

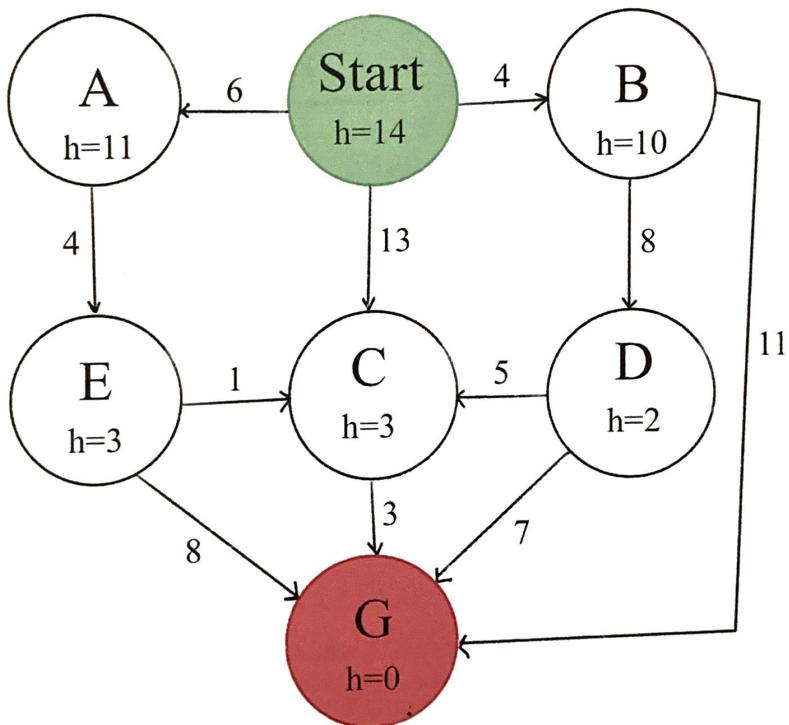
## HW1: Search

CS4300: Artificial Intelligence  
University of Utah

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## 1 Graph Search

Alice the agent really wants to go skiing right after AI class is over. She starts in the lecture hall (the “start” state below) and wants to make it to Park City (the “goal” state) as soon as possible. There are several possible paths she can take denoted in the graph below:



The available actions at each state are denoted by arrows with an edge cost label next to each arrow. For each of the following graph search strategies, describe the order in which states are expanded as well as the path returned by the graph search. Show all work. When expanding states add their children to the data structure in alphabetical order (i.e. when expanding from Start, put A into the data structure before B), using ‘G’ for the goal. Remember that in graph search, states are expanded only once.

1. Depth first search

Visited Set

S  
C

Stack

S  
S → A  
S → B  
S → E  
S → C → G

Answer: S → C → G

2. Breadth first search

Visited Set

S  
A  
B  
C  
E  
D

Queue

S → C → G  
S → A → E → C  
S → A → E → G  
S → B → D → C  
S → B → D → G

Answer: S → B → G

3. Uniform cost search

Visited Set

S  
B  
A  
E  
C  
D

PQ

S → A → E → C, 11  
S → A → E → G, 12  
S → A → E → C → G, 14  
S → B, 4  
S → C, 13  
S → B → D → C, 17  
S → B → D → G, 19

Answer: S → A → E → C → G  
(cost is 14)

4. Greedy search with the heuristic values listed at each state

Visited Set

S  
C

PQ

S, 14  
S → A, 11  
S → B, 10  
S → C, 3  
S → C → G, 0

Answer: S → C → G

5. A\* search with the heuristic values listed at each state

Visited Set

S  
B  
D

PQ

S, 0 (14)  
S → A (6+11=17)  
S → B (4+10=14)  
S → C (13+3=16)  
S → B → D (12+2=14)  
S → B → G (15+0=15)  
S → B → D → C (17+3=20)  
S → B → D → G (19+0=19)

Answer: S → B → G  
(cost of 15)

Answer the following questions based on the above graph:

6. Do the heuristic values above constitute a consistent heuristic? If not, which edge(s) is/are not consistent?

No, the heuristic values above do not constitute a consistent heuristic. One example is, from  $A \rightarrow E$ ,  $h(A) - h(E) \leq \text{cost}(A \text{ to } E)$  is not true.

Answer: \_\_\_\_\_

(8)

(4)

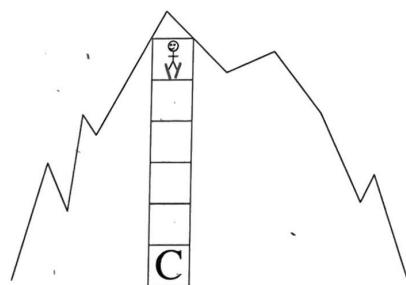
7. If necessary, fix the above heuristic to be consistent by decreasing its values as little as possible, i.e., minimizing the sum of the changes you make. Feel free to leave some values unchanged if changing them is not necessary.

Node	Old Heuristic Value	New Heuristic Value
Start	14	13
A	11	7
B	10	unchanged
C	3	unchanged
D	2	unchanged
E	3	unchanged
Goal	0	unchanged

Only nodes 'Start' and 'A' are changed

## 2 Downhill Skiing

After getting to Park City, Alice takes the lift up to the top of the mountain. Now she wants to ski down, so her only option is to go straight downhill. She begins with a velocity of 0 and can safely maintain a maximum velocity of  $V$ . At any state, she has three actions she can take: accelerate, decelerate or coast. If she accelerates, her velocity increases by 1; if she decelerates, it decreases by 1; if she coasts, it stays the same. After her velocity is adjusted by her action, she moves downhill an equal number of squares to her current velocity.



Consider the above figure. If Alice's first action is "accelerate" then she will end up in the second square down with a velocity of 1. If she then "coasts" then she will end up in the third square down with a velocity of 1. If she "accelerates" again, she will end up in the fifth square down with a velocity of 2.

Alice's goal is to reach the chair lift (marked "C") with a velocity of zero. (No, Alice cannot have negative velocities). She would like to get there as quickly as possible. However, if she ends her time at the chair lift square with a non-zero velocity, she skies into the parking lot and destroys her skis.

1. Describe the components that need to be included in the state definition. Justify your answer. What are the start/goal states?

The states used to represent this problem can be broken down into a coordinate system of position and velocity pairs, or  $(P, V)$ . Using this, the top of the mountain can be at position 0 and the chair lift is at position 5. Therefore, the start state is  $(0, 0)$  and the goal state is  $(5, 0)$ .

2. Give an example of a state that is not reachable and explain why it is not reachable.

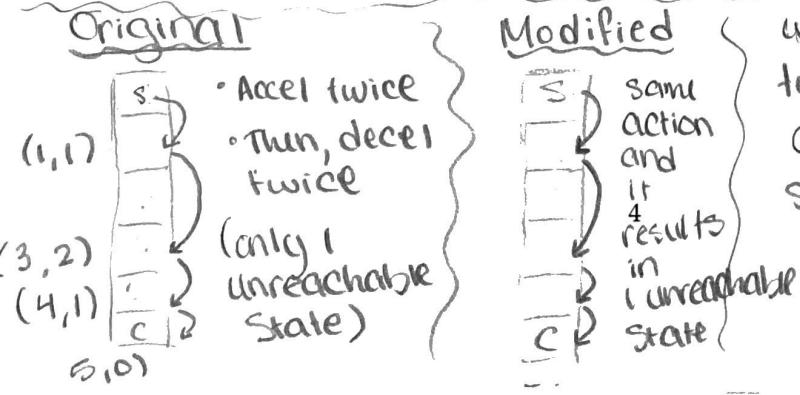
One example of an unreachable state is  $(2, 2)$  or position=2 with velocity=2. This is because when Alice starts at  $(0, 0)$  she can accelerate once, resulting in her new state being  $(1, 1)$ . If she accelerated again, she would be in state  $(3, 2)$ . Thus, she can never reach  $(2, 2)$  from start since she cannot accelerate twice in one

3. Assuming all actions have a cost of 1, is Alice's current velocity an admissible heuristic? Why or why not?

It is an admissible heuristic because, from start,  $(0, 0) \leq (P+V, V)$ . when  $V=1$ . Because of this rule, the estimated cost to reach a state will always be less than or equal to the actual cost to reach a state.

4. Suppose that Alice cannot accelerate after she has decelerated (i.e. all accelerations must occur before all decelerations, but she can coast whenever she wants): does this yield more unreachable states? If so, give an example of one. Justify your answer either way.

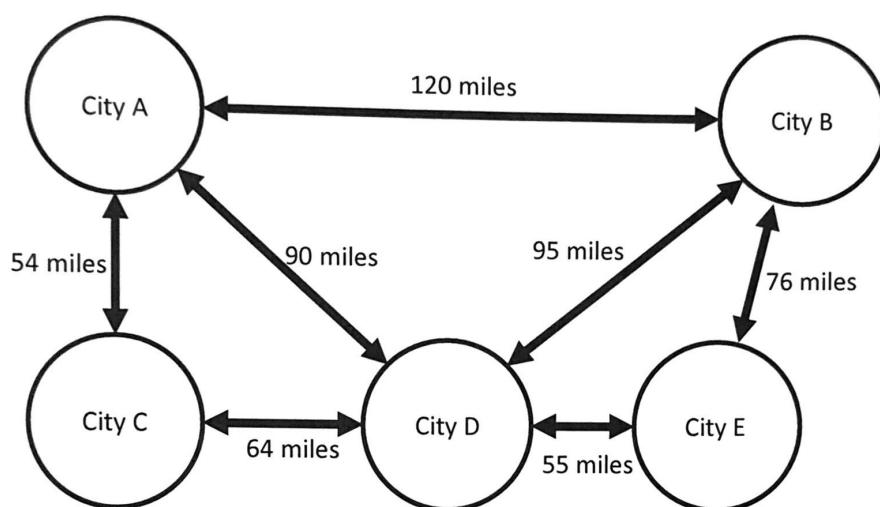
With this change, it does not yield more unreachable states; instead it still results in the same number of unreachable states. For example:



when accelerating twice, then decelerating twice, both the original and modified conditions result in 7 unreachable states which is  $(2, 2)$ . This is while considering the actions result in Alice reaching the goal state.

### 3 Human-Centric Factors

Consider the following graph, which shows the connectivity via roads between five cities. The edges and their labels represent the existence and length of the roadways in between the cities.



Jessie, who resides in City A, needs to travel to City B. Jessie has a medical condition that frequently produces life-threatening symptoms at unpredictable times that requires treatment in an emergency room. There are emergency rooms in each of the five cities, but medical care is non-existent on the roads themselves.

Please consider and thoughtfully answer the following questions:

- From just looking at the graph, what path do you think Jessie should take to get from City A to City B? Explain your choice.

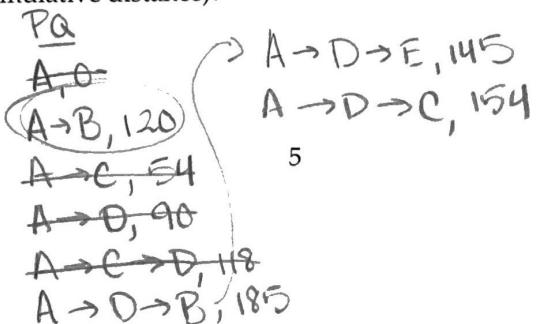
I think Jessie should take the path:  $A \rightarrow C \rightarrow D \rightarrow E \rightarrow B$ .

This is because her medical condition could happen randomly driving, therefore I want to prioritize taking paths with less miles between each city. This way, if she does need medical attention, she'll reach the nearest city (where there is an emergency room).

- (a) Using the edge costs given in the graph, what path is returned using the UCS algorithm with the traditional objective function (shortest cumulative distance)?

Visited Set

A  
C  
D



The path returned using UCS is:

City A → City B  
with cost 120 miles

- (b) Based on your answer to question 1 and (2a), do you think that the shortest cumulative distance on the path from City A to City B is the right objective function to use when considering the optimal route for Jessie? Why or why not?

No, it's not the right objective function. Even though the shortest cumulative path seems like the more efficient option, we need to consider the humanistic angle of Jessie's condition such that we pick options that lessen the risk of her not seeking medical attention at the nearest city. Thus, I would prioritize choosing the minimum outgoing distance. However, it also

3. Imagine you are working with a team that is trying to train assistive AI to help Jessie decide on routes to travel safely between cities. Your teammates propose the following objective functions and you need to decide whether these objective functions will enable good outcomes for Jessie. Carefully critique each objective function by answering the questions provided (note: some of these algorithms require additional information to solve - your team has only proposed the objective function).

**Objective function 1:** Since we do not want Jessie to take longer roads, an objective function could be to minimize the sum of the cube of each distance between cities to punish longer edge traversals, i.e., Edge Cost = Distance<sup>3</sup>.

- (a) Do you think this objective function is appropriate for this context when considering your answer to question 1?

I think this objective function is appropriate for this context because it officially prioritizes choosing the minimum outgoing path and punishing longer ones. It also allows for considering other paths to traverse in choosing the minimum outgoing path.

- (b) Given just the information provided by your teammate, can we use UCS with this objective function? If so, what path is returned? If not, explain why not.

We can still use UCS with it because it is still a linearized problem. Therefore, the path returned by UCS would be:

City: A → C → D → E → B with cost 1,024,959 miles.

- (c) Given just the information provided by your teammate, can we use A\* with this objective function? If so, what path is returned? If not, explain why not.

A heuristic wasn't defined for this problem; therefore, we cannot use A\* search with this objective function. If we started to derive some heuristic for this problem, then we could.

- (d) Using your answers above, which, if any, of the search algorithms above will return the route you think is best for Jessie for this objective function for this specific graph? Justify your answer.

I think using Uniform Cost Search will yield the best route for Jessie. This is because the routes taken from  $A \rightarrow C \rightarrow D \rightarrow E \rightarrow B$  are the shortest paths per city which ensures a safer trip than a more efficient one prior.

**Objective function 2:** An objective function could be to minimize the maximum distance between any two adjacent cities on the path (i.e., we define the total cost of each path to be the longest edge on that path, and the objective is to minimize this value).

- (e) Do you think this objective function is appropriate for this context when considering your answer to question 1?

I think this objective function is still appropriate for this context. This is because choosing the total minimum edge from each maximum edge route essentially provides an estimated safest path among all paths on the graph. Thus, it supports a safer trip.

- (f) Given just the information provided by your teammate, can we use UCS with this objective function? If so, what path is returned? If not, explain why not.

We can use UCS because it is still a linearized that returns a proper shortest path. The path returned would be : City  $A \rightarrow C \rightarrow D \rightarrow E \rightarrow B$  with a cost of 76 miles (since 76 is the max route on that path but the min value to City B).

- (g) Given just the information provided by your teammate, can we use A\* with this objective function? If so, what path is returned? If not, explain why not.

No, we cannot use A\* search because no explicit heuristic was defined. However, if we derived heuristic values by assigning the start state's heuristic with the absolute minimum path cost, we could potentially use A\* search. (let's not consider that though for the sake of objectivity).

- (h) Using your answers above, which, if any, of the search algorithms above will return the route you think is best for Jessie for this objective function for this specific graph? Justify your answer.

I think Uniform Cost Search will provide the best route for Jessie with this objective function. It prioritizes the safest path by estimating the smallest costs between each city for this context.

4. So far, we've been considering a specific graph. However we want our algorithms to work for any arbitrary graph.

- (a) Describe an objective function that you think is most appropriate for this scenario (use or modify one of the objective functions given above or come up with your own).

### Objective Function:

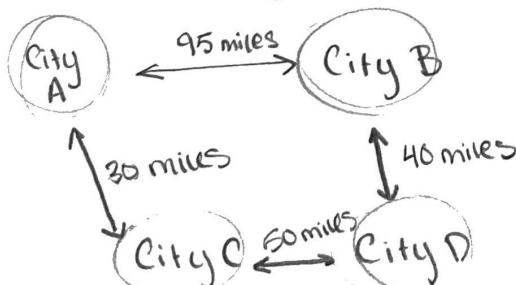
$$\text{Edge Cost } (v_1, v_2) = \begin{cases} \max\left(\frac{\text{distance}(v_1, v_2)}{2}\right) & \text{if } \leq \text{Total Average distance} \\ & \text{else don't consider path} \end{cases}$$

Where Total Average Distance =  $\frac{\text{sum of all distances}}{\text{number of edges}}$

- (b) Determine which, if any, of the search algorithms we've covered will still be optimal for this objective for any arbitrary graph.

Uniform Cost Search will still be optimal because the edge cost still produce a linearized problem. This shouldn't drastically effect its produced results for prioritizing the safest path.

- (c) Justify your answer with a discussion of the kinds of paths the algorithm might choose between in an unknown, arbitrary graph.



$$\text{Edge Cost } A \rightarrow B = \left(\frac{95}{2}\right) \text{ if } \leq 53.75 \\ = 48$$

$$\text{Edge Cost } A \rightarrow C = \frac{30}{2} = 15$$

This kind of path will still enable UCS to choose shortest paths

Optional: you may find it helpful to consider a few specific graphs when considering all arbitrary graphs. I've included a few such graphs at the end of this document to help you think about this.

results for this context (safer trips).

They remain similar to the other objective functions because of its linearized parameters.

5. In this problem, we are modeling this situation solely based on distances between cities and locations of emergency rooms. What is another example of a human-centric factor that we would need to consider when modeling the optimal path for Jessie that demonstrates the complex nature of defining an "optimal" path in real life?

Another human-centric factor we would need to consider is the current gallons of gas the car has along the path. This can be measured either as another static edge weight or, more accurately, as a function relative to the number of miles traveled along a path. This complicates our problem because maybe certain cities don't offer gas stations (while others do) which may drastically change our uniformed search. For instance, using the graph given in section 3, City A → B may be longer but the path City A → C doesn't have a gas station whereas City D does. Yet, if we continue along City A → C → D, we run the risk of losing gas compared to simply driving from City A → D.

