27 June 2018

Notation and Definitions

U Set of possible histories of the universe.
u A history of the universe in U.
X Setup function of an ID that maps $U \to X(U)$. A binary question concerning $\Gamma(u)$.
x A binary question and a member of image $X(U)$.
Y Conclusion function of an ID that maps $U \to \{-1, 1\}$. A binary answer of an ID for $X(u) = x$.
y A single-valued answer, and member of image $Y(U) = \{0, 1\}$.
Γ A function of the actual values of a physical variable over U, equivalent to $\Gamma(u) = S(t_i)(u)$.
γ Possible value of a physical variable, a member of the image $\Gamma(U)$.
δ Probe of any variable V parameterized by $v \in V$ such that :

$$\delta_v(v') = \begin{cases} 1 & \text{if } v = v' \\ -1 & \text{otherwise} \end{cases}$$

$\wp$ Set of probes over $\Gamma(U)$.
$\mathcal{D} = (X, Y)$ An inference device, consisting of functions X and Y.
$\bar{F}$ Inverse. Given a function F over U, $F^{-1} = \bar{F} \equiv \{\{u : F(u) = f\} : f \in F(U)\}$.
$>$ Weak inference: a device $\mathcal{D}$ weakly infers Γ *iff* $\forall \gamma \in \Gamma(U), \exists x \in X(U)$ s.t. $\forall u \in U$, $X(u) = x \implies Y(u) = \delta_\gamma(\Gamma(u))$.
$>>$ Strong inference: a device (X_1, Y_1) strongly infers a device (X_2, Y_2) *iff* $\forall \delta \in \wp(Y_2)$ and all x_2, $\exists x_1$ *such that* $X_1 = x_1 \implies X_2 = x_2, Y_1 = \delta(Y_2)$.

28 June 2018

Turing Machines

We construct an Inference Device (ID) such that it weakly infers a Turing Machine (TM). A TM behaves according to a set of rules (p, s, a, q) corresponding to a starting state, scanned input, action, and end state such that $p, q \in Q$, $s \in S = \{0, 1, B\}$, and $a \in A = \{0, 1, B, L, R\}$. Hence, a TM can be represented by a mapping between $Q \times S \to S \times Q$. Note that Q is a finite set of internal states. Any two distinctive (p, s, a, q) quadruples must differ in the first two elements, such that the Turing machine is deterministic.

Example 1 Consider a Turing machine T with a single internal state q. We construct a set of rules for T:

(p, s)	(a, q)
$(q, 0)$	$(0, q)$
$(q, 1)$	$(1, q)$
(q, B)	(B, q)

Now consider an inference device $\mathcal{D}$ that can weakly infer T.

u	$X(u)$	$Y(u)$	$\Gamma(u)$
$(q, 0)$	$(0, q)$	1	$(0, q)$
$(q, 0)$	$(1, q)$	-1	$(0, q)$
$(q, 1)$	$(1, q)$	1	$(1, q)$
$(q, 1)$	$(0, q)$	-1	$(1, q)$
(q, B)	(B, q)	1	(B, q)
(q, B)	$(1, q)$	-1	(B, q)

We show that weak inference holds. Recall that the definition of weak inference is:

$$\mathcal{D} > \Gamma \text{ iff } \forall \gamma \in \Gamma(U), \exists x \in X(U) \text{ such that } \forall u \in U, X(u) = x \implies Y(u) = \delta_\gamma(\Gamma(u))$$

Then,

$$\Gamma(U) = S \times Q = \{(0, q), (1, q), (B, q)\}$$

$$\Gamma((q, 0)) = (0, q), x = (0, q) \in X(U) \implies Y(u) = 1 = \delta_{(0,q)}((0, q))$$

$$\Gamma((q, 0)) = (0, q), x = (1, q) \in X(U) \implies Y(u) = -1 = \delta_{(0,q)}((1, q))$$

$$\Gamma((1, q)) = (1, q), x = (1, q) \in X(U) \implies Y(u) = 1 = \delta_{(1,q)}((1, q))$$

$$\Gamma((1, q)) = (1, q), x = (0, q) \in X(U) \implies Y(u) = -1 = \delta_{(1,q)}((0, q))$$

$$\Gamma((q, B)) = (B, q), x = (B, q) \in X(U) \implies Y(u) = 1 = \delta_{(B,q)}((B, q))$$

$$\Gamma((q, B)) = (1, q), x = (1, q) \in X(U) \implies Y(u) = -1 = \delta_{(B,q)}((1, q))$$

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Strong Inference of a Turing Machine