I. WAKE FIELD FOR SYNERGIA

The wake field effects in Synergia are introduced using the split operator method. Every step, each macroparticle momentum is kicked according to

$$\beta c \Delta p_z = -qQW^{||}(z), \tag{1}$$

$$\beta c \Delta p_x = -qQ\left(W_X(z)X + W_x(z)x\right) \tag{2}$$

$$\beta c \Delta p_y = -qQ\left(W_Y(z)Y + W_y(z)y\right), \tag{3}$$

where Q, X, Y (q, x, y) represent the charge and horizontal and vertical displacements of the leading (trailing) particle. The wake functions, $W^{||}, W_X, W_Y, W_x, W_y$ depend only on the distance, z, between the leading and the trailing particle.

The Eqs. 1,2 and 3 are valid for vacuum chambers with horizontal and vertical mirror symmetry. For pipes with circular or rectangular symmetries, the detuning wakes $W_x(z), W_y(z)$ are zero in the ultrarelativistic limit .

A. Wake field file

The wake functions $W^{||}(z), W_X(z), W_Y(z), W_x(z), W_y(z)$ in Synergia simulations are read from a file.

1. Reading a wake file

A typical wake file has 6 columns (z, W_X, W_x, W_Y, W_y and $W^{||}$) or less if due to the symmetry some of the wakes are zero. The order of columns is important. **To make sure there is no mistake check the reading order corresponding to your *wake_type* parameter in wake_field.cc.**

Examples:

- wake_type="XLXTYLYTZ", requires 6 columns wake file, in order z, W_X (lead particle X transverse , i.e. XL), W_x (trailing particle X transverse, i.e XT), W_Y, (lead particle Y transverse , i.e. YL), W_y (trailing particle Y transverse, i.e YT) and $W^{||}$ (longitudinal wake, i.e. Z).

- wake_type="XLYLZ" (used for IOTA, see the wake file *IOTA_straight_rw_wake.dat*) requires 3 columns, in order z, $W_X = W_Y$ (lead particle X transverse equal to lead particle Y transverse , i.e. XLYL) and $W^{||}$ (longitudinal wake, i.e. Z).

2. Preparing a wake file

The units of the quantities in the wake file are:

- $z[m]$. $z = \beta ct$
- $\frac{W_X}{Z_0 L}[m^{-2}s^{-1}]$. The transverse wake per unit length, divided by the vacuum impedance $Z_0 \approx 376.7\Omega$. Valid for both leading and trailing transverse wakes.
- $\frac{W^{||}}{Z_0 L}[m^{-1}s^{-1}]$. The longitudinal wake per unit length, divided by the vacuum impedance $Z_0 \approx 376.7\Omega$.

z in the wake file is on a quadratic grid, i.e every row corresponds to an integer i such $z_i = i * |i| * \Delta z + \Delta_0$. i can start from negative values if the wake in front of the source particle is considered (at finite γ). Δz determines the grid resolution and $\Delta_0 \ll \Delta z$ is chosen to avoid the point $z = 0$.

Example:

The wake file for IOTA was calculated as described in *A. Macridin, et. al, FERMILAB-PUB-12-518-CD* for a circular pipe.

B. Impedance in Synergia

The ***Impedance*** object in Synergia is a collective operator and should be implemented in Synergia in the same way the space charge solvers are implemented (via the ***Stepper class***).

The constructor is:

impedance_op=synergia.collective.Impedance(wake_file, wake_type, zgrid, lattice_length, bucket_distance, registred_turns, full_machine, wave_number)

where

- wake_file (string) is the name of the wake file

- wake_type (string) is the type of the file, see the discussion in Sec I A 1. For IOTA, wake_type="XLYLZ".

- zgrid (integer) determines the number of slices the bunch is longitudinally divided.

- lattice_length (double, float) is the ring length.

- bucket_distance (double, float) the distances between buckets

- registred_turns (integer) The number of previous turns considered to produce wakes.

- full_machine (bool). When full_machine=1 one considers that all the buckets are filled with identical bunches. It is a single bunch calculation (since all the bunches are identical), but the bunch feels the wake produced by the other buckets. The multi-bunch instabilities are suppressed in this approximation.

- wave_number (integer type vector of size 3). wave_number=[wnx,wny,wnz]. Relevant when full_machine=1. The buckets are filled with identical bunches which are displaced along the ring with the wave number wn. For example, the horizontal displacement of the buckets along the ring is modulated by $\cos\left(\frac{2\pi \text{ wnx bucket\_index}}{\text{num\_buckets}}\right)$;