ELECTRICITY DATA ANALYSIS BASED ON SPARSE CODING

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INTRODUCTION

- Energy consumption of both residential and commercial buildings has steadily increased, reaching figures between 20% and 40% in developed countries.
- The rapidly growing has already raised concerns over supply difficulties, exhaustion of energy resources and heavy environmental impacts.
- There is an urgent need to develop technologies that examine energy usage in homes.

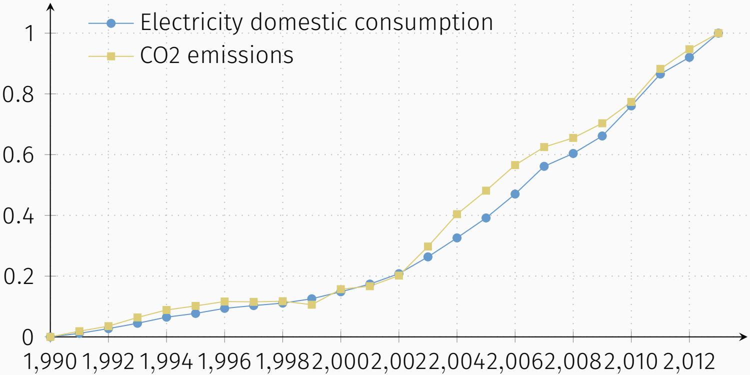


Figure 1: Electricity domestic consumption and CO_2 emissions in China. Source: Enerdata.

THE GOAL

- Find behavioral patterns and outlier detection.
- Helpful for building energy conservation services.
- Give residents feedback on energy consumption.
- Encourage energy efficient behaviors.

Classification and clustering algorithms:

- K-Means [Chen and Cook]
- CART [LIU, CHEN, MORI and KIDA]
- DBSCAN [Khan, Capozzoli, Corgnati and Cerquitelli]

Statistical Approach:

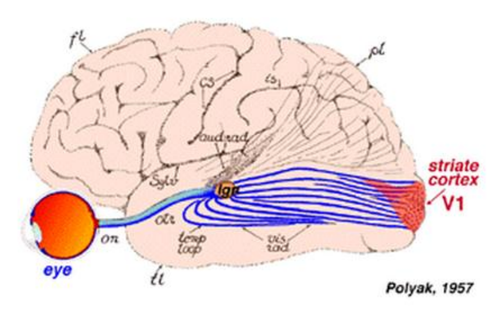
- Generalized Extreme Studentized Deviate (GESD) [John E. Seem]
- boxplot statistical method [John E. Seem]

OUR MODEL

In this report, we present a novel approach for finding outliers and pattern recognition based on **sparse model** that include the follow two steps:

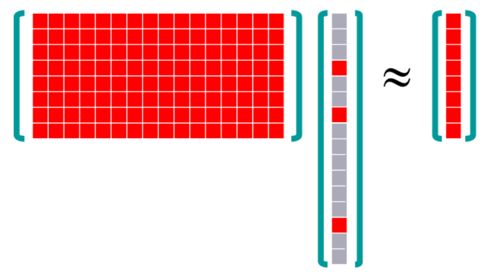
- Sparse Coding
- Dictionary Learning

Applications that use this model are many and include compression, feature extraction, image denoising and more.



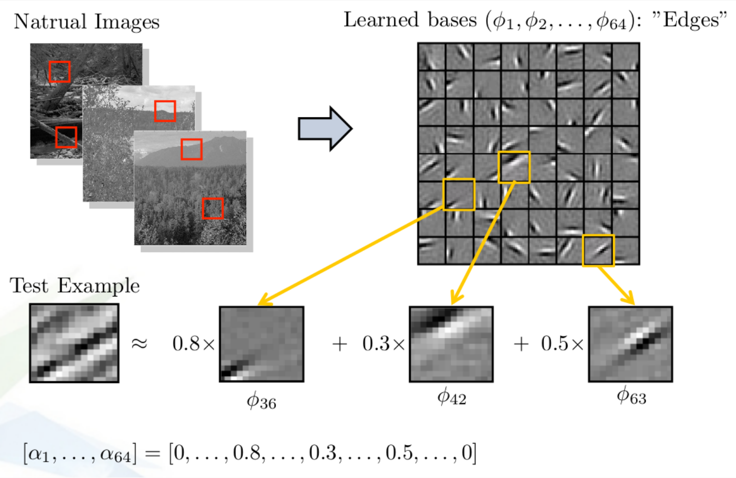
Neuroscience studies suggest that primary visual cortex (area V1) uses a sparse code to efficiently represent natural scenes.

Bruno A Olshausen et al. Emergence of simple-cell receptive field properties by learning a sparse code for natural images. Nature, 381(6583):607–609,1996.



$$D \quad x \quad \approx \quad y$$

x is sparse. **Sparsity** implies many zeros in a vector.


Natrual Images
Learned bases ($\phi_1, \phi_2, \ldots, \phi_{64}$): "Edges"
Test Example
$\approx$ $0.8\times$ $+$ $0.3\times$ $+$ $0.5\times$
ϕ_{36}
ϕ_{42}
ϕ_{63}
$[\alpha_1, \ldots, \alpha_{64}] = [0, \ldots, 0.8, \ldots, 0.3, \ldots, 0.5, \ldots, 0]$

Generally, we expect sparse solutions, a obvious idea is find the solution with the fewest number of non-zero coefficients:

$$\min_{\mathbf{x}} ||\mathbf{x}||_0 \quad \text{subject to} \quad \mathbf{y} = \mathbf{D}\mathbf{x}, \tag{1}$$

or

$$\min_{\mathbf{x}} ||\mathbf{x}||_0 \quad \text{subject to} \quad ||\mathbf{y} - \mathbf{D}\mathbf{x}||_2 < \epsilon \tag{2}$$

Here $||\cdot||_0$ is the l^0 norm, counting the number of non-zeros of a vector.

Unfortunately, (1) is not a convex optimisation problem. In fact, it is an NP-hard problem in general[Davis, Mallat and Avellaneda].

Geoff Davis, Stephane Mallat, and Marco Avellaneda. Adaptive greedy approximations. Constructive approximation, 13(1):57–98, 1997.

Greedy algorithms:

- *Matching Pursuit* (MP)[Mallat and Zhang]
- *Orthogonal Matching Pursuit* (OMP)[Pati, Rezaiifar and Krishnaprasad]

l^1 minimisation or *Basis Pursuit*[Chen, Donoho, and Saunders]:

$$\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{subject to} \quad \mathbf{y} = \mathbf{D}\mathbf{x} \tag{3}$$

Stéphane G Mallat and Zhifeng Zhang. Matching pursuits with time-frequency dictionaries. Signal Processing, IEEE Transactions on, 41(12):3397–3415, 1993.

Yagyensh Chandra Pati, Ramin Rezaiifar, and PS Krishnaprasad. Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition. In Signals, Systems and Computers, 1993. 1993 Conference Record of The Twenty-Seventh Asilomar Conference on, pages 40–44. IEEE, 1993.

Scott Shaobing Chen, David L Donoho, and Michael A Saunders. Atomic decomposition by basis pursuit. SIAM journal on scientific computing, 20(1):33–61, 1998.

If the data has some noises, we should apply *Basis Pursuit Denoising* (BPDN):

$$\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{subject to} \quad ||\mathbf{y} - \mathbf{D}\mathbf{x}||_2 < \epsilon \tag{4}$$

or equivalent

$$\min_{\mathbf{x}} \frac{1}{2}||\mathbf{y} - \mathbf{D}\mathbf{x}||_2^2 + \lambda||\mathbf{x}||_1 \tag{5}$$

(5) is also known as *Lasso*[Tibshirani] in statistics.

Robert Tibshirani. Regression shrinkage and selection via the lasso. Journal of the Royal Statistical Society. Series B (Methodological), pages 267–288, 1996.

The geometric explanation:

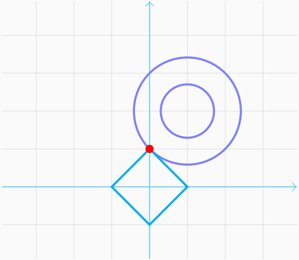


Figure 2: l^1-norm

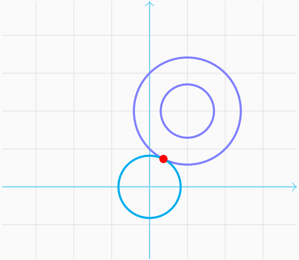


Figure 3: l^2-norm

Rigorous proof by Candès, Romberg and Tao.

Emmanuel J Candès, Justin Romberg, and Terence Tao. Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. Information Theory, IEEE Transactions on, 52(2):489–509, 2006.

How can we get the dictionary $\mathbf{D}$?

1. Predefined dictionary

- DCT Basis
- Wavelet
- ...

2. Adaptive dictionary - learn from data

- K-SVD (l^0-norm)
- Online dictionary learning (l^1-norm)
- ...

K-SVD is a generalization of K-means algorithm and also an iterative method that alternates between sparse coding and The update of the dictionary[Aharon, Elad and Bruckstein].

Objective function:

$$\min_{\mathbf{D},\mathbf{X}} \left\{||\mathbf{Y} - \mathbf{D}\mathbf{X}||_F^2\right\} \qquad \forall i, ||\mathbf{x}_i||_0 \leq T_0. \tag{6}$$

Michal Aharon, Michael Elad, and Alfred Bruckstein. K-svd: An algorithm for designing overcomplete dictionaries for sparse representation. Signal Processing, IEEE Transactions on, 54(11):4311–4322, 2006.

We will use the online dictionary learning algorithm[Mairal, Bach, Ponce and Sapiro] in our model to analyze energy data. This algorithm is also widely used in many libraries such as scikit-learn, SPAMS, etc.

Different from K-SVD, the online dictionary learning algorithm uses l^1 rather than l^0 penalties.

Objective function:

$$\min_{\mathbf{D}\in\mathcal{C},\alpha\in\mathbb{R}^{k\times n}} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{2}||\mathbf{x}_i - \mathbf{D}\alpha_i||_2^2 + \lambda||\alpha_i||_1\right). \tag{7}$$

Julien Mairal, Francis Bach, Jean Ponce, and Guillermo Sapiro. Online dictionary learning for sparse coding. In Proceedings of the 26th Annual International Conference on Machine Learning, pages 689–696. ACM, 2009.

Similar with K-SVD, the Online Dictionary Learning is also an iterative method.

In Sparse Coding, draws one element $\mathbf{x}_t$ at a time, then compute using LARS:

$$\alpha_t \triangleq \operatorname*{argmin}_{\alpha \in \mathbb{R}^k} \frac{1}{2} ||\mathbf{x}_t - \mathbf{D}_{t-1}\alpha||_2^2 + \lambda ||\alpha||_1. \tag{8}$$

In Dictionary Learning, compute $\mathbf{D}_t$ using gradient descent:

$$\begin{aligned} \mathbf{D}_t &\triangleq \operatorname*{argmin}_{\mathbf{D} \in \mathcal{C}} \frac{1}{t} \sum_{i=1}^{t} \frac{1}{2} ||\mathbf{x}_i - \mathbf{D}\alpha_i||_2^2 + \lambda ||\alpha_i||_1, \\ &= \operatorname*{argmin}_{\mathbf{D} \in \mathcal{C}} \frac{1}{t} \left(\frac{1}{2} \mathrm{Tr}(\mathbf{D}^T \mathbf{D} \mathbf{A}_t) - \mathrm{Tr}(\mathbf{D}^T \mathbf{B}_t) \right). \end{aligned} \tag{9}$$

EXPERIMENT

Through some city electric power company, we collect the data of residents electricity consumption range from 2013-1-1 to 2014-6-23 , a total of 538 days.

Table 1: Consumption data

User	Day 1	Day 2	Day 3	Day 4	Day 5	...	Day 535	Day 536	Day 537	Day 538
1	11.37	21.03	12.22	14.79	28.59	...	13.41	10.11	6.71	8.06
2	38.14	84.02	80.92	37.64	2.50	...	5.24	8.54	6.34	6.44
3	15.25	28.51	16.20	18.63	23.72	...	7.10	6.15	6.38	7.51
4	1.80	2.70	19.31	48.26	2.22	...	24.37	17.01	16.39	17.25
5	2.15	11.03	2.63	75.19	87.84	...	15.94	17.15	16.53	13.02
...	...	...	...	...	...	...	...	...	...	...
4314	31.92	22.99	20.86	11.10	17.49	...	12.30	11.60	9.31	8.82
4315	8.48	5.61	5.85	6.19	4.92	...	5.61	4.75	5.57	9.21
4316	2.33	2.15	1.07	1.45	1.43	...	2.07	1.98	3.15	2.90

We transform the formal data of each row to a pattern organized by week and the new row of data is made up of 7 days.

Table 2: Weekly consumption data

User	Week	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1	1	11.37	21.03	12.22	14.79	28.59	16.64	7.80
	2	6.63	23.18	9.17	14.36	21.91	5.11	7.79
	3	5.64	11.45	7.48	10.13	7.27	7.11	7.38
	4	4.96	6.14	9.47	11.08	10.33	5.70	9.94
	5	4.08	4.95	3.97	8.94	6.91	6.26	4.90
	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...
4316	...	...	...	...	...	...	...	...
	72	1.32	1.89	1.91	2.02	1.20	2.97	2.17
	73	2.52	1.39	3.58	2.56	3.52	3.79	6.09
	74	3.54	5.60	5.21	3.66	3.06	3.68	1.92
	75	2.79	2.07	2.14	3.65	2.39	2.18	1.91
	76	6.44	2.58	6.76	2.77	3.01	1.73	1.93

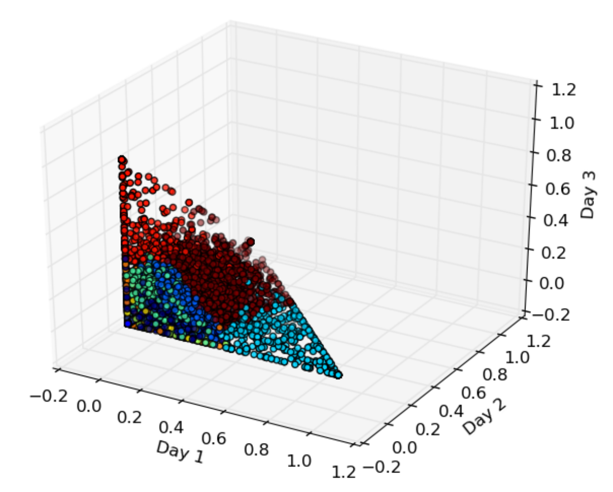


Figure 4: K-Means

Use Sparse Coding first, then apply K-Means to sparse matrix.

Test on digits dataset:

- K-Means: 467/1797
- This method: 280/1797

Test on iris dataset:

- K-Means: 16/150
- This method: 16/150

The each column of **D** is a energy pattern. One resident's consumption data can be represented as a linear combination of some patterns. we consider an energy pattern d_j to be an outlier, if this energy pattern is rarely be used in the linear combination.

We use the score below to describe the pattern's degree of usage:

$$f(j) = \sum_{k=1}^{N} g\left(\frac{\sum_{i \in U_k} \frac{|\alpha_{ij}|}{\sum_{q=1}^{M} |\alpha_{iq}|}}{|U_k|}\right)$$

where U_k is the set of rows belong to user k, $|U_k|$ is the number of elements in the set U_k, N is the number of users, M is the number of basis and $g(x)$ is a function we choose to reduce single user affect in the outliers score. For each pattern d_j, it has a score $f(j)$.

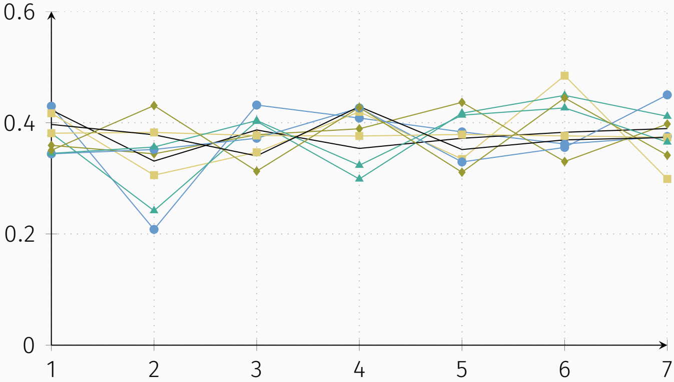


Figure 5: The basis most frequently used

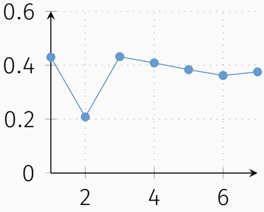


Figure 6: One of most used basis

User	Week	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
200	54	109.29	69.47	107.25	144.24	99.86	95.21	109.30
1537	59	107.50	64.14	103.02	95.95	91.43	106.42	85.58
1675	20	147.68	121.69	156.19	148.74	125.92	127.63	148.35
1769	64	80.19	46.58	81.96	86.65	72.62	82.17	80.93
1474	60	124.23	63.35	128.49	119.88	127.36	104.49	114.25

Table 3: The users use this basis

We can easily find from the two figures that they have the same behavior as the pattern, so we can use the pattern to represent these users' behavior in this week.

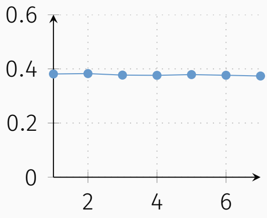


Figure 7: Another basis most used

User	Week	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
739	29	190.28	193.52	182.01	189.68	200.06	194.81	193.41
1469	66	140.33	141.95	140.46	141.91	146.73	140.49	139.42
	39	144.40	145.02	146.36	145.97	148.70	151.53	145.64
1675	32	243.09	239.92	234.66	212.71	229.12	228.68	221.51
	29	239.88	240.09	224.31	236.91	230.27	220.61	218.10

Table 4: The users use this basis

We see that these users are nearly unchangeable in one week and have very large consumption. If you observes these users' data in other weeks, for example, users 1469, you can find that the data is also similar with this pattern.

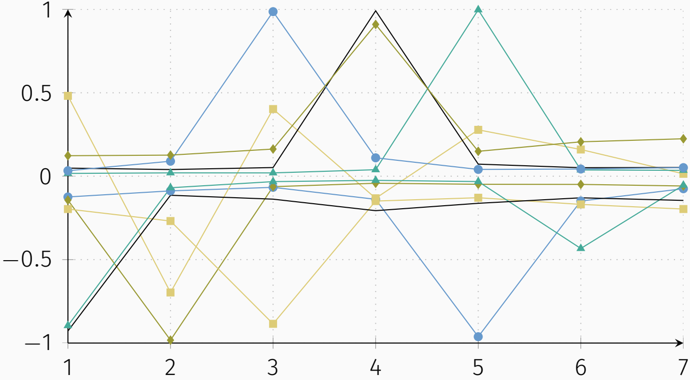


Figure 8: The basis of outliers

EXAMPLE

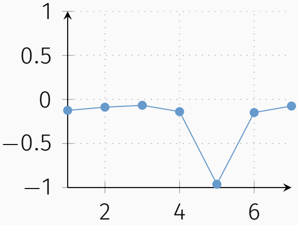


Figure 9: One basis of outliers

User	Week	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1768	57	35.44	21.80	11.23	14.93	51.19	33.71	31.92
1810	33	16.73	9.30	12.57	8.92	31.95	9.42	3.82
19	33	30.90	27.32	10.18	16.00	59.64	26.22	14.84
4	55	32.59	39.48	1.59	33.45	69.56	40.48	32.84
25	6	112.36	0.00	1.94	39.05	84.06	44.72	13.00

Table 5: The users use this basis

CONCLUSION

- In this report, we presented a novel approach for analyzing energy consumption data.
- By using Sparse Coding method, we detect the outliers which less used.
- This will be helpful for building energy conservation services and give residents feedback about energy efficient behaviors.

FUTURE WORK

- Improve model to adapt the energy data.
- Test using more data in more powerful machine or hadoop cluster.

Thanks to:

- My advisor: Prof. Weiguo Gao
- My partner: Li Zhou

Thank You!