# Practice 1

10 декабря 2016 г.

# 1 Задание 1

## 1.1 Генерация точек

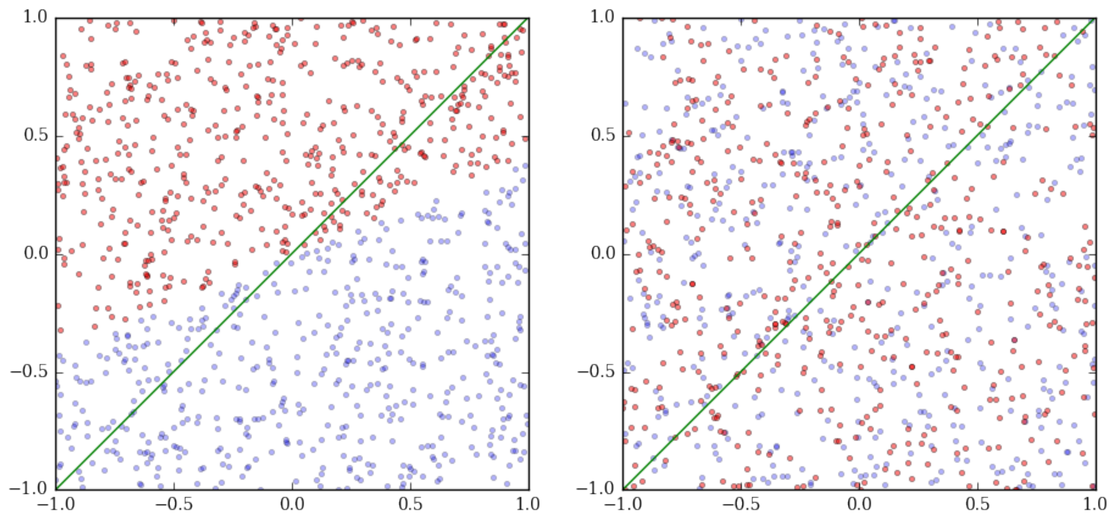
In [380]:
```
import numpy as np
from scipy.stats import norm, uniform, randint
from scipy.special import expit
import matplotlib.pyplot as plt
%matplotlib inline


def get_sample(n=2, size=1000, is_separable=True):
    center = [np.array([1] * n), np.array([-1] * n)]

    var = 0.01
    if not is_separable:
        var = 1 / var

    points = []
    for i in range(size):
        c = randint.rvs(0, 2)

        points.append(np.array([c, -1] + list(center[c] + norm.rvs(scale=var ** 0.5, size=n))))

    return np.array(points)


def get_equation(omega):
    return lambda x: np.sum(x * omega[1:]) - omega[0]


def get_a(omega):
    return np.array([-omega[2], omega[1]])


def get_points_2(omega, var=10):
    point = omega[0] * omega[1:]
    points = [point - var * get_a(omega), point + var * get_a(omega)]
```

```
        return np.array(points)


    def get_uniform_sample(n=2, size=1000, is_separable=True, eps=1e-2):
        x = np.array([uniform.rvs(loc=-1, scale=n, size=size) for i in range(n)])

        c = [0] * size

        if not is_separable:
            c = randint.rvs(0, 2, size=size)
        else:
            omega = np.array([0] + list(uniform.rvs(loc=-1, scale=2, size=n)))
            equation = get_equation(omega)
            for i in range(size):
                if equation(np.array(x[:, i])) > eps:
                    c[i] = 0
                elif equation(np.array(x[:, i])) < -eps:
                    c[i] = 1
                else:
                    c[i] = 2

        return np.array(list(filter(lambda x: x[0] != 2, list(zip(c, [-1] * size, *x)))))


    def print_sample_2(sample):
        class_0 = np.array(list(filter(lambda x: x[0] == 0, sample)))
        class_1 = np.array(list(filter(lambda x: x[0] == 1, sample)))

        if len(class_0):
            plt.plot(class_0[:, 2], class_0[:, 3], 'ob', ms=3, alpha=0.3)

        if len(class_1):
            plt.plot(class_1[:, 2], class_1[:, 3], 'or', ms=3, alpha=0.5)


    def print_line_2(omega, line_par=2):
        omega = omega / (omega[1] ** 2 + omega[2] ** 2) ** 0.5
        equation = get_equation(omega)

        points = get_points_2(omega, line_par)
        plt.plot(points.T[0], points.T[1], 'g')
```

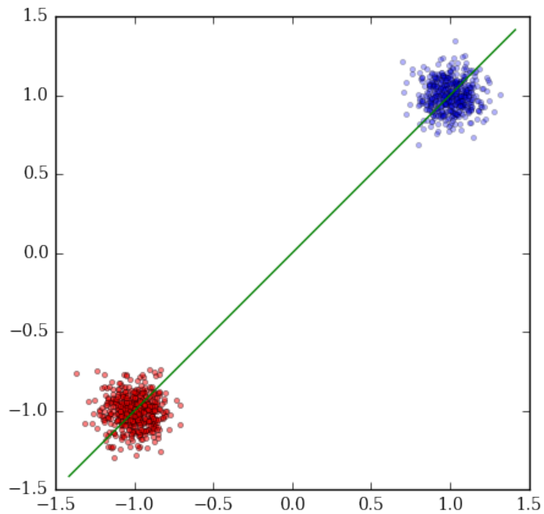
```
In [381]: separable_sample = get_sample(size=1000, is_separable=True)
          nseparable_sample = get_sample(size=1000, is_separable=False)
          uniform_separable_sample = get_uniform_sample(is_separable=True)
          uniform_nseparable_sample = get_uniform_sample(is_separable=False)
```

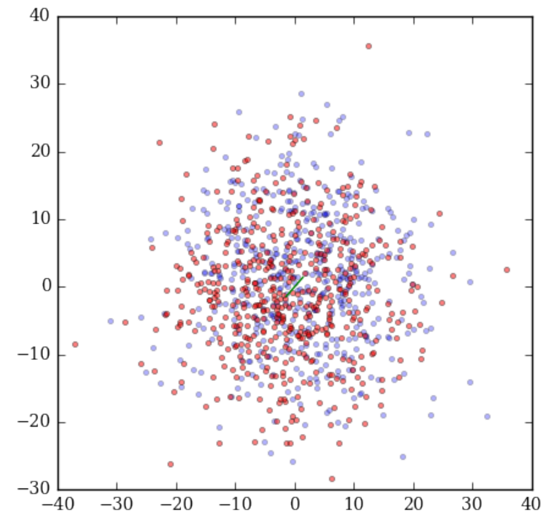
```
In [382]: plt.figure(figsize=(11, 5))
```

```
plt.subplot(1, 2, 1)
print_sample_2(uniform_separable_sample)
print_line_2(np.array([0, -1, 1]), line_par=1.5)
plt.xlim(-1, 1)
plt.ylim(-1, 1)

plt.subplot(1, 2, 2)
print_sample_2(uniform_nseparable_sample)
print_line_2(np.array([0, -1, 1]))
plt.xlim(-1, 1)
plt.ylim(-1, 1)

plt.show()
```




```
In [383]: plt.figure(figsize=(11, 5))

plt.subplot(1, 2, 1)
print_sample_2(separable_sample)
print_line_2(np.array([0, -1, 1]))

plt.subplot(1, 2, 2)
print_sample_2(nseparable_sample)
print_line_2(np.array([0, -1, 1]))

plt.show()
```




## 1.2 Градиентный спуск

In [403]:
```
default_t = lambda t, k, x: t * 0.98


def project(x):
    return x / np.sqrt((x * x).sum())


def vect_abs(x):
    return np.sqrt((x * x).sum())

# Нахождение миниумума методом градиентного спуска
def find_min(x_0, f, grad, next_t=default_t, eps=1e-6, MAX_K = 1000, use_projection=False):
    t = 0.5
    old_x = project(x_0)
    k = 0

    while True:
        k += 1
        t = next_t(t, k, old_x)
        x = old_x - t * grad(old_x)

        if use_projection and vect_abs(x) > 1:
            x = project(x)

        if (np.abs(f(x) - f(old_x)) < eps) or k == MAX_K:
            old_x = x
            break
```

```
        old_x = x

    return (old_x, k)
```

#### 1.2.1 Формулировка для разделяющей гиперплоскости

$\omega$ - уравнение плоскости (нулевую координату имеет свободный член), $x$ - точка с дополнительной нулевой координатой со значением $-1$. Утверждается, что точка минимума функции риска $Q(\omega) = \sum_{i=1}^{m} ln(1 + exp(-y_i \cdot < x_i, \omega >))$ есть искомая разделяющая прямая ($y_i$ - соответствующий класс точки).

$\frac{\delta Q}{\delta \omega_i} = \sum_{i=1}^{m}(1 - \frac{1}{1+exp(-y_i \cdot <x_i,\omega>)}) \cdot (-y_i x_i^k)$

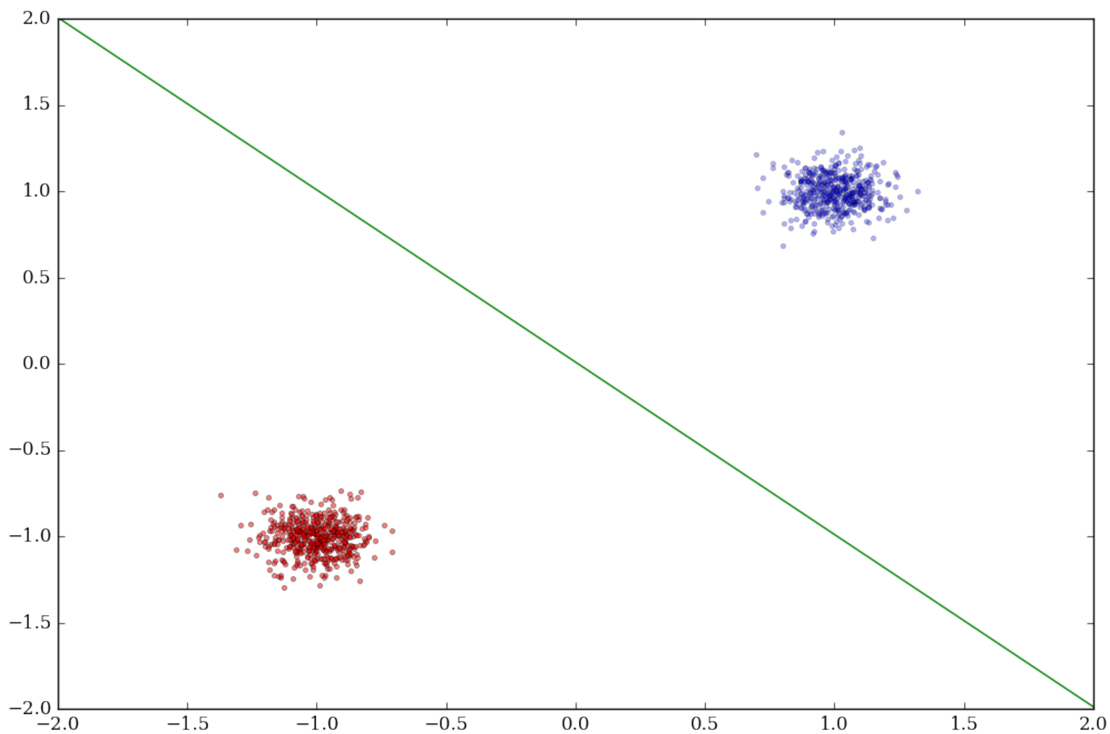
In [404]:
```
def get_class(c):
    if c:
        return 1
    return -1


def raw_grad(omega, X, Y):
    return np.sum((1 - expit(Y * np.sum(X * omega, axis=1))) * (-Y * X.T), axis=1)


def get_grad(X, Y):
    return lambda omega: raw_grad(omega, X, Y)


def raw_fun(X, Y, omega):
    return np.sum(np.logaddexp(0, -Y * np.sum(X * omega, axis=1)))


def get_fun(X, Y):
    return lambda omega: raw_fun(X, Y, omega)


def get_xy(sample):
    return sample[:, 1:], np.array([get_class(y[0]) for y in sample])
```

In [405]:
```
def test_gradient_2(sample, x0, next_t=default_t, max_k=10000, use_projection=False,
                    x_lim=2, y_lim=2, line_par=100):

    X, Y = get_xy(sample)

    omega, k = find_min(x0, get_fun(X, Y), get_grad(X, Y),
                        MAX_K=max_k, use_projection=use_projection)

    print(omega, k)
```

```
    omega = omega / (omega[1] ** 2 + omega[2] ** 2) ** 0.5
    equation = get_equation(omega)

    plt.figure(figsize=(12, 8))
    print_sample_2(sample)
    points = get_points_2(omega, line_par)
    plt.plot(points.T[0], points.T[1], 'g')

    plt.xlim(-x_lim, x_lim)
    plt.ylim(-y_lim, y_lim)
    plt.show()
```

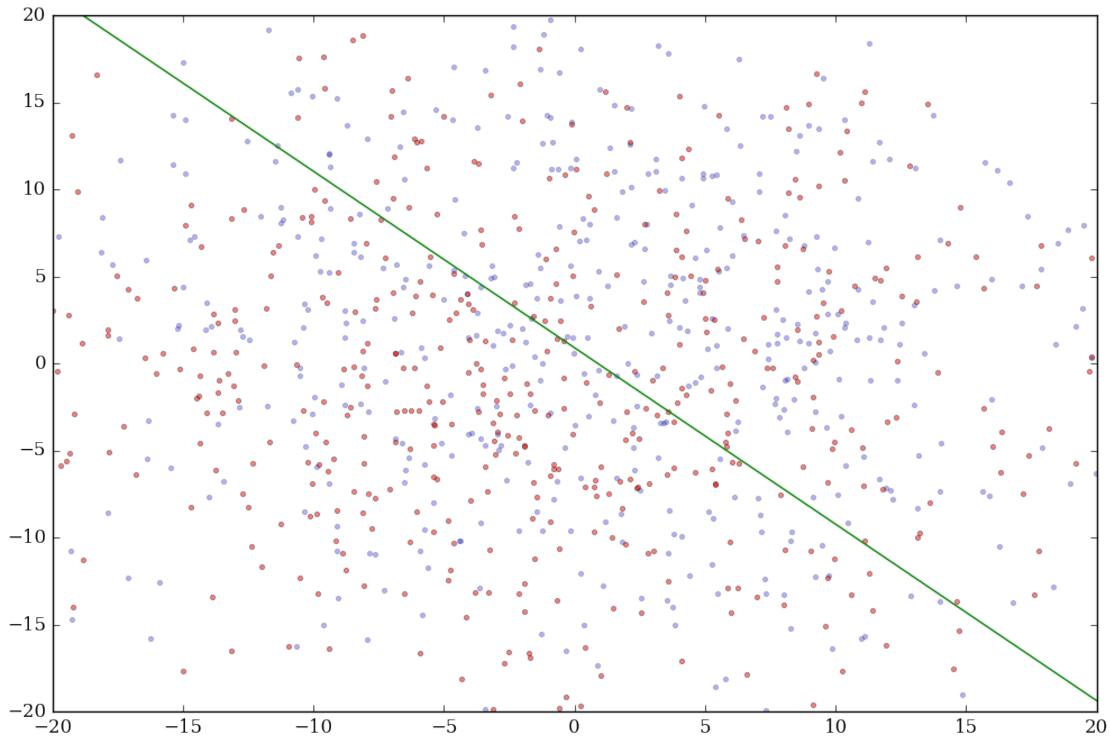
In [406]:
```
test_gradient_2(separable_sample, np.array([0, -1, 1]), use_projection=False)
```

[  -2.86935396 -245.02378034 -245.36647943] 2



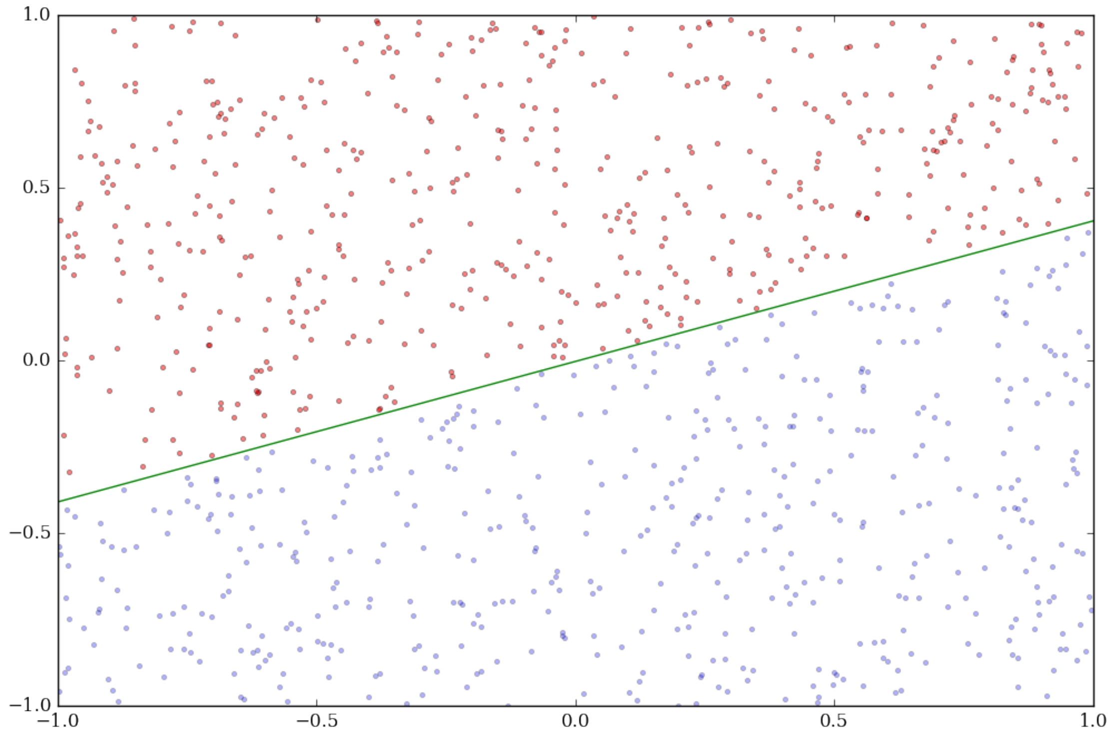

In [407]:
```
test_gradient_2(nseparable_sample, np.array([0, -1, 1]), x_lim=20, y_lim=20)
```

[-0.02181676 -0.02392207 -0.02359634] 683




In [408]: test_gradient_2(uniform_separable_sample, np.array([0, -1, 1]), x_lim=1, y_lim=1, max_k=20000)

[ -0.21576265 -59.35697036 145.8797259 ] 291




### 1.2.2 Количество шагов от точности.

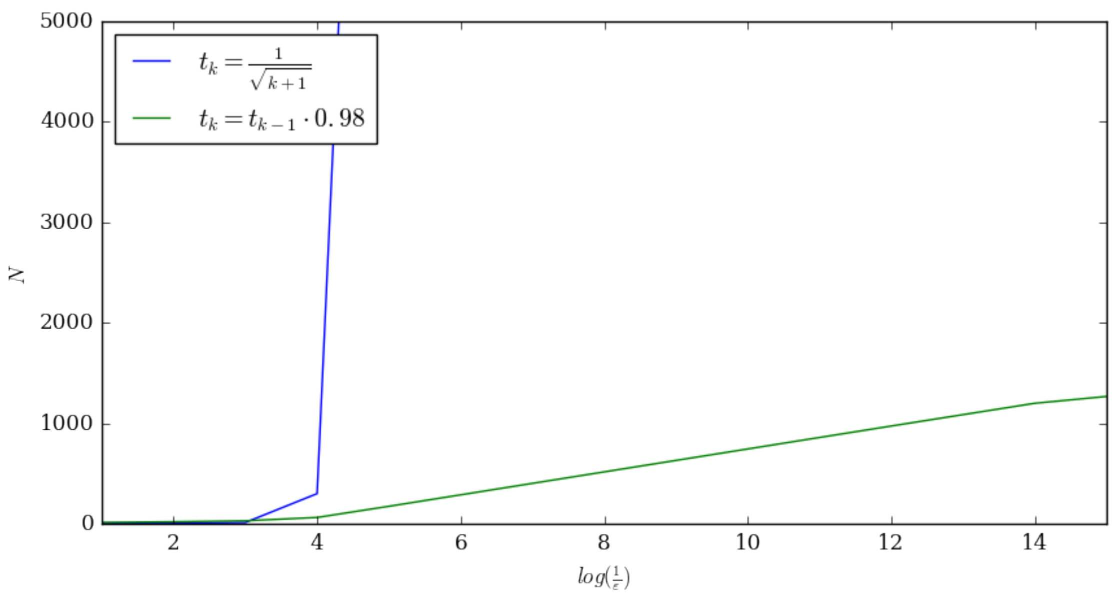
Применяются две стратегии выбора шага:

1. $t_k = t_{k-1} \cdot 0.98$
2. $t_k = \frac{1}{\sqrt{k+1}}$

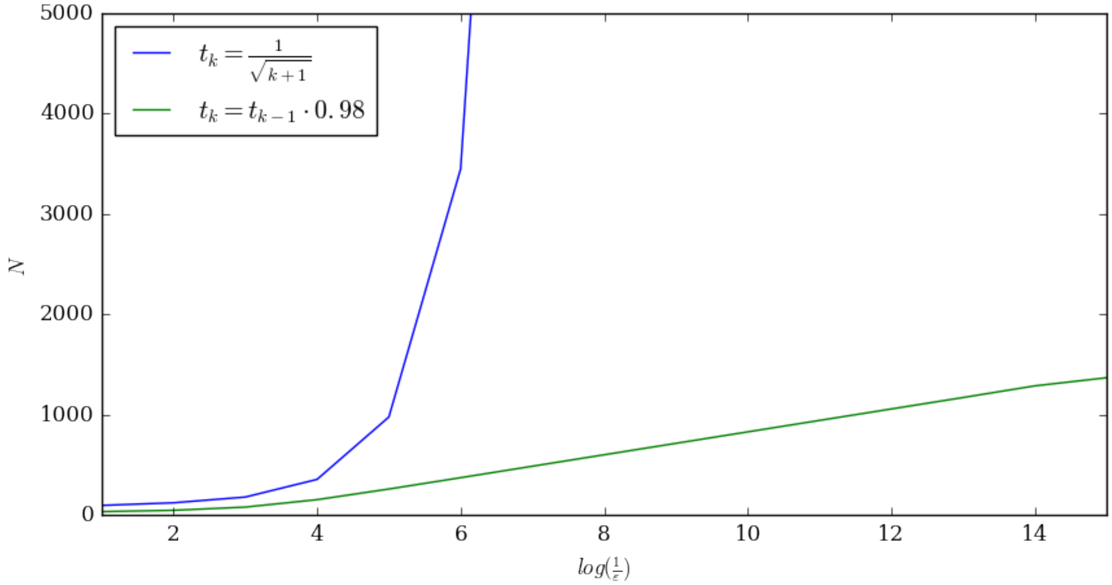
In [414]:
```
plt.rc('font', family='serif', size="12")


def test_t(x0, sample, next_t, label):
    X, Y = get_xy(sample)

    K = []
    N = 15
    for i in range(1, N + 1):
        omega, k = find_min(x0, get_fun(X, Y), get_grad(X, Y), MAX_K=100000,
                            eps=10 ** (-i), next_t=next_t)

        K.append(k)
        if (k > 10000):
            break
```

```
        plt.plot(range(1, N + 1), K + [K[-1]] * (N - len(K)), label=label)


    def test(x0, sample):
        plt.figure(figsize=(10, 5))
        test_t(x0, sample, lambda t, k, x: 1 / np.sqrt(k + 1), r'$t_k = \frac{1}{\sqrt{k + 1}}$')
        test_t(x0, sample, lambda t, k, x: t * 0.98, r'$t_k = t_{k - 1} \cdot 0.98$')

        plt.legend(loc='upper left')
        plt.ylabel(r'$N$')
        plt.ylim(0, 5000)
        plt.xlim(1, 15)
        plt.xlabel(r'$log(\frac{1}{\varepsilon})$')
        plt.show()
```

In [415]:
```
# n = 2
test(np.array([0.001, -1, 1]), uniform_separable_sample)
```




In [416]:
```
# n = 3
uniform_separable_sample_3 = get_uniform_sample(n=3)
test(np.array([0.001, -1, 1, 1]), uniform_separable_sample_3)
```




Видно, что у 2 метода скорость сходимости линейная (от логарифма), а вот в случае 1 очевидна не менее, чем экспоненциальная