Approximating discontinous functions

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Abstract

Calculating discontinous functions can be difficult and computationally expensive. By combining using approximation functions we can lower the computational expensiveness. Defining good approximation function can be a hard to get right. However, defining an approximation that works for some value can be simpler. We show how combining several simple approximations can give us a better approximation and reduce computational requirements.

1 Introduction

Outline

2 Theory

We are trying to approximate a function f:

$$f : X \mapsto \mathbb{R}$$

Although we explain the idea in $\mathbb{R}$ set, the results also apply to any set that has transitive relation $<$ and $+-$ operation (TODO: find the correct algebraic structure). Using $<$ we can define min and max for that set.

$$\min(x,y) := \begin{cases} x & \text{if } x \le y, \\ y & \text{otherwise.} \end{cases}$$

$$\max(x,y) := \begin{cases} y & \text{if } x \le y, \\ x & \text{otherwise.} \end{cases}$$

X can be any set.

2.1 Exact approximation

Let's assume we are interested in range $R \subseteq X$. Let's assume we have functions α and β such that

$$\alpha(x) \le f(x), \forall x \in R$$
$$\beta(x) \le f(x), \forall x \in R$$

Now we can define ϵ function.

$$\alpha(x) = f(x) + \varepsilon_\alpha, \varepsilon_\alpha \ge 0$$
$$\beta(x) = f(x) + \varepsilon_\beta, \varepsilon_\alpha \ge 0$$

It is trivial to derive function γ where ε_γ is smaller than ε_α and ε_β.

$$\begin{aligned} \gamma(x) &= max(\alpha(x), \beta(x)) \\ &= max(f(x) - \varepsilon_\alpha(x), f(x) - \varepsilon_\beta(x)) \\ &= f(x) - min(\varepsilon_\alpha(x), \varepsilon_\beta(x)). \end{aligned}$$

This also gives us a usefulness requirement for α and β:

$$\exists x \in R,\ \alpha(x) < \beta(x)$$
$$\exists x \in R,\ \beta(x) < \alpha(x)$$

This means that function α and β must be complementary. For some inputs one should give better approximations than the other.

2.2 Probablisitic approximation

3 Approximating an unknown function

4 Examples

Although the theory is straightforward the complexity arises from finding the appropriate functions to combine.

Levenshtein Distance

Clustering

5 Results

6 Conclusions

References