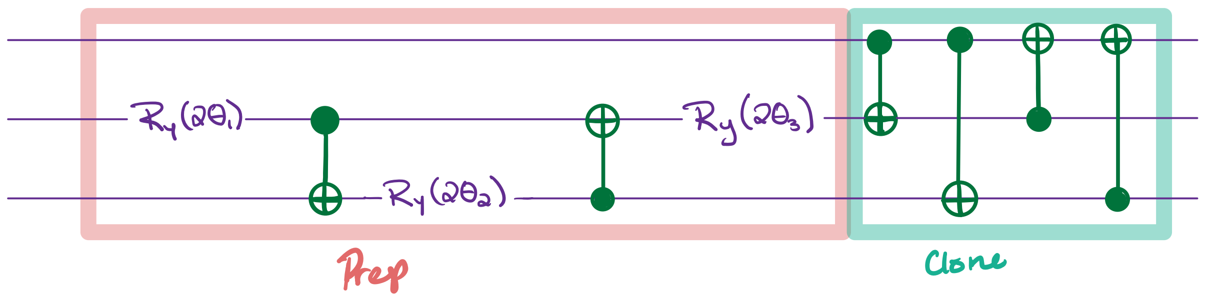
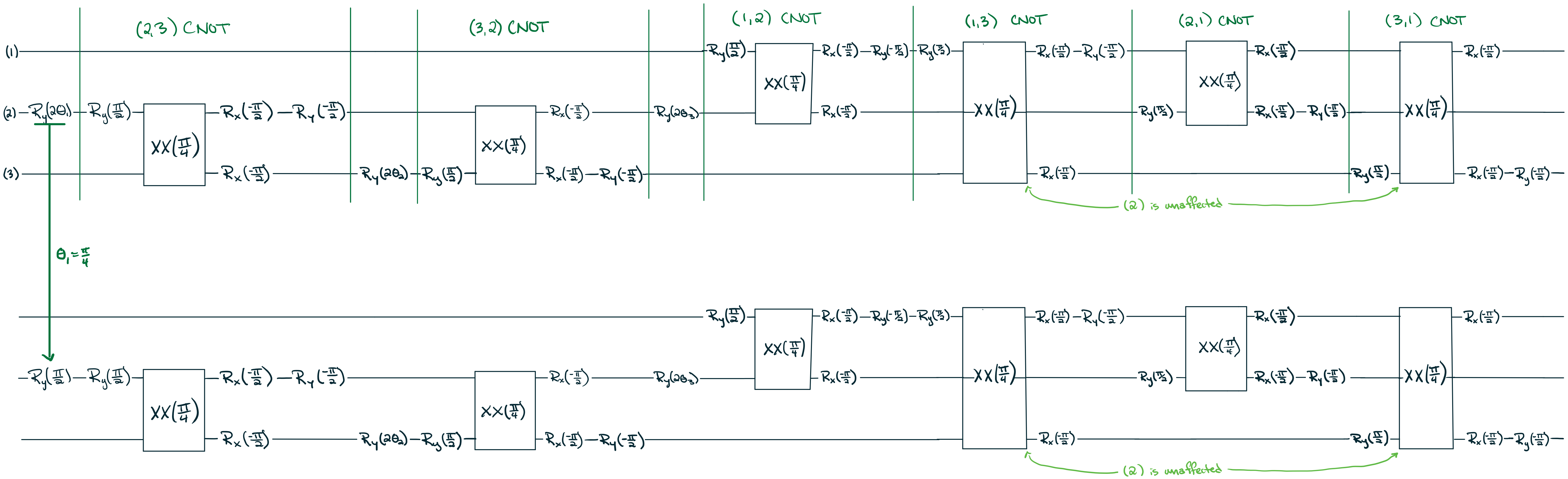
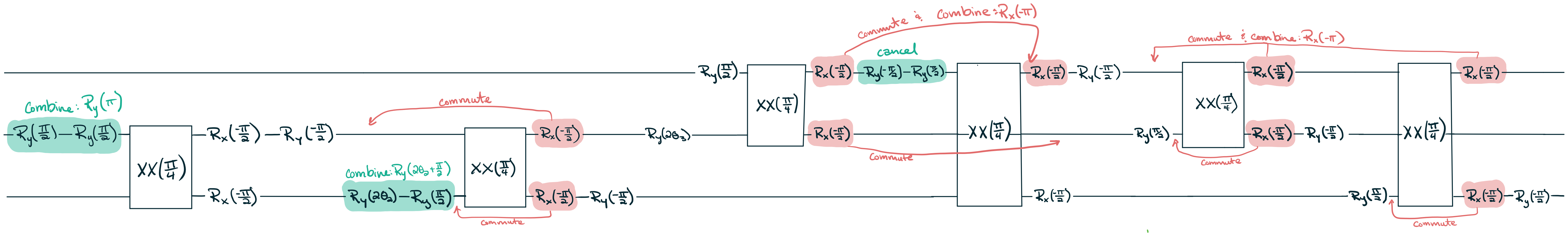
Original:

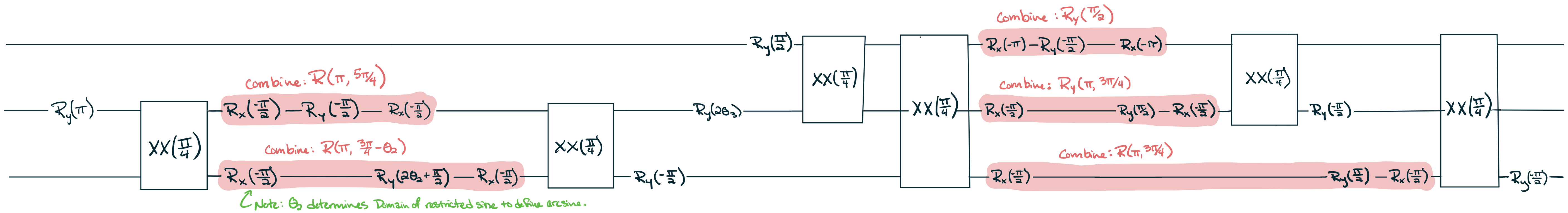


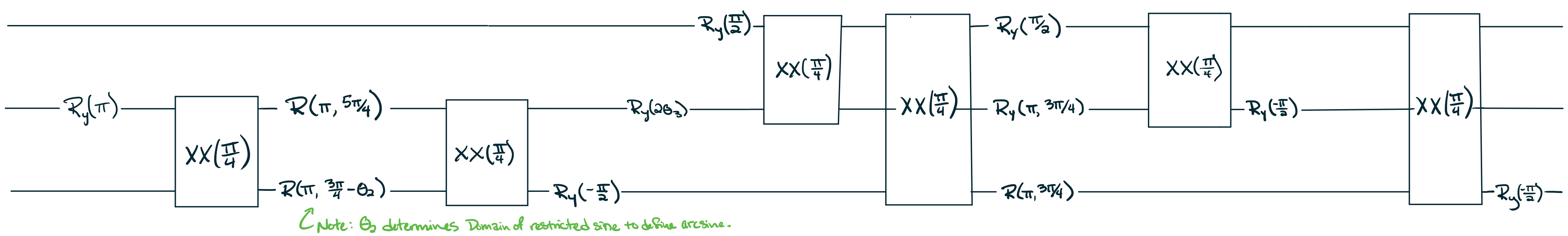
Converting CNOT to XX, R_x, & R_y



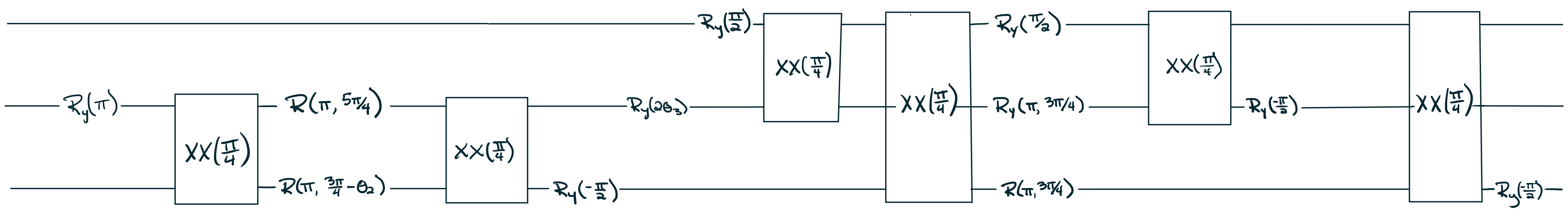
Simplifying using $R(\pi,\theta)$ & Maslov Identities







Converting to IonQ Native Gates



$XX(\frac{\pi}{4}) \longrightarrow MS(0,0)$

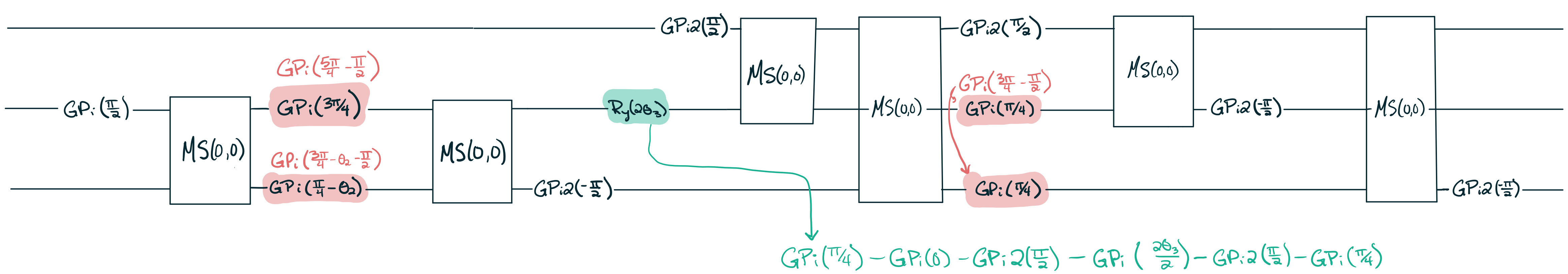
$R(\pi, \theta) \longrightarrow GPi(\theta - \frac{\pi}{2})$

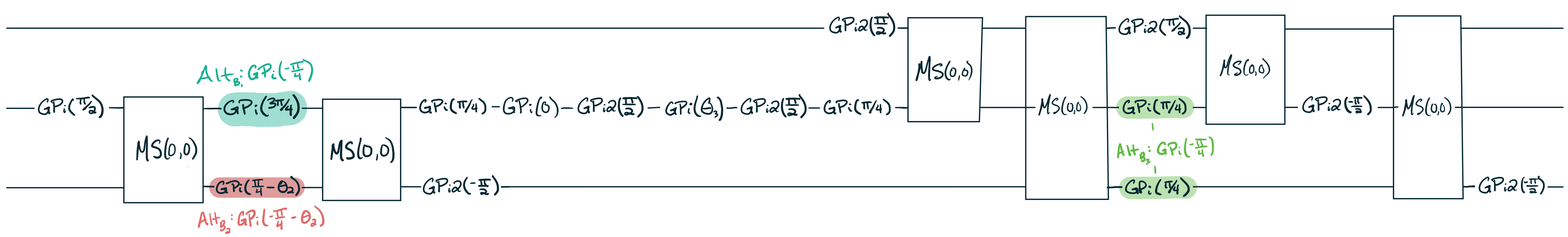
$R_y(\frac{\pi}{2}) \longrightarrow GPi2(\pi/2)$

$R_y(-\frac{\pi}{2}) \longrightarrow GPi2(-\pi/2)$

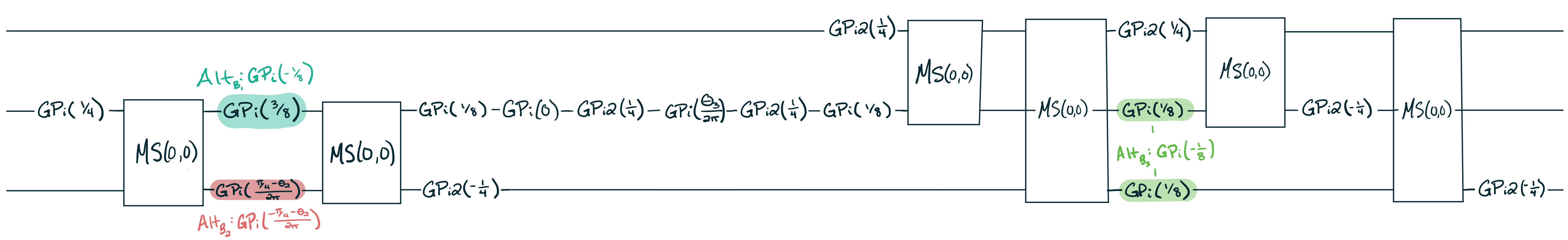
$R_y(\pi) \longrightarrow GPi(\frac{\pi}{2})$

$R_y(\theta) \longrightarrow GPi(\pi/4) - GPi(0) - GPi2(\pi/2) - GPi(\theta/2) - GPi2(\pi/2) - GPi(\pi/4)$





Turns:



Equivalences/Calculations for Alternates:

$$\mathrm{Alt}_{B_1}: \; GP_i\left(-\tfrac{\pi}{4}\right) \equiv R\left(\pi, \tfrac{\pi}{4}\right) = e^{\pi i}\begin{bmatrix} 0 & e^{\pi i/4} \\ -e^{-\pi i/4} & 0 \end{bmatrix} = e^{\pi i}\, GP_i\left(-\tfrac{\pi}{4}\right)$$

Note $R_x\left(\frac{-\pi}{2}\right) \cdot R_y\left(\frac{-\pi}{2}\right) \cdot R_x\left(\frac{-\pi}{2}\right) = \begin{bmatrix} 0 & e^{\pi i/4} \\ -e^{-\pi i/4} & 0 \end{bmatrix} \rightsquigarrow \begin{cases} c = \pi \\ d = \pi - \arcsin\left(-\frac{1}{\sqrt{2}}\right) = \frac{5\pi}{4} \end{cases}$

by Maslov $R_x(a) \cdot R_y(b) \cdot R_x(a) \equiv R(\theta, \phi) = \begin{bmatrix} \cos\frac{\theta}{2} & -ie^{-i\phi}\sin\frac{\theta}{2} \\ -ie^{i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$ where $\theta = \arccos\left(\cos(a)\cos\left(\frac{b}{2}\right)\right)$

& $\phi = \begin{cases} \arcsin\left(\dfrac{\sin(b/2)}{\sqrt{1-\cos^2(a)\cos^2(b/2)}}\right), & a > 0 \\ \pi - \arcsin\left(\dfrac{\sin(b/2)}{\sqrt{1-\cos^2(a)\cos^2(b/2)}}\right), & a < 0 \end{cases}$

$$\mathrm{Alt}_{B_2}: GP_i\left(-\tfrac{\pi}{4} - \theta_2\right) = \begin{bmatrix} 0 & e^{\frac{\pi i}{4} + \theta_2 i} \\ e^{-\frac{\pi i}{4} - \theta_2 i} & 0 \end{bmatrix} \equiv e^{3\pi i/2}\Bigg(\underbrace{e^{\frac{-\pi i}{2}}\begin{bmatrix} 0 & e^{-\frac{3\pi i}{4} + \theta_2 i} \\ e^{\frac{3\pi i}{4} - \theta_2 i} & 0 \end{bmatrix}}_{R\left(\pi, \frac{3\pi}{4} - \theta_2\right) \equiv R_x\left(\frac{-\pi}{2}\right) - R_y\left(2\theta_2 + \frac{\pi}{2}\right) - R_x\left(\frac{-\pi}{2}\right)}\Bigg)$$

$$\begin{cases} c = \pi \\ d = \pi - \arcsin\left(\sin\left(\theta_2 + \frac{\pi}{4}\right)\right) = \frac{3\pi}{4} - \theta_2 \end{cases}$$

$\left(2\theta_2 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow \theta_2 \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \Rightarrow \theta_2 + \frac{\pi}{4} \in \left[0, \frac{\pi}{2}\right]\right.$

& restricted sine def. on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\mathrm{Alt}_{B_3}: GP_i\left(-\tfrac{\pi}{4}\right) = \begin{bmatrix} 0 & e^{\pi i/4} \\ e^{-\pi i/4} & 0 \end{bmatrix} = e^{\frac{\pi i}{2}}\underbrace{\begin{bmatrix} 0 & -e^{-\pi i/4} \\ e^{\pi i/4} & 0 \end{bmatrix}}_{R_x\left(-\frac{\pi}{2}\right) - R_y\left(\frac{\pi}{2}\right) - R_x\left(\frac{-\pi}{2}\right) \equiv R\left(\pi, \frac{3\pi}{4}\right)}$$

$: \; c = \pi, \; d = \pi - \arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$

Basis X:

initial state $|-\rangle$:
(bit value 1)

$$— \overset{-\frac{\pi}{2}}{GPi2\left(-\tfrac{1}{4}\right)} —$$

initial state $|+\rangle$:
(bit value 0)

$$— \overset{\frac{\pi}{2}}{GPi2\left(\tfrac{1}{4}\right)} — GPi(0) —$$

Measurement Prep:

$$— GPi(0) — \overset{-\frac{\pi}{2}}{GPi2\left(-\tfrac{1}{4}\right)} —$$

$$— GPi(0) — GPi2\left(-\tfrac{1}{4}\right) —$$

$$— GPi(0) — GPi2\left(-\tfrac{1}{4}\right) —$$

Hadamard:

$$GPi(0) - GPi2\left(-\tfrac{1}{4}\right)$$

$$= GPi2\left(\tfrac{1}{4}\right) - GPi(0)$$

Basis Y:

initial State $|-i\rangle$:
(bit value 1)

$$— \overset{\frac{\pi}{2}}{GPi2\left(\tfrac{1}{4}\right)} — \overset{-\frac{\pi}{4}}{GPi\left(-\tfrac{1}{8}\right)} —$$

initial State $|i\rangle$:
(bit value 0)

$$— \overset{\frac{\pi}{2}}{GPi2\left(\tfrac{1}{4}\right)} — \overset{\frac{\pi}{4}}{GPi\left(1/8\right)} —$$

Measurement Prep:

$$— \overset{\frac{\pi}{4}}{GPi\left(\tfrac{1}{8}\right)} — \overset{-\frac{\pi}{2}}{GPi2\left(-\tfrac{1}{4}\right)} —$$

$$— GPi\left(1/8\right) — GPi2\left(-\tfrac{1}{4}\right) —$$

$$— GPi\left(1/8\right) — GPi2\left(-\tfrac{1}{4}\right) —$$

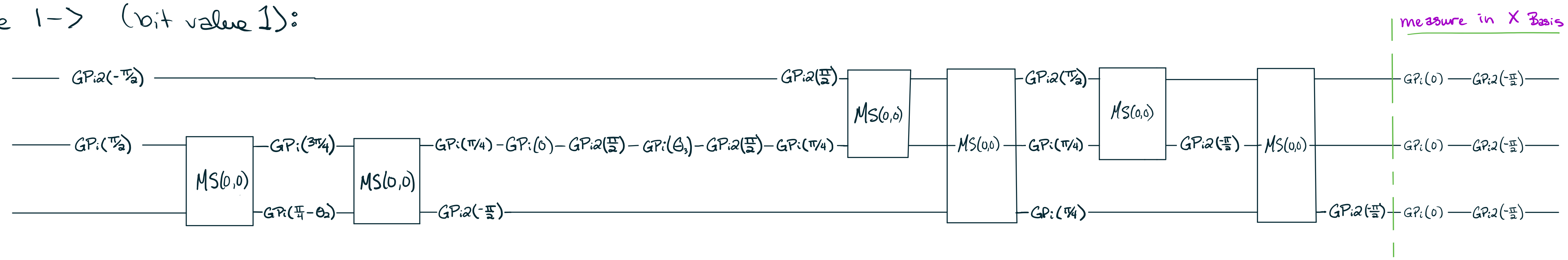
Alt_B: $GPi2(0)$

Since

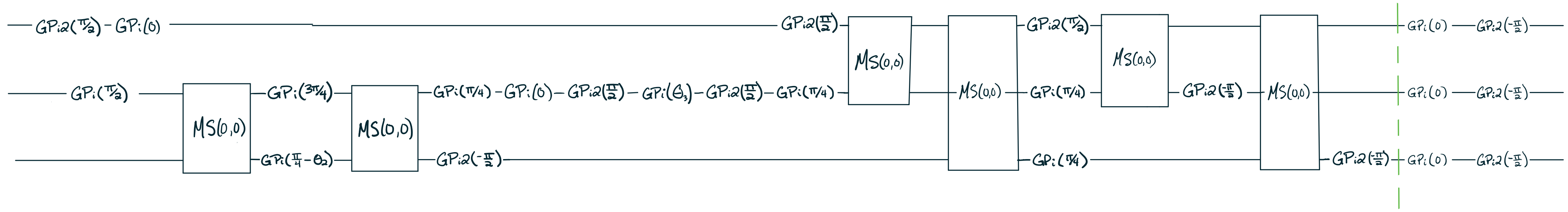
$$GPi2\left(-\tfrac{1}{4}\right) = GPi(0)\,GPi2\left(\tfrac{1}{4}\right)\,GPi(0)$$

Basis X:

initial State |-> (bit value 1):

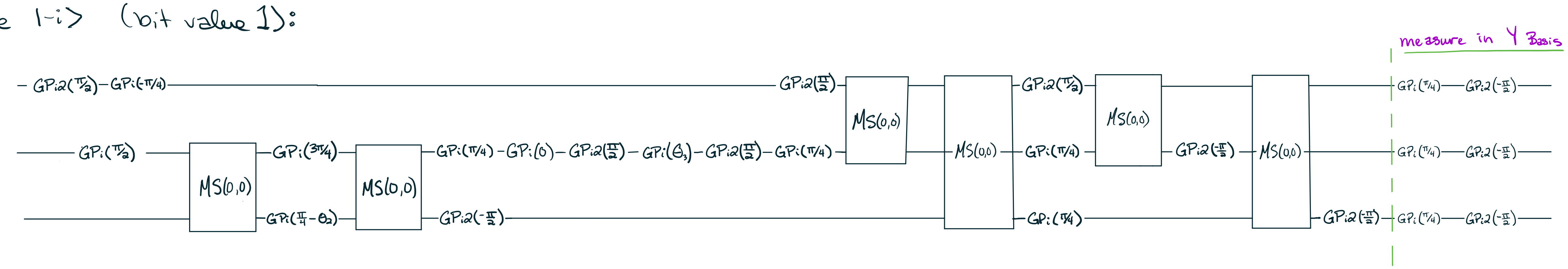


initial State |+> (bit value 0):



Basis Y:

initial State |-i> (bit value 1):



initial State |i> (bit value 0):