IGIMF NOTES
Based on Yan, Jerabkova, & Kroupa (2021) A&A

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1 Method

$$
\begin{aligned}
\delta t &= 10^7 \mathrm{yr} \\
Z_\odot &= 0.0142 \\
\Delta\alpha &= 63 \\
m_{\max *} &= 150 M_\odot \\
M_{\mathrm{ecl},\max *} &= 10^9 M_\odot \quad (10^{10} \mathrm{mightbebetter}) \\
M_{\mathrm{ecl},\min} &= 5 M_\odot
\end{aligned}
$$

$$
\begin{aligned}
\bar{\psi}_{\delta t} &= \mathrm{const?} \equiv 2 M_\odot/\mathrm{yr?} \\
Z(t) &= \frac{M_Z(t)}{M_{\mathrm{gas}}(t)} \\
[Z](t) &= \log_{10}\left(Z(t)/Z_\odot\right)
\end{aligned}
$$

Parameters to compute the stellar IMF

$$
\begin{aligned}
\alpha_1 &= 1.3 + \Delta\alpha \cdot (Z - Z_\odot), \\
\alpha_2 &= 2.3 + \Delta\alpha \cdot (Z - Z_\odot),
\end{aligned} \tag{1}
$$

M_{ecl}-dependent exponent

$$
\alpha_3 = \begin{cases} 2.3, & x < -0.87, \\ -0.41x + 1.94, & x > -0.87. \end{cases} \tag{2}
$$

$$
x = -0.14[\mathrm{Z}] + 0.99 \log_{10}(\rho_{\mathrm{cl}}/10^6) \tag{3}
$$

$$
\log_{10} \rho_{\mathrm{cl}} = 0.61 \log_{10} M_{\mathrm{ecl}} + 2.85, \tag{4}
$$

Stellar IMF for individual embedded clusters

$$
\xi_\star(m, M_{\mathrm{ecl}}, Z) = \mathrm{d}N_\star/\mathrm{d}m = \begin{cases} 2k_\star m^{-\alpha_1(Z)}, & 0.08 \leq m/M_\odot < 0.50\,, \\ k_\star m^{-\alpha_2(Z)}, & 0.50 \leq m/M_\odot < 1.00\,, \\ k_\star m^{-\alpha_3(Z, M_{\mathrm{ecl}})}, & 1.00 \leq m/M_\odot < m_{\max}\,, \end{cases} \tag{5}
$$

extract $k_\star$ and $m_{\rm max}$

$$M_{\rm ecl} = \int_{0.08\ \rm M_\odot}^{m_{\rm max}} m\ \xi_\star(m)\,{\rm d}m, \tag{6}$$

$$1 = \int_{m_{\rm max}}^{m_{\rm max*}} \xi_\star(m)\,{\rm d}m, \tag{7}$$

Parameter to compute the ECMF

$$\beta = -0.106\log_{10}\bar{\psi}_{\delta t} + 2. \tag{8}$$

Embedded cluster mass function

$$\xi_{\rm ecl} = {\rm d}N_{\rm ecl}/{\rm d}M_{\rm ecl} = k_{\rm ecl}M_{\rm ecl}^{-\beta}, \qquad 5M_\odot \leqslant M_{\rm ecl} < M_{\rm ecl,max}. \tag{9}$$

extract $k_{\rm ecl}$ and $M_{\rm ecl,max}$

$$M_{\rm tot} = \int_{M_{\rm ecl,min}}^{M_{\rm ecl,max}} M_{\rm ecl}\,\xi_{\rm ecl}(M_{\rm ecl})\,{\rm d}M_{\rm ecl} = \bar{\psi}_{\delta t}\,\delta t, M_{\rm tot} - \int_{M_{\rm ecl,min}}^{M_{\rm ecl,max}} M_{\rm ecl}\,k_{\rm ecl}(M_{\rm ecl,max})M_{\rm ecl}^{-\beta})\,{\rm d}M_{\rm ecl} = \bar{\psi}_{\delta t}\,\delta t = 0 \tag{10}$$

$$1 = \int_{M_{\rm ecl,max}}^{M_{\rm ecl,max*}} k_{\rm ecl}M_{\rm ecl}^{-\beta}\,{\rm d}M_{\rm ecl} \tag{11}$$

$$k_{\rm ecl}(M_{\rm ecl,max}) = \frac{1}{\int_{M_{\rm ecl,max}}^{M_{\rm ecl,max*}} M_{\rm ecl}^{-\beta}\,{\rm d}M_{\rm ecl}} \tag{12}$$

$$\tag{13}$$

$$k_{\rm ecl} = \frac{1-\beta}{M_{\rm ecl}^{1-\beta}|_{M_{\rm ecl,max}}^{M_{\rm ecl,max*}}} \tag{14}$$

$$\frac{M_{\rm ecl}^{2-\beta}|_{M_{\rm ecl,min}}^{M_{\rm ecl,max}}}{M_{\rm ecl}^{1-\beta}|_{M_{\rm ecl,max}}^{M_{\rm ecl,max*}}} = \frac{2-\beta}{1-\beta}\bar{\psi}_{\delta t}\,\delta t \tag{15}$$

$$\frac{M_{\rm ecl}^{2-\beta}|_{5M_\odot}^{M_{\rm ecl,max}}}{M_{\rm ecl}^{1-\beta}|_{M_{\rm ecl,max}}^{10^9 M_\odot}} = \frac{2-\beta}{1-\beta}\bar{\psi}_{\delta t}\,\delta t \tag{16}$$

$$\frac{{M_{\rm ecl,max}}^{2-\beta} - {5M_\odot}^{2-\beta}}{{M_{\rm ecl,max}}^{1-\beta} - {10^9 M_\odot}^{1-\beta}} = \frac{2-\beta}{1-\beta}\bar{\psi}_{\delta t}\,\delta t \tag{17}$$

$${M_{\rm ecl,max}}^{2-\beta} - {M_{\rm ecl,max}}^{1-\beta}\frac{2-\beta}{1-\beta}\bar{\psi}_{\delta t}\,\delta t = (5M_\odot)^{2-\beta} - (10^9 M_\odot)^{1-\beta}\frac{2-\beta}{1-\beta}\bar{\psi}_{\delta t}\,\delta t \tag{18}$$

$$\tag{19}$$

IGIMF (or equivalently, gwIMF)

$$\xi_{\rm IGIMF}(m;t) = {\rm d}N_\star/{\rm d}m = \int_{5M_\odot}^{M_{\rm ecl,max}} \xi_\star(m, M_{\rm ecl}, [Z/X])\,\xi_{\rm ecl}(M_{\rm ecl}, \bar{\psi}_{\delta t})\,{\rm d}M_{\rm ecl}. \tag{20}$$