MOMENTUM KICK TO A PROTON TRAVERSING AN ELECTRON LENS

Eric G. Stern

Abstract

I calculate the momentum kick for a proton passing through an electron lens with a cylindrically round Gaussian or uniform density profile. This calculation is valid for non-relativistic protons.

INTRODUCTION

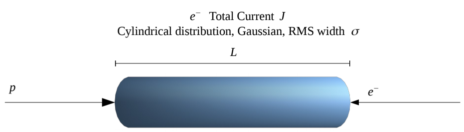

Figure 1: Diagram of electron lens configuration

I consider the momentum kick of a proton passing through an electron lens of length L with a cylindrically round Gaussian density profile. The proton flow direction is opposite to the electron progagation so the EM force is focussing for protons to counteract the defocussing space charge force. The configuration is illustrated in Fig. 1.

Table 1: Parameters and symbols used in the calculation

Parameter	Definition
J	Electron current
L	Electron lens length
e	unit of electric charge
c	speed of light
m	proton mass
r_p	proton classical radius $e^2/4\pi\epsilon_0 mc^2$
σ	RMS radius of current distribution
a	radius of the uniform current distribution
β_e	electron velocity/c
β_p	proton velocity/c
β_b	reference proton beam velocity/c
γ_b	reference proton beam relativistic factor

The names and symbols of parameters used in the calculation are shown in Table 1.

GAUSSIAN ELECTRON BEAM

The electric and magnetic fields experienced by a particle at radius r are determined the amount of charge and current countained within the cylinder of radius r. For the Gaussian profile, this is:

$$\int_0^r e^{-r^2/2\sigma^2} r\,dr = 1 - e^{-r^2/2\sigma^2}$$

Using 2D Gauss's law, the normal electric field in the electron lens at radius r is

$$E_n = -\frac{J}{2\pi\epsilon_0\beta_e c}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}$$

identifying the charge density is $J/\beta_e c$. The electric momentum kick $qE\Delta t$ for a single charge over length L is

$$\Delta p_E = -\frac{Je}{2\pi\epsilon_0\beta_e c}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}\frac{L}{\beta_p c}$$

The magnetic field from the 2D Ampere's law is

$$B_\theta = \frac{J}{2\pi\epsilon_0 c^2}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}$$

where I have used the identity $\epsilon_0\mu_0 = 1/c^2$. The magnetic momentum kick $qv \times B\Delta t$ for a single charge over length L is

$$\Delta p_M = -\frac{Je\beta_p c}{2\pi\epsilon_0 c^2}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}\frac{L}{\beta_p c}$$

The total momentum kick is the sum of the electric and magnetic kicks:

$$\Delta p_{E+M} = -\frac{JLe(1+\beta_e\beta_p)}{2\pi\epsilon_0\beta_e\beta_p c^2}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}$$

In Synergia, the particle momentum is normalized by the reference beam momentum, given by $m\beta_b\gamma_b c$. The momentum kick can be expressed by

$$\frac{\Delta p}{p_b} = -\frac{JLe(1+\beta_e\beta_p)}{2\pi\epsilon_0 mc^2\beta_e\beta_p\beta_b\gamma_b c}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}$$

which can be rewritten using the proton classical radius r_p as

$$\frac{\Delta p}{p_b} = -\frac{2JLr_p(1+\beta_e\beta_p)}{e\beta_e\beta_p\beta_b\gamma_b c}(1 - e^{-r^2/2\sigma^2})\frac{1}{r}$$

In this form, $1/r$ does not have a derivative at $r = 0$ so its behavior at the origin is not manifest. It becomes clear by separating x and y components by multiplying by the unit vector component x/r or y/r.

$$\begin{aligned}
\frac{\Delta p_x}{p_b} &= -\frac{2JLr_p(1+\beta_e\beta_p)}{e\beta_e\beta_p\beta_b\gamma_b c}(1 - e^{-r^2/2\sigma^2})\frac{x}{r^2} \\
\frac{\Delta p_y}{p_b} &= -\frac{2JLr_p(1+\beta_e\beta_p)}{e\beta_e\beta_p\beta_b\gamma_b c}(1 - e^{-r^2/2\sigma^2})\frac{y}{r^2}
\end{aligned}$$

Expanding the factor $1 - e^{-r^2/2\sigma^2}$ near the origin gives $r^2/2\sigma^2$ so the electron lens kick near the origin resembles a focussing lens in both planes.

$$\begin{aligned}
\frac{\Delta p_x}{p_b} &\approx -\frac{2JLr_p(1+\beta_e\beta_p)}{e\beta_e\beta_p\beta_b\gamma_b c}\frac{x}{2\sigma^2} \\
\frac{\Delta p_y}{p_b} &\approx -\frac{2JLr_p(1+\beta_e\beta_p)}{e\beta_e\beta_p\beta_b\gamma_b c}\frac{y}{2\sigma^2}
\end{aligned}$$

UNIFORM ELECTRON BEAM

In the case of a uniform electron beam, the factor $(1 - e^{-r^2/2\sigma^2})$ is replaced by r^2/a^2. The kick becomes

$$\frac{\Delta p}{p_b} = -\frac{2JLr_p(1+\beta_e\beta_p)}{e\beta_e\beta_p\beta_b\gamma_b c}\frac{r}{a^2}$$

In Tevatron electron lens proposals [1], the kick on an ultra-relativistic proton from a uniform distribution electron lens is described as being caused by a potential function $V(r)$ of the form

$$V(r) = r^2\frac{(1+\beta_e)JLr_p}{e\beta_e c a^2\gamma_p}$$

Calculating the kick as $-\partial V/\partial r$, this matches the expression calculated in this paper after setting β_p to 1.

REFERENCES

[1] Shiltsev, et. al, “Considerations on Compensation of Beam-Beam Effects,” Phys. Rev. ST Accel Beams, **2, 171001 (1999).**