9^{th} September, 2025

STK-MAT3700/4700

Mandatory assignment 1 of 1

Submission deadline

Thursday 16^{th} October 2025, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

In your mandatory assignment, you need to report how you came up with the answers to the various questions and the software you used and/or developed for this. You are free to choose software and/or programming language.

Problem 1. In this exercise you are going to study the Markowitz portfolio problem.

Download the 5 data series of daily stock prices from the course homepage (see "Messages"). If you find such on the web, you can instead download daily prices of your 5 favorite stocks and use these in the analysis.

a) Estimate the expected return and the volatility of the return of each asset. Plot the empirical distribution and the fitted normal distribution for the return of each asset.

b) Estimate the correlation between the returns of all five asset returns. What is the covariance matrix?

c) Plot the efficient frontier, and find the minimum-variance portfolio (along with its expected return and risk). Plot also the (σ, μ)-coordinates of each of the fives stocks in the same graphics. Comment on what you see.

d) Plot the efficient frontier if you remove one of your assets from your portfolio universe (that is, you have only 4 stocks to invest in). Explain what happens with the risk and expected return compared to the case of 5 assets.

Problem 2. In this exercise you are going to study the Black & Scholes formula and implied volatility

a) Implement the Black & Scholes formula, and calculate the price of call options written on an asset where the volatility is either $\sigma = 10\%$, or $\sigma = 30\%$ or $\sigma = 50\%$ (annually). For these three different volatility scenarios, plot your calculated options prices for strikes being $K = S_0$, $K = \pm 20\%$ of S_0 and $K = \pm 40\%$ of S_0, and exercise times being 1 month, 3 months and half a year. Measure time in years, and reason yourself what the risk-free interest rate r should be. You need to provide the argument for the choice.

b) Choose a stock or an index which has call options written on it.[1] Collect market prices for the options for a fixed exercise time, but with

[1] You can ignore the difference between American and European options, and treat all as European-style.

different strikes (at least two above and two below the current price, and one close to the current price). Calculate the implied volatilities, and plot these as a function of the strikes. Comment on what you find!

c) How would the implied volatilities look like if the market used the Black & Scholes formula to price options?

Hint to b): Here you can use a ready-made routine if you like. If not, you can implement a solver to find the implied volatility. Use the following idea: If you want to find the x such that $f(x) = 0$ for a function f, do the following iteration. Find two values x_0 and x_1 such that $f(x_0) < 0$ and $f(x_1) > 0$ (or opposite!!). We know that the desired x is in the interval $[x_0, x_1]$. Then define x_m to be the middle point in the interval $[x_0, x_1]$. Calculate $f(x_m)$, and if $f(x_m) > 0$, then choose the interval $[x_0, x_m]$ (since the desired x must be in this interval), or, if $f(x_m) < 0$, then choose the interval $[x_m, x_1]$ (since in this case the desired x must be here). Let us assume that we ended up choosing $[x_m, x_1]$. Relabel this to $[x_0, x_1]$, and find the middle point again x_m. Check the sign of $f(x_m)$, and choose a new interval according to whether the desired zero is in $[x_0, x_m]$ or $[x_m, x_1]$. Continue until the interval becomes so small that you can tell what the zero is with at least two decimals of accuracy (check interval distance, and continue halving the interval in the above manner until the distance is less than 0.005).