Child Care Subsidies and Child Skill Accumulation in One- and Two-Parent Families

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Abstract

I examine the role of family structure and child care subsidies in child skill accumulation. I establish empirically that skill accumulation is more responsive to child care price for one-parent families than for two-parent families. I analyze the effects of child care subsidies in a model featuring endogenous family formation, parental altruism, and a baseline subsidy resembling that of the United States. I find that eliminating this subsidy generates welfare losses of 1.63 percent of lifetime consumption, that equilibrium adjustments act to mitigate these losses, and that increasing uptake among one-parent families yields the highest welfare gains per additional recipient.

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1 Introduction

There is widespread agreement that early childhood is a key phase in the development of skill, and that policies targeted at this age group can improve educational attainment and labor market outcomes later in life.[1] This consensus is founded on evaluations of several small-scale randomized controlled trials (RCTs) with long-run follow-ups, where children under the age of 5 from impoverished backgrounds are exposed to environments structured to foster the development of their skill (e.g., Heckman et al. (2006); Anderson (2008); Garcia et al. (2017)). The evidence from RCTs is complemented by studies of large-scale subsidized child care programs with universal eligibility, which lower the cost of child care services already available in the market for many different kinds of families (e.g., Baker et al. (2008); Berlinski et al. (2009); Gupta and Simonsen (2010)).[2] Recent evaluations of such programs have emphasized heterogeneity in the effects of child care subsidies on child outcomes across family income groups (Havnes and Mogstad (2014)) and family structures (Kottelenberg and Lehrer (2017)). In the United States, these demographic attributes are quantitatively relevant and intertwined: one-parent families raise twenty percent of children under five, and are almost eight times more likely to be poor than couples with children of the same age.[3]

In this paper, I first examine the role of family structure in child skill accumulation. Empirically, the way in which families allocate their children's time shows systematic differences: I find that one-parent families use more non-parental child care, and contribute less quality time from resident parents in the family, than couples do. These choices determine the activities and environments that children experience, and I interpret them as reflecting an investment decision made by families. Building on the observed disparities in these family choices, I allow the technologies which govern how investment in child skill is generated to differ across family structures. I use the family optimization problems to derive estimation equations for the parameters of these technologies, and test whether the two technologies are different. I find that they are.

To highlight why heterogeneity in skill investment technologies is relevant for the design of child care subsidies, I construct investment's implicit price index for each family structure, and derive the elasticity of this price index with respect to the price of child care. This elasticity encapsulates a family's responsiveness to child care subsidies: a subsidy will increase the skill outcomes of children to the extent that it decreases the price of investment in skill. I find that this elasticity is 33 percent higher for one-parent families than it is for two-parent families, which is consistent with research that documents higher gains from child care subsidies for children raised by the former group. I believe this provides a novel perspective on how investment technologies mediate the response of family investments to child care subsidies.

The parameters governing child skill accumulation are estimated using the Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), a family-level data set from the US Department of Education. The ECLS-B is designed to be representative of families raising 9-month old children in the United States in 2001, and provides information on parental and market time inputs, parental wages, child care prices, and family

[1] See Almond and Currie (2011) for a review, and the seminal study of Cunha et al. (2010), recently revisited by Agostinelli and Wiswall (2017). For the perspective of child psychologists, see Shonkoff (2010) and Nisbett et al. (2012).
[2] Examples include programs in Oklahoma (started in 1998), Quebec (1997), Argentina (1993), Norway (1975), and Denmark (1964).
[3] Author's calculations, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B (2007)).

structure for children over three waves of the survey, corresponding to when the child is 9 months, 2 years, and 4 years old. The panel structure of these data allows me to control for unobserved relative parenting productivities by implementing a fixed effects estimator.

In addition to investment inputs, the ECLS-B reports measures of child skill in each wave of the survey. Together with family attributes, these test scores generate empirical moments which I use to calibrate a general equilibrium model for policy analysis. This model features a marriage market, in which each generation of men and women meets via random search and endogenously forms either one- or two-parent families. After the marriage market, single females and couples spend a portion of their lifetime altruistically investing in their children's skill, using the technology of their family structure. In this phase they correctly anticipate how skill affects the lifetime utility of their child. The model framework also incorporates a baseline child care subsidy modeled after the Child Care and Development Fund, one of the largest sources of federal funding for child care subsidies in the United States. Upon calibration, the baseline equilibrium matches untargeted empirical facts on the correlation of parental inputs within couples and the marriage rate over the family income distribution. In addition, the model output quantitatively aligns with untargeted empirical measures of the elasticity of the price of investment with respect to the price of child care, computed from the data using the estimated investment technologies.

Among its attributes, the model allows both the marginal cost and the marginal benefit of skill to adjust in equilibrium to the child care subsidy. The marginal cost's response is mediated by the investment technology. The marginal benefit of skill is the increase in lifetime utility, stemming from the higher consumption and leisure one can finance thanks to higher earnings and better marriage prospects. As a child care subsidy makes skill investment cheaper, the labor market return to skill may decrease as labor income taxes increase to finance the subsidy (which would dampen increases in investment due to the subsidy), or families may form differently when the costs of parenting decrease (thus making skill more or less useful in the marriage market).

In the model's baseline equilibrium, the child care subsidy targets low-income families, who are mostly single parents. Eliminating this baseline policy generates welfare losses of 1.63 percent of lifetime consumption. The general equilibrium framework allows the economy to adjust to the removal of the policy and mitigate the damage due to the resulting shift in the skill distribution. Without such general equilibrium adjustments, welfare losses due to decreases in skill would be 2.54 percent of lifetime consumption. Expansions of the current policy, whether along the dimension of eligibility thresholds or uptake rates, yield welfare gains. The highest return per additional family served by the child care policy comes from expanding uptake among poor, one-parent families. This is exactly what the technology estimation results suggest. Nevertheless, restricting subsidy expansions to impoverished single parent families excludes many families who would also benefit: switching to universal eligibility with guaranteed subsidy receipt yields aggregate welfare gains of more than 15 percent of lifetime consumption.

In my framework, the critical primitive is the set of functions that maps from the time use of the child (chosen by the family) to the child's skill outcomes. When specifying these technologies, I allow for time inputs from each resident parent and from a non-parental care provider; each of these inputs has a different productivity, which families take as given. I am not the first to incorporate both parents separately for the couple's investment technology. Other studies that do so include Abbott (2021) and Del Boca et al. (2014), although neither considers non-parental child care as a distinct input. Those studies also differ from mine in their emphasis: Abbott (2021) analyzes income risk, whereas Del Boca et al. (2014) focus on the changing role of mother time in child skill production at each stage of early childhood. By contrast, I focus on how heterogeneity in investment technologies affects investment's responsiveness to a child care subsidy in the population.

My model features altruistic parents investing in the skill of their young children, an approach to intergenerational transfers which dates back to Barro and Becker (1989) and which I share with Lee and Seshadri (2019) and Daruich (2020), the two studies most closely related to mine. Lee and Seshadri (2019) compare skill subsidies at different stages of life, where child skill responds differently to investment in each stage but in the same way for all families.[4] By contrast, I include multiple family structures whose investment technologies feature different parameters, and focus on early childhood as the phase in which investment in skill occurs. Daruich (2020) develops an environment in which parental time and money (i.e., goods, which includes child care) combine to form investment in children. In this environment, he examines the effects of transfers-in-kind subsidies to the goods input, and the technologies governing how child skill is accumulated are either taken from the literature or calibrated within the model. My approach differs from this study in two ways. First, I specify time in child care as an alternative input to time with parents, which reflects an interpretation of child skill accumulation as being founded on activities and interactions with others (similar to the definition of "active time" in Del Boca et al. (2014)). My specification also reflects the fact that, in the data, money spent on young children is mostly spent on child care.[5] Second, in this paper the skill investment technology parameters are estimated directly.

Another important feature that I model is endogenous family formation, where both one- and two-parent families exist in equilibrium. This is consistent with recent work by Blandin and Herrington (2020), who emphasize the role of family structure, in addition to family income, in explaining children's college attainment outcomes over time. Other macroeconomic studies also incorporate endogenous family formation into their frameworks. For example, Abbott et al. (2019) focus on college financial aid in the United States, but unlike my framework do not feature multiple family structures existing in equilibrium. Instead, the authors emphasize positive assortative matching of couples as a magnifying force for inequality, a more traditional approach to incorporating family formation in macroeconomic analyses of education policies.[6]

My specification explicitly emphasizes family investment in child skill. It is possible that labor supply decisions of parents could also be substantially affected by child care subsidies like those I consider. A study that does focus on labor supply effects is Guner et al. (2020), which allows for heterogeneity in family structures and considers how this heterogeneity leads to different policy responses. Unlike my framework,

[4] Another paper with this approach is Caucutt and Lochner (2020), which emphasizes that the timing of binding borrowing constraints determines the optimal timing for interventions. Restuccia and Urrutia (2004) instead emphasize the relative importance of early parental investments in determining adult outcomes and the implications of this for broader education policy design.

[5] My calculations supporting this claim use the PSID CDS and are reported in Supplementary Appendix B.

[6] Lochner and Monge-Naranjo (2011) also focus on this policy's interaction with intergenerational dynamics, although families do not form endogenously in that study.

however, the authors do not emphasize the effect of policy on the skill accumulation of children.

The paper proceeds as follows. Section 2 presents the investment problems of one- and two-parent families, which form the core of my framework. In section 3, I present the data with which I estimate the parameters of the skill investment technologies—using equations derived from parental investment problems—and the estimation results. Section 4 discusses how heterogeneity in investment technologies matters for investment's response to a child care subsidy. In this section I derive elasticities with respect to the price of child care, both for investment's composite price index and for the marginal cost of child skill. Section 5 presents the general equilibrium model framework, model parameterization, and model fit. Section 6 reports the child care policy experiments. Section 7 concludes.

2 Investment in Child Skill

The critical family choice affecting child skill accumulation, which I focus on here, is how much of the child's time is spent with each resident parent and with a non-parental child care provider. Effectively, parents choose an overall investment level, which ultimately determines child's realized skills, by balancing the child's well-being against their own. Given this desired level of investment, parents decide how much time to spend with their child themselves, doing quality activities—such as reading to them, telling them stories, or playing with them—by weighing the necessary foregone earnings against the cost of using child care services in each period. At the same time, this choice also takes into account that a unit of time with parents can impact child skill differently than the same amount of time spent with a child care provider.

In this section, I formalize the investment problems of one- and two-parent families. The optimality conditions that characterize investment choices are used to derive estimation equations; these equations will be used to identify the parameters of the investment technologies. Hereafter, I refer to one-parent families as single females and two-parent families as married couples. This classification aligns with the empirical characteristics of these groups: one-parent families are by far mostly single women, and two-parent families are usually married.

2.1 Parenting Problems

The structure of a family is identified by the subscript $str \in \{SF, MC\}$, where SF denotes a "single female" and MC denotes a "married couple." Each family takes as given the initial skill of their child, θ_1, and a vector of family attributes, z_{str}. Family attributes include parenting productivities θ_g for each resident parent of gender $g \in \{f, m\}$, where f denotes female and m denotes male. They also include the productivity and price of child care, $\theta_{n,str}$ and $p_{n,str,t}$, and the price of parental time, wages $w_{g,t}$. Here, the productivity of child care depends on family structure; the price of child care and parental wages are allowed to vary at the family level. The family chooses an outcome for their child, θ_{kid}, as well as consumption, savings, and the leisure for each parent, to maximize the family's lifetime utility.

Families invest in their child because they altruistically internalize the lifetime utility of the child associated with the chosen skill outcome, $V_{kid}(\theta_{kid})$.[7] This altruistic motivation is constrained by the fact that the production of child skill is costly: an outcome for the child is associated with a corresponding cost by a structure-specific cost function, $X_{str}(\cdot)$, which depends on the initial skill of the child, parenting productivities for each resident parent, the productivity of child care, and investment input prices. The parenting problem encompasses J_{ec} periods of early childhood, and the parents are all aged j when it begins.

Next, I explain the parenting problems of the two family structures. While these problems are similar, explaining them separately highlights the structural differences which are subsequently reflected in the empirical estimation and the policy analysis.

Single Females Single females are indexed by their type, $z_{SF} \equiv \{\theta_1, \theta_f, \theta_{n,SF}\}$, which, in addition to the initial skill of the child, includes the fixed productivities of parenting time (θ_f) and child care time ($\theta_{n,SF}$). When choosing an outcome for their child, single females solve:

$$\begin{aligned} V_{SF}(z_{SF}) &= \max_{\theta_{kid}, \left\{c_t, \ell_{f,t}, a_{t+1}\right\}_{t=j}^{J_{ec}+j-1}} \sum_{t=j}^{J_{ec}+j-1} \beta^{t-j} u_{SF}(c_t, \ell_{f,t}) + bV_{kid}(\theta_{kid}) \\ c_t + a_{t+1} &\leq (1+r)\, a_t + w_{f,t}(1-\ell_{f,t}) + T - X_{SF,t}(\theta_{kid}, w_{f,t}, p_{n,SF,t}, z_{SF}) \end{aligned} \tag{1}$$

where, in each period, c_t denotes consumption, $\ell_{f,t} \in [0,1]$ is leisure, and a_t represent assets (with $a_j, a_{J_{ec}+j} = 0$). Here, $w_{f,t}$ is the wage of the parent, $p_{n,SF,t}$ is the price of non-parental child care, $1+r$ is the return on savings, and T is the lump-sum transfer the single female may receive from the government. The per-period utility function of the agent, $u_{SF}(\cdot)$, is strictly increasing and concave in its arguments.

Embedded in (1) is a cost minimization problem that identifies the efficient production of the chosen outcome θ_{kid}. This is represented by the cost function $X_{SF,t}(\theta_{kid}, w_{f,t}, p_{n,SF,t}, z_{SF})$. It is useful to decompose this cost minimization into *intratemporal* and *intertemporal* components. For a working single female, the per-period cost of financing a total investment I_t towards the production of the desired outcome θ_{kid} is the result of choosing non-parental child care time and parental time taking her static investment technology, $I_{SF}(\theta_f q_{f,t}, \theta_{n,SF} n_t)$, as given. This intratemporal cost minimization gives rise to the expenditure function:

$$\begin{aligned} \tilde{X}_{SF,t}(w_{f,t}, p_{n,SF,t}, z_{SF}, I_t) &= \min_{n_t, q_{f,t}} p_{n,SF,t} n_t + w_{f,t} q_{f,t} \\ s.t. \quad I_{SF}(\theta_{n,SF} n_t, \theta_f q_{f,t}) &\geq I_t \end{aligned} \tag{2}$$

where, in each period, n_t is the quantity of child care time, $q_{f,t}$ is the quantity of parental time, and $n_t + q_{f,t} \in [0,1]$. Note that, for a linearly homogeneous investment technology (as assumed in the parameterization that follows), the per-period expenditure can be written as $\tilde{X}_{SF,t} = \Lambda_{SF,t}(w_{f,t}, p_{n,SF,t}, z_{SF})\, I_t$, where $\Lambda_{SF,t}$ is the composite price index of investment in period t for single females.

[7] An alternative way of motivating intergenerational transfers is through paternalistic preferences, or "warm glow" returns (Andreoni (1990)). The advantage of an altruism framework is that the returns to investment can respond endogenously to policy, as they will in section 5. This is because parents fully incorporate the economic returns to their investment in terms of their child's lifetime utility, and their behavior changes accordingly. The main benefit of a paternalistic specification is its tractability and flexibility in matching parenting behaviors (De Nardi (2004)).

The per-period level of investment I_t that constrains this problem is, in turn, the result of an intertemporal cost minimization problem where the target outcome θ_{kid} is taken as given. Specifically, the sequence $I^*_{SF,t}$ is the solution to:

$$\begin{aligned}\left\{I^*_{SF,t}\left(\theta_{kid}, w_{f,t}, p_{n,SF,t}, z_{SF}\right)\right\}_{t=j}^{J_{ec}+j-1} &= \underset{\{I_t\}_{t=j}^{J_{ec}+j-1}}{\arg\min} \sum_{t=j}^{J_{ec}+j-1} \left[\frac{\Lambda_{SF,t}\left(\cdot\right) I_t}{\left(1+r\right)^{t-j}}\right] \\ s.t. \quad f_{SF}\left(\{I_t\}_{t=j}^{J_{ec}+j-1}, \theta_1\right) &\geq \theta_{kid}\end{aligned} \tag{3}$$

Here, f_{SF} is the dynamic skill accumulation function which governs how a given sequence of investment levels affects a child's skill outcome. Using the solution to this problem, and the value function defined by the intratemporal cost minimization in (2), the total cost of child care in each period, which appears in (1), is $X_{SF,t}\left(\theta_{kid}, w_{f,t}, p_{n,SF,t}, z_{SF}\right) \equiv \tilde{X}_{SF,t}\left(\cdot, I^*_{SF,t}\right)$.

Married or Cohabiting Couples Couples are indexed by $z_{MC} = \{\theta_1, \theta_f, \theta_m, \theta_{n,MC}\}$. Relative to single females, here I include the individual productivities of the two resident parents θ_f and θ_m. Couples solve a similar problem to that of single females, with their (strictly increasing and concave) per-period utility function depending on joint consumption and separate leisure terms. They solve:

$$\begin{aligned}V_{MC}\left(z_{MC}\right) &= \max_{\theta_{kid}, \left\{a_{t+1}, c_t, \ell_{f,t}, \ell_{m,t}\right\}_{t=j}^{J_{ec}+j-1}} \sum_{t=j}^{J_{ec}+j-1} \beta^{t-j} u_{MC}\left(c_t, \ell_{f,t}, \ell_{m,t}\right) + bV_{kid}\left(\theta_{kid}\right) \\ c_t + a_{t+1} &\leq \left(1+r\right) a_t + w_{f,t}\left(1-\ell_{f,t}\right) + w_{m,t}\left(1-\ell_{m,t}\right) + T - X_{MC,t}\left(\theta_{kid}, w_{f,t}, w_{m,t}, p_{n,MC,t}, z_{MC}\right)\end{aligned} \tag{4}$$

where $\ell_{f,t}, \ell_{m,t} \in [0,1]$, and $a_j, a_{J_{ec}+j} = 0$. The wage in each period for female and male parents is $w_{f,t}$ and $w_{m,t}$, respectively, and $p_{n,MC,t}$ is the price of non-parental child care.

Relative to single females, the intratemporal component of cost minimization features one additional input, as both parents can independently contribute their own time. Within a period, working couples take their static investment technology $I_{MC}\left(\theta_f q_{f,t}, \theta_m q_{m,t}, \theta_{n,MC} n_t\right)$ as given, and solve:

$$\begin{aligned}\tilde{X}_{MC,t}\left(w_{f,t}, w_{m,t}, p_{n,MC,t}, z_{MC}, I_t\right) &= \min_{n_t, q_{f,t}, q_{m,t}} p_{n,MC,t} n_t + w_{f,t} q_{f,t} + w_{m,t} q_{m,t} \\ s.t. \quad I_{MC}\left(\theta_{n,MC} n_t, \theta_f q_{f,t}, \theta_m q_{m,t}\right) &\geq I_t\end{aligned} \tag{5}$$

where here $q_{f,t}$ and $q_{m,t}$ are the quantity of own time supplied by the two parents, and $n_t + \max\{q_{f,t}, q_{m,t}\} \in [0,1]$. This time constraint maintains the assumption that parental quality time is non-rival. For a linearly homogeneous parameterization of the investment technology the per-period expenditure can be written as $\tilde{X}_{MC,t} = \Lambda_{MC,t}\left(w_{f,t}, w_{m,t}, p_{n,MC,t}, z_{MC}\right) I_t$, where $\Lambda_{MC,t}$ is the composite price index of investment in period t for married couples.

As for the intertemporal component of cost minimization, couples solve:

$$\begin{aligned} \left\{I^*_{MC,t}\left(\theta_{kid}, w_{f,t}, w_{m,t}, p_{n,MC,t}, z_{MC}\right)\right\}_{t=j}^{J_{ec}+j-1} &= \underset{\{I_t\}_{t=j}^{J_{ec}+j-1}}{\arg\min} \sum_{t=j}^{J_{ec}+j-1} \left[\frac{\Lambda_{MC,t}\left(\cdot\right) I_t}{\left(1+r\right)^{t-j}}\right] \\ s.t. \quad f_{MC}\left(\left\{I_t\right\}_{t=j}^{J_{ec}+j-1}, \theta_1\right) &\geq \theta_{kid} \end{aligned} \tag{6}$$

where f_{MC} is the dynamic production function that governs how a sequence of investment levels affects a child's skill outcome for married couples. Evaluating the value function of the problem in (5) at the optimal solution for (6) yields the per-period expenditure term used in the lifetime utility maximization in (4), that is $X_{MC,t}\left(\theta_{kid}, w_{f,t}, w_{m,t}, p_{n,MC,t}, z_{MC}\right) \equiv \tilde{X}_{MC,t}\left(\cdot, I^*_{MC,t}\right)$.

2.2 Technology Functional Forms and Optimality Conditions

There are two sets of technologies which determine the cost of investment X_{str} for each of the two family structures $str \in \{SF, MC\}$ in (1) and (4). These are, first, a static technology which combines family and child care time inputs into investment within a period; and, second, a dynamic technology which combines investments across periods, along with the initial stock of child skill, to generate the final outcome θ_{kid}. I assume that the static technologies for both family structures takes a Constant Elasticity of Substitution (CES) functional form, where the structure of the couple's technology nests that of single females.

$$I_{str,t} = \left[\gamma_{str}\left(h_{str,t}\right)^{\frac{\nu_{str}-1}{\nu_{str}}} + \left(1-\gamma_{str}\right)\left(\theta_{n,str} n_t\right)^{\frac{\nu_{str}-1}{\nu_{str}}}\right]^{\frac{\nu_{str}}{\nu_{str}-1}} \tag{7}$$

$$h_{SF,t} = \theta_f q_{f,t} \tag{8}$$

$$h_{MC,t} = \left[\alpha\left(\theta_f q_{f,t}\right)^{\frac{\rho-1}{\rho}} + \left(1-\alpha\right)\left(\theta_m q_{m,t}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{9}$$

Here, ν_{str} is the elasticity of substitution for family time and purchased non-parental child care time, and γ_{str} is the share on family time, for family structure $str \in \{SF, MC\}$. For single females, the total quantity of family time, $h_{SF,t}$, is simply efficiency units of time from the female parent. For couples, the quantity of family time, $h_{MC,t}$, is generated with a CES aggregator of efficiency units of time from both parents. This inner aggregator for couples has an elasticity of substitution between time from each parent, ρ, and a share parameter on the female parent's time, α. Given this nested parameterization, the static investment technologies are the same across family structures if $\gamma_{SF} = \gamma_{MC}$ and $\nu_{SF} = \nu_{MC}$.

The dynamic production function that generates the target outcome θ_{kid} also takes a (recursive) CES form, where only the productivity of investment is allowed to differ across family structures:

$$f_{str}\left(\left\{I_{str,t}\right\}_{t=j}^{J_{ec}+j-1}, \theta_1\right) = \left(\sum_{t=j}^{J_{ec}+j-1} \upsilon\left(1-\upsilon\right)^{J_{ec}+j-1-t}\left(\lambda_{str} I_{str,t}\right)^{\frac{\chi-1}{\chi}} + \left(1-\upsilon\right)^{J_{ec}} \theta_1^{\frac{\chi-1}{\chi}}\right)^{\frac{\chi}{\chi-1}} \tag{10}$$

This parameterization maintains the condition that early investments increase the productivity of later investments. The parameter λ_{str} is a scaling parameter that accounts for units of measurement being different across investment and the stock of skill. The level of this parameter is allowed to differ across family structures; the dynamic technologies are the same across family structures if $\lambda_{MC} = \lambda_{SF}$.

The tangency conditions from the intratemporal cost problems (2) and (5) imply a system of equations which can be used to estimate the parameters of the static investment technology. Specifically, the optimal combination of parental times for married couples satisfies:

$$\ln\left(\frac{q_{MC,f,t}}{q_{MC,m,t}}\right) = -\rho\ln\left(\frac{w_{f,t}}{w_{m,t}}\right) + \rho\ln\left(\frac{\alpha}{1-\alpha}\right) + (\rho-1)\ln\left(\frac{\theta_f}{\theta_m}\right) \tag{11}$$

Next, the optimal combination of family time and child care time, for family structure $str \in \{SF, MC\}$, can be expressed as:

$$\ln\left(\frac{h_{str,t}}{n_{str,t}}\right) = -\nu_{str}\ln\left(\frac{p_{h,str,t}}{p_{n,str,t}}\right) + \nu_{str}\ln\left(\frac{\gamma_{str}}{1-\gamma_{str}}\right) + (\nu_{str}-1)\ln\left(\frac{1}{\theta_{n,str}}\right) \tag{12}$$

where $h_{SF,t}$ and $h_{MC,t}$ are the family time aggregates defined by equations (8) and (9), $p_{h,SF,t} \equiv \frac{w_{f,t}}{\theta_f}$ is the unit cost of family time for a single female, and $p_{h,MC,t}$ is the composite price index for family time in couples (this price aggregator is dual to the production function in (9), and depends on the wages of both parents).

Finally, the tangency conditions of the intertemporal cost minimization problems, (3) and (6), yield an investment equation for the parameters of the dynamic production function in (10):

$$\ln\left(\frac{X_{str,t+1}}{X_{str,t}(1+r)}\right) = (1-\chi)\ln\left(\frac{\Lambda_{str,t+1}}{\Lambda_{str,t}(1+r)}\right) - \chi\ln(1-\upsilon) \tag{13}$$

where $\Lambda_{str,t}$ is the price index of intratemporal investment, for each family structure $str \in \{SF, MC\}$, introduced earlier (these are dual to the nested CES investment functions of equations (7) to (9)).

3 Data and Estimation

In this section, I first explain data sources and describe several empirical patterns that are evident when comparing across family structures in the population. Next, after discussing the estimation sample, I provide the estimation equations, the estimated coefficients, and the implied estimates for technology parameters.

3.1 Data

I combine two datasets to measure parental educational time inputs ($q_{g,i,t}$), non-parental child care time inputs ($n_{i,t}$), hourly wages ($w_{g,i,t}$), and hourly non-parental child care prices ($p_{n,str,i,t}$) (the indexes denote parent gender g in family i with family structure str in period t, while n indexes non-parental child care's price). Specifically, I use the ECLS-B, which is a panel data set, and the American Time Use Survey

(ATUS), which is a repeated cross-section sampled from the Current Population Survey (CPS). Data for hourly wages, hourly price of child care, quality time from the parents in weekly frequency of activities, and non-parental child care time in hours per week can be constructed from information reported in the ECLS-B. The ATUS contributes measures of hours per activity, which I impute to the ECLS-B to transform frequencies of activities into hours per week of parental quality time inputs.

The Early Childhood Longitudinal Study, Birth Cohort The ECLS-B reports labor supply status (full time, part time, or not in the labor force), labor earnings, the period of time over which the labor earnings were accrued (a day, a week, two weeks, etc.), and the hours worked in a week separately for each parent. It also reports frequencies of activities with the child for each resident parent (i.e., reading to the child one a week, twice a week, every day, etc.). In addition, there are assessments of child skill reported in each wave of the survey.[8]

I convert pre-tax labor earnings into pre-tax hourly wages using hours worked per week. If hours worked were not reported for the parent, they are imputed when possible using the response to part-time or full-time status (assigning 30 or 40 hours worked per week, respectively). Next, I convert pre-tax hourly wages into after-tax hourly wages using the slopes (tax rates) from Table 6 of McGrattan and Prescott (2017).[9] The result is hourly after-tax wages for mothers and fathers, conditional on observing labor earnings and some information about the intensity of labor supply. I do not impute wages for non-working parents.

To calculate the hourly price of non-parental child care, I use expenditures on the primary source of non-parental child care, adjusted by the total hours of care that these expenditures purchased.[10]

Hours purchased for the primary source of child care are reported directly in the ECLS-B. However, hours of parental time inputs $q_{g,i,t}$ are not reported directly. Instead, parents report how frequently they engage in different activities with the child in the first three waves of the survey. Some of these activities involve actively engaging with the child and speaking or listening to them. In particular, I consider the following activities to be forms of investment in the child: reading to the child, telling the child stories, and playing with the child indoors.[11] This definition is consistent with the literature for the importance of active time with children (Del Boca et al. (2014)). In turn, these activities have counterparts in the ATUS, so that for each activity occurrence I can impute the amount of time spent in that activity using group-specific time averages for the ATUS counterpart activity.

The American Time Use Survey Data on time per activity come from the 2003-2016 pooled ATUS sample. This dataset provides a time diary along with CPS variables for age, gender, marital status, labor

[8]These are used in the calibration of the general equilibrium model later in the paper. For more information on child skill measures in the ECLS-B, see Supplementary Appendix A.

[9]I use the slope corresponding to the bin of the original level of earnings in Table 6 of McGrattan and Prescott (2017). This adjustment accounts for the progressive nature of the US tax system with respect to labor earnings.

[10]The ECLS-B also reports all sources of child care; the relevant moments for the calibration are not affected by using total hours in child care instead of only the primary source.

[11]See Supplementary Appendix A for further information on measured activities in the ECLS-B and the definition of quality time I apply here.

force status, parental status, and child age. For the purpose of my analysis, observations are restricted to individuals between 18 and 55 years of age, with a child aged 3 years or younger. I use information on parent gender, marital status (married/cohabiting or single), and labor force status (full time or part time). I calculate the survey-weighted average of time spent on an activity (conditional on engaging in it) for each group, for time spent reading to the child, speaking and listening to the child, and playing with the child.[12]

Imputation After linking parents in the ECLS-B with their appropriate group in the ATUS, the ATUS levels of time for a given occurrence of an activity are assigned to their appropriate activities in the ECLS-B. Next, total quality time per parent in each family in each wave of the ECLS-B is calculated by summing across the three activities.

Population Moments in the ECLS-B Table 1, Table 2, and Figure 1 present moments from a population sample. I applied several broad requirements on the ECLS-B raw data to generate these moments. For both couples and single females, observations are only admissible if the resident primary caregiver is a biological parent. I restrict the population sample to families who do not change family structure over the course of the panel, for whom I observe family income in all three waves, and for whom I observe both initial skill (wave 1) and final skill (waves 4 or 5) for the child. Finally, I require that the total time spent with parents in quality time activities or child care providers must not exceed 100 hours a week and not be missing values. This criteria allows me to include families which specifically report not engaging in any of the activities which I include in the category of parental investment time, or who report using zero hours of child care. It also includes observations where the price of child care or the wage of the parent is zero or missing.[13]

Table 1 contains population moments from the ECLS-B, for each family structure and parent gender. Here, couples can be either married or cohabiting, and single females are those who are a primary caregiver and who do not have a significant other living in the household with them. In these summary statistics, I used cross-sectional primary caregiver weights from the primary caregiver survey for the single and married/cohabiting female moments, and cross-sectional resident father weights from the resident father survey for the moments of married/cohabiting males.[14]

By comparing across family structures, three qualitative points are apparent. First, child care represents a more sizable input for one-parent families. Single females in the ECLS-B contribute 9 hours from the family and purchase 35 hours of child care per week, on average. By comparison, couples contribute about 10 and 8 hours from the female and male parents, respectively, and purchase 30 hours of child care per week. Second, purchased child care differs systematically in price for the two family structures: the average hourly price of child care purchased by single females is 43% of the average price of child care purchased by coupled females. The third qualitative point is that one-parent families face lower wages compared to

[12]Summary statistics and tabulations of the ATUS sample are reported in Supplementary Appendix A.
[13]Only prices specifically reported to be 0 are included in price moments reported in the summary statistics tables.
[14]See Supplementary Appendix A for an outline of the survey structure of the ECLS-B. There are several questionnaires at the family level in each wave of the survey, each with its own set of weights. Not all married couple families in the ECLS-B have a completed questionnaire for both the primary caregiver (usually the mother) and the resident father.

coupled parents: average hourly wages for single females are 65 percent of the average for married females, and 60 percent of the average wage of married males.

The fact that the average wage of single females is lower than for married females reflects differences in the fraction with a bachelor's degree or higher in each group (7 percent versus 46 percent, respectively), but is likely not driven by labor force status (single and coupled females have similar distributions of labor force statuses in the population sample, although single females are more likely to be unemployed). Differences in educational attainment may partly be because single females tend to be younger than married females (26 versus 33, respectively). All of these compositional differences are reflected in the family incomes: the average income of single female families is roughly one-third that of couples, even after correcting for taxes.[15] Related to differences in family income, but additionally incorporating family size, the poverty rate for single females is almost eight times higher than it is for coupled females (45 versus 6 percent),[16] and they are more than three times as likely to be below 185 percent of the poverty line (71 versus 19 percent).[17]

Table 1: Population Sample Moments

	Singles: Females			Couples: Females			Couples: Males		
	mean	p50	sd	mean	p50	sd	mean	p50	sd
Parental Time	9.45	7.96	7.27	10.07	8.02	6.88	7.98	6.32	6.79
CC Time	34.57	40.00	13.11	29.67	32.00	14.15	29.99	33.00	14.03
Parent Hourly Wage	9.86	8.55	6.25	15.24	11.48	32.85	16.46	13.30	15.39
CC Hourly Price	1.25	0.23	2.15	2.91	2.14	3.84	2.88	2.17	3.76
BA or higher	0.07			0.46			0.37		
Parent Age	26.06	25.00	5.99	32.54	32.00	5.55	34.53	35.00	6.14
Full Time	0.61			0.67			0.93		
Part Time	0.18			0.23			0.03		
Unemployed	0.09			0.01			0.02		
NILF	0.11			0.08			0.03		
Family Inc.	29918	20000	35402	86985	62500	63415	86453	62500	62474
Family Inc., Aftx	22426	16260	23361	63520	47812	41585	63197	47813	40960
$\leq$ Pov. Line	0.45			0.06			0.05		
$\leq$ 185% Pov. Line	0.71			0.19			0.20		
Observations	1000			4200			3050		

This table: population sample moments using waves 1-3; sample criteria are described in the text. Differences in averages for married females and males are due to the different weights used to generate sample moments and because not all primary caregiver surveys have an associated resident father survey. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Observations rounded to nearest 50 per NCES requirements.

Child Skill Accumulation, Family Income, and Family Structure in the ECLS-B The ECLS-B provides measures of child skill in each wave of the survey. These measures are age-appropriate tests of children's cognitive abilities and so can be used to examine a child's development over time relative to other children of the same age. Using wave 3 of the population sample described above, Table 2 shows that in the

[15] After-tax income is computed using Table 5 in McGrattan and Prescott (2017).

[16] For comparison, a similar statistics computed from the US Census show similar disparities in poverty rates. In the 2000 US Census, single females of children under 5 have a poverty rate of 41%, while for married couples this is 6% (author's calculations).

[17] The reason that the fraction below 185% of the poverty line is reported in particular in Tables 1 and 3 is that this is the income threshold below which families are eligible for the National School Lunch Program and the Special Supplemental Nutrition Programs for Women, Infants, and Children.

ECLS-B child skill accumulation exhibits a systematic relationship with family income and family structure.[18] In particular, although at age 9 months measures of child skill are uncorrelated with family income and family structure, by the time the child is in kindergarten these correlation coefficients have jumped to 0.32 with after-tax family income, Y, and 0.21 with an indicator for two-parent families, $\mathbb{I}_{MC}$. This demonstrates that both richer families and couples tend to see better relative outcomes for their children's skill, despite very similar initial child skills across income levels and family structures. Table 2 also reports the fraction of families who are married, for the same population sample for which the correlation coefficients are computed. It is 0.79.

Table 2: Child Skill Accumulation Moments and Marriage Rate

Corr(θ^*_{kid}, Y)	Corr($\theta^*_{kid}, \mathbb{I}_{MC}$)	Fraction Married
0.32	0.21	0.79
Families	1750	

This table: all families, wave 3, using cross-sectional primary caregiver weights for that wave. Sample criteria described in the text. I do not impose that both the mother and father have to be reporting a valid observation, so observation counts align with 1 wave of the sample in Table 1 using information from mothers. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Observations rounded to nearest 50 per NCES requirements.

At the same time, family structure is also systematically related to family income: in the ECLS-B, the marriage rate is monotonically increasing in the after-tax income quartile of the family (Figure 1). This relationship arises in part because two-parent families have two potential earners, but is also a product of the higher wages of each parent compared to hourly wages of single females (Table 1).[19] I interpret the relationship between family income and family structure as arising from endogenous family formation, and therefore include an endogenous marriage market in the expanded model of section 5.

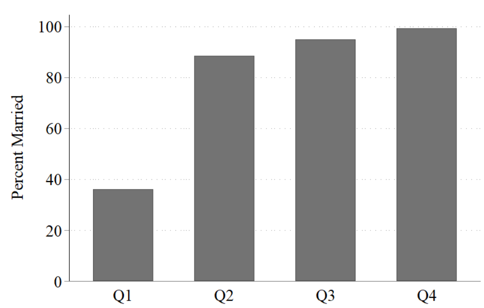

Figure 1: Marriage Rate by Family Income Quartile in the ECLS-B

This figure: percent of families in each family income quartile who are married or cohabiting, wave 3, using cross-sectional primary caregiver weights for that wave. Sample described in the text and is the same as Table 2. Income is corrected for taxes using McGrattan and Prescott (2017), Table 5. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

[18] Child skill outcome θ^*_{kid} is measured in ECLS-B waves 4 or 5, and initial child skill is measured in wave 1. For more information on ECLS-B child skill measures and survey structure, see Supplementary Appendix A.

[19] Correcting income for family size, or using pre-tax income, yields very similar results.

The Estimation Sample The estimation sample puts additional restrictions on the data compared with the population sample described in the previous section. In particular, for both couples and single females, observations are only admissible if the resident primary caregiver is less than 55 years of age, reports working for pay, makes less than 250 dollars an hour, pays at least 1 cent per hour for the primary source of child care, and whose child spends at least 0.01 hours per week in child care. In addition, families are only valid observations if the biological mother had her first child after age 15 and before age 45. For couples, I also restrict attention to families where both parents are valid observations with regards to investment input quantities and prices. This leaves me with 50 one-parent families and 200 two-parent families for the estimation.[20]

Moments from the estimation samples are presented in Table 3. The three qualitative points that were apparent in the population moments presented in Table 1 survive for the most part in Table 3. First, one-parent families put more emphasis on child care time than family time compared to two-parent families. Second, the price per hour of child care time is lower for single females than for couples. Third, single females earn lower wages, have lower incomes, are more likely to be poor and to qualify for government benefits, and are less educated than adults parenting in a couple.

Table 3: Estimation Sample Moments

	Singles: Females			Couples: Females			Couples: Males		
	mean	p50	sd	mean	p50	sd	mean	p50	sd
Parental Time	11.15	9.37	7.04	10.48	8.16	6.49	8.74	6.36	6.68
CC Time	37.31	40.00	10.98	33.14	36.00	12.74	33.31	37.00	12.77
Parent Hourly Wage	12.24	10.42	6.40	15.07	12.68	8.7	17.82	14.12	15.05
CC Hourly Price	2.59	2.00	2.10	4.20	3.20	3.73	4.18	3.13	3.69
BA or higher	0.28			0.67			0.52		
Parent Age	29.54	29.00	5.87	33.83	34.00	4.44	34.88	35.00	5.14
Full Time	0.79			0.79			0.99		
Part Time	0.21			0.21			0.01		
Unemployed	0.00			0.00			0.00		
NILF	0.00			0.00			0.00		
Family Inc.	42611	32500	38730	106134	87500	58776	105216	87500	59419
Family Inc., Aftx	31231	24863	25402	76376	65100	38090	75763	65100	38523
$\leq$ Pov. Line	0.20	0.00	0.40	0.00	0.00	0.04	0.00	0.00	0.04
$\leq$ 185% Pov. Line	0.44	0.00	0.50	0.04	0.00	0.19	0.05	0.00	0.21
Observations	150			600			600		
Families	50			200			200		

This table: estimation sample moments using waves 1-3; sample criteria are described in the text. Differences in group moments for married females and males are due to different weights used for the primary caregiver and resident father survey, respectively. See text for details. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Observations rounded to nearest 50 per NCES requirements.

In this sample definition, I have restricted attention to families for which the estimation equations I implement are valid. This assumes that parents supply labor on the market so that the price of their time is their observed wage. Because of this requirement, if a parent in a given family does not work, then that family

[20]These sample sizes are rounded to the nearest 50, as required by the National Center for Education Statistics (NCES) for the restricted-use ECLS-B.

is not in my estimation sample. Comparing Table 3 to Table 1 shows that this restriction raises the average level of time inputs across both family structures and raises the average parental wage. It also lowers the poverty rate across both family structures.

Differences between Tables 1 and 3 do not present a problem for identifying the technology parameters of the population, as long as the families excluded from the Table 3 sample still use the same technologies to invest in their children as the families which are included. The assumption, therefore, is that I simply do not observe the appropriate relative prices and quantities for those excluded families. For example, in families where a parent does not work, those parents face relative prices of inputs which I cannot observe directly (the price of time for a non-working parent is determined by the marginal utility of leisure for that parent). Although they are not included in the estimation sample, my assumption is that families with non-working parents are still choosing input quantities by solving the same cost-minimization problem, but one which incorporates the appropriate input prices. The sample restrictions I use here focus attention to the set of realizations for relative prices and relative quantities that are directly measured in the ECLS-B.

3.2 Estimation

Given the sequential nature of intratemporal and intertemporal cost minimization problems, and the nested CES form of the couple's technology, estimation proceeds in three steps. First, I use the optimality condition for the allocation of within-family time in married couples, as provided in (11). The resulting estimating equation can be expressed as:

$$\tilde{y}_{i,t} = \tilde{\beta}_0 + \tilde{\beta}_1 \tilde{x}_{i,t} + \tilde{\tau}_i + \tilde{\epsilon}_{i,t} \tag{14}$$

Here, the dependent variable is $\ln\left(\frac{q_{f,i,t}}{q_{m,i,t}}\right)$ and the independent variable is $\ln\left(\frac{w_{f,i,t}}{w_{m,i,t}}\right)$. The term $\tilde{\tau}_i$ is a fixed effect parameter that captures the family-specific but time-invariant input productivities in (11). This fixed effect captures relative productivities, and thus varies at the family level even though child care's productivity is the same within a given family structure. $\tilde{\epsilon}_{i,t}$ is a random term assumed distributed i.i.d. Next, I use the intratemporal condition (12) that governs the optimal combination of family time and child care time for each family structure. This second regression takes the form:

$$y_{str,i,t} = \beta_0 + \beta_{str,1} x_{str,i,t} + \tau_i + \epsilon_{i,t} \tag{15}$$

Here, the dependent variable is $\ln\left(\frac{h_{str,i,t}}{\theta_{f,i} n_t}\right)$, and the independent variable is $\ln\left(\frac{\theta_{f,i} p_{h,str,i,t}}{p_{n,str,i,t}}\right)$. The term τ_i is a fixed effect parameter that captures unobserved family-specific but time-invariant relative productivities of female parent time and non-parental child care time. $\epsilon_{i,t}$ is a random term assumed to be i.i.d..

Note that simply using Ordinary Least Squares (OLS) to estimate (14) and (15) (while conflating the relative productivity terms with the residual) would yield biased estimates of the slopes and intercept parameters of interest if, as seems plausible, the investment productivities of parental time and child care time are correlated with the observed prices of these inputs. Thus, I estimate these models using a Fixed Effects (FE) estimation at the family level. That is, I treat τ_i and $\tilde{\tau}_i$ as family-specific fixed effects to be estimated and, by a suitable normalization, I will assume $E\left[\tau_i|str\right]=0$ for each family structure, and $E\left[\tilde{\tau}_i\right]=0$ for married couples. This means that group-specific intercepts can be constructed using group-specific averages of the fixed effect estimates from the original estimation and the estimated intercept: $\beta_{str,0}=\beta_0+E\left[\tau_i|str\right]$, for $str\in\{SF,MC\}$. This normalization allows me to use the estimated intercept to identify share parameters in each stage for each family structure.

The last estimation step exploits the tangency conditions for the intertemporal component of cost minimization, as in equation (13). The resulting regression equation, estimated by OLS, is:

$$y_{d,i,t} \quad = \quad \beta_{d,0}+\beta_{d,1}x_{d,i,t}+\epsilon_{d,i,t} \tag{16}$$

Here, the dependent variable is log of the ratio of consecutive expenditures of family i with structure str in periods t and $t+1$: $\ln\left(\frac{X_{str,i,t+1}}{X_{str,i,t}(1+r)}\right)$. The independent variable is $\ln\left(\frac{\Lambda_{str,i,t+1}}{\Lambda_{str,i,t}(1+r)}\right)$, where $\Lambda_{str,i,t}$ is the price index that arises from intratemporal cost minimization using the CES parameterization.[21] The random terms $\epsilon_{d,i,t}$ is assumed to be i.i.d..

Estimation Results The estimated coefficients for the three estimation equations are reported in Table 4. From the results in the first column of this table, it is apparent that the slope coefficient $\tilde{\beta}_1$ is positive and not statistically different from zero. This implies that the postulated CES aggregator for married couples, which combines efficiency units of mother and father family times, reduces to a Leontief.[22] In what follows, I will maintain the limiting Leontief structure implied by $\tilde{\beta}_1=0$. This means that, rather than the share parameter α of the couple family aggregator in equation (9), the remaining relevant parameter is the Leontief ratio of time inputs $\alpha_L\equiv\frac{\theta_f q_{f,t}}{\theta_m q_{m,t}}$. With a suitable reformulation of the CES structure in (9),[23] this Leontief parameter is:

$$\alpha_L \quad = \quad \exp\left(\tilde{\beta}_0\right) \tag{17}$$

Using the estimated value of α_L along with the predicted relative productivities of couples implied by the estimated fixed effects, the quantity of family time, $h_{MC,i,t}$, and its price index, $p_{h,MC,i,t}$, can then be constructed. For married couples the dependent variable in (15) is then written as $y_{MC,i,t}=\ln\left(\frac{q_{f,i,t}}{n_t}\right)$, and the independent variable is $x_{MC,i,t}=\ln\left(w_{f,i,t}+\frac{\theta_{f,i}}{\theta_{m,i}\alpha_L}w_{m,i,t}\right)-\ln\left(p_{n,MC,i,t}\right)$. For single females, the corresponding variables in (15) are $y_{SF,i,t}=\ln\left(\frac{q_{f,i,t}}{n_t}\right)$ and $x_{SF,i,t}=\ln\left(w_{f,i,t}\right)-\ln\left(p_{n,SF,i,t}\right)$.

[21] In the estimation I set $r=0.04$, with suitable adjustments to account for the fact that the time interval between wave is not constant in the ECLS-B.

[22] A positive point estimate for $\tilde{\beta}_1$ implies a negative ρ, which is not an admissible value for an elasticity of substitution.

[23] A property of the standard CES functional form, as in (9), is that its limiting case (when the elasticity of substitution approaches zero) implies a ratio of inputs equal to one, regardless of the share parameter α. To allow for a ratio of inputs different from one, as is typically of interest, a suitable re-parameterization of the CES is needed. See Saito (2012) for details.

Table 4: Model Regression Coefficients for Skill Investment Technology Estimation

	Intratemporal		Intertemporal
	(MC)	$(MC+SF)$	$(MC+SF)$
$\tilde{\beta}_0$	0.353		
	(0.040)		
$\tilde{\beta}_1$	0.179		
	(0.107)		
β_0		-0.207	
		(0.208)	
$\beta_{1,MC}$		-0.594	
		(0.118)	
$\beta_{1,SF}$		-0.443	
		(0.178)	
$\beta_{d,0}$			0.255
			(0.047)
$\beta_{d,1}$			0.620
			(0.129)
FEs	Y	Y	N
R^2	0.45	0.43	0.08
No. families MC	200	200	200
No. families SF		50	50

This table: first, second, and third-stage estimated coefficients for investment aggregator. Standard errors in parentheses. Data source: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Sample size rounded to nearest 50 per NCES requirements.

The remaining structural parameters of interest can be retrieved from the estimated coefficients reported in Table 4 as follows:

$$\nu_{str} = -\beta_{str,1} \tag{18}$$

$$\gamma_{str} = \frac{1}{1+\exp\left(\frac{\beta_{str,0}}{\beta_{str,1}}\right)} \tag{19}$$

$$\chi = 1-\beta_{d,1} \tag{20}$$

$$\upsilon = 1-\exp\left(\frac{\beta_{d,0}}{\beta_{d,1}-1}\right) \tag{21}$$

The point estimates of this structural parameters, along with their standard errors (computed using the delta method), are reported in Table 5. The point estimates for elasticity parameters ν_{SF} and ν_{MC} mean that, for both family structures, this estimation rejects a Cobb-Douglas specification for how family and child care time are combined. However, these elasticities are not statistically different across one- and two-parent families. At the same time, the family share γ_{str} is not different from zero for single females but is statistically different from zero for couples (at the 0.01 significance level). As for the point estimates of the dynamic skill accumulation technology parameters, χ and υ, the results reject a Cobb-Douglas specification.[24] In addition, the share on investment is sizable, indicating a significant role for family investments in determine child outcomes.

[24] See Cunha and Heckman (2007) for a discussion of the functional form and its interpretation.

Table 5: Parameters of the Family Investment Technologies

α_L	ν_{SF}	ν_{MC}	γ_{SF}	γ_{MC}	χ	υ
1.42	0.443	0.594	0.171	0.465	0.380	0.490
(0.06)	(0.178)	(0.128)	(0.094)	(0.095)	(0.129)	(0.138)

This table: Point estimates of technology parameters, using estimates of coefficients from Table 4 and equations (17) to (21). Standard errors are in parentheses, calculated using the delta method. Data source: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

Comparing Investment Technologies Across One- and Two-Parent Families The cost-minimizing relative quantities of family time and child care time depend on the relative prices of these two inputs, for a given family structure. Comparing parameter values in Table 5 across family structures, the estimated share on family time, γ_{str}, is larger for couples than for single females because couples tend to choose more family time relative to child care time, given similar relative prices. At the same time, because similar changes in relative prices tend to result in larger changes in relative quantities for couples, the elasticity ν_{MC} is higher in magnitude than ν_{SF}.

With the estimated parameters in hand, I consider the null hypothesis that two-parent families are using the same technology as one-parent families to invest in their children. This requires that $\gamma_{SF} = \gamma_{MC}$ and $\nu_{SF} = \nu_{MC}$. The hypotheses $\gamma_{SF} = \gamma_{MC}$ is rejected by the F test at the 0.01 significance level. While the other hypothesis $\nu_{SF} = \nu_{MC}$ is not rejected, the joint hypothesis that the two parametric restrictions hold simultaneously is also rejected at the 0.01 significance level. These results establish that these technologies have statistically the same elasticity of substitution between child care time and parental time, but very different share parameters on child care time.

4 The Heterogeneous Importance of Child Care's Price

To gain some insight into how family attributes map into the responsiveness of investment to child care subsidies, here I derive the elasticity of skill's marginal cost with respect to the price of child care, for each family structure. I decompose this marginal cost elasticity into two components: the fraction of potential income being invested in the child, and the elasticity of the price of investment with respect to the price of child care. Using the estimated parameters and an assumed functional form for utility, I calculate and compare the empirical distributions of the marginal cost elasticity and its two components. Differences across family structures in these two components will drive heterogeneity in investment responses to child care subsidies.

The Elasticity of the Marginal Cost of Investment With Respect To Child Care's Price From the optimization problems of single females in (1), and of couples in (4), consider the simple case where early childhood lasts one period ($J_{ec} = 1$), there is no borrowing or saving, and the utility functions take the form $u_{SF}(c, \ell_f) = \ln(c) + \psi_{SF} \ln(\ell_f)$ and $u_{MC}(c, \ell_f, \ell_m) = \ln(c) + \psi_{MC,f} \ln(\ell_f) + \psi_{MC,m} \ln(\ell_m)$. Let (c^*_{SF}, ℓ^*_{SF}) and $\left(c^*_{MC}, \ell^*_{MC} = \left\{\ell^*_{MC,f}, \ell^*_{MC,m}\right\}\right)$ denote the optimal consumption and leisure choices conditional on a given final skill for the child, θ_{kid}, for each family structure.[25]

The optimal choice of the final child skill, θ_{kid}, maximizes $u_{str}\left(c^*_{str}, \ell^*_{str}\right) + bV_{kid}(\theta_{kid})$. For single females, this means it satisfies:

$$\frac{(1+\psi_{SF})\Lambda_{SF}}{w_f + T - \Lambda_{SF} I_{SF}} \frac{\partial I_{SF}}{\partial \theta_{kid}} = b\frac{\partial V_{kid}\left(\theta_{kid}\right)}{\partial \theta_{kid}}$$

The left-hand-side of this equation displays the full "marginal cost" of child skill production in terms of utility, which balances the marginal benefit on the right-hand-side. Consider the elasticity of this marginal cost with respect to the price of child care, denoted σ_{SF}:

$$\sigma_{SF} \equiv \epsilon_{SF}\left(\frac{w_f + T}{w_f + T - \Lambda_{SF} I_{SF}\left(\theta_{kid}\right)}\right) \tag{22}$$

where $\epsilon_{SF} = \frac{\partial \Lambda_{SF}}{\partial p_{n,SF}} \frac{p_{n,SF}}{\Lambda_{SF}}$ is the elasticity of the price of investment with respect to the unit cost of child care. Given the technology presented in (7), this elasticity is:

$$\epsilon_{SF} = \frac{\left(1-\gamma_{SF}\right)^{\nu_{SF}}}{\left(1-\gamma_{SF}\right)^{\nu_{SF}} + \gamma_{SF}^{\nu_{SF}}\left[\left(\frac{p_{h,SF}}{p_{n,SF}}\right)\theta_{n,SF}\right]^{1-\nu_{SF}}} \tag{23}$$

where $p_{h,SF} \equiv \frac{w_f}{\theta_f}$ is the price per efficiency unit of family time for single females. Proceeding analogously, for married couples these objects are:

$$\sigma_{MC} \equiv \epsilon_{MC}\left(\frac{w_f + w_m + T}{w_f + w_m + T - \Lambda_{MC} I_{MC}\left(\theta_{kid}\right)}\right) \tag{24}$$

$$\epsilon_{MC} = \frac{\left(1-\gamma_{MC}\right)^{\nu_{MC}}}{\left(1-\gamma_{MC}\right)^{\nu_{MC}} + \gamma_{MC}^{\nu_{MC}}\left[\left(\frac{p_{h,MC}}{p_{n,MC}}\right)\theta_{n,MC}\right]^{1-\nu_{MC}}} \tag{25}$$

where $p_{h,MC} \equiv \frac{w_m}{\alpha_L \theta_m} + \frac{w_f}{\theta_f}$ is the composite price index per efficiency unit of family time for couples.

The higher the elasticity of investment price with respect to the price of child care, the more responsive the marginal cost will be to the child care subsidy, and the larger the expected increase in child skill. This elasticity of the investment price is decreasing in the share on family time, γ_{str}, and, when $\nu_{str} < 1$, decreasing in the ratio of the price per efficiency unit of family time versus chid care time, $\left(\frac{p_{h,str}}{p_{n,str}}\right)\theta_{n,str}$. Family potential income plays an additional role, however, as represented by the income-expenditure ratio multiplying the price elasticity to yield the marginal cost elasticity in equations (22) and (24). This term is increasing in the fraction of potential income spent on investment in children.

[25] Since early childhood lasts one period here, I drop period subscripts on wages and the price of child care.

Empirical Distributions of σ_{SF} and σ_{MC} Table 6 reports the empirical distribution of the elasticities in (22) and (24), computed using the estimated parameters and ECLS-B data.[26] This table yields three qualitative points. First, both the mean and median of the elasticity of investment's price with respect to the price of child care are higher for single females than for couples. This reflects the point estimates of the technology parameters for each family structure. Second, the fraction of potential income spent on investment in children's skill has a slightly higher mean and median for single females. Third, on average single females have a 37% larger marginal cost elasticity than couples do. Most of the differences across family structures in σ_{str} is driven by ϵ_{str}, which is on average 33% higher for single females.

Table 6: Empirical Elasticity of the Marginal Cost of Investment and its Components

	Single Females			Married Couples		
	mean	p50	sd	mean	p50	sd
Investment Price Elasticity ϵ_{str}	0.44	0.43	0.16	0.33	0.33	0.11
$\frac{\text{Potential income}}{\text{Potential income - investment expense}}$	1.19	1.16	0.10	1.14	1.12	0.07
Marginal Cost Elasticity σ_{str}	0.52	0.51	0.19	0.38	0.37	0.13
Observations	150			600		
Families	50			200		

This table: estimation sample moments the elasticity of investment's marginal cost with respect to the price of child care, and its two component parts: the potential income-investment expense ratio and the elasticity of investment's price. Data is waves 1-3, weighted with cross-sectional weights (sample details in Table 3). Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File. Sample size rounded to nearest 50 per NCES requirements.

5 Investment in Child Skill in General Equilibrium

I now embed the parenting problems of section 2 in a general equilibrium framework, which endogenizes the distribution of skill, family formation, the altruism term, labor income tax rates, and the price of child care. I also incorporate a baseline child care subsidy, modeled after the Child Care and Development Fund.

Overview of the Model Environment Besides consumers, there are three other types of agents in the expanded economy: a goods producer, the government, and a non-parental child care provider. The goods producer selects a quantity of efficiency unit of labor subject to a linear production technology with one input (there are no assets in the expanded model). The government chooses a linear labor income tax, τ_y, to finance lump-sum transfers, T, and non-parental child care subsidies. Given a function which determines the intensity of the subsidy upon receipt, $\tau_{n,str}(\cdot)$, a child care policy is defined by a skill threshold, ι_{str}, which is used to determine eligibility, and a probability of subsidy receipt, $\pi_{R,str}$, for each family structure. The non-parental child care sector supplies child care to single females and couples at the amount demanded in equilibrium, at a pre-subsidy price $p_{n,str}$ which is proportional to the distribution of after-tax wages of

[26] For the purpose of these computations, potential income (not directly observed in the data) is backed out using weekly time endowments of 100 hours per week and hourly wages measured in the ECLS-B. Wages are corrected using slopes, and total potential earnings are adjusted using intercepts, from Table 6 of McGrattan and Prescott (2017). In addition, when computing ϵ_{str} I use the estimated fixed effects at the family level to back out relative investment input productivities.

female parents in each family structure. The price of child care is not allowed to be idiosyncratic to the family, as in section 2. Instead, it now depends solely on family structure. The productivity of this child care still depends on family structure; child care's productivity will be calibrated in the model baseline equilibrium. Families parenting in structure str face a post-subsidy child care price $\tilde{p}_{n,str}$.[27]

The Life Cycle of Consumers Figure 2 illustrates the life cycle of a consumer. Each individual lives for L periods. During their childhood, which lasts for the first J periods of life, an individual makes no decisions: they are a passive recipient of consumption and investment chosen by their parents. Just as in problems (1) and (4) of section 2, children are born with some initial skill θ_1 and investments are made only during early childhood. During the investment phase, families solve the problems described in section 2, with three main changes. First, early childhood lasts one period, and occurs at the start of the parent's adulthood (i.e., I set $J_{ec} = 1$ and $j = J + 1$); second, there is no savings decision; and, third, every child has one sibling of the opposite gender, who is born with the same initial skill and receives the same investment from the family.

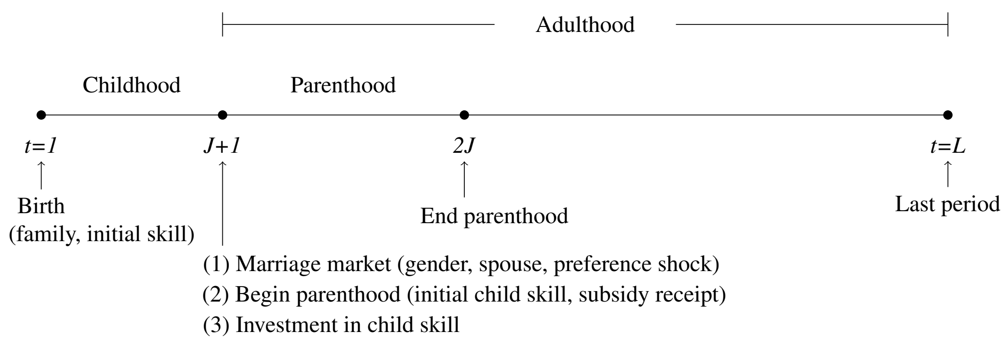

Figure 2: Consumer Life Cycle

This figure illustrates the life cycle of a consumer. At birth, an individual draws their family and their initial skill (shocks at each age are given in parentheses in the figure). Early childhood lasts one period and total childhood lasts J periods. At age $J + 1$, an individual leaves home and participates in a marriage market with their cohort. After forming, families draw their children's initial skill and their family's subsidy receipt outcome. These steps are ordered as listed in the figure above, but instantaneous. Families then invest in their children during age $J + 1$, and split consumption with their children until age $2J$. After that, their children leave home as new adults. Death occurs at age $L + 1$.

Upon independence, at the beginning of age $J + 1$, the individual leaves the family that raised them with the level of skill they have accumulated, θ_{kid}. At this point, an individual with skill θ_{kid} draws their gender $g \in \{f, m\}$, and afterwards their skill is referred to by θ_g. Next, adults participate in a marriage market and form families. The draws of gender and family formation are instantaneous. For the rest of age $J + 1$, single females and couples invest in their children. From ages $J + 1$ to $2J$, families also have to support the children by splitting consumption with them (if a single female or a married couple) or making child support payments (if a single male). Investment and sharing consumption are the two ways families make

[27] In this section, I assume that skill levels are constant in adulthood, so I have removed period subscripts from parental wages and child care prices, compared to the model notation of section 2.

transfers to their children in this model; only investment is a choice. From ages $2J+1$ to L, the problem of the family is a standard lifecycle problem. At age $L+1$, all adults in the family die.

The Marriage Market New adults participate in a marriage market, where individuals randomly draw a potential spouse of the opposite gender from their cohort, as well as a preference shock ξ which is normally distributed with mean μ_ξ and standard deviation σ_ξ. This preference shock additively raises the value of the match. A female young adult with skill θ_f and potential spouse θ_m compares the expected value of being a single female and the expected value of being a married female, given $(\theta_m, \theta_f, \xi)$. A male young adult solves a similar problem. The expected utility of each option is found by taking expectations over the subsidy receipt outcome ($R = 1$ or 0) and the initial skill of their children θ_1, which are unknown when the marriage decision is made and are drawn immediately subsequent to family formation. The initial skill of the children is independent of family attributes, but the child care subsidy receipt probabilities are not. If both members of the potential match agree, a couple is formed. This marriage market happens once.[28] After the marriage market, adults make decisions as part of a family. All of the adults in a family are the same age, so that the age of an individual and the "age" of a family can be referred to interchangeably.

Parenthood Each family raises two children, with the same initial skill draw (single females and couples each raise both a daughter and a son). Whether parenting alone or in a couple, the lifetime utility of any individual contains a term that incorporates rational expectations about the lifetime utility of their children at the level of skill they begin adulthood with. In this altruism term, the child's expected lifetime utility at adulthood, conditional on the level of skill, is taken over outcomes in the marriage market, the initial skill of one's grandchildren, and subsidy receipt outcomes for one's children when they are parents, for both the male and the female child. A child support system exists, enforced by the government, where single males contribute a fixed amount, T_{cs}, that is redistributed lump-sum and equally to all single females.

Parenting families solve cost-minimization problems to determine how to finance investment, where the prices of investment inputs are distorted by labor income taxes, a gender-specific wedge on skill, τ_g, and a child care subsidy policy. The gender-specific wedge is included to avoid confounding the gender wage gap with calibrated parameters of the utility function. It penalizes the efficiency units of labor that an adult sells on the labor market.

Adult skill is the individual's skill outcome from early childhood. This endogenizes the skill distribution in the adult population. Adult wages, rather than being taken as given as in section 2, now arise as linear functions of skill. In the family problems, value functions are indexed by family structure: single females (SF), single males (SM), and married or cohabiting couples (MC). Family type, z_{str}, records the attributes of a family once all of the shocks in the environment have been realized; the attributes of a family differ from section 2 because I have now introduced child care subsidy receipt and the marriage preference shock. The elements of z_{str} are specified along with the corresponding family problem below.

[28] Supplementary Appendix B contains transition matrices across waves in the ECLS-B for the mother's marital status. In the data, it is extremely persistent when the child is under 5 years old.

Single Females A single female of type $z_{SF} \equiv \{\theta_1, \theta_f, \theta_{n,SF}, R\}$, chooses consumption, c_t, leisure $\ell_{f,t}$, and a child outcome θ_{kid} to solve:

$$\begin{aligned} V_{SF}(z_{SF}) &= \max_{\theta_{kid}, \{c_t, \ell_{f,t}\}_{t=J+1}^{L}} \sum_{t=J+1}^{L} \beta^{t-J-1} u_{SF}\left(\frac{c_t}{\Phi_{SF,t}}, \ell_{f,t}\right) + \mathbb{I}_{t=J+1} b V_{kid}(\theta_{kid}) \\ c_t &\leq w_f (1-\ell_{f,t}) + \mathbb{I}_{t\leq 2J} T_{cs} + T - X_{SF,t}(\theta_{kid}, w_f, \tilde{p}_{n,SF}, z_{SF}) \\ w_f &\equiv (1-\tau_y) w (1-\tau_f) \theta_f \end{aligned} \tag{26}$$

where $\ell_{f,t} \in [0,1]$, and w_f is the return to labor supply for single female with skill θ_f. The consumption equivalence (CE) scales $\{\Phi_{SF,t}\}_{t=J+1}^{L}$ vary over the lifecycle as children leave the household. Here, and in all the family problems, w is the return per unit of skill on the labor market before incorporating distortions from taxes and the gender-specific wedge.[29] If the single female works, then the static cost function of investment incorporates her after-tax market wage w_f, as well as the post-subsidy price of child care:

$$\begin{aligned} \tilde{X}_{SF,t}(w_f, \tilde{p}_{n,SF}, I_t) &= \min_{n_t, q_{f,t}} \tilde{p}_{n,SF} n_t + w_f q_{f,t} \\ s.t. \quad I_{SF}(\theta_{n,SF} n_t, \theta_f q_{f,t}) &\geq I_t \\ \tilde{p}_{n,SF} &\equiv (1-\tau_{n,SF}(z_{SF}, \cdot)) p_{n,SF} \end{aligned} \tag{27}$$

where $n_t, q_{f,t}, n_t + q_{f,t} \in [0,1]$, and $t = J+1$. The child care subsidy depends on family type as well as additional arguments which will be specified in section 5.1. The optimal level of investment is determined by solving (3), incorporating expenditures from (27). The resulting cost in is $X_{SF,t}(\theta_{kid}, w_f, \tilde{p}_{n,SF}, z_{SF}) \equiv \tilde{X}_{SF,t}\left(\cdot, I^*_{SF,t}\right)$.

Married or Cohabiting Couples Similarly, a couple with type $z_{MC} = \{\theta_1, \theta_f, \theta_m, \theta_{n,MC}, \xi, R\}$ chooses consumption, c_t, leisure $\ell_{f,t}$ and $\ell_{m,t}$ for each adult, and a child outcome θ_{kid} to solve:

$$\begin{aligned} V_{MC}(z_{MC}) &= \max_{\theta_{kid}, \{c_t, \ell_{f,t}, \ell_{m,t}\}_{t=J+1}^{L}} \sum_{t=J+1}^{L} \beta^{t-J-1} u_{MC}\left(\frac{c_t}{\Phi_{MC,t}}, \ell_{f,t}, \ell_{m,t}\right) + \xi + \mathbb{I}_{t=J+1} b V_{kid}(\theta_{kid}) \\ c_t &\leq w_f (1-\ell_{f,t}) + w_m (1-\ell_{m,t}) + T - X_{MC,t}(\theta_{kid}, w_f, w_m, \tilde{p}_{n,MC}, z_{MC}) \\ w_g &\equiv (1-\tau_y) w (1-\tau_g) \theta_g \quad \forall g \in \{m, f\} \end{aligned} \tag{28}$$

where $\ell_{g,t} \in [0,1] \, \forall g \in \{m, f\}$. The CE scales for couples $\{\Phi_{MC,t}\}_{t=J+1}^{L}$ vary over the lifecycle as children leave the household and also reflect the presence of more than one adult in the family. Market wages for each parent, w_g, are constructed the same way as for single females. If both parents work, then

[29] In fact, w acts to simply change units from skill to real quantities in the budget constraint. Its level is set equal to 1, which is equivalent to assuming linear production technology in labor for a goods producer.

their market wages enter into the cost function, and the optimal combination of inputs is found by solving:

$$\begin{aligned} \tilde{X}_{MC,t}\left(w_f, w_m, \tilde{p}_{n,MC}, z_{MC}, I_t\right) &= \min_{n_t, q_{f,t}, q_{m,t}} \tilde{p}_{n,MC} n_t + w_f q_{f,t} + w_m q_{m,t} \\ s.t. \quad I_{MC}\left(\theta_{n,MC} n_t, \theta_f q_{f,t}, \theta_m q_{m,t}\right) &\geq I_t \\ \tilde{p}_{n,MC} &\equiv \left(1 - \tau_{n,MC}\left(z_{MC}, \cdot\right)\right) p_{n,MC} \end{aligned} \tag{29}$$

where $n_t, q_{f,t}, q_{m,t}, n_t + \max\{q_{f,t}, q_{m,t}\} \in [0,1]$, and $t = J+1$. The optimal level of investment is determined by solving (6), incorporating expenditures from (29). The resulting cost from (28) is $X_{MC,t}\left(\theta_{kid}, w_f, w_m, \tilde{p}_{n,MC}, z_{MC}\right) \equiv \tilde{X}_{MC,t}\left(\cdot, I^*_{MC,t}\right)$.

Single Males A single male of type $z_{SM} = \{\theta_1, \theta_f, \theta_m, \theta_{n,SF}, R\}$ chooses consumption c_t and leisure $\ell_{m,t}$ to solve:

$$\begin{aligned} V_{SM}\left(z_{SM}\right) &= \max_{\{c_t, \ell_{m,t}\}_{t=J+1}^{L}} \sum_{t=J+1}^{L} \beta^{t-J-1} u_{SM}\left(c_t, \ell_{m,t}\right) + \mathbb{I}_{t=J+1} b \mathbb{E}\left(V_{kid}\left(\theta_{kid}\right) | z_{SM}\right) \\ c_t &\leq w_m\left(1 - \ell_{m,t}\right) - \mathbb{I}_{t \leq 2J} T_{cs} + T \\ w_m &\equiv \left(1 - \tau_y\right) w \left(1 - \tau_m\right) \theta_m \end{aligned} \tag{30}$$

where $\ell_{m,t} \in [0,1]$. Here w_m is after-tax wage of the single male with skill θ_m. The CE scales for single males equals 1, to reflect that single males live alone. Single males internalize the expected outcome of their child in an altruism term, given the child's initial skill, the female parent's productivity type, the productivity of child care for single female families, and whether or not the female parent receives the child care subsidy. A single male cannot use his own time to invest in his children, but he is required to make a lump-sum payment to the child's mother, T_{cs}. On the marriage market, where families are formed before θ_1 and R are realized, the single male compares his outside option with the expected value of marrying the women he met, taking expectations over initial child skill and subsidy receipt. Thus, the male makes the marriage decision using the same amount of information about the future as the female. Once a man in this economy rejects his randomly-drawn potential spouse (or is rejected), the child's outcome is no longer affected by his choices, but he does observe the child's initial skill and subsidy receipt outcome, and then correctly predicts his child's adult skill outcome. All of this is reflected in the value function of the single male.

Goods Sector Goods are produced by a representative firm that chooses a quantity of efficiency units of labor, H^d, subject to a linear production technology $Y = H^d$. In equilibrium, the wage rate per unit of skill, w, is equal to one, and the profits of the goods producer are zero.

Government The government collects revenue from labor income taxes τ_y to finance lump-sum transfers T and non-parental child care subsidies $\tau_{n,str}(\cdot)$. In particular, the government sets τ_y such that $\tau_y = \frac{T}{Y} + S_n$, where S_n is aggregate government expenditures on the subsidy for non-parental child care, expressed as a fraction of output Y. Note that output is equal to total pre-tax labor earnings. Because the wage rate per unit of skill is equal to one, aggregate output (pre-tax earnings) is determined by the distribution of skill and the corresponding labor supply decisions of consumers.

Non-parental Child Care Sector The non-parental child care sector provides the demanded aggregate quantity of non-parental child care for each family structure str at price $p_{n,str}$. This pre-subsidy price of non-parental child care is set as a constant fraction $\overline{\phi}_{str}$ of the average return to labor supply for all female parents in family structure $str \in \{SF, MC\}$, $\overline{w}_{str,f}$:

$$p_{n,str} = \overline{\phi}_{str}\overline{w}_{f,str} \tag{31}$$

Because this price depends on the return to labor supply, the price of non-parental child care adjust with the average level of skill in the economy and with the labor income tax. It also responds to the skill composition of parents in one- and two-parent families, which arises endogenously in the marriage market.[30]

Equilibrium Given an initial skill distribution, a distribution of preference shocks, a level of lump-sum transfers, a level of child support transfers, a gender-specific tax on skill, child care productivities for each family structure, and a child care subsidy policy for each family structure, a stationary steady-state equilibrium is:

1. individual marriage decisions, family choices, and family value functions,
2. expectations $V_{kid}(\theta_{kid})$ and $\mathbb{E}(V_{kid}(\theta_{kid})|z_{SM})$,
3. a goods producer choice of H^d,
4. a government choice of labor income tax,
5. a price of child care for each family structure,
6. and a skill distribution $\mu(\theta_{kid})$

such that agents optimize, expectations are rational, the government balances its budget constraint, the child care price is set using (31), the goods market clears, the labor market clears, and the distribution of skill is stationary.

5.1 Parameterization

In order to fully parameterize the general equilibrium model, I assume functional forms for utility and the distribution of initial child skill. I also assume a functional form for the child care subsidy and estimate its parameters. The functional forms for the skill accumulation technologies are specified as described in section 2; the parameter values for these technologies are estimated and reported in section 3.

[30]The productivity of child care will be calibrated in the baseline economy to be proportional to the skill distribution of mothers in each family structure (for details, see Table 11 in the Appendix).

Period Utility Functions Utility functions are seperately defined for single males, single females, and married couples:

$$\begin{aligned} u_{SM} &= \log(c_t) + \psi_{SM}\log(\ell_{m,t}) \\ u_{SF} &= \log\left(\frac{c_t}{\Phi_{SF,t}}\right) + \psi_{SF}\log(\ell_{f,t}) \\ u_{MC} &= \log\left(\frac{c_t}{\Phi_{MC,t}}\right) + \psi_{MC,f}\log(\ell_{f,t}) + \psi_{MC,m}\log(\ell_{m,t}) \end{aligned}$$

Utility functions for single females and married couples are as in section 4, but now correct consumption using age-indexed consumption equivalence scales.

Distribution of Initial Child Skill I assume that initial child skill is drawn independently and identically from $\pi_0(\theta_1)$, which I set as a uniform distribution. The empirical motivation for the i.i.d. assumption is based on the correlation between the cognitive score at 9 months of age and after-tax family income and family structure in the ECLS-B. Neither are significantly different from zero.

Baseline Child Care Subsidy The child care subsidy in the baseline equilibrium is disciplined with information on the Child Care and Development Fund (CCDF), one of the largest source of federal funding for child care subsidies in the United States.[31] This program allocates federal funds to states, which are in turn responsible for setting eligibility requirements within some statutory limits. The CCDF subsidy has three salient properties: there is a cutoff for eligibility based on family income and family structure; the subsidy is not an entitlement; and, the intensity of the subsidy is decreasing in family income (i.e., a poor family receives a higher proportional subsidy that a less poor family when both families receive CCDF aid).

To assign eligibility thresholds and subsidy receipt probabilities, I rely on the statistics reported in Herbst (2008), Table 2. In particular, I use the fraction of one- and two-parent families who are eligible for CCDF aid to discipline ι_{str}, and the fraction who receive aid, conditional on eligibility, to discipline receipt lottery probabilities $\pi_{R,str}$. In the baseline, therefore, I set $\{\iota_{SF}, \pi_{R,SF}\} = \{53, 0.28\}$ and $\{\iota_{MC}, \pi_{R,MC}\} = \{21, 0.09\}$.

I estimate the relationship between subsidy intensity and family income relative to the population median, $\tau_{n,str}(\cdot)$, using administrative data reported by states on CCDF recipients.[32] With this estimated relationship, the subsidy intensity within the model is then determined using the parent skill most closely correlated with family income. In single females this is of course the female parent. In couples, this is the male parent.

[31] Ho and Pavoni (2020) apply a Mirrleesian approach to child care subsidies with a baseline policy that also emphasizes the CCDF. They additionally incorporate the Dependent Care Tax Credit (DCTC), which they consider to be the other large source of price-related child care subsidies at the federal level. I do not model the tax credit programs like the DCTC. Ho and Pavoni (2020) do not include child skill accumulation or heterogeneity in family structures.

[32] Further details on the CCDF and the policy parameterization are in Supplementary Appendix A.

Specifically, the level of the subsidy is assigned in the model using the following rule:

$$\tau_{n,str}\left(z_{str},\iota_{str},\theta_{p50}\right) = \begin{cases} \hat{\beta}_0^{CC}+\hat{\beta}_1^{CC}\left(\frac{\theta_x}{0.85\times\theta_{p50}}\right)+\hat{\beta}_2^{CC}\left(\frac{\theta_x}{0.85\times\theta_{p50}}\right)^2 & if\, pctl\left(\theta_x\right)<\iota_{str}\, and\, R=1 \\ 0 & if\, pctl\left(\theta_x\right)\geq\iota_{str}\, or\, R=0 \end{cases}$$

where $x = f$ for $str = SF$ and $x = m$ for $str = MC$, and θ_{p50} is the median adult skill in the population. Families are eligible for the subsidy if the percentile of the relevant parent skill is below the threshold for their family structure, ι_{str}. Thus, if an ineligible family "wins" the lottery ($R = 1$, where R is an element of z_{str}) the subsidy level is still zero. When I estimate this relationship between subsidy intensity and family income in the data, I find that $\hat{\beta}_0^{CC} = 0.98$, $\hat{\beta}_1^{CC} = -0.29$, and $\hat{\beta}_2^{CC} = 0.03$.

Model Calibration The remaining parameters are divided into three categories: those set exogenously, those set as closed-form functions of other equilibrium objects, and those chosen to bring model moments as close as possible to the data. The first two categories are presented in Table 11 of the Appendix and explained there. The third category contains a set of nine parameters: the altruism coefficient, b, the marginal utility of leisure for each parent in each family structure, $\{\psi_{SF},\psi_{SM},\psi_{MC,m},\psi_{MC,f}\}$, the productivity of investment λ_{str} for each family structure, and the parameters governing the distribution of preference shocks, (μ_ξ,σ_ξ). The nine moments these nine parameters were chosen to match, and the calibration results, are reported in Table 7.

In the data column of Table 7, the first three rows contain the correlation of child skill outcomes with after-tax family income, the correlation of child skill outcomes with an indicator for being raised by a couple, and the fraction of parenting families who are married from Table 2. The fourth row is the set of average labor supplies for single males, single females, married males, and married females from the CPS, expressed as fractions of a weekly time endowment of 100 hours. These averages are computed for observations between the ages of 20 and 65 who are either employed or not in the labor force, and who are the head of household or married to the head of household.[33] The fifth and sixth rows are fractions of the weekly time endowment spent engaging in quality time for each resident parent in a given family structure from Table 1.

Although all of these parameters affect each of the moments in Table 7, in practice this calibration has three phases. First, I fix the marginal utilities of leisure to match labor supplies. Second, I choose b, λ_{SF}, and λ_{MC} to match the amount of parental time and the correlation coefficient for child skill outcomes and family income. Finally, I choose μ_ξ and σ_ξ to match both the correlation of child skill with family structure and the fraction of parenting families who are married couples. Because the altruism term also determines the relative value of marriage, b also affects the marriage rate. In addition, because the productivity of investment determines how efficiently investment expenditures translate to child skill outcomes, λ_{SF} and λ_{MC} also affect the correlation of child skill outcomes with family income and family structure. Thus, I iterate on the last two phases.

[33] The CPS data is from a pooled sample that covers 2000 and 2001. Here, I define "married" as those who are married and whose spouse is reported as being present. Married respondents with absent spouses do not qualify for the CPS sample of coupled parents.

I match all of the targeted moments quite well. The moment furthest from the data is the amount of parental time from single female parents, which is too low in the model. To match this higher amount of time in the model requires either raising the correlation of child skill and family structure or reducing the correlation of child skill and family income, because it requires either reducing λ_{SF} or raising b. Since the family time share parameter is the technology parameter estimated with the lowest degree of precision in section 3, I emphasize matching the correlation coefficients at the expense of the time level for single females.

Table 7: Calibration Moments

Moment	Source	Data	Model
Corr(θ^*_{kid}, Family Income)	ECLS-B	0.32	0.32
Corr(θ^*_{kid}, Family Structure)	ECLS-B	0.21	0.21
Fraction Married	ECLS-B	0.79	0.79
Average labor supply	CPS	$(0.37, 0.32, 0.39, 0.25)$	$(0.37, 0.32, 0.39, 0.25)$
Parental time: MC (m, f)	ECLS-B	(0.08,0.10)	(0.08,0.10)
Parental time: SF	ECLS-B	0.09	0.08

Table 7: model calibration moments, sources, data target, and model value in the baseline equilibrium. Data moments from the ECLS-B are from Tables 1 and 2. Entries in rows concerning labor supply and parental time are expressed as fractions of an individual's time endowment, which in the data is set to 100 hours per week. The parameter values that achieve the equilibrium described in this table are given in Panel C of Table 11 in the Appendix. ECLS-B refers to: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

Untargeted Moments Table 8 and Figure 3 compare the equilibrium moments of this model with untargeted moments in the data. These moments fall into two broad categories: investment elasticities and family inputs, presented in Table 8, and family formation patterns, illustrated in Figure 3.

Table 8 first reports the elasticities of the investment price and the marginal cost of skill, both in the model and from the ECLS-B (as reported in Table 6). These untargeted statistics are important for the responsiveness of the model to a child care subsidy; they are extremely close to the data for both family structures. It was not a foregone conclusion that simply including the estimated investment technologies in the model would yield such a close match. For example, in reality there may be many kinds of non-parental child care available, each with its own price and productivity, but the model presented in this section abstracts from this and has one kind of non-parental child care per family structure.[34] Table 8 indicates that the model response to changes in the child care subsidy will be in line with what we could expect the empirical population to do.

The last row of Table 8 shows that the random search marriage market captures a quantitatively reasonable relationship of assortative matching among spouses, as measured by parenting behavior within the family. That is, couples are formed in the model such that they choose investment inputs with a similar degree of correlation to the data, when both parents have positive labor supply. Figure 3 demonstrates that the model's relationship between family income and family structure is also quantitatively in line with the data. Overall, the model is parsimonious yet generates correlation in investment input choices which aligns

[34] See Supplementary Appendix A for types of non-parental child care used in the ECLS-B.

with its empirical counterparts, while also generating a similar pattern of marriage over the family income distribution.

Table 8: Untargeted Moments

	Moment	Single Females Data	Single Females Model	Married Couples Data	Married Couples Model
Elasticity of Marginal Cost	ϵ_{str}	0.44	0.45	0.33	0.33
and its Components:	$\frac{\text{Potential income}}{\text{Potential income - investment expense}}$	1.19	1.15	1.14	1.13
	σ_{str}	0.52	0.51	0.38	0.37
Investment Choices:	Corr(q_f^*, q_m^*)			0.22	0.26

This table: untargeted moments in the data and in the model. From top to bottom, the moments are: weighted means of the elasticity of investment's price with respect to the price of child care, the ratio of potential income to potential income net of investment expenditures, the elasticity of the marginal cost of investment, and the Pearson correlation coefficient of quality time from each parent within a couple when both parents work. The data uses cross-sectional survey weights and is computed for the estimation sample, and the model uses population weights for each family type. Data moments for the elasticity of marginal cost and its components are from Table 6. Data Source: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

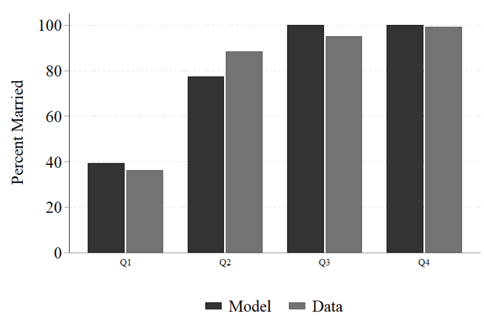

Figure 3: Marriage Rate by Family Income Quartile

This figure: family structure by family income quartile, model and data. Model moment: percent who are two-parent families in each family income quartile. Data moment: percent who are married or cohabiting families by income quartile, using cross-sectional primary caregiver weights, waves 1-3. Data source: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

6 Policy Experiments

How important for aggregate welfare is subsidized child care, in the context of the family heterogeneity emphasized in this study? To address this question, I first consider the effects of the current CCDF program compared to an alternative environment without such a subsidy. I then examine the effects of raising eligibility thresholds, ι_{str}, and subsidy receipt rates, $\pi_{R,str}$, for one family structure at a time, as well as the effects of switching to a child care subsidy program with both universal eligibility and guaranteed receipt. In the baseline, eligibility is restricted to relatively poorer families, and uptake rates are low. Along both of these dimensions, family structure plays a role in determining subsidy receipt. All of these attributes of the baseline economy reflect CCDF program design features.[35] The policy reforms that I examine here, therefore, are a class of practical amendments to the CCDF program, which provides real-world context for the main implications of the analysis.

Welfare gains are computed in the stationary steady state, behind the veil of ignorance at birth. This metric takes into account how the subsidy affects the distribution of families one can be born into, the distribution of potential spouses one may meet at adulthood, and the subsequent different experiences of parenting (both in terms of parenting alone or in a couple, and subsidy receipt). It also means that I evaluate each equilibrium from the perspective of the same "person." Attributes of the economy computed after eliminating the baseline child care subsidy, both in general equilibrium and in several kinds of partial equilibrium, are reported in Table 9. Economies in stationary equilibrium after various reforms to the baseline child care policy are reported in Table 10. Below I highlight the main implications of these policy experiments.

The Value of Child Care Subsidies Eliminating the baseline subsidy implies welfare losses of 1.63 percent of lifetime consumption (column 2 of Table 9). These losses arise because the types drawn at birth, and at adulthood, become more payoff-relevant when child care subsidies are no longer providing partial insurance against bad draws. This increase in the risk faced at birth is reflected in the increase in the correlation of child outcomes with family income and family structure. The drop in the stock of skill due to investment becoming more expensive is reflected in the 1.08 percent drop in output. Considering that eliminating this program removes coverage for 4.59 percent of families raising young children, the consequences are strikingly large.

The Role of General Equilibrium In general equilibrium, the economy adjusts to mitigate the welfare losses due to eliminating the child care subsidy. These losses are incurred through changes in the endogenous distribution of skill. The adjustments which mitigate the losses are: the drop in the labor income tax due to lowered government expenditures, which allows each unit of skill to receive a higher return; the decrease in the price of child care, which tracks the decrease in parental skill in equilibrium; and, finally, adjustments to the altruism term, so that parents internalize the changing return to skill for their children.

To highlight these opposing forces, the relative importance of endogenizing the price of child care, labor income taxes, the altruism term,[36] as well as the skill distribution, can be seen by comparing column 2 with columns 3 to 6 in Table 9. When the price of child care or labor income taxes are held fixed, but the skill distribution and altruism term are endogenized, the economy responds slightly more strongly to the loss of the child care subsidy (columns 3 and 4): welfare losses increase to 1.97 and 2.06 percent, respectively. A larger increase is seen when moving to column 5, where adjustments to the altruism term of parents are shut off. There, welfare losses reach 2.54 percent of lifetime consumption. All of this is due to not allowing the economy to adjust to the changing skill distribution. To see that explicitly, note that if the skill distribution itself is also held fixed, as in column 6, welfare losses are quite small by comparison: 0.83 percent. This case can be interpreted as the partial equilibrium response of the economy to a loss of the baseline child care

[35] See Supplementary Appendix A for an extended description of the CCDF.

[36] This refers to the endogenous component, $V_{kid}(\theta_{kid})$, of the full altruism term, $bV_{kid}(\theta_{kid})$, where b is a fixed parameter.

subsidy. Once children grow up to raise their own families, these losses compound unless prices, taxes, and the altruism term are allowed to adjust. This can be seen by comparing column 6 with columns 5 and 2.

Table 9: The Value of Baseline Child Care Subsidies

	(1)	(2)	(3)	(4)	(5)	(6)
Fixed Objects	Baseline	GE	$p_{n,str}$	$p_{n,str}, \tau_y$	$p_{n,str}, \tau_y, V_{kid}(\theta_{kid})$	$p_{n,str}, \tau_y, V_{kid}(\theta_{kid}), \mu(\theta_{kid})$
Corr(θ^*_{kid}, Family Income)	0.32	0.35	0.35	0.35	0.36	0.35
Corr(θ^*_{kid}, Family Structure)	0.21	0.24	0.24	0.24	0.25	0.24
Δ Output (%)	0	-1.08	-1.30	-1.31	-1.95	-0.89
Δ Welfare	0	-1.63	-1.97	-2.06	-2.54	-0.83

This table: equilibrium attributes and welfare changes after eliminating the baseline child care subsidy. Changes are relative to the baseline economy, which is reported in column 1. Column 2 is the general equilibrium with no subsidy. Columns 3-6 also have no subsidy; moving from left to right, each column holds fixed an additional equilibrium object to highlight its role (fixed objects in a column are listed). Recall that $V_{kid}(\cdot)$ is the endogenous component of the full altruism term for the family investment problems, and $\mu(\cdot)$ is the distribution over skill outcomes for kids. Table rows from top to bottom show the correlation of child skill outcomes with family income, the correlation of child skill outcome with family structure, the percent change in total output compared to the baseline, and the welfare change behind the veil of ignorance in consumption equivalent units relative to the baseline.

Analyzing Alternative Child Care Policies For each policy in Table 10, including the baseline, Panels B and C display additional statistics relative to Table 9. In Panel B, I report the percent of families parenting young children who receive the subsidy, the welfare gain per additional recipient family, and the size of the child care subsidy program as a percent of output. This allows for a more direct comparison of welfare gains across columns. Panel C contains information about non-parental child care levels, the composition of investment inputs, and changes in child skill outcomes, for each family structure. These statistics clarify how child care subsidies affect child skill accumulation.

Attributing Welfare Losses from Eliminating the Baseline Subsidy In columns 2 and 3 of Table 10 I report the effect of removing eligibility for one family structure at a time, while maintaining the baseline subsidy for the other family structure. Compared to removing coverage for couples (column 2), removing the subsidy for single females (column 3) leads to a larger increase in risk and a larger drop in output (Panel A). This larger drop in output for column 3 is one reason that the expense of the program as a fraction of output, S_n, is essentially the same across columns 2 and 3, despite column 2 subsidizing more families (Panel B).[37,38] The other reason is that in this model (as in the data) couples use a more expensive form of child care than single females, so that subsidizing couples is more expensive. The relatively higher expense and lower payoff of subsidizing couples is reflected in the fact that removing couples from coverage generates half the welfare loss per family than removing single females generates.

[37]The equilibrium labor income tax rate, τ_y, is equal to the percent of output spent on the child care subsidy, S_n, plus the fraction of output spent on lump-sum transfers, which is calibrated to 8 percent (see Table 11).

[38]The expense of the program in the baseline economy of column 1, as a fraction of GDP, aligns well with its empirical counterpart: for the 2002 FY, I calculate the size of the CCDF program to be 0.061 percent of 2002 US GDP. See Supplementary Appendix A for details.

In Panel C, one can see that when the subsidy is removed for a given family structure, those families use less child care on average. This occurs for two reasons: first, because families targeting a given level of investment substitute non-parental child care with family time once the relative price of n increases; and, second, because families lower their target θ^*_{kid}. Families lower the target skill level of their child (and thus choose a lower level of investment) not only because investment is more expensive, but also because in equilibrium the skill distribution of parents has a lower mean, thus lowering family income and further contracting the average family's choice set for their child's outcome. The average percent change in child skill outcomes for each family structure, $\Delta\theta^*_{kid}$, thus reflects both the direct effect due to the change in the expense of achieving any level of skill, and the general equilibrium effect of lower family resources due to a downward shift in the skill of the parents. This general equilibrium effect occurs for children raised by couples in column 3, for example, because the skill composition of the entire adult population has fallen relative to the baseline to a sufficient extent (as reflected by the drop in output).

Table 10: Reforms to Baseline Child Care Subsidy Design

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Child Care Subsidy		Baseline	$No\,MC$	$No\,SF$	ι_{MC}	ι_{SF}	$\pi_{R,MC}$	$\pi_{R,SF}$	Full
Panel A:	Corr(θ^*_{kid}, Family Income)	0.32	0.32	0.35	0.32	0.31	0.27	0.23	0.17
	Corr(θ^*_{kid}, Family Structure)	0.21	0.21	0.25	0.22	0.20	0.24	0.13	0.15
	Δ Output (%)	0	-0.19	-0.86	0.39	0.25	2.45	2.59	10.18
	Δ Welfare	0	-0.30	-1.31	0.55	0.33	3.78	3.87	15.37
Panel B:	Subsidized (%)	4.59	3.09	1.50	10.22	7.34	19.72	12.67	100
	$\widehat{\Delta W}$	0	-0.20	-0.42	0.10	0.12	0.25	0.48	0.16
	S_n (%)	0.06	0.03	0.03	0.12	0.09	0.38	0.14	1.25
Panel C:	SF: n^*	0.37	0.37	0.32	0.37	0.40	0.37	0.51	0.61
	$\frac{q^*_f}{n^*}$	0.29	0.29	0.33	0.29	0.28	0.28	0.18	0.15
	$\Delta\theta^*_{kid}$ (%)	0	0.11	-5.52	0.09	1.60	-0.96	15.44	20.70
	MC: n^*	0.30	0.29	0.30	0.31	0.30	0.38	0.29	0.56
	$\frac{q^*_f}{n^*}$	0.39	0.40	0.40	0.37	0.39	0.34	0.37	0.18
	$\Delta\theta^*_{kid}$ (%)	0	-0.37	-0.43	0.63	0.05	2.40	1.28	8.88

This table: general equilibrium attributes and welfare changes for the stationary steady state under various child care policy designs, relative to the baseline economy that is reported in column 1. Column 2 removes the subsidy to couples, and column 3 removes the subsidy to single females. Column 4 is universal eligibility for couples, column 5 is universal eligibility for single females, column 6 is guaranteed receipt for couples, and column 7 is guaranteed receipt for single females. Column 8 is universal eligibility and guaranteed subsidy receipt for all families. Table rows from top to bottom show the following. In Panel A: the correlation of child skill outcomes with family income, the correlation of child skill outcome with family structure, the percent change in total output compared to the baseline, and the welfare change behind the veil of ignorance in consumption equivalent units relative to the baseline. In Panel B: the percent of families raising young children who receive the subsidy in equilibrium, welfare gains per additional subsidy recipient family, and the expense of the child care subsidy as a percent of output. In Panel C: for single females, the average amount of child care used as a fraction of the child's time endowment, the average ratio of female parent time and child care time, and the percent change in the average child skill outcome for the given family structure, relative to the baseline. For married couples, the same statistics are reported in the same order.

Targeted Welfare-Improving Expansions of Current Program If a policy maker is considering expanding the child care subsidy program, and deciding between expanding eligibility or improving uptake for those already eligible, where should they direct their efforts? In columns 4 to 7 of Table 10, I report welfare gains and the attributes of equilibrium after expanding the eligibility percentile cutoff ι_{str} or uptake rate $\pi_{R,str}$ to their highest possible values (100 and 1, respectively), for one family structure at a time. In each column, I hold the other child care policy instruments fixed at their baseline levels and allow all of the equilibrium objects to adjust. In this framework, welfare changes are monotonic in these instruments, so setting them to their highest possible value highlights the qualitative possibilities of an entire class of reforms for generating welfare gains. In column 8, I report the same statistics for the most generous policy I consider: universal eligibility and guaranteed receipt for all families.

Holding uptake rates fixed, columns 4 and 5 of Table 10 show that increasing the eligibility threshold of either family structure has limited potential for increasing welfare: as reported in Panel A, a universal eligibility policy for couples yields a 0.55 percent gain in welfare, while for single females the gain is 0.33 percent. These welfare gains are low because such a policy does not provide much insurance against a bad draw at the beginning of life, and because expanding eligibility sends funds to families who do not substantially change their behavior in response to the policy (the increases in child skill accumulation, and the resulting increase in output, are modest in both columns 4 and 5).

Panel B shows that the welfare gain per additional recipient family are in fact higher for the policy expanding to universal eligibility for single females. This is because the effect of the program on output, while larger in column 4 than in column 5, is not in fact proportional to the expansion of the number of recipient families in column 4. This occurs because the investment levels chosen by couples do not respond very strongly to the subsidy: these families tend to shift the composition of their investment input bundle while only slightly increasing the targeted level of investment. As a consequence, universal eligibility for couples costs 33 percent more as a fraction of output than a similar program directed at single females.

Panel C illustrates that, with universal eligibility, the family structure that is now more likely to receive the subsidy tends to on average increase the use of child care relative to the baseline economy. At the same time, the skill outcomes of children raised by the targeted group increases on average, illustrating that the change in relative prices for investment inputs is accompanied by a decrease in the expense of each level of investment, which raises θ^*_{kid}. Skill outcomes for the untargeted group also increase slightly because, in equilibrium, all parents have higher skill and this leads to higher outcomes for children. These increases in child skill outcomes are modest, which is reflected in the modest increases in output.

Next, in columns 6 and 7 of Table 10, I compare the effects of expanding the probability of subsidy receipt to 1 for couples or single females, respectively. This exercise maintains the eligibility cutoffs at their baseline values. Panel A shows that increasing the rate of subsidy receipt lowers risk in the economy and raises output. In contrast to expanding eligibility, increasing uptake rates with the current eligibility rules has the potential to substantially increase welfare. Guaranteed receipt for couples increases aggregate welfare by 3.78 percent (column 6); guaranteed receipt for single females increases aggregate welfare by 3.87 percent (column 7).

As is apparent in Panel B, the expansion of uptake rates for couples is much more expensive in terms of resources than the analogous expansion for single females: subsidies directed to couples are more expensive due to their higher child care price, and the child skill outcomes for children raised by couples respond less strongly to the subsidy. The fact that column 6 represents a more expensive subsidy, which leads to lower increases in the stock of skill and therefore output in equilibrium, is reflected in the lower welfare gains per additional recipient family in column 6 compared to column 7.

The guaranteed subsidy receipt policies explored in columns 6 and 7 have stark effects on the use of child care, with couples allocating about 8 percentage points more of the child's time endowment to non-parental child care (column 6), and single females allocating about 14 percentage points more of the child's time endowment to non-parental child care (column 7). However, these generous subsidies do not drive family inputs to zero; the average ratio of female parent time to child care time remains well above its lower bound. The large expense of the program in column 6 cancels out increases in skill for the population of parents, and the skill outcomes of the untargeted group's children fall in equilibrium. This stands in contrast to the policies considered in columns 4 and 5, which led skill outcomes to slightly increase even for children of the untargeted group. A guaranteed subsidy to low-skill single females, by contrast, raises the outcomes of their children and raises the overall equilibrium skill composition of parents in the economy to a sufficient extent that children of couples also see skill increases, because parents in those families now tend to have more disposable resources to invest. This happens because, compared to couples, the investment decisions of single female families respond more strongly to child care subsidies due to the technology they use to generate investment. This difference in responsiveness across family structures is summarized by the differences in marginal cost elasticities emphasized in section 4. Among the policies considered in Table 10, the generosity and targeted nature of the programs in columns 6 and 7 make this difference across family structures the most stark.

The most generous child care subsidy in Table 10 is universal eligibility with guaranteed subsidy receipt for all families (column 8). This policy lowers the correlation coefficient of child outcomes and family income the most among the policies I examine here, but does not affect the correlation of child outcome and family structure as much as single female-specific subsidies (column 7). Nevertheless, the gains in output and welfare are the highest of the policies I consider: 10.18 and 15.37 percent, respectively. The magnitude of these welfare gains is not surprising: children in this model are expensive, families are altruistic but varied in their ability to finance investment, and adult skill is very important for one's well-being. Child care subsidies encourage child skill accumulation by making it cheaper to invest in young children; the government has a special technology (the tax system) which allows it to then recoup the cost via labor income taxes.

Panel B illustrates that the resources necessary to finance the program of column 8 are indeed large, so that after correcting for the mass of subsidy recipients the policy of column 8 exhibits lower (although positive) gains. In Panel C, one can see that because universal eligibility and guaranteed receipt is so generous, the subsidy increases the quantity of non-parental child care used in this economy to very high values. Nevertheless, average family inputs are not driven to zero, and child outcomes do improve on average for both family structures.[39]

[39] When the levels of child care being chosen by families are large, as in columns 6, 7, and 8, it is possible that the child's time constraint may be binding. In that case, in the model I solve a system of equations that is consistent with the child's time constraint

In the foregoing analysis, I compared expansions of eligibility cutoffs and expansions of subsidy receipt rates and found that expanding subsidy receipt rates is more beneficial, especially when this is targeted at one-parent families. The interpretation of a policy which increases eligibility cutoffs is relatively straightforward, because this involves an instrument that policymakers can control directly. By comparison, increasing subsidy receipt is a more nuanced policy objective. What does it take to increase uptake of CCDF aid among eligible families? This depends on the reasons for which the measured uptake rates are low. One interpretation of these rates is that they reflect rationing, because aggregate funding caps are too low to subsidize all eligible families. In that case, increasing subsidy receipt in the model maps into increasing aggregate funding for the CCDF. At the same time, low measured uptake could also reflect the complexities of a system which is difficult for families to navigate: they may not know they are eligible, or they may not understand the steps which are necessary to receive aid. In that case, increasing clarity in the rules which govern eligibility and receipt for CCDF aid, as well as funding efforts to inform families about their options, would be approaches by which one could increase uptake rates. In practice, it is likely that both low aggregate funding and information frictions need to be addressed in order to achieve an increase in subsidy receipt.

7 Conclusion

This paper studies child skill accumulation in one- and two-parent families, and how this interacts with child care subsidies. I first specify two skill investment functions and, based on hitherto underused ECLS-B data, empirically test and reject the hypothesis that one- and two-parent families use the same technology to invest in their children's skill. In particular, I find that the investment function of single females emphasizes non-parental child care more than the investment function of married couples. Using the estimated elasticity of investment's price with respect to the price of child care, I highlight an implication of the empirical finding for a widely used policy intervention shown to affect child skill accumulation: child care subsidies. The estimation implies that one-parent families are more responsive to child care subsidies than two-parent families.

To perform policy experiments, I expand the model to include an endogenous skill distribution among adults, endogenous family formation, and a child care subsidy which is financed by a labor income tax. The baseline child care subsidy is modeled to reflect the design of the current Child Care and Development Fund, one of the largest sources of federal funds for price-based child care subsidies in the United States. I find that the current policy yields sizable welfare gains, especially through its coverage of families headed by single females. The general equilibrium framework I construct allows for equilibrium adjustments to dampen the welfare losses from removing the subsidy. I also examine the potential benefits of several program expansions, and find that expanding uptake among single female families yields the highest welfare gains per additional subsidy recipient family. However, because single female families make up about 21 percent of those raising young children in the baseline economy (and marriage rates are insensitive to the subsidy policy in the framework I analyze here) limiting access to only single female families also limits the potential

binding, rather than the standard system corresponding to the interior solution. In terms of feasibility, there is an upper bound on investment that is determined by the time-constrained system of equations for investment inputs.

gains from the policy, because it excludes many families who would also benefit. A policy with universal eligibility and guaranteed subsidy receipt for all families achieves welfare gains of more than 15 percent of lifetime consumption.

Although the estimation, analysis, framework and results of this paper make novel contributions to the literature on skill accumulation during early childhood, some caveats should be noted. For example, in reality parents are able to make wealth transfers to their children, which could complement or substitute transfers through skill investment during early childhood. In addition, the child care producer in this paper's general equilibrium framework is a simplified version of the market for child care. Large-scale policies that affect the use of child care can feasibly have an effect on the economy by redirecting labor towards providing paid child care, and the productivity of child care time (not just its price) may also respond directly to policy. Who decides to make a living looking after other people's children? How can we encourage qualified workers to make this choice? Answering these questions would further inform child care policy design.

Appendix

General Equilibrium Model: Further Details on Parameterization All parameters for the model of section 5, besides those of the investment technologies, are presented in the panels of Table 11: fixed parameters in Panel A, "fixed" calibrated parameters in Panel B, and "free" calibrated parameters in Panel C.

Beginning with Panel A, the discount (patience) factor is set to an annual value of 0.96 to match the presumed risk-free interest rate. There are 13 periods of life, each 5 years long, 4 of which are spent in childhood and 9 in adulthood. The first period of life corresponds to early childhood. The gender-specific skill penalty is set so that $\frac{1-\tau_f}{1-\tau_m}$ is equal to the ratio of weekly earnings for full-time women and full-time men, aged 16 and over (Bureau of Labor Statistics of the United States (2002)), with the male wedge normalized to zero. The CE scales are taken from the 1994 scales from the Organization for Economic Co-operation and Development (OECD). These scales assign a value of 1 for the first adult and 0.5 for the subsequent adults; for each dependent the weight is 0.3.[40] The remaining parameters in Panel A specify the child care subsidy in the baseline economy (see Supplementary Appendix A for details). Although not restated here, the technology parameters estimated in section 3 are also taken as fixed parameter inputs into the model of section 5.

Panel B presents the parameters for which there is a closed-form expression as a function of other equilibrium objects. The level of lump-sum transfers T and the level of child support payments T_{cs} are set to 8% of output and 45% of the average per-family transfer, respectively. The first empirical target is from the ratio of government transfers to persons for federal benefits from social insurance funds, Supplemental Nutrition Assistance Program (SNAP), supplemental security income, refundable tax credits, and "other" (which includes payments to nonprofit institutions and student loans, among other categories) to GDP from the National Income and Production Accounts (NIPA) tabulations for 2001. The second is the ratio of average child support payments owed (per month per capita) to average government transfers (per month per family) from the Census Bureau (Grall (2003)).

Next, the parameters governing the price of child care relative to mother wages are set so that in the baseline equilibrium, the after-tax and post-subsidy ratio of child care prices to mother wages for mothers who work matches moments from the ECLS-B. This implies an average ratio of pre-subsidy child care prices and mother wages for all mothers in equilibrium. This latter ratio is what I consider to be the environment primitive, and I hold it fixed at the baseline value as I vary the child care policy design. The last set of parameters in Panel B are the productivities of child care for single females and married couples, respectively. These are set so that, in the baseline equilibrium, the same moment that I used to identify the technology share parameters holds in the model; this moment assumes that the average log of productivity ratios is zero within each family structure. The resulting calibrated productivity levels for child care are then held fixed as I vary the child care subsidy policy design away from the baseline.

[40]The CE scales adjust money spent on consumption into units of consumption for each member of the household. Once children leave the family, the CE scale for single females goes back to 1, and the scale for couples falls to 1.5.

Panel C reports the calibrate parameters which I set so that model moments match moments from the data. They are assigned using a calibration procedure which is described in the main text.

Table 11: Parameter Values

	Symbol	Name	Source	Value
A. Fixed:	β	Patience	Assumption	0.96^5
	L	Periods of life	Death after 65	13
	J	Periods of childhood	20-year childhood	4
	(τ_f, τ_m)	Gender wage gap 2001	BLS report	$(0.24, 0)$
	$\{\Phi_{SF,t}\}_t$	CE Scales	OECD	$\{1.6\}_{t\leq 2J}, \{1\}_{t>2J}$
	$\{\Phi_{MC,t}\}_t$	CE Scales	OECD	$\{2.1\}_{t\leq 2J}, \{1.5\}_{t>2J}$
	ι_{str}	CCDF Frac. Eligible	Herbst (2008), T.2	(21,53)
	$\pi_{R,str}$	CCDF Receipt	Herbst (2008), T.2	(0.09,0.28)
	$\hat{\beta}_j^{CC},\ j \in \{0,1,2\}$	CCDF Subsidy	CCDF Admin. Data	(0.98,-0.29,0.03)
B. Calibrated: (fixed)	T	Transfers	NIPA	8% of output
	T_{cs}	Child Support	Census, NIPA	45 % of T
	$\overline{\phi}_{SF}, \overline{\phi}_{MCF}$	$\frac{p_{n,str}}{w_{f\,str}}$	Post-subsidy price ratios	0.20, 0.31
	$\theta_{n,SF}, \theta_{n,MC}$	Prod. of child care	Ave. relative prod.	0.27, 0.34
C. Calibrated: (free)	ψ_{SM}, ψ_{SF}	MU_ℓ Singles	See Table 7	$1.455, 1.495$
	$\psi_{MC,m}, \psi_{MC,f}$	MU_ℓ Couples		$0.730, 0.775$
	b	Altruism coefficient		$0.320 \times \beta^J$
	$\lambda_{MC}, \lambda_{SF}$	Prod. of investment		$7.950, 3.890$
	μ_ξ, σ_ξ	Pref. shock distribution		$9.250, 11.100$

Table 11 presents model parameters in 3 groups: (A) fixed parameters whose values are exogenous to attributes of the model equilibrium; (B) calibrated parameters whose values are determined in the model equilibrium but which are perfectly identified by a single model equilibrium moment; and (C) free parameters whose values are chosen so that equilibrium model moments match data moments, but which are not perfectly identified by a single model moment. ECLS-B Data Source: Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Birth Cohort (ECLS-B), Longitudinal 9- Month-Kindergarten 2007 Restricted-Use Data File.

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