Dynamic Contracting with Multidimensional Screening

Egor Malkov
University of Minnesota and FRB Minneapolis

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Introduction

Environment where a principal and an agent with more than one private characteristic are involved into repeated interaction.

- Persistent private information.
- Applications: life insurance contracts, income taxation.

Multidimensional screening problem.

Properties of the optimal contract.

- Intratemporal: quantities, interdependence between characteristics.
- Intertemporal: dynamics of optimal quantities.

This paper: A simple model — benchmark for complex environments with multidimensional persistent private information.

What I Do in This Paper?

Characterization of the optimal contract between monopolist and buyer.

- Monopolist repeatedly sells two nondurable goods.
- Buyer's preferences over goods is a two-dimensional private info.
- Buyer's preferences stochastically evolve over time.

How is the dynamic model with multidimensional private information different from the dynamic unidimensional setting?

How is it different from the static model?

Another application: optimal income taxation of couples.

- Pareto frontier characterization.
- Non-Rawlsian government's taste for redistribution.

Main Conclusions

Monopolistic Nonlinear Pricing

1. Optimal contract is history dependent.
2. Optimal quantities are shaped by the cross-sectional distribution of the buyer's subtypes.
   - If non-negative covariance is high enough, then the optimal quantity of a good does not depend on the report about another good.
   - If non-negative covariance is low enough, then the optimal quantity of a good depends on the report about another good.
3. Persistence of private information accounts for dynamics of contract.

Optimal Income Taxation

1. Cross-sectional distribution of private types and the government's taste for redistribution jointly shape the optimal tax schedule.
2. Generalization of the ABC-formula for multidimensional private info.

Relevant Literature

Monopolistic Nonlinear Pricing

Rustichini and Wolinsky (1995), Armstrong (1996), Armstrong and Rochet (1999), Battaglini (2005, 2007), Pavan, Segal, Toikka (2014), Battaglini and Lamba (2019), Bloedel, Krishna, Leukhina (2020).

Optimal Income Taxation

Mirrlees (1971, 1976), Cremer, Pestieau, Rochet (2001), Kleven, Kreiner, Saez (2007, 2009), Battaglini and Coate (2008), Frankel (2014), Golosov, Troshkin, Tsyvinski (2016), Lehmann, Renes, Spiritus, Zoutman (2018), Moser and Olea de Souza e Silva (2019), Alves, Costa, Moreira (2021).

This Paper: Dynamic Contracting + Multidimensional Screening.

Economic Environment

Buyer (he) & seller's (she) relationship lasts for $T+1$ periods, $T \to \infty$.

Buyer's type: $(\theta_t, \varphi_t) \in \{\theta_L, \theta_H\} \times \{\varphi_L, \varphi_H\}$ with $\theta_H > \theta_L$ & $\varphi_H > \varphi_L$.

Cross-sectional distribution of types: $\psi(\theta_i, \varphi_j) \equiv \psi_{ij}$.

Covariance between subtypes: $\rho \equiv \psi_{HH}\psi_{LL} - \psi_{HL}\psi_{LH}$, assume $\rho \geq 0$.

Buyer's type evolves stochastically over time: $f^{\theta}(\theta_t | \theta_{t-1})$

Persistence: $f^{\theta} \equiv f^{\theta}(\theta_H | \theta_H) \geq f^{\theta}(\theta_H | \theta_L)$

Each period t, buyer learns his type (θ_t, φ_t).

Seller does not observe (θ_t, φ_t), but observes past allocations.

In $t = 0$, seller offers a contract to buyer who can accept or reject it.

Seller commits to the offered contract.

Common discount factor δ.

Economic Environment

Buyer's preferences: $u\left(\theta_t, q_t^{\theta}\right) + v\left(\varphi_t, q_t^{\varphi}\right) - p_t$

Seller's profit: $p_t - c\left(q_t^{\theta}\right) - c\left(q_t^{\varphi}\right)$

Per-period surplus generated by a contract:

$$S\left(\theta_t, \varphi_t, q_t^{\theta}, q_t^{\varphi}\right) = u\left(\theta_t, q_t^{\theta}\right) + v\left(\varphi_t, q_t^{\varphi}\right) - c\left(q_t^{\theta}\right) - c\left(q_t^{\varphi}\right)$$

Buyer's type revealed in period t: $\left(\hat{\theta}_t, \hat{\varphi}_t\right)$.

Buyer's revelation history: $\hat{\theta}^t = \{\hat{\theta}_0, ..., \hat{\theta}_t\}$ and $\hat{\varphi}^t = \{\hat{\varphi}_0, ..., \hat{\varphi}_t\}$.

Seller's strategy is described by a contract

$$\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\boldsymbol{\varphi}} \rangle = \left\{ \left(p\left(\hat{\theta}^t, \hat{\varphi}^t\right), q^{\theta}\left(\hat{\theta}^t, \hat{\varphi}^t\right), q^{\varphi}\left(\hat{\theta}^t, \hat{\varphi}^t\right) \right) \right\}_{t=0}^{T}$$

Given a contract, a buyer's strategy is described by function $\sigma^t(\cdot)$ that maps a history $\left\{ \left(\theta^{t-1}, \varphi^{t-1}\right), \left(\theta_t, \varphi_t\right), \left(\hat{\theta}_{t-1}, \hat{\varphi}_{t-1}\right) \right\}$ into $\left(\hat{\theta}^t, \hat{\varphi}^t\right)$.

Seller's Problem

The optimal contract solves

$$\max_{\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\boldsymbol{\varphi}} \rangle} \mathbb{E}_0 \sum_{t=0}^{T} \delta^t \left[p\left(\hat{\theta}^t, \hat{\varphi}^t\right) - c\left(q^\theta\left(\hat{\theta}^t, \hat{\varphi}^t\right)\right) - c\left(q^\varphi\left(\hat{\theta}^t, \hat{\varphi}^t\right)\right)\right]$$

subject to the incentive constraints (IC)

$$V\left(\theta_i, \varphi_j | \hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right) \geq V\left(\hat{\theta}_i, \hat{\varphi}_j | \left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right), (\theta_i, \varphi_j)\right),$$
$$\forall t, (\theta_i, \varphi_j), \left(\hat{\theta}_i, \hat{\varphi}_j\right), \left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right), (i,j) \in \{L, H\}$$

individual rationality (IR) constraints

$$V\left(\theta_i, \varphi_j | \hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right) \geq 0, \quad \forall t, (\theta_i, \varphi_j), \left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right), (i,j) \in \{L, H\}$$

and non-negativity constraints

$$\boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\boldsymbol{\varphi}} \geq 0$$

Multidimensional Screening

How to approach the problem in a unidimensional case?

- Reduce the state space by assuming that only local ICs bind (‘high’ types want to mimic ’low’ types), and then verify ex-post.

The first-order approach (FOA) can be problematic in some settings.

- Continuation values are important relative to instant payoffs.
- Multidimensional screening.

Armstrong, Rochet (1999): FOA works if types are positively correlated.

- Empirically plausible range of parameters.

Proceed in four steps:

- Take the relaxed seller’s problem (downward ICs & IR for LL-buyer).
- Find conditions under which ICs for HH-buyer are binding.
- Characterize the optimal contract.
- Show that it also solves the full problem.

Steps 1 & 2: Relaxed Seller's Problem

Relaxed problem:

- Downward ICs: after any history, HH-buyer pretends to be LL-, LH-, or HL-buyer; and LH- or HL-buyer pretends to be LL-buyer.
- IR constraints for LL-buyer.

Lemma 1. *Suppose that the menu $\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\boldsymbol{\varphi}} \rangle$ solves the relaxed problem. Then the ICs corresponding to LH- and HL-buyer pretending to be LL-buyer in period $t = 0$ are binding.*

Proposition 1. *Consider $t = 0$. There is a threshold*

$$\bar{\rho} = \psi_{HL}\psi_{LH}/\psi_{LL}$$

such that (i) if $\rho > \bar{\rho}$, then the ICs corresponding to HH-buyer pretending to be HL-, LH-, and LL-buyer are binding, (ii) if $\rho \in [0, \bar{\rho}]$, then the ICs corresponding to HH-buyer pretending to be HL- and LH-buyer are binding.

Proof given in Armstrong and Rochet (1999).

Idea: Check the conditions for the Lagrange multipliers to be strictly positive.

Steps 1 & 2: Relaxed Seller's Problem

It is without loss to assume that in the relaxed problem the relevant downward ICs and IR for LL-buyer hold with equality after any history.

Lemma 2. *Suppose that the menu $\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\varphi} \rangle$ satisfies the constraints of the relaxed problem. Then there exist a price schedule $\tilde{\mathbf{p}}$ such that $\langle \tilde{\boldsymbol{p}}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\varphi} \rangle$ (i) satisfies all the constraints of the relaxed problem, (ii) delivers the same profits as $\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\varphi} \rangle$, (iii) satisfies with equality the ICs corresponding to LH- and HL-buyer pretending to be LL-buyer and the individual rationality constraint for LL-buyer after any history, and (iv-a) satisfies with equality the ICs corresponding to HH-buyer pretending to be HL-, LH-, and LL-buyer after any history if $\rho > \bar{\rho}$; (iv-b) satisfies with equality the ICs corresponding to HH-buyer pretending to be HL- and LH-buyer after any history if $\rho \in [0, \bar{\rho}]$.*

I characterize the optimal contract in this relaxed problem and show that it solves the full problem.

Step 3: Optimal Contract Characterization

Proposition 2. *Suppose that* $u(\theta_t, q_t^{\theta}) = \theta_t q_t^{\theta}$, $v(\varphi_t, q^{\varphi}) = \varphi_t q_t^{\varphi}$, *and* $c(q_t) = q_t^2/2$. *Then the optimal contract has the following characterization.*

1. *If a buyer ever revealed* θ_H *(similarly,* φ_H*) in his history, then the optimal contract in period* t *is efficient and characterized by*

$$\tilde{q}^{\theta}\left(\hat{\theta}_t, \hat{\varphi}_t | \hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right) = \begin{cases} \theta_H & \text{if } \hat{\theta}_t = \theta_H, \forall t, \hat{\theta}^{t-1} \notin \tilde{\Theta}^{t-1} \\ \theta_L & \text{if } \hat{\theta}_t = \theta_L, \forall t, \hat{\theta}^{t-1} \notin \tilde{\Theta}^{t-1} \end{cases}$$

2. *Suppose* $\rho > \bar{\rho}$. *In period* $t = 0$, *if a buyer reports* θ_L, *then*

$$\tilde{q}^{\theta}(\theta_L, \varphi_L) = \tilde{q}^{\theta}(\theta_L, \varphi_H) < \theta_L$$

3. *Suppose* $\rho \in [0, \bar{\rho}]$. *In period* $t = 0$, *if a buyer reports* θ_L, *then*

$$\tilde{q}^{\theta}(\theta_L, \varphi_L) < \tilde{q}^{\theta}(\theta_L, \varphi_H) < \theta_L$$

4. *The optimal contract in periods* $t > 0$ *satisfy*

$$\tilde{q}^{\theta}\left(\hat{\theta}^t, \hat{\varphi}^t\right) = \frac{2f^{\theta} - 1}{f^{\theta}} \tilde{q}^{\theta}\left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right)$$

Step 4 & Discussion

Proposition 3. *Suppose $\rho \geq 0$. Let $\langle \tilde{\boldsymbol{p}}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\boldsymbol{\varphi}} \rangle$ be a menu with the properties described in Lemma 2. This schedule solves the full problem if and only if it solves the relaxed problem where relevant downward ICs and IR for LL-buyer are assumed to hold with equality after any history.*

How is the optimal contract different from the unidimensional case?

- Optimal quantity may depend on the report about the other good.
- Covariance between the subtypes matters if always reports θ_L or φ_L.

How is it different from the static setting?

- Vanishing distortion at the bottom, generalized no distortion at the top (Battaglini, 2005) for any covariance between the subtypes.

The framework can be applied to the other (more general) environments.

Alternative Application: Optimal Income Taxation

Continuum of couples consisting of a male (m) and a female (f).

Types θ_t and φ_t are spousal abilities.

Assume linear production technology, so that $y_t^m = \theta_t n_t^m$ and $y_t^f = \varphi_t n_t^f$.

Preferences: $U\left(c_t, y_t^m, y_t^f, \theta_t, \varphi_t\right) = c_t - \phi\left(\frac{y_t^m}{\theta_t}\right) - \phi\left(\frac{y_t^f}{\varphi_t}\right)$

What is different from the monopoly pricing problem?

- Resource feasibility constraint (pricing problem — one point).
- Planner's taste for redistribution (pricing problem — Rawlsian).

Partial equilibrium: savings technology $1/R$.

The planner evaluates social welfare using weights $\lambda\left(\theta_i, \varphi_j\right) \equiv \frac{\omega_{ij}\psi_{ij}}{\sum_{g,l}\omega_{gl}\psi_{gl}}$

Assumption 1. *Primitive welfare weights are non-negative, $\omega_{ij} \geq 0$, and satisfy (i) $\omega_{HL} = \omega_{LH} \equiv \tilde{\omega}$, (ii) $\tilde{\omega} \geq \omega_{HH}$, and (iii) $\omega_{LL} > 2\tilde{\omega}$.* ▸ KKS (2009)

Planner's Problem

The optimal allocation solves

$$\max_{\langle \boldsymbol{c},\boldsymbol{y^m},\boldsymbol{y^f}\rangle} \sum_{i,j} \lambda(\theta_i,\varphi_j)\mathbb{E}_0\left\{ \sum_{t=0}^{T} \delta^t \left[c_t(\theta,\varphi) - \phi\left(\frac{y_s^m t(\theta,\varphi)}{\theta_t}\right) - \phi\left(\frac{y_s^f t(\theta,\varphi)}{\varphi_t}\right)\right] |(\theta_i,\varphi_j)\right\}$$

subject to the resource feasibility constraint

$$\sum_{t=0}^{T}\left(\frac{1}{R}\right)^t \mathbb{E}_0\left[c_t(\theta,\varphi)|\theta_0,\varphi_0\right] + G \leq \sum_{t=0}^{T}\left(\frac{1}{R}\right)^t \mathbb{E}_0\left[y_t^m(\theta,\varphi) + y_t^f(\theta,\varphi)|(\theta_0,\varphi_0)\right]$$

and ICs

$$V_t\left(\boldsymbol{c},\boldsymbol{y^m},\boldsymbol{y^f}\right) \geq c_t\left(\sigma^t(\theta,\varphi)\right) - \phi\left(\frac{y_t^m\left(\sigma^t(\theta,\varphi)\right)}{\theta_t}\right) - \phi\left(\frac{y_t^f\left(\sigma^t(\theta,\varphi)\right)}{\varphi_t}\right) +$$
$$\delta\mathbb{E}_t\left\{V_{t+1}\left(\left(\boldsymbol{c},\boldsymbol{y^m},\boldsymbol{y^f}\right),(\theta^{t-1},\varphi^{t-1}),\sigma^t(\theta,\varphi),(\theta_{t+1},\varphi_{t+1})\right)|(\theta_t,\varphi_t)=(\theta,\varphi)\right\},$$
$$\forall t,\sigma^t,(\theta^t,\varphi^t)$$

Assortative Mating Threshold & IC Constraints

Proposition 4. *Consider period $t = 0$. There exists a threshold*

$$\bar{\rho} = \frac{(\omega_{LL} + \omega_{HH} - 2\tilde{\omega})\,\psi_{HL}\psi_{LH}}{(\omega_{LL} - \omega_{HH})\,\psi_{LL} + (\tilde{\omega} - \omega_{HH})\,(\psi_{HL} + \psi_{LH})}$$

such that if $\rho > \bar{\rho}$, then the ICs corresponding to HH-couples pretending to be HL-, LH-, and LL-couples hold with equality, (ii) $\rho \in [0, \bar{\rho}]$, then the ICs corresponding to HH-couples pretending to be HL- and LH-couples hold with equality.

Armstrong and Rochet (1999) implicitly assume $\omega_{LL} > \tilde{\omega} = \omega_{HH}$, hence

$$\bar{\rho} = \frac{\psi_{LH}\psi_{HL}}{\psi_{LL}}$$

Define the labor wedge as

$$1 - \tau_t^m(\theta_t, \varphi_t) \equiv -\frac{U_m\left(c_t, y_t^m/\theta_t, y_t^f/\varphi_t\right)}{\theta_t U_c\left(c_t, y_t^m/\theta_t, y_t^f/\varphi_t\right)} = -\frac{U_m\left(c_t, y_t^m/\theta_t, y_t^f/\varphi_t\right)}{\theta_t}$$

Optimal Marginal Tax Rates

Proposition 5. *Suppose that Assumption 1 holds. Then the optimal labor supply distortions have the following characterization.*

1. *The optimal distortions for the spouses who ever reported high ability in their history are zero:*

$$\frac{\tau_t^g(\theta,\varphi)}{1-\tau_t^g(\theta,\varphi)}=0 \qquad \forall t, \theta^t \notin \tilde{\Theta}^t, \varphi^t \notin \tilde{\Phi}^t, g \in \{m,f\}$$

2. *Suppose $\rho > \bar{\rho}$. Then the optimal distortions in $t=0$ for the low-ability males (similarly, females) satisfy* **separability***:*

$$\frac{\tau_1^m(\theta_L,\varphi_L)}{1-\tau_1^m(\theta_L,\varphi_L)}=\frac{\tau_1^m(\theta_L,\varphi_H)}{1-\tau_1^m(\theta_L,\varphi_H)}$$

3. *Suppose $\rho \in [0,\bar{\rho}]$. Then the optimal distortions in $t=0$ for the low-ability males satisfy* **negative jointness***:*

$$\frac{\tau_1^m(\theta_L,\varphi_L)}{1-\tau_1^m(\theta_L,\varphi_L)}>\frac{\tau_1^m(\theta_L,\varphi_H)}{1-\tau_1^m(\theta_L,\varphi_H)}$$

4. *The optimal distortions in periods $t>1$ satisfy*

$$\frac{\tau_t^m(\theta,\varphi)}{1-\tau_t^m(\theta,\varphi)}=\delta R\frac{2f^\theta-1}{f^\theta}\cdot\frac{\tau_{t-1}^m}{1-\tau_{t-1}^m}$$

Intuition: Variational Argument

Consider the best possible separable tax schedule.

Perturb the tax system towards negative jointness ($\varepsilon > 0$ is small enough).

- Spouses in LL-couples work a bit less, $dy^g_{LL} = -\frac{\varepsilon}{\psi_{LL}}$.
- Low-type spouses in mixed couples work more, $dy^f_{HL} = \frac{\varepsilon}{\psi_{HL}}$ & $dy^m_{LH} = \frac{\varepsilon}{\psi_{LH}}$.

Perturbation does not change the aggregate output.

Adjust consumption allocations so that IC constraints are satisfied.

Surplus from LL-, LH-, and HL-couples:

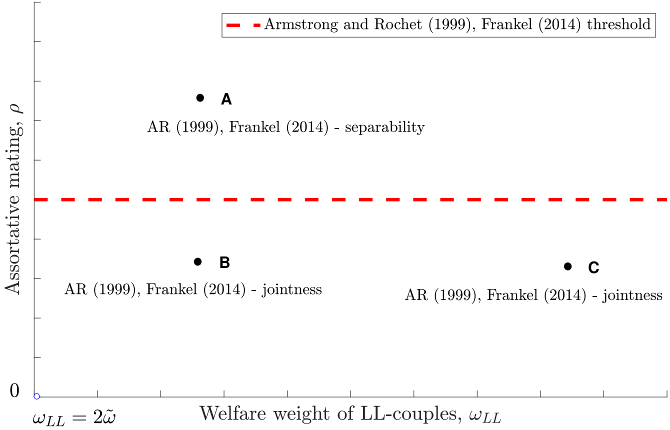
$$\psi_{LL}\Delta^c_{LL} + \psi_{LH}\Delta^c_{LH} + \psi_{HL}\Delta^c_{HL} = -\underbrace{\left[\phi'\left(\frac{y^f_{LL}}{\varphi_L}\right)\frac{1}{\varphi_L} - \phi'\left(\frac{y^f_{LL}}{\varphi_H}\right)\frac{1}{\varphi_H}\right]}_{>0}\frac{\psi_{LH}\varepsilon}{\psi_{LL}} - \underbrace{\left[\phi'\left(\frac{y^m_{LL}}{\theta_L}\right)\frac{1}{\theta_L} - \phi'\left(\frac{y^m_{LL}}{\theta_H}\right)\frac{1}{\theta_H}\right]}_{>0}\frac{\psi_{HL}\varepsilon}{\psi_{LL}} < 0$$

Aggregate change in consumption of HH-couples (similar vs. HL-couples):

$$\psi_{HH}\Delta^c_{HH,LH} = \left[\phi'\left(\frac{y^m_{LH}}{\theta_L}\right)\cdot\frac{1}{\theta_L} - \phi'\left(\frac{y^m_{LH}}{\theta_H}\right)\cdot\frac{1}{\theta_H}\right]\frac{\psi_{HH}\varepsilon}{\psi_{LH}} - \left[\phi'\left(\frac{y^f_{LL}}{\varphi_L}\right)\cdot\frac{1}{\varphi_L} - \phi'\left(\frac{y^f_{LL}}{\varphi_H}\right)\cdot\frac{1}{\varphi_H}\right]\frac{\psi_{HH}\varepsilon}{\psi_{LL}}$$

Higher ρ (less mixed couples, $\psi_{LH}\downarrow$, $\psi_{HL}\downarrow$) $\Rightarrow$ LL/LH/HL-surplus $\downarrow$ and $\psi_{HH}\Delta^c_{HH}\uparrow \Rightarrow$ less resources to satisfy feasibility & for redistribution.

Optimal Tax System: Assortative Mating & Taste for Redistribution



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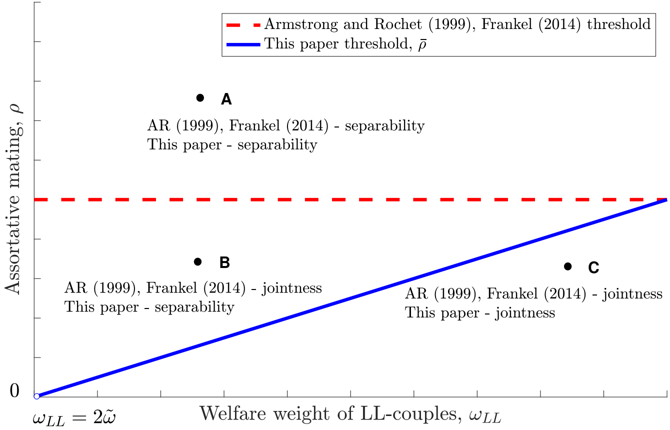

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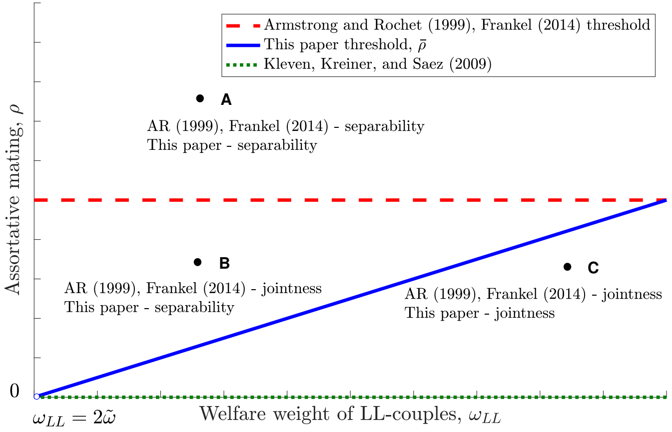


Optimal Labor Supply Distortions (ABC-Formula)

Assume the following disutility of labor: $\phi(n) = \frac{n^{1+1/\eta}}{1+1/\eta}$.

The optimal labor supply distortion in $t = 0$ is given by

$$\frac{\tau_0^m(\theta_L, \varphi)}{1 - \tau_0^m(\theta_L, \varphi)} = \frac{1 - \left(\frac{\theta_L}{\theta_H}\right)^{1+1/\eta}}{\psi_{LH} + \psi_{LL}} \sum_{s=L,H} \psi_{Hs} \left(1 - \frac{\omega_{Hs}}{\sum_{ij} \omega_{ij}\psi_{ij}}\right) + J^m(\varphi) \cdot \mathbb{I}\{\rho \in [0, \bar{\rho}]\}$$

Optimal distortions are driven by several forces:

- Higher elasticity of labor supply (η) $\Rightarrow$ lower optimal marginal tax rates.
- Higher fraction of couples with high-ability males ($\psi_{HH} + \psi_{HL}$) $\Rightarrow$ need stronger incentives for truthful reporting $\Rightarrow$ higher optimal τ.
- Higher fraction of couples with low-ability males ($\psi_{LL} + \psi_{LH}$) or relative low-high productivity (θ_L/θ_H) $\Rightarrow$ lower optimal τ.
- Higher planner's taste for redistribution (ω's) $\Rightarrow$ higher optimal τ.
- Interdependence between the types (separability or jointness).

Generalization of the ABC-formula (Diamond, 1998; Saez, 2001) for the case with multidimensional private information.

Conclusions

Dynamic contracting with multidimensional screening.

Applications:

- **Nonlinear Pricing:** joint life insurance (private info — health).
  - In the United States, strong assortative mating by health.
- **Optimal Taxation:** taxation of couples, taxation of individuals (multidimensional skills).

Driving forces that shape the optimal contract:

- Cross-sectional distribution of privately observed types (covariance).
- Planner's taste for redistribution.
- Persistence of privately observed types.

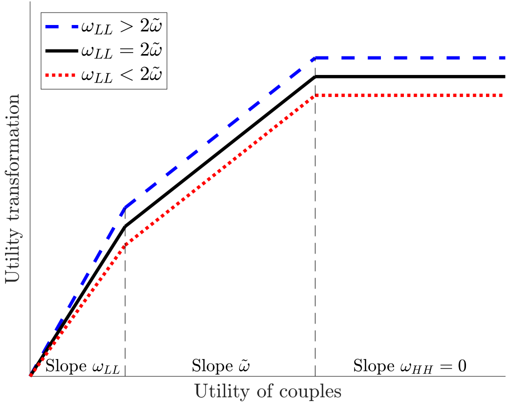
Further work:

- **Nonlinear Pricing:** $n \times m$ types, where $n, m > 2$.
- **Taxation:** within-household redistribution, risk-averse households.

Appendix

Social Welfare

Back



Changes in slopes: $\Delta_1 = \omega_{LL} - \tilde{\omega}$ and $\Delta_2 = \tilde{\omega} \quad \Rightarrow \quad \Delta_1 - \Delta_2 = \omega_{LL} - 2\tilde{\omega}$

Case $\omega_{LL} > 2\tilde{\omega}$ corresponds to Kleven, Kreiner, and Saez (2009).