Affinity Propagation

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General Overview

Affinity Propagation (AP) was proposed by Frey and Dueck (2007).

General idea:

1. AP takes real-valued dissimilarity measures between pairs of data points as input.
2. Real-valued messages are exchanged between data points until a high-quality set of *exemplars* and corresponding clusters gradually emerges.

AP simultaneously considers all data points as potential exemplars & no need to specify the number of clusters beforehand.

- K-means/K-medoids is quite sensitive to the initial selection of exemplars.
- K-means/K-medoids requires to specify K.

AP can take unusual measures of dissimilarity as input.

- K-means requires distances as input.

AP selects clusters with much lower error and much faster than K-medoids.

- Vlasblom and Wodak (2009): Markov clustering works better than AP on protein interaction graph partitioning.

Algorithm Components

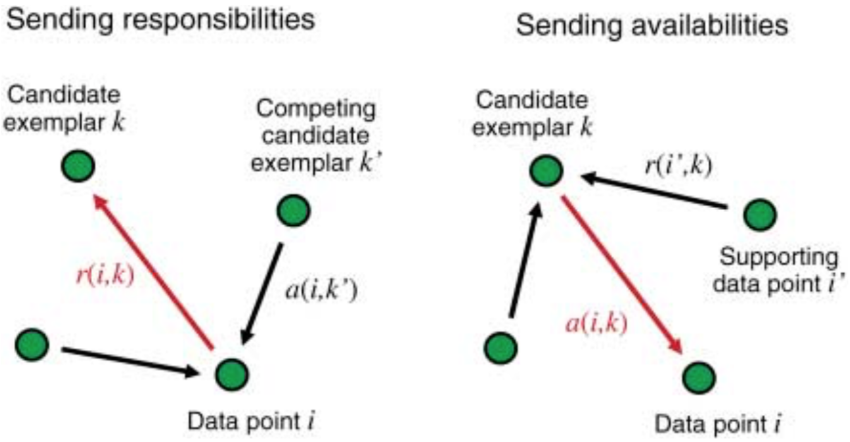
Input: Similarity $s(i,k)$.

- $s(i,j) > s(i,k)$ means *"x_i is more similar to x_j than to x_k"*.
- Euclidean distance: $s(i,k) = -||x_i - x_k||^2$. *Criterion can be more general!*
- "Preferences" $s(k,k)$: Larger values $\rightarrow$ more likely to be chosen as exemplars.
- How to choose a common value for $s(k,k)$ for all k? Median, minimum over $s(i,k)$...

Message Passing

- "Responsibility" $r(i,k)$: How well-suited point k is to serve as the exemplar for point i, taking into account other potential exemplars for point i.
- "Availability"" $a(i,k)$: How appropriate it would be for point i to choose point k as its exemplar, taking into account other points' preference for point k as an exemplar.
- View $r(i,k)$ and $a(i,k)$ as log-probability ratios.
- Combine $r(i,k)$ and $a(i,k)$ to monitor the exemplar decisions (*algorithm termination*).
- To avoid oscillation, add noise to the similarities or use damping factor.

Message Passing



Source: Frey and Dueck (2007).

Algorithm

1. To initialize, set all availabilities to zero, $a(i,k) = 0$.
2. Iterate until either the cluster boundaries remain unchanged over a number of iterations, or some # of iterations is reached:
   2.1 Update responsibility: $\underbrace{r(i,k)}_{\text{“responsibility”}} \leftarrow s(i,k) - \max_{k' \neq k}\{\underbrace{a(i,k')}_{\text{“availability”}} + s(i,k')\}$.
   2.2 Update availability:

$$a(i,k) \leftarrow \min\left\{0, r(k,k) + \sum_{i' \notin \{i,k\}} \max\{0, r(i',k)\}\right\}, \quad \text{for } i \neq k$$

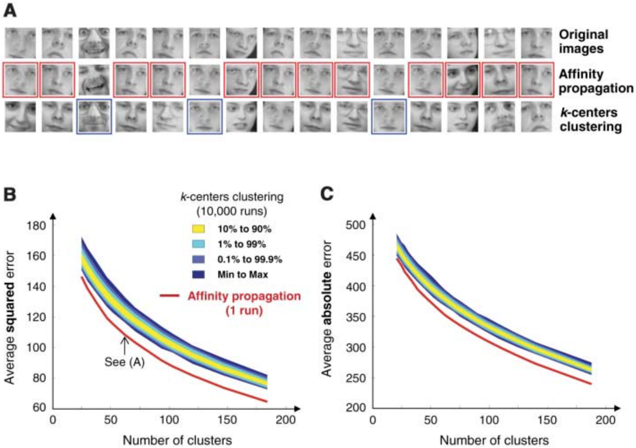
$$a(k,k) \leftarrow \sum_{i' \neq k} \max\{0, r(i',k)\}$$

Positive $r(i,k)$ means that k is a good exemplar to explain i.

When point i is effectively assigned to exemplar k', then $a(i,k)$ is negative.

For point i, the value of k that maximizes $a(i,k) + r(i,k)$ either identifies point i as an exemplar if $k = i$, or identifies the data point that is the exemplar for point i.

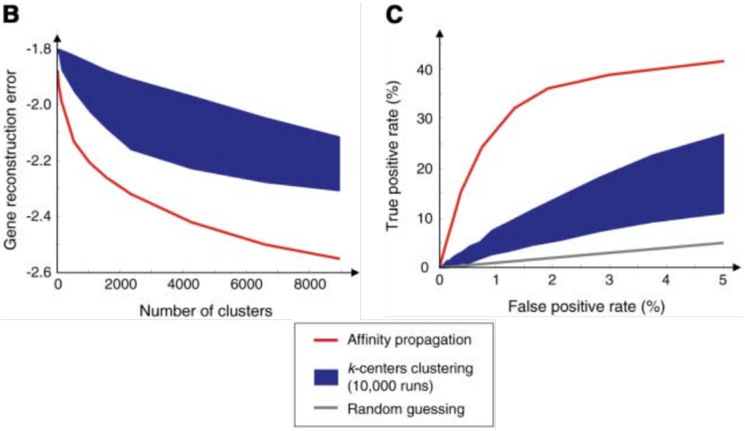
Example: Face Recognition



Source: Frey and Dueck (2007).

Main lesson: Affinity propagation works better and faster than K-centers clustering.

Example: Genes Detection



Source: Frey and Dueck (2007).

Again, affinity propagation works better and faster *(6 minutes vs. 208 hours)*.