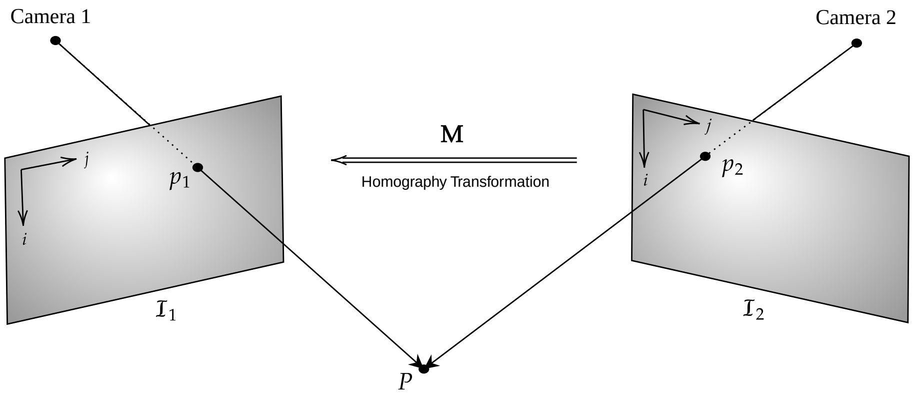
CMPE 482: Assignment 6

Spring 2021

Deadline: July 2, 2021, 20:00

In this assignment, you are given 2 images ("img_left.jpg", "img_right.jp") from the same 3D scene, and you are expected to stitch them using some of their corresponding (matching) points ("points_left.npy", "points_right.npy"). In the first question, you will estimate the **homography transformation** between these images. As for the second and third questions, you will map the pixel coordinates of the both images onto the same image plane by using your estimated homography transformation, and perform a 2D linear interpolation operation on this image plane. For each question, we provide some example outputs in order for you to be able to compare your results.



Let $\mathcal{I}_1$ and $\mathcal{I}_2$ be two different images of a scene from different camera angles, and let $\mathcal{C}_1$ and $\mathcal{C}_2$ be their coordinate systems. We assume that they are represented via 3-way integer tensors of shape $H \times W \times 3$, i.e. $\mathcal{I}_1, \mathcal{I}_2 \in \{0, 1 \ldots, 255\}^{H \times W \times 3}$, where H is the height, and W is the width of the images. Assume that $P = (x, y, z) \in \mathbb{R}^3$ is a point in 3D space, and let $p_1 = (i_1, j_1) \in \mathcal{C}_1$ and $p_2 = (i_2, j_2) \in \mathcal{C}_2$ be its projections onto the image planes of $\mathcal{I}_1$ and $\mathcal{I}_2$, respectively. Then, p_1 and p_2 are called **corresponding points**, and they are assumed to satisfy the following **homogeneous transformation** relation:

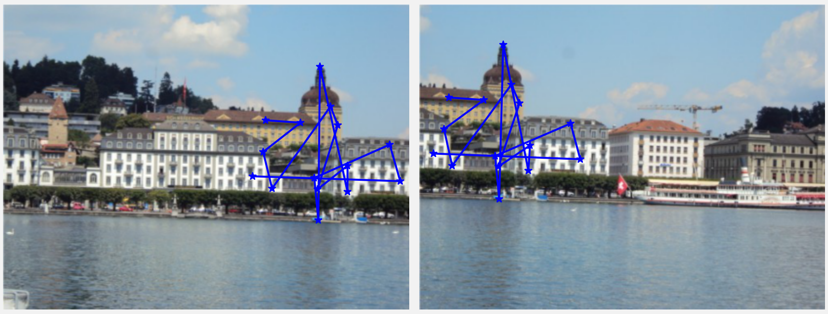
$$c \begin{bmatrix} i_1 \\ j_1 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} m_{00} & m_{01} & m_{02} \\ m_{10} & m_{11} & m_{12} \\ m_{20} & m_{21} & m_{22} \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} i_2 \\ j_2 \\ 1 \end{bmatrix} \tag{1}$$

for a scalar $c \in \mathbb{R}$ and for a matrix $\mathbf{M} \in \mathbb{R}^{3\times3}$ with the condition $\|\text{vec}(\mathbf{M})\|_2 = 1$. Here $\mathbf{M}$ is called a **homography matrix** whose purpose is mapping the points in the second coordinate system $\mathcal{C}_2$ to the first coordinate system $\mathcal{C}_1$. Note that there can be many corresponding point pairs between $\mathcal{I}_1$ and $\mathcal{I}_2$, and for each such pair the equality above should be satisfied by the same homography matrix $\mathbf{M}$, while the value of c may differ.

Question 1 (Homography estimation) *In this question, our goal is to estimate a homography matrix from the corresponding point pairs between two images.*

(a) *Read $\mathcal{I}_1$ ("img_left.jpg") and $\mathcal{I}_2$ ("img_right.jpg") into the memory, and plot them using matplotlib.pyplot.imshow. Read the corresponding point pairs "points_left.npy" and "points_right.npy", and also plot them onto the pictures.*

Output



(b) *Show that if* (1) *is satisfied for two corresponding points, then so is the following equality:*

$$\underbrace{\begin{bmatrix} i_2 & j_2 & 1 & 0 & 0 & 0 & -i_1 i_2 & -j_2 i_1 & -i_1 \\ 0 & 0 & 0 & i_2 & j_2 & 1 & -j_1 i_2 & -j_2 j_1 & -j_1 \end{bmatrix}}_{\mathbf{a}} \underbrace{\begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{20} \\ m_{21} \\ m_{22} \end{bmatrix}}_{\mathrm{vec}(\mathbf{M})} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2}$$

(c) *For each corresponding point pair, you will have a different equation in the form of $\mathbf{a} \cdot \mathrm{vec}(\mathbf{M}) = \mathbf{0}$. In order to estimate $\mathbf{M}$ from these equations, you need to solve them simultaneously for $\mathrm{vec}(\mathbf{M})$:*

$$\underbrace{\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \\ \vdots \end{bmatrix}}_{\mathbf{A}} \mathrm{vec}(\mathbf{M}) = \mathbf{0} \tag{3}$$

with the constraint $\|\mathrm{vec}(\mathbf{M})\|_2 = 1$. Note that we don't expect to find an exact solution, since $\mathbf{A}$ is a tall matrix. Show that the vector with the smallest singular *value in the row space of* $\mathbf{A}$*, i.e. the last column of* $\mathbf{V}$ *in the singular value decomposition of* $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$*, is the least squares solution of this system of constrained linear equations.*

(d) *Construct the matrix* $\mathbf{A}$ *for the corresponding point pairs you plotted in the first part, and calculate the least squares solution* $\mathrm{vec}(\mathbf{M})$*. Print the resulting homography matrix* $\mathbf{M}$*.*

Output

$$\mathbf{M} = \begin{bmatrix} 3.4129610e-03 & 1.8847009e-04 & 9.9513519e-01 \\ -9.9461991e-05 & 3.5539148e-03 & 9.8332070e-02 \\ -3.1538366e-07 & 3.5065750e-07 & 3.4879083e-03 \end{bmatrix}$$

Question 2 (2D linear interpolation) *In this part, our goal is to interpolate the colors of arbitrary coordinates in an image. Let* $\mathcal{I} \in \{0, 1 \ldots, 255\}^{H\times W\times 3}$ *be an* $H \times W$ *image, whose pixels are located on the integer grid* $\mathcal{G} = \{0, 1, \ldots, H-1\} \times \{0, 1, \ldots, W-1\}$*. We denote the color of a pixel at coordinate* $p = (i, j)$ *by the vector* $\mathcal{I}(i, j) \in \{0, 1 \ldots, 255\}^3$*, where* $0 \le i \le H-1$ *and* $0 \le j \le W-1$*. In order to interpolate the color of a single point* $q = (y, x)$ *on this image, first we need to locate the closest pixel coordinate on its upper left side, whose formula is given by the equation:*

$$(i_q, j_q) = (\lfloor y \rfloor, \lfloor x \rfloor) \tag{4}$$

For interpolating the color of the point q*, we make use of the color information of its 4 surrounding pixels. We assume a linear relationship between the colors and the coordinates of these pixels,*

$$\begin{bmatrix} 1 & i_q & j_q \\ 1 & i_q & j_q+1 \\ 1 & i_q+1 & j_q \\ 1 & i_q+1 & j_q+1 \end{bmatrix} \mathbf{v} = \begin{bmatrix} \mathcal{I}(i_q, j_q) \\ \mathcal{I}(i_q, j_q+1) \\ \mathcal{I}(i_q+1, j_q) \\ \mathcal{I}(i_q+1, j_q+1) \end{bmatrix} \tag{5}$$

and estimate the coefficient matrix $\mathbf{v}$ *by solving the equation above. Then, the color of the coordinate* q *can be interpolated linearly by*

$$\hat{\mathcal{I}}(y, x) = \begin{bmatrix} 1 & y & x \end{bmatrix} \mathbf{v} \tag{6}$$

(a) *Create the following mesh-grids:*

- *A mesh-grid on* $[0, 374] \times [0, 499]$ *with* 75×100 *equispaced mesh points.*
- *A mesh-grid on* $[0, 250] \times [0, 300]$ *with* 501×601 *equispaced mesh points.*
- *A mesh-grid on* $[-100, 475] \times [-100, 600]$ *with* 250×250 *equispaced mesh points.*

(b) *For each mesh-grid, we will produce an interpolated image. For each point in each mesh-grid, linearly interpolate the color of the point using* (5) *and* (6)*. If a point falls outside of the range of the image, its color will be assumed black. Combine the resulting colors in a 3-way integer (uint8) tensor, and plot them.*

Output

Question 3 (Image stitching) *For this question, you are expected to map the pixels in $\mathcal{I}_2$ into the coordinate system $\mathcal{C}_1$, and stitch the resulting image onto $\mathcal{I}_1$.*

(a) *Map the corner coordinates of the image $\mathcal{I}_2$ into $\mathcal{C}_1$ using the homography transformation in* (1) *and the homography matrix you calculated in the first question. Find the bounding rectangle of the image to be transformed by using the resulting coordinates. Print the upper left, and lower right coordinates of the bounding rectangle.*

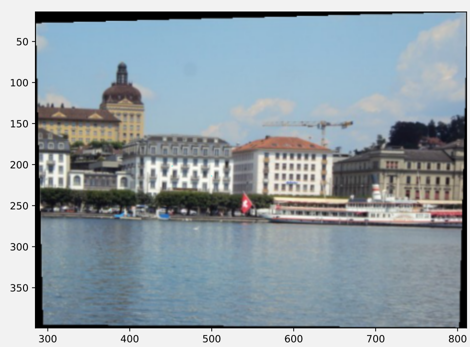
Output

Upper-left: (y, x) = (14, 285)
Lower-right: (y, x) = (398, 810)

(b) *Construct a mesh-grid on the bounding rectangle with unit distance between neighbouring points. Note that these mesh points will correspond to the coordinates of new pixels in this rectangle.*

(c) *Map mesh points into $\mathcal{C}_2$ using the inverse of the homography matrix* $\mathbf{M}$. *Determine the colors of the mapped points using 2D linear interpolation on the image $\mathcal{I}_2$. Color the pixels of the bounding rectangle accordingly, to obtain the homography transformation of the image $\mathcal{I}_2$. Plot the transformed $\mathcal{I}_2$.*

Output



(d) *Stitch $\mathcal{I}_1$ and the transformed $\mathcal{I}_2$ together, i.e. place them into a panoramic image.*

Output