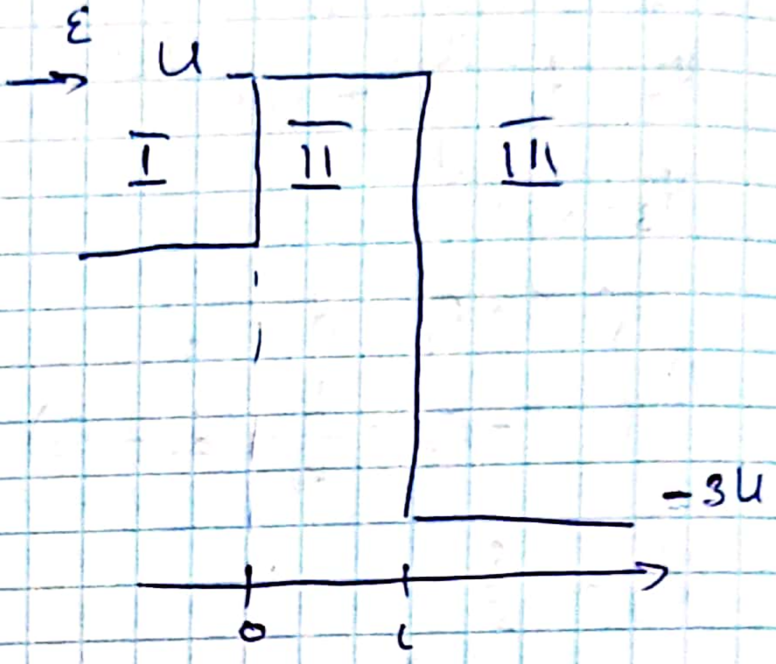
№ 3

$\varepsilon = 1\,эВ = U$

$l = 7{,}8\ \mathring{A}$

$D - ?$




$$k_1 = \frac{\sqrt{2m\varepsilon}}{\hbar} = k$$

$$k_2 = \frac{\sqrt{2m(\varepsilon - U)}}{\hbar} = 0$$

$$k_3 = \frac{\sqrt{2m(\varepsilon-(-3U))}}{\hbar} = 2k_1$$

Для (2): $-\frac{\hbar}{2m}\psi'' + \underbrace{(U-\varepsilon)}_{0}\psi = 0 \rightarrow \psi'' = 0$

$\psi' = const \rightarrow \psi = ax + b$

$$(1-2)\cdot \begin{cases} 1 + r_1 = b \\ ik(1 - r_1) = a \end{cases}$$

$$(2-3) \begin{cases} a\cdot l + b = de^{i\cdot 2kl} \\ a = i\cdot 2kde^{i2kl} \end{cases}$$

$$\frac{al+b}{a} = \frac{1}{2ik}$$

$$ikl(1-r) + 1 + r = \frac{1-r}{2}$$

$$\frac{3}{2}r - iklr = -ikl - \frac{1}{2}$$

Сканировано с CamScanner

$$r = \frac{-ikl - \frac{1}{2}}{\frac{3}{2} - ikl} \Rightarrow a = ik\left(1 + \frac{ikl + \frac{1}{2}}{\frac{3}{2} - ikl}\right) =$$

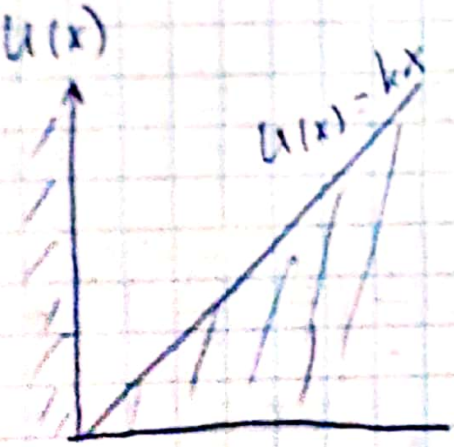
$$= ik \frac{2}{\frac{3}{2} - ikl}$$

$$d = \frac{1}{2ik e^{2ikl}} \cdot \frac{2ik}{\frac{3}{2} - ikl} = \frac{1}{\left(\frac{3}{2} - ikl\right) e^{ikl}}$$

$$D = \frac{2k}{k}|d|^2 = \frac{2}{\left(\frac{3}{2}\right)^2 + (kl)^2} = \frac{2}{\frac{9}{4} + \frac{2mEl^2}{\hbar^2}} = \frac{2}{\frac{9}{4} + \frac{64}{4}} =$$

$$= \boxed{\frac{8}{73}}$$

N 3.5




$\Psi = x e^{-ax}$

$E_{..} - ?$

$$\langle E \rangle = \frac{\int_0^\infty \Psi^* \hat{H} \Psi dx}{\int_0^\infty \Psi^* \Psi dx}$$

$$\int_0^\infty \Psi^* \Psi dx = \int_0^\infty x^2 e^{-2ax} dx = \frac{1}{4a^3}$$

$$\hat{H}\Psi = -\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + kx\Psi = \frac{1}{8}$$

$$= -\frac{\hbar^2}{2m}\left(2(-a)e^{-ax} + xa^2e^{-ax}\right) + kx^2e^{-ax}$$

$$\int_0^\infty \psi^* \hat{H} \psi \, dx = \int_0^\infty x e^{-2ax}\left(\frac{\hbar^2 a}{m} - \frac{\hbar^2 x a^2}{2m} + kx^2\right) dx =$$

$$= \frac{\hbar^2 a}{m} \cdot \frac{1}{4a^2} - \frac{\hbar^2 a^2}{2m} \cdot \frac{1}{4a^3} + k \cdot \frac{3}{8a^4}$$

$$\langle E \rangle = \frac{\hbar^2 a^2}{m} - \frac{\hbar^2 a^2}{2m} + \frac{3k}{2a} = \frac{\hbar^2 a^2}{2m} + \frac{3k}{2a}$$

$$\langle E \rangle_a' = \frac{2\hbar^2 a}{2m} - \frac{3k}{2a^2} = 0$$

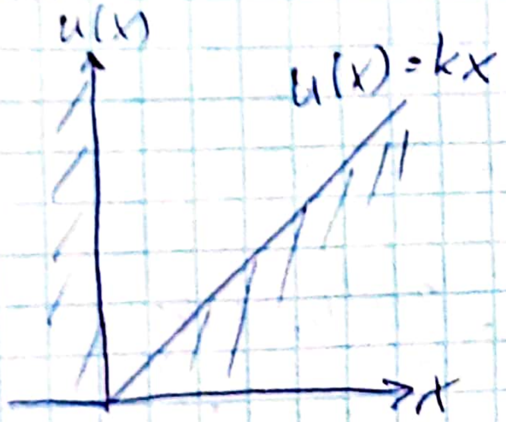
$$\frac{\hbar^2}{m} a^3 = \frac{3k}{2} \rightarrow a^3 = \frac{3km}{2\hbar^2}$$

$$\langle E \rangle_{min} = \frac{\hbar^2}{2m}\left(\frac{3km}{2\hbar}\right)^{\frac{2}{3}} + \frac{3k}{2} \cdot \left(\frac{2\hbar^2}{3km}\right)^{1/3} =$$

$$= \frac{1}{2}\left(\frac{9}{4} \cdot \frac{k^2\hbar^2}{m}\right)^{1/3} + \left(\frac{9}{4} \cdot \frac{k^2\hbar^2}{m}\right)^{1/3} =$$

$$= \boxed{\frac{3}{2}\left(\frac{9}{4} \cdot \frac{k^2\hbar^2}{m}\right)^{1/3}}$$

№ 3.6




$E_n$ - ?

$$\oint \bar{p}\,dl = nh$$

$$2px = nh$$

$$p = \frac{nh}{2x} = m v_n$$

$$x = n\frac{\lambda}{2} = \frac{n}{2}\frac{h}{p}$$

$$T_n = \frac{p_n^2}{2m} = \frac{n^2h^2}{8mx^2} \qquad E_n = \frac{n^2h^2}{8mx^2} + kx$$

$$\frac{dE_n}{dx} = 0 \rightarrow k - \frac{n^2h^2}{4mx^3} = 0 \rightarrow x^3 = \frac{n^2h^2}{4mk}$$

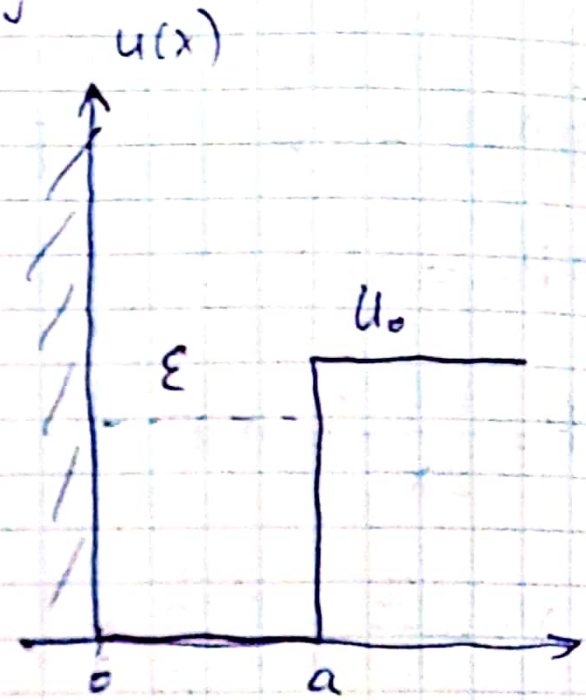
$$\boxed{E_n = \frac{3}{2}\left(\frac{h^2k^2}{4m}\right)^{1/3} n^{2/3}}$$

№ 3.49

$\mathcal{E} = \frac{3}{4} U_0$

$\psi'' + \overset{k}{\frac{2m\mathcal{E}}{\hbar^2}}\,\psi = 0$

$\psi'' - \underbrace{\frac{2m}{\hbar^2}(U_0 - \mathcal{E})}_{\varkappa}\,\psi = 0$




$A \sin ka = B e^{-\varkappa a}$

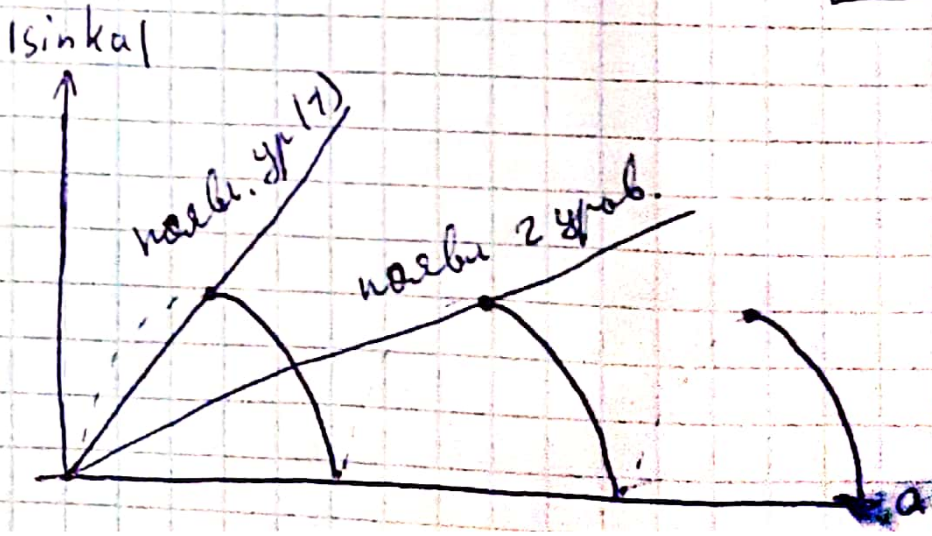
$A k \cos ka = -B \varkappa e^{-\varkappa a}$

$k \,\mathrm{ctg}\, ka = -\varkappa$

$\mathrm{ctg}^2 ka = \frac{\varkappa^2}{k^2} = \frac{U_0 - \mathcal{E}}{\mathcal{E}} = \frac{U_0}{\mathcal{E}} - 1$

~~tek~~ $1 + \mathrm{ctg}^2 ka = \frac{U_0}{\mathcal{E}}$

$|\sin ka| = \sqrt{\frac{\mathcal{E}}{U_0}} = \sqrt{\frac{\hbar^2 k^2 a^2}{2m U_0 a^2}} = \underbrace{\sqrt{\frac{\hbar^2}{2m U_0 a^2}}}_{\gamma} \cdot ka = \gamma ka$



$1^{й}$ уровень при $ka = \frac{\pi}{2}$, т.е. $U_0 = \mathcal{E}$

$$k^2a^2 = \frac{\pi^2}{4} = \frac{2mU_0}{\hbar^2}a^2 \rightarrow \boxed{U_0 a^2 > \frac{\pi^2\hbar^2}{8m}}$$

$$ctg^2 ka = \frac{1}{3}$$

$$ka = \frac{2\pi}{3} \qquad k^2a^2 = \frac{4\pi^2}{9} = \frac{3mU_0}{2\hbar^2}$$

$$U_0 a^2 = \frac{8\pi^2\hbar^2}{27m}$$

После сжатия: $U_0 a_1^2 = \frac{\pi^2\hbar^2}{8m}$

$$\boxed{\frac{a}{a_1} = \frac{8}{3\sqrt{2}} \approx 1{,}54}$$

N 0-5-1

$\Psi$ на границах ямы должна $= 0$

$\Psi(x) = \sin(kx)$ :

$kl = \pi n$ $\qquad n = 1$ - основное сост.

$$k = \sqrt{\frac{2mE}{\hbar^2}} \rightarrow E = \frac{k^2\hbar^2}{2m} = \left(\frac{\pi}{l}\right)^2\frac{\hbar^2}{2m}$$

$$l_1 = l/2 \rightarrow E_1 = \left(\frac{\pi}{l_1}\right)^2\frac{\hbar^2}{2m} = 4E$$

$$\Delta E = A = E_1 - E = 3E = \boxed{3\left(\frac{\pi}{l}\right)^2 \frac{\hbar^2}{2m}}$$

№ 0-5-2

В ямке: $\Psi = \sin kx$

$k = \frac{\pi n}{l}$ $\qquad l = 3\,\text{Å}$ $\qquad \Delta E = E_3 - E_1 = 5$ эВ

$m - ?$

$$k = \sqrt{\frac{2mE}{\hbar^2}} \rightarrow k^2 = \frac{2mE}{\hbar^2}$$

$$\frac{2m}{\hbar^2}(E_3 - E_1) = k_3^2 - k_1^2 = (9-1)\left(\frac{\pi}{l}\right)^2$$

$$\boxed{m = 4\left(\frac{\pi}{l}\right)^2 \frac{\hbar^2}{\Delta E} \approx 6{,}1 \cdot 10^{-27}\ г}$$

№3.14

Канал: $L \times l \times d$

$L \gg l \qquad l \gg d$

$d = 10^{-3}$ см

Рассматриваем канал как резонатор:

Первый энерг. уровень при $d \ll l$ и $d \ll L$:

$$E_1 = \frac{\pi^2 \hbar^2}{2md^2} \qquad E_1 \leq K = \frac{mv^2}{2}$$

$$\boxed{v \geq \frac{\pi \hbar}{md} = \frac{h}{2md} \approx 2 \text{ см/с}}$$

Канал: $L \times d \times d \qquad L \gg d$

Аналогично

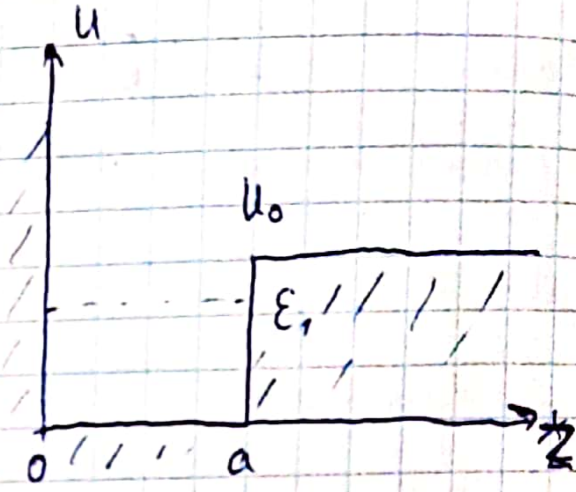
$$E = \frac{k^2 \hbar^2}{2m} = \frac{\hbar^2}{2m}\left(k_1^2 + k_2^2\right) = \frac{\hbar^2}{2m}\left(\left(\frac{\pi}{d}\right)^2 + \left(\frac{\pi}{d}\right)^2\right) =$$

$$= \frac{\hbar^2 \pi^2}{md^2}$$

$$\cancel{v \geq \frac{\sqrt{2}\pi\hbar}{md}} \qquad \boxed{v \geq \frac{\sqrt{2}\,\pi \hbar}{md} \approx 2{,}8 \text{ см/с}}$$

Сканировано с CamScanner

№ 3.21




$U_0 = \mathcal{E}_1 = 1\,K$

$a = 6\,\mathring{A}$

$0 < x < a$:

$$\psi'' + \underbrace{\frac{2m\mathcal{E}_1}{\hbar^2}}_{k^2}\psi = 0$$

$a < x$; $\psi'' - \underbrace{\frac{2m}{\hbar^2}(U_0 - \mathcal{E}_1)}_{\varkappa^2}$

$$\begin{cases} A\sin ka = Be^{-\varkappa a} \\ Ak\sin ka = -B\varkappa e^{-\varkappa a} \end{cases} \rightarrow k\,\mathrm{ctg}\,ka = -\varkappa,$$

$$ka\,\mathrm{ctg}\,ka = \cancel{-\frac{\sqrt{2ma}}{\hbar^2 a}(U_0 - \mathcal{E}_1)} - \frac{\sqrt{2m(U_0 - \mathcal{E}_1)}}{\hbar}a \approx -1{,}22$$

$ka = \frac{2\pi}{3}$ - из усл.

$$k = \frac{\sqrt{2m\mathcal{E}_1}}{\hbar} \rightarrow \boxed{\mathcal{E}_1 = \left(\frac{2\pi}{3}\frac{\hbar}{a}\right)^2 \cdot \frac{1}{2m} \approx 3\,k}$$

~~$U_0$~~

$$\boxed{U_0 = 4_k}$$

$$\langle z \rangle = \frac{\int_0^{+\infty} \psi^* z \psi \, dz}{\int_0^{+\infty} \psi^* \psi \, dz}$$

$\cancel{\int_0^{+\infty} \psi^* \psi \, dz =}$ Пусть $A = 1$

$$B = \frac{\sin ka}{e^{-\varkappa a}} \approx 2{,}93$$

$$\int_0^{+\infty} \psi^* \psi \, dz = \int_0^a \sin^2 kz \, dz + B \int_a^{+\infty} e^{-2\varkappa z} \, dz =$$

$$= \frac{1}{2} \int_0^a \left( (2 \sin^2 kz - 1) + 1 \right) dz + B \cdot \frac{1}{2\varkappa} e^{-2a\varkappa} =$$

$$= \frac{1}{2} \left( a - \int_0^a \cos 2kz \, dz \right) + \frac{B}{2\varkappa} e^{-2a\varkappa} =$$

$$= \frac{1}{2} a - \frac{1}{2} \cdot \frac{1}{2k} \sin 2ka + \frac{B}{2\varkappa} e^{-2a\varkappa} =$$

$$\approx 0{,}71 a$$

$$\int_0^{+\infty} \psi^* z \psi \, dz = \int_0^a \sin^2 kz \cdot z \, dz + B \int_a^{+\infty} e^{-2\varkappa z} \cdot z \, dz =$$

$$= \frac{1}{2} \int_0^a z (1 - \cos 2kz) \, dz - \frac{B}{2\varkappa} \int_a^{+\infty} z \, d(e^{-2\varkappa z}) =$$

$$= \frac{a^2}{4} - \frac{1}{2}\cdot\frac{1}{2k}\int_0^a z\, d(\sin 2kz) +$$

$$+ \frac{Ba}{2\varkappa}e^{-2\varkappa a} + \frac{B}{2\varkappa}\int_a^{+\infty} e^{-2\varkappa z}\,dz =$$

$$= \frac{a^2}{4} - \frac{1}{4k}\, a\cdot\sin 2ka + \frac{1}{4k}\int_0^a \sin 2kz\, dz +$$

$$+ \frac{Ba}{2\varkappa}e^{-2\varkappa a} + \frac{B}{4\varkappa^2}e^{-2\varkappa a} =$$

$$= \frac{a^2}{4} - \frac{a\sin 2ka}{4k} - \frac{1}{8k^2}\left(\cos 2ka - 1\right) + \frac{Ba}{2\varkappa}e^{-2\varkappa a} +$$

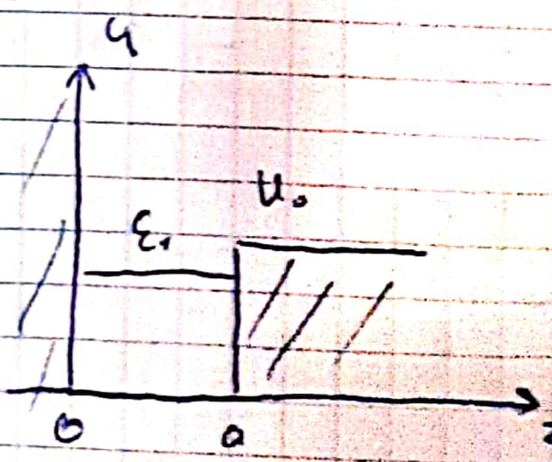
$$+ \frac{B}{4\varkappa^2}e^{-2\varkappa a} \approx 0{,}54\, a^2$$

$$\boxed{\langle z\rangle \approx 0{,}76\, a \approx 4{,}56\ \text{Å}}$$

## №3.23

$a = 2\ \text{Å}$ $\qquad \alpha = \frac{\sqrt{3}}{2}$

$\psi(a) = \alpha\, \psi_{max}$




$0 < z < a$:

$\psi = \sin kz$

$\psi_{max} = 1$

$\psi(a) = \alpha = \sin ka \rightarrow ka = \arcsin\alpha = \frac{2\pi}{3}$

$\varepsilon = U_0 - \varepsilon_1$ $\quad\quad$ $\varkappa = -k\,ctg\,ka$ (из № 3.21)

$k = \frac{\sqrt{2m\varepsilon_1}}{\hbar}$ $\quad\quad$ $\varkappa = \frac{\sqrt{2m\varepsilon}}{\hbar}$

$$\boxed{\varepsilon = \frac{\varkappa^2\hbar^2}{2m} \approx 1,42 \text{ эВ}}$$

$\varepsilon_1 = \frac{k^2\hbar^2}{2m}$

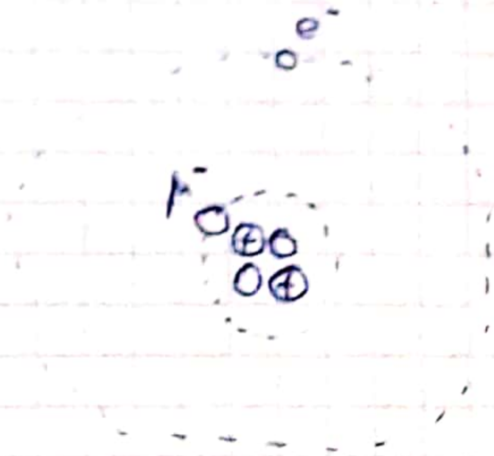
$$\boxed{U_0 = \varepsilon + \varepsilon_1 = \varepsilon + \frac{k^2\hbar^2}{2m} \approx 5,61 \text{ эВ}}$$

N 4.45

$m_\mu c^2 = 106,6$ МэВ

$3p \to 2s$

$\varepsilon_{3,2}$ - ? $\quad$ $\lambda_{3,2}$ - ?



$n = 3 \to n = 2$

$R_\mu = \frac{\hbar^2}{m_\mu Z e^2} = r_Б \frac{m_e}{m_\mu} \cdot \frac{1}{Z} \approx 1,27 \cdot 10^{-11}$ см

$\frac{1}{\lambda_{32}} = \frac{1}{R_\infty}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36} R_\infty = 1,52 \cdot 10^4 \text{ см}^{-1}$

$\boxed{\lambda_{32} = 656 \text{ нм}}$

$$\boxed{\varepsilon_{32} = \frac{hc}{\lambda_{32}} \approx 1{,}89\ эВ}$$

№ 4.38

$Z < 50$

$T_e - ?$

$$E_n = -R_y (z - \sigma)^2 \frac{1}{n^2}$$

↑

постоянная экранирование

$_{30}Zn \rightarrow e^-$

$\uparrow \gamma$

$_{47}Ag$

$$E = \hbar\omega = R_y (z - \sigma)^2 \left( \frac{1}{n_1^2} + \frac{1}{n_2^2} \right)$$

$n_1 = 1 \qquad n_2 = 2$

$$E = R_y (z - \sigma)^2 \cdot \frac{3}{4}$$

$$\sigma = z - \sqrt{\frac{4E}{3R_y}} \approx 1$$

$$E_{иониз} = \hbar\omega_{Zn} = R_y (z - \sigma)^2 \approx 11{,}4\ кэВ$$

$$\boxed{T_e = E_\gamma - E_{иониз} \approx 10{,}2\ кэВ}$$

№5.25

$\lambda = 4{,}61$ мкм

$A_0 - ?$

$T_1 - ?$

$$\mu = \frac{m_0 m_c}{m_0 + m_c} = 11{,}4 \cdot 10^{-24} \text{ г}$$

$$\omega = \frac{2\pi c}{\lambda}$$

$$\langle E \rangle = \langle T \rangle + \langle U \rangle$$

$$\langle T \rangle = \langle U \rangle = \frac{\mu \omega^2 \langle x^2 \rangle}{2}$$

$$\langle x^2 \rangle = \frac{1}{2} A_0^2$$

$$E_0 = \frac{\hbar\omega}{2} = 2 \frac{\mu \omega^2}{2} \frac{A_0^2}{2}$$

$$\boxed{A_0 = \sqrt{\frac{\hbar}{\mu\omega}} = \sqrt{\frac{\hbar\lambda}{2\pi c \mu}} = 4{,}7 \cdot 10^{-10} \text{ см}}$$

$T_1$ при $kT \geq \hbar\omega$

$$\boxed{T_1 \geq \frac{\hbar\lambda}{2\pi c \cdot k} \approx 3100 \text{ K}}$$