

12S

- (a) Determine whether the following differential forms are exact. For each one that is exact, find a function f such that the differential form is equal to df .

- (i) $\exp(x+y) dx + \exp(x+y) dy$,
- (ii) $\sin x \sin y dx + \cos x \cos y dy$,
- (iii) $2xy^3z^4 dx + 3x^2y^2z^4 dy + 4x^2y^3z^3 dz$.

[9]

- (b) Find and classify all the stationary points of the function

$$g(x, y) = 1 - \cos x + \frac{1}{2}y^2,$$

and calculate the stationary values of g .

Sketch the contours of $g(x, y)$ in the region $-2\pi < x < 2\pi$, $-3 < y < 3$, paying particular attention to any contour lines that pass through the stationary points, and labelling the important features of the plot.

[11]