PINNs for Option Pricing

From Black-Scholes to Stochastic Volatility Calibration

FYS5429: Advanced Machine Learning and Data Analysis for the Physical Sciences

Project 1

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Abstract

Financial markets exhibit pronounced non-stationarities driven by structural breaks, macroeconomic shocks, and evolving investor behavior. This project investigates the use of machine learning methods for identifying and modeling regime changes in financial time series. We consider supervised and unsupervised approaches to regime detection, including clustering-based methods and probabilistic state models, and evaluate their ability to capture shifts in volatility, return dynamics, and cross-asset dependencies. Model performance is assessed using historical market data under realistic out-of-sample settings. The results highlight both the potential and limitations of machine-learning-based regime modeling in quantitative finance, with implications for risk management, portfolio allocation, and adaptive trading strategies.

1 Introduction

This is an intro lorem ipsum with a reference Hastie et al. (2009) and such (Hastie et al. 2009).

The classical starting point for many partial differential equation (PDE) methods in mathematical finance is the heat equation, which describes the diffusion of temperature in a homogeneous medium. In one spatial dimension, the heat equation is given by

$$\frac{\partial u}{\partial t}(x,t) = \alpha \frac{\partial^2 u}{\partial x^2}(x,t),$$

where $u(x,t)$ represents temperature, t denotes time, x is the spatial coordinate, and $\alpha > 0$ is the thermal diffusivity constant. The equation states that the rate of change in temperature is proportional to the spatial curvature of the temperature field, capturing the fundamental mechanism of diffusion.

The importance of the heat equation in option pricing arises from the fact that the Black–Scholes PDE can be transformed into a diffusion equation of this form through a suitable change of variables.

Under the standard geometric Brownian motion model for an asset price S_t,

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

the value $V(S,t)$ of a European option satisfies the Black–Scholes PDE

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0.$$

By introducing the log-price transformation

$$x = \ln\left(\frac{S}{K}\right), \quad \tau = T - t,$$

together with an exponential rescaling of the dependent variable, the Black–Scholes equation can be reduced to the standard diffusion equation

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2}.$$

This connection provides both analytical insight and a natural bridge to modern numerical approaches. In this thesis, Physics-Informed Neural Networks (PINNs) are used to solve the Black–Scholes PDE by embedding the governing differential equation directly into the loss function, allowing neural networks to approximate option prices while respecting the underlying diffusion structure inherited from the heat equation.

While the Black–Scholes model assumes constant volatility, empirical financial markets exhibit stochastic volatility dynamics. A widely used extension is the Heston stochastic volatility model, in which both the asset price and its variance evolve as stochastic processes:

$$dS_t = rS_t dt + \sqrt{v_t}\, S_t dW_t^S,$$
$$dv_t = \kappa(\theta - v_t)dt + \xi\sqrt{v_t}\, dW_t^v,$$

with correlation

$$dW_t^S dW_t^v = \rho dt.$$

In this setting, the option price becomes a function $V(S,v,t)$ and satisfies the two-dimensional Heston PDE

$$\frac{\partial V}{\partial t} + \frac{1}{2}vS^2\frac{\partial^2 V}{\partial S^2} + \rho\xi vS\frac{\partial^2 V}{\partial S \partial v} + \frac{1}{2}\xi^2 v\frac{\partial^2 V}{\partial v^2}$$
$$+ \quad rS\frac{\partial V}{\partial S} + \kappa(\theta - v)\frac{\partial V}{\partial v} - rV = 0.$$

Compared to the Black–Scholes equation, the Heston PDE introduces an additional spatial dimension and a mixed derivative term, making classical numerical solution methods more computationally demanding. This increased complexity makes the model particularly well-suited for physics-informed machine learning approaches. In this thesis, PINNs are therefore applied not only to the Black–Scholes equation but also to the Heston PDE, demonstrating how neural-network-based solvers can handle higher-dimensional option pricing problems governed by diffusion-type equations.

1.1 Black Scholes

Abdelmessih (2026).

$$S\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = V$$

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV - rS\frac{\partial V}{\partial S}$$

$$d_1 = \frac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Where:

- S = stock price
- K = strike price
- r = risk-free rate
- σ = volatility
- T = time to maturity
- $N(\cdot)$ = standard normal CDF
- $\phi(\cdot)$ = standard normal PDF

Call option:

$$C = SN(d_1) - Ke^{-rT}N(d_2)$$

Put option:

$$P = Ke^{-rT}N(-d_2) - SN(-d_1)$$

$$\Delta = \frac{\partial V}{\partial S}$$

$$\Gamma = \frac{\partial^2 V}{\partial S^2}$$

$$\Theta = \frac{\partial V}{\partial t}$$

$$\text{Vega} = \frac{\partial V}{\partial \sigma}$$

$$\rho = \frac{\partial V}{\partial r}$$

Black–Scholes Equation in Greek Form

$$\Theta + \frac{1}{2}\sigma^2 S^2 \Gamma + rS\Delta - rV = 0$$

$$\Theta + \frac{1}{2}\sigma^2 S^2 \Gamma = r(V - S\Delta)$$

2 Methods

This is the methods section, lorem ipsum.

Modern quantitative finance sits at the intersection of stochastic modeling, numerical methods, and machine learning. Classical stochastic volatility modeling builds on multiscale diffusion techniques, where volatility evolves on multiple time scales, as described by Fouque et al. (2011). Analytical approaches to option pricing under stochastic dynamics are further developed in tractable model frameworks such as those presented in Gulisashvili (2012), while probabilistic interpretations of derivative pricing link option values to risk-neutral expectations (Prefetta n.d.).

In parallel, machine learning methods have become increasingly relevant for solving high-dimensional approximation problems arising from partial differential equations and financial data modeling. Foundational deep learning theory and architectures are presented in Goodfellow et al. (2016), while practical implementations using modern software frameworks are discussed in Raschka et al. (2022). Reinforcement learning methods provide an additional computational paradigm for sequential decision-making and dynamic optimization problems in finance (Sutton and Barto 2018). Finally, time-change techniques offer an alternative stochastic modeling framework that connects diffusion processes with subordinated dynamics in financial markets, as discussed in Swishchuk (2008). Together, these perspectives motivate the use of physics-informed machine learning methods for solving option pricing problems governed by stochastic differential equations and partial differential equations.

3 Results

These are the results

4 Conclusion

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5 Appendix