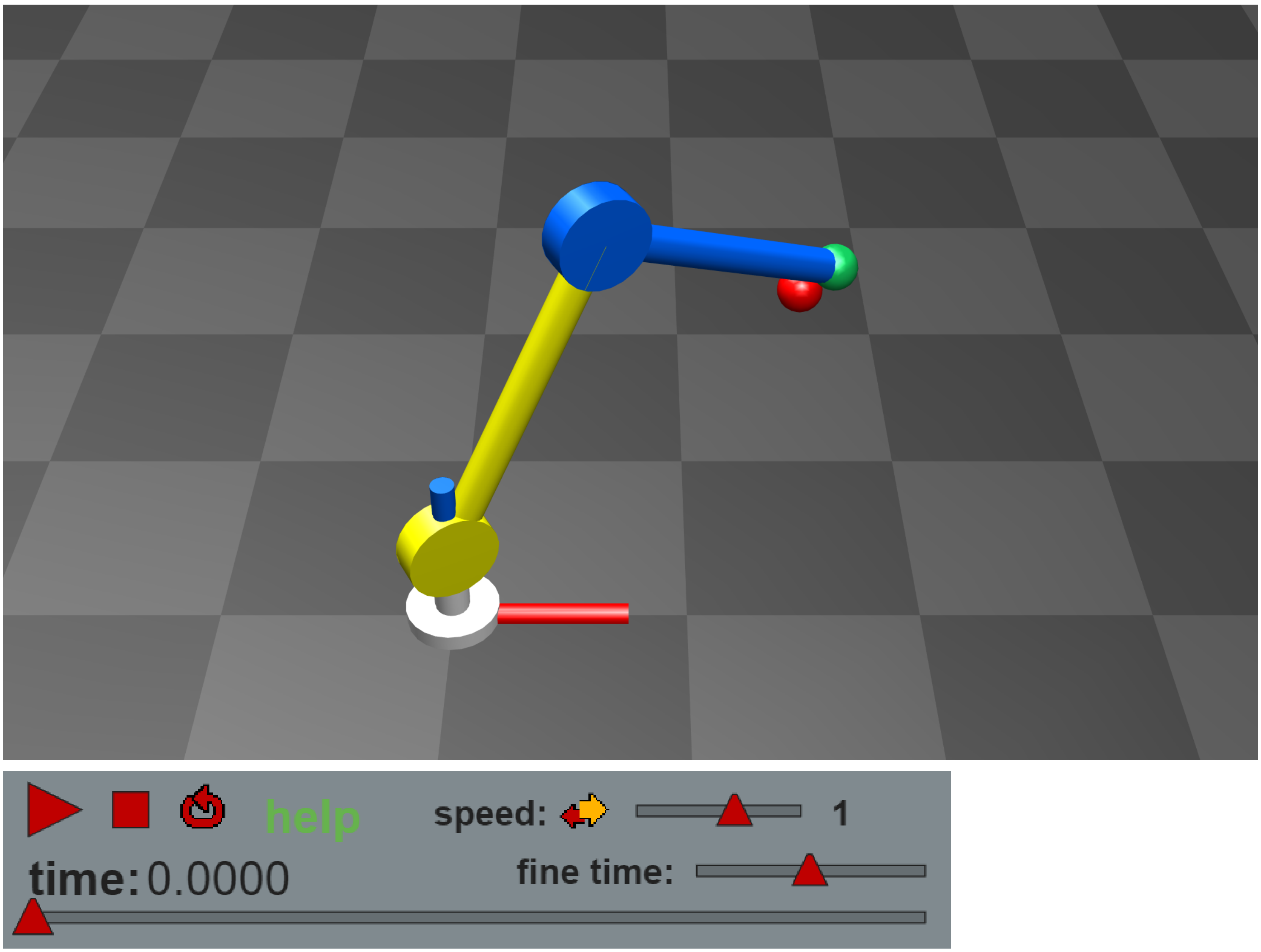
```
fprintf('Simulation complete, use showmotion to animate the robot!')
```

```
Simulation complete, use showmotion to animate the robot!
```

Please make a recording of your animation and submit with the videos named:

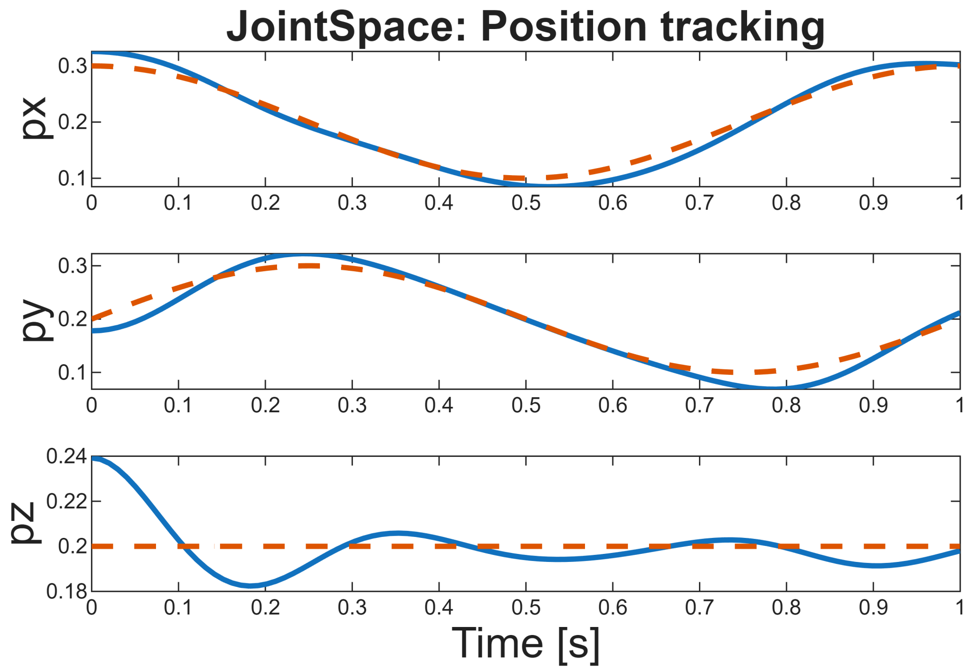
- "JointSpace"

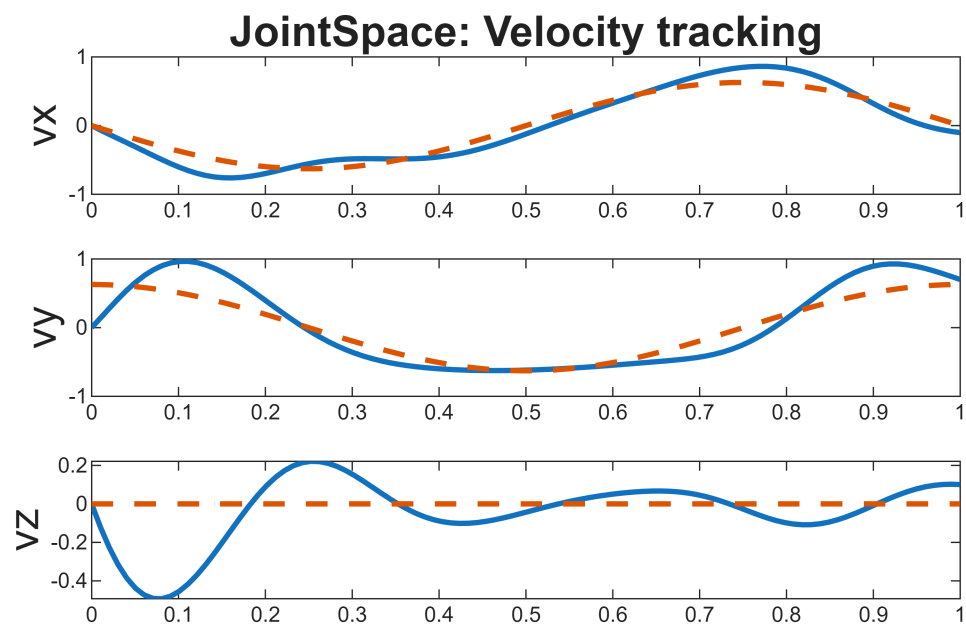
```
% Animation
t_viz = tout;
q_viz = [Xout(:,1:3), XDesout(:,1:3)]';
% copy the following line of code to the command window to animate
close all force;
showmotion(p.robot_viz,t_viz,q_viz);
```




```
% Plots
```

```
makePlots_assignment1(tout, Xout, p, FLAG_controller)
```






```
FLAG_controller = 'TaskSpace';
[tout, Xout, XDesout] = run_controller(FLAG_controller, p);
```

```
sol = 6×1
   40.4704
   14.5912
   27.9594
   -1.4894
    3.6342
   -3.7618
tau = 3×1
    1.4487
    1.4655
```

```
    0.8099
sol = 6×1
   40.8447
   14.9401
   21.5198
   -1.2615
    3.8039
   -3.3286
tau = 3×1
    1.4628
    1.3503
    0.6802
sol = 6×1
   40.0335
   15.4047
   14.1157
   -0.9911
    3.8809
   -2.8427
tau = 3×1
    1.4371
    1.2153
    0.5355
sol = 6×1
   38.0803
   15.7944
    6.2132
   -0.7173
    3.8571
   -2.3098
tau = 3×1
    1.3751
    1.0557
    0.3817
sol = 6×1
   35.0632
   15.8825
   -1.6565
   -0.4803
    3.7244
   -1.7327
tau = 3×1
    1.2815
    0.8657
    0.2250
sol = 6×1
   31.1011
   15.4499
   -8.9536
   -0.3142
    3.4755
   -1.1132
tau = 3×1
    1.1615
    0.6407
    0.0717
sol = 6×1
   26.3586
   14.3233
  -15.1803
   -0.2409
    3.1063
   -0.4548
tau = 3×1
```

```
    1.0206
    0.3790
   -0.0722
sol = 6×1
   21.0458
   12.4058
  -19.9231
   -0.2679
    2.6168
    0.2343
tau = 3×1
    0.8640
    0.0835
   -0.2010
sol = 6×1
   15.4100
    9.6916
  -22.8852
   -0.3884
    2.0131
    0.9394
tau = 3×1
    0.6969
   -0.2380
   -0.3097
sol = 6×1
    9.7182
    6.2656
  -23.9060
   -0.5846
    1.3077
    1.6384
tau = 3×1
    0.5239
   -0.5738
   -0.3943
sol = 6×1
    4.2339
    2.2871
  -22.9688
   -0.8323
    0.5195
    2.3045
tau = 3×1
    0.3496
   -0.9092
   -0.4520
sol = 6×1
   -0.8083
   -2.0375
  -20.1937
   -1.1065
   -0.3274
    2.9098
tau = 3×1
    0.1784
   -1.2291
   -0.4814
sol = 6×1
   -5.2238
   -6.4852
  -15.8191
   -1.3847
   -1.2055
```

```
    3.4296
tau = 3×1
    0.0146
   -1.5199
   -0.4831
sol = 6×1
   -8.8909
  -10.8433
  -10.1734
   -1.6499
   -2.0859
    3.8460
tau = 3×1
   -0.1376
   -1.7712
   -0.4592
sol = 6×1
  -11.7538
  -14.9280
   -3.6433
   -1.8899
   -2.9403
    4.1496
tau = 3×1
   -0.2742
   -1.9761
   -0.4135
sol = 6×1
  -13.8170
  -18.5916
    3.3588
   -2.0970
   -3.7421
    4.3396
tau = 3×1
   -0.3919
   -2.1318
   -0.3509
sol = 6×1
  -15.1324
  -21.7232
   10.4230
   -2.2658
   -4.4667
    4.4221
tau = 3×1
   -0.4884
   -2.2381
   -0.2768
sol = 6×1
  -15.7849
  -24.2430
   17.1651
   -2.3921
   -5.0921
    4.4077
tau = 3×1
   -0.5623
   -2.2971
   -0.1967
sol = 6×1
  -15.8770
  -26.0969
   23.2455
```

```
   -2.4719
   -5.5991
    4.3096
tau = 3×1
   -0.6134
   -2.3118
   -0.1161
sol = 6×1
  -15.5169
  -27.2533
   28.3845
   -2.5017
   -5.9718
    4.1418
tau = 3×1
   -0.6428
   -2.2861
   -0.0395
sol = 6×1
  -14.8091
  -27.7015
   32.3744
   -2.4790
   -6.1991
    3.9178
tau = 3×1
   -0.6523
   -2.2237
    0.0292
sol = 6×1
  -13.8486
  -27.4536
   35.0883
   -2.4032
   -6.2756
    3.6506
tau = 3×1
   -0.6444
   -2.1286
    0.0875
sol = 6×1
  -12.7185
  -26.5457
   36.4839
   -2.2765
   -6.2030
    3.3520
tau = 3×1
   -0.6222
   -2.0052
    0.1335
sol = 6×1
  -11.4886
  -25.0388
   36.6014
   -2.1040
   -5.9904
    3.0327
tau = 3×1
   -0.5890
   -1.8580
    0.1668
sol = 6×1
  -10.2171
```

```
  -23.0166
   35.5537
   -1.8933
   -5.6542
    2.7021
tau = 3×1
   -0.5481
   -1.6919
    0.1877
sol = 6×1
   -8.9514
  -20.5807
   33.5104
   -1.6539
   -5.2164
    2.3684
tau = 3×1
   -0.5024
   -1.5121
    0.1973
sol = 6×1
   -7.7304
  -17.8437
   30.6771
   -1.3962
   -4.7025
    2.0384
tau = 3×1
   -0.4550
   -1.3239
    0.1971
sol = 6×1
   -6.5863
  -14.9208
   27.2735
   -1.1301
   -4.1395
    1.7178
tau = 3×1
   -0.4083
   -1.1326
    0.1890
sol = 6×1
   -5.5463
  -11.9232
   23.5141
   -0.8645
   -3.5529
    1.4111
tau = 3×1
   -0.3641
   -0.9431
    0.1749
sol = 6×1
   -4.6342
   -8.9515
   19.5933
   -0.6065
   -2.9658
    1.1219
tau = 3×1
   -0.3242
   -0.7599
    0.1563
```

```
sol = 6×1
   -3.8709
   -6.0927
   15.6756
   -0.3616
   -2.3975
    0.8534
tau = 3×1
   -0.2896
   -0.5868
    0.1348
sol = 6×1
   -3.2753
   -3.4182
   11.8918
   -0.1333
   -1.8630
    0.6082
tau = 3×1
   -0.2607
   -0.4272
    0.1114
sol = 6×1
   -2.8647
   -0.9843
    8.3391
    0.0761
   -1.3735
    0.3887
tau = 3×1
   -0.2380
   -0.2835
    0.0872
sol = 6×1
   -2.6538
    1.1670
    5.0845
    0.2656
   -0.9368
    0.1975
tau = 3×1
   -0.2213
   -0.1580
    0.0626
sol = 6×1
   -2.6552
    3.0064
    2.1691
    0.4349
   -0.5577
    0.0370
tau = 3×1
   -0.2102
   -0.0521
    0.0379
sol = 6×1
   -2.8780
    4.5158
   -0.3870
    0.5841
   -0.2386
   -0.0902
tau = 3×1
   -0.2041
```

```
    0.0333
    0.0135
sol = 6×1
   -3.3275
    5.6866
   -2.5797
    0.7139
    0.0199
   -0.1818
tau = 3×1
   -0.2023
    0.0977
   -0.0108
sol = 6×1
   -4.0044
    6.5196
   -4.4180
    0.8250
    0.2185
   -0.2355
tau = 3×1
   -0.2042
    0.1411
   -0.0348
sol = 6×1
   -4.9045
    7.0237
   -5.9199
    0.9184
    0.3594
   -0.2496
tau = 3×1
   -0.2089
    0.1640
   -0.0587
sol = 6×1
   -6.0182
    7.2165
   -7.1112
    0.9953
    0.4458
   -0.2233
tau = 3×1
   -0.2157
    0.1674
   -0.0823
sol = 6×1
   -7.3312
    7.1237
   -8.0236
    1.0569
    0.4818
   -0.1567
tau = 3×1
   -0.2238
    0.1529
   -0.1059
sol = 6×1
   -8.8247
    6.7790
   -8.6937
    1.1047
    0.4728
   -0.0513
```

```
tau = 3×1
   -0.2325
    0.1225
   -0.1292
sol = 6×1
  -10.4764
    6.2241
   -9.1622
    1.1404
    0.4250
    0.0898
tau = 3×1
   -0.2414
    0.0790
   -0.1522
sol = 6×1
  -12.2618
    5.5078
   -9.4734
    1.1658
    0.3457
    0.2614
tau = 3×1
   -0.2499
    0.0254
   -0.1748
sol = 6×1
  -14.1549
    4.6853
   -9.6737
    1.1830
    0.2430
    0.4566
tau = 3×1
   -0.2577
   -0.0347
   -0.1965
sol = 6×1
  -16.1294
    3.8166
   -9.8110
    1.1942
    0.1258
    0.6660
tau = 3×1
   -0.2646
   -0.0974
   -0.2173
sol = 6×1
  -18.1588
    2.9653
   -9.9332
    1.2023
    0.0034
    0.8783
tau = 3×1
   -0.2704
   -0.1584
   -0.2365
sol = 6×1
  -20.2166
    2.1968
  -10.0867
    1.2105
```

```
   -0.1143
    1.0802
tau = 3×1
   -0.2751
   -0.2133
   -0.2539
sol = 6×1
  -22.2744
    1.5766
  -10.3141
    1.2224
   -0.2175
    1.2562
tau = 3×1
   -0.2789
   -0.2576
   -0.2687
sol = 6×1
  -24.3004
    1.1683
  -10.6527
    1.2428
   -0.2960
    1.3892
tau = 3×1
   -0.2821
   -0.2867
   -0.2805
sol = 6×1
  -26.2558
    1.0323
  -11.1327
    1.2769
   -0.3402
    1.4608
tau = 3×1
   -0.2848
   -0.2961
   -0.2884
sol = 6×1
  -28.0913
    1.2235
  -11.7747
    1.3311
   -0.3407
    1.4514
tau = 3×1
   -0.2874
   -0.2816
   -0.2917
sol = 6×1
  -29.7417
    1.7888
  -12.5885
    1.4124
   -0.2885
    1.3413
tau = 3×1
   -0.2900
   -0.2393
   -0.2894
sol = 6×1
  -31.1223
    2.7648
```

```
  -13.5722
    1.5279
   -0.1760
    1.1118
tau = 3×1
   -0.2928
   -0.1656
   -0.2809
sol = 6×1
  -32.1243
    4.1730
  -14.7106
    1.6846
    0.0031
    0.7469
tau = 3×1
   -0.2953
   -0.0583
   -0.2652
sol = 6×1
  -32.6143
    6.0154
  -15.9759
    1.8875
    0.2526
    0.2361
tau = 3×1
   -0.2968
    0.0837
   -0.2418
sol = 6×1
  -32.4361
    8.2684
  -17.3282
    2.1382
    0.5731
   -0.4226
tau = 3×1
   -0.2959
    0.2596
   -0.2105
sol = 6×1
  -31.4197
   10.8759
  -18.7169
    2.4331
    0.9608
   -1.2189
tau = 3×1
   -0.2908
    0.4657
   -0.1716
sol = 6×1
  -29.3975
   13.7439
  -20.0833
    2.7613
    1.4060
   -2.1267
tau = 3×1
   -0.2788
    0.6952
   -0.1265
sol = 6×1
```

```
  -26.2292
   16.7373
  -21.3631
    3.1035
    1.8929
   -3.1016
tau = 3×1
   -0.2573
    0.9375
   -0.0770
sol = 6×1
  -21.8353
   19.6818
  -22.4906
    3.4325
    2.3987
   -4.0808
tau = 3×1
   -0.2237
    1.1782
   -0.0263
sol = 6×1
  -16.2323
   22.3730
  -23.4019
    3.7150
    2.8943
   -4.9875
tau = 3×1
   -0.1765
    1.4002
    0.0218
sol = 6×1
   -9.5642
   24.5951
  -24.0395
    3.9168
    3.3463
   -5.7382
tau = 3×1
   -0.1158
    1.5848
    0.0630
sol = 6×1
   -2.1167
   26.1480
  -24.3569
    4.0083
    3.7200
   -6.2557
tau = 3×1
   -0.0439
    1.7146
    0.0933
sol = 6×1
    5.6968
   26.8784
  -24.3234
    3.9706
    3.9842
   -6.4828
tau = 3×1
    0.0347
    1.7761
```

```
    0.1095
sol = 6×1
   13.3810
   26.7089
  -23.9286
    3.7996
    4.1169
   -6.3945
tau = 3×1
    0.1138
    1.7627
    0.1098
sol = 6×1
   20.4321
   25.6554
  -23.1858
    3.5070
    4.1101
   -6.0044
tau = 3×1
    0.1867
    1.6760
    0.0946
sol = 6×1
   26.4206
   23.8263
  -22.1322
    3.1178
    3.9724
   -5.3620
tau = 3×1
    0.2479
    1.5259
    0.0662
sol = 6×1
   31.0573
   21.4021
  -20.8256
    2.6651
    3.7273
   -4.5424
tau = 3×1
    0.2941
    1.3285
    0.0283
sol = 6×1
   34.2256
   18.6011
  -19.3367
    2.1837
    3.4086
   -3.6305
tau = 3×1
    0.3245
    1.1027
   -0.0148
sol = 6×1
   35.9720
   15.6420
  -17.7394
    1.7050
    3.0529
   -2.7053
tau = 3×1
```

```
    0.3406
    0.8672
   -0.0586
sol = 6×1
   36.4659
   12.7135
  -16.1024
    1.2529
    2.6934
   -1.8297
tau = 3×1
    0.3456
    0.6372
   -0.0997
sol = 6×1
   35.9454
    9.9572
  -14.4818
    0.8430
    2.3548
   -1.0453
tau = 3×1
    0.3431
    0.4239
   -0.1358
sol = 6×1
   34.6675
    7.4631
  -12.9183
    0.4828
    2.0526
   -0.3735
tau = 3×1
    0.3364
    0.2337
   -0.1655
sol = 6×1
   32.8715
    5.2761
  -11.4375
    0.1735
    1.7937
    0.1805
tau = 3×1
    0.3282
    0.0693
   -0.1883
sol = 6×1
   30.7599
    3.4061
  -10.0518
   -0.0879
    1.5781
    0.6225
tau = 3×1
    0.3204
   -0.0695
   -0.2045
sol = 6×1
   28.4916
    1.8392
   -8.7635
   -0.3069
    1.4021
```

```
    0.9645
tau = 3×1
    0.3139
   -0.1844
   -0.2146
sol = 6×1
   26.1838
    0.5479
   -7.5681
   -0.4900
    1.2596
    1.2210
tau = 3×1
    0.3092
   -0.2782
   -0.2196
sol = 6×1
   23.9185
   -0.5023
   -6.4566
   -0.6439
    1.1440
    1.4071
tau = 3×1
    0.3064
   -0.3540
   -0.2201
sol = 6×1
   21.7499
   -1.3475
   -5.4180
   -0.7745
    1.0489
    1.5365
tau = 3×1
    0.3051
   -0.4147
   -0.2169
sol = 6×1
   19.7113
   -2.0227
   -4.4400
   -0.8873
    0.9686
    1.6214
tau = 3×1
    0.3051
   -0.4632
   -0.2108
sol = 6×1
   17.8206
   -2.5603
   -3.5107
   -0.9869
    0.8985
    1.6720
tau = 3×1
    0.3060
   -0.5018
   -0.2024
sol = 6×1
   16.0856
   -2.9890
   -2.6185
```

```
   -1.0769
    0.8348
    1.6968
tau = 3×1
    0.3075
   -0.5328
   -0.1922
sol = 6×1
   14.5067
   -3.3339
   -1.7533
   -1.1605
    0.7745
    1.7026
tau = 3×1
    0.3093
   -0.5578
   -0.1805
sol = 6×1
   13.0793
   -3.6163
   -0.9062
   -1.2401
    0.7153
    1.6948
tau = 3×1
    0.3110
   -0.5784
   -0.1678
sol = 6×1
   11.7962
   -3.8542
   -0.0696
   -1.3174
    0.6556
    1.6778
tau = 3×1
    0.3126
   -0.5958
   -0.1544
sol = 6×1
   10.6480
   -4.0627
    0.7624
   -1.3939
    0.5941
    1.6548
tau = 3×1
    0.3139
   -0.6109
   -0.1404
sol = 6×1
    9.6244
   -4.2539
    1.5944
   -1.4707
    0.5297
    1.6284
tau = 3×1
    0.3148
   -0.6246
   -0.1261
sol = 6×1
    8.7144
```

```
   -4.4375
    2.4296
   -1.5483
    0.4618
    1.6006
tau = 3×1
    0.3151
   -0.6376
   -0.1116
sol = 6×1
    7.9073
   -4.6210
    3.2696
   -1.6272
    0.3897
    1.5727
tau = 3×1
    0.3149
   -0.6502
   -0.0972
sol = 6×1
    7.1923
   -4.8100
    4.1151
   -1.7074
    0.3130
    1.5459
tau = 3×1
    0.3141
   -0.6630
   -0.0828
sol = 6×1
    6.5589
   -5.0085
    4.9657
   -1.7889
    0.2314
    1.5207
tau = 3×1
    0.3125
   -0.6761
   -0.0686
sol = 6×1
    5.9973
   -5.2191
    5.8198
   -1.8714
    0.1445
    1.4978
tau = 3×1
    0.3102
   -0.6898
   -0.0548
sol = 6×1
    5.4980
   -5.4430
    6.6751
   -1.9543
    0.0522
    1.4774
tau = 3×1
    0.3071
   -0.7042
   -0.0413
```

```
sol = 6×1
    5.0524
   -5.6804
    7.5286
   -2.0372
   -0.0456
    1.4596
tau = 3×1
    0.3030
   -0.7193
   -0.0283
sol = 6×1
    4.6525
   -5.9307
    8.3766
   -2.1192
   -0.1492
    1.4444
tau = 3×1
    0.2980
   -0.7351
   -0.0158
sol = 6×1
    4.2907
   -6.1922
    9.2148
   -2.1995
   -0.2583
    1.4318
tau = 3×1
    0.2919
   -0.7515
   -0.0038
sol = 6×1
    3.9606
   -6.4631
   10.0386
   -2.2770
   -0.3728
    1.4217
tau = 3×1
    0.2846
   -0.7685
    0.0075
sol = 6×1
    3.6560
   -6.7406
   10.8431
   -2.3507
   -0.4925
    1.4138
tau = 3×1
    0.2761
   -0.7859
    0.0182
sol = 6×1
    3.3718
   -7.0218
   11.6230
   -2.4196
   -0.6170
    1.4080
tau = 3×1
    0.2663
```

```
   -0.8036
    0.0283
```

```
fprintf('Simulation complete, use showmotion to animate the robot!')
```

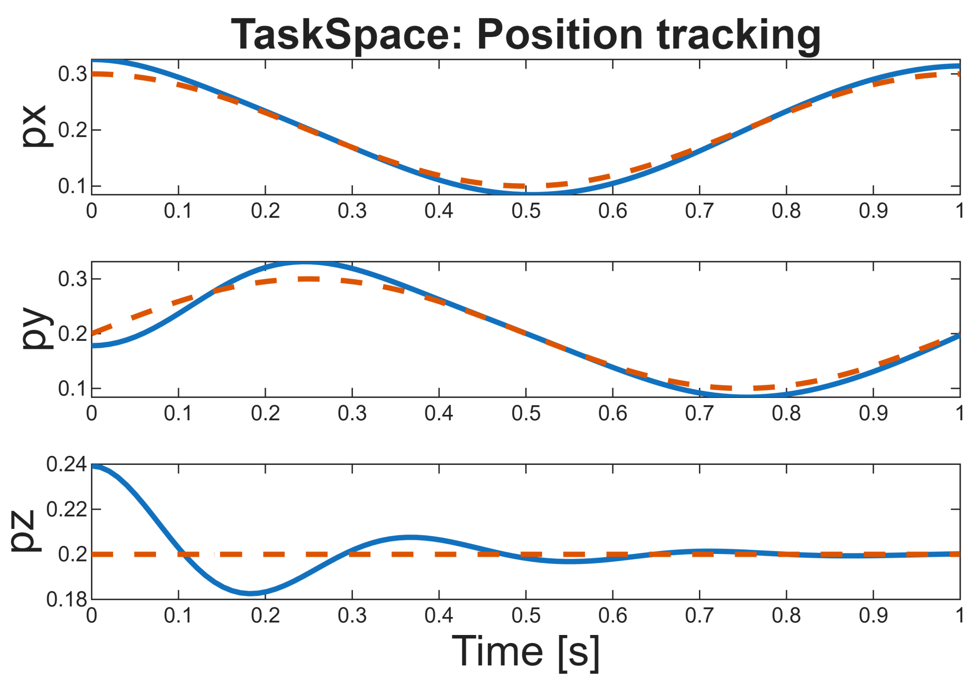
```
Simulation complete, use showmotion to animate the robot!
```

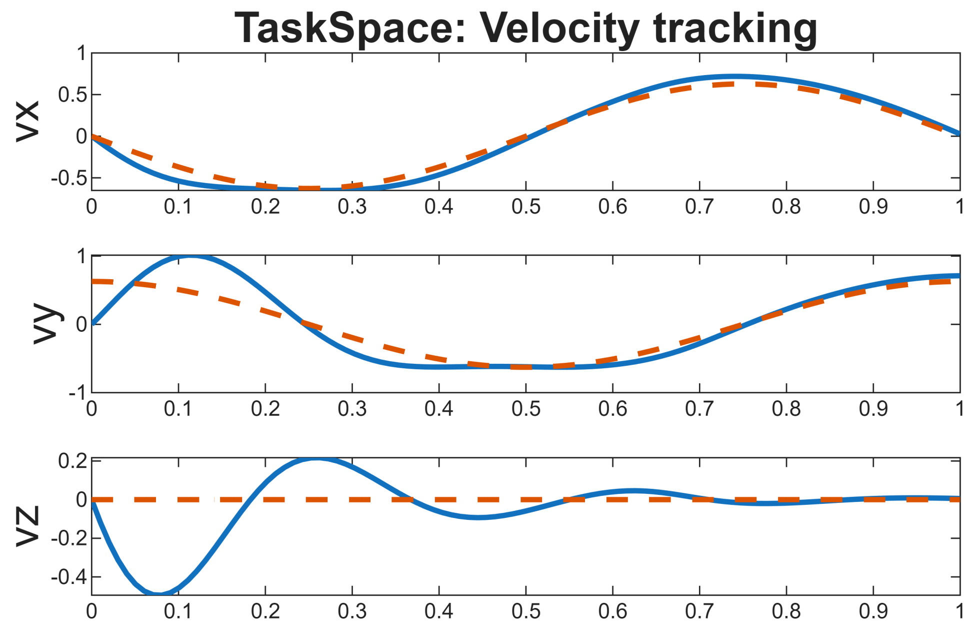
Please make a recording of your animation and submit with the videos named:

- "TaskSpace_method1"
- "TaskSpace_method2"

```
% Animation
t_viz = tout;
q_viz = [Xout(:,1:3), XDesout(:,1:3)]';
% copy the following line of code to the command window to animate
% close all force;
% showmotion(p.robot_viz,t_viz,q_viz);

% Plots
makePlots_assignment1(tout, Xout, p, FLAG_controller)
```





Function to be implemented

```
function [tau, qDes] = JointSpace_controller(rs, rsDes, qGuess, p)
q = rs.q;
dq = rs.dq;
J = rs.J_ee;
posDes = rsDes.pos;
velDes = rsDes.vel;
M = rs.M;
bias = rs.bias;

Kp = diag(300 * [1,1,1]);
Kd = diag(10 * [1,1,1]);

% Perform IK to get the desired joint angle and velocity
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% YOUR CODE STARTS
% - Define the residual function e = pos - posDes
residual = @(qVar) fcn_pos_ee(qVar, p.linkLength) - posDes;
% YOUR CODE ENDS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% YOUR CODE STARTS
% - use lsqnonlin function to solve for qDes
qDes = lsqnonlin(residual, qGuess);
% YOUR CODE ENDS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% YOUR CODE STARTS
% - solve for dqDes using velocity IK
dqDes = pinv(fcn_J_ee(qDes, p.linkLength)) * velDes;
% YOUR CODE ENDS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

ddqDes = Kp * (qDes - q) + Kd * (dqDes - dq);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% YOUR CODE STARTS
% - define tau to achieve joint space controller that uses partial feedback
% linearization (PFL)
tau = M * ddqDes + bias;
% YOUR CODE ENDS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end
```

Implement the Task-Space controller using 2 methods:

1. Projecting dynamics to the task space and get the task-space inertia matrix and bias term
2. Solving the TSC problem as a system of linear equations

```
function tau = TaskSpace_controller(rs, rsDes, p)
M = rs.M;
bias = rs.bias;
J = rs.J_ee;
dJdq = rs.dJdq_ee;
Jinv = pinv(J);
Lambda = Jinv' * M * Jinv;

Kp = diag(300 * [1,1,1]);
Kd = diag(10 * [1,1,1]);

dvDes =  Kp * (rsDes.pos - rs.pos) + Kd * (rsDes.vel - rs.vel);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% YOUR CODE STARTS
% Method 1: project dynamics to the task space
% - Calculate task space bias term mu
% - Perform TSC by defining joint torque tau to achieve partial feedback
% linearization (PFL)
mu = inv(J / M * J') * ( J * (M \ bias) - dJdq );
tau = J' * ( inv(J / M * J') * dvDes + mu );

% YOUR CODE ENDS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% YOUR CODE STARTS
```

```
% Method 2: solve TSC as a system of linear equations
n = size(M,1);
m = size(J,1);
A = [ M,       -J';
      J,  zeros(m, m) ];
b = [-bias;
     dvDes - dJdq * rs.dq];
sol = A \ b;
tau = J' * sol(n+1:n+m);
% YOUR CODE ENDS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end
```

Assignment 2: Contact Simulation [6 pts]

We are going to simulate the scenario where the robot falls due to gravity, hit the ground and bounce back.

Specifically, we will implement the functions ***impact_event*** and ***impact_map*** in the **Function to be implemented** section below:

```
tDuration = 3;
tstart = 0;
q0 = [-1 0.2 0.5]';
dq0 = [5 0 1]';
X0 = [q0; dq0];

[tout, Xout, Uout] = deal([]);

for ii = 1:5
tau = zeros(3,1);

opts = odeset('Events',@(t,X)impact_event(t,X,p));
[t, X, te] = ode45(@(t,X)dynamics(t, X, tau, p), [tstart, tDuration], X0, opts);

% impact map
tstart = te;
Xpre = X(end,:)';
X0 = impact_map(Xpre,p);

% logging
tout = [tout; t(2:end)];
Xout = [Xout; X(2:end,:)];
end
```

```
vc_new = 3×1
   -0.5440
    0.2811
    0.9606
vc_new = 3×1
   -0.5094
   -0.0727
    0.6472
```

```
vc_new = 3×1
    0.1172
   -0.4809
    0.4201
vc_new = 3×1
    1.3777
   -0.5683
    0.3356
vc_new = 3×1
   -0.7982
    0.5095
    0.4312
```

```
fprintf('Simulation complete, use showmotion to animate the robot!')
```

```
Simulation complete, use showmotion to animate the robot!
```

Animation

Please make a recording of your animation and submit with the videos named:

- "ContactSim"

```
t_ = tout;
q_ = Xout(:,1:3)';
% copy the following line of code to the command window to animate
% close all force;
% showmotion(p.robot,t_,q_)
```

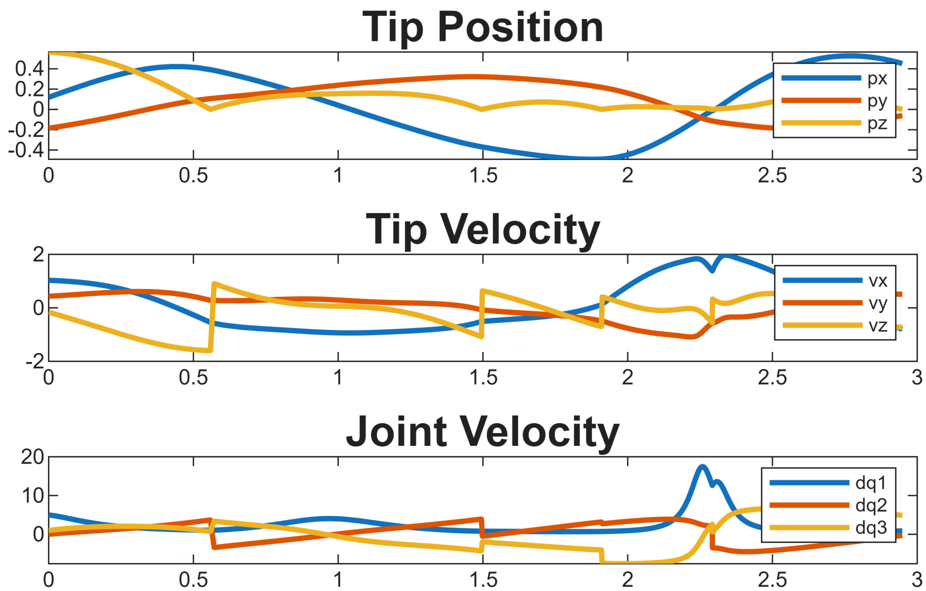
Plots

```
N = length(t_);
qLog  = Xout(:,1:3);
dqLog = Xout(:,4:6);
for k = 1:N
    qk  = qLog(k,:)';
    dqk = dqLog(k,:)';
    posAct(:,k) = fcn_pos_ee(qk,  p.linkLength);
    velAct(:,k) = fcn_vel_ee(qk,dqk,p.linkLength);
end

figure
subplot(3,1,1)
plot(t_,posAct,'LineWidth',2)
legend('px','py','pz')
title('Tip Position','FontSize',16)

subplot(3,1,2)
plot(t_,velAct,'LineWidth',2)
legend('vx','vy','vz')
title('Tip Velocity','FontSize',16)
subplot(3,1,3)
plot(t_,dqLog,'LineWidth',2)
```

```
legend('dq1','dq2','dq3')
title('Joint Velocity','FontSize',16)
```



Functions to be implemented

```
function [value, isterminal, direction] = impact_event(t, X, p)
    q = X(1:3);
    linkLength = p.linkLength;

    pos = fcn_pos_ee(q,linkLength);

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % YOUR CODE STARTS
    % - define value as the signed distance function
    value = pos(3);
    % YOUR CODE ENDS
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    isterminal = 1;
    direction = -1;
end

function [Xp, F] = impact_map(X, p)
    q = X(1:3);
    dq = X(4:6);
    linkLength = p.linkLength;
    eta = 0.6;          % coefficient of restitution

    Jc = fcn_J_ee(q, linkLength);
    M = HandC(p.robot, q, dq);

    % M * (dq_new - dq_old) = Jc' * F;
    % Jc * dq_new = vel_new
```

```
    dq_old = dq;

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % YOUR CODE STARTS
    % - Define the post-impact EE velocity
    % vel_new = Matrix(eta) * Jc * dq_old
    % think about how the Matrix should be defined if:
    % the vx = eta*vx, vy = eta*vy, vz = -eta*vz
    vc_new = diag([1, 1, -eta]) * Jc * dq_old
    % YOUR CODE ENDS
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % YOUR CODE STARTS
    % - Formulate the linear system for the impact map
    Amat = [M, -Jc';
            Jc, zeros(3,3)];

    bvec = [M * dq_old;
            vc_new];

    % YOUR CODE ENDS
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % YOUR CODE STARTS
    % - Solve the linear system to get dqnew_F
    dqnew_F = Amat \ bvec;
    % YOUR CODE ENDS
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    dq_new = dqnew_F(1:3);
    F = dqnew_F(4:6);

    Xp = [q; dq_new];
end
```

Helper Functions

```
function [tout, Xout, XDesout] = run_controller(ctrl_type, p)

% parameters
tstart = 0;
tDuration = 1;
dt_sim = 0.01;
MAX_ITER = floor(tDuration / dt_sim);

[tout, Xout, Uout, XDesout] = deal([]);
```

```
tend = dt_sim;
q0 = [0.5 0.5 1.5]';
dq0 = [0 0 0]';
X0 = [q0; dq0];
qDes = q0;
% simulate robot
for ii = 1:MAX_ITER

    rs = getRobotState(X0, p);

    rsDes = getRobotStateDes(tstart, p);

    % controllers
    if strcmp(ctrl_type, 'JointSpace')
        [tau, qDes] = JointSpace_controller(rs, rsDes, qDes, p);
    elseif strcmp(ctrl_type, 'TaskSpace')
        tau = TaskSpace_controller(rs, rsDes, p);
    end

    % simulation
    [t, X] = ode45(@(t,X)dynamics(t, X, tau, p), [tstart, tend], X0);

    % update
    X0 = X(end, :)';
    tstart = tend;
    tend = tstart + dt_sim;

    % logging
    tout = [tout; t(1:end-1)];
    Xout = [Xout; X(1:end-1,:)];
    nt = length(t)-1;
    Uout = [Uout; repmat(tau(:,1)',[nt,1])];
    XDesout = [XDesout; repmat(rsDes.pos(:,1)',[nt,1])];
end

end

function dXdt = dynamics(t, X, tau, p)
q = X(1:3);dq = X(4:6);

% M * ddq + bias = tau
[M, bias] = HandC(p.robot,q,dq);
ddq = M \ (-bias + tau);
dXdt = [dq; ddq];

end

function rs = getRobotState(X, p)
linkLength = p.linkLength;
q = X(1:3);dq = X(4:6);
```

```
rs.q = q;
rs.dq = dq;
rs.X = [q;dq];

rs.pos = fcn_pos_ee(q,linkLength);
rs.vel = fcn_vel_ee(q,dq,linkLength);
rs.J_ee = fcn_J_ee(q,linkLength);
rs.dJdq_ee = fcn_dJdq_ee(q,dq,linkLength);

[M, bias] = HandC(p.robot,q,dq);
rs.M = M;
rs.bias = bias;

end

function rsDes = getRobotStateDes(t, p)

center = [0.2; 0.2; 0.2];
omega = 2 * pi;
radius = 0.1;

rsDes.pos = [center(1) + radius * cos(omega*t);
             center(2) + radius * sin(omega*t);
             center(3)];

rsDes.vel = [-radius * omega * sin(omega*t);
              radius * omega * cos(omega*t);
              0];

end

function R = rot(th, a)
    % Create coordinate rotation matrix
    c = cos(th);
    s = sin(th);
    if a == 'x'
        R = [ 1  0  0;
              0  c  -s;
              0  s  c];
    elseif a == 'y'
        R = [ c  0  s;
              0  1  0;
             -s  0  c];
    elseif a == 'z'
        R = [ c -s  0;
              s  c  0;
              0  0  1];
    else
        disp('specify rotation axis\n')
```

```
    end
end

function makePlots_assignment1(tout, Xout, p, FLAG_controller)
t_ = linspace(tout(1), tout(end), 100)';
X_ = interp1(tout, Xout, t_);

N = length(t_);
posAct = zeros(3,N);
velAct = zeros(3,N);
posDes = zeros(3,N);
velDes = zeros(3,N);

qLog  = X_(:,1:3);
dqLog = X_(:,4:6);
for k = 1:N
    qk  = qLog(k,:)';
    dqk = dqLog(k,:)';
    posAct(:,k) = fcn_pos_ee(qk,  p.linkLength);
    velAct(:,k) = fcn_vel_ee(qk,dqk,p.linkLength);

    rsDes_k = getRobotStateDes(t_(k), p);
    posDes(:,k) = rsDes_k.pos;
    velDes(:,k) = rsDes_k.vel;
end

figure
subplot(3,1,1)
plot(t_,posAct(1,:), '-', t_, posDes(1,:), '--', 'LineWidth',2)
ylabel('px','FontSize',16)
title([FLAG_controller ': Position tracking'],'FontSize',16)

subplot(3,1,2)
plot(t_,posAct(2,:), '-', t_, posDes(2,:), '--', 'LineWidth',2)
ylabel('py','FontSize',16)

subplot(3,1,3)
plot(t_,posAct(3,:), '-', t_, posDes(3,:), '--', 'LineWidth',2)
ylabel('pz','FontSize',16)
xlabel('Time [s]','FontSize',16)

figure
subplot(3,1,1)
plot(t_,velAct(1,:), '-', t_, velDes(1,:), '--', 'LineWidth',2)
ylabel('vx','FontSize',16)
title([FLAG_controller ': Velocity tracking'],'FontSize',16)

subplot(3,1,2)
plot(t_,velAct(2,:), '-', t_, velDes(2,:), '--', 'LineWidth',2)
ylabel('vy','FontSize',16)
```

```
subplot(3,1,3)
plot(t_,velAct(3,:), '-', t_, velDes(3,:), '--', 'LineWidth',2)
ylabel('vz','FontSize',16)
end
```