Child Skill Accumulation in One- and Two-Parent Families

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Abstract

In this paper, I examine the role of family structure in child skill accumulation. I establish empirically that one- and two-parent families use different technologies to invest in their children: the choices of one-parent families are more sensitive to child care's price. I analyze the effect of child care subsidies using a general equilibrium framework with endogenous family formation. I find that subsidies improve skill outcomes for children, especially those raised by one-parent, low-income families, thereby narrowing the gap in child outcomes across family structures. Exogenous attributes of family structure drive between 28 and 40 percent of this gap.

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1 Introduction

There is widespread concensus that early childhood is a key phase in the development of skill, and that policies targeted at this age group can improve educational attainment and labor market outcomes later in life. This concensus is founded on evaluations of several small-scale randomized controlled trials (RCTs) with long-run follow-ups, where children under the age of 5 from impoverished backgrounds are exposed to active engagement with adults in environments structured to foster the development of child skill (see Almond and Currie (2011) for a review), and Heckman, Stixrud, and Urzua (2006), Anderson (2008), Cunha, Heckman, and Schennach (2010), and Garcia, Heckman, and Ziff (2017), among many).[1] The evidence from RCTs is complemented by studies of large-scale subsidized child care programs with universal eligibility (e.g. Baker, Gruber, and Milligan (2008), Berlinski, Galiani, and Gertler (2009), and Gupta and Simonsen (2010).[2] Recent evaluations of such programs have emphasized heterogeneity in the effects of child care subsidies on child outcomes across family income and family structure (Havnes and Mogstad (2014) and Kottelenberg and Lehrer (2017)).[3] In the United States, these demographic attributes are quantitatively relevant and intertwined: single mothers are much more likely to be poor than parenting couples,[4] and they raise 20 percent of children under 5.

In this paper I examine the role of family structure in child skill accumulation. The way in which families allocate their children's time differs systematically across family structures: I find that one-parent families use more child care, and contribute less quality time from parents, than couples do (despite facing similar relative prices). These choices determine the activities and environments that children experience, and I interpret them as reflecting an investment decision made by families. Building on the observed disparities in these family choices, I establish that there is a significant difference in how one- and two-parent families combine their own time and purchased child care to generate investment in their children's skill.

Next, I incorporate investment in child skill into a general equilibrium framework with endogenous family formation and evaluate the effects of a universal child care subsidy in this setting. Without a subsidy, children of one-parent families have lower levels of skill at the beginning of adulthood than children raised by couples. I find that the subsidy increases child skill outcomes, especially for one-parent families, thereby narrowing the gap across family structures. The model also allows me to shut off endogenous sorting of parents into family structures, so that I can decompose the gap in child outcomes into components due to endogenous and exogenous family attributes. I find that heterogeneity in exogenous family attributes, such as the technologies families use to generate investment in their children, drives a sizeable portion of the gap.

To establish that one- and two-parent families use different investment technologies, I analyze the problem

[1]For the perspective of child psychologists, see Shonkoff (2010) and Nisbett, Aronson, Blair, Dickens, Flynn, Halpern, and Turkheimer (2012).

[2]Examples include programs in Oklahoma (started in 1998), Quebec (1997), Argentina (1993), Norway (1975), and Denmark (1964).

[3]Unlike these studies of universal child care subsidies, De Hann and Leuven (2020) use data from the means-tested Head Start program in the United States to show similar results: there are positive gains from the program, concentrated at the lower end of the counterfactual outcome distribution. This program is targeted to children who are older than the ones I focus on, but the approach and results are similar to Havnes and Mogstad (2014) and Kottelenberg and Lehrer (2017).

[4]In the 2000 U.S. census, single mothers with children under 5 had a poverty rate of 41 percent, versus 6 percent for married couples (author's calculations).

of families investing in their children's skill by choosing time from parents and child care bought on the market. I allow the parameters which govern how investment is generated to differ across family structures. I then use the problem of the parents to derive estimation equations for these parameters, and test whether the two technologies are different. I find that they are.

To highlight why heterogeneity in investment technologies matters for policy design, I construct the implied price of investment in child skill for each family structure, and derive the elasticity of the price of investment with respect to the price of child care. A child care subsidy will increase children's skill outcomes to the extent that it decreases the price of investment in skill. I find in the data that this elasticity is three times higher for one-parent families than it is for two-parent families, which is consistent with research that documents higher gains from child care subsidies for children of one-parent families. I believe this provides a novel perspective on how investment technologies mediate the response of parental investments to child care subsidies.

My main data source is the Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), a family-level data set from the US Department of Education (Department of Education (2001)). Unlike other data sets commonly used to estimate skill accumulation technologies during early childhood, the ECLS-B is designed to be representative of families raising 9-month old children in the United States.[5] The ECLS-B provides information on parental and market time inputs, parental wages, child care prices, and family structure for children over three waves of the survey, corresponding to when the child is 9 months, 2 years, and 4 years old. This allows me to control for unobserved parenting productivities by implementing a fixed effects estimator. In addition, I observe measures of child skill in each of these waves. Together with family attributes, these test scores generate empirical moments which I use to calibrate a general equilibrium model.

The general equilibrium framework I use features a marriage market, in which each generation of men and women meets via random search and endogenously forms one- or two-parent families. After forming, families spend a portion of their lifetime altruistically investing in their children's skill using the technology of their family structure. This phase of life matches the optimization problem used in the estimation. Skill investment is the only form of transfer across generations—at the end of life, families die without leaving financial bequests. Among its attributes, this model allows both the marginal cost and the marginal benefit of skill to adjust to the subsidy in equilibrium. In particular, th emarginal benefit of skill operates through increased earnings and better marriage prospects. As a child care subsidy makes skill investment cheaper, the labor market return to skill may decrease as labor income taxes increase to finance the subsidy (which would dampen increases in investment due to the subsidy), or families may form differently when the costs of parenting decrease (thus making skill more or less useful on the marriage market).

I calibrate the model to match empirical moments of child skill, family income, and family formation. The model also qualitatively matches untargeted empirical facts on the gap in child outcomes across family structures, the positive assortative matching by hourly wage within couples, and the marriage rate over the family income distribution. In addition, the model output quantitatively aligns with empirical measures of

[5] Two common sources of data used in similar studies are the Panel Study of Income Dynamics, Child Development Supplement (PSID CDS) and the Children of the National Longitudinal Survey of Youth 1979 cohort (NLSY79 Child).

the elasticity of the price of investment with respect to the subsidy, computed from the ECLS-B using the estimated investment technologies.

I examine the equilibrium effects of a universal child care subsidy with the fully parameterized model. I find that such a subsidy increases aggregate welfare by up to 5.6% in consumption equivalent units. It does so by mimicking the contract which children would like to make with their parents and others in society, promising a portion of their future earnings to fund investment in their skill during childhood. The subsidy makes investment cheaper, and then taxes future earnings to recoup the cost.

I find that my model framework delivers a gap in child skill outcomes between children of one- and two-parent families, which is reduced by the child care subsidy. This is qualitatively in line with the findings of Kottelenberg and Lehrer (2017). Although the child care subsidy is the same level for all families, the effect it has on parental investment in children is heterogeneous: the subsidy has a larger effect on children of one-parent, low-income families. Both endogenous and exogenous differences across family structures cause this gap. Besides differences in income composition due to endogenous family formation, the two family structures differ in how their time inputs affect their children's skill.

Using the model, I perform a counterfactual exercise to separate the role of family income from family structure in determining child skill outcomes. I then examine how this decomposition changes in response to a child care subsidy. To do this I construct a child outcomes in an environment in which one- and two-parent families form exogenously, by assigning family structure randomly while keeping the marriage rate the same. The resulting counterfactual gap in child skill outcomes is driven entirely by differences in exogenous attributes of family structure (in particular, investment technologies), while the difference between the equilibrium gap and counterfactual gap is driven by the relative composition of parental skill and family income in one-parent families. I find that between 28% and 40% of the gap in child skill outcomes across family structures is driven by exogenous factors, depending on the level of the child care subsidy.

In my framework, the critical primitive is the function that maps from family time use decisions to child skill outcomes. The distinctive feature of my specification for this technology is that I only allow for time inputs from parental and non-parental care givers. I am not the first to include both parents for the couple's technology; several other studies estimate an investment technology for couples with these inputs. These studies include Abbott (2020) and Del Boca, Flinn, and Wiswall (2014), although conceptually the investment inputs in those specifications differ from mine.[6] Those studies also differ in their emphasis: Abbott (2020) analyzes income risk while Del Boca, Flinn, and Wiswall (2014) focus on the changing role of mother time in child skill production at each stage of early childhood. By contrast, I focus on how heterogeneity in investment technologies affects investment's responsiveness to a child care subsidy.

My derivation of estimation equations is similar to that of Lee and Seshadri (2019), despite different specifications for the skill accumulation process. In particular, I do not include multiple phases of childhood, each with different technology parameters. Instead, I model multiple family structures with different parameters, and a single phase of investment which lasts multiple periods (early childhood). I also differ from studies,

[6]Neither considers non-parental child care as a distinct input.

like Daruich (2020), which calibrate parameters of the skill investment technology within the model instead of performing a structural estimation.[7] Instead, I use child care time as an alternative input to parental time, which reflects an interpretation of child skill accumulation as being founded on activities and interactions with others (similar to the definition of Del Boca, Flinn, and Wiswall (2014) for "active time"). This also reflects the fact that in the data money spent on young children is mostly spent on child care.[8]

This study features altruistic parents investing in the skill of their young children, an approach to intergenerational transfers which dates back to Barro and Becker (1989). I have this in common with both Lee and Seshadri (2019) and Daruich (2020), the two studies most closely related to mine. However, these papers examine different aspects of subsidies to skill accumulation. Lee and Seshadri (2019) compares skill subsidies at different stages of life.[9] Daruich (2020) examines the effects of transfers-in-kind child care subsidies on the aggregate economy, focusing on the scaling up of free high-quality child care programs, and the economy's resulting transition to a new equilibrium. Both of these studies are consident with Cunha, Heckman, and Schennach (2010), and Agostinelli and Wiswall (2017) in highlighting the role of families and early childhood interventions. Building on these results, I narrow skill investment decisions to being made by parents, and consider subsidies targeted at early childhood.

Another important feature of my model is endogenous family formation, where both one- and two-parent families exist in equilibrium. Other macroeconomic studies also incorporate endogenous family formation into their frameworks, for example Abbott, Gallipolli, Meghir, and Violante (2019), who focus on college financial aid in the United States,[10] and Gayle, Limor, and Soytas (2017), who seek to attribute the observed intergenerational correlation earnings to various sources (including parental investments). However, the frameworks of these papers do not feature multiple family structures in equilibrium. Instead, the authors emphasize positive assortative matching of couples as a magnifying force for inequality.

In my model I do not focus on analyzing the labor force participation of parents. The extensive margin could be important for policy evaluation if the purpose of child care subsidies were to affect the labor supply and earnings of parents. A study which focuses on this potential effect is Guner, Kaygusuz, and Ventura (2016), who allow for heterogeneity in family structures and consider how this heterogeneity leads to different policy responses. However, the focus is on the labor supply of parents, not the skill accumulation of children, and family structure is specified to emphasize that decision. My specification explicitly emphasizes invesment in child skill rather than how having a spouse affects labor supply choices of adults. The specification I use for the married couple problem is based on Guvenen and Rendall (2015)—as in that study, I assume that individual leisures are perfectly complementary within a couple, an assumption motivated with time use data as documented in Aguiar and Hurst (2007). In my model, differences in labor supply of men and women who live together are driven by differences in parenting time devoted to children.

[7] In Daruich (2020), parent time and money (goods) are combined to form investment, and subsidized child care is like a transfer-in-kind of the goods input.

[8] My calculations supporting this claim use the PSID CDS and are reported in Appendix II.

[9] Another paper with this approach is Caucutt and Lochner (2020), which emphasizes that the timing of binding borrowing constraints determines the optimal timing for interventions. Restuccia and Urrutia (2004) instead emphasize the relative importance of early parental investments in determining adult outcomes and the implications of this for broader education policy design.

[10] Lochner and Monge-Naranjo (2011) also focus on this policy, although families do not form endogenously in that study.

The paper proceeds as follows. Section 2 presents the investment problem of one- and two-parent families, which form the core of my framework. From these problems I derive estimation equations. In Section 3, I present the data used to estimate the parameters of the skill investment technologies, and the estimation results. Section 4 discusses how heterogeneity in investment technologies matters for investment's response to a child care subsidy. In this section I derive elasticities, both for the price of investment and for the marginal cost of child skill, with respect to the price of child care. Section 5 presents the general equilibrium model framework, model parameterization, and model fit. Section 6 reports the child care policy experiment and decomposition results. Section 7 concludes.

2 Investment in Child Skill

The critical family decision that identifies the parameters governing investment in children is the allocation of child time between child care and time with parents. I interpret this decision as determining investment in child skill, and thereby child skill accumulation. When parents choose how to allocate their child's time, they internalize how these activities and environments affect child skill through an investment technology which they take as given, while balancing the relative prices and productivities of these different sources of inputs. For example, parents make the decision of how much time to personally spend with their child, doing activities such as reading to them or playing with them, by weighing the necessary foregone earnings against the cost of using child care instead. If child care is very high quality, this lowers the cost of using it as a substitute for parental time, and vice versa.

In this section, I formalize the problems of parents in one- and two-parent families. The purpose of this section is to highlight how parents combine their own time and child care time to produce investment in their child's skill, and how this decision depends on attributes of their investment technology. The optimization conditions for the parenting problems are used to derive estimation equations for the investment technology parameters.

In what follows, I refer to one-parent families as single mothers and two-parent families as couples. This aligns with the observed characteristics of one- and two-parent families: one-parent families are by far mostly single mothers, and two-parent families may be either married or cohabiting couples.

2.1 Parenting Problems

Parents make skill investment decisions for their child for each of the J periods of childhood. The child is born with some initial skill θ_1. In each period t, the parent affects the child's stock of skill in that period, θ_t, by their choice of investment, I_t^{type}, according to the skill production function $\theta_{t+1} = f\left(\theta_t, I_t^{type}\right)$, where $type \in \{SM, MC\}$ for single mothers (SM) and married couples (MC), respectively.

Investment is generated from parental time and time purchased on the market in the form of non-parental child care. Time contributed from parent type j is denoted $q_{j,t}$, where j can refer to a single mother ($j = 1$), a married or cohabiting mother ($j = 2$), or a married or cohabiting father ($j = 3$). Child care time is denoted n_t. The price of parental time is the wage of that parent, and the price of child care time is denoted p_t.

Single mothers are indexed by the initial skill of their child, θ_1, the productivity of child care they use, ϕ_n, the parenting productivity ϕ_1, and their wages in each time period, $\{w_{1,t}\}$. During parenthood, a single mother chooses non-parental child care time n_t, and her own time investments in her child $q_{1,t}$, in each period, to produce her child's skill accumulation. The single mother also chooses consumption, c_t, and savings, a_{t+1}, to solve the following problem:

$$\begin{aligned} \max_{\{c_t,\ell_t,a_{t+1},n_t,q_{1,t}\}_{t=1}^{J}} & \quad \left[\sum_{t=1}^{J} \beta^{t-1} u^{SM}\left(c_t,\ell_t\right)\right] + \beta^{J-1} b V^{child}\left(\theta_{J+1}\right) \qquad (1) \\ s.t.\ \theta_{t+1} & = f\left(\theta_t, I_t^{SM}(\phi_n n_t, \phi_1 q_{1,t})\right) \\ c_t + a_{t+1} + p_t n_t & \leq w_{1,t}\left(1-\ell_t-q_{1,t}\right) + \left(1+r_t\right) a_t + T \ , \ \forall t \end{aligned}$$

where, in each period, $n_t, \ell_t, q_{1,t} \geq 0$, and $a_1, a_{J+1} = 0$. The time constraints for the mother and child are $\ell_t + q_{1,t} \leq 1$ and $n_t + q_{1,t} \leq 1$, respectively. Here, T is the lump sum transfer the single mother receives from the government, and $1 + r_t$ is the return on savings. This formulation assumes that investment in children is motivated by altruistic returns to the parent from the child's expected lifetime utility, expressed as a function of their skill at adulthood.[11] Parents can affect this outcome through the skill accumulation technology. Specifically, parental and non-parental child care time generate investment in child skill according to the investment function $I_t^{SM}\left(\phi_n n_t, \phi_1 q_{1,t}\right)$.

Married couples are similarly indexed by the initial skill of their child, θ_1, the productivity of child care ϕ_n, the parenting productivity of the female and male parent, ϕ_2 and ϕ_3, and the wages of each parent in each period, $\{w_{2,t}, w_{3,t}\}_{t=1}^{J}$. Both parents and the child have a one-unit time endowment in each period. The couple jointly chooses consumption, c_t, savings, a_{t+1}, child care purchased on the market, n_t, and quality time from the female and male parents, $q_{2,t}$ and $q_{3,t}$, to solve the following problem:

$$\begin{aligned} \max_{\{c_t,\ell_t,a_{t+1},n_t,q_{2,t},q_{3,t}\}_{t=1}^{J}} & \quad \left[\sum_{t=1}^{J} \beta^{t-1} u^{MC}\left(c_t,\ell_t\right)\right] + \beta^{J-1} b V^{child}\left(\theta_{J+1}\right) \qquad (2) \\ s.t.\ \theta_{t+1} & = f\left(\theta_t, I_t^{MC}\left(\phi_n n_t, \phi_2 q_{2,t}, \phi_3 q_{3,t}\right)\right) \\ c_t + a_{t+1} + p_t n_t & \leq w_{2,t}\left(1-\ell_t-q_{2,t}\right) + w_{3,t}\left(1-\ell_t-q_{3,t}\right) + \left(1+r_t\right) a_t + T \ , \ \forall t \end{aligned}$$

where, in each period, $n_t, \ell_t, q_{2,t}, q_{3,t} \geq 0$, $a_1, a_{J+1} = 0$, and the time constraints are $\ell_t + q_{2,t} \leq 1$, $\ell_t + q_{3,t} \leq 1$, and $n_t + \max\{q_{2,t}, q_{3,t}\} \leq 1$. The last constraint embeds the assumption that parents are allowed to invest in the child at the same time (their quality time investments are non-rival).

2.2 Cost Minimization

Implicit in these optimization problems is intra-temporal cost-minimization problems, where families select the cheapest combination of inputs to finance a desired level of investment, subject to their skill investment technology.

[11] For a discussion of this modelling choice, see Appendix II.

For single mothers, for a given level of investment I_t the time inputs from the mother and child care provider solve:

$$\begin{aligned}\min_{\{q_{1,t},n_t\}} \quad & w_{1,t}q_{1,t} + p_t n_t \\ s.t. \quad & I_t^{SM}\left(\phi_n n_t, \phi_1 q_{1,t}\right) \geq I_t\end{aligned} \tag{3}$$

where the time constraint $n_t + q_{1,t} \leq 1$ is assumed not binding.

In what follows I will specify a linear homogeneous investment function $I_t^{SM}(.)$, implying that the value function of the cost minimization problem satisfies $C\left(w_{1,t}, p_t; I_t\right) = I_t \Lambda^{SM}\left(w_{1,t}, p_t\right)$. Hence, $\Lambda^{SM}\left(w_{1,t}, p_t\right)$ can be interpreted as the implicit price index of investment in period t (a composite of the prices of the two inputs, the mother's wage and the price of child care).

Similarly, for a given level of investment I_t couples choose inputs $\{q_{2,t}, q_{3,t}, n_t\}$ to solve:

$$\begin{aligned}\min_{\{q_{2,t},q_{3,t},n_t\}} \quad & w_{2,t}q_{2,t} + w_{3,t}q_{3,t} + p_t n_t \\ s.t. \quad & I_t^{MC}\left(\phi_n n_t, \phi_2 q_{2,t}, \phi_3 q_{3,t}\right) \geq I_t\end{aligned} \tag{4}$$

where the time constraint $n_t + \max\left\{q_{2,t}, q^{3,t}\right\} \leq 1$ is assumed to be not binding. Again, provided the investment function is homogeneous of degree one, the value function of the cost minimization problem satisfies $C\left(w_{2,t}, w_{3,t}, p_t; I_t\right) = I_t \Lambda^{MC}\left(w_{2,t}, w_{3,t}, p_t\right)$, where $\Lambda^{MC}\left(w_{2,t}, w_{3,t}, p_t\right)$ is the relevant price of investment for the married couple (a composite of the prices of the three inputs).

2.3 Optimal Time Choices

The skill investment functions for the two family types are assumed to have a Constant Elasticity of Subsitution (CES) structure:

$$I_t^{SM} = \left[\alpha_1\left(\phi_1 q_{1,t}\right)^{\frac{\eta_1-1}{\eta_1}} + \left(1-\alpha_1\right)\left(\phi_n n_t\right)^{\frac{\eta_1-1}{\eta_1}}\right]^{\left(\frac{\eta_1}{\eta_1-1}\right)} \tag{5}$$

$$I_t^{MC} = \left[\alpha_2\left(\phi_2 q_{2,t}\right)^{\frac{\eta_2-1}{\eta_2}} + \alpha_3\left(\phi_3 q_{3,t}\right)^{\frac{\eta_2-1}{\eta_2}} + \left(1-\alpha_2-\alpha_3\right)\left(\phi_n n_t\right)^{\frac{\eta_2-1}{\eta_2}}\right]^{\left(\frac{\eta_2}{\eta_2-1}\right)} \tag{6}$$

Here, η_1 is the elasticity of substitution for the single-mother function, η_2 is the elasticity of substitution for the married couple function, and α_j $(j = 1, 2, 3)$ are share parameters. The heterogeneity in the technology of skill investment functions is apparent in the structure of these two functions: for single mothers, only two inputs are available, where married couple can draw on three distinct inputs. Note, however, that the MC skill investment function reduces to the SM function if $\alpha_2 = \alpha_1$, $\alpha_3 = 0$, and $\eta_2 = \eta_1$.

I derive the estimation equations for the parameters of the skill investment functions using the optimality conditions of the cost minimization problems defined in 3 and 4. Specifically, the cost-minimizing solutions for the SM problem, $q^*_{1,t}$ and n^*_t, satisfy the tangency condition:

$$\frac{\frac{\partial I_t^{SM}\left(\phi_1 q^*_{1,t}, \phi_n n^*_t\right)}{\partial n_t}}{\frac{\partial I_t^{SM}\left(\phi_1 q^*_{1,t}\phi_n n^*_t\right)}{\partial q_{1,t}}} = \frac{p_t}{w_{1,t}}$$

Similarly, the optimal solutions of the MC cost minimization problem, $q^*_{1,t}$, $q^*_{3,t}$ and n^*_t, satisfy:

$$\frac{\frac{\partial I_t^{MC}\left(\phi_2 q^*_{2,t}, \phi_3 q^*_{3,t}, \phi_n n^*_t\right)}{\partial n_t}}{\frac{\partial I_t^{MC}\left(\phi_2 q^*_{2,t}, \phi_3 q^*_{3,t}, \phi_n n^*_t\right)}{\partial q_{j,t}}} = \frac{p_t}{w_{j,t}} \quad j = 2, 3$$

Given the CES parameterization, the optimality conditions for the two family structures can then be written as:

$$\ln\left(\frac{q^*_{1,t}}{n^*_t}\right) = \eta_1 \ln\left(\frac{\alpha_1}{1-\alpha_1}\right) - \eta_1 \ln\left(\frac{w_{1,t}}{p_t}\right) - (\eta_1 - 1)\ln\left(\frac{\phi_n}{\phi_1}\right) \tag{7}$$

$$\ln\left(\frac{q^*_{j,t}}{n^*_t}\right) = \eta_2 \ln\left(\frac{\alpha_j}{1-\alpha_2-\alpha_3}\right) - \eta_2 \ln\left(\frac{w_{j,t}}{p_t}\right) + \left(\frac{1}{\eta_2 - 1}\right)\ln\left(\frac{\phi_j}{\phi_n}\right) \quad j = 2, 3 \tag{8}$$

In the next section, I describe how these equations are applied to the data to find point estimates for the parameters of the two skill investment technologies.

3 Estimation

The cost-minimizing solutions of equations (7) and (8) provide the structural representation for a panel regression with which I estimate the investment technology parameters. Specifically, I observe the relevant variables measured in each period t for each family i with parental type j (recall that $j = 1$ for a single mother, $j = 2$ for a married or cohabiting mother, and $j = 3$ for a married or cohabiting father). The estimation equations are written as:

$$y_{j,i,t} = \beta_{0,j} + \beta_j x_{j,i,t} + \tau_{j,i} + \nu_{j,i,t} \tag{9}$$

Here, the dependent variable is $y_{j,i,t} \equiv \ln\left(\frac{q_{j,i,t}}{n_{j,i,t}}\right)$, the independent variable is $x_{j,i,t} \equiv \ln\left(\frac{w_{j,i,t}}{p^n_{j,i,t}}\right)$, and the residual error term $\nu_{j,i,t}$ is assumed to be i.i.d. Note that, in view of (8), the slope parameters for the MC parents are the same, that is $\beta_2 = \beta_3$ —a restriction that is maintained in estimation.

In the panel regression, $\tau_{j,i}$ captures the unobserved and time-invariant relative efficiency coefficients of inputs, which are evident in equations (7) and (8). If the investment productivities of parental time and child care time are correlated with the observed prices of these inputs, which seems plausible, then the coefficient on the price ratio regressor will be biased in an Ordinary Least Squares (OLS) estimation because of the omitted variable in the residual. In light of this, I estimate the model using a Fixed Effects estimation at the parent level. That is, I treat $\tau_{j,i}$ as fixed effects to be estimated and, by a suitable normalization, I will assume $E[\tau_{1,i}] = E[\tau_{2,i}] = E[\tau_{3,i}] = 0$. This provides the family-specific intercepts $\beta_{0,j}$ that are needed to retrieve the structural CES parameters. The parameters of equation (9) map into the structural parameters of interest as follows:

$$\eta_j = -\beta_j \ , \ j = 1, 2 \tag{10}$$

$$\alpha_1 = \frac{\exp\left(-\frac{\beta_{0,1}}{\beta_1}\right)}{1 + \exp\left(-\frac{\beta_{0,1}}{\beta_1}\right)} \tag{11}$$

$$\alpha_j = \frac{\exp\left(-\frac{\beta_{0,j}}{\beta_2}\right)}{1 + \exp\left(-\frac{\beta_{0,2}}{\beta_2}\right) + \exp\left(-\frac{\beta_{0,3}}{\beta_2}\right)} \ , \ j = 2, 3 \tag{12}$$

3.1 Data

I combine two datasets to measure parental educational time inputs ($q_{j,i,t}$), non-parental child care time inputs ($n_{j,i,t}$), hourly wages ($w_{j,i,t}$), and hourly non-parental child care prices ($p_{j,i,t}$) (recall that the indexes denote parental type j in family i in period t). Specifically, I use the ECLS-B, which is a panel data set, and the American Time Use Survey (ATUS), which is a repeated cross-section sampled from the Current Population Survey (CPS). Data for hourly wages, hourly price of child care, quality time from the parents in weekly frequency of activities, and non-parental child care time in hours per week can be constructed from information reported in the ECLS-B. The ATUS contributes measures of hours per activity, which I impute to the ECLS-B to transform frequencies into hours per week of parental educational time inputs.

The Early Childhood Longitudinal Study, Birth Cohort The ECLS-B reports labor earnings, the period of time over which the labor earnings were accrued (a day, a week, two weeks, etc.), and the hours worked in a week separately for each parent. In addition, there are assessments of child skill reported in each wave of the survey. These are used when I compare the model's moments to untargeted moments in the data.[12]

After-tax labor earnings are converted into hourly wages using hours worked per week.[13] If hours worked were not reported for the parent, they are imputed using the response to part-time or full-time status (assigning 30 or 40 hours worked per week, respectively). The result is hourly after-tax wages for mothers and fathers, conditional on observing labor earnings and some information about the intensity of labor supply.[14]

[12] For more information on child skill measures in the ECLS-B, see Appendix I.

[13] To account for taxes, labor earnings are corrected using the slopes (tax rates) from Table 2 of McGrattan and Prescott (2017). This adjustment accounts for the progressive nature of the US tax system.

[14] I do not impute hourly wages for observations without this information.

To calcuate hourly price of non-parental child care, I use spending on the primary source of non-parental child care, adjusted by the number of weeks that cost represents and the hours per week the source of child care is used by the family.[15] The resulting price per hour for non-parental child care, and hourly wages of mothers and fathers, are observed for each family in the first three waves of the survey when the variables necessary to construct them are reported.[16]

Hours purchased for the primary source of child care are reported directly in the ECLS-B. Quantities of parental educational time, the other source of investment inputs, are constructed using information reported in both the ECLS-B and the ATUS. Various activities that parents do with their children are reported in the first three waves of the ECLS-B; these activities are reported in units of frequencies (every day, once a week, etc.). Only some of these activities are considered to be educational time: these are activities with the child that include talking and reading and spending time outside.[17] This definition is consistent with the literature for the importance of active time with children (Del Boca, Flinn, and Wiswall (2014)). To convert observed quality time from frequencies into hours per week, the next step is to impute time per activity from the ATUS.

The American Time Use Survey Data on levels of time per activity for a parent with a given set of characteristics come from the 2003-2016 pooled ATUS sample. This dataset provides a time diary along with CPS variables on age, gender, marital status, labor force status, educational attainment, parental status, and child age. Individuals are restricted to be between 15 and 55 years of age, with a child 3 years or younger. I use information on gender, marital status (married/cohabiting or single), labor force status (participating or not), and educational attainment, where educational attainment is discretized into those with a high school degree or less, and those with more than a high school degree. I calculate the survey-weighted average of time spent on an activity (conditional on engaging in it) for each group, for both time spent reading to the child and time spent playing with the child.[18]

Imputation After linking parents in the ECLS-B with their appropriate group in the ATUS along dimensions of age, gender, marital status, and educational status, the ATUS levels of time spent reading and time spent playing are assigned to reading activities and playing outside activities in the ECLS-B. All ECLS-B observatiosn with the same age, gender, marital status, and education bin are assigned the same number of hours to each time they report engaging in an acitvity with their child. Next, total quality time per parent in each family in each wave is calculated by summing across the two activities.

[15] The ECLS-B also reports all sources of child care; the relevant moments for the calibration are not affected by using total hours in child care instead of only the primary source.

[16] To see how the sample selection procedure affected the composition of child care sources used by families, see part I of the Appendix.

[17] This definition is founded in part on an exercise in which the definition of quality time was varied to identify which definition led the model estimation equations to best fit the data.

[18] Summary statistics and tabulations of the ATUS sample, along with summary statistics of the raw ECLS-B sample, are reported in Appendix I.

Restrictions on the Estimation Sample The estimation sample for mothers and fathers parenting in couples pools those who are married and those who are cohabiting. The single mother estimation sample is composed of mothers in the sample who are a primary caregiver and who do not have a significant other living in the household with them. For both couples and single mothers, observations are only admissable if the resident primary caregiver is a biological parent, is less than 55 years of age, reports working for pay, makes less than 200 dollars an hour, pays at least 1 cent per hour for the primary source of child care, and whose child spends at least 0.1 hours per week in child care. Families are only valid observations if the biological mother had her first child after age 15 and before age 45. Finally, I only use families I observe in all of the first three waves of the ECLS-B. This leaves me with 50 single mothers and 300 couples (with sample sizes rounded to the nearest 50, as per NCES requirements).

In the sample selection I restrict attention to families for which the estimation equation is valid. With regards to non-participation, I seek to use families that satisfy the assumptions of my estimation equation. These families I interpret as obeying the tangency conditions of the cost-minimization problem. If a parent did not participate in the workforce, or if the child's time constraint was binding, the estimation equation would not hold, although I assume the investment technology would be the same.

Estimation Sample Moments Moments from the estimation samples for each parental type are presented in Table 1. By comparing across parental types, three qualitative points are apparent.

First, couples contribute more parental time, and purchase less child care, than single mothers do. Mothers in a couple spend on average 9 hours per week engaging in educational activities with their children, while fathers spend an average of about 5 hours per week, and an average couple purchases about 33 hours per week of child care time. From Table 1, single mothers spend on average almost 2.5 hours in educational activities with their children, and purchase 36 hours per week of non-parental child care time. The ratio of mother's time to non-parental child care time is correspondingly lower than for married mothers, at 0.09 for the former compared to 0.37 for the latter.

Second, single mothers make on average about seven dollars per hour less than mothers parenting in couples. However, single mothers tend to use cheaper child care than couples do, so that the ratio of the price of child care to the mothers hourly wage is about the same as it is for married and cohabiting mothers, at 35% relative to 39% for the latter.

Finally, single mothers have an average income less than half that of married mothers or fathers. Differences in income are echoed by differences in other socioeconomic attributes across the two family structures. Starting with couples, both parents are on average in their mid-30s, with mothers who are married or cohabiting having had their first child at the age of 29 on average. Parents raising young children in a couple are well-educated: 65% of mothers have a bachelors degree or higher, and 54% of fathers do. Single mothers, by comparison, are less educated than parents raising children in a couple (21% of single mothers have a college degree) and are younger both at the time of data collection and at the age at which they had their first child. Men and women parenting in couples are not poor: only 1% of mothers in the sample are below the poverty line (0% of the fathers are), while only 6% of mothers and 5% of fathers are below 185% of that threshold.[19] The poverty rate of single mothers is nearly thirty time higher than couples, at 28%, while the percent of single mothers below 185% of the poverty line is more than eight times higher, at 51%.[20]

Table 1: Estimation Sample Moments (Waves 1-3, Unweighted)

	Singles: Mothers			Couples: Mothers			Couples: Fathers		
	mean	p50	sd	mean	p50	sd	mean	p50	sd
Parental Time	2.53	2.74	1.13	9.33	9.05	6.84	4.80	4.72	2.39
Child Care Time	36.61	40.00	12.10	33.96	40.00	12.54	33.38	37.00	12.58
Parental Hourly Wage	8.72	7.91	4.16	15.80	12.70	13.00	16.11	12.77	11.19
CC Hourly Price	2.77	2.00	3.08	4.09	3.05	4.24	4.06	3.00	4.34
Ratio (Prices)	0.35	0.22	0.43	0.39	0.24	2.80	0.31	0.22	0.57
Ratio (Quantities)	0.09	0.07	0.11	0.37	0.23	0.56	0.20	0.14	0.22
Fam. Inc. (Thous.)	35.99	27.50	40.16	106.71	87.50	67.09	106.43	87.50	64.50
Fam. Inc. (Thous. After Tax)	39.53	34.02	28.06	88.72	75.73	46.13	88.55	75.73	44.31
Age of Parent	28.76	28.00	5.82	34.17	34.00	5.16	35.63	35.00	5.45
Age First Child (Mother)	23.61	22.50	5.42	29.04	29.00	5.37			
BA or higher	0.21			0.65			0.54		
Poor (100% Pov. Line)	0.28			0.01			0.00		
Poor (185% Pov. Line)	0.51			0.06			0.05		
Observations	200			1450			1000		
Families	50			500			350		

Data source: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Sample size rounded to nearest 50 per NCES requirements.

3.2 Estimation Results

Table 2 reports the estimation results for the model in equation (9). I used wave 3 longitudinal weights for the primary caregiver survey for the single and married/cohabiting mothers samples, and wave 3 longitudinal weights for observations in the resident father survey for the married/cohabiting fathers sample.[21]

Using the estimates of Table 2, the structural parameters can be retrieved from equations (10) - (12). These estimated parameters are presented in Table 3.

I consider the null hypothesis that two-parent families are using the same technology as one-parent families to invest in their children. This requires that $\alpha_1 = \alpha_2$, $\alpha_3 = 0$, and $\eta_1 = \eta_2$ (hypotheses 1, 2, and 3, respectively). The results of the test of this hypothesis are presented in Table 4 (the number of observations used for the degrees of freedom is rounded to the nearest 50, following NCES requirements).

[19] 185% of the poverty line is the income threshold below which families are eligible for the National School Lunch Program and the Special Supplemental Nutrition Programs for Women, Infants, and Children.

[20] For comparison, a similar statistics computed from the US Census show even higher income disparities. Single mothers of children under 5 have a poverty rate of 41%, while for married couples this is 6% (author's calculations). Discrepancies are due to my sample selection procedure.

[21] See the appendix for an outline of the survey structure of the ECLS-B. There are several questionnaires, each with its own set of weights.

Table 2: Model Regression Coefficients for Skill Investment Technology Estimation (FE Model)

	Estimated Value
$\beta_{0,1}$	-1.957*** (0.0858)
$\beta_{0,2}$	-0.556*** (0.0858)
$\beta_{0,3}$	-0.984*** (0.0858)
β_1	-0.449*** (0.127)
β_2	-0.485*** (0.0639)
R^2	0.5834
Observations	2650
SM families	50
MC families: mothers	500
MC families: fathers	350

Standard errors in parentheses. * p<0.05, ** p<0.01, *** p<0.001 Data source: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9-Month-Kindergarten 2007 Restricted-Use Data File. Sample size rounded to nearest 50 per NCES requirements.

Table 3: Parameters of the Human Capital Accumulation Technology

η_1	α_1	η_2	α_2	α_3
0.449	0.0126	0.485	0.219	0.0908
(0.127)	(0 .0159)	(0.0639)	(0.0468)	(0.0299)

Note: Standard errors are in parentheses, calculated for those parameters that are a function of estimated coefficients using the delta method.

Table 4: Hypothesis Test Results

	F	df	p-value
$H_{0,1}$	20.19	(1,1750)	0.0000
$H_{0,2}$	9.25	(1,1750)	0.0024
$H_{0,3}$	0.06	(1,1750)	0.7992
Joint:	406.99	(3,1750)	0.0000

Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Sample size rounded to nearest 50 per NCES requirements.

These results establish three conclusions. First, one- and two-parent families use different technologies to invest in their children: the null hypothesis for the joint test is rejected at a 0.01% significance level. Second, these technologies have statistically the same elasticity of substitution between child care time and parental time. And third, the technologies have very different share parameters on child care time versus parental time.

The estimated elasticities indicate that a 1% increase in the ratio of a single mother's hourly wage to the price of non-parental child care causes her to adjust the ratio of her time input to non-parental child care by 0.45%. She does not heavily readjust her investment input choices because of the price change. For married mothers this number is very similar, at 0.48%. For married fathers, this statistic is by assumption the same as for the married mothers.[22] These elasticities are informative about what one can expect parents to do with their time in the presence of a subsidy to the price of child care. All parents will shift inputs away from their own time and toward non-parental child care if the latter's price decreases. Ceteris paribus, this shift in input composition will not be dramatic; to achieve large changes in time use of the parents, the change in relative prices induced by a child care subsidy would have to be large.

The stark difference in the share on child care will drive differences in how families respond to a child care subsidy. Single mothers have a much higher share on the child care time input than couples do. In both family structures, however, the share on child care time is significant and larger than the shares on other inputs.

To summarize, if the price of child care decreases due to a subsidy, families will substitute towards child care in order to generate any given level of investment, and as they do so they will adjust the ratio of inputs to a similar extent. They will also increase the target level of investment that they choose, because the price of each level of investment has gone down. Single mothers will increase their investment more than couples, ceteris paribus, because for single mothers the price of child care plays a larger role in determing the price of investment.

4 Heterogeneous Importance of Child Care Price

The price of child care—and therefore the effect of a child care subsidy—has different impacts across one- and two-parent families because these family types use different technologies to invest in their children, and because they face different income constraints. Different incomes arise because of the number of adults present in the household, and because of endogenous sorting by adult skill into family structures in the marriage market.

To gain some insight into the extent to which each of these differences may determine differences in responsiveness to child care subsidies, in this section I derive a an expression for the elasticity of skill's marginal cost with respect to the price of child care for each family structure. I decompose this into two components: a component that depends on the fraction of income being invested in the child, and the elasticity of the

[22] Relaxing this assumption and allowing the elasticity to be different for mothers and fathers yields point estimates which are not statistically different from those presented here.

price of investment. Using the estimated parameters presented in the foregoing and ECLS-B data, as well as an assumption about the functional form of utility, I calculate and compare the empirical distributions of the marginal cost elasticity and its two components across family structures. Differences across family structures in two components will drive heterogeneity in response to the subsidy.

4.1 The Elasticity of the Marginal Cost of Investment With Respect To Child Care's Price

From optimization problem of single mothers in (1), and of couples in (2), consider the simple case where childhood lasts one period ($J = 1$), there is no borrowing or saving, and suppose the utility function in problems (1) and (2) takes the form $u(c,\ell) = \ln(c) + \psi^{type}\ln(\ell)$, where $type \in \{SM, MC\}$. Let (c_t^*, ℓ_t^*) denote the optimal solutions conditional on a given final skill for the child, θ_{J+1}. For single mothers this yields $c_t^* = \frac{w_{1,t}+T-\Lambda_t^{SM} I_t^{SM}}{1+\psi^{type}}$ and $\ell_t^* = \frac{\psi^{type}\left(w_{1,t}+T-\Lambda_t^{SM} I_t^{SM}\right)}{(1+\psi^{type})(w_{1,t}+T)}$. Then, the optimal choice of the final child skill, θ_{J+1}, maximizes $u(c^*, \ell^*) + bV(\theta_{J+1})$ and thus satisfies:

$$\frac{(1+\psi)\Lambda_t^{SM}}{\left(w_{1,t}+T-\Lambda_t^{SM} I_t^{SM}\right)} \frac{\partial I_t^{SM}}{\partial \theta_{J+1}} = b\frac{\partial V(\theta_{J+1})}{\partial \theta_{J+1}}$$

The left-hand-side of this equation displays the full "marginal cost" of child skill production in terms of utility, which balances the marginal benefit on the right-hand-side. Let MC^{SM} denote this marginal cost of single mothers in period t, and consider its elasticity with respect to the price of child care:

$$\rho_t^{SM} \equiv \frac{\partial MC_t^{SM}}{\partial p_t}\frac{p_t}{MC_t^{SM}} = \epsilon_t^{SM}\left(\frac{w_{1,t}+T}{w_{1,t}+T-\Lambda_t^{SM} I_t(\theta_{J+1})}\right) \tag{13}$$

where $\epsilon_{SM} = \frac{\partial \Lambda_t^{SM}}{\partial p_t}\frac{p_t}{\Lambda_t^{SM}}$ is the elasticity of the price of investment with respect to the price of child care. Given (5), this elasticity is:

$$\epsilon_t^{SM} = \frac{(1-\alpha_1)^{\eta_1}}{(1-\alpha_1)^{\eta_1} + \alpha_1^{\eta_1}\left[\left(\frac{w_{1,t}}{p_t}\right)\left(\frac{\phi_n}{\phi_1}\right)\right]^{1-\eta_1}} \tag{14}$$

Proceeding analogously for married couples, given (6) these objects are:

$$\rho_t^{MC} = \epsilon_t^{MC}\left(\frac{w_{2,t}+w_{3,t}+T}{w_{2,t}+w_{3,t}+T-\Lambda_t I_t^{MC}(\theta_{J+1})}\right) \tag{15}$$

$$\epsilon_t^{MC} = \frac{(1-\alpha_2-\alpha_3)^{\eta_2}}{(1-\alpha_2-\alpha_3)^{\eta_2} + \alpha_2^{\eta_2}\left[\left(\frac{w_{2,t}}{p_t}\right)\left(\frac{\phi_n}{\phi_2}\right)\right]^{1-\eta_2} + \alpha_3^{\eta_2}\left[\left(\frac{w_{3,t}}{p_t}\right)\left(\frac{\phi_n}{\phi_3}\right)\right]^{1-\eta_2}} \tag{16}$$

The higher the elasticity of investment price with respect to the price of child care, the more responsive the marginal cost will be to the child care subsidy, and the larger the expected increase in child skill, ceteris paribus. This elasticity is increasing in the share on child care time, and decreasing in the elasticity of substitution across inputs. Family income plays an additional role, however, as represented by the term multiplying the price elasticity to yield the investment elasticity. This income-composition term is increasing in the fraction of income spent on investment in children's skill.

4.2 Empirical Distributions of ρ_t^{SM} and ρ_t^{MC}

Table 5 reports the empirical distribution of the elasticities in (13) and (15), computed using the estimated parameters and ECLS-B data. Potential income is not measured (I only observe realized income), and so in order to calculate investment elasticities I use the estimated fixed effects at the parent level, which are necessarily not very precise.[23] Nevertheless, the table does yield two qualitative points. First, the mean and median of the elasticity of investment's price with respect to the price of child care are higher for single mothers than for couples, as anticipated from the point estimates for each family structure. Second, the fraction of income spent on investment in children's skill has a very similar mean and median for the two family structures. It is the empirical differences between ϵ_t^{SM} and ϵ_t^{MC} that drive the differences between the implied elasticity of the marginal cost of investment to the price of child care. Specifically, in the ECLS-B single mothers have on average a much larger marginal cost elasticity than couples do: it is almost three times higher for the former group.

Table 5: Empirical Elasticities: ρ_t^{type} and its Components

		mean	p50	sd
Single Mothers:	ϵ_t^{SM}	0.87	0.89	0.09
	$\frac{income}{\text{income - inv. expense}}$	1.21	1.16	0.29
	ρ_t^{SM}	1.06	1.02	0.31
	Observations	200		
	Families	50		
Couples:	ϵ_t^{MC}	0.29	0.27	0.12
	$\frac{income}{\text{income - inv. expense}}$	1.29	1.25	0.20
	ρ_t^{MC}	0.36	0.34	0.16
	Observations	900		
	Families	300		

Estimation Sample, Waves 1-3, Unweighted. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Sample size rounded to nearest 50 per NCES requirements.

[23] Each family fixed effect is identified based on three observations from the first three waves of the ECLS-B. Single mothers in Table 5 are the same group as the sample. Couples in Table 5 are those families for whom both the father and mother are in the estimation sample. This is an additional restriction relative to the investment technology estimation. It is necessary to impose this restriction because the statistics reported here vary at the family level, not at the level of the parent.

In the following section, I develop a general equilibrium framework which I use for policy analysis. When I evaluate the model's fit using untargeted moments, I also compute an associated set of model statistics with their counterparts in Table 5. This comparison illustrates that the model population's sensitivity to the child care subsidy is in accordance with the actual population's sensitivity, as measured in the ECLS-B.

5 Investment in Child Skill in General Equilibrium

In this section I build a general equilibrium framework to perform counterfactual policy analysis. In this expanded framework, the parenting problem for couples and single mothers during early childhood remains the same. The life cycle of the parents, however, now includes periods after children leave home. In addition, this framework endogenizes the family formation decision via marriage market. This necessitates specifying the problem of the single father to make his outside option to marriage concrete. To model the family formation decision, a small adjustment of notation is needed: the productivity of parents is now indexed by their gender, and refers to productivity in both the labor market and as a skill investment input. Accordingly, the symbol θ (with appropriate subscripts) is used to refer to this general productivity of the parent.

5.1 The Expanded Model

There are four sets of agents in the economy: consumers, a representative firm, the government, and a non-parental child care provider.

Given prices for labor and capital, the firm chooses labor and capital inputs to maximize profits. This firm produces with a constant returns to scale technology and takes prices as given. The government chooses labor income taxes to finance lump-sum transfers and non-parental child care subsidies. A child support system exists, enforced by the government, where single fathers contribute a lump-sum amount that is re-distributed lump-sum and equally to all single mothers. Finally, the non-parental child care sector supplies child care at the amount demanded in equilibrium. Consistent with the identifying assumptions in the estimation, I assume that the productivity of child care is proportional to the average mother's labor productivity for one-parent families and two-parent families, respectively. To reflect that one- and two-parent families face similar relative prices, while single mothers also exhibit lower hourly wages, I allow the price of child care to differ by family structure.

The Life Cycle of Consumers Figure 1 illustrates the life cycle of the consumer. Each individual lives for $J+L$ periods. During childhood, which lasts for the first J periods of life, an individual makes no decisions: she is a passive recipient of consumption and investment chosen by her family. Upon independence, at the beginning of age $J+1$, the individual leaves with the level of skill she has accumulated by then to start the remaining L periods of her life as an independent decision-making adult. As an adult, an individual operates as part of a family: families are either married/cohabiting couples (MC), single mothers (SM), and single fathers (SF). These families are formed in a marriage market that occurs at the start of period $J+1$. The first J periods of adulthood are spent either actively parenting children (if a single mother or a married couple) or making child support payments (if a single father). From periods $2J+1$ to $J+L$, the problem of the consumer is a standard lifecycle problem. As noted earlier, there are no wealth bequests—families make transfers to their children by increasing their skill.

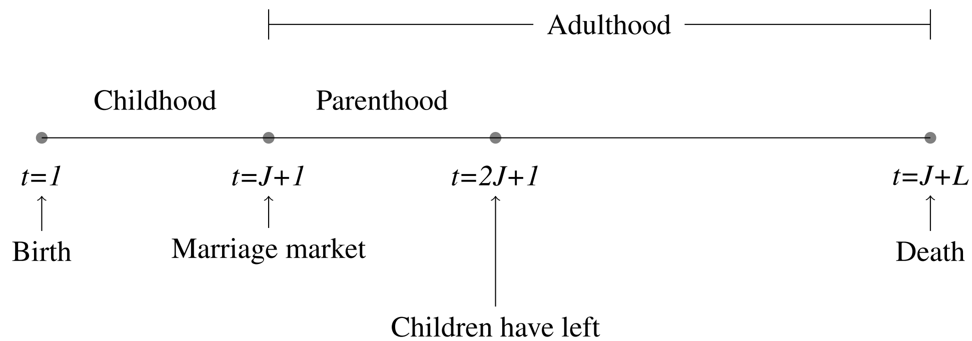


Figure 1: Life Cycle of the Consumer

Family Problems The family structures differ in three ways. First, they differ in their efficiencies of consumption: for single mothers and couples, these are denoted $\left\{\Phi_t^{SM}\right\}_{t=1}^{L}$ and $\left\{\Phi_t^{MC}\right\}_{t=1}^{L}$, which represent consumption equivalence (CE) scales that vary over the lifecycle as children leave the household. The CE scales of the single father are always equal to 1. Second, the different types of families are allowed to have different marginal utilities of leisure (which will be reflected in the parameters of the period utility functions $\left(u^{SF}, u^{SM}, u^{MC}\right)$. Third, families that raise children—single mothers and married/cohabiting couples—use a skill accumulation technology specific to their family structure to invest in their child. This is consistent with the framework used in Section 2, but with a shift of notation for the parenting productivities and hourly wage.[24]

The solution to a family's life-cycle problem is a set of choices and lifetime utilities, for every potential couple $\{\theta_m, \theta_f\}$ and each possible draw of the initial child skill θ_1. Here, θ_f and θ_m denote the female and male parent's productivities, respectively. The lifetime utility for a single mother of type θ_f and child of type θ_1 is $V^{SM}\left(\theta_1, \theta_f\right)$, the lifetime utility for a single father of type θ_m with partner θ_f and child θ_1 is $V^{SF}\left(\theta_1, \theta_f, \theta_m\right)$, and the lifetime utility of a couple with types θ_f and θ_m, whose child has type θ_1, is $V^{MC}\left(\theta_1, \theta_f, \theta_m\right)$.

The Marriage Decision Families are formed at the beginning of adulthood, when everyone participates in a marriage market that occurs instantaneously at start of period $J+L$.[25] On the marriage market, a potential match is drawn randomly from the skill distribution of the other gender in the same generation. Once assigned a potential spouse, and knowing that parenthood is certain in the environment, the agent compares the expected present discounted value of parenting alone or in a couple. A marriage is formed

[24]In particular, here the productivity of individuals as parents and in the labor market is assumed to be the same, and there is a common return per efficiency unit across individuals.

[25]Appendix I contains transition matrices across waves in the ECLS-B for the mother's marital status. In the data, it is extremely persistent.

if both the husband and wife accept the match (the two individuals remain single otherwise). In particular, a female young adult with skill θ_f and potential spouse θ_m compares the expected value of being a single mother, $\mathbb{E}_{\theta_1}\left[V^{SM}\left(\theta_1,\theta_f\right)\right]$ and the expected value of being a married mother, $\mathbb{E}_{\theta_1}\left[V^{MC}\left(\theta_1,\theta_f,\theta_m\right)\right]$. A male young adult with skill θ_m and potential spouse θ_f compares the expected value of being a single father, $\mathbb{E}_{\theta_1}\left[V^{SF}\left(\theta_1,\theta_f,\theta_m\right)\right]$ and the expected value of being a married father, $\mathbb{E}_{\theta_1}\left[V^{MC}\left(\theta_1,\theta_f,\theta_m\right)\right]$.[26]

After the marriage market, single mothers and couples draw the initial skill of their two children, which is the same for both children (single mothers and couples each raise both a daughter and a son). Whether parenting alone or in a couple, the lifetime utility of any individual contains a term that incorporates rational expectations about the lifetime utility of one's child at the level of skill they begin adulthood with.[27] In the altruism term, the expected lifetime utility at adulthood, conditional on the level of skill, is taken over the potential spouses one might meet as an adult (using the distribution of skill in the economy, which is endogenous) and also over the distribution of initial skill one's child may be born with (which is exogenous).

The decision rule that maps from the type of the spouse to "no" or "yes" marriage market decision is represented by $D_g\left(\theta_m,\theta_f\right)\in\{0,1\}$. It takes as given the skill θ_m or θ_f and gender g of the decision maker, which can be either male, m, or female ,f, and solves:

$$D_f\left(\theta_m,\theta_f\right) = \underset{\delta\in\{0,1\}}{\arg\max}\left[\delta\int_{\theta_1}V^{MC}\left(\theta_1,\theta_f,\theta_m\right)\pi\left(\theta_1\right)d\theta_1+(1-\delta)\int_{\theta_1}V^{SF}\left(\theta_1,\theta_f,\theta_m\right)\pi\left(\theta_1\right)d\theta_1\right] \quad (17)$$

$$D_m\left(\theta_m,\theta_f\right) = \underset{\delta\in\{0,1\}}{\arg\max}\left[\delta\int_{\theta_1}V^{MC}\left(\theta_1,\theta_f,\theta_m\right)\pi\left(\theta_1\right)d\theta_1+(1-\delta)\int_{\theta_1}V^{SM}\left(\theta_1,\theta_f\right)\pi\left(\theta_1\right)d\theta_1\right] \quad (18)$$

where the distribution of initial child skill θ_1 has a probability density function given by $\pi\left(\theta_1\right)$. The optimal marriage decision takes the form of a threshold strategy in the space of the potential spouse's skill. The value functions that young adults compare when deciding whether to marry depend on the return to skill in terms of lifetime utility, which is constructed next.

Expected Lifetime Utility The expected lifetime utility of the family's two children (one boy and one girl) enters into the parent problem. It is:

$$V^{child}\left(\theta\right) = \sum_{g\in\{m,f\}}\frac{1}{2}\tilde{V}_g\left(\theta\right) \quad (19)$$

[26]The outside option to parenting in a couple differs by gender in the model because, empirically, the vast majority of single parents who are raising young children in their home are women. For a discussion of what the ECLS-B offers in terms of discipline on contributions of parental time from single fathers, see Appendix G.

[27]This is what makes parents altruistic. An alternative way of motivating intergenerational transfers is through paternalistic preferences, or "warm glow" returns (Andreoni (1990)). The advantage of an altruism framework is that the returns to investment can respond endogenously to policy, because parents fully incorporate the economic returns to their investment in terms of their child's lifetime utility, and their behavior changes accordingly. The main benefit of a paternalistic specification is its tractability and flexibility in matching parenting behaviors. Some models include both altruism and a paternalistic preference for, say, college attainment which is distinct from its monetary returns. For an application of paternalistic preferences to intergenerational transfers of wealth, see De Nardi (2004).

This function sums the expected lifetime utility given skill θ across potential spouses, potential children, and the gender of the child. That is, each family internalizes the average return to skill in terms of lifetime utility across men and women in equilibrium when making investment decisions. For each gender the expected return to skill in terms of lifetime utility is:

$$\tilde{V}_f(\theta) = \int_{\theta_m} \left[\mathbb{I}_d \int_{\theta_1} V^{MC}(\theta_1, \theta, \theta_m)\, \pi(\theta_1)\, d\theta_1 + (1 - \mathbb{I}_d) \int_{\theta_1} V^{SM}(\theta_1, \theta)\, \pi(\theta_1)\, d\theta_1 \right] \mu(\theta_m)\, d\theta_m \quad (20)$$

$$\tilde{V}_m(\theta) = \int_{\theta_f} \left[\mathbb{I}_d \int_{\theta_1} V^{MC}(\theta_1, \theta_f, \theta)\, \pi(\theta_1)\, d\theta_1 + (1 - \mathbb{I}_d) \int_{\theta_1} V^{SF}(\theta_1, \theta_f, \theta)\, \pi(\theta_1)\, d\theta_1 \right] \mu(\theta_f)\, d\theta_f \quad (21)$$

where $\mathbb{I}_d \equiv D_f(\theta_m, \theta_f) \times D_m(\theta_m, \theta_f)$ indicates a mutual acceptance of the match. In addition, $\mu(\theta)$ is the endogenous distribution over adult skill θ, which is the same for each gender because parents are constrained to not targeting investments by gender.

Once families are formed, the family solves a sequential life-cycle problem. Just as in problems (1) and (2), the family has J periods to invest in their child. The child is born with some initial skill θ_1. At each age during adulthood, t, the parent affects the child's stock of skill in that period, θ_t, by their choice of investment, I_t^{type}, where $type \in \{SM, MC\}$ for single mothers and married couples, respectively. Note that age t of adulthood is age $t + J$ of the parents' life, so that in total one lives for $J + L$ periods. The way investment interacts with the child's skill in each period is defined by the skill accumulation function $f\left(\theta_t, I_t^{type}\right)$. In turn, the way investment in each period is generated depends on the skill investment technology, which is indexed by family structure. Investment is generated by time contributed from the parent(s) (q_t^m and q_t^f) and from time purchased on the market in the form of non-parental child care n_t.[28] In all the family problems, w is the return per unit of skill on the labor market, r is the interest rate earned from the stock of savings, τ_y is the labor income tax, τ_n is the subsidies to child care, T are lump-sum transfers, and T_{cs} are child support payments.

Single Mothers The problem of a single mother, articulated in (1) but with the aforementioned updated notation, can now be stated as:

$$\begin{aligned}
V^{SM}(\theta_1, \theta_f) &= \max_{\left\{c_t, \ell_t, a_{t+1}, n_t, q_t^f\right\}_{t=1}^{L}} \left[\sum_{t=1}^{L} \beta^{t-1} u^{SM}\left(\frac{c_t}{\phi_t^{SM}}, \ell_t \right) \right] + \beta^{J-1} b V^{child}(\theta_{J+1}) \quad (22)\\
&s.t.\\
c_t + a_{t+1} + (1-\tau_n)\, p_t^{SM} n_t &\leq w\theta_f \left(1 - \ell_t - q_t^f\right) + (1+r)\, a_t + T + \mathbb{I}_{t \leq J} T_{cs}\\
\theta_{t+1} &= f\left(\theta_t, I_t^{SM}\right)\\
I_t^{SM} &= I_t^{SM}\left(\phi_n^{SM} n_t, \theta_f q_t^f\right)
\end{aligned}$$

where, as before, $a_1, a_{L+1} = 0, n_t, \ell_t, q_{1,t} \geq 0$, and the time constraints are $\ell_t + q_{1,t} \leq 1$ and $n_t + q_{1,t} \leq J_{ec}$. The mother has one unit of time per period, and the child has J_{ec} units, which can be less than 1 to reflect

[28] Recall that the superscripts m and f now refer to the male and female parent.

that early chidhood is only a fraction of the time the child spends at home with his or her family. Here, b is the altruism parameter, $w\theta_f$ denotes the wage of the parent in period t, and p_t^{SM} denotes the price of non-parental child care in period t. As before, parental and non-parental child care time investments in child skill affect skill in the next period according to the skill production function $f\left(\theta_t, I_t^{SM}\right)$ and the investment function $I_t^{SM}\left(\phi_n n_t, \theta_f q_t^f\right)$. Note that the investment function I_t^{SM} is indexed to the family structure, while the production function f is the same for both types of families. The final child skill θ_{J+1} enters the objective function of the mother through an altruism term $bV^{child}\left(\theta_{J+1}\right)$, which weights the expected lifetime utility of the child $V^{child}\left(\theta_{J+1}\right)$ with the altruism coefficient b.

Married or Cohabiting Couples Similarly, in the expanded framework's notation the problem of couples who are married or cohabiting is stated as:

$$
\begin{aligned}
V^{MC}\left(\theta_1, \theta_f, \theta_m\right) &= \max_{\left\{c_t, \ell_t, a_{t+1}, n_t, q_t^m, q_t^f\right\}_{t=1}^{L}} \left[\sum_{t=1}^{L} \beta^{t-1} u^{MC}\left(\frac{c_t}{\phi_t^{MC}}, \ell_t\right)\right] + \beta^{J-1} bV^{child}\left(\theta_{J+1}\right) \quad (23) \\
s.t. & \\
c_t + a_{t+1} + \left(1-\tau_n\right) p_t^{MC} n_t &\leq w\theta_f\left(1-\ell_t-q_t^f\right) + w\theta_m\left(1-\ell_t-q_t^m\right) + (1+r) a_t + T \\
\theta_{t+1} &= f\left(\theta_t, I_t^{MC}\right) \\
I_t^{MC} &= I_t^{MC}\left(\phi_n^{MC} n_t, \theta_f q_t^f, \theta_m q_t^m\right)
\end{aligned}
$$

where $a_1, a_{L+1} = 0$, $n_t, \ell_t, q_{2,t}, q_{3,t} \geq 0$, and the time constraints are $\ell_t + q_{2,t} \leq 1$, $\ell_t + q_{3,t} \leq 1$, and $n_t + \max\left\{q_{2,t}, q_{3,t}\right\} \leq J_{ec}$. Parents are allowed to invest in the child at the same time (their quality time investments are non-rival). Leisure of spouses is assumed to be perfectly complementary, so that a single level is chosen for both members of a couple (as in Guvenen and Rendall (2015)).

Single Fathers A single father chooses consumption c_t, savings a_{t+1}, and leisure ℓ_t to solve the following problem.

$$
\begin{aligned}
V^{SF}\left(\theta_1, \theta_f, \theta_m\right) &= \max_{\left\{c_t, \ell_t, a_{t+1}\right\}_{t=1}^{L}} \left[\sum_{t=1}^{L} \beta^{t-1} u^{SF}\left(c_t, \ell_t\right)\right] + \beta^{J-1} bV^{child}\left(\mathbb{E}\left(\theta_{J+1} | \theta_1, \theta_f\right)\right) \quad (24) \\
c_t + a_{t+1} + \mathbb{I}_{t \leq J} T_{cs} &\leq w\theta_m\left(1-\ell_t\right) + (1+r) a_t + T
\end{aligned}
$$

where $\ell \in [0,1]$, and $a_1, a_{L+1} = 0$. Here $w\theta_m$ is wage of the single father with skill θ_m. Single fathers take their child's outcome θ_{J+1} as given conditional on the female parent's type, θ_f, as well as the child's initial skill θ_1. A single father cannot use his own time to invest in his children, but he is required to make a lump-sum payment to the child's mother, T_{cs}. The expected outcome of a single father's children affects his outcome directly through the altruism term, $bV^{child}\left(\mathbb{E}\left(\theta_{J+1} | \theta_1, \theta_f\right)\right)$.

Government The government collects revenue from labor income taxes τ_y to finance lump-sum transfers T and non-parental child care subsidies τ_n. The variable H_t is the aggregate supply of labor efficiency units at each age during adulthood t, N_t^{type} is the aggregate demand for non-parental child care from parents in family structure $type \in \{SM, MC\}$.

$$\tau_y w \sum_{t=1}^{L} H_t \quad = \quad T + \tau_n \sum_{type} p_n^{type} \sum_{t=1}^{J} N_t^{type} \tag{25}$$

Representative Firm The firm chooses capital K_F and labor inputs H_F to maximize profits, taking prices r and w as given. The parameter δ_F is the depreciation rate of capital.

$$\max_{K_F, H_F} \quad \left\{ K_F^{\alpha_F} H_F^{1-\alpha_F} - wH_F - (r + \delta_F) K_F \right\} \tag{26}$$

Non-parental Care Sector The non-parental child care sector provides N units of non-parental child care at price p_n^{type}. The price of non-parental child care is set as a constant fraction κ^{type} of the average return to labor supply for female parents in family structure $type \in \{SM, MC\}$.

$$p_n^{type} \quad = \quad \kappa^{type} (1 - \tau_y) w \int_{\theta_f} \theta_f \mathbb{I}_{type} \mu(\theta_f) d\theta_f \tag{27}$$

This allows the price of non-parental child care to adjust with the average level of skill in the economy, but without specifying a production function for non-parental child care.[29]

Equilibrium Given a government policy τ_n, transfers T, and child support T_{cs}, a stationary equilibrium is defined as factor prices and a labor income tax, individual marriage decisions, family decisions and lifetime utilities, expected lifetime utilities at a given level of adult skill, and single father expectations about single mother decisions such that markets clear, the government balances its budget constraint, consumers and firms optimize and expectations are rational.

5.2 Parameterization

In order to fully parameterize the general equilibrium model, I assume functional forms for utility, the initial distribution of child skill and its correlation with parental attributes, and $f\left(\theta_t, I_t^{type}\right)$, the function that governs how investment and skill today combine to produce tomorrow's skill. I assign parameters to these functions via calibration and estimation, respectively.

[29]Because parents cannot target investment to the child based on their gender, the skill distribution is the same for both genders. That means that, within the model, using the skill distribution of fathers to pin down the price of child care would be equivalent.

Period Utility Functions Utility functions are defined separately for one- and two-parent families:

$$\begin{aligned} u^{SM}(c,\ell) &= \log(c) + \psi_s \log(\ell) \\ u^{SF}(c,\ell) &= \log(c) + \psi_s \log(\ell) \\ u^{MC}(c,\ell) &= \log(c) + \psi_{mc} \log(\ell) \end{aligned}$$

Distribution of Initial Child Skill I assume that initial child skill is drawn independently and identically from $\pi(\theta_1)$, which I set as a uniform distribution. The empirical motivation for the i.i.d. assumption is based on the correlation between the cognitive score at 9 months of age and after-tax family income in the ECLS-B, which is 0.01 with a p-value of 0.12. This correlation is not statistically different from 0.

Skill Production Function The child's current stock of skill, θ_t, and chosen level of investment I_t^{type} combine according to the skill production function $f\left(\theta_t, I_t^{type}\right)$ to produce the stock of skill in the next period. For both family structures, I assume the same CES functional form:

$$\theta_{t+1} = f\left(\theta_t, I_t^{type}\right) = \left[\upsilon\left(\lambda_{type} I_t^{type}\right)^{\frac{\chi-1}{\chi}} + (1-\upsilon)(\theta_t)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}$$

In a one-period parenting problem parameter, the elasticity of substitution parameter χ can be interpreted as governing how easily investment can compensate for initial skill. If this elasticity is low, it takes more investment to change initial skill than if the elasticity is high. In a multi-period setting where $J > 1$, the elasticity of substitution χ also governs how easily investment can be shifted across periods of childhood in response to changes in its price across periods. If χ is low, the technology does not allow investment to be easily reallocated across periods in response to a change in the price of investment in one period. In a one-period parenting problem, the share parameter υ can be interpreted as determining the importance of investment in determining the final skill of the child. If υ is low, it takes more investment to affect the stock of skill, and vice versa. In a model where $J > 1$, the share parameter υ also determines how investment with a constant price across consecutive periods would be allocated. If υ is low, then investment in each period of childhood will have to be more uniform, whereas if υ is high then investment could be more lumpy. Finally, λ_{type} is a scaling parameter that accounts for units of measurement being different across investment and the stock of skill (this parameter is allowed to differ across family structures).

Estimating χ and υ To estimate the parameters of the production function $f\left(\theta_t, I_t^{type}\right)$, I rely on the tangency conditions from inter-temporal cost minimization. First, note that the child's final skill depends on the initial condition and the sequence of investments chosen by the family. The implied function governing the mapping from initial to final skill can be constructed recursively as:

$$\overline{\theta}_{J+1}\left(\theta_1, \left\{I_t^{type}\right\}_{t=1}^{J}\right) = \left[\sum_{t=1}^{J} \upsilon(1-\upsilon)^{J-t}\left(\lambda I_t^{type}\right)^{\frac{\chi-1}{\chi}} + (1-\upsilon)^J(\theta_1)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}$$

Next, by the (conditional) cost minimization implicit in the optimization problems (22) and (23), one can derive an inter-temporal Euler equation which balances the relative price of investment in consecutive periods with the relative price of moving value across those periods. Specifically:

$$\frac{1}{1+r_{t+1}}\frac{\Lambda_t}{\Lambda_{t+1}} = \frac{\frac{\partial\bar{\theta}_{J+1}\left(\theta_1,\left\{I_t^{type}\right\}_{t=1}^J\right)}{\partial I_t}}{\frac{\partial\bar{\theta}_{J+1}\left(\theta_1,\left\{I_t^{type}\right\}_{t=1}^J\right)}{\partial I_{t+1}^{type}}} \tag{28}$$

Substituing the partial derivatives of $\overline{\theta}_{J+1}$ into (28) and rearranging yields the following estimation equation:

$$\ln\left(\frac{I_{i,t+1}^{type}}{I_{i,t}^{type}}\right) = -\chi\ln\left(\frac{\Lambda_{i,t+1}^{type}}{\Lambda_{i,t}^{type}\left(1+r_{t+1}\right)}\right)+\chi\ln\left(1-\upsilon\right) \tag{29}$$

where the parameters of the unit cost functions $\Lambda_{i,t}^{type}$ were estimated earlier in the paper. This is the structural portion of a linear regression of the form $y_{i,t} = \gamma_0 + \gamma_1 x_{i,t} + e_{i,t}$, which I estimate by OLS. The last two skill production technology parameters can then be retrieved as $\chi = -\gamma_1$ and $\upsilon = 1 - \exp\left(-\frac{\gamma_0}{\gamma_1}\right)$. The results of this estimation are reported in Table 6, yielding a point estimate for χ of 0.23 and a point estimate for υ of 0.27.

Table 6: Model Regression Coefficients

	Estimated Value
γ_1	-0.227*
	(0.0956)
γ_0	0.0723*
	(0.0336)
R^2	0.0189
Obs.	700
SM families	50
MC families	300

Standard errors in parentheses. * p<0.05, ** p<0.01, *** p<0.001. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Sample size rounded to nearest 50 per NCES requirements.

Remaining Parameters The remaining parameters of this model are divided into three groups and presented in the panels of Table 7: fixed parameters in Panel A, estimated parameters in Panel B, and calibrated parameters in Panel C.

Beginning with Panel A, there are 3 periods of adulthood and 1 period of childhood. The length of a lifetime and of each phase are proportional to 20 years of childhood and 60 years of adulthood (death at age 80).

Early childhood therefore involves a time endowment of 0.25 units for the child; the remaining time during that period is not available to affect the child's skill, but the child still lives at home and affects the CE scales of his or her family. This reflects 5 years of early childhood for 20 years living at home during the period of childhood. The discount (patience) factor is set to a yearly value of 0.96 to match the presumed risk-free interest rate. The share on capital,α_F, is set to 0, thereby preserving the assumption of constant returns to scale in production. Because of the long length of a period in this model, effectively this shuts off long-term borrowing for parents. The depreciation rate of capital, δ_F is set to 0. Finally, the CE scales are taken from the 1994 scales from the Organisation for Economic Co-operation and Development (OECD). These scales assign a value of 1 for the first adult, and 0.5 for the subsequent adults; for each dependent the weight is 0.3. They adjust money spent on consumption into units of consumption for each member of the household. Once children leave the family, the CE scale for single mothers goes back to 1, and the scale for couples falls to 1.5.

Table 7: Fixed, Estimated, and Calibrated Parameter Values

	Symbol	Name	Source	Value
A. Fixed:	β	Patience	Assumption	0.96^{20}
	L	Periods of adult life	3 20-year periods	3
	J	Periods of childhood	1 20-year period	1
	J_{ec}	Time endowment of child	$\frac{\text{5 year early childhood}}{\text{20 year period}}$	0.25
	$\{\alpha_F, \delta_F\}$	Production technology	Assumption	$\{0, 0\}$
	$\{\phi_1^{SM}\}$	CE Scales: 1 ad., 2 ch.	OECD	$\{1.6\}$
	$\{\phi_1^{MC}, \phi_{2,3}^{MC}\}$	CE Scales: 2 ad., 2 ch.	OECD	$\{2.1, 1.5\}$
B. Estimated:	T	Transfers	NIPA	8% of output
	T_{cs}	Child Support	Census, NIPA	45 % of T
	κ^{type}	p_n^{type} coefficient	ECLS-B	35% $\overline{w}_{mother}^{SM}$, 39% $\overline{w}_{mother}^{MC}$
C. Calibrated:	b	Altruism coefficient		$0.63 \times \beta$
	ψ_s, ψ_{mc}	MU_ℓ singles and couples		$2.55, 1.9$
	$\lambda_{SM}, \lambda_{MC}$	Prod. of Inv. (SM,MC)		$13, 25$

Moving to Panel B, the level of lump-sum transfers T, the level of child support payments T_{cs}, and the price of p_n are set to 8% of output, 45% of the average per-family transfer, and 35% (single mothers) or 39% (married/cohabiting mothers) of the average mother's wage, respectively. The first empirical target is from the ratio of government transfers to persons for federal benefits from social insurance funds, Supplemental Nutrition Assistance Program (SNAP), supplemental security income, refundable tax credits, and other (which includes payments to nonprofit instutitions and student loans, among other categories) to GDP from the National Income and Production Accounts (NIPA) tabulations. The second is the ratio of average child support payments owed per month per capita to average monthly government transfers per family. The third is the average ratio of hourly price of non-parental child care to hourly wages of mothers in the ECLS-B. See the appendix for further details on the targets for these parameters.

Panel C presents the values for internally calibrated parameters —the coefficients $b, \psi_s, \psi_{mc}, \lambda_{SM}$ and λ_{MC} are chosen to bring the model moments in the baseline as close as possible to the moments in Table 8. The coefficient b controls the degree of altruism; ψ_S, ψ_{MC} are the marginal utilities of leisure for singles and married couples, respectively. The parameters λ_{SM} and λ_{MC} are shifters in the skill technology that scales up investment into efficiency units in the production of skill. The moments these parameters were chosen to match, and the fit of the calibration, are reported in Table 8.

Table 8: Calibration Targeted Moments

Moment	Source	Data	Model
$Corr$ (Skill $_{\text{child}}$, Inc. $_{\text{family}}$) age 5	ECLS-B	0.32	0.32
Ave. labor supply	CPS	0.31	0.31
Pct. families MC	ECLS-B	0.81	0.81
Parental time: MC ($\frac{Hours}{Week}$)	ECLS-B	(9.3,4.8)	(9.4 ,8.0) (MM,MF)
Parental time: SM ($\frac{Hours}{Week}$)	ECLS-B	2.5	4.7 SM

The moments in Table 8 are the correlation of child skill with family income, the average labor supply of parents with children under 5 who are between 15 and 55 years old, the percent of single mothers raising children under 5, and the average time invested by parents of each family structure type. The target moments from the ECLS-B are from the pooled estimation sample whose moments are given in Table 1, except for the marriage rate, which is from the unrestricted weighted sample described in 12 in the appendix. [30]

The moment most poorly matched in this calibration is the hours of parental time provided by single mothers and married/cohabiting fathers, which are too high. This is driven by the fact that λ_{SM} has to be adjusted downward relative to λ_{MC} to match the marriage rate in equilibrium.

Untargeted Moments Table 9 compares the equilibrium moments of this model with untargeted moments in the data. These moments fall into two broad categories: family formation and inputs, presented in Table 9a, and investment elasticities, presented in Table 9b.

Table 9a shows that the random search marriage market captures a quantitatively reasonable degree of assortative matching among spouses: the correlation of wages within a couple is slightly lower than the data at 0.22 versus 0.27. The lower equilibrium value could be due to unmodelled types which couples sort by

[30] Note that the correlation of child skill and family income grows to 0.32 by the time the child is 5 years old; by contrast, when children are 9 months old, the measures of skill available in the data are uncorrelated with family income, although they do have predictive power for later child test scores at age 4. For regressions supporting these points, especially the assumption that initial child skill is independent of family attributes, see Appendix II.

in the data but not the model, such as college attainment, or to an unmodeled segmentation of the marriage market. The second moment shows that the correlation of time inputs in the model and the data are quite close, although again the model demonstrates a slightly lower value for the statistic at 0.35 instead of 0.39. Finally, the ratio of time input levels from mothers and child care is again slightly lower (0.33) than the data (0.36). Overall, the model is parsimonious but generates correlation in parental types and investment input choices which align with their empirical counterparts.

Table 9b reports the elasticities of the investment price and the marginal cost of skill. These statistics are important for the responsiveness of the model to a child care subsidy, and they are untargeted in the model calibration. Comparing the model with the data, the elasticities are quite close for single mothers but slightly higher for couples. This is because of the higher investment elasticity in the model versus the data for couples, which in turn is a result of stronger sorting by parent skill into marital status than is present empirically. In the next section, I examine the relationship between family income, family structure, and child outcomes more closely.

Table 9: Untargeted Moments

(a) Family Formation and Input Choices

Moment	Data	Model
Correlation of wages within a couple	0.27	0.22
Correlation of time inputs within a couple	0.39	0.35
Average $\frac{\text{mother time}}{\text{non-parental child care time}}$	0.36	0.33

(b) Elasticity of Marginal Cost of Investment ρ_t^{type} and its Components

	Moment (means)	Data	Model
Single Mothers:	ϵ_t^{SM}	0.87	0.86
	$\frac{\text{income}}{\text{income - inv. expense}}$	1.21	1.28
	ρ_t^{SM}	1.06	1.10
Couples:	ϵ_t^{MC}	0.29	0.39
	$\frac{\text{income}}{\text{income - inv. expense}}$	1.29	1.10
	ρ_t^{MC}	0.36	0.43

Note: Table 9a correlations are Pearson correlation coefficients, weighted with survey weights in the ECLS-B and using the analogous distribution in the model. Note: Data moments in Table 9b are from Table 5; model moments are computed from model output.

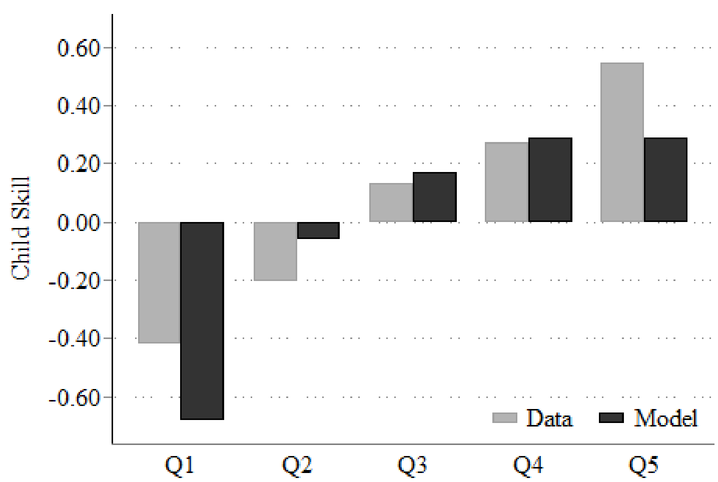
6 Policy Experiment: Universal Child Care Subsidy

Before proceeding to analyzing the effects of universal child care subsidies in general equilibrium, I review how family income, family structure, and child outcomes are related in the baseline equilibrium of the model.[31] In my analysis of the policy experiment, I return to these statistics and evaluate how the subsidy affects them.

6.1 Family Income, Family Structure, and Child Outcomes

In the model, initial child skill is independent of family attributes, but child outcomes are not. In the ECLS-B, when children are 9 months old, measures of skill and after-tax family income exhibit a correlation coefficient of 0.01 (with a p-value of 0.12). When they are 4 years old, however, this correlation jumps to 0.32 and is significant at the 0.01 level. The correlation of child skill with family income is a targeted moment in the calibration of the model, which generates an untargeted pattern of child skill outcomes being increasing by family income quintiles, as demonstrated in Figure 2. For each family income quintile, the figure reports the average child skill in units of standard deviations from the population mean, for both the ECLS-B and the model output. The two are a qualitative match, but quantitatively the model is somewhat more extreme than the data.

Figure 2: Child Skill Outcomes and Family Income: Data and Model

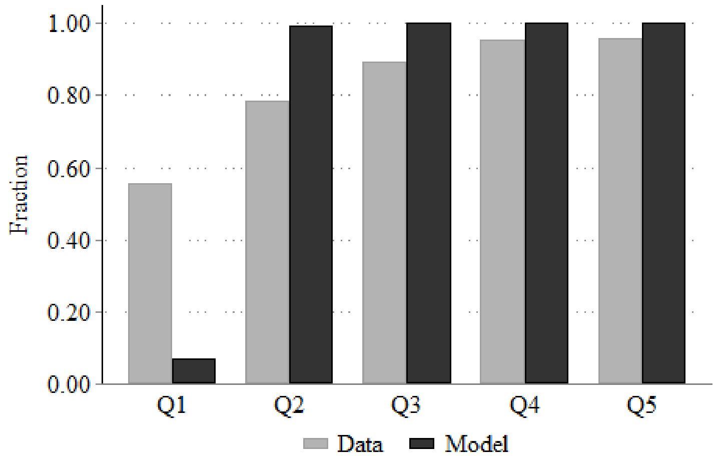


Data moment: average child skill at age 4, units of standard deviations from the population mean, using cross-sectional resident primary caregiver weights. Model moment: average child skill outcome within each family income quintile, units of standard deviations from the population mean. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

Family structure is correlated with income both in the model and in the data. In Figure 3, the marriage rate within each income quintile is shown to be strongly increasing in income in both the data and the model. However, the model exhibits income differences across family structures which are more extreme than what is present in the data.

[31] See Appendix I for a description of skill assessments for children in the ECLS-B. Measures of child skill, using age-appropriate assessments, were collected in each wave of the survey.

Figure 3: Marriage Rate by Family Income Quintile: Data and Model

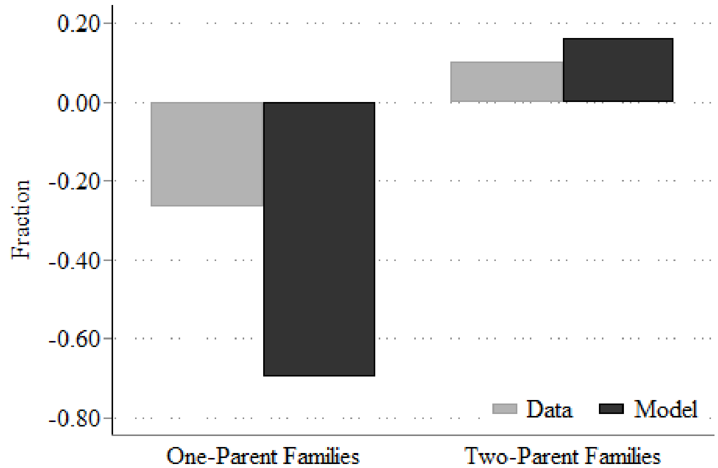


Data moment: fraction of families in each family income quintile who are married. Model moment: fraction of families in each family income quintile who are married or cohabiting, using cross-sectional primary caregiver weights. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

Finally, the relationship between child skill outcomes and family structure, on the other hand, is not targeted in the calibration and is an equilibrium result. Figure 4 presents the average child skill in equilibrium by family structure in the ECLS-B and the baseline equilibrium of the model. In the model, children of single mothers have an average skill outcome that is about 0.9 standard deviations lower than the average child outcome of couples. The data has the same pattern but quantitatively is more subdued: children of single mothers are on average 0.37 standard deviations lower than the average child outcome for couples. This is in keeping with the previous two comparisons made in this section.

There are several reasons why the model's relationship between family attributes and child outcomes would be more extreme than what is present in the data. For example, the concept of child outcome in the model is directly linked with adult outcomes. It is a more long-term and permanent concept than the child skill score recorded in the ECLS-B. A direct comparison, while informative, does not correct for the difference in interpretation of these statistics. In addition, the model exhibits a much stronger link between family income and family structure than what is present in the data. Since family formation is not driven by preference shocks in this model (which would provide some mixing) it is not surprising that the link would be strong along the only dimension by which people sort: parent skill, and via that, family income.

Figure 4: Child Skill Outcomes and Family Structure: Data and Model



Data moment: child skill at age 4, units of standard deviations from the population mean, using cross-sectional resident primary caregiver weights. Model moment: average child skill outcome within each family income quintile, units of standard deviations from the population mean. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

6.2 Universal Child Care Subsidies

With the parameterized model in hand, I vary the child care subsidy rate τ_n between 0 and 99 percent, solve for the stationary equilibrium, and record welfare, child skill outcomes, family income, and choices of child care and parental quality time. I also record labor income taxes that balance the government's budget constraint at every level of the child care subsidy. Because marriage is endogenous, the model keeps track of family decision and child outcome variables conditional on the marital status, so that I can construct the counterfactual outcome for parents and children if the couple had formed (or not) instead of what was optimally chosen in equilibrium.

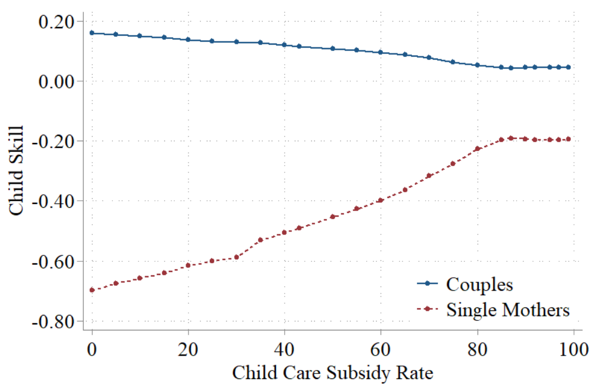
This model exhibits heterogeneity in the effects of child care subsidies across family income and family structures. The model also proposes a mechanism by which family income and family structure are correlated endogenously via the marriage market decision. Using the model to construct a counterfactual scenario where sorting into family structure is random, I decompose the difference in child outcomes across family structures into a component due to the family structure technology, and a portion due to the compositional differences across family types. As the gap narrows due to the child care subsidy, the role of technology in driving outcome differences is computed and compared over different levels of the subsidy.

6.2.1 Child Care Subsidies and Child Skill Accumulation in Equilibrium

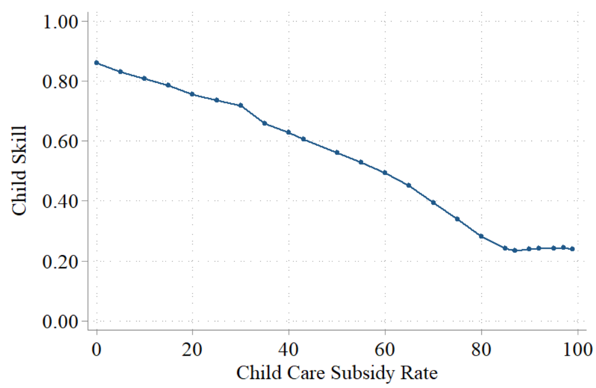
Child care subsidies have three effects in this environment. First, the subsidy narrows the gap in child outcomes between one- and two-parent families. This occurs because the subsidy leads to a larger increase in the outcomes of children from one-parent families relative to two-parent families, due to both the lower income of single mothers and the technology they use to invest in their children. Second, the subsidy does not strongly affect family formation decisions or the family structure composition of different income quintiles, and it leads to an increase in the labor income tax in order to fund it, which lowers the return to skill in the market and decreases the marginal benefit of skill. This force offsets the decrease in the marginal cost of skill due to the subsidy, and eventually dominates it. Thus, at a very large subsidy, average child outcomes are decreasing in the level of the subsidy. Third, welfare gains are maximized where gains in child skill reach their highest level.

Levels of child skill, in units of standard deviations from the population mean, are presented in Figure 5a. These gains are due to the subsidy lowering the cost of investment. As a result, children do better, and become more productive adults, both as workers and as parents. These effects are stronger for one-parent families, which is in line with the attributes of their investment technology as summarized by the elasticity of marginal cost with respect to the price of child care. It is also in line with the findings of Kottelenberg and Lehrer (2017) and Havnes and Mogstad (2014), who document higher gains for children of one-parent, poor households. Because of this difference in child skill gains, the gap in child outcomes across one- and two-parent families narrows as the subsidy increases (Figure 5b).

Figure 5: Heterogeneity in Effects of Child Care Subsidy Decreases the Gap in Child Outcomes



(a) Effect on Level of Child Skill



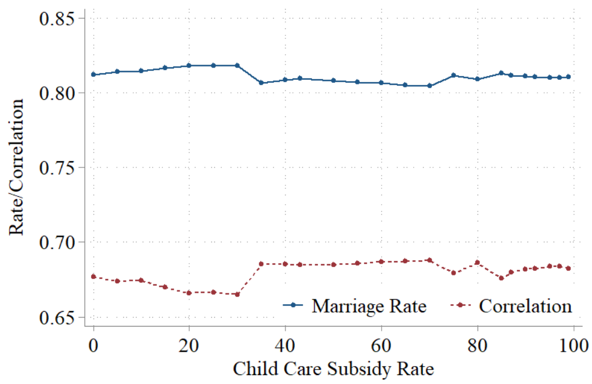
(b) Effect on Gap in Child Skill

Figure 6a presents the percent of families who are married/cohabiting as well as the correlation between marital status and family income quintile, at each level of the child care subsidy. Neither statistic is strongly affected by the subsidy. In this model, endogenous family formation will matter in determining the skill composition of single mothers versus couples. However, the marriage decision is not a margin of adjustment in the face of changes to the price of child care, and the distribution of family structures over income quintiles is correspondingly stable.

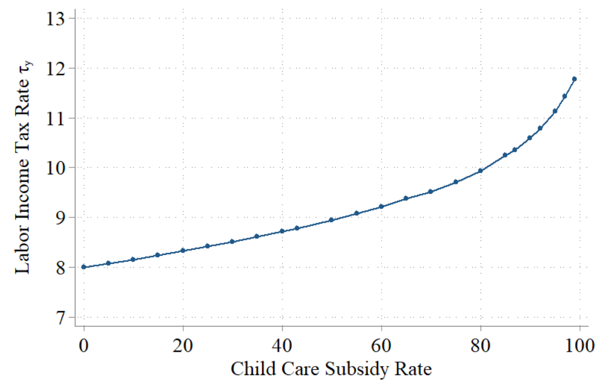
As the child care subsidy increases, labor income taxes also increase to fund it. This is shown in Figure 6b. Eventually, the resulting losses in the return to skill on the market lead aggregate welfare to start decreasing in the child care subsidy.

As the child care subsidy level increases, thereby decreasing the marginal cost of investment, the labor

Figure 6: Effect of a Child Care Subsidy on Family Formation and the Labor Income Tax



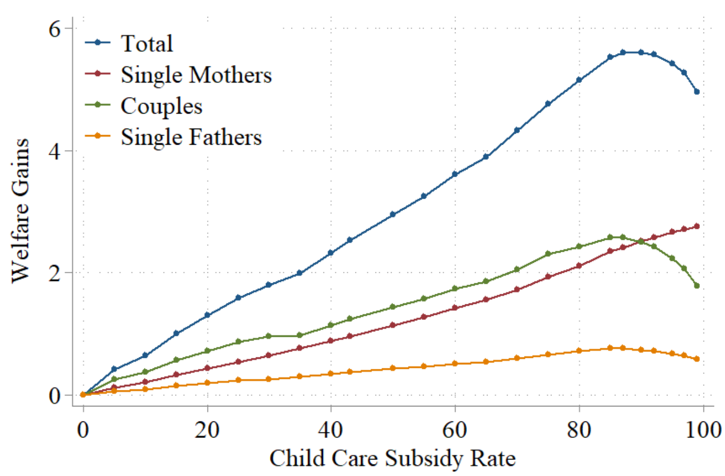
(a) Percent of Families Married/Cohabiting



(b) Labor Income Tax τ_y

income tax increases as well in order to fund the subsidy. This increase in the labor income tax decreases the return to skill, $(1 - \tau_y) w$, so that the marginal benefit of skill in terms of lifetime utility is also decreasing in the subsidy. It is possible for these changes to offset one another, and they are more likely to do so for couples and single fathers at lower levels of the subsidy. This is because the price of investment is less affected by the price of child care for couples than for single mothers, due to the lower share on child care in their investment technology. Single fathers benefit vicariously from increases in child outcomes, but do not have to contribute from their budget constraint to finance child care of parental time. Instead, their contributions are lump-sum in the form of child support. Figure 7 shows the total welfare gains. It also reports additive weighted contributions from couples, single mothers, and single fathers to the total. It is evident that the three family types in the economy experience different patterns of welfare gains due to the subsidy.

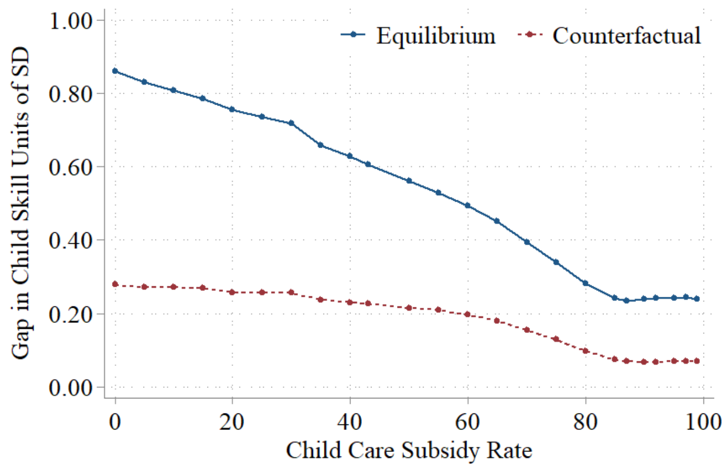
Figure 7: Effect of a Child Care Subsidy on Aggregate Welfare



6.2.2 Counterfactual Effects of a Child Care Subsidy: Randomized Family Type

Using the model output, I can construct counterfactual child outcomes in the case where family structure type is exogenously and randomly assigned, rather than being an endogenous choice. This eliminates sorting into family structure by parent skill. The result is that the gap in child skill narrows between single mothers and couples even without a subsidy, because the average income of single mothers increases relative to couples. Performing the same experiment in this environment with random marital status assignment, while holding fixed the total marriage rate at each level of the subsidy, I record the child skill outcomes across family structures. Figure 8 displays counterfactual and equilibrium gaps across family structures in child skill outcomes, over different levels of the subsidy.

Figure 8: Countefactual Effect of a Child Care Subsidy on Child Skill



In this model, single parenthood is usually the result of a high-skilled man meeting a low-skilled woman and rejecting the match. Women rarely reject men, because single motherhood turns out to be such a difficult state. Rejected matches, in equilibrium, would have counterfactually formed a couple with only slightly lower income than the average couple which does form. These counterfactual couples would have funded similar investment in children at the expense of lower consumption. On the other hand, single parent households that do not arise in equilibrium would have counterfactually resulted in highly skilled women parenting alone, with much higher income than the one-parent families which arise endogenously. These counterfactual single mothers would have invested more in their children, due to their higher income and parenting productivity.

Thus, randomizing the marriage market outcome does not affect the composition of child outcomes among couples, but it does affect the composition of child outcomes among single mothers. Accordingly, controlling for endogenous sorting into family structure reduces the gap in child outcomes across family structures because it raises the outcome of children raised by single mothers. In Figure 8, this is reflected in the lower gap at any level of the subsidy in the counterfactual economy.

Using the definition of the marginal cost elasticities presented in equations (13) and (15), in equilibrium the skill composition of single mothers works to reinforce the elasticity of their technology to the price of child care. Randomly assigning parents to family structures breaks the endogenous link between parent skill and single parenthood: the remaining gap is due to exogenous attributes of the two family structures, rather than endogenous composition differences.

This contribution from technology, calculated as the ratio of the countefactual gap and the equilibrium gap, has a value of about 33 percent at low levels of the subsidy. As the subsidy increases, this fraction at first also increases, because the counterfactual gap is less affected by the subsidy than the equilibrium (lower slope), and peaks at about 40 percent. Then, as the subsidy reaches higher levels, the counterfactual gap also begins to increase at a high rate, again lowering the the ratio, to reach a minimum of 28 percent. Accounting for the skill composition of single mothers in this way, it is evident that at any level of the subsidy exogenous attributes of family structure, such as the number of parents and the investment technology, play a large role in explaining the gap in child outcomes across family structures.

7 Conclusion

In this paper, I examine the role of family structure in determining the effects of a child care subsidy. In particular, I first specify two investment technologies and empirically test and reject the hypothesis that one- and two-parent families use the same technology to invest in their child's skill. I find that the investment technology used by single mothers emphasizes non-parental child care more than the one used by couples, although the elasticity of substitution between inputs is statistically the same. Using the elasticity of investment's price with respect to the price of child care, I highlight an implication of the empirical finding for a widely used policy intervention intended to increase child skill: child care subsidies. The estimation implies that one-parent families will have a larger response to child care subsidies than two-parent families will.

To perform policy experiments, I expand the model to include endogenous family formation and a child care subsidy which is financed by a labor income tax. I document the effects of a child care subsidy in the stationary equilibrium of this expanded model, and verify the implications of the previous estimation: single mothers see higher gains in child skill. This is driven both by their lower income and by the technology they use to invest in their children. Single mothers have lower income in part because on average they have lower skill than married mothers in the model, due to endogenous sorting on the marriage market. This model outcome is qualitatively in line with the data. To isolate the role of technology in generating the gap in child outcomes seen in the model, I use the general equilibrium framework to perform a counterfactual exercise in which endogenous sorting into family structures is shut off. This allows me to decompose the gap in child outcomes into a component due to investment technologies, which incorporate exogenous differences in the number of parents, and a residual component due to composition differences in parental skill across the two groups.

The framework presented in this paper offers insight into the role of family structure in child skill accumulation. Heterogeneity in investment technologies has not been emphasized in the macroeconomic literature on subsidies for early childhood education, so that the ability of subsidies to close gaps in child outcomes may be overestimated when family structures are not incorporated into the analysis.

Although the framework and results of this paper make novel contributions to the literature on skill accumulation during early childhood, some limitations are apparent. First, parents are unable to make wealth transfers to their children. These could complement or substitute transfers through skill during early childhood, as well as affect outcomes in the marriage market. Second, the role of gender in determining earnings in this model may not sufficiently capture how payoff-relevant this attribute is in the labor market, especially for parents of young children. Across many countries with very different sets of policies, mothers suffer scarring to their earnings upon the birth of their child which fathers do not experience (Kleven, Landais, Posch, Steinhauer, and Zweimuller (2019)). The model presented here relies on differences in the role of parent time in child skill investment to generate differences in earnings, and these differences are not enough to explain the empirical gap. Third, the model simplifies the relationship between labor market productivity and parenting productivity. The potential of policies to affect these two attributes of parents may not be equal: high-earning parents are not necessarily going to be more productive at playing with their children. Finally, the child care producer in my general equilibrium framework is a simplified version of the market for child care. Large-scale policies that affect child care use can feasibly have a distortive effect on the economy through redirection of labor towards paid child care, which my framework does not capture. In addition, the productivity of child care time is assumed to have an exogenous relationship with the productivity of the average parent. Although disciplined by moments from the data, this does not account for selection into provision of child care in the labor market. Who decides to make a living looking after other people's children? How can we encouraged qualified workers to make this choice? Although they are very relevant for the policies considered here, answering these questions is outside the scope of this paper.

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