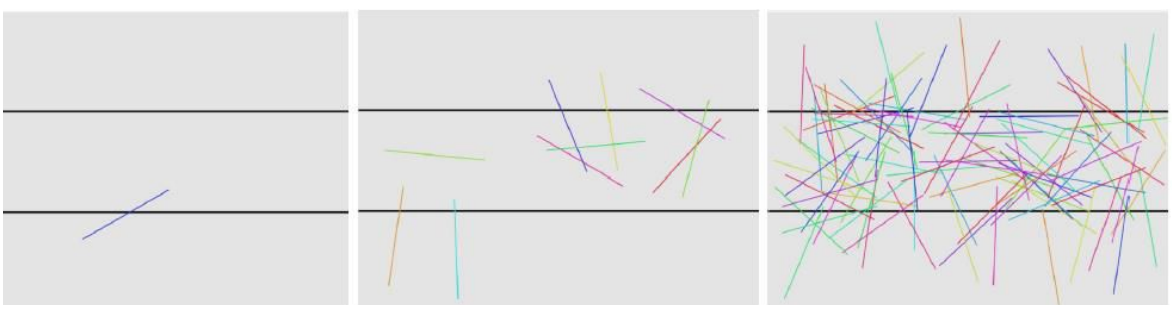
Cmpe 49G : Buffon's Needle

Emre Girgin - 2016400099

Problem Definition:

Buffon's needle is a probability problem which can be solved by using Monte Carlo Simulations. Suppose you have N needles with length L. You put sticks parallel to each other with distance D. The question is, what is the probability of dropping a needle into one of those sticks ? In fact, it is approximated to **2 * L / D * PI**. This problem can be solved geometrically. However, we are going to use Monte Carlo simulations to verify the equation above.



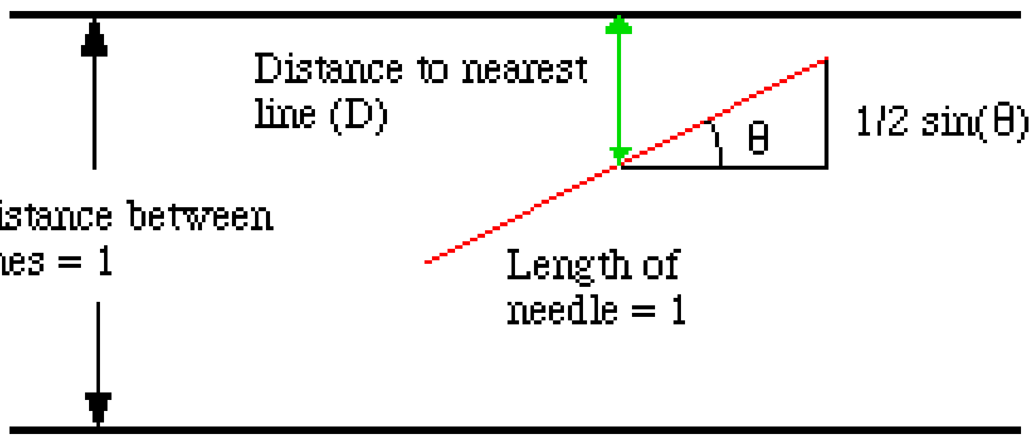
From the description of Project 1

Approach:

When we drop a needle, it has two features: position and orientation. Position can be represented in a cartesian plane with x and y coordinates of its center. In fact, we do not need to use a dimension for horizontal position since the environment doesn't change horizontally. Thus, we will store the position only with its height in the environment. On the other hand, the orientation can be represented in radians. We also don't need to store it in a range from 0 to 2*PI. By only looking for the positive angle the needle made with the x-axis, we can represent its orientation. <u>At the end, we have a height and angle ranging between 0 and PI for each needle.</u>

On the other hand, the environment can be represented in a simpler manner, too. We already removed its horizontal axis due to its stability in that dimension. We can also bound it between 0 and D/2. When we drop a needle, its center can either drop the upper part of the gap between two lines or the lower part. Since the distribution of the needles is uniform, simulating only for the lower part of a gap is exactly the same for doing it for the upper part. The same thing applies for the downside of the line. Thus, it is sufficient to simulate only the part from 0 to D/2 for a single line located at 0.

Finally, we need to count the number of crossing needles to our single line located at 0. When a needle is dropped, if its center is located lower than the vertical distance between its center and lower end then we have a crossing. We can easily calculate the vertical distance between its center and lower point by taking the sin of its angle and multiply it with L/2.

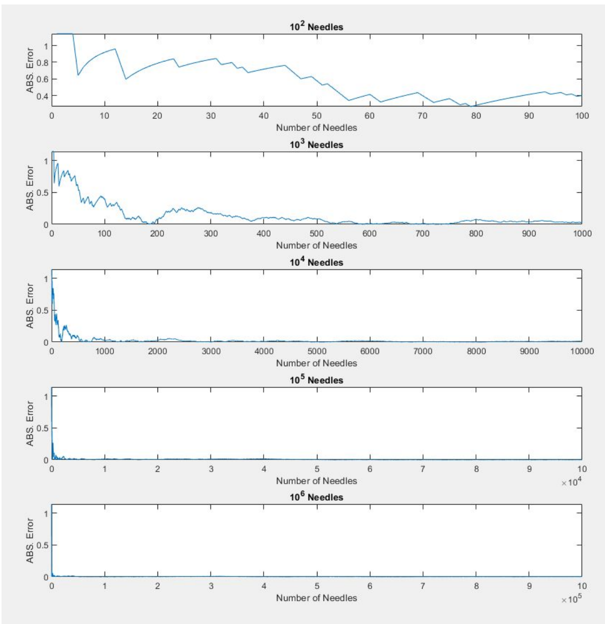


Buffon's Needle, MSTE, University of Illinois
(Both L and D is 1 here so L/2 * sin(theta) = ½ * sin(theta). Also notice that in the figure above, D is the vertical distance from center to closest line, different than our notation.)

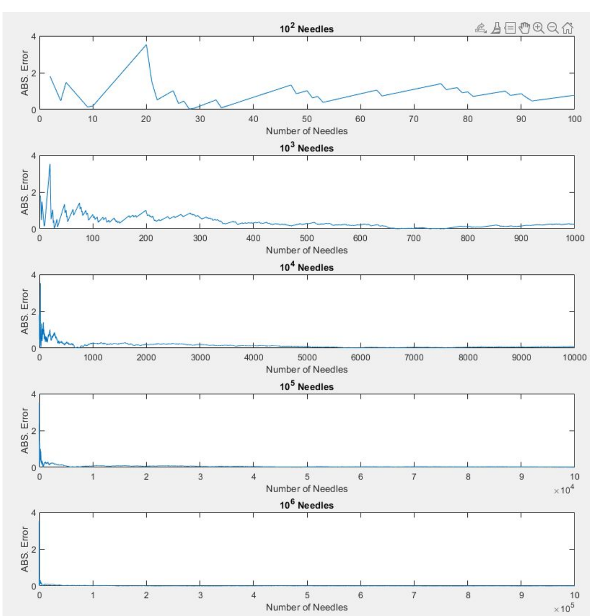
Thus, the crossing is occurred iff height <= L/2 * sin(theta), in our simulation environment.

Some of the Results: (X-Axis : # of needles, Y-Axis: ABS error)

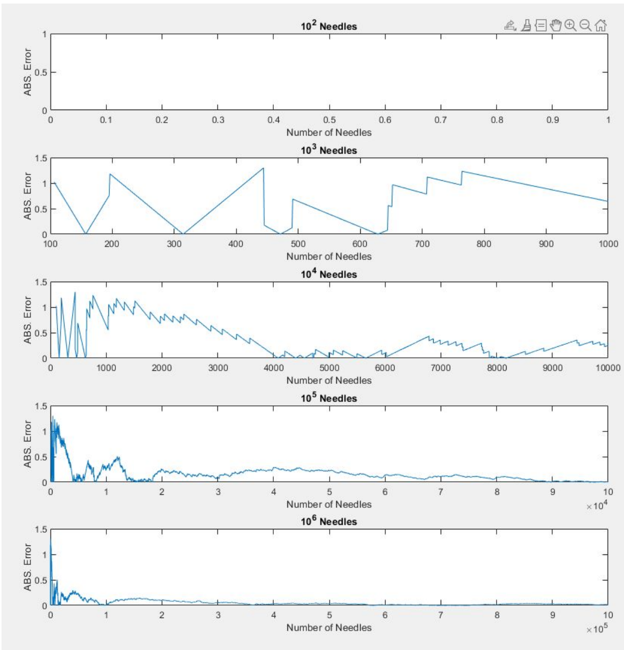
When L = 5 and D = 5



When L = 1 and D = 3



When L = 1 and D = 100



Conclusion:

The absolute error between PI and estimated PI decreases as the number of needles increases. Even in sparse cases like L = 1 and D = 100, the number error converges to 0 when there are a million needles. This simulation is valid only if L <= D.