（計算したいもの）（2）

(＊) $\widetilde{CSW}_{4m+3} = \sum_{\sigma \in \mathfrak{S}_7} \mathrm{sgn}(\sigma)\, \bar{R}_{\sigma_1 l_1 ①}{}^{l_0}\, \bar{R}_{\sigma_2 \sigma_3 l_2}{}^{l_1}$
$\times \bar{R}_{\sigma_4 \sigma_5 l_3}{}^{l_2}\, \bar{R}_{\sigma_6 \sigma_7 l_0}{}^{l_3}$

$\sigma = (\sigma_1, \cdots, \sigma_7) \in \mathfrak{S}_7$ （1〜7の置換群）

（↙件）
（(＊)の右辺の最初の $\bar{R}$ には $\underline{1}$ が入っている（赤丸））

index の上げ下げは $g_{ij} = \delta_{ij}$, $g^{ij} = \delta^{ij}$

〈index の使い方〉

$\alpha, \beta, \gamma = 1, 2, 3$

$a, b, c = 4, 5, 6, 7$

$i, j, k, l = 1, 2, 3, \cdots, 7$

$l_1, l_2, l_3 = 1, 2, 3, \cdots, 7$

$$\bar{R}_{abcd} = R_{abcd} - p_\alpha^2 \sum_{\alpha=1}^{3} A_{\alpha,bc} A_{\alpha,ad} + p_\alpha^2 \sum_{\alpha=1}^{3} A_{\alpha,ac} A_{\alpha,bd} + 2p_\alpha^2 \sum_{\alpha=1}^{3} A_{\alpha,ab} A_{\alpha,cd}$$

$$\bar{R}_{abc\alpha} = -p_\alpha \nabla_a A_{\alpha,bc} + p_\alpha \nabla_b A_{\alpha,ac}$$

$$\bar{R}_{ab\alpha\beta} = -2p_\alpha p_\beta A_{\beta,a}{}^{c} A_{\alpha,b}{}^{d} g_{cd} + 2p_\alpha \sum_{\beta=1}^{3} A_{\beta,ab} \varepsilon_{\beta\alpha}{}^{\gamma} \xi_\gamma$$

$$\bar{R}_{a\alpha\beta b} = -p_\alpha p_\beta A_{\alpha,b}{}^{c} A_{\beta,ac} + \sum_{\gamma=1}^{3} p_\gamma \varepsilon_{\alpha\beta}{}^{\gamma} A_{\gamma,ab}$$

$\bar{R}_{abcd}$, R_{abcd} は curvature の性質をもつ

$\bar{R}_{abcd} = \bar{R}_{cdab}$, $\bar{R}_{abcd} = -\bar{R}_{bacd}$,

$\bar{R}_{abcd} + \bar{R}_{bcad} + \bar{R}_{cabd} = 0$ (Jacobi identity)

....

- $p_1, p_2, p_3 \ (= p_4)$ は parameter

- $A_1 \simeq \left(\begin{array}{cc|c} 0 & 1 & \\ -1 & 0 & 0 \\ \hline & 0 & \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \end{array}\right)$, $A_2 = \left(\begin{array}{cc|c} & & \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \\ \hline 0 & & \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \end{array}\right)$

$A_3 = \left(\begin{array}{cc|c} & 0 & \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \\ \hline \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} & & 0 \end{array}\right)$