Graphics Homework Assignment 4

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1 General Information

Operating system: Windows 11 (I usually use a Mac to complete these assignments, and came to office hours with some questions about setting up this assignment on Mac. Ultimately, I was able to borrow a Windows machine from a friend a few days before the assignment was due.)
IDE: Visual Studio Community 2022

2 PART I: PARTICLES – NUMERICAL INTEGRATION SCHEMES

2.1 TASK 1 – INTEGRATION SCHEMES

For the Euler and Euler-Cromer integration schemes, I updated each particle's position and velocity at each timestep using the given equations:
Euler:

$$x_{t+h} = x_t + hv_t \ v_{t+h} = v_t + h\frac{1}{m}F_t,$$

Euler-Cromer:

$$x_{t+h} = x_t + hv_{t+h} \ v_{t+h} = v_t + h\frac{1}{m}F_t.$$

For the Verlet integration scheme, I updated each particle's position and velocity at each timestep using the given equations as well, with the velocity being defined implicitly:

$$x_{t+h} = 2x_t - x_{t-h} + h^2\frac{F_t}{m} + O(h^4) \ v_{t+h} = \frac{x_{t+h} - x_t}{h}.$$

I used Euler's method to compute the t_{-1} position value (the "old position" when current position = initial position), as the Verlet equation for position would require knowledge of an indefinite number of previous positions (i.e. to compute x_{-1} I would need to know x_{-2} and x_{-3}, but to compute x_{-2} I would need to know x_{-3} and x_{-4}, and so on). I also used h^4 as the value of $O(h^4)$.

Integration Scheme	Number of Particles	Total Energy	Time to Converge
Euler	2	-147.66	43.48
Euler	20	-1545.51	50.28
Euler	200	-15478.72	73.03
Euler-Cromer	2	-147.66	45.11
Euler-Cromer	20	-1545.52	54.89
Euler-Cromer	200	-15485.71	65.95
Verlet	2	-176.00	14.44
Verlet	20	-1760.00	14.53
Verlet	200	-17600.00	15.52

Table 1: The time to converge and total energy at time of convergence for each integration scheme and number of particle.

2.2 TASK 2 – ENERGY COMPUTATION

To compute system energies, we first need to compute the kinetic and potential energies for each particle in the system using the following formulas:
Kinetic Energy:

$$E_{kinetic} = \frac{1}{2}mv^2,$$

where m is the mass of the given particle and v is the velocity of the particle.
Potential Energy:

$$E_{potential} = mgh,$$

where m is the mass of the particle, g is the force of gravity, and h is the height of the particle computed using particlePosition.y + halfWorld.getY().
The sum of the kinetic energy for each particle at a given timestep is the kinetic energy of the system at that timestep (and the same applies to potential energy).

2.3 TASK 3 – COMPARISON

I ran the simulation for 2, 20, and 200 particles for each of the three integration schemes (see table 1). For each simulation, I noted the "time to converge," or the time at which the potential energy of the system stopped changing (the kinetic energy continued to vary by less than 1, as the particles moved slightly on the "floor" of the simulation box). I also noted the total energy of the system at this convergence time. Based on these simulations, we see that the Verlet integration scheme converged the fastest, and that Euler and Euler-Cromer converged at similar speeds (though Euler Cromer proved to be faster for a large number of particles). We also see that the magnitude of the total energy of the system increases with the number of particles within it. The energy computation and the time to converge shows us that Verlet is the most stable system, followed by Euler-Cromer and Euler.

3 PART II: RIGID BODY CUBE

3.1 TASK 1 – FORCES

First, we apply gravity by applying a force vector (0, -9.8, 0) to each particle. Then, we apply the spring force to the particles at the end of each spring using Hooke's Law:

$$F_s = kx,$$

where k is the spring constant and x is equivalent to

$$initialSpringLength - (p2.getPosition() - p1.getPosition),$$

that is, the distance the spring has been compressed. We apply this force to $p1$ in the direction of $p1 - p2$, and vice versa.
We also apply the damping force to the pair of particles at the end of each spring using the formula

$$F_d = -dv,$$

where d is the damping constant and v is the velocity of the particle.

3.2 TASK 2 – ENERGY COMPUTATION

We compute kinetic and potential energies using the same formulas listed in Part 1 Section 2. We also compute the spring energy of the system by taking the sum of the energies from each spring using the following formula:

$$E_{spring} = \frac{1}{2}kx^2,$$

where k and x are the same spring constant and distance measure used in Hooke's Law.

3.3 TASK 3 – COMPARISON

I ran the simulation for three different combinations of spring and damping constants (spring = 1000 and damping = 10; spring = 2000 and damping = 10; spring = 10 and damping = 1000) for each of the three integration schemes (see table 2). For each simulation, I noted the "time to converge," or the time at which the spring energy of the system stopped changing. I also noted the total energy of the system at this convergence time. When the spring constant was 10 and the damping constant was 1000, the system failed to stabilize and the energy did not converge to a single value (see demo video - the cube falls apart due to the uneven forces). For the other two constant value combinations, we see that the Euler scheme converged the fastest, followed by Euler Cromer and the Verlet.

Integration Scheme	Spring Constant	Damping Constant	Total Energy	Time to Converge
Euler	1000	10	537.90	4.55
Euler	2000	10	537.52	5.22
Euler	10	1000	N/A	N/A
Euler-Cromer	1000	10	537.62	4.66
Euler-Cromer	2000	10	536.68	5.42
Euler-Cromer	10	1000	N/A	N/A
Verlet	1000	10	536.47	5.67
Verlet	2000	10	536.33	5.92
Verlet	10	1000	N/A	N/A

Table 2: The time for the energy computation to converge and the amount of energy in the system at time of convergence for each integration scheme and spring/damping constant value.