18X The notation $\left(\frac{\partial a}{\partial b}\right)_c$ indicates the partial derivative of a with respect to b at constant c.

(a) If $v \equiv v(s,t)$ express $\left(\frac{\partial u}{\partial s}\right)_v$ in terms of the partial derivatives of $u(s,t)$ and $t(s,v)$. To gain full marks you must make clear which variables, if any, are held constant in any partial derivatives which feature in your answer. [3]

(b) Find the function $\mu(x)$ satisfying $\mu(1) = 1$ for which

$$df = 2\mu(x)\sin y\ dx - \mu(x)x\cos y\ dy$$

is an exact differential. For the function $\mu(x)$ just found, determine $f(x,y)$. [7]

(c) Three variables a, b and c are related by

$$a = \frac{bc(b^2 - c^2)}{(b^2 + c^2)^2}.$$

Prove that

$$\left(\frac{\partial a}{\partial b}\right)_c \left(\frac{\partial b}{\partial c}\right)_a \left(\frac{\partial c}{\partial a}\right)_b = -1.$$

[10]