Dynamic Contracting with Multidimensional Screening

PRELIMINARY AND INCOMPLETE

Egor Malkov*

December 2020

How to design an optimal contract under the long-term principal-agent interactions when the agent's type is described by more than one characteristic? I study a principal-agent problem where a monopolist repeatedly sells two non-durable goods to a buyer. A buyer's type, that captures the preferences over the goods, is two-dimensional private information that stochastically evolves over time according to a Markov process. I characterize the optimal contract and describe how it is shaped by the history of the buyer's reports, cross-sectional distribution of the buyer's characteristics, and their persistence. In particular, I show that there exists a non-negative threshold such that (i) if covariance between the buyer's subtypes is above this threshold, then the optimal quantity of a good does not depend on the report about the marginal valuation of another good, and (ii) if covariance is between zero and this threshold, then the optimal quantity of a good depends on the report about the marginal valuation of another good. The behavior of the optimal contract over time is shaped by persistence of the buyer's type. Furthermore, I apply this framework to the environment with income taxation of couples. I find that the optimal tax schedule crucially depends on the interaction between the cross-sectional distribution of spousal types in the economy and the government's taste for redistribution. In addition, I obtain a generalization of the ABC-formula for the optimal labor supply distortions under multidimensional private information.

Keywords: Dynamic Contracting, Multidimensional Screening, Nonlinear Pricing, Persistent Private Information, Optimal Taxation, Assortative Mating.

JEL: D42, D82, H21, H31.

* University of Minnesota and the Federal Reserve Bank of Minneapolis. E-mail: malko017@umn.edu. I would like to thank Fatih Guvenen, Larry Jones, and Fabrizio Perri for their valuable advice and constant support. I thank Job Boerma, V.V. Chari, George-Levi Gayle, Musab Kurnaz, Monica Tran Xuan, and participants at several conferences for excellent comments and insightful discussions. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

Consider an environment where a principal and an agent, whose type is described by more than one characteristic, are involved into repeated interaction. How to design an optimal contract in this setting? This problem has a broad range of applications in the real world such as life insurance contracts and income taxation. Despite recent advances in the literature, there are many open questions about the optimal dynamic contracting under multidimensional private information.

In this paper, I study a principal-agent problem where a monopolist repeatedly sells two non-durable goods to a buyer. A buyer's type, that captures the preferences over the goods, is private information and has two dimensions. Moreover, it stochastically evolves over time according to a Markov process. To characterize the optimal contract, I solve a multidimensional screening problem. This is a nontrivial task because the standard techniques, like the 'first-order approach', are, in general, not valid in this setting. To the best of my knowledge, this is the first paper that embeds a multidimensional screening problem into dynamic context with persistent private information in an analytically tractable way. Despite simplicity, the model allows to get nontrivial theoretical results and may serve as a useful benchmark for more complex economic environments with multidimensional screening.

The main results of the paper are the following. I show that the optimal contract is history-dependent and has infinite memory. In each period of time, the optimal quantities depend on the full history of past buyer's reports about his type, the current report, and the cross-sectional distribution of the buyer's type. In particular, I demonstrate that there exists a non-negative threshold on covariance between the buyer's subtypes (dimensions of the buyer's type) such that (i) if covariance is above this threshold, then the optimal quantity of a good does not depend on the report about the marginal valuation of another good, and (ii) if covariance is between zero and this threshold, then the optimal quantity of a good depends on the report about the marginal valuation of another good. The behavior of the optimal contract over time is shaped by persistence of the buyer's type. Furthermore, I apply my framework to the problem of optimal income taxation of couples, and show how the cross-sectional distribution of spousal types, government's taste for redistribution, and persistence of the spousal types jointly shape the optimal tax schedule. In particular, I obtain a generalization of the ABC-formula (Diamond, 1998; Saez, 2001) for the optimal labor supply distortions under multidimensional private information. In the setting with taxation of couples, I interpret covariance between types as a degree of assortative mating in the economy.

This paper is related to several strands of literature. First, my results complement the papers that characterize the solution to the multidimensional screening problem. The applications include optimal monopoly pricing, (Armstrong, 1996; Rochet and Choné, 1998; Armstrong and Rochet, 1999) and optimal taxation, (Cremer et al., 2001; Lehmann et al., 2018; Moser and Olea de Souza e Silva, 2019; Alves et al., 2021). I build on the studies by Armstrong and Rochet (1999) and Frankel (2014) who show that covariance between types is an important sufficient statistic for understanding what incentive constraints are binding. In the optimal taxation setting, I generalize their result by allowing for more general government's taste for redistribution. Carroll (2017) studies a robust version of the principal's problem where she observes the marginal distribution of each component of the agent's multidimensional type, but does not know the joint distribution. He shows that it is optimal to screen along each component separately. On a computational side, Judd et al. (2018) and Moser and Olea de Souza e Silva (2019) discuss the algorithms for solving the multidimensional screening problems. Similarly to Moser and Olea de Souza e Silva (2019), I also study multidimensional screening problem in dynamic setting with persistent private information, but my framework allows to characterize the optimal schedule in an analytically tractable way.

Second, this paper contributes to the literature on dynamic contracting. I extend the results from Battaglini (2005) to the environment with multidimensional types. One of the first papers that embeds stochastic types into the principal-agent model is Townsend (1982) who assumes the serially independent types. In a subsequent literature, the models were extended to have persistent private information (Baron and Besanko, 1984; Rustichini and Wolinsky, 1995; Laffont and Tirole, 1996; Williams, 2011; Fu and Krishna, 2019; Bloedel et al., 2020). Furthermore, in a similar framework, Battaglini (2007) characterizes the optimal renegotiation-proof contract. It turns out that the techniques that are applicable in a static environment, do not necessarily work in dynamic settings. Battaglini and Lamba (2019) discusses the limitations of the 'first-order approach' in dynamic models.

Finally, my findings from applying the aforementioned framework to the optimal income taxation problem in the spirit of Mirrlees (1971), contribute to the literature on optimal taxation in dynamic settings (Battaglini and Coate, 2008; Farhi and Werning, 2013; Golosov et al., 2016). A detailed discussion of this topic is provided by Stantcheva (2020). Similarly to Battaglini and Coate (2008), I consider risk neutral individuals, while the other cited papers assume more general preferences. Traditionally, the design of optimal individual tax schedule implies one dimension of private information, namely, productivity of a person. However, once we study the optimal taxation of couples, we naturally need to work with two-dimensional heterogeneity. The main feature of my paper is that I allow for this heterogeneity and study the multidimensional screening problem. On the theoretical side, my paper generalizes the results from Battaglini and Coate (2008) to the case of two-dimensional asymmetric information. I show how the interaction between the cross-sectional distribution of spousal types and the government's taste for redistribution shapes the optimal tax schedule. In particular, the result from Kleven et al. (2009) about optimality of negative jointness is a special case that holds under the assumptions that the spousal types are independently distributed and that the government's taste for redistribution to the couples, where both spouses have low ability, is high enough. In a different framework, using an equilibrium collective marriage market model, Gayle and Shephard (2019) also show that the optimal tax system features negative jointness. Furthermore, Rothschild and Scheuer (2013), Alves et al. (2021), and Kurnaz (2021) study the optimal tax problems in environments where agents have multidimensional characteristics. Differently from my work, all these papers study the optimal taxation problem in static setting. My paper also complements the work by Wu and Krueger (2021) who study optimal taxation of couples in a life-cycle setting and use restricted tax instruments. In addition, my paper contributes to the literature on the within-household inequality (Blundell et al., 2005; Lise and Seitz, 2011). In particular, I study the design of optimal tax system in an environment where the government cares not only about between- but also within-household inequality.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3, I characterize the optimal contract. In Section 4, I apply my framework to the optimal taxation problem. Finally, Section 5 concludes.

2 Model

2.1 Environment

Consider an economic environment with two players, a buyer (consumer, he) and a seller (monopolist, she). The buyer repeatedly buys two non-durable goods from the seller. I assume that time is discrete and the relationship between the buyer and the seller lasts for $T \to \infty$ periods. Each period, a buyer's type is characterised by a two-dimensional vector (θ, φ). A buyer of type (θ, φ) enjoys a per-period utility $u\left(\theta, q^{\theta}\right) + v\left(\varphi, q^{\varphi}\right) - p$ where q^{θ} and q^{φ} denote the number of units bought, and p denotes the total price. The utility functions $u\left(\theta, q^{\theta}\right)$ and $v\left(\varphi, q^{\varphi}\right)$ are increasing and differentiable in both arguments with $u(\theta, 0) = v(\varphi, 0) = 0$, and concave in q^{θ} and q^{φ} correspondingly. In every period, the seller produces the goods with cost function $\mathcal{C}\left(q^{\theta}, q^{\varphi}\right) = c\left(q^{\theta}\right) + c\left(q^{\varphi}\right)$. The cost function is increasing, convex, and differentiable, with $c'(0) = 0$ and $\lim_{q \to \infty} c'(q) = \infty$. The per-period profit of the monopolist who sells quantities $\left(q^{\theta}, q^{\varphi}\right)$ to a buyer of type (θ, φ) is given by $p - c\left(q^{\theta}\right) - c\left(q^{\varphi}\right)$. Define the per-period surplus generated by a contract between the buyer and the seller as

$$s\left(\theta, \varphi, q^{\theta}, q^{\varphi}\right) = u\left(\theta, q^{\theta}\right) + v\left(\varphi, q^{\varphi}\right) - c\left(q^{\theta}\right) - c\left(q^{\varphi}\right) \tag{1}$$

The subtypes θ_t and φ_t evolve over time according to a Markov process. I assume that each period there are two possible realizations of each subtype: $\theta_t \in \Theta = \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L$, and $\varphi_t \in \Phi = \{\varphi_L, \varphi_H\}$ with $\varphi_H > \varphi_L$. Hence there are four types, $(\theta_t, \varphi_t) \in \Theta \times \Phi$, with corresponding distribution $\psi(\theta_i, \varphi_j) \equiv \psi_{ij}$ where $i, j \in \{L, H\}$. The probability of reaching state k if the current state is l is given by $f^{\theta}(\theta_k|\theta_l) \equiv f^{\theta}_{kl}$ and $f^{\varphi}(\varphi_k|\varphi_l) \equiv f^{\varphi}_{kl}$. I assume that the subtypes are persistent, or positively correlated *over time*, i.e. $f^{\theta} \equiv f^{\theta}(\theta_H|\theta_H) \geq f^{\theta}(\theta_H|\theta_L)$ and $f^{\varphi} \equiv f^{\varphi}(\varphi_H|\varphi_H) \geq f^{\varphi}(\varphi_H|\varphi_L)$. I also assume that the subtypes are positively correlated in *cross-section*. In particular, following Armstrong and Rochet (1999), I define covariance between subtypes as follows:

$$\rho \equiv \psi_{HH}\psi_{LL} - \psi_{HL}\psi_{LH} \tag{2}$$

and assume $\rho \geq 0$. Later I discuss the case of $\rho < 0$.

In each period, the buyer observes the realization of his type. The seller, in turn, does not observe it, and can observe past allocations only. At date $t = 0$, the seller has a prior $\boldsymbol{\mu} = \left(\boldsymbol{\mu}^{\boldsymbol{\theta}}, \boldsymbol{\mu}^{\boldsymbol{\varphi}}\right) = \left(\left(\mu_H^\theta, \mu_L^\theta\right), \left(\mu_H^\varphi, \mu_L^\varphi\right)\right)$ on the buyer's type. I assume that the prior has full support.

At date $t = 1$, the seller offers a supply contract to the buyer. The buyer can accept or reject it. If the buyer accepts the offer, he can leave the relationship at any date $t \geq 1$ if the expected continuation utility of the contract falls below the reservation utility $\underline{U} = 0$. The common discount factor is $\delta \in (0, 1)$. I assume that the seller commits to the offered contract.

Denote by $\left(\hat{\theta}_t, \hat{\varphi}_t\right)$ the buyer's type revealed at time t. Define a revelation history of the buyer at time t to be the sequence of his past and current type revelations, i.e. $\hat{\theta}^t = \{\hat{\theta}_1, ..., \hat{\theta}_t\}$ and $\hat{\varphi}^t = \{\hat{\varphi}_1, ..., \hat{\varphi}_t\}$. Alternatively, we can define it recursively: $\hat{\theta}^t = \{\hat{\theta}^{t-1}, \hat{\theta}_t\}$, $\hat{\theta}^0 = \varnothing$ and $\hat{\varphi}^t = \{\hat{\varphi}^{t-1}, \hat{\varphi}_t\}$, $\hat{\varphi}^0 = \varnothing$. Denote by $\hat{\Theta}^t$ and $\hat{\Phi}^t$ the sets of all possible revelation histories for subtypes θ and φ at time t. Denote by $\tilde{\Theta}^\tau$ (similarly, $\tilde{\Phi}^\tau$) the set of histories when $\hat{\theta}_t = \theta_L$ (similarly, $\hat{\varphi}_t = \varphi_L$), $\forall t = 1, ..., \tau$. In this environment, a form of the revelation principle (Myerson, 1986) is valid, therefore, without loss of generality, I only consider contracts that in period t depend on the history of type revelations and the type revealed at date t. Formally, the contract can be written as

$$\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\boldsymbol{\varphi}} \rangle = \left\{ \left(p\left(\hat{\theta}^t, \hat{\varphi}^t\right), q^\theta\left(\hat{\theta}^t, \hat{\varphi}^t\right), q^\varphi\left(\hat{\theta}^t, \hat{\varphi}^t\right)\right) \right\}_{t=1}^T$$

A strategy for the seller consists of offering a direct mechanism $\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\boldsymbol{\varphi}} \rangle$ described above. In period t, the buyer knows his true type realizations for the periods up until the current one, i.e. $(\theta^t, \varphi^t) \in \Theta^t \times \Phi^t$, where (θ^t, φ^t) denotes the history of the true type realizations, Θ^t and Φ^t denote the sets of all possible true-type histories at time t. For a given contract, a strategy for the buyer is described by a function $\sigma^t(\cdot)$ that maps a history $\left\{ \left(\theta^{t-1}, \varphi^{t-1}\right), (\theta_t, \varphi_t), \left(\hat{\theta}_{t-1}, \hat{\varphi}_{t-1}\right) \right\}$ into a revealed type $\left(\hat{\theta}^t, \hat{\varphi}^t\right)$.

The seller's problem consists of choosing a contract $\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\boldsymbol{\varphi}} \rangle$ that maximizes the expected discounted profits subject to the incentive compatibility constraints and the individual rationality constraints. The expected discounted profits are given by

$$\Pi = \mathbb{E}_0 \sum_{t=1}^{T} \delta^{t-1} \left[p\left(\hat{\theta}^t, \hat{\varphi}^t\right) - c\left(q^\theta\left(\hat{\theta}^t, \hat{\varphi}^t\right)\right) - c\left(q^\varphi\left(\hat{\theta}^t, \hat{\varphi}^t\right)\right)\right] \tag{3}$$

where expectation is taken over the cross-section of the types and time.

The incentive compatibility constraints imply that, after any history, the buyer does not want to report a false type. Denote by $V\left(\hat{\theta}_i, \hat{\varphi}_j | \left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right), (\theta_i, \varphi_j)\right)$ the expected utility of a buyer with type (θ_i, φ_j) who reports to be of type $\left(\hat{\theta}_i, \hat{\varphi}_j\right)$ at time t after history $\left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right)$, and always reports his true type thereafter. Next, I denote by $V\left(\theta_i, \varphi_j | \hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right)$ the expected utility of a buyer with type (θ_i, φ_j) who truthfully reports his type at time t after history $\left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right)$, and always reports his true type thereafter. Using the one-shot deviation principle, I describe the incentive compatibility constraints for type (θ_i, φ_j), after history $\left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right)$ at time t as

$$V\left(\theta_i, \varphi_j | \hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right) \geq V\left(\hat{\theta}_i, \hat{\varphi}_j | \left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right), (\theta_i, \varphi_j)\right) \tag{4}$$

$\forall\,(\theta_i, \varphi_j)$, $\left(\hat{\theta}_i, \hat{\varphi}_j\right)$, $\left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right)$, $(i, j) \in \{L, H\}$. Denote the incentive compatibility constraint described in (4) by $IC_t\,(\theta_i, \varphi_j)$. Note that the buyer reports his type along two dimensions, and hence this is a multidimensional screening problem.

The individual rationality constraints imply that, after any history, the buyer receives at least his reservation utility $\underline{U} = 0$:

$$V\left(\theta_i, \varphi_j | \hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right) \geq 0 \tag{5}$$

$\forall\,(\theta_i, \varphi_j)$, $\left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right)$, $(i, j) \in \{L, H\}$. Denote the individual rationality constraint described in (5) by $IR_t\,(\theta_i, \varphi_j)$. The contract that satisfies all the incentive compatibility and individual rationality constraints is said to be *implementable*.

To summarize, the seller chooses a contract $\langle \boldsymbol{p}, \boldsymbol{q^\theta}, \boldsymbol{q^\varphi} \rangle$ that maximizes (3) subject to $\boldsymbol{q^\theta} \geq 0$, $\boldsymbol{q^\varphi} \geq 0$, $IC_t\,(\theta_i, \varphi_j)$ described in (4), $IR_t\,(\theta_i, \varphi_j)$ described in (5), $\forall i, j \in \{L, H\}, t$, $\left(\hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right)$.

2.2 Multidimensional Screening

The standard approach to characterize the optimal contract in a setting with one dimension of private information follows two steps. First, we need to solve a relaxed problem where keep the local downward incentive compatibility constraints for H-type and the individual rationality constraints for L-type only. Second, following the first step, we ex-post verify the remaining constraints. This 'first-order approach' is widely used in the literature (Baron and Besanko, 1984;

Kapička, 2013; Pavan et al., 2014; Farhi and Werning, 2013). Battaglini and Lamba (2019) discuss the applicability of the 'first-order approach' in various environments, and conclude that it can be problematic in the settings where expected continuation values are important relative to instant payoffs.

In this paper, I consider an environment with multidimensional screening where applicability of the 'first-order approach' can be problematic as well (Moser and Olea de Souza e Silva, 2019). However, as shown by Armstrong and Rochet (1999), under some conditions on the cross-sectional distribution of types, the 'first-order approach' may be applicable to this class of problems. Furthermore, as I discuss later, these conditions are empirically plausible. In particular, we need to assume that subtypes θ and φ are positively correlated. The current buyer-seller environment can be applied to the joint life insurance contracts for spouses, hence positive correlation between θ and φ can be interpreted as positive correlation between health conditions of the spouses. When I map the problem into the setting with the optimal taxation of couples, it has an interpretation about positive assortative mating in education or earning ability between spouses. To tackle this problem, I combine the techniques from Armstrong and Rochet (1999) and Battaglini (2005).[1] First, I consider a problem with the downward incentive compatibility constraints only, i.e. when, after any history, HH-buyer pretends to be either LL-, LH-, or HL-buyer, and LH- or HL-buyer pretends to be LL-buyer. Hence I do not consider the upward incentive compatibility constraints as well as the constraints where HL-buyer pretends to be LH-buyer and vice versa. Second, I show that the incentive compatibility constraints, corresponding to LH- and HL-buyer pretending to be LL-buyer, are always binding. Third, I show the conditions under which the incentive compatibility constraints, corresponding to HH-buyer pretending to be LL-, LH-, or HL-buyer, hold with equality. Throughout the proofs, I use the result from Battaglini (2005) that in a dynamic setting, although the constraints are not necessarily binding in every optimal scheme, it is without loss of generality to assume that constraints in the relaxed problem hold with equality. Finally, I show that the relaxed problem solves the full problem.

[1] If the buyer has type (θ_i, φ_j), I call him ij*-buyer*. For instance, the buyer of type (θ_H, φ_H) is called HH-buyer.

3 Optimal Contract

3.1 Characterization

I begin by stating a useful lemma.

Lemma 1. *Suppose that the menu $\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\varphi} \rangle$ solves the 'relaxed' problem. Then the incentive compatibility constraint corresponding to LH- and HL-buyer pretending to be LL-buyer at period $t = 1$ is binding.*

Proof. *See Appendix.*

Next, I refer to the the proposition formulated in Armstrong and Rochet (1999).

Proposition 1. *Consider the first period, $t = 1$. Suppose $\rho \geq 0$. Then there exists a threshold $\bar{\rho}$ given by*

$$\bar{\rho} = \frac{\psi_{HL}\psi_{LH}}{\psi_{LL}} \tag{6}$$

such that if (i) $\rho > \bar{\rho}$, then the incentive compatibility constraints corresponding to HH-buyer pretending to be HL-, LH-, and LL-buyer hold with equality, (ii) $\rho \in [0, \bar{\rho}]$, then the incentive compatibility constraints corresponding to HH-buyer pretending to be HL- and LH-buyer hold with equality.

Proof. *See Armstrong and Rochet (1999).*

In Section 4.1, I generalize Proposition 1 to an environment with optimal taxation and general government's taste for redistribution. Next, consider the following lemma.

Lemma 2. *Suppose that the menu $\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\varphi} \rangle$ satisfies the constraints of the 'relaxed' problem. Then there exist a price schedule $\tilde{\boldsymbol{p}}$ such that $\langle \tilde{\boldsymbol{p}}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\varphi} \rangle$ (i) satisfies all the constraints of the 'relaxed' problem, (ii) delivers the same profits as $\langle \boldsymbol{p}, \boldsymbol{q}^{\boldsymbol{\theta}}, \boldsymbol{q}^{\varphi} \rangle$, (iii) satisfies with equality the incentive compatibility constraints corresponding to LH- and HL-buyer pretending to be LL-buyer and the individual rationality constraint for LL-buyer after any history, and (iv) satisfies with equality the incentive compatibility constraints corresponding to HH-buyer pretending to be HL-, LH-, and LL-buyer after any history if $\rho > \bar{\rho}$; satisfies with equality the incentive compatibility constraints corresponding to HH-buyer pretending to be HL- and LH-buyer after any history if $\rho \in [0, \bar{\rho}]$, where $\bar{\rho}$ is given in Proposition 1.*

Proof. *See Appendix.*

Thus, the downward incentive compatibility constraints and the individual rationality constraints for LL-buyer are assumed to hold with equality after any history without loss of generality. Note that the configuration of the incentive compatibility constraints for HH-buyer depends on the cross-sectional distribution of the buyer's type. The next proposition characterizes the optimal contract. For illustration, I assume the version of the model proposed by Mussa and Rosen (1978), where $u(\theta, q^\theta) = \theta q^\theta$, $v(\varphi, q^\varphi) = \varphi q^\varphi$, and $c(q) = q^2/2$. Hence θ and φ account for the marginal valuations of the goods.

Proposition 2. *Suppose that $u(\theta, q^\theta) = \theta q^\theta$, $v(\varphi, q^\varphi) = \varphi q^\varphi$, and $c(q) = q^2/2$. Then the optimal contract has the following characterization.*

1. *If a buyer ever revealed θ_H or φ_H in his history, then the optimal contract in period t is efficient and characterized by*

$$\tilde{q}^\theta\left(\hat{\theta}_t, \hat{\varphi}_t | \hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right) = \begin{cases} \theta_H & \text{if } \hat{\theta}_t = \theta_H, \forall t, \hat{\theta}^{t-1} \notin \tilde{\Theta}^{t-1} \\ \theta_L & \text{if } \hat{\theta}_t = \theta_L, \forall t, \hat{\theta}^{t-1} \notin \tilde{\Theta}^{t-1} \end{cases} \tag{7}$$

$$\tilde{q}^\varphi\left(\hat{\theta}_t, \hat{\varphi}_t | \hat{\theta}^{t-1}, \hat{\varphi}^{t-1}\right) = \begin{cases} \varphi_H & \text{if } \hat{\varphi}_t = \varphi_H, \forall t, \varphi^{t-1} \notin \tilde{\Phi}^{t-1} \\ \varphi_L & \text{if } \hat{\varphi}_t = \varphi_L, \forall t, \varphi^{t-1} \notin \tilde{\Phi}^{t-1} \end{cases} \tag{8}$$

2. *Suppose $\rho > \bar{\rho}$. In period $t = 1$, if a buyer reports θ_L or φ_L, then the optimal contract is characterized by*

$$\tilde{q}^\theta\left(\theta_L, \varphi_L\right) = \tilde{q}^\theta\left(\theta_L, \varphi_H\right) < \theta_L \tag{9}$$

$$\tilde{q}^\varphi\left(\theta_L, \varphi_L\right) = \tilde{q}^\varphi\left(\theta_H, \varphi_L\right) < \varphi_L \tag{10}$$

3. *Suppose $\rho \in [0, \bar{\rho}]$. In period $t = 1$, if a buyer reports θ_L or φ_L, then the optimal contract is characterized by*

$$\tilde{q}^\theta\left(\theta_L, \varphi_L\right) < \tilde{q}^\theta\left(\theta_L, \varphi_H\right) < \theta_L \tag{11}$$

$$\tilde{q}^\varphi\left(\theta_L, \varphi_L\right) < \tilde{q}^\varphi\left(\theta_H, \varphi_L\right) < \varphi_L \tag{12}$$

4. *The optimal contract in periods $t > 1$ satisfy*

$$\tilde{q}^{\theta}\left(\hat{\theta}^{t},\hat{\varphi}^{t}\right)=\frac{2f^{\theta}-1}{f^{\theta}}\tilde{q}^{\theta}\left(\hat{\theta}^{t-1},\hat{\varphi}^{t-1}\right) \tag{13}$$

$$\tilde{q}^{\varphi}\left(\hat{\theta}^{t},\hat{\varphi}^{t}\right)=\frac{2f^{\varphi}-1}{f^{\varphi}}\tilde{q}^{\varphi}\left(\hat{\theta}^{t-1},\hat{\varphi}^{t-1}\right) \tag{14}$$

Proof. *See Appendix.*

Proposition 2 generalizes the results from Battaglini (2005) to the framework with multidimensional types. First, I show that the optimal contract is history-dependent and has infinite memory. Despite this, we can easily characterize it. If the buyer has ever reported θ_H (similarly, φ_H) in his history, then the quantity q^{θ} (similarly, q^{φ}) is at the efficient level and depends on the reported type in a given period. This is consistent with the so-called 'generalized no-distortion at the top principle' (Battaglini, 2005). Next, if the buyer has always reported θ_L (similarly, φ_L), then the optimal contract is jointly shaped by cross-sectional covariance between θ and φ as well as their persistence. In particular, if $\rho > \bar{\rho}$, then quantity q^{θ} (similarly, q^{φ}) does not depend on reported φ (similarly, θ). Hence it is optimal for the principal to screen along each component separately. In turn, if $\rho \in [0, \bar{\rho}]$, then quantity q^{θ} (similarly, q^{φ}) decreases in reported φ (similarly, θ). Hence, it is optimal for the principal to condition the optimal quantity of one good on the marginal valuation of the other good. In Section 4.1, I provide an intuition behind this result in more general setting with optimal taxation by considering a variational argument. Finally, the optimal contract converges over time to an efficient contract along any history. This result corresponds to the 'vanishing distortion at the bottom principle' (Battaglini, 2005). The speed of convergence depends on the degree of persistence of private information.

3.2 Application: Joint Insurance Contracts

One of the natural candidates for application of my results is the design of joint life insurance contracts. These contracts offer coverage for two people (spouses) for a single premium payment each month. If I interpret θ and φ as the health conditions of the spouses, and, following Guner et al. (2018) who document positive assortative mating in spousal health, assume that they are positively correlated, I can directly apply the results from Section 3.1. To the best of my knowledge, the joint life insurance contracts are not extensively explored in the literature (Youn and Shemyakin, 1999; Luciano et al., 2008; Gourieroux and Lu, 2015). In a related work, Hendel and Lizzeri (2003) use the data on individual life insurance contracts to study the properties of long-term contracts with the lack of commitment by buyers.

4 Extensions and Alternative Applications

The framework considered in Section 2 can be further extended and applied to the other environments. In particular, I pursue three directions. First, I discuss the optimal contract under the negative cross-sectional covariance between the subtypes. Second, I apply the techniques from Section 3 to characterize the solution to the optimal income taxation problem. One possible interpretation of the environment with two dimensions of private information is the taxation of couples. Finally, within the optimal taxation framework, I study the setting where the government cares about both between- and within-family redistribution.

4.1 Optimal Taxation

How should the optimal income taxes for married couples be designed? Over the last decades, one of the stable features of the U.S. and many European economies is a presence of positive assortative mating between spouses, i.e. people more likely match and marry partners with similar characteristics.[2] In turn, this phenomenon is often considered as one of the driving forces of income inequality.[3] To address this problem, one of the options for the government is to use tax policy as a means of redistribution. Hence the question about optimal income tax design for couples is of crucial importance for the policymakers.

In this section, I apply my framework to study the optimal income taxation of couples in a dynamic Mirrlees setting. I study how the cross-sectional distribution of spousal types, government's taste for redistribution, and persistence of the spousal types jointly shape the optimal tax schedule. It is important to emphasize that, according to the previous paragraph, the assump-

[2] See Schwartz (2010) on assortative mating by earnings, and Eika et al. (2019) on assortative mating by education.

[3] Using the data on household surveys from 34 countries, Fernandez et al. (2005) show that there is a positive relationship between sorting in skills and income inequality. However, Eika et al. (2019) argue that changes in educational assortative mating over time barely move the trends in household income inequality.

tion about positive covariance between spousal types is empirically relevant in this environment. The problems of monopoly pricing and optimal taxation differ along several aspects. First, the solution to the monopoly pricing problem is one point that corresponds to the maximum profit. In turn, in the optimal taxation setting, I characterize the part of Pareto frontier. Furthermore, the individual rationality constraint in the buyer-seller framework implies that we assume the Rawlsian welfare function, while in this section I allow for more general government's taste for redistribution.

Consider an economy populated by a continuum of couples. Each couple consists of two spouses—a male (denoted by m) and a female (denoted by f). Spouses differ in their abilities to produce. The ability of a male is $\theta \in \Theta = \{\theta_L, \theta_H\}$ and the ability of a female is $\varphi \in \Phi = \{\varphi_L, \varphi_H\}$ with $\theta_H > \theta_L > 0$ and $\varphi_H > \varphi_L > 0$. Let y^g_{ij} be the earnings or output of individual $g \in \{m, f\}$ where $i, j \in \{L, H\}$. Assume linear production technology, so that $y^m_{ij} = \theta n^m_{ij}$ and $y^f_{ij} = \varphi n^f_{ij}$ where n^g_{ij} are working hours. The rest of notation follows Section 2.

Assume the following per-period utility function of a couple:

$$U\left(c_t, y^m_t, y^f_t, \theta_t, \varphi_t\right) = c_t - \phi\left(\frac{y^m_t}{\theta_t}\right) - \phi\left(\frac{y^f_t}{\varphi_t}\right) \tag{15}$$

where $\phi(\cdot)$ is increasing, strictly convex, and twice continuously differentiable. Risk neutrality in preferences implies that the only source of distortions in this environment is the desire of the government to redistribute resources.

The government observes consumption and spousal outputs, but not their abilities. It evaluates total welfare using the weights $\lambda\left(\theta_i, \varphi_j\right)$ assigned to the couples that have type $\left(\theta_i, \varphi_j\right)$ in period $t = 1$, where $i, j \in \{L, H\}$. I normalize the weights such that

$$\lambda\left(\theta_i, \varphi_j\right) \equiv \frac{\omega_{ij}\psi_{ij}}{\sum_{g,l}\omega_{gl}\psi_{gl}} \tag{16}$$

With the risk-neutral agents, the utilitarian government will set all the marginal taxes to zero, and thus I assume that it has a taste for redistribution that is different from utilitarian. In what follows, I make the following assumption.

Assumption 1. *The primitive welfare weights are non-negative,* $\omega_{ij} \geq 0$*, and satisfy the following conditions: (i)* $\omega_{HL} = \omega_{LH} \equiv \tilde{\omega}$*, (ii)* $\tilde{\omega} \geq \omega_{HH}$*, and (iii)* $\omega_{LL} > 2\tilde{\omega}$*.*

Part (i) accounts for 'anonymity', so that the planner assigns equal weight to the mixed couples. Part (iii) states that the government has a strong enough taste for redistribution towards LL-couples. This is equivalent to promising some level of reservation utility $\underline{U}$ to LL-buyer in Section 2. Note that Assumption 1 implies that $\sum_{i,j} \omega_{ij}\psi_{ij} \equiv \mathbb{E}(\omega) > \tilde{\omega} \geq \omega_{HH}$, or, in words, that the mean primitive welfare weight is strictly greater than the weights assigned to HL-, LH-, and HH-couples.

An allocation in this economy is given by

$$\langle \boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f} \rangle = \left\{ \left(c_t\left(\theta^t, \varphi^t\right), y_t^m\left(\theta^t, \varphi^t\right), y_t^f\left(\theta^t, \varphi^t\right) \right) \right\}_{t=1}^{T}$$

To simplify notation, in this section, I omit explicit dependence on the past history. Whenever it does not cause confusion, a notation $x_t^g(\theta, \varphi)$ denotes the value of a random variable x_t^g at a history $(\theta^{t-1}, \varphi^{t-1}, \theta_t, \varphi_t)$, and x_{t-1}^g denotes $x_{t-1}^g(\theta^{t-1}, \varphi^{t-1})$.

Define expected discounted utility of a couple as

$$V_t\left(\boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f}\right) = \mathbb{E}_t \left\{ \sum_{s=t}^{T} \delta^{s-t} \left[c_s(\theta, \varphi) - \phi\left(\frac{y_s^m(\theta, \varphi)}{\theta_s}\right) - \phi\left(\frac{y_s^f(\theta, \varphi)}{\varphi_s}\right) \right] |(\theta_t, \varphi_t) \right\} \quad (17)$$

An allocation is said to be *resource feasible* if it satisfies the aggregate resource constraint:

$$\sum_{t=1}^{T} \left(\frac{1}{R}\right)^{t-1} \mathbb{E}_1\left[c_t(\theta, \varphi) |\theta_1, \varphi_1\right] + G \leq \sum_{t=1}^{T} \left(\frac{1}{R}\right)^{t-1} \mathbb{E}_1\left[y_t^m(\theta, \varphi) + y_t^f(\theta, \varphi) |(\theta_1, \varphi_1)\right] \quad (18)$$

I study the partial equilibrium where the government can transfer aggregate resources across periods at a gross rate of return R. Note that the aggregate resource constraint is an additional constraint that we do not have in the monopoly pricing model.

An allocation is said to be *incentive compatible* if it satisfies the following set of incentive constraints for each couple's report σ^t, history (θ^t, φ^t), and t:

$$V_t\left(\boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f}\right) \geq c_t\left(\sigma^t(\theta, \varphi)\right) - \phi\left(\frac{y_t^m\left(\sigma^t(\theta, \varphi)\right)}{\theta_t}\right) - \phi\left(\frac{y_t^f\left(\sigma^t(\theta, \varphi)\right)}{\varphi_t}\right) + \delta\mathbb{E}_t\left\{V_{t+1}\left(\left(\boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f}\right), (\theta^{t-1}, \varphi^{t-1}), \sigma^t(\theta, \varphi), (\theta_{t+1}, \varphi_{t+1})\right) | (\theta_t, \varphi_t) = (\theta, \varphi)\right\} \quad (19)$$

The government solves the following dynamic mechanism design problem:

$$\max_{\langle \boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f}\rangle} \sum_{i,j} \lambda(\theta_i, \varphi_j) V_1\left(\boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f}\right)$$

subject to the aggregate resource constraint (18) and the incentive compatibility constraints (19).

First, I state the proposition that generalizes Proposition 1 to the environment with more general welfare weights.

Proposition 3. *Consider the first period, $t = 1$. Suppose $\rho \geq 0$. Then there exists a threshold $\bar{\rho}$ given by*

$$\bar{\rho} = \frac{(\omega_{LL} + \omega_{HH} - 2\tilde{\omega})\,\psi_{HL}\psi_{LH}}{(\omega_{LL} - \omega_{HH})\,\psi_{LL} + (\tilde{\omega} - \omega_{HH})\,(\psi_{HL} + \psi_{LH})} \quad (20)$$

such that if $\rho > \bar{\rho}$, then the incentive compatibility constraints corresponding to HH-couples pretending to be HL-, LH-, and LL-couples hold with equality, (ii) $\rho \in [0, \bar{\rho}]$, then the incentive compatibility constraints corresponding to HH-couples pretending to be HL- and LH-couples hold with equality.

Proof. *See Appendix.*

Proposition 3 is a generalization of the result from Armstrong and Rochet (1999) and Frankel (2014).[4] In these papers, the authors assume that $\omega_{LL} > \omega_{LH} = \omega_{HL} = \omega_{HH} \geq 0$. This results in the threshold $\bar{\rho} = \psi_{HL}\psi_{LH}/\psi_{LL}$.

[4] Without Assumption 1, where I set $\omega_{HL} = \omega_{LH} \equiv \tilde{\omega}$, the threshold is given by

$$\bar{\rho} = \frac{(\omega_{LL} + \omega_{HH} - \omega_{HL} - \omega_{LH})\,\psi_{HL}\psi_{LH}}{(\omega_{LL} - \omega_{HH})\,\psi_{LL} + (\omega_{LH} - \omega_{HH})\,\psi_{LH} + (\omega_{HL} - \omega_{HH})\,\psi_{HL}}$$

I use Lemmas 1 and 2 from Section 3 to argue the following. Suppose that the allocation $\langle \boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f} \rangle$ satisfies the constraints of the relaxed problem. Then there exist a consumption allocation $\tilde{\boldsymbol{c}}$ such that $\langle \tilde{\boldsymbol{c}}, \boldsymbol{y^m}, \boldsymbol{y^f} \rangle$ (i) satisfies all the constraints of the 'relaxed' problem, (ii) provides the same welfare as $\langle \boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f} \rangle$, (iii) satisfies with equality the incentive compatibility constraints corresponding to LH- and HL-couples pretending to be LL-couples after any history, and (iv-a) satisfies with equality the incentive compatibility constraints corresponding to HH-couples pretending to be HL-, LH-, and LL-couples after any history if $\rho > \bar{\rho}$, or (iv-b) satisfies with equality the incentive compatibility constraints corresponding to HH-couples pretending to be HL- and LH-couples after any history if $\rho \in [0, \bar{\rho}]$, where $\bar{\rho}$ is given in Proposition 3. Next, following the logic from Section 3, I argue that the solution to the relaxed problem solves the full government's problem, i.e. one with all the incentive compatibility constraints included.

Turning to the distortions, I define the optimal labor wedges τ_{ij}^m and τ_{ij}^f as

$$MRS_{n,c}^{m,ij} = (1 - \tau_{ij}^m)\theta_i \tag{21}$$

$$MRS_{n,c}^{f,ij} = (1 - \tau_{ij}^f)\varphi_j \tag{22}$$

Proposition 4 characterizes the optimal labor supply distortions.

Proposition 4. *Suppose that the couple's preferences are given by (15), and Assumption 1 holds. Then the optimal labor supply distortions have the following characterization.*

1. *The optimal distortions for the spouses who ever had high ability in their history are zero:*

$$\frac{\tau_t^g(\theta, \varphi)}{1 - \tau_t^g(\theta, \varphi)} = 0 \qquad \forall t, \theta^t \notin \tilde{\Theta}^t, \varphi^t \notin \tilde{\Phi}^t, g \in \{m, f\} \tag{23}$$

2. *Suppose $\rho > \bar{\rho}$. Then the optimal distortions at $t = 1$ for the low-ability spouses satisfy*

$$\frac{\tau_1^m(\theta_L, \varphi_L)}{1 - \tau_1^m(\theta_L, \varphi_L)} = \frac{\tau_1^m(\theta_L, \varphi_H)}{1 - \tau_1^m(\theta_L, \varphi_H)} \tag{24}$$

$$\frac{\tau_1^f(\theta_L, \varphi_L)}{1 - \tau_1^f(\theta_L, \varphi_L)} = \frac{\tau_1^f(\theta_H, \varphi_L)}{1 - \tau_1^f(\theta_H, \varphi_L)} \tag{25}$$

3. *Suppose $\rho \in [0, \bar{\rho}]$. Then the optimal distortions at $t = 1$ for the low-ability spouses satisfy*

$$\frac{\tau_1^m(\theta_L, \varphi_L)}{1 - \tau_1^m(\theta_L, \varphi_L)} > \frac{\tau_1^m(\theta_L, \varphi_H)}{1 - \tau_1^m(\theta_L, \varphi_H)} \tag{26}$$

$$\frac{\tau_1^f(\theta_L, \varphi_L)}{1 - \tau_1^f(\theta_L, \varphi_L)} > \frac{\tau_1^f(\theta_H, \varphi_L)}{1 - \tau_1^f(\theta_H, \varphi_L)} \tag{27}$$

4. *The optimal distortions in periods $t > 1$ satisfy*

$$\frac{\tau_t^m(\theta, \varphi)}{1 - \tau_t^m(\theta, \varphi)} = \delta R \frac{2f^\theta - 1}{f^\theta} \cdot \frac{\tau_{t-1}^m}{1 - \tau_{t-1}^m} \tag{28}$$

$$\frac{\tau_t^f(\theta, \varphi)}{1 - \tau_t^f(\theta, \varphi)} = \delta R \frac{2f^\varphi - 1}{f^\varphi} \cdot \frac{\tau_{t-1}^f}{1 - \tau_{t-1}^f} \tag{29}$$

Proof. *See Appendix.*

Proposition 4 generalizes the results from Battaglini and Coate (2008) to the framework with multidimensional types. The optimal taxes are history-dependent. In particular, the first part states that if a person has ever had high type, then her/his earnings are undistorted and the optimal marginal tax rate for her/him is zero irrespective of the history of types of her/his spouse. The 'no distortion at the top' result (23) is by construction because I only have two types. Furthermore, it also crucially depends on the assumption of risk neutrality because in all the periods the individuals have the same marginal utility of consumption equal to 1. In the case of general $u(c)$, the government's generalized welfare weights in period t also depend on the marginal utility of consumption in that period, and the result about zero optimal distortions for the individuals who ever had high ability in their history no longer holds.

In turn, positive distortions exist only for those who are currently and have always had low ability. What is crucially different from the *individual* optimal taxation literature, is the intratemporal, or cross-sectional, component of the optimal distortions because of interdependency between the spousal types. The second part of the proposition states that if $\rho > \bar{\rho}$, then, in period $t = 1$, the optimal distortions for low-ability individuals whose spouses also have low ability are equal to the optimal distortions for low-ability individuals with high-ability spouses. Hence I say that there is *separability* in the marginal tax rates, i.e. earnings and a marginal tax rate for an individual are determined by her/his type and are not affected by the type of her/his spouse. Next, if $\rho \in [0, \bar{\rho}]$, then (26) and (27) show that in period $t = 1$ the optimal distortions for low-ability individuals whose spouses also have low ability are greater than the optimal distortions for low-ability individuals with high-ability spouses. Hence I say that there is *negative jointness* in the marginal tax rates, i.e. the optimal distortion of an individual decreases in the earnings of her/his spouse. This tax schedule is proposed to be optimal in Kleven et al. (2009) and Gayle and Shephard (2019).

To provide the intuition behind the results described in the previous paragraph, it is instructive to refer to a variational argument. Consider an allocation that corresponds to the best possible separable tax schedule. Next, perturb the tax system towards negative jointness, so that low ability spouses in LL-couples work and produce slightly less, $dy^g_{LL} = -\varepsilon/\psi_{LL}$, and low ability spouses in HL- and LH-couples work and produce slightly more, $dy^f_{HL} = \varepsilon/\psi_{HL}$ and $dy^m_{LH} = \varepsilon/\psi_{LH}$, where $\varepsilon > 0$ is small enough. In what follows, I show that, under low enough degree of assortative mating, this perturbation does not violate the aggregate resource constraint and incentive compatibility constraints. Furthermore, given low enough level of assortative mating, higher planner's taste for redistribution towards LL-couples makes it welfare-improving.

First, note that the perturbation does not change the aggregate output:

$$dY = \psi_{LL} \underbrace{(-2\varepsilon/\psi_{LL})}_{dy^m_{LL} + dy^f_{LL}} + \psi_{LH} \underbrace{(\varepsilon/\psi_{LH})}_{dy^m_{LH}} + \psi_{HL} \underbrace{(\varepsilon/\psi_{HL})}_{dy^f_{HL}} = 0$$

Before exploring the changes in the aggregate consumption, I make sure that the incentive compatibility constraints are not violated. Following the perturbation, the change in utility of LL-couples is given by

$$dU_{LL} = dc_{LL} + \left[\phi'\left(\frac{y^m_{LL}}{\theta_L}\right) \cdot \frac{1}{\theta_L} + \phi'\left(\frac{y^f_{LL}}{\varphi_L}\right) \cdot \frac{1}{\varphi_L}\right] \frac{\varepsilon}{\psi_{LL}}$$

Define the change in consumption that makes LL-couples indifferent to perturbation:

$$dc_{LL}|_{dU_{LL}=0} \equiv \Delta^c_{LL} = -\left[\phi'\left(\frac{y^m_{LL}}{\theta_L}\right) \cdot \frac{1}{\theta_L} + \phi'\left(\frac{y^f_{LL}}{\varphi_L}\right) \cdot \frac{1}{\varphi_L}\right] \frac{\varepsilon}{\psi_{LL}} \tag{30}$$

Next, consider the incentive compatibility constraint that keeps LH-couples from pretending to be LL-couples:

$$U_{LH} \geq c_{LL} - \phi\left(\frac{y_{LL}^{m}}{\theta_L}\right) - \phi\left(\frac{y_{LL}^{f}}{\varphi_H}\right)$$

By construction, low-ability males in LH-couples work more, while low-ability males and females in LL-couples work less. Hence, the perturbation does not violate this constraint if

$$dc_{LH} - \phi'\left(\frac{y_{LH}^{m}}{\theta_L}\right) \cdot \frac{1}{\theta_L} \cdot \frac{\varepsilon}{\psi_{LH}} \geq dc_{LL} + \left[\phi'\left(\frac{y_{LL}^{m}}{\theta_L}\right) \cdot \frac{1}{\theta_L} + \phi'\left(\frac{y_{LL}^{f}}{\varphi_H}\right) \cdot \frac{1}{\varphi_H}\right] \frac{\varepsilon}{\psi_{LL}}$$

Setting $dc_{LL} = \Delta_{LL}^{c}$ where Δ_{LL}^{c} is from (30), I obtain the smallest change in consumption of LH-couples that keeps them from mimicking LL-couples:

$$\Delta_{LH}^{c} = \phi'\left(\frac{y_{LH}^{m}}{\theta_L}\right) \cdot \frac{1}{\theta_L} \cdot \frac{\varepsilon}{\psi_{LH}} - \left[\phi'\left(\frac{y_{LL}^{f}}{\varphi_L}\right) \cdot \frac{1}{\varphi_L} - \phi'\left(\frac{y_{LL}^{f}}{\varphi_H}\right) \cdot \frac{1}{\varphi_H}\right] \frac{\varepsilon}{\psi_{LL}} \tag{31}$$

Similarly, for HL-couples:

$$\Delta_{HL}^{c} = \phi'\left(\frac{y_{HL}^{f}}{\varphi_L}\right) \cdot \frac{1}{\varphi_L} \cdot \frac{\varepsilon}{\psi_{HL}} - \left[\phi'\left(\frac{y_{LL}^{m}}{\theta_L}\right) \cdot \frac{1}{\theta_L} - \phi'\left(\frac{y_{LL}^{m}}{\theta_H}\right) \cdot \frac{1}{\theta_H}\right] \frac{\varepsilon}{\psi_{LL}} \tag{32}$$

Finally, I consider the incentive compatibility constraints that keep HH-couples from pretending to be the other couples. Begin from the constraint that connects HH-couples and LH-couples:

$$U_{HH} \geq c_{LH} - \phi\left(\frac{y_{LH}^{m}}{\theta_H}\right) - \phi\left(\frac{y_{LH}^{F}}{\varphi_H}\right)$$

The perturbation does not violate this constraint if

$$dc_{HH} \geq dc_{LH} - \phi'\left(\frac{y_{LH}^{m}}{\theta_H}\right) \cdot \frac{1}{\theta_H} \cdot \frac{\varepsilon}{\psi_{LH}}$$

Setting $dc_{LH} = \Delta^c_{LH}$ where Δ^c_{LH} is from (31), I obtain the smallest change in consumption of HH-couples that keeps them from mimicking LH-couples:

$$\Delta^c_{HH,LH} = \left[\phi'\left(\frac{y^m_{LH}}{\theta_L}\right)\cdot\frac{1}{\theta_L} - \phi'\left(\frac{y^m_{LH}}{\theta_H}\right)\cdot\frac{1}{\theta_H}\right]\frac{\varepsilon}{\psi_{LH}} - \left[\phi'\left(\frac{y^f_{LL}}{\varphi_L}\right)\cdot\frac{1}{\varphi_L} - \phi'\left(\frac{y^f_{LL}}{\varphi_H}\right)\cdot\frac{1}{\varphi_H}\right]\frac{\varepsilon}{\psi_{LL}} \tag{33}$$

Similarly, obtain the smallest change in consumption of HH-couples that keeps them from pretending to be HL-couples:

$$\Delta^c_{HH,HL} = \left[\phi'\left(\frac{y^f_{HL}}{\varphi_L}\right)\cdot\frac{1}{\varphi_L} - \phi'\left(\frac{y^f_{HL}}{\varphi_H}\right)\cdot\frac{1}{\varphi_H}\right]\frac{\varepsilon}{\psi_{HL}} - \left[\phi'\left(\frac{y^m_{LL}}{\theta_L}\right)\cdot\frac{1}{\theta_L} - \phi'\left(\frac{y^m_{LL}}{\theta_H}\right)\cdot\frac{1}{\theta_H}\right]\frac{\varepsilon}{\psi_{LL}} \tag{34}$$

Furthermore, note that the perturbation relaxes the constraint that keeps HH-couples from mimicking LL-couples.

The aggregate change in consumption of LL-, LH-, and HL-couples resulting from the perturbation is given by

$$\begin{aligned}
\psi_{LL}\Delta^c_{LL} + \psi_{LH}\Delta^c_{LH} + \psi_{HL}\Delta^c_{HL} = &-\left[\phi'\left(\frac{y^m_{LL}}{\theta_L}\right)\cdot\frac{1}{\theta_L} + \phi'\left(\frac{y^f_{LL}}{\varphi_L}\right)\cdot\frac{1}{\varphi_L}\right]\varepsilon + \\
&\phi'\left(\frac{y^m_{LH}}{\theta_L}\right)\cdot\frac{\varepsilon}{\theta_L} - \left[\phi'\left(\frac{y^f_{LL}}{\varphi_L}\right)\cdot\frac{1}{\varphi_L} - \phi'\left(\frac{y^f_{LL}}{\varphi_H}\right)\cdot\frac{1}{\varphi_H}\right]\frac{\psi_{LH}\varepsilon}{\psi_{LL}} + \\
&\phi'\left(\frac{y^f_{HL}}{\varphi_L}\right)\cdot\frac{\varepsilon}{\varphi_L} - \left[\phi'\left(\frac{y^m_{LL}}{\theta_L}\right)\cdot\frac{1}{\theta_L} - \phi'\left(\frac{y^m_{LL}}{\theta_H}\right)\cdot\frac{1}{\theta_H}\right]\frac{\psi_{HL}\varepsilon}{\psi_{LL}} = \\
-\left[\underbrace{\phi'\left(\frac{y^f_{LL}}{\varphi_L}\right)\frac{1}{\varphi_L} - \phi'\left(\frac{y^f_{LL}}{\varphi_H}\right)\frac{1}{\varphi_H}}_{>0}\right]\frac{\psi_{LH}\varepsilon}{\psi_{LL}} &- \left[\underbrace{\phi'\left(\frac{y^m_{LL}}{\theta_L}\right)\frac{1}{\theta_L} - \phi'\left(\frac{y^m_{LL}}{\theta_H}\right)\frac{1}{\theta_H}}_{>0}\right]\frac{\psi_{HL}\varepsilon}{\psi_{LL}} < 0
\end{aligned} \tag{35}$$

where the second equality comes from using $y^m_{LL} = y^m_{LH}$ and $y^f_{LL} = y^f_{HL}$. When high-ability spouses pretend to be low-ability individuals, their marginal disutility of labor is lower than under true reporting, hence the terms in square brackets are strictly positive. Overall, the perturbation towards negative jointness creates a surplus from the couples with at least one low-ability spouse since the aggregate change in their consumption is negative.

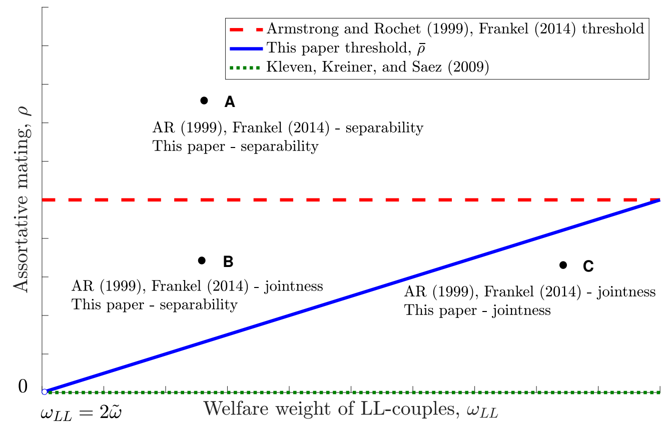

Figure 1: Thresholds for the measure of assortative mating, ρ, and the optimal tax systems.

On the other hand, the aggregate change in consumption of HH-couples is given by

$$\psi_{HH}\Delta^c_{HH,LH} = \left[\underbrace{\phi'\left(\frac{y^m_{LH}}{\theta_L}\right)\cdot\frac{1}{\theta_L} - \phi'\left(\frac{y^m_{LH}}{\theta_H}\right)\cdot\frac{1}{\theta_H}}_{>0}\right]\frac{\psi_{HH}\varepsilon}{\psi_{LH}} - \left[\underbrace{\phi'\left(\frac{y^f_{LL}}{\varphi_L}\right)\cdot\frac{1}{\varphi_L} - \phi'\left(\frac{y^f_{LL}}{\varphi_H}\right)\cdot\frac{1}{\varphi_H}}_{>0}\right]\frac{\psi_{HH}\varepsilon}{\psi_{LL}} \tag{36}$$

For the new allocation to be resource feasible, the change in the aggregate consumption cannot be greater than 0. The planner can use the surplus collected from LL-, LH-, and HL-couples, (35), to compensate the change in consumption of HH-couples, (36). However, the degree of assortative mating may limit this redistribution. In particular, on the one hand, higher assortative mating, ρ, that translates into lower fraction of mixed couples, ψ_{LH} and ψ_{HL}, reduces the surplus (35). On the other hand, it increases the change in consumption of HH-couples as follows from the first square bracket in (36). Overall, the perturbation towards negative jointness is resource feasible under low enough degree of assortative mating. Furthermore, when we turn to social welfare, Proposition 3 shows that, given low enough level of assortative mating, the planner needs to have strong enough taste for redistribution towards LL-couples to make the perturbation welfare-improving.

In Figure 1, to highlight the findings about the optimal tax schedule under different parameter values, I compare the threshold on ρ from Armstrong and Rochet (1999) and Frankel (2014) with the threshold from Proposition 3 assuming $\tilde{\omega} > \omega_{HH} = 0$. It reports several interesting features.

First, the threshold from Armstrong and Rochet (1999) and Frankel (2014), the red line, is weakly greater than the threshold from Proposition 1, the blue line. The only case, when they coincide, corresponds to $\omega_{LH} = \omega_{HL} = \omega_{HH}$. Second, as $\omega_{LL} \to 2\tilde{\omega}$, the threshold from Proposition 3 goes to zero. Third, the planner's taste for redistribution have important implication for the optimal tax schedule. To illustrate the idea, consider three countries: A, B, and C. From Figure 1, we observe that the assortative mating in country A is above both thresholds for any configuration of the welfare weights. Hence in this case my conclusion coincides with those from Armstrong and Rochet (1999) and Frankel (2014): the optimal tax system in country A should feature separability. We also come to a similar conclusion about country C where the assortative mating is below both thresholds. However, turning to country B, we can see that restricting the welfare weights to $\omega_{LH} = \omega_{HL} = \omega_{HH}$, we should conclude that the optimal tax system features negative jointness. However, with more general welfare weights we conclude that it should be separable like in country A. I also show how my results correspond to Kleven et al. (2009) who assume $\rho = 0$. In particular, they consider the government that maximizes the sum of increasing and concave transformations $\Psi(\cdot)$ of the couples' utilities with $\Psi'(\cdot)$ strictly convex. In Appendix, I show that their assumption is consistent with Assumption 1. Overall, a simple example from Figure 1 illustrates the joint importance of cross-sectional distribution of spousal types in the economy and the government's taste for redistribution.

Finally, the fourth part of Proposition 4 that states the result about the dynamics of optimal distortions is fully consistent with the individual taxation papers by Battaglini and Coate (2008), Farhi and Werning (2013), and Golosov et al. (2016) for the case of risk neutrality. The size of the optimal distortions converges to zero over time since I assume $f^{\theta} \in (0.5, 1)$ and $f^{\varphi} \in (0.5, 1)$.

For those spouses whose output is distorted, the optimal marginal taxes can be described as the sum of two terms: an intratemporal (cross-sectional) component and an intertemporal (time-series) component (Golosov et al., 2016). Since the individuals are risk neutral, they do not need insurance against the life-cycle shocks, and thus the intratemporal components are equal to zero in all periods $t > 1$. However, as I show, at period $t = 1$, the intratemporal component crucially depends on the degree of assortative mating in the economy and the government's taste for redistribution. In turn, the intertemporal component is zero at period $t = 1$, and positive in subsequent periods.

To conclude this section, I want to map the results from Proposition 4 onto the results from the new dynamic public finance literature. Assume that disutility of labor takes the following form:

$$\phi(n) = \frac{n^{1+1/\eta}}{1+1/\eta} \tag{37}$$

With this functional form, η is the Frisch elasticity of labor supply. I want to compare the optimal distortions from Proposition 4 with the results from Diamond (1998) and Saez (2001), or, in the dynamic context, Golosov et al. (2016). In particular, applying equation (17) from their paper to the risk neutral case, Golosov et al. (2016) find that, in the first period, the optimal labor distortion is given by the following ABC-formula:

$$\frac{\tau_1(\theta)}{1-\tau_1(\theta)} = \underbrace{\frac{1+1/\eta}{\theta f_1(\theta)}}_{A_1(\theta)\frac{B_1(\theta)}{1-F_1(\theta)}} \underbrace{\int_\theta^\infty (1-\alpha(x))\, f_1(x)dx}_{(1-F_1(\theta))\,C_1(\theta)} \equiv A_1(\theta)B_1(\theta)C_1(\theta)$$

In my paper, if $\rho \geq 0$, the optimal labor distortions in period $t = 1$, or the intratemporal component, are given by

$$\frac{\tau_1^m(\theta_L,\varphi)}{1-\tau_1^m(\theta_L,\varphi)} = \frac{1-\left(\frac{\theta_L}{\theta_H}\right)^{1+1/\eta}}{\psi_{LH}+\psi_{LL}} \sum_{s=L,H} \psi_{Hs}\left(1-\frac{\omega_{Hs}}{\sum_{ij}\omega_{ij}\psi_{ij}}\right) + J^m(\varphi)\cdot \mathbb{I}\{\rho\in[0,\bar{\rho}]\} \tag{38}$$

$$\frac{\tau_1^f(\theta,\varphi_L)}{1-\tau_1^f(\theta,\varphi_L)} = \frac{1-\left(\frac{\varphi_L}{\varphi_H}\right)^{1+1/\eta}}{\psi_{HL}+\psi_{LL}} \sum_{s=L,H} \psi_{sH}\left(1-\frac{\omega_{sH}}{\sum_{ij}\omega_{ij}\psi_{ij}}\right) + J^f(\theta)\cdot \mathbb{I}\{\rho\in[0,\bar{\rho}]\} \tag{39}$$

where $\mathbb{I}\{\cdot\}$ is an indicator function that takes value 1 if $\rho \in [0,\bar{\rho}]$ and 0 otherwise, and J^m and J^f are the terms that capture jointness. I explicitly emphasize that these terms depend on the spousal types.

The size of the optimal labor supply distortions is shaped by several forces. First, more elastic labor supply, i.e. higher η, translates into higher labor supply distortions. Therefore, higher elasticity of labor supply reduces the size of the optimal marginal tax rates. Second, distribution of types and relative productivity of types also affect the distortions. Without loss, consider the distortions for males in (38). A positive marginal tax on type θ_L prevents couples with θ_H from pretending to be couples with a low-ability male. If the fraction of couples with high-ability males, $\psi_{HL}+\psi_{HH}$, is high, an optimal distortion on low-ability males should provide stronger incentives for these couples to report their type truthfully. Therefore, higher fraction of couples with high-ability males tends to increase the optimal labor supply distortions. Furthermore, if the fraction of couples with low-ability males, $\psi_{LL}+\psi_{LH}$, or the relative productivity of low-ability males, θ_L/θ_H, is high, then the size of the optimal distortions should be lower. Next, the distortions are affected by the curvature of the social welfare function, captured by weights ω_{ij}. Higher planner's taste for redistribution, i.e. lower welfare weights assigned to couples with high-ability spouses, tends to increase the size of the optimal distortions. Finally, the last term captures the possibility of interdependence between the types. Overall, equations (38)-(39) is a generalization of the ABC-formula for the case with multidimensional private information.[5]

In period $t=1$, the intertemporal component is zero. However, for $t>1$, Golosov et al. (2016) show that in the risk neutral case:

$$\frac{\tau_t(\theta)}{1-\tau_t(\theta)}=\delta R\upsilon\frac{\tau_{t-1}}{1-\tau_{t-1}}$$

where υ measures the persistence of ability shocks. This is the intertemporal component of optimal labor distortions. If $\upsilon=0$, and hence the current type carries no information about the previous period type, then $\tau_t(\theta)=0$, $\forall t$. If $\upsilon\in(0,1)$, then the size of distortions converges to zero over time. Finally, if $\upsilon=1$, i.e. the types are constant, and the planner essentially solves a sequence of static problems, then the distortions are constant over time as well. Note that this is exactly what equations (28) and (29) show.

4.2 Taxation of Couples: Within-Family Redistribution

The literature emphasize that within-household inequality can account for a sizeable part of inequality between individuals (Lise and Seitz, 2011). My framework is flexible enough to consider

[5] For comparison, in a unidimensional case with two types, θ_H and θ_L, that have fractions ψ_H and ψ_L, and primitive welfare weights ω_H and ω_L, the formula is given by

$$\frac{\tau\left(\theta_L\right)}{1-\tau\left(\theta_L\right)}=\frac{1-\left(\frac{\theta_L}{\theta_H}\right)^{1+1/\eta}}{\psi_L}\psi_H\left(1-\frac{\omega_H}{\omega_H\psi_H+\omega_L\psi_L}\right)$$

the optimal taxation and allowing for within-household redistribution.

Suppose that the government wants to redistribute both between and within households. Denote by $\kappa_{ij} \in [0,1]$ the welfare weight that the government assigns to the male in ij-couple, $i,j \in \{L,H\}$. Next, denote by $\xi_{ij} \in [0,1]$ the male's consumption share.

The couple's expected utility is given by

$$V_t\left(\boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f}\right) = \mathbb{E}_t\left\{\sum_{s=t}^{T} \delta^{s-t}\left[\kappa_{ij}\left(\xi_{ij}\cdot c_s\left(\theta,\varphi\right) - \phi\left(\frac{y_s^m\left(\theta,\varphi\right)}{\theta_s}\right)\right) + \right.\right.$$
$$\left.\left.\left(1-\kappa_{ij}\right)\left(\left(1-\xi_{ij}\right)\cdot c_s\left(\theta,\varphi\right) - \phi\left(\frac{y_s^f\left(\theta,\varphi\right)}{\varphi_s}\right)\right)\right] | (\theta_t, \varphi_t)\right\} \quad (40)$$

In this setting, the Lagrange multipliers, corresponding to the incentive compatibility constraints in the government's problem, can be decomposed into two terms: one accounts for between-household redistribution (as in the previous section) and another one stands for within-household redistribution. The threshold for assortative mating is now a function of not only cross-sectional distribution of couples and between-household taste for redistribution, but also within-household taste for redistribution.

To illustrate, first, consider the case when husbands and wives split consumption equally, i.e. $\xi_{ij} = 1/2$, $i,j \in \{L,H\}$. Under separability in the marginal tax rates, the Lagrange multipliers corresponding to the incentive constraints for HH-couples that want to mimic HL- and LH-couples in $t=1$:

$$\gamma_2 = \underbrace{\frac{1}{2}\cdot\frac{\psi_{HL}}{\psi_{HL}+\psi_{LL}}\cdot\left[\frac{\omega_{LL}\psi_{LL}+\omega_{HL}\psi_{HL}}{\sum_{s,r}\omega_{sr}\psi_{sr}} - \psi_{HL} - \psi_{LL}\right]}_{\text{between-household redistribution}} +$$

$$\underbrace{\frac{\phi'\left(\frac{y_L^f}{\varphi_L}\right)/\varphi_L}{\phi'\left(\frac{y_L^f}{\varphi_L}\right)/\varphi_L - \phi'\left(\frac{y_L^f}{\varphi_H}\right)/\varphi_H}\left[\psi_{LL}\left(\frac{1}{2}-\kappa_{HL}\right)\frac{\omega_{HL}\psi_{HL}}{\sum_{s,r}\omega_{sr}\psi_{sr}} + \psi_{HL}\left(\kappa_{LL}-\frac{1}{2}\right)\frac{\omega_{LL}\psi_{LL}}{\sum_{s,r}\omega_{sr}\psi_{sr}}\right]}_{\text{within-household redistribution}}$$

$$\gamma_3 = \underbrace{\frac{1}{2}\cdot\frac{\psi_{LH}}{\psi_{LH}+\psi_{LL}}\cdot\left[\frac{\omega_{LL}\psi_{LL}+\omega_{LH}\psi_{LH}}{\sum_{s,r}\omega_{sr}\psi_{sr}} - \psi_{LH} - \psi_{LL}\right]}_{\text{between-household redistribution}} +$$

$$\underbrace{\frac{\phi'\left(\frac{y_L^m}{\theta_L}\right)/\theta_L}{\phi'\left(\frac{y_L^m}{\theta_L}\right)/\theta_L-\phi'\left(\frac{y_L^m}{\theta_H}\right)/\theta_H}\left[\psi_{LL}\left(\frac{1}{2}-\kappa_{LH}\right)\frac{\omega_{LH}\psi_{LH}}{\sum_{s,r}\omega_{sr}\psi_{sr}}+\psi_{LH}\left(\kappa_{LL}-\frac{1}{2}\right)\frac{\omega_{LL}\psi_{LL}}{\sum_{s,r}\omega_{sr}\psi_{sr}}\right]}_{\text{within-household redistribution}}$$

Note that if the government assigns equal welfare weights for each spouse, $\kappa_{ij}=1/2$, then the second term in both equations is equal to zero, and we are back to the original model with no within-household redistribution. Finish.

5 Conclusion

In this paper, I study a principal-agent problem where a monopolist repeatedly sells two non-durable goods to a buyer. A two-dimensional buyer's type, that captures his preferences over the goods, is private information and stochastically evolves over time according to a Markov process. I characterize the optimal contract in this environment. I show that it is history-dependent and has infinite memory. In each period of time, the optimal quantities depend on the full history of past buyer's reports about his type, the current report, and the cross-sectional distribution of the buyer's type. In particular, I show that there exists a threshold on covariance between the buyer's subtypes that determines whether the quantity of one good depends on the report about the marginal valuation of another good. The behavior of the optimal contract over time is shaped by persistence of the buyer's type. In addition, I apply this framework to the problem of optimal income taxation of couples, and show how the cross-sectional distribution of spousal types, government's taste for redistribution, and persistence of the spousal types jointly shape the optimal tax schedule. I obtain a generalization of the ABC-formula for the optimal labor supply distortions under multidimensional private information.

To the best of my knowledge, this is the first paper that embeds a multidimensional screening problem into dynamic context with persistent private information in an analytically tractable way. Despite its simplicity, it allows to get nontrivial theoretical results and may serve as a benchmark for more complex models of multidimensional screening in dynamic settings. The results of this paper can be applied to various settings, including the joint insurance contracts and taxation of couples.

References

Alves, C. B. M., C. E. d. Costa, and H. A. Moreira (2021): "Intrahousehold Inequality and the Joint Taxation of Household Earnings," *FGV EPGE Working Paper No. 825*.

Armstrong, M. (1996): "Multiproduct Nonlinear Pricing," *Econometrica*, 64, 51–75.

Armstrong, M. and J.-C. Rochet (1999): "Multi-Dimensional Screening: A User's Guide," *European Economic Review*, 43, 959–979.

Baron, D. P. and D. Besanko (1984): "Regulation and Information in a Continuing Relationship," *Information Economics and Policy*, 1, 267–302.

Battaglini, M. (2005): "Long-Term Contracting with Markovian Consumers," *American Economic Review*, 95, 637–658.

——— (2007): "Optimality and Renegotiation in Dynamic Contracting," *Games and Economic Behavior*, 60, 213–246.

Battaglini, M. and S. Coate (2008): "Pareto Efficient Income Taxation with Stochastic Abilities," *Journal of Public Economics*, 92, 844–868.

Battaglini, M. and R. Lamba (2019): "Optimal Dynamic Contracting: the First-Order Approach and Beyond," *Theoretical Economics*, 14, 1435–1482.

Bloedel, A. W., R. V. Krishna, and O. Leukhina (2020): "Insurance and Inequality with Persistent Private Information," *Working Paper*.

Blundell, R., P.-A. Chiappori, and C. Meghir (2005): "Collective Labor Supply with Children," *Journal of Political Economy*, 113, 1277–1306.

Carroll, G. (2017): "Robustness and Separation in Multidimensional Screening," *Econometrica*, 85, 453–488.

Cremer, H., P. Pestieau, and J.-C. Rochet (2001): "Direct Versus Indirect Taxation: The Design of the Tax Structure Revisited," *International Economic Review*, 42, 781–800.

Diamond, P. A. (1998): "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates," *American Economic Review*, 88, 83–95.

Eika, L., M. Mogstad, and B. Zafar (2019): "Educational Assortative Mating and Household Income Inequality," *Journal of Political Economy*, 127, 2795–2835.

Farhi, E. and I. Werning (2013): "Insurance and Taxation over the Life Cycle," *Review of Economic Studies*, 80, 596–635.

Fernandez, R., N. Guner, and J. Knowles (2005): "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality," *Quarterly Journal of Economics*, 120, 273–344.

Frankel, A. (2014): "Taxation of Couples under Assortative Mating," *American Economic Journal: Economic Policy*, 6, 155–177.

Fu, S. and R. V. Krishna (2019): "Dynamic Financial Contracting with Persistent Private Information," *RAND Journal of Economics*, 50, 418–452.

Gayle, G.-L. and A. Shephard (2019): "Optimal Taxation, Marriage, Home Production, and Family Labor Supply," *Econometrica*, 87, 291–326.

Golosov, M., M. Troshkin, and A. Tsyvinski (2016): "Redistribution and Social Insurance," *American Economic Review*, 106, 359–86.

Gourieroux, C. and Y. Lu (2015): "Love and Death: A Freund Model with Frailty," *Insurance: Mathematics and Economics*, 63, 191–203.

Guner, N., Y. Kulikova, and J. Llull (2018): "Marriage and Health: Selection, Protection, and Assortative Mating," *European Economic Review*, 109, 162–190.

Hendel, I. and A. Lizzeri (2003): "The Role of Commitment in Dynamic Contracts: Evidence from Life Insurance," *Quarterly Journal of Economics*, 118, 299–328.

Judd, K., D. Ma, M. A. Saunders, and C.-L. Su (2018): "Optimal Income Taxation with Multidimensional Taxpayer Types," *Mimeo*.

Kapička, M. (2013): "Efficient Allocations in Dynamic Private Information Economies with Persistent Shocks: A First-Order Approach," *Review of Economic Studies*, 80, 1027–1054.

Kleven, H. J., C. T. Kreiner, and E. Saez (2009): "The Optimal Income Taxation of Couples," *Econometrica*, 77, 537–560.

Kurnaz, M. (2021): "Optimal Taxation of Families: Mirrlees Meets Becker," *Economic Journal*.

Laffont, J.-J. and J. Tirole (1996): "Pollution Permits and Environmental Innovation," *Journal of Public Economics*, 62, 127–140.

Lehmann, E., S. Renes, K. Spiritus, and F. Zoutman (2018): "Optimal Tax and Benefit Policies with Multiple Observables," *Mimeo*.

Lise, J. and S. Seitz (2011): "Consumption Inequality and Intra-Household Allocations," *Review of Economic Studies*, 78, 328–355.

Luciano, E., J. Spreeuw, and E. Vigna (2008): "Modelling Stochastic Mortality for Dependent Lives," *Insurance: Mathematics and Economics*, 43, 234–244.

Mirrlees, J. A. (1971): "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 175–208.

Moser, C. and P. Olea de Souza e Silva (2019): "Optimal Paternalistic Savings Policies," *Opportunity and Inclusive Growth Institute Working Paper 17*.

MUSSA, M. AND S. ROSEN (1978): "Monopoly and Product Quality," *Journal of Economic Theory*, 18, 301–317.

MYERSON, R. B. (1986): "Multistage Games with Communication," *Econometrica*, 54, 323–358.

PAVAN, A., I. SEGAL, AND J. TOIKKA (2014): "Dynamic Mechanism Design: A Myersonian Approach," *Econometrica*, 82, 601–653.

ROCHET, J.-C. AND P. CHONÉ (1998): "Ironing, Sweeping, and Multidimensional Screening," *Econometrica*, 66, 783–826.

ROTHSCHILD, C. AND F. SCHEUER (2013): "Redistributive Taxation in the Roy Model," *Quarterly Journal of Economics*, 128, 623–668.

RUSTICHINI, A. AND A. WOLINSKY (1995): "Learning about Variable Demand in the Long Run," *Journal of Economic Dynamics and Control*, 19, 1283–1292.

SAEZ, E. (2001): "Using Elasticities to Derive Optimal Income Tax Rates," *Review of Economic Studies*, 68, 205–229.

SCHWARTZ, C. R. (2010): "Earnings Inequality and the Changing Association between Spouses' Earnings," *American Journal of Sociology*, 115, 1524–1557.

STANTCHEVA, S. (2020): "Dynamic Taxation," *Annual Review of Economics*, 12, 801–831.

TOWNSEND, R. M. (1982): "Optimal Multiperiod Contracts and the Gain from Enduring Relationships under Private Information," *Journal of Political Economy*, 90, 1166–1186.

WILLIAMS, N. (2011): "Persistent Private Information," *Econometrica*, 79, 1233–1275.

WU, C. AND D. KRUEGER (2021): "Consumption Insurance against Wage Risk: Family Labor Supply and Optimal Progressive Income Taxation," *American Economic Journal: Macroeconomics*, 13, 79–113.

YOUN, H. AND A. SHEMYAKIN (1999): "Statistical Aspects of Joint Life Insurance Pricing," *Proceedings of the American Statistical Association*, 34–38.

Appendix

Proof of Proposition 3

Denote $c(\theta_i, \varphi_j) \equiv c_{ij}$, $y^m(\theta_i, \varphi_j) \equiv y^m_{ij}$, $y^f(\theta_i, \varphi_j) \equiv y^f_{ij}$, and $\lambda(\theta_i, \varphi_j) \equiv \lambda_{ij}$. Since the model is static, I omit the time indices. The government solves the following problem:

$$\max_{\langle \boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f} \rangle} \lambda_{HH}\left[c_{HH} - \phi\left(\frac{y^m_{HH}}{\theta_H}\right) - \phi\left(\frac{y^f_{HH}}{\varphi_H}\right)\right] + \lambda_{HL}\left[c_{HL} - \phi\left(\frac{y^m_{HL}}{\theta_H}\right) - \phi\left(\frac{y^f_{HL}}{\varphi_L}\right)\right] +$$
$$\lambda_{LH}\left[c_{LH} - \phi\left(\frac{y^m_{LH}}{\theta_L}\right) - \phi\left(\frac{y^f_{LH}}{\varphi_H}\right)\right] + \lambda_{LL}\left[c_{LL} - \phi\left(\frac{y^m_{LL}}{\theta_L}\right) - \phi\left(\frac{y^f_{LL}}{\varphi_L}\right)\right] \tag{A.1}$$

$$\text{s.t.} \quad \psi_{HH}\left[y^m_{HH} + y^f_{HH} - c_{HH}\right] + \psi_{HL}\left[y^m_{HL} + y^f_{HL} - c_{HL}\right] +$$
$$\psi_{LH}\left[y^m_{LH} + y^f_{LH} - c_{LH}\right] + \psi_{LL}\left[y^m_{LL} + y^f_{LL} - c_{LL}\right] - G \geq 0 \tag{A.2}$$

$$c_{HH} - \phi\left(\frac{y^m_{HH}}{\theta_H}\right) - \phi\left(\frac{y^f_{HH}}{\varphi_H}\right) \geq c_{LL} - \phi\left(\frac{y^m_{LL}}{\theta_H}\right) - \phi\left(\frac{y^f_{LL}}{\varphi_H}\right) \tag{A.3}$$

$$c_{HH} - \phi\left(\frac{y^m_{HH}}{\theta_H}\right) - \phi\left(\frac{y^f_{HH}}{\varphi_H}\right) \geq c_{HL} - \phi\left(\frac{y^m_{HL}}{\theta_H}\right) - \phi\left(\frac{y^f_{HL}}{\varphi_H}\right) \tag{A.4}$$

$$c_{HH} - \phi\left(\frac{y^m_{HH}}{\theta_H}\right) - \phi\left(\frac{y^f_{HH}}{\varphi_H}\right) \geq c_{LH} - \phi\left(\frac{y^m_{LH}}{\theta_H}\right) - \phi\left(\frac{y^f_{LH}}{\varphi_H}\right) \tag{A.5}$$

$$c_{HL} - \phi\left(\frac{y^m_{HL}}{\theta_H}\right) - \phi\left(\frac{y^f_{HL}}{\varphi_L}\right) \geq c_{LL} - \phi\left(\frac{y^m_{LL}}{\theta_H}\right) - \phi\left(\frac{y^f_{LL}}{\varphi_L}\right) \tag{A.6}$$

$$c_{LH} - \phi\left(\frac{y^m_{LH}}{\theta_L}\right) - \phi\left(\frac{y^f_{LH}}{\varphi_H}\right) \geq c_{LL} - \phi\left(\frac{y^m_{LL}}{\theta_L}\right) - \phi\left(\frac{y^f_{LL}}{\varphi_H}\right) \tag{A.7}$$

where (A.2) is the aggregate resource constraint, (A.3)-(A.7) is the set of the incentive compatibility constraints.

First, notice that (A.6) and (A.7) hold with equalities. In what follows, I prove that (A.6) holds with equality, and the proof for (A.7) follows similar arguments. Consider a contract $\langle \boldsymbol{c}, \boldsymbol{y^m}, \boldsymbol{y^f} \rangle$ that solves the government's problem. Suppose that (A.6) holds with strict inequality. Consider an alternative contract $\langle \tilde{\boldsymbol{c}}, \tilde{\boldsymbol{y}}^{\boldsymbol{m}}, \tilde{\boldsymbol{y}}^{\boldsymbol{f}} \rangle$ such that $\tilde{\boldsymbol{y}}^{\boldsymbol{m}} = \boldsymbol{y^m}$, $\tilde{\boldsymbol{y}}^{\boldsymbol{f}} = \boldsymbol{y^f}$, and

$$(\tilde{c}_{HH}, \tilde{c}_{HL}, \tilde{c}_{LH}, \tilde{c}_{LL}) = (c_{HH} + \varepsilon, c_{HL} - \delta, c_{LH} + \varepsilon, c_{LL} + \varepsilon)$$

with $\varepsilon > 0$ and $\delta > 0$ small enough such that (A.6) is still satisfied. Choose $\delta = (1 - \psi_{HL})\,\varepsilon / \psi_{HL}$, so that the aggregate resource constraint is also satisfied. The change in welfare is given by

$$\Delta W = (\lambda_{HH} + \lambda_{LH} + \lambda_{LL})\,\varepsilon - \lambda_{HL}\frac{(1-\psi_{HL})\,\varepsilon}{\psi_{HL}} = (\lambda_{HH} + \lambda_{HL} + \lambda_{LH} + \lambda_{LL})\,\varepsilon - \frac{\lambda_{HL}}{\psi_{HL}}\varepsilon = \left[1 - \frac{\lambda_{HL}}{\psi_{HL}}\right]\varepsilon = \left[1 - \frac{\tilde{\omega}}{\sum_{i,j}\omega_{ij}\psi_{ij}}\right]\varepsilon > 0$$

where I use normalization $\sum_{i,j}\lambda_{ij} = 1$ in the third equality, and definition of λ_{ij} from (16) in the fourth equality. By Assumption 1, $\tilde{\omega} < \sum_{i,j}\omega_{ij}\psi_{ij} \equiv \mathbb{E}(\omega)$, a new contract delivers strictly greater welfare. This is a contradiction to the fact that the original contract is a solution to the problem. Hence the incentive compatibility constraint (A.6) holds with equality. Q.E.D.

Next, it is convenient to change the variables. Denote $U_{ij} \equiv c_{ij} - \phi\left(\frac{y^m_{ij}}{\theta_i}\right) - \phi\left(\frac{y^f_{ij}}{\varphi_j}\right)$, and rewrite the government's problem as

$$\max_{\langle \boldsymbol{U}, \boldsymbol{y^m}, \boldsymbol{y^f}\rangle} \lambda_{HH}U_{HH} + \lambda_{HL}\left[U_{LL} + \phi\left(\frac{y^m_{LL}}{\theta_L}\right) - \phi\left(\frac{y^m_{LL}}{\theta_H}\right)\right] + \lambda_{LH}\left[U_{LL} + \phi\left(\frac{y^f_{LL}}{\varphi_L}\right) - \phi\left(\frac{y^f_{LL}}{\varphi_H}\right)\right] + \lambda_{LL}U_{LL}$$

$$\text{s.t.}\quad \psi_{HH}\left[y^m_{HH} + y^f_{HH} - \phi\left(\frac{y^m_{HH}}{\theta_H}\right) - \phi\left(\frac{y^f_{HH}}{\varphi_H}\right) - U_{HH}\right] + \psi_{HL}\left[y^m_{HL} + y^f_{HL} - \phi\left(\frac{y^m_{HL}}{\theta_H}\right) - \phi\left(\frac{y^f_{HL}}{\varphi_L}\right) - U_{LL} - \phi\left(\frac{y^m_{LL}}{\theta_L}\right) + \phi\left(\frac{y^m_{LL}}{\theta_H}\right)\right] + \psi_{LH}\left[y^m_{LH} + y^f_{LH} - \phi\left(\frac{y^m_{LH}}{\theta_L}\right) - \phi\left(\frac{y^f_{LH}}{\varphi_H}\right) - U_{LL} - \phi\left(\frac{y^f_{LL}}{\varphi_L}\right) + \phi\left(\frac{y^f_{LL}}{\varphi_H}\right)\right] + \psi_{LL}\left[y^m_{LL} + y^f_{LL} - \phi\left(\frac{y^m_{LL}}{\theta_L}\right) - \phi\left(\frac{y^f_{LL}}{\varphi_L}\right) - U_{LL}\right] - G \geq 0$$

$$U_{HH} \geq U_{LL} + \phi\left(\frac{y^m_{LL}}{\theta_L}\right) - \phi\left(\frac{y^m_{LL}}{\theta_H}\right) + \phi\left(\frac{y^f_{LL}}{\varphi_L}\right) - \phi\left(\frac{y^f_{LL}}{\varphi_H}\right) \tag{A.8}$$

$$U_{HH} \geq U_{LL} + \phi\left(\frac{y^m_{LL}}{\theta_L}\right) - \phi\left(\frac{y^m_{LL}}{\theta_H}\right) + \phi\left(\frac{y^f_{HL}}{\varphi_L}\right) - \phi\left(\frac{y^f_{HL}}{\varphi_H}\right) \tag{A.9}$$

$$U_{HH} \geq U_{LL} + \phi\left(\frac{y^m_{LH}}{\theta_L}\right) - \phi\left(\frac{y^m_{LH}}{\theta_H}\right) + \phi\left(\frac{y^f_{LL}}{\varphi_L}\right) - \phi\left(\frac{y^f_{LL}}{\varphi_H}\right) \tag{A.10}$$

where I use $U_{HL} = U_{LL} + \phi\left(\frac{y^m_{LL}}{\theta_L}\right) - \phi\left(\frac{y^m_{LL}}{\theta_H}\right)$ and $U_{LH} = U_{LL} + \phi\left(\frac{y^f_{LL}}{\varphi_L}\right) - \phi\left(\frac{y^f_{LL}}{\varphi_H}\right)$ that follow from (A.6) and (A.7) holding with equalities.

Denote by ζ the Lagrange multiplier corresponding to the aggregate resource constraint. Denote by γ_1, γ_2, and γ_3 the Lagrange multipliers corresponding to the incentive compatibility constraints (A.8), (A.9), and (A.10) correspondingly. I obtain the following first-order conditions:

$$[U_{HH}] \qquad \lambda_{HH} - \psi_{HH}\zeta + \gamma_1 + \gamma_2 + \gamma_3 = 0$$

$$[U_{LL}] \qquad \lambda_{HL} + \lambda_{LH} + \lambda_{LL} - (\psi_{HL} + \psi_{LH} + \psi_{LL})\,\zeta - \gamma_1 - \gamma_2 - \gamma_3 = 0$$

$$[y^m_{HH}] \qquad \psi_{HH}\zeta\left[1 - \tfrac{1}{\theta_H}\phi'\left(\tfrac{y^m_{HH}}{\theta_H}\right)\right] = 0$$

$$[y^f_{HH}] \qquad \psi_{HH}\zeta\left[1 - \tfrac{1}{\varphi_H}\phi'\left(\tfrac{y^f_{HH}}{\varphi_H}\right)\right] = 0$$

$$[y^m_{HL}] \qquad \psi_{HL}\zeta\left[1 - \tfrac{1}{\theta_H}\phi'\left(\tfrac{y^m_{HL}}{\theta_H}\right)\right] = 0$$

$$[y^f_{HL}] \qquad \psi_{HL}\zeta\left[1 - \tfrac{1}{\varphi_L}\phi'\left(\tfrac{y^f_{HL}}{\varphi_L}\right)\right] - \gamma_2\left[\tfrac{1}{\varphi_L}\phi'\left(\tfrac{y^f_{HL}}{\varphi_L}\right) - \tfrac{1}{\varphi_H}\phi'\left(\tfrac{y^f_{HL}}{\varphi_H}\right)\right] = 0$$

$$[y^m_{LH}] \qquad \psi_{LH}\zeta\left[1 - \tfrac{1}{\theta_L}\phi'\left(\tfrac{y^m_{LH}}{\theta_L}\right)\right] - \gamma_3\left[\tfrac{1}{\theta_L}\phi'\left(\tfrac{y^m_{LH}}{\theta_L}\right) - \tfrac{1}{\theta_H}\phi'\left(\tfrac{y^m_{LH}}{\theta_H}\right)\right] = 0$$

$$[y^f_{LH}] \qquad \psi_{LH}\zeta\left[1 - \tfrac{1}{\varphi_H}\phi'\left(\tfrac{y^f_{LH}}{\varphi_H}\right)\right] = 0$$

$$[y^m_{LL}] \qquad \psi_{LL}\zeta\left[1 - \tfrac{1}{\theta_L}\phi'\left(\tfrac{y^m_{LL}}{\theta_L}\right)\right] + (\lambda_{HL} - \zeta\psi_{HL} - \gamma_1 - \gamma_2)\left[\tfrac{1}{\theta_L}\phi'\left(\tfrac{y^m_{LL}}{\theta_L}\right) - \tfrac{1}{\theta_H}\phi'\left(\tfrac{y^m_{LL}}{\theta_H}\right)\right] = 0$$

$$[y^f_{LL}] \qquad \psi_{LL}\zeta\left[1 - \tfrac{1}{\varphi_L}\phi'\left(\tfrac{y^f_{LL}}{\varphi_L}\right)\right] + (\lambda_{LH} - \zeta\psi_{LH} - \gamma_1 - \gamma_3)\left[\tfrac{1}{\varphi_L}\phi'\left(\tfrac{y^f_{LL}}{\varphi_L}\right) - \tfrac{1}{\varphi_H}\phi'\left(\tfrac{y^f_{LL}}{\varphi_H}\right)\right] = 0$$

First, from the first-order conditions for U_{HH} and U_{LL}, we obtain $\zeta = 1$. Next, from the first-order conditions for y^m_{HH}, y^f_{HH}, y^m_{HL}, and y^f_{LH}, we obtain:

$$\phi'\left(\frac{y^m_{HH}}{\theta_H}\right) = \phi'\left(\frac{y^m_{HL}}{\theta_H}\right) = \theta_H \tag{A.11}$$

$$\phi'\left(\frac{y^f_{HH}}{\varphi_H}\right) = \phi'\left(\frac{y^f_{LH}}{\varphi_H}\right) = \varphi_H \tag{A.12}$$

or, alternatively,

$$y_{HH}^{m} = y_{HL}^{m} = \theta_H \left(\phi'\right)^{-1} \left(\theta_H\right) \equiv y_H^m \tag{A.13}$$

$$y_{HH}^{f} = y_{LH}^{f} = \varphi_H \left(\phi'\right)^{-1} \left(\varphi_H\right) \equiv y_H^f \tag{A.14}$$

Next, following the procedure from Armstrong and Rochet (1999), consider two cases. First, $\gamma_1 > 0$, $\gamma_2 > 0$, and $\gamma_3 > 0$. Second, $\gamma_1 = 0$, $\gamma_2 > 0$, and $\gamma_3 > 0$.

Case $\gamma_1 > 0$*,* $\gamma_2 > 0$*, and* $\gamma_3 > 0$

We have incentive compatibility constraints (A.8)-(A.10) holding with equality. From these equalities, we obtain that $y_{LH}^m = y_{LL}^m \equiv \tilde{y}_L^m$ and $y_{HL}^f = y_{LL}^f \equiv \tilde{y}_L^f$. Using this result together with $\zeta = 1$, from the first-order conditions for y_{HL}^f and y_{LL}^f we obtain

$$\frac{\psi_{HL}}{\psi_{LL}} = \frac{\gamma_2}{\gamma_1 + \gamma_3 - \lambda_{LH} - \psi_{LH}}$$

Inserting the first-order condition for U_{LL}, we get

$$\frac{\psi_{HL}}{\psi_{LL}} = \frac{\gamma_2}{\lambda_{HL} + \lambda_{LL} - \psi_{HL} - \psi_{LL} - \gamma_2}$$

Finally, we solve for γ_2 verify that $\gamma_2 > 0$:

$$\gamma_2 = \frac{\psi_{HL}}{\psi_{HL} + \psi_{LL}} \left(\lambda_{HL} - \psi_{HL} + \lambda_{LL} - \psi_{LL}\right) =$$
$$\frac{\psi_{HL}}{\psi_{HL} + \psi_{LL}} \left[\left(\omega_{LL} - \tilde{\omega}\right) \psi_{LH}\psi_{LL} + \left(\omega_{LL} - \omega_{HH}\right) \psi_{HH}\psi_{LL} + \left(\tilde{\omega} - \omega_{HH}\right) \psi_{HH}\psi_{HL}\right] > 0 \tag{A.15}$$

Following the similar steps, we obtain

$$\gamma_3 = \frac{\psi_{LH}}{\psi_{LH} + \psi_{LL}} \left(\lambda_{LH} - \psi_{LH} + \lambda_{LL} - \psi_{LL}\right) =$$
$$\frac{\psi_{LH}}{\psi_{LH} + \psi_{LL}} \left[\left(\omega_{LL} - \tilde{\omega}\right) \psi_{HL}\psi_{LL} + \left(\omega_{LL} - \omega_{HH}\right) \psi_{HH}\psi_{LL} + \left(\tilde{\omega} - \omega_{HH}\right) \psi_{HH}\psi_{LH}\right] > 0 \tag{A.16}$$

Inserting (A.15) and (A.16) into the first-order condition for U_{HH}, we get

$$\gamma_1 = \psi_{HH} - \lambda_{HH} - \tfrac{\psi_{HL}}{\psi_{HL}+\psi_{LL}} \left(\lambda_{HL} - \psi_{HL} + \lambda_{LL} - \psi_{LL}\right) - \tfrac{\psi_{LH}}{\psi_{LH}+\psi_{LL}} \left(\lambda_{LH} - \psi_{LH} + \lambda_{LL} - \psi_{LL}\right)$$

After doing some algebra and using the definition of ρ from (2), we obtain

$$\gamma_1 = \frac{\pi_{LL}}{(\pi_{HL} + \pi_{LL})(\pi_{LH} + \pi_{LL})} \cdot \\ \{[(\omega_{LL} - \omega_{HH})\psi_{LL} + (\tilde{\omega} - \omega_{HH})(\psi_{HL} + \psi_{LH})]\rho - (\omega_{LL} + \omega_{HH} - 2\tilde{\omega})\psi_{HL}\psi_{LH}\} \quad \text{(A.17)}$$

It follows from (A.17) that $\gamma_1 > 0$ if

$$\rho > \frac{(\omega_{LL} + \omega_{HH} - 2\tilde{\omega})\psi_{HL}\psi_{LH}}{(\omega_{LL} - \omega_{HH})\psi_{LL} + (\tilde{\omega} - \omega_{HH})(\psi_{HL} + \psi_{LH})} \equiv \bar{\rho} > 0 \quad \text{(A.18)}$$

where the last inequality follows from Assumption 1.

Summing up, the incentive compatibility constraints (A.8)-(A.10) hold with equality if $\rho > \bar{\rho}$ where $\bar{\rho}$ is defined in (A.18).

Case $\gamma_1 = 0$, $\gamma_2 > 0$, *and* $\gamma_3 > 0$

We have incentive compatibility constraints (A.9) and (A.10) holding with equality. Incentive compatibility constraint (A.8) holds with strict inequality. Finish

Assumption 1 and Government Objective in Kleven et al. (2009)

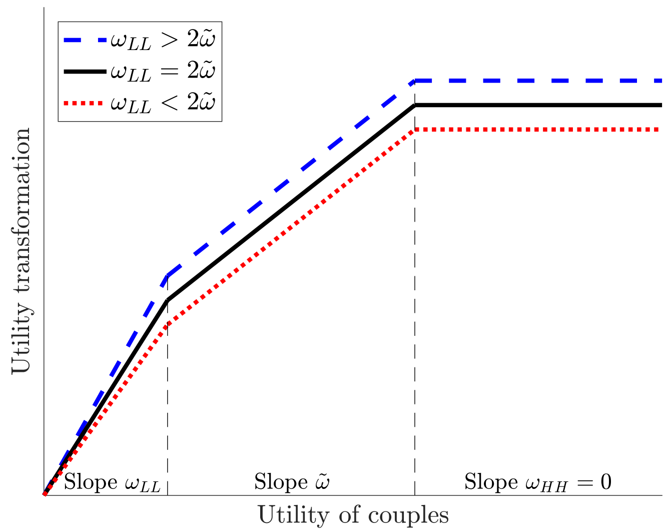

Figure A.1: Transformation of couples' utilities.

Kleven et al. (2009) assume that the government maximizes the sum of increasing and concave transformations $\Psi(\cdot)$ of the couples' utilities with $\Psi'(\cdot)$ strictly convex (see page 542 and Assumption 2 in their paper). In what follows, I show that Assumption 1 from my paper is consistent with their assumptions on the government objective.

Figure A.1 illustrates transformations of the couples' utilities under $\omega_{LH} = \omega_{HL} \equiv \tilde{\omega} \geq 0$, $\omega_{LL} > \tilde{\omega}$, and, without loss of generality, $\omega_{HH} = 0$. They are the analogues of $\Psi(\cdot)$ from Kleven et al. (2009). First, we immediately observe that parts (i) and (ii) of Assumption 1 are satisfied. Furthermore, the transformations are increasing and concave as in Kleven et al. (2009).

Next, the change in the slope between the first two regions is given by $\Delta_1 = \omega_{LL} - \tilde{\omega}$, and the change in the slope between the second and the third regions is given by $\Delta_2 = \tilde{\omega} - \omega_{HH} = \tilde{\omega}$. The difference between Δ_1 and Δ_2 is an analogue of the second derivative of $\Psi'(\cdot)$ from Kleven et al. (2009). In particular, $\Delta_1 - \Delta_2 = \omega_{LL} - 2\tilde{\omega}$. The sign of this difference depends on the relative weight that the planner puts on LL-couples relative to the mixed couples. Figure A.1 reports three possible cases. Part (iii) of Assumption 1 states that $\omega_{LL} > 2\tilde{\omega}$ (blue dashed line), and this consistent with the assumption from Kleven et al. (2009) about strict convexity of $\Psi'(\cdot)$.