CMPT 732-G200 Practices for Visual Computing

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Images

- An array of pixels

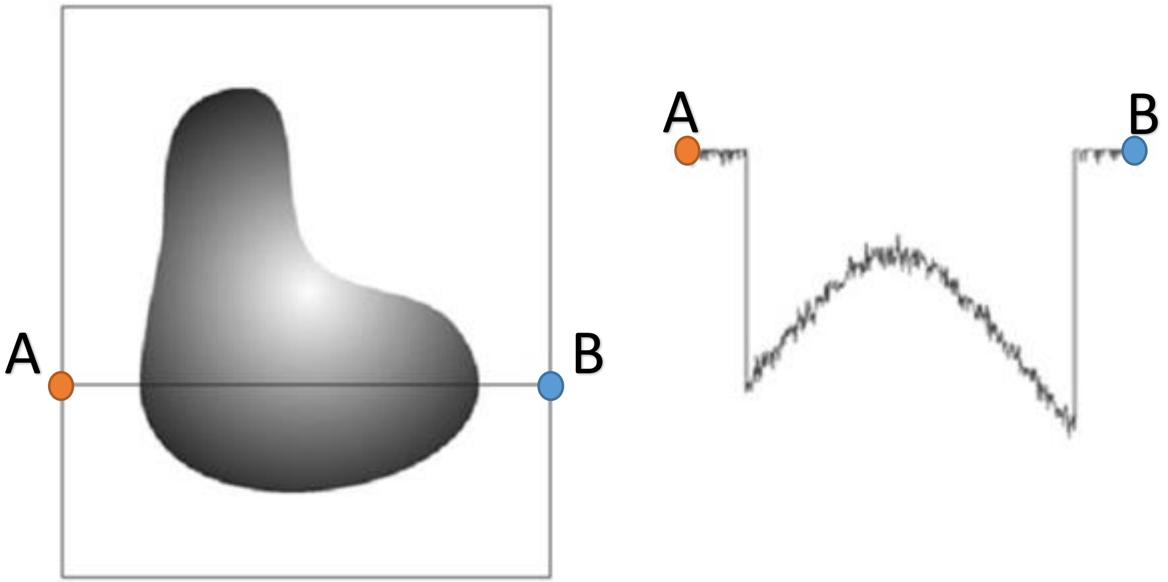
Images

- An array of pixels

- Each pixel has integer values
  - 1 value for grey scale
  - 3 values for RGB images

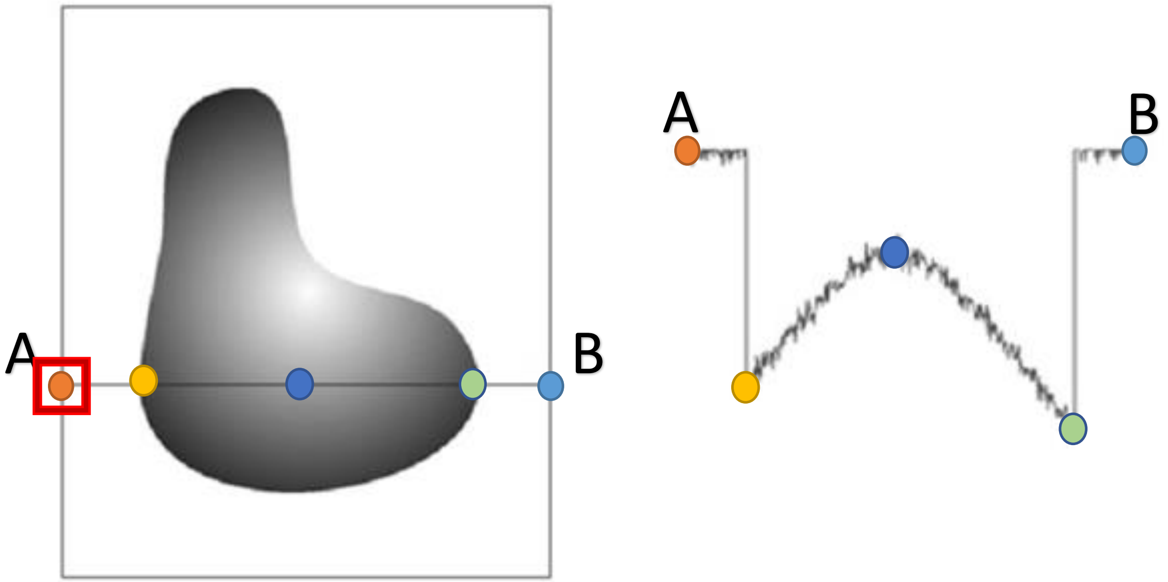
Digitizing Images

- Images are capturing continuous phenomena

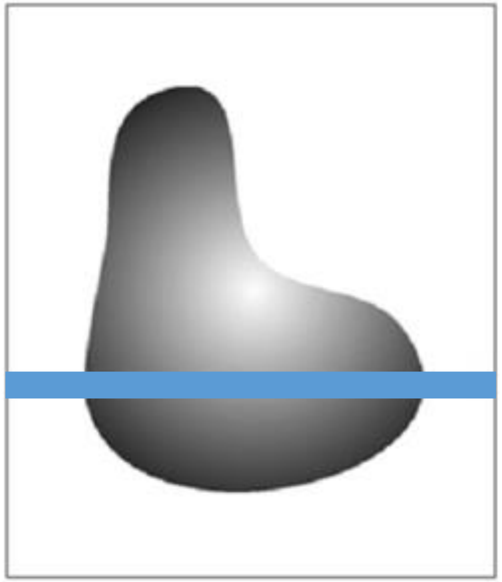


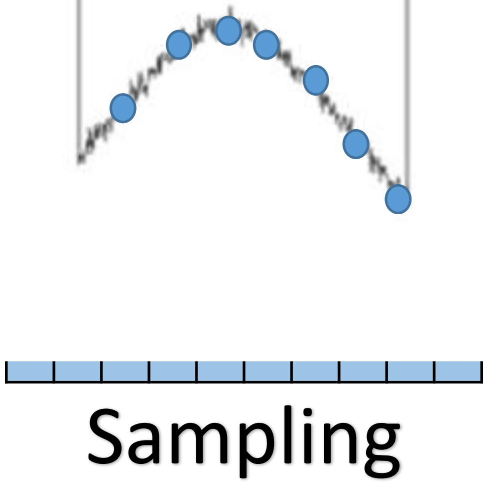
Digitizing Images

- Images are capturing continuous phenomena

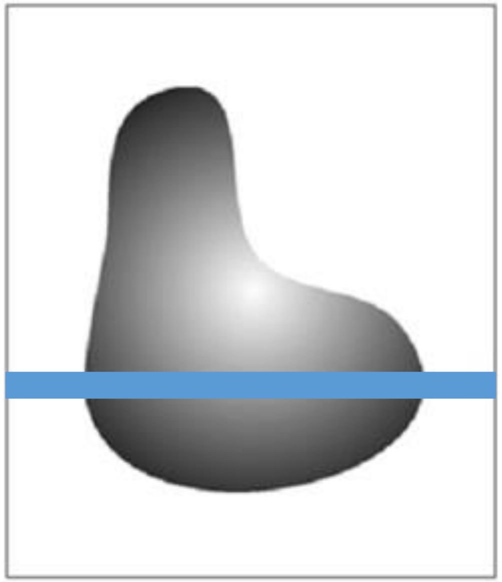


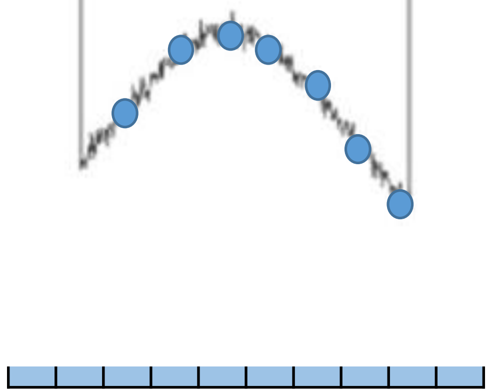
Sampling



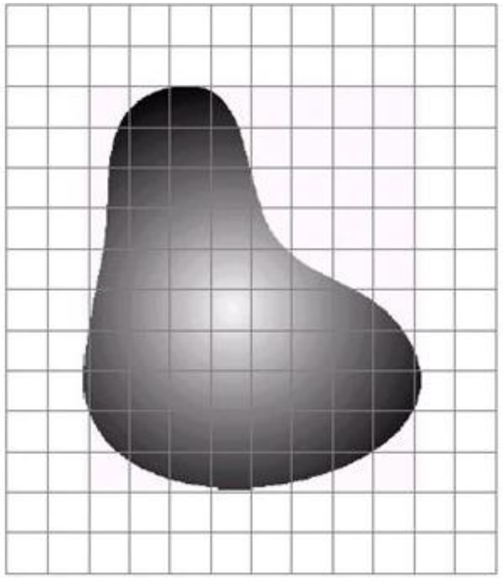


Sampling

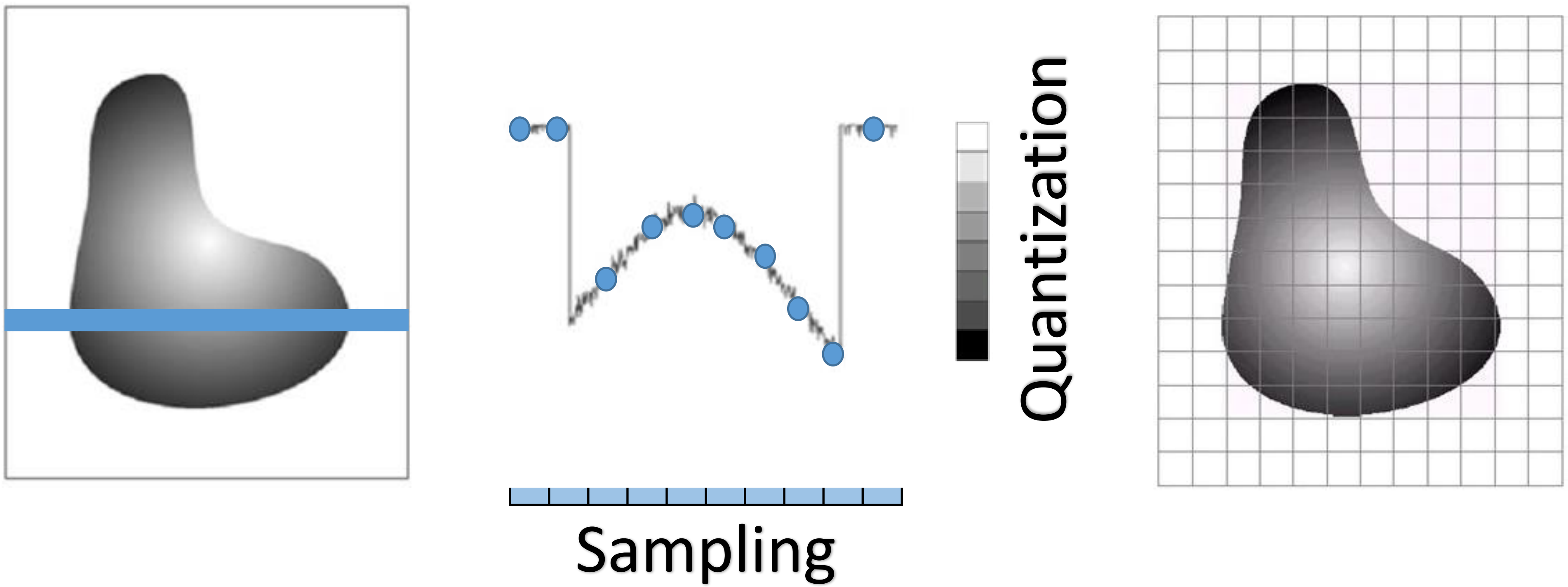




Sampling



Quantization



Quantization

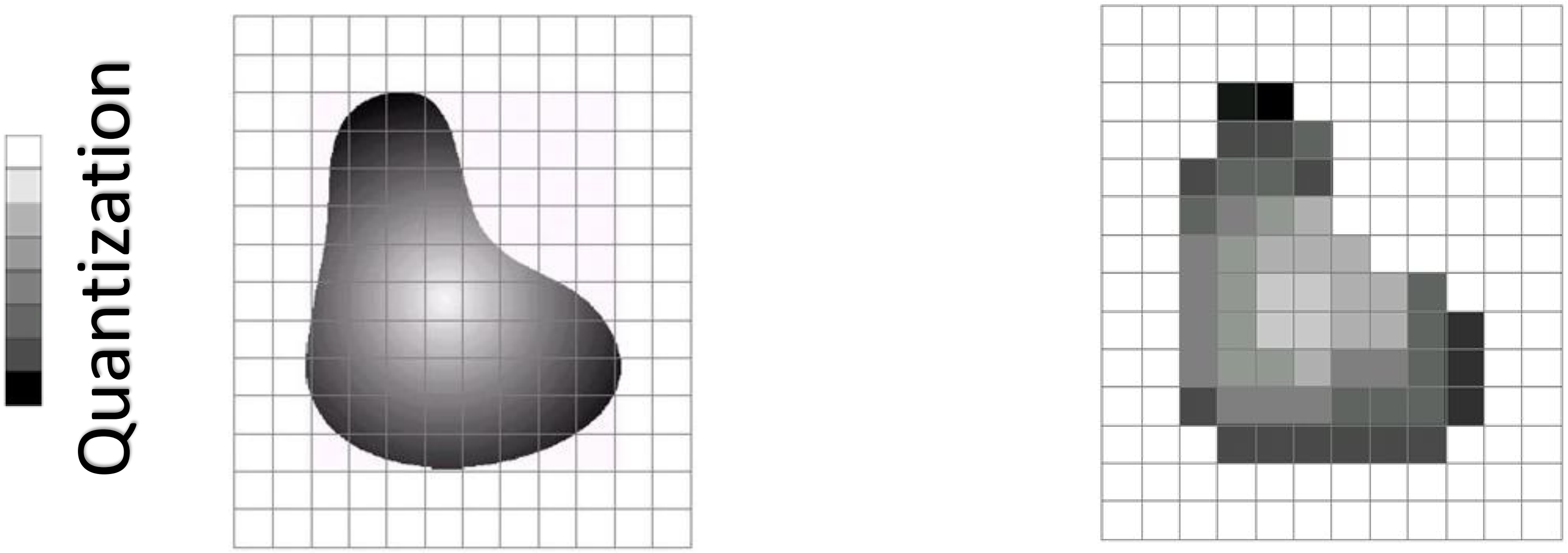


Image Artifacts

- What kind of image artifact is the result of quantization?

Image Artifacts

- Saturation

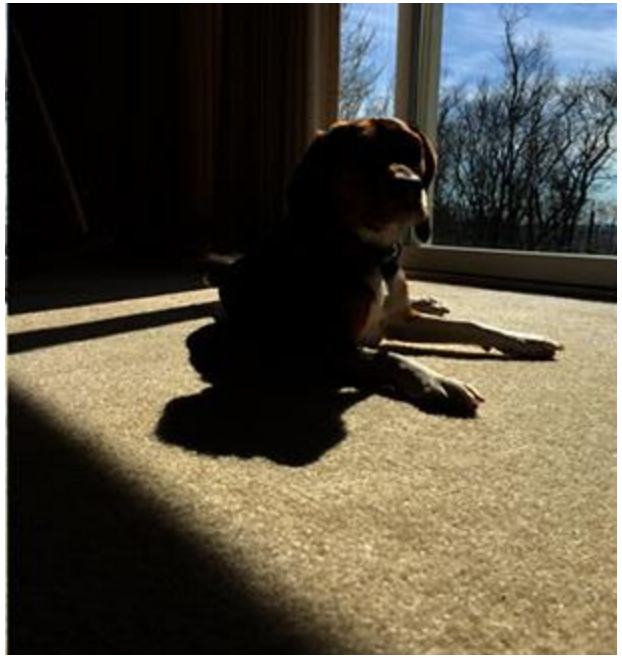
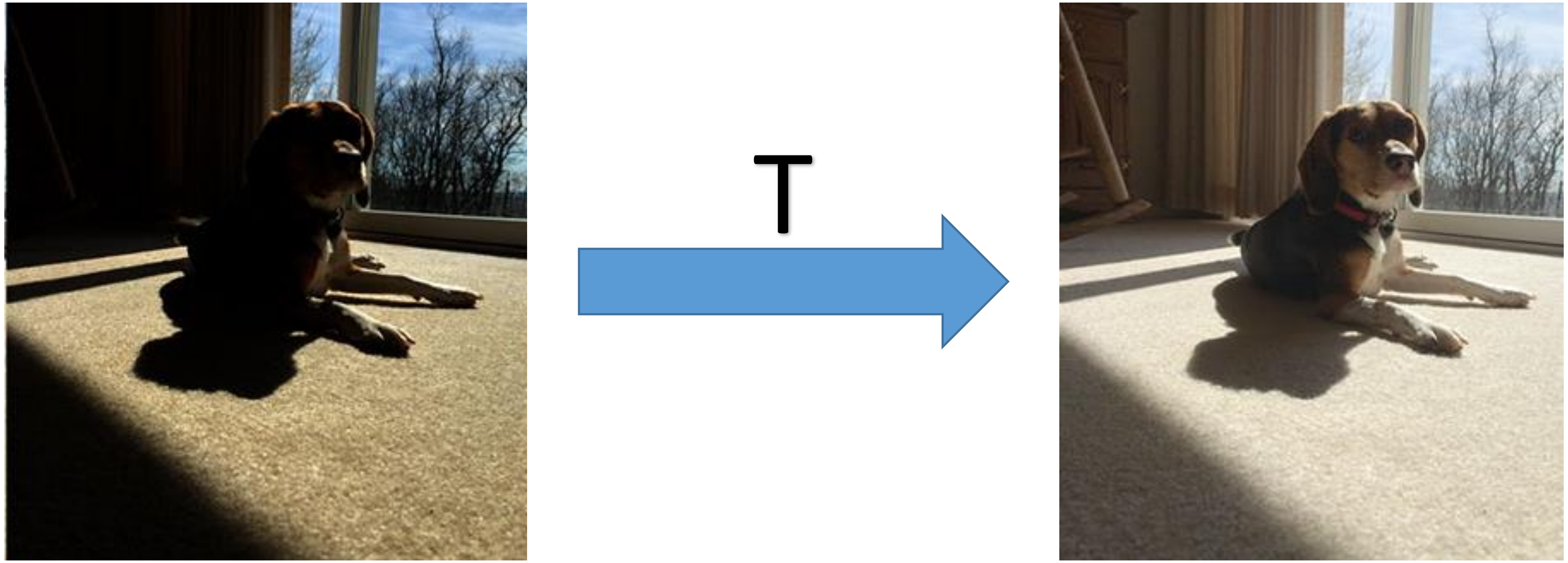


Image Enhancement

- Can we turn on the lights?

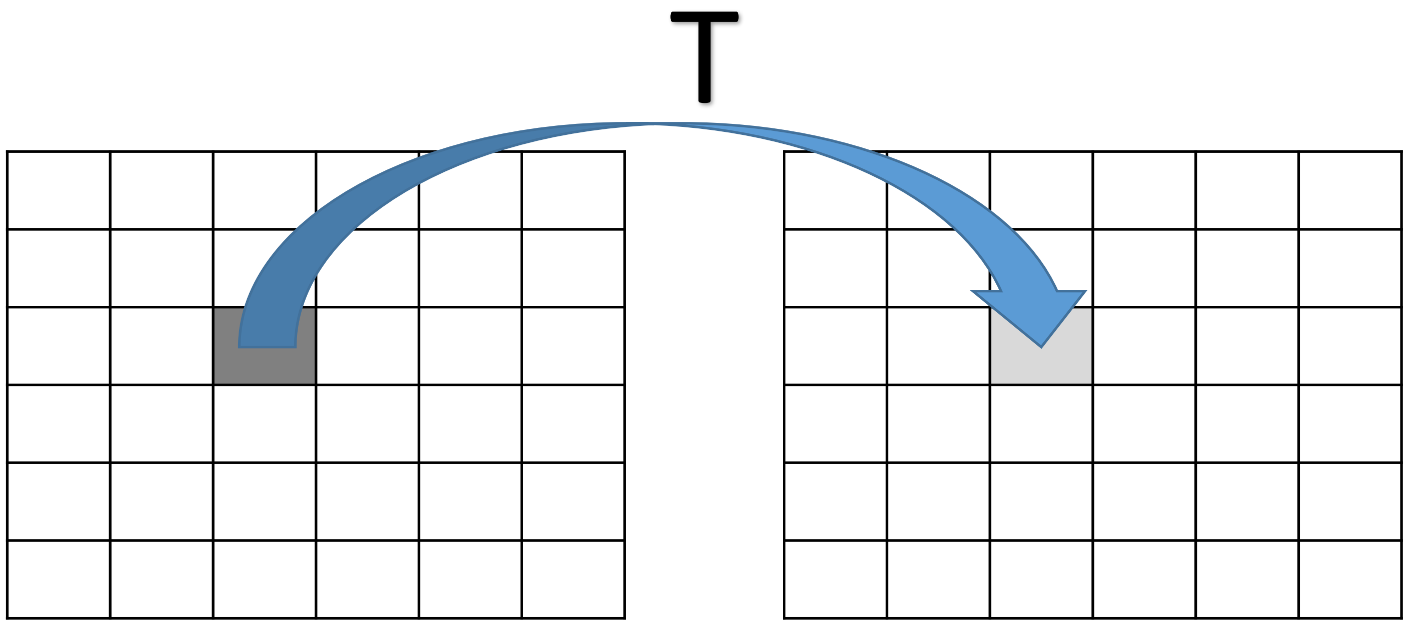
Image Enhancement

- Finding a transformation to re-arrange the pixel value distribution.



Transformations

- Pixel to pixel
  - No information about neighborhood



Transformations

- A transformation function to manipulate the intensity values of an image

Transformations

- A transformation function to manipulate the intensity values of an image

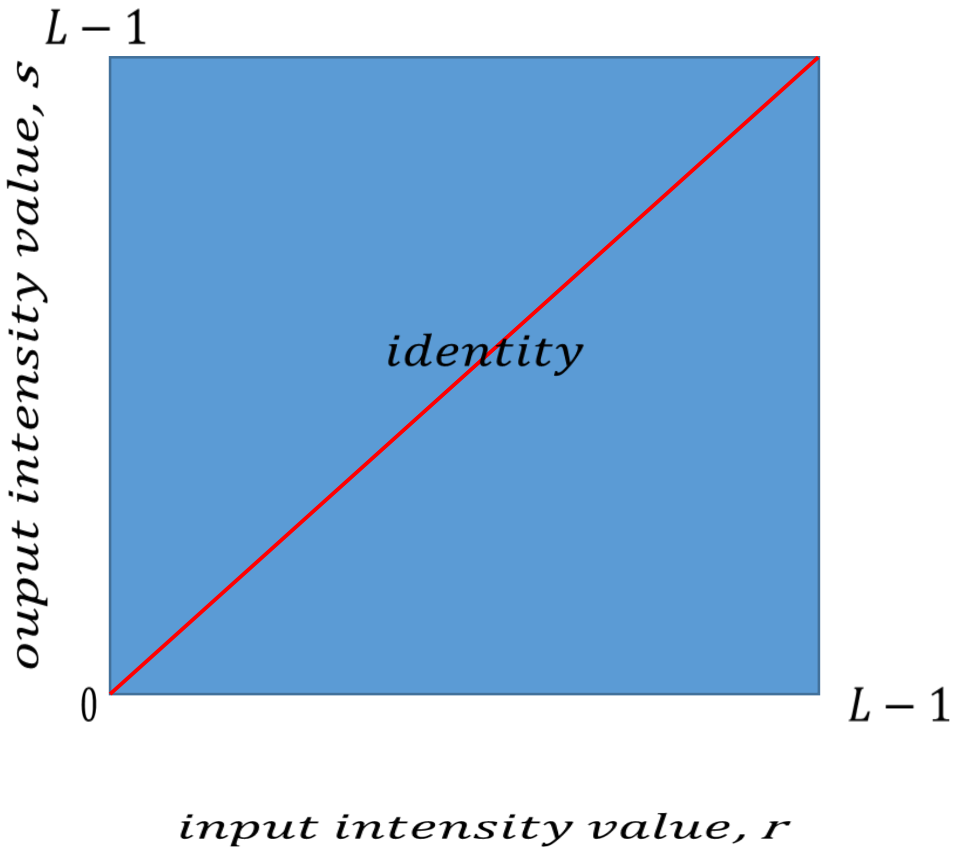
Original Intensity value

$$s = T(r)$$

New Intensity value

Transformation Options

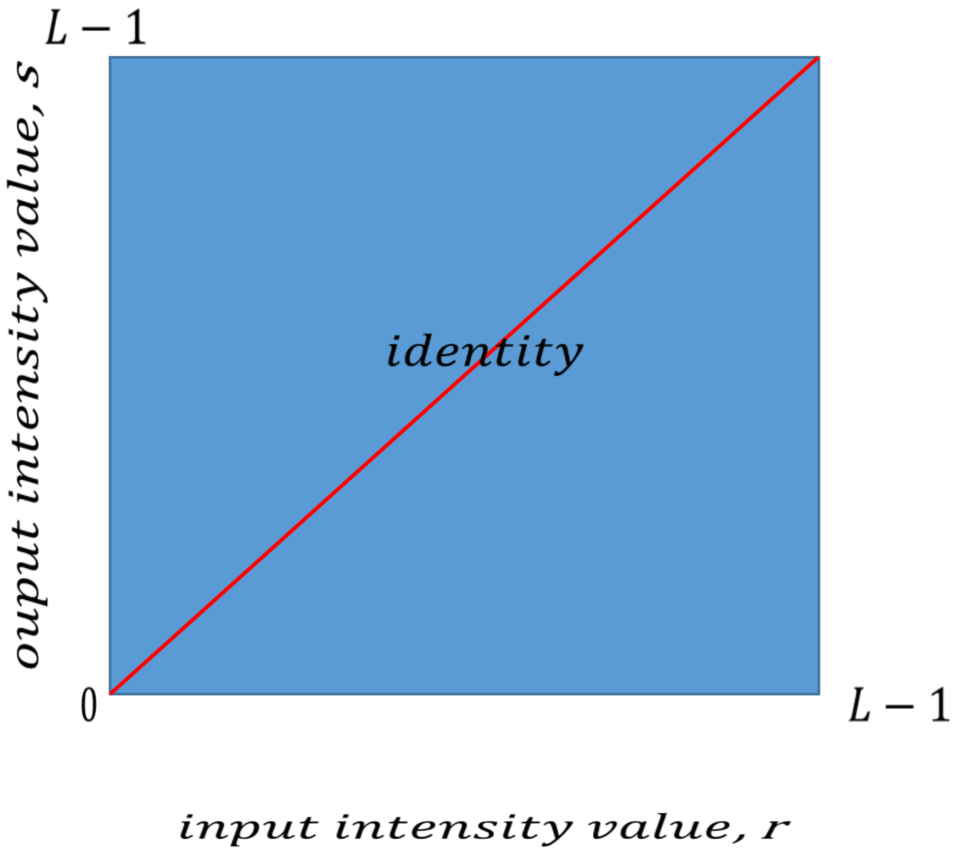
- What type of functions are candidates for T?



$$s = T(r) = ?$$

Transformation Options

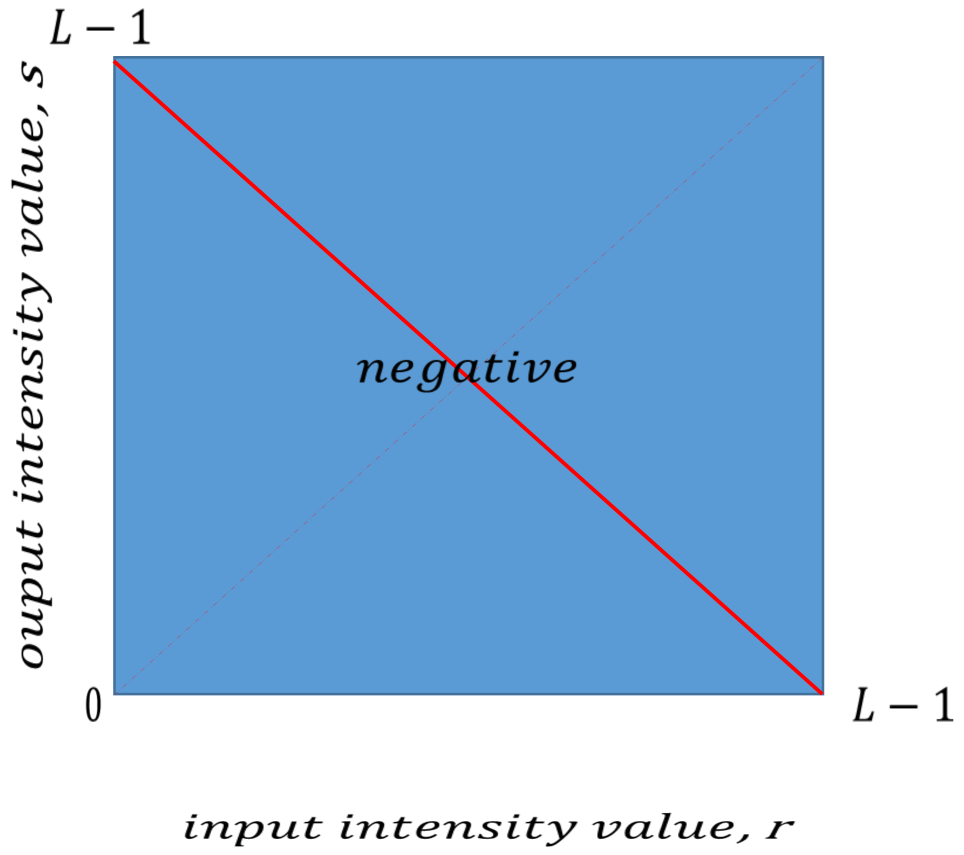
- Identity function



$$s = r$$

Transformation Options

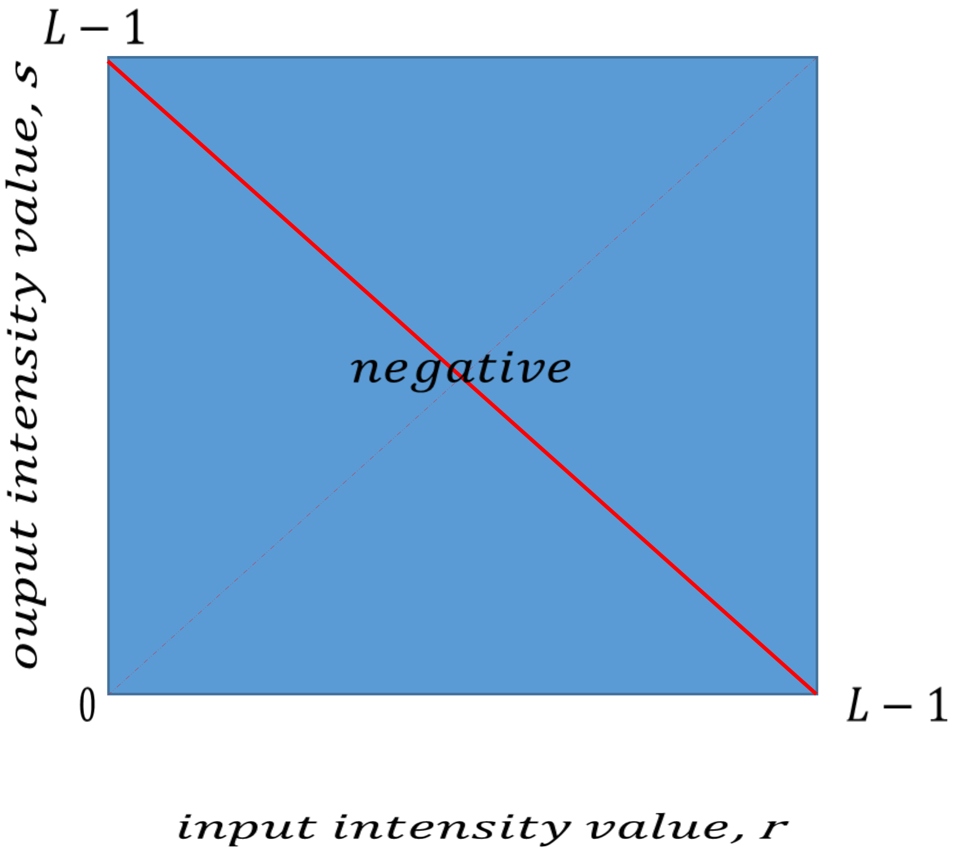
- Negative function



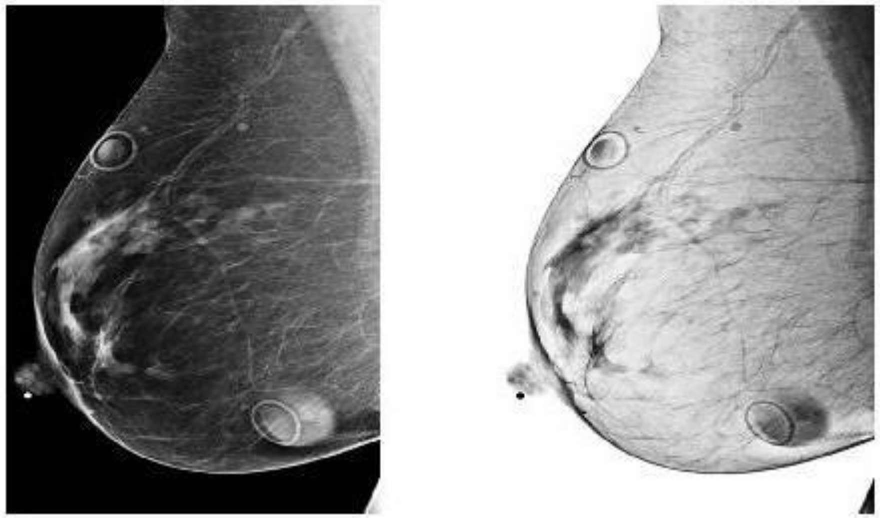
$$s = T(r) = ?$$

Transformation Options

- Negative function

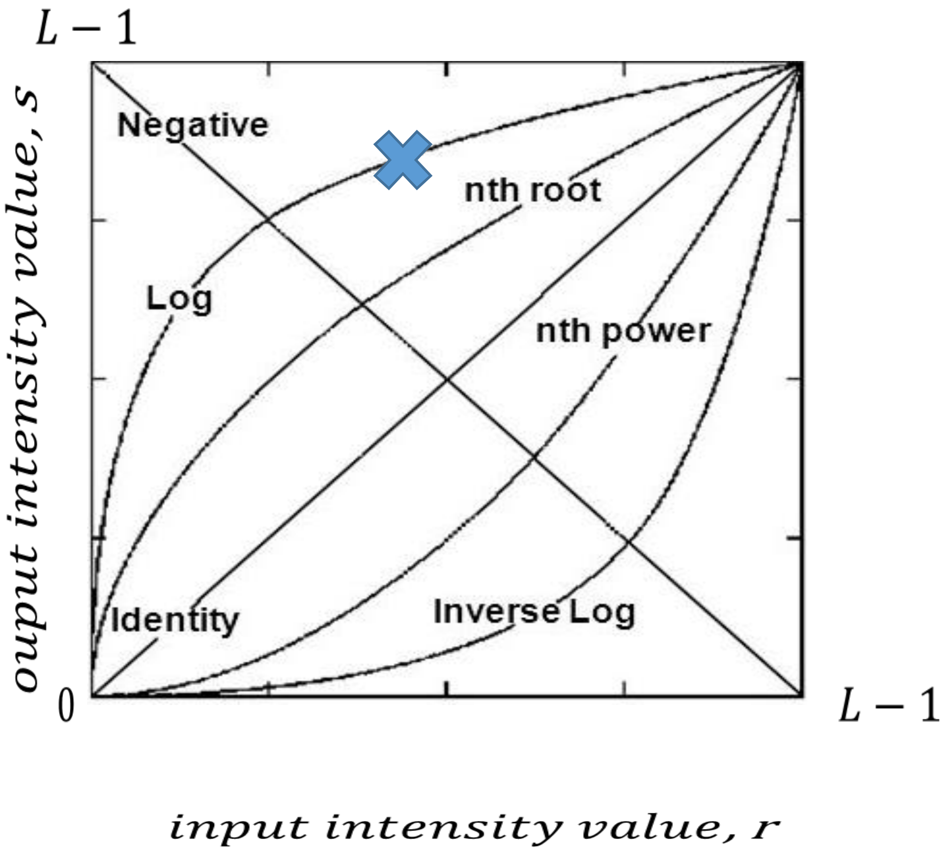


$$s = L - 1 - r$$

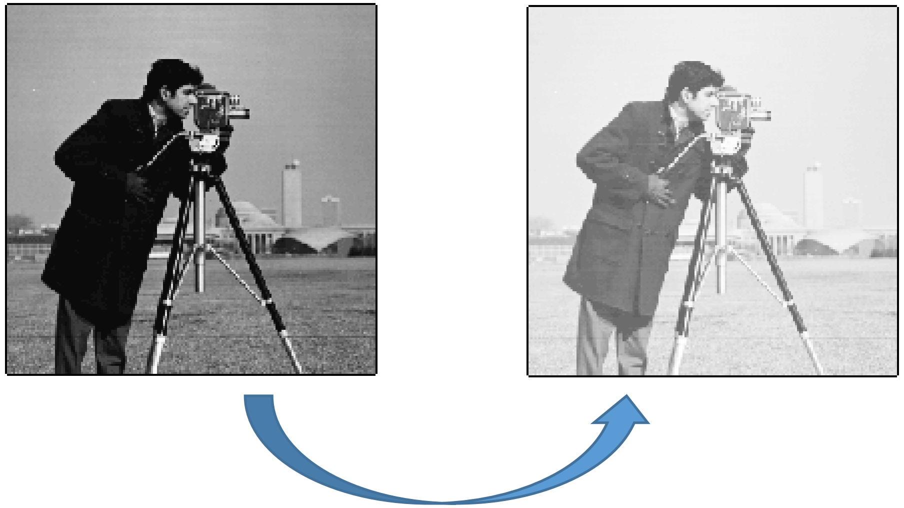


Transformation Options

- Log function

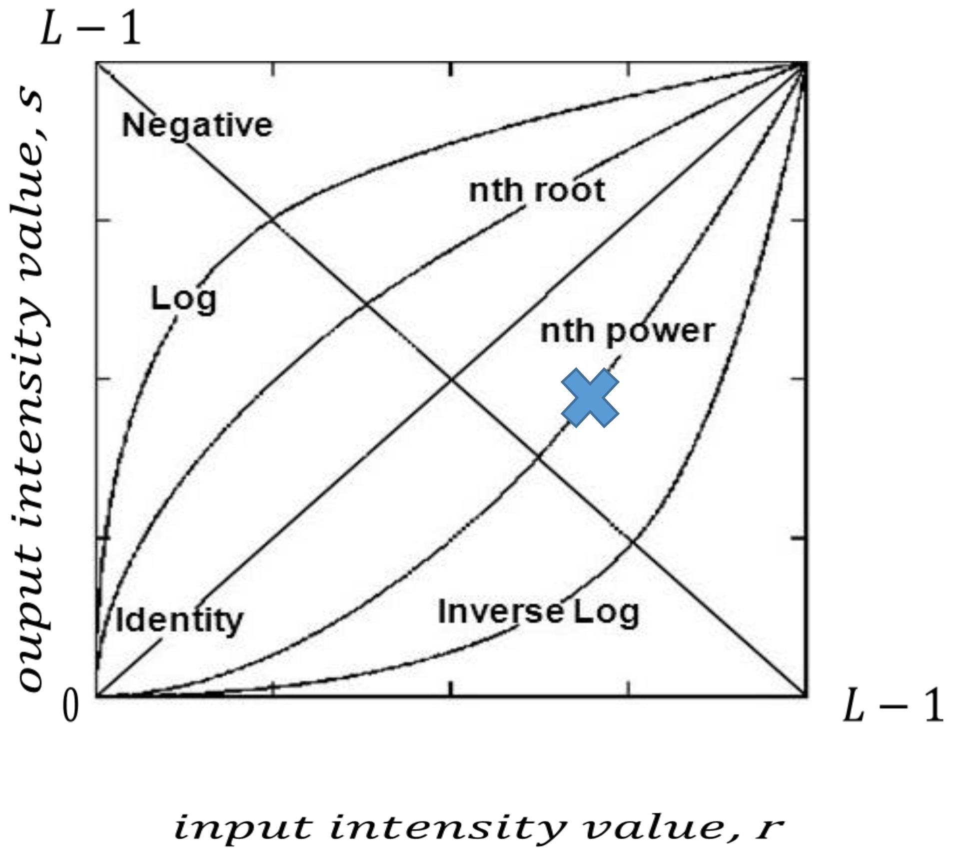


$$s = c\log(r+1)$$



Transformation Options

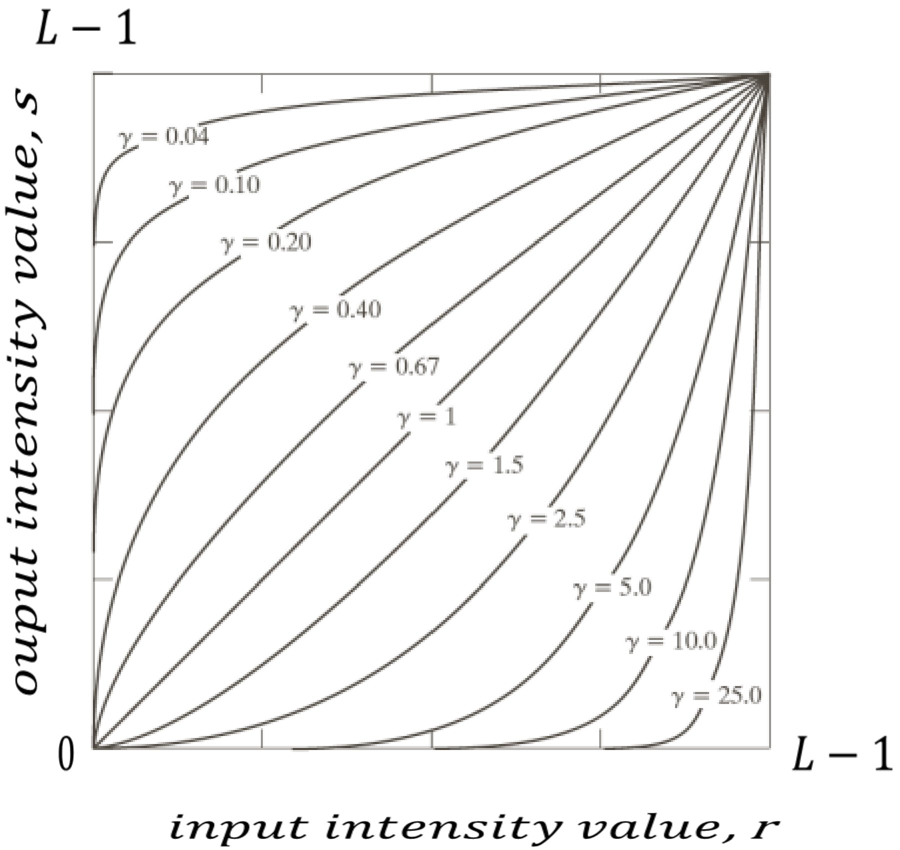
- Power-Law (Gamma) transformation



$$s = cr^{\gamma}$$

Transformation Options

- Changing gamma makes a family of functions



$$s = cr^{\gamma}$$

Transformation Options

- Power-Law (Gamma) transformation

Original image
MRI of a fractured human spine

$c = 1$

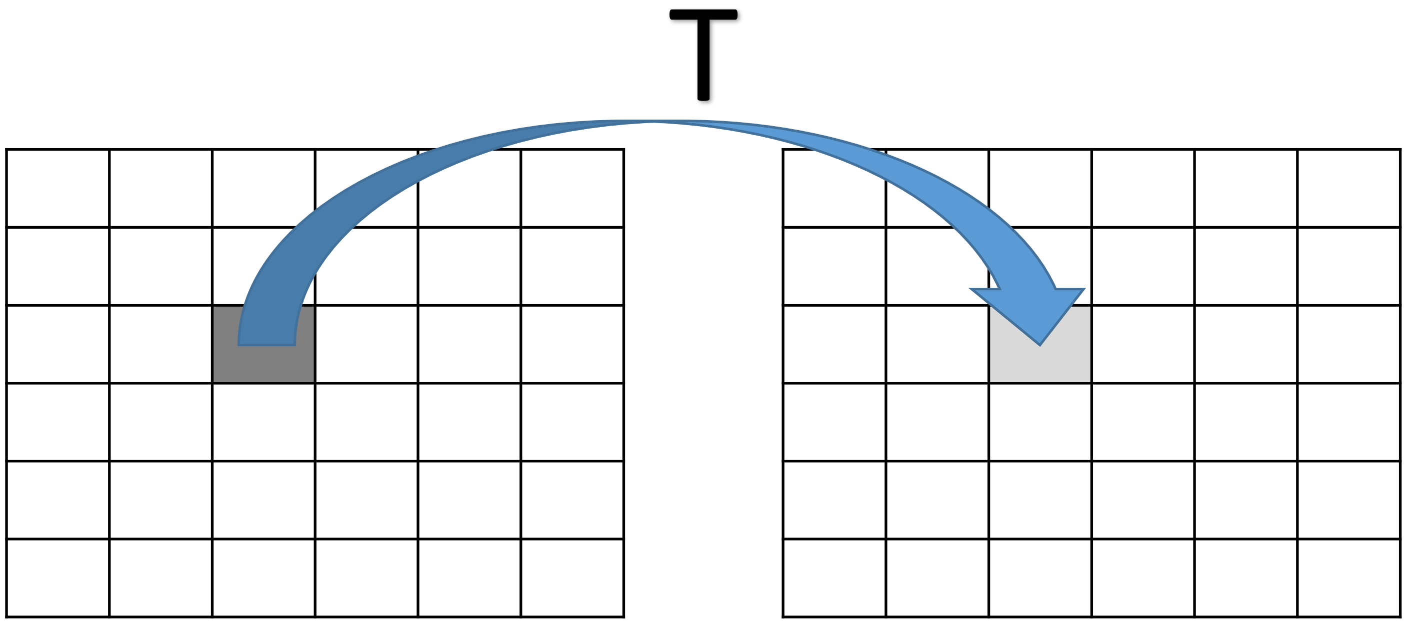
$\gamma = 0.6$

$\gamma = 0.4$

$\gamma = 0.3$

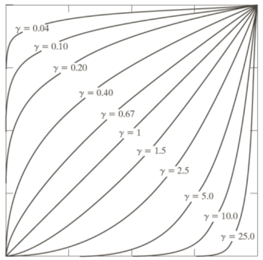
Recap (so far)

- We defined a set of simple pixel to pixel transformations that enhance the quality of images to reveal more details/information.



Cons

- Hard to find the best transformation function.

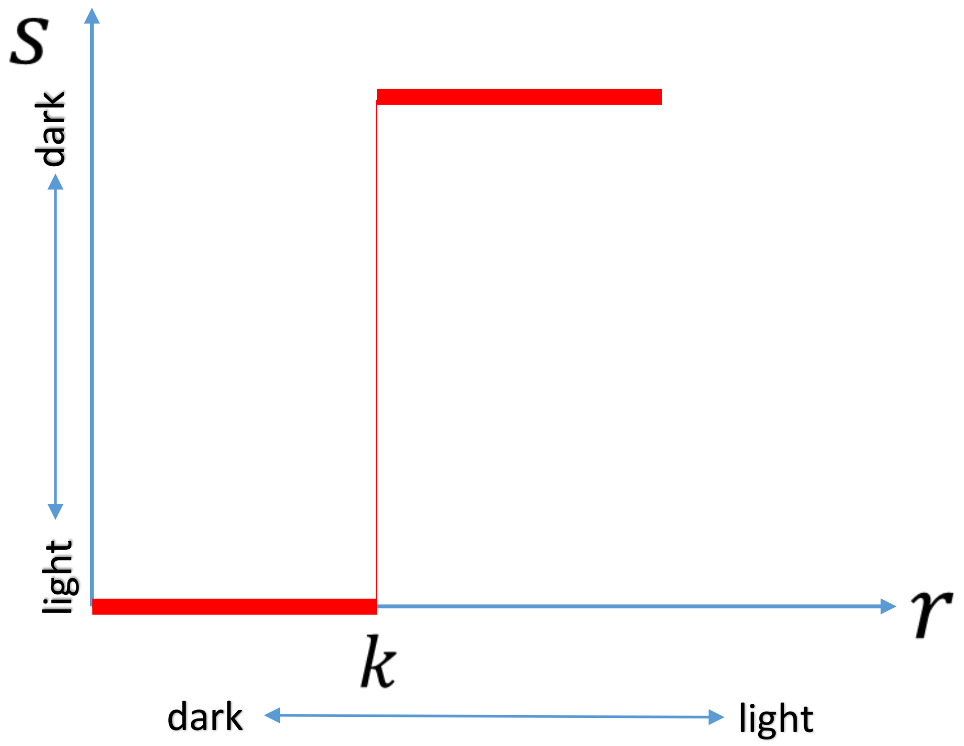


- It is blind to the distribution of pixels.
  - What if all the pixels are between 0-32?

- What is an alternative?

Thresholding Transformations

- 1D function that maps all the pixels smaller than a value to zero, larger than a value to (e.g., 1 or k).

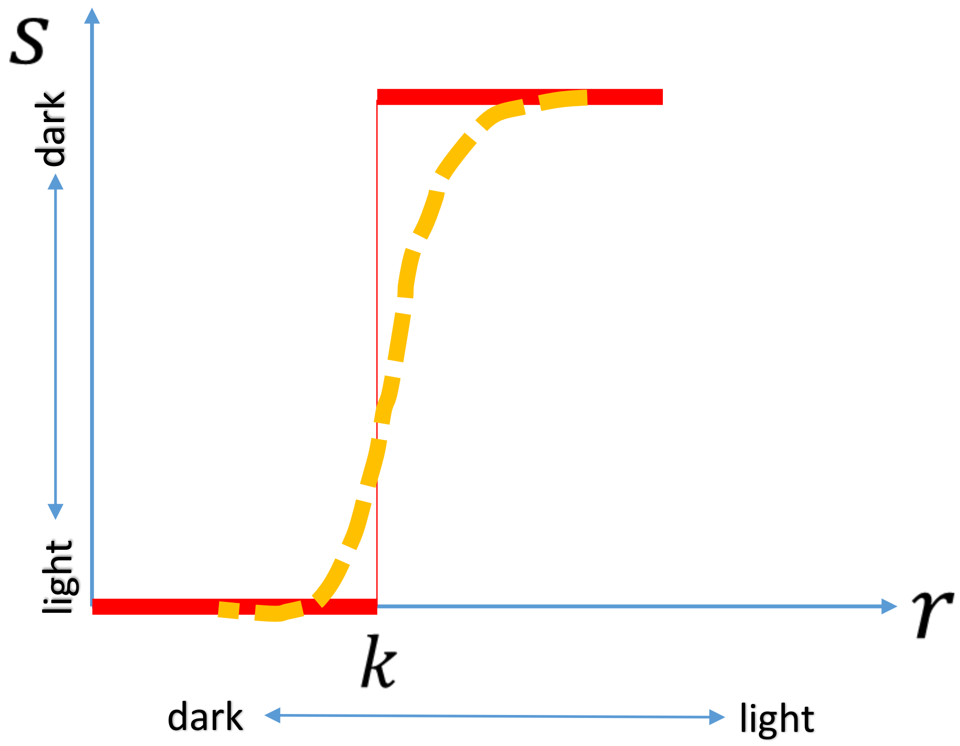


$$s = T(r)$$

Thresholding function

Stretching Transformations

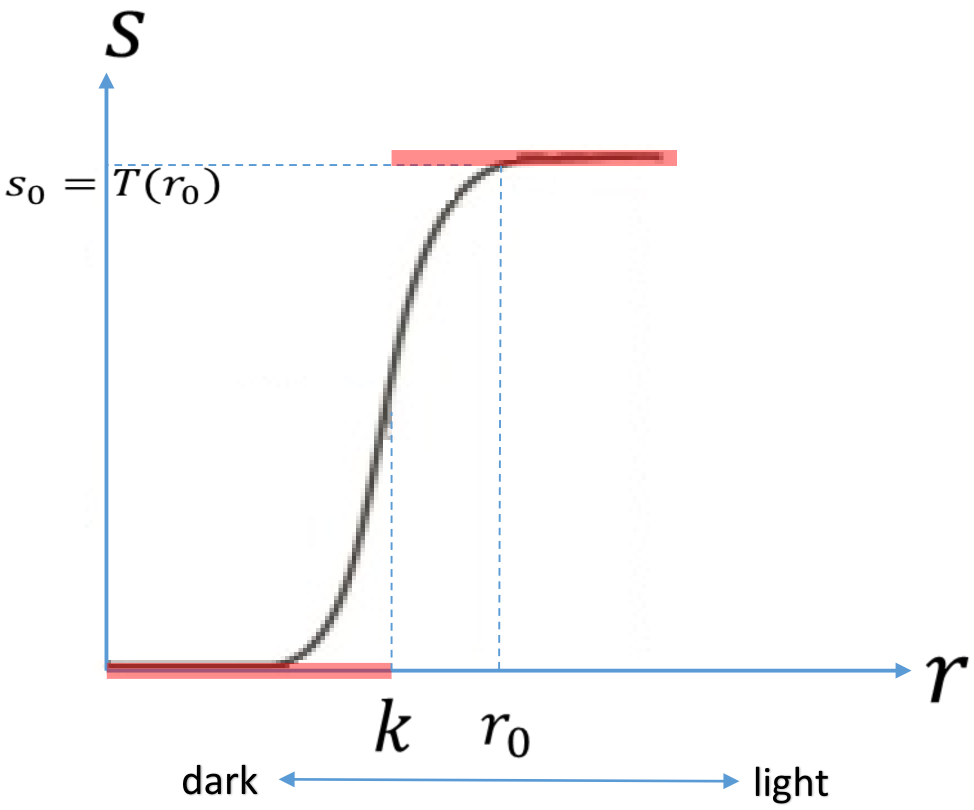
- We want to somehow smooth the middle to stretch the contrast.



$$s = T(r)$$

Stretching Transformations

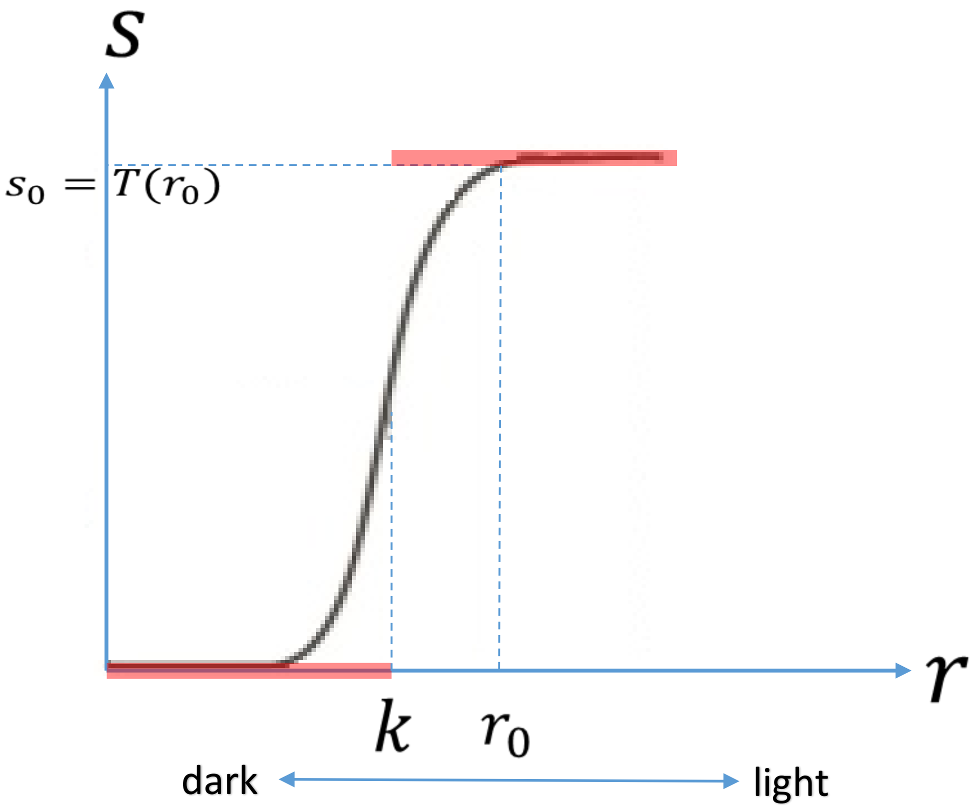
- We want to somehow smooth the middle to stretch the contrast.



$$s = T(r)$$

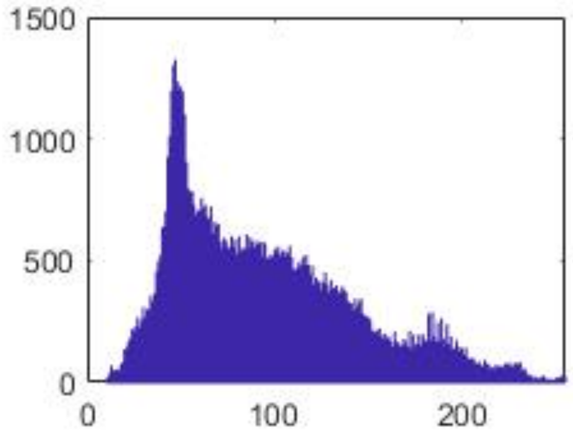
Stretching Transformations

- One way to define such a function is to use histograms equalization

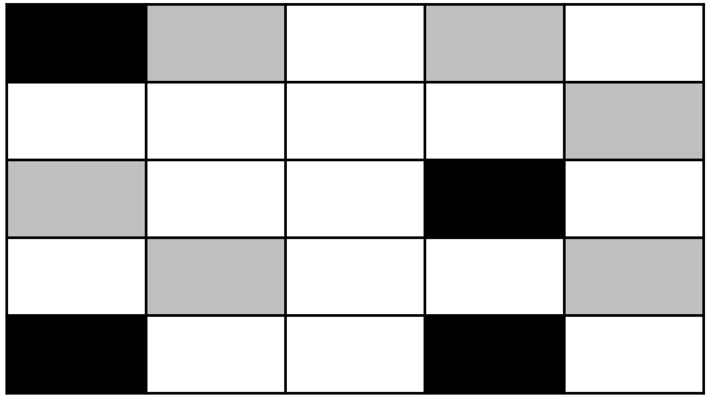


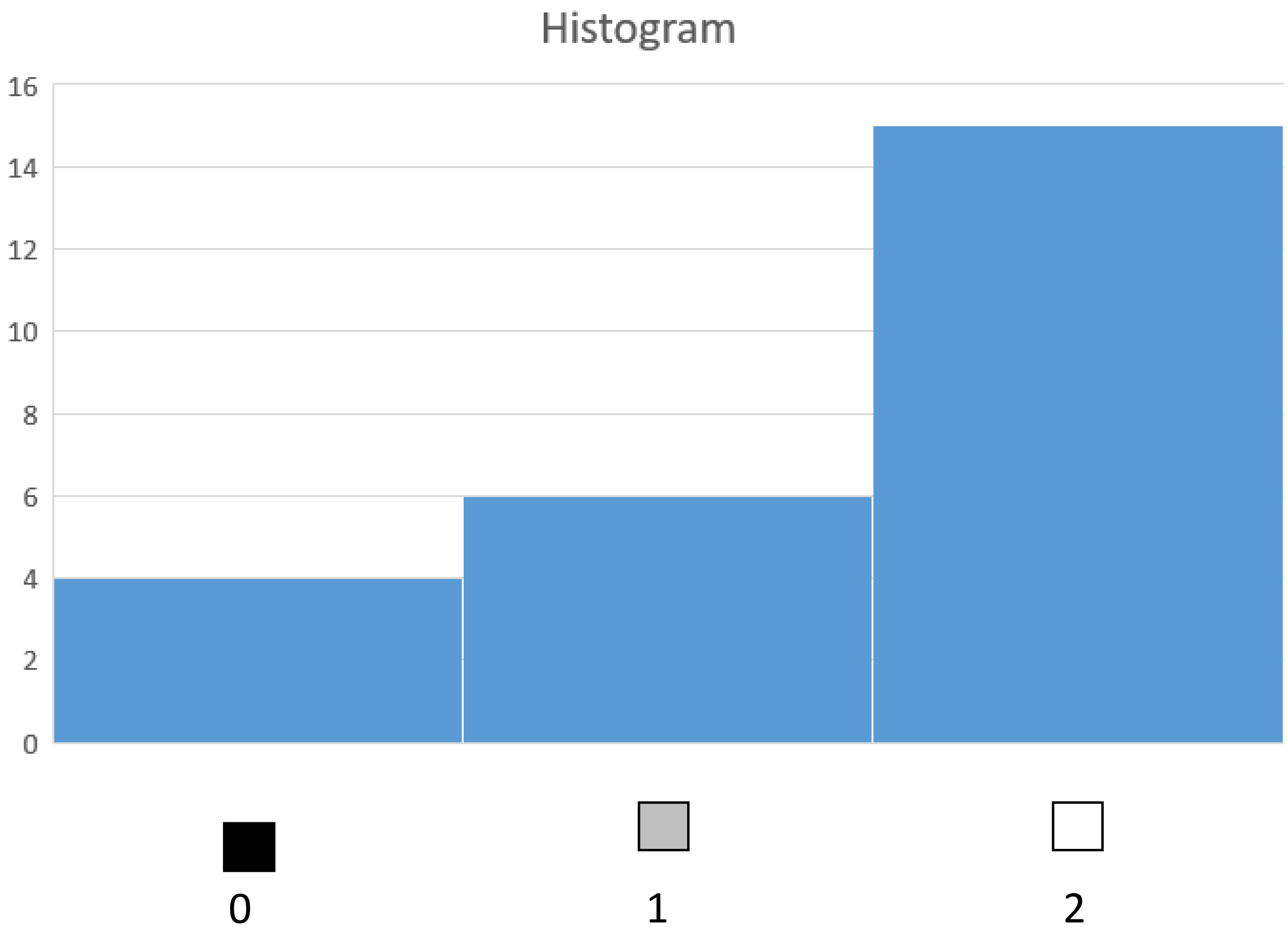
$$s = T(r)$$

Histogram



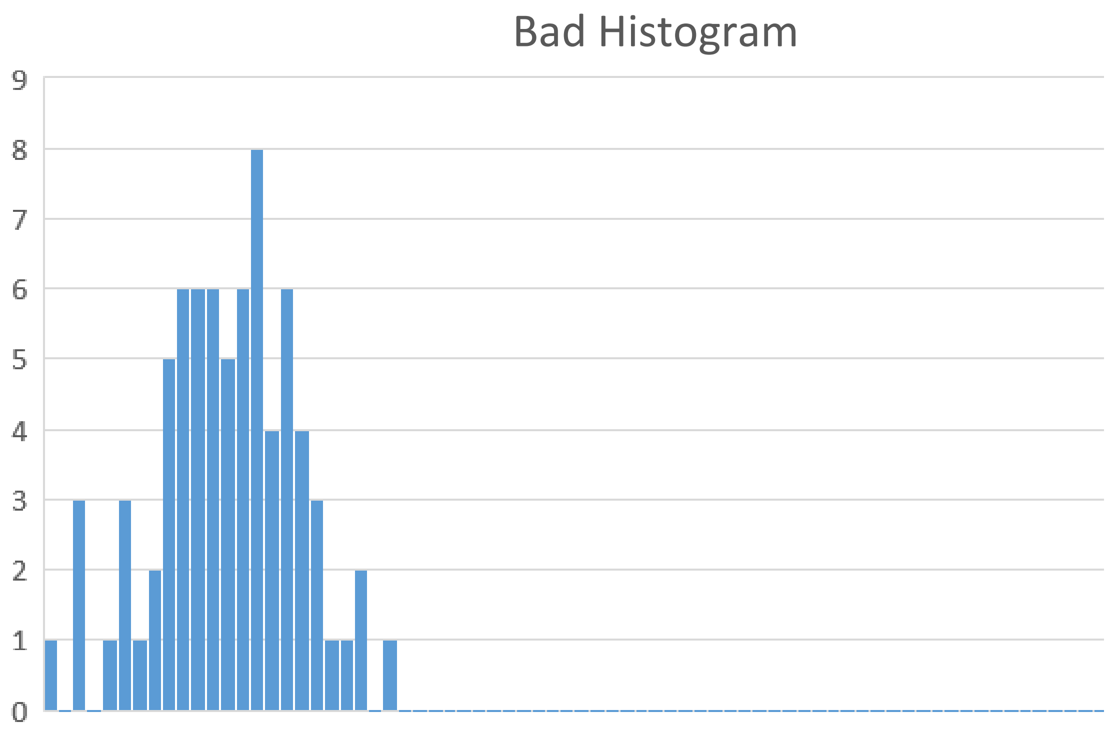
Histogram

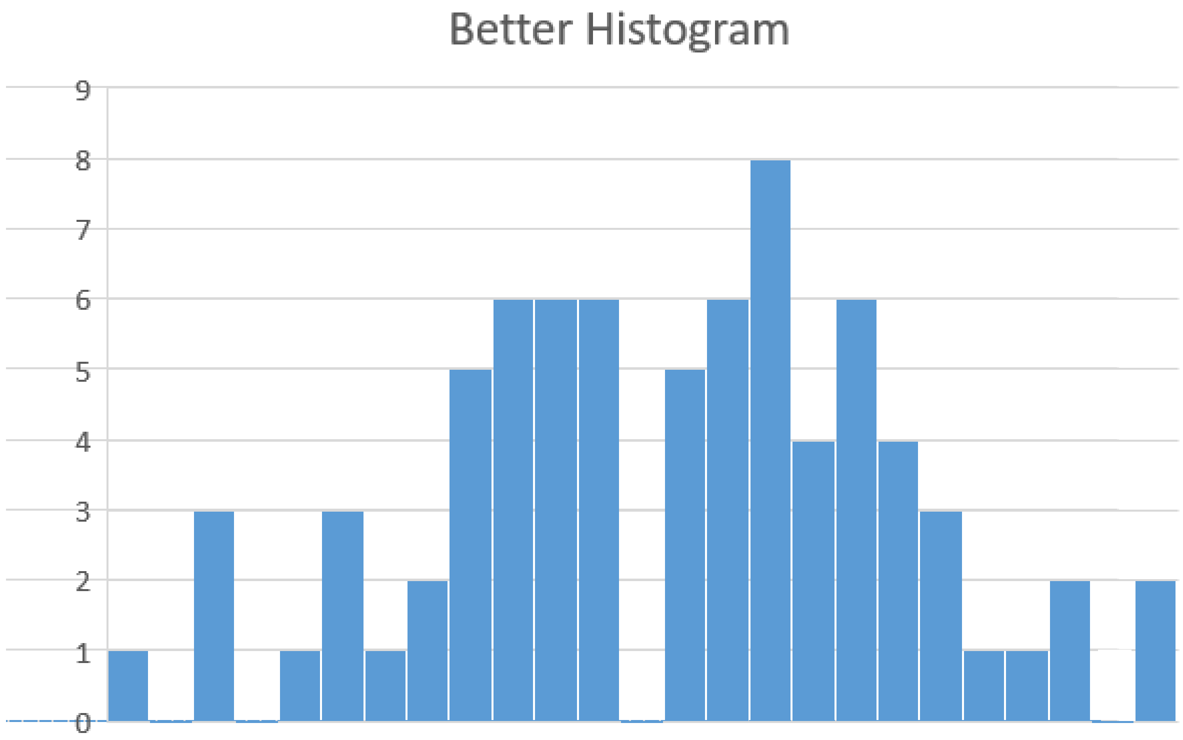




Histogram Equalization

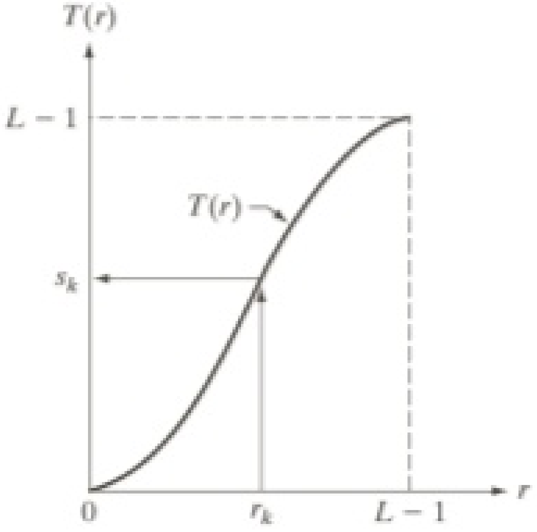
- We are looking for a transformation to map the histogram of an image to a well distributed histogram.





Histogram Equalization

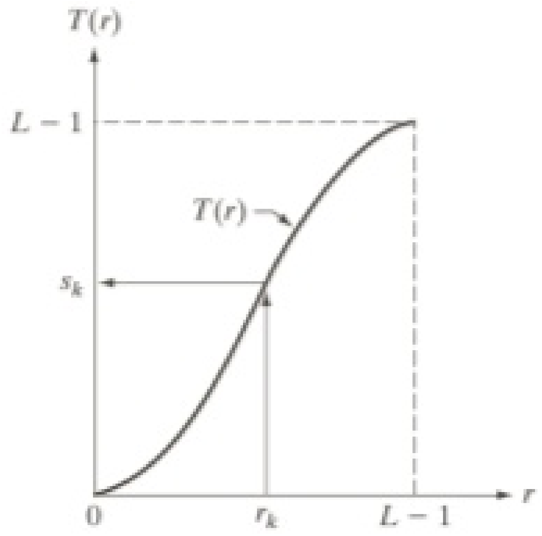
- Function should be monotonically increasing (why?)



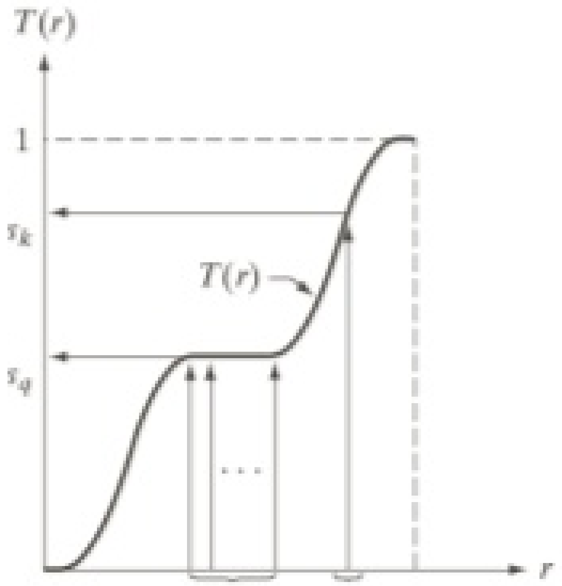
Strictly monotonically increasing

Histogram Equalization

- Function should be monotonically increasing (why?)



Strictly monotonically increasing



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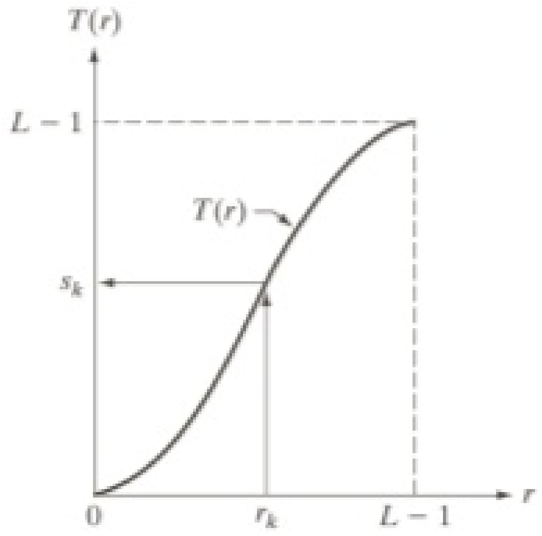
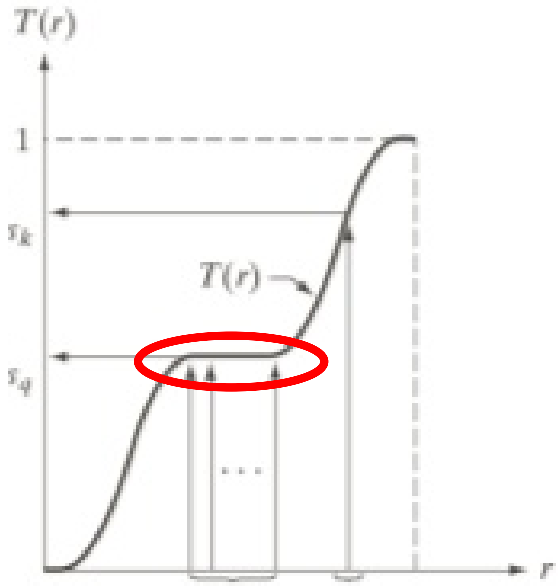
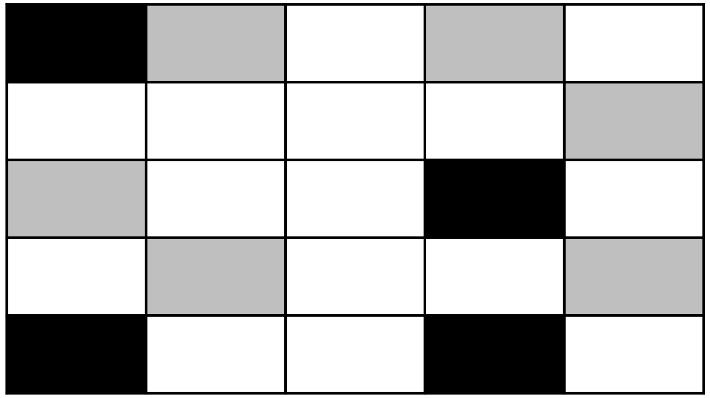


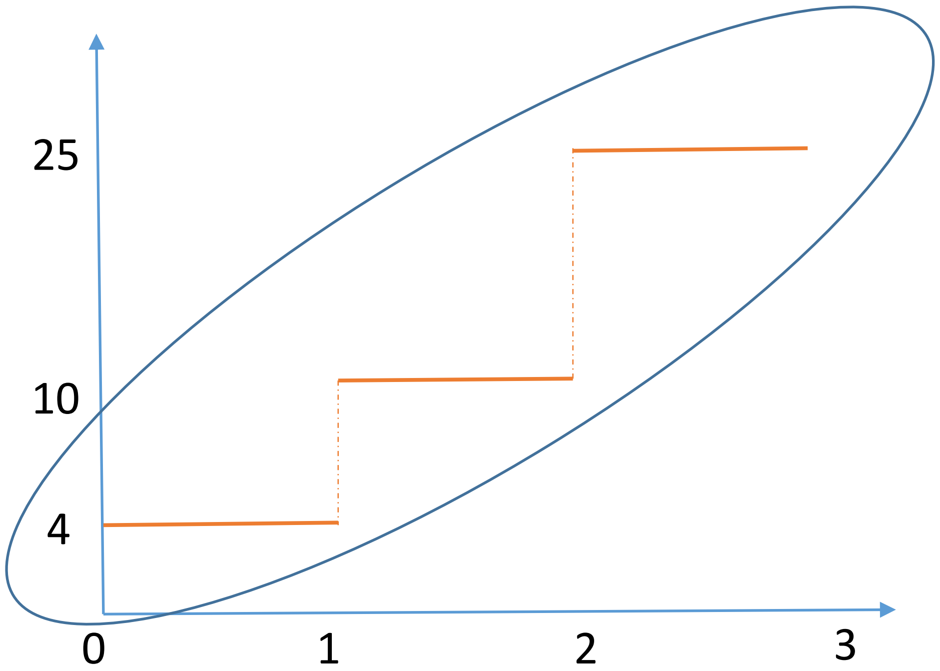
Strictly monotonically increasing



Histogram Equalization

- One way to define such a function is to use **cumulative distribution function** or cdf.

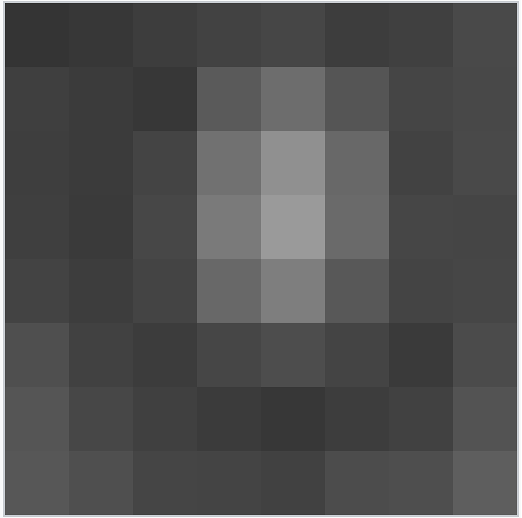




Monotonically increasing

Histogram Equalization

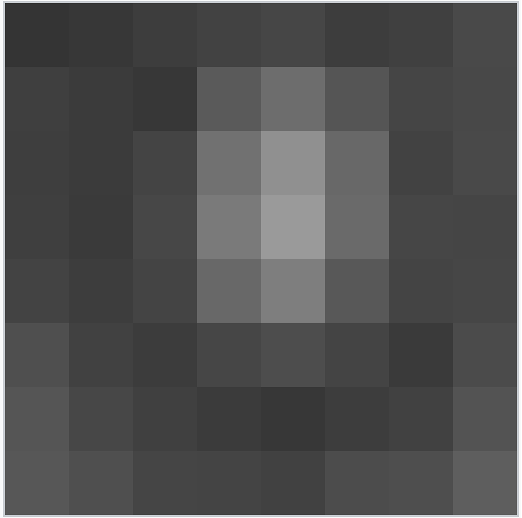
- Example



$$\begin{bmatrix} 52 & 55 & 61 & 59 & 70 & 61 & 76 & 61 \\ 62 & 59 & 55 & 104 & 94 & 85 & 59 & 71 \\ 63 & 65 & 66 & 113 & 144 & 104 & 63 & 72 \\ 64 & 70 & 70 & 126 & 154 & 109 & 71 & 69 \\ 67 & 73 & 68 & 106 & 122 & 88 & 68 & 68 \\ 68 & 79 & 60 & 79 & 77 & 66 & 58 & 75 \\ 69 & 85 & 64 & 58 & 55 & 61 & 65 & 83 \\ 70 & 87 & 69 & 68 & 65 & 73 & 78 & 90 \end{bmatrix}$$

Histogram Equalization

- Example

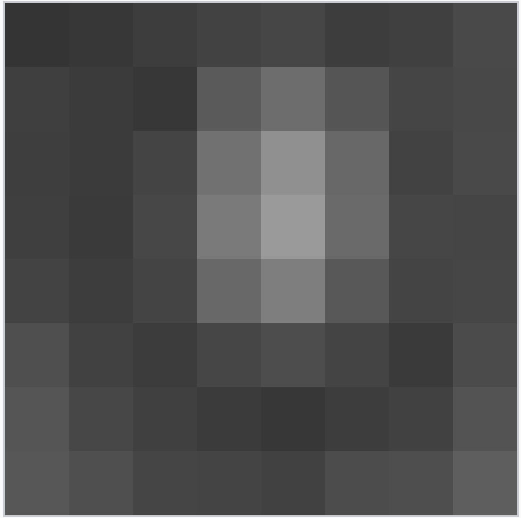


Smallest value

$$\begin{bmatrix} \boxed{52} & 55 & 61 & 59 & 70 & 61 & 76 & 61 \\ 62 & 59 & 55 & 104 & 94 & 85 & 59 & 71 \\ 63 & 65 & 66 & 113 & 144 & 104 & 63 & 72 \\ 64 & 70 & 70 & 126 & 154 & 109 & 71 & 69 \\ 67 & 73 & 68 & 106 & 122 & 88 & 68 & 68 \\ 68 & 79 & 60 & 79 & 77 & 66 & 58 & 75 \\ 69 & 85 & 64 & 58 & 55 & 61 & 65 & 83 \\ 70 & 87 & 69 & 68 & 65 & 73 & 78 & 90 \end{bmatrix}$$

Histogram Equalization

- Example



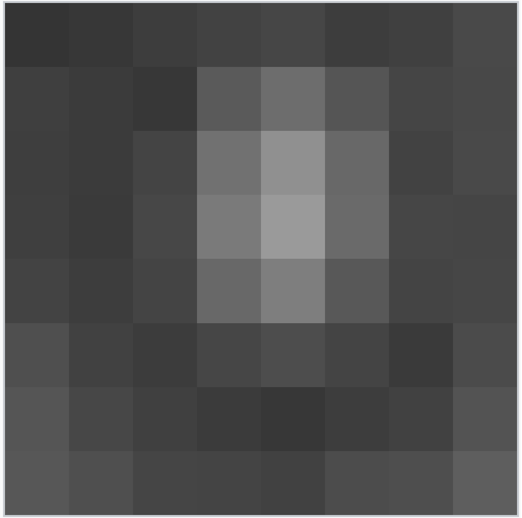
Smallest value

$$\begin{bmatrix} \boxed{52} & 55 & 61 & 59 & 70 & 61 & 76 & 61 \\ 62 & 59 & 55 & 104 & 94 & 85 & 59 & 71 \\ 63 & 65 & 66 & 113 & 144 & 104 & 63 & 72 \\ 64 & 70 & 70 & 126 & 154 & 109 & 71 & 69 \\ 67 & 73 & 68 & 106 & 122 & 88 & 68 & 68 \\ 68 & 79 & 60 & 79 & 77 & 66 & 58 & 75 \\ 69 & 85 & 64 & 58 & 55 & 61 & 65 & 83 \\ 70 & 87 & 69 & 68 & 65 & 73 & 78 & 90 \end{bmatrix}$$

r	cdf
52	1

Histogram Equalization

- Example



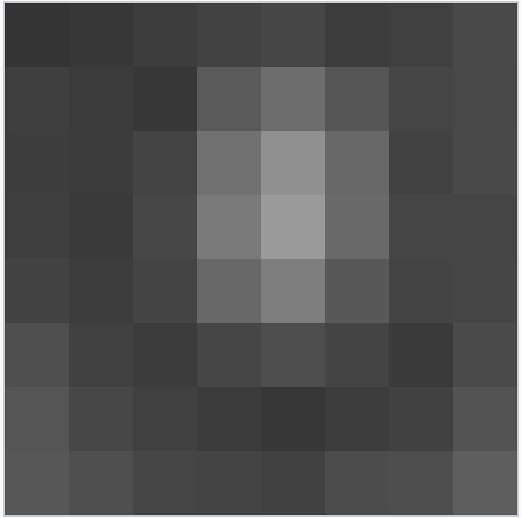
Next Smallest value

$$\begin{bmatrix} 52 & \boxed{55} & 61 & 59 & 70 & 61 & 76 & 61 \\ 62 & 59 & \boxed{55} & 104 & 94 & 85 & 59 & 71 \\ 63 & 65 & 66 & 113 & 144 & 104 & 63 & 72 \\ 64 & 70 & 70 & 126 & 154 & 109 & 71 & 69 \\ 67 & 73 & 68 & 106 & 122 & 88 & 68 & 68 \\ 68 & 79 & 60 & 79 & 77 & 66 & 58 & 75 \\ 69 & 85 & 64 & 58 & \boxed{55} & 61 & 65 & 83 \\ 70 & 87 & 69 & 68 & 65 & 73 & 78 & 90 \end{bmatrix}$$

value	cdf
52	1
55	3+1=4

Histogram Equalization

- Example



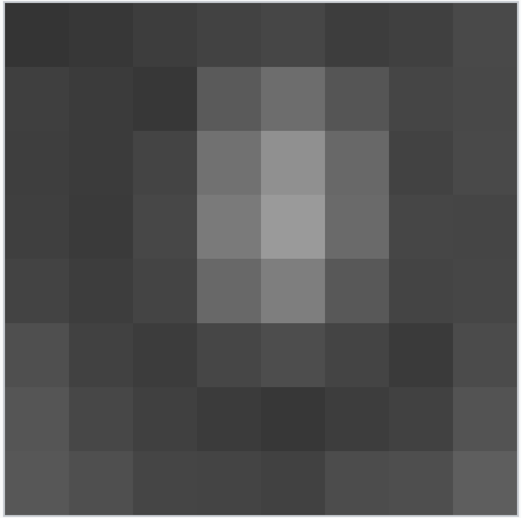
Next Smallest value

$$\begin{bmatrix} 52 & 55 & 61 & 59 & 70 & 61 & 76 & 61 \\ 62 & 59 & 55 & 104 & 94 & 85 & 59 & 71 \\ 63 & 65 & 66 & 113 & 144 & 104 & 63 & 72 \\ 64 & 70 & 70 & 126 & 154 & 109 & 71 & 69 \\ 67 & 73 & 68 & 106 & 122 & 88 & 68 & 68 \\ 68 & 79 & 60 & 79 & 77 & 66 & \boxed{58} & 75 \\ 69 & 85 & 64 & \boxed{58} & 55 & 61 & 65 & 83 \\ 70 & 87 & 69 & 68 & 65 & 73 & 78 & 90 \end{bmatrix}$$

r	cdf
52	1
55	4
58	4+2=6

Histogram Equalization

- Example



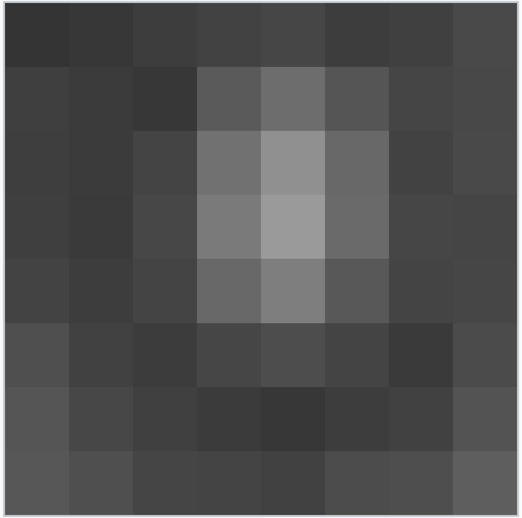
Next Smallest value

$$\begin{bmatrix} 52 & 55 & 61 & \boxed{59} & 70 & 61 & 76 & 61 \\ 62 & \boxed{59} & 55 & 104 & 94 & 85 & \boxed{59} & 71 \\ 63 & 65 & 66 & 113 & 144 & 104 & 63 & 72 \\ 64 & 70 & 70 & 126 & 154 & 109 & 71 & 69 \\ 67 & 73 & 68 & 106 & 122 & 88 & 68 & 68 \\ 68 & 79 & 60 & 79 & 77 & 66 & 58 & 75 \\ 69 & 85 & 64 & 58 & 55 & 61 & 65 & 83 \\ 70 & 87 & 69 & 68 & 65 & 73 & 78 & 90 \end{bmatrix}$$

r	cdf
52	1
55	4
58	6
59	6+3=9

Histogram Equalization

- Example



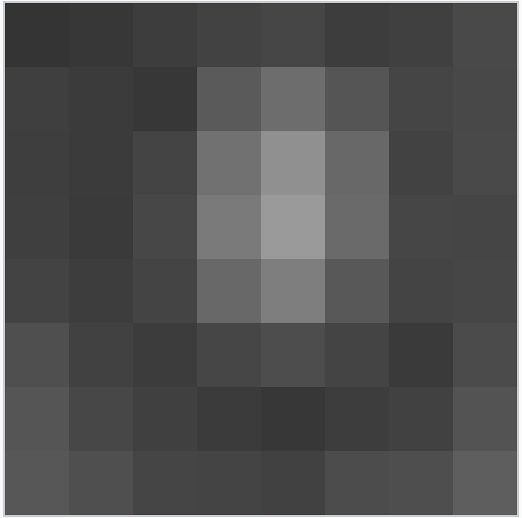
Largest value

$$\begin{bmatrix} 52 & 55 & 61 & 59 & 70 & 61 & 76 & 61 \\ 62 & 59 & 55 & 104 & 94 & 85 & 59 & 71 \\ 63 & 65 & 66 & 113 & 144 & 104 & 63 & 72 \\ 64 & 70 & 70 & 126 & \boxed{154} & 109 & 71 & 69 \\ 67 & 73 & 68 & 106 & 122 & 88 & 68 & 68 \\ 68 & 79 & 60 & 79 & 77 & 66 & 58 & 75 \\ 69 & 85 & 64 & 58 & 55 & 61 & 65 & 83 \\ 70 & 87 & 69 & 68 & 65 & 73 & 78 & 90 \end{bmatrix}$$

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

Histogram Equalization

- Example



Largest value

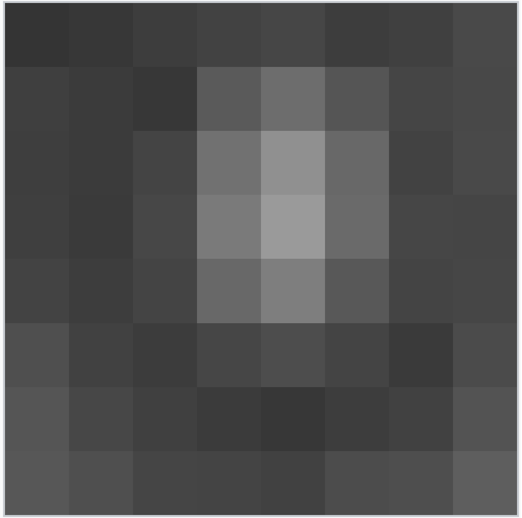
$$
\begin{bmatrix}
52 & 55 & 61 & 59 & 70 & 61 & 76 & 61 \\
62 & 59 & 55 & 104 & 94 & 85 & 59 & 71 \\
63 & 65 & 66 & 113 & 144 & 104 & 63 & 72 \\
64 & 70 & 70 & 126 & 154 & 109 & 71 & 69 \\
67 & 73 & 68 & 106 & 122 & 88 & 68 & 68 \\
68 & 79 & 60 & 79 & 77 & 66 & 58 & 75 \\
69 & 85 & 64 & 58 & 55 & 61 & 65 & 83 \\
70 & 87 & 69 & 68 & 65 & 73 & 78 & 90
\end{bmatrix}
$$

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

s
0
. . .
255

Histogram Equalization

- Example



Largest value

$$\begin{bmatrix} 52 & 55 & 61 & 59 & 70 & 61 & 76 & 61 \\ 62 & 59 & 55 & 104 & 94 & 85 & 59 & 71 \\ 63 & 65 & 66 & 113 & 144 & 104 & 63 & 72 \\ 64 & 70 & 70 & 126 & 154 & 109 & 71 & 69 \\ 67 & 73 & 68 & 106 & 122 & 88 & 68 & 68 \\ 68 & 79 & 60 & 79 & 77 & 66 & 58 & 75 \\ 69 & 85 & 64 & 58 & 55 & 61 & 65 & 83 \\ 70 & 87 & 69 & 68 & 65 & 73 & 78 & 90 \end{bmatrix}$$

r	cdf
52	1
55	4
58	6
59	9
⋮	
144	63
154	64

s
0
⋮
255

We get help from cdf

Histogram Equalization

- Example

$$s = \left\lfloor \frac{cdf(r) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L-1) \right\rfloor$$

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

s
0
. . .
255

We get help from cdf

Histogram Equalization

- Example

cdf of a pixel with value r

$$s = \left\lfloor \frac{cdf(r) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L - 1) \right\rfloor$$

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

s
0
. . .
255

We get help from cdf

Histogram Equalization

- Example

$$s = \left\lfloor \frac{cdf(r) - cdf_{min}}{(\underbrace{M}_{\text{height}} \times \underbrace{N}_{\text{width}}) - cdf_{min}} \times (L-1) \right\rfloor$$

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

s
0
. . .
255

We get help from cdf

Histogram Equalization

- Example

$$s = \left\lfloor \frac{cdf(r) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L - 1) \right\rfloor$$

Probability distribution of pixels

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

s
0
. . .
255

Histogram Equalization

- Example

To map pixel with minimum intensity to 0

$$s = \left\lfloor \frac{cdf(r) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L - 1) \right\rfloor$$

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

s
0
. . .
255

Histogram Equalization

- Example

$$s = \left\lfloor \frac{cdf(r) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L - 1) \right\rfloor$$

To map pixel with maximum intensity to L-1
Why?

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

s
0
. . .
255

Histogram Equalization

- Example

$$s = \left\lfloor \frac{cdf(r) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L-1) \right\rfloor$$

$$v: 58 \rightarrow \left\lfloor \frac{cdf(58) - cdf_{min}}{8 \times 8 - cdf_{min}} \times 255 \right\rfloor$$

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

s
0
. . .
255

Histogram Equalization

- Example

$$s = \left\lfloor \frac{cdf(r) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L-1) \right\rfloor$$

$$v: 58 \rightarrow \left\lfloor \frac{cdf(58) - cdf_{min}}{8 \times 8 - cdf_{min}} \times 255 \right\rfloor = \left\lfloor \frac{6-1}{64-1} \times 255 \right\rfloor$$

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

s
0
. . .
255

Histogram Equalization

- Example

$$s = \left\lfloor \frac{cdf(r) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L-1) \right\rceil$$

$$v: 58 \rightarrow \left\lfloor \frac{cdf(58) - cdf_{min}}{8 \times 8 - cdf_{min}} \times 255 \right\rceil = \left\lfloor \frac{6-1}{64-1} \times 255 \right\rceil = \lfloor 20.2380 \rceil = 20$$

r	cdf
52	1
55	4
58	6
59	9
. . .	
144	63
154	64

s
0
20
. . .
255

Histogram Equalization

- Example

$$s = \left\lfloor \frac{cdf(r) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L-1) \right\rfloor$$

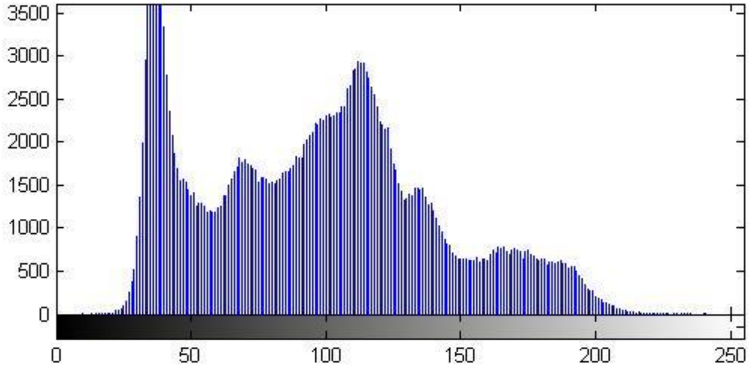
r	cdf
52	1
55	4
58	6
59	9
⋮	
144	63
154	64

s
0
12
20
32
⋮
251
255

Following the same formula produces a better distribution for pixel values

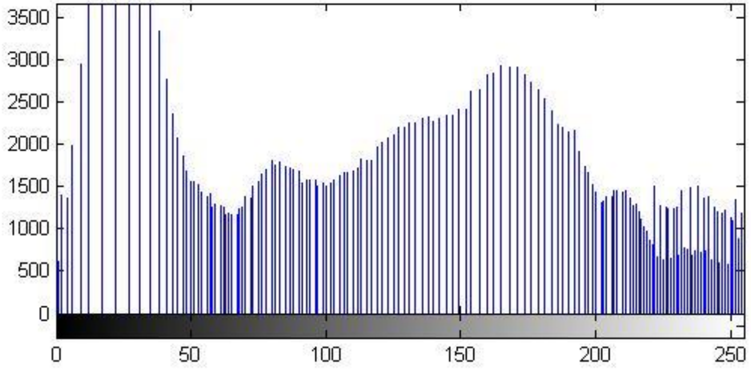
Histogram Equalization

Original Image



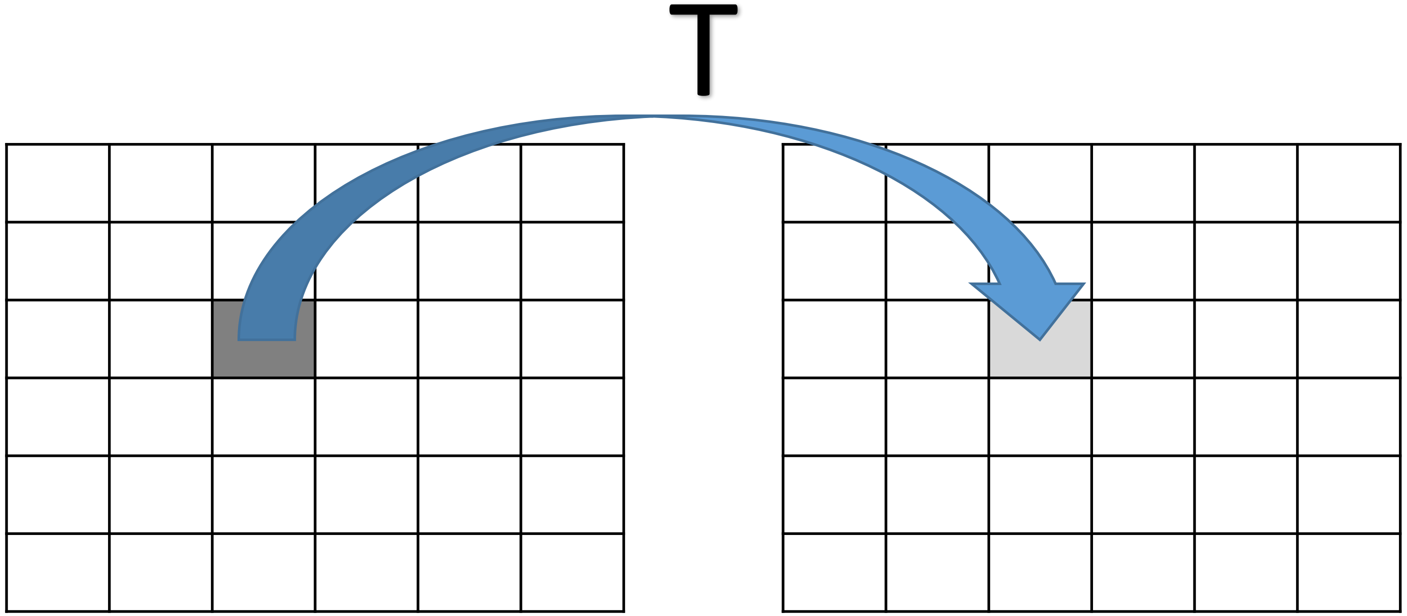
Enhanced Image

Histogram equalization



Local Neighbourhood Operations

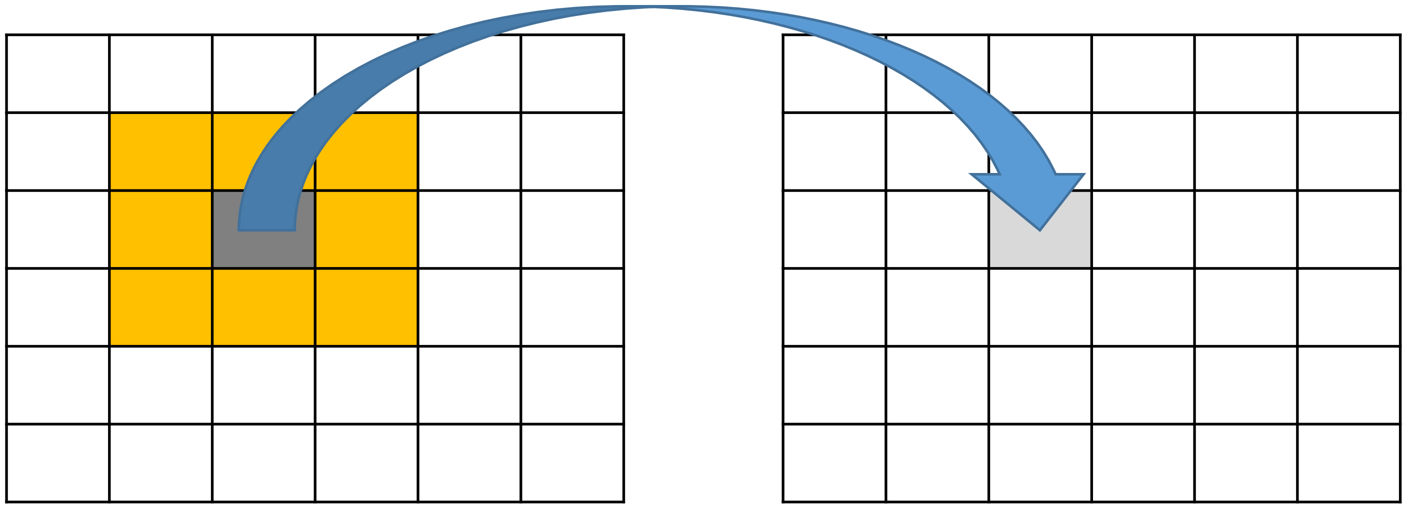
- So far, the modification was only based on the value of pixels
- We did not consider neighbouring pixels



Local Neighbourhood Operations

- Define a transformation via a local mask

w_0	w_1	w_2
w_3	w_4	w_5
w_6	w_7	w_8

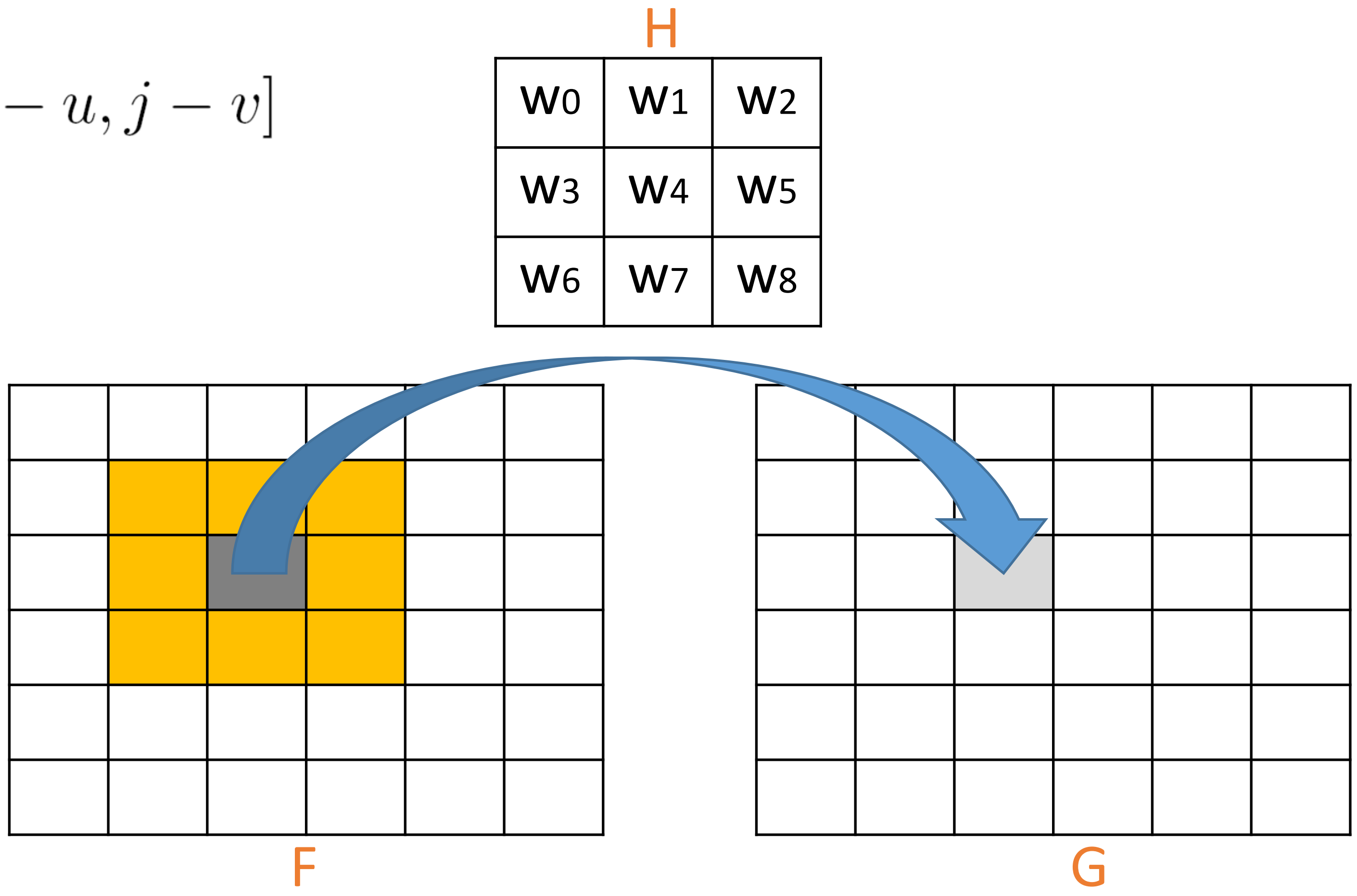


Local Neighbourhood Operations

- Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H * F$$



Local Neighbourhood Operations

- What does this filter do?

Original

$$*$$

0	0	0
0	1	0
0	0	0

Local Neighbourhood Operations

- Identity

Original

*

0	0	0
0	1	0
0	0	0

=

Identical image

Local Neighbourhood Operations

- What does this filter do?

Original

$*$

0	0	0
1	0	0
0	0	0

Local Neighbourhood Operations

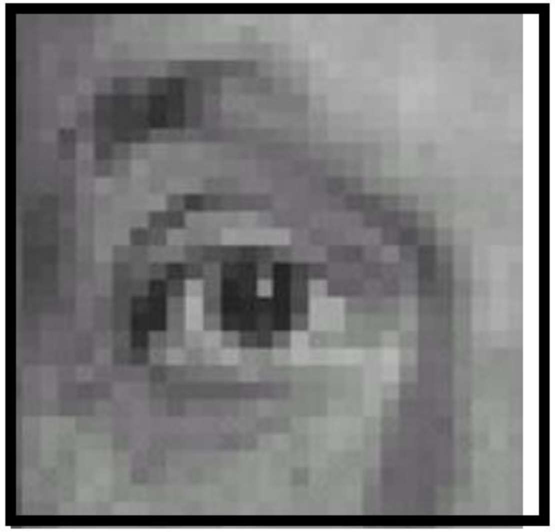
- Shift to left by one pixel.

Original

$*$

0	0	0
1	0	0
0	0	0

$=$



Local Neighbourhood Operations

- What does this filter do?

Original

$$* \ \frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Local Neighbourhood Operations

- Average (blur)

Original

$$* \ \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} =$$

Local Neighbourhood Operations

- Weighted Average (blur)

Original

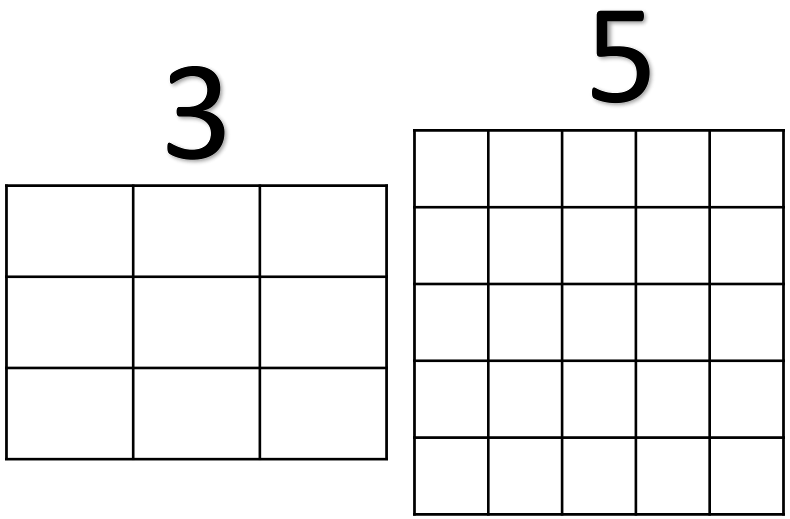
$*$

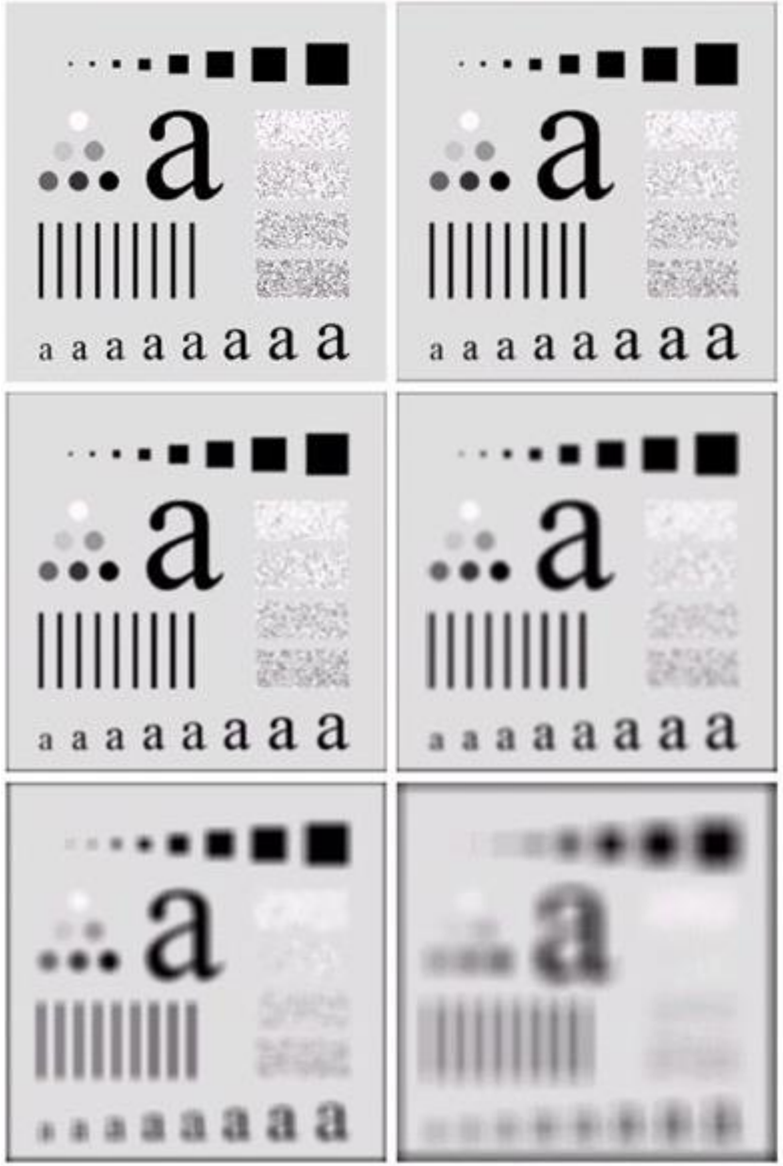
$\frac{1}{16}$	$\frac{8}{16}$	$\frac{1}{16}$
$\frac{1}{16}$	$\frac{8}{16}$	$\frac{1}{16}$
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

$=$

Local Neighbourhood Operations

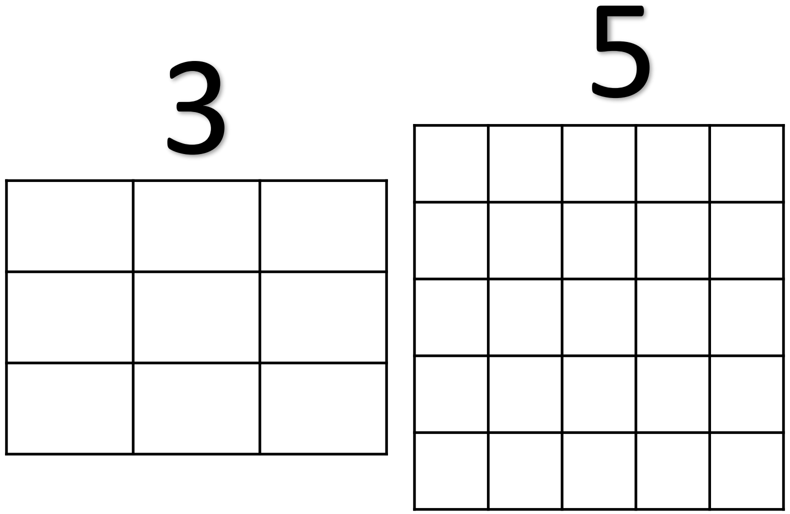
- Filter size



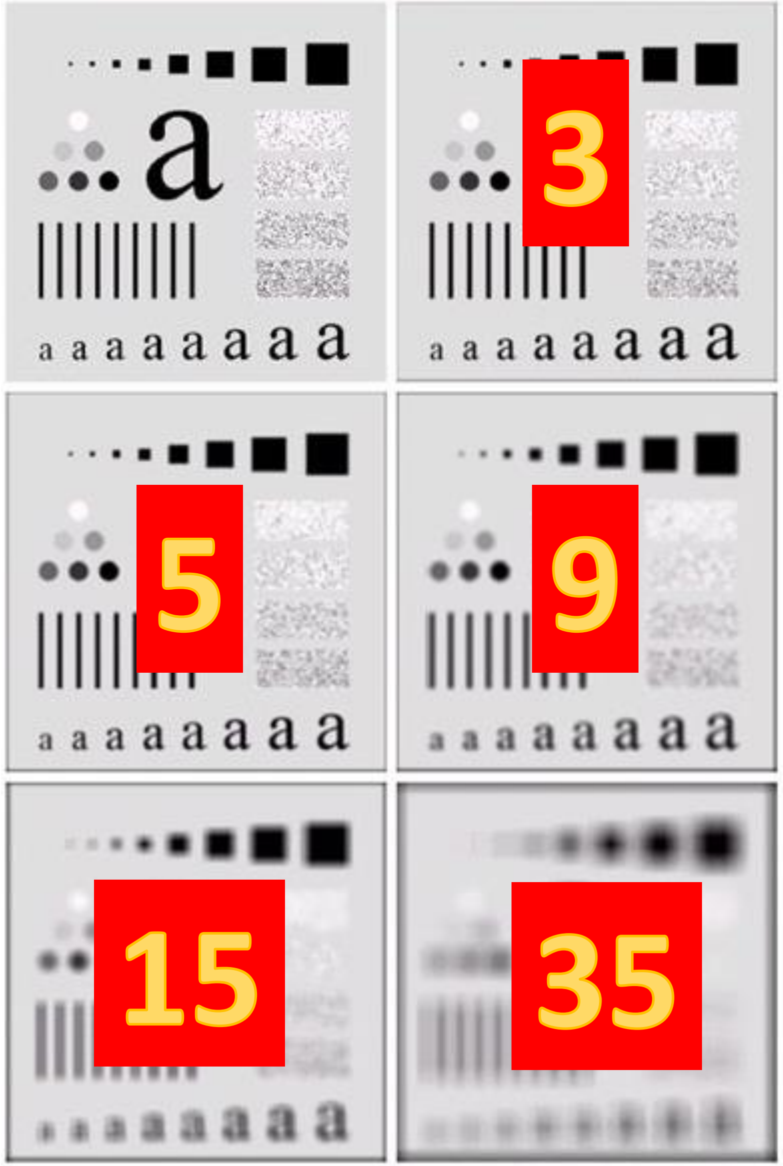


Local Neighbourhood Operations

- Filter size



Why do we get more blurriness with larger filter size?



Local Neighbourhood Operations

- Can we get sharpening artifacts using filters we learned?

Original

*

?

=

Local Neighbourhood Operations

- Sharpening

−

=

Local Neighbourhood Operations

- Adding more details will sharpen the image.
  - You can control the sharpness by alpha

$+ \alpha$

$=$

Local Neighbourhood Operations

- Adding more details will sharpen the image.
  - You can control the sharpness by alpha

$+ \alpha$

$=$

$$\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{9} \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Local Neighbourhood Operations

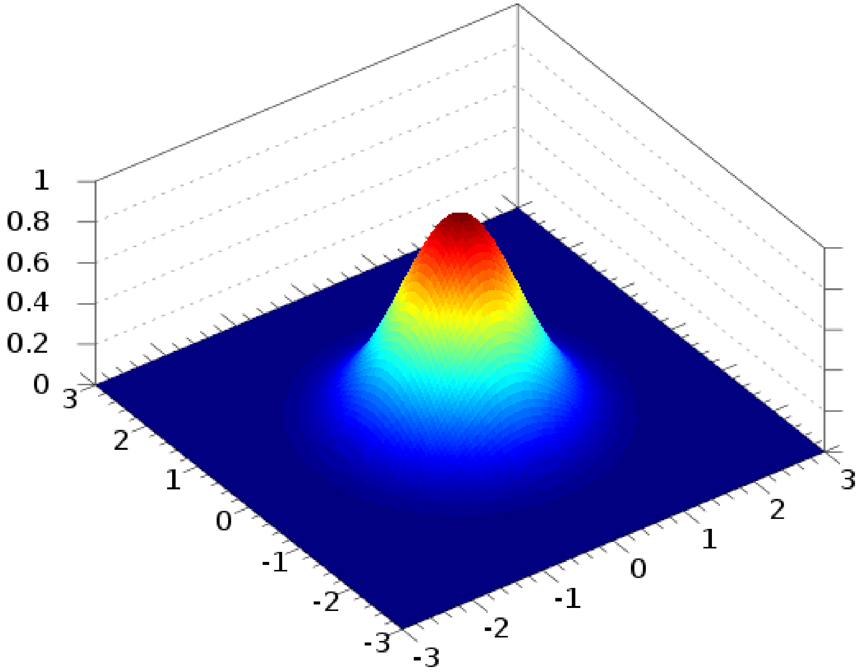
- Can we get sharpening artifacts using filters we learned?

Original

$$* \left(\begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \right) =$$

Local Neighbourhood Operations

- Another smoothing (blur) operation is Gaussian



$$f(x,y) = A\exp\left(-\left(\frac{(x-x_o)^2}{2\sigma_x^2} + \frac{(y-y_o)^2}{2\sigma_y^2}\right)\right)$$

Local Neighbourhood Operations

- If we use Gaussian as the convolution function, we can find a filter (convolution mask) by integration. (filter is not unique)

$$\frac{1}{273}\begin{bmatrix} 1 & 4 & 7 & 4 & 1 \\ 4 & 16 & 26 & 16 & 4 \\ 7 & 26 & 41 & 26 & 7 \\ 4 & 16 & 26 & 16 & 4 \\ 1 & 4 & 7 & 4 & 1 \end{bmatrix}$$

Edge Detection

- Similar to finding the derivative
  - Why?

Edge Detection

- Similar to finding the derivative
  - First derivative

$$\frac{\partial I}{\partial x} \approx I(x+1, y) - I(x, y)$$

0	0	0
0	-1	1
0	0	0

Edge Detection

- Similar to finding the derivative
  - First derivative

$$\frac{\partial I}{\partial x} \approx I(x+1, y) - I(x, y)$$

0	0	0
0	-1	1
0	0	0

  - Second derivative

$$\frac{\partial^2 I}{\partial x^2} \approx I(x+1, y) - 2I(x, y) + I(x-1, y)$$

0	0	0
1	-2	1
0	0	0

Edge Detection

- Similar to finding the derivative
  - First derivative

$$\frac{\partial I}{\partial x} \approx I(x+1, y) - I(x, y)$$

0	0	0
0	-1	1
0	0	0

  - Second derivative

$$\frac{\partial^2 I}{\partial x^2} \approx I(x+1, y) - 2I(x, y) + I(x-1, y)$$

0	0	0
1	-2	1
0	0	0

$$\frac{\partial^2 I}{\partial x^2} \approx \frac{\partial I(x,y)}{\partial x} - \frac{\partial I(x+1,y)}{\partial x} = I(x+1,y) - I(x,y) - \big(I(x,y) - I(x-1,y)\big) = I(x+1,y) - 2I(x,y) + I(x-1,y)$$

Edge Detection

- Similar to finding the derivative
  - Derivative on both axes (Laplacian)

$$\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} \approx I(x+1,y) - 2I(x,y) + I(x-1,y) + I(x,y+1) - 2I(x,y) + I(x,y-1)$$

0	1	0
1	-4	1
0	1	0

Edge Detection

- Another version

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

Edge Detection

Edge Detection

- Sobel operator

$$G_x = \frac{1}{2}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$G_y = \frac{1}{2}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$G = \sqrt{G_x^2 + G_y^2}$$

Or

$$G = |G_x| + |G_y|$$

faster

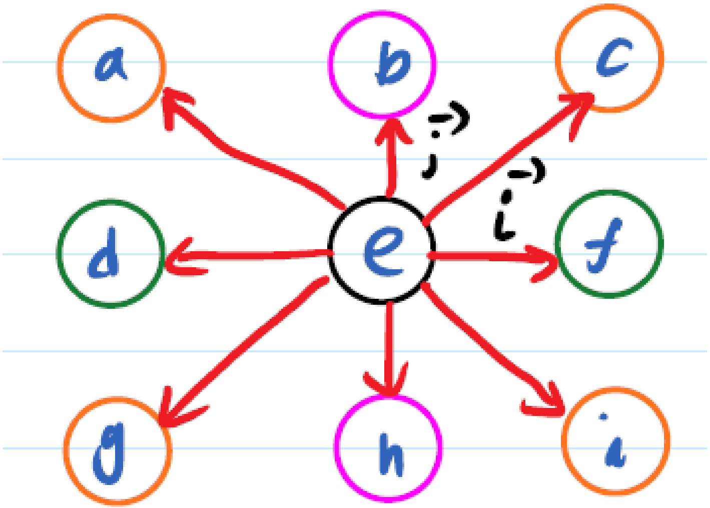
Edge Detection

- Sobel operator

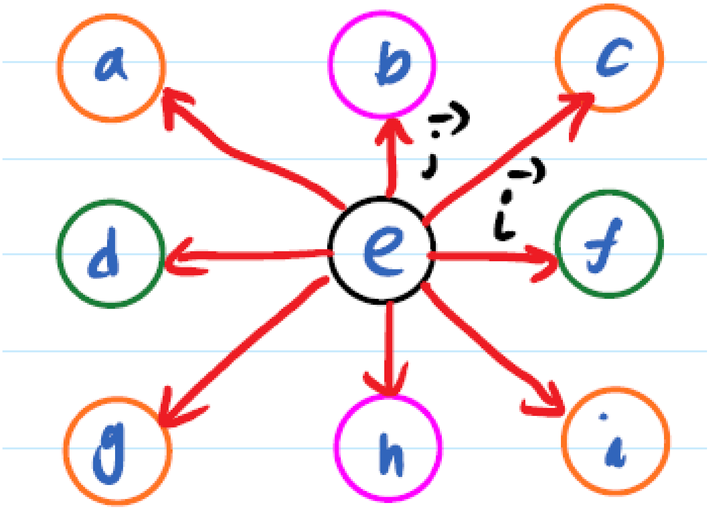
$$G_x = \frac{1}{2}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$G_y = \frac{1}{2}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

How did we find these filters?

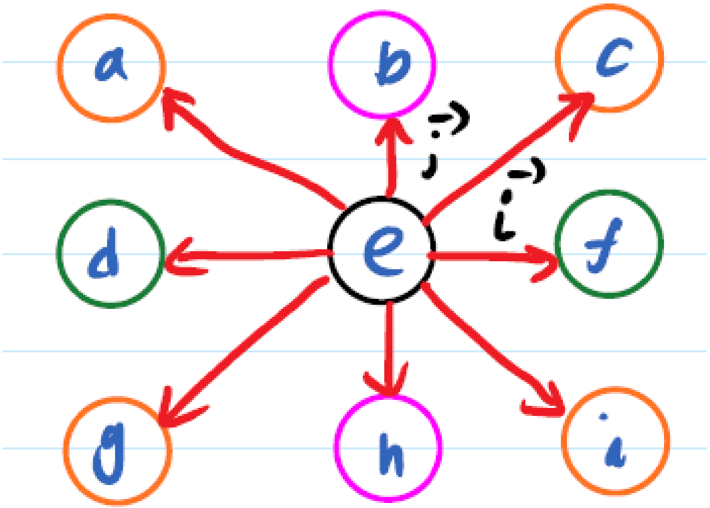


We take the derivative along these eight vectors



We take the derivative along these eight vectors

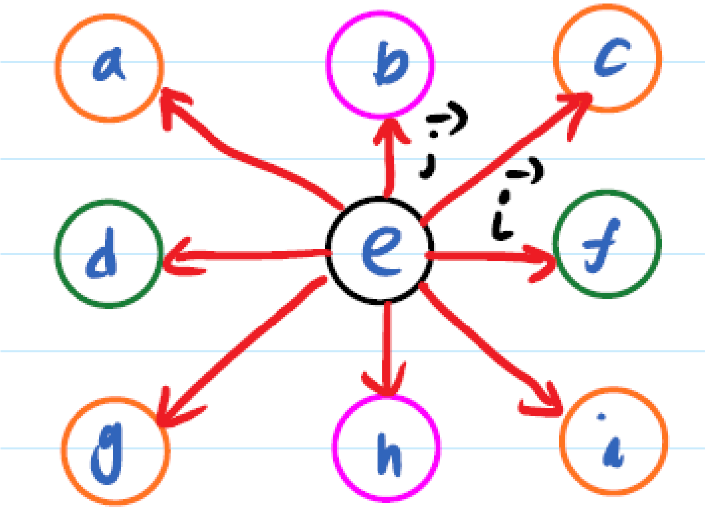
Derivatives along $\vec{\imath}$ and $\vec{\jmath}$ are easy:



We take the derivative along these eight vectors

Derivatives along $\vec{\imath}$ and $\vec{\jmath}$ are easy:

$$(f-e)\vec{\imath}+(\mathrm{d}-\mathrm{e})(-\vec{\imath})+(b-e)\vec{\jmath}+(\mathrm{h}-\mathrm{e})(-\vec{\jmath})$$

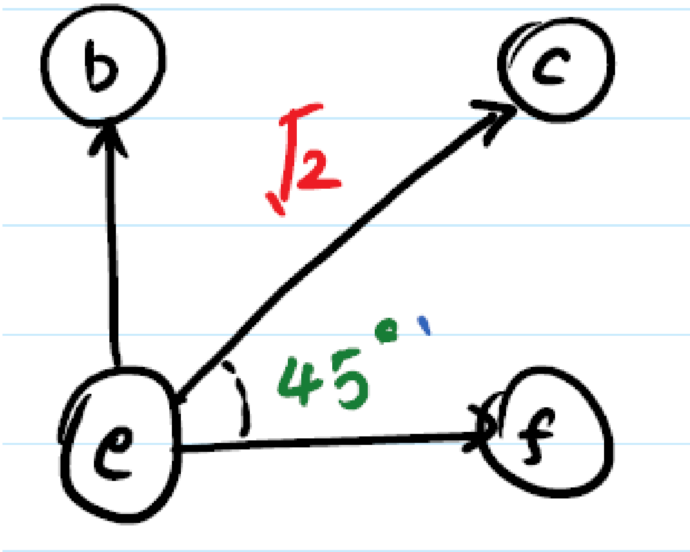


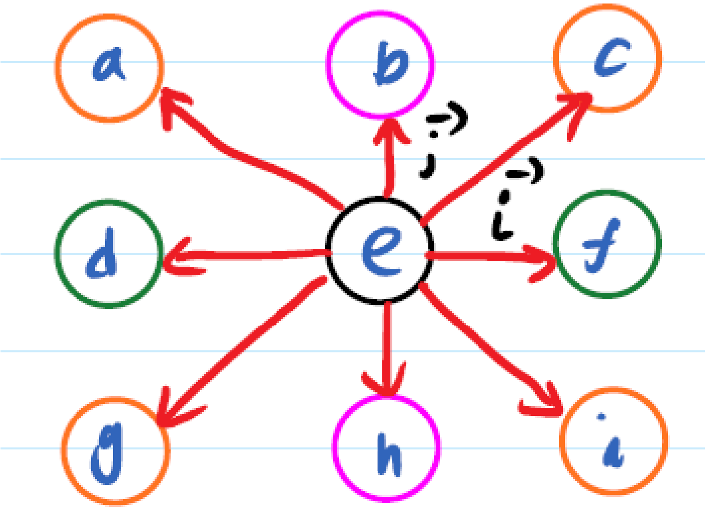
We take the derivative along these eight vectors

Derivatives along $\vec{\imath}$ and $\vec{\jmath}$ are easy:

$$(f-e)\vec{\imath}+(\mathrm{d}-\mathrm{e})(-\vec{\imath})+(b-e)\vec{\jmath}+(\mathrm{h}-\mathrm{e})(-\vec{\jmath})$$

Let’s find the derivative along diagonals:





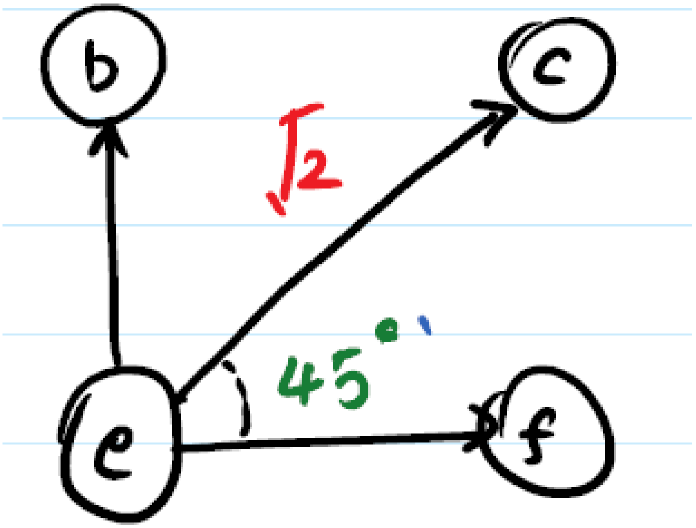
We take the derivative along these eight vectors

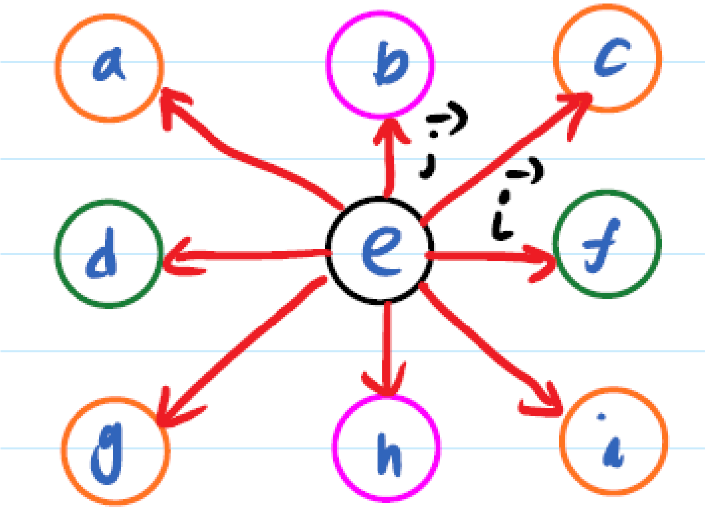
Derivatives along $\vec{\imath}$ and $\vec{\jmath}$ are easy:

$$(f - e)\vec{\imath} + (\mathrm{d} - \mathrm{e})(-\vec{\imath}) + (b - e)\vec{\jmath} + (\mathrm{h} - \mathrm{e})(-\vec{\jmath})$$

Let's find the derivative along diagonals:

$$\frac{(c - e)}{\sqrt{2}}$$ Normalize the length





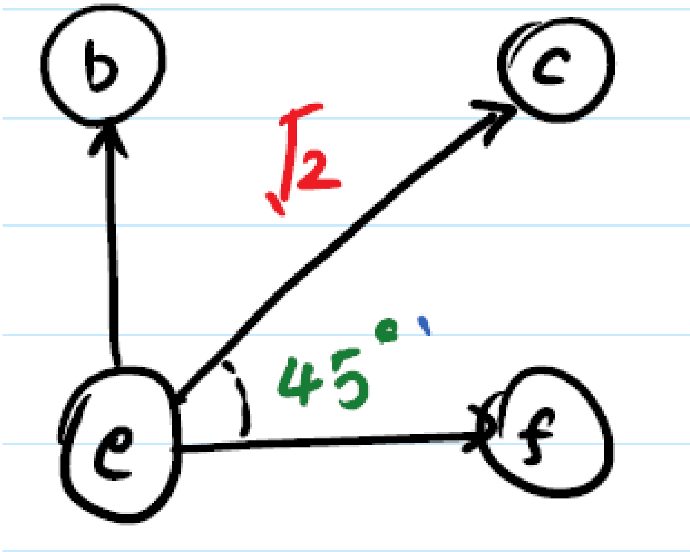
We take the derivative along these eight vectors

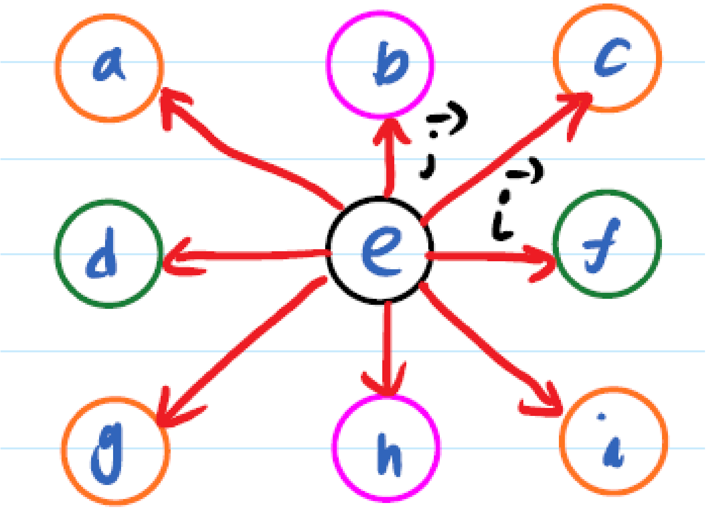
Derivatives along $\vec{\imath}$ and $\vec{\jmath}$ are easy:

$$(f-e)\vec{\imath}+(\mathrm{d}-\mathrm{e})(-\vec{\imath})+(b-e)\vec{\jmath}+(\mathrm{h}-\mathrm{e})(-\vec{\jmath})$$

Let's find the derivative along diagonals:

$$\frac{(c-e)}{\sqrt{2}}$$ Normalize the length, project on $\vec{\imath}$ and $\vec{\jmath}$



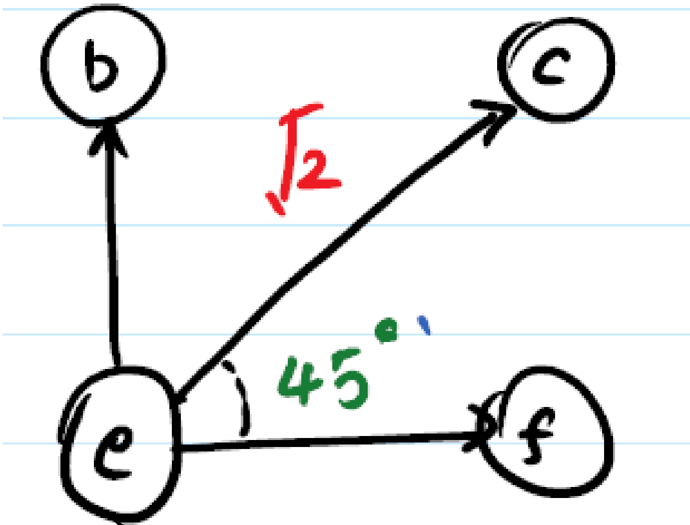


We take the derivative along these eight vectors

Derivatives along $\vec{\imath}$ and $\vec{\jmath}$ are easy:

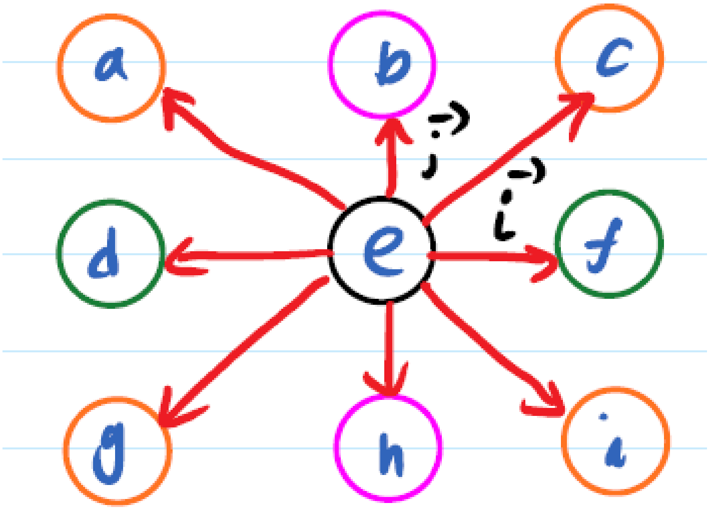
$$(f-e)\vec{\imath}+(\mathrm{d}-\mathrm{e})(-\vec{\imath})+(b-e)\vec{\jmath}+(\mathrm{h}-\mathrm{e})(-\vec{\jmath})$$

Let's find the derivative along diagonals:



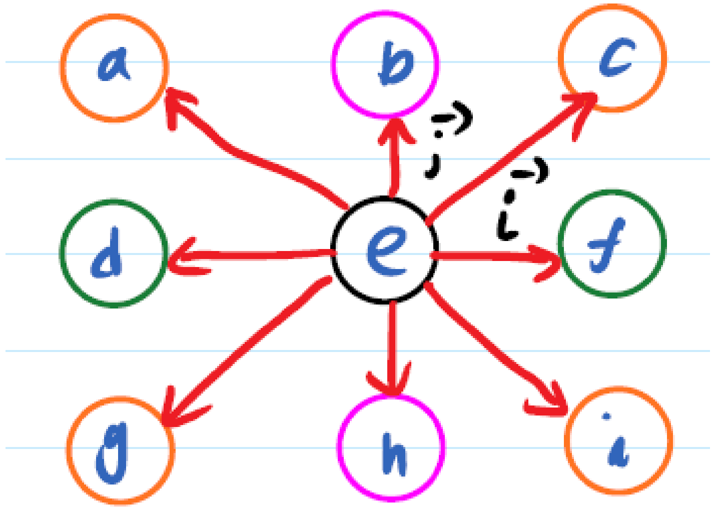
$\dfrac{(c-e)}{\sqrt{2}}$ Normalize the length, project on $\vec{\imath}$ and $\vec{\jmath}$

$$\frac{(c-e)}{\sqrt{2}}\cos(45)\,\vec{\imath}+\frac{(c-e)}{\sqrt{2}}\sin(45)\,\vec{\jmath}=\frac{1}{2}(c-e)(\vec{\imath}+\vec{\jmath})$$



Similarly, we find all the diagonal derivatives

$$(f-e)\vec{\imath}+(\mathrm{d}-\mathrm{e})(-\vec{\imath})+(b-e)\vec{\jmath}+(\mathrm{h}-\mathrm{e})(-\vec{\jmath})+$$
$$\frac{1}{2}(c-e)(\vec{\imath}+\vec{\jmath})+\frac{1}{2}(a-e)(\vec{\jmath}-\vec{\imath})+\frac{1}{2}(g-e)(-\vec{\jmath}-\vec{\imath})+\frac{1}{2}(i-e)(\vec{\imath}-\vec{\jmath})$$



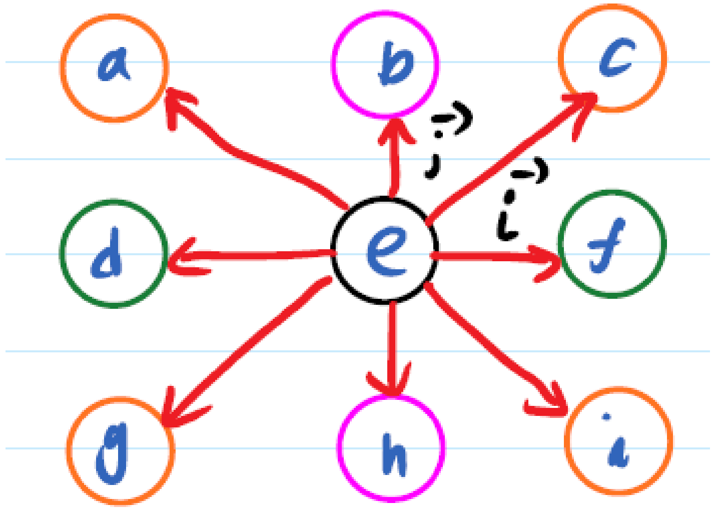
Add all them up

$$(f-e)\vec{\imath}+(\mathrm{d}-\mathrm{e})(-\vec{\imath})+(b-e)\vec{\jmath}+(\mathrm{h}-\mathrm{e})(-\vec{\jmath})+$$

$$\frac{1}{2}(c-e)(\vec{\imath}+\vec{\jmath})+\frac{1}{2}(a-e)(\vec{\jmath}-\vec{\imath})+\frac{1}{2}(g-e)(-\vec{\jmath}-\vec{\imath})+\frac{1}{2}(i-e)(\vec{\imath}-\vec{\jmath})$$

$$=\left(f-d+\frac{1}{2}c-\frac{1}{2}a-\frac{1}{2}g+\frac{1}{2}i\right)\vec{\imath}$$

$$+\left(b-h+\frac{1}{2}a-\frac{1}{2}g-\frac{1}{2}i+\frac{1}{2}c\right)\vec{\jmath}$$



Correspond values to find filters

$$(f-e)\vec{\imath}+(\mathrm{d}-\mathrm{e})(-\vec{\imath})+(b-e)\vec{\jmath}+(\mathrm{h}-\mathrm{e})(-\vec{\jmath})+$$

$$\frac{1}{2}(c-e)(\vec{\imath}+\vec{\jmath})+\frac{1}{2}(a-e)(\vec{\jmath}-\vec{\imath})+\frac{1}{2}(g-e)(-\vec{\jmath}-\vec{\imath})+\frac{1}{2}(i-e)(\vec{\imath}-\vec{\jmath})$$

$$=\left(f-d+\frac{1}{2}c-\frac{1}{2}a-\frac{1}{2}g+\frac{1}{2}i\right)\vec{\imath}$$

$$+\left(b-h+\frac{1}{2}a-\frac{1}{2}g-\frac{1}{2}i+\frac{1}{2}c\right)\vec{\jmath}$$

$\frac{-1}{2}$	0	$\frac{1}{2}$
-1	0	1
$\frac{-1}{2}$	0	$\frac{1}{2}$

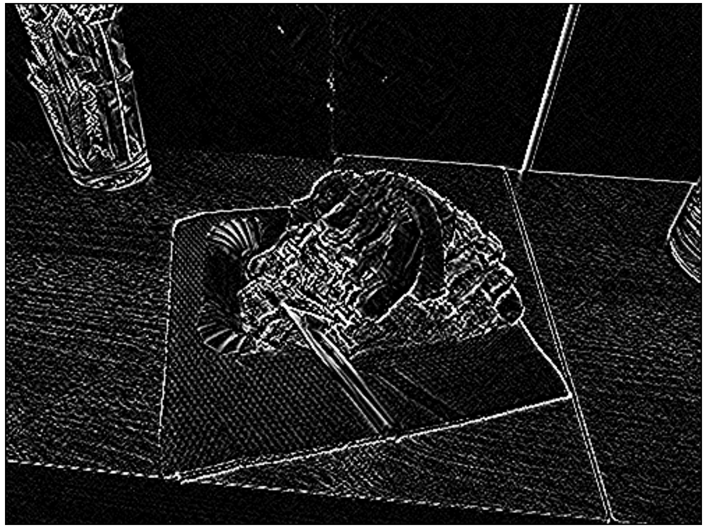
$\vec{\imath}$

$\frac{1}{2}$	1	$\frac{1}{2}$
0	0	0
$\frac{-1}{2}$	-1	$\frac{-1}{2}$

$\vec{\jmath}$

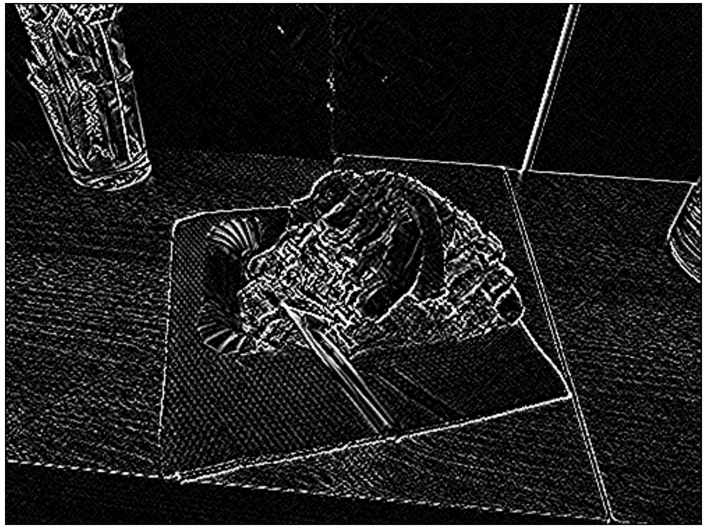
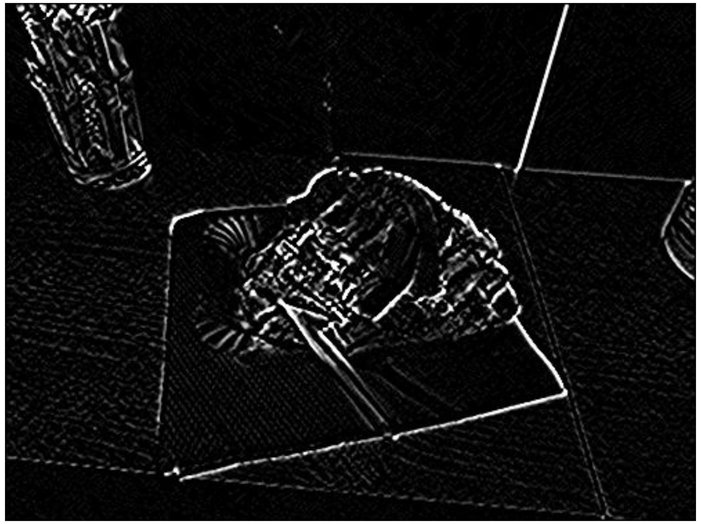
Edge Detection

- Sobel operator
  - One of the most common edge detection filters



Edge Detection

- Notice that there are many artifacts. To avoid these artifacts, we can first smooth the image by Gaussian and then apply the filter.

Edge Detection

- Sobel operator is itself a combination of a smoothing and an edge detection. How?

Edge Detection

- Sobel operator is itself a combination of a smoothing and an edge detection. How?

- hint: vertical smoothing (weighted average), horizontal edge detection (derivative)

-1	0	1
-2	0	2
-1	0	1

Edge Detection

- Sobel operator is itself a combination of a smoothing and an edge detection. How?
- hint: vertical smoothing, horizontal edge detection (derivative)

-1	0	1
-2	0	2
-1	0	1

1
2
1

-1	0	1

Edge Detection

- Sobel operator is itself a combination of a smoothing and an edge detection. How?

- hint: vertical smoothing, horizontal edge detection (derivative)

-1	0	1
-2	0	2
-1	0	1

1
2
1

-1	0	1

f(x+h)-f(x-h)

Image Features

- You can define many different convolution kernels (filters).

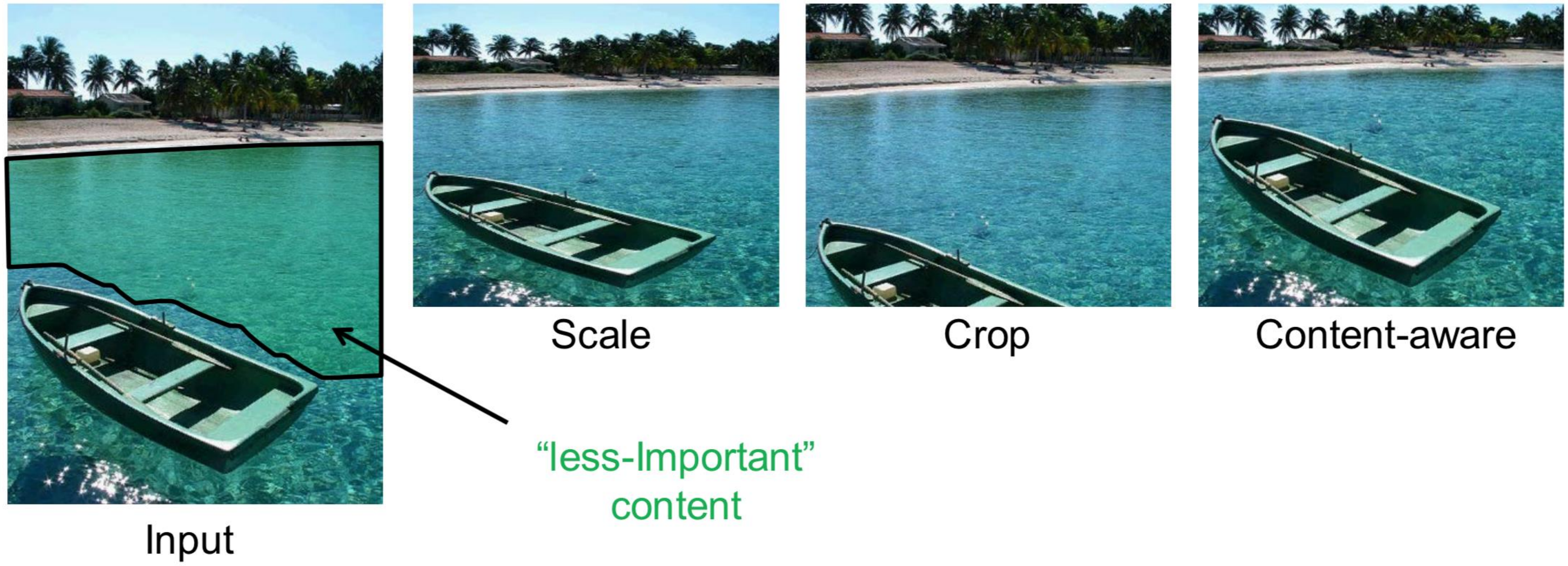
- Applying these filters may result a set of image features (e.g., corners, edges, etc), such filters are called feature detectors.

Image Features

- Find a list of feature detector filters and try to implement them in Matlab.

Application

- Image Resizing
  - An application of edge detection aside from Active Contours



Application

- How can we define “less important”?
  - The parts that have less visible edges or they have a lower energy

$$e_1(\mathbf{I}) = |\frac{\partial}{\partial x}\mathbf{I}| + |\frac{\partial}{\partial y}\mathbf{I}|$$

Application

- How can we define "less important"?
  - The parts that have less visible edges or they have a lower energy

$$e_1(\mathbf{I}) = |\frac{\partial}{\partial x}\mathbf{I}| + |\frac{\partial}{\partial y}\mathbf{I}|$$

reminder: $\frac{\partial I}{\partial x} \approx I(x+1, y) - I(x, y)$

Application

- We cannot remove single pixels

1	2	1	2
1	2	3	4
2	4	1	0
0	1	2	1
1	2	3	1

Application

- We cannot remove single pixels

How can we shift the pixels?

1	2	1	2
1	2	3	4
2	4	1	0
0	1	2	1
1	2	3	1

Application

- We can remove a column or row that has the smaller sum of energy

1	2	1	2
1	2	3	4
2	4	1	0
0	1	2	1
1	2	3	1

Application

- We can remove a column or row that has the smaller sum of energy

1	2	1	2
1	2	3	4
2	4	1	0
0	1	2	1
1	2	3	1
5	11	10	8

Application

- We can remove a column or row that has the smaller sum of energy

remove the column

1	2	1	2
1	2	3	4
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5	11	10	8

Application

- Problem with column removing
  - The chance of removing important features is high

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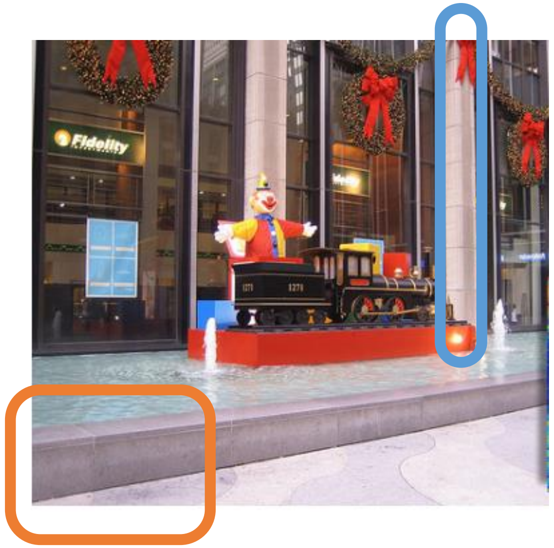
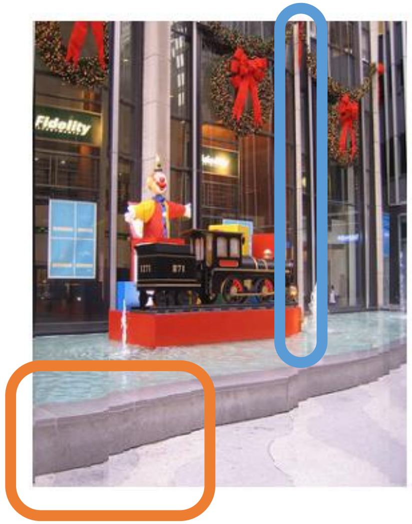




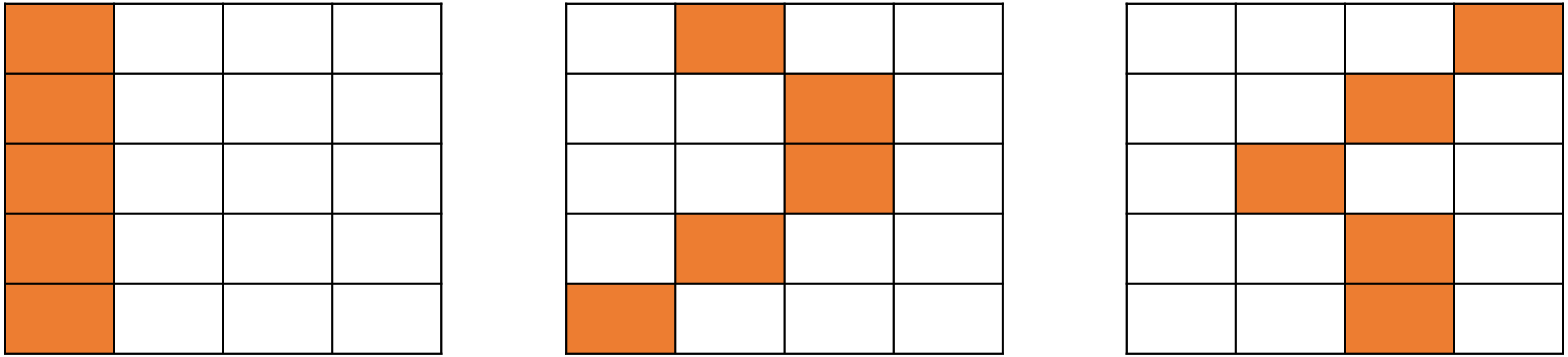
Application

- Instead of a column, we can remove a seam

Application

- How do you define a seam?
  - vertical seam: it has a pixel at each row; two consecutive rows have pixels with a difference with maximum one.

$$\mathbf{s}^{\mathbf{x}} = \{s_i^x\}_{i=1}^{n} = \{(x(i), i)\}_{i=1}^{n}, \text{ s.t. } \forall i, |x(i) - x(i-1)| \leq 1$$



Application

- How do you select a seam?
  - Dynamic programming

Application

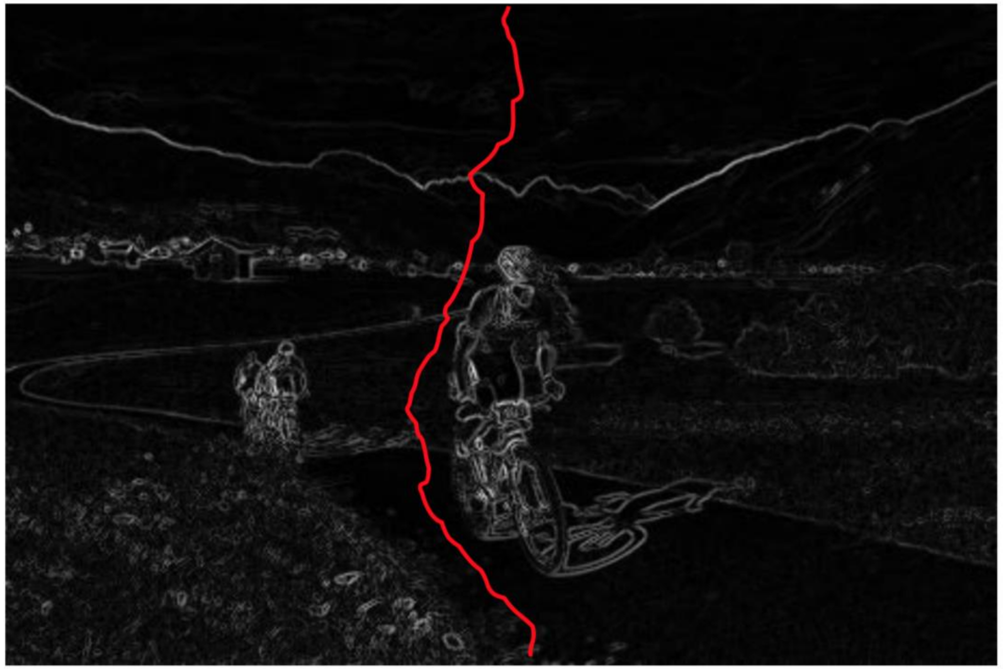
- How do you select a seam?
  - First find the energy of each pixel using discrete derivative

$$e_1(\mathbf{I}) = |\frac{\partial}{\partial x}\mathbf{I}| + |\frac{\partial}{\partial y}\mathbf{I}|$$

Application

- How do you select a seam?
  - First find the energy of each pixel using discrete derivative

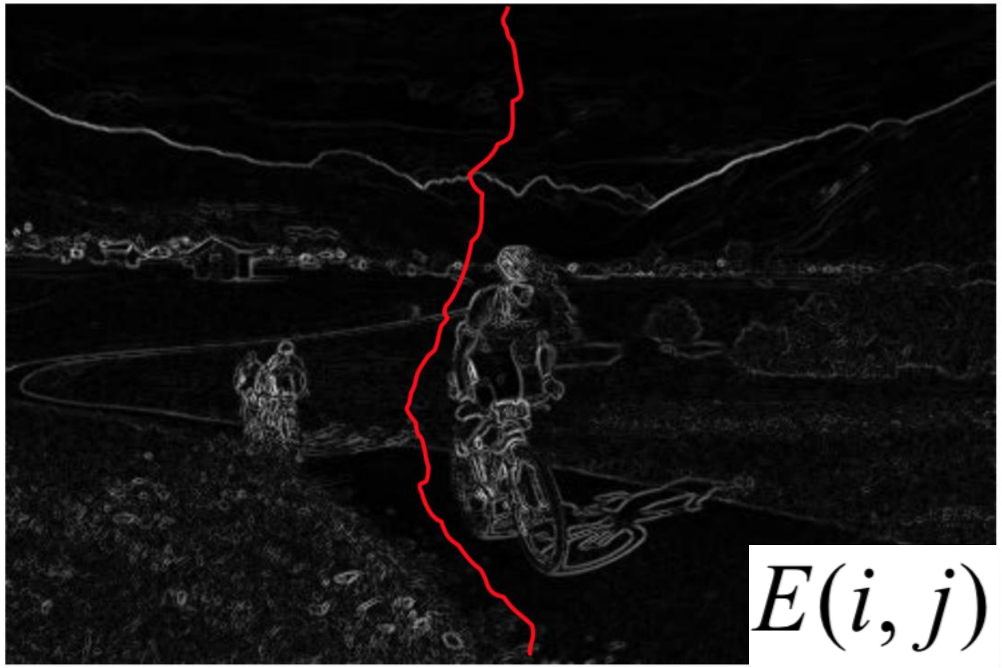
$$E(i, j)$$



Application

- How do you select a seam?
  - First find the energy of each pixel using discrete derivative
  - Then use dynamic programming with this recursive equation

			$\mathbf{M}(i,j)$



$$\mathbf{M}(i,j) = E(i,j) + \min\big(\mathbf{M}(i-1,j-1), \mathbf{M}(i-1,j), \mathbf{M}(i-1,j+1)\big)$$

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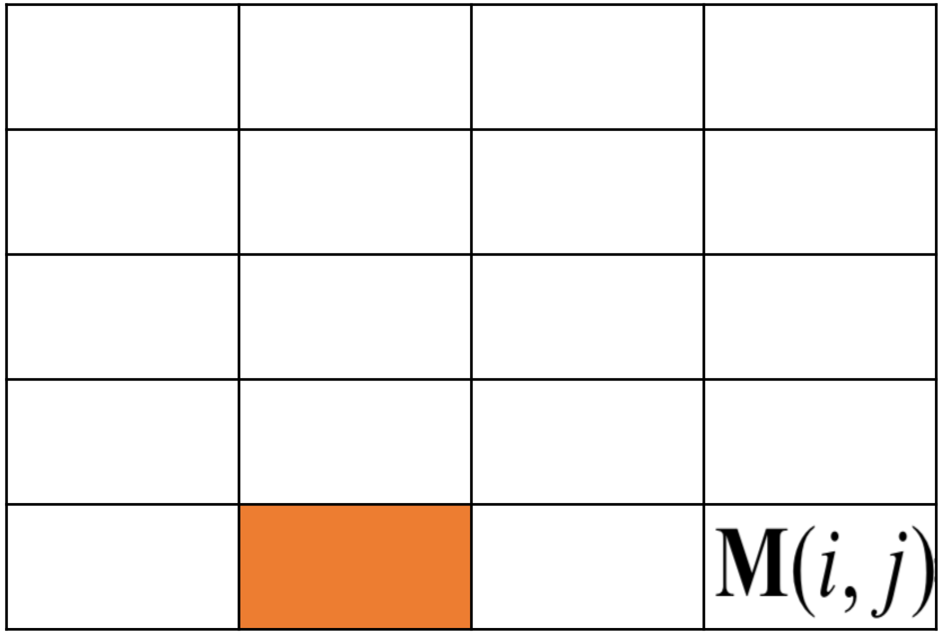

Find the pixel with smaller M(i,j) value
and backtrack to find the seam

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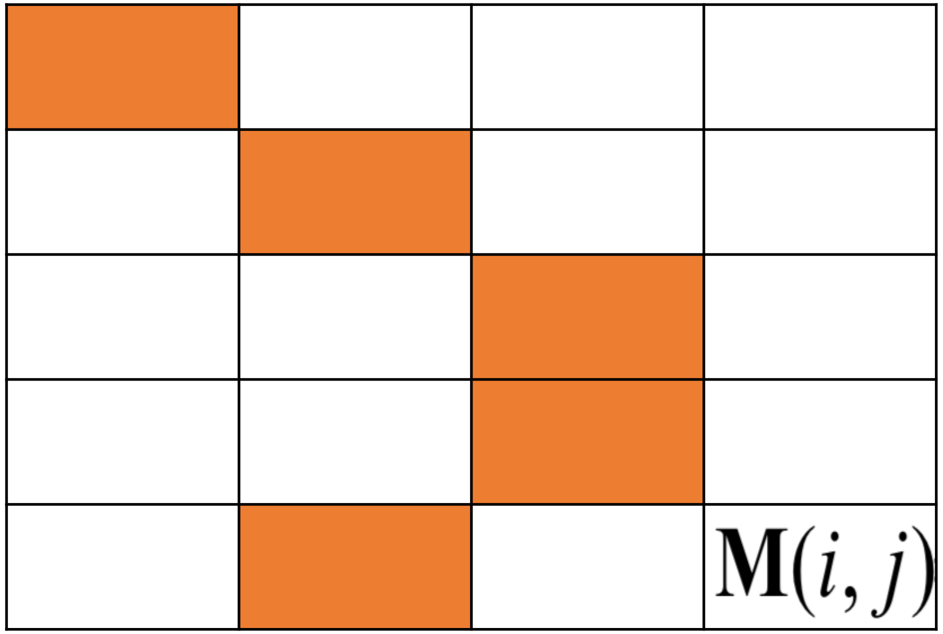

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M(i,j)

E(i,j)

5	8	12	3
9	2	3	9
7	3	4	2
4	5	7	8

Example

$$\mathbf{M}(i,j) = E(i,j) + \min\Big(\mathbf{M}(i-1,j-1), \mathbf{M}(i-1,j), \mathbf{M}(i-1,j+1)\Big)$$

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M(i,j)
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Example

- Remove the seam on the actual image

5	8	12	3
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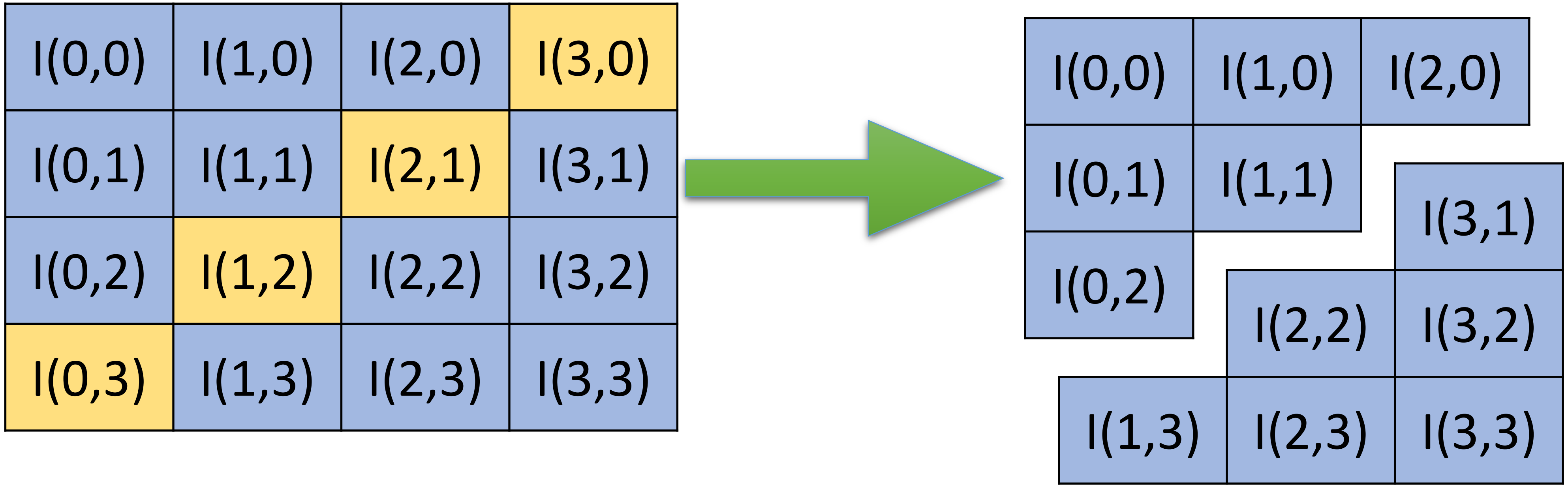
14	9	10	8
13	14	15	16

I(0,0)	I(1,0)	I(2,0)	I(3,0)
I(0,1)	I(1,1)	I(2,1)	I(3,1)
I(0,2)	I(1,2)	I(2,2)	I(3,2)
I(0,3)	I(1,3)	I(2,3)	I(3,3)

Example

- Remove the seam on the actual image



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I(0,0)	I(1,0)	I(2,0)
I(0,1)	I(1,1)	I(3,1)
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I(1,3)	I(2,3)	I(3,3)

Results

Results

- Adding seams