PINNs for Option Pricing

From Black-Scholes to Stochastic Volatility Calibration

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1 Introduction

Financial markets exhibit pronounced non-stationarities driven by structural breaks, macroeconomic shocks, and evolving investor behavior. Traditional quantitative finance relies on Partial Differential Equations (PDEs) to price derivatives, but these often struggle with the non-linearities and calibration demands of modern assets.

This project investigates the use of Physics-Informed Neural Networks (PINNs) to solve and calibrate option pricing models, bridging the gap between deep learning and financial domain knowledge.

In addition to solving option pricing PDEs (the forward problem), the project also considers the inverse problem of model calibration, where neural networks are used to infer model parameters from observed option prices.

2 Methodology

The primary objective is to develop PINN-based solvers that progress from the classical Black-Scholes equation to the complex Heston stochastic volatility model. Unlike standard "black-box" neural networks, PINNs incorporate physical laws directly into the training process by adding PDE residual terms to the loss function.

Both forward and inverse problems are considered:

- **Forward problem:** Given model parameters, solve for option prices $V(S,t)$.
- **Inverse problem:** Given observed option prices, calibrate model parameters.

2.1 Physics-Informed Loss Function

For an option price $V(S,t)$, the network is trained to minimize a composite loss function:

$$L = L_{PDE} + L_{BC} + L_{IC}. \tag{1}$$

In the Black-Scholes case, the PDE residual L_{PDE} is derived from

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0. \tag{2}$$

The Heston model extends this framework by introducing stochastic volatility. The variance process follows

$$dv_t = \kappa(\theta - v_t)\,dt + \xi\sqrt{v_t}\,dW_t, \tag{3}$$

and the corresponding option pricing PDE becomes

$$\frac{\partial V}{\partial t} + \frac{1}{2}vS^2\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} + \kappa(\theta - v)\frac{\partial V}{\partial v} - rV = 0. \tag{4}$$

3 Tentative Timeline

The project follows a structured timeline from initial research to final documentation:

- **February:** Literature review, data simulation for geometric Brownian motion and Heston paths, and setup of the training pipeline.
- **March:** Implementation of the Black-Scholes PINN baseline and validation against analytical benchmarks.
- **April:** Development of the Heston PINN and implementation of calibration utilities for inverse problems.
- **May:** Comprehensive experiments, result interpretation, and performance assessment under out-of-sample settings.
- **June:** Final write-up of results and preparation of repository demos.

4 Project Focus

We consider both the forward problem of solving option pricing PDEs and the inverse problem of model calibration using market data. The results will highlight the potential and limitations of physics-informed machine learning methods in quantitative finance, particularly for stochastic volatility modeling and parameter estimation.