Notes on Pairs Trading

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1 Introduction

A basic pairs trading strategy consists of exploiting an out-of-equilibrium market; if two assets typically trade at some spread, then the narrowing/widening of the spread between the assets may be exploited for profit For example, if the spread widens, then one should buy the low asset and short the high asset. On the other hand, if the spread narrows, than one should short the higher asset and buy the lower asset.

Consider a state process $\{x_k\}$, where x_k denotes the value of a real variable at the time $t_k = k\tau$, where τ is the separation between times and $k = 0, 1, 2, \ldots$. We assume that $\{x_k\}$ is mean-reverting, meaning we have the following relation between the spread of susequent pairs of x_k:

$$x_{k+1} - x_k = (a - bx_k)\tau + \sigma\sqrt{\tau}\varepsilon_{k+1}, \tag{1.1}$$

where $\sigma \geq 0, b > 0, a \in \mathbf{R}$, and $\{\varepsilon_k\}$ is iid Gaussian $\mathcal{N}(0,1)$ and independent of x_k. With this definition, the process reverts to $\mu = a/b$ with *strength b*. This implies that

$$x_k \sim \mathcal{N}(\mu_k, \sigma_k^2), \tag{1.2}$$

where

$$\mu_k = \frac{a}{b} + \left[\mu_0 - \frac{a}{b}\right](1 - b\tau)^k, \tag{1.3}$$

and

$$\sigma_k^2 = \frac{\sigma^2\tau}{1 - (1 - b\tau)^2}\left[1 - (1 - b\tau)^{2k}\right] + \sigma_0^2(1 - b\tau)^{2k} \tag{1.4}$$

We can also express the mean reversion condition in the form

$$x_{k+1} = A + Bx_k + C\varepsilon_{k+1}, \tag{1.5}$$

where $A = a\tau \geq 0, 0 < B = 1 - b\tau$ and $C = \sigma\sqrt{\tau}$. Furthermore, we may also consider the stochastic process $X(k\tau) = x_k$ where $\{X(t), t \geq\}$ satisfies the stochastic differential equation

$$\dot{X}(t) = (a - b\,X(t))\dot{t} + \sigma\dot{W}(t), \tag{1.6}$$

where $W(t)$ is a standard Brownian motion.

2 Coitegration

Two time series are cointegrated if some linear combination of both datasets has a constant mean and standard deviation. That is, the time series resulting from the linear combination of the individual