MOMENTUM KICK TO A PROTON TRAVERSING AN ELECTRON LENS

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Abstract

I calculate the momentum kick for a proton passing through an electron lens with a cylindrically round Gaussian density profile. This calculation is valid for non-relativistic protons.

INTRODUCTION

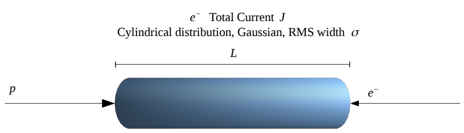

Figure 1: Diagram of electron lens configuration

I consider the momentum kick of a proton passing through an electron lens of length L with a cylindrically round Gaussian density profile. The proton flow direction is opposite to the electron progagation so the EM force is focussing for protons to counteract the defocussing space charge force. The configuration is illustrated in Fig. 1.

Table 1: Parameters and symbols used in the calculation

Parameter	**Definition**
J	Electron current
L	Electron lens length
e	unit of electric charge
c	speed of light
m	proton mass
r_p	proton classical radius $e^2/4\pi\epsilon_0 mc^2$
σ	RMS radius of current distribution
β_e	electron velocity/c
β_p	proton velocity/c
β_b	reference proton beam velocity/c
γ_b	reference proton beam relativistic factor

The names and symbols of parameters used in the calculation are shown in Table 1.

CALCULATION

The electric and magnetic fields experienced by a particle at radius r are determined the amount of charge and current countained within the cylinder of radius r. For the Gaussian profile, this is:

$$\int_0^r e^{-r^2/2\sigma^2} r dr = 1 - e^{-r^2/2\sigma^2}$$

Using 2D Gauss's law, the normal electric field in the electron lens at radius r is

$$E_n = -\frac{J}{2\pi\epsilon_0\beta_e c}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}$$

identifying the charge density is $J/\beta_e c$. The electric momentum kick $qE\Delta t$ for a single charge over length L is

$$\Delta p_E = -\frac{Je}{2\pi\epsilon_0\beta_e c}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}\frac{L}{\beta_p c}$$

The magnetic field from the 2D Ampere's law is

$$B_\theta = \frac{J}{2\pi\epsilon_0 c^2}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}$$

where I have used the identity $\epsilon_0\mu_0 = 1/c^2$. The magnetic momentum kick $qv \times B\Delta t$ for a single charge over length L is

$$\Delta p_M = -\frac{Je\beta_p c}{2\pi\epsilon_0 c^2}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}\frac{L}{\beta_p c}$$

The total momentum kick is the sum of the electric and magnetic kicks:

$$\Delta p_{E+M} = -\frac{JLe(1+\beta_e\beta_p)}{2\pi\epsilon_0\beta_e\beta_p c^2}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}$$

In Synergia, the particle momentum is normalized by the reference beam momentum, given by $m\beta_b\gamma_b c$. The momentum kick can be expressed by

$$\frac{\Delta p}{p_b} = -\frac{JLe(1+\beta_e\beta_p)}{2\pi\epsilon_0 mc^2\beta_e\beta_p\beta_b\gamma_b c}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}$$

which can be rewritten using the proton classical radius r_p as

$$\frac{\Delta p}{p_b} = -\frac{2JLr_p(1+\beta_e\beta_p)}{e\beta_e\beta_p\beta_b\gamma_b c}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}$$