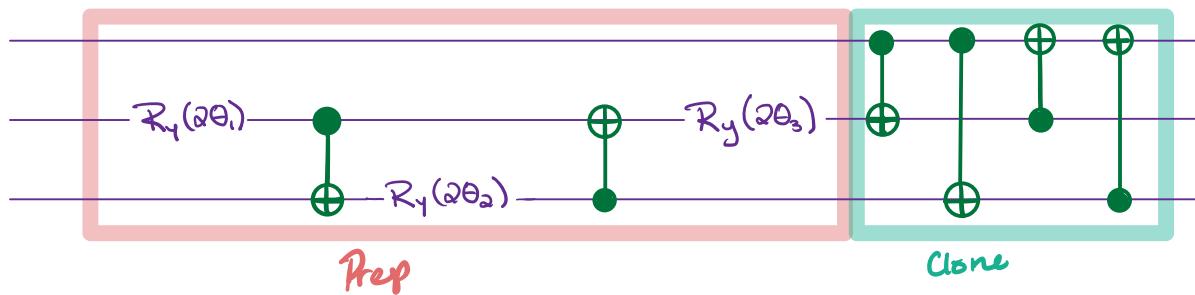
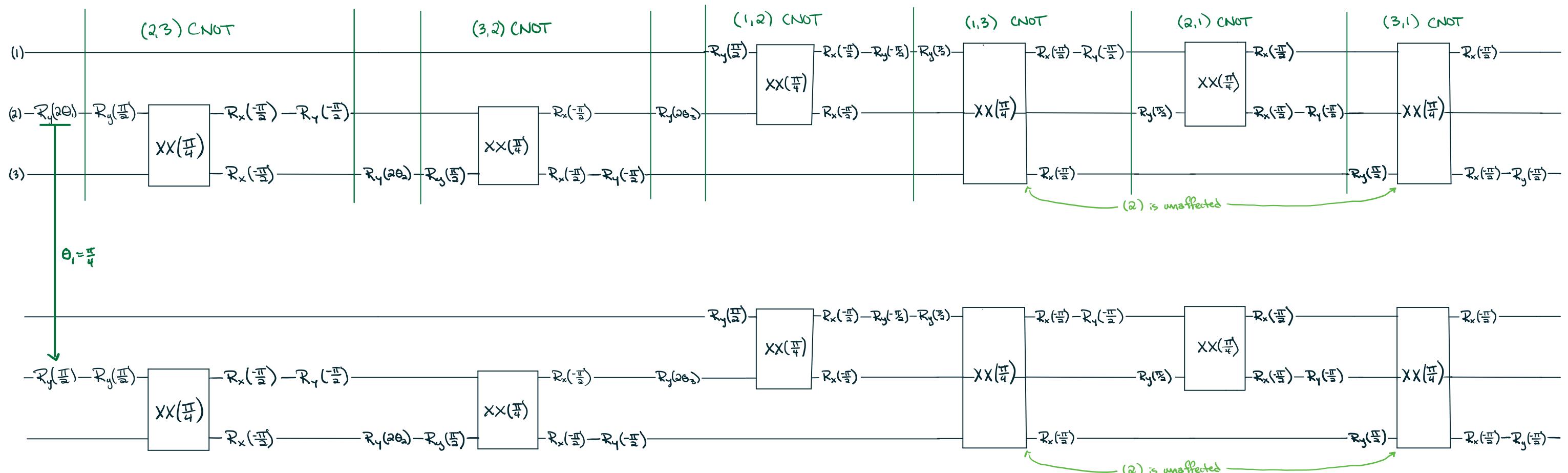


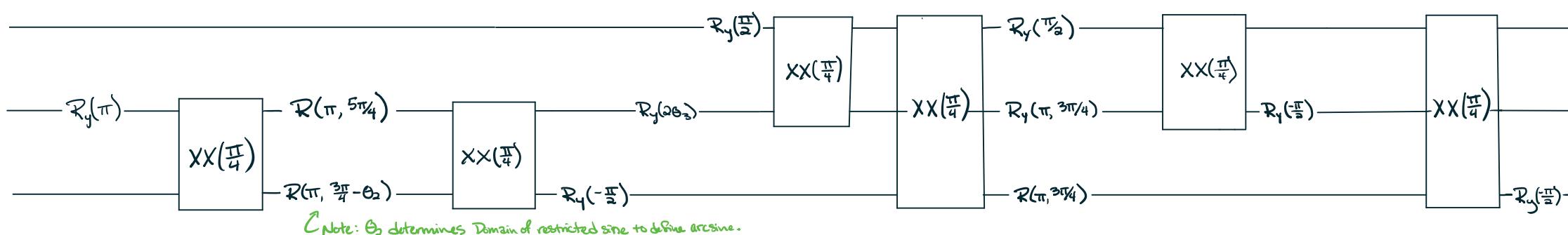
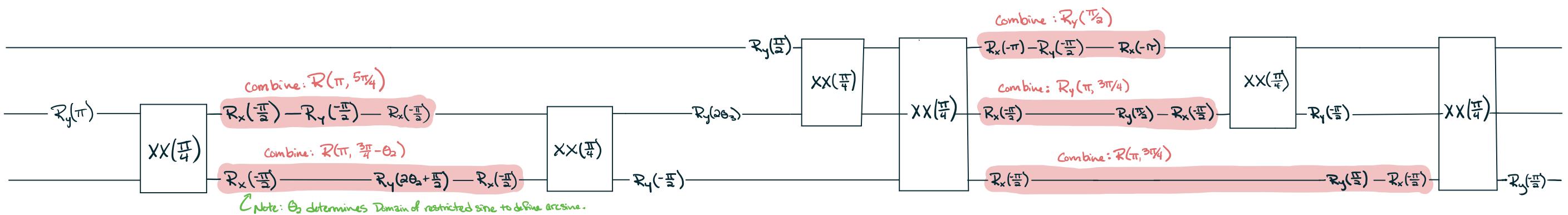
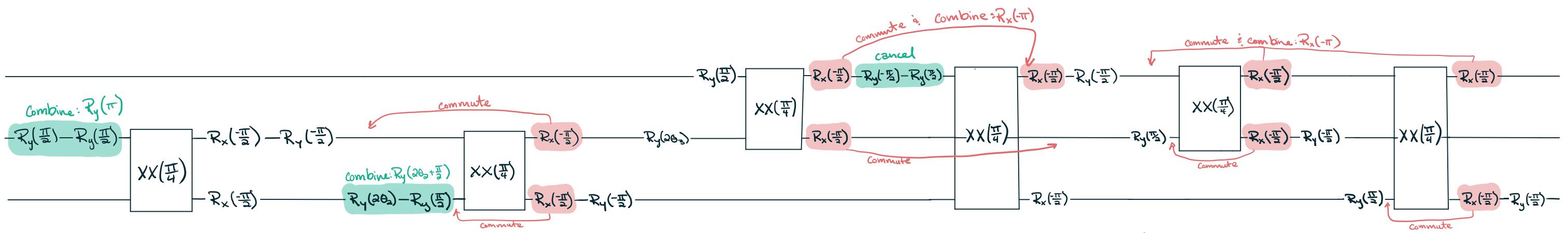
Original:



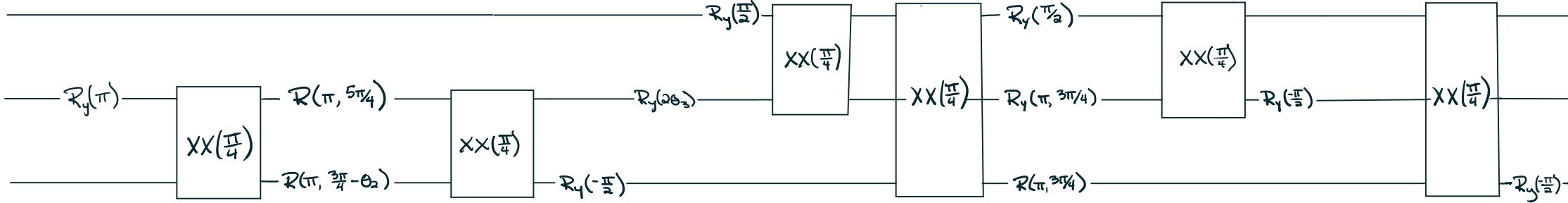
Converting CNOT to XX, R_x, & R_y



Simplifying using $R(\pi, \theta)$ & Maslov Identities



Converting to IonQ Native Gates



$$XX\left(\frac{\pi}{4}\right) \rightarrow MS(0,0)$$

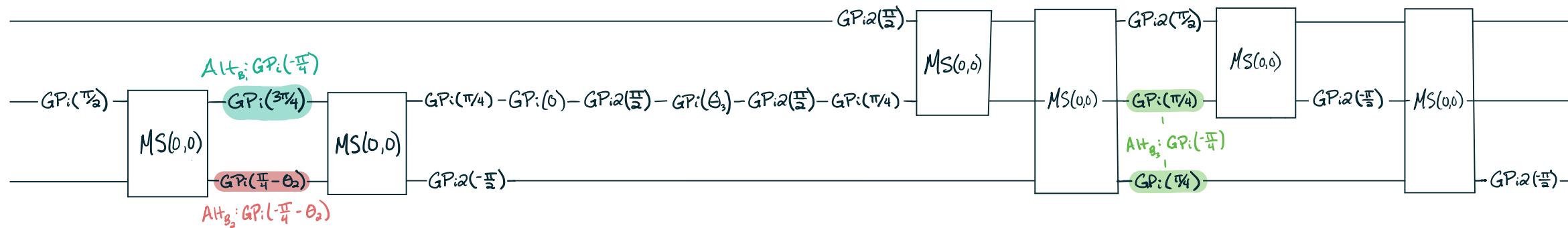
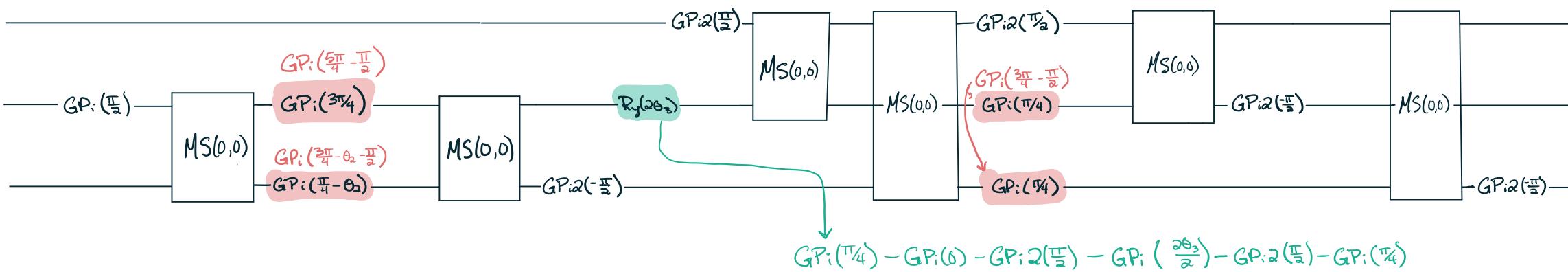
$$R(\pi, \theta) \rightarrow GP_i(\theta - \frac{\pi}{2})$$

$$R_y\left(\frac{\pi}{2}\right) \rightarrow GP:2\left(\frac{\pi}{2}\right)$$

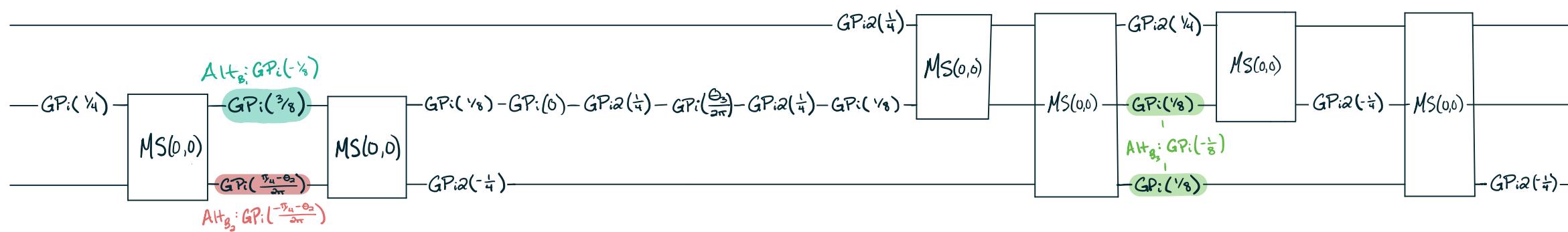
$$R_y(-\frac{\pi}{2}) \rightarrow GP:2(-\pi/2)$$

$$R_y(\pi) \rightarrow GP; (\frac{\pi}{2})$$

$$R_y(\theta) \rightarrow GP_i(\frac{\pi_1}{4}) - GP_i(0) - GP_i(2(\frac{\pi_1}{2})) - GP_i(\theta_2) - GP_i(2(\frac{\pi_1}{2})) - GP_i(\frac{\pi_1}{4})$$



Turns:



Equivalences/Calculations for Alternates:

$$\text{Alt}_{B_1}: GP_i\left(-\frac{\pi}{4}\right) \equiv R\left(\pi, \frac{\pi}{4}\right) = e^{\pi i} \begin{bmatrix} 0 & e^{\pi i/4} \\ -e^{-\pi i/4} & 0 \end{bmatrix} = e^{\pi i} GP_i\left(\frac{\pi}{4}\right)$$

Note $R_x\left(-\frac{\pi}{2}\right) \cdot R_y\left(-\frac{\pi}{2}\right) \cdot R_x\left(-\frac{\pi}{2}\right) = \begin{bmatrix} 0 & e^{\pi i/4} \\ -e^{-\pi i/4} & 0 \end{bmatrix}$ $\longrightarrow \begin{cases} c = \pi \\ d = \pi - \arcsin(-\frac{1}{\sqrt{2}}) = \frac{5\pi}{4} \end{cases}$

by Maslov $R_x(a) \cdot R_y(b) \cdot R_x(a) \equiv R(\theta, \phi) = \begin{bmatrix} \cos \frac{a}{2} & -ie^{i\frac{a}{2}} \sin \frac{a}{2} \\ ie^{i\frac{a}{2}} \sin \frac{a}{2} & \cos \frac{a}{2} \end{bmatrix}$ where $\theta = \arccos(\cos(a) \cos(\frac{b}{2}))$
 $\therefore \phi = \begin{cases} \arcsin\left(\frac{\sin(\frac{b}{2})}{\sqrt{1-\cos^2(a)\cos^2(\frac{b}{2})}}\right), a > 0 \\ \pi - \arcsin\left(\frac{\sin(\frac{b}{2})}{\sqrt{1-\cos^2(a)\cos^2(\frac{b}{2})}}\right), a < 0 \end{cases}$

$$\text{Alt}_{B_2}: GP_i\left(-\frac{\pi}{4} - \theta_2\right) = \begin{bmatrix} 0 & e^{\pi i/4 + \theta_2 i} \\ e^{-\pi i/4 - \theta_2 i} & 0 \end{bmatrix} \equiv e^{3\pi i/2} \underbrace{\left(e^{-\frac{\pi i}{2}} \begin{bmatrix} 0 & e^{-\frac{3\pi i}{4} + \theta_2 i} \\ e^{3\pi i/4 - \theta_2 i} & 0 \end{bmatrix} \right)}_{R\left(\pi, \frac{3\pi}{4} - \theta_2\right)} \equiv R_x\left(-\frac{\pi}{2}\right) - R_y\left(2\theta_2 + \frac{\pi}{2}\right) - R_x\left(-\frac{\pi}{2}\right)$$

$$\begin{cases} c = \pi \\ d = \pi - \arcsin\left(\sin(\theta_2 + \frac{\pi}{4})\right) = \frac{3\pi}{4} - \theta_2 \end{cases}$$

$(2\theta_2 \in [-\frac{\pi}{2}, \frac{\pi}{2}] \Rightarrow \theta_2 \in [-\frac{\pi}{4}, \frac{\pi}{4}] \Rightarrow \theta_2 + \frac{\pi}{4} \in [0, \frac{\pi}{2}] \text{ is restricted sine def. on } [\frac{\pi}{2}, \frac{\pi}{4}])$

$$\text{Alt}_{B_3}: GP_i\left(-\frac{\pi}{4}\right) = \begin{bmatrix} 0 & e^{\pi i/4} \\ e^{-\pi i/4} & 0 \end{bmatrix} = e^{\pi i/2} \underbrace{\begin{bmatrix} 0 & -e^{-\pi i/4} \\ e^{\pi i/4} & 0 \end{bmatrix}}_{R_x\left(-\frac{\pi}{2}\right) - R_y\left(\frac{\pi}{2}\right) - R_x\left(-\frac{\pi}{2}\right)} \equiv R\left(\pi, \frac{3\pi}{4}\right) : \begin{cases} c = \pi \\ d = \pi - \arcsin(\frac{1}{\sqrt{2}}) = \frac{3\pi}{4} \end{cases}$$

Basis X:

initial state $|-\rangle$:
(bit value 1)

$$\text{GPi2}(-\frac{\pi}{4})$$

$\frac{-\pi}{2}$

initial state $|+\rangle$:
(bit value 0)

$$\text{GPi2}(\frac{\pi}{4}) - \text{GPi}(0)$$

$\frac{\pi}{2}$

Measurement Prep:

$$\begin{aligned} & \text{GPi}(0) - \text{GPi2}(-\frac{\pi}{4}) \\ & \text{GPi}(0) - \text{GPi2}(-\frac{\pi}{4}) \\ & \text{GPi}(0) - \text{GPi2}(-\frac{\pi}{4}) \end{aligned}$$

$\frac{-\pi}{2}$

Basis Y:

initial State $|-\rangle$:
(bit value 1)

$$\text{GPi2}(\frac{1}{4}) - \text{GPi}(-\frac{1}{8})$$

$\frac{\pi}{2}$ $\frac{-\pi}{4}$

initial State $|+\rangle$:
(bit value 0)

$$\text{GPi2}(\frac{1}{4}) - \text{GPi}(\frac{1}{8})$$

$\frac{\pi}{2}$ $\frac{\pi}{4}$

Measurement Prep:

Hadamard:

$$\begin{aligned} & \text{GPi}(0) - \text{GPi2}(-\frac{1}{4}) \\ & = \text{GPi2}(\frac{1}{4}) - \text{GPi}(0) \end{aligned}$$

[

$$\begin{aligned} & \text{GPi}(\frac{1}{8}) - \text{GPi2}(-\frac{1}{4}) \\ & \text{GPi}(\frac{1}{8}) - \text{GPi2}(-\frac{1}{4}) \\ & \text{GPi}(\frac{1}{8}) - \text{GPi2}(-\frac{1}{4}) \end{aligned}$$

$\frac{\pi}{4}$ $\frac{-\pi}{2}$

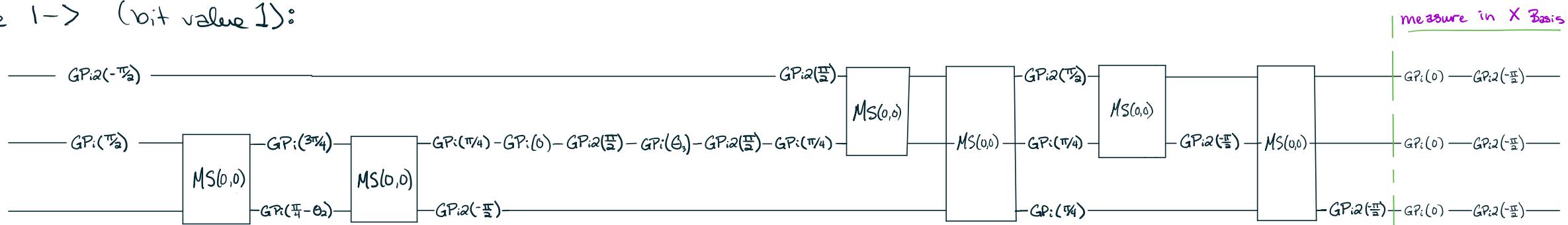
↑
Alt_B: GPi2(0)

Since

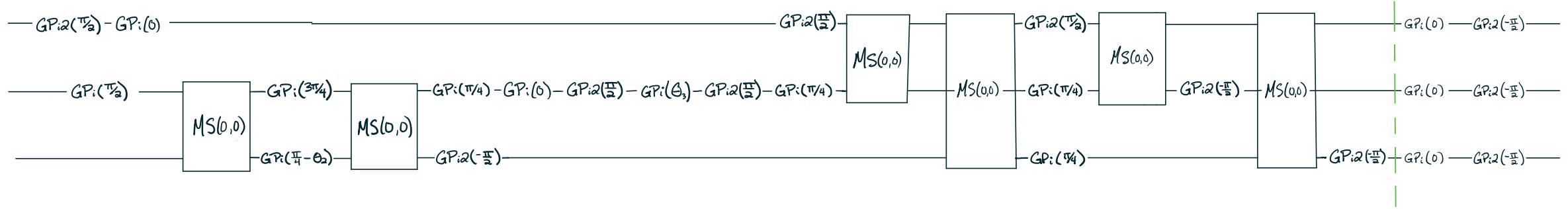
$$\text{GPi2}(-\frac{1}{4}) = \text{GPi}(0)\text{GPi2}(\frac{1}{4})\text{GPi}(0)$$

Basis X :

initial state $|1\rangle \rightarrow$ (bit value 1):

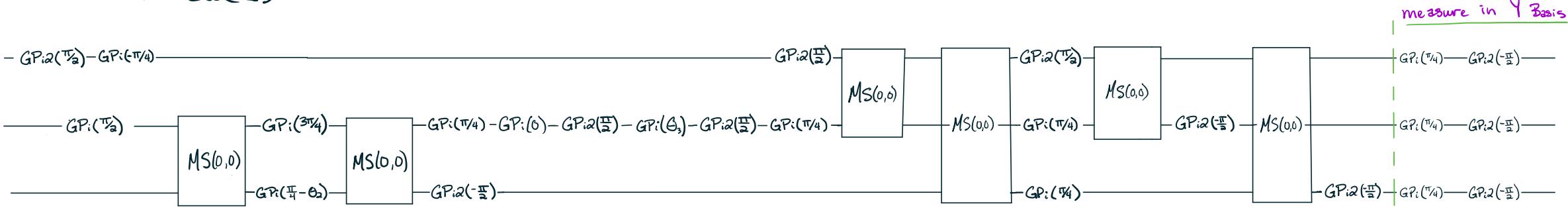


initial state $|1\rangle \rightarrow$ (bit value 0):



Basis Y :

initial state $|1\rangle \rightarrow$ (bit value 1):



initial state $|1\rangle \rightarrow$ (bit value 0):

