14Y

(a) Show that the differential $(4x + xy^2)dx + (y + x^2y)dy$ is exact and find a function $f(x, y)$ such that

$$df = (4x + xy^2)dx + (y + x^2y)dy\,.$$

Use your result to solve $df = 0$ for $y(x)$. [4]

Find the particular solution for $y(x)$ given that $y(1) = 2$. [2]

(b) It is given that

$$M(x, y)dx + N(x, y)dy = 0$$

has an integrating factor μ which depends only on x. By writing μ in the form

$$\mu = \exp\left(\int f(x)dx\right),$$

find an expression for $f(x)$. [6]

If instead the integrating factor is $\psi(y)$, a function of y only, what is the expression for ψ? [2]

(c) The equation

$$(3xy^2 + 2y)dx + (2x^2y + x)dy = 0$$

has an integrating factor $\mu(x)$ which depends only on x. Find $\mu(x)$ and hence solve the equation for $y(x)$ explicitly. [6]