

Question 1. Simulation of a multi-compartment Hodgkin-Huxley model.

1.1 The Hodgkin-Huxley model for an active neurite with input current $I_e(j, t)$ for a single cylindrical compartment j is given by equations 1-12.

$$V' = \frac{V}{mV} \quad (1)$$

$$I_e(t) = C_m \frac{dV'_j}{dt} + g_L(V'_j - E_L) + g_{Na,j}(V'_j - E_{Na}) + g_{K,j}(V'_j - E_K) + g_{ax}(V'_j - V'_{j-1}) + g_{ax}(V'_j - V'_{j+1}) \quad (2)$$

$$g_{Na,j} = \bar{g}_{Na} m_j^3 h_j, \quad g_{K,j} = \bar{g}_K n_j^4 \quad (3)$$

Sodium channels (not writing indices j):

$$\frac{dm}{dt} = \alpha_m(1 - m) - \beta_m m \quad (4)$$

$$\alpha_m = \begin{cases} 0.1 \frac{V' - 25}{1 - \exp(-\frac{V' - 25}{10})} & V \neq 25mV \\ 1 & V = 25mV \end{cases} \quad (5)$$

$$\beta_m = 4 \exp\left(-\frac{V'}{18}\right) \quad (6)$$

$$\frac{dh}{dt} = \alpha_h(1 - h) - \beta_h h \quad (7)$$

$$\alpha_h = 0.07 \exp\left(-\frac{V'}{20}\right) \quad (8)$$

$$\beta_h = \frac{1}{1 + \exp\left(-\frac{V' - 30}{10}\right)} \quad (9)$$

Potassium channel (not writing indices j):

$$\frac{dn}{dt} = \alpha_n(1 - n) - \beta_n n \quad (10)$$

$$\alpha_n = \begin{cases} 0.01 \frac{V' - 10}{1 - \exp(-\frac{V' - 10}{10})} & V \neq 10mV \\ 0.1 & V = 10mV \end{cases} \quad (11)$$

$$\beta_n = 0.125 \exp\left(-\frac{V'}{80}\right) \quad (12)$$

The electrical and geometrical properties are

- $C_m = 1 \mu F$, membrane capacitance
- $E_{Na} = 115 mV$, sodium equilibrium potential
- $E_K = -12 mV$, potassium equilibrium potential
- $E_L = 10.6 mV$, leak equilibrium potential
- $V(t = 0) = 0 mV$, initial (and equilibrium) membrane potential
- $\bar{g}_{Na} = 120 mS$, maximum sodium channel conductance
- $\bar{g}_K = 36 mS$, maximum potassium channel conductance

- $g_L = 0.3 \text{ mS}$, leak conductance
- $g_{ax} = 0.5 \text{ mS}$, axial conductance
- $N = 100$, number of compartments

Given the Hodgkin-Huxley model described above, we approximate the DEQs using the Forward Euler method and derive an equation for $V(j, t)$ for an arbitrary compartment j . We assume that the first compartment is terminated as a 'sealed end' and the last compartment is terminated as a 'killed end', as shown in Figure 1.

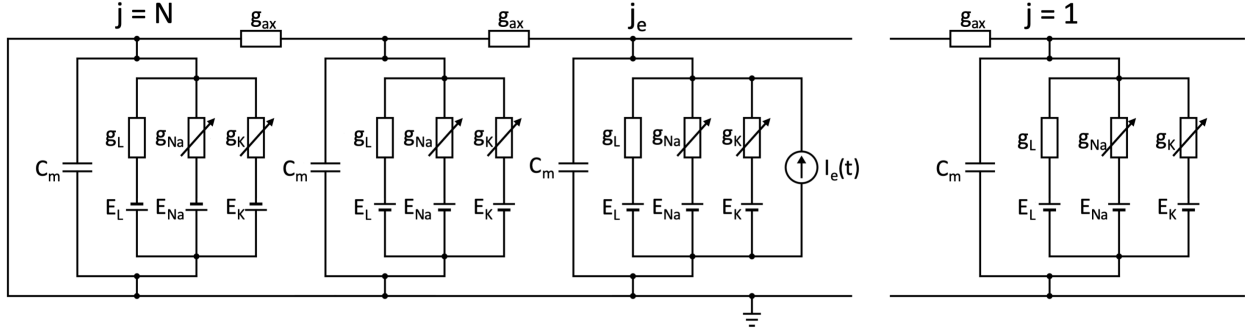


Figure 1: Circuit diagram of a multi-compartment Hodgkin-Huxley model. Compartment 1 is terminated as a 'sealed end' and compartment N is terminated as a 'killed end'. Note that the connection between compartment 1 and compartment j_e is not shown for simplicity.

The approximated DEQ for voltage response of compartment j into which current is injected looks as shown below

$$V'_j(t + \Delta t) = V'_j(t) + \frac{\Delta t}{C_m} (I_e(t) + g_L (E_L - V'_j(t)) + \bar{g}_{Na} m_j^3(t) h_j(t) (E_{Na} - V'_j(t)) \\ + \bar{g}_K n_j^4(t) (E_K - V'_j(t)) + g_{ax} (V'_{j-1}(t) - V'_j(t)) \\ + g_{ax} (V'_{j+1}(t) - V'_j(t)))$$

Following Homework 4, voltage response for compartment i where $0 < i < j$ is

$$V'_i(t + \Delta t) = V'_i(t) + \frac{\Delta t}{C_m} (g_L (E_L - V'_i(t)) + \bar{g}_{Na} m_i^3(t) h_i(t) (E_{Na} - V'_i(t)) \\ + \bar{g}_K n_i^4(t) (E_K - V'_i(t)) + g_{ax} (V'_{i-1}(t) - V'_i(t)) \\ + g_{ax} (V'_{i+1}(t) - V'_i(t)))$$

As for compartment i where $j < i < N$, we have

$$V'_i(t + \Delta t) = V'_i(t) + \frac{\Delta t}{C_m} (g_L (E_L - V'_i(t)) + \bar{g}_{Na} m_i^3(t) h_i(t) (E_{Na} - V'_i(t)) \\ + \bar{g}_K n_i^4(t) (E_K - V'_i(t)) + g_{ax} (V'_{i+1}(t) - V'_i(t)) \\ + g_{ax} (V'_{i-1}(t) - V'_i(t)))$$

As regards the termini, membrane potential for compartment $j = N$ is set to 0 and for compartment $j = 1$ we assume no axial current flowing out, and hence all membrane potential difference is due to current flowing through the membrane

$$V_1'(t + \Delta t) = V_1'(t) + \frac{\Delta t}{C_m} (g_{ax} (V_2'(t) - V_1'(t)) + g_L (E_L - V_1'(t)) + \bar{g}_{Na} m_1^3(t) h_1(t) (E_{Na} - V_1'(t)) + \bar{g}_K n_1^4(t) (E_K - V_1'(t)))$$

For the approximations of m, h, n gating variables please refer to Homework 4.