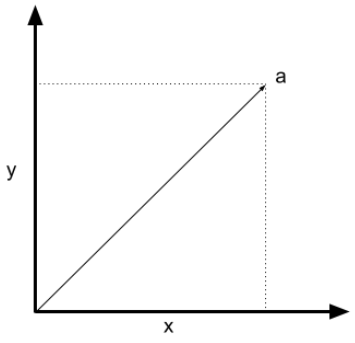
Topic 1 - Graphics Overview

Vectors - 1

Magnitud | Norm



2D

$a = (x_a, y_a)$ $\quad\quad$ $\|a\| = \sqrt{x_a^2 + y_a^2}$

3D

$a = (x_a, y_a, z_a)$ $\quad\quad$ $\|a\| = \sqrt{x_a^2 + y_a^2 + z_a^2}$

Unit Vector

$\hat{a} = (x_u, y_u)$ $\quad\quad$ $\sqrt{x_u^2 + y_u^2} = 1$

$\frac{x_u}{y_u} = \frac{x_a}{y_a}$ $\quad\quad$ $x_u = \frac{x_a}{\|a\|}$ $\quad\quad$ $y_u = \frac{y_a}{\|a\|}$

- The magnitude of this vector is equal to 1
- It's used to calculate the direction of a vector
- Setting a vector to length 1 is called **normalizing** it. Dividing the vector by its length

Example:

$a = (3, 4)$

$\|a\| = 5$ The **length** of the vector is 5

$\hat{a} = (0.6, 0.8)$ The **direction** of the vector is (0.6,0.8)

Defining a vector with unit vector

A vector can be expressed as its unit vector (a') **scaled** by its magnitude ($\|a\|$)

$a = (x_a, y_a)$

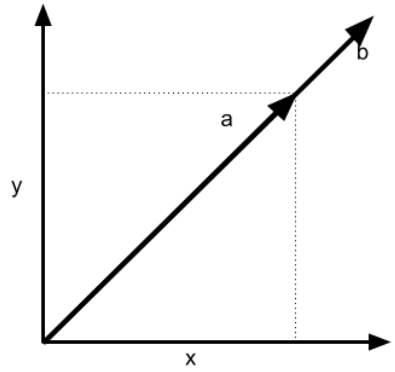
$a' = a / \|a\|$

$a = a' * \|a\|$

Vectors -2

Scaling

- Scaling a vector creates another vector with the **same direction** but **different magnitude**



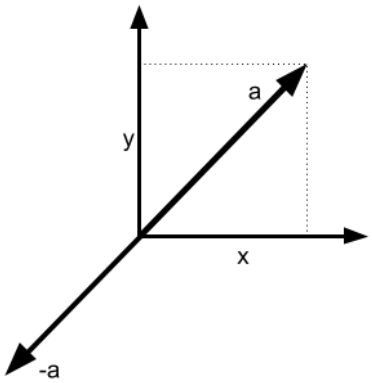
$b = 1.5 * a$

$x_b = 1.5 * x_a$

$y_b = 1.5 * y_a$

Negating

- If the vector its scaled by -1, the vector will point to the opposite direction

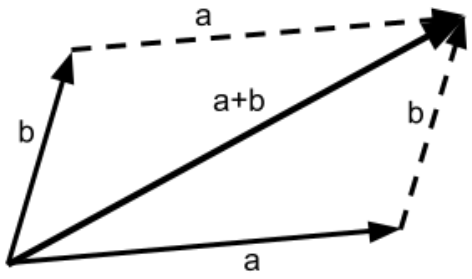


$-a = -1 * a$

$-a = (-x_a, -y_a)$

Vector Addition

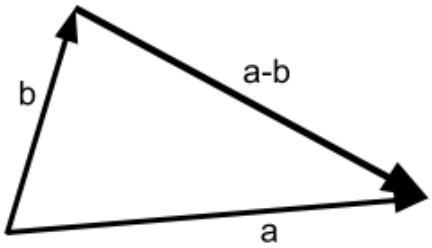
- Adding two vectors creates a new vector with a **new magnitude and direction**
- It's **commutative**



$a + b = (x_a + x_b, y_a + y_b)$

Vector Subtraction

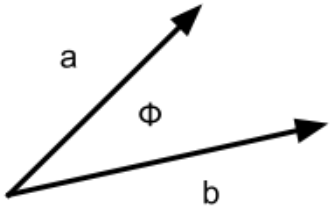
- Subtracting two vectors creates a new vector with a **new magnitude and direction**
- It's **not commutative**



$a - b = (x_a - x_b, y_a - y_b)$

Vectors - 3

Dot Product

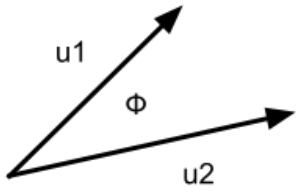


- Happens between **two vectors** and the result is a **scalar**
- It is **commutative**

$a \bullet b = x_a * x_b + y_a * y_b$

$a \bullet b = \|a\| * \|b\| * cos(\theta)$

Dot Product of unit vectors



- The dot product of a unit vector by itself is equal to one
- The angle between two unit vectors increases as it reaches 90 degrees the dot product gets closer to 0

$u_1 \bullet u_2 = cos(\theta)$

Cross product

- The angle between two unit vectors increases as it reaches 90 degrees the cross product gets closer to 1
- It is **not commutative**

$a \times b = \|a\| * \|b\| * sin(\theta) * n$ where n is the unit vector

$\|a \times b\| = \|a\| * \|b\| * sin(\theta)$

$a \times b = [(y_a * z_b - z_a * y_b), (z_a * x_b - x_a * z_b), (x_a * y_b - y_a * x_b)]$

Formula:

$(a1, a2, a3) \times (b1, b2, b3) = (a2b3 - a3b2, \quad a3b1 - a1b3, \quad a1b2 - a2b1)$

Objects and Meshes

- Objects are made by points
- Point join to create edges
- Edges join to create polygons
- Polygons joint to create Meshes (surfaces)

Polygons are good building block for 3D graphics because they **are simple** and a computer can **manage a lot of them**

Primitives are a cube, sphere, cylinders, planes.