IE 306
SYSTEM SIMULATION
Homework 1
Group 4

Group Members

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IE 306 System Simulation - Homework 1

PROJECT DESC.

The task desired to be accomplished in this assignment is to design a queuing system of a “call center”. The system is mainly composed of three main sections.

The first section is an automated answering system that responses to the customers in the first place. This answering system consists of 100 parallel servers whose queueing capacity is exactly 0. Meaning that when all the 100 servers are busy, if a new customer arrives then he drops from the system entirely. We call those customers as **dropped** customers. The arrival rate of the customers (lambda) is exponentially distributed with a mean of 6. Meaning that a new customer arrives every 6 minutes on the average. On the other hand, the automated answering system records the personal details of the caller and routes the caller to either operator 1 or operator 2 by voice recognition. The duration of voice recognition is distributed exponentially with a mean of 5. Meaning that customers arrive at answering system with lambda 1/6 customers per minute and leaves from answering system to be routed to operators with lambda 1/5 customers per minute. In our design, we design the system described above as an M/M/100 queue with 0 queue capacity. (lambda is 1/6, mu is 1/5)

Customers routed to either operator 1 or operator 2. 30% of the customers routed to operator 1 and 70% to the other. In addition, 10% of overall routed customers are misrouted and they automatically leave the system. We call that customers as **misrouted** customers.

After a customer is routed to either of the operators, they join the queue of that operator. If the operator is busy they wait in the queue. A customer waits in the queue for up to 10 minutes. At the end of the 10 minutes, if he does not get a service, he hangs up and leaves the system. We call that customers as **reneged** customers. If a customer does not renege, meaning that he wait less than 10 minutes in the queue, he gets the service and leaves the system.

Each operator can take 3 minute breaks during their 8 hour shifts. The number of breaks an operator wishes to take during an 8-hour shift is known to be distributed according to a Poisson distribution with a mean of 8 breaks per shift. Break times in a shift are sampled from uniform distribution and for each operator in each shift number of breaks are sampled then that many uniform samples are sampled for the time of the breaks. Operators cannot take a break until processing all the customers waiting for them.

PARAMETERS

RANDOM_SEED	= 9, 8, 7, 6, 5, 4, 3, 2, 1, 99
SERVICE_RANGE	= [1,7]
INTERARRIVAL_RATE	=1/6
NUM_MACHINES	=100
RECOGNITION_RATE	=1/5
PATIENCE	=0
PATIENCE2	=10
MEAN	=12
STD	=6

RANDOM_SEED

This parameter is needed to recreate the same random variable sequences. So that we can use the same random variables with different parameters.

SERVICE_RANGE

It is the range for the uniform distribution to create random variables for the service duration of operator 2.

PATIENCE

It is the patience for the voice recognition system. If the system is full, then the customer drops immediately.

PATIENCE2

It is the patience for the queues of operators. If a customer waits as much as patience constant, then s/he reneges.

CALCULATING LOGNORMAL

Calculating mu and sigma for lognormal distribution using given mean and standard deviation

$$\mu = \ln\left(\frac{\mu_L^2}{\sqrt{\mu_L^2 + \sigma_L^2}}\right)$$

$$\sigma^2 = \ln\left(\frac{\mu_L^2 + \sigma_L^2}{\mu_L^2}\right)$$

Using these formulas, the parameters of lognormal distribution is calculated.

```
mean = 12
std = 6
```

```
MU = math.log(mean**2/math.sqrt(mean**2 + std**2))

VARIANCE = math.log((mean**2 + std**2) / mean**2 )

SIGMA = math.sqrt(VARIANCE)
```

CLASS STRUCTURE

AnswerMachine

Fields

machine is a simpy resource whose capacity is determined by *NUM_MACHINES* parameter.

recognize_rate is determined by *RECOGNITION_RATE* parameter.

Functions

recognize(..) is called when a customer is answered by the voice recognition system. The duration of the recognition is distributed exponentially with *recognize_rate* parameter.

Customer

Fields

name is the name of the customer.

arrival_t is the arrival time to the system.

redirection_time is the time when the customer finished the conversation with the voice recognition system and is redirected to an operator.

leave_time is the time the customer leaves the system.

action is to call *firstCall(..)* function automatically.

total_waiting_time is the time passed in a queue.

total_service_time is the time passed talking with a voice recognition system and an operator.

Functions

ask_question(..) is to ask a question to an operator given as a parameter. If the customer asks to operator 1, the duration is distributed as lognormally with calculated mu and sigma. If the customer asks to operator 2, the duration is distributed as normal with *SERVICE_RANGE* parameter.

redirect(..) with this method, the customer enters the queue of an operator given as parameter and starts to wait. If the customer waits as much as *PATIENCE2* parameter, s/he reneges. If the turn comes to her/him, s/he asks a question to the desired operator with *ask_question(..)* function.

probability(..) with this method, which operator the customer asks is determined by a uniformly distributed random value. The customer may be misrouted by a 10% chance. Then the customer hangs up.

firstCall(..) method represents the arrival of a customer to the answering system. The customer requests for an empty server of the answering system. If there is an available one, then the *recognize(..)* function is invoked. If not the customer drops. After that, *probability(..)* function is called.

A sample flow of functions for a ordinary customer:

firstCall(..) -> recognize(..) -> probability(..) -> redirect(..) -> ask_question(..)

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generate_breaks(...) With this method we modelled the process of generation of the breaks for each operator at the beginning of each shift.

take_break_1(...) With this method we modelled the process of the operator 1 trying to take a break at the generated break times.

take_break_2(...) With this method we modelled the process of the operator 1 trying to take a break at the generated break times.

simulation_results() With this method we calculate the metrics of interest after completing a simulation

STATISTICS

PS: The percentages of utilizations are out of 100, meaning that 44 utilization is 44%.

General Statistics for 1000 Customers:

	Seed	Answer_System_Util	Operator1_Util	Operator2_Util	Avg_queue_len_1	Avg_queue_len_2	Max_total_wait_time	Max_total_system_time	MTWT/MTST	avg_wait_time	Num_Of_Unsatisfied_Customers	Simulation_time
0	9	0.788641	44.766676	41.131224	0.365499	0.199143	1780.102830	11830.760778	15.046394	2.000116	162	6100.106421
1	8	0.864075	48.845112	42.202846	0.375994	0.279054	2107.360894	12601.589548	16.722977	2.308172	138	5913.724598
2	7	0.892891	48.707365	44.304568	0.430205	0.277755	2168.933119	12215.628641	17.755395	2.412606	167	5511.047383
3	6	0.855937	44.515552	42.466448	0.357070	0.227717	1873.927049	12250.045612	15.297307	2.061526	143	6012.501178
4	5	0.764848	44.452163	42.188343	0.393729	0.258601	1925.787356	11888.388999	16.198893	2.123250	156	6107.328698
5	4	0.838588	41.950022	40.675488	0.383868	0.203067	1647.614339	11682.553645	14.103204	1.874419	164	6027.557166
6	3	0.807514	45.809346	41.386881	0.311904	0.222411	1918.974472	12020.552104	15.964113	2.090386	142	6014.720182
7	2	0.843235	42.722403	42.787651	0.412428	0.261767	1917.963580	12043.843329	15.924847	2.112295	157	5962.236896
8	1	0.786278	43.140918	41.660686	0.364361	0.221695	1871.003606	11752.727701	15.919739	2.067407	151	6046.479640
9	10	0.812896	46.978923	40.368514	0.389268	0.255671	1984.780962	12092.882657	16.412803	2.188292	150	5993.997495

	Answer_System_Util	Operator1_Util	Operator2_Util	Avg_queue_len_1	Avg_queue_len_2	Max_total_wait_time	Max_total_system_time	MTWT/MTST	avg_wait_time	Num_Of_Unsatisfied_Customers	Simulation_time
count	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000
mean	0.825490	45.188848	41.917265	0.378433	0.240688	1919.644821	12037.897301	15.934567	2.123847	153.000000	5968.969966
std	0.040138	2.386984	1.145962	0.032439	0.029515	148.919940	272.957663	0.988018	0.151788	9.899495	171.005205
min	0.764848	41.950022	40.368514	0.311904	0.199143	1647.614339	11682.553645	14.103204	1.874419	138.000000	5511.047383
25%	0.793359	43.468729	41.195139	0.364645	0.221874	1871.734467	11845.167833	15.452915	2.062996	144.750000	5970.177046
50%	0.825742	44.641114	41.924514	0.379931	0.241694	1918.469026	12032.197716	15.944480	2.101340	153.500000	6013.610680
75%	0.852762	46.686528	42.400547	0.392614	0.260975	1970.032561	12184.942145	16.359325	2.172031	160.750000	6041.749021
max	0.892891	48.845112	44.304568	0.430205	0.279054	2168.933119	12601.589548	17.755395	2.412606	167.000000	6107.328698

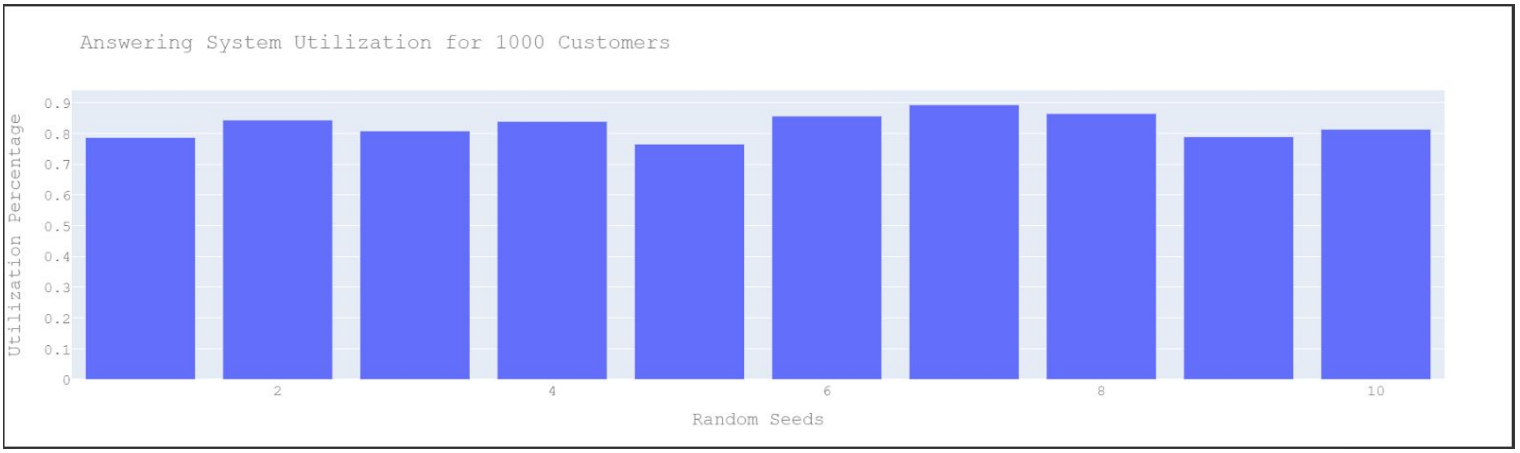
General Statistics for 5000 Customers:

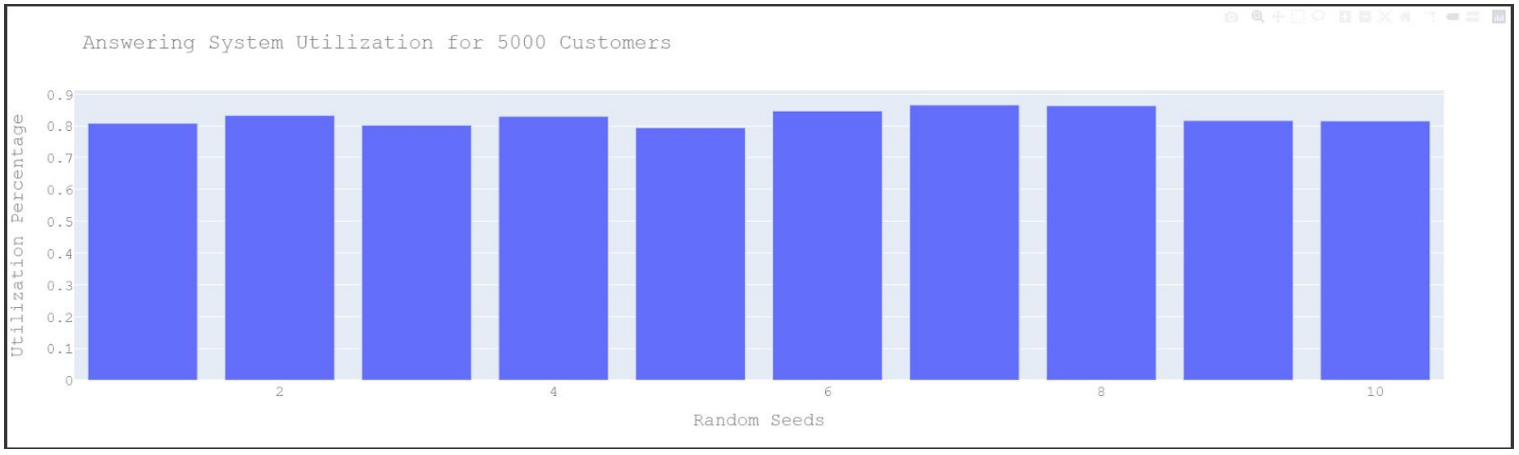
Seed	Answer_System_Util	Operator1_Util	Operator2_Util	Avg_queue_len_1	Avg_queue_len_2	Max_total_wait_time	Max_total_system_time	MTWT/MTST	avg_wait_time	Num_Of_Unsatisfied_Customers	Simulation_time
9	0.816389	45.295703	40.146492	0.375693	0.228977	9544.663234	60554.905307	15.761998	2.115395	774	30530.235427
8	0.863237	45.530369	41.182325	0.421645	0.263155	10263.098073	61202.092178	16.769195	2.279169	811	29438.304469
7	0.865630	46.316251	42.444939	0.362172	0.258059	9949.399575	60835.563701	16.354578	2.204120	779	29024.045583
6	0.846834	45.000056	41.086000	0.387272	0.232007	9446.518270	60940.434309	15.501232	2.094107	775	30154.056226
5	0.794023	43.829267	41.544748	0.346484	0.251297	9122.199506	59894.263477	15.230506	2.010624	717	30812.712688
4	0.829542	44.940208	40.584073	0.374683	0.229048	9279.651433	60474.783548	15.344663	2.070426	781	30386.747064
3	0.801684	43.836094	40.446680	0.363539	0.232733	9454.318219	59665.136921	15.845632	2.096301	787	30532.362053
2	0.832389	46.019526	41.080493	0.402647	0.242375	9849.632142	61013.186382	16.143448	2.173352	790	30036.334473
1	0.807802	43.083977	41.617893	0.359713	0.213068	8821.645999	58782.671088	15.007222	1.964294	758	30191.207789
10	0.815339	44.747231	41.372703	0.363415	0.249846	9682.323718	59927.059047	16.156848	2.139740	780	29969.330768

	Answer_System_Util	Operator1_Util	Operator2_Util	Avg_queue_len_1	Avg_queue_len_2	Max_total_wait_time	Max_total_system_time	MTWT/MTST	avg_wait_time	Num_Of_Unsatisfied_Customers	Simulation_time
count	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000	10.000000
mean	0.827287	44.859868	41.150635	0.375726	0.240057	9541.345017	60329.009596	15.811532	2.114753	775.200000	30107.533654
std	0.024898	1.022427	0.661072	0.022516	0.015549	419.134279	753.919047	0.552148	0.091463	24.484916	536.401874
min	0.794023	43.083977	40.146492	0.346484	0.213068	8821.645999	58782.671088	15.007222	1.964294	717.000000	29024.045583
25%	0.809686	44.063878	40.708178	0.362483	0.229788	9321.368142	59902.462369	15.383805	2.076347	774.250000	29986.081694
50%	0.822965	44.970132	41.134163	0.369111	0.237554	9499.490726	60514.844427	15.803815	2.105848	779.500000	30172.632008
75%	0.843223	45.471703	41.501737	0.384377	0.250934	9807.805036	60914.216657	16.153498	2.164949	785.500000	30494.363336
max	0.865630	46.316251	42.444939	0.421645	0.263155	10263.098073	61202.092178	16.769195	2.279169	811.000000	30812.712688

1-) Utilization of Answering System

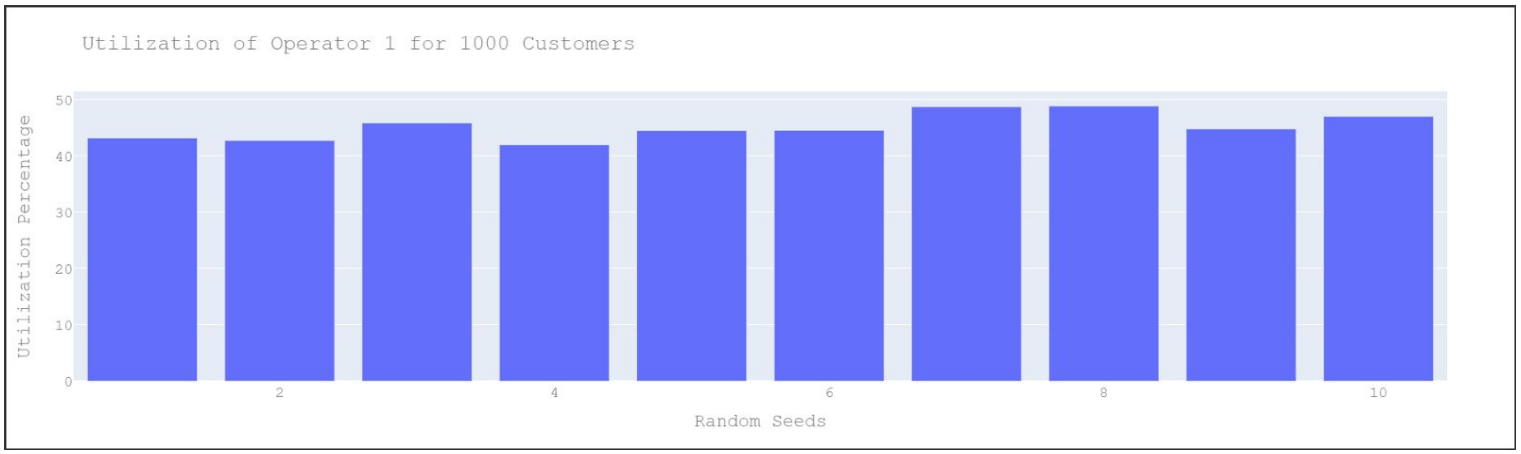
Server utilization is defined as the proportion of time that a server is busy. In order to calculate the utilization of the answering system, we summed up the time of all the customers spent in answering system and divided into the number of answering services. (100 in our case) This leads us to the average amount of time a single server is busy, during the simulation, which is the service time of an answering machine server on the average. Then we divided this time to total simulation time so that we end up the proportion of time a single answering system server is busy on the average.

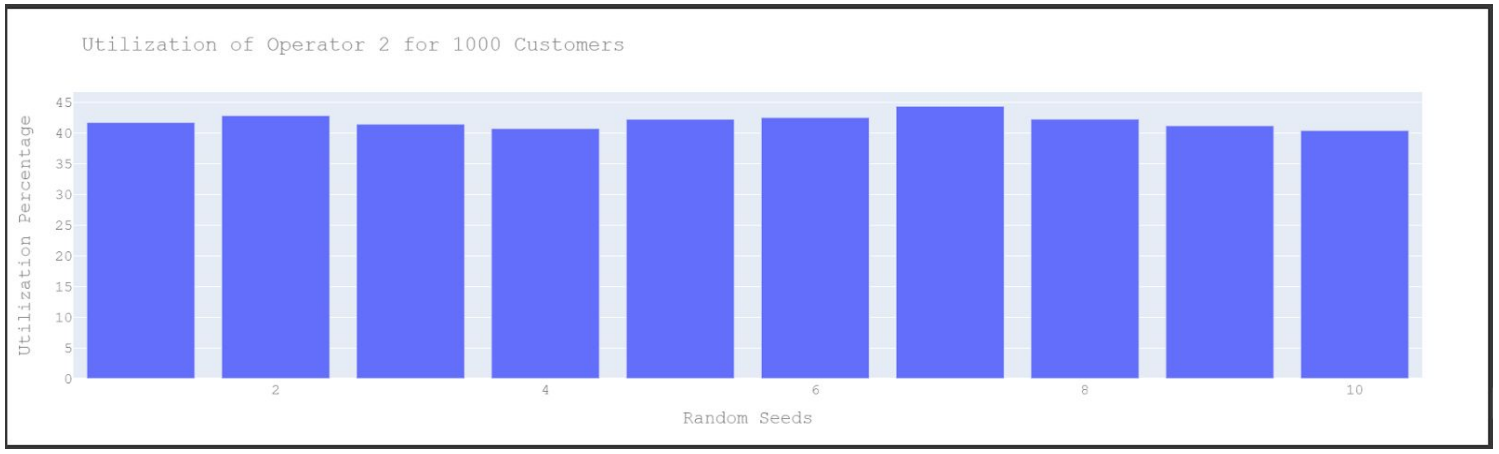


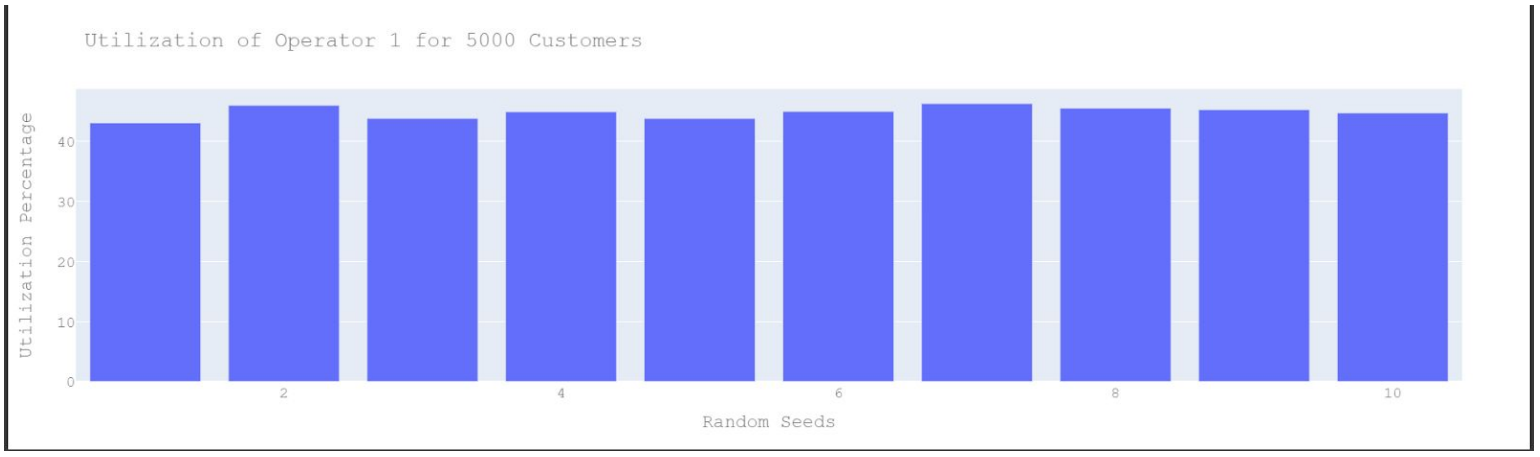


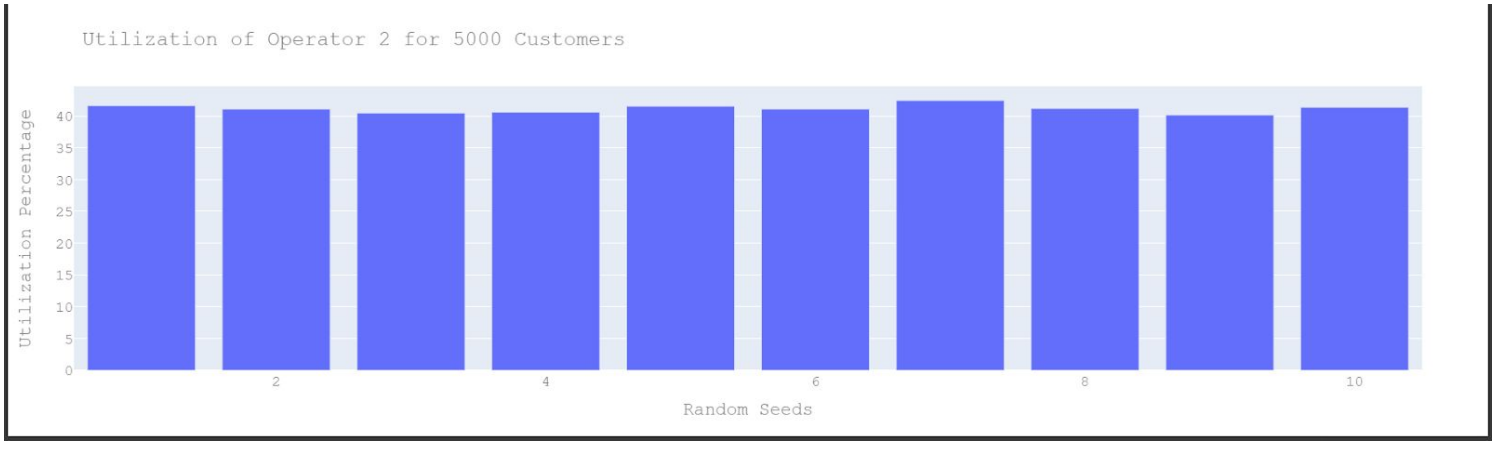
2-) Utilization of Operators

We divided the total amount of time an operator is busy, to the total simulation time.


Utilization of Operator 1 for 1000 Customers
Utilization Percentage
0
10
20
30
40
50
2
4
6
8
10
Random Seeds

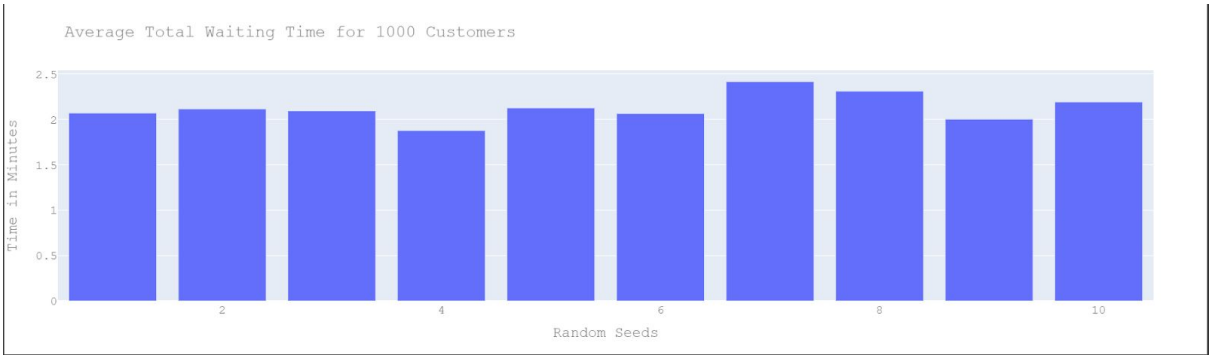

Utilization of Operator 2 for 1000 Customers
Utilization Percentage
0
5
10
15
20
25
30
35
40
45
2
4
6
8
10
Random Seeds

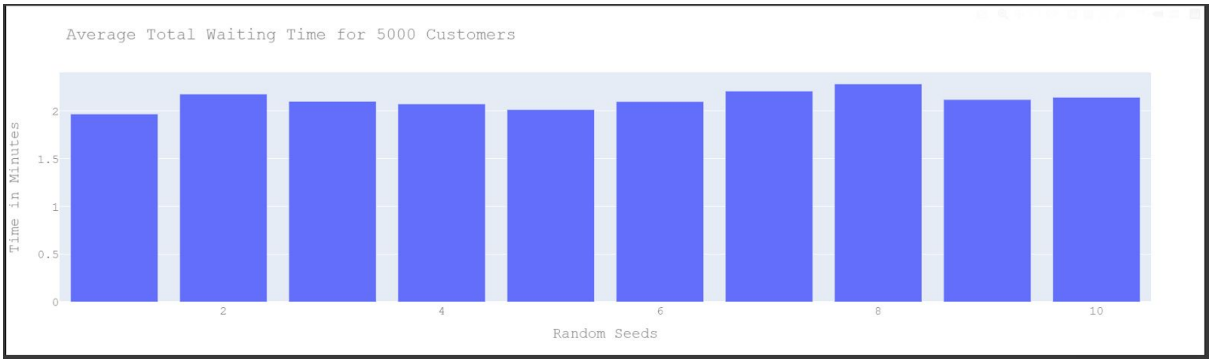

Utilization of Operator 1 for 5000 Customers
Utilization Percentage
0
10
20
30
40
2
4
6
8
10
Random Seeds


Utilization of Operator 2 for 5000 Customers
Utilization Percentage
0
5
10
15
20
25
30
35
40
2
4
6
8
10
Random Seeds

3-) Average Total Waiting Time

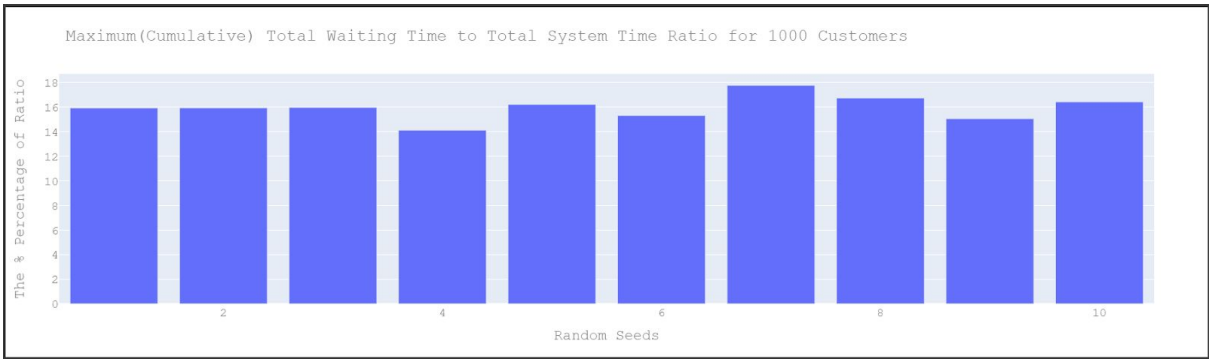
We count the amount of time each customer spent in the queue for both queues **including the reneged ones**. Then summed them up for each queue and divided the number of customers joined to that queue during the whole simulation.

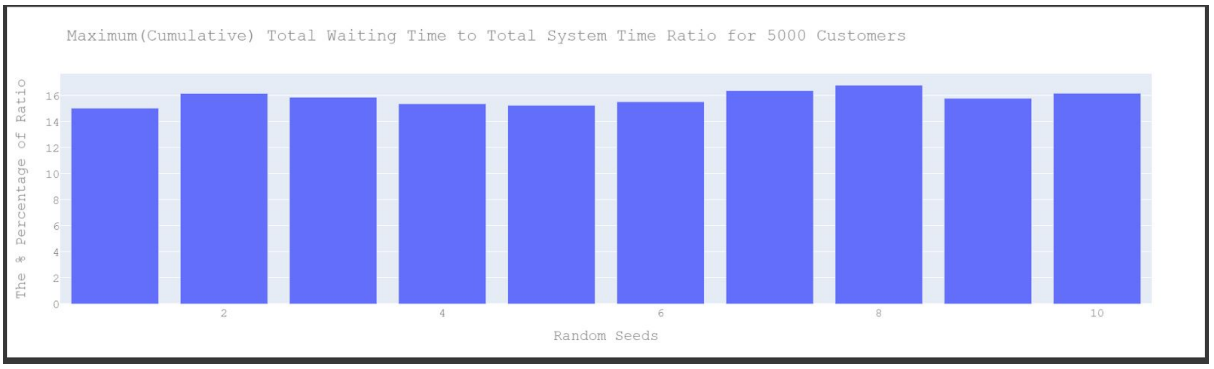




4-) Maximum Total Waiting Time to Total System Time Ratio

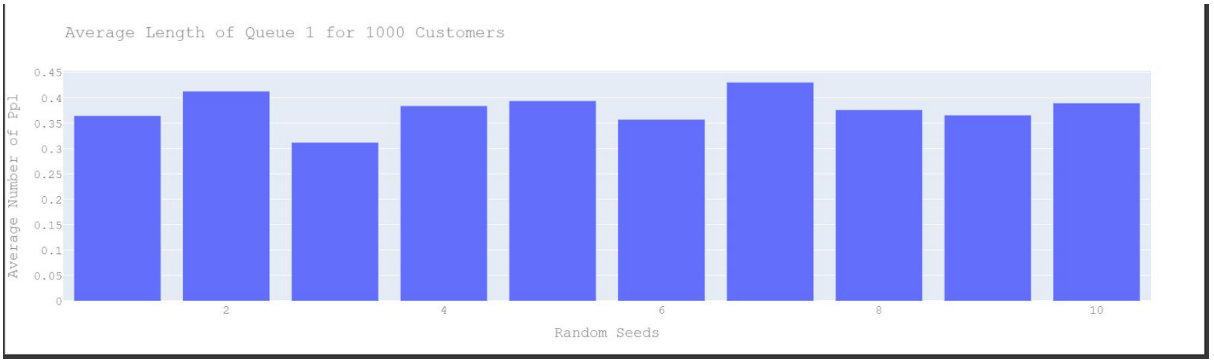
We count the amount of time spent in the queue and the duration of the time spent in the system for each customer. **We included the ones reneged after waiting 10 minutes in the queue.** Then we summed up all the times spent in the queue and the system (answering system + queue + service). Then divided the total queue time to total system time.

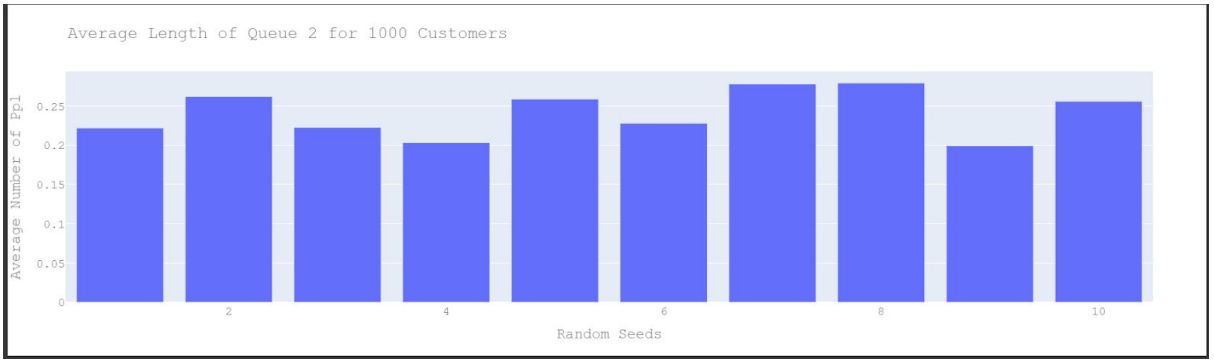


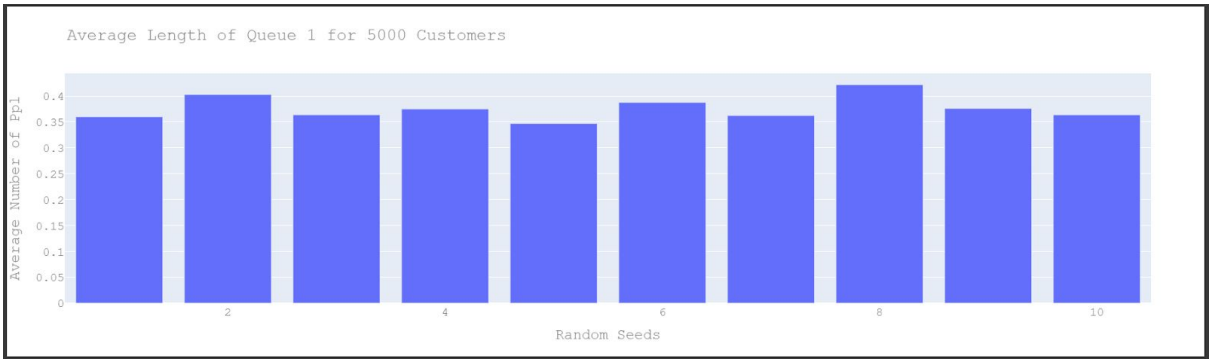


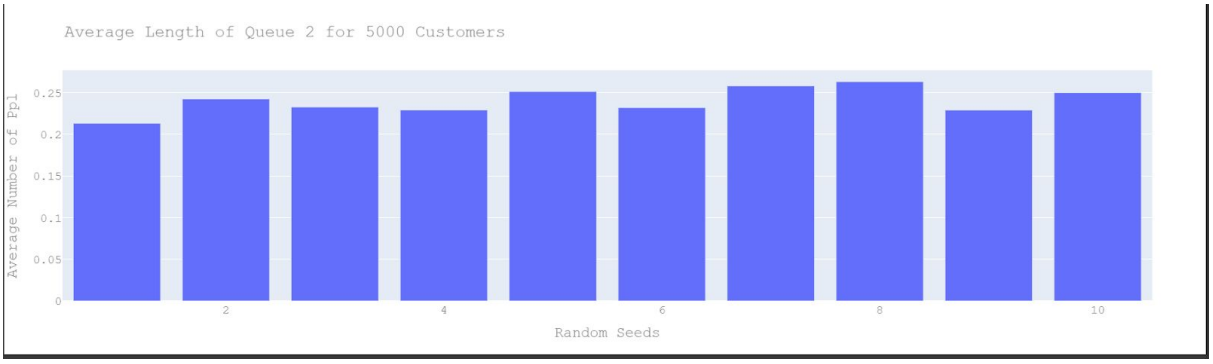
5-) Average number of people waiting to be served by each operator

This average is the same as the average queue length of each operator. The formula for this metric for a simulation is the integral of queue length from zero to the end of the simulation over total simulation time.

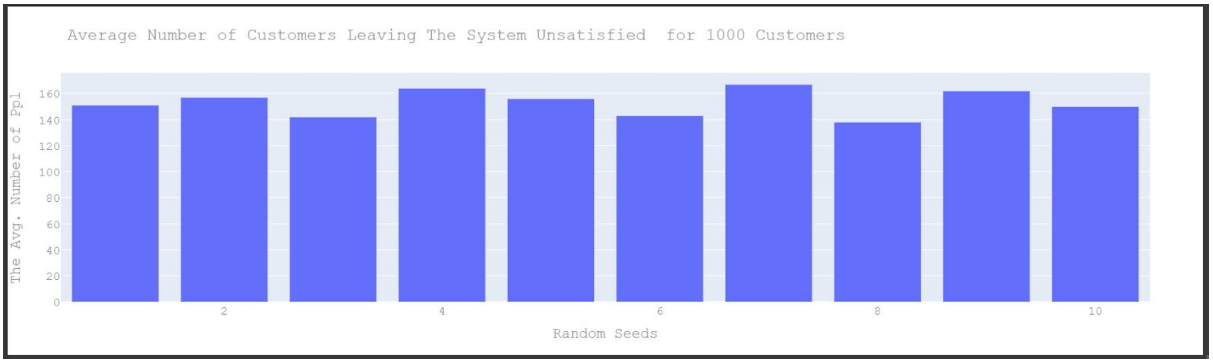


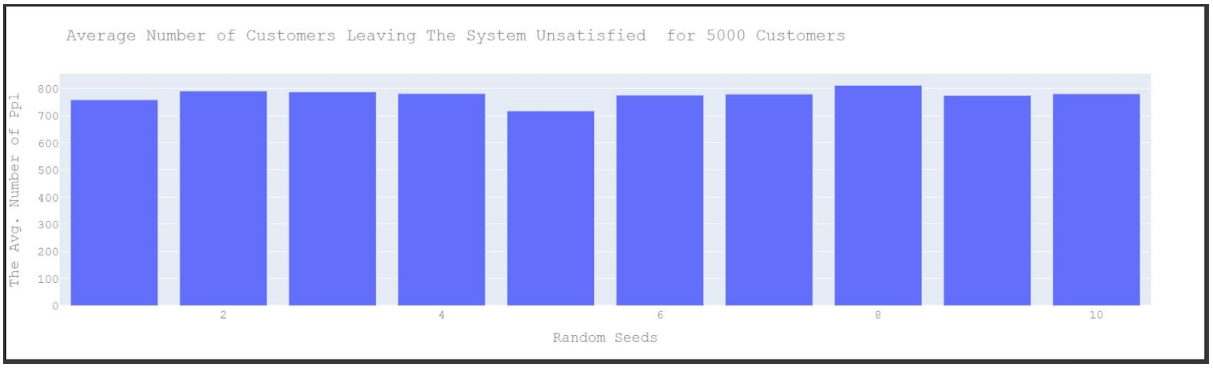






6-) Average number of customers leaving the system unsatisfied either due to incorrect routing or due to long waiting times





count	10.000000
mean	153.000000
std	9.899495
min	138.000000
25%	144.750000
50%	153.500000
75%	160.750000
max	167.000000

count	10.000000
mean	775.200000
std	24.484916
min	717.000000
25%	774.250000
50%	779.500000
75%	785.500000
max	811.000000

The average number of unsatisfied customers for 1000(right), and 5000(left) customers.