Child Care Subsidies and Child Skill Accumulation in One- and Two-Parent Families

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May 27, 2021

Abstract

I examine the role of family structure and child care subsidies in child skill accumulation. I establish empirically that skill accumulation is more responsive to child care price for one-parent families than for two-parent families. I analyze the effects of child care subsidies in a model featuring endogenous family formation, parental altruism, and a baseline subsidy resembling that of the United States. I find that eliminating this subsidy generates welfare losses of 1.75 percent of lifetime consumption, that equilibrium adjustments act to mitigate these losses, and that increasing uptake among one-parent families yields the highest welfare gains per additional recipient.

Keywords: Early Childhood Education, Family Structure, Households, Skills
JEL Classification Numbers: J24, J12, I21, D13

*College of William & Mary. *email:* egmoschini@wm.edu. *Acknowledgments*: I am grateful for the feedback provided by Anmol Bhandari, Adam Blandin, Elizabeth Caucutt, Diego Daruich, Mariacristina De Nardi, Christopher Herrington, Loukas Karabarbounis, Karen Kopecky, Jeremy Lise, Lance Lochner, Ellen McGrattan, Peter McHenry, Joseph Mullins, Sergio Ocampo Díaz, Arthur Rolnick, and Pengpeng Xiao, as well as seminar participants at the University of Toronto, Auburn University, the College of William & Mary, Ryerson University, Virginia Commonwealth University, Midwest Macro 2019 (Athens, GA), the 2019 SED (St. Louis, MO), the 2020 Barcelona Summer Forum, the 2021 Annual MEA meetings, and the 2021 Annual SOLE meetings. All errors are mine. A previous version of this paper was circulated under the title “Child Skill Investment in One- and Two-Parent Families”.

1 Introduction

There is widespread agreement that early childhood is a key phase in the development of skill, and that policies targeted at this age group can improve educational attainment and labor market outcomes later in life.[1] This consensus is founded on evaluations of several small-scale randomized controlled trials (RCTs) with long-run follow-ups, where children under the age of 5 from impoverished backgrounds are exposed to environments structured to foster the development of their skill (e.g., Heckman et al. (2006); Anderson (2008); Garcia et al. (2017)). The evidence from RCTs is complemented by studies of large-scale subsidized child care programs with universal eligibility, which lower the cost of child care services already available in the market for many different kinds of families (e.g., Baker et al. (2008); Berlinski et al. (2009); Gupta and Simonsen (2010)).[2] Recent evaluations of such programs have emphasized heterogeneity in the effects of child care subsidies on child outcomes across family income groups (Havnes and Mogstad (2014)) and family structures (Kottelenberg and Lehrer (2017)). In the United States, these demographic attributes are quantitatively relevant and intertwined: one-parent families raise twenty percent of children under five, and are almost eight times more likely to be poor than couples with children of the same age.[3]

In this paper, I first examine the role of family structure in child skill accumulation. Empirically, the way in which families allocate their children's time shows systematic differences: I find that one-parent families use more non-parental child care, and contribute less quality time from resident parents in the family, than couples do. These choices determine the activities and environments that children experience, and I interpret them as reflecting an investment decision made by families. Building on the observed disparities in these family choices, I allow the technologies which govern how investment in child skill is generated to differ across family structures. I use the family optimization problems to derive estimation equations for the parameters of these technologies, and test whether the two technologies are different. I find that they are.

To highlight why heterogeneity in skill investment technologies is relevant for the design of child care subsidies, I construct investment's implicit price index for each family structure, and derive the elasticity of this price index with respect to the price of child care. This elasticity encapsulates a family's responsiveness to child care subsidies: a subsidy will increase the skill outcomes of children to the extent that it decreases the price of investment in skill. I find that this elasticity is 33 percent higher for one-parent families than it is for two-parent families, which is consistent with research that documents higher gains from child care subsidies for children raised by the former group. I believe this provides a novel perspective on how investment technologies mediate the response of family investments to child care subsidies.

The parameters governing child skill accumulation are estimated using the Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), a family-level data set from the US Department of Education. The ECLS-B is designed to be representative of families raising 9-month old children in the United States in 2001, and provides information on parental and market time inputs, parental wages, child care prices, and family

[1]See Almond and Currie (2011) for a review, and the seminal study of Cunha et al. (2010), recently revisited by Agostinelli and Wiswall (2017). For the perspective of child psychologists, see Shonkoff (2010) and Nisbett et al. (2012).
[2]Examples include programs in Oklahoma (started in 1998), Quebec (1997), Argentina (1993), Norway (1975), and Denmark (1964).
[3]Author's calculations, Early Childhood Longitudinal Study, Birth Cohort.

structure for children over three waves of the survey, corresponding to when the child is 9 months, 2 years, and 4 years old. The panel structure of these data allows me to control for unobserved relative parenting productivities by implementing a fixed effects estimator.

In addition to investment inputs, the ECLS-B reports measures of child skill in each wave of the survey. Together with family attributes, these test scores generate empirical moments which I use to calibrate a general equilibrium model for policy analysis. This model features a marriage market, in which each generation of men and women meets via random search and endogenously forms either one- or two-parent families. After the marriage market, single females and couples spend a portion of their lifetime altruistically investing in their children's skill, using the technology of their family structure. In this phase they correctly anticipate how skill affects the lifetime utility of their child. The model framework also incorporates a baseline child care subsidy modeled after the Child Care and Development Fund, one of the largest sources of federal funding for child care subsidies in the United States. Upon calibration, the baseline equilibrium matches untargeted empirical facts on the correlation of parental inputs within couples and the marriage rate over the family income distribution. In addition, the model output quantitatively aligns with untargeted empirical measures of the elasticity of the price of investment with respect to the price of child care, computed from the data using the estimated investment technologies.

Among its attributes, the model allows both the marginal cost and the marginal benefit of skill to adjust in equilibrium to the child care subsidy. The marginal cost's response is mediated by the investment technology. The marginal benefit of skill is the increase in lifetime utility, stemming from the higher consumption and leisure one can finance thanks to higher earnings and better marriage prospects. As a child care subsidy makes skill investment cheaper, the labor market return to skill may decrease as labor income taxes increase to finance the subsidy (which would dampen increases in investment due to the subsidy), or families may form differently when the costs of parenting decrease (thus making skill more or less useful in the marriage market).

In the model's baseline equilibrium, the child care subsidy targets low-income families, who are mostly single parents. Eliminating this baseline policy generates welfare losses of 1.75 percent of lifetime consumption. The general equilibrium framework allows the economy to adjust to the removal of the policy and mitigate the damage due to the resulting shift in the skill distribution. Without such general equilibrium adjustments, welfare losses due to decreases in skill would be 2.66 percent of lifetime consumption. Expansions of the current policy, whether along the dimension of eligibility thresholds or uptake rates, yield welfare gains. The highest return per additional family served by the child care policy comes from expanding uptake among poor, one-parent families. This is exactly what the technology estimation results suggest. Nevertheless, restricting subsidy expansions to impoverished single parent families excludes many families who would also benefit: switching to universal eligibility with guaranteed subsidy receipt yields aggregate welfare gains of more than 15 percent of lifetime consumption.

In my framework, the critical primitive is the set of functions that maps from the time use of the child (chosen by the family) to the child's skill outcomes. When specifying these technologies, I allow for time inputs from each resident parent and from a non-parental care provider; each of these inputs has a different productivity, which families take as given. I am not the first to incorporate both parents separately for the couple's investment technology. Other studies that do so include Abbott (2021) and Del Boca et al. (2014), although neither considers non-parental child care as a distinct input. Those studies also differ from mine in their emphasis: Abbott (2021) analyzes income risk, whereas Del Boca et al. (2014) focus on the changing role of mother time in child skill production at each stage of early childhood. By contrast, I focus on how heterogeneity in investment technologies affects investment's responsiveness to a child care subsidy in the population.

My model features altruistic parents investing in the skill of their young children, an approach to intergenerational transfers which dates back to Barro and Becker (1989) and which I share with Lee and Seshadri (2019) and Daruich (2020), the two studies most closely related to mine. Lee and Seshadri (2019) compare skill subsidies at different stages of life, where child skill responds differently to investment in each stage but in the same way for all families.[4] By contrast, I include multiple family structures whose investment technologies feature different parameters, and focus on early childhood as the phase in which investment in skill occurs. Daruich (2020) develops an environment in which parental time and money (i.e., goods, which includes child care) combine to form investment in children. In this environment, he examines the effects of transfers-in-kind subsidies to the goods input, and the technologies governing how child skill is accumulated are either taken from the literature or calibrated within the model. My approach differs from this study in two ways. First, I specify time in child care as an alternative input to time with parents, which reflects an interpretation of child skill accumulation as being founded on activities and interactions with others (similar to the definition of "active time" in Del Boca et al. (2014)). My specification also reflects the fact that, in the data, money spent on young children is mostly spent on child care.[5] Second, in this paper the skill investment technology parameters are estimated directly.

Another important feature that I model is endogenous family formation, where both one- and two-parent families exist in equilibrium. This is consistent with recent work by Blandin and Herrington (2020), who emphasize the role of family structure, in addition to family income, in explaining children's college attainment outcomes over time. Other macroeconomic studies also incorporate endogenous family formation into their frameworks. For example, Abbott et al. (2019) focus on college financial aid in the United States, but unlike my framework do not feature multiple family structures existing in equilibrium. Instead, the authors emphasize positive assortative matching of couples as a magnifying force for inequality, a more traditional approach to incorporating family formation in macroeconomic analyses of education policies.[6]

My specification explicitly emphasizes family invesment in child skill. It is possible that labor supply decisions of parents could also be substantially affected by child care subsidies like those I consider. A study that does focus on labor supply effects is Guner et al. (2020), which allows for heterogeneity in family structures and considers how this heterogeneity leads to different policy responses. Unlike my framework,

[4]Another paper with this approach is Caucutt and Lochner (2020), which emphasizes that the timing of binding borrowing constraints determines the optimal timing for interventions. Restuccia and Urrutia (2004) instead emphasize the relative importance of early parental investments in determining adult outcomes and the implications of this for broader education policy design.

[5]My calculations supporting this claim use the PSID CDS and are reported in Supplementary Appendix B.

[6]Lochner and Monge-Naranjo (2011) also focus on this policy's interaction with intergenerational dynamics, although families do not form endogenously in that study.

however, the authors do not emphasize the effect of policy on the skill accumulation of children.

The paper proceeds as follows. Section 2 presents the investment problems of one- and two-parent families, which form the core of my framework. In section 3, I present the data with which I estimate the parameters of the skill investment technologies—using equations derived from parental investment problems—and the estimation results. Section 4 discusses how heterogeneity in investment technologies matters for investment's response to a child care subsidy. In this section I derive elasticities with respect to the price of child care, both for investment's composite price index and for the marginal cost of child skill. Section 5 presents the general equilibrium model framework, model parameterization, and model fit. Section 6 reports the child care policy experiments. Section 7 concludes.

2 Investment in Child Skill

The critical family choice affecting child skill accumulation, which I focus on here, is how much of the child's time is spent with each resident parent and with a non-parental child care provider. Effectively, parents choose an overall investment level, which ultimately determines child's realized skills, by balancing the child's well-being against their own. Given this desired level of investment, parents decide how much time to spend with their child themselves, doing quality activities—such as reading to them, telling them stories, or playing with them—by weighing the necessary foregone earnings against the cost of using child care services in each period. At the same time, this choice also takes into account that a unit of time with parents can impact child skill differently than the same amount of time spent with a child care provider.

In this section, I formalize the investment problems of one- and two-parent families. The optimality conditions that characterize investment choices are used to derive estimation equations; these equations will be used to identify the parameters of the investment technologies. Hereafter, I refer to one-parent families as single females and two-parent families as married couples. This classification aligns with the empirical characteristics of these groups: one-parent families are by far mostly single women, and two-parent families are usually married.

2.1 Parenting Problems

The structure of a family is identified by the subscript $str \in \{SF, MC\}$, where SF denotes a "single female" and MC denotes a "married couple." Each family takes as given the initial skill of their child, θ_1, and a vector of family attributes, z_{str}. Family attributes include parenting productivities θ_g for each resident parent of gender $g \in \{f, m\}$, where f denotes female and m denotes male. They also include the productivity and price of child care, $\theta_{n,str}$ and $p_{n,str,t}$, and the price of parental time, wages $w_{g,t}$. Here, the productivity of child care depends on family structure; the price of child care and parental wages are allowed to vary at the family level. The family chooses an outcome for their child, θ_{kid}, as well as consumption, savings, and the leisure for each parent, to maximize the family's lifetime utility.

Families invest in their child because they altruistically internalize the lifetime utility of the child associated with the chosen skill outcome, $V_{kid}(\theta_{kid})$.[7] This altruistic motivation is constrained by the fact that the production of child skill is costly: an outcome for the child is associated with a corresponding cost by a structure-specific cost function, $X_{str}(\cdot)$, which depends on the initial skill of the child, parenting productivities for each resident parent, the productivity of child care, and investment input prices. The parenting problem encompasses J_{ec} periods of early childhood, and the parents are all aged j when it begins.

Next, I explain the parenting problems of the two family structures. While these problems are similar, explaining them seperately highlights the structural differences which are subsequently reflected in the empirical estimation and the policy analysis.

Single Females Single females are indexed by their type, $z_{SF} \equiv \{\theta_1, \theta_f, \theta_{n,SF}\}$, which, in addition to the initial skill of the child, includes the fixed productivities of parenting time (θ_f) and child care time ($\theta_{n,SF}$). When choosing an outcome for their child, single females solve:

$$\begin{aligned} V_{SF}(z_{SF}) &= \max_{\theta_{kid}, \left\{c_t, \ell_{f,t}, a_{t+1}\right\}_{t=j}^{J_{ec}+j-1}} \sum_{t=j}^{J_{ec}+j-1} \beta^{t-j} u_{SF}(c_t, \ell_{f,t}) + bV_{kid}(\theta_{kid}) \\ c_t + a_{t+1} &\leq (1+r)\, a_t + w_{f,t}(1-\ell_{f,t}) + T - X_{SF,t}(\theta_{kid}, w_{f,t}, p_{n,SF,t}, z_{SF}) \end{aligned} \tag{1}$$

where, in each period, c_t denotes consumption, $\ell_{f,t} \in [0,1]$ is leisure, and a_t represent assets (with $a_j, a_{J_{ec}+j} = 0$). Here, $w_{f,t}$ is the wage of the parent, $p_{n,SF,t}$ is the price of non-parental child care, $1+r$ is the return on savings, and T is the lump-sum transfer the single female may receive from the government. The per-period utility function of the agent, $u_{SF}(\cdot)$, is strictly increasing and concave in its arguments.

Embedded in (1) is a cost minimization problem that identifies the efficient production of the chosen outcome θ_{kid}. This is represented by the cost function $X_{SF,t}(\theta_{kid}, w_{f,t}, p_{n,SF,t}, z_{SF})$. It is useful to decompose this cost minimization into intratemporal and intertemporal components. For a working single female, the per-period cost of financing a total investment I_t towards the production of the desired outcome θ_{kid} is the result of choosing non-parental child care time and parental time taking her static investment technology, $I_{SF}(\theta_f q_{f,t}, \theta_{n,SF} n_t)$, as given. This *intratemporal* cost minimization gives rise to the expenditure function:

$$\begin{aligned} \tilde{X}_{SF,t}(w_{f,t}, p_{n,SF,t}, z_{SF}, I_t) &= \min_{n_t, q_{f,t}} p_{n,SF,t} n_t + w_{f,t} q_{f,t} \\ s.t. \quad I_{SF}(\theta_{n,SF} n_t, \theta_f q_{f,t}) &\geq I_t \end{aligned} \tag{2}$$

where, in each period, n_t is the quantity of child care time, $q_{f,t}$ is the quantity of parental time, and $n_t + q_{f,t} \in [0,1]$. Note that, for a linearly homogeneous investment technology (as assumed in the parameterization that follows), the per-period expenditure can be written as $\tilde{X}_{SF,t} = \Lambda_{SF,t}(w_{f,t}, p_{n,SF,t}, z_{SF}) I_t$, where

[7] An alternative way of motivating intergenerational transfers is through paternalistic preferences, or "warm glow" returns (Andreoni (1990)). The advantage of an altruism framework is that the returns to investment can respond endogenously to policy, as they will in section 5. This is because parents fully incorporate the economic returns to their investment in terms of their child's lifetime utility, and their behavior changes accordingly. The main benefit of a paternalistic specification is its tractability and flexibility in matching parenting behaviors. For an application of paternalistic preferences to intergenerational transfers of wealth, see De Nardi (2004).

$\Lambda_{SF,t}$ is the composite price index of investment in period t for single females.

The per-period level of investment I_t that constrains this problem is, in turn, the result of an *intertemporal* cost minimization problem where the target outcome θ_{kid} is taken as given. Specifically, the sequence $I^*_{SF,t}$ is the solution to:

$$\begin{aligned} \left\{I^*_{SF,t}\left(\theta_{kid}, w_{f,t}, p_{n,SF,t}, z_{SF}\right)\right\}_{t=j}^{J_{ec}+j-1} &= \underset{\{I_t\}_{t=j}^{J_{ec}+j-1}}{\arg\min} \sum_{t=j}^{J_{ec}+j-1} \left[\frac{\Lambda_{SF,t}\left(\cdot\right) I_t}{\left(1+r\right)^{t-j}}\right] \\ s.t. \quad f_{SF}\left(\left\{I_t\right\}_{t=j}^{J_{ec}+j-1}, \theta_1\right) &\geq \theta_{kid} \end{aligned} \tag{3}$$

Here, f_{SF} is the dynamic skill accumulation function which governs how a given sequence of investment levels affects a child's skill outcome. Using the solution to this problem, and the value function defined by the intratemporal cost minimization in (2), the total cost of child care in each period, which appears in (1), is $X_{SF,t}\left(\theta_{kid}, w_{f,t}, p_{n,SF,t}, z_{SF}\right) \equiv \tilde{X}_{SF,t}\left(\cdot, I^*_{SF,t}\right)$.

Married or Cohabiting Couples Couples are indexed by $z_{MC} = \{\theta_1, \theta_f, \theta_m, \theta_{n,MC}\}$. Relative to single females, here I include the individual productivities of the two resident parents θ_f and θ_m. Couples solve a similar problem to that of single females, with their (monotonic and concave) per-period utility function depending on joint consumption and separate leisure terms. They solve:

$$\begin{aligned} V_{MC}\left(z_{MC}\right) &= \max_{\left\{a_{t+1}, c_t, \ell_{f,t}, \ell_{m,t}, \theta_{kid}\right\}_{t=j}^{J_{ec}+j-1}} \sum_{t=j}^{J_{ec}+j-1} u_{MC}\left(c_t, \ell_{f,t}, \ell_{m,t}\right) + bV_{kid}\left(\theta_{kid}\right) \\ c_t + a_t &\leq \left(1+r\right) a_{t+1} + w_{f,t}\left(1-\ell_{f,t}\right) + w_{m,t}\left(1-\ell_{m,t}\right) + T - X_{MC,t}\left(\theta_{kid}, w_{f,t}, w_{m,t}, p_{n,MC,t}, z_{MC}\right) \end{aligned} \tag{4}$$

where $\ell_{f,t}, \ell_{m,t} \in [0,1]$, and $a_j, a_{J_{ec}+j} = 0$. The wage in each period for female and male parents is $w_{f,t}$ and $w_{m,t}$, respectively, and $p_{n,MC,t}$ is the price of non-parental child care.

Relative to single females, the intratemporal component of cost minimization features one additional input, as both parents can independently contribute their own time. Within a period, working couples take their static investment technology $I_{MC}\left(\theta_f q_{f,t}, \theta_m q_{m,t}, \theta_{n,MC} n_t\right)$ as given, and solve:

$$\begin{aligned} \tilde{X}_{MC,t}\left(w_{f,t}, w_{m,t}, p_{n,MC,t}, z_{MC}, I_t\right) &= \min_{n_t, q_{f,t}, q_{m,t}} p_{n,MC,t} n_t + w_{f,t} q_{f,t} + w_{m,t} q_{m,t} \\ s.t. \quad I_{MC}\left(\theta_{n,MC} n_t, \theta_f q_{f,t}, \theta_m q_{m,t}\right) &\geq I_t \end{aligned} \tag{5}$$

where here $q_{f,t}$ and $q_{m,t}$ are the quantity of own time supplied by the two parents, and $n_t + \max\{q_{f,t}, q_{m,t}\} \in [0,1]$. This time constraint maintains the assumption that parental quality time is non-rival. For a linearly homogeneous parameterization of the investiment technology the per-period expenditure can be written as $\tilde{X}_{MC,t} = \Lambda_{MC,t}\left(w_{f,t}, w_{m,t}, p_{n,MC,t}, z_{MC}\right) I_t$, where $\Lambda_{MC,t}$ is the composite price index of investment in period t for married couples.

As for the intertemporal component of cost minimization, couples solve:

$$\begin{aligned}
\left\{I^*_{MC,t}\left(\theta_{kid}, w_{f,t}, w_{m,t}, p_{n,MC,t}, z_{MC}\right)\right\}_{t=j}^{J_{ec}+j-1} &= \underset{\{I_t\}_{t=j}^{J_{ec}+j-1}}{\arg\min} \sum_{t=j}^{J_{ec}+j-1} \left[\frac{\Lambda_{MC,t}\left(\cdot\right) I_t}{\left(1+r\right)^{t-j}}\right] \qquad (6) \\
s.t. \quad f_{MC}\left(\{I_t\}_{t=j}^{J_{ec}+j-1}, \theta_1\right) &\geq \theta_{kid}
\end{aligned}$$

where f_{MC} is the dynamic production function that governs how a sequence of investment levels affects a child's skill outcome for married couples. Evaluating the value function of the problem in (5) at the optimal solution for (6) yields the per-period expenditure term used in the lifetime utility maximization in (4), that is $X_{MC,t}\left(\theta_{kid}, w_{f,t}, w_{m,t}, p_{n,MC,t}, z_{MC}\right) \equiv \tilde{X}_{MC,t}\left(\cdot, I^*_{MC,t}\right)$.

2.2 Technology Functional Forms and Optimality Conditions

There are two sets of technologies which determine the cost of investment X_{str} for each of the two family structures $str \in \{SF, MC\}$ in (1) and (4). These are, first, a static technology which combines family and child care time inputs into investment within a period; and, second, a dynamic technology which combines investments across periods, along with the initial stock of child skill, to generate the final outcome θ_{kid}. I assume that the static technologies for both family structures takes a Constant Elasticity of Substitution (CES) functional form, where the structure of the couple's technology nests that of single females.

$$I_{str,t} = \left[\gamma_{str}\left(h_{str,t}\right)^{\frac{\nu_{str}-1}{\nu_{str}}} + \left(1-\gamma_{str}\right)\left(\theta_{n,str} n_t\right)^{\frac{\nu_{str}-1}{\nu_{str}}}\right]^{\frac{\nu_{str}}{\nu_{str}-1}} \qquad (7)$$

$$h_{SF,t} = \theta_f q_{f,t} \qquad (8)$$

$$h_{MC,t} = \left[\alpha\left(\theta_f q_{f,t}\right)^{\frac{\rho-1}{\rho}} + \left(1-\alpha\right)\left(\theta_m q_{m,t}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \qquad (9)$$

Here, ν_{str} is the elasticity of substitution for family time and purchased non-parental child care time, and γ_{str} is the share on family time, for family structure $str \in \{SF, MC\}$. For single females, the total quantity of family time, $h_{SF,t}$, is simply efficiency units of time from the female parent. For couples, the quantity of family time, $h_{MC,t}$, is generated with a CES aggregator of efficiency units of time from both parents. This inner aggregator for couples has an elasticity of substitution between time from each parent, ρ, and a share parameter on the female parent's time, α. Given this nested parameterization, the static investment technologies are the same across family structures if $\gamma_{SF} = \gamma_{MC}$ and $\nu_{SF} = \nu_{MC}$.

The dynamic production function that generates the target outcome θ_{kid} also takes a (recursive) CES form, where only the productivity of investment is allowed to differ across family structures:

$$f_{str}\left(\{I_{str,t}\}_{t=j}^{J_{ec}+j-1}, \theta_1\right) = \left(\sum_{t=j}^{J_{ec}+j-1} \upsilon\left(1-\upsilon\right)^{J_{ec}+j-1-t}\left(\lambda_{str} I_{str,t}\right)^{\frac{\chi-1}{\chi}} + \left(1-\upsilon\right)^{J_{ec}+j-1}\theta_1^{\frac{\chi-1}{\chi}}\right)^{\frac{\chi}{\chi-1}} \qquad (10)$$

This parameterization maintains the condition that early investments increase the productivity of later investments. The parameter λ_{str} is a scaling parameter that accounts for units of measurement being different across investment and the stock of skill. The level of this parameter is allowed to differ across family structures; the dynamic technologies are the same across family structures if $\lambda_{MC} = \lambda_{SF}$.

The tangency conditions from the intratemporal cost problems (2) and (5) imply a system of equations which can be used to estimate the parameters of the static investment technology. Specifically, the optimal combination of parental times for married couples satisfies:

$$\ln\left(\frac{q_{MC,f,t}}{q_{MC,m,t}}\right) = -\rho\ln\left(\frac{w_{f,t}}{w_{m,t}}\right) + \rho\ln\left(\frac{\alpha}{1-\alpha}\right) + (\rho-1)\ln\left(\frac{\theta_f}{\theta_m}\right) \tag{11}$$

Next, the optimal combination of family time and child care time, for family structure $str \in \{SF, MC\}$, can be expressed as:

$$\ln\left(\frac{h_{str,t}}{n_{str,t}}\right) = -\nu_{str}\ln\left(\frac{p_{h,str,t}}{p_{n,str,t}}\right) + \nu_{str}\ln\left(\frac{\gamma_{str}}{1-\gamma_{str}}\right) + (\nu_{str}-1)\ln\left(\frac{1}{\theta_{n,str}}\right) \tag{12}$$

where $h_{SF,t}$ and $h_{MC,t}$ are the family time aggregates defined by equations (8) and (9), $p_{h,SF,t} \equiv \frac{w_{f,t}}{\theta_f}$ is the unit cost of family time for a single female, and $p_{h,MC,t}$ is the composite price index for family time in couples (this price aggregator is dual to the production function in (9), and depends on the wages of both parents).

Finally, the tangency conditions of the intertemporal cost minimization problems, (3) and (6), yield an investment equation for the parameters of the dynamic production function in (10):

$$\ln\left(\frac{X_{str,t+1}}{X_{str,t}\,(1+r)}\right) = (1-\chi)\ln\left(\frac{\Lambda_{str,t+1}}{\Lambda_{str,t}\,(1+r)}\right) - \chi\ln(1-\upsilon) \tag{13}$$

where $\Lambda_{str,t}$ is the price index of intratemporal investment, for each family structure $str \in \{SF, MC\}$, introduced earlier (these are dual to the nested CES investment functions of equations (7) to (9)).

3 Data and Estimation

In this section, I first explain data sources and describe several empirical patterns that are evident when comparing across family structures in the population. Next, after discussing the estimation sample, I provide the estimation equations, the estimated coefficients, and the implied estimates for technology parameters.

3.1 Data

I combine two datasets to measure parental educational time inputs ($q_{g,i,t}$), non-parental child care time inputs ($n_{i,t}$), hourly wages ($w_{g,i,t}$), and hourly non-parental child care prices ($p_{n,str,i,t}$) (the indexes denote parental gender g in family i with family structure str in period t, while n indexes non-parental child care's price). Specifically, I use the ECLS-B, which is a panel data set, and the American Time Use Survey

(ATUS), which is a repeated cross-section sampled from the Current Population Survey (CPS). Data for hourly wages, hourly price of child care, quality time from the parents in weekly frequency of activities, and non-parental child care time in hours per week can be constructed from information reported in the ECLS-B. The ATUS contributes measures of hours per activity, which I impute to the ECLS-B to transform frequencies of activities into hours per week of parental quality time inputs.

The Early Childhood Longitudinal Study, Birth Cohort The ECLS-B reports labor supply status (full time, part time, or not in the labor force), labor earnings, the period of time over which the labor earnings were accrued (a day, a week, two weeks, etc.), and the hours worked in a week separately for each parent. It also reports frequencies of activities with the child for each resident parent (i.e., reading to the child one a week, twice a week, every day, etc.). In addition, there are assessments of child skill reported in each wave of the survey.[8]

I convert after-tax labor earnings into hourly wages using hours worked per week.[9] If hours worked were not reported for the parent, they are imputed when possible using the response to part-time or full-time status (assigning 30 or 40 hours worked per week, respectively). The result is hourly after-tax wages for mothers and fathers, conditional on observing labor earnings and some information about the intensity of labor supply. I do not impute wages for non-working parents.

To calculate the hourly price of non-parental child care, I use expenditures on the primary source of non-parental child care, adjusted by the total hours of care that these expenditures purchased.[10]

Hours purchased for the primary source of child care are reported directly in the ECLS-B. However, hours of parental time inputs $q_{g,i,t}$ are not reported directly. Instead, parents report how frequently they engage in different activities with the child in the first three waves of the survey. Some of these activities involve actively engaging with the child and speaking or listening to them. In particular, I consider the following activities to be forms of investment in the child: reading to the child, telling the child stories, and playing with the child indoors.[11] This definition is consistent with the literature for the importance of active time with children (Del Boca et al. (2014)). In turn, these activities have counterparts in the ATUS, so that for each activity occurrence I can impute the amount of time spent in that activity using group-specific time averages for the ATUS counterpart activity.

The American Time Use Survey Data on time per activity come from the 2003-2016 pooled ATUS sample. This dataset provides a time diary along with CPS variables for age, gender, marital status, labor force status, parental status, and child age. Observations are restricted to individuals between 18 and 55

[8] These are used in the calibration of the general equilibrium model later in the paper. For more information on child skill measures in the ECLS-B, see Supplementary Appendix A.

[9] To account for taxes, wages are corrected using the slopes (tax rates) from Table 6 of McGrattan and Prescott (2017). This adjustment accounts for the progressive nature of the US tax system with respect to labor earnings.

[10] The ECLS-B also reports all sources of child care; the relevant moments for the calibration are not affected by using total hours in child care instead of only the primary source.

[11] See Supplementary Appendix A for further information on measured activities in the ECLS-B and the definition of quality time I apply here.

years of age, with a child aged 3 years or younger. I use information on parent gender, marital status (married/cohabiting or single), and labor force status (full time or part time). I calculate the survey-weighted average of time spent on an activity (conditional on engaging in it) for each group, for time spent reading to the child, speaking and listening to the child, and playing with the child.[12]

Imputation After linking parents in the ECLS-B with their appropriate group in the ATUS, the ATUS levels of time for a given occurrence of an activity are assigned to their appropriate activities in the ECLS-B. Next, total quality time per parent in each family in each wave of the ELCS-B is calculated by summing across the three activities.

Population Moments in the ECLS-B Table 1, Table 2, and Figure 1 present moments from a population sample. I applied several broad requirements on the ECLS-B raw data to generate these moments. For both couples and single females, observations are only admissable if the resident primary caregiver is a biological parent. I restrict the population sample to families who do not change family structure over the course of the panel, for whom I observe family income in all three waves, and for whom I observe both initial skill (wave 1) and final skill (waves 4 or 5) for the child. Finally, I require that the total time spent with parents in quality time activities or child care providers must not exceed 100 hours a week and not be missing values. This criteria allows me to include families which specifically report not engaging in any of the activities which I include in the category of parental investment time, or who report using zero hours of child care. It also includes observations where the price of child care or the wage of the parent is zero or missing.[13]

Table 1 contains population moments from the ECLS-B, for each family structure and parent gender. Here, couples can be either married or cohabiting, and single females are those who are a primary caregiver and who do not have a significant other living in the household with them. In these summary statistics, I used cross-sectional primary caregiver weights from the primary caregiver survey for the single and married/cohabiting female moments, and cross-sectional resident father weights from the resident father survey for the moments of married/cohabiting males.[14]

By comparing across family structures, three qualitative points are apparent. First, child care represents a more sizable input for one-parent families. Single females in the ECLS-B contribute 9 hours from the family and purchase 35 hours of child care per week, on average. By comparison, couples contribute about 10 and 8 hours from the female and male parents, respectively, and purchase 30 hours of child care per week. Second, purchased child care differs systematically in price for the two family structures: on average, the child care that single females purchase has a price per hour that is \$1.66 lower than the average price for coupled females. The third qualitative point is that one-parent families face lower wages compared to coupled parents: average hourly wages for single females are 65 percent of the average for married females, and 60 percent of the average wage of married males.

[12] Summary statistics and tabulations of the ATUS sample are reported in Supplementary Appendix A.

[13] Only prices specifically reported to be 0 are included in price moments reported in the summary statistics tables.

[14] See Supplementary Appendix A for an outline of the survey structure of the ECLS-B. There are several questionnaires at the family level in each wave of the survey, each with its own set of weights. Not all married couple families in the ECLS-B have a completed questionnaire for both the primary caregiver (usually the mother) and the resident father.

The fact that the average wage of single females is lower than for married females reflects differences in the fraction with a bachelor's degree or higher in each group (7 percent versus 46 percent, respectively), but is likely not driven by labor force status (single and coupled females have similar distributions of labor force statuses in the population sample, although single females are more likely to be unemployed). Differences in educational attainment may partly be because single females tend to be younger than married females: 26 versus 33, respectively. All of these compositional differences are reflected in the family incomes: the average income of single female families is roughly one-third that of couples.[15] Related to differences in family income, but additionally incorporating family size, the poverty rate for single females is almost eight times higher than it is for coupled females (45 versus 6 percent), and they are more than three times as likely to be below 185 percent of the poverty line (71 versus 19 percent).[16]

Table 1: Population Sample Moments

	Singles: Females			Couples: Females			Couples: Males		
	mean	p50	sd	mean	p50	sd	mean	p50	sd
Parental Time	9.45	7.96	7.27	10.07	8.02	6.88	7.98	6.32	6.79
CC Time	34.57	40.00	13.11	29.67	32.00	14.15	29.99	33.00	14.03
Parent Hourly Wage	9.86	8.55	6.25	15.24	11.48	32.85	16.46	13.30	15.39
CC Hourly Price	1.25	0.23	2.15	2.91	2.14	3.84	2.88	2.17	3.76
BA or higher	0.07			0.46			0.37		
Parent Age	26.06	25.00	5.99	32.54	32.00	5.55	34.53	35.00	6.14
Full Time	0.61			0.67			0.93		
Part Time	0.18			0.23			0.03		
Unemployed	0.09			0.01			0.02		
NILF	0.11			0.08			0.03		
Family Inc.	29918	20000	35402	86985	62500	63415	86453	62500	62474
Family Inc., Aftx	22426	16260	23361	63520	47812	41585	63197	47813	40960
$\leq$ Pov. Line	0.45			0.06			0.05		
$\leq$ 185% Pov. Line	0.71			0.19			0.20		
Observations	1000			4200			3050		

This table: population sample moments using waves 1-3; sample criteria are described in the text. Differences in averages for married females and males are due to the different weights used to generate sample moments and because not all primary caregiver surveys have an associated resident father survey. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Observations rounded to nearest 50 per NCES requirements.

Child Skill Accumulation, Family Income, and Family Structure in the ECLS-B The ECLS-B provides measures of child skill in each wave of the survey. These measures are age-appropriate tests of children's cognitive abilities and so can be used to examine a child's development over time relative to other children of the same age. Using wave 3 of the population sample described above, Table 2 shows that in the ECLS-B child skill accumulation exhibits a systematic relationship with family income and family structure.[17] In particular, although at age 9 months measures of child skill are uncorrelated with family income

[15] After-tax income is computed using Table 5 in McGrattan and Prescott (2017).

[16] For comparison, a similar statistics computed from the US Census show similar disparities in poverty rates. In the 2000 US Census, single females of children under 5 have a poverty rate of 41%, while for married couples this is 6% (author's calculations). The reason the fraction below 185% of the poverty line is reported is that this is the income threshold below which families are eligible for the National School Lunch Program and the Special Supplemental Nutrition Programs for Women, Infants, and Children.

[17] For more information on child skill measures in the ECLS-B, see Supplementary Appendix A.

and family structure, by the time the child is in kindergarten these correlations have jumped to 32 percent with family income, Y, and 21 percent with an indicator for two-parent families, $\mathbb{I}_{MC}$.[18] This demonstrates that both richer families and couples tend to see better relative outcomes for their children's skill, despite very similar initial child skills across income levels and family structures. Table 2 also reports the marriage rate for families in the same population sample for which the correlation coefficients are computed. It is 79 percent.[19]

Table 2: Child Skill Accumulation Moments and Marriage Rate

Corr(θ_{kid}, Y)	Corr($\theta_{kid}, \mathbb{I}_{MC}$)	Fraction Married
0.32	0.21	0.79
Families	1750	

This table: all families, wave 3, using cross-sectional primary caregiver weights for that wave. Sample criteria described in the text. I do not impose that both the mother and father have to be reporting a valid observation, so observation counts align with 1 wave of the sample in Table 1 using information from mothers. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Observations rounded to nearest 50 per NCES requirements.

At the same time, family structure is also systematically related to family income: in the ECLS-B, the marriage rate is monotonically increasing in the income quartile of the family (Figure 1). This relationship arises in part because two-parent families have two potential earners, but is also a product of the higher wages of each parent compared to hourly wages of single females (Table 1).[20] I interpret the relationship between family income and family structure as arising from endogenous family formation, and therefore include and endogenous marriage market in the expanded model of section 5.

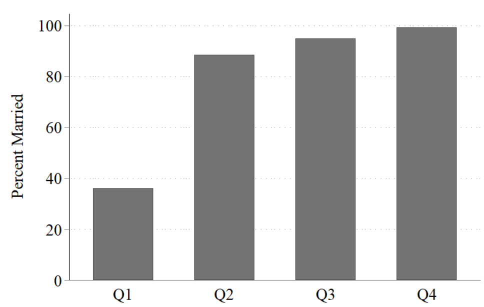

Figure 1: Marriage Rate by Family Income Quartile in the ECLS-B

This figure: fraction of families in each family income quartile who are married or cohabiting, wave 3, using cross-sectional primary caregiver weights for that wave. Sample described in the text and is the same as Table 2. Income is corrected for taxes using **?**, Table 5. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

[18]Child skill outcome θ_{kid} is measured in ECLS-B waves 4 or 5. For the structure of the ECLS-B, see Supplementary Appendix A.

[19]Because I will later evaluate the model fit against moments computed in the population sample presented in Table 2, I target this marriage rate as well as the two correlation coefficients presented in Table 2 in my calibration of the expanded model of section 5.

[20]Correcting income for family size, or using pre-tax income reported directly in the ECLS-B, yields very similar results.

The Estimation Sample The estimation sample puts additional restrictions on the data compared with the population sample described in the previous section. In particular, for both couples and single females, observations are only admissable if the resident primary caregiver is less than 55 years of age, reports working for pay, makes less than 250 dollars an hour, pays at least 1 cent per hour for the primary source of child care, and whose child spends at least 0.01 hours per week in child care. In addition, families are only valid observations if the biological mother had her first child after age 15 and before age 45. For couples, I also restrict attention to families where both parents are valid observations with regards to investment input quantities and prices. This leaves me with 50 one-parent families and 200 two-parent families for the estimation (these sample sizes are rounded to the nearest 50, as required by the National Center for Education Statistics (NCES) for the restricted-use ECLS-B).

Moments from the estimation samples are presented in Table 3. The three qualitative points that were apparent in the population moments presented in Table 1 survive for the most part in Table 3. First, one-parent families put more emphasis on child care time than family time compared to two-parent families. Second, the price per hour of child care time is lower for single females than for couples. Third, single females earn lower wages, have lower incomes, are more likely to be poor and to qualify for government benefits, and are less educated than adults parenting in a couple.

Table 3: Estimation Sample Moments

	Singles: Females			Couples: Females			Couples: Males		
	mean	p50	sd	mean	p50	sd	mean	p50	sd
Parental Time	11.15	9.37	7.04	10.48	8.16	6.49	8.74	6.36	6.68
CC Time	37.31	40.00	10.98	33.14	36.00	12.74	33.31	37.00	12.77
Parent Hourly Wage	12.24	10.42	6.40	15.07	12.68	8.7	17.82	14.12	15.05
CC Hourly Price	2.59	2.00	2.10	4.20	3.20	3.73	4.18	3.13	3.69
BA or higher	0.28			0.67			0.52		
Parent Age	29.54	29.00	5.87	33.83	34.00	4.44	34.88	35.00	5.14
Full Time	0.79			0.79			0.99		
Part Time	0.21			0.21			0.01		
Unemployed	0.00			0.00			0.00		
NILF	0.00			0.00			0.00		
Family Inc.	42611	32500	38730	106134	87500	58776	105216	87500	59419
Family Inc., Aftx	31231	24863	25402	76376	65100	38090	75763	65100	38523
$\leq$ Pov. Line	0.20	0.00	0.40	0.00	0.00	0.04	0.00	0.00	0.04
$\leq$ 185% Pov. Line	0.44	0.00	0.50	0.04	0.00	0.19	0.05	0.00	0.21
Observations	150			600			600		
Families	50			200			200		

This table: estimation sample moments using waves 1-3; sample criteria are described in the text. Differences in group moments for married females and males are due to different weights used for the primary caregiver and resident father survey, respectively. See text for details. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Observations rounded to nearest 50 per NCES requirements.

In this sample definition, I have restricted attention to families for which the estimation equations I implement are valid. This assumes that parents supply labor on the market so that the price of their time is their observed wage. If a parent does not work, that family is not in my estimation sample, and comparing Table 3 and Table 1 shows that this raises the average level of time inputs across both family structures and raises the average parental wage. It also lowers the poverty rate across both one- and two-parent families.

Differences between Tables 1 and 3 do not present a problem for identifying the technology parameters of the entire population, as long as the families excluded from Table 3 still use the same technologies to invest in their children as the families which are included. Under this assumption, I simply do not observe the appropriate relative prices and quantities for those excluded families. For example, in families where a parent does not work, those parents face relative prices of inputs which I cannot observe directly (the price of time for a non-working parent is determined by the marginal utility of leisure for that parent). If I could observe these input prices and the resulting choice of quantities, I would find that families with non-working parents are still choosing input quantities by solving the same cost-minization problem, but one which incorporates the appropriate input prices. The sample restrictions I impose here focus attention the set of realizations for relative prices and relative quantities that are directly measured in the ECLS-B.[21]

3.2 Estimation

Given the sequential nature of intratemporal and intertemporal cost minimization problems, and the nested CES form of the couple's technology, estimation proceeds in three steps. First, I use the optimality condition for the allocation of within-family time in married couples, as provided in (11). The resulting estimating equation can be expressed as:

$$\tilde{y}_{i,t} \quad = \quad \tilde{\beta}_0 + \tilde{\beta}_1 \tilde{x}_{i,t} + \tilde{\tau}_i + \tilde{\epsilon}_{i,t} \tag{14}$$

Here, the dependent variable is $\ln\left(\frac{q_{f,i,t}}{q_{m,i,t}}\right)$ and the independent variable is $\ln\left(\frac{w_{f,i,t}}{w_{m,i,t}}\right)$. The term $\tilde{\tau}_i$ is a fixed effect parameter that captures the family-specific but time-invariant input productivities in (11). This fixed effect captures relative productivities, and thus varies at the family level even though child care's productivity is the same within a given family structure. $\tilde{\epsilon}_{i,t}$ is a random term assumed distributed i.i.d. Next, I use the intratemporal condition (12) that governs the optimal combination of family time and child care time for each family structure. This second regression takes the form:

$$y_{str,i,t} \quad = \quad \beta_0 + \beta_{str,1} x_{str,i,t} + \tau_i + \epsilon_{i,t} \tag{15}$$

Here, the dependent variable is $\ln\left(\frac{h_{str,i,t}}{\theta_{f,i} n_t}\right)$, and the independent variable is $\ln\left(\frac{\theta_{f,i} p_{h,str,i,t}}{p_{n,str,i,t}}\right)$. The term τ_i is a fixed effect parameter that captures unobserved family-specific but time-invariant relative productivities of female parent time and non-parental child care time. $\epsilon_{i,t}$ is a random term assumed to be i.i.d..

Note that simply using Ordinary Least Squares (OLS) to estimate (14) and (15) (while conflating the relative productivity terms with the residual) would yield biased estimates of the slopes and intercept parameters of interest if, as seems plausible, the investment productivities of parental time and child care time are

[21] A similar assumption holds for children whose time constraint binds: if these families are using all of the child's time endowment to invest in skill, they still use the same technology as families in my estimation sample, but the system of equations that governs how inputs are chosen is different from the estimation equation I implement. This does not imply that the parameters of the technologies being used by observations within the sample or outside of it are different: this alternative system of equations still depends on the set of technology parameters in the estimation equations I apply in the ECLS-B.

correlated with the observed prices of these inputs. Thus, I estimate these models using a Fixed Effects (FE) estimation at the family level. That is, I treat τ_i as family-specific fixed effects to be estimated and, by a suitable normalization, I will assume $E[\tau_i|str] = 0$ for each family structure, and $E[\tilde{\tau}_i] = 0$ for married couples. This means that group-specific intercepts can be constructed using group-specific averages of the fixed effect estimates from the original estimation and the estimated intercept: $\beta_{str,0} = \beta_0 + E[\tau_i|str]$, for $str \in \{SF, MC\}$. This normalization allows me to use the estimated intercept to identify share parameters in each stage for each family structure. These share parameters are needed to retrieve the structural CES equations from the dynamic estimation equation.

The last estimation step exploits the tangency conditions for the intertemporal component of cost minimization, as in equation (13). The resulting regression equation, estimated by OLS, is:

$$y_{d,i,t} = \beta_{d,0} + \beta_{d,1} x_{d,i,t} + \epsilon_{d,i,t} \tag{16}$$

Here, the dependent variable is the ratio of consecutive expenditures of family i with structure str in periods t and $t+1$: $\ln\left(\frac{X_{str,i,t+1}}{X_{str,i,t}(1+r)}\right)$. The independent variable is $\ln\left(\frac{\Lambda_{str,i,t+1}}{\Lambda_{str,i,t}(1+r)}\right)$, where $\Lambda_{str,i,t}$ is the price index that arises from intratemporal cost minimization using the CES parameterization. The random terms $\epsilon_{d,i,t}$ is assumed to be i.i.d..

Estimation Results The estimated coefficients for the three estimation equations are reported in Table 4. From the results in the first column of this table, it is apparent that the slope coefficient $\tilde{\beta}_1$ is not statistically different from zero. This implies that the postulated CES aggregator for married couples, which combines efficiency units of mother and father family times, reduces to a Leontief. In what follows, I will maintain the limiting Leontief structure implied by $\tilde{\beta}_1 = 0$. This means that, rather than the share parameter α of the couple family aggregator in equation (9), the remaining relevant parameter is the Leontief ratio of time inputs $\alpha_L \equiv \frac{\theta_f q_{f,t}}{\theta_m q_{m,t}}$. With a suitable reformulation of the CES structure in (9), this Leontief parameter is:[22]

$$\alpha_L = \exp\left(\tilde{\beta}_0\right) \tag{17}$$

Using the estimated value of α_L along with the predicted relative productivities of couples implied by the estimated fixed effects, the quantity of family time, $h_{MC,i,t}$, and its price index, $p_{h,MC,i,t}$, can then be constructed. For married couples the dependent variable in (15) is then written as $y_{MC,i,t} = \ln\left(\frac{q_{f,i,t}}{n_t}\right)$, and the independent variable is $x_{MC,i,t} = \ln\left(w_{f,i,t} + \frac{\theta_{f,i}}{\theta_{m,i}\alpha_L} w_{m,i,t}\right) - \ln\left(p_{n,MC,i,t}\right)$. For single females, the corresponding variables in (15) are $y_{SF,i,t} = \ln\left(\frac{q_{f,i,t}}{n_t}\right)$ and $x_{SF,i,t} = \ln\left(w_{f,i,t}\right) - \ln\left(p_{n,SF,i,t}\right)$.

[22] A property of the standard CES functional form, as in (9), is that its limiting case (when the elasticity of substitution approaches zero) implies a ratio of inputs equal to one, regardless of the share parameter α. To allow for a ratio of inputs different from one, as is typically of interest, a suitable reparameterization of the CES is needed. See Saito (2012) for details.

Table 4: Model Regression Coefficients for Skill Investment Technology Estimation

	Intratemporal		Intertemporal
	(MC)	$(MC + SF)$	$(MC + SF)$
$\tilde{\beta}_0$	0.353		
	(0.040)		
$\tilde{\beta}_1$	0.179		
	(0.107)		
$\beta_{1,MC}$		-0.594	
		(0.118)	
$\beta_{1,SF}$		-0.443	
		(0.178)	
β_0		-0.207	
		(0.208)	
$\beta_{d,1}$			0.620
			(0.129)
$\beta_{d,0}$			0.255
			(0.047)
FEs	Y	Y	N
R^2	0.45	0.43	0.08
No. families MC	200	200	200
No. families SF		50	50

This table: first, second, and third-stage estimated coefficients for investment aggregator. Standard errors in parentheses. Data source: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Sample size rounded to nearest 50 per NCES requirements.

The remaining structural parameters of interest can be retrieved from the estimated coefficients reported in Table 4 as follows:

$$\nu_{str} = -\beta_{str,1} \tag{18}$$

$$\gamma_{str} = \frac{1}{1 + \exp\left(\frac{\beta_{str,0}}{\beta_{str,1}}\right)} \tag{19}$$

$$\chi = 1 - \beta_{d,1} \tag{20}$$

$$\upsilon = 1 - \exp\left(\frac{\beta_{d,0}}{\beta_{d,1} - 1}\right) \tag{21}$$

The point estimates of this structural parameters, along with their standard errors (computed using the delta method), are reported in Table 5. The elasticity parameters ν_{SF} and ν_{MC} are both statistically different from zero, so this estimation rejects a Cobb-Douglas specification for how family and child care time are combined. However, these elasticities are not statistically different across one- and two-parent families. At the same time, the family share γ_{str} is not different from zero for single females but is statistically different from zero for couples (at the 0.01 significance level). As for the point estimates of the dynamic skill accumulation technology parameters, χ and υ, the results reject a Cobb-Douglas specification, and the share on investment is sizable, indicating a significant role for family investments in determine child outcomes.

Table 5: Parameters of the Family Investment Technologies

α_L	ν_{SF}	ν_{MC}	γ_{SF}	γ_{MC}	χ	υ
1.42	0.443	0.594	0.171	0.465	0.380	0.490
(0.06)	(0.178)	(0.128)	(0.094)	(0.095)	(0.129)	(0.138)

This table: Point estimates of technology parameters, using estimates of coefficients from Table 4 and equations (17) to (21). Standard errors are in parentheses, calculated using the delta method. Data source: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

Comparing Investment Technologies Across One- and Two-Parent Families The cost-minimizing relative quantities of family time and child care time depend on the relative prices of these two inputs, for a given family structure. Comparing parameter values in Table 5 across family structures, the estimated share on family time, γ_{str}, is larger for couples than for single females because couples tend to choose more family time relative to child care time for the similar relative prices. At the same time, because similar changes in relative prices then to result in larger changes in relative quantities for couples, the elasticity ν_{MC} is higher in magnitude than ν_{SF}.

With the estimated parameters in hand, I consider the null hypothesis that two-parent families are using the same technology as one-parent families to invest in their children. This requires that $\gamma_{SF} = \gamma_{MC}$ and $\nu_{SF} = \nu_{MC}$. The hypotheses $\gamma_{SF} = \gamma_{MC}$ is rejected by the F test at the 0.01 significance level. While the other hypothesis $\nu_{SF} = \nu_{MC}$ is not rejected, the joint hypothesis that the two parametric restrictions hold simultaneously is also rejected at the 0.01 significance level. These results establish that these technologies have statistically the same elasticity of substitution between child care time and parental time, but very different share parameters on child care time.

Turning to to the dynamic equation, the elasticity of substitution χ is identified by the responsiveness of expenditures to the price of investment over time, and the share parameter υ is identified by variation in expenditures over time that is not explained by variation in investment prices. As υ increases, the percent change in output for a percent change in investment increases. I find that υ is quite sizable. Given an initial level of skill, the parameter χ determines the elasticity of final skill θ_{kid} with respect to investment. When $\chi < 1$, as I find, then this final skill elasticity is decreasing in the level of investment.

To summarize, the estimation results analyzed here imply that single females will increase their investment more than couples in the face of a child care subsidy, because for single females the price of child care plays a larger role in determining the composite price of investment. This is due to the smaller share on family time for single females. The dynamic equation estimation results indicate that changes in investment levels will have a sizable affect on the skill accumulation of children.

4 The Heterogeneous Importance of Child Care's Price

To gain some insight into how family attributes map into the responsiveness of investment to child care subsidies, here I derive the elasticity of skill's marginal cost with respect to the price of child care, for each family structure. I decompose this marginal cost elasticity into two components: the fraction of potential income being invested in the child, and the elasticity of the price of investment with respect to the price of child care. Using the estimated parameters and an assumed functional form for utility, I calculate and compare the empirical distributions of the marginal cost elasticity and its two components. Differences across family structures in these two components will drive heterogeneity in investment responses to child care subsidies.

The Elasticity of the Marginal Cost of Investment With Respect To Child Care's Price From the optimization problems of single females in (1), and of couples in (4), consider the simple case where early childhood lasts one period ($J_{ec} = 1$), there is no borrowing or saving, and the utility functions take the form $u_{SF}(c, \ell_f) = \ln(c) + \psi_{SF} \ln(\ell_f)$ and $u_{MC}(c, \ell_f, \ell_m) = \ln(c) + \psi_{MC,f} \ln(\ell_f) + \psi_{MC,m} \ln(\ell_m)$. Let (c^*_{SF}, ℓ^*_{SF}) and $\left(c^*_{MC}, \ell^*_{MC} = \left\{\ell^*_{MC,f}, \ell^*_{MC,m}\right\}\right)$ denote the optimal consumption and leisure choices conditional on a given final skill for the child, θ_{kid}, for each family structure.[23]

The optimal choice of the final child skill, θ_{kid}, maximizes $u_{str}(c^*_{str}, \ell^*_{str}) + bV_{kid}(\theta_{kid})$. For single females, this means it satisfies:

$$\frac{(1 + \psi_{SF})\Lambda_{SF}}{w_f + T - \Lambda_{SF} I_{SF}} \frac{\partial I_{SF}}{\partial \theta_{kid}} = b \frac{\partial V_{kid}(\theta_{kid})}{\partial \theta_{kid}}$$

The left-hand-side of this equation displays the full "marginal cost" of child skill production in terms of utility, which balances the marginal benefit on the right-hand-side. Consider the elasticity of this marginal cost with respect to the price of child care, denoted σ_{SF}:

$$\sigma_{SF} \equiv \epsilon_{SF} \left(\frac{w_f + T}{w_f + T - \Lambda_{SF} I_{SF}(\theta_{kid})} \right) \tag{22}$$

where $\epsilon_{SF} = \frac{\partial \Lambda_{SF}}{\partial p_{n,SF}} \frac{p_{n,SF}}{\Lambda_{SF}}$ is the elasticity of the price of investment with respect to the unit cost of child care. Given the technology presented in (7), this elasticity is:

$$\epsilon_{SF} = \frac{(1 - \gamma_{SF})^{\nu_{SF}}}{(1 - \gamma_{SF})^{\nu_{SF}} + \gamma_{SF}^{\nu_{SF}} \left[\left(\frac{p_{h,SF}}{p_{n,SF}} \right) \theta_{n,SF} \right]^{1-\nu_{SF}}} \tag{23}$$

where $p_{h,SF} \equiv \frac{w_f}{\theta_f}$ is the price per efficiency unit of family time for single females. Proceeding analogously, for married couples these objects are:

$$\sigma_{MC} \equiv \epsilon_{MC} \left(\frac{w_f + w_m + T}{w_f + w_m + T - \Lambda_{MC} I_{MC}(\theta_{kid})} \right) \tag{24}$$

$$\epsilon_{MC} = \frac{(1 - \gamma_{MC})^{\nu_{MC}}}{(1 - \gamma_{MC})^{\nu_{MC}} + \gamma_{MC}^{\nu_{MC}} \left[\left(\frac{p_{h,MC}}{p_{n,MC}} \right) \theta_{n,MC} \right]^{1-\nu_{MC}}} \tag{25}$$

[23] Since early childhood lasts one period here, I drop period subscripts on wages and the price of child care.

where $p_{h,MC} \equiv \frac{w_m}{\alpha_L \theta_m} + \frac{w_f}{\theta_f}$ is the composite price index per efficiency unit of family time for couples.

The higher the elasticity of investment price with respect to the price of child care, the more responsive the marginal cost will be to the child care subsidy, and the larger the expected increase in child skill. This elasticity of the investment price is decreasing in the share on family time, γ_{str}, and, when $\nu_{str} < 1$, decreasing in the ratio of the price per efficiency unit of family time versus chid care time, $\left(\frac{p_{h,str}}{p_{n,str}}\right)\theta_{n,str}$. Family potential income plays an additional role, however, as represented by the income-expenditure ratio multiplying the price elasticity to yield the marginal cost elasticity in equations (22) and (24). This term is increasing in the fraction of potential income spent on investment in children.

Empirical Distributions of σ_{SF} and σ_{MC} Table 6 reports the empirical distribution of the elasticities in (22) and (24), computed using the estimated parameters and ECLS-B data.[24] This table yields three qualitative points. First, the mean and median of the elasticity of investment's price with respect to the price of child care are both higher for single females than for couples, as anticipated from the technology point estimates for each family structure. Second, the fraction of potential income spent on investment in children's skill has a slightly higher mean and median for single females. Third, single females have on average a 37% larger marginal cost elasticity than couples do, and ϵ_{SF} is 33% higher than ϵ_{MC}. Most of the difference across family structures in σ_{str} is driven by ϵ_{str}.

Table 6: Empirical Elasticity of the Marginal Cost of Investment and its Components

	Single Females			Married Couples		
	mean	p50	sd	mean	p50	sd
Investment Price Elasticity ϵ_{str}	0.44	0.43	0.16	0.33	0.33	0.11
$\frac{\text{Potential income}}{\text{Potential income - investment expense}}$	1.19	1.16	0.10	1.14	1.12	0.07
Marginal Cost Elasticity σ_{str}	0.52	0.51	0.19	0.38	0.37	0.13
Observations	150			600		
Families	50			200		

This table: estimation sample moments the elasticity of investment's marginal cost with respect to the price of child care, and its two component parts: the potential income-investment expense ratio and the elasticity of investment's price. Data is waves 1-3, weighted with cross-sectional weights (sample details in Table 3). Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Sample size rounded to nearest 50 per NCES requirements.

5 Investment in Child Skill in General Equilibrium

I now embed the parenting problems of Section 2 in a general equilibrium framework, which endogenizes the distribution of skill, family formation, the altruism term, labor income tax rates, and the price of child care. I also incorporate a baseline child care subsidy, modeled after the CCDF.

[24]For the purpose of these computations, full income inclusive of transfers (not directly observed in the data) is backed out using weekly time endowments of 100 hours per week and hourly wages measured in the ECLS-B. Wages are corrected using slopes, and total potential earnings are adjusted using intercepts, from Table 6 of McGrattan and Prescott (2017). In addition, I use the estimated fixed effects at the family level for relative productivities.

Overview of the Model Environment Besides consumers, there are three other types of agents in the expanded economy: a goods producer, the government, and a non-parental child care provider. The goods producer selects a quantity of efficiency unit of labor subject to a linear production technology with one input (there are no assets in the expanded model). The government chooses a linear labor income tax, τ_y, to finance lump-sum transfers, T, and non-parental child care subsidies. Given a function which determines the intensity of the subsidy upon receipt, $\tau_{n,str}(\cdot)$, a child care policy is defined by a skill threshold, ι_{str}, which is used to determine eligibility, and a probability of subsidy receipt, $\pi_{R,str}$, for each family structure. The non-parental child care sector supplies child care to single females and couples at the amount demanded in equilibrium, at a pre-subsidy price $p_{n,str}$ which is proportional to the distribution of after-tax wages of female parents in each family structure. The price of child care is not allowed to be idiosyncratic to the family, as in section 2. Instead, it now depends solely on family structure. The productivity of this child care still depends on family structure; child care's productivity will be calibrated in the model baseline equilibrium. Families parenting in structure str face a post-subsidy child care price $\tilde{p}_{n,str}$.[25]

The Life Cycle of Consumers Figure 2 illustrates the life cycle of a consumer. Each individual lives for L periods. During their childhood, which lasts for the first J periods of life, an individual makes no decisions: they are a passive recipient of consumption and investment chosen by their parents. Just as in problems (1) and (4) of section 2, children are born with some initial skill θ_1 and investments are made only during early childhood. During the investment phase, families solve the problems described in section 2, with three main changes. First, early childhood lasts one period, and occurs at the start of the parent's adulthood (i.e., I set $J_{ec} = 1$ and $j = J + 1$); second, there is no savings decision; and, third, every child has one sibling of the opposite gender, who is born with the same initial skill and receives the same investment from the family.

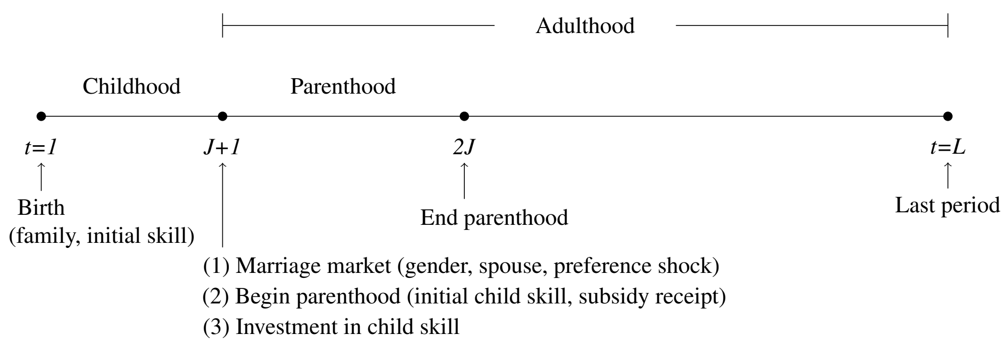

Figure 2: Consumer Life Cycle

This figure illustrates the life cycle of a consumer. At birth, an individual draws their family and their initial skill (shocks at each age are given in parentheses in the figure). Early childhood lasts one period and total childhood lasts J periods. At age $J + 1$, an individual leaves home and participates in a marriage market with their cohort. After forming, families draw their children's initial skill and their family's subsidy receipt outcome. These steps are ordered as listed in the figure above, but instantaneous. Families then invest in their children during age $J + 1$, and split consumption with their children until age $2J$. After that, their children leave home as new adults. Death occurs at age $L + 1$.

[25] In this section, I assume that skill levels are constant in adulthood, so I have removed period subscripts from parental wages and child care prices, compared to the model of section 2.

Upon independence, at the beginning of age $J+1$, the individual leaves the family that raised them with the level of skill they have accumulated, θ_{kid}. At this point, an individual with skill θ_{kid} draws their gender $g \in \{f, m\}$, and afterwards their skill is referred to by θ_g. Next, adults participate in a marriage market and form families. The draw of gender and family formation are instantaneous. For the rest of age $J+1$, single females and couples invest in their children. From ages $J+1$ to $2J$, families also have to support the children by splitting consumption with them (if a single female or a married couple) or making child support payments (if a single male). Investment and sharing consumption are the two ways families make transfers to their children in this model; only investment is a choice. From ages $2J+1$ to L, the problem of the family is a standard lifecycle problem. At age $L+1$, all adults in the family die.

The Marriage Market New adults participate in a marriage market, where individuals randomly draw a potential spouse of the opposite gender from their cohort, and a preference shock ξ which is normally distributed, with mean μ_ξ and standard deviation σ_ξ. This preference shock additively raises the value of the match. A female young adult with skill θ_f and potential spouse θ_m compares the expected value of being a single female and the expected value of being a married female, given $(\theta_m, \theta_f, \xi)$. A male young adult solves a similar problem. The expected utility of each option is found by taking expectations over subsidy receipt ($R = 1$ or 0) and initial skill of their children θ_1, which are unknown when the decision is made and drawn immediately subsequent to family formation. The initial skill of the children is independent of family attributes, but the child care subsidy receipt probabilities are not. If both members of the potential match agree, a couple is formed. This marriage market only happens once.[26] After the marriage market, adults make decisions as part of a family. All of the adults in a family are the same age, so that the age of an individual and the "age" of a family can be referred to interchangeably.

Parenthood Each family raises two children, with the same initial skill draw (single females and couples each raise both a daughter and a son). Whether parenting alone or in a couple, the lifetime utility of any individual contains a term that incorporates rational expectations about the lifetime utility of their children at the level of skill they begin adulthood with. In this altruism term, the expected lifetime utility at adulthood, conditional on the level of skill, is taken over outcomes in the marriage market, the initial skill of one's grandchildren, and subsidy receipt outcomes for one's children when they are parents, for both the male and the female child. A child support system exists, enforced by the government, where single males contribute a fixed amount, T_{cs}, that is redistributed lump-sum and equally to all single females.

Parenting families solve cost-minimization problems to determine how to finance investment, where the prices of investment inputs are distorted by labor income taxes, a gender-specific wedge on the wage, τ_g, and a child care subsidy policy. The gender-specific wedge is included to avoid confounding the gender wage gap with calibrated parameters of the utility function.

Adult skill is the individual's skill outcome from early childhood. This endogenizes the skill distribution in the adult population. Adult wages, rather than being taken as given as in section 2, now arise as linear

[26] Supplementary Appendix B contains transition matrices across waves in the ECLS-B for the mother's marital status. In the data, it is extremely persistent.

functions of skill. In the family problems, value functions are indexed by family structure: single females (SF), single males (SM), and married or cohabiting couples (MC). Family type, z_{str}, records the attributes of a family once all of the shocks in the environment have been realized; the attributes of a family differ from section 2 because I have now introduced child care subsidy receipt and the marriage preference shock. The elements of z_{str} are specified along with the corresponding family problem below.

Single Females A single female of type $z_{SF} \equiv \{\theta_1, \theta_f, \theta_{n,SF}, R\}$, where R is an indicator for subsidy receipt, chooses consumption, c_t, leisure $\ell_{f,t}$, and a child outcome θ_{kid} to solve:

$$\begin{aligned} V_{SF}(z_{SF}) &= \max_{\theta_{kid}, \{c_t, \ell_{f,t}\}_{t=J+1}^{L}} \sum_{t=J+1}^{L} \beta^{t-J-1} u_{SF}\left(\frac{c_t}{\Phi_{SF,t}}, \ell_{f,t}\right) + \mathbb{I}_{t=J+1} b V_{kid}(\theta_{kid}) \\ c_t &\leq w_f (1-\ell_{f,t}) + \mathbb{I}_{t\leq 2J} T_{cs} + T - X_{SF,t}(\theta_{kid}, w_f, \tilde{p}_{n,SF}, z_{SF}) \\ w_f &\equiv (1-\tau_f)(1-\tau_y) w \theta_f \end{aligned} \tag{26}$$

where $\ell_{f,t} \in [0,1]$, and w_f is the return to labor supply for single female with skill θ_f. The consumption equivalence (CE) scales $\{\Phi_{SF,t}\}_{t=J+1}^{L}$ vary over the lifecycle as children leave the household. Here, and in all the family problems, w is the return per unit of skill on the labor market before incorporating distortions from taxes and the gender-specific wedge.[27] If the single female works, then the static cost function of investment incorporates her after-tax market wage w_f, as well as the post-subsidy price of child care:

$$\begin{aligned} \tilde{X}_{SF,t}(w_f, \tilde{p}_{n,SF}, I_t) &= \min_{n_t, q_{f,t}} \tilde{p}_{n,SF} n_t + w_f q_{f,t} \\ s.t. \quad I_{SF}(\theta_{n,SF} n_t, \theta_f q_{f,t}) &\geq I_t \\ \tilde{p}_{n,SF} &\equiv (1-\tau_{n,SF}(z_{SF}, \iota_{SF}, \theta_{p50}))\, p_{n,SF} \end{aligned} \tag{27}$$

where $n_t, q_{f,t}, n_t + q_{f,t} \in [0,1]$. The optimal sequence of investments is determined by solving (3), incorporating expenditures from (27), and the resulting cost in each period is $X_{SF,t}(\theta_{kid}, w_f, \tilde{p}_{n,SF}, z_{SF}) \equiv \tilde{X}_{SF,t}\left(\cdot, I^*_{SF,t}\right)$.

Married or Cohabiting Couples Similarly, a couple with type $z_{MC} = \{\theta_1, \theta_f, \theta_m, \theta_{n,MC}, \xi, R\}$ chooses consumption, c_t, leisure $\ell_{f,t}$ and $\ell_{m,t}$ for each adult, and a child outcome θ_{kid} to solve:

$$\begin{aligned} V_{MC}(z_{MC}) &= \max_{\theta_{kid}, \{c_t, \ell_{f,t}, \ell_{m,t}\}_{t=J+1}^{L}} \sum_{t=J+1}^{L} \beta^{t-J-1} u_{MC}\left(\frac{c_t}{\Phi_{MC,t}}, \ell_{f,t}, \ell_{m,t}\right) + \xi + \mathbb{I}_{t=J+1} b V_{kid}(\theta_{kid}) \\ c_t &\leq w_f(1-\ell_{f,t}) + w_m(1-\ell_{m,t}) + T - X_{MC,t}(\theta_{kid}, w_f, w_m, \tilde{p}_{n,MC}, z_{MC}) \\ w_g &\equiv (1-\tau_g)(1-\tau_y) w \theta_g \quad \forall g \in \{m,f\} \end{aligned} \tag{28}$$

where $\ell_{g,t} \in [0,1]\, \forall g \in \{m,f\}$. The CE scales for couples $\{\Phi_{MC,t}\}_{t=J+1}^{L}$ vary over the lifecycle as

[27] In fact, w acts to simply change units from skill to real quantities in the budget constraint. Its level is set equal to 1, which is equivalent to assuming linear production technology in labor for a goods producer.

children leave the household and also reflect the presence of more than one adult in the family. Market wages for each parent, w_g, are constructed the same way as for single females. If both parents work, then their market wages enter into the cost function, and the optimal combination of inputs is found by solving:

$$\begin{aligned} \tilde{X}_{MC,t}\left(w_f, w_m, \tilde{p}_{n,MC}, z_{MC}, I_t\right) &= \min_{n_t, q_{f,t}, q_{m,t}} \tilde{p}_{n,MC} n_t + w_f q_{f,t} + w_m q_{m,t} \\ s.t. \quad I_{MC}\left(\theta_{n,MC} n_t, \theta_f q_{f,t}, \theta_m q_{m,t}\right) &\geq I_t \\ \tilde{p}_{n,MC} &\equiv \left(1 - \tau_{MC,n}\left(z_{MC}, \iota_{MC}, \theta_{p50}\right)\right) p_{n,MC} \end{aligned} \tag{29}$$

where $n_t, q_{f,t}, q_{m,t}, n_t + \max\{q_{f,t}, q_{m,t}\} \in [0,1]$. The optimal sequence of investments is determined by solving (6), incorporating expenditures from (29). The resulting cost in each period from (28) is $X_{MC,t}\left(\theta_{kid}, w_f, w_m, \tilde{p}_{n,MC}, z_{MC}\right) \equiv \tilde{X}_{MC,t}\left(\cdot, I^*_{MC,t}\right)$.

Single Males A single male of type $z_{SM} = \{\theta_1, \theta_f, \theta_m, \theta_{n,SF}, R\}$ chooses consumption c_t and leisure $\ell_{m,t}$ to solve:

$$\begin{aligned} V_{SM}\left(z_{SM}\right) &= \max_{\{c_t, \ell_{m,t}\}_{t=J+1}^{L}} \sum_{t=J+1}^{L} \beta^{t-J-1} u_{SM}\left(c_t, \ell_{m,t}\right) + \mathbb{I}_{t=J+1} b \mathbb{E}\left(V_{kid}\left(\theta_{kid}\right) | z_{SM}\right) \\ c_t &\leq w_m\left(1 - \ell_{m,t}\right) - \mathbb{I}_{t \leq 2J} T_{cs} + T \\ w_m &\equiv \left(1 - \tau_m\right)\left(1 - \tau_y\right) w \theta_m \end{aligned} \tag{30}$$

where $\ell_{m,t} \in [0,1]$. Here w_m is after-tax wage of the single male with skill θ_m. The CE scales for single males equals 1, to reflect that single males live alone. Single males internalize the expected outcome of their child in an altruism term, given the child's initial skill, the female parent's productivity type, the productivity of child care for single female families, and whether or not the female parent receives the child care subsidy. A single male cannot use his own time to invest in his children, but he is required to make a lump-sum payment to the child's mother, T_{cs}. On the marriage market, where families are formed before θ_1 and R are realized, the single male compares his outside option with the expected value of marrying the women he met, taking expectations over initial child skill and subsidy receipt. Thus, the male makes the marriage decision using the same amount of information about the future as the female. Once a man in this economy rejects his randomly-drawn potential spouse (or is rejected), the child's outcome is no longer affected by his choices, but he does observe the child's initial skill, the subsidy receipt outcome, and the child's outcome as they are realized. All that is reflected in his value function.

Goods Sector Goods are produced by a representative firm that chooses a quantity of efficiency units of labor, H^d, subject to a linear production technology $Y = H^d$. In equilibrium, the wage rate per unit of skill, w, is equal to one, and the profits of the goods producer are zero.

Government The government collects revenue from labor income taxes τ_y to finance lump-sum transfers T and non-parental child care subsidies $\tau_{n,str}(\cdot)$. In particular, the government sets τ_y such that:

$$\tau_y = \frac{T + \overline{\tau}_{n,SF} p_{n,SF} + \overline{\tau}_{n,MC} p_{n,MC}}{E} \tag{31}$$

where $\overline{\tau}_{n,str} p_{n,str}$ are aggregate equilibrium expenditures on the child care subsidy for family structure str, and E is aggregate pre-tax labor earnings. Because the wage rate per unit unit of skill is always equal to one, aggregate earnings are determined by the distribution of skill and the corresponding labor supply decisions of consumers.

Non-parental Child Care Sector The non-parental child care sector provides the demanded aggregate quantity of non-parental child care for each family structure str at price $p_{n,str}$. This pre-subsidy price of non-parental child care is set as a constant fraction $\overline{\phi}_{str}$ of the average return to labor supply for all female parents in family structure $str \in \{SF, MC\}$, $\overline{w}_{str,f}$.

$$p_{n,str} = \overline{\phi}_{str} \overline{w}_{f,str} \tag{32}$$

Because this price depends on the return to labor supply, the price of non-parental child care adjust with the average level of skill in the economy and with the labor income tax. It also responds to the skill composition of parents in one- and two-parent families, which arises endogenously in the marriage market.[28]

Equilibrium Given an initial skill distribution, a distribution of preference shocks, a level of lump-sum transfers, a level of child support transfers, a gender-specific tax on skill, child care productivities for each family structure, and a child care subsidy policy for each family structure, a stationary steady-state equilibrium is:

1. individual marriage decisions, family choices, and family value functions,
2. expectations $V_{kid}(\theta_{kid})$ and $\mathbb{E}(V_{kid}(\theta_{kid}) | z_{SM})$,
3. a goods producer choice of H^d,
4. a government choice of labor income tax,
5. a price of child care for each family structure,
6. and a skill distribution $\mu(\theta_{kid})$

such that agents optimize, expectations are rational, the government balances its budget constraint as in (31), the child care price is set using (32), the goods market clears, the labor market clears, and the distribution of skill is stationary.

[28]The productivity of child care will be calibrated in the baseline economy to be proportional to the skill distribution of mothers in each family structure (for details, see Table 11 in the Appendix).

5.1 Parameterization

In order to fully parameterize the general equilibrium model, I assume functional forms for utility and the distribution of initial child skill. I also assume a functional form for the child care subsidy and estimate its parameters. The functional forms for the skill accumulation technologies are specified as described in Section 2; the parameter values for these technologies are estimated and reported in section 3.

Period Utility Functions Utility functions are defined for single males, single females, and married couples separately:

$$\begin{aligned} u_{SM}\left(c_t, \ell_{m,t}\right) &= \log\left(c_t\right) + \psi_{SM} \log\left(\ell_{m,t}\right) \\ u_{SF}\left(c_t, \ell_{f,t}\right) &= \log\left(\frac{c_t}{\Phi_{SF,t}}\right) + \psi_{SF} \log\left(\ell_{f,t}\right) \\ u_{MC}\left(c_t, \ell_{f,t}, \ell_{m,t}\right) &= \log\left(\frac{c_t}{\Phi_{MC,t}}\right) + \psi_{MC,f} \log\left(\ell_{f,t}\right) + \psi_{MC,m} \log\left(\ell_{m,t}\right) \end{aligned}$$

Distribution of Initial Child Skill I assume that initial child skill is drawn independently and identically from $\pi_0\left(\theta_1\right)$, which I set as a uniform distribution. The empirical motivation for the i.i.d. assumption is based on the correlation between the cognitive score at 9 months of age and after-tax family income and family structure in the ECLS-B. Neither are significantly different from zero.

Baseline Child Care Subsidy The child care subsidy in the baseline equilibrium is disciplined with information on the Child Care and Development Fund (CCDF), one of the largest source of federal funding for child care subsidies in the United States.[29] This program allocates federal funds to states, which are in turn responsible for setting eligibility requirements within some statutory limits. The CCDF subsidy has three salient properties: there is a cutoff for eligibility based on family income and family structure; the subsidy is not an entitlement; and, the intensity of the subsidy is decreasing in family income (i.e., poor families see a higher proportional subsidy that less poor families when both families receive CCDF aid).

To assign eligibility thresholds and receipt probabilities, I rely on the statistics reported in Herbst (2008), Table 2. In particular, I use the fraction of one- and two-parent families who are eligibile for CCDF aid to discipline ι_{str}, and the fraction who receive aid, conditional on eligibility, to discipline receipt lottery probabilities $\pi_{R,str}$. In the baseline, therefore, I set $\{\iota_{SF}, \pi_{R,SF}\} = \{53, 0.28\}$ and $\{\iota_{MC}, \pi_{R,MC}\} = \{21, 0.09\}$.

I estimate the relationship between subsidy intensity and family income relative to the population median, $\tau_{n,str}\left(\cdot\right)$, using administrative data reported by states on CCDF recipients.[30] With this estimated relationship, the subsidy intensity within the model is then determined using the parent skill most closely correlated

[29] Ho and Pavoni (2020) apply a Mirrleesian approach to child care subsidies with a baseline policy that also emphasizes the CCDF. They additionally incorporate the Dependent Care Tax Credit (DCTC), which they consider to be the other large source of price-related child care subsidies at the federal level. I do not model the tax credit programs like the DCTC. Ho and Pavoni (2020) do not include child skill accumulation or heterogeneity in family structures.

[30] Further details on the CCDF and the policy parameterization are in Supplementary Appendix A.

with family income. In single females this is of course the female parent. In couples, this is the male parent. Specificaly, the level of the subsidy is assigned in the model using the following rule:

$$\tau_{n,str}\left(z_{str},\iota_{str},\theta_{p50}\right) = \begin{cases}\hat{\beta}_0^{CC}+\hat{\beta}_1^{CC}\left(\frac{\theta_x}{0.85\times\theta_{p50}}\right)+\hat{\beta}_2^{CC}\left(\frac{\theta_x}{0.85\times\theta_{p50}}\right)^2 & if\, pctl\left(\theta_x\right)<\iota_{str}\, and\, R=1\\ 0 & if\, pctl\left(\theta_x\right)\geq\iota_{str}\, or\, R=0\end{cases}$$

where $x = f$ for $str = SF$ and $x = m$ for $str = MC$, and θ_{p50} is the median skill in the population across all families. Families are eligible for the subsidy if the percentile of the relevant parent skill is below the threshold for their family structure, ι_{str}. Thus, if an ineligible family "wins" the lottery ($R = 1$, where R is an element of z_{str}) the subsidy level is still zero. When I estimate this relationship between subsidy intensity and family income in the data, I find that $\hat{\beta}_0^{CC} = 0.98$, $\hat{\beta}_1^{CC} = -0.29$, and $\hat{\beta}_2^{CC} = 0.03$.

Model Calibration The remaining parameters are divided into three categories: those set exogenously, those set as closed-form functions of other equilibrium objects, and those chosen to bring model moments as close as possible to the data. The first two categories are presented in Table 11 of the Appendix and explained there. The third category contains a set of nine parameters: the altruism coefficient, b, the marginal utility of leisure for each parent in each family structure, $\{\psi_{SF}, \psi_{SM}, \psi_{MC,m}, \psi_{MC,f}\}$, the productivity of investment λ_{str} for each family structure, and the parameters governing the distribution of preference shocks, (μ_ξ, σ_ξ). The nine moments these nine parameters were chosen to match, and the calibration results, are reported in Table 7.

The first two rows of Table 7 contain the correlation of child skill outcomes with after-tax family income and the correlation of child skill outcomes with an indicator for being raised by a couple.[31] The third row is the set of average labor supplies for single males, single females, married/cohabiting males, and married/cohabiting females from the CPS. These averages are computed for observations between the ages of 20 and 65 who are either employed or not in the labor force, and who are the head of household or married to the head of household. The fourth row is the marriage rate among families with young children, and the fifth and sixth rows are the hours per week spent (data) or fraction of time endowment (model) engaging in quality time for each resident parent in a given family structure. Because I am matching the model population with the entire population of ECLS-B parents, the target moments are from Tables 1 and 2.

Although all of these parameters affect each of the moments in Table 7, in practice this calibration has three phases. First, I fix the marginal utilities of leisure to match labor supplies. Second, I choose b, λ_{SF}, and λ_{MC} to match the levels of parental time and the correlation coefficient for child skill outcomes and family income. Finally, I choose μ_ξ and σ_ξ to match both the correlation of child skill with family structure and the fraction of parenting families who are married couples. Because the altruism term also determines the relative value of marriage, b also affects the marriage rate. In addition, because the productivity of investment determines how efficiently investment expenditures translate to child skill outcomes, λ_{SF} and

[31] See Supplementary Appendix A for a description of skill assessments for children in the ECLS-B. Measures of child skill, using age-appropriate assessments, were collected in each wave of the survey.

λ_{MC} also affect the correlation of child skill outcomes with family income and family structure. Thus, I iterate on the last two phases.

I match all of the targeted moments quite well. The moment furthest from the data is the level of parental time from single female parents, which is too low in the model. To match this higher level of time requires either raising the correlation of child skill and family structure or reducing the correlation of child skill and family income, because it requires either reducing λ_{SF} or raising b. Since the family time share parameter is the technology parameter estimated with the lowest degree of precision in section 3, I emphasize matching the correlation coefficients at the expense of the time level for single females.

Table 7: Calibration Moments

Moment	Source	Data	Model
Average labor supply	CPS	$(0.37, 0.33, 0.39, 0.25)$	$(0.37, 0.33, 0.39, 0.25)$
Corr(θ_{kid}, Family Income)	ECLS-B	0.32	0.32
Corr(θ_{kid}, Family Structure)	ECLS-B	0.21	0.21
Fraction Married	ECLS-B	0.79	0.79
Parental time: MC (m, f)	ECLS-B	(8.0,10.1)	(7.6,10.4)
Parental time: SF	ECLS-B	9.5	7.8

Table 7: model calibration moments, sources, data target, and model value in the baseline equilibrium. The parameter values that achieve the equilibrium described in this table are given in Panel C of Table 11 in the Appendix. The CPS data is from a pooled sample that covers 2000 and 2001. ECLS-B refers to: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

Untargeted Moments Table 8 and Figure 3 compare the equilibrium moments of this model with untargeted moments in the data. These moments fall into two broad categories: investment elasticities and family inputs, presented in Table 8, and family formation patterns, illustrated in Figure 3.

Table 8 first reports the elasticities of the investment price and the marginal cost of skill, both in the model and from the ECLS-B (as reported in Table 6). These untargeted statistics are important for the responsiveness of the model to a child care subsidy; they are extremely close to the data for both family structures. It was not a foregone conclusion that simply including the estimated investment technologies in the model would yield such a close match. For example, in reality there may be many kinds of child care available, each with its own price and productivity, but the model presented in this section has one kind of child care per family structure. Table 8 indicates that the model response to changes in the child care subsidy will be in line with what we could expect the empirical population to do.[32]

The last row of Table 8 shows that the random search marriage market captures a quantitatively reasonable relationship of assortative matching among spouses, as measured by parenting behavior within the family. That is, couples are formed in the model such that they choose investment inputs with a similar degree of correlation to the data, when both parents have positive labor supply. Figure 3 demonstrates that the model's relationship between family income and family structure is also quantitatively in line with the

[32]See Supplementary Appendix A for types of child care used in the ECLS-B.

data. Overall, the model is parsimonious yet generates correlation in investment input choices which aligns with its empirical counterparts, while also generating a similar pattern of marriage over the family income distribution.

Table 8: Untargeted Moments

	Moment	Single Females Data	Single Females Model	Married Couples Data	Married Couples Model
Elasticity of Marginal Cost	ϵ_{str}	0.44	0.45	0.33	0.32
and its Components:	$\frac{\text{Potential income}}{\text{Potential income - investment expense}}$	1.19	1.14	1.14	1.12
	σ_{str}	0.52	0.51	0.38	0.36
Investment Choices:	Corr(q_f, q_m)			0.22	0.25

This table: untargeted moments in the data and in the model. From top to bottom, the moments are: elasticity of investment's price with respect to the price of child care, the ratio of potential income to potential income net of investment expenditures, the elasticity of the marginal cost of investment, and the correlation coefficient of quality time from each parent within a couple when both parents work. For the first three rows of statistics, both data and model moments are weighted means. The last row contains weighted Pearson correlation coefficients. The data uses cross-sectional survey weights and is computed for the estimation sample, and the model uses population weights. Data moments for the elasticity of marginal cost and its components are from Table 6. Data Source: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

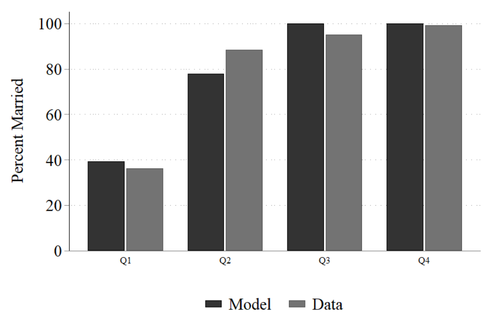

Figure 3: Marriage Rate by Family Income Quartile

This figure: Model moment: fraction of two-parent families in each family income quartile. Data moment: fraction of married or cohabiting families by income quartile, using cross-sectional primary caregiver weights, waves 1-3. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

6 Policy Experiments

How important for aggregate welfare is subsidized child care, in the context of the family heterogeneity emphasized in this study? To address this question, I first consider the effects of the current CCDF program compared to an alternative environment without such a subsidy. I then examine the effects of raising eligibility thresholds and uptake rates for each family structure, as well as the effects of a child care subsidy with universal eligibility and guaranteed receipt. In the baseline, eligibility is restricted to relatively poorer families, and uptake rates are low. Along both of these dimensions, family structure plays a role in determining subsidy receipt. All of these attributes of the baseline economy reflect CCDF program design features. The policy reforms that I examine here are, therefore, a class of practical amendments to the CCDF program, which provides real-world context for the main implications of the analysis.

Welfare gains are computed in the stationary steady state, behind the veil of ignorance at birth. This metric takes into account how the subsidy affects the distribution of families one can be born into, the distribution of potential spouses one may meet at adulthood, and the subsequent different experiences of parenting (both in terms of parenting alone or in a couple, and subsidy receipt). It also means that I evaluate each equilibrium from the perspective of the same "person." Attributes of the economy computed after eliminating the baseline child care subsidy, both in general equilibrium and in several kinds of partial equilibrium, are reported in Table 9. Economies in stationary equilibrium after various reforms to the current child care policy are reported in Table 10. Below I highlight the main implications from these policy experiments.

The Value of Child Care Subsidies Eliminating the baseline subsidy implies welfare losses of 1.75 percent of lifetime consumption (column 2 of Table 9). These losses arise because the types drawn at birth, and at adulthood, become more payoff-relevant when child care subsidies are no longer providing partial insurance against bad draws. This is reflected in the increase in the correlation of child outcomes with family income and family structure. The drop in the stock of skill due to investment becoming more expensive is reflected in the 1.14 percent point drop in output. Considering that eliminating this program removes coverage for fewer than 5 percent of families raising young children, the consequences are strikingly large.

The Role of General Equilibrium In general equilibrium, the economy adjusts to mitigate the welfare losses due to eliminating the child care subsidy. These losses are incurred through changes in the endogenous distribution of skill. The adjustments which mitigate the losses are: the drop in the labor income tax due to lowered government expenditures, which allows each unit of skill to receive a higher return; the decrease in the price of child care, which tracks the decrease in parental skill in equilibrium; and, finally, adjustments to the altruism term so that parents internalize the changing return to skill for their children.

To highlight these opposing forces, the relative importance of endogenizing the price of child care, labor income taxes, the returns to skill for lifetime utility (the altruism term), as well as the skill distribution, can be seen by comparing columns 3 to 6 with column 2 in Table 9. When the price of child care or labor income taxes are held fixed, but the skill distribution and altruism term are endogenized, the economy responds slightly more strongly to the loss of the child care subsidy (columns 3 and 4): welfare losses increase to 2.12 and 2.21 percent, respectively. A larger increase is seen when moving to column 5, where adjustments to the altruism term of parents are shut off. There, welfare losses reach 2.66 percent of lifetime consumption. All of this is due to not allowing the economy to adjust to the changing skill distribution. To see that explicitly, note that if the skill distribution itself is also held fixed, as in column 6, welfare losses are quite small by comparison: 0.83 percent. This case can be interpreted as the partial equilibrium response of the economy to a loss of the current child care subsidy. Once children grow up to raise their own families, these losses compound, which can be seen in column 5. However, family investment behavior also changes in response to the new reality, so that in general equilibrium the losses are mitigated because objects which aggregate these changes adjust. This is seen in column 2.

Table 9: The Value of Current Child Care Subsidies

Fixed Objects	(1) Baseline	(2) GE	(3) $p_{n,str}$	(4) $p_{n,str}, \tau_y$	(5) $p_{n,str}, \tau_y, V_0$	(6) $p_{n,str}, \tau_y, V_0, \mu(\theta_{kid})$
Corr(θ_{kid}, Y)	0.32	0.35	0.36	0.36	0.36	0.36
Corr($\theta_{kid}, \mathbb{I}_{MC}$)	0.21	0.24	0.24	0.24	0.25	0.24
Δ Output (%)	0	-1.14	-1.40	-1.42	-2.04	-0.89
Subsidized (%)	4.58	0	0	0	0	0
Δ Welfare	0	-1.75	-2.12	-2.21	-2.66	-0.83

This table: equilibrium attributes and welfare changes, both in GE and holding various equilibrium objects fixed, after eliminating the baseline child care subsidy. Changes are relative to the baseline economy, which is reported in column 1. Column 2 is the general equilibrium with no subsidy. Columns 3-6 also have no subsidy; moving from left to right, each column holds fixed an additional equilibrium object to highlight its role. These objects are p_n, p_n and τ_y, p_n, τ_y and V_0, and p_n, τ_y, V_0 and $\mu(\theta_{kid})$, respectively (recall that $\mu(\cdot)$ is the distribution of skill outcomes for kids). Table rows from top to bottom show the correlation of child skill outcomes with family income, the correlation of child skill outcome with family structure, the percent change in total output compared to the baseline, the percent (mass) of families raising young children who receive the subsidy in equilibrium, and the welfare change behind the veil of ignorance in consumption equivalent units, relative to the baseline.

Attributing Welfare Losses from Eliminating the Baseline Subsidy The first two policy changes reported in Table 10 indicate that the baseline subsidy for single females yields the highest return per recipient family. To see why, note that in the baseline child care subsidy for couples reach 1.49 percent of families raising young children, and the subsidy for single females reaches 3.12 percent of that group: the mass of single female subsidy recipients is twice the corresponding mass of couples. Yet, columns 2 and 3 of Table 10 illustrate that eliminating the baseline subsidy to single females (column 3) leads to welfare losses which are four times higher than those incurred from eliminating the baseline subsidy to couples (column 2).[33] This is a much larger effect than the relative masses of recipients would suggest. To see the sources of these welfare losses, note that eliminating the subsidy to couples has little effect on risk in the economy.[34] At the same time, eliminating it leads to a 0.25 percent drop in output, because of slight decreases in skill outcomes, which explains the slight welfare loss. By comparison, eliminating the subsidy for single females causes a significant rise in risk in the environment and nearly a one percent drop in output (column 3).

Targeted Welfare-Improving Expansions of Current Program If a policy maker is considering expanding the child care subsidy program, and deciding between expanding eligiblity or improving uptake for those already eligible, where should they direct their efforts? In columns 4 to 7 of Table 10, I report welfare gains and the attributes of equilibrium after expanding the eligibility percentile cutoff ι_{str} or uptake rate $\pi_{R,str}$ to their highest possible values (100 and 1, respectively), for either family structure. In column 8, I report the same statistics for the most generous policy I consider: universal eligibility and guaranteed receipt for all families. In each case, I hold the other child care policy instruments fixed at their baseline levels, but

[33]Note that the change in the mass of recipients takes into account any shift in the marriage rate. This shift is usually very small.

[34]The mass of subsidized single females is slightly lower in column 2 than in the baseline. This is due to discretization in the quantitative analysis.

allow all of the equilibrium objects to adjust. In this framework, welfare changes are monotonic in these instruments, so setting them to their highest possible value highlights the qualitative possibilities of an entire class of reforms for generating welfare gains.

Table 10: Reforms to Baseline Child Care Subsidy Design

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Child Care Subsidy	Baseline	SF	MC	ι_{MC}	ι_{SF}	$\pi_{R,MC}$	$\pi_{R,SF}$	Full
Corr(θ_{kid}, Y)	0.32	0.32	0.35	0.32	0.31	0.27	0.23	0.18
Corr($\theta_{kid}, \mathbb{I}_{MC}$)	0.21	0.21	0.25	0.22	0.20	0.24	0.13	0.15
Δ Output (%)	0	-0.25	-0.98	0.35	0.22	2.56	2.54	10.31
Subsidized (%)	4.58	3.08	1.50	10.21	7.32	19.73	12.67	100
Δ Welfare	0	-0.38	-1.49	0.48	0.29	3.85	3.86	15.41

This table: equilibrium attributes and welfare changes for the stationary steady state under various child care policy designs, relative to the baseline economy that is reported in column 1. Column 2 is the general equilibrium with only the baseline subsidy for single females, and column 3 is the general equilibrium with only the baseline subsidy for couples. Column 4 is the general equilibrium with universal eligibility for couples, column 5 is the general equilibrium universal eligibility for single females, column 6 is the general equilibrium with guaranteed receipt for couples, and column 7 is the general equilibrium with guaranteed receipt for single females. Column 8 is universal eligibility and guaranteed subsidy receipt for all families. Table rows from top to bottom show the correlation of child skill outcomes with family income, the correlation of child skill outcome with family structure, the percent change in total output compared to the baseline, the percent (mass) of families raising young children who receive the subsidy in equilibrium, and the welfare change behind the veil of ignorance in consumption equivalent units, relative to the baseline.

Holding uptake rates fixed, columns 4 and 5 of Table 10 show that increasing the eligibility threshold of either family structure has limited potential for increasing welfare: a universal eligibility policy for couples yields a 0.48 percent gain in welfare, and for single females the gains are 0.29 percent of lifetime consumption. These low welfare gains are due to the fact that such a policy does not provide much insurance against a bad draw at the beginning of life, and that expanding eligibility sends funds to families who respond less to the policy. By contrast, increasing uptake rates with the current eligibility rules has the potential to substantially increase welfare. Guaranteed receipt for couples increases aggregate welfare by 3.85 percent (column 6); guaranteed receipt for single females increases aggregate welfare by 3.86 percent (column 7).

Raising eligibility thresholds for one-parent families yields higher welfare gains per additional recipient family. Specifically, a comparison of welfare gains in Table 10 is facilitated by taking into account the mass of families that receive the subsidy under each policy (fourth row of the table). While the welfare gains from a universal eligibility for single females are slightly more than one-half of the gains from a universal eligibility rule for couples, the former policy results in far fewer families being covered (7.32 versus 10.21 percent of parenting families, respectively). This is because single females represent a much smaller fraction of the families raising young children. In fact, the welfare gain per additional family served is slightly higher in column 5 than in column 4 (0.11 and 0.09, respectively).

This pattern is even more apparent when comparing the gains from increasing the receipt probability to 1 for couples or single females (columns 6 and 7, respectively), while maintaining baseline eligibility cutoffs. Here, the welfare gains per additional family served are 1.92 times higher in column 7 compared to column 6 (0.48 and 0.25, respectively). Thus, targeting child care subsidy expansions to low-skill single females raising young children is more efficient at improving welfare than a similarly designed policy targeted at low-skill couples. This finding reflects the greater sensitivity of one-parent families to child care's price when selecting how much to invest in their children. On the other hand, the calibrated model I analyze here indicates that marriage rates are unlikely to change substantially in response to these subsidies; this lowers the upper bound on the benefits of targeting to single female families, because the mass of this type of family structure is limited in equilibrium.

The most generous child care subsidy I consider in Table 10 is universal eligibility with guaranteed subsidy receipt for all families (column 8). This policy lowers the correlation coefficient of child outcomes and family income the most among the policies I report here, but does not affect the correlation of child outcome and family structure as much as single female-specific subsidies (column 7). Nevertheless, the gains in output and welfare are the highest of the policies I consider: 10.31 and 15.41 percent, respectively. The magnitude of these welfare gains is not surprising: children in this model are expensive, families are altruistic but varied in their ability to finance investment, and adult skill is very important for one's wellbeing. Child care subsidies encourage child skill accumulation by making it cheaper to invest in young children; the government has a special technology (the tax system) which allows it to then recoup the cost via labor income taxes.

The interpretation of a policy which increases eligibility cutoffs is relatively straightforward, because this involves an instrument that policymakers can control directly. By comparison, increasing subsidy receipt is a more nuanced policy objective. What does it take to increase uptake of CCDF aid among eligible families? This depends on the reasons for which the measured uptake rates are low. One interpretation of these rates is that they reflect rationing, because aggregate funding caps are too low. In that case, increasing subsidy receipt in the model maps into increasing aggregate funding for the CCDF. At the same time, low measured uptake could also reflect the complexities of a system which is difficult for families to navigate: they may not know they are eligibile, or they may not understand the steps which are necessary to receive aid. In that case, increasing clarity in the rules which govern eligibility and receipt for CCDF aid, as well as funding efforts to inform families about their options, would be approaches by which one could increase uptake rates. In practice, it is likely that both low aggregate funding and information frictions need to be addressed in order to achieve an increase in subsidy receipt.

7 Conclusion

This paper studies child skill accumulation in one- and two-parent families, and how this interacts with child care subsidies. I first specify two skill investment functions and, based on hitherto underused ECLS-B data, empirically test and reject the hypothesis that one- and two-parent families use the same technology to invest in their children's skill. In particular, I find that the investment function of single females emphasizes non-parental child care more than the investment function of married couples. Using the estimated elasticity of investment's price with respect to the price of child care, I highlight an implication of the empirical finding for a widely used policy intervention intended to increase child skill: child care subsidies. The estimation implies that one-parent families are more responsive to child care subsidies than two-parent families.

To perform policy experiments, I expand the model to include endogenous family formation and a child care subsidy which is financed by a labor income tax. The baseline child care subsidy is modeled to reflect the design of the current Child Care and Development Fund, one of the largest sources of federal funds for price-based child care subsidies in the United States. I find that the current policy yields sizable welfare gains, especially through its coverage of families headed by single females. The general equilibrium framework I construct allows for equilibrium adjustments to dampen the welfare losses from removing the subsidy. I also examine the potential benefits of several program expansions, and find that expanding uptake among single female families yields the highest welfare gains per additional family covered. However, because single female families make up about 21 percent of those raising young children in the baseline economy, and marriage rates are insensitive to the subsidy policy in the framework I analyze here, limiting access to single female families alone also limits the potential gains from the policy, because it excludes many families who would also benefit. A policy with universal eligibility and guaranteed subsidy receipt for all families achieves welfare gains of more than 15 percent of lifetime consumption.

Although the framework and results of this paper make novel contributions to the literature on skill accumulation during early childhood, some caveats should be noted. For example, in reality parents are able to make wealth transfers to their children, which could complement or substitute transfers through skill investment during early childhood. In addition, the child care producer in this paper's general equilibrium framework is a simplified version of the market for child care. Large-scale policies that affect the use of child care can feasibly have an effect on the economy through redirection of labor towards providing paid child care, and the productivity of child care time (not just its price) may also respond directly to policy. Who decides to make a living looking after other people's children? How can we encouraged qualified workers to make this choice? Answering these questions would further inform child care policy design.

Appendix

General Equilibrium Model: Further Details on Parameterization All parameters for the model of section 5, besides those of investment technologies, are presented in the panels of Table 11: fixed parameters in Panel A, "fixed" calibrated parameters in Panel B, and "free" calibrated parameters in Panel C.

Beginning with Panel A, the discount (patience) factor is set to an annual value of 0.96 to match the presumed risk-free interest rate. There are 13 periods of life, each 5 years long, 4 of which are spent in childhood and 9 in adulthood. The first period of life corresponds to early childhood. The gender-specific tax is set so that $\frac{1-\tau_f}{1-\tau_m}$ is equal to the ratio of weekly earnings for full-time women and full-time men, aged 16 and over (Bureau of Labor Statistics of the United States (2002)), with the male tax normalized to zero. The CE scales are taken from the 1994 scales from the Organization for Economic Co-operation and Development (OECD). These scales assign a value of 1 for the first adult and 0.5 for the subsequent adults; for each dependent the weight is 0.3.[35] The remaining parameters in Panel A specify the child care subsidy in the baseline economy (see Supplementary Appendix A for details). Although not restated here, the technology parameters estimated in section 3 are also taken as fixed parameter inputs into the model of section 5.

Panel B presents the parameters for which there is a closed-form expression as a function of other equilibrium objects. The level of lump-sum transfers T and the level of child support payments T_{cs} are set to 8% of output and 45% of the average per-family transfer, respectively. The first empirical target is from the ratio of government transfers to persons for federal benefits from social insurance funds, Supplemental Nutrition Assistance Program (SNAP), supplemental security income, refundable tax credits, and "other" (which includes payments to nonprofit instutitions and student loans, among other categories) to GDP from the National Income and Production Accounts (NIPA) tabulations for 2001. The second is the ratio of average child support payments owed (per month per capita) to average government transfers (per month per family) from the Census Bureau (Grall (2003)).

Next, the parameters governing the price of child care relative to mother wages are set so that in the baseline equilibrium, the after-tax and post-subsidy ratio of child care prices to mother wages for mothers who work matches moments from the ECLS-B. This implies an average ratio of pre-subsidy child care prices and mother wages for all mothers in equilibrium. This latter ratio is what I consider to be the environment primitive, and I hold it fixed at the baseline value as I vary the child care policy design. The last set of parameters in Panel B are the productivities of child care for single females and married couples, respectively. These are set so that, in the baseline equilibrium, the same moment that I used to identify the technology share parameters holds in the model; this moment assumes that the average log of productivity ratios is zero within each family structure. The resulting calibrated productivity levels for child care are then held fixed as I vary the child care subsidy policy design away from the baseline.

Panel C reports the calibrate parameters which I set so that model moments match moments from the data. They are assigned using a calibration procedure which is described in the main text.

[35]The CE scales adjust money spent on consumption into units of consumption for each member of the household. Once children leave the family, the CE scale for single females goes back to 1, and the scale for couples falls to 1.5.

Table 11: Parameter Values

	Symbol	Name	Source	Value
A. Fixed:	β	Patience	Assumption	0.96^5
	L	Periods of life	Death after 65	13
	J	Periods of childhood	20-year childhood	4
	(τ_f, τ_m)	Gender wage gap 2001	BLS report	$(0.24, 0)$
	$\{\Phi_{SF,t}\}_t$	CE Scales	OECD	$\{1.6\}_{t\leq 2J}, \{1\}_{t>2J}$
	$\{\Phi_{MC,t}\}_t$	CE Scales	OECD	$\{2.1\}_{t\leq 2J}, \{1.5\}_{t>2J}$
	ι_{str}	CCDF Frac. Eligible	Herbst (2008), T.2	(21,53)
	$\pi_{R,str}$	CCDF Receipt	Herbst (2008), T.2	(0.09,0.28)
	$\hat{\beta}_j^{CC}$, $j \in \{0,1,2\}$	CCDF Subsidy	CCDF Admin. Data	(0.98,-0.29,0.03)
B. Calibrated: (fixed)	T	Transfers	NIPA	8% of output
	T_{cs}	Child Support	Census, NIPA	45 % of T
	$\overline{\phi}_{SF}, \overline{\phi}_{MCF}$	$\frac{p_{n,str}}{w_{f_{str}}}$	Post-subsidy price ratios	0.20, 0.31
	$\theta_{n,SF}, \theta_{n,MC}$	Prod. of child care	Ave. relative prod.	0.27, 0.34
C. Calibrated: (free)	ψ_{SM}, ψ_{SF}	MU_ℓ Singles	See Table 7	$1.45, 1.48$
	$\psi_{MC,m}, \psi_{MC,f}$	MU_ℓ Couples		$0.73, 0.77$
	b	Altruism coeff.		$0.315 \times \beta^{J-1}$
	$\lambda_{SF}, \lambda_{MC}$	Prod. of inv.		$8, 3.9$
	μ_ξ, σ_ξ	Pref. shock dist.		$6.7, 7.9$

Table 11 presents model parameters in 3 groups: (A) fixed parameters whose values are exogenous to attributes of the model equilibrium; (B) calibrated parameters whose values are determined in the model equilibrium but which are perfectly identified by a single model equilibrium moment; and (C) free parameters whose values are chosen so that equilibrium model moments match data moments, but which are not perfectly identified by a single model moment. ECLS-B Data Source: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

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