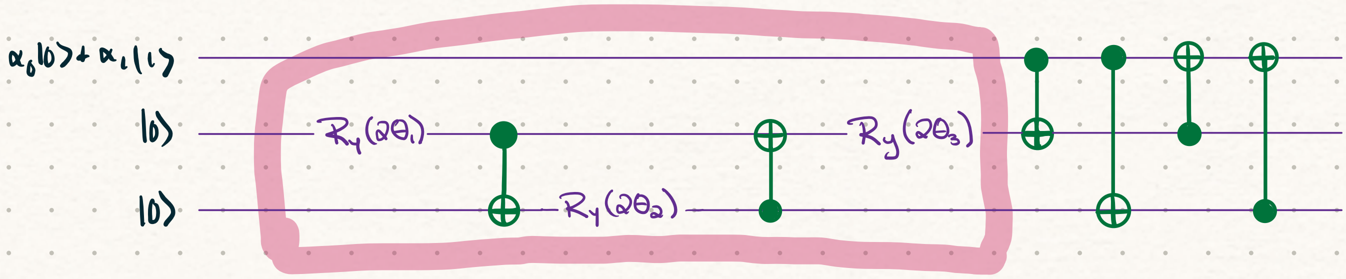
$P_{kl}|0\rangle_k|0\rangle_l = |0\rangle_k|0\rangle_l$ $\quad$ $P_{kl}|0\rangle_k|1\rangle_l = |0\rangle_k|1\rangle_l$

$P_{kl}|1\rangle_k|0\rangle_l = |1\rangle_k|1\rangle_l$ $\quad$ $P_{kl}|1\rangle_k|1\rangle_l = |1\rangle_k|0\rangle_l$

$|\,\rangle_k$ is control qubit so $|\,\rangle_l$ changes if we have $|1\rangle_k$

$R_j(\theta)|0\rangle_j = \cos\theta|0\rangle_j + \sin\theta|1\rangle_j$

$R_j(\theta)|1\rangle_j = -\sin\theta|0\rangle_j + \cos\theta|1\rangle_j$



Prep

$$R_2(\theta_1)|0\rangle = \cos\theta_1|0\rangle + \sin\theta_1|1\rangle$$

$$P_{23}\,(\cos\theta_1|0\rangle + \sin\theta_1|1\rangle)|0\rangle_3 = \cos\theta_1|0\rangle|0\rangle + \sin\theta_1|1\rangle|1\rangle$$

$$R_3(\theta_2)(\cos\theta_1|0\rangle|0\rangle + \sin\theta_1|1\rangle|1\rangle) = \cos\theta_1|0\rangle(\cos\theta_2|0\rangle + \sin\theta_2|1\rangle) + \sin\theta_1|1\rangle(-\sin\theta_2|0\rangle + \cos\theta_2|1\rangle)$$

$$= \cos\theta_1\cos\theta_2|0\rangle|0\rangle + \cos\theta_1\sin\theta_2|0\rangle|1\rangle - \sin\theta_1\sin\theta_2|1\rangle|0\rangle + \sin\theta_1\cos\theta_2|1\rangle|1\rangle$$

$$P_{32}(\cos\theta_1\cos\theta_2|0\rangle|0\rangle + \cos\theta_1\sin\theta_2|0\rangle|1\rangle - \sin\theta_1\sin\theta_2|1\rangle|0\rangle + \sin\theta_1\cos\theta_2|1\rangle|1\rangle)$$

$$= \cos\theta_1\cos\theta_2|0\rangle|0\rangle + \cos\theta_1\sin\theta_2|1\rangle|1\rangle - \sin\theta_1\sin\theta_2|1\rangle|0\rangle + \sin\theta_1\cos\theta_2|0\rangle|1\rangle$$

$$R_2(\theta_3)(\cos\theta_1\cos\theta_2|0\rangle|0\rangle + \cos\theta_1\sin\theta_2|1\rangle|1\rangle - \sin\theta_1\sin\theta_2|1\rangle|0\rangle + \sin\theta_1\cos\theta_2|0\rangle|1\rangle)$$

$$\cos\theta_1\cos\theta_2(\cos\theta_3|0\rangle + \sin\theta_3|1\rangle)|0\rangle + \cos\theta_1\sin\theta_2(-\sin\theta_3|0\rangle + \cos\theta_3|1\rangle)|1\rangle$$

$$- \sin\theta_1\sin\theta_2(-\sin\theta_3|0\rangle + \cos\theta_3|1\rangle)|0\rangle + \sin\theta_1\cos\theta_2(\cos\theta_3|0\rangle + \sin\theta_3|1\rangle)|1\rangle$$

So out from prep is

$$\cos\theta_1\cos\theta_2\cos\theta_3|00\rangle + \cos\theta_1\cos\theta_2\sin\theta_3|10\rangle - \cos\theta_1\sin\theta_2\sin\theta_3|01\rangle$$

$$+ \cos\theta_1\sin\theta_2\cos\theta_3|11\rangle + \sin\theta_1\sin\theta_2\sin\theta_3|00\rangle - \sin\theta_1\sin\theta_2\cos\theta_3|10\rangle$$

$$+ \sin\theta_1\cos\theta_2\cos\theta_3|01\rangle + \sin\theta_1\cos\theta_2\sin\theta_3|11\rangle$$

$$C_1|00\rangle + C_2|01\rangle + C_3|10\rangle + C_4|11\rangle$$

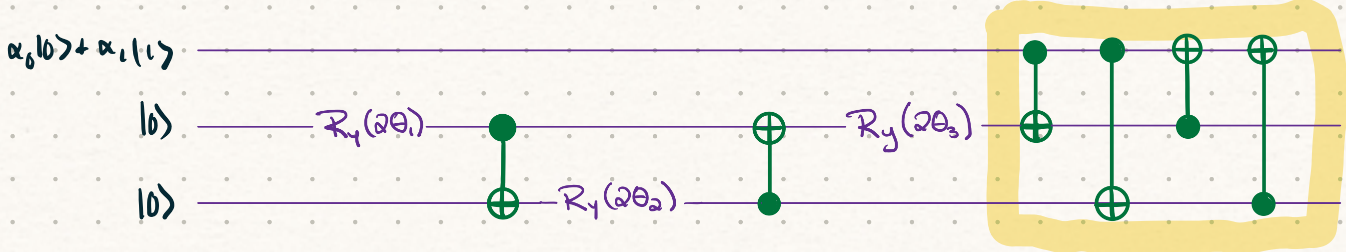
Prep output verified ✓

$C_1 = \cos\theta_1\cos\theta_2\cos\theta_3 + \sin\theta_1\sin\theta_2\sin\theta_3$

$C_2 = \sin\theta_1\cos\theta_2\cos\theta_3 - \cos\theta_1\sin\theta_2\sin\theta_3$

$C_3 = \cos\theta_1\cos\theta_2\sin\theta_3 - \sin\theta_1\sin\theta_2\cos\theta_3$

$C_4 = \cos\theta_1\sin\theta_2\cos\theta_3 + \sin\theta_1\cos\theta_2\sin\theta_3$



Cloning $\alpha_0|0\rangle + \alpha_1|1\rangle$

$$P_{12}\left((\alpha_0|0\rangle+\alpha_1|1\rangle)(c_1|00\rangle+c_2|01\rangle+c_3|10\rangle+c_4|11\rangle)\right)$$

$$\alpha_0 c_1|0\rangle|00\rangle + \alpha_0 c_2|0\rangle|01\rangle + \alpha_0 c_3|0\rangle|10\rangle + \alpha_0 c_4|0\rangle|11\rangle$$
$$+ \alpha_1 c_1|1\rangle|10\rangle + \alpha_1 c_2|1\rangle|11\rangle + \alpha_1 c_3|1\rangle|00\rangle + \alpha_1 c_4|1\rangle|01\rangle$$

$$\Big\downarrow P_{13}$$

$$\alpha_0 c_1|0\rangle|00\rangle + \alpha_0 c_2|0\rangle|01\rangle + \alpha_0 c_3|0\rangle|10\rangle + \alpha_0 c_4|0\rangle|11\rangle$$
$$+ \alpha_1 c_1|1\rangle|11\rangle + \alpha_1 c_2|1\rangle|10\rangle + \alpha_1 c_3|1\rangle|01\rangle + \alpha_1 c_4|1\rangle|00\rangle$$

$$\Big\downarrow P_{21}$$

$$\alpha_0 c_1|0\rangle|00\rangle + \alpha_0 c_2|0\rangle|01\rangle + \alpha_0 c_3|1\rangle|10\rangle + \alpha_0 c_4|1\rangle|11\rangle$$
$$+ \alpha_1 c_1|0\rangle|11\rangle + \alpha_1 c_2|0\rangle|10\rangle + \alpha_1 c_3|1\rangle|01\rangle + \alpha_1 c_4|1\rangle|00\rangle$$

$$\Big\downarrow P_{31}$$

$$\alpha_0 c_1|0\rangle|00\rangle + \alpha_0 c_2|1\rangle|01\rangle + \alpha_0 c_3|1\rangle|10\rangle + \alpha_0 c_4|0\rangle|11\rangle$$
$$+ \alpha_1 c_1|1\rangle|11\rangle + \alpha_1 c_2|0\rangle|10\rangle + \alpha_1 c_3|0\rangle|01\rangle + \alpha_1 c_4|1\rangle|00\rangle$$

$$= \boxed{\begin{aligned}&\alpha_0 c_1|000\rangle + \alpha_0 c_2|101\rangle + \alpha_0 c_3|110\rangle + \alpha_0 c_4|011\rangle \\ &+ \alpha_1 c_1|111\rangle + \alpha_1 c_2|010\rangle + \alpha_1 c_3|001\rangle + \alpha_1 c_4|100\rangle\end{aligned}}$$

Cloner output ✓

Note that running cloner backwards switches c_2 & c_3

Rezakhani: $|i\rangle_A |0\rangle_B |\Sigma\rangle_X \rightarrow \mu |i\rangle_A |i\rangle_B |i\rangle_X + \nu |i\rangle_A |j\rangle_B |j\rangle_X + \xi |j\rangle_A |i\rangle_B |j\rangle_X$

$i=0$: $\mu |000\rangle + \nu |011\rangle + \xi |101\rangle$

$i=1$: $\mu |111\rangle + \nu |100\rangle + \xi |010\rangle$

$$\alpha_0 C_1 |000\rangle + \alpha_0 C_2 |101\rangle + \alpha_0 C_3 |110\rangle + \alpha_0 C_4 |011\rangle$$
$$+ \alpha_1 C_1 |111\rangle + \alpha_1 C_2 |010\rangle + \alpha_1 C_3 |001\rangle + \alpha_1 C_4 |100\rangle$$

$\therefore \quad C_1 = \mu \quad C_2 = \xi \quad C_3 = 0 \quad C_4 = \nu$