Family Policies and Child Skill Accumulation*

Emily G. Moschini† Monica Tran-Xuan‡

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Abstract

We analyze the economic effects of two major family policies in the United States, the Child Tax Credit and the Child Care and Development Fund, in an overlapping-generations framework where altruistic parents invest in their child's skill using parent time and purchased childcare. We incorporate realistic differences in the design and uptake rates of these two policies, endogenizing the lower uptake rates for childcare subsidies with a fixed cost of access incurred by the household. We find that eliminating the cost of uptake for the childcare subsidy yields higher welfare gains in the long run than a spending-equivalent increase in the tax credit level. We decompose these welfare gains and find that, for childcare subsidies, they stem from raising the average level of consumption and leisure in the economy, whereas welfare gains from the tax credit are due to changes in the distribution of these quantities across households. We then compare the effects of the 2018 child tax credit expansion with a counterfactual spending-equivalent expansion of the childcare subsidy implemented by lowering the uptake cost. The childcare subsidy yields higher welfare gains in the long run. These gains, however, accrue more slowly over the transition than those of the tax credit expansion because they are driven by the dynamics of child skill accumulation.

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Keywords: Childcare subsidy, Child tax credit, Early childhood, General equilibrium, Skill investment.

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†William & Mary. E-mail: egmoschini@wm.edu.
‡SUNY at Buffalo. E-mail: monicaxu@buffalo.edu

1 Introduction

Economists have long acknowledged that aid to families with children can bolster children's accumulation of skill and improve the child's labor market outcomes later in life (Leibowitz, 1974). Such government interventions may be welfare improving because, for example, parents of children incur expenses which are not present at other points in the life cycle while facing borrowing constraints. Additionally, families may not internalize the social benefits of creating more productive future taxpayers and parents (Folbre, 1994). As one would expect given this environment, "family policies"—by which we mean policies that provide aid to families with children—have been widespread across countries for some time (Kamerman, 2000; Olivetti and Petrongolo, 2017). In practice, implementation of family policies is accomplished either through the tax system or through a spending program that disburses transfers-in-kind or cash benefits. In the United States, there tend to be much higher uptake rates among the eligible for policies implemented through the tax system compared to policies implemented through a spending program (Sommartino, Toder, and Maag, 2002). In the economic analysis of family policies, the macroeconomic approach has emphasized how government policy can address inefficiencies present in a competitive equilibrium. It has also sought to uncover the consequences of scaling up interventions previously studied with small treatment populations (Guner et al., 2020; Daruich, 2022; Zhou, 2022; Moschini, 2023). Among microeconomic analyses, as well as government reports on program spending, more attention is paid to the fact that not all of the eligible receive aid from a program (Johnson, Martin, and Brooks-Gunn, 2011; Crandall-Hollick, 2018; Chien, 2019a,b, 2020). By combining the insights of these two literatures within a macroeconomic framework, as we do in this paper, the economic analysis of family policies can offer additional insights for policy makers.

In this paper we study two major family policies in the United States—the child tax credit and the federal childcare subsidy program—in the presence of child skill accumulation. In practice, these two policies have large and partially-overlapping eligible populations. They differ qualitatively in the form of the transfer to families (in cash or in kind), and in the method of their implementation (via the tax system or a spending program). They also differ quantitatively in uptake rates among the eligible and the level of aggregate spending. To understand the potential benefits of these two policies, consider an environment where altruistic parents use some of their resources to invest in their child's skill. This represents a kind of intergenerational transfer. On the one hand, cash transfers to families with children via tax credits could direct resources to a point in the lifecycle where the marginal utility of consumption is high. In the presence of consumer borrowing constraints, such a policy can yield welfare gains. On the other hand, childcare subsidies can be beneficial to parents because they lower the cost of achieving a given child outcome and free up resources to finance more consumption or leisure for parents. Both programs may also encourage investment in child skill and thereby improve labor market outcomes later in the child's life. For example, the tax credit provides income which may be partially directed to investment in child skill, whereas the childcare subsidy lowers the relative price of investment in child skill and thereby leads altruistic parents to substitute towards child skill investment. We study how increases in government spending on each of these two policies impacts the economy.

We proceed by embedding a childcare subsidy and a child tax credit into a general equilibrium, overlapping generations environment in which altruistic parents invest in their child's skill. This skill investment combines parent time with time purchased on the market in the form of non-parental childcare. We endogenize uptake of subsidized childcare by including a fixed cost for access. By contrast, uptake of the tax credit via the tax system is frictionless. We calibrate the model to target the correlation of child skill outcomes and family income, average labor supply, hours of childcare, childcare subsidy receipt, and income inequality observed in US data in the period from 2015 to 2017. With the estimated model, we perform two counterfactual experiments in which we vary the amount of spending on the child tax credit or the childcare subsidy and document the economic effects. Changes in the size of the childcare subsidy are implemented by the government paying a portion of the fixed cost of uptake, which can be interpreted as making the subsidy easier to access by spending more on overall funding to reduce waiting list times and/or raising the efficiency of bureaucratic infrastructure. Spending on the child tax credit is increased by raising the level of the credit, widening the income range for eligibility, or both.

In our first counterfactual, we expand one policy at a time in order to examine the mechanisms behind the policy's aggregate effects. We find that both policies are qualitatively similar in that they redistribute from old to young adults. However, childcare subsidies act to raise output by subsidizing the relative price of skill investment, whereas increasing the child tax credit level bolsters leisure for adults with children at the expense of older consumers, with a smaller pass-through to investments in child skill. Each of the two policies yields welfare gains across steady states, but welfare gains for childcare subsidies are larger. To examine the sources of these gains, we implement a welfare decomposition following the method of Guvenen, Kambourov, Kuruscu, Ocampo, and Chen (2023). We find that welfare changes from expansions of the child tax credit are driven by changes in the distribution of consumption and leisure across households in the economy, whereas expansions of spending on the childcare subsidy yield welfare gains driven by an increase in the average level of consumption and leisure. Lastly, we examine which general equilibrium adjustments drive the relatively higher welfare gains exhibited by a childcare subsidy expansion. We find that equilibrium adjustments in the skill distribution of adults, which reflect changes in child outcomes, are most important for this result.

In our second counterfactual, we evaluate the effects of the 2018 expansion of the US child tax credit passed into law by the 2017 Tax Cuts and Jobs Act, and compare it with a budget-equivalent expansion of uptake rates for childcare subsidies. The 2018 expansion both increased the per-child credit amount and expanded the income range that received a positive child tax credit (Crandall-Hollick, 2018). We model this reform as a permanent change and examine the effects over the transition to the new steady state. Compared with this CTC expansion, a budget-equivalent expansion of subsidized childcare yields larger welfare gains for new households in the long run, although these gains take longer to fully accrue. Consumers at older ages suffer welfare losses under either policy, but for the childcare expansion these losses are mitigated over time as the economy converges to the new steady state and the tax base expands.

The properties of the baseline equilibrium's childcare subsidy are modeled after the Child Care and Development Fund (CCDF), a US spending program. The CCDF is a means-tested childcare subsidy with a work requirement for parents, available to families with children under 13. Only 15 percent of children who are eligible according to federal rules receive the subsidy, and overall only 3.5 percent of all households with young children receive CCDF-subsidized childcare.[1] We endogenize uptake of the CCDF by means of a fixed cost that households must incur in order to access the childcare subsidy. This expense appears in the budget constraint of parenting households, and represents transaction costs associated with accessing subsidized childcare such as proving eligibility or being placed on a waiting list. The value of this fixed cost is calibrated to match the observed share of households with children under 5 who receive the subsidy.

The properties of the child tax credit reflect those of the Child Tax Credit/Additional Child Tax Credit (CTC/ACTC, hereafter CTC) of the US tax code. This tax credit offsets a household's tax liability by a fixed amount per dependent child aged under 17, and can be partially refunded to taxpayers as a cash transfer. Most government spending on the CTC is received by middle-income households because the level of the credit is a function of household income: only households with taxable income between a lower and upper bound are eligible for a positive credit amount, with phase-in and phase-out regions at either end of the eligible income interval. The CTC is implemented via the tax system, and exhibits uptake rates among the eligible higher than 90 percent (Crandall-Hollick, 2018).

This paper builds on previous work in Moschini (2023) by broadening the set of family policies under consideration to include a child tax credit similar to that of the US tax code, by endogenizing childcare subsidy uptake, and by examining the transition path dynamics due to policy changes. We also abstract from heterogeneity in family structures because our focus here is on the comparison of policies that differ in their design and method of implementation, rather than comparing the

[1] Uptake rates among children are based on (Chien, 2019a,b, 2020); the mass of families receiving CCDF funds reflect author calculations described in Section 4.

same policy targeted towards one- versus two-parent families.

Two other studies closely related to ours are Guner, Kaygusuz, and Ventura (2020) and Zhou (2022).[2] Guner, Kaygusuz, and Ventura (2020) examines a battery of family policies including childcare subsidies and child tax credits. The emphasis is on policy effects on parental labor supply; consistent with this purpose, the analysis accounts for the skill accumulation of parents rather than children. By contrast, we focus on child skill accumulation. We model childcare as an input into skill investment along with parental quality time, representing the time use of children as a primary determinant of their skill accumulation early in life. Childcare subsidies in our model therefore lower the relative price of investment in child skill and thereby incentivize altruistic parents to substitute towards spending on child skill accumulation, as in Moschini (2023). In this respect we also differ from Zhou (2022), which examines aggregate fertility responses to stylized family policies. Because of our interest in comparing specific policy reforms in the United States, we explicitly model attributes of the child tax credit in the US tax code (such as the phase-in and phase-out rates and income thresholds) which make the CTC a transfer whose level is hump-shaped in household income (Goldin and Michelmore, 2022). These attributes are relevant for our policy analysis: the phase-out region of the CTC distorts labor supply as households adjust their behavior to qualify for more of the credit, an effect that is mitigated when the phase-out income threshold is raised and magnified when the maximum level of the credit is increased. Changes along these margins are part of the 2018 CTC expansion that we simulate in our second counterfactual experiment. Importantly, we also differ from both Guner, Kaygusuz, and Ventura (2020) and Zhou (2022) in that we endogenize uptake of subsidized childcare, making explicit an additional distinction between the two policies that we examine based on how they are implemented in the United States (i.e., the tax system versus a spending program).

The paper proceeds as follows. Section 2 overviews the policy environment and Section 3 describes the model environment. Section 4 reports model parameterization; Section 5 examines properties of the baseline equilibrium. The model experiments and results are presented in Section 6. Section 7 concludes.

2 The Policy Environment

In this Section, we describe in more detail the attributes of the Child Tax Credit, a feature of the US tax code, and the Child Care and Development Fund, a US spending program.

[2] Other structural studies of childcare subsidies which allow for endogenous skill accumulation of children include Bastani, Blomquist, and Micheletto (2020) and Ho and Pavoni (2020). Both perform a Mirrleesian analysis of optimal childcare subsidies; the former focuses on childcare quality choices, whereas the latter focuses on parental labor supply.

The Child Tax Credit The CTC is a partially-refundable federal tax credit that began in 1997, after ten years of public debate, and was subsequently expanded by legislation passed in 2001, 2010, 2017, and 2021 (Steuerle, 1990; Crandall-Hollick, 2021; Goldin and Michelmore, 2022). Broadly speaking, tax credits are subtracted from a household's tax liability to reduce it and thereby raise disposable income. If the tax credit level is greater than the tax liability, then a refundable tax credit rebates the residual to the taxpayer (sometimes partially, as with the CTC); a nonrefundable tax credit does not.[3] To be eligible for a positive CTC amount, households must have a dependent child under the age of 17, and taxable household income must be within a certain range. The level of the credit, which combines nonrefundable and refundable components, is lower at both ends of the eligible income interval, because the maximum credit level phases out at higher incomes and the portion of the credit that is refunded to the taxpayer phases in at lower incomes.[4]

The CTC is similar to the better-studied Earned Income Tax Credit (EITC), in that eligibility is highly correlated with the presence of children (Marr, Huang, Sherman, and DeBot, 2015).[5] However, the EITC is several decades older than the CTC, and it is fully refundable. Another difference is that, whereas the credit levels of the CTC and the EITC are both hump-shaped in the taxable income of the household, the income threshold where the credit level begins to decrease is much higher for the CTC than for the EITC (Crandall-Hollick, 2018; Crandall-Hollick et al., 2021). In addition, only households with income above a threshold level qualify for the refundable portion of the CTC. Because of these properties, more families receive the CTC compared with the EITC, and on average CTC recipients have higher incomes. However, total spending on the CTC is lower than spending on the EITC because the credit level per recipient household is lower (Ackerman, Cooper, Rachel, and Tong, 2016).

Uptake of the CTC is possible when an eligible taxpayer correctly completes their income tax form. Such a process identifies eligible households who then face a reduced tax liability or even receive a check for the refundable portion of the credit. Uptake among the eligible is estimated at greater than 90 percent (Crandall-Hollick, 2018).

The Child Care and Development Fund The CCDF in its current form began when the Personal Responsibility and Work Opportunity Reconciliation Act of 1996 consolidated four programs

[3]Tax credits are thus similar to cash transfers, especially if they are fully refundable (Goldin and Michelmore, 2022).

[4]In this paper, the program we refer to as the CTC is the sum of the nonrefundable and refundable components of the child tax credit (Crandall-Hollick, 2018, 2021). In US government reports, the nonrefundable component of the tax credit for children is often called the "CTC", while the refundable component is often referred to as the Additional Child Tax Credit, or "ACTC".

[5]Technically, a much-reduced EITC is available for low-income households without children.

into one (Lynch, 2022).[6] The rules governing how funding is spent were authorized initially until 2020, after which they have been re-authorized annually by Congress.[7] The CCDF gives block grants from the federal government to states; states then use some of these funds to pay portions of childcare expenses for households with children under 13 years of age, although most children receiving the subsidy are aged five and under (Chien, 2019a,b, 2020). Conditional on CCDF receipt, households contribute a fixed copayment towards the cost of childcare, with the government financing the residual. The portion of childcare expenses shouldered by the CCDF is decreasing in household income because copayment amounts are increasing in household income.[8] Besides having young children, to qualify for the CCDF parents in the household must engage in an approved work-related activity and household income must be below a certain threshold. The specific eligibility requirements used by states must be at least as stringent as federal guidelines; for example, the federal government requires that household income be at or below 85 percent of the state median income for households of a similar size and composition, but most states have set lower income cutoffs (Chien, 2019a,b, 2020). In practice, the CCDF reaches eligible households who successfully take up the subsidy, which may require significant resources to accomplish.

Households hoping to receive the subsidy must contact a social services agency, provide documentation of their work-related activity and income level, and complete an interview with a social worker. Once eligibility is established it may be re-evaluated at regular intervals thereafter, so that maintaining eligibility involves the household incurring these transaction costs multiple times. Eligible households may also be placed on a waiting list, to be contacted when funds become available. Taken together, these frictions lower the net value of the subsidy for recipients. It is important to note that, because the CCDF is not an entitlement, if an eligible household does not receive the CCDF this is entirely in keeping with the mandate of the program in its current form. Indeed, over the 2015-2017 period, government reports estimate uptake among federally eligible children at 15 percent on average, or 23 percent of eligible children according to more restrictive state eligibility rules (Chien, 2019a,b, 2020). In 2017, eligibility rules and low uptake rates combined meant that only 3.5 percent of households with children aged under 5 received CCDF subsidies.[9]

[6]Since then, temporary extensions in funding continued until 2021, when the American Rescue Plan Act provided permanent annual appropriations.

[7]The funding given to states by the federal government has a mandatory component and a discretionary component. The discretionary component fluctuates more over time. States may then complement these funds with state money, sometimes from other federal funding streams such as Temporary Assistance for Needy Families (TANF).

[8]Some CCDF funds are earmarked for enforcement of quality control of the childcare industry in the state, and childcare providers paid with CCDF funds must meet certain criteria that are intended to keep provider quality comparable to the market average.

[9]The mass of families receiving CCDF funds reflect author calculations described in Section 4.

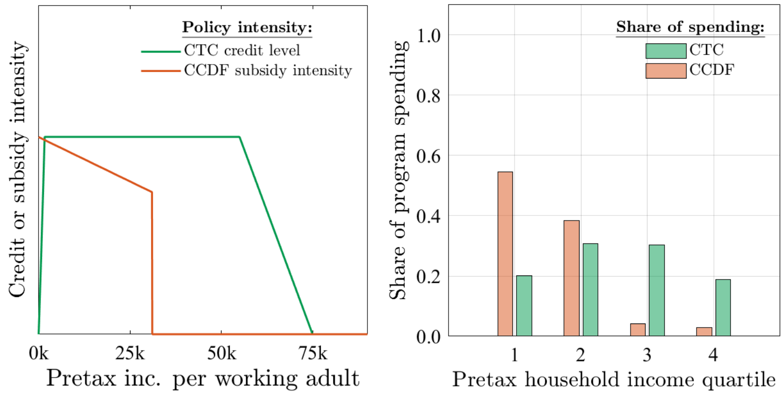
Comparison of policy intensity and spending patterns Figure 1a shows a representation of the in CTC and CCDF intensities as a function of household income for recipient households with working parents (i.e., abstracting from the uptake and labor supply decision). This representation reflects 2016 government policy and uses the parameterization of our model baseline. In the model's approximation, the CTC is humped-shaped in income and the CCDF subsidy intensity is decreasing in household income, reflecting the qualitative properties of the two policies discussed earlier in this Section.

Moving to aggregate spending on each policy, Figure 1b reports the share of total spending by household income quartile for the CTC and the CCDF. Here, households are assigned to quartiles using the distribution of Adjusted Gross Income in the 2016 tax year reported by IRS SOI Tax Stats (2016). Spending on the CTC is computed by aggregating government expenditures on the nonrefundable and refundable components of the credit, for income levels within each income quartile using information from all tax returns in the 2016 tax year IRS SOI (2016), Table 3.3. Spending on the CCDF is computed by aggregating the value of subsidies received by households within a given income quartile, as measured in CCDF Administrative Data 2016 FY.

Consistent with the properties of the policies shown in Figure 1a, aggregate CTC spending is humped-shaped in household income, and is received by households with higher income levels than the CCDF. CCDF spending is decreasing in household income, with some spending going to the top two quartiles because of household size variation and other heterogeneity in the data that is abstracted from in the policy representation of Figure 1a. Nevertheless, the policy representations align well with aggregate spending patterns.[10]

Discussion of policy implementation methods Using the tax system to implement family policy objectives reaches a greater fraction of the eligible compared to a spending program. The properties of these alternative implementation methods are well-documented: selecting one or the other is a policy choice (Steuerle, 1990; Sommartino, Toder, and Maag, 2002). In the United States, there has been an increasing trend since the 1980s towards using the tax system to implement social policy objectives such as transfers to families with children. Our second experiment models the 2018 CTC expansion passed into law by the 2017 Tax Cuts and Jobs Act—a case in point of this legislative trend.

[10]The functions representing the CTC and CCDF policies in Figure 1a are described in Section 3.1, and their parameterization is described in Section 4. For a more detailed description of the data and methodology underlying Figure 1b, see Appendix A.3.



(a) Policy intensity by income level

(b) Spending per income quartile

Figure 1: CTC and CCDF policy intensity and spending share: relationship with income

Notes: Figure 1 shows the model's approximation of the intensity of the CTC and CCDF as a function of income (Figure 1a) and the share of total CTC or CCDF spending by household income quartile for the 2016 fiscal year (Figure 1b). Source: policy intensity values are computed using values from Crandall-Hollick (2018) and author estimations using CCDF administrative data. Income thresholds for quartiles are from IRS SOI Tax Stats (2016); government spending is from CCDF Administrative Data 2016 FY for the CCDF and IRS SOI (2016), Table 3.3 for the CTC.

3 The Model Environment

The model features heterogeneous consumers organized into households, the government, a representative final goods producer, and a representative childcare provider. Time is discrete and runs forever; each period lasts for five years.

Consumers live for $J = 13$ periods, corresponding to a lifespan of 65 years; Figure 2 illustrates the phases of a consumer's life. During the "childhood" phase, which lasts for the first $j_a = 4$ periods of life, consumers are children who do not make their own decisions. In period $j_a + 1$, consumers become adults indexed by productivity θ_a, and draw their child's initial skill draw, θ_k, i.i.d. from a distribution $\mu_k(\theta_k)$ (mnemonic: $k =$"kid"). The "parenthood" phase lasts the first j_a periods of adulthood, and during this time parents split consumption with their child. During the first period of parenthood, consumers decide how much to spend on consumption, c, and non-parental childcare, n, as well as how to allocate their time between hours supplied on the market as labor, ℓ, quality time with their child, q, and leisure. The also decide whether or not to take up the childcare subsidy by setting $R = 1$. For the rest of parenthood after $j_a + 1$, q and n are no longer in the choice set. Beginning at age 40-45, during the "empty nester" phase, the child has left the household and the consumer solves a standard consumption-leisure decision problem.

Parent choices about their child's time use, q and n, imply a level of skill investment which together

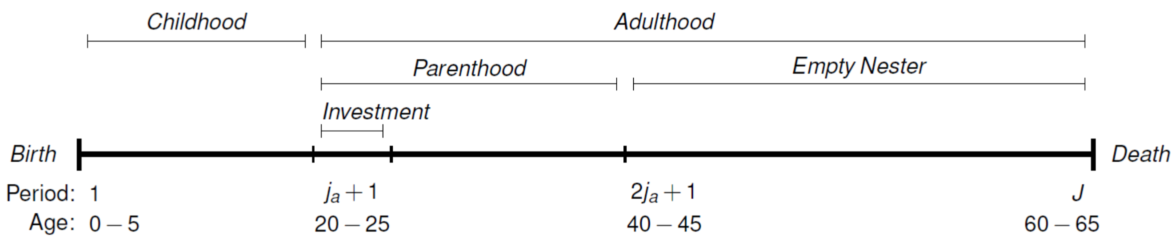


Figure 2: Consumer lifecycle

Notes: Figure 2 shows the consumer's lifecycle from birth to death. At birth, the individual draws an initial skill endowment and a parent. Childhood lasts the first j_a periods, corresponding to the first twenty years of life, where the first period of childhood corresponds to early childhood. Adulthood begins in period $j_a + 1$ and lasts until death. The first j_a periods of adulthood correspond to parenthood. At the start of the first period of parenthood, when the consumer is aged 20-25, the adult draws their child's skill endowment and then invests in their child's skill for one period, splitting consumption with their child for the remainder of parenthood. After their child leaves their household, the consumer is an empty nester who lives alone and works until they die at age 65 after period J of their lifecycle.

with the child's skill endowment determines the child's adult level of skill, θ_k^a, via child skill accumulation during early childhood. When choosing their child's time use, parents weigh the relative productivities of the two investment inputs while minimizing the cost of financing a desired skill level outcome for their child. The marginal product of each investment input is mediated by the skill investment technology, defined in Section 3.1.3. Taking cost minimization as given, the optimal level of investment is in turn chosen to balance the marginal benefit of a better future outcome for the child against the marginal cost in terms of foregone consumption and leisure for the parent. Parents invest in their child's skill because they are altruistic—they care about their child's expected lifetime utility, which is increasing in the child's level of skill as an adult and appears in the parent's objective function weighted by the altruism parameter, b, as the term $bV_k^a\left(\theta_k^a\right)$.[11] Via this investment decision, the distribution of adult skill in the population, $\mu_a\left(\theta_a\right)$, is endogenous in the sense that it reflects the skill investment decisions of parents in the previous generation.

The final goods producer uses a linear production technology that takes efficiency units of labor, H, as its sole input: $Y = H$.

The childcare provider offers a perfectly elastic supply of childcare with productivity θ_n at price p. The price of childcare is set as a constant fraction ϕ of the average pre-tax wage of adults with young children: $p = \phi \int_{\theta_a} w\mu_a\left(\theta_a\right) d\theta_a$.

Finally, the government raises revenue from a flat consumption tax, τ_c, and a progressive income tax explained in Section 3.1.1. It uses this revenue to purchase a fixed fraction of output for government consumption and to finance a system of transfers to families with young children who meet

[11]The expectation is taken over the initial skill draw of the adults' grandchildren.

eligibility requirements. The government also finances a system of means-tested childcare subsidies, $\tau_n(y)$; eligible households must pay a fixed cost ξ in consumption units to access the subsidy, although this fixed cost payment does not contribute to government revenue. The government can make uptake less costly for eligible families by subsidizing the fixed cost with τ_ξ.

3.1 Functional forms

3.1.1 Government policy

Government tax policy determines the mapping from pre-tax to after-tax income. It is specified by the income tax progressivity parameter, τ_y^p, as well as vectors of parameters for the CTC, $\vec{\pi}_C$, and the EITC, $\vec{\pi}_E$. The government also sets a vector of parameters governing TANF, which we model as a transfer to non-working parents of young children, $\vec{\pi}_T$. Lastly, the government sets a vector of parameters for the non-parental childcare subsidy (the CCDF), $\vec{\pi}_N$. The variable τ_y, which determines the average income tax rate, adjusts to balance the government's budget constraint.

The income tax and transfer system Given a level of labor supply ℓ, an adult with skill θ_a at age j has pre-tax income $y = w\theta_a\ell$ and after-tax disposable income given by

$$y_{AT,j}(y, \vec{\pi}_C, \vec{\pi}_E, \vec{\pi}_T) \equiv \begin{cases} (1-\tau_y)\, y^{1-\tau_y^p} + CTC_j(y, \vec{\pi}_C) + EITC_j(y, \vec{\pi}_E) & \text{if } y > 0 \\ TANF_j(y, \vec{\pi}_T) & \text{if } y = 0 \end{cases} \tag{1}$$

where the functional form of the income tax system follows Heathcote, Storesletten, and Violante (2017). The policies on the right-hand side of (1) depend on the earned income of the consumer, their age, and the relevant vector of government policy parameters. If a household's income or age is such that they are not eligible for a transfer, then the value of that transfer is set to zero.

Note that we do not allow τ_y^p to vary by consumer age; instead, we separate out the CTC and EITC in the budget constraint because they depend on the presence and age of children, and also because their net effect on the disposable income of the consumer may make it higher than pre-tax income that is equal to zero (i.e., the EITC is a fully refundable tax credit that is much more generous to families with children).[12]

TANF as we model it here offers cash transfers to non-working families with young children. This represents the cash assistance component of the US funding stream of the same name that is partially spent on such lump-sum transfers. Because children are present at the same ages in all

[12]This differentiates the EITC and CTC from the Child Care and Dependent Tax Credit, which is nonrefundable and is left embedded in the estimation of τ_y^p.

households of the model, TANF is an age-dependent transfer policy in the model. We therefore model it separately from the initial income tax system.

Next, we provide the functional form for the CTC and its vector of parameters, followed by a description of the CCDF and its parameterization. Details for the EITC and TANF policies are available in Appendix A.1.

The Child Tax Credit The CTC is calculated per qualifying child as the sum of a nonrefundable component and a refundable component (Crandall-Hollick, 2018, 2021). The nonrefundable component offsets the tax liability; the refundable component of a tax credit can be received even by a household with zero tax liability. Both components depend on pre-tax income and whether there is a child under 17 in the house (i.e., the parents are aged $j_a - 1$ or less).

$$CTC_j\left(y, \vec{\pi}_C\right) = \begin{cases} CTC_j^{nref}\left(y, \vec{\pi}_C\right) + CTC_j^{ref}\left(y, \vec{\pi}_C\right) & \text{if } j \in [1, j_a - 1] \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The non-refundable tax credit the household is eligible for, κ_{aux}^{nref}, is the difference between the maximum nonrefundable amount, $\overline{\kappa}_{nref} = \Theta_{\kappa,nref} y_{p50}$, and a term that phases out the tax credit at a rate β_{out} once income is above the phase-out threshold $\underline{y}_{nref} = \Theta_{y,nref} y_{p50}$. Note that both the transfer level and the income cutoffs are set proportional to the median income in the population y_{p50}.

$$\kappa_{aux}^{nref} \equiv \begin{cases} \overline{\kappa}_{nref} & \text{if } y < \underline{y}_{nref} \\ \overline{\kappa}_{nref} - \beta_{out}\left(y - \underline{y}_{nref}\right) & \text{if } y \geq \underline{y}_{nref} \end{cases}$$

The non-refundable component is bounded above by the income tax liability, $y - (1 - \tau_y)\, y^{1-\tau_y^p}$,

$$CTC_j^{nref}\left(y, \vec{\pi}_C\right) \equiv \min\left\{\kappa_{aux}^{nref}, y - (1 - \tau_y)\, y^{1-\tau_y^p}\right\}$$

The difference between the tax credit the household is eligible for and the household's nonrefundable component is the upper bound for the refundable portion of the CTC and is denoted κ_{aux}^{ref}, which can be at most the maximum refundable credit $\overline{\kappa}_{ref} = \Theta_{\kappa,ref} y_{p50}$:

$$\kappa_{aux}^{ref} \equiv \min\left\{\overline{\kappa}_{ref}, \kappa_{aux}^{nref} - CTC_j^{nref}\left(y, \vec{\pi}_C\right)\right\}$$

For households with income above the refundability threshold $\underline{y}_{ref} = \Theta_{y,ref} y_{p50}$, the refundable portion of the CTC is the minimum of κ_{aux}^{ref} and a fraction β_{in} of income above the refundability threshold, where β_{in} is the phase-in rate.

$$CTC_j^{ref}(y, \vec{\pi}_C) \equiv \begin{cases} \min\left\{\kappa_{aux}^{ref}, \beta_{in}\left(y - \underline{y}_{ref}\right)\right\} & \text{if } y \geq \underline{y}_{ref} \\ 0 & \text{otherwise} \end{cases}$$

The vector of parameters governing this policy is therefore given by
$\vec{\pi}_C = [\Theta_{\kappa,nref}, \Theta_{\kappa,ref}, \beta_{out}, \Theta_{y,nref}, \beta_{in}, \Theta_{y,ref}]$.

The Child Care and Development Fund From the perspective of the government, the CCDF transfer to a given household depends on: $R \in \{0, 1\}$, the subsidy uptake decision of the household; n, the number of hours of childcare the household chooses; and y, the household's pre-tax income which determines their eligibility for the subsidy as well as the subsidy's intensity. Specifically, if the household chooses to take out the subsidy, the CCDF pays a proportion τ_ξ of the fixed cost of subsidy uptake plus a proportion $\tau_n(y) = \beta_{0,n} + \beta_{1,n}\left(\frac{y}{y_{p50}}\right)$ of childcare expenses, which depends on household income.[13] The subsidy only takes a positive value for household income levels below a cutoff, $\overline{y}_n$, set to the percentile $\Theta_{y,n}$ of income for parents raising young children.

$$CCDF_j(R, n, y, \vec{\pi}_N) = \begin{cases} R\left[\tau_\xi \xi + \tau_n(y)\, pn\right] & \text{if } y \in (0, \overline{y}_n] \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The vector of parameters governing this policy is given by $\vec{\pi}_N = [\tau_\xi, \beta_{0,n}, \beta_{1,n}, \Theta_{y,n}]$.

3.1.2 Consumer preferences

The utility function u_j takes as inputs consumption, c, and leisure, $1 - \ell - q$, where ℓ is labor supply and q is quality time the parent spends with the child. It also is affected by a consumption equivalence scale, Ψ_j, which reflects the presence of a child in the household at certain ages:

$$u_j\left(\frac{c}{\Psi_j}, 1 - \ell - q\right) = (1 - \psi)\ln\left(\frac{c}{\Psi_j}\right) + \psi \ln(1 - \ell - q) \tag{4}$$

3.1.3 Skill investment technology

The skill investment technology consists of a dynamic and a static function. The dynamic function combines investment, I, and the child's stock of skill today, θ_k, to generate the stock of skill for

[13] We model the quality of subsidized childcare as identical to that of unsubsidized childcare; subsidized care providers are not fundamentally different from unsubsidized ones in our model environment. Note that, in the meta-analysis of Mullins (2020), the average treatment effect across studies of subsidized formal childcare is found to be positive (with the caveat that the programs being analyzed are usually targeted and not universal in eligibility).

the child when they become an adult, θ_k^a:

$$\theta_k^a = f\left(\theta_k, I\left(q, n|\theta_a\right)\right) = \left[\left(1-\upsilon\right)\left(\theta_k\right)^{\frac{\chi-1}{\chi}} + \upsilon\left(\lambda_I I\left(q, n|\theta_a\right)\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}} \tag{5}$$

Here, υ and χ are the CES share parameter for investment and the elasticity of substitution between the stock of skill and investment, respectively. The parameter λ_I scales the units of investment into the space of skill.

The static function combines investment inputs (parent time and market time) within a period to generate investment:

$$I\left(q, n|\theta_a\right) = \left[\gamma\left(\theta_a q\right)^{\frac{\nu-1}{\nu}} + \left(1-\gamma\right)\left(\theta_n n\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}} \tag{6}$$

In this specification, γ is the CES share parameter on the efficiency units of parent quality time investment input, and ν is the elasticity of substitution between efficiency units of parent time and efficiency units of childcare time.

3.1.4 The initial skill distribution

The distribution $\mu_k\left(\theta_k\right)$ is assumed to be uniform. The difference between the upper and lower bounds of the support for this distribution, $\overline{\Delta\theta_k}$, determines the variance of the initial skill distribution.

3.2 Agent problems

The childcare provider and production firm solve simple problems provided in the model overview earlier in this section; below we detail the consumer and government problems.

3.2.1 Consumers

In the first period of adulthood, when $j = j_a + 1$, the consumer is indexed by their own skill and their child's initial skill endowment. Given their type and the static and dynamic investment technologies given in (5) and (6), consumers choose a feasible adult skill outcome for their child, a level of household consumption, labor supply, and whether or not to take out the childcare subsidy. Note that, after paying the fixed cost, households only receive a positive subsidy if their pre-tax income makes them eligible.

$$
\begin{aligned}
V_j(\theta_a, \theta_k) &= \max_{\theta_k^a, c, \ell, R} u\left(\frac{c}{\Psi_j}, 1 - \ell - q(\theta_k^a|\theta_a, \theta_k)\right) + bV_k^a(\theta_k^a) + \beta V_{j+1}(\theta_a) \\
(1+\tau_c)c + pn(\theta_k^a|\theta_a, \theta_k) + R\xi &\leq y_{AT,j}(y, \vec{\pi}_C, \vec{\pi}_E, \vec{\pi}_T) + CCDF_j(R, n(\theta_k^a|\theta_a, \theta_k), y, \vec{\pi}_N) \quad (7)
\end{aligned}
$$

where $q + n \leq 1$, $q + \ell \leq 1$, $c, \ell, q, n \geq 0$, and $R \in \{0, 1\}$. Any candidate adult skill level for the child implies levels of parent quality time and childcare time, implicitly chosen to minimize the cost of achieving the targeted level of skill given the investment technology and household type (this dependence of q and n on θ_k^a, θ_a, and θ_k is explicit in our notation). These quantities are therefore not modeled as separate choices in (7).

In later periods of life, the consumer chooses consumption and labor supply, although they continue to split consumption with their child up to and including period $j = 2j_a$.[14]

$$
\begin{aligned}
V_j(\theta_a) &= \max_{c,\ell} u\left(\frac{c}{\Psi_j}, 1 - \ell\right) + \mathbb{I}_{j<J}\beta V_{j+1}(\theta_a) \quad (8) \\
(1+\tau_c)c &\leq y_{AT,j}(y, \vec{\pi}_C, \vec{\pi}_E, \vec{\pi}_T)
\end{aligned}
$$

In the last period of life, when $j = J$, the continuation value drops out of the objective function. Note that, for these ages, the child's initial skill endowment no longer affects the choice set or objective function of the adult parent and therefore no longer indexes the value function of the household.

3.2.2 The government

The government implements the CCDF, CTC, EITC, and TANF, and also finances government consumption expenditure that is set as a fraction Θ_G of output. In the model, receipt of the CTC and EITC is automatic, reflecting the fact that these programs are implemented through the tax system (Falk and Landers, 2022). Receipt of TANF is also assumed to be frictionless and only for non-working households with young children; this is a simplification of the broader US funding stream with the same name.[15] The net revenue from an optimizing household of type z_j is

$$
\begin{aligned}
Rev(z_j|\lambda_y, \vec{\pi}_C, \vec{\pi}_N, \vec{\pi}_E, \vec{\pi}_T) \equiv\ & y_j^*(z_j) - (1-\tau_y)\left(y_j^*(z_j)\right)^{1-\tau_y^p} \\
& - CTC_j\left(y_j^*(z_j), \vec{\pi}_C\right) - CCDF_j\left(R_j^*(z_j), n_j^*(z_j), y_j^*(z_j), \vec{\pi}_N\right) \\
& - EITC_j\left(y_j^*(z_j), \vec{\pi}_E\right) - TANF_j\left(y_j^*(z_j), \vec{\pi}_T\right)
\end{aligned}
$$

[14]Because the CCDF takes a value of 0 in these periods, it is omitted from the budget constraint.

[15]Spending on the TANF program as we model it is small in the data, and not the focus of our analysis in this paper.

where a superscript $*$ denotes the optimal choice of the corresponding variable by a consumer aged j with type z_j, where z_j is (θ_a, θ_k) for age $j = 1$ and θ_a otherwise. Government budget balancing requires τ_y^* such that in equilibrium the following equation holds:

$$\frac{\sum_{j=j_a+1}^{J} \int_{z_j} \left[Rev\left(z_j|\tau_y^*, \vec{\pi}_C, \vec{\pi}_E, \vec{\pi}_T, \vec{\pi}_N\right)\right] \Omega\left(j, z_j\right) dj\, dz_j}{\sum_{j=j_a+1}^{J} \int_{z_j} \left[y_j^*\left(z_j\right)\right] \Omega\left(j, z_j\right) dj\, dz_j} = \Theta_G \tag{9}$$

Here, the joint distribution over household types, $\Omega\left(j, z_j\right)$, is given by:

$$\Omega\left(j, z_j\right) = \begin{cases} \mu_a\left(\theta_a\right) \mu_k\left(\theta_k\right) \frac{1}{J-j_a} & \text{if } j = j_a + 1 \\ \mu_a\left(\theta_a\right) \frac{1}{J-j_a} & \text{if } j > j_a + 1 \end{cases}$$

3.3 Stationary steady-state equilibrium

Given an initial distribution of skill endowments $\mu_k\left(\theta_k\right)$, a goods production technology $Y = H$, a childcare pricing rule ϕ, and a government policy $\{\vec{\pi}_C, \vec{\pi}_N, \vec{\pi}_E, \vec{\pi}_T\}$, a stationary steady-state equilibrium is defined as:

1. agent choices and objective functions (consumers, goods producer, and childcare provider)
2. consumer expectations about lifetime utility given a child's adult skill outcome, $V_k^a\left(\theta_k^a\right)$
3. an adult skill distribution $\mu_a\left(\theta_a\right)$
4. an average income tax rate τ_y
5. a wage rate w

such that

1. agents optimize taking prices as given
2. consumer expectations are rational
3. the distribution of adult skill is stationary
4. the government balances its budget constraint
5. the labor market clears

When all of these conditions hold, the goods market clears by construction. Assumptions about the production firm's technology imply that the wage rate, w, is always equal to 1. Assumptions about the childcare provider's pricing rule implies that the price of childcare depends entirely on the distribution of skill among parenting adults, while its perfectly elastic supply allows the market for childcare to clear simultaneously.

4 Model Parameterization

The model parameters can be separated into those estimated outside of the model, shown in Table 1, and those calibrated inside of the model, shown in Table 2.

Panel A of Table 1 reports parameters related to government revenue and consumption, as well as demographics. The consumption tax rate, τ_c, is estimated using Organization of Economic Cooperation and Development (OECD) data for the years 2015-2017, applying the method of Mendoza, Razin, and Tesar (1994). The income tax progressivity parameter, τ_y^p, is estimated using microdata from PSID (2015-2017).[16] This parameter represents the progressivity of the tax system after netting out the CTC and the EITC, the two largest family policies implemented through the tax system in terms of spending.[17] Details of the estimation for the consumption and income tax parameters are provided in Appendix A.2. The third row of Panel A reports the share of output that goes towards government consumption, estimated using data from the Bureau of Economic Analysis for the period 2015-2017. The discount rate, β, is set to an annual value of 0.96. Finally, the consumption-equivalence scales, Ψ_j, are set to 1.3 when there is a child in the household and 1 thereafter, following the 1994 OECD scales based on the findings of Hagenaars, de Vos, and Zaidi (1994).

Panel B of Table 1 contains values for the parameters governing the CTC for the 2016 Fiscal Year, chosen to match the value reported in Crandall-Hollick (2018) converted into per-parent terms.[18] Dollar values are in 2016 US dollars (USD), and are mapped into the model by multiplying their level by the ratio of median income in the model to median household income in the US in 2016, $y_{p50,US} = \$57,617$, from Guzman (2017).

Panel C contains the CCDF policy parameters for the 2016 Fiscal Year; the government is assumed to offer no subsidy for the uptake cost in the baseline. For eligible recipients, the subsidy on childcare expenses is decreasing in household income, reflecting estimation results presented in

[16] We also estimate this parameter using aggregate data underlying the figures for the 2016 fiscal year in U.S. Congressional Budget Office (2019), applying the method of Heathcote, Storesletten, and Violante (2017). These two estimation methods yield very similar progressivity parameter values.

[17] Specifically, the progressivity parameter in our specification represents the effects of deductions, federal tax rates, and the Supplemental Nutrition Assistance Program, as well as the effect of state tax rates and Federal Insurance Contributions Act (i.e., payroll taxes). In our approach, refundable tax credits are not embedded in the tax rate but are explicitly modeled.

[18] When values are different for single parents and couples, a weighted sum using the population shares from Table 1 of Moschini (2023) is used for the parameterization, as we do not model family structure heterogeneity.

Appendix A.2.3. The eligibility threshold percentile is set to the 27th percentile of income for households raising young children to match the across-state average of income eligibility cutoffs, a value computed for 2015, 2016, and 2017 by scaling federal income eligibility thresholds using the relative size of state-to-federal eligible populations of children as reported in Chien (2019a,b, 2020). We then take the average across the three years.

The parameters for the two remaining policies, the EITC and TANF (Panel D), are estimated using information from Crandall-Hollick et al. (2021) and Office of Family Assistance (2015), respectively; specific parameter values are reported in Appendix A.1.1 and Appendix A.1.2.

Finally, in Panel E, the skill accumulation technology parameters are set to values estimated in Moschini (2023) or to averages of the parameters estimated there as described in Appendix A.2.5. Qualitatively, the investment technology exhibits CES complementarity between investment inputs (governed by ν), as well as between investment and the current stock of child skill (governed by χ). The complementarity between initial child skill and investment indicates that the higher the child's time endowment, the larger the effect of a unit of investment on skill accumulation. Because high- and low-skill parents are equally likely to have a high-skill child in this model, high returns to investment are spread across the population of parenting households rather than being concentrated at one end of the income distribution (the assumption that all parents draw their child's initial skill endowment from the same distribution is based on the findings of Moschini (2023)). The CES shares on investment inputs, governed by γ, indicate that nonparental childcare plays a sizable role in generating investment, so that subsidies to childcare's price have the potential to reduce investment's composite price index significantly. This motivates a substitution towards skill investment by the household and a consequent improvement in skill outcomes for the child.

Five parameters are calibrated jointly inside of the model and presented in Table 2: the marginal utility of leisure, ψ, the investment productivity, λ_I, the degree of altruism, b, the fixed cost of taking up the child care subsidy, ξ, and the variance of the initial skill distribution which is determined by $\overline{\Delta\theta_k}$, the difference between the lower and upper bound of the distribution of initial skill endowments. [19] Although all of these parameters affect the five target moments to some extent, in Panel A they are presented in the same row as the moment they are chosen to match. The first three target moments are from Moschini (2023), and the fourth is computed using information on the number of households with children under five, the number of households receiving CCDF funds, and the share of CCDF recipient households with children under five for the 2017 fiscal year (Federal Interagency Forum on Child and Family Statistics, 2017; Office of Child Care, Administration for Children and Families, 2023a,b). The last moment is the average of the annual difference be-

[19]This method of disciplining properties of the initial skill distribution using moments on earnings inequality is based on Huggett, Ventura, and Yaron (2006).

Table 1: Externally estimated parameters

Symbol	Parameter description	Data source	Parameter value
Panel A: Government policy, preferences, and demographics			
τ_c	Consumption tax rate	See Appendix A.2.1	0.043
τ_y^p	Income tax progressivity	See Appendix A.2.2	0.110
Θ_G	Gov. consumption (% GDP)	BEA (2022a) and BEA (2022b)	0.140
$\{\Psi_j\}_{j=1}^{J}$	Consumption-equiv. scales	Hagenaars et al. (1994)	$1 + 0.3 \times \mathbb{I}_{j \leq j_a}$
β	Patience	Assumption	0.96^5
j_a, J	Length of childhood and lifetime	Assumption (20 and 65 years)	4, 13
Panel B: CTC policy $\vec{\pi}_C$			
$\Theta_{\kappa,nref}$	Max. non-refundable credit ratio	Crandall-Hollick (2018) and Guzman (2017)	$\frac{\$1,000}{y_{p50,US}}$
$\Theta_{\kappa,ref}$	Max. refundable credit ratio		$\frac{\$1,000}{y_{p50,US}}$
β_{out}	Phase-out rate		0.050
$\Theta_{y,nref}$	Phase-out threshold ratio		$\frac{\$59,200}{y_{p50,US}}$
β_{in}	Phase-in rate		0.150
$\Theta_{y,ref}$	Refundability threshold ratio		$\frac{\$1,815}{y_{p50,US}}$
Panel C: CCDF policy $\vec{\pi}_N$			
τ_ξ	Uptake cost subsidy	Assumption	0
$\beta_{0,n}, \beta_{1,n}$	Childcare subsidy function	See Appendix A.2.3	(0.999, −0.517)
$\Theta_{y,n}$	Eligibility threshold percentile	Chien (2019a,b, 2020)	27
Panel D: other tax system policies			
$\vec{\pi}_E$	EITC policy	Crandall-Hollick et al. (2021)	See Appendix A.1.1
$\vec{\pi}_T$	TANF policy	Office of Family Assistance (2015)	See Appendix A.1.2
Panel E: Skill accumulation technology parameters			
Dynamic equation			
χ	Elasticity of substitution	Moschini (2023)	0.380
υ	Share on investment		0.490
Static equation			
ν	Elasticity of substitution	See Appendix A.2.5	0.560
γ	Share on parent time		0.410

Notes: Table 1 reports externally estimated parameter symbols, descriptions, data sources, and values. Median household income, $y_{p50,US}$, is set to \$57,617 in 2016 USD.

tween the 50th and 10th percentiles of the pre-tax log income distribution for the United States from 2015 to 2017, using values from GRID (2015-2017). The calibrated model aligns well with the data.[20]

Panel B of Table 2 reports parameters whose values are set proportional to other equilibrium quantities in the baseline and held fixed in the counterfactual experiments. The childcare productivity,

[20]Note that targeting the share of households receiving the CCDF in the baseline, as we do, yields a household-level uptake rate of about 14 percent in the baseline equilibrium. This statistic is lower than the child-level uptake rate of 23 percent for those eligible according to state rules reported in Chien (2019a,b, 2020). This is not a contradiction: in the data, the household-level uptake rate implied by the share of households receiving CCDF funds will be lower than child-level uptake rate if eligible households with more children are more likely to receive CCDF funds.

θ_n, is calibrated so that the expected logged ratio of parental and nonparental childcare productivites is equal to zero for working parents. The pre-subsidy price ratio between non-parental and parental time, ϕ, is chosen so that in the baseline equilibrium the average post-subsidy and after-tax price ratio of investment inputs chosen by working parents matches what is observed in the data from Table 2 of Moschini (2023).

Table 2: Internally calibrated parameters

	Parameter		**Moments**		
Symbol	**Description**	**Value**	**Description**	**Data**	**Model**
Panel A: Parameters calibrated jointly					
ψ	Marginal utility of leisure	0.639	Average hours worked	0.330	0.330
λ_I	Investment productivity	7.858	Average non-parental childcare hours	0.307	0.307
b	Altruism parameter	$0.371 \times \beta$	Corr. child skill outcomes and family income	0.320	0.320
ξ	Fixed cost of CCDF uptake (% y_{p50})	0.128	Share of Hhs receiving CCDF	0.035	0.036
$\overline{\Delta\theta_k}$	Variance of μ_k (θ_k)	9.169	p50-p10 log income	1.403	1.404
Panel B: Parameters set proportional to other equilibrium objects					
θ_n	Child care productivity	4.014	Average productivity of working parents		
ϕ	Pricing ratio for n	0.160	Post-policy input price ratio		

Notes: Table 2 presents internally calibrated parameters.

The calibrated fixed cost of subsidized childcare uptake warrants particular attention. We find a value of 12.8% of median income, which corresponds to $7,375 in 2016 USD. Average childcare expenses in the model's baseline population are on the order of 15% of median household income, which falls within the range of 8 to 19 percent documented for the US in 2018 by Landivar, Graf, and Rayo (2023) and corresponds to a value of $8,643 in 2016 US dollars for average childcare expenses of households raising young children in the model (eligible and ineligible). The economic magnitude of this calibrated fixed cost indicates that, in order to match observed low CCDF receipt rates, the costs of barriers to uptake (e.g., demonstrating eligibility and waiting lists) must be high enough to offset the value of the subsidy for many eligible households. This is a model quantification of uptake costs which are difficult to measure directly in the data: for example, to our knowledge there is no public dataset on CCDF waitlist length or average wait times. As we will show in the following Section, in the baseline equilibrium the eligible households who are willing to pay this uptake cost have more resources and higher childcare expenses, which makes paying the fixed cost worthwhile for them.

5 Properties of the Baseline Equilibrium

The calibrated baseline equilibrium exhibits several properties that are relevant to our policy experiments. In this section we begin by examining the composition of baseline childcare subsidy recipients, and, by implication, the composition of households on the margin for childcare subsidy uptake. Next, we turn to the distribution of program spending across household income quartiles in the model and compare it with the data. The way each program affects individual consumer problems in the model allows the distribution of spending over income quartiles to capture the main properties of the data as presented in Section 2. Finally, we report the size of spending on the CTC, CCDF, EITC, and TANF as a fraction of output in the baseline equilibrium to show that the size of each is very similar to the data despite being untargeted. This indicates that the household-level modeling of eligibility and program intensity, combined with the extent of heterogeneity across households, is sufficient to capture the relative size of each program.

CCDF uptake patterns In Figure 3, we show which household types (indexed by adult skill, θ_a, and child skill endowment, θ_k) are eligible for the childcare subsidy (light blue region) compared to those who choose to pay the fixed cost and take up the subsidy (dark blue region). In the baseline equilibrium, eligible households with higher child skill tend to take up the subsidy, because the complementarity of skill and investment means that these are the eligible households investing the most in their children and therefore planning to spend the most on childcare. This uptake pattern is also qualitatively consistent with the results of Johnson, Martin, and Brooks-Gunn (2011), which indicate that those receiving the CCDF have relatively more resources compared to the pool of eligible non-recipients. Through the lens of the model, this corresponds to eligible households with relatively higher θ_a values comprising the pool of CCDF recipients. The close relationship between adult skill and income in the model, combined with the negative relationship between CCDF subsidy intensity and recipient income, leads aggregate spending patterns to align with our empirical findings in Section 2.[21]

The composition of CCDF recipients in the baseline equilibrium provides intuition for the counterfactual experiment results. For example, subsidizing the fixed cost of uptake will gradually expand the pool of CCDF recipients, starting with the marginal families who lie closer in the type space to the baseline subsidy recipients as shown in Figure 3. At the same time, the highest welfare gains are achieved from reaching the families with low θ_a and high θ_k, due to complementarity between investment and skill in the dynamic skill production function combined with limited re-

[21] Appendix B.1 compares our baseline specification with an alternative model framework featuring a utility cost of uptake: besides offering a better fit with baseline government spending patterns, our preferred specification provides a lower bound for the welfare gains due to expansions of the childcare subsidy.

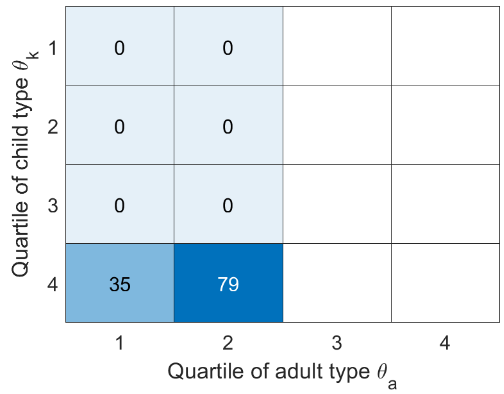


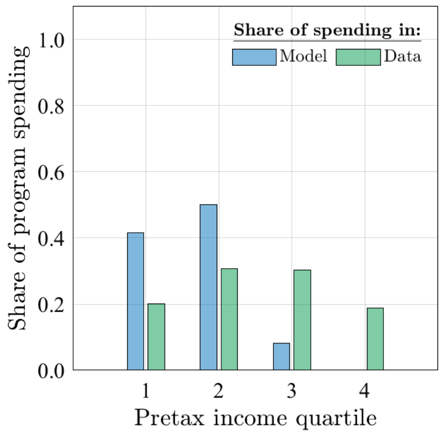
Figure 3: CCDF uptake rates by household type

Notes: Figure 3 shows the uptake rate, conditional on eligibility, for the CCDF. Uptake rates are reported for the discretized joint distribution over household types in period $j_a + 1$ of the lifecycle, with adult skill type θ_a on the x-axis and child skill endowment θ_k on the y-axis. Shaded cells denote positive within-cell eligibility rates using realized pretax income, with the uptake rate (percentage of eligible taking up the CCDF) written inside of the cell.

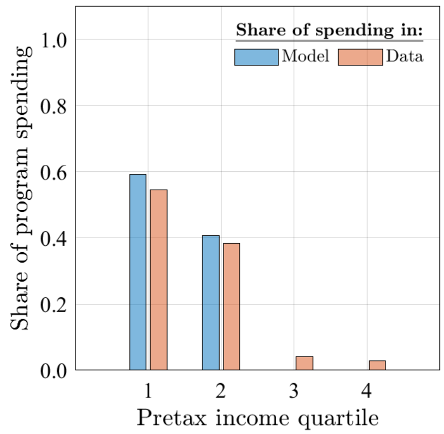
sources among low-skill parents. Recall that the fact that there is always a sizable mass of low-skill parents with high-skill children is a consequence of the zero correlation coefficient between child skill at 9 months and parent income reported in Moschini (2023). Figure 3 indicates these are the families relatively close to the baseline recipients in the type space, so that it will not be necessary to eliminate uptake costs to affect these households. Another implication of this CCDF uptake pattern in the baseline equilibrium is that, when the fixed cost is subsidized, inflows into subsidy uptake will arrive from families with relatively lower income who will receive a more generous CCDF subsidy. This increases the expense of such an expansion not just through the fixed cost subsidy but also through the higher cost to the government of subsidizing childcare for the new recipient households.

Family policy spending patterns Figure 4 shows how spending on the CTC and CCDF is allocated across pretax income quartiles in the population. It also includes empirical counterparts for these spending patterns, replicated from Figure 1 of Section 2.

For the CTC (left panel), spending in the model baseline is hump-shaped in income. Compared to the data, the model also captures the property that the second and third income quartiles together receive most of CTC spending, although it understates the extent to which higher income households receive spending from the CTC. Spending on the CCDF (right panel) in the model baseline equilibrium is decreasing in household income and concentrated in the first and second income quartiles, qualitatively aligning with aggregate spending in the data. Although eligible households who take up the subsidy have relatively higher adult skill levels in the model's baseline equilibrium (Figure 3), poor households who take up CCDF aid tend to receive large amounts of it because the subsidy intensity is higher for lower-income households (Panel C, Table 2). This is why aggregate spending on the first income quartile is quite high in the baseline, despite uptake rates for the lowest-skill eligible adults being low.



(a) Child Tax Credit



(b) Child Care and Development Fund

Figure 4: Share of program spending by income quartile of recipients

Notes: Share of program spending going to each income quartile for the Child Tax Credit (Figure 4a) and the Child Care and Development Fund (Figure 4b) in the model baseline equilibrium. Data values for both programs are replicated from Figure 1. The y-axis of each graph is the share of program spending going to a given income quartile; the x-axis is the income quartile assigned using the distribution of income for all households.

Table 3 reports the size of government spending on the CTC, CCDF, EITC, and TANF programs in the baseline model equilibrium and compares them with their empirical counterparts. For both data and model moments, size is measured by aggregate spending on the policy normalized by gross domestic product. Although these moments are untargeted, the model aligns well with the data. However, spending on the CCDF is slightly larger in the model than in the data because we assume that there is a single price of childcare; this inflates the size of the CCDF because in the data the average childcare price of those receiving the CCDF is lower than the average price in the population due to selection by family structure, as shown in Moschini (2023). Details on the computation of data moments are in Appendix A.2.4.

6 Experiments and Results

We perform two sets of experiments with the calibrated model. Section 6.1 evaluates the effects of expanding the CTC or the CCDF, holding the other program fixed at its baseline. The purpose of this exercise is to illustrate how each policy affects the model economy when spending on it is

Table 3: Government spending in model baseline and in US data

Program	Program size		Data source
	Model	**Data**	
Child Tax Credit (CTC)	0.301	0.287	IRS SOI (2017), Table 3.3
Child Care and Development Fund (CCDF)	0.079	0.053	Chien (2019a,b, 2020)
Earned Income Tax Credit (EITC)	0.347	0.347	IRS SOI (2017), Table 2.5
Temporary Assistance for Needy Families (TANF)	0.023	0.022	Office of Family Assistance (2015)

Notes: Table 3 reports the size of each program in the baseline economy and the size of the program in US data during the period 2015-2017, as well as the source of this empirical moment. All program sizes are in percentage of output (model) or of average US GDP from 2015 to 2017 (data), from BEA (2022a).

increased.[22] Section 6.2 reports the effects of the 2018 expansion in the CTC, a component of the 2017 Tax Cuts and Jobs Act, and compares it with an expansion of the CCDF that uses the same additional percentage of output.

Implementation In Section 6.1, the government expands the CTC by raising both the nonrefundable and the refundable per-child maximum amounts ($\overline{\kappa}_{nref}$ and $\overline{\kappa}_{ref}$, respectively) by the same proportion relative to their baseline values. We use this policy margin in this exercise because it allows for a larger range of total spending on the program to be implemented (i.e., compared to raising the phase-out threshold). This flexibility allows us to calibrate the CTC policy to match a desired level of total spending on the CTC and CCDF, combined, for comparability across program expansions. In Section 6.2, however, the CTC is expanded in a way that reflects the actual changes to the program as described in Crandall-Hollick (2018). This expansion combines an increase in the credit level with an increase in the phase-out threshold for income, as well as a slight decrease in the refundability threshold for income.

In both experiments, expansions of the CCDF are implemented by subsidizing the fixed cost of uptake using τ_ξ from (3), while leaving unchanged the intensity of the subsidy at a given level of income and the income eligibility threshold.

Measuring welfare changes When analyzing welfare changes across equilibria, we report two main measures: CE_1 and $\overline{CE}_2$. CE_1 is defined as

$$CE_1 \equiv \exp\left[\frac{W_{eq} - W_0}{(1-\lambda)\sum_{j=1}^{J}\beta^{j-1}}\right]$$

[22] Appendix B.2 reports the effects of reallocating economic resources towards either the CTC or the CCDF, holding fixed the total share of output spent on both programs; this illuminates the opportunity cost of funds spent on either program, in terms of resources not available for the other.

where W_{eq} is the expected lifetime utility of adult households with type $z_{j_a+1} = (\theta_a, \theta_k)$, using the distribution over types in the referenced equilibrium eq to compute W_{eq}. $\overline{CE}_2$, meanwhile, is the weighted average across types of $CE_2(j, z_j) \equiv \exp\left[\frac{V_j^{eq}(z_j) - V_j^0(z_j)}{(1-\lambda)\sum_{age=j}^{J} \beta^{age-j}}\right]$, evaluated at $j = j_a + 1$ and using the distribution over types in the new equilibrium eq to aggregate $CE_2(j, z_j)$. Specifically:

$$\overline{CE}_2 \equiv \int_{z_{j_a+1}} CE_2(j_a + 1, z_{j_a+1})\, \Omega(j_a + 1, z_{j_a+1})\, dz_{j_a+1}$$

When reporting age-specific welfare changes, we leave the age argument as a variable in the expectation and denote the result $\overline{CE}_2(j)$, defined as:

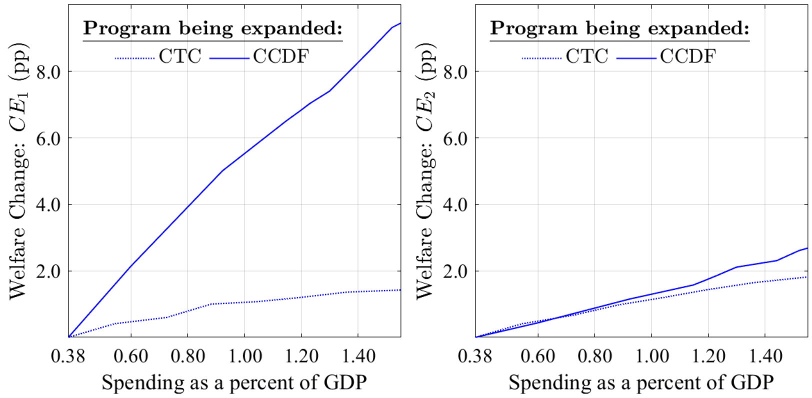
$$\overline{CE}_2(j) \equiv \int_{z_j} CE_2(j, z_j)\, \Omega(j, z_j)\, dz_j$$

When decompositions of welfare changes are reported, we use the method of Guvenen, Kambourov, Kuruscu, Ocampo, and Chen (2023). Appendix C provides more details on the computation of welfare changes and their decomposition.

6.1 The effects of expanding each program

Figure 5 shows the welfare gains of gradually expanding the CTC tax credit per child and gradually subsidizing the uptake cost for the CCDF, measured with welfare metrics CE_1 (left) and $\overline{CE}_2$ (right). The percent of output spent on the two programs combined is affected directly by raising the per-child credit for the CTC or increasing the subsidy on fixed costs for the CCDF; using the spending percentage value for the x-axis allows expansions of both programs to be presented on the same graph. The upper bound on the x-axis is the share of GDP spent on the two programs, combined, when the fixed cost of the CCDF is eliminated. As the Figure indicates, welfare gains behind the veil of ignorance are always higher after expansions of the CCDF, while the average welfare change across types is at first slightly larger after CTC expansions.

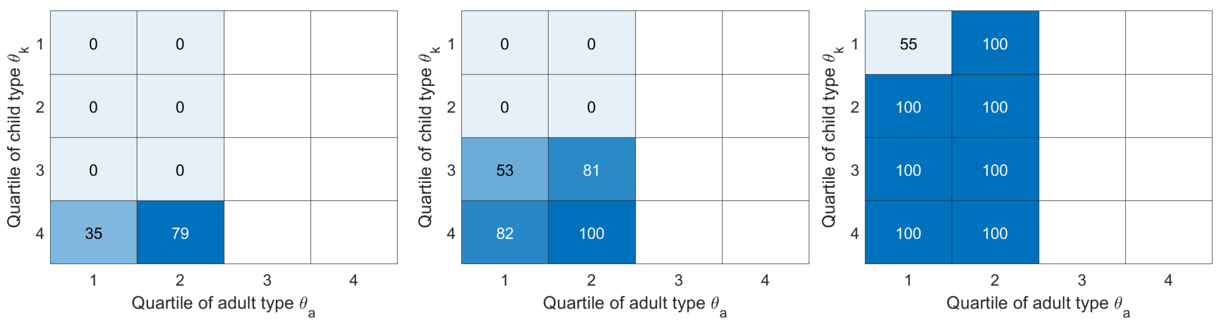
In the baseline equilibrium, households who are at the margin for childcare subsidy uptake are characterized by relatively higher adult and child skill, compared to the pool of eligible households. As the subsidy rate on the fixed cost of uptake increases (and the share of GDP spent on the subsidy increases), the marginal households change in composition: they are worse off financially (lower parent skill) and their children are lower-skilled. To demonstrate this point, Figure 6 shows how CCDF uptake decisions respond as the subsidy on uptake fixed costs gradually increases. At small subsidy rates, inflows into subsidy receipt arrive from households with relatively lower θ_k's (Figure 6b). Households with low θ_a and high θ_k, however, do change their uptake decisions to a greater extent at intermediate subsidy rates (Figure 6c).



(a) Welfare change: CE_1 (b) Welfare change: $\overline{CE}_2$

Figure 5: Welfare changes as each program is expanded

Notes: Welfare changes due to raising CTC credit amount and subsidizing the CCDF uptake cost, using welfare metric CE_1 (Figure 5a) and $\overline{CE}_2$ (Figure 5b). The x-axis is the percent of GDP spent on the CTC and CCDF, combined.



(a) No subsidy (b) 10 % subsidy (c) 50 % subsidy

Figure 6: CCDF uptake rates as fixed-cost subsidy increases

Notes: Figure 6 shows the uptake rate, conditional on eligibility, for the CCDF with no subsidy on the fixed cost of uptake (Figure 6a), a 10 percent subsidy (Figure 6b), and a 50 percent subsidy (Figure 6c). Uptake rates are reported for the discretized joint distribution over household types at $j = j_a + 1$, (θ_a, θ_k), with θ_a on the x-axis and θ_k on the y-axis. Shaded cells denote positive within-cell eligibility rates using realized pretax income, with the uptake rate (percentage of eligible taking up the CCDF) written inside of the cell.

We next examine changes in aggregate quantities after expansions of each program so that the change in government spending on the CTC and CCDF combined is equivalent to the change after a 100-percent subsidy of the fixed cost for CCDF uptake.[23] For the CTC, this requires a 4.36-fold

[23]Eliminating this fixed cost of CCDF uptake for households with a single large expense (rather than the government paying the fixed cost each period) is conceptually similar to switching from the spending program implementing the childcare subsidy to a fully refundable tax credit for childcare expenses (this statement abstracts from the timing of transfer disbursements). In the US tax system, there is already a tax credit for dependent care, but it is nonrefundable. Because such a switch would be likely less costly than subsidizing the fixed cost of uptake in each period, our implementation gives lower bounds on welfare gains from eliminating the fixed cost of uptake.

expansion in the CTC credit level. Table 4 reports changes across stationary steady states in various equilibrium quantities relative to the baseline economy, as well as the resulting welfare changes.

Table 4: Effects of expanding each program

Comparison across steady states			$\triangle$ after expanding:	
Category	**Variable**	**Baseline**	**(1) CTC**	**(2) CCDF**
Panel A: Program size and tax rate	Child Tax Credit (% Y)	0.30	1.24	0.03
$\triangle$ units: percentage point change	Child Care and Development Fund (% Y)	0.08	0.01	1.22
	CTC + CCDF (% Y)	0.38	1.25	1.25
	Income tax rate (at ave. inc. initial ss)	9.06	1.41	0.84
Panel B: Aggregate quantities	Quality time (parents only)	0.09	-1.13	-1.58
$\triangle$ units: percent change	Non-parental childcare (parents only)	0.31	-1.77	37.38
	Consumption	1.41	-1.66	4.99
	Leisure	0.66	0.40	0.39
	Labor supply	0.33	-0.77	-0.72
	Output	1.67	-1.67	6.21
Panel C: Welfare changes	CE_1		1.50	9.78
$\triangle$ units: percent change	$\overline{CE}_2$		1.91	2.87

Notes: Table 4 reports changes in aggregate quantities and welfare for expansions of the CTC (column 1) and CCDF (column 2) that result in the same percent of output being spent on both programs as eliminating the uptake cost of the CCDF. Specifically, in column (1) the per-child credit is $\overline{\kappa}_{nref} = \overline{\kappa}_{ref} = 4.678 \times \overline{\kappa}_{baseline}$, and in column (2) $\tau_{\xi} = 1$. For both columns, the change in spending on both programs as a percent of GDP is the same. Changes are relative to the baseline equilibrium ("Baseline").

When the CTC per-child credit is expanded, the percent of output spent on the CTC increases by 1.24 percentage points (column 1 of Panel A in Table 4). The CCDF also expands slightly, as more eligible families can pay the fixed cost or finance more spending on childcare with CTC funds. The increase in government spending on the CTC and CCDF contributes to the sizable increase in the average tax rate, defined as $1 - (1 - \tau_y)\, y^{-\tau_y^p}$ evaluated at the initial equilibrium's average income level. Equilibrium adjustments in spending on the EITC and TANF, as well as shifts in labor supply, also contribute to this tax rate change.[24] Note that the CTC is by its nature a transfer only to parents raising children; older consumers face a higher tax rate without the higher transfer. Parental quality time decreases and non-parental childcare decreases as well (Panel B). These changes reflect a decrease in skill investment, resulting in a slightly lower stock of skill in the new steady state and compounding the increase in income taxes necessary to finance the CTC expansion. The lower relative price of leisure along with the distortionary effect of the child tax credit leads consumers to reduce labor supply by 0.77 percent. In the new steady state, the drop in labor supply compounds the decrease in the stock of skill, which leads to a fall in output of 1.67 percent. The effect of these changes in quantities on welfare can be seen in Panel C: whether

[24] Instead of equalizing spending on the CTC and CCDF across policies, one could alternatively equalize government spending overall. When we do this, we find that the qualitative results of the policy experiments in this section are unchanged, and the quantitative results are very similar.

measured behind the veil of ignorance (CE_1) or using the average change across types for new adult households ($\overline{CE}_2$), the CTC expansion generates welfare gains because it raises leisure even as it lowers output overall.

In column (2) of Table 4, we see that when the CCDF is expanded by lowering uptake costs, this causes an increase in the size of both programs. The CCDF expands in size because the government is subsidizing the fixed cost of uptake for eligible families, which raises uptake rates; the CTC, meanwhile, expands because family incomes move above the refundability threshold $\underline{y}_{ref}$. The effect of subsidizing non-parental childcare is that it increases investment in children and changes the composition of investment inputs: non-parental childcare increases dramatically, by more than 37 percent, while parental quality time slightly decreases. The tax rate increases to finance the increased government spending, but the higher level of skill in the economy leads to an increase in both consumption and leisure. Although labor supply falls by 0.72 percent, output nevertheless rises by 6.21 percent because the increase in the stock of skill offsets the adjustment in consumer time away from labor supply. Overall, welfare increases by a larger amount behind the veil of ignorance (a 9.78 percent change) when the CCDF is expanded compared to an expansion in the CTC. Welfare gains measured by $\overline{CE}_2$ are also higher after the CCDF expansion. The reason CE_1 is so much higher than $\overline{CE}_2$ in column (2) is that the CCDF expansion provides insurance against poor outcomes for one's children, which is only internalized before the revelation of household type. This is illustrated further in the welfare decomposition, which we turn to next.[25]

To clarify differences in the source of welfare gains from each policy expansion, Table 5 decomposes changes from expanding the CTC, in column (1), or the CCDF, in column (2), into three components: welfare changes due to changes in the average level of consumption and leisure only, in the altruism term, and in the distribution of consumption and leisure (Appendix C.3 explains the calculations in more detail). Panel A reports the decomposition for CE_1; Panel B shows the decomposition results for $\overline{CE}_2$. In both panels, it is evident that gains from CTC expansions are mainly due to changes in the distribution of consumption and leisure across households, while gains from expansions in the CCDF are mainly stemming from changes in the average levels of consumption and leisure. Changes in the altruism term, reflecting improved child lifetime outcomes, play qualitatively similar roles in welfare changes across columns (1) and (2). Unsurprisingly, the effect of changes in the altruism term is larger for the expansion in the CCDF, as child outcomes improve more in that counterfactual.

The main takeaway of the preceding analysis is that raising the child tax credit level acts to redistribute from the old to the young but contracts the economy. Ultimately, this reduces consumption and labor supply in the long run even as it raises leisure. Welfare gains from this program stem

[25] Appendix B.3 reports the effects of changing margins other than the uptake cost for the childcare subsidy policy.

Table 5: Decomposing welfare changes

Welfare measure	Component	(1) CTC	(2) CCDF
Panel A: Components of CE_1	Levels of consumption and leisure	-0.96	5.71
	Altruism term	0.17	9.41
	Distribution	2.31	-5.08
	Total	1.50	9.78
Panel B: Components of $\overline{CE}_2$	Levels of consumption and leisure	-0.96	5.71
	Altruism term	0.46	2.56
	Distribution	2.44	-5.15
	Total	1.91	2.87

Notes: Table 5 decomposes welfare changes from Panel C of Table 4 into three components. The total welfare change corresponds to the values in Panel C of Table 4.

from redistributing from the old to the young; Figure 7 illustrates the redistribution across ages, reporting values for $\overline{CE}_2\,(j)$ for each adult age group. Welfare changes are very negative for consumers who are not investing in children or who do not receive the tax credit.

By comparison, childcare subsidies stimulate investment in child skill. In the new steady state, this results in a larger stock of skill, and therefore higher output even as labor supply decreases slightly. Altruistic links across generations make welfare gains behind the veil of ignorance from this program quite high. Expanding childcare subsidies also partially pays for itself by expanding the income tax base, while expanding the child tax credit level has the opposite effect. Because of this difference, the same increase in government spending requires a smaller increase in the average income tax rate in column (2) of Table 4, compared to column (1). Accordingly, Figure 7 shows that welfare losses in the new steady state for older consumers are lower after an expansion of the CCDF compared to losses for these ages after a large expansion in the CTC.

To examine the role of general-equilibrium adjustments in generating the results of Table 4, Table 6 shows how welfare changes after each program's expansion differ when the skill distribution $\mu_a\,(\theta_a)$ is held fixed at the baseline distribution, the altruism term $V_k^a\,(\theta_k^a)$ is held fixed at its baseline value, and the average labor income tax parameter τ_y is held fixed at its baseline value. If the welfare gain is higher when the general equilibrium object is held fixed (relative to the general equilibrium welfare change), this indicates the adjustments in that object act to mitigate welfare gains in general equilibrium and vice-versa.

Beginning with the first row of 6, which reports welfare changes behind the veil of ignorance as measured by CE_1, we find that when the CTC is expanded, all three general equilibrium adjustments act to mitigate its benefits to some extent, but adjustments in the tax rate have the greatest effect. Without accounting for such adjustments in the income tax rate, welfare gains would be more

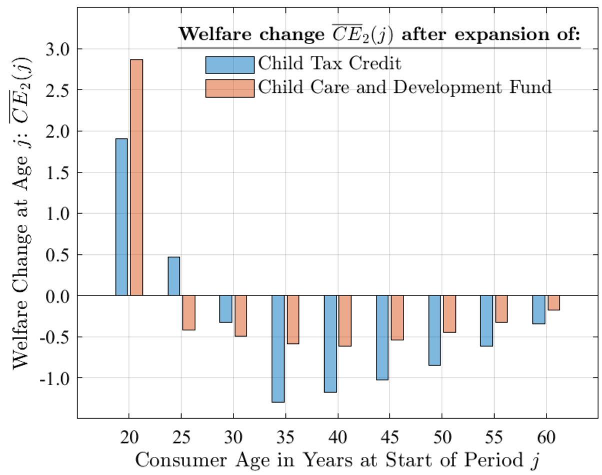


Figure 7: Welfare changes by age

Notes: Figure 7 shows the age-specific welfare changes averaged across types, computed across steady states after expanding the CTC or the CCDF. These welfare changes reflect the same exercise as is analyzed in Table 4 and Table 5.

than twice as high. When the CCDF is expanded, welfare changes are magnified by adjustments in the skill distribution (and to a lesser extent the altruism term), but mitigated by adjustments in the income tax rate.

Moving to the second row of Table 6, which reports population-weighted averages of type-specific changes for new households as measured by $\overline{CE}_2$, the results differ in that for expansions of the CTC equilibrium adjustments in the altruism term act to magnify rather than mitigate general-equilibrium welfare gains. For CCDF expansions, the altruism term now acts to magnify welfare gains, whereas results for the other equilibrium objects are unchanged qualitatively although the magnitudes differ.

Comparing outcomes across policies, the results of Table 6 indicate that the skill distribution is the most important object to endogenize in general equilibrium, in the sense that if this was held fixed the ranking of the two programs by welfare gains would be reversed. Although adjustments in the other two general equilibrium objects do play a significant role in terms of the magnitudes of welfare changes, the welfare ranking of the two policy changes would not be affected if either object was held fixed.

6.2 The 2018 expansion of the Child Tax Credit

The final policy experiment evaluates an actual policy change: the 2018 expansion of the Child Tax Credit. Implemented by the 2017 Tax Cuts and Jobs Act, this expansion was intended to be temporary, and was accompanied by other changes to the tax code. Here, we model it as a

Table 6: Role of general equilibrium adjustments

Comparison across equilibria	**Δ after expanding the CTC**				**Δ after expanding the CCDF**			
Equ. object held fixed	**None (GE)**	$\mu_a(\theta)$	$V_k^a(\cdot)$	τ_y	**None (GE)**	$\mu_a(\theta)$	$V_k^a(\cdot)$	τ_y
CE_1	1.50	1.91	2.16	3.87	9.78	1.51	9.34	11.67
$\overline{CE}_2$	1.91	1.94	1.37	4.10	2.87	1.65	1.99	4.59

Notes: Table 6 reports welfare changes of the experiments in Table 4, when the economy moves to a new equilibrium where the skill distribution $\mu_a(\theta)$ is held fixed at the baseline distribution, the altruism term's value function $V_k^a(\cdot)$ is held fixed at the baseline function, and the average labor income tax τ_y is held fixed at its baseline value, for each program's expansion.

permanent change made in isolation. This expansion raised the maximum nonrefundable credit level per child, $\overline{\kappa}_{nref}$, from \$1,000 to \$2,000 and the maximum refundable credit per child, $\overline{\kappa}_{ref}$, from \$1,000 to \$1,400. The change also lowered the refundability threshold $\underline{y}_{ref}$ from \$1,815 to \$1,513 and raising the phase-out threshold $\underline{y}_{nref}$ from \$59,200 to \$200,000 (per resident parent). Figure 8 illustrates how the 2018 expansion in the CTC affected the income levels eligible for a positive credit and the magnitude of the credit at each income level, using the model representation of the CTC.

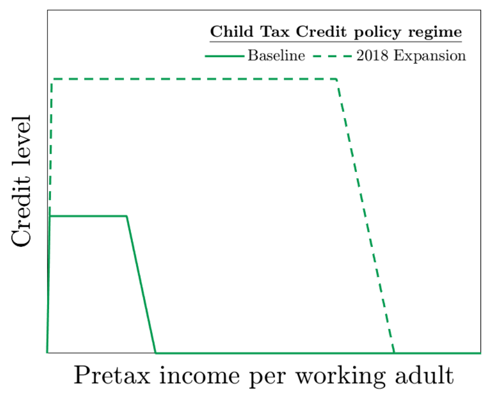


Figure 8: Model Parameterization: the Child Tax Credit before and after the 2018 CTC expansion

Notes: Figure 8 shows how the 2018 CTC expansion affected the properties of the Child Tax Credit in terms of the income levels eligible for the credit and the magnitude of the credit. The figure shows the effect using the model representation of the CTC.

In column (1) of Table 7 we report changes in economic variables after the economy has moved to the new steady state with the 2018 CTC policy in place. The CTC expansion results in an increase in the size of the child tax credit of about 0.75 percent of output, and a slight increase in the size of the CCDF. The income tax must increase to finance this rise in expenditures. Investment in child skill rises slightly, because the CTC provides financing for parents to shift towards non-parental childcare time while reducing quality time. Consumers substitute from consumption towards leisure, in light of higher tax rates, and labor supply falls. The increase in skill slightly dominates the fall in labor supply, so that output rises by 0.03 percent. The resulting welfare gains are 1.66 percent of lifetime consumption behind the veil of ignorance, with slightly smaller gains of 1.09 percent when measured as the average across household types for new adults.

Column (2) reports the effects of subsidizing the fixed cost of uptake for the CCDF until the total expenditures on the CCDF and CTC combined are the same as in column (1). The CTC policy in column (2), however, is held at its baseline parameterization. To meet this requirement, the subsidy on uptake costs for the CCDF must be 56 percent. The resulting expansion of the childcare subsidy uptake raises both programs' size relative to the baseline and increases the percentage of families with young children who receive it to 25 percent. Despite requiring the same fraction of output in additional government spending, the CCDF expansion in column (2) requires a smaller rise in the average income tax rate compared to the CTC expansion in column (1). Unsurprisingly, households on average shift away from parent time and towards non-parental childcare time when investing in their children, because the relative price of childcare has decreased for many parents. Overall, investment increases, allowing both consumption and leisure to increase as the stock of skill in the economy expands. Labor supply falls, but not by enough to offset the expansion in the stock of skill, so that overall output rises by 4.99 percent. Welfare in this alternative policy increases by much more than in column (1): behind the veil of ignorance, welfare gains are 6.82 percent of lifetime consumption, while averaged across household types in the first period of adulthood the gains are 1.66 percent.

To emphasize the relevance of our modeling choices, we note at this juncture that expanding CCDF spending without lowering uptake costs would yield more ambiguous a welfare ranking when compared to the 2018 CTC expansion. For example, consider expanding the CCDF by raising the value of $\beta_{1,n}$ while leaving eligibility rules unchanged. This makes the subsidy intensity $\tau_n(y)$ in equation (3) decline less steeply with income, which corresponds to lowering the required copayment for eligible households with higher incomes.[26] Such a policy change yields welfare gains that are larger behind the veil of ignorance, but lower when averaging across new households, compared to the 2018 CTC expansion. Results of this exercise and supporting analysis are discussed in detail in Appendix B.4.

The net effect of the 2018 CTC expansion combines the effect of an increase in the child tax credit with a decrease in the refundability income threshold and an increase in the phase-out income threshold. The effects of an increase in the level of the credit were explored in Section 6.1, while the effects of the other two changes in the 2018 expansion are examined in more detail in Appendix B.5, which studies each policy margin's change in isolation. The main takeaway of that analysis

[26]Eligible households with the lowest income levels already have very high subsidy rates.

is that the increase in the credit level and the increase in the phase-out threshold drive almost all of the total effects of the 2018 CTC expansion. The effect of the phase-out threshold increase is to raise investment and output in the long run, while raising the credit level has the opposite effect. The effects differ because raising the credit level distorts the labor supply decision of households downward as they seek to qualify for more of the credit, while raising the income threshold mitigates this effect at higher income levels and raises labor supply. Taken together, the two changes almost cancel out with regards to changes in output. This is why the level of output slightly increases in column (1) of Table 7 whereas it decreases in column (1) of Table 4 in Section 6.1, although in both cases spending on the CTC is increased.

Table 7: Specific reform: 2018 Child Tax Credit expansion

Comparison across steady states		**△ after expanding:**	
Category	**Variable**	**(1) CTC**	**(2) CCDF**
Panel A: Program size and tax rate	Child Tax Credit (% Y)	0.75	0.04
△ units: percentage point change	Child Care and Development Fund (% Y)	0.07	0.78
	CTC + CCDF (% Y)	0.81	0.81
	Income tax rate (at ave. inc. initial ss)	0.89	0.52
Panel B: Aggregate quantities	Quality time (parents only)	-0.59	-0.50
△ units: percent change	Non-parental childcare (parents only)	3.26	25.50
	Consumption	-0.09	4.00
	Leisure	0.27	0.26
	Labor supply	-0.52	-0.51
	Output	0.03	4.89
Panel C: Welfare changes	CE_1	1.66	6.80
△ units: percent change	$\overline{CE}_2$	1.09	1.73

Notes: Table 7 reports changes in aggregate quantities and welfare predicted by the model after the 2018 expansion of the CTC (column 1) and an equivalent expansion of government spending where the additional funds are spent on the CCDF instead (column 2). Specifically, in column (1) changes are as described in the main text, and in column (2) $\tau_\xi = 0.558$.

Comparisons of steady states can mask transitional dynamics which would reveal that seemingly appealing policies are not viable alternatives to the status quo. To address this, Table 8 reports welfare changes over the transition to the two new steady states discussed in Table 7, averaged across new adult households ("New") and all households alive during the transition ("All"), computed as reported in Appendix C.2. Welfare changes for new households 100 years into the transition match the values for $\overline{CE}_2$ in Panel C of Table 7.

Five years after the 2018 expansion in the CTC, new adult households see welfare gains averaged across household types of 1.05 percent. However, all households alive during the transition suffer welfare losses of 0.55 percent, because this measure includes effects on the elderly whose well-being does not incorporate the transfer and who face higher taxes. Over the course of the transition to the new steady state, moving down each column within (1), these welfare changes are largely unchanged. By contrast, welfare gains take longer to reach their full level after a spending-equivalent expansion of the CCDF, shown in (2). In fact, welfare gains for all households alive during the transition are slightly more negative than the expansion of the CTC in the initial periods of the transition. Unlike the CTC, however, the welfare losses for all ages due to the CCDF expansion are reduced over the transition, to 0.19 percent of lifetime consumption after 20 years, and to 0.13 percent after 100 years. This is because the stock of skill in the economy sizeably increases after one generation, raising output and dampening the increase in the income tax rate. Average across types, welfare gains for new households increase within 20 years, from 1.21 to 1.71 percent of lifetime consumption, reaching most of their long-run value within the same medium-run time frame.

Table 8: Welfare changes over the transition

Program being expanded:	**(1) CTC**		**(2) CCDF**	
Years into transition	**New**	**All**	**New**	**All**
5	1.05	-0.55	1.21	-0.71
20	1.08	-0.49	1.71	-0.19
50	1.09	-0.48	1.73	-0.13
100	1.09	-0.48	1.73	-0.13

Notes: Table 8 reports welfare changes over the transition path to the new steady-states for the experiments in Table 7, both for the across-type averages for new households ("New") and averaging across all household ages and types alive at a given period of the transition ("All"). The results are reported 5, 20, 50, and 100 years into the transition as the economy adjusts to the new steady state. The values for new households 100 years into the transition in Table 8 correspond to those for $\overline{CE}_2$ in Panel C of Table 7.

The results of Tables 7 and 8 indicate that, initially, the 2018 expansion of the CTC generated smaller welfare losses for all households alive during the transition, but smaller welfare gains for new households, compared to the spending-equivalent CCDF expansion. After several decades, however, the CCDF expansion causes the stock of skill in the economy to expand, lessening the tax burden on older consumers and raising the welfare gains for new households.

7 Conclusion

In this paper, we compare two major family policies in the United States: the Child Tax Credit and the Child Care and Development Fund. Our model features realistic representations of these two family policies and embed them into an overlapping generations framework that accounts for endogenous child skill accumulation. Uptake of the childcare subsidy is endogenously low in our framework because of an uptake cost, which we calibrate to match CCDF receipt in the model baseline.

In counterfactual experiments, we establish that expanding the childcare subsidy by reducing uptake costs for CCDF-eligible households would yield welfare gains that are larger than expansions of the CTC at similar government expense, while also sizably raising output in the long run. Consistent with this, we find that gains from a CTC expansion arise mostly because of changes in the distribution of consumption and leisure across households; gains from a CCDF expansion arise from changes in the average level of these two quantities in the economy. Examining a specific policy change (the 2018 CTC expansion), we again find that welfare gains from the CCDF expansion are larger, but they take a generation to fully accrue because they stem from expansions in the stock of skill in future generations. By contrast, most of the welfare gains from the CTC expansion are realized immediately.

In the United States, childcare subsidies exhibit low uptake rates among the eligible. Increasing uptake rates by reducing uptake costs for childcare subsidies represents spending to improve bureaucracy and/or to reduce underfunding which gives rise to waiting lists. Given the larger welfare gains from reducing CCDF uptake costs that we find in our analysis, policy makers should consider measures aimed at making uptake of the childcare subsidy less costly for eligible households.

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Online Appendix

"Family Policies and Child Skill Accumulation"

by Emily G. Moschini and Monica Tran-Xuan

Contents

A Parameterization

A.1 The EITC and TANF

The details of the Child Care and Development Fund (CCDF) and the Child Tax Credit/ Additional Child Tax Credit (CTC) are described in the main text. The baseline model also incorporates the fully refundable Earned Income Tax Credit (EITC) and cash transfers to unemployed parents from Temporary Assistance for Needy Families (TANF), whose parameters of these policies are held at their baseline values in counterfactual experiments, although spending on each program may change as the distribution of income shifts. Next, we describe the specification of the EITC and TANF policies in the model and explain how they are parameterized.

A.1.1 The EITC

The EITC provides a fully refundable tax credit to families who have positive but low taxable income in a way that depends on the presence of children in the household (Crandall-Hollick, 2018, 2021). For a household with pretax income y, the EITC takes the form presented in equation (10) below.

$$EITC_j\left(y,\vec{\pi}_E\right) = \begin{cases} \beta_{in,j}\left(j\right)\times y & \text{if } y \leq \underline{y}_{EI,j} \\ \overline{\kappa}_{max,j} & \text{if } y \in \left(\underline{y}_{EI,j}, \underline{y}_{out,j}\right] \\ \max\left\{\overline{\kappa}_{max,j} - \beta_{out,j}\times\left(y-\underline{y}_{out,j}\right), 0\right\} & \text{if } y \in \left(\underline{y}_{out,j}, \overline{y}_{max,j}\right] \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Here, the transfer level and income thresholds are expressed as fractions of median income, so that $\overline{\kappa}_{max,j} = \Theta_{\kappa,j}\times y_{p50}$, $\underline{y}_{EI,j} = \Theta_{EI,j}\times y_{p50}$, $\underline{y}_{out,j} = \Theta_{out,j}\times y_{p50}$, and $\underline{y}_{max,j} = \Theta_{max,j}\times y_{p50}$. The vector of parameters governing this policy is a set of parameters for each age j, $\vec{\pi}_E\left(j\right) = \left[\Theta_{\kappa,j}, \beta_{in}, \beta_{out}, \Theta_{EI,j}, \Theta_{out,j}, \Theta_{max,j}\right]$. The values of the policy parameters for the EITC depend on whether or not there are children in the taxpayer's household under the age of 19 and are reported in Table A.9. Specifically, the parameterization uses values from Table 1 of Crandall-Hollick et al. (2021), for single filers and married joint filers with 0 or 1 child, averaged using population weights for each family structure from Moschini (2023).

Table A.9: EITC policy parameters for 2015-17

Symbol	Parameter description	Parameter value Children under 18 ($j \leq j_a$)	No children under 18 ($j > j_a$)
$\Theta_{\kappa,j}$	Max. credit amount	$\frac{\$3{,}091}{y_{p50,US}}$	$\frac{\$325}{y_{p50,US}}$
$\beta_{in,j}$	Credit (phase-in) rate	0.40	0.0765
$\beta_{out,j}$	Phase-out rate	0.2106	0.0765
$\Theta_{EI,j}$	Earned income threshold	$\frac{\$8{,}954}{y_{p50,US}}$	$\frac{\$4{,}253}{y_{p50,US}}$
$\Theta_{out,j}$	Phase-out threshold	$\frac{\$14{,}021}{y_{p50,US}}$	$\frac{\$7{,}645}{y_{p50,US}}$
$\Theta_{max,j}$	Zero credit threshold	$\frac{\$31{,}028}{y_{p50,US}}$	$\frac{\$11{,}898}{y_{p50,US}}$

Notes: Table A.9 reports parameter values $\vec{\pi}_{EITC}(j)$ for the EITC in the baseline economy. Source: Table 1, Crandall-Hollick et al. (2021), single filers and married joint filers with 0 or 1 child, averaged using population weights for each family structure from Moschini (2023). Numerator values are converted from 2020 dollars to 2016 dollars using the Consumer Price Index. The 2016 median household income in the US, $y_{p50,US}$, is from Guzman (2017).

A.1.2 TANF

Even by the standards of the United States, TANF in reality is an especially complex funding stream. In this paper, "TANF" refers to the direct cash payment component of the constellation of uses for TANF funding in practice: In reality, not only do TANF funds provide direct cash payments, but they can also fund state tax credits and childcare expense aid. However, the main categories that TANF money is spent on are basic assistance (cash transfers), child care, and "other services" (Figure 1, Office of Family Assistance (2016)). In fact, there are no federal rules governing how TANF funding is spent by states, but all states use some portion of these funds for cash transfers (Office of Family Assistance, 2015).

Although TANF uptake rates of less than one are abstracted from in the main text, it seems that the "eligibile" are those the state defines as needy families, and that uptake among this population is quite low on average. For example, Table A-1 in Falk and Landers (2022) shows that as a percentage of all poor children TANF child recipients make up about 16 percent in 2021, but 35 percent in 2001, indicating that not only is coverege far below 100% but that it has fluctuated over time.

As noted in Falk and Landers (2022), in practice if families have income other than TANF then they receive a reduced benefit. In the main text, the policy referred to as "TANF" plays the role of supporting non-working families with young children with cash transfers, so we are abstracting from any TANF benefit receipt when there are other sources of income present. On paper, TANF transfers are meant to facilitate family adults working for pay. In practice, the "all families work participation standard" does not require that all recipients of TANF work, just some fraction of them at the state level. If a given state does not meet this work participation standard, that state is *at risk* of being penalized with reduced TANF funds. It is very common for states to not meet this work participation requirement (see Figure 4 in Falk and Landers (2022) for the all-family work participation rate).

In the model of the main text, the transfer function $TANF$ gives a lump-sum amount κ_{TANF} to non-working families with young children:

$$TANF_j(y, \vec{\pi}_T) = \begin{cases} \kappa_{TANF} & \text{if } y = 0 \text{ and } j = 1 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Here, the transfer level is set as a fraction of median income, so that $\kappa^{TANF} = \Theta_{TANF} y_{p50}$. The vector of parameters governing this policy is $\vec{\pi}_{TANF} = [\Theta_{TANF}]$, which is estimated using annualized monthly cash transfer amounts from Office of Family Assistance (2015) and national median household income from Guzman (2017)

Table A.10: TANF policy parameters for 2015-17

Symbol	**Parameter description**	**No children under 17** ($j > 3$)
Θ_{TANF}	Cash transfer ratio	$\frac{\$4{,}776}{y_{p50,US}}$

Notes: Table A.10 reports parameter values $\vec{\pi}_{TANF}$ for TANF in the baseline economy. The numerator is the monthly amount of cash assistance received by families in the 2015 FY converted to annual amount. The denominator is the median income for US households in the period 2015-2017. Sources: Office of Family Assistance (2015) and Guzman (2017).

A.2 Externally estimated parameters

This Section provides more details on how the values for the externally estimated parameters of Table 1 are computed.

A.2.1 Consumption taxes

The consumption tax rate is estimated by applying the method of Mendoza, Razin, and Tesar (1994) to data for the United States in the period 2015 to 2017 (the referenced paper only provides estimates until 1988). Specifically, we apply equation (5) from Mendoza, Razin, and Tesar (1994) to data from OECD Stat (2015-2017) and OECD GRSD (2015-2017), for each year of the period of interest, and then average the annual tax rate across the three years.

$$\tau_{c,t} = 100 \times \frac{5110_t + 5121_t}{C_t + G_t - GW_t - 5110_t - 5121_t}$$

The t in the subscript of each variable denotes the year. The 4-digit numbers in this expression are variable names in the OECD data corresponding to "General taxes on goods and services" (5110) and "Excises" (5121).

Table A.11: Consumption tax rate estimation

		Year			
Variable	**Description**	**2015**	**2016**	**2017**	**Source**
Panel A: Total tax revenue (all levels of government)					
5110	General taxes on goods and services	374	385	406	Manual extraction
5121	Excises	157	159	162	Manual extraction
Panel B: Final consumption expenditure					
C	Private	11,892	12,292	12,819	Table 5
G	Government	2,609	2,663	2,727	Table 11
Panel C: Compensation of employees by source					
GW	Paid by producers of gov't services	1,758	1,799	1,846	Table 11
$\tau_{c,t}$	Annual estimated tax rate (pp)	4.35	4.31	4.32	
τ_c	Estimated consumption tax rate (pp)	4.33			

Notes: Table A.11 reports the components for annual estimations of the consumption tax rate. Data in Panel A are pulled from OECD GRSD (2015-2017), while Panels B and C are from OECD Stat (2015-2017). Dollar values are in billions of current USD for that year in this table (pulled in units of millions of current USD). Note that because the dollar values in this table are rounded the annual estimated tax rate for 2017 reported in the Table is 0.01 smaller than the implied value from table numbers reported here.

A.2.2 Income tax progressivity

We estimate the degree of income tax progressivity, not including transfers to the old, Medicaid, or TANF transferred to households with positive income from other sources, and net of the CTC and EITC, using two methods that yield the same result.

Using PSID data Our preferred method for estimating the degree of income tax progressivity suitable for our model specification uses data from the Panel Study of Income Dynamics (PSID) (Survey Research Center, Institute for Social Research, 2023). The tax function on total income is $T(y) = y - (1-\tau_y)y^{1-\tau_y^p}$, where τ_y^p captures the degree of progressivity. This tax specification implies that the relationship between pre-tax income and post-tax income is

$$\log(y - T(y)) = \log(1-\tau_y) + (1-\tau_y^p)\log(y)$$

Using the above equation, we regress the logarithm of post-tax household income on a constant and the logarithm of pre-tax household income. The coefficient on the logarithm of pre-tax household income gives us the estimated tax progressivity τ_y^p.

We use the PSID data from 2015 to 2017 and the TAXSIM program by the NBER to estimate tax progressivity (Feenberg and Coutts, 1993). We extend the program written by Kimberlin et al. (2015) that prepares the inputs for TAXSIM for households in the PSID for the survey years 2015 and 2017. The program identifies multiple tax units within each PSID family unit.[27] Finally, the total PSID family unit tax is the sum of taxes for all tax units present within the family unit. We follow this method instead of a more simplified approach of treating family unit as a whole tax unit because this method allows us to have a more accurate measure of income taxes and child-related tax credits for low-income households. Low-income households generally face a small income tax liability, pay high payroll taxes, and receive significantly large amounts of tax credits that are child related, such as the EITC and the CTC. The method carefully assigns children to appropriate tax units using relationship codes. The method also accounts for self-employment and immigrant legal status, which is important for determining work eligibility in receiving tax credits.

We estimate the tax progressivity excluding the effect of child-related policies. To do so, we compute measures of pre-tax income and post-tax income that exclude the effect of children. We exclude the effect of children by comparing taxes in the benchmark case, in which the TAXSIM inputs for the number of children and child care amount are from the PSID data, to taxes in the no-dependent case, in which the TAXSIM inputs for the number of children and the child care amount are zero for all tax units.

Pre-tax income is the federal adjusted gross income computed by TAXSIM in the no-dependent case. Gross income includes labor earnings, self-employment income, asset income from business, income from interests, dividends, and rents, plus taxable transfer income (Social Security, pension, other retirement income, annuities, and unemployment income).

Post-tax income is pre-tax income minus tax liabilities plus nontaxable transfers. Taxes include federal and state income taxes and FICA tax, adjusted for the amount of CTC and EITC. The adjustment of CTC and EITC for federal and state taxes is the difference between the benchmark and no-dependent amounts. This calculation implies that we only exclude the amounts of CTC and EITC that are child related. Nontaxable transfers include welfare, Social Security, VA pension, worker's compensation and child support. We exclude TANF in the transfers.

Our sample selection focuses on the working population. In particular, we restrict our sample to households with heads aged 25-60 and at least the head or the cohabitator of the household earns more than the equivalent of part-time work at the minimum wage. We exclude households in the top 1% and bottom 1% of pre-tax income. Table A.12 reports the sample size after each step of

[27] The tax units are head (with head dependents), cohabitor (with cohabitor dependents), joint/married (with head and legal wife dependents), and ofum (with all adults besides the head, cohabitor, and all children not assigned to the head or cohabitor).

sample selection. We begin with 165,146 households for the 2015-2017 surveys. The selection results in a sample of 38,160 households.

Table A.12: Estimating income tax progressivity in the PSID: estimation sample

Category	**Sample size**
Raw sample	165,146
After sample selection	
Ages 25-60	44,599
Part-time mininum wage income	39,189
Excluding pre-tax income outliers	38,160

Notes: Table A.12 reports the sample size before and after each step of sample selection; sample counts refer to number of households. The first row shows the original PSID sample for 2015-2017. The final row gives the size of the sample used in the tax progressivity estimation.

Table A.13 reports the results for the tax progressivity estimation including and excluding child-related policies. Both estimates for tax progressivity are statistically significant. The estimate of tax progressivity including all child-related polices is $\tau_y^p = 0.20$, which is qualitatively consistent to the findings of Heathcote et al. (2017). We find that the estimate of tax progressivity excluding child-related policies is $\tau_y^p = 0.11$. Our results are consistent with the estimates using CBO data, report in the next section.

Table A.13: Estimating income tax progressivity in the PSID: estimation results

	When child-related policies are:	
Parameter	**Included**	**Excluded**
Implied $\hat{\tau}_y^p$	0.20 (0.001)	0.11 (0.001)

Notes: Table A.13 reports the estimation results of tax progressivity for the cases including and excluding the effects of child-related policies from after-tax income. Standard errors are in parentheses. Data is from PSID 2015-2017 and calculated with NBER TAXSIM. The dependent variable is logarithm of post-tax income and the independent variable is the logarithm of pre-tax income.

Using CBO data An alternative method for estimating the degree of income tax progressivity uses aggregate data published by the Congressional Budget Office. To estimate the income tax progressivity, τ_y^p, with this method we apply the robustness method of Heathcote, Storesletten, and Violante (2017) to data underlying the exhibits for U.S. Congressional Budget Office (2019), specifically data underlying Figures 1, 3, 4, and 14 of that publication as reported in U.S. Congressional Budget Office (2016). We use the data underlying figures from the CBO report to find the baseline federal tax rate (column 1 in Table A.14), as well as the transfer rate from refundable tax credits (column 2), and TANF, Medicaid, Supplemental Nutrition Assistance Program (SNAP), and Supplemental Security Income (SSI) in columns (3), (4), (5), and (6), respectively. We compute the empirical equivalent of the net tax rate for our model as the federal tax rate (which includes refundable credits as reported in column 1) plus the transfer rate from refundable credits, minus the transfer rate from SNAP and SSI and report this net tax rate in column (7). Average pretax income in column (8) is logged in column (9) and logged after-tax income reported in column (10) is computed by taking the log of the net tax rate in column (7) applied to the pretax income of column (8). The specific figures in U.S. Congressional Budget Office (2019) whose underlying data provides the empirical moments are: for column (1), Figure 4; for column (2), Figure 14; For columns (3)-(6), Figure 3; and, for column (8), Figure 1.

Table A.14: Estimating income tax progressivity using CBO data: estimation data

Percentiles		**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**
Min	Max	Fed. tax	Ref. tax credits	TANF	Medicaid	SNAP	SSI	Net tax	Ave. Y	$\log(Y)$	$\log(Y_{AT})$
99	100	33.3						33.3	1789	3.25	3.08
96	99	26.8						26.8	360	2.56	2.42
91	95	23.6						23.6	218	2.34	2.22
81	90	21.2						21.2	160	2.20	2.10
60	80	17.9						17.9	110	2.04	1.96
40	60	13.9	1.7	0.5	3.7			15.6	72	1.86	1.78
20	40	9.4	3.8	2.0	11.2	1.2	0.9	11.1	45	1.65	1.60
0	20	1.7	12	10.1	46.7	8.4	6.4	-1.1	21	1.32	1.33

Notes: Table A.14 reports the components for the estimation of the income tax progressivity parameter τ_y. Data is from 2016 and dollar values in column (8) are in millions of current USD. After-tax income is defined as $Y_{AT} \equiv (1 - \text{Net tax})$, where the net tax rate is defined as $(7) \equiv (1) + (2) - (5) - (6)$.

To estimate τ_y^p, we derive the estimation equation from the relationship $Y_{AT} = (1 - \tau_y)\, Y^{1-\tau_y^p}$. Taking the log of both sides yields $\log(Y_{AT}) = \log(1 - \tau_y) + \left(1 - \tau_y^p\right)\log(Y)$. This yields the estimation equation:

$$\log(Y_{AT}) \quad = \quad \beta_0 + \beta_1 \log(Y)$$

where $\beta_1 = \left(1 - \tau_y^p\right)$. We therefore regress column (10) from Table A.14 on column (9), using population shares for each row as weights (which are implied by percentiles in that row). The results are presented in Table A.15. The estimated value for τ_y^p is 0.11, whose standard error is computed using the delta method.

Table A.15: Income tax progressivity estimation results

Coefficient	$\log(Y_{AT})$
β_1	0.890 (0.009)
β_0	0.139 (0.018)
Implied $\hat{\tau}_y^p$	0.110 (0.009)

Notes: Table A.15 reports estimation results. Standard errors are in parentheses; coefficients are significant at the 0.1 percent significance level.

A.2.3 The CCDF subsidy rate

The largest federal childcare subsidy program is CCDF. The federal government sets broad criteria for eligibility and states are allowed to apply additional requirements. Federal guidelines require that family income be below 85 percent of state median income to be eligible for subsidy receipt (Lynch, 2022; Stevens et al., 2017); state eligibility requirements, which determine actual eligibility, are usually more stringent.

Conditional on eligibility and uptake, the average subsidy is 81 percent of childcare expenses as calculated using CCDF Administrative Data 2016 FY and reported in Table A.16. The subsidy rate is constructed by taking the ratio of the copayment to the sum of total payments made for the family to the childcare provider and the copayment, and subtracting that from 1. In the estimation sample, observations are included for households earning income from employment only, whose subsidy-recipient child is under 5 years old, and for whom we observe income, state of residency, and family structure. We drop observations where the reported hours in childcare exceed the assumed time endowment of the child (100 hours), where the copayment exceeds family income, and where the reason for receiving the subsidy is something other than allowing the parent to work. We also restrict attention to families who are eligible according to federal income cutoffs.

Table A.16: Estimation Sample Moments

	mean	median	sd
Subsidy rate	0.81	0.84	0.17
Monthly income	1,888	1,810	812
Number of kids	2.04	2.00	1.10
Single mothers (share)	0.88		
Observations	29,549		

Notes: Table A.16 reports summary statistics for the CCDF policy estimation sample. Income is in 2016 USD. Source: CCDF Administrative Data 2016 FY.

To account for the intensity of the subsidy varying with family income in our specification of this policy (due to larger copayments for richer families), we discipline the relationship between subsidy intensity and family income by performing the following estimation. The independent variable in the regression is given by the ratio of household income and state median income; the denominator is from US Census Bureau and Social Explorer (2023). The regression equation is

$$\text{Subsidy rate}_i \quad = \quad \beta_{0,n} + \beta_{1,n}\left(\frac{\text{Monthly income}_i}{\text{State median income}_i}\right) + \epsilon_i$$

where ϵ_i is normally distributed with mean zero. The results of estimating the above model on the sample summarized in Table A.16 are presented in Table A.17.

Table A.17: Estimation Results

Coefficient	**Value**
$\beta_{1,n}$	-0.517 (0.006)
$\beta_{0,n}$	1.000 (0.002)
Observations	29,549

Notes: Table A.17 reports estimation results for the CCDF's baseline policy. Standard errors are in parentheses. Source: CCDF Administrative Data 2016 FY.

A.2.4 Program size

In Table 3 of the main text we report the size of the CTC, CCDF, EITC and TANF programs in US data. Each of these program sizes is reported as a percentage of the average annual US GDP from 2015 to 2017 in 2016 dollars, which is $\$18,724$ billion. The remainder of this section explains in more detail how each program size is computed.

Child Tax Credit Using moments from Figure 2 in Crandall-Hollick (2018), the money spent on the CTC in 2015 is reported as 53.7 billion in 2015 USD. After converting to 2016 dollars using the Consumer Price Index or CPI (1.01), the size of this program is 0.29 percent of GDP.

Child Care Development Fund The size of the CCDF is computed using the reported size of the program from the 2015, 2016, and 2017 fiscal year reported in Chien (2019a,b) and Chien (2020): 10.2 billion, 9.9 billion, and 9.7 billion for the 3 fiscal years, respectively. From each of these reports we convert these dollar amounts into 2016 dollars using the CPI and express them as a percentage of 2016 US GDP in each fiscal year. Lastly, we average across fiscal years and find the result: 0.053 percent of GDP.

Earned Income Tax Credit Using IRS SOI (2017), Table 2.5 , the size of the EITC is 66.4 billion in 2017 USD. After converting to 2016 dollars using the CPI (0.98), the size of this program is 0.35 percent of GDP.

Temporary Assistance for Needy Families Using Row 1 of Figure 1-A in Office of Family Assistance (2015), (federal TANF and State MOE expenditures and transfers: basic assistance, all funds) the size of TANF is 7.9 billion in 2015 dollars. Converting to 2016 dollars with CPI (1.01), we account for the fact that we model the portion of TANF spent on non-working adults only by multiplying this by 1 minus the participation rate for TANF recipients (100-48.4 = 51.6 percent). The result is the size of the TANF program we model in the data: 0.022 percent of GDP.

A.2.5 Parameterization of skill investment technology

Our method for parameterizing the skill investment (static) technology capitalizes on household optimization and draws on moments and point estimates for one- and two-parent families in Moschini (2023). Consider the cost-minimization problem of a parent choosing q and n to finance a level of investment I, where w_q is the price of parent time and p_n the price of non-parental childcare time faced by the family.

$$\begin{array}{ll} \min\limits_{q,n} & w_q q + p_n n \\ s.t. & I(q,n) \geq I \end{array}$$

where the investment technology is given in equation (6) of the main text. From the first-order conditions of this problem we obtain:

$$\ln\left(\frac{n}{q}\right) = \nu \ln\left(\frac{w_q}{p_n}\right) + \nu \ln\left(\frac{1-\gamma}{\gamma}\right) + (1-\nu)\ln\left(\frac{\theta_q}{\theta_n}\right)$$

Most datasets do not allow the econometrician to control for $\frac{\theta_q}{\theta_n}$ at the household level. If one assumes that for the average household using childcare in the population the parent and childcare productivity are the same, then for the average household the term $(1-\nu)\ln\left(\frac{\theta_q}{\theta_n}\right)$ is equal to 0 and the following equation holds:

$$\ln\left(\frac{n}{q}\right) = \nu \ln\left(\frac{w_q}{p_n}\right) + \nu \ln\left(\frac{1-\gamma}{\gamma}\right)$$

One can then express γ in terms of ν and average quantities and prices:

$$\gamma = \frac{1}{1+\exp\left(\frac{1}{\nu}\ln\left(\frac{n}{q}\right) - \ln\left(\frac{w_q}{p_n}\right)\right)}$$

Because in Moschini (2023) the elasticity parameter ν is not statistically different across family structures, we assign ν as the population-weighted average of estimates from Moschini (2023), 0.56. We then use population-weighted averages for prices and quantities reported in Table 2 of Moschini (2023) and evaluate the above expression to find the implied value for γ given ν and observed averages for prices and quantities, which is 0.32. This parameterization approach allows us capture the qualitative takeaways of Moschini (2023)'s in our current framework, which abstracts from heterogeneity in family structure.

A.3 Distribution of government spending by income quartile

The Child Tax Credit We measure the spending of CTC/ACTC using data from Internal Revenue Service of the United States, Statistics of Income (2023a) for the 2016 year (2017 tax returns). Using quartile thresholds for AGI reported in IRS SOI Tax Stats (2016), we aggregate rows of in Internal Revenue Service of the United States, Statistics of Income (2023a) within each quartile to find the total combined spending on the CTC and ACTC for that quartile. When the income bins in Internal Revenue Service of the United States, Statistics of Income (2023a) do not align closely with the AGI quartile thresholds from IRS SOI Tax Stats (2016), we assign a proportional fraction of the spending from the adjacent bin to the quartile. This assumes that spending is distributed evenly within an income bin. We then compute share of total spending for the CTC/ACTC by income quartiles; Table A.18 reports the income cutoffs and fractions of spending per income quartile. Note that in the main text we use "CTC" to refer to CTC and ACTC, combined.

Table A.18: Distribution of CTC/ACTC spending

Income quartile	Income range (2016 USD)	Percentage of CTC spending
1	0–19,989	20.11
2	19,989–40,919	30.79
3	40,919–81,629	30.33
4	81,629 and above	18.78

Notes: Table A.18 reports the income cutoffs and the percentage of CTC.ACTC spending for each income bin. Data sources are Internal Revenue Service of the United States, Statistics of Income (2023a) and IRS SOI Tax Stats (2016).

The Child Care and Development Fund To estimate the distribution of CCDF spending over income quartiles, we first assign households from CCDF Administrative Data 2016 FY to income bins using thresholds from IRS SOI Tax Stats (2016). Next, we restrict to households where reported hours in childcare do not exceed 100 hours a week, who receive a positive subsidy, who report nonmissing income, and whose required copayment does not exceed their reported income. We begin with 98,478 observations, and the selection results in a sample of 96,204 valid observations. Table A.19 reports the income cutoffs and fractions of CCDF spending per income bin, where dollars are aggregated using family-child weights.

Table A.19: Distribution of CCDF spending: results

Income quartile	Income range (2016 USD)	Percentage of CCDF spending
1	0–19,989	54.59
2	19,989–40,919	38.46
3	40,919–81,629	4.11
4	81,629 and above	2.84

Notes: Table A.19 reports the income cutoffs and the percentage of CCDF spending for each income bin.

B Model and Results

B.1 Fixed cost of childcare uptake

One modeling decision that affects our results is how to motivate the low uptake rates for the Child Care and Development Fund (childcare subsidy) in the baseline equilibrium. In the main text, we use a fixed cost of uptake that appears in the budget constraint of the consumer in units of consumption. An alternative would be to use a utility fixed cost that appears in the objective function and penalizes utility when the childcare subsidy is used. This modeling choice is sometimes referred to as a "stigma cost" in the literature. To see how it contrasts with our preferred specification, consider a modified model environment where the consumer problem is expanded to include a utility fixed cost of childcare subsidy uptake. Specifically, taking investment technologies as given, the parenting problem for consumers age $j = j_a + 1$ given by:

$$\begin{aligned} V_j(\theta_a, \theta_k) &= \max_{\theta_k^a, c, \ell, R} u\left(\frac{c}{\Psi_j}, 1 - \ell - q\right) + bV_k^a(\theta_k^a) + \beta V_{j+1}(\theta_a) - \xi_u R \\ (1+\tau_k)c + pn + R\xi &\leq y_{AT,j}(y, \vec{\pi}_C, \vec{\pi}_E, \vec{\pi}_T) + CCDF_j(R, n, y, \vec{\pi}_N) \end{aligned} \tag{12}$$

where $q + n \leq 1$, $q + \ell \leq 1$, $c, \ell, q, n \geq 0$, and $R \in \{0, 1\}$. To examine the implications of choosing one specification over the other, here we add a utility cost ξ_u that households must pay if they take out the subsidy. We recalibrate the model with $\xi = 0$, choosing ξ_u to match the CCDF uptake rate; the results are shown in Table B.1.

Table B.1: Internally calibrated parameters with utility cost of uptake

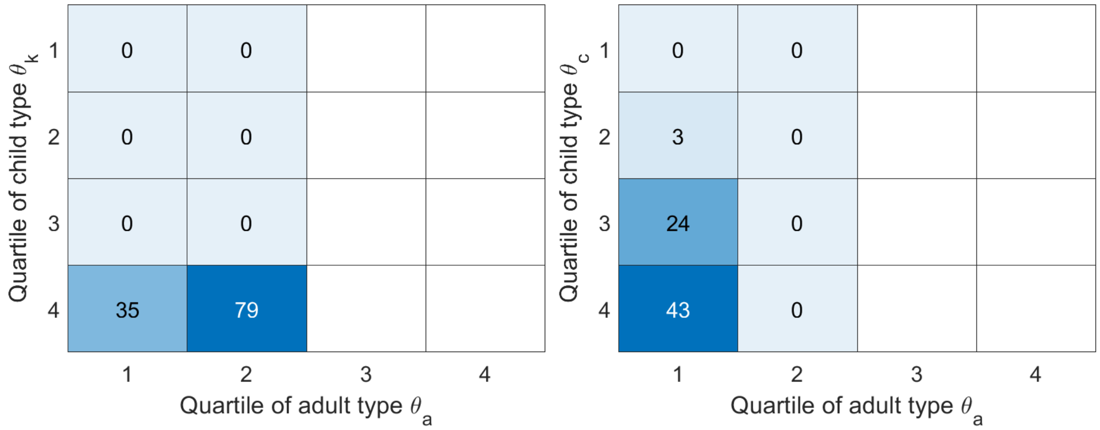
	Parameter		Moments		
Symbol	**Description**	**Value**	**Description**	**Data**	**Model**
Panel A: Parameters calibrated jointly					
ψ	Marginal utility of leisure	0.639	Average hours worked	0.330	0.330
λ_I	Investment productivity	7.699	Average non-parental childcare hours	0.307	0.307
b	Altruism parameter	$0.330 \times \beta$	Corr. child skill outcomes and family income	0.320	0.321
ξ_u	Fixed cost of CCDF uptake	0.225	Share families receiving CCDF	0.035	0.035
$\overline{\Delta\theta_k}$	Variance of $\mu_k(\theta_k)$	9.078	p50-p10 log income	1.403	1.406
Panel B: Parameters set proportional to other equilibrium objects					
θ_n	Child care productivity	3.876	Average productivity of working parents		
ϕ	Pricing ratio for n	0.161	Post-policy input price ratio		

Notes: Table B.1 presents internally calibrated parameters using the framework with a utility cost for CCDF uptake.

As mentioned in the main text, one consequence of how we model the fixed cost of uptake is the composition of families using the childcare subsidy in the baseline economy, which has implications for the effects of policy changes. Figure B.1 compares the household types who use the CCDF in the baseline (left panel) versus an alternative model using a utility cost and calibrated to match the same moments (right panel). The two frameworks differ in terms of the adult skill endowment θ_a that characterizes households using the subsidy in the baseline equilibrium: in the preferred specification they are mostly high-skill, while in the alternative specification they are low-skill. We chose the consumption-units fixed cost of uptake in light of our empirical evidence, presented in Figure 1 of the main text. This also qualitatively aligns with the findings of Johnson, Martin, and Brooks-Gunn (2011), who show that eligible families who take up the subsidy tend to have higher levels of education. Through our model's lens, the findings of the latter study translate to relatively higher levels of parent skill among eligible subsidy recipients.

The differing type-composition for CCDF subsidy recipients means that, when low uptake is motivated by a utility fixed cost, the CCDF goes to the lowest income quartile in the baseline equilibrium. This can be seen in Figure B.2 which compares the baseline income composition of CCDF recipients with the same statistics for the model in the main text. Note that, comparing across the two specifications, the model of the main text is a closer match with the distribution of CCDF spending over income quartiles, replicated here from Figure 1b in the main text.

Comparing welfare benefits of eliminating subsidy uptake costs across the two model frameworks in Table B.2, we see that welfare gains from eliminating CCDF uptake frictions are slightly larger when low uptake rates are motivated with a fixed cost in utils, because there is no cost to society from eliminating the friction in column (2) while there is in column (1). Although the mechanisms



(a) Consumption fixed cost

(b) Utility fixed cost

Figure B.1: Uptake rates and eligibility across model specifications

Notes: Figure B.1 shows uptake rates and eligibility by adult skill quartile (x-axis) and child skill quartile (y-axis) in baseline equilibrium using a fixed cost in foregone consumption (Figure B.1a) and a fixed cost in utils (Figure B.1b).

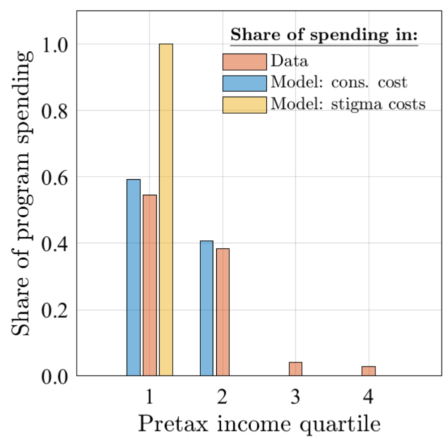


Figure B.2: Program spending by income quartile across model specifications versus data

Notes: Figure B.2 reports the share of program spending by income quartile in the baseline equilibrium using a consumption fixed cost and a utility fixed cost in utils. It also displays spending by income quartile in the data for comparison, replicated from Figure 1b in the main text.

at work are evidently similar as shown by Panels B and C of Table B.2, the two specifications differ in their implications is in the welfare gains for a given expansion in government spending on child-related policies. This insight combines information from Panels A and C, which show that for very similar welfare gains the utility cost specification requires a smaller increase in government spending.

Figure B.3 shows welfare changes across the two model frameworks as government spending on

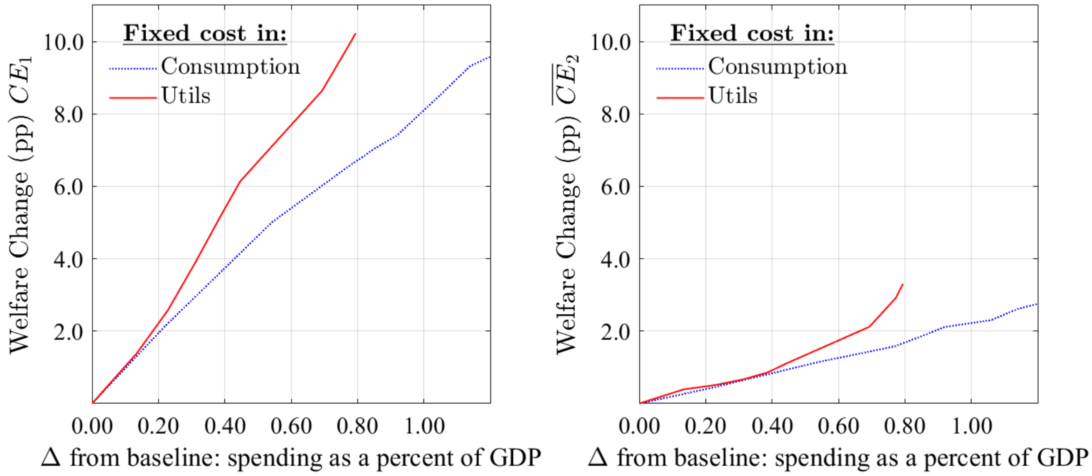
Table B.2: Effects of eliminating the CCDF fixed cost of uptake

Comparison across steady-states		Consumption cost		Utility cost	
Category	**Variable**	**Baseline**	**(1) Δ**	**Baseline**	**(2) Δ**
Panel A: Program size and tax rate	Child Tax Credit (% Y)	0.30	0.03	0.31	0.03
Δ units: percentage point change	Child Care and Development Fund (% Y)	0.08	1.22	0.13	0.76
	CTC + CCDF (% Y)	0.38	1.25	0.44	0.79
	Income tax rate (at ave. inc. initial ss)	9.06	0.84	9.50	0.29
Panel B: Aggregate quantities	Quality time (parents only)	0.09	-1.58	0.09	-1.17
Δ units: percent change	Non-parental childcare (parents only)	0.31	37.38	0.31	34.47
	Consumption	1.41	4.99	1.35	5.86
	Leisure	0.66	0.39	0.66	0.31
	Labor supply	0.33	-0.72	0.33	-0.59
	Output	1.67	6.21	1.60	6.59
Panel C: Welfare changes	CE_1		9.78		10.23
Δ units: percent change	$\overline{CE}_2$		2.87		3.30

Notes: Table B.2 reports changes in aggregate quantities and welfare for eliminating the fixed cost of uptake for the CCDF when it is in terms of foregone consumption (column 1) and utils (column 2), relative to the baseline equilibrium. Levels in the baseline equilibrium are also reported for the two models since they represent different calibrations.

the CTC and CCDF combined is gradually increased. Figure B.3a reports welfare changes across specifications for CE_1, while Figure B.3b reports welfare changes using $\overline{CE}_2$. At a given level of spending, welfare changes are always higher for the specification where the fixed cost of uptake is in utils, compared to our preferred specification where it is in units of consumption. In the framework with a fixed cost in utils, reducing the cost of uptake reaches the neediest families immediately because subsidy recipients were relatively poorer in the baseline. Welfare gains are accordingly higher as the subsidy is reduced in this alternative specification. One thing to keep in mind when comparing these welfare gains is that, with a fixed cost in units of consumption, the government assumes a portion of this fixed cost when reducing it. Reducing or eliminating a utility cost is “free” because the model is silent on how much it would cost to make this change.

Another reason the welfare gains are higher at a given level of spending in a model with utility costs is the composition of households that switch to subsidy uptake as the fixed cost is gradually reduced. This is illustrated in Figure B.4, which compares the CCDF uptake response to small changes in the uptake cost across the two model specifications. The first row of the figure is the model of the main text where the fixed cost is in units of foregone consumption. The second row is the alternative specification where the fixed cost is a utility cost in utils. As the fixed cost of uptake gradually decreases (moving from left to right within each row) the uptake rate gradually increases, but the composition of new households using subsidized childcare is characterized by lower θ_a and higher θ_k for the utility cost model, compared to the consumption cost framework of the main text.



(a) Change in CE_1 across specifications

(b) Change in $\overline{CE}_2$ across specifications

Figure B.3: Welfare changes across specifications as CCDF fixed cost is gradually reduced.

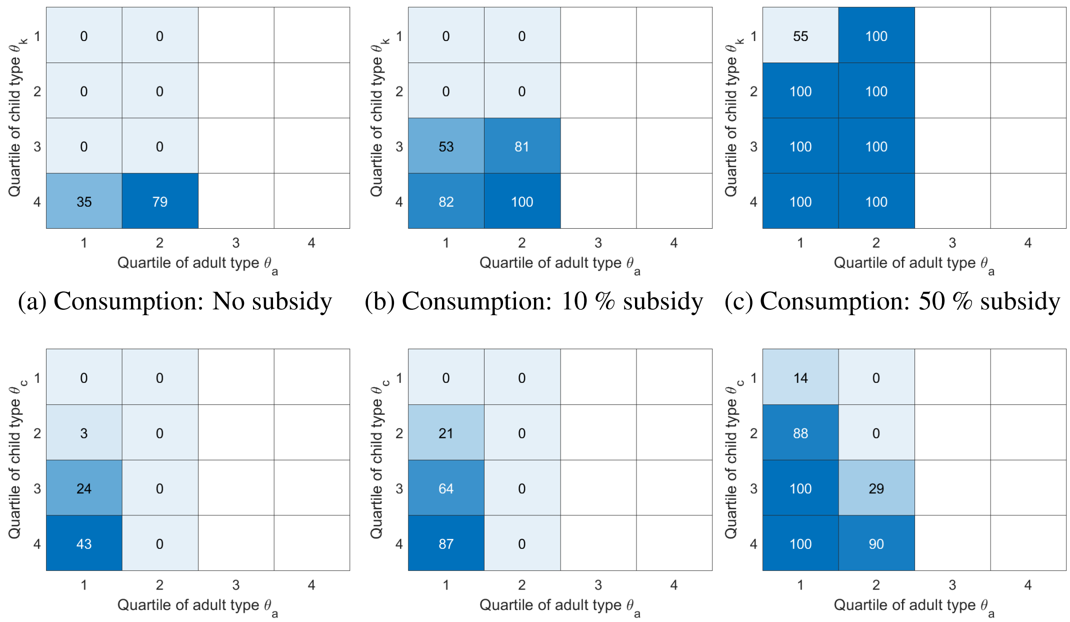
Notes: Welfare changes due to reducing the CCDF uptake cost when it is in terms of foregone consumption or in utils, using CE_1 (Figure B.3a) and $\overline{CE}_2$ (Figure B.3b).

To summarize, compared to an alternative framework with utility costs for CCDF uptake, the model specification of the main text offers a better fit to untargeted moments and provides a lower bound on welfare gains from expansions in government spending on the CCDF.

B.2 The effects of reallocating resources across programs

What is the opportunity cost of expenditures on either the CTC or the CCDF in the baseline economy, in terms of spending on the other program instead? Table B.3 presents results from two exercises that address this question, where the total percent of output spent on the two programs combined is held fixed at its baseline value, but only one program is financed at a time. To meet this restriction, the CTC per-child credits must be expanded by a factor of 1.25 times their baseline values, and the subsidy on uptake costs for the CCDF must be 15 percent.

As shown in column (1), when the childcare subsidy is eliminated the CTC is expanded by a proportional amount, the average income tax rate has to *increase* to finance the change. This happens because eliminating the childcare subsidy causes a slight a drop in childcare use which parent quality time does not compensate for, leading to a drop in skill and a contraction of the income tax base in the new steady state. The slightly higher tax rate causes consumers to shift from consumption to leisure as the latter's relative price falls. The drop in skill is compounded by a fall in labor supply, so that output also falls by 1.11 percent. Overall, behind the veil of ignorance this causes welfare losses of 0.42 percent, because of drops in child outcomes and levels



(a) Consumption: No subsidy (b) Consumption: 10 % subsidy (c) Consumption: 50 % subsidy

(d) Utils: Baseline cost (e) Utils: 10 % reduction (f) Utils: 50 % reduction

Figure B.4: CCDF uptake rates as fixed-cost of CCDF uptake gradually decreases

Notes: The first row of Figure B.4 replicates Figure 6 in the main text; it shows the uptake rate, conditional on eligibility, for the CCDF with no subsidy on the fixed cost of uptake (Figure B.4a), a 10 percent subsidy (Figure B.4b), and a 50 percent subsidy (Figure B.4c). The second row shows CCDF uptake rates from the model with utility costs, in the baseline equilibrium (Figure B.4d), a 10 percent reduction in the utility cost (Figure B.4e), and a 50 percent reduction in the utility cost (Figure B.4f). Uptake rates are reported for the discretized joint distribution over household types at $j = 1$, (θ_a, θ_k), with θ_a on the x-axis and θ_k on the y-axis. Shaded cells denote positive within-cell eligibility rates using realized pretax income, with the uptake rate (percentage of eligible taking up the CCDF) written inside of the cell.

of consumption. However, the average across type-specific welfare changes indicates small welfare gains of 0.21 percent of lifetime consumption.

Column (2), meanwhile, shows that the corresponding reallocation of resources to the CCDF allows the income tax rate to fall by 0.23 percentage points. Although not reported in the table, this increase in funding for the childcare subsidy program corresponds to expanding the share of all families with young children who receive the subsidy from 3.6 percent to 15 percent. The resulting rise in non-parental childcare time indicates a higher level of investment and a rise in the stock of skill (parent quality time falls slightly as investment input composition adjusts, but overall investment increases). Consumption rises while leisure falls, and labor supply rises. Overall, the rise in skill combined with an increase in labor supply drives an increase in output of 3.53 percent. Welfare effects reflect the better expected adult outcomes in the new economy: behind the veil of ignorance consumers gain 2.13 percent of lifetime consumption. Nevertheless, averaging across household types shows a small welfare loss of -0.30 percent, because families that benefited from the CTC in the baseline equilibrium no longer do so.

To summarize, redistributing funds from the CTC towards a policy which more specifically targeted the skill investment margin is beneficial behind the veil of ignorance, while a redistribution of resources away from the childcare subsidy program causes small welfare losses behind the veil of ignorance, despite the small mass of childcare subsidy recipients in the baseline equilibrium.

Table B.3: Government spending-neutral reallocation of resources

Comparison across steady-states		**△ after reallocating to:**	
Category	**Variable**	**(1) CTC**	**(2) CCDF**
Panel A: Program size and tax rate	Child Tax Credit (% Y)	0.08	-0.30
△ units: percentage point change	Child Care and Development Fund (% Y)	-0.08	0.30
	CTC + CCDF (% Y)	0	0
	Income tax rate (at ave. inc. initial ss)	0.13	-0.23
Panel B: Aggregate quantities	Quality time (parents only)	-0.01	-0.70
△ units: percent change	Non-parental childcare (parents only)	-3.21	12.11
	Consumption	-0.99	3.12
	Leisure	0.08	-0.41
	Labor supply	-0.16	0.85
	Output	-1.11	3.53
Panel C: Welfare changes	CE_1	-0.42	2.13
△ units: percent change	$\overline{CE}_2$	0.21	-0.30

Notes: Table B.3 reports changes in aggregate quantities and welfare when government spending on both the CTC and CCDF in the baseline is reallocated entirely to the CTC (column 1) or the CCDF (column 2). Specifically, in column (1) the per-child credit is $\overline{\kappa}_{nref} = \overline{\kappa}_{ref} = 1.249 \times \overline{\kappa}_{baseline}$, and in column (2) $\tau_{\xi} = 0.147$.

B.3 Examining the mechanism: CCDF policy margins

In the main text, expansions of the CCDF are implemented by subsidizing the uptake cost for subsidized childcare. Leaving the fixed cost at its baseline value and increasing CCDF subsidy spending in other ways is explored further in Table B.4. If one makes all levels of income eligible for the subsidy while holding fixed the subsidy's relationship with income (column 1), only 4.19 percent of families with young children take up the subsidy. This small expansion in uptake is due to the decreasing subsidy intensity as a function of income which remains unchanged from the baseline in this experiment. The fixed cost is too large to be offset by the value of the subsidy for higher-income families that are newly eligible in this counterfactual. Alternatively, if one eliminates the subsidy's relationship with income by setting it a flat rate equal to the average value in the data, while leaving the income threshold unchanged (column 2), spending increases by more because uptake of the CCDF rises to 8.98 percent. Combining these two changes (column 3) raises uptake to 64.8 percent, because the increased generosity of the subsidy at higher, newly eligible income levels incentivizes uptake. However, low-income families who were always eligible still face a substantial barrier to childcare subsidy uptake, and now pay higher taxes (as do older consumers), making welfare gains averaged across household types ($\overline{CE}_2$) negative.

Table B.4: Alternative CCDF policy margins

Comparison across steady-states			**△ with new CCDF policy:**		
Category	**Variable**	**Baseline**	**(1) $\Theta_{y,n}$**	**(2) $\beta_{0,n}$**	**(3) Both**
Panel A: Program size, tax rate	CTC (% Y)	0.3	0	-0.01	-0.02
△ units: percentage point change	CCDF (% Y)	0.08	0.01	0.18	2.18
	CTC + CCDF (% Y)	0.38	0.01	0.17	2.16
	Inc. tax rate (at ave. inc. initial ss)	9.06	0	0.03	1.54
Panel B: Agg. quantities	Quality time (parents only)	0.09	0.01	-0.52	-4.72
△ units: percent change	Non-parental cc (parents only)	0.31	0.41	7.47	86.63
	Consumption	1.41	0.09	1.01	4.67
	Leisure	0.66	0	0	-0.01
	Labor supply	0.33	0	0.02	0.17
	Output	1.67	0.11	1.24	7.49
Panel C: Welfare changes	CE_1		0.06	0.81	4.74
△ units: percent change	$\overline{CE}_2$		-0.01	0.01	-0.44

Notes: Table B.4 reports changes in aggregate quantities and welfare predicted by the model after changing three changes to the baseline CCDF policy. Column (1) shows the effect of eliminating the income threshold so that all working families are eligible ("$\Theta_{y,n}$"); column (2) shows the effects of eliminating the means-testing for subsidy intensity and setting the flat subsidy equal to its average value, $\beta_{0,n} = 0.81$, ("β_0");; column (3) shows the effects of combining the changes of both (1) and (2)' ("Both').

B.4 Alternative expansions of CCDF spending-equivalent to 2018 CTC

To see the relative potential of more traditional policy margins for expansions of the CCDF, in Table B.5 we compare the effects of the 2018 CTC expansion (column 1) with those of a subsidy to the fixed cost of CCDF uptake that is spending-equivalent (column 2) or an spending-equivalent expansion of the CCDF that uses a more traditional policy margin (column 3). Specifically, in column (3) we make the subsidy intensity upon receipt independent of income and recalibrate the level $\beta_{0,n}$ until spending reaches the desired level. No other changes are made to the CCDF policy in the economy of column (3).

We select the margin used in column (3) of Table B.5 using the following reasoning. As discussed in Appendix B.3, eliminating the income threshold for eligibility or setting the subsidy level to a flat rate equal to its average value have comparatively little effect. As is evident from the results presented in Appendix B.3, doing both of these changes at the same time increases spending by too much for the exercise at hand. In experiments not reported here, we establish that if the income eligibility threshold is eliminated, re-calibrating the flat subsidy rate to match the increase in spending due to the 2018 CTC expansion yields $CE_1 = 1.09$ and $\overline{CE}_2 = -0.54$. Using this result in column (3) would artificially deprecate traditional margins for CCDF expansion. This leads us to focus on reducing the magnitude of $\beta_{0,n}$ setting $\beta_{1,n}$ to 0, or reducing the magnitude of $\beta_{1,n}$

holding $\beta_{0,n}$ at its baseline value (in both cases holding the income eligibility threshold fixed). We implemented both methods and found them to yield almost equivalent welfare changes. In Table B.5 we present the effects of recalibrating $\beta_{1,n}$;the new value is -0.06.

As shown in Panel C, a traditional method for expanding the CCDF yields welfare gains which compare more ambiguously with the 2018 CTC expansion, in contrast to a CCDF expansion financed by a fixed-cost subsidy. Specifically, the welfare gains measured behind the veil of ignorance (CE_1) still exceed those of the CTC expansion, but averaging type-specific gains across households ($\overline{CE}_2$) yields comparatively smaller gains (Panel C, column 3).

Table B.5: Alternative CCDF Expansions: Spending-Equivalent to 2018 CTC

Comparison across steady-states			**△ after CTC exp.:**	**△ after CCDF exp.:**	
Category	**Variable**	**Baseline**	**(1) 2018**	**(2) τ_ξ**	**(3) Trad.**
Panel A: Program size, tax rate	CTC **(% Y)**	0.3	0.75	0.04	0.04
△ units: percentage point change	CCDF **(% Y)**	0.08	0.07	0.78	0.78
	CTC + CCDF **(% Y)**	0.38	0.81	0.81	0.81
	Inc. tax rate **(at ave. inc. initial ss)**	9.06	0.89	0.52	0.55
Panel B: Agg. quantities	Quality time **(parents only)**	0.09	-0.59	-0.50	-1.48
△ units: percent change	Non-parental cc **(parents only)**	0.31	3.26	25.50	34.30
	Consumption	1.41	-0.09	4.00	3.38
	Leisure	0.66	0.27	0.26	0.19
	Labor supply	0.33	-0.52	-0.51	-0.33
	Output	1.67	0.03	4.89	4.34
Panel C: Welfare changes	CE_1		1.66	6.80	4.75
△ units: percent change	$\overline{CE}_2$		1.09	1.73	0.85

Notes: Table B.5 reports changes in aggregate quantities and welfare predicted by the model after (1) the 2018 CTC expansion, (2) - (3) two alternative policy changes to the CCDF that have the same spending as the 2018 CTC expansion.

B.5 Breakdown of policy margins in the 2018 CTC expansion

The 2018 expansion of the Child Tax Credit involved modifications of more than the level of the credit: the eligibility threshold and phase-out threshold in income also changed. In the main text we compare a full subsidy of the fixed cost of childcare subsidy uptake with a spending-equivalent increase in the child tax credit level. We changed only the tax credit level for that exercise because the range of its effect on government spending is wider than an expansion in the range of income levels eligible for the CTC—at some point spending no longer increases as the income range that defines eligibility widens.

For the 2018 CTC expansion, one may wonder how much of the aggregate impact of the policy is driven by each component of the change. Table B.6 aims to shed light on this question, by isolating the effect of the rise in the credit level ($\overline{\kappa}$ in the table), the decrease in the income refundability threshold ($\underline{y}$ in the table), and the phase-out income threshold ($\overline{y}$ in the table). The stylized notation is a slight shift from the main text, but it allows all the counterfactuals to be presented in the same table. In column (1), the table shows steady-state changes for the full policy change. Columns (2) through (4) reflect changes when one component at a time of the reform is held fixed at its baseline value. The effect this has on changes in quantities is proportional to the role of that component in generating the full effect of the policy change. Another way to see this is to shift one component at a time to its new value, as shown in column (5) through (7).

The results in Table B.6 indicate that the increase in the phase-out threshold and the rise in the credit level together drive almost all of the impact of the reform. Although the increase in the phase-out threshold is not how we expand the CTC in Section 6.1 of the main text, it does play a major role in driving welfare gains from the 2018 CTC expansion. Incorporating this threshold in the model's representation of the Child Tax Credit is therefore useful because it allows us to incorporate the effect of a policy change along this margin.

Another takeaway of the results from these exercises is that raising the income threshold without changing the the other two instruments yields an increase in output via an expansion in the stock of skill (column 7). This occurs because removing the upper bound on income for eligibility removes a distortion on labor supply; households can now raise labor hours and still be eligible for the CTC. This contrasts with the dampening effect on output due to a rise in the credit level (column 5), which raises tax rates while also making the labor supply margin worse. The opposing forces of these two margins on output almost cancel out in the full CTC expansion, as shown in column (1). Welfare gains from raising the credit level and expanding the income phase-out threshold correspondingly stem from different sources: the former raises leisure while the latter raises consumption. They also have opposite effects on labor supply (raising the credit level lowers labor supply). In the model, the two changes together raise leisure and lower consumption while lowering labor supply, so that in this respect the rise in the credit level dominates.

C Measuring Welfare

All of the analytic expressions for consumption equivalent welfare changes impose that the utility function is $u(c,\ell) = (1-\psi)\ln(c) + \psi\ln(\ell)$. Also recall that

$$z_j \equiv \begin{cases} (\theta_a, \theta_k) & \text{if } j = j_a + 1 \\ \theta_a & \text{otherwise} \end{cases}$$

where θ_a is the adult skill endowment and θ_k is the child's initial skill endowment.

Table B.6: Breakdown of policy changes in the 2018 Child Tax Credit expansion

Comparison across steady-states			**△ after holding fixed:**			**△ after changing only:**		
Category	**Variable**	**(1) Full**	**(2) $\overline{\kappa}$**	**(3) $\underline{y}$**	**(4) $\overline{y}$**	**(5) $\overline{\kappa}$**	**(6) $\underline{y}$**	**(7) $\overline{y}$**
Panel A: Program size, tax rate	CTC (% Y)	0.75	0.25	0.75	0.30	0.30	0	0.25
△ units: percentage point change	CCDF (% Y)	0.07	0.06	0.07	0	0	0	0.06
	CTC + CCDF (% Y)	0.81	0.31	0.81	0.30	0.30	0	0.31
	Inc. tax rate (at ave. inc. initial ss)	0.89	0.28	0.90	0.38	0.38	0	0.28
Panel B: Agg. quantities	Quality time (parents only)	-0.59	-0.12	-0.67	-0.46	-0.46	0	-0.12
△ units: percent change	Non-parental cc (parents only)	3.26	2.66	3.23	-0.56	-0.56	0	2.66
	Consumption	-0.09	0.80	-0.12	-0.76	-0.76	0	0.81
	Leisure	0.27	-0.02	0.25	0.26	0.25	0.02	-0.04
	Labor supply	-0.52	0.05	-0.47	-0.51	-0.48	-0.03	0.08
	Output	0.03	0.89	0	-0.76	-0.75	0	0.89
Panel C: Welfare changes	CE_1	1.66	1.09	1.60	0.27	0.25	0.01	1.06
△ units: percent change	$\overline{CE}_2$	1.09	0.55	1.06	0.46	0.45	0.01	0.54

Notes: Table B.6 reports changes in aggregate quantities and welfare predicted by the model after the 2018 expansion of the CTC (column 1) and policies where the credit level $\overline{\kappa}$, eligibility income threshold $\underline{y}$, and phase-out income threshold $\overline{y}$ are held fixed at their baseline value as the others are changed to their 2018 values (columns 2, 3, and 4), or each is changed to its 2018 value holding the other margins fixed (columns 5, 6, and 7 for the three margins, respectively).

Notation Define additional notation used in this appendix as follows.

- Let $V_j^{eq}(z_j)$ denote the value function for type z_j at age j in equilibrium eq.
- Let $x_j^{eq}(z_j)$ be the policy function of type z_j for choice variable x at age j in equilibrium eq.
- Let $V_k^{a,eq}(\theta_k^{a,eq}(\theta_a, \theta_k))$ be the expected lifetime utility in equilibrium eq evaluated at the policy choice of type $z_{j_a+1} = (\theta_a, \theta_k)$ for child outcome in equilibrium eq.
- Let $V_j^{eq,t}(z_j)$ denote the value function for type z_j in period j of the lifecycle, at time t of the transition from the baseline steady-state to equilibrium eq.
- Let X_{eq} as the aggregate value for quantity X in equilibrium eq.
- Let $\hat{x}_j^{eq}(z_j) \equiv \frac{X_{eq}}{X_{eq=0}} x_j^{eq=0}(z_j)$ for choice variables $x \in \{c, \ell\}$

C.1 Welfare changes across stationary steady-state equilibria

C.1.1 CE_1: Welfare change behind the veil of ignorance

Define

$$W_{eq} \equiv \int_{\theta_a,\theta_k} \left[V_{j_a+1}^{eq}(\theta_a, \theta_k)\right] \mu_a^{eq}(\theta_a)\, \mu_k^{eq}(\theta_k)\, d\theta_a d\theta_k$$

Then the consumption-equivalent welfare change behind the veil of ignorance of moving to equilibrium eq from equilibrium 0 is:

$$CE_1 \equiv \exp\left[\frac{W_{eq} - W_0}{(1-\lambda)\sum_{j=j_a+1}^{J}\beta^{j-j_a-1}}\right]$$

C.1.2 $CE_2(j, z_j)$: Welfare change given type z_j and age j

Define the type and age-specific consumption-equivalent welfare change of moving to equilibrium eq from equilibrium 0 as:

$$CE_2(j, z_j) \equiv \exp\left[\frac{V_j^{eq}(z_j) - V_j^{0}(z_j)}{(1-\lambda)\sum_{age=j}^{J}\beta^{age-j}}\right]$$

Note that the denominator incorporates the fact that the lifespan is shorter at older ages. Averaging over type-specific changes for new households yields:

$$\overline{CE}_2 \equiv \int_{z_{j_a+1}} CE_2(j_a+1, z_{j_a+1})\,\Omega(j_a+1, z_{j_a+1})\,dz_{j_a+1}$$

C.2 Welfare changes over the transition

The consumption equivalent welfare change for type z_j at period j of the lifecycle in period t of a transition from equilibrium 0 to equilibrium eq is:

$$CE_2^{TP}(j, z_j, t) \equiv \exp\left[\frac{V_j^{eq,t}(z_j) - V_j^{0,t}(z_j)}{(1-\lambda)\sum_{age=j}^{J}\beta^{age-j}}\right]$$

In the main text, we hold t fixed and average this metric across types and ages when reporting the welfare gains of all households alive at period t of the transition. We fix t and set $j = j_a + 1$, then take the average across types, to find the welfare gains for new households at period t of the transition.

C.3 Decomposing welfare changes

Following Guvenen et al. (2023), construct two intermediate counterfactual value functions as follows:

$$
\begin{aligned}
\hat{V}_j^{eq}(z_j) &= \sum_{age=j}^{J} \beta^{age-j}\left[(1-\lambda)\log\left(\frac{\hat{c}_j^{eq}(z_j)}{\Psi_j}\right)+\lambda\log\left(\hat{\ell}_j^{eq}(z_j)\right)\right] \\
&\quad +\mathbb{I}_{j=j_a+1} b V_k^{a,eq=0}\left(\theta_k^{a,0}(z_j)\right) \\
&= \sum_{age=j}^{J} \beta^{age-j}\left[(1-\lambda)\log\left(\frac{C_{eq}}{C_0}\right)+\lambda\log\left(\frac{L_{eq}}{L_0}\right)\right]+V_{j_a+1}^{eq=0}(z_j) \\
\tilde{V}_j^{eq}(z_j) &= \sum_{age=j}^{J} \beta^{age-j}\left[(1-\lambda)\log\left(\frac{c_j^{eq}(z_j)}{\Psi_j}\right)+\lambda\log\left(\ell_j^{eq}(z_j)\right)\right] \\
&\quad +\mathbb{I}_{j=j_a+1} b V_k^{a,eq=0}\left(\theta_k^{a,0}(z_j)\right) \\
&= V_j^{eq}(z_j)+\mathbb{I}_{j=j_a+1} b\left[V_k^{a,eq=0}\left(\theta_k^{a,0}(z_j)\right)-V_k^{a,eq}\left(\theta_k^{a,eq}(z_j)\right)\right]
\end{aligned}
$$

Also construct the expected lifetime utility of each of these intermediate counterfactuals:

$$
\begin{aligned}
\hat{W}_{eq} &\equiv \int_{\theta_a,\theta_k}\left[\hat{V}_{j_a+1}^{eq}(\theta_a,\theta_k)\right]\mu_a^{eq=0}(\theta_a)\,\mu_k^{eq=0}(\theta_k)\,d\theta_a d\theta_k \\
\tilde{W}_{eq} &\equiv \int_{\theta_a,\theta_k}\left[\tilde{V}_{j_a+1}^{eq}(\theta_a,\theta_k)\right]\mu_a^{eq}(\theta_a)\,\mu_k^{eq}(\theta_k)\,d\theta_a d\theta_k
\end{aligned}
$$

C.3.1 CE_1: Decomposing welfare change behind the veil of ignorance

Consider the welfare effects $CE_1$of moving from the baseline equilibrium $eq=0$ to a new steady-state, eq. These welfare changes can be decomposed into three components:

- Welfare gains from level changes: $CE_1^L \equiv \exp\left[\frac{\hat{W}_{eq}-W_0}{(1-\lambda)\sum_{j=j_a+1}^{J}\beta^{j-j_a-1}}\right]$
- Welfare gains from distributional changes: $CE_1^D \equiv \exp\left[\frac{\tilde{W}_{eq}-\hat{W}_{eq}}{(1-\lambda)\sum_{j=j_a+1}^{J}\beta^{j-j_a-1}}\right]$
- Welfare gains from the altruism term: $CE_1^A \equiv \exp\left[\frac{W_{eq}-\tilde{W}_{eq}}{(1-\lambda)\sum_{j=j_a+1}^{J}\beta^{j-j_a-1}}\right]$

Then we can write $CE_1^L \times CE_1^D \times CE_1^A = \exp\left[\frac{W_{eq}-W_0}{(1-\lambda)\sum_{j=j_a+1}^{J}\beta^{j-j_a-1}}\right] \equiv CE_1$.

C.3.2 $\overline{CE}_2$: Decomposing welfare changes of new households averaged across types

Consider the welfare effects $CE_2(j, z_j)$ of moving from the baseline equilibrium $eq = 0$ to a new steady-state, eq, for a consumer age j of type z_j. These type-specific welfare changes can be decomposed into three components:

- Welfare gains from level changes: $CE_2^L(j, z_j) \equiv \exp\left[\frac{\hat{V}_j^{eq}(z_j) - V_j^0(z_j)}{(1-\lambda)\sum_{age=j}^{J} \beta^{age-j}}\right]$
- Welfare gains from distributional changes: $CE_2^D(j, z_j) \equiv \exp\left[\frac{\tilde{V}_j^{eq}(z_j) - \hat{V}_k^{eq}(z_j)}{(1-\lambda)\sum_{age=j}^{J} \beta^{age-j}}\right]$
- Welfare gains from the altruism term: $CE_2^A(j, z_j) \equiv \exp\left[\frac{V_j^{eq}(z_j) - \tilde{V}_j^{eq}(z_j)}{(1-\lambda)\sum_{age=j}^{J} \beta^{age-j}}\right]$

We can then write $CE_2^L(j, z_j) \times CE_2^D(j, z_j) \times CE_2^A(j, z_j) = \exp\left[\frac{V_j^{eq}(z_j) - V_j^0(z_j)}{(1-\lambda)\sum_{age=j}^{J} \beta^{age-j}}\right] \equiv CE_2(j, z_j)$. The component of $\overline{CE}_2$ is constructed from type-specific changes by taking a weighted average using the endogenous distribution of adult skill in equilibrium eq.

C.3.3 Approximate additive separability of decomposition components

Consumption equivalent shifter CE_1 implies a percent change $\Delta = CE_1 - 1$. For the decomposition components, denote the percent change as Δ_x such that $CE_1^x = 1 + \Delta_x$. Then:

$$\begin{aligned}
\log\left(CE_1^L \times CE_1^D \times CE_1^A\right) &= \log(CE_1) = \log(1+\Delta) \\
\log\left(CE_1^L \times CE_1^D \times CE_1^A\right) &= \log\left(CE_1^L\right) + \log\left(CE_1^D\right) + \log\left(CE_1^A\right) \\
\log(1+\Delta) &= \log(1+\Delta_L) + \log(1+\Delta_D) + \log(1+\Delta_A)
\end{aligned}$$

and if Δ_x is small for each x then the standard log approximation for growth rates applies, so one can write the last line above as:

$$\Delta \approx \Delta_L + \Delta_D + \Delta_A$$

To conclude, although this decomposition is not strictly additive, it is approximately additive for small welfare changes. The approximation also holds for $CE_2(1, z_j)$.