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CS3200
Assignment 1 - Report

Question 1 Answered

The answer to the coffee cup problem analytically: T = 76.36℃

Work Shown:

1) Solve the coffee cup problem analytically for 5 minutes based upon Newton's Law of Cooling

$$\frac{dT_c}{dt} = -r(T_c - T_s)$$

$T_s = 19°C$ $\quad T_c = 84°C$ $\quad r = 0.025/\text{second}$

Work

$$\frac{dT_c}{\cancel{dt}\,(T_c - T_s)} = \frac{-r(T_c - T_s)}{T_c - T_s} \cdot dt \Rightarrow \int \frac{dT_c}{T_c - T_s} = \int -r\,dt$$

$$\Rightarrow \ln(T_c - T_s) = -rt + C \Rightarrow T_c - T_s = e^{-rt+C} \quad + T_s$$

$$T_c = e^{-rt+c} + T_s \Rightarrow T_c = e^{-rt} \cdot C_1 + T_s \Rightarrow T = T_s + (T_c - T_s)e^{-rt}$$

$T_c = T_s + e^0 \cdot C_1$
@ $t = 0$
$C_1 = T_c - T_s$

$$\boxed{T = T_s + (T_c - T_s)e^{-rt}}$$

Using this solution, we plug in to find the temperature of the coffee after 5 minutes

$$T = 19 + (84 - 19)e^{-0.025 \cdot 5}$$

$$\boxed{T = 76.36°C}$$

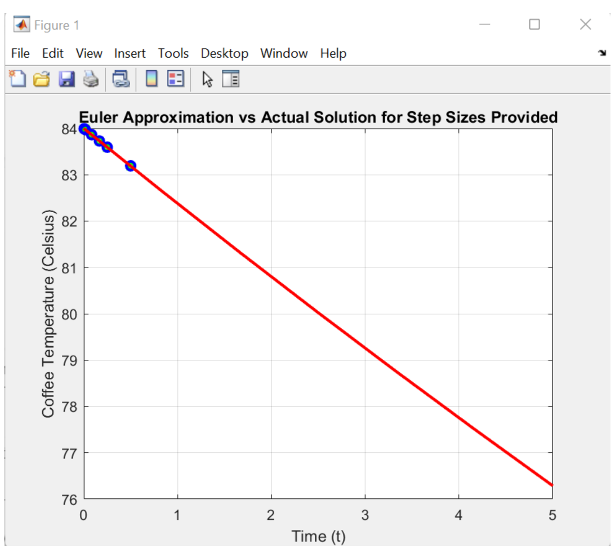
Methods Used & Results

In my submitted code “a1.m”, **the function takes in parameters for computing both the Euler Approximation and Actual Solution of Newton’s Law of Cooling on the coffee cup problem.** For context on the results, I simply ran the code (instructions provided in README file) using the values provided in Question 1 in my parameters.

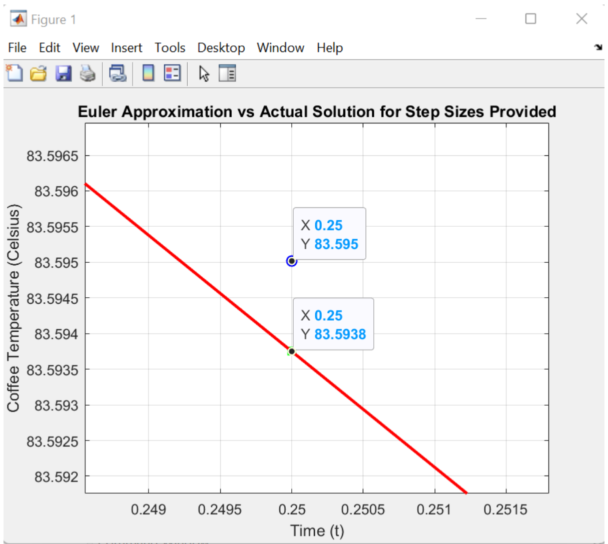
To achieve the correctness in the graphed data, I looked up variables to use when coding for both formulas which are set prior to running the formulas. The method primarily sets up two formulas: Euler Approximation (within the for-loop) and the Actual Solution, those will then be plotted under two different figures.

Figure 1 is responsible for answering Question 3 on the assignment. It plots the various step sizes (h) by looping through a pre-built array containing those times, which were converted to minutes (to accurately plot on the x-axis). It then plots both the Euler Approximation and an Actual Solution formulas’ data under separate loops (for better readability). **As a side note, you’ll have to zoom in to see both the plots for the Approximation and Analytical data**. **The Blue Circle represents Analytical Data and Green Square represents Approximation Data.** The results are as followed (with zoomed in samples):

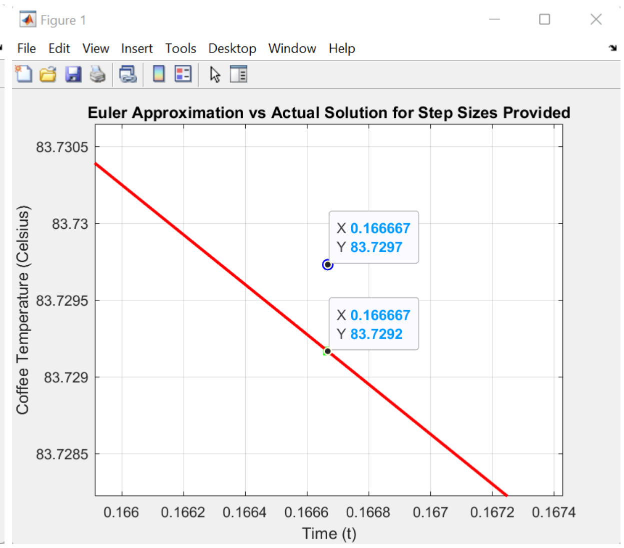
(Full graph)



(At 30 seconds)



(At 15 seconds)



(At 10 seconds)

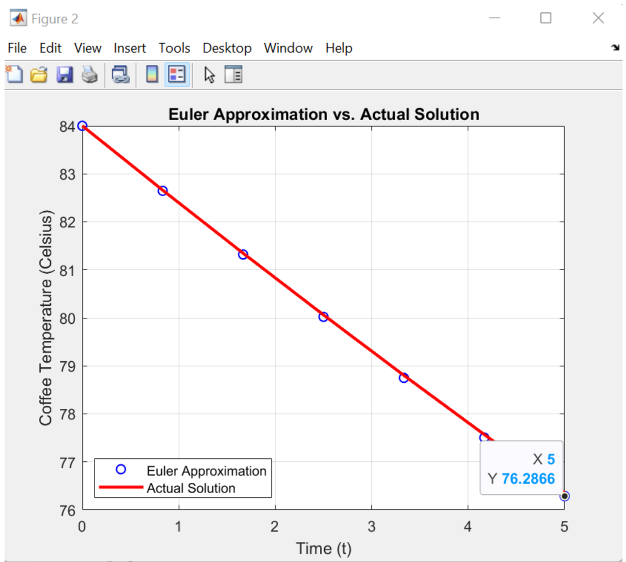
The results of Figure 1 resulted in values that were so close together and so miniscule that they practically became the same value. Personally, this shows that the Euler Approximation formula is a great way to represent the step sizes for Newton's Law of Cooling because it is much more simple and direct. This makes it beneficial too because it is a faster formula computationally.

Figure 2 is responsible for comparing the Euler Approximation and Actual Solution formulas that solve Newton's Law of Cooling over a period of time. It plots the formulas down individually by utilizing the formulas that were created here:

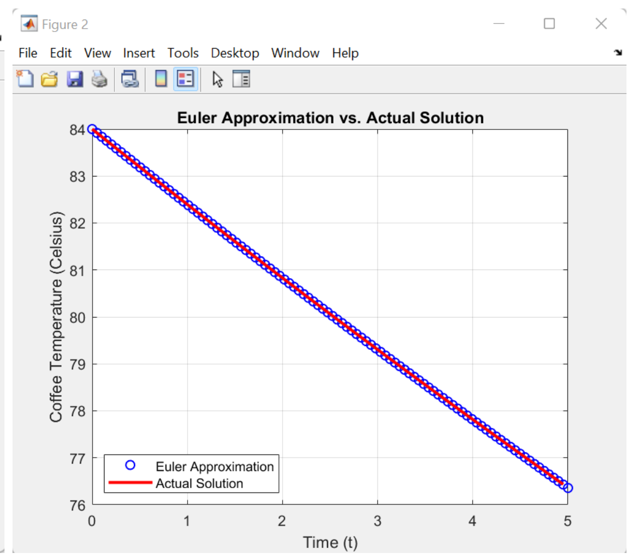
```
%Euler Approximation computation of temperature over time
    for i = 1:numOfTimeSteps-1
        T(i+1) = T(i) - K * (T(i) - Ts);
    end

%Actual Solution for Newton's Law of Cooling for Analysis
    Tsol = Ts + (Ti - Ts) * exp(-r*t);
```

To specify how many plots will be needed for a much more accurate graphing of Euler's Approximation formula, the parameter for "numOfTimeSteps" can be set to large value, like 100. These were the results for various step sizes:

(When Step Size is 7)



(When Step Size is 100)



The results of Figure 2 has first graph detail the Approximation for the temperature of the coffee at 5 minutes. This makes the difference between the Approximation and the Actual Solution: 0.0734℃ which is very small considering it's an approximation. Again, **like I stated in figure 2**, the Euler Approximation is a better formula because it's faster. However, here in figure 2, it proves that Euler Approximation error is very tiny. Even though the Actual Solution is accurate, it is much slower to compute.

<u>Sources</u>

1. https://www.geeksforgeeks.org/newtons-law-of-cooling/