

Welfare Effects of Labor Income Tax Changes on Married Couples: A Sufficient Statistics Approach

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Introduction

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What are the welfare effects of income tax changes on couples?

Literature: another group sensitive to tax changes — single mothers.

- Single mothers — lower end of the income distribution.
- Household = single person \Rightarrow no within-household interaction.

What I Do?

1. Develop a framework for assessing the welfare effects of labor income tax changes on married couples.
 - Model of couples' labor supply with intensive and extensive margins.
 - Tractable expression for welfare gains as a function of labor supply elasticities, policy parameters, and labor income shares.

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2. Use the expression with CPS data & NBER TAXSIM to estimate the welfare effects of the 1986, 1993, 2001, and 2017 U.S. tax reforms.
 - Welfare gains are from -0.16% to 0.62% of aggregate labor income.
 - Quantitative importance of extensive margin & spousal cross-effects.
 - Aggregate welfare measures mask significant heterogeneity.
 - Welfare gains and income distribution: monotonically increasing (1986, 1993, 2017) and U-shaped (1993 and 2001) patterns.

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3. Sensitivity analysis.
 - Elasticity parameterization, initial income distribution and tax policy.
 - Bias in welfare gain estimates from assuming linear tax function?
 - Under a tax progressivity reform, it is given by the ratio between progressivity parameter and inverse elasticity of taxable income.
 - In the United States, linearization bias is in the range 3.6-18.1%.

Welfare Effects of Policy Reforms

Harberger (1964), Feldstein (1999), Kleven and Kreiner (2006), Eissa, Kleven, and Kreiner (2008), Chetty (2009), Immervoll, Kleven, Kreiner, and Verdellin (2009), Hotchkiss, Moore, and Rios-Avila (2012, 2021), Blomquist and Simula (2019), Hendren and Sprung-Keyser (2020).

Taxation of Couples

Bar and Leukhina (2009), Guner, Kaygusuz, and Ventura (2012), Bick and Fuchs-Schündeln (2017, 2018), Borella, De Nardi, and Yang (2021).

Aggregate and Heterogeneous Effects of Tax Reforms

Bitler, Gelbach, and Hoynes (2006), Barro and Redlick (2011), Mertens and Ravn (2012, 2013), Barro and Furman (2018), Holter, Krueger, and Stepanchuk (2019), Zidar (2019).

This paper: Welfare analysis of tax reforms in a framework with couples.

Model

Environment

Static model of couples, no marriage.

Males choose hours, females — participation and hours. ▶ Annual Hours

Utility of couple $i = 1, \dots, N$:

$$U_i(c, h^m, h^f) = v_i(c, h^m, h^f) - q_i \cdot 1\{h^f > 0\}, \quad q_i \sim F_i(q_i)$$

Budget constraint:

$$c \leq w_i^m h^m + w_i^f h^f - T \left(w_i^m h^m, w_i^f h^f; \underbrace{\theta}_{d\theta \approx 0: \text{tax reform}} \right)$$

Marginal tax rate: $\tau_i^j(\theta) \equiv \partial T / \partial (w_i^j h_i^j)$, $j = m, f$

Participation tax rate:

$$a_i(\theta) \equiv \frac{T(w_i^m h_i^{m,2}, w_i^f h_i^f, \theta) - T(w_i^m h_i^{m,1}, 0, \theta)}{w_i^m (h_i^{m,2} - h_i^{m,1}) + w_i^f h_i^f}$$

Compensated Functions

Expenditure minimization of dual-earner couples:

$$\min_{c, h^m, h^f} c - w_i^m h^m - w_i^f h^f + T(w_i^m h^m, w_i^f h^f; \theta) \quad \text{s.t.} \quad v_i(c, h^m, h^f) \geq \bar{U}_i + q_i$$

\Rightarrow compensated \tilde{c}_i^2 , $\tilde{h}_i^{m,2}$, and \tilde{h}_i^f .

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Compensated participation cost threshold: $E_i^2(\bar{U}_i + \tilde{q}_i, \theta) = E_i^1(\bar{U}_i, \theta)$.

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Compensated participation cost threshold: $E_i^2(\bar{U}_i + \tilde{q}_i, \theta) = E_i^1(\bar{U}_i, \theta)$.

Compensated aggregate labor supply:

$$\tilde{L} = \sum_{i=1}^N \left[\underbrace{F_i(\tilde{q}_i)}_{\text{affected by } a_i} \underbrace{(\tilde{h}_i^{m,2} + \tilde{h}_i^f)}_{\text{affected by } \tau_i^m \text{ and } \tau_i^f} + \underbrace{(1 - F_i(\tilde{q}_i))}_{\text{affected by } a_i} \underbrace{\tilde{h}_i^{m,1}}_{\text{affected by } \tau_i^m} \right]$$

Compensated Elasticities

Female participation elasticity:

$$\eta_i \equiv \frac{\partial F_i(\tilde{q}_i)}{\partial (1 - a_i)} \frac{1 - a_i}{F_i(\tilde{q}_i)}$$

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Male hours-of-work elasticity:

$$\varepsilon_i^{m,\iota} \equiv \frac{\partial \tilde{h}_i^{m,\iota}}{\partial(1-\tau_i^m)} \frac{1-\tau_i^m}{\tilde{h}_i^{m,\iota}}, \quad \iota = 1, 2$$

Female hours-of-work elasticity, ε_i^f : similar definition.

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Female hours-of-work elasticity, ε_i^f : similar definition.

Cross-elasticities of working hours:

$$\varepsilon_i^{mf} \equiv \frac{\partial \tilde{h}_i^{m,2}}{\partial (1 - \tau_i^f)} \cdot \frac{1 - \tau_i^f}{\tilde{h}_i^{m,2}}$$
$$\varepsilon_i^{fm} \equiv \frac{\partial \tilde{h}_i^f}{\partial (1 - \tau_i^m)} \cdot \frac{1 - \tau_i^m}{\tilde{h}_i^f}$$

Reform-Induced Change in Aggregate Efficiency

Aggregate excess burden from a tax and transfer system θ :

$$D = \sum_{i=1}^N \int_0^{\infty} \left[\underbrace{E_i(\bar{U}_i, q_i, \theta) - E_i(\bar{U}_i, q_i, 0)}_{\text{equivalent variation}} - \underbrace{R(\bar{U}_i, q_i, \theta)}_{\text{govt. revenue}} \right] dF_i(q_i)$$

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By the envelope theorem, reform-induced behavioral responses do not affect the expenditure function.

Effect of any arbitrary small tax reform $d\theta \approx 0$ on economic efficiency = behavioral revenue effect (“fiscal externality”) = difference between mechanical revenue effect ($\partial T_i / \partial \theta$) and total revenue effect ($dT_i / d\theta$).

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$$\begin{aligned} \frac{dD}{d\theta} = & - \sum_{i=1}^N \left[\tau_i^m w_i^m \frac{\partial \tilde{h}_i^{m,2}}{\partial \theta} F_i(\tilde{q}_i) + \tau_i^m w_i^m \frac{\partial \tilde{h}_i^{m,1}}{\partial \theta} (1 - F_i(\tilde{q}_i)) + \right. \\ & \left. \tau_i^f w_i^f \frac{\partial \tilde{h}_i^f}{\partial \theta} F_i(\tilde{q}_i) + a_i \left[w_i^m (h_i^{m,2} - h_i^{m,1}) + w_i^f \tilde{h}_i^f \right] \frac{\partial F_i(\tilde{q}_i)}{\partial \theta} \right] \end{aligned}$$

Reform-Induced Change in Economic Efficiency

Marginal aggregate excess burden as a share of aggregate labor income:

$$\frac{dD/d\theta}{W} = \sum_{i=1}^N \left[\underbrace{\left(\frac{\tau_i^m}{1 - \tau_i^m} \cdot \frac{d\tau_i^m}{d\theta} \epsilon_i^{m,2} \right)}_{\text{2E male hours}} + \underbrace{\left(\frac{\tau_i^m}{1 - \tau_i^f} \cdot \frac{d\tau_i^f}{d\theta} \epsilon_i^{mf} \right)}_{\text{2E male cross-effect}} \right] s_i^{m,2} + \underbrace{\left(\frac{\tau_i^m}{1 - \tau_i^m} \cdot \frac{d\tau_i^m}{d\theta} \epsilon_i^{m,1} \right)}_{\text{1E male hours}} s_i^{m,1} +$$

$$\left(\underbrace{\left(\frac{\tau_i^f}{1 - \tau_i^f} \cdot \frac{d\tau_i^f}{d\theta} \epsilon_i^f \right)}_{\text{female hours}} + \underbrace{\left(\frac{\tau_i^f}{1 - \tau_i^m} \cdot \frac{d\tau_i^m}{d\theta} \epsilon_i^{fm} \right)}_{\text{female cross-effect}} \right) s_i^f + \underbrace{\left(\frac{a_i}{1 - a_i} \cdot \frac{da_i}{d\theta} \eta_i \left(s_i^f + s_i^{m,2} - \frac{F_i(\tilde{q}_i)}{1 - F_i(\tilde{q}_i)} s_i^{m,1} \right) \right)}_{\text{female participation}}$$

$s_i^{m,j}$ and s_i^f are (expected) labor income shares: $s_i^f \equiv w_i^f \tilde{h}_i^f F_i(\tilde{q}_i) / W$

Red terms (elasticities): Use (bounds on) estimates from the literature.

Blue terms: Use the microdata and tax calculator to obtain labor income shares, pre-reform tax rates, and reform-induced changes in tax rates.

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Framework with couples:

- Cross-elasticities ($\neq 0$).
- Change in husband's working hours (≈ 0).

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Quantitative Results

Four tax reforms implemented in the United States: [▶ Tax Parameters](#)

- Tax Reform Act of 1986 (TRA86).
- Omnibus Budget Reconciliation Act of 1993 (OBRA93).
- Economic Growth And Tax Relief Reconciliation Act 2001 (EGTRRA01).
- Tax Cuts and Jobs Act of 2017 (TCJA17).

Data: [▶ Summary Statistics](#)

- Current Population Survey, Annual Social & Economic Supplement.
- Married couples, spouses aged 25-54.
- Earnings = wage and salary income + self-employment income.
- Husbands have strong labor market attachment ($\text{income} \geq 0.5 \times \text{min. wage} \times 520 \text{ hours}$).

NBER TAXSIM tax calculator (Feenberg and Coutts, 1993).

- **Goal:** Calculate tax liabilities under U.S. Federal and State income tax laws from individual data.
- **Input:** Wage and salary income (including self-employment), age, marital status, number of dependents, state, income from various sources, expenditures.
- **Output:** Federal, state, and the Federal Insurance Contributions Act (FICA) tax liabilities and marginal tax rates.
- **Microdata + Tax Calculator:** Capture heterogeneous effects of tax reforms on taxpayers and nonlinearities of the income tax code.

Marginal and Participation Tax Rates

Marginal tax rate (e.g., for a woman):

$$\tau_{it}^f = \frac{T(y_{it}^m, \hat{y}_{it}^f + \$0.1, \text{Dem}_{it}) - T(y_{it}^m, \hat{y}_{it}^f, \text{Dem}_{it})}{\$0.1}$$

Participation tax rate:

$$a_{it} = \frac{T(y_{it}^m, \hat{y}_{it}^f, \text{Dem}_{it}) - T(y_{it}^m, 0, \text{Dem}_{it})}{\hat{y}_{it}^f}$$

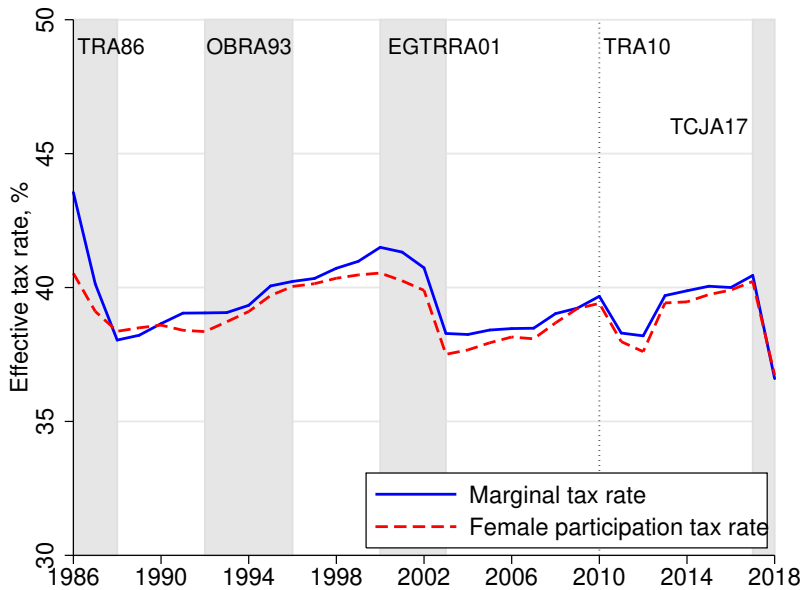
Assume that workers bear the full incidence of employer payroll taxes.

Self-selection of married women into employment

- Two-stage Heckman to impute earnings of non-working women.
- Exclusion restrictions: spousal earnings and the number of kids aged 0-5 do not directly affect the woman's wage (Mulligan and Rubinstein, 2008).

Marginal and Participation Tax Rates

► Representative Couple



Reform-Induced Changes in Tax Rates

► Representative Couple

Isolate the changes in federal tax rates from the other tax changes, behavioral responses, and time and macroeconomic effects on income:

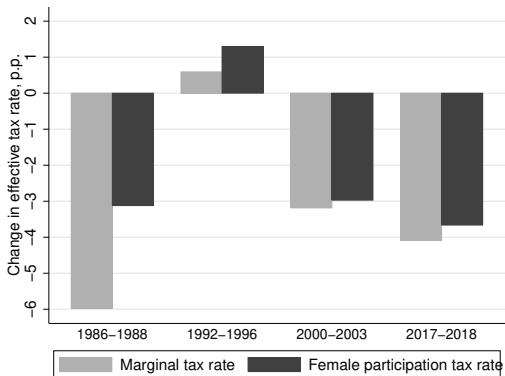
	Real Income _{<i>t</i>}	Federal tax liability _{<i>t</i>}	Federal tax liability _{<i>t+k</i>}
Spouse <i>i</i>	Y_{it}	$T_t(Y_{it}, \cdot)$	$T_{t+k}(Y_{it}, \cdot)$

Reform-Induced Changes in Tax Rates

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Spouse i	Real Income _{t} Y_{it}	Federal tax liability _{t} $T_t(Y_{it}, \cdot)$	Federal tax liability _{$t+k$} $T_{t+k}(Y_{it}, \cdot)$
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Sufficient Statistics Formula

$$\frac{dD/d\theta}{W} = \sum_{i=1}^N \left[\left(\frac{\tau_i^m}{1 - \tau_i^m} \cdot \frac{d\tau_i^m}{d\theta} \epsilon^{m,2} + \frac{\tau_i^m}{1 - \tau_i^f} \cdot \frac{d\tau_i^f}{d\theta} \epsilon^{mf} \right) s_i^{m,2} + \frac{\tau_i^m}{1 - \tau_i^m} \cdot \frac{d\tau_i^m}{d\theta} \epsilon^{m,1} s_i^{m,1} + \right. \\ \left. \left(\frac{\tau_i^f}{1 - \tau_i^f} \cdot \frac{d\tau_i^f}{d\theta} \epsilon^f + \frac{\tau_i^f}{1 - \tau_i^m} \cdot \frac{d\tau_i^m}{d\theta} \epsilon^{fm} \right) s_i^f + \frac{a_i}{1 - a_i} \cdot \frac{da_i}{d\theta} \eta s_i^f \right]$$

Elasticities: Blau, Kahn (2007), Meghir, Phillips (2010), Bargain, Orsini, Peichl (2014), Attanasio, Levell, Low, Sánchez-Marcos (2018), etc.

- **Baseline:** $\epsilon^m = 0.05$, $\epsilon^{mf} = -0.05$, $\epsilon^f = 0.15$, $\epsilon^{fm} = -0.1$, $\eta = 0.6$.

Blue terms: CPS + NBER TAXSIM.

Caveats:

- Elasticity parameterization (heterogeneity, changes over time).
- Initial income distribution and pre-reform tax rates can matter.
- Linear tax function.

Benchmark: Representative Couple

Useful benchmark: a representative couple model.

Assumptions:

- No heterogeneity in income, tax rates, and tax rate changes.
- Because of tax system jointness, $\tau^m = \tau^f \equiv \tau$.
- The pre-reform tax rates, τ and a , are given by the mean effective marginal and participation tax rates. ► Tax Rates
- The reform-induced tax changes, $d\tau/d\theta$ and $da/d\theta$, are given by the mean changes in the tax rates. ► Tax Rate Changes

$$\frac{dD/d\theta}{W} = \frac{\tau}{1-\tau} \cdot \frac{d\tau}{d\theta} [(\varepsilon^m + \varepsilon^{mf}) s^m + (\varepsilon^f + \varepsilon^{fm}) s^f] + \frac{a}{1-a} \cdot \frac{da}{d\theta} \eta s^f$$

Aggregate Welfare Gains

Reform	Welfare gain, % of aggregate labor income						RC	Tax Liab. Reduc., %	Δ Welfare/ \$ Spent
	Intensive Males (1)	Intensive Females (2)	Extensive Females (3)	Cross- Effects (4)	Total w/o C.E. (5)	Total (6)			
TRA86	0.19	0.18	0.45	-0.27	0.82	0.55	0.44	7.20	1.08
OBRA93	-0.01	-0.02	-0.15	0.03	-0.18	-0.16	-0.16	0.27	0.63
EGTRRA01	0.09	0.12	0.40	-0.17	0.61	0.44	0.42	7.19	1.07
TCJA17	0.10	0.17	0.57	-0.22	0.84	0.62	0.58	6.58	1.10

NOTES: The pre-reform tax rates and reform-induced changes in tax rates are calculated using NBER TAXSIM applied to the ASEC CPS data. Column (5) shows total welfare gains when the cross-effects are ignored, and calculated as (1) + (2) + (3). Column (6) shows total welfare gains, and calculated as (4) + (5). Column (7) shows the welfare gains in a representative-couple economy. Column (9) is calculated as $(8)/[(8) - (6)]$, where (8) is the decrease in tax liabilities as a share of labor income before behavioral responses.

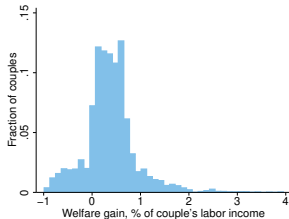
Alternative elasticity parameterizations:

- Lower and upper bounds on welfare gains.
- Blau and Kahn (2007) and Heim (2007): shrinking elasticities of married female labor supply in the 1970-2000s.
- Elasticities shrinking along the income distribution.

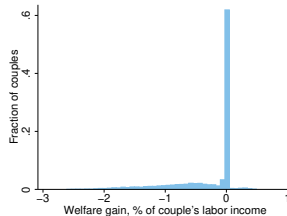
Alternative Elasticity Parameterizations

Reform	Welfare gain, % of aggregate labor income							Tax Liab. Reduc., %	Δ Welfare/ \$ Spent
	Intensive Males	Intensive Females	Extensive Females	Cross-Effects	Total w/o C.E.	Total	RC		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)		
"Upper-Bound" Parameterization: $\epsilon^m = 0.1, \epsilon^f = 0.2, \epsilon^{mf} = 0, \epsilon^{fm} = -0.05, \eta = 0.8$									
TRA86	0.39	0.24	0.60	-0.08	1.23	1.15	1.03	7.20	1.19
OBRA93*	0.00	-0.01	-0.10	0.04	-0.12	-0.07	-0.25	0.27	0.79
EGTRRA01	0.18	0.16	0.54	-0.04	0.88	0.84	0.77	7.19	1.13
TCJA17	0.19	0.23	0.76	-0.06	1.18	1.12	1.03	6.58	1.21
"Lower-Bound" Parameterization: $\epsilon^m = 0, \epsilon^f = 0.1, \epsilon^{mf} = -0.1, \epsilon^{fm} = -0.15, \eta = 0.4$									
TRA86	0.00	0.12	0.30	-0.47	0.42	-0.05	-0.14	7.20	0.99
OBRA93*	-0.02	-0.03	-0.20	0.01	-0.25	-0.25	-0.07	0.27	0.53
EGTRRA01	0.00	0.08	0.27	-0.30	0.35	0.05	0.06	7.19	1.01
TCJA17	0.00	0.12	0.38	-0.37	0.49	0.12	0.13	6.58	1.02
"High-Elasticity" Parameterization: $\epsilon^m = 0.1, \epsilon^f = 0.2, \epsilon^{mf} = -0.1, \epsilon^{fm} = -0.15, \eta = 0.8$									
TRA86	0.39	0.24	0.60	-0.47	1.23	0.75	0.57	7.20	1.12
OBRA93	-0.02	-0.03	-0.20	0.04	-0.25	-0.21	-0.22	0.27	0.57
EGTRRA01	0.18	0.16	0.54	-0.30	0.88	0.57	0.54	7.19	1.09
TCJA17	0.19	0.23	0.76	-0.37	1.18	0.81	0.76	6.58	1.14
"Low-Elasticity" Parameterization: $\epsilon^m = 0, \epsilon^f = 0.1, \epsilon^{mf} = 0, \epsilon^{fm} = -0.05, \eta = 0.4$									
TRA86	0.00	0.12	0.30	-0.08	0.42	0.34	0.32	7.20	1.05
OBRA93	0.00	-0.01	-0.10	0.01	-0.12	-0.11	-0.11	0.27	0.72
EGTRRA01	0.00	0.08	0.27	-0.04	0.35	0.31	0.29	7.19	1.05
TCJA17	0.00	0.12	0.38	-0.06	0.49	0.44	0.40	6.58	1.07
Baseline Parameterization + Participation Elasticity Varies by Income Quintile									
TRA86	0.19	0.18	0.23	-0.27	0.61	0.33	-	7.21	1.05
OBRA93	-0.01	-0.02	-0.21	0.03	-0.24	-0.21	-	0.27	0.56
EGTRRA01	0.09	0.12	0.28	-0.17	0.49	0.32	-	7.19	1.05
TCJA17	0.10	0.17	0.34	-0.22	0.61	0.39	-	6.58	1.06

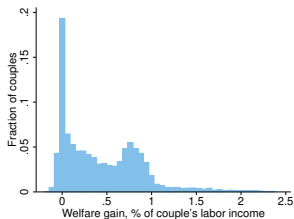
Distribution of Welfare Gains



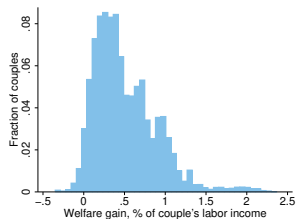
(a) TRA 1986



(b) OBRA 1993



(c) EGTRRA 2001



(d) TCJA 2017

Distribution of Welfare Gains

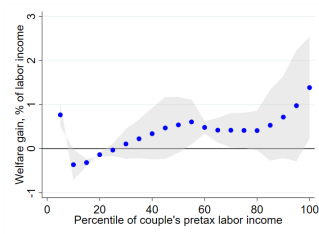
Table 1: Distribution of welfare gains for couples, % of couple's labor income

Reform	P10	P25	P50	P75	P90
TRA86	-0.21	0.13	0.37	0.61	0.94
OBRA93	-1.09	-0.40	0.00	0.00	0.00
EGTRRA01	0.00	0.06	0.36	0.76	0.95
TCJA17	0.10	0.23	0.42	0.74	1.02

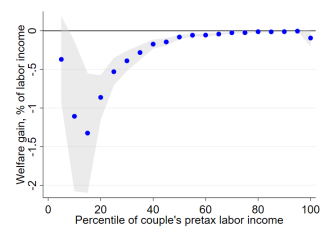
Table 2: Fractions of winners, losers, and welfare-neutral couples

Reform	Winners, %	Losers, %	Neutral, %
TRA86	78.7	12.3	9.1
OBRA93	1.4	31.2	67.4
EGTRRA01	69.6	0.3	30.1
TCJA17	90.3	0.6	9.0

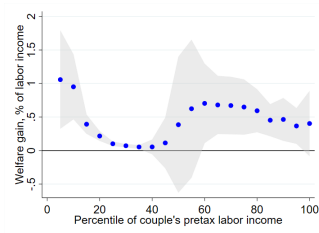
Welfare Gains and Income Distribution



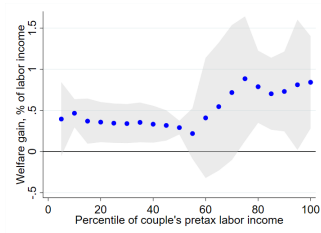
(a) TRA 1986



(b) OBRA 1993



(c) EGTRRA 2001



(d) TCJA 2017

Counterfactual Reforms

How does the pre-reform income distribution matter for my results?

How do the initial conditions — pre-reform income distribution and tax law — jointly matter for the estimates of welfare gains?

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Counterfactual Reforms 1

- Take the income distribution in pre-reform year t (e.g., 1986), and apply the pre- and post-reform X 's (e.g., TCJA 2017) tax laws.
- Capture the differences in income distribution.

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Counterfactual Reforms 1

- Take the income distribution in pre-reform year t (e.g., 1986), and apply the pre- and post-reform X 's (e.g., TCJA 2017) tax laws.
- Capture the differences in income distribution.

Counterfactual Reforms 2

- Take the income distribution and tax law in pre-reform year t (e.g., 1986) and apply the post-reform X 's (e.g., TCJA 2017) tax law.
- Welfare consequences of moving from the pre-TRA 1986 economy to the post-TCJA 2017 economy.

Counterfactual Reforms 1

Example: If TRA 1986 were to be applied to the 2017 distribution, welfare gain per \$ spent would be 5.48% higher than from actual reform.

Reform	Welfare gain, % of aggregate labor income						Tax Liab. Reduc., %	Δ Welfare/ \$ Spent	Diff., %
	Intensive Males	Intensive Females	Extensive Females	Cross-Effects	Total	RC			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A: Tax Reforms Applied to Pre-TRA86 Distribution of Couples									
TRA86	0.19	0.18	0.45	-0.27	0.55	0.44	7.20	1.08	0.00
OBRA93	-0.01	-0.02	-0.13	0.02	-0.14	-0.14	0.29	0.68	+7.54
EGTRRA01	0.09	0.11	0.36	-0.16	0.40	0.37	7.46	1.06	-0.80
TCJA17	0.09	0.12	0.36	-0.18	0.40	0.37	5.76	1.07	-2.68
Panel B: Tax Reforms Applied to Pre-OBRA93 Distribution of Couples									
TRA86	0.19	0.22	0.53	-0.30	0.63	0.51	7.38	1.09	+1.09
OBRA93	-0.01	-0.02	-0.15	0.03	-0.16	-0.16	0.27	0.63	0.00
EGTRRA01	0.08	0.12	0.39	-0.16	0.43	0.40	7.38	1.06	-0.32
TCJA17	0.09	0.14	0.41	-0.18	0.45	0.42	5.87	1.08	-1.88
Panel C: Tax Reforms Applied to Pre-EGTRRA01 Distribution of Couples									
TRA86	0.33	0.31	0.82	-0.48	0.97	0.76	10.23	1.11	+2.11
OBRA93	-0.04	-0.04	-0.18	0.07	-0.19	-0.20	-0.97		
EGTRRA01	0.09	0.12	0.40	-0.17	0.44	0.42	7.19	1.07	0.00
TCJA17	0.10	0.14	0.44	-0.20	0.48	0.45	6.19	1.08	-1.80
Panel D: Tax Reforms Applied to Pre-TCJA17 Distribution of Couples									
TRA86	0.29	0.42	1.13	-0.52	1.32	1.05	10.62	1.14	+5.48
OBRA93	-0.03	-0.05	-0.22	0.07	-0.24	-0.25	-0.96		
EGTRRA01	0.08	0.13	0.48	-0.17	0.52	0.49	7.15	1.08	+1.18
TCJA17	0.10	0.17	0.57	-0.22	0.62	0.58	6.58	1.10	0.00

Counterfactual Reforms 2

Example: Move from pre-TRA 1986 to post-1993/2001/2017 economies.

Reform	Welfare gain, % of aggregate labor income					RC	Tax Liab. Reduc., %	Δ Welfare/ \$ Spent
	Intensive Males	Intensive Females	Extensive Females	Cross- Effects	Total			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Tax Reforms Applied to Pre-TRA86 Distribution of Couples and Tax Law								
TRA86	0.19	0.18	0.45	-0.27	0.55	0.44	7.20	1.08
OBRA93	0.19	0.17	0.35	-0.27	0.44	0.29	7.73	1.06
EGTRRA01	0.27	0.27	0.75	-0.41	0.88	0.74	17.85	1.05
TCJA17	0.36	0.38	1.02	-0.58	1.19	0.96	22.28	1.06
Panel B: Tax Reforms Applied to Pre-OBRA93 Distribution of Couples and Tax Law								
TRA86	—	—	—	—	—	—	—	—
OBRA93	-0.01	-0.02	-0.15	0.03	-0.16	-0.16	0.27	0.63
EGTRRA01	0.06	0.09	0.26	-0.12	0.29	0.27	10.09	1.03
TCJA17	0.13	0.19	0.51	-0.25	0.57	0.52	14.69	1.04
Panel C: Tax Reforms Applied to Pre-EGTRRA01 Distribution of Couples and Tax Law								
TRA86	0.09	0.08	0.25	-0.15	0.27	0.22	-0.74	—
OBRA93	—	—	—	—	—	—	—	—
EGTRRA01	0.09	0.12	0.40	-0.17	0.44	0.42	7.19	1.07
TCJA17	0.15	0.23	0.69	-0.31	0.76	0.70	12.16	1.07
Panel D: Tax Reforms Applied to Pre-TCJA17 Distribution of Couples and Tax Law								
TRA86	0.03	0.02	0.05	-0.05	0.05	-0.02	-6.40	—
OBRA93	-0.03	-0.06	-0.26	0.07	-0.27	-0.29	-7.38	—
EGTRRA01	—	—	—	—	—	—	—	—
TCJA17	0.10	0.17	0.57	-0.22	0.62	0.58	6.58	1.10

Efficiency Loss and Nonlinear Taxation of Couples

Concern: Tax and transfer function is assumed to be [linear](#).

How sizable is the bias in the estimates of welfare gains resulting from the linearity assumption?

- Extend Blomquist and Simula (2019) to the framework with couples.
- Abstract from participation margin.

Efficiency Loss and Nonlinear Taxation of Couples

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- Extend Blomquist and Simula (2019) to the framework with couples.
- Abstract from participation margin.

(v_m, v_f) -type couple's preferences:

$$v(c, y_m, y_f, v_m, v_f), \quad (v_m, v_f) \sim I(\cdot)$$

Budget constraint:

$$c \leq \underbrace{y_m + y_f}_{\text{taxable income}} - T(y_m, y_f, \theta)$$

Idea: Approximate $T(\cdot)$ with linear $T^L(\cdot)$ that gives the same allocation:

$$T^L(y_m, y_f, \tau_m, \tau_f) = \tau_m(\theta)y_m + \tau_f(\theta)y_f + T^*$$

Efficiency Loss and Nonlinear Taxation of Couples

Proposition 2 (Efficiency Loss and Nonlinear Taxation of Couples).

Under nonlinear tax function T , efficiency loss from any arbitrary small tax reform $d\theta \approx 0$ is given by

$$\frac{dD}{d\theta} = - \int \sum_{j=m,f} \frac{T'_j \left[(\psi''_{mf} + T''_{mf}) T''_{-j,\theta} - (\psi''_{-j,-j} + T''_{-j,-j}) T''_{j\theta} \right]}{\underbrace{(\psi''_{mm} + T''_{mm})(\psi''_{ff} + T''_{ff}) - (\psi''_{mf} + T''_{mf})^2}_{\text{Marginal DWL of spouse } j}} d\Gamma(v_m, v_f)$$

Under linearized tax function T^L , efficiency loss from any arbitrary small tax reform $d\theta \approx 0$ is given by

$$\frac{dD^L}{d\theta} = - \int \left[\frac{T'_m (\psi''_{mf} T''_{f\theta} - \psi''_{ff} T''_{m\theta})}{\psi''_{mm} \psi''_{ff} - (\psi''_{mf})^2} + \frac{T'_f (\psi''_{mf} T''_{m\theta} - \psi''_{mm} T''_{f\theta})}{\psi''_{mm} \psi''_{ff} - (\psi''_{mf})^2} \right] d\Gamma(v_m, v_f)$$

ψ -terms capture utility curvature, T -terms — tax function curvature.

Efficiency Loss with HSV Tax Function

Quasilinear preferences:

$$v(c, y_m, y_f, v_m, v_f) = c - \frac{v_m}{\sigma + 1} \left(\frac{y_m}{v_m} \right)^{\sigma+1} - \frac{v_f}{\sigma + 1} \left(\frac{y_f}{v_f} \right)^{\sigma+1}$$

Use log-linear (HSV, 2017) tax function that yields a good approximation of the actual tax and transfer system in the U.S.

- Heathcote, Storesletten, and Violante (2017).

Joint taxation of spousal income

$$T(y_m, y_f, \theta) = y_m + y_f - \lambda (y_m + y_f)^{1-\theta}$$

Separate taxation of spousal income

$$T(y_m, y_f, \theta) = y_m + y_f - \tilde{\lambda} y_m^{1-\theta} - \tilde{\lambda} y_f^{1-\theta}$$

Parameter θ stands for tax progressivity.

Solve for λ (level of tax rates) from the government budget constraint.

Define the linearization bias as

$$\Delta = \left(\frac{dD^L}{d\theta} - \frac{dD}{d\theta} \right) / \frac{dD}{d\theta}$$

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Proposition 3 (Linearization Bias with HSV Tax Function). *Consider a small reform that changes tax progressivity, $d\theta \approx 0$. Under both joint and separate taxation of spouses, the linearization bias is given by*

$$\Delta = \theta / \sigma$$

Linearization bias = $\frac{\text{progressivity parameter (tax function curvature)}}{\text{inverse elasticity of taxable income (utility curvature)}}$

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Proposition 3 (Linearization Bias with HSV Tax Function). *Consider a small reform that changes tax progressivity, $d\theta \approx 0$. Under both joint and separate taxation of spouses, the linearization bias is given by*

$$\Delta = \theta / \sigma$$

Linearization bias = $\frac{\text{progressivity parameter (tax function curvature)}}{\text{inverse elasticity of taxable income (utility curvature)}}$

HSV (2017) estimate $\theta = 0.181$ for the United States in 2000-2006.

Neisser (2021): meta-analysis of 1720 estimates of $1/\sigma$ from 61 papers.

- Majority in $[0, 1]$ with peak around 0.3 and excess mass in $[0.7, 1]$.

Under $1/\sigma \in [0.2, 1]$, the upward bias is in the range of 3.6-18.1%.

Concluding Remarks

Framework to study welfare effects of income tax changes on couples.

- Expression for efficiency gains as a function of (i) labor supply elasticities, (ii) tax policy parameters, and (iii) labor income shares.
- Transparent decomposition of welfare gains.

Welfare effects of labor income tax changes induced by U.S. tax reforms.

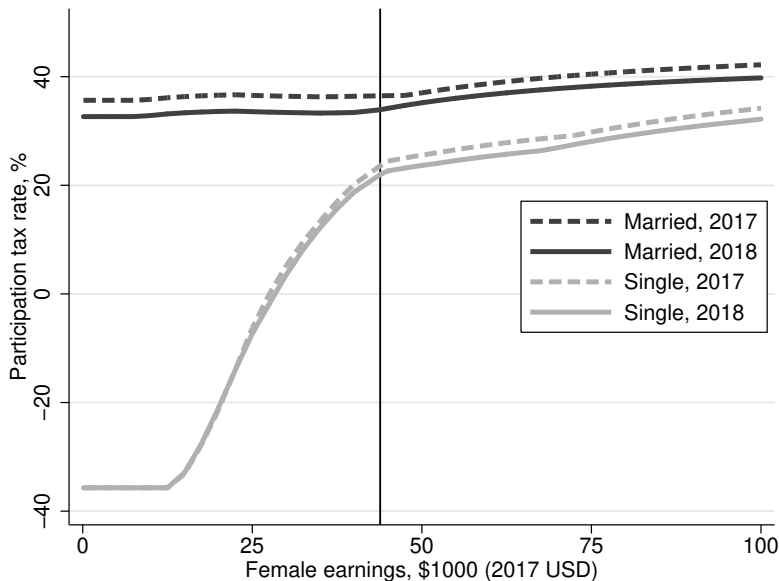
- Aggregate gains range from -0.16% to 0.62% of aggregate earnings.
- Participation margin & spousal cross-effects of working hours matter.
- Heterogeneity in welfare gains (winners/losers, by income).

How reliable are the estimates from this sufficient statistics approach?

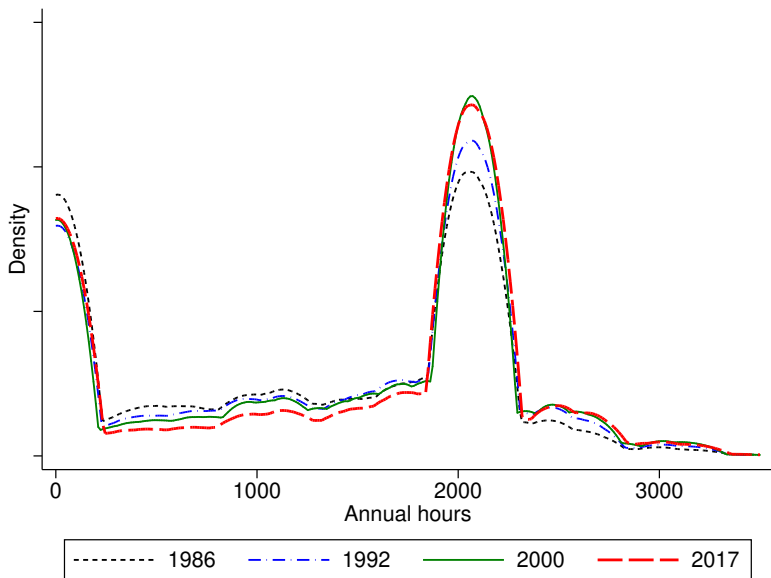
- Alternative parameterizations of elasticities.
- Role of initial income distribution & tax policy.
- Linearization bias under progressivity reform = progressivity rate / inverse elasticity of taxable income (3.6-18.1% for the U.S.).

Appendix

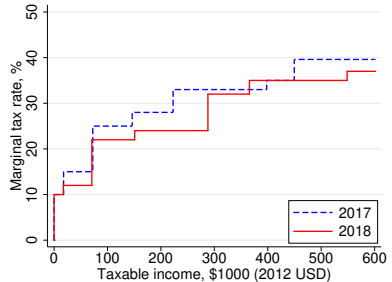
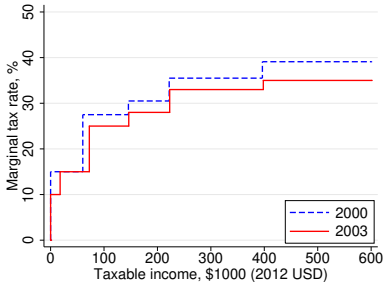
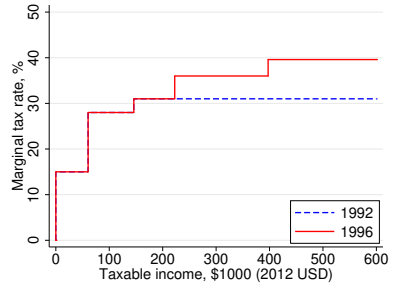
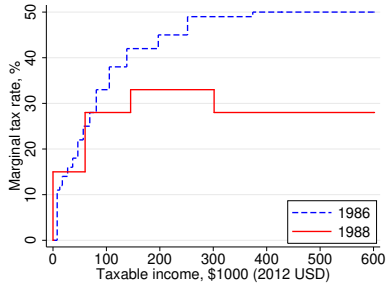
Participation Tax Rate under Joint Taxation of Spouses

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Annual Hours of Married Women in the United States

[◀ Back](#)

U.S. Federal Income Tax Schedule, Married Filing Jointly

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Earned Income Tax Credit Parameters [◀ Back](#)

Year	Eligible Children	Phase-in Rate	First Kink	Maximum Credit	Second Kink	Phase-out Rate	Exhaustion Point
1986	any	11	5000	550	6500	12.22	11000
1988	any	14	6240	874	9840	10	18576
1992	1	17.6	7520	1324	11840	12.57	22370
	2+	18.4	7520	1384	11840	13.14	22370
1996	0	7.65	4220	323	5280	7.65	9500
	1	34	6330	2152	11610	15.98	25078
	2+	40	8890	3556	11610	21.06	28495
2000	0	7.65	4610	353	5770	7.65	10380
	1	34	6920	2353	12690	15.98	27413
	2+	40	9720	3888	12690	21.06	31152
2002	0	7.65	4910	376	7150	7.65	12060
	1	34	7370	2506	14520	15.98	30201
	2+	40	10350	4140	14520	21.06	34178
2017	0	7.65	6670	510	13930	7.65	20600
	1	34	10000	3400	23930	15.98	45207
	2	40	14040	5616	23930	21.06	50597
	3+	45	14040	6318	23930	21.06	53930
2018	0	7.65	6780	519	14170	7.65	20950
	1	34	10180	3461	24350	15.98	46010
	2	40	14290	5716	24350	21.06	51492
	3+	45	14290	6431	24350	21.06	54884

Standard Deductions and Personal Exemptions

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Year	Standard Deduction	Personal Exemption
1986	3670	1080
1988	5000	1950
1992	6000	2300
1996	6700	2550
2000	7350	2800
2002	7850	3000
2017	13000	4050
2018	24000	0

NOTE: For married couples filing jointly two personal exemptions are allowed. The Tax Cuts and Jobs Act of 2017 eliminated personal exemptions for tax years 2018-2025.

Summary Statistics

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	1986			1992		
	Mean	Median	St. Dev.	Mean	Median	St. Dev.
Males						
Age	38.94	38	7.88	39.48	39	7.61
White	0.896	1	0.305	0.892	1	0.311
College degree	0.291	0	0.454	0.311	0	0.463
Annual hours	2201	2080	588	2217	2080	606
Earnings (2012 USD)	52873	47893	30218	53919	47610	33521
Females						
Age	36.66	36	7.55	37.47	37	7.37
White	0.895	1	0.306	0.891	1	0.312
College degree	0.211	0	0.408	0.259	0	0.438
Employment	0.732	1	0.443	0.764	1	0.425
Annual hours	1214	1400	940	1330	1664	939
Earnings (2012 USD)	25946	22104	19507	29740	25293	21906
Number of children	1.62	2	1.22	1.54	2	1.19
Number of children under 6	0.50	0	0.78	0.49	0	0.77
Female — secondary earner	0.834	1	0.372	0.788	1	0.409
Number of observations	17127			18032		

Summary Statistics

[◀ Back](#)

	2000			2017		
	Mean	Median	St. Dev.	Mean	Median	St. Dev.
Males						
Age	40.63	41	7.63	40.76	41	7.77
White	0.865	1	0.341	0.812	1	0.391
College degree	0.351	0	0.477	0.440	0	0.496
Annual hours	2294	2080	558	2229	2080	532
Earnings (2012 USD)	72918	53688	74811	76318	56644	81251
Females						
Age	38.78	39	7.57	38.96	39	7.78
White	0.862	1	0.345	0.804	1	0.397
College degree	0.324	0	0.468	0.493	0	0.500
Employment rate	0.777	1	0.416	0.747	1	0.435
Annual hours	1393	1820	947	1388	1872	971
Earnings (2012 USD)	37659	31332	37063	49817	37763	54504
Number of children	1.55	2	1.23	1.61	2	1.27
Number of children under 6	0.46	0	0.76	0.51	0	0.79
Female — secondary earner	0.775	1	0.417	0.720	1	0.449
Number of observations	26883			17415		

Joint Taxation

Efficiency loss from a small change in tax progressivity $d\theta \approx 0$:

$$\frac{dD_{joint}}{d\theta} = \int \left[1 - \lambda^{\frac{\sigma}{\sigma+\theta}} (1-\theta)^{\frac{\sigma}{\sigma+\theta}} (v_m + v_f)^{-\frac{\sigma\theta}{\sigma+\theta}} \right] \frac{[\lambda(1-\theta)(v_m + v_f)^\sigma]^{\frac{1}{\sigma+\theta}}}{\sigma + \theta} \left[\frac{1}{1-\theta} + \frac{\log(\lambda(1-\theta)(v_m + v_f)^\sigma)}{\sigma + \theta} \right] d\Gamma(v_m, v_f)$$

Efficiency loss under linearized tax function:

$$\frac{dD_{joint}^L}{d\theta} = \int \left[1 - \lambda^{\frac{\sigma}{\sigma+\theta}} (1-\theta)^{\frac{\sigma}{\sigma+\theta}} (v_m + v_f)^{-\frac{\sigma\theta}{\sigma+\theta}} \right] \frac{[\lambda(1-\theta)^{1-\sigma-\theta}(v_m + v_f)^\sigma]^{\frac{1}{\sigma+\theta}}}{\sigma} \left[1 + \frac{(1-\theta) \log(\lambda(1-\theta)(v_m + v_f)^\sigma)}{\sigma + \theta} \right] d\Gamma(v_m, v_f)$$

Separate Taxation

Efficiency loss from a small change in tax progressivity $d\theta \approx 0$:

$$\frac{dD_{sep}}{d\theta} = \int \sum_{j=m,f} \left[1 - \tilde{\lambda}^{\frac{\sigma}{\sigma+\theta}} (1-\theta)^{\frac{\sigma}{\sigma+\theta}} v_j^{-\frac{\sigma\theta}{\sigma+\theta}} \right] \frac{[\tilde{\lambda}(1-\theta)^{1-\sigma-\theta} v_j^\sigma]^{\frac{1}{\sigma+\theta}}}{\sigma + \theta} \left[1 + \frac{(1-\theta) \log(\tilde{\lambda}(1-\theta) v_j^\sigma)}{\sigma + \theta} \right] d\Gamma(v_m, v_f)$$

Efficiency loss under linearized tax function:

$$\frac{dD_{sep}^L}{d\theta} = \int \sum_{j=m,f} \left[1 - \tilde{\lambda}^{\frac{\sigma}{\sigma+\theta}} (1-\theta)^{\frac{\sigma}{\sigma+\theta}} v_j^{-\frac{\sigma\theta}{\sigma+\theta}} \right] \frac{[\tilde{\lambda}(1-\theta)^{1-\sigma-\theta} v_j^\sigma]^{\frac{1}{\sigma+\theta}}}{\sigma} \left[1 + \frac{(1-\theta) \log(\tilde{\lambda}(1-\theta) v_j^\sigma)}{\sigma + \theta} \right] d\Gamma(v_m, v_f)$$