

AN EFFICIENT APPROACH TO ESTIMATE FRACTAL DIMENSION OF TEXTURAL IMAGES

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Abstract—Fractal dimension is an interesting parameter to characterize roughness in an image. It can be used in texture segmentation, estimation of three-dimensional (3D) shape and other information. A new method is proposed to estimate fractal dimension in a two-dimensional (2D) image which can readily be extended to a 3D image as well. The method has been compared with other existing methods to show that our method is both efficient and accurate.

Fractal dimension Texture analysis Image segmentation Computer vision
Image roughness measure

1. INTRODUCTION

Most of the objects surrounding us are very complex and erratic in nature. A man-made environment can be described by the ideal shape primitives, such as cubes, cones or cylinders. There exist many objects in nature that are so complex and erratic that they cannot be described by classical geometry. We need a model to describe the high degree of erratic behaviour of surface complexity in some controlled way. The controlled erraticism was first handled by Mandelbrot,⁽¹⁾ who introduced the concept of *fractals* in this connection.

Many workers attempted to use fractals in different fields of research. Pentland^(2,3) developed a three-dimensional (3D) fractal model which has been used in texture segmentation and classification, estimation of 3D shape information and to distinguish between perpetually smooth and perpetually textured surfaces in the scene. Among other applications, Orford and Whalley⁽⁴⁾ used *fractal dimension* (FD) in sedimentology. Kaye⁽⁵⁾ applied this concept in particle morphology. Also, Rigaut⁽⁶⁾ used FD for image segmentation.

In the area of texture analysis, Peleg *et al.*⁽⁷⁾ derived a set of 48 features using the ε -blanket method of estimation of FD suggested by Mandelbrot.⁽¹⁾ They used these features as global characteristics to recognize large patches of natural texture. Keller *et al.*⁽⁸⁾ used FD as a scale insensitive ruggedness measure of picture. They used a model based on fractional Brownian motion (fBM) to recover two characteristics of silhouettes. Pickover and Khorasani⁽⁹⁾ used box-counting method to find FD to characterize speech wave graphs. Gangepain and Roques-Carmes⁽¹⁰⁾ developed the box-counting method for measuring FD of image intensity surfaces. Keller *et al.*⁽¹¹⁾ made some modifi-

cation to the method due to Voss⁽¹²⁾ and used FD as one of the tools for image segmentation.

Clearly, there exists several approaches of determining FD. It is useful to compare the approaches and suggest, if possible, a modified approach that is computationally attractive and gives accurate results. This paper is motivated to this end. The basics of FD are described in Section 2, while different approaches of determining FD are described in Section 3. In Section 4, the modified approach is proposed and it is compared with the other approaches both computationally and experimentally.

2. BASICS OF FRACTAL DIMENSION

A set is called a fractal set if its Hausdorff–Besicovitch dimension is strictly greater than its topological dimension. Mandelbrot⁽¹⁾ coined the term fractal from the Latin word *fractus*, which means irregular segments.

Mandelbrot first described an approach to calculate FD while estimating length of coastline. Consider all points with distances to the coastline of no more than ε . These points form a strip of width 2ε , and the suggested length $l(\varepsilon)$ of the coastline is the area of the strip divided by 2ε . As ε decreases $l(\varepsilon)$ increases. Mandelbrot studied that for many coastlines the following formula holds good:

$$l(\varepsilon) = F\varepsilon^{1-D} \quad (1)$$

where F and D are constants for a specific coastline. He called D the fractal dimension (FD) of the line. D can be derived from least square linear fit of log–log plot of $l(\varepsilon)$ and ε . If m is the slope of the fitted line then the FD of curve (coastline) will be $1 - m$. Note that m is always negative.

The FD defined in this way supplies only global information about the analysed images and is sufficient only for an ideal, fully self-similar fractal object and

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such FD is called a *total fractal dimension*. The recent results in the field of fractal model and the fractal geometry point out that most of the natural objects are not ideal but semi-fractal. Creutzberg and Ivanov⁽¹³⁾ observed that often two linear regions appear in the plot as two fractional elements. Quantitative measure of first one is called *textural fractal dimension* and the second one is called *structural fractal dimension*. Such distinction is not necessary for our purpose since we deal here with textural images, and their FD will be predominantly textural fractal dimension. It is to be noted that FD of fractals in 1D topological space, i.e. curves and lines lie between 1.0 and 2.0. Similarly image intensity surfaces or natural surfaces which are in 2D topological space, have FD between 2.0 and 3.0.

3. SOME APPROACHES FOR ESTIMATING FD IN IMAGE

Peleg *et al.*⁽⁷⁾ adopted Mandelbrot's idea and extended it to surface area calculation. In this extension, the image can be viewed as a hilly terrain surface whose height from the normal ground is proportional to the image gray value. Then all points at distance ε from the surface on both sides create a blanket of thickness 2ε . The estimated surface area is the volume of the blanket divided by 2ε . For different ε , blanket area can be iteratively estimated as follows. The covering blanket is defined by its upper surface u_ε and the lower surface b_ε . Initially, given the gray level function $g(i, j)$, $u_0(i, j) = b_0(i, j) = g(i, j)$. For $\varepsilon = 1, 2, 3, \dots$, the blanket surfaces are defined as follows:

$$u_\varepsilon(i, j) = \max \{u_{\varepsilon-1}(i, j) + 1, \max_{d(i, j, m, n) \leq 1} u_{\varepsilon-1}(m, n)\}$$

$$b_\varepsilon(i, j) = \min \{b_{\varepsilon-1}(i, j) - 1, \min_{d(i, j, m, n) \leq 1} b_{\varepsilon-1}(m, n)\}$$

where $d(i, j, m, n)$ is the distance between pixels (i, j) and (m, n) .

Volume of the blanket is given by

$$v_\varepsilon = \sum_{i, j} (u_\varepsilon(i, j) - b_\varepsilon(i, j))$$

while the surface area is measured as

$$A(\varepsilon) = \frac{(v_\varepsilon - v_{\varepsilon-1})}{2\varepsilon}.$$

The area of fractal surface behaves according to the expression

$$A(\varepsilon) = F\varepsilon^{2-D} \quad (2)$$

Fractal dimension can be derived from least square linear fit of log-log plot of $A(\varepsilon)$ and ε , with the help of equation (2).

Pentland⁽²⁾ suggested a method of estimating FD by using Fourier power spectrum of image intensity surface. It can be shown that Fourier power spectrum $P(f)$ of fractal Brownian function (f) is proportional to f^{-2h-1} , where $h = 2 - D$, and D is the FD. From least square fit of log-log of $P(f)$ and f , one can estimate FD of

an image intensity surface, provided image intensity surface can be modelled as a fractal Brownian function.

Mandelbrot stated that one criterion of a surface being fractal is its self-similarity. Self-similarity can be explained as follows. Consider a bounded set A in Euclidean n -space. The set is said to be self-similar when A is the union of N_r distinct (non-overlapping) copies of itself each of which is similar to A scaled down by a ratio r . Fractal dimension D of A can be derived from⁽¹⁾

$$1 = N_r r^D \text{ or } D = \frac{\log(N_r)}{\log(1/r)} \quad (3)$$

However, natural scenes practically do not exhibit deterministic self-similarity. Instead, they exhibit some statistical self-similarity. Thus, if a scene is scaled down by a ratio r in all n dimensions, then it becomes statistically identical to the original one, so that equation (3) is satisfied.

It is difficult to compute D using (3) directly. An approximate method used by Gangepain and Roques-Carmes,⁽¹⁰⁾ called the *reticular cell counting* approach is as follows. Consider the 3D space where two coordinates (x, y) represent 2D position and the third (z) coordinate represents the image intensity. For a given scale L , partition the 3D space into boxes of sides $L \times L \times L'$, where L can be multiple of sidelength of a pixel in (x, y) and L' can be multiple of gray level unit in the z -direction. If G is total gray levels and $M \times M$ is the size of image then $L' = \lfloor L \times G/M \rfloor$. Let for $L = 1$ the box be called *space-intensity cell* or *spicel*. Then for $L = 3$, the box contains $3 \times 3 \times 3 = 27$ spicels. Suppose we can cover the 3D space by a 3D box of size L_{\max} . Then $L = r \times L_{\max}$. Changing parameter from r to L we have, from equation (3)

$$N_L = \frac{1}{r^D} = \left[\frac{L_{\max}}{L} \right]^D$$

i.e.

$$N_L \propto L^{-D}. \quad (4)$$

Count the number N_L of boxes in the space that contain at least one sample of gray level intensity surface. Several values of L are chosen and least square linear fit of $\log N_L$ versus $\log L$ gives the value of $-D$. But when the actual FD of an image is very high, points on image intensity surface become widely spaced in the z -direction, effectively lowering the estimated FD. From Fig. 3 it is seen that FD estimated by this method saturates at about 2.5. However, this method is faster than Pentland's⁽²⁾ method since no Fourier transform computation is included.

Keller *et al.*⁽¹¹⁾ proposed a modification of a method due to Voss.⁽¹²⁾ Let $P(m, L)$ be the probability that there are m intensity points within a box of size L centred about an arbitrary point of image intensity surface. For any value of L we have

$$\sum_{i=1}^N P(m, L) = 1$$

where N is the number of possible points in the box of size L . In fact, N equals the number of spicels the box contains. Suppose that the image is of size $M \times M$. If one overlays the image intensity surface with boxes of side L , then the number of boxes needed to cover the whole image is

$$N_L = M^2 \sum_{i=1}^N (1/m) P(m, L).$$

Since M^2 is constant for an image, let it be dropped from the expression, i.e.

$$N_L = \sum_{i=1}^N (1/m) P(m, L). \quad (5)$$

Using equations (4) and (5) we can estimate D . This method has the same limitation as in the method of Gangepain and Roques-Carmes. To avoid this Keller *et al.*⁽¹¹⁾ devised a new version of probability estimation. In this refinement, the fractal surface between the centre point of a box and its neighbours are approximated by linear interpolation. The newly interpolated surface is intersected with the box and number of points m in the box of side L is recorded. N_L is calculated using equation (5). This method takes a little more time but gives satisfactory results except for the image intensity surfaces whose FD are very high. Figure 3 shows that FD estimated by this method saturates at 2.75.

4. PROPOSED METHOD, RESULTS AND COMPUTER COMPLEXITY

We have a basic equation of FD given by

$$D = \frac{\log(N_r)}{\log(1/r)}$$

In our proposed method, N_r is counted in a different manner than those in Gangepain and Roques-Carmes⁽¹⁰⁾ and Keller *et al.*⁽¹¹⁾ Consider that the image of size $M \times M$ pixels has been scaled down to a size $s \times s$ where $M/2 \geq s > 1$ and s is an integer. Then we have an estimate of $r = s/M$. Now, as in previous techniques, consider the image as a 3D space with (x, y) denoting 2D position and the third coordinate (z) denoting gray level. The (x, y) space is partitioned into grids of size $s \times s$. On each grid there is a column of boxes of size $s \times s \times s'$. If the total number of gray levels is G then $\lfloor G/s' \rfloor = \lfloor M/s \rfloor$. See for example Fig. 1, where $s = s' = 3$. Assign numbers 1, 2, ... to the boxes as shown. Let the minimum and maximum gray level of the image in (i, j) th grid fall in box number k and l , respectively. Then $n_r(i, j) = l - k + 1$ is the contribution of N_r in (i, j) th grid. For example, in Fig. 1, $n_r(i, j) = 3 - 1 + 1$. Taking contributions from all grids, we have

$$N_r = \sum_{i,j} n_r(i, j) \quad (6)$$

where N_r is counted for different values of r (i.e. different values of s). Then using (3) we can estimate D , the fractal dimension, from the least square linear fit of $\log(N_r)$ against $\log(1/r)$.

The reason for counting N_r in this manner is that it gives a better approximation to the boxes intersecting the image intensity surface, which is quantized in space and gray value. This is particularly so when there is sharp gray level variation in neighbouring pixels in the image. Box counting in the other methods^(10,11) does not cover the image surface so well and hence cannot capture the fractal dimension for rough textured surface.

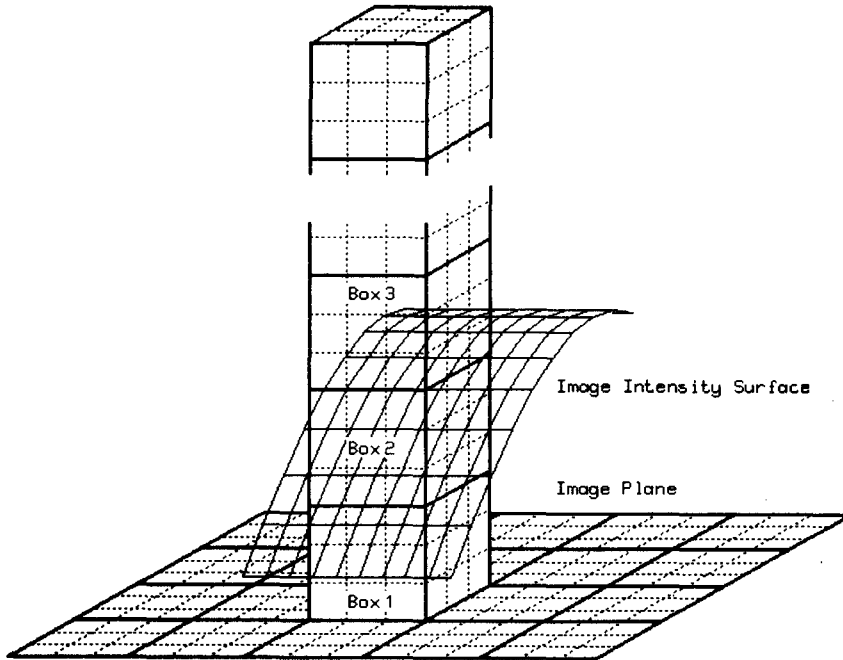


Fig. 1. Determination of n_r by proposed method.

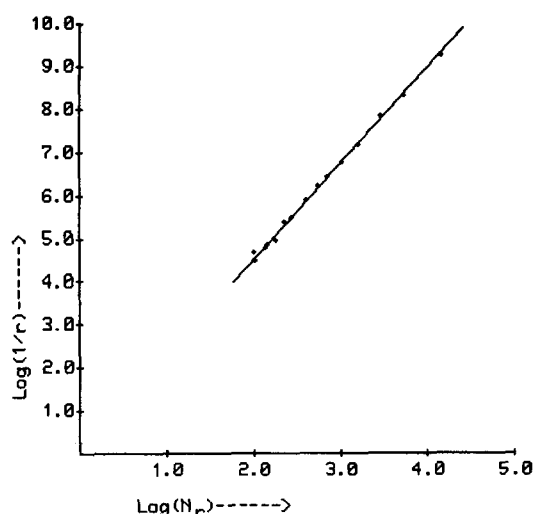


Fig. 2. Plot of $\log N_r$ vs $\log(1/r)$ of texture image (D33 in Brodatz⁽¹⁴⁾).

A typical plot of $\log(N_r)$ vs $\log(1/r)$ of the image D33 (Brodatz⁽¹⁴⁾) is shown in Fig. 2. Let $y = mx + c$ be the fitted straight line, where y denotes $\log(N_r)$ and x denotes $\log(1/r)$. Then error of fit E can be expressed as the r.m.s. distance of the points from the fitted line

$$E = \frac{\sqrt{\left(\sum_{i=1}^n \frac{(mx_i + c - y_i)^2}{(1 + m^2)} \right)}}{n} \quad (7)$$

The error provides a measure of fit so that the lower the value of E , the better is the fit.

For our experiment we took 12 images from Brodatz⁽¹⁴⁾ and 36 synthetic textured images. The synthetic images are actually noise added to an absol-

utely smooth image surface at gray level 128. Zero-mean Gaussian noise with different standard deviation σ has been added to this smooth image surface so that the resulting gray levels lie in the range 0–255. For all of these synthetic images the size is equal to 128×128 .

We choose five algorithms including ours for comparative study. The other four algorithms are due to Pentland,⁽²⁾ Peleg *et al.*,⁽⁷⁾ Gangepain and Roques-Carnes⁽¹⁰⁾ and Keller *et al.*⁽¹¹⁾ At first, the algorithms are tested on the synthetic images. It is expected that the fractal dimension will increase if the σ of additive noise increases and beginning at 2.0 it will asymptotically go towards a value of 3.0. The results are plotted in Fig. 3. The methods due to Pentland, Peleg *et al.*, and ours give a satisfactory result, i.e. (a) FD lies in the full dynamic range of 2.0–3.0. FD of a very rough image approximates 3.0. (b) Increment in noise level, i.e. σ and hence roughness of image monotonically reflects on the calculated FD.

Figure 3 also shows that methods due to Gangepain and Roques-Carnes and Keller *et al.* give a satisfactory result up to a certain level of roughness of the image intensity surface. It is seen that after a certain value of σ , the slope of the curve nearly goes to zero so that the results do not truly indicate the roughness. These methods do not cover the full dynamic range of FD. In the method due to Keller *et al.* the range is 2.0–2.75, while in Gangepain and Roques-Carnes' method the range is 2.0–2.5.

Next we compare the computational complexity of different methods. Our method is readily comparable with methods due to Gangepain and Roques-Carnes⁽¹⁰⁾ as well as Keller *et al.*,⁽¹¹⁾ because in all these cases the computations are done taking boxes of different sizes. Relation between r in our method and L in the methods due to Gangepain and Roques-Carnes as

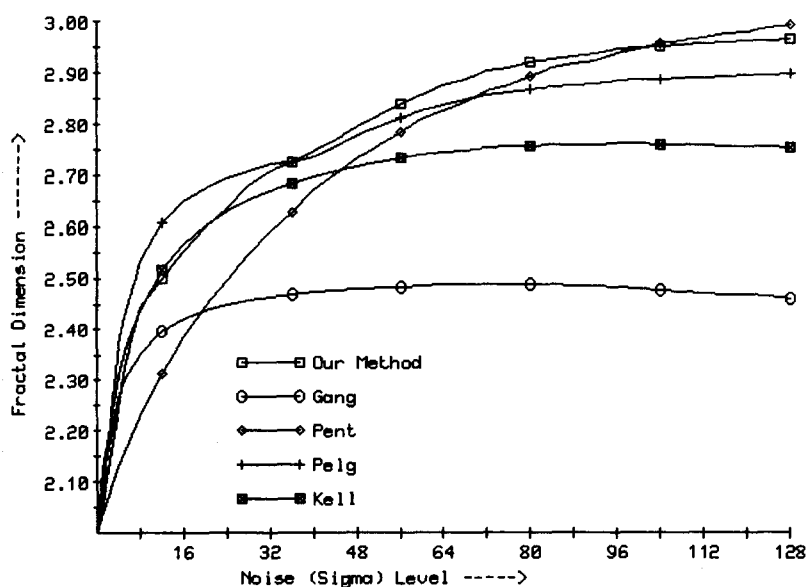


Fig. 3. Fractal dimensions of synthetic images by different methods.

Table 1. Comparison of complexity of three methods

	Ours	Gang	Kell
Comparison	$2M^2$	$1/r^4$	r^2M^4
Addition	$4/r^2$	$M/r + 4/r^2$	$2M^2 + M^2L^2$
Subtraction	$2/r^2$		M^2
Multiplication	$3/r^2$	$2/r^2$	
Division	$3/r^2$	M^2	$r^3M^3 + M^2L^2$

well as Keller *et al.* is given by $L = s = r \times M$, where $M \times M$ is the size of the image. The number of computations required in terms of M and r for each method is shown in Table 1. Computational superiority of our method is readily observed from this table.

The idea of finding FD using Peleg *et al.*'s⁽⁷⁾ method is a little different. Here in each iteration, the upper and

lower blanket are computed. To calculate each blanket, one needs $10M^2$ comparisons, $6M^2$ additions, $6M^2$ subtractions, one multiplication and one division. Initially, one needs $4M^2$ subtractions to calculate the zeroth blanket, before the start of the iteration.

Computational complexity of Pentland's⁽²⁾ method is very high. It needs Fourier transform to find Fourier power spectra. To calculate FFT of an $M \times M$ image, one needs $2M \times \log_2 M$ operations, where each of such operations consists of $M + M \log_2 M$ additions, $M + M \log_2 M$ subtractions, $4M$ multiplications, three divisions, one sine and one cosine operation. As other overheads one needs $4M^2$ comparisons and M^2 square roots.

The computation required for regression is not considered in any of the methods because regression

Table 2. Comparison of number of computations

	Ours	Gang	Kell	Pelg	Pent
Comparison	464,142	417,040	2,687,181	2,457,600	45,056
Addition	37,524	445,183	418,935	1,474,560	933,376
Subtraction	18,762	0	144,060	1,540,096	636,928
Multiplication	28,143	18,762	0	15	825,856
Division	28,143	232,071	261,630	15	101,888
Square root	0	0	0	0	16,384
Sine/cosine	0	0	0	0	3584

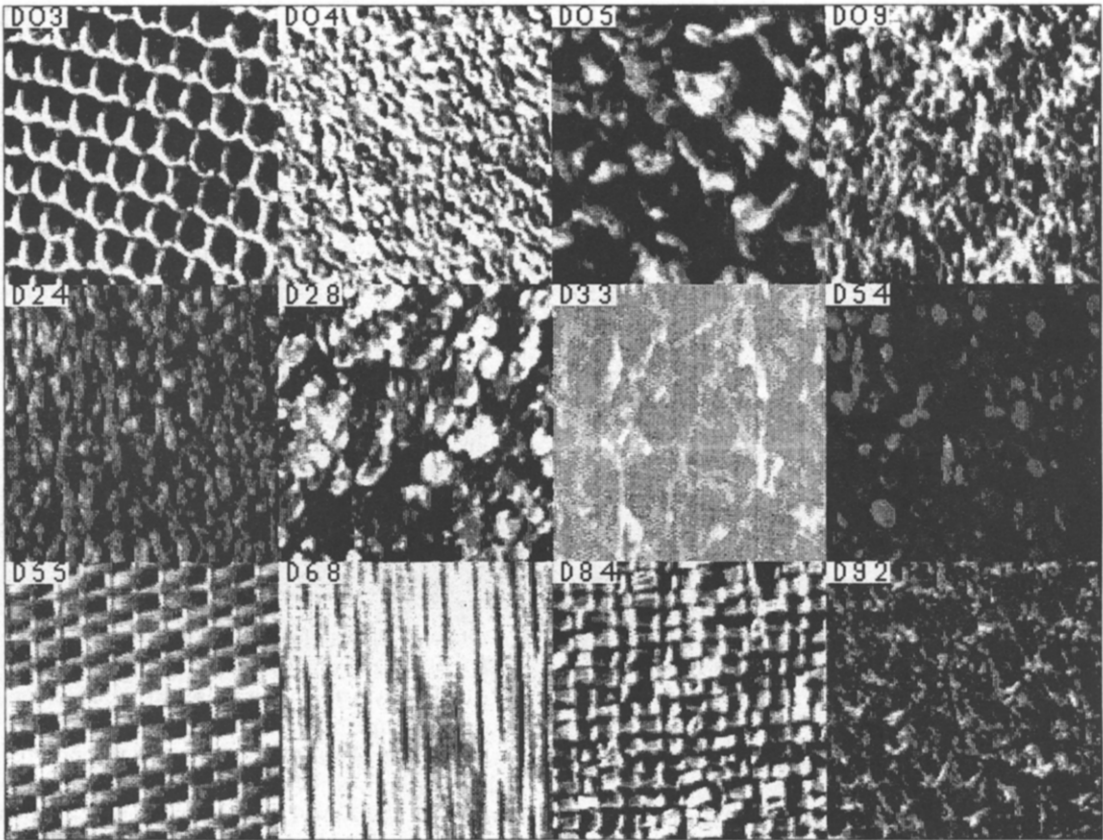


Fig. 4. Natural textures (Brodatz⁽¹⁴⁾).

Table 3. Fractal dimension of natural textures using different methods (image numbers correspond to Brodatz's book)

Texture images	Ours		Pent		Pelg		Gang		Kell	
	FD	E	FD	E	FD	E	FD	E	FD	E
D03	2.60	0.032	2.54	0.004	2.69	0.011	2.40	0.074	2.63	0.036
D04	2.66	0.026	2.55	0.012	2.72	0.008	2.45	0.072	2.68	0.030
D05	2.45	0.032	2.38	0.003	2.52	0.012	2.38	0.057	2.57	0.025
D09	2.59	0.028	2.49	0.003	2.65	0.009	2.43	0.066	2.65	0.026
D24	2.45	0.022	2.36	0.002	2.59	0.007	2.39	0.048	2.57	0.014
D28	2.55	0.033	2.48	0.007	2.61	0.012	2.41	0.066	2.62	0.031
D33	2.23	0.007	2.21	0.002	2.34	0.003	2.26	0.024	2.36	0.008
D54	2.39	0.023	2.31	0.011	2.53	0.008	2.35	0.044	2.51	0.015
D55	2.48	0.031	2.37	0.006	2.60	0.010	2.39	0.057	2.59	0.023
D68	2.52	0.024	2.44	0.008	2.63	0.007	2.40	0.054	2.60	0.019
D84	2.60	0.029	2.47	0.001	2.68	0.009	2.43	0.067	2.65	0.028
D92	2.50	0.023	2.38	0.007	2.59	0.007	2.41	0.052	2.59	0.018

Gang, method due to Gangepain and Roques-Carmes;⁽¹⁰⁾ Kell, method due to Keller *et al.*;⁽¹¹⁾ Pent, method due to Pentland;⁽³⁾ Pelg, method due to Peleg *et al.*⁽⁷⁾

is common to all. A comparative study on the actual number of operations required for real texture image (D03 of Brodatz⁽¹⁴⁾) is shown in Table 2, where 15 iterations are taken in methods other than Pentland's. Note that iterations are not necessary in Pentland's method. Again one can see that our method is computationally cheaper than other methods.

Next, we have taken a set of 12 texture images from Brodatz's⁽¹⁴⁾ album to compare the FD obtained by different methods. The images are shown in Fig. 4, while results are presented in Table 3. It can be seen that the methods due to Pentland,⁽²⁾ Peleg *et al.*⁽⁷⁾ and ours give identical results. Also it may be noted that the FD due to our method is intermediate between that due to Pentland and Peleg *et al.* for all texture images. In Fig. 3 also, FD due to our method is intermediate between that due to Pentland and Peleg *et al.* in the range 2.23–2.58 which is roughly the range of FD for these texture images. Finally, the comparative study of error of fit shows that the error in our method is less than that due to Gangepain and Roques-Carmes⁽¹⁹⁾ as well as Keller *et al.*,⁽¹⁰⁾ but somewhat more than that due to Pentland and Peleg *et al.* However, since the results are consistent throughout, the error has negligible effect on the computed FD, which has been checked on 20 other texture images.

5. CONCLUSION

A simple method of finding FD of 2D image has been proposed. As compared to other known methods, it is computationally attractive and yet consistently gives satisfactory results on synthetic and practical data. The method can be readily extended to compute FD of 3D images.

6. SUMMARY

Fractal dimension is an important measure of roughness and self-similarity in pictures. It has been used in several image processing and pattern recogni-

tion applications. Several authors have proposed different techniques of estimating fractal dimension, that are summarized in this paper. Some of the methods, e.g. those due to Pentland⁽²⁾ and Peleg *et al.*⁽⁷⁾ are accurate and cover the full dynamic range of fractal dimensions, but are computationally expensive. Others, e.g. those due to Gangepain and Roques-Carmes⁽¹⁰⁾ as well as Keller *et al.*⁽¹¹⁾ are computationally attractive but do not cover the full dynamic range. More specifically, these methods are insensitive and inaccurate for a rough texture. The purpose of this paper is to propose a method that is computationally more attractive than these methods as well as accurate and covers the full dynamic range of fractal dimension.

The proposed method is based on the equation of fractal dimension D given by⁽¹⁾

$$D = \frac{\log(N_r)}{\log(1/r)}$$

If an image of size $M \times M$ pixels is scaled down to a size $s \times s$ where $M/2 \geq s > 1$ and s is an integer then we have $r = s/M$. Let $s' = \lfloor G \times r \rfloor$. The image is considered as a 3D space with (x, y) denoting the image plane and (z) denoting the gray level. The (x, y) space is partitioned into grids of size $s \times s$ on each of which there is a column of boxes of size $s \times s \times s'$. If the minimum and maximum gray level of the image in (i, j) th grid fall in box number k and l , respectively, then $n_r(i, j) = l - k + 1$ is the contribution of N_r in (i, j) th grid. Hence we have

$$N_r = \sum_{i,j} n_r(i, j).$$

For different values of r the quantity N_r is computed and D is obtained from the plot $\log(N_r)$ versus $\log(1/r)$.

It can be shown that the proposed method is computationally cheaper than four other methods^(2,7,10,11) described here. Also, the accuracy of the proposed method has been tested and compared with others on 36 synthetic and 12 real texture images. The synthetic images are noise added to a smooth image surface so that with increased level of noise the roughness of

image also increases. The results show that the proposed method gives consistent results and cover the full dynamic range of fractal dimension.

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