Decomposing the Impact of Child Care Subsidies on One- and Two-Parent Families

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First version: February 26, 2020
This version: March 25, 2020

Abstract

Child care subsidies have the potential to improve long-run labor market outcomes for children by encouraging investment in their skill. Studies of large-scale subsidized child care programs have shown that the resulting gains in child skill are higher for children raised by one-parent families compared to two-parent families, and are decreasing in family income (which is in turn correlated with family structure). This paper decomposes the gap in child skill gains across one- and two-parent families into two components: a portion due to exogenous differences in the technologies used to invest in child skill, and a portion due to endogenous differences in family income. To do this, a model is constructed in which one- and two-parent families form endogenously and altruistically invest in their children's skill, using both their own time and time purchased on the market in the form of child care. Besides differences in income composition arising from endogenous family formation, the two family structures differ in how their time inputs affect their children's skill through an investment technology. This allows for both exogenous and endogenous differences across family structures to cause child care subsidies to have different effects. The two skill investment technologies are estimated using longitudinal survey data from the US Department of Education; the results indicate that attributes of their investment technology make the skill investment decisions of one-parent families more sensitive to the price of child care. With these estimates, the model is calibrated to match aggregate moments on marriage and child skill accumulation in equilibrium, so that the model captures the empirical correlation between family structure, family income, and investment in children's skill. The estimated model framework is used to predict each family's response to a proportional child care subsidy of between 0 and 100 percent, and to decompose the difference across one- and two-parent families in the subsidy's impact on child skill. The main result is that exogenous technological differences drive between 63 and 92 percent of the gap in skill gains, depending on the level of the subsidy.

Keywords: Skills, Children, Family Structure, Intergenerational Transfers
JEL Classification Numbers: J24, J13, J12, D6

*William & Mary. *email:* egmoschini@wm.edu. *Acknowledgements*: I would like to thank Anmol Bhandari, Elizabeth Caucutt, Loukas Karabarbounis, Karen Kopecky, Jeremy Lise, Lance Lochner, Ellen McGrattan, Peter McHenry, Joseph Mullins, and Sergio Ocampo Diaz for their comments. A previous version of this paper was circulated under the title "Child Care Subsidies with One- and Two-Parent Families". All errors are mine. The Early Childhood Longitudinal Study data used in this paper were obtained through a restricted-use data license from the U.S. Department of Educations National Center for Education Statistics.

1 Introduction

Early childhood is widely accepted as an important time in the development of skill, with long-run repercussions for labor market outcomes. In light of this, child care subsidies are often suggested as one avenue by which policy makers can encourage families to invest in their young children. At the same time, recent studies of large-scale child care subsidies in Canada and Norway have uncovered heterogeneity in the effects of these programs on gains in child skill, with one-parent families and poor families seeing larger gains than two-parent or high-income families (Kottelenberg and Lehrer (2017), Havnes and Mogstad (2015)). In the United States, one-parent families with young children are overrepresented among the poor: family structure and family income are correlated [1]. In this paper, I use a model framework to compute and then decompose differences in the effects of child care subsidies on child skill accumulation across one- and two-parent families. The decomposition attributes the gap to two components: a component due to exogenous differences in the technologies used to invest in child skill, and a component due to endogenous differences in family income.

To do this, I construct a model in which one- and two-parent families form endogenously and altruistically invest in their children's skill, using both their own time and time purchased on the market in the form of child care. Besides differences in income composition arising from endogenous family formation, the two family structures differ in how their time inputs affect their children's skill as inputs into an investment technology. This allows for both exogenous and endogenous differences across family structures to cause child care subsidies to have differing impacts.

I estimate the parameters of the two skill investment technologies using longitudinal survey data from the US Department of Education, and find that attributes of their investment technology mean one-parent families are more sensitive than couples to the price of child care when making skill investment decisions for their children. My main data source for this estimation is the Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), which is an individual-level panel data set from the US Department of Education. This data set allows me to observe parental and market time inputs, parental wages, child care prices, and family structure for children aged 9 months through kindergarten entry. Unlike other data sets commonly used to estimate skill accumulation technologies during early childhood, the ECLS-B is designed to be representative of families raising 9-month old children in the United States. The panel structure of the ECLS-B allows me to use a fixed effects estimator, which accounts for time-invariant, family-specific, and unobserved input productivities that may be correlated with input prices.

With these estimates, I calibrate model to match aggregate moments on marriage and child skill accumulation in equilibrium, so that the model captures the empirical correlation between family

[1] Single mothers are 40% of families below the poverty line, although they are only about 20% of all families with children under 5 (author's calculations, ECLS-B).

structure, family income, and investment in children's skill. I use the estimated model framework to predict each family's equilibrium response to a proportional child care subsidy of between 0 and 100 percent, and to decompose the difference across one- and two-parent families in the subsidy's impact on child skill. I find that exogenous technological differences drive between 63 and 92 percent of the gap, depending on the level of the subsidy.

This paper makes three contributions to the literature. First, it contributes to the reduced-form literature on the heterogeneous effects of universal child care by decomposing these differences into two components: technology and composition. Second, it contributes to the literature on skill accumulation technologies by providing novel estimates of how these technologies differ across family structures, using data especially suited to the task. Third, this paper contributes to the macroeconomic literature on gains from large-scale child care subsidies by extending a standard framework to include endogenous family formation and heterogeneity in the technologies used to produce child skill. With this extension I am able to incorporate the empirical finding from the reduced-form literature that one-parent families are a group especially sensitive to the price of child care when making investment decisions. I can then use the model to find that it is their technology - not their income composition due to marriage market sorting - that is primarily driving this higher sensitivity. Future macroeconomic models seeking to incorporate a sufficient degree of sensitivity to child care prices can build on this by incorporating several types of investment technologies. According to my findings, the way that agents sort into using these technologies to parent (via the marriage market) is not as important as incorporating the heterogeneity itself.

The paper proceeds as follows. Section 2 is a brief review of the related literature to put this study in a larger context. In Section 3, I lay out the model. Section 4 presents the skill investment technologies, as well as the estimation equations used to find values for their parameters. It also contains a description of the main data sources, the results of the estimation, and a brief discussion of the results. In Section 5, I lay out how the remaining parameters of the model are identified. In Section 6, I explain the policy exercise and discuss its results. Section 7 concludes.

2 Related Literature

The literature on child care subsidies as a tool for improving child skill is founded on small-scale randomized controlled trials (RCTs), especially the Abecedarian program. The Abecedarian is a randomized trial of child care, with an adult follow up, where the child care was provided for 10 hours a day and the children were in their first year of life at entry. Although many studies have analyzed the Abecedarian program, one of the most recent is Garcia, Heckman, and Ziff (2018). In that study, the authors document statistically significant gain in IQ for both female and male children from the program, although girls benefit slightly more (Table 4 of that study). Although these results are encouraging, the targeted nature of the program and smaller sample sizes means that inference to the effects of a scaled-up policy from these results is not straightforward.

Large-scale subsidized child care programs have been implemented around the world. Examples include Oklahoma (started in 1998), Quebec (1997), Argentina (1993), Norway (1975), and Denmark (1964). Empirical evaluations of these large-scale child care subsidies include Baker, Gruber, and Milligan (2008), Gupta and Simonsen (2010), Berlinski, Galiani, and Gertler (2009), Havnes and Mogstad (2015), and Kottelenberg and Lehrer (2017). The two most recent of these papers used variation across geographic locations in access to subsidized child care to identify its effects, in Norway and Canada respectively. Both papers emphasized heterogeneity in the treatment effects of these programs: Havnes and Mogstad (2015) showing that the effects on lifetime earnings largest for children of poor families, and Kottelenberg and Lehrer (2017) showing that the effects on child development scores are higher for children of single mothers than for children of couples.

This paper contributes to the macroeconomic literature on gains from child care subsidies by extending a standard framework to include endogenous family formation and heterogeneity in the technologies used to produce child skill. This extension incorporates heterogeneity in both family structure and family income, qualitatively incorporating the empirical finding from the reduced-form literature that one-parent families are a group especially sensitive to the price of child care when making investment decisions. It is an augmented framework that is standard in the macroeconomic literature on families investing in their children: other studies with similar model environments include Restuccia and Urrutia (2004), Lochner and Monge-Naranjo (2011), Guner, Kaygusuz, and Ventura (2016), Gayle, Limor, and Soytas (2017), Daruich (2019), Caucutt and Lochner (2017), Abbott, Gallipolli, Meghir, and Violante (2018), and Lee and Seshadri (2019). The decomposition exercise performed in this paper highlights that it is their technology - not their income composition due to marriage market sorting - that is primarily driving this higher sensitivity. Future models seeking to incorporate a sufficient degree of sensitivity to child care prices can build on this by incorporating several types of investment technologies. According to my findings, the way that agents sort into using these technologies to parent (via the marriage market) is not as important as incorporating the heterogeneity itself.

The skill accumulation technologies determine the law of motion for child skill, and they are the critical primitives in this paper's framework. Here, investment combines with current skill in the same way for all families, but investment is generated with a different aggregation function across family structures. The specification of inputs into investment used in this paper is unusual in that, instead of allowing money to be an input into investment, all of the inputs are in units of time. This specification reflects the empirical fact that most of the money spent on children is actually spent on child care, which takes up the child's time and leaves less time with his or her parents. Such tradeoffs are not incorporated in a specification that includes only money and one source of time input. Theoretically, allowing for the child's time constraint to bind is not trivial, and in this paper a solution is provided for the cost-minimization problem of parents when this is the case. Besides the novel input specification and the corresponding interpretive usefulness of the estimates, this paper's estimation contributes to the literature on skill accumulation technologies by providing estimates of how these technologies differ across family structures, using data especially suited to the task.

3 The Model

There are four sets of agents in the economy: consumers, a representative firm, the government, and a non-parental child care provider. Consumers are grouped into families with either one or two parents, who altruistically invest in their child's skill with their own time and purchased child care time, as well as choosing consumption, savings, labor supply and leisure. The way time inputs affect children's skill is determined by a technology, which is indexed by the number of parents in the household. For each family structure, the technology takes inputs that determine the time use of the child, as well as the current stock of the child's skill, and produces the skill of the child in the next period.

Given prices for labor and capital, the firm chooses labor and capital inputs to maximize profits. This firm produces with a Constant Returns to Scale (CRS) technology and takes prices as given. The government chooses labor income taxes to finance lump-sum transfers and non-parental child care subsidies. A child support system exists, enforced by the government, where single fathers contribute a lump-sum amount that is redistributed lump-sum and equally to all single mothers. Finally, the non-parental child care sector supplies child care at the amount demanded in equilibrium, at a price equal to some fraction of the average hourly wage.

The Life Cycle of Consumers

Each individual for $T + J$ periods. During childhood, which lasts for the first J periods of life, an individual makes no decisions: she is a passive recipient of consumption and investment chosen by her family. Upon independence, at the end of age J, the individual leaves with the level of skill she has accumulated by then to start the remaining T periods of her life as an independent decision-making adult. The first J periods of adulthood are spent either actively parenting children (if a single mother or a married couple) or making child support payments (if a single father). From periods $J + 1$ to T, the problem of the consumer is a standard lifecycle problem.

Families are formed at the beginning of adulthood, when everyone participates in a marriage market that occurs instantaneously after the end of period J and before the start of period $J + 1$. On the marriage market, a potential match is drawn randomly from the skill distribution of the other gender in the same generation. Once assigned a potential spouse, and knowing that parenthood is certain in the environment, the agent compares the expected present discounted value of parenting alone or in a couple.

The gains from joining a couple are reflected by higher efficiencies in translating income into consumption (introduced with consumption equivalence scales) while the costs are reflected by the fact that spouses must compromise on time use. This compromise occurs because the level of leisure for couples must be the same, and both parents must contribute time to generating investment in their children using the two-parent technology. Being a single parent, meanwhile, is an outside option to marriage that differs by gender. For a woman, single parenthood means that she keeps her children with her, using the one-parent technology to invest in them, and receives lump-sum child support transfers from single fathers. For a man, single parenthood means that he cannot directly affect his child's skill with his time use, but does have to pay a lump-sum child support tax to the single mother.[2] Single father's have accurate expectations about the sort of children their potential spouse will raise, conditional on the child's initial skill, and altruistically internalize the expected outcome of their children. Likewise, the expected value of joining a couple is the expectation of that marriage's lifetime utility over the initial skill of the child, which is drawn from an exogenous distribution and is unknown when the marriage decision is made. The predictive power of a child's initial skill for lifetime utility, however, is endogenously determined: this is the channel by which policy affects the family formation decision.

Once each member of a potential couple has compared their two alternatives, a marriage is formed if both the husband and wife accept the match (the two individuals remain single otherwise). After the marriage market, single mothers and couples draw the initial skill of their two children, which is the same for both children (single mothers and couples each raise both a son and a daughter). Whether parenting alone or in a couple, the lifetime utility of any individual contains a term that incorporates rational expectations about the lifetime utility of one's child at the level of skill they begin adulthood with.[3] In the altruism term, the expected lifetime utility at adulthood, conditional on the level of skill, is taken over the potential spouses one might meet (using the distribution of skill in the economy, which is endogenous) and also over the distribution of initial skill one's child may be born with (which is exogenous).

Each family starts life with no wealth and dies with no wealth, because there are no financial bequests in this economy. Instead, families make transfers to their children by increasing their skill. Before the last period of life, there are no borrowing constraints: borrowing constraints on the parents are not a source of market incompleteness in this model. To summarize, the shocks in the life of an individual are their own initial skill, their gender, the family that raises them, their potential spouse, and the draw of their children's initial skill. Figure 1 illustrates the timing of these shocks over the lifetime of the consumer.

[2] The outside option to parenting in a couple differs by gender in the model because, empirically, the vast majority of single parents who are raising young children in their home are women. For a discussion of what the ECLS-B offers in terms of discipline on contributions of parental time from single fathers, see Appendix O.

[3] This is what makes parents altruistic. An alternative way of incentivizing intergenerational transfers is through paternalistic preferences, or "warm glow" returns (Andreoni (1990)). The benefit of motivating parents with altruism is that the returns to investment can respond endogenously to policy, because parents fully incorporate the economic returns to their investment in terms of their child's lifetime utility, and their behavior changes accordingly. The main benefit of a paternalistic specification is its tractability and flexibility in matching parenting behaviors. Some models combine the two, and include both altruism and a paternalistic preference for, say, college attainment which is distinct from its monetary returns. For an application of paternalistic preferences to intergenerational transfers of wealth, see De Nardi (2004).

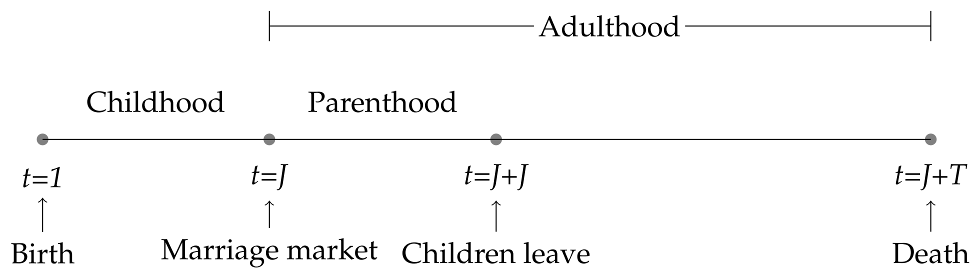

Figure 1: Life Cycle of the Consumer

One- and Two-Parent Families

The three family structures differ in their efficiencies of consumption, $\{\phi_t^{SM}\}_{t=1}^T$ and $\{\phi_t^{MC}\}_{t=1}^T$, which are consumption equivalence scales that vary over the lifecycle as children leave the household (with those of the single father are always equal to 1). The different types of families are also allowed to have different marginal utilities of leisure (which will be reflected in the parameters of the period utility functions $\left(u^{SF}, u^{SM}, u^{MC}\right)$, whose functional forms are defined in the model parameterization section. For single mothers and married/cohabiting couples, who raise children, each family structure uses a skill accumulation technology specific to two-parent families to invest in their children, $\theta_{t+1} = f\left(\theta_t, I_t^{type}\right)$), where $type$ is either SM or MC. The terminology "skill investment technology" refers to the function that governs how I_t^{type} is generated from the chosen levels of inputs. The term "skill accumulation technology" refers to the function that governs how child skill θ_t and investment I_t^{type} in period t combine to generate child skill in period $t+1$.

Family Problems

The solution to a family's life-cycle problem is a set of choices and lifetime utilities. For every potential couple $\{\theta_m, \theta_f\}$ and each possible draw of the initial child skill θ_1. They are indexed by family structure type:

- For single mothers: $V^{SM}(\theta_m, \theta_f, \theta_1)$
- For single fathers: $V^{SF}(\theta_m, \theta_f, \theta_1)$
- For married couples: $V^{MC}(\theta_m, \theta_f, \theta_1)$

A female young adult with skill θ_m and potential spouse θ_f compares are the value of being a single mother, $\mathbb{E}_{\theta_1}\left[V^{SM}(\theta_m, \theta_f, \theta_1)\right]$ and the value of being a married mother, $\mathbb{E}_{\theta_1}\left[V^{MC}(\theta_m, \theta_f, \theta_1)\right]$. A male young adult with skill θ_f and potential spouse θ_m compares are the value of being a single father, $\mathbb{E}_{\theta_1}\left[V^{SF}(\theta_m, \theta_f, \theta_1)\right]$ and the value of being a married father, $\mathbb{E}_{\theta_1}\left[V^{MC}(\theta_m, \theta_f, \theta_1)\right]$.

The Marriage Decision

The decision rule that maps from the type of the spouse to a yes or no marriage market decision represented by $d_{mm}^{g}\left(\theta_m, \theta_f, g\right) \in \{0,1\}$. It takes as given the skill θ_m or θ_f and gender g of the decision maker). It solves:

$$
\begin{aligned}
d_{mm}^{f}\left(\theta_m, \theta_f\right) &= \underset{\delta \in \{0,1\}}{\arg\max}\left[\delta \int_{\theta_1} V^{MC}\left(\theta_m, \theta_f, \theta_1\right)\pi\left(\theta_1\right)d\theta_1 + (1-\delta)\int_{\theta_1} V^{SF}\left(\theta_m, \theta_f, \theta_1\right)\pi\left(\theta_1\right)d\theta_1\right] \quad (1)\\
d_{mm}^{m}\left(\theta_m, \theta_f\right) &= \underset{\delta \in \{0,1\}}{\arg\max}\left[\delta \int_{\theta_1} V^{MC}\left(\theta_m, \theta_f, \theta_1\right)\pi\left(\theta_1\right)d\theta_1 + (1-\delta)\int_{\theta_1} V^{SM}\left(\theta_m, \theta_f, \theta_1\right)\pi\left(\theta_1\right)d\theta_1\right] \quad (2)
\end{aligned}
$$

where the distribution of initial child skill θ_1 has a probability density function given by $\pi\left(\theta_1\right)$. The optimal marriage decision takes the form of a threshold strategy in the space of the potential spouse's skill. The value functions that young adults compare when deciding whether to marry depend on the return to skill in terms of lifetime utility, which is constructed next.

Expected Lifetime Utility

The expected lifetime utility of a child enters into the parent problem. It is:

$$
V^{child}\left(\theta\right) = \frac{1}{2}\sum_{g \in \{m,f\}} \tilde{V}_{g}^{child}\left(\theta\right) \quad (3)
$$

This function averages the expected lifetime utility given skill θ across potential spouses, potential children, and the gender of the child. That is, each family internalizes the average return to skill in terms of lifetime utility across men and women in equilibrium when making investment decisions. For each gender the expected return to skill in terms of lifetime utility is:

$$
\begin{aligned}
\tilde{V}_{child}^{f}\left(\theta\right) &= \int_{\theta_m}\left[\mathbb{I}_d \int_{\theta_1} V^{MC}\left(\theta_f, \theta_m, \theta_1\right)\pi\left(\theta_1\right)d\theta_1 + \left(1-\mathbb{I}_d\right)\int_{\theta_1} V^{SF}\left(\theta_m, \theta_f, \theta_1\right)\pi\left(\theta_1\right)d\theta_1\right]\mu\left(\theta_m\right)d\theta_m \quad (4)\\
\tilde{V}_{child}^{m}\left(\theta\right) &= \int_{\theta_f}\left[\mathbb{I}_d \int_{\theta_1} V^{MC}\left(\theta_f, \theta_m, \theta_1\right)\pi\left(\theta_1\right)d\theta_1 + \left(1-\mathbb{I}_d\right)\int_{\theta_1} V^{SM}\left(\theta_m, \theta_f, \theta_1\right)\pi\left(\theta_1\right)d\theta_1\right]\mu\left(\theta_f\right)d\theta_f \quad (5)
\end{aligned}
$$

where $\mathbb{I}_d \equiv d_{mm}^{f}\left(\theta_m, \theta_f\right) \times d_{mm}^{m}\left(\theta_m, \theta_f\right)$ indicates a mutual acceptance of the match. In addition, $\mu\left(\theta\right)$ is the endogenous distribution over adult skill θ, which is the same for each gender because parents are constrained to not targeting investments by gender.

Once families are formed, the family solves a sequential life-cycle problem. In the following subsections, I define the life-cycle problem for each of the three family types: single fathers, single mothers, and married (cohabiting) couples.

3.1 Consumer Problems

In the family problems which I specify below, the family has J periods to invest in their child. The child is born with some initial skill θ_1. At each age during adulthood, t, the parent affects the child's stock of skill in that period, θ_t, by their choice of investment, I_t^{type}, where $type \in \{SM, MC\}$ for single mothers and married couples respectively. Note that age t of adulthood is age $t+J$ of the parents' life. The way investment interacts with the child's skill in each period is defined by the function $f\left(\theta_t, I_t^{type}\right)$. In turn, the way investment in each period is generated depends on family structure $type$, where $type$ can be SM, for single mothers, or MC, for married couples. Investment is generated by time contributed from the parent(s) (q_t^m and q_t^f) and from time purchased on the market in the form of non-parental child care n_t.[4] In all the family problems, w the return per unit of skill on the labor market, r is the interest rate earned from the stock of savings, τ_y is the labor income tax, τ_n is the subsidies to child care, T are lump-sum transfers, and T_{cs} are child support payments.

The Problem of a Single Mother

During parenthood, a single mother chooses consumption c_t, leisure ℓ_t, savings a_{t+1}, non-parental child care time n_t, and her own time investments in her child q_t^m in each period to solve the following problem:

$$
\begin{aligned}
V^{SM}\left(\theta_1, \theta_f, \theta_m\right) &= \max_{\{c_t, \ell_t, a_{t+1}, n_t, q_t^m\}_{t=1}^T} \left[\sum_{t=1}^T \beta^{t-1} u^{SM}\left(\frac{c_t}{\phi_t^{SM}}, \ell_t\right)\right] + \beta^{J-1} b V^{child}\left(\theta_{J+1}\right) \quad (6)\\
s.t. & \\
c_t + a_{t+1} + (1-\tau_n)\, p_t^n n_t &\leq w\theta_m\left(1 - \ell_t - q_t^m\right) + (1+r)\, a_t + T + \mathbb{I}_{t\leq J} T_{cs}\\
\ell_t, n_t, q_t^m \in [0,1] & \qquad \ell_t + q_t^m \leq 1 \quad n_t + q_t^m \leq 1\\
a_1, a_{J+1} &= 0\\
\theta_{t+1} &= f\left(\theta_t, I_t^{SM}\right)\\
I_t^{SM} &= I_t^{SM}\left(\phi_n n_t, \theta_m q_t^m\right)
\end{aligned}
$$

From the time constraints in problem (6), it is clear that in each period the mother and child have a unit of time to dispose of. This means that the total time invested in the child cannot exceed the child's time endowment. Here, and in what follows, b is the altruism parameter, $w\theta_m$ denotes the wage of the parent in period t, and p_t^n denotes the price of non-parental child care in period t. Parental and non-parental child care time investments in child skill affect skill in the next period according to $f\left(\theta_t, I_t^{SM}\right)$ and $I_t^{SM}\left(\phi_n n_t, \theta_m q_t^m\right)$. The production function, $f\left(\theta_t, I_t^{SM}\right)$, aggregates the current stock of capital with investment to form the stock of capital tomorrow. The investment function, I_t^{SM}, aggregates efficiency units of child care time ($\phi_n n_t$) and efficiency units of

[4] The superscripts m and f on q (or subscripts on θ) denote contributions from (or attributes of) the mother and father, respectively, and the superscript n in p_t^n refers to non-parental child care.

time contributed by the mother ($\theta_m q_t^m$) to form investment, which then enters as an argument in $f\left(\theta_t, I_t^{SM}\right)$. Note that the investment function I_t^{SM} is indexed to the family structure, while the function f is the same for both types of families. The final child skill θ_{J+1} enters the objective function of the mother through an altruism term $bV^{child}\left(\theta_{J+1}\right)$, which weights the expected lifetime utility of the child $V^{child}\left(\theta_{J+1}\right)$ with the altruism coefficient b.

The Problem of a Married Couple

During the parenting phase, married couples choose consumption c_t, leisure ℓ_t, savings a_{t+1}, non-parental child care time n_t, and parental time inputs q_t^f and q_t^m. Married couples solve one problem jointly, and enjoy an altruistic return from the lifetime utility of their children $V^{child}\left(\theta_{J+1}\right)$, as a function of their skill at adulthood θ_{J+1}. Both parents and the child have a one-unit time endowment in each period. Parental and non-parental child care time investments in child skill affect skill in the next period according to $f\left(\theta_t, I_t^{MC}\right)$ and $I_t^{MC}\left(\phi_n n_t, \theta_f q_t^f, \theta_m q_t^m\right)$. The first function, $f\left(\theta_t, I_t^{MC}\right)$, aggregates the current stock of capital with investment to form the stock of capital tomorrow - this is the same function as that in the single mother problem. The second function, I_t^{MC}, aggregates efficiency units of child care time ($\phi_n n_t$) and efficiency units of time contributed by the mother ($\theta_m q_t^m$) and from the father ($\theta_f q_t^f$) to form investment, which then enters as argument I_t^{MC} in $f\left(\theta_t, I_t^{MC}\right)$.

$$
\begin{aligned}
V^{MC}\left(\theta_1, \theta_f, \theta_m\right) &= \max_{\left\{c_t, \ell_t, a_{t+1}, n_t, q_t^m, q_t^f\right\}_{t=1}^{T}} \left[\sum_{t=1}^{T} \beta^{t-1} u^{MC}\left(\frac{c_t}{\phi_t^{MC}}, \ell_t\right)\right] + \beta^{J-1} b V^{child}\left(\theta_{J+1}\right) \quad (7) \\
s.t. \quad \forall t & \\
c_t + a_{t+1} + \left(1-\tau_n\right) p_t^n n_t &\leq w\theta_f\left(1-\ell_t-q_t^f\right) + w\theta_m\left(1-\ell_t-q_t^m\right) + (1+r)\, a_t + T \\
\ell_t, n_t, q_t^f, q_t^m \in [0,1] & \quad \ell_t + q_t^f \leq 1 \quad \ell_t + q_t^m \leq 1 \quad n_t + \max\left\{q_t^f, q_t^m\right\} \leq 1 \\
\theta_{t+1} &= f\left(\theta_t, I_t^{MC}\right) \\
I_t^{MC} &= I_t^{MC}\left(\phi_n n_t, \theta_f q_t^f, \theta_m q_t^m\right) \\
a_1, a_{J+1} &= 0
\end{aligned}
$$

In the couple problem, parents are allowed to invest in the child at the same time (their quality time investments are non-rival). Therefore, the maximum of the parental time inputs is what determines whether the time constraint binds, not their sum. More generally, the specification used here for the married couple problem is based on Guvenen and Rendall (2015). As in that study, here the perfect complementarity in leisure of the spouses is motivated with time use data as documented in Aguiar and Hurst (2007), Table V. This is reflected in the parenting problem (7) by the fact that the couple only chooses one level of leisure which they both enjoy. [5]

[5] Unlike Guvenen and Rendall (2015), here the marginal utility of leisure is deterministic.

The Problem of a Single Father

A single father chooses consumption c_t, savings a_{t+1}, and leisure ℓ_t to solve the following problem.

$$
\begin{aligned}
V^{SF}\left(\theta_1, \theta_f, \theta_m\right) &= \max_{\{c_t, \ell_t, a_{t+1}\}_{t=1}^{T}} \left[\sum_{t=1}^{T} \beta^{t-1} u^{SF}\left(c_t, \ell_t\right)\right] + \beta^{J-1} b V^{child}\left(\mathbb{E}\left(\theta_{J+1} | \theta_1, \theta_m\right)\right) \qquad (8) \\
c_t + a_{t+1} + \mathbb{I}_{t \leq J} T_{cs} &\leq w \theta_f \left(1 - \ell_t\right) + \left(1 + r\right) a_t + T \\
\ell &\in [0, 1] \\
a_1, a_{J+1} &= 0
\end{aligned}
$$

Here $w\theta_f$ is wage of the single father with skill θ_f. Single fathers take their child's outcome θ_{J+1} as given conditional on the mother's type, θ_m, as well as the child's initial skill (θ_1. A single father cannot use his own time to invest in his children, but he is required to make a lump-sum payment to the child's mother, T_{cs}. The expected outcome of a single father's children affects his outcome directly through the altruism term, $bV^{child}\left(\mathbb{E}\left(\theta_{J+1} | \theta_1, \theta_m\right)\right)$.

Government

The government collects revenue from labor income taxes τ_y to finance lump-sum transfers T and non-parental child care subsidies τ_n. The variable H_t is the aggregate supply of labor efficiency units at each age during adulthood t, N_t is the aggregate demand for non-parental child care.

$$
\tau_y w \sum_{t=J+1}^{T} H_t = T + \sum_{t=J+1}^{J+J} \left[\tau_n p_n N_t\right] \qquad (9)
$$

Representative Firm

The firm chooses capital K_F and labor inputs H_F to maximize profits, taking prices r and w as given. The parameter δ_F is the depreciation rate of capital.

$$
\max_{K,H} \quad \left\{K_F^{\alpha_F} H_F^{1-\alpha_F} - w H_F - \left(1 + r - \delta_F\right) K_F\right\} \qquad (10)
$$

Non-parental Care Sector

The non-parental child care sector provides N units of non-parental child care at price p_n. The price of non-parental child care is set as a constant fraction κ of the average earnings per unit of time:

$$p_n = \kappa \sum_{t=J+1}^{T} wH_t \tag{11}$$

This allows the price of non-parental child care to adjust with the average level of skill in the economy, but without specifying a production function for non-parental child care.

Equilibrium

Given a government policy $\{\tau_n, \tau_{qm}, \tau_{qf}\}$, transfers T, and child support T_{cs}, a stationary equilibrium is defined as:

- Factor prices and a labor income tax,
- Individual marriage decision rules for each skill and gender
- Consumer choices for each period of adulthood, and lifetime utilities for each family type for parent and child combination
- Expected lifetime utility at a given level of adult skill, $V^{child}(\theta)$
- Single father expectations about single mother decisions
- A non-parental child care price

such that:

- Capital and labor markets clear and the government's budget constraint holds
- Marriage market participants optimize
- Families optimize
- Parental have rational expectations about $V^{child}(\theta)$
- Single fathers have rational expectations about single mother decisions
- The non-parental child care price is a constant fraction κ of average hourly earnings.

Relation to the Literature

The family formation channel in my model is similar to that of Abbott, Gallipolli, Meghir, and Violante (2018), who examines how college tuition subsidies can affect the composition of marriages in the economy, but who do not allow for single-parent families to exist in equilibrium. The baseline framework into which I incorporate and endogenize heterogeneous family structures has been used to analyze the interaction of policy and skill investment in many studies, including Restuccia and Urrutia (2004), Lochner and Monge-Naranjo (2011), Guner, Kaygusuz, and Ventura (2016), Gayle, Limor, and Soytas (2017), Daruich (2019), Caucutt and Lochner (2017), Abbott, Gallipolli, Meghir, and Violante (2018), and Lee and Seshadri (2019).

4 Estimation of $f\left(\theta_t, I_t^{type}\right)$, I_t^{SM}, and I_t^{MC}

The estimation equations for the skill accumulation technology, $f\left(\theta_t, I_t^{type}\right)$, which includes the equations that estimate the skill investment technologies I_t^{SM}, and I_t^{MC}, are derived from the optimizaton conditions of families making investment decisions in the model. The derivation of these estimation equations requires only an additional assumptions on the functional forms of $f\left(\theta_t, I_t^{type}\right)$, I_t^{SM}, and I_t^{MC}, the functions that govern how inputs into investment affect skill. Using these assumptions, and the first order conditions of the cost-minimization portion of the parenting problems described above, the estimation equations applied to the data can be derived.

4.1 Functional Form Assumptions

The current stock of skill, θ_t, and chosen level of investment I_t^{type} combine according to the function $f\left(\theta_t, I_t^{type}\right)$ to produce the stock of skill in the next period, θ_{t+1}. Here, $type \in \{SM, MC\}$. The two family structures both use $f\left(\theta_t, I_t^{type}\right)$ to aggregate their child's skill with the family's investment.

$$\theta_{t+1} = f\left(\theta_t, I_t^{type}\right) \quad = \quad \left[\upsilon\left(\lambda_{type} I_t^{type}\right)^{\frac{\chi-1}{\chi}} + (1-\upsilon)\left(\theta_t\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}$$

The function f is assumed to be a constant elasticity of substitution (CES) function, with parameters χ and υ. In a one-period parenting problem, the elasticity of substitution parameter χ can be interpreted as governing how easily investment can compensate for initial skill. If this elasticity is low, it takes more investment to change initial skill than if the elasticity is high. In a model where $J > 1$, so that parenting is a multi-period investment problem, the elasticity of substitution χ governs how easily investment can be shifted across periods of childhood in response to changes in its price across periods. If χ is low, it means that the technology does not allow investment to be reallocated across periods easily in response to a change in the price of investment in one period. In a one-period parenting problem, the share parameter υ can be interpreted as determining the importance of investment in determining the final skill of the child. If υ is low, it takes more investment to affect the stock of skill, and vice versa. In a model where $J > 1$, the share parameter υ

determines how investment with a constant price across consecutive periods would be allocated. If υ is low, then investment in each period of childhood will have to be smoothed out across periods, whereas if υ was high investment could be more lumpy. Finally, λ_{type} is simply a scaling parameter that accounts for units of measurement being different for investment and the stock of skill.

The two family structures differ in the way that parental time and time purchased on the market in the form of child care combine to generate investment. For single mothers, the functional form for investment in each period is a CES aggregator:

$$I_t^{SM}\left(\phi_n n_t, \theta_m q_t^m\right) = \left[\alpha_s\left(\theta_m q_t^m\right)^{\frac{\eta-1}{\eta}} + \left(1-\alpha_s\right)\left(\phi_n n_t\right)^{\frac{\eta-1}{\eta}}\right]^{\left(\frac{\eta}{\eta-1}\right)}$$

For married couples the investment function is also a CES, but with different inputs:

$$I_t^{MC}\left(\phi_n n_t, \theta_f q_t^f, \theta_m q_t^m\right) = \left[\alpha_1\left(\theta_f q_t^f\right)^{\frac{\epsilon-1}{\epsilon}} + \alpha_2\left(\theta_m q_t^m\right)^{\frac{\epsilon-1}{\epsilon}} + \left(1-\alpha_1-\alpha_2\right)\left(\phi_n n_t\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\left(\frac{\epsilon}{\epsilon-1}\right)}$$

With this CES functional form, the share parameters α_1, α_2 and α_s control the relative level of inputs given a price ratio. The values of η, and ϵ control the percentage change in the ratio of inputs for a percentage change in the ratio of their prices. This parameterization allows for parental time and non-parental child care time to interact differently across household structures. Another way of specifying the investment aggregator would be to include money (goods) and parental time inputs. In appendix V, an exercise is presented in which the contribution of child care costs to expenditures on children is measured for different age groups and definitions of total expenditures, using the 2001 PSID and the 2002 PSID CDS. This exercise demonstrates that child care costs are a sizeable component of money spent on children by any measure of spending on children considered (the share ranges from 50 to 70 percent of total spending). The specification used in this study makes explicit how expenditures on children affect child skill accumulation through child time use.

4.2 Estimation Equations

There are four estimation equations: (12), (13), (14), and (15). For the derivation of these equations, see Appendix I. Evidently, the notation has adjusted slightly from that of the model, to allow for greater clarity in how variables in the equation map into variables in a panel. The panel reports variables measured in each period t for each family i; this is reflected in the addition of time and family subscripts to all of the variables.

The estimation equations must be implemented in a particular sequence. This is because, to measure the father's share for the couple's technology, a necessary input is the ratio of the mother's share and that of child care. In addition, to construct investment levels and thereby investment prices, which are inputs into (15), one needs the all the parameters estimated in (12), (13), and (14). The models I estimate are below. The error term in each equation, $\nu_{j,i,t} \quad j \in \{1,2,3,4\}$, is assumed to be i.i.d..

$$\ln\left(\frac{q^m_{i,t}}{n_{i,t}}\right) = \beta_{sm,0} + \beta_{sm,1}\ln\left(\frac{w^m_{i,t}}{p^n_{i,t}}\right) + (1-\eta)\ln\left(\frac{\phi_{n,i}}{\theta_{m,i}}\right) + \nu_{1,i,t} \quad (12)$$

$$\ln\left(\frac{q^m_{i,t}}{n_{i,t}}\right) = \beta_{m,0} + \beta_{m,1}\ln\left(\frac{w^m_{i,t}}{p^n_{i,t}}\right) + (1-\epsilon)\ln\left(\frac{\phi_{n,i}}{\theta_{m,i}}\right) + \nu_{2,i,t} \quad (13)$$

$$\ln\left(\frac{q^f_{i,t}}{n_{i,t}}\right) = \beta_{f,0} + \beta_{f,1}\ln\left(\frac{w^f_{i,t}}{p^n_{i,t}}\right) + (1-\epsilon)\ln\left(\frac{\phi_{n,i}}{\theta_{f,i}}\right) + \nu_{3,i,t} \quad (14)$$

$$\ln\left(\frac{X_{type,i,t+1}}{X_{type,i,t}}\right) - \ln\left(\frac{\Lambda^{type}_{i,t+1}}{\Lambda^{type}_{i,t}}\right) = \gamma_0 + \gamma_1\left(\ln\left(\frac{1}{1+r_t}\right) - \ln\left(\frac{\Lambda^{type}_{i,t+1}}{\Lambda^{type}_{i,t}}\right)\right) + \nu_{4,i,t} \quad (15)$$

In equations (12), (13), and (14), the unobserved component is the second-to-last term on the right-hand side, followed by the random component. The unobserved component is the ratio of relative productivities for the two inputs, which can vary across families but is time-invariant within a family. If the investment productivities of parental time and child care time are correlated with the observed prices of these inputs, which seems reasonable, then the coefficient on the price ratio regressor will be biased in an Ordinary Least Squares (OLS) estimation because of the omitted variable in the residual. In light of this, I estimate using Fixed Effects (FE) at the family level for these equations. As long as the ratio of productivities for the inputs remains constant over time for a given family, the FE estimator accounts for this omitted variable by cancelling it out. In the last equation, (15), I use an OLS estimator. [6] From the estimation equations, one can see that the fixed effect correction adjusts for the logged ratio of parenting productivity to child care productivity. When constructing investment for the estimation of the outermost aggregator in equation (15), I incorporate parental fixed effects recovered from (12), (13), and (14).

The mapping from regression coefficients to the parameters of the skill accumulation technologies is given below. First, the set of parameters for the single mother investment technology is a

[6] I use parenting productivities as inputs into this estimation equation indirectly. I back them out of the first three equations in order to construct the ratio of investment prices over time at the family level, which are then used in estimating (15).

function of the slope and intercept coefficients of equation (12).

$$\begin{aligned}\beta_{sm,0} &= \eta \ln \left[\frac{\alpha_s}{1-\alpha_s}\right] \\ \beta_{sm,1} &= -\eta\end{aligned}$$

The parameters of the investment technology used by married couples are functions of the slope and intercept coefficients of equations (13) and (14).

$$\begin{aligned}\beta_{m,0} &= \epsilon \ln \left(\frac{\alpha_2}{1-\alpha_1-\alpha_2}\right) \\ \beta_{f,0} &= \epsilon \ln \left(\frac{\alpha_1}{1-\alpha_1-\alpha_2}\right) \\ \beta_{m,1} &= -\epsilon \\ \beta_{f,1} &= -\epsilon\end{aligned}$$

Finally, the parameters of the function that aggregates investment with skill are functions of the slope and intercept coefficients from equation (15).

$$\begin{aligned}\gamma_0 &= \chi \ln \left(\frac{1}{1-\upsilon}\right) \\ \gamma_1 &= \chi\end{aligned}$$

To estimate equations (12) - (15), it is necessary to construct empirical counterparts at the family level for the dependent and independent variables. This is done using several data sources, as the next section explains.

4.3 Description of Estimation Data Sources

The variables that needed are the ones needed to construct the dependent and independent variables in the regression equations (12) to (15). The indexes denote family i in period t, with father f and mother m. Non-parental child care's price has a superscript n.

- Parental educational time inputs ($q^f_{i,t}, q^m_{i,t}$)
- Non-parental child care time inputs ($n_{i,t}$)
- Hourly wages ($w^m_{i,t}, w^f_{i,t}$)

- Hourly non-parental child care prices ($p^n_{i,t}$)

Two datasets are combined to measure these variables: the ECLS-B, which is a panel data set, and the American Time Use Survey (ATUS), which is a repeated cross-section sampled from the Current Population Survey (CPS).

The Early Childhood Longitudinal Study, Birth Cohort Data for hourly wages, hourly price of child care, quality time from the parents in weekly frequency of activities, and non-parental child care time in hours per week come from the ECLS-B.[7]

The ECLS-B reports labor earnings, the period of time over which the labor earnings were accrued (a day, a week, two weeks, etc.), and the hours worked in a week. To correct for taxes, labor earnins are corrected using the slopes (tax rates) from Table 2 of McGrattan and Prescott (2017). This correction accounts for progressive nature of the US tax system. Next, labor earnings are converted into weekly earnings, and then into hourly wages using hours worked per week. If hours worked were not reported for the parent, they are imputed using the response to part-time or full-time status (assigning 30 or 40 hours worked per week, respectively). The result is hourly after-tax wages for mothers and fathers, conditional on observing labor earnings and some information about the intensity of labor supply.[8]

For the price of non-parental child care, the ECLS-B collects information on weekly spending for each of the three main kinds of non-parental child care providers: relative, nonrelative, and center-based. The survey also reports which of these sources is the primary source of non-parental child care. To calcuate hourly price of non-parental child care, the total cost per time unit for the primary source of non-parental child care is adjusted by the number of weeks that cost represents and the hours per week the child spends in that form of non-parental child care. The resulting price per hour for non-parental child care, in addition to hourly wages of mothers and fathers, are observed

[7] As was evident from the model specification section, single fathers do not contribute their time to raising children in this study's specification. The motivation for this modelling decision is partly empirical and partly due to data limitations. Empirically, single fathers usually don't raise children under five alone unless the mother has died; instead, they play a visiting role. In the ECLS-B there is a separate questionnaire for non-resident fathers, which is completed if the resident parent allows the survey-giver to contact the non-resident father, and the father agrees to participate. Appendix O provides one justification for the choice not to include include time contributions from non-resident fathers: in these families, there is a selection bias in the fathers who complete the non-resident father survey (the most involved fathers do so) and in addition, conditional on completing the non-resident father survey, single mothers and non-resident fathers disagree strongly about how much the fathers are contributing (fathers claim they contribute more time and have more influence than the single mothers acknowledge). Besides concerns about eggregious measurement error in the single father sample, as well as severe selection bias, there is also the problem of different variables being reported for the single father survey than on the resident parent questionnaire (in regards to activities done with children). Even if the single father sample were representative and the reporting accurate given the questionnaire single fathers receive, the available variables only allow an approximation of what is reported for resident parents.

[8] I do not impute hourly wages for observations without this information. In previous versions of this estimation, if they did not report earnings because they were out of the labor force (an issue confined mostly to mothers) I imputed their hourly wages using a regression of hourly wages on education, age, and age squared for the sample on which I could construct wages. I then evaluated this regression for observations where I could see education and age to get imputed hourly wages. This step meant that previously I needed to cluster standard errors at the state of residence.

for each family in each wave of the survey.[9]

Activities parents do with their children are reported in the first three waves of the ECLS-B. These activities are reported at frequencies (every day, once a week, etc.). Quality time the parents spend with the child is defined as activities with the child that include talking and reading and spending time outside. This definition is founded on an exercise in which the definition of quality time was varied to identify which definition led the model estimation equations to best fit the data.[10] This definition is consistent with the literature for the importance of active time with children (Del Boca, Flinn, and Wiswall (2014)). In order to convert observed quality time from frequencies into of hours per week, the next step is to impute time per activity from the ATUS.

The American Time Use Survey Data on levels of time per activity for a parent with a given set of characteristics come from the 2003-2016 pooled ATUS sample. This dataset provides a time diary along with CPS variables on age, gender, marital status, labor force status, educational attainment, parental status, and child age. Observations are restrict to be between 15 and 55, with a child 4 years or younger. I use information on gender, marital status (married/cohabiting or single), labor force status (participating or not), and educational attainment, where educational attainment is discretized into those with a high school degree or less, and those with more than a high school degree. For each group, the survey-weighted average for of time spent on an activity (conditional on engaging in it) are calculated. These activities are time spent reading to the child and time spent playing with the child. [11]

Imputation After linking parents in the ECLS-B with their appropriate group in the ATUS, the ATUS levels of time spent reading and time spent playing are assigned to reading activities and playing outside activities in the ECLS-B. Next, total quality time per parent in each family in each wave is calculated by summing across activities. This yields measures of quality time investments, non-parental time investments, after-tax hourly wages, and non-parental child care prices at the child-family pair and wave level.

Restrictions on the Estimation Sample The married couple estimation sample pools married and cohabiting couples. The single mother estimation sample is composed of mothers in the sample who are a primary caregiver and who do not have a significant other living in the household with them. For both couples and single mothers, observations are only admissable if the resident primary caregiver is a biological parent, is less than 55 years of age, reports working for pay (I do not use imputed wages in the estimation), makes less than 200 dollars an hour, pays at least 50 cents an hour for the primary source of child care, and whose child spends at least 0.1 hours per week in child care. In addition, families are only valid observations if the biological mother had

[9]To see how the sample selection procedure affected the composition of child care sources used by families, see V

[10]In Table 13 of Appendix V I show that this measure of parental time inputs predicts child skill accumulation.

[11]Tabulations of the ATUS sample, along with the raw ECLS-B sample, are reported in Appendix V.

her first child after age 15 and before age 45.

Estimation Sample Moments Moments from the estimation samples for couples and single mothers are presented in Table 4.3 and Table 2, respectively. These summary statistics report averages for the two groups of families (couples and single mothers), averaged across the first 3 waves of the sample. The moments are weighted with wave 3 survey weights provided by the ECLS-B, which are designed to correct for attrition in the sample.

Table 1: Married/Cohabiting Parents in the ECLS-B (Waves 1-3, Unweighted)

	Levels						
	mean	p10	p50	p90	sd	min	max
Education Time Mother: Hours per Week	10.34	2.74	9.05	22.76	6.94	1.10	22.76
Education Time Father: Hours per Week	4.63	1.17	4.50	7.65	2.40	1.17	8.82
Non-parental Care: Hours per Week (Primary)	30.23	9.00	33.00	45.00	13.82	1.00	80.00
Non-parental Care: Hours per Week (All)	31.63	9.00	35.00	47.00	14.09	1.00	80.00
Hourly Price Child Care (Primary)	3.96	1.42	3.02	6.88	4.05	0.54	68.75
Hourly Price Child Care (All)	3.89	1.37	3.00	6.67	4.03	0.39	68.75
Hourly Pay Mother, After Tax	14.24	5.30	11.30	24.16	12.04	0.93	177.30
Hourly Pay Father, After Tax	15.79	6.68	12.72	26.59	11.86	0.60	197.00
Ratio: Hourly Pay Mother/CC Price	0.36	0.12	0.26	0.64	0.48	0.02	12.61
Ratio of Time: Mother/Child Care	0.53	0.09	0.29	1.13	0.83	0.02	21.66
Ratio of Time: Father/Child Care	0.24	0.04	0.15	0.51	0.29	0.02	4.10
Family Income	96336	35001	87500	150000	63026	1600	300000
Family Income, After Tax	81579	39351	75730	117460	43301	13267	223092
Age Resident Mother	33.17	26.00	33.00	40.00	5.49	17.00	50.00
Mother: Age First Child	27.78	20.00	28.00	35.00	5.76	16.00	43.00
Age Resident Father	35.12	27.00	35.00	43.00	5.98	19.00	54.00
	Rates						
Mother: BA or higher	0.57						
Father: BA or higher	0.49						
Below 100 % Poverty Line	0.02						
Below 185 % Poverty Line	0.11						
Obs_rounded	2700						

Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

Based on Table 4.3, the following qualitative points are apparent. First, fathers and mothers spend substantial time per week engaged in education time with their children. Reported hours per week in non-parental child care are 30 on average, but there is a high variation in this level. The relative quantities of the two time inputs measured in this sample, along with hourly prices for each, will translate into large estimated CES shares on non-parental child care. The average ratio of mother time to non-parental child care is on average 0.53, but with a high variation. Married or cohabiting mothers and fathers have hourly wages that are about four times higher than the hourly price of child care they use, although there is large variation in all of these prices within my sample. On average, the price of child care is 36% of the mother's hourly wage. Poverty in the sample of married or cohabiting parents is not common: only 2% are below the poverty line, while only 11% are below 185% of that threshold. Both parents are on average in their 30s, and the fraction in this group with a college degree or more is about half for each gender - especially mothers.

Table 2: Single Mothers in the ECLS-B (Waves 1-3, Unweighted)

	Levels mean	p10	p50	p90	sd	min	max
Education Time Mother: Hours per Week	2.59	1.10	2.74	3.84	1.66	1.10	22.76
Non-parental Care: Hours per Week (Primary)	34.61	12.00	40.00	48.00	12.49	3.00	80.00
Non-parental Care: Hours per Week (All)	37.21	15.00	40.00	50.00	13.98	3.00	96.00
Hourly Price Child Care (Primary)	2.78	0.79	2.00	4.81	2.75	0.51	23.21
Hourly Price Child Care (All)	2.69	0.74	2.00	4.59	2.76	0.32	23.21
Hourly Pay Mother, After Tax	9.72	4.56	7.52	13.95	12.44	1.71	188.50
Ratio: Hourly Pay Mother/CC Price	0.36	0.11	0.26	0.66	0.39	0.01	4.33
Ratio of Time: Mother/Child Care	0.10	0.03	0.07	0.19	0.12	0.01	1.28
Family Income	36307	8000	27500	62500	40648	1	300000
Family Income, After Tax	39727	18851	34021	58984	28377	11762	223092
Age Resident Mother	28.04	21.00	27.00	37.00	6.20	18.00	49.00
Resident Mother: Age First Child	22.47	17.00	21.00	31.00	5.22	16.00	42.00
	Rates						
Mother: BA or higher	0.15						
Below 100 % Poverty Line	0.29						
Below 185 % Poverty Line	0.60						
Obs_rounded	900						

Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

Table 2 presents a similar set of statistics for single mothers. Single mothers spend only slightly fewer hours than married mothers in educational activities with their children, but non-parental child care time is on average five hours higher than for couples. The ratio of mother's time to non-parental child care time is correspondingly lower than for couples, at 0.10 (compared to 0.53 for the latter). Within a family, the ratio of mother time to non-parental child care time is almost half that of married or cohabiting families. The poverty rate of single mothers is fifteen times higher than couples, at 29%, while the percent of single mothers below 185% of the poverty line is about five times higher, at 60%. On average, single mothers make about five dollars per hour less than mothers parenting in couples. The ratio of the price of child care to the mothers hourly wage is about the same as it is for married and cohabiting mothers. Finally, the age of single mothers is on average 5 years lower than married or cohabiting mothers, and their educational attainment is one third that of married mothers: only 15% of single mothers have a college degree or more.

Comparing the two estimation samples described in Tables and 2 helps to establish priors about what the estimates of the skill accumulation technologies of each should look relative to one another. Because single mothers are the only source of parental time for their child, and the amount of non-parental time purchased is so large, one expects to see a larger CES share for non-parental child care in the single mother problem, ceteris paribus.

Table 3: Pooled Sample Moments (Waves 1-3, Unweighted)

	Levels mean	p10	p50	p90	sd	min	max
Education Time Mother: Hours per Week	8.43	1.10	6.30	21.66	6.93	1.10	22.76
Non-parental Care: Hours per Week (Primary)	31.31	9.00	35.00	45.00	13.63	1.00	80.00
Non-parental Care: Hours per Week (All)	33.01	10.00	38.00	50.00	14.27	1.00	96.00
Hourly Price Child Care (Primary)	3.67	1.25	2.75	6.43	3.80	0.51	68.75
Hourly Price Child Care (All)	3.59	1.19	2.66	6.25	3.79	0.32	68.75
Family Income	81517	18000	62500	150000	63789	1.00	300000
Family Income, After Tax	71247	27091	58984	117460	44004	11762	223092
	Rates						
Mother: BA or higher	0.46						
Below 100 % Poverty Line	0.09						
Below 185 % Poverty Line	0.23						
Single Mother	0.25						
	Weighted Moments						
Corr. (Child Skill, Family Inc.): child aged 9 mo.	0.04						
Corr. (Child Skill, Family Inc.): child aged 4	0.32						
Obs_rounded	3600						

Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

Table 3 reports selected unweighted moments for the pooled estimation sample of married/cohabiting couples and single mothers. On average, mothers spend about 8 hours a week in educational activities and families purchase about 31 hours of child care. The price per hour of the primary source of child care is 3.67 dollars, on average. The pooled, unweighted estimation sample is 25% single mothers, with 9% of all families living in poverty and 23% of families below 185% of the poverty line. About half of all mothers raising young children in the estimation sample have a BA or higher.

The bottom half of Table **??** reports the weighted correlation coefficients for child skil land family income, when the child is 9 months old (wave 1) and when the child is 4 years old (wave 3). The correlation coefficient is weighted using wave 3 survey weights. The initial correlation is not significantly different from 0, while the final correlation is statistically significant at the 1 percent level.[12]

For estimation weights, I used wave 3 weights for the primary caregiver survey sample to estimate $\{\eta, \alpha_s\}$, and wave 3 weights for the resident father survey sample to estimate ϵ and α_1 and α_2. [13] When using a fixed effects estimator, the requirement for clustering standard errors is heterogeneity in treatment effects, which is not the case here (see Section 4 of Abadie, Athey, Imbens, and Wooldridge (2017)).

4.4 Estimation Results

The estimation results for equations (12), (13), (14), and (15), are reported in Columns (1) to (4) of Table 4, respectively. To reiterate, the first three estimations use a FE estimator, but the last uses an OLS estimator. This is because it is not possible to linearly separate the parenting productivities in the last estimation equation. In addition, Table 4 reports the unconstrained estimates of the reduced-form linear equations derived above. As evidenced from the structural counterparts of the estimated coefficients, however, these unconstrained estimates ignore possible parametric restrictions. Specifically, the coefficients $\beta_{m,1}$ and $\beta_{f,1}$ in equations (13) and (14) respectively, both equal the negative of the structural parameter ϵ. There is not a statistically significant difference between the estimates of ϵ from equations (2) and (3) in Table 4, and ϵ is assigned its value from equation (2). These results yield a set of parameters for the skill accumulation technologies, reported in Table 5.

[12]These correlation coefficients are reported in the appendix with their p-values in Table12. Table 12 also reports regression results showing that while family income in the first wave does not predict standardized measures of the child's skill in that wave, when the child is 9 months old, family income does predict standardized measures of skill for the child when he or she is 4 years old, even controlling for initial skill. Importantly, variation in initial skill as reported in the ECLS-B for 9-month old children is predictive of variation reported for children when they are 4 years old. Although measuring a child's skill at such an early age is challenging, the fact that skill as it is measured in the ECLS-B is persistent indicates that the measure of skill at that age is not noise.

[13]See the appendix for an outline of the survey structure of the ECLS-B. There are several questionnaires, each with their own set of weights.

Table 4: Skill Accumulation Technology Estimation by Measure of Parental Time Input:

	(1)	(2)	(3)	(4)
$\ln\left(\frac{\text{AT wage mother}}{\text{Price CC}}\right)$	-0.375***	-0.531***		
	(0.102)	(0.0692)		
$\ln\left(\frac{\text{AT wage father}}{\text{Price CC}}\right)$			-0.517***	
			(0.0658)	
$\ln\left(\frac{1}{1+r_t}\right) - \ln\left(\frac{\Lambda_{i,t+1}^{type}}{\Lambda_{i,t}^{type}}\right)$				0.524***
				(0.0319)
Constant	-2.139***	-0.512***	-1.156***	0.126***
	(0.133)	(0.0867)	(0.0913)	(0.0156)
r2	.1177	.0958	.1296	.2495
r2_b	.0759	.0705	.037	
r2_o	.0838	.0745	.0489	
Obs_rounded	900	2700	2700	1300
Groups_rounded	650	1600	1600	
share	.0033	.2563	.0718	.2134
share_se	.0063	.0085	.0307	.0237
complementarity	.3746	.5312	.5312	.5235
complementarity_se	.1021	.0692	.0692	.0319

Standard errors in parentheses

Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Except for the single mother share α_s and the married father's share α_1, the parameter values reported in Table 5 are statistically different from zero at the 0.1% significance level. The married father's share is significantly different from 0 at the 5% level. As for the single mother share, it is not significantly different from 0. The ordering of the share parameters by magnitude is consistent with the intuition based on sample summary statics: the largest share is on market-bought child care, then mother time, then time from the father (for married couples), and the share on child care is larger than the share on mother time for single mothers.

The elasticity parameters is interpreted as the percent change in input ratios if the ratio of their prices changes by one percent. The fact that this elasticity parameter for inputs into investment is lower for single mothers than for couples means that the ratio of her inputs is less sensitive to a change in relative prices than it is for couples. However, this does not mean that the level of investment is insensitive to changes in prices of inputs, just that the ratio of inputs will not respond very strongly to such changes. Specifically, for married mothers, a 1% increase in the ratio of the mother's hourly wage to the price of non-parental child care decreases the ratio of her time to non-parental time by 0.53%. For married fathers, the statistic is the same as for the married mothers by assumption. Single mothers are less responsive in their input ratios: they would adjust the ratio of their time to non-parental child care time by only 0.37% in the face a 1% change in the price ratio. The share values, meanwhile, indicate that the level of her time would already be very low compared to the time inputs from couples raising children in this economy.

Table 5: Parameters of the Human Capital Accumulation Technology

η	α_s	ϵ	α_1	α_2	χ	υ
0.375	0.0033	0.531	0.072	0.256	0.524	0.21
(0.102)	(0 .0063)	(0.0692)	(0.0307)	(0.0085)	(0.0319)	(0.0389)

Note: Standard errors are in parentheses, calculated for those parameters that are a function of estimated coefficients using the delta method.

Discussion of Estimation Results

There are three main takeaways from the skill technology estimation. First, the share parameters on father time and single mother time inputs are very low. Even if their hourly wages were the same as the prices of the other inputs in the family's investment technology, the corresponding ratios of investment inputs would still show a relatively low level of inputs coming from their quality time. This is independent of the composition of hourly wages across family structures. By contrast, non-parental child care has a large share in the investment technologies of both one- and two-parent families. If this share were zero, changes in the price of non-parental child care due to a subsidy would have no effect on the price of investment. In turn, this would mean that families did not change their choice of θ_{J+1} and skill accumulation in children would remain unaffected by the subsidy.

The second takeaway concerns the relative elasticities of the three parental inputs with respect to non-parental child care. A 1% increase in the ratio of a single mother's hourly wage to the price of non-parental child care causes her to adjust the ratio of her time input to non-parental child care by 0.31%. She does not heavily readjust her investment input choices because of the price change. Married mothers, by comparison, adjust their input choices more—by 0.53%. For married fathers, this statistic is by assumption the same as for the married mothers. This matters for policy analysis, because it is informative about what one can expect parents in the economy to do with their time in the presence of a subsidy to the price of child care. All parents will shift inputs away from their own time and toward non-parental child care if the latter's price decreases. Ceteris paribus, this shift in input composition will not be dramatic; to achieve large changes in time use of the parents, the change in relative prices induced by a child care subsidy will have to be large. Time use is relevant because if parents easily substituted away from parenting and (at least partially) into time spent working, this would be a source of expansion in the current tax base used to finance the child care subsidy. My estimates indicate that expansions in the tax base from parents substituting away from parenting quality time and into labor should be expected to be small. Expansions in the tax base will have to come instead from changes in the population distribution of labor market skill and parenting productivities in the long run, rather than changes in the composition parental time use.

The third and final takeaway is the role of investment in the outermost aggregator, which takes investment and the current stock of skill to produce tomorrow's skill. The share parameter on investment is significantly different from 0: the role of investment role in producing tomorrow's skill is sizeable. In addition, the elasticity of substitution between investment and skill is $\chi = 0.52$, and its statistical significant allows me to reject a Cobb-Douglas specification in favor of one with greater complementarity (less substitutability). To build intuition for interpreting this result, consider the intertemporal cost minimization equation ((19) or (22)) that I use to derive my estimation equation for χ in Appendix I. If the ratio of investment prices today and tomorrow changes by 1%, this estimate says that the ratio of investments today and tomorrow will change by 0.52%. The distribution of investments over time is not very sensitive to changes in the price of that investment across periods. If you think of early childhood as lasting for multiple periods, where only one of those periods is subsidized, you should not expect to see parents being able to shift their investment to that period to take advantage of the subsidy.

Comparison with Other Findings

There is a large body of work on the estimation of skill technologies (e.g., see Todd and Wolpin (2003) and Cunha and Heckman (2008) for a discussion of specification and estimation issues). Direct comparisons of point estimates are problematic because specifications of the skill accumulation technology vary across studies, but such comparisons can nevertheless be qualitatively informative. [14]

For a general comparison, consider one widely cited estimation of a skill accumulation technology due to Cunha, Heckman, and Schennach (2010). In that study, the authors use the NLSY79 Children and Young Adults datasets to estimate a CES skill accumulation technology for a two-dimensional skill vector containing both cognitive and non-cognitive skills. Each of these dimensions of skill is allowed to affect the evolution of the other dimension. This skill technology is very general; it nests several specifications examined in other studies. Cunha, Heckman, and Schennach (2010) find substitutability between investment and skill during early childhood. In that study, the authors are not specific about which inputs aggregate into investment in skill.

Lee and Seshadri (2019) estimate a skill technology with a single dimension of skill, but assume that there is no initial draw of skill (or rather, that it is identical for everyone and equal to 0). They thereby impose by assumption a property that Cunha, Heckman, and Schennach (2010) found as a result: that in the first period of life, initial investment and the initial stock of skill are

[14] Specifications vary for several reasons; one is that the data requirements vary with the estimation equations. The equations derived here require information on time inputs from each resident parent and market-based child care, as well as their prices.

substitutable. For other periods, however, Lee and Seshadri (2019) are unable to reject a Cobb-Douglas specification. This is a higher level of substitutability than what I find, but lower than that in Cunha, Heckman, and Schennach (2010).[15]

The complementarity that I find between parental time inputs is relevant for the structure of my model because it is interpreted as complementarity in home production for the skill level of spouses. Complementarity leads to positive assortative matching in the marriage market (Becker (1974)), which is what allows my calibrated framework to achieve a realistic degree of correlation between the wages of spouses in equilibrium (this moment is documented in the Model Parameterization Section).

My estimates show complementarity between investments and skill early in life. I do not ignore heterogeneity in the first measure I have of skill at 9 months, as in the specification of Lee and Seshadri (2019), because it doesn't appear to be noise in my data and because I have no observations of the investment that generated that initial stock (motivating regressions for these two claims are in the appendix). If the investment were solely responsible for the skill I observe at 9 months, the skill would already be correlated with family income. That is not what I find. See the appendix of Cunha, Heckman, and Schennach (2010) for a discussion of the implications of substitutability versus complementarity between skill and investment for policy design.

The Role of Technology

Using the definition of the price of investment, and the expressions for optimal ratios of inputs into investments for the two family structures, one can derive the elasticity of the price of investment with respect to the price of child care for the case where the time constraint of the child is slack. This statistic, which differs across family structures, is useful to understand how the estimated parameters of the skill accumulation technologies map into differing sensitivities to the price of child care across the two family structures. For the derivation of (16) and (17), see Appendix III.

For single mothers, this elasticity is:

$$\frac{\partial \Lambda_t^{SM}}{\partial p_t^n} \frac{p_t^n}{\Lambda_t^{SM}} = \frac{(1-\alpha_s)^\eta}{(1-\alpha_s)^\eta + \alpha_s^\eta \left[\frac{(w)}{\left(\frac{p_t^n}{\phi_n}\right)} \right]^{1-\eta}} \tag{16}$$

[15]In their 2016 working paper, Agostinelli and Wiswall (2016) the authors show that the assumptions made in Cunha, Heckman, and Schennach (2010) (specifically re-normalization of the latent skill variables) impose over-identifying restrictions which can bias the estimation of the complementarity parameter. The direction of the bias is explored in Monte Carlo simulations which demonstrate that the direction of the bias depends on several attributes of the estimation procedure. My estimation method does not re-normalize skill distributions in each period. This is because by using intertemporal cost minimization to derive estimation equation ((19) or (22)), I am able to avoid using measures of skill in the estimation of the skill accumulation technology.

For married couples, it is:

$$\frac{\partial \Lambda_t^{MC}}{\partial p_t^n}\frac{p_t^n}{\Lambda_t^{MC}} \quad = \quad \frac{(1-\alpha_1-\alpha_2)^{\epsilon}}{(1-\alpha_1-\alpha_2)^{\epsilon}+\alpha_1^{\epsilon}\left[\frac{(w)}{\left(\frac{p_t^n}{\phi_n}\right)}\right]^{1-\epsilon}+\alpha_2^{\epsilon}\left[\frac{(w)}{\left(\frac{p_t^n}{\phi_n}\right)}\right]^{1-\epsilon}} \tag{17}$$

Examining equations (16) and (17), there are two points that should be emphasized. First, note that the assumption that the labor market and parenting productivities are the same means that $\frac{w_m}{\theta_m}$ is just the wage level w, which does not vary across families. Neither does the productivity of child care time. Therefore the elasticity of the price of investment with respect to price of child care is constant within a family structure. What does vary both within and across family structures is the family income available to finance investment in the child's skill: this is lower in single-mother families due to endogenous sorting in the marriage market.

Second, consider the role of the technological primitive the two structures use to invest. In the equations above, the opposing forces of the share parameters and the elasticity of substitution parameters are evident. Consider (16). As the share on child care, $1-\alpha_s$, increases, the numerator and the denominator get closer together in value for a given elasticity of substitution parameter η. This means the sensitivity of the investment price to the price of child care is increasing in $1-\alpha_s$ and decreasing in η. However, the point estimates presented in this paper indicate that, since for single mothers the share on parental time α_s is not statistically different from 0, the elasticity defined above is 1. For couples, on the other hand, the elasticity is less than 1 and approaches 1 as the relative price per efficiency unit of parental time relative to child care time approaches 0. The model lens therefore yields intuition that is consistent with the intuition of the summary statistics for single mothers and married couples, while adding interpretive clarity and a policy implication for child care subsidy design.

In the summary statistics, it was clear that the ratio of parental time to child care time was much lower, on average, for single mothers than for either parent in a married or cohabiting couple. This is reflected in the large share my estimation finds for the child care input in the single mother technology. What is not as apparent from cross-sectional summary statistics is how this ratio reacts to changes in the ratio of prices for the inputs into investment. My estimation uses variation within a family over time to pin down the parameter that governs that: I find that single mothers are less able than couples to adjust their input composition when the ratio of input prices changes. The fact that child care plays such a bigger role for single mothers in the data, and that they are not able to easily substitute with parental time, makes the price of investment for single mothers very sensitive to the price of child care. In turn the price of investment governs how much it costs to change the skill of a child, given her initial endowment. In this framework, the larger role of child care for single mothers relative to couples means that a child care subsidy will have a greater effect on the skill accumulation of children being raised by single mothers than those being raised by couples.

5 Model Parameterization

After assigning functional forms, the full model is implemented by grouping the model parameters into those estimated outside the model, those chosen externally, and those calibrated inside the model. Parameters chosen externally are standard in the literature. Parameters estimated outside the model refers to the set of parameters disciplined with data, but for which it is not necessary to solve for the model equilibrium to check how the model moment compares with the moment in the data. This group consists of the skill accumulation technology parameters, which are estimated using first-order conditions of the parenting problems. Finally, the set of parameters calibrated inside the model refers to those parameters for which it is necessary to solve for the model equilibrium in order to generate a model moment to compare with the data. These parameters include the altruism coefficient b, the productivities of investment, λ_{SM} and λ_{MC}, and the marginal utility of leisure for singles (ψ_s) and couples (ψ_{mc}).

5.1 Functional Form Assumptions

Period Utility Functions

Utility functions are defined separately for one- and two-parent families:

$$\begin{aligned} u^{SM}(c,\ell) &= \log(c) + \psi_s \log(\ell) \\ u^{SF}(c,\ell) &= \log(c) + \psi_s \log(\ell) \\ u^{MC}(c,\ell) &= \log(c) + \psi_{mc} \log(\ell) \end{aligned}$$

Distribution of Initial Child Skill

I assume that initial child skill is drawn independently and identically from $\pi(\theta_1)$, which I set as a uniform distribution. For motivating regressions for the i.i.d. assumption, see Appendix IV.

5.2 Externally Chosen Parameters

The length of a lifetime and of each phase are proportional to 20 years of childhood and 60 years of adulthood (death at age 80). The discount (patience) factor is set to a yearly value of 0.96 to match the risk-free interest rate. The time endowment for early childhood is 5 years, which is 0.25 the length of a period (20 years). The share on capital,α_F, is set to 0, thereby preserving the assumption of constant returns to scale in production. Because of the long length of a period in this model, this is in effect shutting off long-term borrowing for parents. The depreciation rate of capital, δ_F is set to 0. Finally, the consumption equivalence (CE) scales are set using the 1994 scales from the Organisation for Economic Co-operation and Development (OECD). These scales assign a value of 1 for the first adult, and 0.5 for the subsequent adults; for each dependent the weight is

0.3. They adjust money spent on consumption into units of consumption for each member of the household. Once children leave the family, the equivalence scale for single mothers goes back to 1, and the scale for couples falls to 1.5. This is summarized in Table 6.

Table 6: Externally Chosen Parameters

Symbol	Name	Value
β	Patience	0.96^{20}
J	Duration Early Childhood	0.25
$\{\alpha_F, \delta_F\}$	Production Technology	$\{0, 0\}$
$\{\phi_1^{SM}\}$	OECD CE Scales: 1 adult, 2 children	$\{1.6\}$
$\{\phi_1^{MC}, \phi_{2,3}^{MC}\}$	OECD CE Scales: 2 adult, 2 children	$\{2.1, 1.5\}$

5.3 Other Externally Estimated Parameters

In Table 7, the level of lump-sum transfers T, the level of child support payments T_{cs}, and the price of p_n are set to 8% of output, 45% of the average per-family transfer, and 36% of the average mother's wage, respectively. The first empirical target is from the ratio of government transfers to persons for federal benefits from social insurance funds, Supplemental Nutrition Assistance Program (SNAP), supplemental security income, refundable tax credits, and other (which includes payments to nonprofit instutitions and student loans, among other categories) to GDP from the National Income and Production Accounts (NIPA) tabulations. The second is the ratio of average child support payments owed per month per capita to average monthly government transfers per family. The third is the average ratio of hourly price of non-parental child care to hourly wages of mothers in the ECLS-B. See the appendix for further details on the targets for these parameters.

Table 7: Externally Estimated Parameters

Symbol	Name	Source	Value
T	Transfers	NIPA	8% of output
T_{cs}	Child Support	Census and NIPA	45 % of T
κ	p_n coefficient	ECLS-B	36% ave. mother's wage

5.4 Internally Calibrated Parameters

Table 8 presents the internally calibrated parameters and the moments targeted to find their values. The parameters $b, \psi_s, \psi_{mc}, \lambda_{SM}$ and λ_{MC} are chosen to bring the model moments in the baseline as close as possible to the moments in the "Data" column of Table 8. The coefficient b controls the degree of altruism; ψ_s, ψ_{mc} are the marginal utilities of leisure for singles and married couples, respectively. The parameters λ_{SM} and λ_{MC} are shifters in the skill technology that scales up investment into efficiency units in the production of skill.

The moments I chose these parameters to match in the model compared with the data are the correlation of child skill with family income, the average labor supply of parents with children under 5 who are between 15 and 55 years old, the percent of single mothers raising children under 5, and the average time invested by parents of each family structure type. The moments from the ECLS-B in Table 8 are from the pooled estimation sample whose moments are given in Table 3. Note that the correlation of child skill and family income grows to 0.32 by the time the child is 5 years old; by contrast, when children are 9 months old, the measures of skill available in the data are uncorrelated with family income, although they do have predictive power for later child test scores at age 4. For regressions supporting these points, especially the assumption that initial child skill is independent of family attributes, see Appendix IV.

The moment most poorly matched in this calibration is the hours of parental time provided by single mothers, which is too high. This is driven by the single price of child care in the model; in Tables 2 and 4.3 it is clear that single mothers are choosing cheaper child care, which allows them to shift from their own time towards child care. This margin of adjustment is not present in the model.

Table 8: Internal Calibration: Targeted Moments + Parameters

Moment	Source	Data	Model
Corr. (child skill, family income) age 5	ECLS-B	0.32	0.32
Ave. pct. of time endowment in labor supply	CPS	0.31	0.33
Pct. parents married or cohabiting	ECLS-B	0.81	0.81
Parental time levels couples ($\frac{Hours}{Week}$)	ECLS-B	(10,4.6) (MM,MF)	(10,5.7)
Parental time levels singles ($\frac{Hours}{Week}$)	ECLS-B	2.6 SM	4.1

Notes: Internal calibration targeted moments (data v. model). ECLS-B data moments are from the raw sample.

Time investments are in units of hours per week.

Parameter	Name	Value
b	Altruism coefficient	$0.78 \times \beta$
ψ_{SM}	Marginal utility of leisure SM	2.1
λ_{SM}	Productivity of investment SM	6
ψ_{MC}	Marginal utility of leisure MC	1.82
λ_{SM}	Productivity of investment MC	23

Notes: Parameter names and values for internal calibration.

5.5 Untargeted Moments

Table 9 comapres the equilibrium moments of this model with six untargeted moments from the data: the correlation of hourly wages in spouse (or cohabiting partners), the correlation of parental time inputs within a couple, the ratio of average hourly wages in married or cohabiting mothers with single mothers, the average ratio of mother and father time, the average ratio of mother time and non-parental child care time, and the marriage rate of the poor.

Table 9 shows that the random search marriage market captures an appropriate degree of assortative matching among spouses: the correlation of wages within couples is a close fit with the data. However, within a couple, the model implies a much lower correlation of time inputs than seen in the data. This could reflect estimation bias in the point estimate for input complementarity parameter of married couples. It may also be due to the model assumption that parenting productivities are perfectly correlated with hourly wages. In the data, spouses may be sorting on unmodelled attributes which also determine parental time inputs.

Table 9: Untargeted Moments (Model Fit)

Moment	Source	Data	Model
Correlation of wages within a couple	ECLS-B	0.28	0.27
Correlation of time inputs within a couple	ECLS-B	0.39	0.25
$\frac{\text{ave. hourly wages, single mothers}}{\text{ave. hourly wages, mc mothers}}$	ECLS-B	0.68	0.67
Average $\frac{\text{father time}}{\text{mother time}}$	ECLS-B	0.77	0.59
Average $\frac{\text{mother time}}{\text{non-parental child care time}}$	ECLS-B	0.37	0.30
Marriage rate of the poor	ECLS-B (waves 1-3)	40%	25 %

Notes: Table 9 shows relevant moments for the implications of the model for sorting across family structures, assortative matching in marriage, patterns of time inputs for investments in child skill and the marital status composition of the poor. For the last moment, the poor in the model are defined as the bottom 25% of the after-tax family income distribution corrected with CE scales. Correlations are Pearson correlation coefficients, weighted with survey weights in the ECLS-B and using the analogous distribution in the model.

To examine how mothers sort into the family structures by skill, the third moment looks at the ratio of their wages across family structures. In the data, single mothers have lower hourly wages than married mothers. TThis paper's model endogenously captures the qualitative attribute of the data, and is also quantitatively in lign with it: mothers with higher labor market productivity tend to parent in a couple.

The fourth and fifth moments in Table 9 show the average ratios of time inputs in the model and in the data. The average ratio of time inputs within a couple is lower than the data, but the average ratio of mother time with non-parental child care time is closer to its empirical counterpoint. The former point is in lign with the correlation of time inputs within a couple being far from its empirical counterpart; these two moments are consistent with one another and to some extent convey redundant information. However, the average ratio does not convey the distribution of ratios in the population, while the correlation does add some information about that. The fifth moment of Table 9 reflects the lack of a quality margin for families in the model relative to the data: just as single mothers in the internal calibration oversupply their own time due to this lack, richer families overuse child care because they cannot adjust the quality instead.

Finally, the last row of Table 9 shows the marriage rate of the poor in the model compared with the data. The poor in the model are defined as the bottom 25% of the pre-tax family income distribution. The close link between skill and marriage in this model means that poor parents (low earners) raising young children are very rarely married compared with the data. Qualitatively the sorting in the model matches what is observed in the data; quantitatively, however, it is too extreme. Marriage in this model is not driven by preference shocks that determine the surplus of a potential match, so there are several degrees of freedom that could be used to better fit this moment. Considering the parsimonious nature of the model specification, the last row of Table 9 is neither surprising nor discouraging.

6 Decomposition Exercise

In this exercise, the model is solved at levels of the child care subsidy, τ_n, between 0 and 1. The model framework allows for the skill distribution, the return to skill, the labor income tax,and the levels of child support transfers, child support payments, productivity of child care, price of child care, and marriage decisions to adjust in light of the subsidy. First, the effects of the child care subsidy on the aggregate economy are discussed. Next, the gap in child skill gains across family structures is documented and decomposed into the two components of interest: composition and technology.

Subsidy Effect on the Aggregate Economy

In the aggregate, varying the child care subsidy encourages increased investments in skill, raising the supply of labor efficiency units but decreasing the return to each unit of skill, due to both a reduction in the wage rate and an increase in labor income taxes. As can be seen in Figure 2, although the return per unit of skill decreases, the hourly wage increases due to the increase in efficiency units of labor in the economy (left). As this relative price of leisure goes up, the time used in labor in decreases (right). That is, the income effect is dominating the substitution effect.

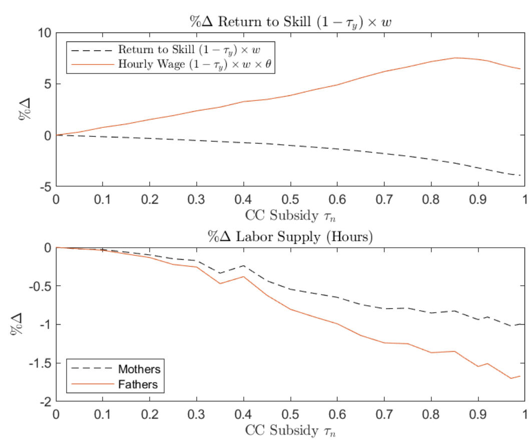

Figure 2

On average, consumers substitute away from labor supply but not enough to fully cancel out gains in earnings due to increases in skill. This is reflected in Figure 3 (left). In the same figure, the gender wage gap is reported. It is notable that the burden of child care disproportionately faced by women in this economy (which is the only difference in gender introduced into this environment) gives rise to a gap in earnings across the genders. This gap decreases due to the child care subsidy, as mothers are compensated by society for the inescable burden of parenthood associated with their gender type.

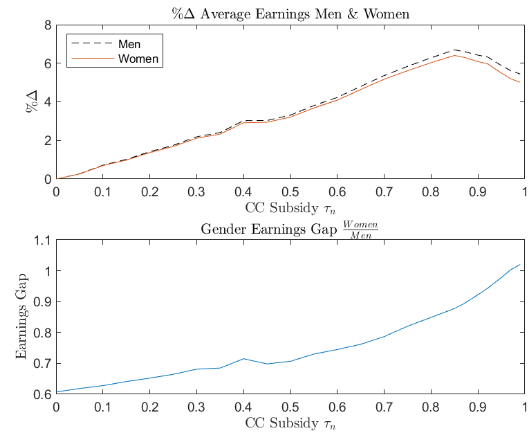

Figure 3

Subsidy Effect on Child Skill Accumulation

As the subsidy varies, the level of final skill for children changes with it. This can be seen in Figure 4, which shows that the levels of skill are higher for children born to two-parent families at any level of the subsidy: this subsidy does not close the gap between children born to the two family structures.

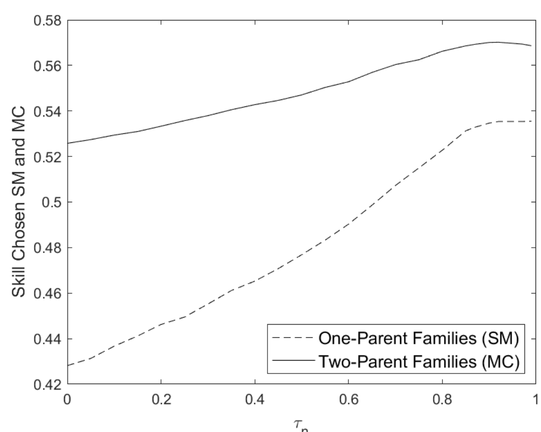

Figure 4

Figure 4 also illustrates that the change in levels and percentage points is higher for one-parent families (single mothers) than it is for two-parent families. This is due to the lower income of single mothers relative to couples, which arises from equilibrium sorting into family structures by labor market productivity. Initially, with no subsidy present, single mothers are choosing lower levels of final skill for their children, because that is what they can afford. Under the subsidy, single mothers also respond more strongly to changes in the price of investment in skill due to changes in the price of child care: evidently, gains in child skill are higher for single mothers than for couples. This reflects the different roles that child care plays as an input into investment: it's larger role for single mothers makes changes to the price of child care have a larger effect on the cost of financing any level of child skill. It also reflects that single mothers have lower labor market productivities than married mothers, and thus lower incomes, and so were choosing relatively lower levels of investment and child outcomes before the subsidy. Consequently, the marginal cost of increasing investment is lower for one-parent families.

Figure 5 shows the percent change in the final skill of children for single mothers and couples. As mentioned from the discussion of 4, the gains for children of single mothers are higher at ever level of the subsidy. Also notable is that the two lines in Figure 5 asymptote to upper bounds, which reflects the fact that the final skill of children is bounded. This is a property inherited from the inputs into investment being bounded as well by time endowments. This property of environment means that, even if the price of an input gets very close to 0, gains in child skill are not limitless.

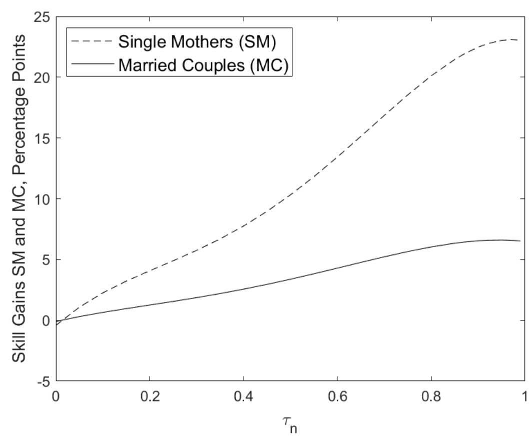

Figure 5

In Figure 5, the gap in child skill gains from child care subsidies across family structures is the difference between the dotted and solid lines plotted in the figure. This gap in skill gains can be attributed to the differences in technologies used by the two family structures, and to the fact that single mothers tend to have lower incomes than one-parent families, making their initial choices lower to begin with and more easily affected by the subsidy. In 6, counterfactual gains in child skill for one-parent families are added to the figure, this time controlling for income composition. To do this, the model is used to find the counterfactual average gains in child skill for single mothers, if they were a representative sample of the population rather than selected to be poorer than average. Evidently, the counterfactual gains for children of single mothers are lower than those of single mothers in equilibrium: the relative poverty of one-parent families magnifies the gains from child care subsidies but does not drive them completely.

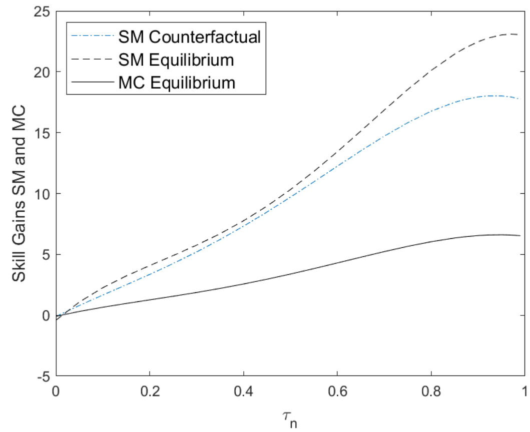

Figure 6

In terms of levels of child skill, Figure 7 shows how the level of child skill is affected by the composition of single mothers (as opposed to the percent change in this level). If single mothers did not endogenously tend to be lower-skilled women, child skill levels without the subsidy would be higher, and the gains in skill from the subsidy are lower. The gap between the outcomes of children from one- and two-parent families would be smaller to begin with, although subsidies would still act to narrow it.

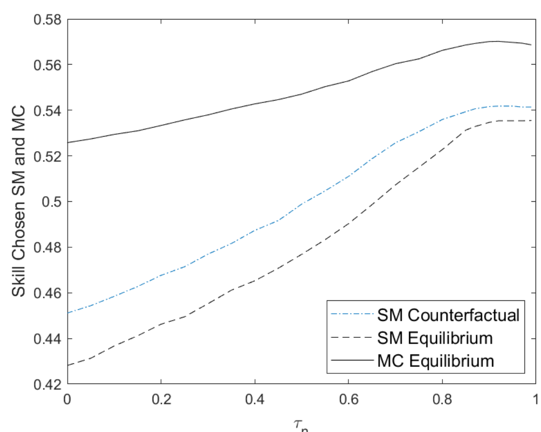

Figure 7

Figure 8 presents the percent of the gain in child skill gains across family structures that is explained by the income composition of single mothers. It is clearly not linear: instead, it first decreases from 40% to 8% and then increases back to about 38% as the subsidy gets larger. This pattern is due to the fact that at first, as the contribution of composition is a roughly constant level of skill gains in this portion, the increasing gap in skill is like increasing the denominator with a constant numerator. However, the counterfactual distribution of single mothers hits the upper bound of child skill gains faster than the equilibrium distribution of single mothers: before the subsidy they were already investing more, so they were closer to the upper bound and the upper bound on gains was correspondingly lower. The increase in the contribution of composition reflects the fact that the counterfactual distribution has hit the upper bound on skill gains at lower levels of the subsidy, while the equilibrium distribution of single mothers at the same subsidy level has not yet hit the upper bound.

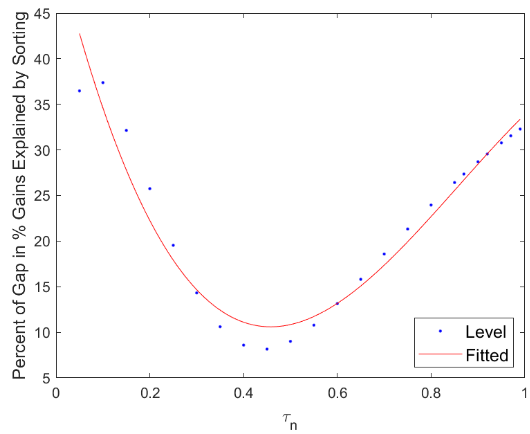

Figure 8

The finding presented in Figure 8 is the main result of the policy exercise conducted in this paper: the contribution of endogenous income composition is between 8% and 40% of the total gap across one- and two-parent families, with the contribution from exogenous differences in technologies being in the residual range of 92% to 60%. The large role of the skill investment technology in driving the higher sensitivity of one-parent families to the subsidy has an implication for the design of child care subsidies. When targeting these subsidies to those families that have the highest sensitivity, targeting by family structure rather than family income is a more direct way to select the most responsive families, whose children will see the highest gains in skill.

7 Conclusion

This paper quantifies and decomposes the gap across family structures in chid skill gains, due to a universally available child care subsidy. The gap is attributed to two components: first, exogenous differences in the way time inputs generate investment in child skill across the two family structures, and second, endogenous differences in family income due to sorting in the marriage market. Using a model which allows for labor income taxes, wages, family formation, the price and productivity of child care, and the effect of skill on lifetime utility to adjust to the child care subsidy, it is possible to control for the latter source of differences and quantify the contribution of each component. The main finding of this paper is that the contribution of endogenous income composition is between 8% and 40% of the total gap across one- and two-parent families, with the contribution from exogenous differences in technologies being in the residual range of 92% to 60%.

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Appendix

Contents (Appendix)

Part I

Deriving the Estimation Equations from the 2-Stage Parenting Problems

A Presenting the Parenting Problem in Two Stages

In what follows I change the notation slightly to allow the solutions to be generalizable. In the model presented in this paper, the hourly wage for a parent of gender $g \in \{f, m\}$ and labor market productivity θ_g is $w\theta_g$. The parenting productivity is assumed to be θ_g, the same as labor market productivity. Here, I denote the wage $w_g = w\theta_g$, and the parenting productivity ϕ_g. This relaxes the assumption that the parenting productivity and labor market productivity are the same.

The Parenting Problem of a Single Mother as a Two-Stage Decision Process

The parenting problem of the single mother can equivalently be solved in two stages: cost minimization and utility maximization. This two-stage representation of the parenting problems allows for a more transparent derivation of the estimation equations, while the previous formulation represented in (6) afforded a clearer interpretation of the problem parents face.

Cost minimization has two component problems: intra-temporal cost minimization and inter-temporal cost minimization. In the intra-temporal component, for a given level of investment the parent chooses the cheapest combination of inputs that achieve it. In the subsequent inter-temporal component, given an initial child skill the parent chooses the cost-minimizing sequence of investments necessary to achieve it some final child skill level, $\overline{\theta}_{J+1}$. This inter-temporal component problem uses investment prices constructed from the solution to the intra-temporal cost minimization problem. This general description will also hold true for the married couples problem, whose parenting problem is specified later.

For the single mother family, for any level of investment I_t^{SM}, the cost-minimizing time inputs from the mother and commercial child care provider solve the following intra-temporal cost minimization problem:

$$
\begin{aligned}
\left\{q_t^{m,*}\left(I_t\right), n_t^*\left(I_t\right)\right\} &= \arg \min_{\left\{q_t^m, n_t\right\}} X_t\left(I_t\right) \\
s.t. \quad \forall t & \\
X_t\left(I_t\right) &= w_t^m q_t^m + p_t^n n_t \\
I_t &= I_t^{SM}\left(\phi_n n_t, \phi_m q_t^m\right) \\
n_t, q_t^m \in [0,1] & \qquad n_t + q_t^m \leq 1
\end{aligned} \tag{18}
$$

The solution to this problem allows the implicit price index of investment to be defined as $\Lambda_t = \frac{X_t^*(I_t)}{I_t}$, where $X_t^*\left(I_t\right)$ is the cost of investment level I_t evaluated at optimal inputs for that level of investment, $q_t^{m,*}\left(I_t\right)$ and $n_t^*\left(I_t\right)$. Note that the price of investment will be a composite of the prices of the two inputs, the mother's wage and the price of child care. Using the sequence of investment prices $\{\Lambda_t\}_{t=1}^J$, for a given final skill of the child, $\overline{\theta}_{J+1}$, the parent find the cost-minimizing sequence of investment levels by solving the second (inter-temporal) component problem of the cost minimization stage:

$$
\begin{aligned}
\left\{I_t^{SM,*}\left(\theta_{J+1}\right)\right\}_{t=1}^J &= \arg \min_{\{I_t\}_{t=1}^J} \Lambda_1 I_1 + \sum_{t=2}^J \left(\Pi_{j=1}^{t-1} \frac{1}{1+r_{j+1}}\right) \Lambda_t I_t \\
s.t. & \\
\theta_{J+1} &= \overline{\theta}_{J+1}\left(\theta_1, \{I_t\}_{t=1}^J\right)
\end{aligned} \tag{19}
$$

The function $\overline{\theta}_{J+1}\left(\theta_1, \{I_t\}_{t=1}^J\right)$ is constructed by working backwards from $\theta_{J+1} = f\left(\theta_J, I_J^{type}\right)$ and substituting in each period t for θ_{t-1}, and so on, until the expression is in terms of the initial stock of skill θ_1 and a sequence of investment levels $\{I_t\}_{t=1}^J$. [16]

In the second stage of the two-stage specification, the family solves the utility maximization problem. Here, parents take as given the price and cost-minimizing level of investment necessary to finance a final skill of the child, θ_{J+1}, and choose the final skill of the child, θ_{J+1}, taking into account the financial cost in terms of foregone earnings and child care, represented by $\Lambda_t I_t^{SM,*}\left(\theta_{J+1}\right)$, and also the loss of time that can be devoted to leisure, represented by $q_t^{m,*}\left(I_t^{SM}\right)$.

[16] I will derive $\overline{\theta}_{J+1}\left(\theta_1, \{I_t\}_{t=1}^J\right)$ later on in this paper, using the assumed functional forms.

$$
\begin{aligned}
V^{SM}\left(a_1,\theta_1,\phi_n,\phi_m,\{w_t^m\}_{t=1}^J\right) &= \max_{\theta_{J+1},\{c_t,\ell_t,a_{t+1}\}_{t=1}^J} \sum_{t=1}^J \beta^{t-1}u^{SM}(c_t,\ell_t)+\beta^{J-1}bV^{child}(\theta_{J+1}) \qquad (20)\\
\text{s.t.} \quad \forall t & \\
c_t+a_{t+1}+\Lambda_t I_t^{SM,*}(\theta_{J+1}) &\leq w_t^m(1-\ell_t)+(1+r_t)a_t+T\\
\ell_t &\in \left[0,1-q_t^{m,*}\left(I_t^{SM}\right)\right]\\
\theta_{J+1} &\in \left[\underline{\theta}^{SM},\overline{\theta}^{SM}\right]\\
a_1,a_{J+1} &= 0
\end{aligned}
$$

The choice of final skill for the child has to be within the range of $\left[\underline{\theta}^{SM},\overline{\theta}^{SM}\right]$, where $\underline{\theta}^{SM}$ is the level of skill achieved if investment is 0 for all periods of childhood, and $\overline{\theta}^{SM}$ is the level of skill achieved if all of the child's time endowment is used for investment in each period of childhood. Finally, single mothers begin life with no assets, and are constrained to consume all of their income in the final period of life.

The Parenting Problem of a Married Couple as a Two-Stage Decision Process

The parenting problem of the married couple, like that of the single mother, can also be represented as being solved in two stages: cost minimization and utility maximization.

The two component problems of cost minimization stage for a married couple investing in their child's skill are similar to those of a single mother. First, within a period the married couple solves an intra-temporal cost minimization problem, where for a given level of investment I_t they choose the combination of inputs $\left\{q_t^f,q_t^m,n_t\right\}$ that achieve it with at lowest cost, subject to their skill production technology. That is, they solve:

$$
\begin{aligned}
\left\{q_t^{f,*}(I_t),q_t^{m,*}(I_t),n_t^*(I_t)\right\} &= \arg\min_{\left\{q_t^f,q_t^m,n_t\right\}} X_t(I_t) \qquad (21)\\
s.t. \quad \forall t & \\
X_t(I_t) &= w_t^f q_t^f+w_t^m q_t^m+p_t^n n_t\\
I_t &= I_t^{MC}\left(\phi_n n_t,\phi_f q_t^f,\phi_m q_t^m\right)\\
n_t,q_t^f,q_t^m &\in [0,1]\\
n_t+\max\left\{q_t^f,q_t^m\right\} &\leq 1
\end{aligned}
$$

Next, in the inter-temporal cost minimization problem, given an initial skill θ_1, the couple chooses the cost-minimizing sequence of investments necessary to achieve a target final skill θ_{J+1}. From the component problem of the cost minimization stage (21), the composite price index of a given level of investment is constructed as $\Lambda_t = \frac{X_t^*(I_t)}{I_t}$, where $X_t^*(I_t)$ is the cost of investment level I_t evaluated at optimal input levels $q_t^{f,*}(I_t), q_t^{m,*}(I_t)$ and $n_t^*(I_t)$. Note that the price of investment is now a composite of the prices of the three inputs: the father's wage, the mother's wage, and the price of child care. Using the sequence of investment prices $\{\Lambda_t\}_{t=1}^J$, for a given final skill of the child the parent find the cost-minimizing sequence of investment levels by solving the following inter-temporal cost minimization problem:

$$\begin{aligned}
\{I_t^*(\theta_{J+1})\}_{t=1}^J &= \arg\min_{\{I_t\}_{t=1}^J} \Lambda_1 I_1 + \sum_{t=2}^J \left(\Pi_{j=1}^{t-1}\frac{1}{1+r_{j+1}}\right)\Lambda_t I_t \\
s.t.& \\
\theta_{J+1} &= \overline{\theta}_{J+1}\left(\theta_1, \{I_t\}_{t=1}^J\right)
\end{aligned} \tag{22}$$

Here, the function $\overline{\theta}_{J+1}\left(\theta_1, \{I_t\}_{t=1}^J\right)$ is constructed as described when the single mother's cost minimization problem was presented above.

For couples, the utility maximization problem uses the investment price and level in each period (given θ_{J+1}) determined in the cost minimization stage. The married couple then choose θ_{J+1} to maximize the family's utility:

$$\begin{aligned}
V_1^{MC}\left(\theta_1, \phi_n, \phi_m, \phi_f, \left\{w_t^m, w_t^f\right\}_{t=1}^J\right) &= \max_{\theta_{J+1}, \{c_t, \ell_t, a_{t+1}\}_{t=1}^J} \left[\sum_{t=1}^J \beta^{t-1} u^{MC}(c_t, \ell_t)\right] + \beta^{J-1} b V^{child}(\theta_{J+1}) \\
s.t. \quad \forall t& \\
c_t + a_{t+1} + \Lambda_t I_t^{MC,*}(\theta_{J+1}) &\leq w_t^f(1-\ell_t) + w_t^m(1-\ell_t) + (1+r_t)a_t + T \\
\ell_t &\in \left[0, 1 - \max\left\{q_t^{f,*}(I_t), q_t^{m,*}(I_t)\right\}\right] \\
\theta_{J+1} &\in \left[\underline{\theta}^{MC}, \overline{\theta}^{MC}\right] \\
a_1, a_{J+1} &= 0
\end{aligned} \tag{23}$$

Here, the bounds on final skill for the child, $\underline{\theta}^{MC}$ and $\overline{\theta}^{MC}$, are indexed to the family structure type.

B Estimation Equations

In this section I assume functional forms for $f\left(\theta_t, I_t^{type}\right)$, I_t^{SM}, and I_t^{MC}, the functions that govern how inputs into investment affect skill. Using these assumptions, and the first order conditions of the cost-minimization portion of the parenting problems described above, I can derive the estimation equations used to estimate the parameters of these functions from the data.

B.1 Investment Technologies

The current stock of skill, θ_t, and chosen level of investment I_t^{type} combine according to the function $f\left(\theta_t, I_t^{type}\right)$ to produce the stock of skill in the next period. The two family structures both use $f\left(\theta_t, I_t^{type}\right)$ to aggregate their child's skill with the family's investment.

$$\theta_{t+1} = f\left(\theta_t, I_t^{type}\right) = \left[\upsilon\left(\lambda_{type} I_t^{type}\right)^{\frac{\chi-1}{\chi}} + (1-\upsilon)\left(\theta_t\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}$$

Here I assume that this function is a constant elasticity of substitution (CES) function, with parameters χ and υ. In a one-period parenting problem parameter, the elasticity of substitution parameter χ can be interpreted as governing how easily investment can compensate for initial skill. If this elasticity is low, it takes more investment to change initial skill than if the elasticity is high. In a model where $J > 1$, so that parenting is a multi-period investment problem, the elasticity of substitution χ governs how easily investment can be shifted across periods of childhood in response to changes in its price across periods. If χ is low, it means that the technology does not allow investment to be reallocated across periods easily in response to a change in the price of investment in one period. In a one-period parenting problem, the share parameter υ can be interpreted as determining the importance of investment in determining the final skill of the child. If υ is low, it takes more investment to affect the stock of skill, and vice versa. In a model where $J > 1$, the share parameter υ determines how investment with a constant price across consecutive periods would be allocated. If υ is low, then investment in each period of childhood will have to be consistent, whereas if υ was high investment could be more lumpy. Finally, λ is simply a scaling parameter that accounts for units of measurement being different across investment and the stock of skill.

The two family structures differ in the way that parental time and time purchased on the market in the form of child care combine into investment. For single mothers, the functional form for investment in each period is:

$$I_t^{SM}\left(\phi_n n_t, \phi_m q_t^m\right) = \left[\alpha_s\left(\phi_m q_t^m\right)^{\frac{\eta-1}{\eta}} + (1-\alpha_s)\left(\phi_n n_t\right)^{\frac{\eta-1}{\eta}}\right]^{\left(\frac{\eta}{\eta-1}\right)}$$

For married couples the investment function is:

$$I_t^{MC}\left(\phi_f q_t^f, \phi_m q_t^m, \phi_n n_t\right) = \left[\alpha_1\left(\phi_f q_t^f\right)^{\frac{\epsilon-1}{\epsilon}} + \alpha_2\left(\phi_m q_t^m\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha_1-\alpha_2)\left(\phi_n n_t\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\left(\frac{\epsilon}{\epsilon-1}\right)}$$

The partial derivatives of the investment technologies, as defined here, will be substituted into the first order conditions of the parenting problem during the derivation of estimation equations in the next section. The partial derivatives of the investment functions above with respect to their arguments are:

$$\begin{aligned}
\frac{\partial I_t^{SM}\left(\phi_n n_t,\phi_m q_t^m\right)}{\partial q_t^m} &= \phi_m\alpha_s\left(\phi_m q_t^m\right)^{\frac{\eta-1}{\eta}-1}\left[\alpha_s\left(\phi_m q_t^m\right)^{\frac{\eta-1}{\eta}}+\left(1-\alpha_s\right)\left(\phi_n n_t\right)^{\frac{\eta-1}{\eta}}\right]^{\left(\frac{1}{\eta-1}\right)-1}\\
\frac{\partial I_t^{SM}\left(\phi_n n_t,\phi_m q_t^m\right)}{\partial n_t} &= \phi_n\left(1-\alpha_s\right)\left(\phi_n n_t\right)^{\frac{\eta-1}{\eta}-1}\left[\alpha_s\left(\phi_m q_t^m\right)^{\frac{\eta-1}{\eta}}+\left(1-\alpha_s\right)\left(\phi_n n_t\right)^{\frac{\eta-1}{\eta}}\right]^{\left(\frac{\eta}{\eta-1}\right)-1}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial I_t^{MC}\left(\phi_f q_t^f,\phi_m q_t^m\phi_n n_t\right)}{\partial q_t^f} &= \phi_f\alpha_1\left(\phi_f q_t^f\right)^{\frac{\epsilon-1}{\epsilon}-1}\left[\alpha_1\left(\phi_f q_t^f\right)^{\frac{\epsilon-1}{\epsilon}}+\alpha_2\left(\phi_m q_t^m\right)^{\frac{\epsilon-1}{\epsilon}}+\left(1-\alpha_1-\alpha_2\right)\left(\phi_n n_t\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\left(\frac{\epsilon}{\epsilon-1}\right)-1}\\
\frac{\partial I_t^{MC}\left(\phi_f q_t^f,\phi_m q_t^m\phi_n n_t\right)}{\partial q_t^m} &= \phi_m\alpha_2\left(\phi_m q_t^m\right)^{\frac{\epsilon-1}{\epsilon}-1}\left[\alpha_1\left(\phi_f q_t^f\right)^{\frac{\epsilon-1}{\epsilon}}+\alpha_2\left(\phi_m q_t^m\right)^{\frac{\epsilon-1}{\epsilon}}+\left(1-\alpha_1-\alpha_2\right)\left(\phi_n n_t\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\left(\frac{\epsilon}{\epsilon-1}\right)-1}\\
\frac{\partial I_t^{MC}\left(\phi_f q_t^f,\phi_m q_t^m\phi_n n_t\right)}{\partial n_t} &= \phi_n\left(1-\alpha_1-\alpha_2\right)\left(\phi_n n_t\right)^{\frac{\epsilon-1}{\epsilon}-1}\left[\alpha_1\left(\phi_f q_t^f\right)^{\frac{\epsilon-1}{\epsilon}}+\alpha_2\left(\phi_m q_t^m\right)^{\frac{\epsilon-1}{\epsilon}}+\left(1-\alpha_1-\alpha_2\right)\left(\phi_n n_t\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\left(\frac{\epsilon}{\epsilon-1}\right)-1}
\end{aligned}$$

B.2 Deriving the Estimation Equations

Given an initial child skill, θ_1, a target final skill for the child, θ_{J+1} , an exogenous sequence of prices $\{p_t^n, w_t^m\}_{t=1}^J$ for single mothers or $\left\{p_t^n, w_t^m, w_t^f\right\}_{t=1}^J$ for married couples, a child care productivity ϕ_n, and parenting productivities $\{\phi_m\}$ for single mothers or $\{\phi_m, \phi_f\}$ for couples, the cost minimization stage of the parenting problem has two components. First, for any level of investment in each period, the family minimizes the cost of producing that investment level by picking child care and parental time inputs. From this the price of investment can be constructed. Second, using the price of investment in each period, the family minimizes the cost of achieving θ_{J+1} by allocating investment levels across periods.

B.2.1 Single Mothers Investment

To derive the estimation equations for the parameters of the function I_t^{SM}, I rely on the optimality conditions of the intra-temporal component problem of the cost minimization stage for single mothers (problem (18)). These first order conditions are:

$$\begin{aligned}
[n_t] &\qquad \frac{\partial X_t}{\partial n_t}-\mu\frac{\partial I_t^{SM}\left(\phi_m q_t^m\phi_n n_t\right)}{\partial n_t}=0\\
[q_t^m] &\qquad \frac{\partial X_t}{\partial q_t^m}-\mu\frac{\partial I_t^{SM}\left(\phi_m q_t^m\phi_n n_t\right)}{\partial q_t^m}=0
\end{aligned}$$

Here, μ is the Lagrangian multiplier on the production level constraint. Taking the ratio of these two equations yields:

$$\frac{p_t^n}{w_t^m} = \frac{\frac{\partial I_t^{SM}(\phi_m q_t^m \phi_n n_t)}{\partial n_t}}{\frac{\partial I_t^{SM}(\phi_m q_t^m \phi_n n_t)}{\partial q_t^m}}$$

Substituting for the partial derivatives of the function I^{SM}, and changing notation to be at the level of observation in the data (family i):

$$\begin{aligned}
\frac{p_{i,t}^n}{w_{i,t}^m} &= \frac{\frac{\partial I_{i,t}}{\partial n_{i,t}}}{\frac{\partial I_{i,t}}{\partial q_{i,t}^m}} \implies \\
\ln\left(\frac{p_{i,t}^n}{w_{i,t}^m}\right) &= \ln\left(\frac{\frac{\partial I_{i,t}^{SM}\left(\phi_m q_{i,t}^m \phi_n n_{i,t}\right)}{\partial n_{i,t}}}{\frac{\partial I_{i,t}^{SM}\left(\phi_m q_{i,t}^m \phi_n n_{i,t}\right)}{\partial q_{i,t}^m}}\right) \implies \\
\ln\left(\frac{q_{i,t}^m}{n_{i,t}}\right) &= \eta \ln\left(\frac{\alpha_s}{1-\alpha_s}\right) - \eta \ln\left(\frac{w_{i,t}^m}{p_{i,t}^n}\right) - (\eta - 1)\ln\left(\frac{\phi_{n,i}}{\phi_{m,i}}\right) \qquad (24)
\end{aligned}$$

Estimating (24) will allow me to map from regression coefficients to the single-mother specific skill technology parameters η and α_s.

B.2.2 Married Couple Investment

To derive the estimation equations for the parameters of the function I^{MC}, I rely on the optimality conditions of the intra-temporal cost minimization component problem for the cost minimization stage of the married couple problem (problem (21)). These first order conditions are:

$$\begin{aligned}
[n_t] \qquad & \frac{\partial X_t}{\partial n_t} - \mu \frac{\partial I_t^{SM}\left(\phi_m q_t^m \phi_n n_t\right)}{\partial n_t} = 0 \\
\left[q_t^f\right] \qquad & \frac{\partial X_t}{\partial q_t^f} - \mu \frac{\partial I_t^{SM}\left(\phi_m q_t^m \phi_n n_t\right)}{\partial q_t^f} = 0 \\
[q_t^m] \qquad & \frac{\partial X_t}{\partial q_t^m} - \mu \frac{\partial I_t^{SM}\left(\phi_m q_t^m \phi_n n_t\right)}{\partial q_t^m} = 0
\end{aligned}$$

Taking ratios of the last two equations with respect to the first one yields the two tangency conditions:

$$\frac{p_t^n}{w_t^m} = \frac{\frac{\partial I_t^{SM}(\phi_m q_t^m \phi_n n_t)}{\partial n_t}}{\frac{\partial I_t^{SM}(\phi_m q_t^m \phi_n n_t)}{\partial q_t^m}}$$

$$\frac{p_t^n}{w_t^f} = \frac{\frac{\partial I_t^{SM}(\phi_m q_t^m \phi_n n_t)}{\partial n_t}}{\frac{\partial I_t^{SM}(\phi_m q_t^m \phi_n n_t)}{\partial q_t^f}}$$

Substituting for the partial derivatives of the investment function, and changing notation to be at the level of observation in the data (family i):

$$\ln\left(\frac{q_{i,t}^m}{n_{i,t}}\right) = \epsilon \ln\left(\frac{\alpha_2}{1-\alpha_1-\alpha_2}\right) - \epsilon \ln\left(\frac{w_{i,t}^m}{p_{i,t}^n}\right) + \left(\frac{1}{\epsilon-1}\right) \ln\left(\frac{\phi_{m,i}}{\phi_{n,i}}\right) \tag{25}$$

$$\ln\left(\frac{q_{i,t}^f}{n_{i,t}}\right) = \epsilon \ln\left(\frac{\alpha_1}{1-\alpha_1-\alpha_2}\right) - \epsilon \ln\left(\frac{w_{i,t}^f}{p_{i,t}^n}\right) + \left(\frac{1}{\epsilon-1}\right) \ln\left(\frac{\phi_{f,i}}{\phi_{n,i}}\right) \tag{26}$$

Estimating (25) and (26) will allow me to map from regression coefficients to the single-mother specific skill technology parameters ϵ, α_1, and α_2.

B.2.3 Common aggregation function

The parameters of the function $f\left(\theta_t, I_t^{type}\right)$, which aggregates the current stock of skill with investment, are the same for single mothers and married couples (problems (19) and (22)). The single mother problem will differ from the married couple problem in the way that the price of investment and total expenditure are constructed. The first order condition of this inter-temporal cost minimization problem are, for period 1,

$$\left[I_1^{type}\right] \qquad \Lambda_1 - \mu \frac{\partial \overline{\theta}_{J+1}\left(\theta_1, \left\{I_t^{type}\right\}_{t=1}^{J}\right)}{\partial I_t} = 0$$

For any period $t > 1$,

$$\left[I_t^{type}\right] \qquad \left(\Pi_{j=1}^{t-1} \frac{1}{(1+r_{j+1})}\right) \Lambda_t - \mu \frac{\partial \overline{\theta}_{J+1}\left(\theta_1, \left\{I_t^{type}\right\}_{t=1}^{J}\right)}{\partial I_t} = 0$$

For any two consecutive periods, the ratio of the first order conditions yields an inter-temporal

Euler equation, which balances the relative price of investment in consecutive periods with the relative price of moving value across those periods:

$$\frac{1}{1+r_t}\frac{\Lambda_t}{\Lambda_{t+1}} = \frac{\frac{\partial\overline{\theta}_{J+1}\left(\theta_1,\{I_t\}_{t=1}^{J}\right)}{\partial I_t}}{\frac{\partial\overline{\theta}_{J+1}\left(\theta_1,\{I_t\}_{t=1}^{J}\right)}{\partial I_{t+1}}} \tag{27}$$

This equation determines how investment in child skill is optimally spread over periods of childhood.

Given my functional form assumption, the function $\overline{\theta}_{J+1}$ can be constructed, and its partial derivatives found:

$$\begin{aligned}
f\left(\theta_t, I_t^{type}\right) &= \left[\upsilon\left(\lambda I_t^{type}\right)^{\frac{\chi-1}{\chi}} + (1-\upsilon)\left(\theta_t\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}} \qquad type \in \{SM, MC\} \\
&\Longrightarrow \\
\overline{\theta}_{J+1}\left(\theta_1, \left\{I_t^{type}\right\}_{t=1}^{J}\right) &= \left[\sum_{t=1}^{J}\upsilon\left(1-\upsilon\right)^{J-t}\left(\lambda I_t^{type}\right)^{\frac{\chi-1}{\chi}} + (1-\upsilon)^J\left(\theta_1\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}
\end{aligned}$$

Then:

$$\begin{aligned}
\frac{\partial\overline{\theta}_{J+1}}{\partial I_t^{type}} &= \lambda\upsilon\left(1-\upsilon\right)^{J-t}\left(\lambda I_t^{type}\right)^{\frac{\chi-1}{\chi}-1}\left[\sum_{t=1}^{J}\upsilon\left(1-\upsilon\right)^{J-t}\left(\lambda I_t^{type}\right)^{\frac{\chi-1}{\chi}} + (1-\upsilon)^J\left(\theta_1\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}-1} \\
\frac{\partial\overline{\theta}_{J+1}}{\partial I_{t+1}^{type}} &= \lambda\upsilon\left(1-\upsilon\right)^{J-t+1}\left(\lambda I_{t+1}^{type}\right)^{\frac{\chi-1}{\chi}-1}\left[\sum_{t=1}^{J}\upsilon\left(1-\upsilon\right)^{J-t}\left(\lambda I_t^{type}\right)^{\frac{\chi-1}{\chi}} + (1-\upsilon)^J\left(\theta_1\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}-1}
\end{aligned}$$

Substituting into (27), I adjust notation in two ways for greater clarity in mapping the function to the data. First, I am specific about which family type is generating investment (I add this index to the subscript for I_t). This will be useful because I have to construct investment in the data (more on this in the next section). The other adjustment I make to notation is changing the subscript notation to be at the level of observation in the data (family i):

$$
\begin{aligned}
\frac{\frac{\partial f\left(\theta_{1,i},\{I_{i,t}^{type}\}_{t=1}^{J}\right)}{\partial I_{i,t}^{type}}}{\frac{\partial f\left(\theta_{1,i},\{I_{i,t}^{type}\}_{t=1}^{J}\right)}{\partial I_{i,t+1}^{type}}} &= \frac{\Lambda_{i,t}^{type}}{\Lambda_{i,t+1}^{type}}\frac{1}{1+r_t} \\
&\Longrightarrow \\
\ln\left(\frac{I_{i,t+1}^{type}}{I_{i,t}^{type}}\right) &= -\chi\ln\left(\frac{\Lambda_{i,t+1}^{type}}{\Lambda_{i,t}^{type}}\right)+\chi\ln\left(\frac{1}{1-\upsilon}\right)+\chi\ln\left(\frac{1}{1+r_t}\right)
\end{aligned}
$$

To build all the variables in this estimation equation, construct $I_{i,t}^{type}$ for all $\{type, i, t\}$ (where $type \in \{SM, MC\}$) in the data using the functional form assumed for $I_{i,t}^{type}$, and then compute $\Lambda_{i,t}^{type}$ from the definition of the cost of investment: $X_{i,t}^{type} = \Lambda_{i,t}^{type} I_{i,t}^{type}$. Here, $X_{i,t}^{type}$ is expenditures on investment in period t.

For single mothers, this is defined as $X_{i,t}^{SM} = p_{i,t}^{n} n_{i,t} + w_{i,t}^{m} q_{i,t}^{m}$, and for couples it is defined as $X_{i,t}^{MC} = p_{i,t}^{n} n_{i,t} + w_{t}^{m} q_{t}^{m} + w_{i,t}^{f} q_{i,t}^{f}$

Using this definition of the price of investment in each period, the estimation equation can be rewritten in terms of expenditures and the price of investment:

$$
\ln\left(\frac{X_{i,t+1}^{type}}{X_{i,t}^{type}}\right)-\ln\left(\frac{\Lambda_{i,t+1}^{type}}{\Lambda_{i,t}^{type}}\right) = \chi\ln\left(\frac{1}{1-\upsilon}\right)+\chi\left[\ln\left(\frac{1}{1+r_t}\right)-\ln\left(\frac{\Lambda_{i,t+1}^{type}}{\Lambda_{i,t}^{type}}\right)\right] \tag{28}
$$

To conclude, by estimating equation (28), I can identify the last two skill production technology parameters υ and χ. It is necessary to construct the price of investment for each family in each period in order to construct this equation; this is done as described above.

Part II

Solutions to the Parenting Problem: Allowing for the Time Constraint of the Child to Bind

In this section, I explain how the potentially binding time constraint of the child does not affect achievable levels of investment or skill (feasibility) but does affect the optimal composition of inputs that are used to achieve that level of investment. In particular, the marginal cost of increasing investment is constant when the time constraint of the child is slack, and increasing when the time constraint is binding.

As the prices of inputs that parents face diverge due to market forces or a policy intervention such as a subsidy, the level of investment that is achievable with a slack time constraint of the child decreases. Accordingly, region of investment where the time constraint is binding gets larger. When using a multiple-input CES with a time constraint, and subsidies to one or more of the inputs (such as I do) it is necessary to account for this so that counterfactuals will be correct in simulation exercises. In the data used for the estimation, few children had more than a reasonable amount of time reported as in either child care or participating in educational activities with parents. This case is not empirically relevant but it is relevant for counterfactuals. Therefore, I go into detail here.

C Married or Cohabiting Couples

Primitives: The skill accumulation technology functional form

For clarity of presentation and to facilitate interpretation, I impose the following transformation of notation to the problem presented in this paper:

$$\begin{aligned}
\omega_1 &\equiv \alpha_1 \phi_f^{\eta} \\
\omega_2 &\equiv \alpha_2 \phi_f^{\eta} \\
\omega_3 &\equiv \alpha_3 \phi_f^{\eta} \\
w_f &= (1-\tau_y)(1-\tau_{qf})\, w\theta_f \\
w_m &= (1-\tau_y)(1-\tau_{qm})\, w\theta_m \\
p_n &= (1-\tau_n)\, \tilde{p}_n
\end{aligned}$$

Which makes the primitives:

$$\begin{aligned}
\theta_{k,t+1} &= \left[\upsilon \left(\lambda_{MC,t} I_t\right)^{\frac{\chi-1}{\chi}} + (1-\upsilon)\, \theta_{k,t}^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}} \\
I_{couple} &= \left[\omega_1 q_f^{\frac{\epsilon-1}{\epsilon}} + \omega_2 q_m^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}} \right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)}
\end{aligned}$$

Restrictions on parameters:

$$\begin{aligned}
\epsilon &\geq 0 \\
\kappa &\geq 0 \\
\chi &\geq 0
\end{aligned}$$

Bounds on inputs

$$\begin{aligned} q_f &\in [0,1] \\ q_m &\in [0,1] \\ n &\in [0,1] \\ n + \max\{q_m, q_f\} &\leq J \end{aligned}$$

D The Couple's Cost Minimization Problem

The Karusch Kuhn-Tucker Formulation of the Couple's Problem

The couple's problem is 2-staged: in the first stage, for a given level of investment the family determines whether it is feasible and what the combination of inputs should be to finance it. In the second stage, the family optimally chooses their level of investment in their child. The first stage is a cost-minimization problem with the time constraint of the child limiting the total inputs into investment that can be used. This implies some restrictions on the levels of investment that are feasible.

Here I focus on the first stage of the couple's parenting problem. I want to characterize the optimal combination of inputs for any feasible level of investment; this includes investment levels for which the time constraint is slack and also for which it is binding.

$$\begin{aligned}
\mathbb{L}_{q_m,q_f,n} \quad & -[w_m q_m + w_f q_f + p_n n] + \mu\left[J - \max\{q_m, q_f\} - n\right] + \lambda\left[\left[\omega_1 q_f^{\frac{\epsilon-1}{\epsilon}} + \omega_2 q_m^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} - I\right] \\
\text{FOCs:} & \\
[q_m] \quad & -w_m - \mu \mathbb{I}_{q_m^* > q_f^*} + \lambda\left(1 - \frac{1}{\epsilon}\right)\omega_1 q_f^{-\frac{1}{\epsilon}} = 0 \\
[q_f] \quad & -w_f - \mu \mathbb{I}_{q_f^* > q_m^*} + \lambda\left(1 - \frac{1}{\epsilon}\right)\omega_2 q_m^{-\frac{1}{\epsilon}} = 0 \\
[n] \quad & -p_n - \mu + \lambda\left(1 - \frac{1}{\epsilon}\right)\omega_3 n^{-\frac{1}{\epsilon}} = 0 \\
[\mu] \quad & J - \max\{q_m, q_f\} - n = 0 \quad or \quad \mu = 0 \\
[\lambda] \quad & \left[\omega_1 q_f^{\frac{\epsilon-1}{\epsilon}} + \omega_2 q_m^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} - I = 0 \quad or \quad \lambda = 0
\end{aligned}$$

$\Longrightarrow$

$$
\begin{aligned}
w_f + \mu \mathbb{I}_{q_m^* > q_f^*} &= \lambda \omega_1 q_f^{-\frac{1}{\epsilon}} \\
w_m + \mu \mathbb{I}_{q_f^* > q_m^*} &= \lambda \omega_2 q_m^{-\frac{1}{\epsilon}} \\
p_n + \mu &= \lambda \omega_3 n^{-\frac{1}{\epsilon}}
\end{aligned}
$$

To take first order conditions, go by cases. Assume that q_m is bigger than q_f, solve for the implied quantities assuming both this and that the time constraint is satisfied, and then check that both are true. If the assumption that $q_m > q_f$ is falsified, proceed to assuming that $q_f > q_m$. If the assumption that the time constraint is falsified, proceed to solving for the n, q_f, and q_m that produce the desired level of investment with a binding time constraint using only the production technology. Here, you will have to relate q_f and q_m in some way.

The maximum possible level of investment

$$
\begin{aligned}
\overline{I}_{mc} &= \max_{n \in [0,J]} \left[\left[\omega_1 (J-n)^{\frac{\epsilon-1}{\epsilon}} + \omega_2 (J-n)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}} \right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} \right] \\
0 &= \kappa \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\left[-\frac{\epsilon-1}{\epsilon} \omega_1 (J-n)^{\frac{\epsilon-1}{\epsilon}-1} - \frac{\epsilon-1}{\epsilon} \omega_2 (J-n)^{\frac{\epsilon-1}{\epsilon}-1} + \frac{\epsilon-1}{\epsilon} \omega_3 n^{\frac{\epsilon-1}{\epsilon}-1} \right]}{\left[\omega_1 (J-n)^{\frac{\epsilon-1}{\epsilon}} + \omega_2 (J-n)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}} \right]} \overline{I}_{mc} \\
\frac{J}{n} &= 1 + \left[\frac{\omega_3}{\omega_1 + \omega_2} \right]^{-\epsilon} = 1 + \left[\frac{\omega_1 + \omega_2}{\omega_3} \right]^{\epsilon} \\
n &= \frac{1}{1 + \left[\frac{\omega_1 + \omega_2}{\omega_3} \right]^{\epsilon}} J = \frac{\omega_3^{\epsilon}}{\omega_3^{\epsilon} + [\omega_1 + \omega_2]^{\epsilon}} J
\end{aligned}
$$

$$
\begin{aligned}
\overline{I}_{mc} &= \left[\omega_1 (J-n)^{\frac{\epsilon-1}{\epsilon}} + \omega_2 (J-n)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}} \right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} \\
\overline{I}_{mc} &= J^{\kappa} \left[(\omega_1 + \omega_2)^{\epsilon} + \omega_3^{\epsilon} \right]^{\kappa\left(\frac{1}{\epsilon-1}\right)}
\end{aligned}
$$

Minimum Investment-level threshold such that the time constraint binds

There is a minimum level of investment such that the time constraint binds. This level of investment is such that the unconstrained solution has a binding time constraint, while the lagrangian multiplier is still 0. That is: $n^* + \max\left\{ q_f^*, q_m^* \right\} = J$.

To know whether this will imply $n^* + q_f^* = J$ or $n^* + q_m^* = J$, check for the condition that $q_f^* \leq q_m^*$

Optimal Input Levels When Time Constraint is Slack

$$\begin{aligned}
n^* &= I \times \frac{\left(\frac{\omega_3}{p_n}\right)^{\epsilon}}{\left(\left(\frac{\omega_1}{w_f}\right)^{\epsilon} w_f + \left(\frac{\omega_2}{w_m}\right)^{\epsilon} w_m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}} \\
q_f^* &= I \times \frac{\left(\frac{\omega_1}{w_f}\right)^{\epsilon}}{\left(\left(\frac{\omega_1}{w_f}\right)^{\epsilon} w_f + \left(\frac{\omega_2}{w_m}\right)^{\epsilon} w_m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}} \\
q_m^* &= I \times \frac{\left(\frac{\omega_2}{w_m}\right)^{\epsilon}}{\left(\left(\frac{\omega_1}{w_f}\right)^{\epsilon} w_f + \left(\frac{\omega_2}{w_m}\right)^{\epsilon} w_m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}}
\end{aligned}$$

Which parental time input binds depends on the relative wages of the parents, with a correction for their parenting productivities and the factor shares .

$$\begin{aligned}
q_f^* &\leq q_m^* \\
\frac{\left(\frac{\omega_1}{w_f}\right)^{\epsilon}}{\left(\left(\frac{\omega_1}{w_f}\right)^{\epsilon} w_f + \left(\frac{\omega_2}{w_m}\right)^{\epsilon} w_m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}} &\leq \frac{\left(\frac{\omega_2}{w_m}\right)^{\epsilon}}{\left(\left(\frac{\omega_1}{w_f}\right)^{\epsilon} w_f + \left(\frac{\omega_2}{w_m}\right)^{\epsilon} w_m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}} \\
\frac{\omega_1}{\omega_2} &\leq \frac{w_f}{w_m}
\end{aligned}$$

Assume that $\max\left\{q_m^*, q_f^*\right\} + n^* < J \implies \mu = 0$

$$\begin{aligned}
w_f &= \lambda \omega_1 q_f^{-\frac{1}{\epsilon}} \\
w_m &= \lambda \omega_2 q_m^{-\frac{1}{\epsilon}} \\
p_n &= \lambda \omega_3 n^{-\frac{1}{\epsilon}} \\
I &= \left[\omega_1 \left(J-n\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_2 \left(J-n\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} \\
&\implies \\
\frac{\omega_2 q_m^{-\frac{1}{\epsilon}}}{\omega_3 n^{-\frac{1}{\epsilon}}} &=
\end{aligned}$$

$$
\begin{aligned}
n^* &= I \times \frac{\left(\frac{\omega_3}{p_n}\right)^\epsilon}{\left(\left(\frac{\omega_1}{w_f}\right)^\epsilon w_f + \left(\frac{\omega_2}{w_m}\right)^\epsilon w_m + \left(\frac{\omega_3}{p_n}\right)^\epsilon p_n\right)^{\frac{\epsilon}{\epsilon-1}}} \\
q_f^* &= I \times \frac{\left(\frac{\omega_1}{w_f}\right)^\epsilon}{\left(\left(\frac{\omega_1}{w_f}\right)^\epsilon w_f + \left(\frac{\omega_2}{w_m}\right)^\epsilon w_m + \left(\frac{\omega_3}{p_n}\right)^\epsilon p_n\right)^{\frac{\epsilon}{\epsilon-1}}} \\
q_m^* &= I \times \frac{\left(\frac{\omega_2}{w_m}\right)^\epsilon}{\left(\left(\frac{\omega_1}{w_f}\right)^\epsilon w_f + \left(\frac{\omega_2}{w_m}\right)^\epsilon w_m + \left(\frac{\omega_3}{p_n}\right)^\epsilon p_n\right)^{\frac{\epsilon}{\epsilon-1}}}
\end{aligned}
$$

Investment Expenditure, Price, and Price Elasticity w.r.t. Price of Child Care

$$
\begin{aligned}
Cost\,(I) &= w_f q_f + w_m q_m + (1-\tau_n)\, p_n n \\
&= I \times \left(\omega_1^\epsilon w_f^{1-\epsilon} + \omega_2^\epsilon w_m^{1-\epsilon} + \omega_3^\epsilon p_n^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \\
\text{price of investment: } \Lambda_I &= \left(\omega_1^\epsilon w_f^{1-\epsilon} + \omega_2^\epsilon w_m^{1-\epsilon} + \omega_3^\epsilon p_n^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \\
\frac{\partial \Lambda_I}{\partial p_n} &= \omega_3^\epsilon p_n^{-\epsilon}\left(\omega_1^\epsilon w_f^{1-\epsilon} + \omega_2^\epsilon w_m^{1-\epsilon} + \omega_3^\epsilon p_n^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}-1} > 0 \\
\epsilon_{\Lambda_I, p_n} &= \frac{\partial \Lambda_I}{\partial p_n}\frac{p_n}{\Lambda_I} = \frac{\omega_3^\epsilon p_n^{1-\epsilon}}{\omega_1^\epsilon w_f^{1-\epsilon} + \omega_2^\epsilon w_m^{1-\epsilon} + \omega_3^\epsilon p_n^{1-\epsilon}}
\end{aligned}
$$

Finding Lower Bound on I

Since the larger parental input just depends on prices and parameters, not on the level of investment you are targeting, you can know ex-ante which is the relevant problem to solve for $\underline{I}$. Here I solve both problems, and the solutions can be chosen amongst by comparing input prices according to the above derived inequality.

Assuming $q_f^* > q_m^*$

$$
\begin{aligned}
J &= q_f^* + n^* \\
J &= \underline{I}\left[\frac{\left(\frac{\omega_1}{w_f}\right)^\epsilon + \left(\frac{\omega_3}{p_n}\right)^\epsilon}{\left(\left(\frac{\omega_1}{w_f}\right)^\epsilon w_f + \left(\frac{\omega_2}{w_m}\right)^\epsilon w_m + \left(\frac{\omega_3}{p_n}\right)^\epsilon p_n\right)^{1-\frac{1}{\epsilon}}}\right] \\
\underline{I} &= \frac{\left(\left(\frac{\omega_1}{w_f}\right)^\epsilon w_f + \left(\frac{\omega_2}{w_m}\right)^\epsilon w_m + \left(\frac{\omega_3}{p_n}\right)^\epsilon p_n\right)^{1-\frac{1}{\epsilon}}}{\left(\frac{\omega_1}{w_f}\right)^\epsilon + \left(\frac{\omega_3}{p_n}\right)^\epsilon} J
\end{aligned}
$$

Assuming $q_m^* > q_f^*$

$$\begin{aligned}
J &= q_m^* + n^* \\
\underline{I} &= \frac{\left(\left(\frac{\omega_1}{w_f}\right)^{\epsilon} w_f + \left(\frac{\omega_2}{w_m}\right)^{\epsilon} w_m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{1-\frac{1}{\epsilon}}}{\left(\frac{\omega_2}{w_m}\right)^{\epsilon} + \left(\frac{\omega_3}{p_n}\right)^{\epsilon}} J
\end{aligned}$$

Assume that $\max\left\{q_m^*, q_f^*\right\} + n^* = J \implies \mu > 0$**, and that** $q_f^* > q_m^*$

$$\begin{aligned}
w_f + \mu &= \lambda \omega_1 q_f^{-\frac{1}{\epsilon}} \\
w_m &= \lambda \omega_2 q_m^{-\frac{1}{\epsilon}} \\
p_n + \mu &= \lambda \omega_3 n^{-\frac{1}{\epsilon}} \\
J &= q_f + n \\
I^{\eta} &= \alpha_1 \left(\phi_f q_f\right)^{\eta} + \alpha_2 \left(\phi_m q_m\right)^{\eta} + \left(1 - \alpha_1 - \alpha_2\right)\left(\phi_n n\right)^{\eta} \\
\implies &
\end{aligned}$$

$$\begin{aligned}
\frac{w_f - p_n}{w_m} &= \frac{\omega_1 \left(J - n\right)^{\eta - 1} - \omega_3 n^{\eta - 1}}{\omega_2 q_m^{\eta - 1}} \\
I^{\eta} &= \alpha_1 \left(\phi_f \left(J - n\right)\right)^{\eta} + \alpha_2 \left(\phi_m q_m\right)^{\eta} + \left(1 - \alpha_1 - \alpha_2\right)\left(\phi_n n\right)^{\eta} \\
\implies & \\
q_m &= \left(\left(\frac{w_m}{w_f - p_n}\right)\left[\frac{\omega_1 \left(J - n\right)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_2}\right]\right)^{-\epsilon} \\
I^{1-\frac{1}{\epsilon}} &= \omega_1 \left(J - n\right)^{1-\frac{1}{\epsilon}} + \omega_2 q_m^{1-\frac{1}{\epsilon}} + \omega_3 n^{1-\frac{1}{\epsilon}}
\end{aligned}$$

This yields a single equation with one unknown:

$$I^{1-\frac{1}{\epsilon}} = \omega_1 \left(J - n\right)^{1-\frac{1}{\epsilon}} + \omega_2 \left(\left(\frac{w_m}{w_f - p_n}\right)\left(\frac{\omega_1 \left(J - n\right)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_2}\right)\right)^{1-\epsilon} + \omega_3 n^{1-\frac{1}{\epsilon}}$$

Solving for the bounds on n **such that the implied** $q_m \in \mathbb{R}_+$ When searching for an n that solves the above system of equations, there is an admissable interval of guesses for n you search through. This interval is potentially empty. It is found by checking that the q_m implied by the guess of n is within the bounds assumed when costructing the problem. Specifically, for $q_m \in [0, J - n]$, the implicit bounds on n need to be inferred from the bounds on q_m:

$$q_m = \left(\left(\frac{w_m}{w_f - p_n}\right)\left[\frac{\omega_1 \left(J - n\right)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_2}\right]\right)^{-\epsilon} \in [0, J - n]$$

There are 2 cases I can see, where case 1 has $w_f - p_n > 0$ and case 2 has $w_f - p_n < 0$

Case 1 In case 1, $w_f - p_n > 0$, so the for the lower bound of $q_m \geq 0$ implies a lower bound on n:

$$\begin{aligned}
\left(\left(\frac{w_m}{w_f - p_n}\right)\left[\frac{\omega_1 (J-n)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_2}\right]\right)^{-\epsilon} &\geq 0 \\
\omega_1 (J-n)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}} &\geq 0 \\
\frac{1}{1+\left(\frac{\omega_1}{\omega_3}\right)^{\epsilon}} J &\leq n
\end{aligned}$$

For the upper bound on q_m, $J - n$, there is another implied lower bound on n:

$$\begin{aligned}
q_m &\leq J - n \\
\frac{1}{1+\left(\frac{\omega_1}{\omega_3} - \frac{\omega_2}{\omega_3}\left(\frac{w_f - p_n}{w_m}\right)\right)^{\epsilon}} J &\geq n
\end{aligned}$$

The bound that holds is:

$$\Theta_2 = \frac{1}{1+\left(\frac{\omega_1}{\omega_3} - \frac{\omega_2}{\omega_3}\left(\frac{w_f - p_n}{w_m}\right)\right)^{\epsilon}} \geq \frac{1}{1+\left(\frac{\omega_1}{\omega_3}\right)^{\epsilon}} = \Theta_1$$

Which is the condition assumed for this case. So, $n \in [\Theta_2, J]$. The condition that the assumption be self-fulfilling is more strict than the condition that the implied $q_m \in \mathbb{R}_+$. That is, Θ_2 is the tighter bound in case 1:

$$\Theta_2 = \frac{1}{1+\left[\frac{\omega_1}{\omega_3} - \left(\frac{w_f - p_n}{w_m}\right)\frac{\omega_2}{\omega_3}\right]^{\epsilon}} \geq \frac{1}{1+\left(\frac{\omega_1}{\omega_3}\right)^{\epsilon}} = \Theta_1$$

Case 2 In case 2, $w_f - p_n < 0$ so the q_m to be a real number implies an upper bound on n:

$$\begin{aligned}
\left(\left(\frac{w_m}{w_f - p_n}\right)\left[\frac{\omega_1 (J-n)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_2}\right]\right)^{-\epsilon} &\leq 0 \\
\omega_1 (J-n)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}} &\geq 0 \\
\frac{1}{1+\left(\frac{\omega_1}{\omega_3}\right)^{\epsilon}} J &\geq n
\end{aligned}$$

For the upper bound on q_m, $J-n$, the implied upper bound on n is:

$$\begin{aligned}
\left(\left(\frac{w_m}{w_f-p_n}\right)\left[\frac{\omega_1\left(J-n\right)^{-\frac{1}{\epsilon}}-\omega_3 n^{-\frac{1}{\epsilon}}}{\omega_2}\right]\right)^{-\epsilon} &\leq J-n \\
\frac{J}{n} &\geq 1+\left[\frac{\omega_1}{\omega_3}-\left(\frac{w_f-p_n}{w_m}\right)\frac{\omega_2}{\omega_3}\right]^{\epsilon} \\
\frac{1}{1+\left[\frac{\omega_1}{\omega_3}-\left(\frac{w_f-p_n}{w_m}\right)\frac{\omega_2}{\omega_3}\right]^{\epsilon}}J &\geq n
\end{aligned}$$

Θ_2 is the tighter bound in case 2, like case 1:

$$\Theta_2=\frac{1}{1+\left[\frac{\omega_1}{\omega_3}-\left(\frac{w_f-p_n}{w_m}\right)\frac{\omega_2}{\omega_3}\right]^{\epsilon}} \quad\leq\quad \frac{1}{1+\left(\frac{\omega_1}{\omega_3}\right)^{\epsilon}}=\Theta_1$$

Discussion of the bounds on n w.r.t. algorithm

- The set of admissable n depends on HCT parameters and prices
- It can be empty
- If it is empty, then the case for $q_f > q_m$ is not possible as a solution, and you don't have to compute the inputs under that assumption and then check expenditures.
- If n is in the admissable set you also don't have to check that $q_f > q_m$ after you calculate the inputs for the case. It's built into the definition of the admissable set.

Finding Set of Levels that are achievable for the case $q_f^* > q_m^*$:

There is a relationship between admissable n and admissable I. Use the set of admissable n to define the admissable $I's$.

Here is the sytem of equations that define the solution when $q_f^* > q_m^*$:

$$\begin{aligned}
q_m &= \left(\left(\frac{w_m}{w_f-p_n}\right)\left[\frac{\omega_1\left(J-n\right)^{-\frac{1}{\epsilon}}-\omega_3 n^{-\frac{1}{\epsilon}}}{\omega_2}\right]\right)^{-\epsilon} \\
I^{1-\frac{1}{\epsilon}} &= \omega_1\left(J-n\right)^{1-\frac{1}{\epsilon}}+\omega_2 q_m^{1-\frac{1}{\epsilon}}+\omega_3 n^{1-\frac{1}{\epsilon}}
\end{aligned}$$

For q_m^* to be real, with the assumption of this case, investment has to be for case 1:

$$\begin{aligned}
I^{1-\frac{1}{\epsilon}} &\geq \omega_1 (J-n)^{1-\frac{1}{\epsilon}} + \omega_2 q_m^{1-\frac{1}{\epsilon}} + \omega_3 n^{1-\frac{1}{\epsilon}} \\
I &\geq [\omega_1^\epsilon + \omega_3^\epsilon]^{\frac{1}{\epsilon-1}} J \\
\implies & \quad \forall I \in \left[[\omega_1^\epsilon + \omega_3^\epsilon]^{\frac{1}{\epsilon-1}} J, \overline{I}\right] \; can\, solve
\end{aligned}$$

Assume that $\max\left\{q_m^*, q_f^*\right\} + n^* = J \implies \mu > 0$**, and that** $q_f^* < q_m^*$

For this case, the system of equations that characterizes the input choices is derived as follows:

$$\begin{aligned}
w_f &= \lambda \omega_1 q_f^{-\frac{1}{\epsilon}} \\
w_m + \mu &= \lambda \omega_2 q_m^{-\frac{1}{\epsilon}} \\
p_n + \mu &= \lambda \omega_3 n^{-\frac{1}{\epsilon}} \\
J &= q_m + n \\
I &= \left[\omega_1 q_f^{\frac{\epsilon-1}{\epsilon}} + \omega_2 q_m^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} \\
\implies & \\
\frac{w_m - p_n}{w_f} &= \frac{\omega_2 (J-n)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_1 q_f^{-\frac{1}{\epsilon}}} \\
I &= \left[\omega_1 q_f^{\frac{\epsilon-1}{\epsilon}} + \omega_2 q_m^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} \\
\implies & \\
q_f &= \left[\frac{w_f}{w_m - p_n}\left(\frac{\omega_2 (J-n)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_1}\right)\right]^{-\epsilon} \\
I &= \left[\omega_1 q_f^{\frac{\epsilon-1}{\epsilon}} + \omega_2 q_m^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)}
\end{aligned}$$

Condition so that $q_f \in \mathbb{R}^+$

Case 1: $w_m - p_n > 0$

$$\begin{aligned}
\omega_2 (J-n)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}} &\geq 0 \\
\frac{1}{1+\left(\frac{\omega_2}{\omega_3}\right)^\epsilon} J &\leq n
\end{aligned}$$

and

$$\begin{aligned}
\left[\frac{w_f}{w_m - p_n}\left(\frac{\omega_2\left(J-n\right)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_1}\right)\right]^{-\epsilon} &\leq J-n \\
\frac{1}{1+\left[\frac{\omega_2}{\omega_3} - \frac{\omega_1[w_m - p_n]}{\omega_3 w_f}\right]^{\epsilon}} J &\leq n
\end{aligned}$$

Again, Θ_2 is the tighter bound in case 1:

$$\Theta_2 = \frac{1}{1+\left[\frac{\omega_2}{\omega_3} - \frac{\omega_1[w_m - p_n]}{\omega_3 w_f}\right]^{\epsilon}} \quad \geq \quad \frac{1}{1+\left[\frac{\omega_2}{\omega_3}\right]^{\epsilon}} = \Theta_1$$

Case 2: $w_m - p_n < 0$

$$\begin{aligned}
\omega_2\left(J-n\right)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}} &\leq 0 \\
\frac{1}{1+\left(\frac{\omega_2}{\omega_3}\right)^{\epsilon}} J &\geq n
\end{aligned}$$

and

$$\begin{aligned}
\left[\frac{w^f}{w^m - p_n}\left(\frac{\omega_2\left(J-n\right)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_1}\right)\right]^{-\epsilon} &\geq J-n \\
\frac{1}{1+\left[\frac{\omega_2}{\omega_3} - \frac{\omega_1[w^m - p_n]}{\omega_3 w^f}\right]^{\epsilon}} J &\geq n
\end{aligned}$$

Θ_2 is the tighter bound in case 2 as well as case 1:

$$\Theta_2 = \frac{1}{1+\left[\frac{\omega_2}{\omega_3} - \frac{\omega_1[w^m - p_n]}{\omega_3 w^f}\right]^{\epsilon}} \quad \geq \quad \frac{1}{1+\left[\frac{\omega_2}{\omega_3}\right]^{\epsilon}} = \Theta_1$$

Optimal Input Levels When: $\max\left\{q^{m,*},q^{f,*}\right\}+n^{*}=J\implies\mu>0$ **and** $q^{f,*}=q^{m,*}$

$$
\begin{aligned}
w^{f}+\mu &= \lambda\omega_{1}\left(q^{f}\right)^{-\frac{1}{\epsilon}}\\
w^{m}+\mu &= \lambda\omega_{2}\left(q^{m}\right)^{-\frac{1}{\epsilon}}\\
p_{n}+\mu &= \lambda\omega_{3}n^{-\frac{1}{\epsilon}}\\
J &= q^{f}+n\\
I &= \left[\omega_{1}\left(q^{f}\right)^{\frac{\epsilon-1}{\epsilon}}+\omega_{2}\left(q^{m}\right)^{\frac{\epsilon-1}{\epsilon}}+\omega_{3}n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)}\\
\implies & \\
\frac{w^{f}-p_{n}}{w^{m}-p_{n}} &= \frac{\omega_{1}\left(J-n\right)^{-\frac{1}{\epsilon}}-\omega_{3}n^{-\frac{1}{\epsilon}}}{\omega_{2}\left(J-n\right)^{-\frac{1}{\epsilon}}-\omega_{3}n^{-\frac{1}{\epsilon}}}\\
I &= \left[\omega_{1}\left(J-n\right)^{\frac{\epsilon-1}{\epsilon}}+\omega_{2}\left(J-n\right)^{\frac{\epsilon-1}{\epsilon}}+\omega_{3}n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)}\\
\implies & \\
n &= \frac{1}{1+\left[\frac{\left(w^{f}-p_{n}\right)\omega_{2}-\left(w^{m}-p_{n}\right)\omega_{1}}{\left(w^{f}-w^{m}\right)\omega_{3}}\right]^{\epsilon}}\\
I &= \left[\left(\omega_{1}+\omega_{2}\right)\left(J-n\right)^{\frac{\epsilon-1}{\epsilon}}+\omega_{3}n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)}
\end{aligned}
$$

From this result I infer that the level of investment for which the two parental time inputs are equal and the time constraint binds is weakly lower than when parental time inputs are equal and the time constraint does not bind. In addition this case pins down the level of investment that requires it, and this level of investment is unique. I know this because I solve above for the appropriate n that generates it and this n is unique.

E Single Mothers

F Skill Accumulation Technologies

For clarity, the following notational transformations are convenient:

$$
\begin{aligned}
\omega_{2} &\equiv \alpha_{s}\left(\phi^{m}\right)^{\eta}\\
\omega_{3} &\equiv \left(1-\alpha_{s}\right)\phi_{n}^{\eta}\\
w^{m} &= \left(1-\tau_{y}\right)\left(1-\tau_{qm}\right)w\theta^{m}\\
p_{n} &= \left(1-\tau_{n}\right)p_{n}
\end{aligned}
$$

Which makes the primitives:

$$\begin{aligned}
\theta_{k,t+1} &= \left[\upsilon\left(\lambda_{SM,t}I_t^{SM}\right)^{\frac{\chi-1}{\chi}} + (1-\upsilon)\,\theta_{k,t}^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}} \\
I_t^{SM} &= \left[\omega_2\left(q^m\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)}
\end{aligned}$$

Restrictions on parameters:

$$\begin{aligned}
\epsilon &\geq 0 \\
\kappa &\geq 0 \\
\chi &\geq 0
\end{aligned}$$

Bounds on inputs

$$\begin{aligned}
q^m &\in [0,1] \\
n &\in [0,1] \\
n + q^m &\leq J
\end{aligned}$$

G The Karusch Kuhn-Tucker Formulation of the Single Mother's Problem

The single mother's problem is 2-staged: in the first stage, for a given level of investment the family determines whether it is feasible and what the combination of inputs should be to finance it. In the second stage, the family optimally chooses their level of investment in their child. The first stage is a cost-minimization problem with the time constraint of the child limiting the total inputs into investment that can be used. This implies some restrictions on the levels of investment that are feasible.

Here I focus on the first stage of the couple's parenting problem. I want to characterize the optimal combination of inputs for any feasible level of investment; this includes investment levels for which the time constraint is slack and also for which it is binding.

$$
\begin{aligned}
\mathbb{L}_{q^m,q_f,n} \quad & -\left[w^m q^m + p_n n\right] + \mu\left[J - q^m - n\right] + \lambda\left[\omega_2\left(q^m\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}} - I^{\frac{1}{\kappa}\left(\frac{\epsilon-1}{\epsilon}\right)}\right] \\
\text{FOCs:} \quad & \\
[q^m] \quad & -w^m - \mu + \lambda\left(1 - \frac{1}{\epsilon}\right)\omega_2\left(q^m\right)^{-\frac{1}{\epsilon}} = 0 \\
[n] \quad & -p_n - \mu + \lambda\left(1 - \frac{1}{\epsilon}\right)\omega_3 n^{-\frac{1}{\epsilon}} = 0 \\
[\mu] \quad & J - q^m - n = 0 \quad or \quad \mu = 0 \\
[\lambda] \quad & \left[\omega_2\left(q^m\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} - I = 0 \quad or \quad \lambda = 0
\end{aligned}
$$

$$
\begin{aligned}
& \Longrightarrow \\
w^m + \mu &= \lambda\omega_2\left(q^m\right)^{-\frac{1}{\epsilon}} \\
p_n + \mu &= \lambda\omega_3 n^{-\frac{1}{\epsilon}}
\end{aligned}
$$

To take first order conditions, go by cases. Solve for the implied quantities assuming that the time constraint is slack, and then check that both are true. If the assumption that the time constraint is slack is falsified, proceed to solving for the n, and q^m that produce the desired level of investment with a binding time constraint using only the production technology.

The Maximum Possible Level of Investment

$$
\begin{aligned}
\overline{I}^{SM} &= \max_{n\in[0,J]}\left[\left[\omega_2\left(J-n\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)}\right] \\
0 &= \kappa\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\left[-\frac{\epsilon-1}{\epsilon}\omega_2\left(J-n\right)^{\frac{\epsilon-1}{\epsilon}-1} + \frac{\epsilon-1}{\epsilon}\omega_3 n^{\frac{\epsilon-1}{\epsilon}-1}\right]}{\left[\omega_2\left(J-n\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]}\overline{I}^{SM} \\
n &= \left(\frac{1}{\left(\frac{\omega_2}{\omega_3}\right)^{\epsilon} + 1}\right) J
\end{aligned}
$$

$$
\overline{I}^{SM} = J^{\kappa}\left[\omega_2^{\epsilon} + \omega_3^{\epsilon}\right]^{\kappa\left(\frac{1}{\epsilon-1}\right)}
$$

The Minimum Level of Investment $\underline{I}$ with a Binding Time Constraint

There is a minimum level of investment such that the time constraint binds. This level of investment is such that the unconstrained solution has a binding time constraint, while the lagrangian multiplier is still 0. That is: $n^* + q^{m,*} = J$.

To know whether this will imply $n^* + q_{f,*} = J$ or $n^* + q^{m,*} = J$, check for the condition that $q_{f,*} \leq q^{m,*}$

The policy functions are:

$$
\begin{aligned}
\left(\frac{p_n}{w^m}\frac{\omega_2}{\omega_3}\right)^{\epsilon} n &= q^m \\
I &= n^{\kappa}\left[\omega_2\left(\frac{p_n}{w^m}\frac{\omega_2}{\omega_3}\right)^{\epsilon-1} + \omega_3\right]^{\left(\frac{\epsilon}{\epsilon-1}\right)} \\
n &= I^{\frac{1}{\kappa}}\left[\omega_2\left(\frac{p_n}{w^m}\frac{\omega_2}{\omega_3}\right)^{\epsilon-1} + \omega_3\right]^{\left(\frac{\epsilon}{1-\epsilon}\right)} \\
n &= I^{\frac{1}{\kappa}}\left(\frac{\omega_3}{p_n}\right)^{\epsilon}\left[\left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right]^{\left(\frac{\epsilon}{1-\epsilon}\right)} \\
q^{m,*} &= I^{\frac{1}{\kappa}}\left(\frac{\omega_2}{w^m}\right)^{\epsilon}\left[\left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right]^{\left(\frac{\epsilon}{1-\epsilon}\right)}
\end{aligned}
$$

Finding $\underline{I}$

Since the larger parental input just depends on prices and parameters, not on the level of investment you are targeting, you can know ex-ante which is the relevant problem to solve for $\underline{I}$. Here I solve both problems, and the solutions can be chosen amongst by comparing input prices according to the above derived inequality.

$$
\begin{aligned}
J &= q_{f,*} + n^* \\
J &= \underline{I}^{\frac{1}{\kappa}} \times \frac{\left(\frac{\omega_3}{p_n}\right)^{\epsilon} + \left(\frac{\omega_2}{w^m}\right)^{\epsilon}}{\left(\left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}} \\
\underline{I} &= \left[\frac{\left(\left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\kappa\epsilon}{\epsilon-1}}}{\left(\left(\frac{\omega_3}{p_n}\right)^{\epsilon} + \left(\frac{\omega_2}{w^m}\right)^{\epsilon}\right)^{\kappa}}\right] J^{\kappa} \\
\omega_2 &\equiv \alpha_s \left(\phi^m\right)^{1-\frac{1}{\epsilon}} \\
\omega_3 &\equiv (1-\alpha_s)\,\phi_n^{1-\frac{1}{\epsilon}} \\
\underline{I} &= \left[\frac{\left(\alpha_s^{\epsilon}\left(\frac{w^m}{\phi^m}\right)^{1-\epsilon} + (1-\alpha_s)^{\epsilon}\left(\frac{p_n}{\phi_n}\right)^{1-\epsilon}\right)^{\frac{\kappa\epsilon}{\epsilon-1}}}{\left(\alpha_s^{\epsilon}\left(\frac{w^m}{\phi^m}\right)^{1-\epsilon}(w^m)^{-1} + (1-\alpha_s)^{\epsilon}\left(\frac{p_n}{\phi_n}\right)^{1-\epsilon} p_n^{-1}\right)^{\kappa}}\right] J^{\kappa}
\end{aligned}
$$

Solving For Inputs Into Investment With and Without a Slack Time Constraint

Assume that $q^{m,*} + n^* < J \implies \mu = 0$

$$
\begin{aligned}
w^m &= \lambda \omega_2 (q^m)^{-\frac{1}{\epsilon}} \\
p_n &= \lambda \omega_3 n^{-\frac{1}{\epsilon}} \\
I &= \left[\omega_2 (q^m)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} \\
\implies & \\
\frac{\omega_2 (q^m)^{-\frac{1}{\epsilon}}}{\omega_3 n^{-\frac{1}{\epsilon}}} &= \frac{w^m}{p_n}
\end{aligned}
$$

$$
\begin{aligned}
n^* &= I^{\frac{1}{\kappa}} \times \frac{\left(\frac{\omega_3}{p_n}\right)^{\epsilon}}{\left(\left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}} \\
q^{m,*} &= I^{\frac{1}{\kappa}} \times \frac{\left(\frac{\omega_2}{w^m}\right)^{\epsilon}}{\left(\left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}}
\end{aligned}
$$

Investment Expenditure, Price, and Price Elasticity w.r.t. Price of Child Care

$$
\begin{aligned}
Cost\,(I) &= w^m q^m + p_n n \\
&= I^{\frac{1}{\kappa}} \times \left[\frac{\left(\left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)}{\left(\left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}}\right] \\
&= I^{\frac{1}{\kappa}} \times \left(\omega_2^{\epsilon} (w^m)^{1-\epsilon} + \omega_3^{\epsilon} p_n^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \\
\text{price of investment: } \Lambda_I &= I^{\frac{1}{\kappa}-1} \times \left(\omega_2^{\epsilon} (w^m)^{1-\epsilon} + \omega_3^{\epsilon} p_n^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \\
\frac{\partial \Lambda_I}{\partial p_n} &= I^{\frac{1}{\kappa}-1} \times \omega_3^{\epsilon} p_n^{-\epsilon} \left(\omega_2^{\epsilon} (w^m)^{1-\epsilon} + \omega_3^{\epsilon} p_n^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}-1} > 0 \\
\epsilon_{\Lambda_I, p_n} &= \frac{\partial \Lambda_I}{\partial p_n} \frac{p_n}{\Lambda_I} = \frac{\left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n}{\left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n} \\
\omega_2 &\equiv \alpha_s (\phi^m)^{\frac{\epsilon-1}{\epsilon}} \\
\omega_3 &\equiv (1-\alpha_s)\, \phi_n^{\frac{\epsilon-1}{\epsilon}} \\
\epsilon_{\Lambda_I, p_n} &= \frac{1}{\left(\frac{\alpha_s}{1-\alpha_s}\right)^{\epsilon} \left[\frac{\left(\frac{w^m}{\phi^m}\right)}{\left(\frac{p_n}{\phi_n}\right)}\right]^{1-\epsilon} + 1}
\end{aligned}
$$

The larger the price per efficiency unit of the mother's time relative to the price per efficiency unit of child care, the smaller the pass-through of a child care subsidy to the price of investment. If the reverse is true, that the relative price per efficiency unit is a small value, the pass-through of child care subsidies is larger.

In addition, the larger the weight on the mother's time input α_s, the larger the denominator and the smaller the the elasticity of the investment price with respect to the price of child care. The share parameters mediate the role of price per efficiency unit in determining the sensitivity of the investment price with respect to the child care subsidy.

Assume that $q^{m,*} + n^* = J \implies \mu > 0$ **(The time constraint binds)**

$$\begin{aligned}
w^m + \mu &= \lambda\omega_2 \left(q^m\right)^{-\frac{1}{\epsilon}} = \lambda\omega_2 \left(J - n\right)^{-\frac{1}{\epsilon}} \\
p_n + \mu &= \lambda\omega_3 n^{-\frac{1}{\epsilon}} \\
J &= q^m + n \\
\implies & \\
\mu &= \frac{p_n\omega_2 \left(\frac{J}{n} - 1\right)^{-\frac{1}{\epsilon}} - \omega_3 w^m}{\left[\omega_3 - \omega_2 \left(\frac{J}{n} - 1\right)^{-\frac{1}{\epsilon}}\right]} \\
I^{\frac{1}{\kappa}\left(1 - \frac{1}{\epsilon}\right)} &= \omega_2 \left(J - n\right)^{1 - \frac{1}{\epsilon}} + \omega_3 n^{1 - \frac{1}{\epsilon}}
\end{aligned}$$

This is a system of two equations with two unknowns, μ and n. You can find the value of μ after you find the level of n. The second equation gets n, and since it's a functional equaiton you can't solve it directly but have to iterate on the guess for n until the equation holds.

Investment Expenditure, Price, and Price Elasticity w.r.t. Price of Child Care

$$\begin{aligned}
Cost\,(I) &= w^m\,(J-n)+p_n n \\
\text{price of investment: } \Lambda_I &= \frac{n^{1-\kappa}\left[w^m\left(\frac{J}{n}-1\right)+p_n\right]}{\left[\omega_2\left(\frac{J}{n}-1\right)^{1-\frac{1}{\epsilon}}+\omega_3\right]^{\kappa\frac{\epsilon}{\epsilon-1}}} \\
&= n^{1-\kappa}\left[w^m\left(\frac{J}{n}-1\right)+p_n\right]\left[\omega_2\left(\frac{J}{n}-1\right)^{1-\frac{1}{\epsilon}}+\omega_3\right]^{\kappa\frac{\epsilon}{1-\epsilon}} \\
\frac{\partial\Lambda_I}{\partial p_n} &= n^{1-\kappa}\left[\omega_2\left(\frac{J}{n}-1\right)^{1-\frac{1}{\epsilon}}+\omega_3\right]^{\kappa\frac{\epsilon}{1-\epsilon}} = \frac{\Lambda_I}{\left[w^m\left(\frac{J}{n}-1\right)+p_n\right]} \\
\epsilon_{\Lambda_I,p_n} &= \frac{\partial\Lambda_I}{\partial p_n}\frac{p_n}{\Lambda_I} = \frac{1}{\left[\frac{w^m}{p_n}\left(\frac{J}{n}-1\right)+1\right]} \\
\epsilon_{\Lambda_I,p_n} &= \frac{1}{\left[\frac{w^m}{p_n}\left(\frac{J}{n}-1\right)+1\right]}
\end{aligned}$$

Since $n \leq J$, this means $\frac{w^m}{p_n}\left(\frac{J}{n}-1\right) \geq 0$, so $\epsilon_{\Lambda_I,p_n} \leq 1$. As n decreases or $\frac{w^m}{p_n}$ increases the elasticity of the investment price with respect to the price of child care decreases. The lower the mother's after-tax wage is relative to the after-subsidy level of the child care price, the higher the pass-through of the child care subsidy to the price of investment.

H Computational Algorithm for Couples Parenting Problem

In this section I explain the computational algorithm I use to solve the couples problem (which is more complex than the single mother's problem). Couples have 3 ways of solving the binding-time constraint case which need to be checked; by comparison, the lagorithm for the single mother's problem has only one way of finding the optimal input combination when the time constraint of the child binds.

1. Compute $\overline{I} = J^\kappa\left[(\omega_1+\omega_2)^\epsilon+\omega_3^\epsilon\right]^{\kappa\left(\frac{1}{\epsilon-1}\right)}$
2. Compute $\underline{I}$ and verify that $\underline{I} \leq \overline{I}$
   (a) If $\frac{w^f}{w^m} \geq \frac{\omega_1}{\omega_2}$ then $q^{f,*} \geq q^{m,*}$ and $\underline{I} = \frac{\left(\left(\frac{\omega_1}{w^f}\right)^\epsilon w^f+\left(\frac{\omega_2}{w^m}\right)^\epsilon w^m+\left(\frac{\omega_3}{p_n}\right)^\epsilon p_n\right)^{1-\frac{1}{\epsilon}}}{\left(\frac{\omega_1}{w^f}\right)^\epsilon+\left(\frac{\omega_3}{p_n}\right)^\epsilon}J$
   (b) If $\frac{w^f}{w^m} < \frac{\omega_1}{\omega_2}$ then $q^{f,*} < q^{m,*}$ and $\underline{I} = \frac{\left(\left(\frac{\omega_1}{w^f}\right)^\epsilon w^f+\left(\frac{\omega_2}{w^m}\right)^\epsilon w^m+\left(\frac{\omega_3}{p_n}\right)^\epsilon p_n\right)^{1-\frac{1}{\epsilon}}}{\left(\frac{\omega_2}{w^m}\right)^\epsilon+\left(\frac{\omega_3}{p_n}\right)^\epsilon}J$
3. If $I \in [0,\underline{I}]$, solve system 0, where the time constraint is slack. If not, proceed.

$$
\begin{aligned}
n^* &= I \times \frac{\left(\frac{\omega_3}{p_n}\right)^{\epsilon}}{\left(\left(\frac{\omega_1}{w^f}\right)^{\epsilon} w^f + \left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}} \\
q^{f,*} &= I \times \frac{\left(\frac{\omega_1}{w^f}\right)^{\epsilon}}{\left(\left(\frac{\omega_1}{w^f}\right)^{\epsilon} w^f + \left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}} \\
q^{m,*} &= I \times \frac{\left(\frac{\omega_2}{w^m}\right)^{\epsilon}}{\left(\left(\frac{\omega_1}{w^f}\right)^{\epsilon} w^f + \left(\frac{\omega_2}{w^m}\right)^{\epsilon} w^m + \left(\frac{\omega_3}{p_n}\right)^{\epsilon} p_n\right)^{\frac{\epsilon}{\epsilon-1}}}
\end{aligned}
$$

4. If $I \in \left[\underline{I}, \overline{I}\right]$ solve for the admissable region of n^* in systems 1 and 2:
   - $n^* \in [\Theta_2, J]$ if $w^f > p_n$ in system 1 or $w^m > p_n$ in system 2
   - $n^* \in [0, \Theta_2]$ if $w^f < p_n$ in system 1 or $w^m < p_n$ in system 2
   - System 1 $\Theta_2 = \frac{1}{1+\left[\frac{\omega_1}{\omega_3} - \left(\frac{w^f - p_n}{w^m}\right)\frac{\omega_2}{\omega_3}\right]^{\epsilon}}$, system 2 $\Theta_2 = \frac{1}{1+\left[\frac{\omega_2}{\omega_3} - \left(\frac{w^m - p_n}{w^f}\right)\frac{\omega_1}{\omega_3}\right]^{\epsilon}}$
   - If the appropriate region is empty for a system, skip solving that system.

   (a) System 1: $J - n^* = q^{f,*} > q^{m,*}$, where n^* solves:

$$
\begin{aligned}
q^m &= \left(\left(\frac{w^m}{w^f - p_n}\right)\left[\frac{\omega_1 (J-n)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_2}\right]\right)^{-\epsilon} \\
res &= I - \left(\omega_1 (J-n)^{\frac{\epsilon-1}{\epsilon}} + \omega_2 (q^m)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right)^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)}
\end{aligned}
$$

   (a) System 2: $q^{f,*} < q^{m,*} = J - n^*$, where n^* solves:

$$
\begin{aligned}
q^f &= \left[\frac{w^f}{w^m - p_n}\left(\frac{\omega_2 (J-n)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_1}\right)\right]^{-\epsilon} \\
res &= I - \left[\omega_1 \left(q^f\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_2 (J-n)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)}
\end{aligned}
$$

5. Compute and compare the total cost of financing the target level of investment in each of the two systems. Select the cheapest method for generating the target level of investment.

6. Record $n^*, q^{m,*}, q^{f,*}$, the system of equations you are using (0,1,2), and the total cost of investment.

$$
\begin{aligned}
\left[\frac{w_f}{w_m - p_n}\left(\frac{\omega_2 \left(J-n\right)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_1}\right)\right]^{-\epsilon} &\geq J - n \\
\frac{1}{1+\left[\frac{\omega_2}{\omega_3} - \frac{\omega_1 [w_m - p_n]}{\omega_3 w_f}\right]^{\epsilon}} J &\geq n
\end{aligned}
$$

Θ_2 is the tighter bound in case 2 as well as case 1:

$$
\Theta_2 = \frac{1}{1+\left[\frac{\omega_2}{\omega_3} - \frac{\omega_1 [w_m - p_n]}{\omega_3 w_f}\right]^{\epsilon}} \quad \geq \quad \frac{1}{1+\left[\frac{\omega_2}{\omega_3}\right]^{\epsilon}} = \Theta_1
$$

Assume that $\max\left\{q_m^*, q_f^*\right\} + n^* = J \implies \mu > 0$**, and that** $q_f^* = q_m^*$

$$
\begin{aligned}
w_f + \mu &= \lambda \omega_1 q_f^{-\frac{1}{\epsilon}} \\
w_m + \mu &= \lambda \omega_2 q_m^{-\frac{1}{\epsilon}} \\
p_n + \mu &= \lambda \omega_3 n^{-\frac{1}{\epsilon}} \\
J &= q_f + n \\
I &= \left[\omega_1 q_f^{\frac{\epsilon-1}{\epsilon}} + \omega_2 q_m^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} \\
\implies & \\
\frac{w_f - p_n}{w_m - p_n} &= \frac{\omega_1 \left(J-n\right)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}}{\omega_2 \left(J-n\right)^{-\frac{1}{\epsilon}} - \omega_3 n^{-\frac{1}{\epsilon}}} \\
I &= \left[\omega_1 \left(J-n\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_2 \left(J-n\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)} \\
\implies & \\
n &= \frac{1}{1+\left[\frac{(w_f - p_n)\omega_2 - (w_m - p_n)\omega_1}{(w_f - w_m)\omega_3}\right]^{\epsilon}} \\
I &= \left[\left(\omega_1 + \omega_2\right)\left(J-n\right)^{\frac{\epsilon-1}{\epsilon}} + \omega_3 n^{\frac{\epsilon-1}{\epsilon}}\right]^{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)}
\end{aligned}
$$

From this result I infer that the level of investment for which the two parental time inputs are equal and the time constraint binds is weakly lower than when parental time inputs are equal and the time constraint does not bind. In addition this case pins down the level of investment that requires it, and this levl of investment is unique. I know this because I solve above for the appropriate n that generates it and this n is unique.

Part III

Elasticity of Investment Price with respect to p_n

In this section I derive the expressions for the elasticity of the price of investment with respect to the price of child care. I start with the expression for the investment price for each family structure (single mothers and married couples) and take the partial derivative of this price with respect to the price of child care, p_t^n. This comparative static represents any decrease in the price of child care faced by families, for example due to a child care subsidy. Although I consider other subsidies in this paper (such as a subsidy to mother time which would reduce the w_m that goes into this expression), this elasticity is sufficient to demonstrate how the differences in estimated parameters across the two family structures affects the relative sensitivity to the price of different inputs across those family structures.

I Elasticity of the Price of Investment with respect to the Price of Child Care

I.1 Single Mothers

The price of investment is found by taking the ratio of the cost of a level of investment and the level of investment. This, combined with the optimal input combination for the case where the time constraint of the child is slack, can be combined to express the price of investment as a combination of prices for the inputs - this substitution removes quantitities from the expression for the investment price.

$$\Lambda_t^{SM} = \frac{Cost_t^{SM}}{I_t^{SM}}$$

$$= \frac{p_t^n + w_t^m \left(\frac{q_t^m}{n_t}\right)}{\phi_n \left[\alpha_s \left(\frac{\phi_m q_t^m}{\phi_n n_t}\right)^{\frac{\eta-1}{\eta}} + 1 - \alpha_s\right]^{\left(\frac{\eta}{\eta-1}\right)}}$$

and

$$\frac{q_t^m}{n_t} = \frac{\phi_n}{\phi_m}\left[\left(\frac{p_t^n}{w_t^m}\right)\left(\frac{\alpha_s}{1-\alpha_s}\right)\left(\frac{\phi_m}{\phi_n}\right)\right]^{\eta} \Longrightarrow$$

$$\Lambda_t^{SM} = \frac{p_t^n}{\phi_n}\left((1-\alpha_s)^{\eta} + \alpha_s^{\eta}\left[\left(\frac{p_t^n}{w_t^m}\right)\left(\frac{\phi_m}{\phi_n}\right)\right]^{\eta-1}\right)^{\frac{1}{1-\eta}}$$

Next, take the partial derivative of the price of investment with respect to the price of child care. The resulting expression can be expressed as the level of investment multiplied by a function of prices:

$$
\begin{aligned}
\frac{\partial \Lambda_t^{SM}}{\partial p_t^n} &= \frac{1}{\phi_n}\left((1-\alpha_s)^\eta + \alpha_s^\eta\left[\left(\frac{p_t^n}{w_t^m}\right)\left(\frac{\phi_m}{\phi_n}\right)\right]^{\eta-1}\right)^{\frac{1}{1-\eta}} \\
&- \frac{p_t^n}{\phi_n}\frac{1}{w_t^m}\left(\frac{p_t^n}{w_t^m}\right)^{\eta-2}\alpha_s^\eta\left[\left(\frac{\phi_m}{\phi_n}\right)\right]^{\eta-1}\left((1-\alpha_s)^\eta + \alpha_s^\eta\left[\left(\frac{p_t^n}{w_t^m}\right)\left(\frac{\phi_m}{\phi_n}\right)\right]^{\eta-1}\right)^{\frac{1}{1-\eta}-1} \\
&= \frac{\Lambda_t^{SM}}{p_t^n}\left[\frac{(1-\alpha_s)^\eta}{\left((1-\alpha_s)^\eta + \alpha_s^\eta\left[\left(\frac{p_t^n}{w_t^m}\right)\left(\frac{\phi_m}{\phi_n}\right)\right]^{\eta-1}\right)}\right]
\end{aligned}
$$

Finally, the expression for the elasticity of the investment price demonstrates that it is independent of the level of investment when the time constraint of the child is slack:

$$
Elasticity_{\{\Lambda^{SM},p_n\}} = \frac{\partial \Lambda_t^{SM}}{\partial p_t^n}\frac{p_t^n}{\Lambda_t^{SM}} \quad = \quad \left[\frac{(1-\alpha_s)^\eta}{(1-\alpha_s)^\eta + \alpha_s^\eta\left[\frac{\left(\frac{w_t^m}{\phi_m}\right)}{\left(\frac{p_t^n}{\phi_n}\right)}\right]^{1-\eta}}\right]
$$

I.2 Married Couples

The analogous exercise to find the price of investment for married/cohabiting couples is also shown here for the case where the time constraint of the child is slack:

$$\begin{aligned}
\Lambda_t^{MC} &= \frac{Cost_t^{MC}}{I_t^{MC}} \\
&= \frac{p_t^n\left[1+\frac{w_t^m}{p_t^n}\frac{q_t^m}{n_t}+\frac{w_t^f}{p_t^n}\frac{q_t^f}{n_t}\right]}{\phi_n\left[\alpha_1\left(\frac{\phi_f q_t^f}{\phi_n n_t}\right)^{\frac{\epsilon-1}{\epsilon}}+\alpha_2\left(\frac{\phi_m q_t^m}{\phi_n n_t}\right)^{\frac{\epsilon-1}{\epsilon}}+1-\alpha_1-\alpha_2\right]^{\left(\frac{\epsilon}{\epsilon-1}\right)}}
\end{aligned}$$

and

$$\begin{aligned}
\left(\frac{\phi_m q_t^m}{\phi_n n_t}\right)^{\frac{\epsilon-1}{\epsilon}} &= \frac{w_t^m q_t^m}{p_t^n n_t}\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)=\left[\left(\frac{p_t^n}{w_t^m}\right)\left(\frac{\alpha_2}{1-\alpha_1-\alpha_2}\right)\left(\frac{\phi_m}{\phi_n}\right)\right]^{\epsilon-1} \\
\left(\frac{\phi_f q_t^f}{\phi_n n_t}\right)^{\frac{\epsilon-1}{\epsilon}} &= \frac{w_t^f q_t^f}{p_t^n n_t}\left(\frac{1-\alpha_1-\alpha_2}{\alpha_1}\right)=\left[\left(\frac{p_t^n}{w_t^f}\right)\left(\frac{\alpha_1}{1-\alpha_1-\alpha_2}\right)\left(\frac{\phi_f}{\phi_n}\right)\right]^{\epsilon-1} \\
\Longrightarrow & \\
\Lambda_t^{MC} &= \frac{p_t^n\left[1+\frac{w_t^m}{p_t^n}\frac{q_t^m}{n_t}+\frac{w_t^f}{p_t^n}\frac{q_t^f}{n_t}\right]^{\frac{1}{1-\epsilon}}}{\phi_n\left(1-\alpha_1-\alpha_2\right)^{\left(\frac{\epsilon}{\epsilon-1}\right)}} \\
&= \frac{p_t^n}{\phi_n}\left[(1-\alpha_1-\alpha_2)^\epsilon+\alpha_2^\epsilon\left[\left(\frac{p_t^n}{w_t^m}\right)\left(\frac{\phi_m}{\phi_n}\right)\right]^{\epsilon-1}+\alpha_1^\epsilon\left[\left(\frac{p_t^n}{w_t^f}\right)\left(\frac{\phi_f}{\phi_n}\right)\right]^{\epsilon-1}\right]^{\frac{1}{1-\epsilon}}
\end{aligned}$$

Taking the partial derivative with respect to the price of child care:

$$\begin{aligned}
\frac{\partial\Lambda_t^{MC}}{\partial p_t^n} &= \Lambda_t^{MC}\left(\frac{1}{p_t^n}-\left[\frac{\alpha_2^\epsilon\frac{1}{w_t^m}\left(\frac{p_t^n}{w_t^m}\right)^{\epsilon-2}\left(\frac{\phi_m}{\phi_n}\right)^{\epsilon-1}+\alpha_1^\epsilon\frac{1}{w_t^f}\left(\frac{p_t^n}{w_t^f}\right)^{\epsilon-2}\left(\frac{\phi_f}{\phi_n}\right)^{\epsilon-1}}{\left[(1-\alpha_1-\alpha_2)^\epsilon+\alpha_2^\epsilon\left[\left(\frac{p_t^n}{w_t^m}\right)\left(\frac{\phi_m}{\phi_n}\right)\right]^{\epsilon-1}+\alpha_1^\epsilon\left[\left(\frac{p_t^n}{w_t^f}\right)\left(\frac{\phi_f}{\phi_n}\right)\right]^{\epsilon-1}\right]}\right]\right) \\
&= \frac{\Lambda_t^{MC}}{p_t^n}\left[\frac{(1-\alpha_1-\alpha_2)^\epsilon}{\left[(1-\alpha_1-\alpha_2)^\epsilon+\alpha_2^\epsilon\left[\left(\frac{p_t^n}{w_t^m}\right)\left(\frac{\phi_m}{\phi_n}\right)\right]^{\epsilon-1}+\alpha_1^\epsilon\left[\left(\frac{p_t^n}{w_t^f}\right)\left(\frac{\phi_f}{\phi_n}\right)\right]^{\epsilon-1}\right]}\right]
\end{aligned}$$

Finally, the expression for the elasticity of the investment price demonstrates that it is independent of the level of investment when the time constraint of the child is slack:

$$Elasticity_{\{\Lambda^{MC},p_n\}} = \frac{\partial \Lambda_t^{MC}}{\partial p_t^n}\frac{p_t^n}{\Lambda_t^{MC}} \quad = \quad \frac{(1-\alpha_1-\alpha_2)^{\epsilon}}{\left[\alpha_1^{\epsilon}\left[\frac{\left(\frac{w_t^f}{\phi_f}\right)}{\left(\frac{p_t^n}{\phi_n}\right)}\right]^{1-\epsilon} + \alpha_2^{\epsilon}\left[\frac{\left(\frac{w_t^m}{\phi_m}\right)}{\left(\frac{p_t^n}{\phi_n}\right)}\right]^{1-\epsilon} + (1-\alpha_1-\alpha_2)^{\epsilon}\right]}$$

J Discussion

The two elasticities have similar forms, and their properties can be compared:

$$Elasticity_{\{\Lambda^{SM},p_n\}} = \frac{\partial \Lambda_t^{SM}}{\partial p_t^n}\frac{p_t^n}{\Lambda_t^{SM}} \quad = \quad \frac{(1-\alpha_s)^{\eta}}{\alpha_s^{\eta}\left[\frac{\left(\frac{w_t^m}{\phi_m}\right)}{\left(\frac{p_t^n}{\phi_n}\right)}\right]^{1-\eta} + (1-\alpha_s)^{\eta}}$$

$$Elasticity_{\{\Lambda^{MC},p_n\}} = \frac{\partial \Lambda_t^{MC}}{\partial p_t^n}\frac{p_t^n}{\Lambda_t^{MC}} \quad = \quad \frac{(1-\alpha_1-\alpha_2)^{\epsilon}}{\alpha_1^{\epsilon}\left[\frac{\left(\frac{w_t^f}{\phi_f}\right)}{\left(\frac{p_t^n}{\phi_n}\right)}\right]^{1-\epsilon} + \alpha_2^{\epsilon}\left[\frac{\left(\frac{w_t^m}{\phi_m}\right)}{\left(\frac{p_t^n}{\phi_n}\right)}\right]^{1-\epsilon} + (1-\alpha_1-\alpha_2)^{\epsilon}}$$

In particular, the numerator is the share on child care, adjusted for the elasticity of substitution between child care and other inputs in the respective family structure. The denominator has a term that incorporates this elasticity-adjusted share of child care, plus terms that reflect the relative price per efficiency unit of parental time inputs and child care inputs into investment. Consider the case when the elasticity of substitution parameter for either family structure (η for SM, ϵ for MC) is less than 1. Then, the higher the relative price per efficiency unit of parental time versus child care time, the larger the denominator, and the smaller the elasticity of investment's price with respect to the price of child care. The importance of this price ration in the denominator is mediated by the share assigned to the relevant parental time input: the larger that share, the more effect it's relative price per efficiency unit is in determining the sensitivity of the investment price to a change in the price of child care.

For intuition about the role of input shares, consider the case where the share on mother time for the single mother is 0. Then the pass-through of the child care subsidy to the price of investment (the elasticity) is 1. As the share on mother time increases, this elasticity decreases. Once the share on mother time reaches the upper bound of 1, the elasticity is 0.

For couples, if the share on father's time is 0 but the share on mother's time is positive, then the pass-through of a subsidy to child care to the price of investment is less than 1. This is an exaggeration of the case I find in my estimation.

For intuition about the role of elasticities of substitution, set the ratio of price per efficiency units to 1, and assume that the shares at the same across the two family structures (the share on father time is 0 for couples). The only parameter that differs is the elasticity of substitution across inputs. If $\eta < \epsilon$, then:

$$\begin{aligned} Elasticity_{\{\Lambda^{SM},p_n\}} = \frac{(1-\alpha)^{\eta}}{\alpha^{\eta}+(1-\alpha)^{\eta}} &< \frac{(1-\alpha)^{\epsilon}}{\alpha^{\epsilon}+(1-\alpha)^{\epsilon}} = Elasticity_{\{\Lambda^{MC},p_n\}} \\ \alpha^{\epsilon-\eta} &< (1-\alpha)^{\epsilon-\eta} \\ \alpha &< (1-\alpha) \end{aligned}$$

Evidently, when the elasticity of substitution across inputs is lower for single mothers, the elasticity of investment's price with respect to the child care subsidy is lower for single mothers than for couples if and only if the share on child care is larger than the share on mother's time. If the magnitude of the shares on inputs is reversed, then the single mother's elasticity of substitution parameter makes her more sensitive to the child care subsidy than couples. Effectively the share on child care and the elasticity of substitution across inputs work in opposite directions when it comes to their effect on the elasticity of investment's price with respect to child care's price.

K Elasticities when the child's time constraint binds

When the child's time constraint binds, there is no closed-form solution for the optimal ratio of inputs into investment. Therefore, in the first step of calculating the elasticities derived above, the substitution for quantities as a function of prices is no longer possible. Instead, a numerical exercise illustrates that the price of investment is increase in the level of investment for the case where the child's time constraint binds.

Part IV

Motivating Regressions from the ECLS-B

In Tables 10 and 11, I report regression analyses I use to motivate two modelling assumptions: parents do not target investments by child gender, and initial skill endowments (at 9 months) affect skill outcomes later in life (at 4 years of age). Table 10 reports four models, each with a time input choice as the dependent variable. The first two are for married couples, the second two for single mothers. Time investments are predicted by attributes of the parents (hourly wages and educational attainment) and attributes of the child (current skill). Child gender is not a statistically significant predictor of parental time inputs, according to Table 10 . There is some evidence in other studies that parenting behavior and treatment effects of the program vary by the gender of the child (see **?**, Kottelenberg and Lehrer (2014)), but I do not see parenting investment decisions depending on gender in my empirical analysis.

Table 11 reports three models: for married couples, for single mothers, and for the pooled sample. The dependent variable in all three models is the final skill of the child at age 4. Explanatory variables include the initial skill of the child, gender of the child, indicators for parental educational attainment (BA or higher), and parental hourly wages. Initial test scores are statistically significant predictors for final test scores, and so are parental attributes related to their skill. This motivates including heterogeneity in initial skill endowments in my model, and supports the assumption concept of child skill evolving endogenously in response to parental decisions, which in turn depend on their individual attributes (measured indirectly here with wages and education).

L Measures of Skill in the ECLS-B

In wave 1, the ECLS-B reports test scores from the Bayley Short Form - Research Edition, which is a shortened version of the Bayley Scaled of Infant Development, Second Edition (BSF-R and BSID-II, respectively). The latter exam is the standard one for measuring development in children under 42 months of age. The BSF-R is a shortened version of the BSID-II, asking only some of the questions. Its scores are then re-scaled to make them comparable with scores from children who receive the BSID-II. For the initial test score variables used in the regressions reported in this appendix, I take the scale scores of the BSF-R at 9 months (in the first wave of the ECLS-B), which are reported both for mental and motor development. I then take the average of the two, and next I standardize them to lie between 0 and 1. By age 5, when the child is 48 months, the BSID-II and its subset exam the BSF-R are no longer an age-appropriate measures of development for children. Instead, the ECLS-B reports a new assessment battery that covers cognitive development in the domains of language, literacy, color knowledge, and mathematics. This is reported as the ECLS-B Direct Cognitive Assessment in several formats. I use the overall scale score of the Direct Cognitive Assessment and standardize it to lie between 0 and 1. This is the final test score used in the regressions of this appendix.

M Assumptions on Gender and Initial Skill

Table 10: Time Investments by Child Gender

	Married Couples		Single Mothers	
	(1)	(2)	(3)	(4)
	Tot. Parental Time	N Time	N Time	Total Time
Child is Female	-0.0845	-0.216	-1.156	0.0434
	(0.212)	(0.782)	(0.850)	(0.499)
Child Test Score [0,1]	8.528***	8.281***	1.023	9.945***
	(0.598)	(2.219)	(2.477)	(1.389)
B.A.: Father	2.104***	-0.895		
	(0.250)	(0.904)		
B.A.: Mother	1.850***	2.745**	1.578	3.484***
	(0.247)	(0.881)	(1.473)	(0.556)
Hourly Wage: Father	0.0105	-0.0248*		
	(0.00588)	(0.0118)		
Hourly Wage: Mother	0.00922*	-0.0146	-0.0638**	-0.0122
	(0.00369)	(0.0131)	(0.0244)	(0.0107)
Constant	-2.921***	10.59***	29.68***	18.85***
	(0.824)	(3.134)	(3.443)	(1.905)
R^2	.13	1.1e-02	5.0e-03	1.7e-02

Standard errors in parentheses. N stands for non-parental child care.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

Table 11: Predicting Final Skill with Initial Skill + Parental Attributes

	(1) Married Couples	(2) Single Mothers	(3) All
Initial Test Score (9 Mo.): Stdzd	0.138***	0.139***	0.130***
	(0.0389)	(0.0350)	(0.0373)
Hourly Wage: Mother	0.0565	0.217***	0.123**
	(0.0359)	(0.0500)	(0.0383)
Hourly Wage: Father	0.0971**		
	(0.0312)		
Child is Female	0.108	0.135	0.0941
	(0.0619)	(0.0738)	(0.0636)
B.A.: Mother	0.291***	0.662***	0.546***
	(0.0744)	(0.113)	(0.0651)
B.A.: Father	0.441***		
	(0.0788)		
Constant	0.851***	0.654***	1.005***
	(0.183)	(0.160)	(0.180)
R^2	0.1695	0.1273	0.1237
Observations	2900	1400	2900

Initial skill has predictive power. Units: standard deviations, except for indicators

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

Correlation of Initial Child Skill and Final Child Skill with Family Income

Table 12: Correlations of Skill and Family Income

	(1) Test Score W1	(2) Test Score W3
Family Income W1	0.000114	0.000651*
	(0.000103)	(0.000254)
Flag: Present in model 2 sample	-0.00677	0
	(0.00754)	(.)
Family Income W3		0.000693***
		(0.000204)
Test Score W1 (SD)		0.141**
		(0.0482)
Constant	1.453***	1.004***
	(0.00733)	(0.0705)
R^2	.003	.125
Observations	1300	1500
Correlation	0.04	0.33
Correlation p-value	.35	0

Income in thousands of dollars. Test scores in standard deviation units.
Standard errors in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

Table 12 reports a slightly different version of the same qualitative points made with Tables 12, 10, and 11. The dependent variables in the two models are initial skill and final skill, with both family structures pooled. Initial income at 9 months has no predictive power for the initial skill score. At age 4, however, final skill can be predicted with income (both at age 9 months and 4 years) and the initial test score. In this table I also report the correlations of the dependent variable for the model (initial skill for model 1, final skill for model 2) with family income in the same period (initial and final, respectively). These correlations jump from zero to 0.33. Note that the measure of family income I use here is income before labor income taxes but including transfers.

Predicting Outputs with Inputs in the ECLS-B

The measure I have selected for parental time inputs predicts the outcomes of children in the ECLS-B. This is document in Table 13. Evidently, variation in demeaned and standardized parental education time at the family level predict variation in the demeaned and standardized skill measure for the child, even controlling for lagged values of the child's skill.

Table 13: Predicting Outputs (Child Outcomes) with Inputs by Definition of Education Time

	(1) MC	(2) MC Given Prices	(3) SM	(4) SM Given Prices
L.demeaned_std_score	0.288***	0.268***	0.242***	0.220**
	(0.0306)	(0.0304)	(0.0669)	(0.0678)
dmnd_std_tot_hours_np_care	0.0251	0.0631*	-0.0108	0.0622
	(0.0295)	(0.0318)	(0.0511)	(0.0603)
dmnd_std_total_ed_time_fth	0.146***	0.132***		
	(0.0290)	(0.0288)		
dmnd_std_total_ed_time_mth	0.000758	-0.00194	0.419***	0.376**
	(0.0306)	(0.0300)	(0.112)	(0.115)
AT_pay_per_hour_mth		0.00408		0.00650*
		(0.00312)		(0.00323)
AT_pay_per_hour_fth		0.00874**		
		(0.00281)		
tot_cost_perhr_np_care		0.0152		0.0471*
		(0.00905)		(0.0208)
Constant	0.298***	0.0587	0.424***	0.169
	(0.0290)	(0.0646)	(0.103)	(0.134)
r2	.1573	.1764	.2116	.2291
Obs_rounded	1850	1850	550	550

Standard errors in parentheses
Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Part V

Further Description of Data Sources

N The Early Childhood Education Longitudinal Study, Birth Cohort

The ECLS-B follows a nationally representative sample of families with a child who was 9 months old in 2001. It was designed and collected by the United States Department of Education. Using birth-certificate data from the National Center for Health Statistics, over 14,000 births were selected within Primary Sampling Units. Children of mothers younger than 15 were excluded from the sampling frame. There are 5 waves: wave 1 is the 9-month old data collection round, wave 2 occurs at 2 years, wave 3 at 4 years, and waves 4 and 5 at kindergarten entry. If the focal child was not in kindergarten when wave 4 was collected, the surveyors went back and collected data the next year when they were enrolled. In addition, if a child repeated kindergarten, their scores were also collected in wave 5 in addition to wave 4. Each wave contains several instruments; these are different self-administered questionnaires (SAQs) for different people in the child's life, in addition to the child-level data. Table 22 summarizes these instruments in each wave of the survey.

Table 14: The Structure of the ECLS-B

Instrument	Wave 1	Wave 2	Wave 3	Wave 4+5
1.	Parent Interview	Parent Interview + SAQ	Parent SAQ	Parent SAQ
2.	Resident Father	Resident Father SAQ	Resident Father SAQ	ECEP Interview[1]
3.	Nonresident Father	Nonresident Father SAQ	Preschool Center Director SAQ	Teacher
4.		Child Care Provider	Preschool ECEP SAQ[1]	WECEP Interview [2]
5.		Center Director		

[1] Early Care and Education Provider
[2] Wrap-around Care Early Care and Education Provider

In each wave of the survey, the primary care provider (usually the mother) and the resident father fill out detailed questionnaires on the activities they do with their kids and at what frequency (once a week, twice a week, once a month, etc.). In addition, they report age, educational attainment, income, hours worked, the number of hours the child spent in non-parental care, what type of care that was (relative, non-relative, center-based), and the cost of that care.

I define quality time as the total amount of time spent (1) reading to the child (2) playing outside with the child. To map from frequencies of activities to levels of quality time supplied by parents, I impute amount of time per activity using data from the ATUS. The imputation uses common characteristics observed across both samples: gender, marital status (married/cohabiting or single), labor force status, and educational attainment. Here educational attainment is less than a college degree, or a college degree or more. For hourly wages, I use time spent working and income to compute the pre-tax levels, and then Table 2 of McGrattan and Prescott (2017) to correct for labor income taxes. For hourly prices of non-parental care, I use total cost of child care and total hours in child care for the primary source of non-parental care reported by the primary caregiver of the survey child subject.

In the following two subsections I report sample summary statistics for the raw ECLS-B sample, before I impose restrictions on it for the estimation sample. The population moments I use in the internal calibration for the fraction of parents who are single mothers comes from this sample. The fraction in the sample that also reports variables necessary for estimation is larger than these population moments. Notice that the fraction below 185% of the poverty line in the pooled sample is quite high, at 50%. This drop to 40% by the time the child is age 4. Averages here include observations for which the response is 0. This explains why the average age of the father is now lower than the mother's.

N.1 Summary Statistics for Raw Sample in ECLS-B

N.1.1 Sumstats Raw ECLS-B Sample

Table 15: Raw ECLS-B Data Moments (Waves 1-3)

	Levels							
	count	mean	p10	p50	p90	sd	min	max
total_ed_time_mth	32050	5.83	0.00	3.84	18.91	6.51	0.00	22.76
tot_hours_np_care	32050	15.00	0.00	6.00	40.00	17.99	0.00	120.00
tot_hours_np_care_combined	31200	17.07	0.00	8.00	45.00	19.87	0.00	144.00
tot_cost_perhr_np_care	32050	1.12	0.00	0.00	3.75	2.85	0.00	99.50
tot_cost_perhr_np_care_combined	17550	2.08	0.00	1.11	5.00	3.58	0.00	99.50
AT_pay_per_hour_mth	28250	11.21	4.60	8.89	18.55	17.65	0.00	1063.80
ratio_pn_wm	28250	76.95	0.00	0.00	0.35	8703.86	0.00	1345064.75
ratio_qm_n1	17100	0.45	0.02	0.16	1.08	0.99	0.00	22.76
income	29450	56407	10000	37500	150000	558423	1	300000
income_aftx	29450	53753	20359	41413	117460	38769	11763	223092
Age primary caregiver	29350	30.21	21.00	30.00	39.00	6.92	15.00	82.00
age_first_child_mth	28400	23.85	17.00	23.00	32.00	6.04	10.00	50.00
	Rates							
indicator_gteBA_mth	32050	0.33						
indicator_poor	32050	0.23						
indicator_poor185	32050	0.44						
indicator_single_biomth	32050	0.17						
Obs_rounded	32050							

Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

In Table 15, the unweighted summary statistics for a pooled sample of waves 1-3 are presented. Over these three waves, mothers on average invest 6 hours of their time per week investing in their children. The mean of this variable is higher than the median, which is about 3.8 hours per week. The mean is more sensitive to outliers - in this case, the high-value outliers are driving the higher value for the mean. Most families don't spend large numbers of hours per week investing parental time in their children. Meanwhile, the total hours spent in the primary source of child care are on average 15, which is much higher than the median of 6 hours per week. This reflects the fact that in the raw sample some families report that their child spend 120 hours per week in child care. Such observations are of interest because they represent the other corner of the investment decision: not zero investment, but rather investment at a binding time constraint of the child. This table demonstrates that very few families are at this constrain in the current equilibrium, which generates the data documented in the ECLS-B. If, instead of restricting to the primary source of child care, I looked at the hours spent in any form of non-parental care, the average moves up to 17 and the median to 8. The maximum value for this category is 144 hours per week, which requires spending more than 20 hours a day in non-parental child care. In the estimation sample I only use observations for families that use less than 100 hours total of their child's time in parental investment activities or non-parental child care. This is because the estimation equations I use are only valid if the time constraint of the child is slack. Moving on the the next category of variables, the price per unit of the time inputs into investment is documented in rows 5-7 of this table. The price for the primary source of child care is lower than for all child care combined. The latter variable is the average across all sources of chidl care - it is higher because some families use small amounts of expensive child care. So, even if the hours in child care don't increase much moving from primary to total sources of child care, the price does jump. The median price per hour of child care is 0. This is because many families use no child care and many also use child care that is completely free. My estimation assumptions effectively assume that child care which is free is not an input into investment. The ratio of the price of child care to the mother's wage is high on average. This ratio is found using imputed wages for mother's who don't work, whereas in the estimation the ratio is only for mothers who report pay earned and hours worked. In the imputation, I used age, age squared, and education level to approximate the wage the mother would make if she worked. Sometimes this approximated wage is positive but very small. The summary statistics reflect this. Moving on to income, before-tax income is on average $\$56,000$. [17] Once I implement the method I use to correct for taxes, using **?**, the redistributive nature of the tax system that I assume is evideent from the fact that, while the median decreases, the median increases and so does the minimum income in the sample. When I report the correlation of child skill and family income over time during the calibration section of the model parameterization section in the main body of this paper, I am using after-tax income. Finally, the last section of the levels statistics for the ECLS-B shows statistics on the age of the primary caregive at the time of the interview, and on the mother's age when her first child was born. If the primary caregiver is the biological mother, as I impose in my estimation sample, then comparing these two rows would give some intuition for whether the child in the ECLS-B sample is the mother's first child. Here, however, they can be different people.

The second part of Table 15 deals with rates in the raw sample. The rate of BA attainment for the primary caregiver is about 1 in 3. The poverty rate is about 1 in 4, and the rate of single motherhood is about 17%..

[17] This is between the mean and median family income for families with children under 18 in current dollars for the years 2001-2004, as reported by the United States Census Bureau in Table F-9 here: `census.gov/data/tables/time-series/demo/income-poverty/historical-income-families.html`

N.1.2 Sumstats Raw ECLS-B Sample, Weighted

Table 16: Weighted ECLS-B Data Moments (Waves 1-3)

	Levels							
	count	mean	p10	p50	p90	sd	min	max
total_ed_time_mth	29450	6.52	1.10	3.84	20.01	6.58	0.00	22.76
tot_hours_np_care	29450	16.04	0.00	9.00	40.00	17.77	0.00	120.00
tot_hours_np_care_combined	29450	17.71	0.00	10.00	45.00	19.50	0.00	144.00
tot_cost_perhr_np_care	29450	1.27	0.00	0.00	4.00	2.92	0.00	99.50
tot_cost_perhr_np_care_combined	17500	2.16	0.00	1.25	5.14	3.51	0.00	99.50
AT_pay_per_hour_mth	28200	10.92	4.56	8.60	18.21	16.53	0.00	1063.80
ratio_pn_wm	28200	144.23	0.00	0.00	0.37	12790.67	0.00	1345064.75
ratio_qm_n1	17050	0.47	0.03	0.17	1.13	0.96	0.00	22.76
income	29450	55624	10500	37500	150000	54483	1	300000
income_aftx	29450	53233	20736	41413	117460	37820	11763	223092
age_res_mth	29300	30.01	22.00	30.00	39.00	6.72	15.00	82.00
age_first_child_mth	26200	23.73	17.00	23.00	32.00	5.82	10.00	50.00
	Rates							
indicator_gteBA_mth	29450	0.25						
indicator_poor	29450	0.24						
indicator_poor185	29450	0.47						
indicator_single_biomth	29450	0.19						
Obs_rounded	29450							

Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

In Table 16, the weighted summary statistics for a pooled sample of waves 1-3 are presented. I use cross-section survey weights for each observation and report pooled moments in this table. Most of the statistics are very similar after weighting; only slight changes to means are noticeable. One exception is the rate of single motherhood, which is 19% in the weighted sample (versus 17% in the unweighted sample). In the model calibration section, I use the marriage rate of the weighted total sample as a target, although I use the time investment levels of the estimation sample for targets as well.

Effect of sample cleaning on composition of child care type used in the sample

In the process of sample cleaning, the composition of child care sources shifts. In Tables 17 and 18 I document how over the three waves included in the estimation sample the restrictions I put on the data to qualify as a valid observation affect the sample composition. This happens because some types of child care are more likely to be completely free than others. It also happens because families who do not use any child care at all are not included in the estimation sample. Table 17 demonstrates that this makes up a sizeable fraction of families when the child is less than 4 years of age (in the first 2 waves of the sample).

Table 17: Primary Source of Child Care: Raw Sample

	(1) Wave 1 pct	(2) Wave 2 pct	(3) Wave 3 pct
NOT ASCERTAINED	0.19	0.13	0.22
NO NONPARENTAL CARE	50.04	50.55	18.54
RELATIVE CARE IN CHILDS HOME	12.61	8.60	5.39
RELATIVE CARE IN ANOTHER HOME	12.52	9.96	6.39
RELATIVE CARE, LOCATION VARIES	1.51	0.95	0.77
NONRELATIVE CARE IN CHILDS HOME	4.00	3.11	1.64
NONRELATIVE CARE IN ANOTHER HOME	10.38	11.02	5.47
NONRELATIVE CARE, LOCATION VARIES	0.12	0.23	0.18
CENTER-BASED PROGRAM	7.86	14.95	46.34
EQUAL TIME IN MULTIPLE ARRANGEMENTS	0.79	0.49	1.99
HEAD START PROGRAM			13.06
Total	100.00	100.00	100.00
Obs_rounded	10700	9850	8950

Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

Table 18: Primary Source of Child Care: Estimation Sample

	(1) Wave 1 pct	(2) Wave 2 pct	(3) Wave 3 pct
RELATIVE CARE IN CHILDS HOME	7.66	6.97	2.59
RELATIVE CARE IN ANOTHER HOME	14.05	10.74	4.42
RELATIVE CARE, LOCATION VARIES	0.78	0.56	0.23
NONRELATIVE CARE IN CHILDS HOME	12.02	7.53	4.19
NONRELATIVE CARE IN ANOTHER HOME	41.09	36.70	15.77
NONRELATIVE CARE, LOCATION VARIES	0.39	0.56	0.38
CENTER-BASED PROGRAM	24.03	36.94	69.99
HEAD START PROGRAM			2.44
Total	100.00	100.00	100.00
Obs_rounded	1050	1250	1300

Source: U.S. Department of Education, National Center for Education Statistics,
Early Childhood Longitudinal Study, Birth Cohort (ECLS-B),
Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.
All sample counts have been rounded to the nearest 50 in accordance with NCES requirements.

N.2 Summary Statistics for Imputation Sample in American Time Use Survey

The time per activity for each demographic bin, as computed in the ATUS, is reported in Tables 19 and 20. The number of observations per bins is also reported in Table 21.

Table 19: ATUS Imputation Sample (Unweighted)

	count	mean	p10	p50	p90	sd	min	max
Edited: sex	8395	1.69	1.00	2.00	2.00	0.46	1.00	2.00
Edited: age	8395	34.26	27.00	34.00	41.00	5.76	15.00	54.00
Number of own children < 18 years of age	8395	2.12	1.00	2.00	3.00	1.03	1.00	10.00
time_cc_play_hrs	4616	1.82	0.50	1.50	3.92	1.44	0.08	10.30
time_cc_reading_hrs	6729	0.49	0.17	0.42	1.00	0.36	0.03	4.50
indicator_gtHS	8395	0.83	0.00	1.00	1.00	0.38	0.00	1.00
indicator_gteBA	8395	0.60	0.00	1.00	1.00	0.49	0.00	1.00
indicator_couple	8395	0.93	1.00	1.00	1.00	0.25	0.00	1.00
income	3562	123795.94	22500.00	87500.00	383150.00	121383.93	2500.00	383150.00
Observations	8395							

These moments are from the pooled 2003-2016 ATUS sample. Moments are weighted with ATUS final weights.
For parents aged 15-55, living with own child aged 3 or less

Table 20: ATUS Imputation Sample (Weighted)

	count	mean	p10	p50	p90	sd	min	max
Edited: sex	8395	1.67	1.00	2.00	2.00	0.47	1.00	2.00
time_cc_reading_hrs	6729	0.48	0.17	0.42	1.00	0.35	0.03	4.50
time_cc_reading_hrs	6729	0.48	0.17	0.42	1.00	0.35	0.03	4.50
time_cc_play_hrs	4616	1.72	0.50	1.25	3.75	1.45	0.08	10.30
indicator_gtHS	8395	0.76	0.00	1.00	1.00	0.42	0.00	1.00
indicator_gteBA	8395	0.53	0.00	1.00	1.00	0.50	0.00	1.00
indicator_couple	8395	0.92	1.00	1.00	1.00	0.27	0.00	1.00
income	3562	120722.31	22500.00	87500.00	383150.00	121987.69	2500.00	383150.00
Observations	8395							

These moments are from the pooled 2003-2016 ATUS sample. Moments are weighted with ATUS final weights.
For parents aged 15-55, living with own child aged 3 or less

Table 21: ATUS Time per Activity Averages by Demographic Bins

Bin	Couple	Gender	gt HS	Reading Hrs	Ed Hrs	Talk. Hrs	Play Hrs	CC Hrs	N Reading	N Ed	N Talk.	N Play	N CC
1	0	1	0	0.793	0.801	0.745	4.222	4.115	7	21	15	11	21
2	0	1	0	.	0.477	0.477	.	3.229	0	4	4	0	4
3	0	1	1	0.519	0.434	0.271	0.566	1.600	12	17	7	4	17
4	0	1	1	0.500	0.500	.	4	5	1	1	0	1	1
5	0	2	0	0.746	0.745	0.672	1.322	2.926	85	147	74	62	147
6	0	2	0	0.387	0.453	0.491	2.353	3.300	39	75	40	37	75
7	0	2	1	0.416	0.592	0.780	1.287	2.766	185	245	76	109	245
8	0	2	1	0.664	0.721	0.651	1.766	4.303	45	69	34	36	69
9	1	1	0	0.503	0.560	0.557	1.622	2.104	232	367	157	133	367
10	1	1	0	0.310	0.471	0.690	1.617	5.031	10	18	8	8	18
11	1	1	1	0.454	0.474	0.415	1.508	2.256	1745	2101	482	1117	2101
12	1	1	1	0.585	0.656	0.685	1.574	3.638	71	77	14	63	77
13	1	2	0	0.474	0.569	0.596	1.562	2.904	239	390	182	153	390
14	1	2	0	0.494	0.608	0.582	1.743	4.210	302	447	212	269	447
15	1	2	1	0.451	0.528	0.550	1.680	3.324	2397	2814	704	1597	2814
16	1	2	1	0.549	0.634	0.533	2.101	4.554	1359	1602	500	1016	1602

O Single Fathers in the ECLS-B

The ECLS-B provides a non-resident father questionnaire (NRQ) in the first two waves of the survey. In this section, I document six points about the sample of non-resident fathers that complete this survey as well as attributes of single mothers in the data. Sample counts for these tabulations reflect response rates for the questions of interest; here, I am not restricting by whether I also observe variables necessary for the skill accumulation technology estimation. In the statistics presented below, I use survey weights for the primary caregiver sample in wave 2. The main purpose of this section is to establish that relatiely few single fathers complete the survey, that those who do are not representative of the sample of single fathers, and that when they do complete the survey their answers and the answer's of their child's mother do not coincide (where comparable). In addition, Table 23 makes an additional point about the marital status composition of single mothers: most were never married. This coincides with the timing and nature of the marriage market in my model.

Selection in the Single Father Sample, Nature of Relationship with Child's Mother by Response Status of SF The first three points are made in Tables 22 and 23. First, Table 22 shows that the response rate of non-resident fathers in each wave is about 1 in 3. Second, Table 23 shows that the marital status of the corresponding single mother is about the same for the group of families with a completed NRQ and without a completed NRQ. Third, Table 23 also shows that most single mothers were never married (about 70% and 65% in the first and second wave of the survey, respectively). Since I do not model divorce, the composition of marital status in single mothers is important to check.

Table 22: Response Rate NR Questionnaire

	(1) Wave 1	(2) Wave 2
Yes	0.300	0.309
No: Refusal	0.292	0.179
No: Not Permission	0.194	0.270
No: Ineligible, Lack of Contact	0.184	0.179
No: no NR	0.0290	0.0596
No: P not Biomother	0.000628	0.00303
Total	1	1
Obs.	2000	2000

Table 22 displays response rates of non-resident fathers to the non-resident father survey in the ECLS-B. Slightly less than one-third of non-resident fathers respond. Sample sizes rounded to nearest 50, following NCES requirements.

Table 23: Marital Status Composition of Mothers with NR fathers, by Questionnaire Response status

	Wave 1		Wave 2	
	(1) Completed NRQ	(2) No NRQ	(3) Completed NRQ	(4) No NRQ
Not Reported	0	0.00369	0	0.000118
Married	0.0640	0.0520	0.0921	0.0980
Separated	0.107	0.119	0.105	0.0939
Divorced	0.0909	0.0929	0.111	0.140
Widowed	0.00320	0.0142	0.00195	0.0169
Never Married	0.734	0.717	0.691	0.647
Not Biomother or Adoptive Parent	0	0.000897	0	0.00439
Total	1	1	1	1
Obs.	650	1350	650	1400

Table 23 displays the marital status composition of families where the biological parents are not cohabiting (single-parent families). The compositions are broken down by response status for the non-resident father questionnaire. Sample sizes rounded to nearest 50, following NCES requirements.

Single Father Visitation Frequency by NFQ Response Status, Influence of Single Father by NFQ Response Status SF vs. SM opinion The next three points are made in Tables 24-28. For point four, Table 24 tabulates the days since the non-resident father last saw the child. Fathers who complete the NRQ have seen the child on average 1.5 days more recently than fathers who do not. Fifth, in Table 24 I tabulate responses to the question "In a typical week, does [the child's] father spend a lot, some, very little, or no time taking care of [the child]?", for families without a completed NRQ's (first column) and for those with an NRQ (second column). Fathers who completed the NRQ are almost 3 times more likely to be parenting with a resident primary caregiver who responds "A lot" to this question (35% compared to 12%). Relatedly, Table 26 shows that fathers who complete the NRQ are almost twice as likely to have seen their child in the last month than fathers who did not complete the NRQ (90% versus 46%). Sixth, in Tables 27 and 28 I tabulate the wave 2 responses to the question "When it comes to making major decisions, please tell me if [child's] father has no influence, some influence, or a great deal of inclucence on such matters as child care?", separately for mothers (Table 27) in families without an NRQ (column 1) and those with an NRQ (column 2) and fathers (Table 28) who completed the NRQ. Fathers who completed the NRQ think they have a lot of influence; mothers with children whose fathers completed the NRQ say they have less influence than the fathers claim, although they report more influence more than do mothers in families without a completed NRQ.

Table 24: Wave 1: Number of Days since NRF last saw child

	(1) No NRQ	(2) Completed NRQ
No. Days	3.860	2.353
Obs.	1300	650

Table 24 displays the average number of days since a non-resident father saw his child in the first wave of the survey, by response status to the non-resident father questionnaire. Sample sizes rounded to nearest 50, following NCES requirements.

Table 25: Wave 2: Frequency NRF last provides child care

	(1) No Completed NRQ	(2) Completed NRQ
Not Applicable	0.541	0.104
A lot	0.121	0.350
Some	0.117	0.280
Very little	0.0912	0.146
No time	0.130	0.120
Total	1	1
Obs.	1350	650

Table 25 displays the response to the question: "In a typical week, does [the child's] father spend a lot, some, very little, or no time taking care of [the child]?", for families without a completed NRQ's (first column) and for those with an NRQ (second column). Sample sizes rounded to nearest 50, following NCES requirements.

Table 26: Wave 2: Number of Days since NRF last saw child

	(1) No NRQ	(2) Completed NRQ
Don't Know	0.01	0
Refused	0.01	0
Not Applicable	0.06	0
Less than 1 month	0.459	0.896
More than 1 month, less than 1 yr	0.238	0.0718
More than 1 yr	0.0975	0.0198
No contact since birth/separation	0.133	0.0118
Total	1	1
Obs.	1350	650

Table 26 compares the amount of time since non-resident fathers last saw their child, by response status to the non-resident father questionnaire. Sample sizes rounded to nearest 50, following NCES requirements.

Table 27: Wave 2: Mother's Opinion of Father's Influence on CC

	(1) No NRQ	(2) Completed NRQ
Not Applicable	0.373	0.0207
No Influence	0.341	0.395
Some Influence	0.138	0.260
A Great Deal of Influence	0.148	0.324
Total	1	1
Obs.	1400	650

Table 27 tabulates mother's responses to the question: "When it comes to making major decisions, please tell me if [child's] father has no influence, some influence, or a great deal of incluence on such matters as child care?", by response status for the non-resident father questionnaire. Sample sizes rounded to nearest 50, following NCES requirements.

Table 28: Wave 2: NRQ Father's Opinion of Father's Influence on CC

	Frequency
Not Ascertained	0.0318
No Influence	0.146
Some Influence	0.328
A Great Deal of Influence	0.494
Total	1
Obs.	650

Table 28 tabulates the response of father's who completed the non-resident father questionnaire to the question "When it comes to making major decisions, please tell me if you have has no influence, some influence, or a great deal of incluence on such matters as child care?". Sample sizes rounded to nearest 50, following NCES requirements.

P Spending on Child Care in the PSID

In the literature on estimating skill accumulation technologies during early childhood, it is common practice to include money spent on goods as one of the components of investment (examples include **?**, Daruich (2019) and Abbott (2018). By contrast, my specification includes time spent in child care instead of money spent on the child. In this section, I use tabulations from the 2001 PSID and 2002 PSID CDS to show how child care expenses contribute to total expenditures on the child. To do this, I construct four different measures of total expenditures on the child (Definitions

1 to 4 in the tables below, with each definition specified in the table footnote). Next, I find the fraction of each measure of total expenditures that comes from spending on child care. I report these fractions in Tables 29 - 32. My conclusion from this exercise is that child care represents the main component (or at least, a large component) of the expenditures on children in the PSID. In that sense, using time in non-parental child care as an input, and including expenditures on child care in the budget constraint of parents, can be viewed as narrowing in on the main component of expenditures on children and being specific about how it contributes to child skill accumulation. More specifically, I have money spent on the child in the form of child care affect child skill accumulation through changing how the child uses her time.

Table 29: Definition 1

	mean	sd	count
Ages [0,3]	0.67	0.29	84
Ages [0,5]	0.68	0.28	146
Ages [0,7]	0.71	0.26	223
Ages [0,9]	0.70	0.27	260
Ages [0,11]	0.70	0.27	275

Notes: Table 29 presents averages by age group for the fraction of total expenditure on children spent on child care. Definition 1 of total expenditures on children includes child care, money spent on toys, and money spent on school supplies

Table 30: Definition 2

	mean	sd	count
Ages [0,3]	0.55	0.28	84
Ages [0,5]	0.58	0.28	146
Ages [0,7]	0.61	0.26	223
Ages [0,9]	0.60	0.27	260
Ages [0,11]	0.60	0.27	275

Notes: Table 30 presents averages by age group for the fraction of total expenditure on children spent on child care. Definition 2 of total expenditures on children includes child care, money spent on toys, and money spent on school supplies.

Table 31: Definition 3

	mean	sd	count
Ages [0,3]	0.49	0.27	83
Ages [0,5]	0.52	0.27	144
Ages [0,7]	0.55	0.26	220
Ages [0,9]	0.53	0.26	256
Ages [0,11]	0.53	0.26	271

Notes: Table 31 presents averages by age group for the fraction of total expenditure on children spent on child care. Definition 3 of total expenditures on children includes child care, money spent on toys, money spent on school supplies, money spent on vacations, and money spent on clothes.

Table 32: Definition 4

	mean	sd	count
Ages [0,3]	0.42	0.24	71
Ages [0,5]	0.43	0.24	126
Ages [0,7]	0.46	0.24	194
Ages [0,9]	0.44	0.23	229
Ages [0,11]	0.43	0.23	243

Notes: Table 32 presents averages by age group for the fraction of total expenditure on children spent on child care. Definition 3 of total expenditures on children includes child care, money spent on toys, money spent on school supplies, money spent on vacations, money spent on clothes, and money spent on food.