Against Gold: A Study

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This is an automatic translation. I've only adjusted some paragraphs that could be explained better. For the original text please refer this link.

1 Introduction and Motivation

I have been interested in human behavior since my childhood. In this context, I delved into various disciplines during my growth process. Each one incomparably expanded my horizons. Daniel Kahneman's book, "Thinking, Fast and Slow," was entirely different. In the studies within the book, people behaved irrationally, and Kahneman demonstrated this quite concretely. Moreover, the concepts he introduced to the literature, such as System 1, System 2, and the Peak-End Rule, overlapped with my background in biology. After encountering this book, I quickly turned my interest to the field of economics. I found it very enjoyable. This science was both a social science and provided the opportunity for numerical analysis. It reached a point where pursuing a master's degree in economics became one of my dreams during my undergraduate education.

After starting my master's in Data Science, I continued to develop myself in the field of finance. I read many finance books. If you ask me what I learned from all these books, it is that the saying "We know nothing" has much deeper meanings than it seems, and that buying an index fund and holding it for as long as possible is the most prudent move in the investment field. The first lesson is not today's topic, so I will pass it over for another time. The second, the advice on stock market investment, forms the basis of this study because this teaching is contrary to our traditional investment method of buying gold and putting it under the pillow. People close to me generally avoid investing in stocks because they think that the stock market is the place where only gamblers take place. Gold, on the other hand, is a self haven. It never loses its value.

So how accurate is this belief? Does the teaching ingrained in our culture tell the truth, or are the books written on investment? In this study, I wanted to analyze this. To make my study more comprehensive, I also addressed whether trading is more advantageous than the passive investment. The data I used mainly consists of the S&P 500 index from 1950 to 2023 [1][2] and gold prices [3]. Additionally, I utilized some data, such as the Consumer Price Index (CPI) [4], SPY ETF prices [5], and S&P 500 dividend yields [6] to aid in the research. For the trading simulations, I considered the method called the "Exponential Moving Average (EMA) Crossover Strategy". Finally, those who want to examine the codes I used during my research can access them via THIS link.

My research questions (RQ) are as follows:

- RQ1: Are the annual returns of gold and stock investments statistically different from each other?
- **RQ2:** Does the purchase price matter in 20-year investments?
- **RQ3**: Are the average returns of investment instruments over a 20-year period different from each other?
- **RQ4:** Does the trading method provide more profit than the passive investment?

1.1 Data Sets

In this research, the price records of gold and the S&P 500 index from 1950 to 2023, the price records of the SPY ETF since its inception, the annual S&P 500 dividend yield, and the annual CPI values have been used. Gold, SPY, and S&P 500 prices are available in four different forms: open, close, high, and low (Table 1). Although there are small differences among them, only the closing price was used in the calculations to eliminate the effect of intraday price fluctuations.

Table 1: S&P 500 Prices

Date	Close	Open	High	Low
04/25/2024	5048.42	5019.88	5057.75	4990.58
04/24/2024	5071.63	5084.86	5089.48	5047.02

CPI data is a monthly published data used to measure inflation [7]. The CPI dataset contains the CPI value for each year, which includes the average of the CPI values announced during that year (Table 2).

Companies can distribute a portion of their profits to shareholders at certain times of the year to attract investors. This is called a dividend. Since each company's share price is different, it is not correct to compare the distributed dividends on a nominal basis. This has led to the emergence of a measure called dividend yield. It is also possible to receive dividends when investing in the S&P 500 index. The dividend yield dataset shows the historical dividend yields of the S&P 500 index (Table 3).

Table 2: CPI Data

Year	CPI
2022	292.7
2023	304.7

Table 3: Dividend Yield Data

Year	Dividend Yield
Dec 31, 2018	2.09%
Dec 31, 2017	1.84%

2 Preliminary

In this section, the statistical concepts used during the study will be discussed to provide the necessary theoretical background for the reader.

Normal Distribution

The normal distribution forms the basis of the science of statistics. This is because it is quite good at explaining uncomplicated phenomena like height and has a series of properties for making inferences. These can be summarized as follows:

- The normal distribution is symmetric.
- To describe a normal distribution, the mean and standard deviation are sufficient.
- 68% of the area under the normal distribution curve lies within one standard deviation from the mean, and 95% lies within two standard deviations.
- The mode, median, and mean values of a normal distribution are the same.

Paired T-test

The paired t-test is a parametric test used to compare the means of two dependent groups to determine if they are significantly different from each other [8]. To calculate the test statistic, first, the mean difference and then the standard deviation of the differences need to be found. Let D represent the differences of all paired samples, with the sample mean calculated as in (1) for a sample size of n.

$$\bar{d} = \frac{D}{n} \tag{1}$$

To calculate the unbiased sample standard deviation, each paired sample difference is compared to this mean:

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1}}$$
 (2)

After calculating the sample mean difference and standard deviation, the final step is to calculate the test statistic:

$$t = \frac{\bar{d} - 0}{\hat{\sigma} / \sqrt{n}} \tag{3}$$

The p-value is automatically obtained from the t-distribution using the test statistic and degrees of freedom. For a two-tailed paired t-test, the relevant hypotheses are as follows:

- H_0 : The mean difference between the compared groups is 0.
- H_1 : The mean difference between the compared groups is not 0.

If the p-value obtained from the test is below the critical p-value, H_0 is rejected. In this study, the critical p-value is set at 0.05 for all tests.

Shapiro-Wilk Test

This test compares whether a distribution is normal or not [9]. As a distribution approaches normality, the test statistic approaches 1. The hypotheses for this test are as follows:

- H_0 : The given distribution is normal.
- H_1 : The given distribution is not normal.

Wilcoxon Signed-Rank Test

The Wilcoxon Signed-Rank test is used for comparing paired samples when the normality condition is not met [10]. It assumes that the distributions being compared are similar and makes a comparison based on their medians. The relevant hypothesis tests are as follows:

- H_0 : Median_A Median_B = 0
- H_1 : Median_A Median_B $\neq 0$

3 Methods

In this section, the mathematical methods behind three different calculations have been examined in detail: annual return calculations, long-term return analyses, and the EMA Crossover strategy.

3.1 Annual Return Calculations

The chosen starting month affects the annual return calculations, resulting in different outcomes (Table A1). For example, the real return of the S&P 500 Index between January 1950 and January 1951 is 23.9%, while the real return between June 1950 and June 1951 is only 14%. To standardize the variability in returns, the returns for each month can be calculated and averaged. Alternatively, using the average annual closing prices yields results very similar to calculating the annual returns for each month and then averaging them (Pearson correlation coefficient = 0.999 for both investment instruments). As a result, annual average closing prices (AAC) were considered when calculating the returns.

The annual proportional nominal returns (APR) of different investment instruments can be calculated using (4).

$$APR_{i,i-1} = \frac{AAC_i - AAC_{i-1}}{AAC_{i-1}} \times 100$$
(4)

It is known that inflation erodes the value of money over time. Therefore, to calculate the annual proportional real returns (ARR), prices need to be adjusted for inflation. The inflation coefficient (IC) can be used for this purpose and is found by the ratio of the CPI values of the two years being examined.

$$IC_{i,j} = \frac{CPI_i}{CPI_j}$$
 (5)

In this context, (4) can be adjusted by IC for the calculation of real returns.

$$ARR_{i,i-1} = \frac{AAC_i - AAC_{i-1} \times IC_{i,i-1}}{AAC_{i-1} \times IC_{i,i-1}} \times 100$$
(6)

As mentioned in the Data Sets section, dividend income contributes to the income earned by a stock investor. The dividend

yield (DY), which is revealed for comparison, is found by the ratio of the gross dividend to the share price.

$$DY_i = \left(\frac{Gross\ Dividend}{Share\ Price}\right)_i \times 100 \tag{7}$$

It is possible to receive dividends when investing in the S&P 500 index. Information on dividend yields for years is available in databases. If an investor does not add the dividends received to the investment portfolio, the annual proportional real returns are calculated using (6); if added, (8) is used.

$$ARR_{i,i-1} = \left(\frac{AAC_i + AAC_{i-1} \times IC_{i,i-1} \times DY_{i-1} - AAC_{i-1} \times IC_{i,i-1}}{AAC_{i-1} \times IC_{i,i-1}}\right) \times 100$$
(8)

Since it is not possible for gold investors to receive dividends, (6) was used to calculate the annual returns for these investors.

3.2 Calculating Long-Term Investment Returns

In this section, the proportional real returns of investors who invested in gold and the S&P 500 index via SPY ETF over twenty-year periods are examined. Before moving on to the methodology followed in long-term investment simulations, it is necessary to mention the assumptions underlying the study:

- Since it is not possible for investors to invest directly in the S&P 500 index, investors use the SPY ETF for this purpose.
- Although the SPY ETF was established in 1993, it can be used in simulations for all years, and its unit price is always 0.1 of the index price ($\mu_{ratio} = 0.1 \& \sigma_{ratio} = 0.00027$).
- The deduction coefficient for management fees (MF) for the SPY ETF is 0.00095, and this coefficient is constant for all years.
- No deductions are applied to the dividends received by the index investor.
- There are no buying or selling commissions for investment instruments.
- An investor trying to profit from trading adopts the EMA Crossover strategy and never deviates from this strategy.
- An investor trying to profit from trading does not evaluate their money elsewhere when not in a position.
- An investor adopting the trading strategy does not take a short position when the strategy is in a sell position.

3.2.1 Twenty-Year ETF Buy-Hold Simulation

The twenty-year buy-hold simulation is based on the scenario where investors buy 2 ETFs on 10 different random dates in the starting year. Accordingly, the total nominal costs (NC) of investors in all simulations can be calculated using (9). The variable F in the equation corresponds to the unit price of the ETF.

$$NC_i = 2 \times \sum_{k=1}^{10} F_{k,random}$$
 (9)

The annual growth rate (AGR) of a portfolio can be found using (10) when dividends are not reinvested.

$$AGR_i = \frac{AAC_i}{AAC_{i-1}} - MF \tag{10}$$

During the study, the dividend-free nominal portfolio value (PV) obtained as a result of simulations was calculated using (11).

$$PV_{i+20,\text{no dividend}} = PV_i \times \prod_{i+1}^{i+20} (AGR_i - MF) \qquad i = 1950, 1951..., 2003$$
(11)

On the other hand, by making a slight adjustment to (11), the nominal portfolio value with dividends was also calculated during the simulations. This approach is based on the assumption that dividends can be fully reinvested regardless of the

ETF price.

$$PV_{i+20,\text{with dividend}} = PV_i \times \prod_{i+1}^{i+20} (AGR_i + DY_{i-1} - MF) \qquad i = 1950, 1951..., 2003$$
(12)

3.2.2 Twenty-Year Gold Buy-Hold Simulation

The steps followed in the twenty-year gold buy-hold simulation are similar to those followed in the twenty-year ETF buy-hold simulation. In this simulation, it is assumed that the gold investor buys 2 ounces of gold on 10 different random dates, so the nominal cost of these investors is calculated using (9). The only difference is that this time the variable F corresponds to the ounce price of gold.

The nominal value of gold after twenty years is found by multiplying the average gold price twenty years later by the amount of gold held. Since it is assumed that the investor bought a total of 20 ounces at the beginning of the simulation, the nominal portfolio value of the gold investor is calculated using (13).

$$PV_{i+20} = 20 \times AAC_{i+20}$$
 $i = 1950, 1951..., 2003$ (13)

Using the relevant CPI values, the nominal values obtained as a result of (9), (11), (12), and (13) can be adjusted for inflation. In this study, the total portfolio values and costs in all simulations are adjusted according to 2023 for standardization.

3.3 EMA Crossover Trading Simulation

3.3.1 EMA Crossover Strategy

The EMA Crossover strategy is based on the comparison of two exponential moving averages (EMA) of different lengths. The main logic is to take a position by measuring the buying or selling appetite of recent buyers or sellers compared to longer-term buyers or sellers. In this study, the EMA Crossover strategy was applied at (3-5) and (5-8) settings (Figure 1). The reason for preferring short-term moving averages is to ensure that investors traders move as differently as possible from those making longer-term investments. Since the simulations are conducted based on daily closing prices, the EMAs were also created based on daily closing prices. A buy signal is generated when the short-term EMA crosses above the long-term EMA, and a sell signal is generated when the opposite occurs.

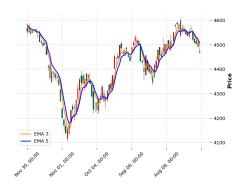


Figure 1: EMA Crossover Strategy

3.3.2 EMA Crossover Simulations

The EMA Crossover simulation starts with the investor buying 20 ETFs when the first buy signal occurs. Conversely, the investor sells all ETFs when a sell signal is received. When ETFs are sold, management fees are deducted proportional to the number of days held, and the remaining money forms the capital for subsequent trades (Trading Breakdown Example). At the end of the simulations, it is assumed that an investor with an open position closes it at the year's last day's price.

The simulation period is one year and has been repeated from 1950 to 2023. To compare the returns of the trading strategy, a control group was created where an investor buys 20 ETFs at the average price of the first month of the year and sells

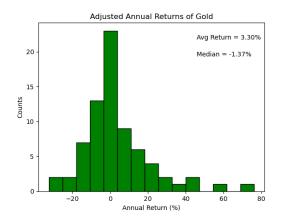
all ETFs at the average price of the last month of the year. The control group's returns are calculated by subtracting the purchase cost and management fees from the revenue obtained from the sale. Since the primary goal of the trading strategy is to profit from price fluctuations, dividend income is not included in the total return calculations for both groups.

4 Test Results and Findings

In this section, the research questions mentioned in the Introduction and Motivation section are addressed one by one, and the findings on these questions are shared with the reader.

4.1 Investment Instruments and Annual Returns

To calculate the annual returns of gold and the S&P 500 index, the formulas mentioned in the Annual Return Calculations section were applied to the data, and the histograms of the results obtained are shown in Figure 2. As can be seen from the histograms, although the average returns of the investment instruments are close to each other, their return distributions are quite different. In other words, the annual return of the S&P 500 index is much more stable than that of gold. Therefore, it can be concluded that investing in the S&P 500 index is more suitable for investors who are uncomfortable with extreme price fluctuations.



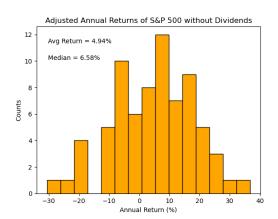


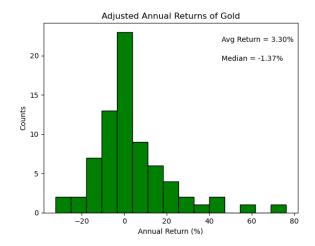
Figure 2: Real Returns of Investment Instruments

RQ1: Are the annual returns of gold and stock investments statistically different from each other?

The histograms in Figure 2 show the distribution of annual returns for both gold and the S&P 500 index. While the average returns for these investment vehicles may appear similar, their distributions indicate a key difference in volatility. The annual returns for the S&P 500 index exhibit a more stable pattern compared to the fluctuating returns of gold. This suggests that the S&P 500 index may be a preferable investment for those who prioritize stability over the potential high volatility associated with gold.

In the study, the Wilcoxon Signed-Rank Test was used to test whether the annual returns of the investment instruments used were different from each other. According to the test results, it was found that the returns from the price movements of gold and the S&P 500 index were not statistically different from each other (Table 4).

As mentioned earlier, investors in the S&P 500 index gain not only from price movements but also from dividends distributed by companies at regular intervals. When dividends are taken into account, the annual real return of the S&P 500 index rises to 8.07% (Figure 3).



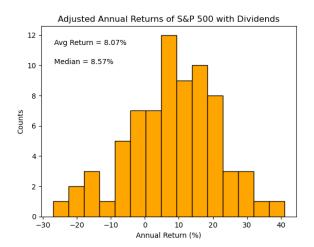


Figure 3: Real Returns of Investment Instruments (2)

When dividends are taken into account, it was found that the annual return of the S&P 500 index is different from that of gold and the S&P 500 without dividends.

Table 4: Annual Return Comparisons

Comparison	Test Used	Accept/Reject H ₀
Gold - S&P 500 Without Dividends	Wilcoxon	Accept
Gold - S&P 500 With Dividends	Wilcoxon	Reject
S&P 500 With Dividends - S&P 500 Without Dividends	Paired T-test	Reject

4.2 Long-Term Returns of Investment Instruments

In the previous section, the annual returns of gold and the S&P 500 were compared, and it was found that, when only price movements are considered, the annual returns of these two investment instruments are not different from each other. However, when dividends are taken into account, the opposite is true. In this section, the returns of an investor who invested in the S&P 500 via the SPY ETF for 20 years will be compared with the returns of a gold investor.

RQ2: Does the purchase price matter in 20-year investments?

Before comparing the returns, it is necessary to determine the extent to which the purchase price affects the total gain obtained by the investor. For this purpose, 20-year investments made in gold and the SPY ETF from 1950 to 2023 were simulated 30 times each, and the standard deviations of the proportional returns obtained were examined. Table 5 shows the periods with the highest standard deviations for 20-year investments in gold and the SPY ETF.

Table 5: Standard Deviations of Gold and S&P 500 Proportional Returns

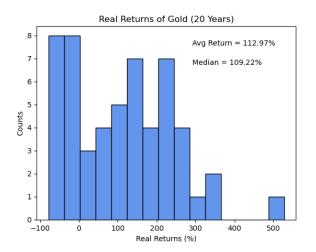
Period	Standard Deviation
(1972, 1992)	28.45
(2003, 2023)	19.87
(1968, 1988)	17.84
(1973, 1993)	17.30
(1969, 1989)	16.77

Period	Standard Deviation
(1980, 2000)	2.72E-13
(1979, 1999)	2.32E-13
(1981, 2001)	2.16E-13
(1982, 2002)	1.96E-13
(1978, 1998)	1.92E-13

The results show that in 20-year investments, the purchase price of gold has a much greater effect on the profit obtained compared to the purchase price of the S&P 500. Therefore, when comparing the long-term returns of investment instruments, only one simulation was performed for each period for stock investments. On the other hand, when calculating gold returns, 30 simulations were performed for each period, and the averages of the returns were considered during the comparison.

RQ3: Are the average returns of investment instruments over a 20-year period different from each other?

To find the returns of 20-year investments in gold and the S&P 500 index, the formulas mentioned in the Calculating Long-Term Investment Returns section were applied, and the results are shown in Figure 4. Consistent with the conclusions reached in the Investment Instruments and Annual Returns section, although the averages are similar, it was observed that the range of returns for gold is wider than for the S&P 500 ([-78%, 523%] and [-42%, 465%], respectively).



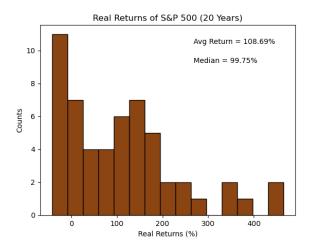
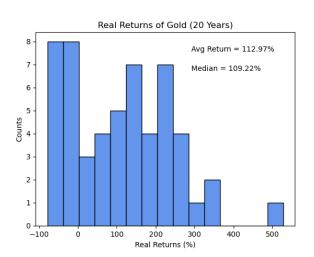


Figure 4: Long-Term Real Returns of Investment Instruments

When dividends are taken into account, the average real return of the S&P 500 increases to 261% (Figure 5). The test results show that the difference in profits due to only price movements is not statistically significant, while the return of the S&P 500 with dividends is different from gold and the S&P 500 without dividends (Table 6).



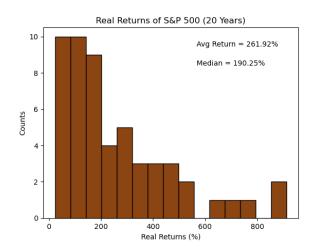


Figure 5: Long-Term Real Returns of Investment Instruments (2)

Table 6: Long-Term Return Results

Comparison	Test Used	Accept/Reject H ₀
Gold vs S&P 500 Without Dividends	Wilcoxon	Accept
Gold vs S&P 500 With Dividends	Wilcoxon	Reject
S&P 500 Without Dividends vs S&P 500 With Dividends	Wilcoxon	Reject

Finally, it was observed that 20-year investments in the S&P 500 index provided real returns in all periods.

4.3 Profiting from Trading

In this section, it is examined whether the profit obtained from daily transactions using the EMA Crossover Trading strategy is statistically different from the profit obtained by buying and holding. The methodology explained in the EMA Crossover Trading Simulation section was followed during the trades.

4.3.1 Trading S&P 500 Index

RQ4: Does the trading provide more profit than the passive investment?

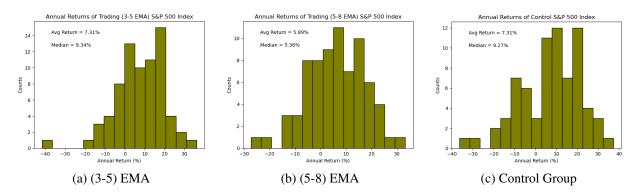


Figure 6: Comparison of Different Trading Strategies in S&P 500

The return histograms for the (3-5) EMA, (5-8) EMA, and control group are shown in Figure 6. The results for the (3-5) EMA are very close to each other, while the (5-8) EMA results are slightly lower compared to the other two.

Table 7: Comparison of Different Trading Strategies in the S&P 500

Comparison	Test Used	Accept/Reject H ₀
(3-5) EMA vs (5-8) EMA	Wilcoxon	Reject
Control vs (3-5) EMA	Wilcoxon	Accept
Control vs (5-8) EMA	Paired T-test	Accept

The findings on whether the distributions are different from each other are summarized in Table 7. A statistically significant difference was found only between the (3-5) and (5-8) EMA strategies. On the other hand, neither strategy showed a significant advantage over the control group.

4.3.2 Trading Gold

The application of the simulations from the previous section to gold is shown in Figure 7. The annual returns of the three strategies are very close to each other and are not statistically significant (Table 8).

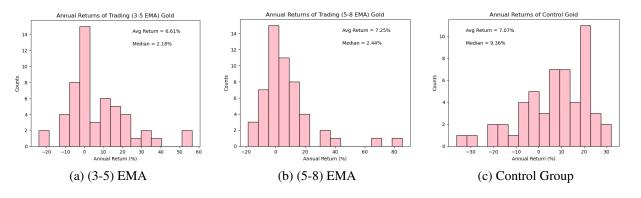


Figure 7: Annual Returns of Different Trading Strategies in Gold

Table 8: Comparison of Different Trading Strategies in Gold

Comparison	Test Used	Accept/Reject H ₀
(3-5) EMA vs (5-8) EMA	Wilcoxon	Accept
Control vs (3-5) EMA	Wilcoxon	Accept
Control vs (5-8) EMA	Wilcoxon	Accept

5 Conclusion and Potential Improvements

This study has examined the returns of investments in gold and the S&P 500 index in various ways. It has been found that in the medium and long term, the return on the S&P 500 index is more stable and higher when dividends are considered (RQ1, RQ3). While investing in gold, the purchase price has been observed to be more crucial than in stock market investments (RQ2). It has been determined that the revenues from buying and selling are not significant for these two investment tools (RQ4).

In the study, it is assumed that long-term investors fully benefit from dividend incomes. However, in reality, a certain amount of tax is deducted from dividend incomes. Moreover, the dividend incomes obtained may not fully reflected in the ETF prices. This situation has shown the revenues obtained by long-term investors to be higher than they are.

For those who engage in buying and selling, different options exist that may be more suitable for their goals. Additionally, it is known that those who buy and sell frequently use leverage and trade over shorter periods. Besides these, individuals who make money from buying and selling typically adopt more complex trading strategies and test how successful these strategies have been in the past. These factors can be considered in a future study to create more realistic simulations. Lastly, the finding in this study that the S&P 500 offers higher returns does not make gold a poor investment option. I will discuss the role of gold in an investment portfolio in another study.

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 Table A1: Variation Introduced by Month Selection

Period	Month	(%)Reel Getiri
(1950, 1951)	1	23.909
(1950, 1951)	2	25.936
(1950, 1951)	3	22.955
(1950, 1951)	4	21.114
(1950, 1951)	5	17.749
(1950, 1951)	6	14.03
(1950, 1951)	7	24.382
(1950, 1951)	8	22.555
(1950, 1951)	9	21.501
(1950, 1951)	10	16.506
(1950, 1951)	11	13.781
(1950, 1951)	12	17.389

Trading Breakdown Example

Date: 1950-01-04

Bought 20 ETFs at a price of: 1.685 Capital invested to this trade: 33.7

Cash in hand: 0
Date: 1950-01-13

Sold 20 ETF's at a price of: 1.66 Capital gained from trade: 33.34 Expense payment: 0.00078

Net cash: 33.33 Date: 1950-01-19

Bought 19.0 ETF's at a price of: 1.68 Capital invested to this trade: 32.05

Cash in hand: 1.28