Elastic Beams

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1 Summary

The local curvature of a simply supported beam was measured at intervals along its length. The measured curvature can be used to calculate the bending stiffness of the beam and, through numerical integration, the deflection at each point along the beam. Comparing this to the measured deflection at one point along the beam shows that the calculated value is within the margin of error for the instruments used.

2 Introduction

When a simply supported is loaded at a point, the beam elastically deforms and thus deflects. The deflection at each point can be calculated by numerically integrating the local curvature measured at each point. The deflection can also be simply measured by using deflection gauges at each point, but this requires more time and effort to set up and calibrate. Instead, the curvature method can be used, and the deflection measured directly at a single point to confirm the accuracy of the numerical integration method. The curvature and moment applied at each point can also be used to calculate the bending moment of the beam, and this compared to the theoretical value calculated from the geometry and loading.

3 Method

An aluminium beam was set up with simple supports at one end and at 15 cm from the other end, as shown in Figure 1. The initial curvature was measured at points spaced 5 cm apart along the length of the beam. A displacement gauge was zeroed at the point 30 cm from one end, at the midpoint of the central section. A loading of 50 N was then applied to the midpoint of the central section of the beam to cause elastic deformation of the beam. The curvature and displacement were then measured at the same points and the difference from initial readings calculated.

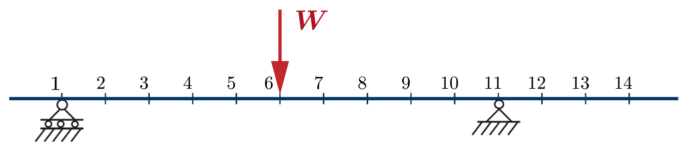

Figure 1: Measurement points and loading of the beam

More detailed information can be found in the lab handout.

4 Results, Observations and Calculations

After collecting the initial and final curvature measurements, the difference between them can be calculated. This avoids any effects due to the beam bending from self-weight or the beam not being manufactured straight. The curvature due to loading (κ) was then plotted as shown in Figure 2.

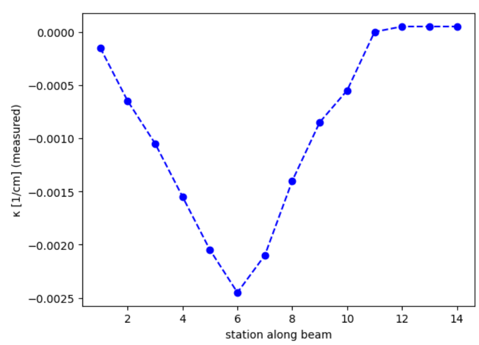

Figure 2: Measurement points and loading of the beam

Based on the loading, the bending moment can be calculated along the beam. The beam can be separated into three sections, starting from the left edge, splitting at each point load. This results in the following bending moments, measuring x from the left end:

$$\overset{\frown}{M} = \begin{cases} -25x & 0 \le x < 25 \\ 50\,(x-25) - 25x & 25 \le x < 50 \\ 50\,(x-25) - 25x - 25\,(x-50) & 50 \le x \le 65 \end{cases}$$

Plotting this results in the same shape as the plot of κ against distance. Plotting the ratio of M to κ shows a linear relationship (Figure 3). The constant of proportionality is the bending stiffness (B) of the beam, which can be estimated from the gradient.

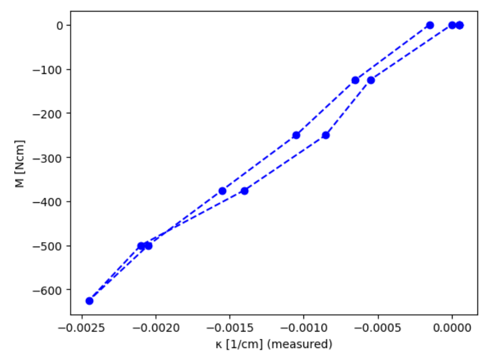

Figure 3: Calculated bending moment against measured κ

B was estimated to be $250\,000\,\mathrm{Ncm}^2$.

To remove any experimental error in further calculations, new linear values of κ were calculated using the calculated values of M and the value of B averaged across the entire beam.

The angle (ψ) that the beam is instantaneously curving at each point is related to the curvature by $\psi = \int \kappa dx + c$, where x is the distance along the beam. This can be calculated through numerical integration, taking $\Delta\psi = \kappa\Delta x = 5\kappa$], and summing consecutive terms from a known starting value of ψ. To get a more accurate value, the value of κ used is the mean of the two values either side of the segment being integrated. Additionally, the value of ψ is known to be 0 at midspan, so other values can be calculated by working outwards from the centre, using $\psi_{n+1} = \psi_n + 5\kappa_{\mathrm{mean}}$. This is shown in Figure 4.

The deflection at each point can subsequently be calculated, as at each point $\frac{dy}{dx} = \sin(\psi)$. Taking a small angle approximation, and using the databook sign convention, this results in $\frac{dy}{dx} = -\psi$. $y = 0$ at each support, so a similar approach can be applied again to calculate y following the relationship $y_{n+1} = y_n + 5\psi_{\mathrm{mean}}$. This is shown in Figure 5, along with the deflection measured directly at station 6.

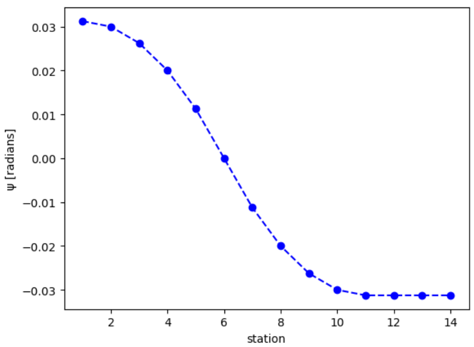

Figure 4: Calculated values of ψ

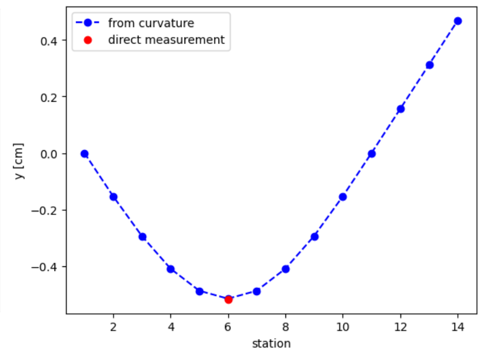

Figure 5: Calculated values of y

5 Discussion

There is likely some error in the measured deflection value. The deflection gauge can be arbitrarily rotated around the horizontal axis, and so was probably not measuring direct vertical displacement but instead displacement along a slight angle. Since this was adjusted by eye, the error in angle is likely around ± 5 deg, resulting in an error in the displacement of around $\pm 0.38\,\%$. Additionally, there is an error of ± 0.001 cm in the measured value, giving a percentage error of $0.19\,\%$.

Comparing the measured deflection value (-0.518 cm) to the calculated value (-0.516 cm) shows the error as $0.39\,\%$. Taking into account the sources of error in the deflection measurement, the two values are within margin of error. This should be confirmed by comparing the values at at least two more places, but this indicates that calculating the deflection from the local curvature is an accurate method to determine the deflection along a beam.

6 Conclusions

- The bending stiffness of a beam can be calculated from the local curvature and bending moment
- The bending angle can be calculated through numerical integration of the local curvature
- Numerically integrating the bending angle results in a good approximation of the deflection of the beam