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Financial Market Modeling using Hidden Markov Models

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Abstract: This project applies stochastic models in quantitative research and analysis to accurately price options and assess associated risks. Leveraging historical financial data, it encompasses data preprocessing, model selection, parameter estimation, option pricing, risk analysis, and sensitivity analysis. Selected models like Black-Scholes-Merton and Heston are calibrated using optimization techniques. Accurate option prices and key option Greeks are calculated based on estimated parameters and underlying asset prices. Risk analysis utilizes Monte Carlo simulation to evaluate metrics such as VaR and ES. Sensitivity analysis explores the impact of changing model parameters. The project emphasizes documentation, resulting in a comprehensive report. It showcases expertise in option valuation, risk assessment, and quantitative analysis, demonstrating proficiency in data preprocessing, model implementation, and advanced quantitative techniques.

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2 Data Collection

3 Stochastic Models: Black-Scholes-Merton and Heston Models

The Black-Scholes-Merton model results from a solution from the stochastic differential equation

$$\d{S}(t) = \mu S(t)\ dt + \sigma S(t)\ dW(t) \tag{3.1}$$

where $S(t)$ is the price of the asset at time t, μ is the drift or expected return of the asset per unit time, σ is the volatility of the asset per unit time, and $W(t)$ is a Wiener process.

It is important to note that the the Black-Scholes-Merton stochastic differential equation makes several key assumtions:

- Efficient Markets: Markets are efficient and there are no opportunities for arbitrage.
- Constant Parameters: The parameters, specifically the risk-free interest rate r, the volatility of the asset σ, and the dividen yield q, of this model are constant with time
- Continuous Time: Asset prices and other factors change continuously rather than discretely.
- Log-Normal Distribution: The asset price follows a log-normal distribution.
- No Transaction Costs: There are no brokerage fees, taxes, or other costs associated with the underlying asset or option.
- No Dividends: The underlying asset does not pay any dividends during the option's lifetime. If the asset does pay dividends, they are assumed to be continuous and accounted for by adjusting the drift term in the stochastic differential equation describing the Black-Scholes model.

- No Market Frictions: There are no liquidity constraints, market impact, trading restrictions, or other market frictions.

- Risk-Neutral Pricing: The expected return for the asset is equal to the risk-free rate. This assumption allows for discounting future payoffs at the discounted rate.

For a European call option, the theoretical price C is given by

$$C = S_t N(d_1) - X e^{-rT} N(d_2) \tag{3.2}$$

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and the theoretical price P of a European put option is given by

$$P = X e^{-rT} N(d_2) - S_t N(d_1) \tag{3.3}$$

where S_t is the current price of the underlying asset, X is the option's strike price, r is the risk-free interest rate, T is the time to expiration in years, $N(x)$ is the cumulative standard normal distribution function, and

$$d_1 = \frac{\ln \frac{S_t}{X} + (r + \sigma^2/2) * T}{\sigma\sqrt{T}} \tag{3.4}$$

$$d_2 = d_1 - \sigma\sqrt{t} \tag{3.5}$$

where σ is the volatility or standard deviation of the asset's returns.

The Heston model for option pricing extends the Black-Scholes-Merton model by incorporating a dynamic process for volatility.

4 Conclusions